Abstract

In the practical design applications the evaluation of seismic response is usually based on linear elastic structural behaviour. However this approach may be not effective in limiting the damage levels of the buildings. To this purpose more accurate methods of analyses, which can predict the real behaviour under strong seismic actions, are required. The non-linear dynamic analysis is the most rigorous method, but it is still too complex for design use. The non-linear static pushover analysis seems to be a more rational method for estimating the lateral strength and the distribution of inelastic deformations. In the last years several simplified non-linear procedures were developed in order to predict the seismic demand by using the results of pushover analysis. These methods were also implemented in recent guidelines based on the new performance based engineering concepts.

In the present research the pushover analysis was applied for studying the response of a RC building. Pushover analyses were performed on three RC buildings designed for earthquake zone 3, 4, and 5 and results were compared.

1. INTRODUCTION

A key component of performance-based seismic evaluation is the estimation of seismic demands. In FEMA-356, which is now recognized as the model for future performance-based seismic codes, these demands are evaluated at the component level in terms of ductility demands or plastic rotations when using non-linear procedures. Since acceptance criteria for various performance objectives are assessed in terms of local component demands, it is essential that a rational basis be established for determining such demands. Of the non-linear procedures advocated in FEMA-356, pushover procedures are becoming increasingly popular in engineering practice.

Pushover analysis is a static, non linear procedure in which the magnitude of the structural loading is incrementally increased in accordance with a certain predefined pattern. With the increase in the magnitude of the loading, weak links and failure modes of the structures are found. The loading is monotonic with the cyclic behavior and load reversals being estimated by using modified monotonic force-deformation criteria and with damping approximations.

2. PRIMARY ELEMENTS OF THE PUSHOVER ANALYSIS

Simplified non-linear analysis procedures using pushover method, such as capacity spectrum method, and displacement coefficient method, requires determination of three primary elements are briefly discussed below:

Capacity: The overall capacity of the structure depends on the strength and deformation capacities of the individual components of the structure. In order to determine the capacities beyond the elastic limits, some form of non-linear analysis, such as the pushover procedure, is required.

Demand (displacement): Ground motions during an earthquake produce complex horizontal displacement patterns in structures that may vary with time. Tracking this motion at every time-step to determine structural design requirements are judged impractical. Traditional linear analysis methods use lateral forces to represent a design condition. For non-linear method it is easier and more direct to use set of lateral displacement as a design condition. For a given structure and ground motion, the displacement demand is an estimate of the maximum expected response of the building during the ground motion.

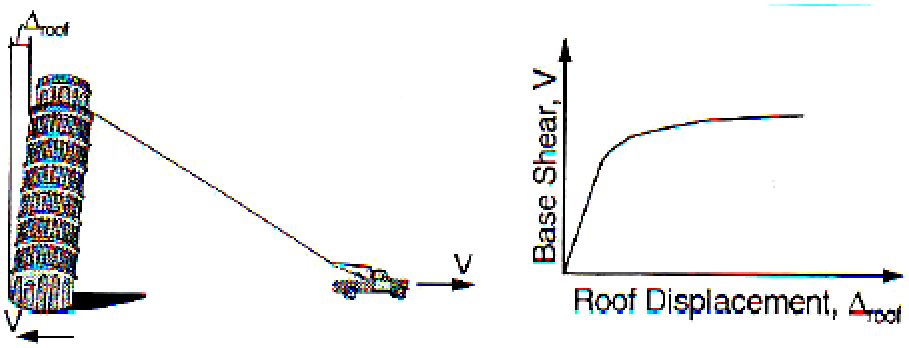
Performance: once a capacity curve and demand displacement are defined a performance check can be done. A performance check verifies that structural and non-structural components are not damaged beyond the acceptable limits of the performance objective for the force and displacement demand.

3. PROCEDURE TO DETERMINE THE CAPACITY

Structure capacity is represented by a pushover curve. The most convenient way to plot the force-displacement curve is by tracking the base shear and the roof displacement.

The capacity curve is generally constructed to represent the first mode response of the structure based on the assumption that the fundamental mode of vibration is the predominant response of the structure. This is generally valid for the buildings with fundamental periods of the vibration up to about one second. For more flexible buildings with a fundamental period of vibration greater than one second, the analyst should consider addressing higher mode effects in the analysis.

The following procedure can be used to construct a pushover curve:



1. Create a computer model of the structure according to chapter 9 of ATC-40 and if the foundation is modeled following the modeling rule in chapter 10 of ATC-40.
2. Classify each element in the model as either primary or secondary.
3. Apply lateral storey forces to the structure in proportion to the product of the mass and fundamental mode shape. This analysis should also include the gravity loads.
4. Calculate the member forces for the required combinations of vertical and lateral load.
5. Adjust the lateral load so that some element (or group of elements) is stressed to within 10 percent of its member strength.
6. Record the base shear and the roof displacement. It is also important o record the member forces and rotations because they will be needed for the performance check.
7. Revise the model using zero (or very small) stiffness for the yielding elements.
8. Apply a new increment of lateral load to the revised structure such that another element (or group of elements) yields.
9. Add the increment of the lateral load and the corresponding increment of roof displacement to the previous totals to give the accumulated values of base shear and roof displacement.
10. Repeat 7, 8 and 9 until the structure reaches an ultimate limit, such as: instability from P-Δ effects; distortions considerably beyond the desired performance level; an element (or group of elements) reaching a lateral deformation level at which significant strength degradation begins or an element (or group of elements) reaching a lateral deformation level at which loss of gravity load carrying capacity occurs. Above figure shows the typical capacity curve.
11. Explicitly model global strength degradation. If the incremental loading was stopped in step 10 as a result of reaching a lateral deformation level at which all or a significant portion of an

element's (or group of elements) load can no longer be resisted, that is, its strength has significantly degraded, then the stiffness of that element(s) is reduced, or eliminated. A new capacity curve is then created, starting with step 3 of this procedure. Create as many additional pushover curves as necessary to adequately define overall loss of the strength.

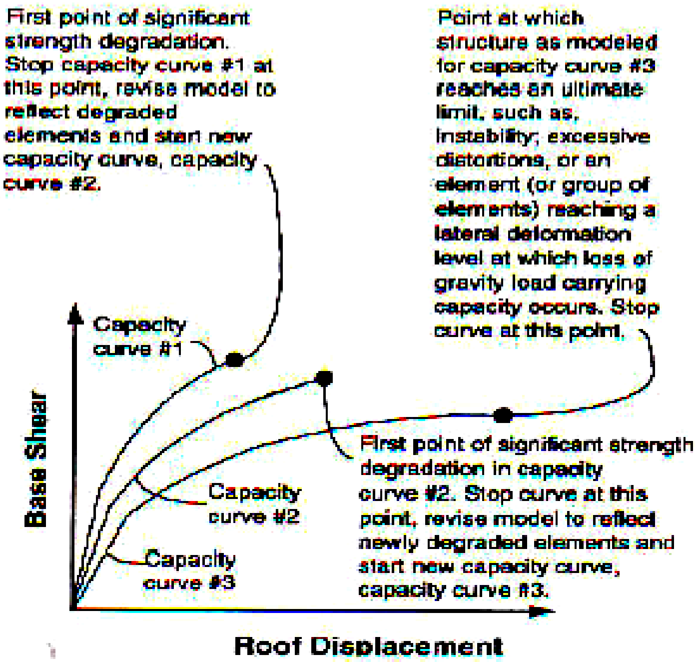


Figure 1. *Multiple capacity curves required to model strength degradation.*

Figure 1 illustrates the process, for an example where three different capacity curves are required.

Plot the final capacity curve to initially follow the first curve, then transition to the second curve at the displacement corresponding to the initial strength degradation, and so on. This curve will have a saw tooth shape as shown in figure 2.

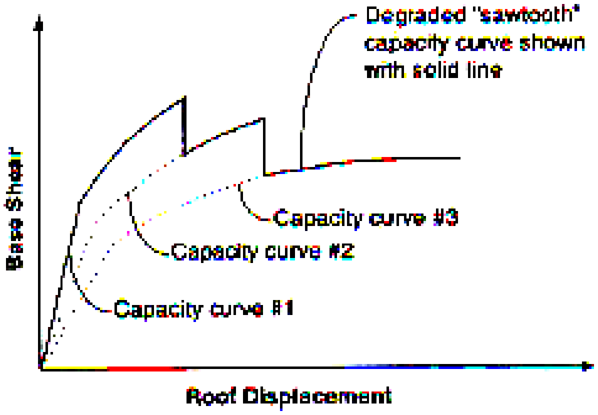


Figure 2. *Capacity curve with global strength degradation modeled.*

4. PROCEDURE TO DETERMINE DEMAND

In order to determine compliance with a given performance level, a displacement along the capacity curve must be determined that is consistent with the seismic demand. Two methodologies are presented in this session.

4.1. Capacity spectrum method

It is based on finding a point on the capacity spectrum that also lies on the appropriate demand response spectrum, reduced for non linear effects, and is most consistent in terms of graphical representation and terminology with the balance of this document.

The demand displacement in the capacity spectrum method occurs at a point on the capacity spectrum called the performance point. This performance point represents the condition for which the seismic capacity of the structure is equal to the seismic demand imposed on the structure by the specified ground motion.

The location of the performance point must satisfy two relationships: 1) the point must lie on the capacity spectrum curve in order to represent the structure at a given displacement, and 2) the point must lie on spectral demand curve, reduced from the elastic, 5 percent damped design spectrum, that represent the non-linear demand at the same structural displacement, for this methodology, spectral reduction factors are given in terms of effective damping. An approximate damping is calculated based on the shape of the capacity curve, the estimated displacement demand, and the resulting hysteresis loop. Probable imperfections in real building hysteresis loops, including degradation and duration effects, are accounted for by reductions in theoretically calculated equivalent viscous damping values.

In general case, determination of the performance point requires a trial and error search for satisfaction of the two criterions specified above. However, here three different procedures have been included which standardize and simplify this iterative process. These alternate procedures are all based on the same concepts and mathematical relationship but vary in there dependence on analytical versus graphical techniques.

Procedure A: This is the most direct application of the concepts and relationship described below. This procedure is truly iterative, but is formula-based and easily can be programmed into spread sheet. It is more an analytical method than graphical method. It may be the best method for beginners because it is the most direct application of the methodology, and consequently is the easiest procedure to understand.

4.1.1. Conceptual development of the capacity spectrum method

4.1.1.1. Conversion of the capacity curve to the capacity spectrum

To use the capacity spectrum method it is necessary to convert the capacity curve, which is in terms of base shear and roof displacement to what is called capacity spectrum, which is the representation of the capacity curve in Acceleration-Displacement Response Spectra (ADRS) format (i.e. S_a versus S_d). The required equations to make the transformations are:

$$PF_1 = \left[\frac{\sum_{i=1}^{N} (w_i \phi_{i1}) / g}{\sum_{i=1}^{N} (w_i \phi^2{}_{i1}) / g} \right] \tag{1}$$

$$\alpha_1 = \frac{\left[\sum_{i=1}^{N} (w_i \phi_{i1}) / g \right]^2}{\left[\sum_{i=1}^{N} w_i / g \right] \left[\sum_{i=1}^{N} (w_i \phi^2{}_{i1}) / g \right]} \tag{2}$$

$$S_a = \frac{V / W}{\alpha_1} \tag{3}$$

$$S_d = \frac{\Delta_{roof}}{PF_1 \phi_{roof,1}} \tag{4}$$

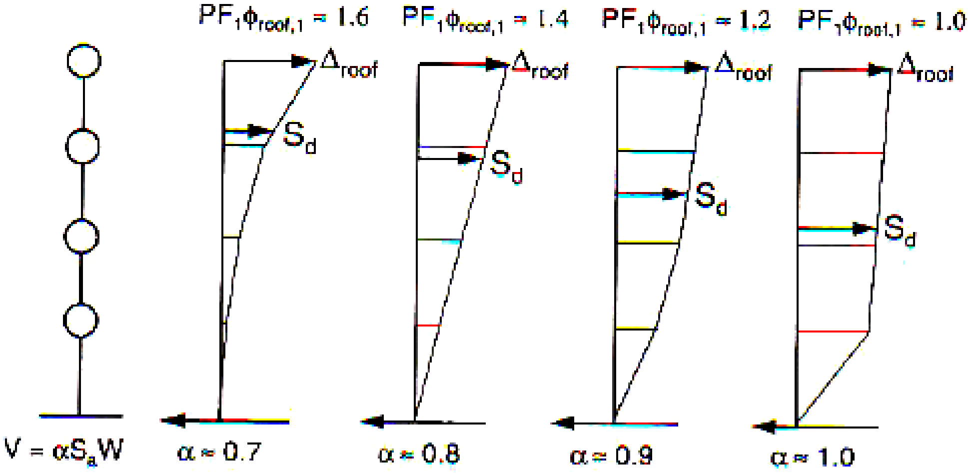


Figure 3. *Example modal participation factors and modal mass coefficients*

Where:

PF_1 = modal participation factor for the first mode.

α_1 =modal mass coefficient for the first natural mode.

w_i/g = mass assigned to level i.

ϕ_{i1} = amplitude of mode 1 at level i.

N = level N, the level which is the uppermost in the main portion of the superstructure.

V = base shear.

W = building dead weight plus likely live loads

Δ_{roof} = roof displacement (V and the associated Δ_{roof} make up points on the capacity curve).

S_a = spectral acceleration.

S_d = spectral displacement (S_a and the associated S_d make up points on the capacity spectrum).

The general process for converting the capacity spectrum, i.e. converting the capacity curve into the ADRS format, is to first calculate modal participation factor PF_1 and the modal mass coefficient α_1 using the equation (1) and (2). Then for each point on the capacity curve, V, Δ_{roof}, calculate the associated point S_a, S_d on the capacity spectrum using equation (3) and (4).

Most engineers are familiar with the traditional S_a versus S_d (ADRS) representation. Figure 4 shows the same spectrum in each format. In the ADRS format, lines radiating from the origin have constant period. For any point on ADRS spectrum, the period, T, can be computed using the relationship $T = 2\pi(S_d/S_a)^{1/2}$. Similarly, for any point on the traditional spectrum, the spectral displacement, S_d, can be computed using the relationship $Sd = S_a T^2/4\pi^2$. These two relationships are the same formula arranged in different way.

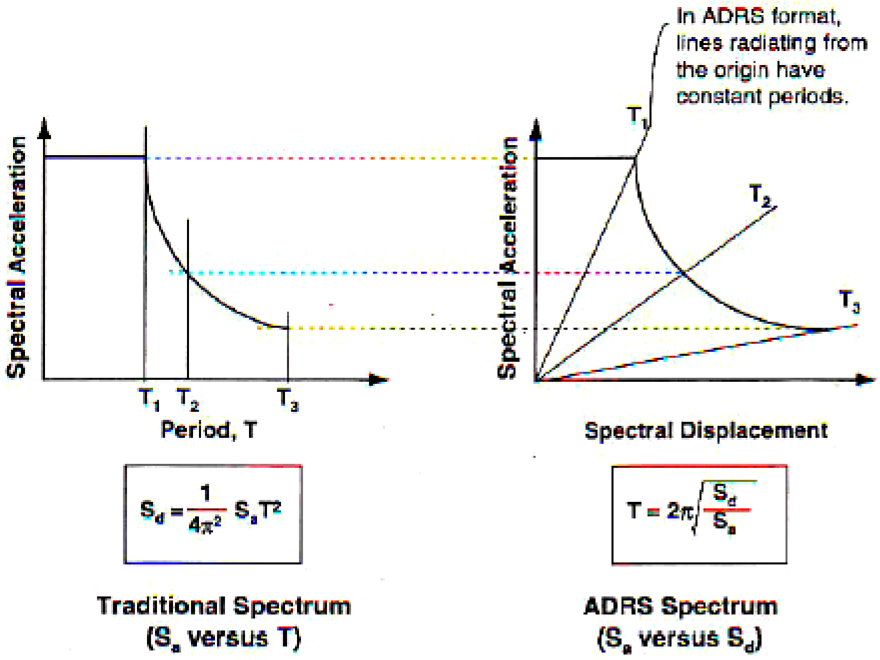


Figure 4. *Response spectra in traditional and ADRS formats*

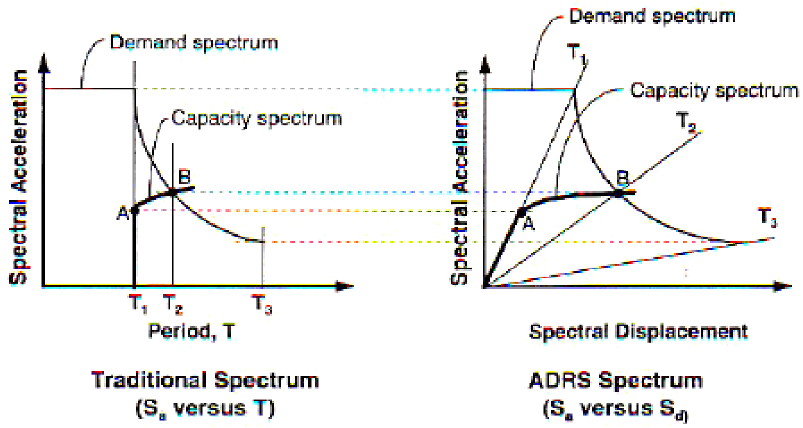


Figure 5. *Capacity spectrum super imposed over response spectra in traditional and ADRS formats*

Figure 5 shows the same capacity spectrum superimposed on each of the response spectra plots shown in figure 4 Following along the capacity spectrum, the period is constant, at T_1, up until point A. when point B is reached, the period is T_2. This indicates that a structure undergoes inelastic displacement, the period lengthens. The lengthening period is most apparent on the traditional spectrum plot, but it is also clear on the ADRS plot, remembering that lines of constant period radiate from the origin.

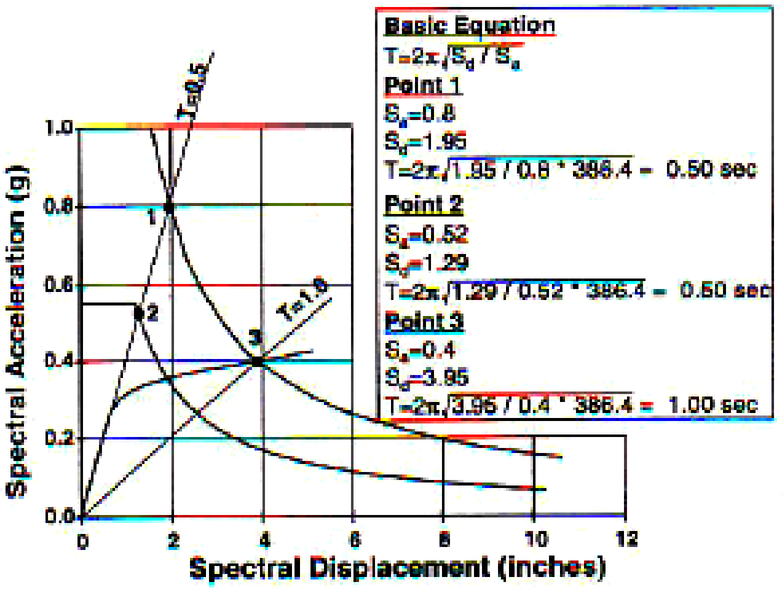


Figure 6. *Lines of constant period and period lengthening in ADRS format*

Figure 6 helps illustrate that, in ADRS format, lines radiating from the origin have constant period, and that the period lengthens as the structure undergoes inelastic displacement. Point 1 and 2 in the figure lie on two different response spectra. And on single line radiating from the origin, as shown by the calculation included in the figure, they both are associated with a period of 0.5 seconds. Point 3 has 1.0 second period. Thus, for capacity spectrum (pushover) shown, the elastic period of the structure is 0.5 seconds, and when the structure is pushed to point 3, with a spectral displacement of 3.95 inches, and considerable inelastic displacement, the period has lengthened to 1.0 seconds.

4.1.1.2. Bilinear representation of the Capacity Spectrum

A bilinear representation of the capacity spectrum is needed to estimate the effective damping and appropriate reduction of spectral demand. Construction of the bilinear representation requires definition of the points a_{pi}, d_{pi},. This point is a trial performance point which is estimated by the engineer to develop a reduced demand response spectrum. If the reduced response spectrum is found to intersect the capacity spectrum at the estimated a_{pi}, d_{pi} point. The first estimate of point a_{pi}, d_{pi} is designated a_{p1}, d_{p1}, the second a_{p1}, d_{p2}, and so on. Guidance on a first estimate of point a_{p1}, d_{p1} is given in the step by step process for each of the three procedures. Oftentimes, the equal displacement approximation can be used as an estimate of a_{p1}, d_{p1}.

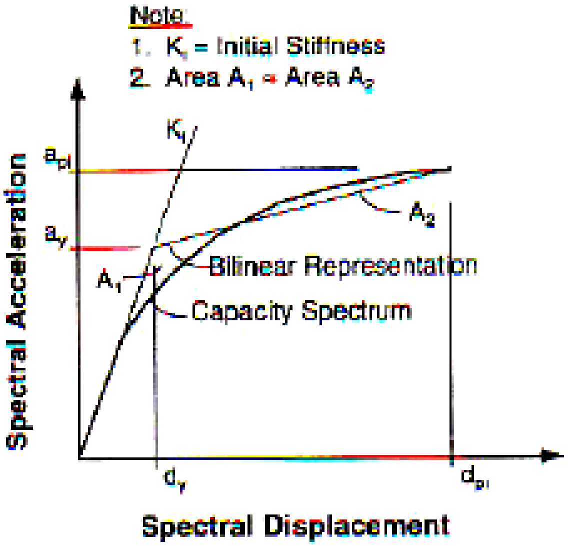


Figure 7. *Bilinear representation of capacity spectrum for capacity spectrum method*

Refer figure 7 for an example bilinear representation of a capacity spectrum. To construct the bilinear representation draw one line up from the origin at the initial stiffness of the building using element stiffnesses. Draw a second line back from the trial performance point, a_{pi}, d_{pi}. Slope the second line such that when it intersects the first line, at point a_y,d_y, the area designated A_1 in the figure is approximately equal to the area designated A_2, the intent of setting area A_1 equal to A_2 is to have equal area under the capacity spectrum and its bilinear representation, that is to have equal energy associated with each curve.

4.1.1.3. Estimation of damping and reduction of 5 percent damped response spectrum

The damping that occurs when earthquake ground motion drives a structure into the inelastic range can be viewed as a combination of viscous damping that is inherent in the structure and hysteretic damping. Hysteretic damping is related to the area inside the loops that are formed when the earthquake force (base shear) is plotted against the structure displacement. Hysteretic damping can be represented as equivalent viscous damping using equations that are available in the literature.

The equivalent viscous damping, β_{eq}, associated with maximum displacement of d_{pi}, can be estimated from the following equation:

$$\beta_{eq} = \beta_o + 0.05 \tag{5}$$

Where,

β_o = hysteretic damping represented as equivalent viscous damping

0.05 = 5% viscous damping inherent in the structure (assumed to be constant)

The term β_o can be calculated as (Chopra 1995):

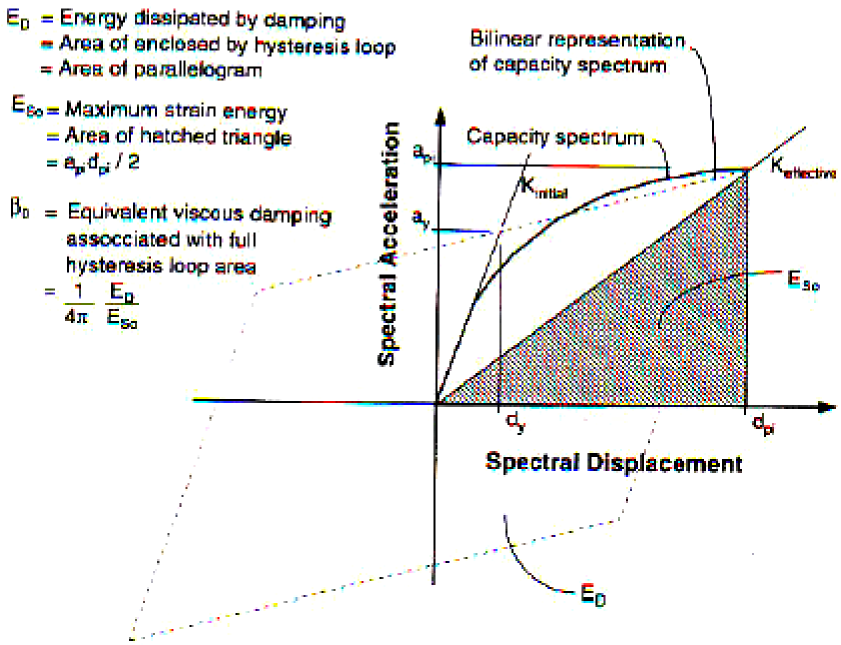


Figure 8. *Derivation of damping for spectral reduction.*

$$\beta_0 = \frac{E_D}{4\pi E_{so}} \tag{5a}$$

Where,

E_D = energy dissipated by damping

E_{so} = maximum strain energy

The physical significance of the terms E_D and E_{so} in equation 5a is illustrated in figure 8 E_D is the energy dissipated by the structure in a single cycle of motion, that is, the area enclosed by a single hysteresis loop. E_{so} is the maximum strain energy associated with that cycle of motion that is the area of the hatched triangle.

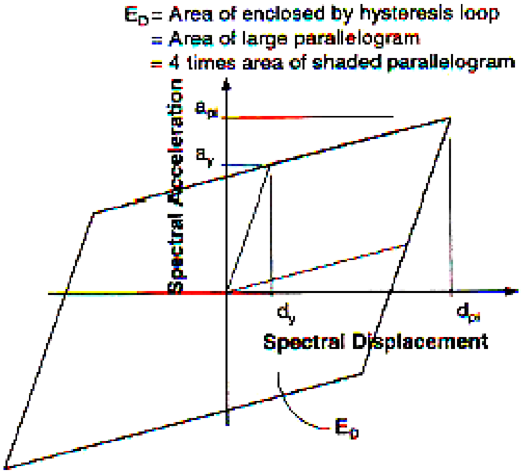


Figure 9. *Derivation of energy dissipation by damping, E_D*

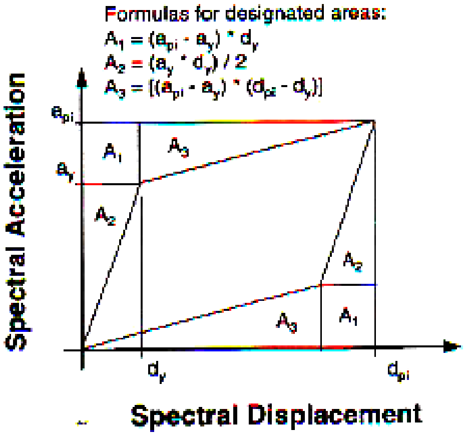


Figure 10. *Derivation of energy dissipation by damping, E_D*

E_D = 4(shaded area in figure 9 or 10)

$= 4(a_{pi}d_{pi} – 2A_1 – 2A_2 – 2A_3)$

$= 4[a_{pi}d_{pi} – a_yd_y – (d_{pi} – d_y)(a_{pi} - a_y) – 2d_y(a_{pi} – a_y)]$

$= 4(a_yd_{pi} - d_ya_{pi})$

Referring figure 8, the term E_{so} can be derived as

$E_{so} = a_{pi}d_{pi}/2$

Thus, β_o can be written as:

$$\beta_0 = \frac{1}{4\pi}\frac{4\left(a_y d_{pi} - d_y a_{pi}\right)}{a_{pi} d_{pi}/2} = \frac{2}{\pi}\frac{a_y d_{pi} - d_y a_{pi}}{a_{pi} d_{pi}} = \frac{0.637(a_y d_{pi} - d_y a_{pi})}{a_{pi} - d_{pi}}$$

(6)

$$\beta_0 = \frac{63.7(a_y d_{pi} - d_y a_{pi})}{a_{pi} d_{pi}}\%$$

And

$$\beta_{eq} = \beta_0 + 5 = \frac{63.7(a_y y_{pi} - d_y a_{pi})}{a_{pi} d_{pi}} + 5 \tag{7}$$

The equivalent viscous damping values obtained from equation (7) can be used to estimate spectral reduction factors using relationship developed by Newmark and Hall. As shown in figure 11, spectral reduction factors are used to decrease the elastic (5% damped) response spectrum to reduce response spectrum with damping greater than 5% of critical damping. For damping values less than about 25%, spectral reduction factors calculated using the β_{eq} from equation (7) and Newmark and Hall equations are consistent with similar factors contained in base isolation code and in the FEMA guidelines.

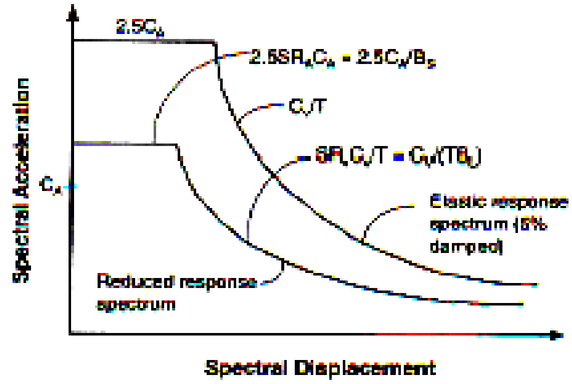


Figure 11. *Reduced response spectrum*

4.1.2. Development of demand spectrum

The 5 percent response spectrum can be developed as shown in following figure;

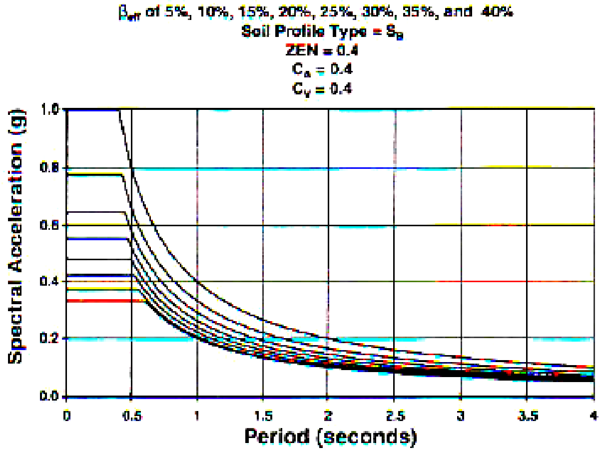


Figure 12. *Family of demand spectra in traditional S_a vs T format*

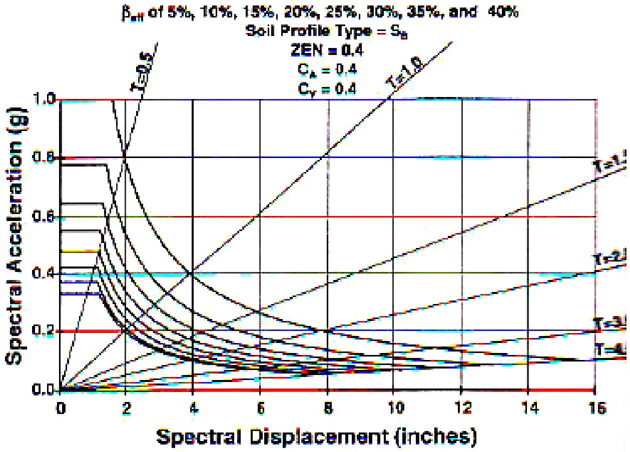


Figure 13. *Family of demand spectra in ADRS format*

4.1.3. Intersection of capacity spectrum and demand spectrum

When the displacement at the intersection of the demand spectrum and the capacity spectrum, d_i is within 5% ($0.95d_{pi} \leq d_i \leq 1.05d_{pi}$) of the displacement of the trial performance point, d_{pi} becomes the performance point. If the intersection of the demand spectrum and the capacity spectrum is not within the acceptable point is selected and tolerance, then a new a_{pi}, d_{pi} point is selected and the process is repeated. Figure 14 illustrates the concept. The performance point represents the maximum structural displacement expected for the demand earthquake ground motion.

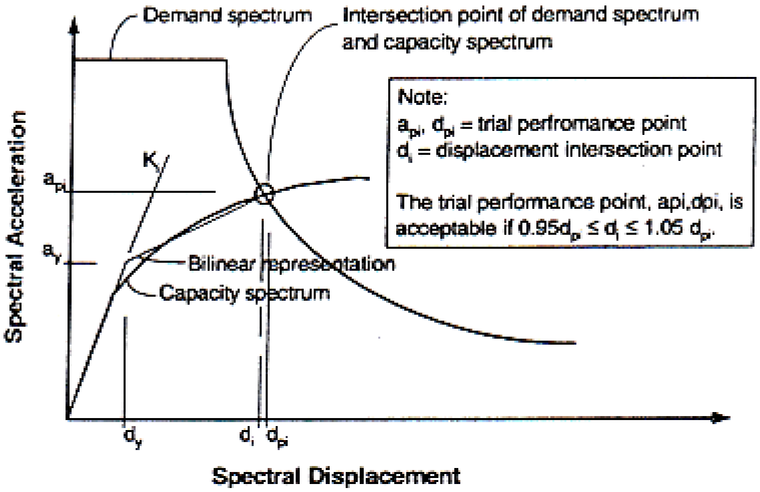


Figure 14. *Intersection point of demand and capacity spectrum within acceptable tolerance*

When a capacity spectrum is a saw tooth curve, that is, the final composite capacity spectrum is constructed from several different capacity spectra which accounts for strength degradation of elements, special care must be taken in determining the performance point. Bilinear representation of the capacity spectrum, that is used to determine the reduction factors for the 5% damped spectrum, is constructed for a single capacity spectrum where the intersection point occurs. Figure 15 illustrate the concept for a saw tooth capacity spectrum.

4.1.4. Calculating the performance point using procedure A

In this procedure, iteration is done by hand or by spreadsheet methods to converge on the performance point. This procedure is the most direct application of the principles described above. The following steps are involved:

1. Develop the 5 percent damped (elastic) response spectrum appropriate for the site.
2. Transform the capacity curve into a capacity spectrum. Plot the capacity curve on the same chart as the 5% damped response spectra as shown in figure 16.

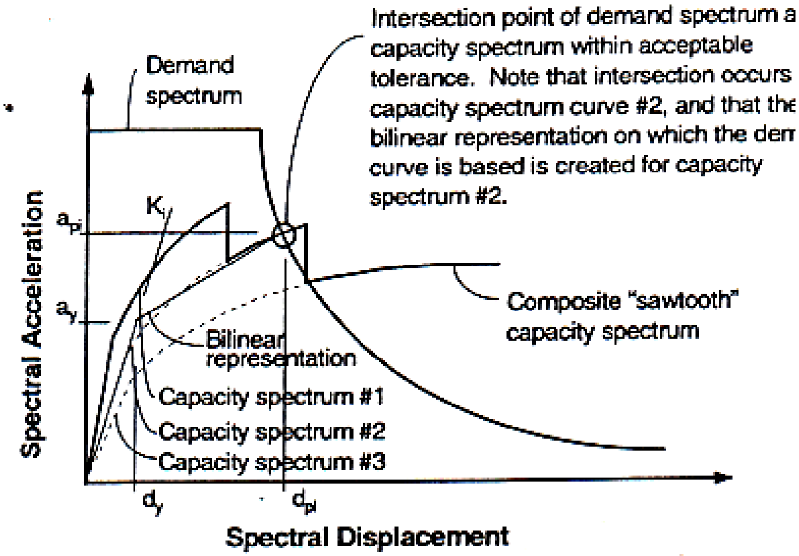


Figure 15. *Intersection point of demand spectrum and saw tooth capacity spectrum*

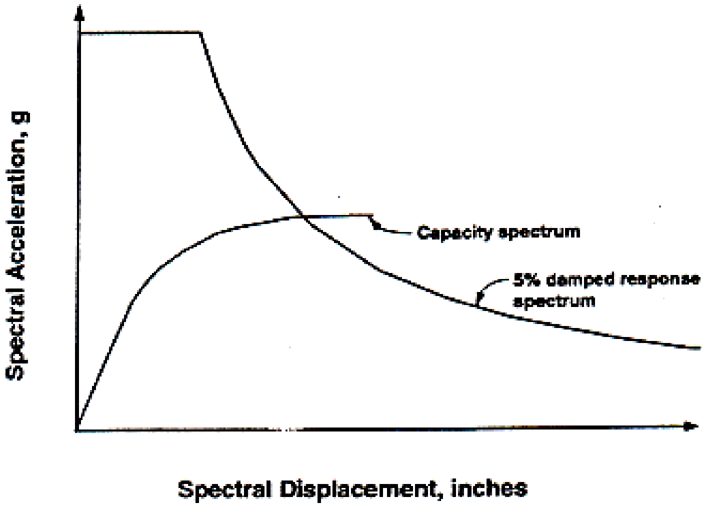


Figure 16. *Capacity spectrum procedure A after step 2.*

3. Select a trial performance point, a_{pi}, d_{pi} as shown in figure 17**.**
4. Develop a bilinear representation of the capacity spectrum

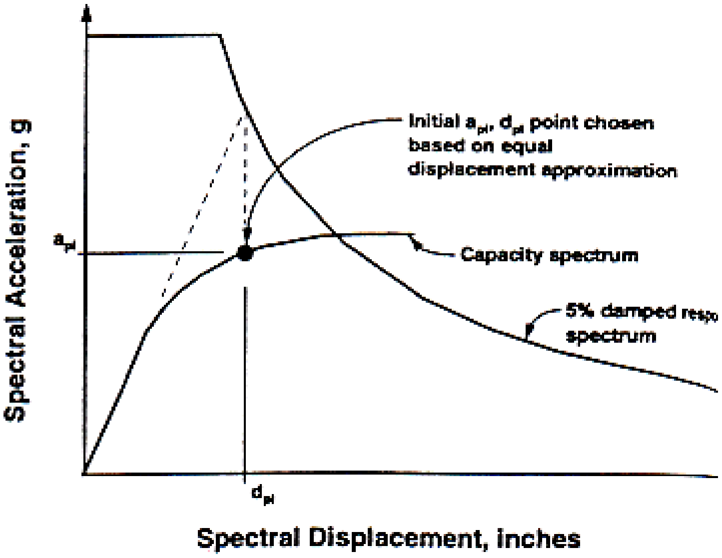


Figure 17. *Capacity spectrum procedure A after step 3*

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5. Calculate the spectral reduction factor as shown below

$$SR_A = \frac{1}{B_S} \approx \frac{3.21 - 0.68\ln(\beta_{eff})}{2.12}$$

$$= \frac{3.21 - 0.68\ln\left[\frac{63.7k(a_y d_{pi} - d_y a_{pi})}{a_{pi} d_{pi}}\right]}{2.12} + 5$$

$$SR_v = \frac{1}{B_L} \approx \frac{2.31 - 0.41\ln(\beta_{eff})}{1.65}$$

$$= \frac{2.31 - 0.41\ln\left[\frac{63.7k(a_y d_{pi} - d_y a_{pi})}{a_{pi} d_{pi}} + 5\right]}{1.65}$$

And these values should be greater than given in following table:

Structural behavior type	**SR_A**	**SR_V**
Type A	0.33	0.50
Type B	0.44	0.56
Type C	0.56	0.67

Develop the demand spectrum. Draw the demand spectrum on the same plot as the capacity spectrum.

6. Refer to Figure 18. Determine if the demand septum intersects the capacity spectrum at the point, api, dpi. Or if the displacement at which the demand spectrum intersects the capacity spectrum, di, is within acceptable tolerance of dpi. The acceptable tolerance is illustrated in Figure 14.

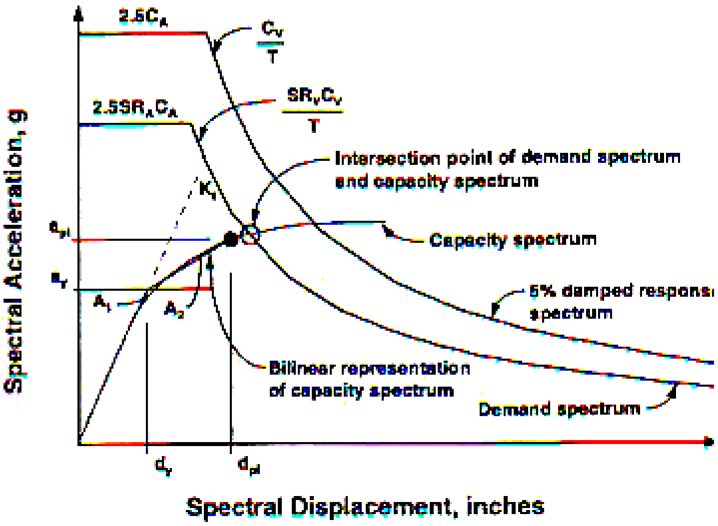

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Figure 18. *Capacity spectrum procedure A after step 6.*

7. If the demand spectrum does not intersect the capacity spectrum within acceptable tolerance. Then select a new api, dpi point and return to step 4.
8. If the demand spectrum intersects the capacity spectrum within acceptable tolerance, then the trial performance point. api, dpi, is the performance point, a_p, d_p, and the displacement, d_p, represents the maximum structural displacement expected for the demand earthquake.

4.1.5. Calculating the performance point using procedure B

This procedure makes a simplifying assumption that is not made in the other two procedures. It assumes that not the initial slope of the bilinear representation of the capacity curve remains constant, but also the point a_y, d_y and the post yield slope remains constant. This simplifying assumption allows a direct solution without drawing multiple curves because it forces the effective damping, β_{eff}, to depend only on d_{pi}. The following steps are involved:

1. Develop the 5% damped response spectrum appropriate for the site
2. Draw the 5% damped response and spectrum and draw a family of reduced spectra on the same chart. It is convenient if the spectra plotted correspond to effective damping values (β_{eff}) ranging from 5 % to the maximum value allowed for the building's structural behavior type.

The maximum β_{eff} for type A construction is 40%, type B construction 29% and type C is 20%. Figure 14 shows an example family of demand spectra.

Shaking duration[1]	**Essentially new buildings**[2]	**Average existing building**[3]	**Poor existing building**[4]
Short	Type A	Type B	Type C
Long	Type A	Type B	Type C

Table 1. *Structural behavior type*

1. Site with a near source distance, N ≥ 1.2, may be assumed to have short duration ground shaking and N ≤ 1.2 assumed to have long duration ground shaking
2. Building whose primary elements make up an essentially new lateral system and little strength or stiffness is contributed by noncomplying elements.
3. Building whose primary elements are combinations of existing and new elements, or better than average existing systems.
4. Building whose primary elements make up complying lateral force systems with poor or unreliable hysteretic behavior.

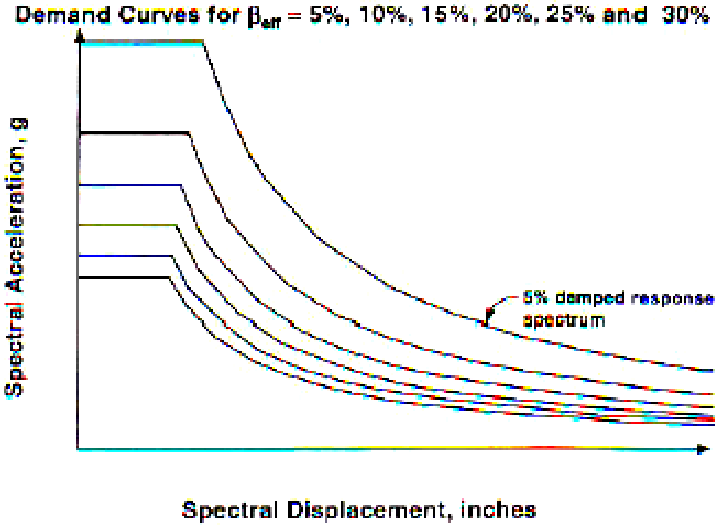


Figure 19. *Capacity spectra procedure "B" after step 2*

3. Transform the capacity curve into a capacity spectrum as described earlier using equations 1, 2, 3 and 4. Plot the capacity spectrum on the same chart as the family of demand spectra, as shown in figure 20.

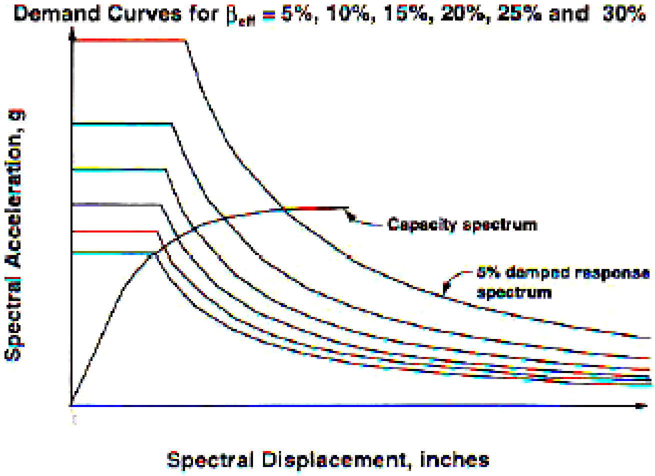


Figure 20. *Capacity spectra procedure "B" after step 3*

4. Develop a bilinear representation of the capacity spectrum is illustrated in figure 21. The initial slope of the bilinear curve is equal to the initial stiffness of the building. The post yield segment of the bilinear representation should be run through the capacity spectrum at a displacement equal to the spectral displacement of the 5% damped spectrum at the initial pre-yield stiffness (equal displacement rule), point a*, d*. The post yield segment should be rotated about this point to balance the areas A_1 and A_2 as shown in figure 21.

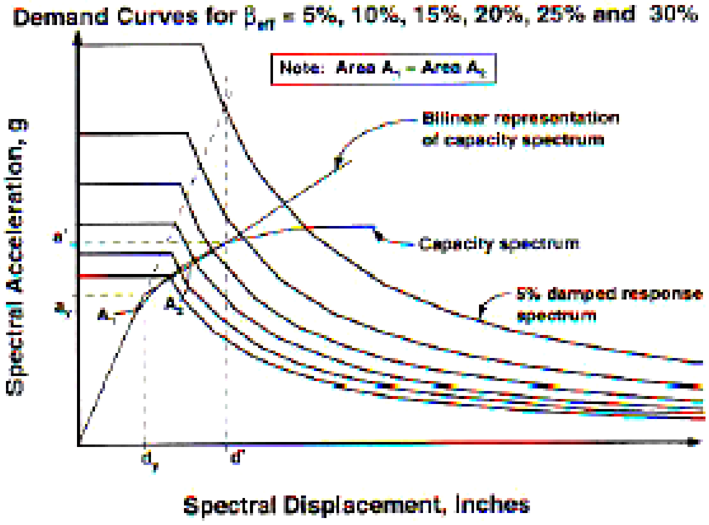


Figure 21. *Capacity spectrum procedure "B" after step 4*

5. Calculate the effective damping for various displacement near the point a*, d*. The slope of the post yield segment of the bilinear representation of the capacity spectrum is given by:

$$\text{Post yield slope} \quad = \frac{a^* - a_y}{d^* - d_y} \tag{9}$$

For any point a_{pi}, d_{pi}, on the post yield segment of the bilinear representation, the slope is given by:

$$\text{Post yield slope} \quad = \frac{a_{pi} - a_y}{d_{pi} - d_y} \tag{10}$$

Since the slope is constant, equation 11 and 12 can be equated:

$$\frac{a^* - a_y}{d^* - d_y} = \frac{a_{pi} - a_y}{d_{pi} - d_y} \tag{11}$$

Solving equation 13 for a_{pi} we get;

$$a_{pi} = \frac{(a^* - a_y)(d_{pi} - d_y)}{d^* - d_y} + a_y \tag{12}$$

This value can be substituted for a_{pi} in equation 7 to obtain the equation for β_{eff} that in terms of only the unknown, d_{pi}.

$$\beta_{eff} = \frac{63.7k(a_y d_{pi} - d_y a_{pi})}{a_{pi} d_{pi}} + 5 \tag{13}$$

Where, *k* = the factor which is the measure of the extent to which the actual building hysteresis is well represented by the parallelogram of figure 8, either initially or after degradation. The *k* factor depends on the structural behavior of the building, which in terns depends on the quality of the seismic resistance system and the duration of the ground shaking. For structure type A, B, C *k* values are 1.0, 2/3 and 1/3 respectively.

6. For each d_{pi} value considered in step 5, plot the resulting d_{pi}, β_{eff} point in the same chart as the family of demand spectra and the capacity spectrum. Figure 22 shows five of these points.
7. Connect the points created in step 6, to form a line. The intersection of this line with the capacity spectrum defines the performance point. This procedure provides the same results as the other procedures if the performance point is at point a*, d*. If the performance point is found to be distant from point a*, d*, then the engineer may want to verify the results using procedure A or C.

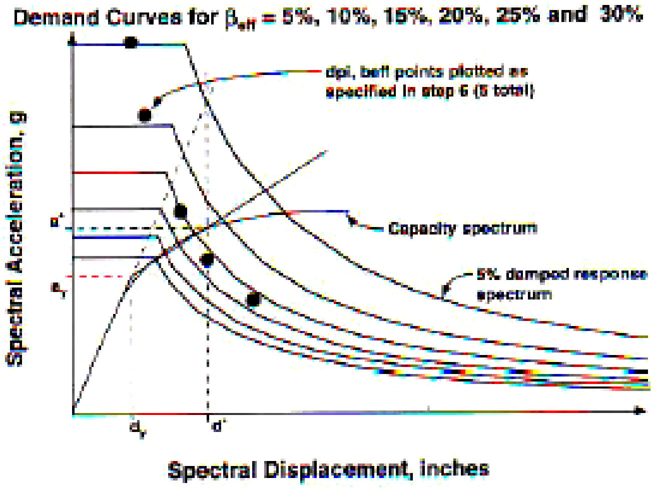


Figure 22. *Capacity spectrum procedure "B" after step 6*

4.1.6. Calculating the performance point using procedure C

This procedure has been developed to provide a graphical solution using hand methods. It has been found to often be reasonably close to the performance point on the first try. The following steps are involved:

1. Develop the 5% damped response spectrum appropriate for the site.
2. Draw the 5% damped response spectrum and draw a family of reduced spectra on the same chart, as illustrated in figure 23. It is convenient if the spectra plotted correspond to effective damping values (β_{eff}) ranging from 5% to the maximum value allowed for the building's structural behavior type.

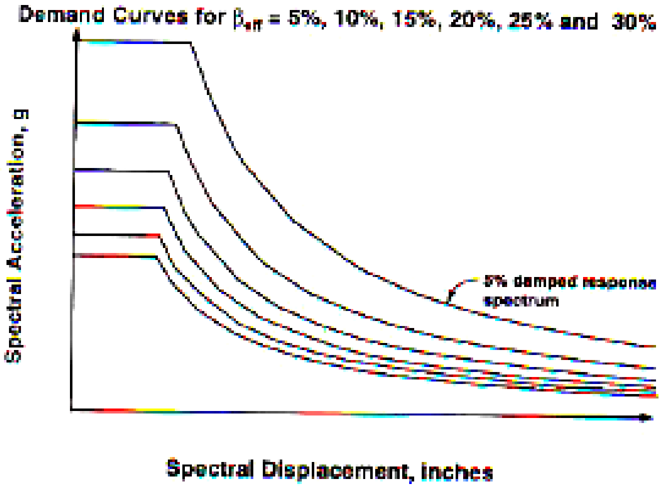


Figure 23. *Capacity spectra procedure "C" after step 2*

3. Transform the capacity curve into a capacity spectrum, and plot it on the same chart as the family of the demand spectra, as illustrated in figure 24.

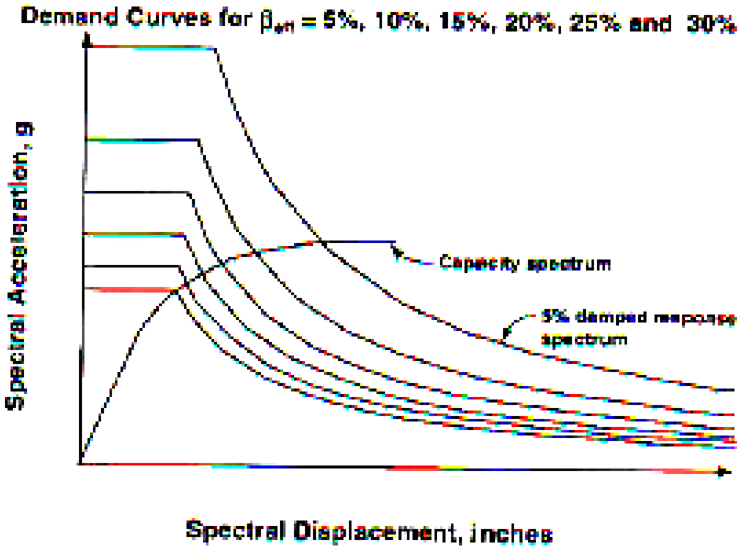


Figure 24. *Capacity spectra procedure "C" after step 3*

4. Develop a bilinear representation of the capacity spectrum. Select the initial point a_{pi}, d_{pi} at the furthest point out on the capacity spectrum or at the intersection with the 5% damped spectrum, whichever is less. A displacement slightly larger than that calculated using the equal displacement approximation (say 1.5 times larger) may also be reasonable estimate for the initial d_{pi}, see figure 25 for as an illustration of this step.

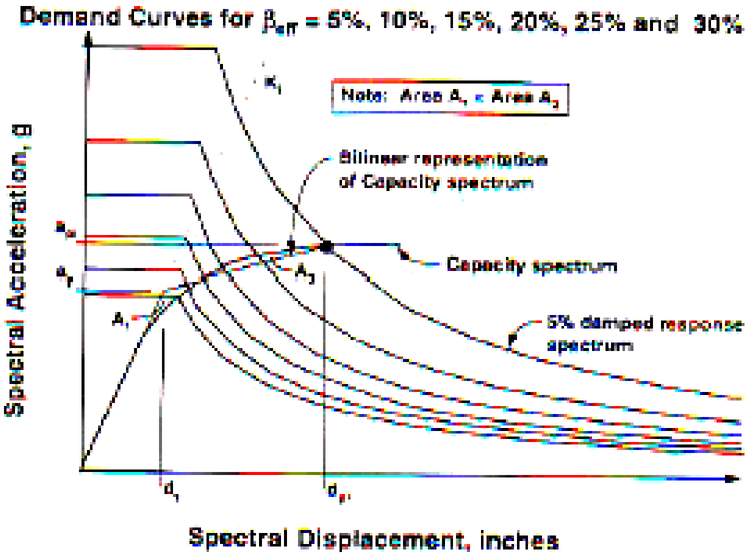


Figure 25. *Capacity spectra procedure "C" after step 4*

5. Determine the ratio of d_{pi}/d_y and $[(a_{pi}/a_y)-1]/[(d_{pi}/d_y)-1]$. Note that the second term is the ratio of the post yield stiffness to the initial stiffness.

6. Based on the ratios obtained in step 5, depending on the building's structural type and find the effective damping value β_{eff}.

7. Refer to figure 26. Extend the initial stiffness line, labeled line 1 in the figure, up to intersect the 5% damped curve. Also draw a line, labeled 2 in the figure, from origin to point api, dpi.

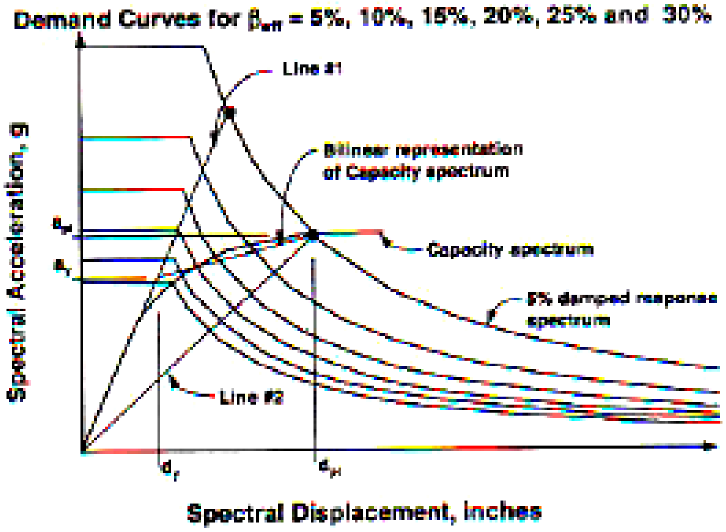


Figure 26. *Capacity spectrum procedure "C" after step 7*

8. Refer to figure 27. Draw a line, labeled 3 in the figure, from the intersection point of line 1 and the 5% damped response spectrum to the intersection point of line 2 and the reduced spectrum correspond to the β_{eff} of approximately 24%.

9. Refer to figure 28. The point where line 3 intersects the capacity spectrum is taken as the estimated performance point a_{p2}, d_{p2} point.

10. If the displacement is within ± 5% of displacement d_{p1}, then the point a_{p2}, d_{p2} is the performance point or in more general terms, if displacement $d_{p(i+1)}$ is within ± 5% of displacement d_{pi}, then the point $a_{p(i+1)}$, $d_{p(i+1)}$ is the performance point, if the displacements are not within the specified tolerance, then proceed to step 11.

11. Repeat the procedure starting at step 4, incrementing i by 1. Thus in the second iteration, line 2 is drawn from the origin to point a_{p2}, d_{p2}.

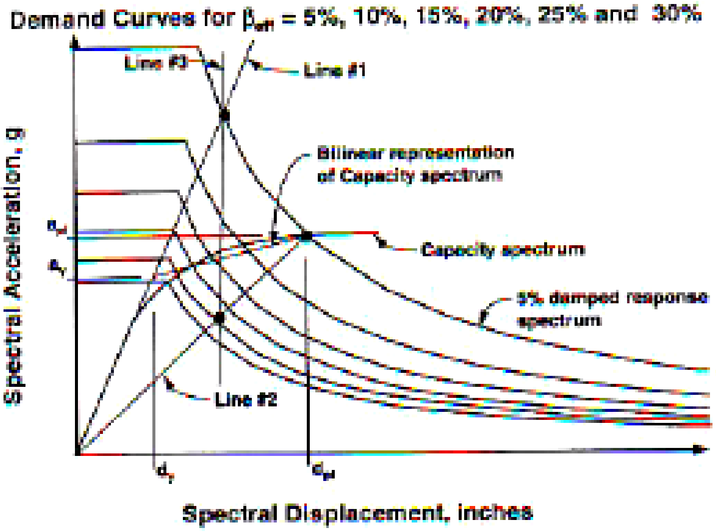


Figure 27. *Capacity spectrum procedure "C" after step 8*

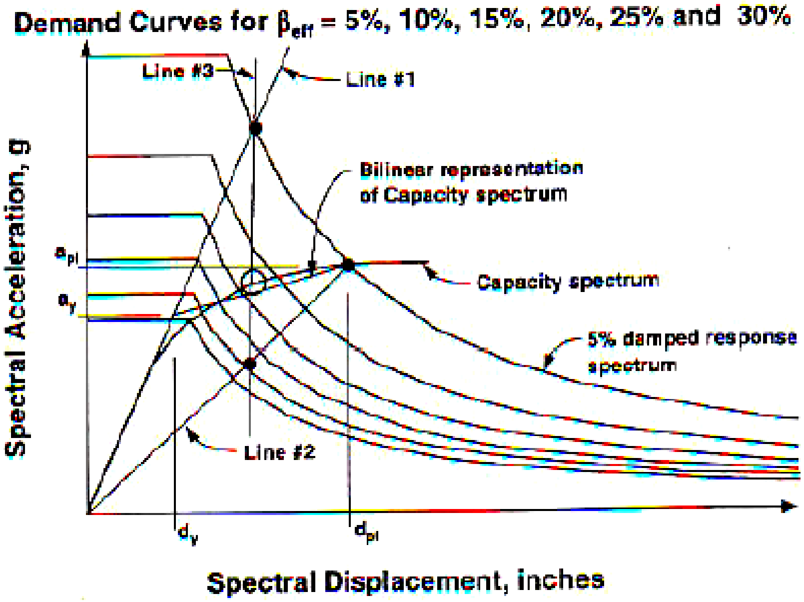


Figure 28. *Capacity spectrum procedure "C" after step 9*

4.2. Calculating demand displacement using the displacement coefficient method

The displacement coefficient method provides a direct numerical process for calculating the displacement demand. It does not require converting the capacity curve to spectral coordinates. The following procedure is accepted from the FEMA 273 guidelines.

The provisions included in this excerpt are limited in application to buildings that are regular and do not have adverse torsional or multimode effects. If the engineer uses this method for any structure that does not meet the limitations described above, then the full source document should be used.

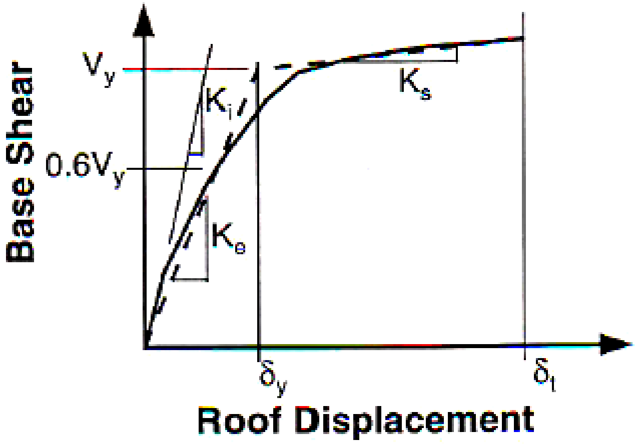


Figure 29. *Bilinear representation of capacity curve for displacement coefficient method*

Before applying this method, the user is encouraged to review the current version of FEMA 273 to determine if any of the criteria described below have been updated.

1. Construct a bilinear capacity curve as follows (refer to figure 29);
   - Draw the post elastic stiffness, K_s, by judgment to represent an average stiffness in the range in which the structure strength has leveled off.
   - Draw the effective elastic stiffness, K_e, by constructing a secant line passing through the point on the capacity curve corresponding to a base shear of $0.6V_y$, where V_y is defined by the intersection of the K_e and K_s lines,
2. Calculate the effective fundamental period (T_e) as:

$$T_e = T_i \sqrt{\frac{K_i}{K_e}} \quad (14)$$

Where:

T_i = elastic fundamental period (in seconds) in the direction under consideration calculated by elastic dynamic analysis.

K_i = elastic lateral stiffness of the building in the direction under consideration (refer to figure 29).

K_e = effective lateral stiffness of the building in the direction under consideration (refer to figure 29).

3. Calculate the target displacement, (δ_t) as:

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^{\,2}}{4\pi^2} \quad (15)$$

Where:

T_e =effective fundamental period as calculated in step 2 above.

C_0 = modification factor to relate spectral displacement and likely building roof displacement; estimated for C_0 can be calculating using either:

- The first modal participation factor at the roof level.
- The modal participation factor at the roof level calculated using the shape vector corresponding to the deflected shape of the building at the target displacement.
- The appropriate value from table 2.

Number of stories	**Modification factor**
1	1.0
2	1.2
3	1.3
5	1.4
10+	1.5

Table 2. *Values for modification factor* c_0

C_1 = modification factor to relate expected maximum elastic displacements to displacements calculated for linear elastic response.

= 1.0 for $T_e \geq T_0$

= $[1.0+ (R-1)\, T_0/T_e]/R$ for $T_e < T_0$ C_1 need not exceed 2.0 for $T_e < 0.1$ second

T_0 = a characteristic period of the response spectrum, defined as the period associated with the transition from the constant acceleration segment of the spectrum to the constant velocity segment of the spectrum.

R = ratio of inelastic strength demand to calculated yield strength coefficient calculated as follows:

$$R = \frac{S_a / g}{V_y / W} \cdot \frac{1}{C_0}$$

(16)

C_2 = modification factor to represent the effect of hysteresis shape on the maximum displacement response. Value of C_2 for different framing systems and performance levels are listed in table 3. Linear interpolation shall be used to estimate values of T_e.

	T= 0.1 second		**T $\geq T_0$ second**	
Structural performance level	**Framing type 1**	**Framing type 2**	**Framing type 1**	**Framing type 2**
Immediate occupancy	1.0	1.0	1.0	1.0
Life safety	1.3	1.0	1.1	1.0
Collapse prevention	1.5	1.0	1.2	1.0

Table 3. *Values of modification factor $c_{2.}$*

C_3 = modification factor to represent increased displacements due to second-order effects. For buildings with positive post-yield stiffness, C_3 shall be set equal to 1.0. For buildings with negative post-yield stiffness, C_3 shall be calculated as

$$C_3 = 1 + \frac{|\alpha|(R-1)^{3/2}}{T_e} \tag{17}$$

Where R and T_e are defined above and α is the ratio of post-yield stiffness to elastic stiffness when the non-linear force-displacement relation is characterized by a bilinear relation.

S_a = response spectrum acceleration, at the effective fundamental period of the building.

V_y = yield strength calculated using the capacity curve, where the capacity curve is characterized by a bilinear relation.

5. STEP BY STEP PROCEDURE FOR CHECKING PERFORMANCE AT THE EXPECTED MAXIMUM DISPLACEMENT

The following steps should be followed in the performance check:

1. For global building response verify the following:

   - ➢ The lateral force resistance has not degraded by more than 20% of the peak resistance.

- The lateral drifts satisfy the limits given in table 4.

	Performance level			
Inter storey drift limit	**Immediate occupancy**	**Damage control**	**Life safety**	**Structural stability**
Maximum total drift	0.01	0.01-0.02	0.02	$0.33\frac{V_i}{P_i}$
Maximum inelastic drift	0.005	0.005-0.015	No limit	No limit

Table 4. *deformation limits*

2. Identify and classify the different elements in the building. Any of the following element types may be present: beam column frames, slab column frames, solid walls, punched walls, floor diaphragms and foundations.

3. Identify all primary and secondary components. This classification is needed for the deformation check in step 5.

4. For each element, use the guidelines to identify the critical components and actions to be checked.

5. The strength and deformation demands at the structure's performance point shall be equal to or less than the capacities considering all co-existing forces acting with the demand spectrum.

6. The performance of structural elements not carrying vertical load shall be reviewed for acceptability for the specified performance level.

7. Non-structural elements shall be checked for acceptability for the specified performance level.

6. ILLUSTRATION OF PUSHOVER ANALYSIS USING SAP 2000

A three story building (G+2) has been taken which is designed for different loadings as follows:

a. Earthquake Zone 3

b. Earthquake Zone 4

c. Earthquake Zone 5

A. Geometry

Plan dimension 7m × 7m

Height of building 12.5 m

Load on Beams:-

at Each floor – 19 kN/m

Lateral Load – 20 kN at 12.5 m from base

– 15 kN at 9 m from base

– 10 kN at 5.5 m from base

– 5 kN at 2 m from base

Displacement control of top story = 0.5 m

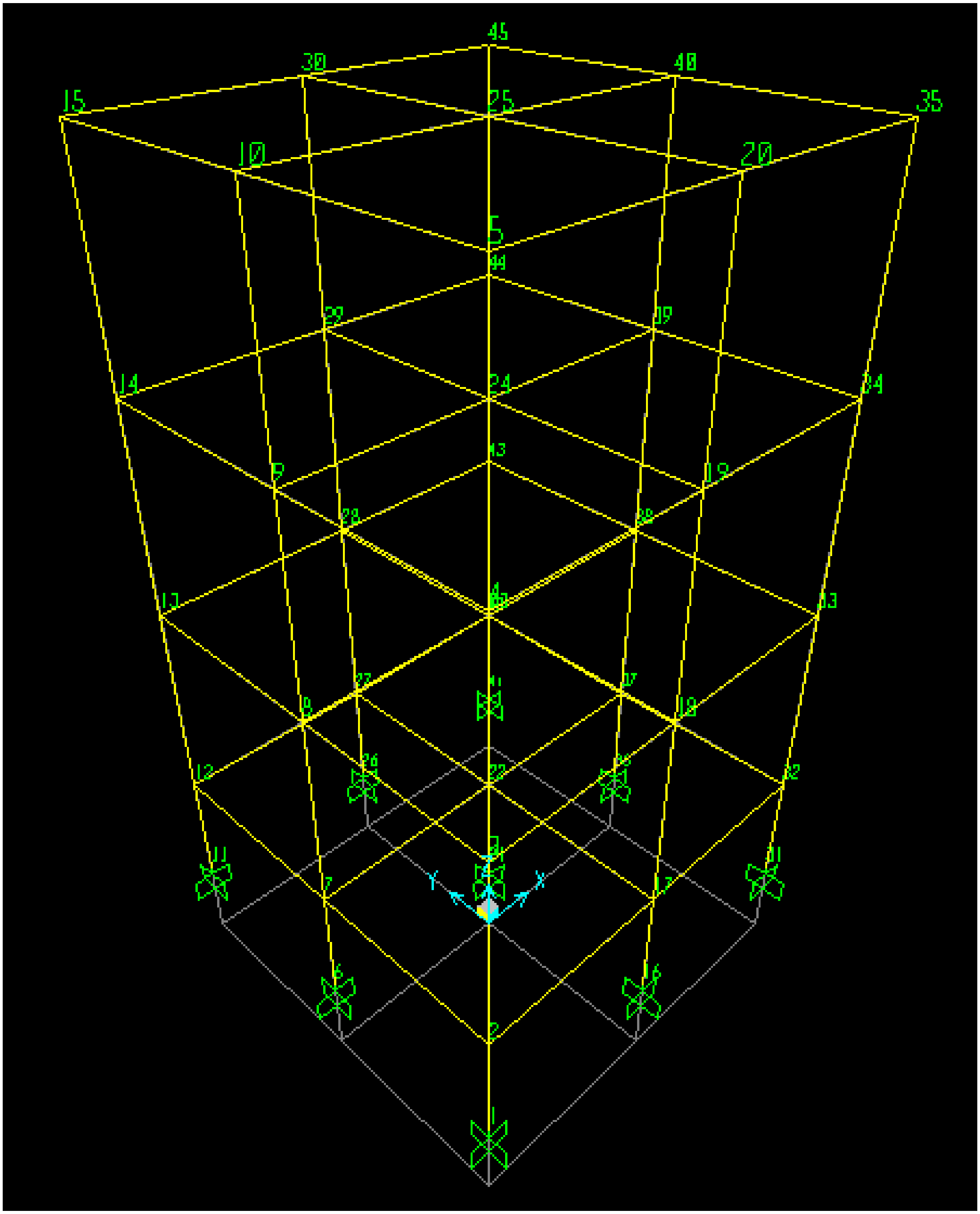


Figure 30. *Geometry of Building (Elevation)*

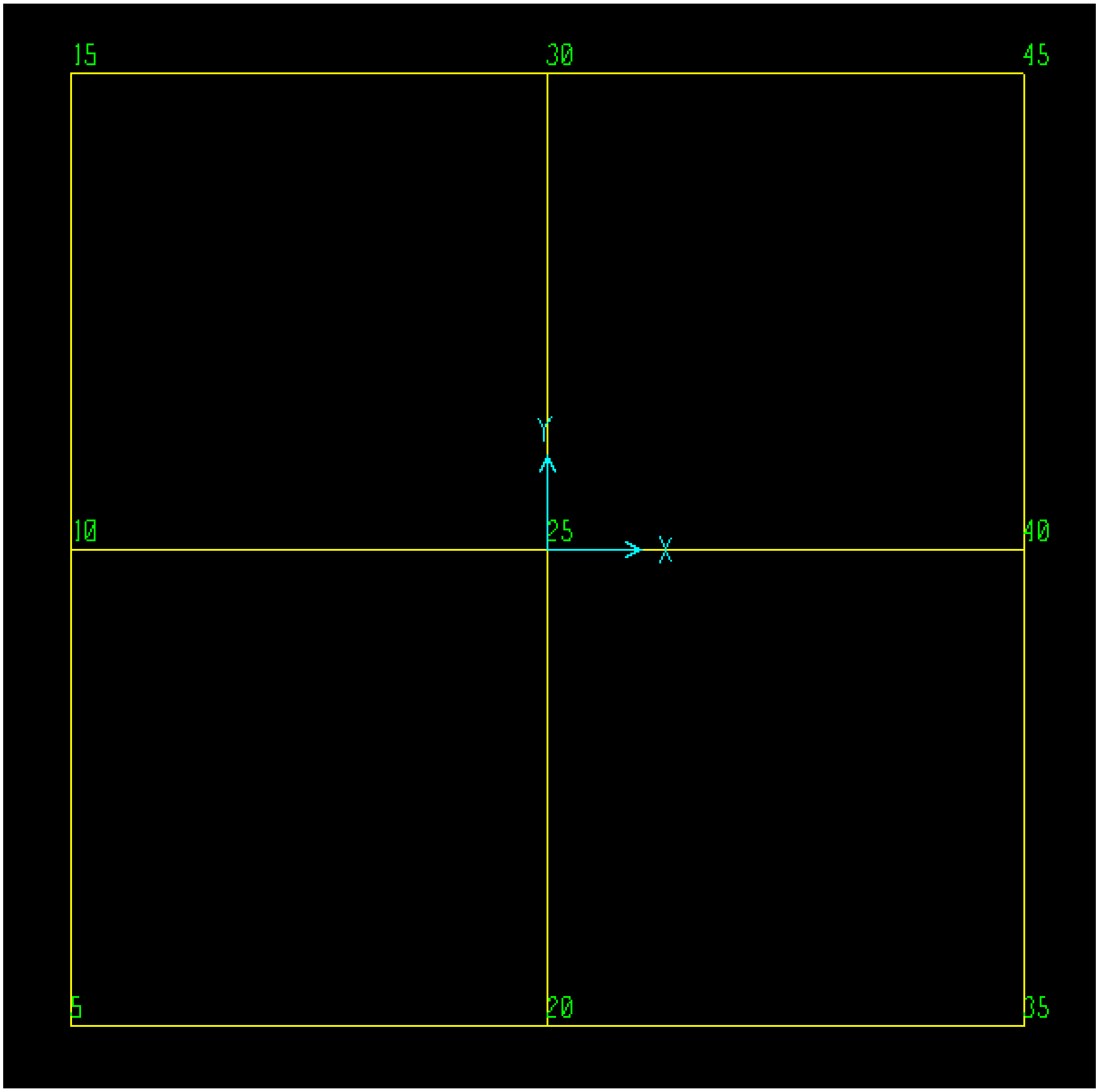


Figure 31. *Plan*

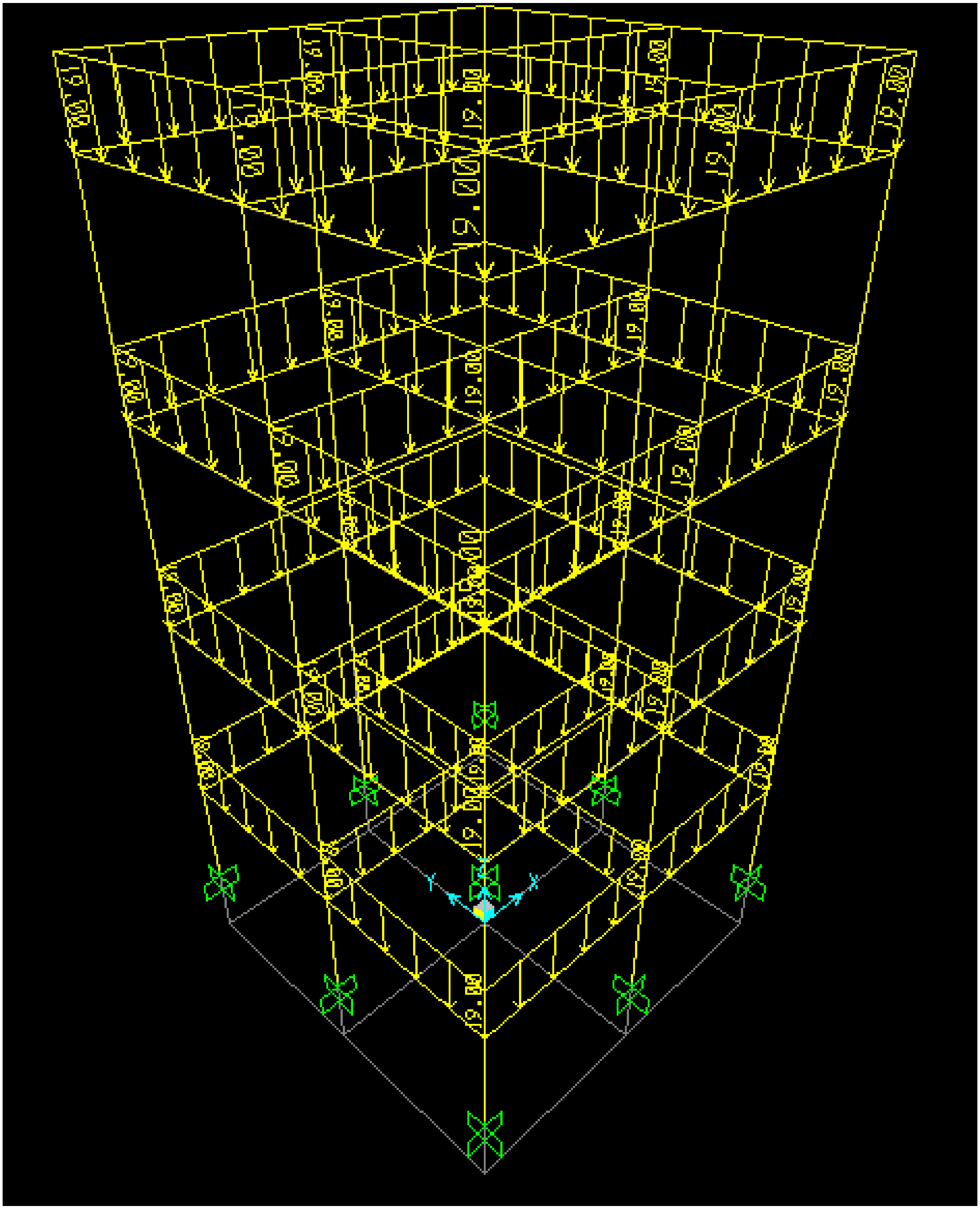


Figure 32. *Dead Load*

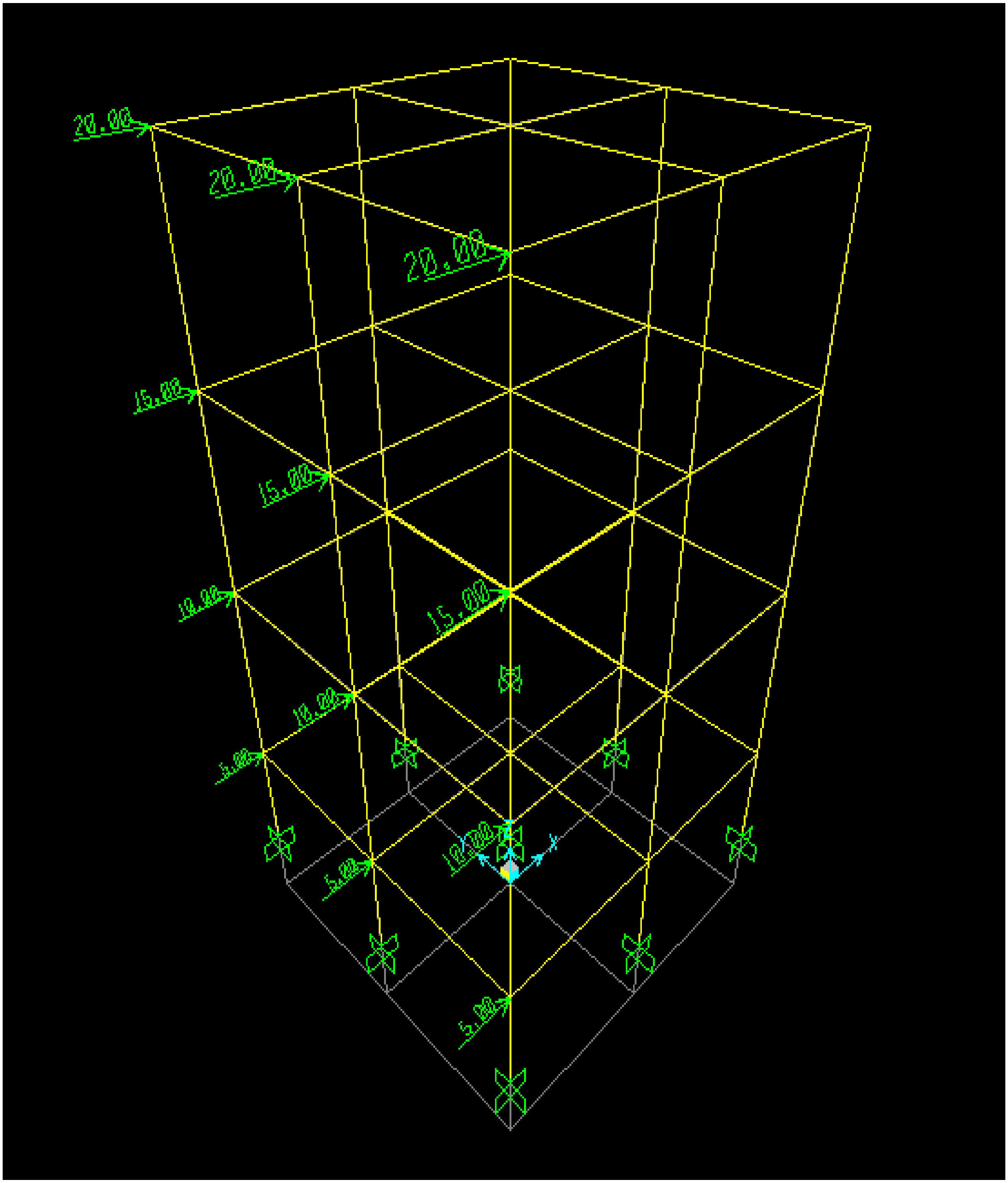
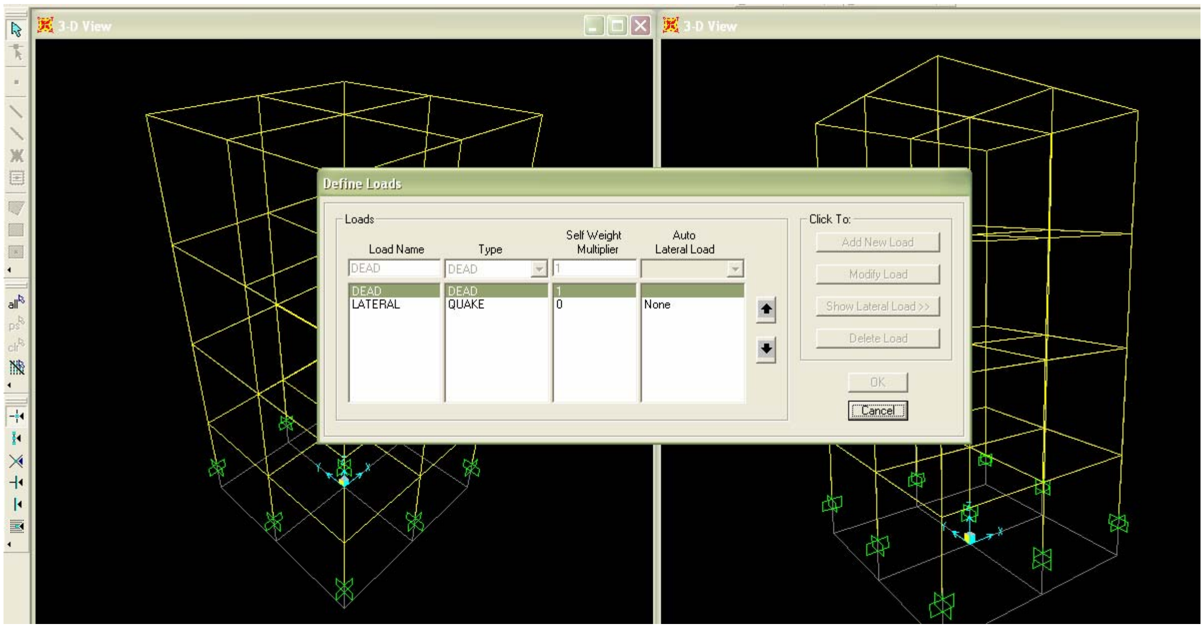


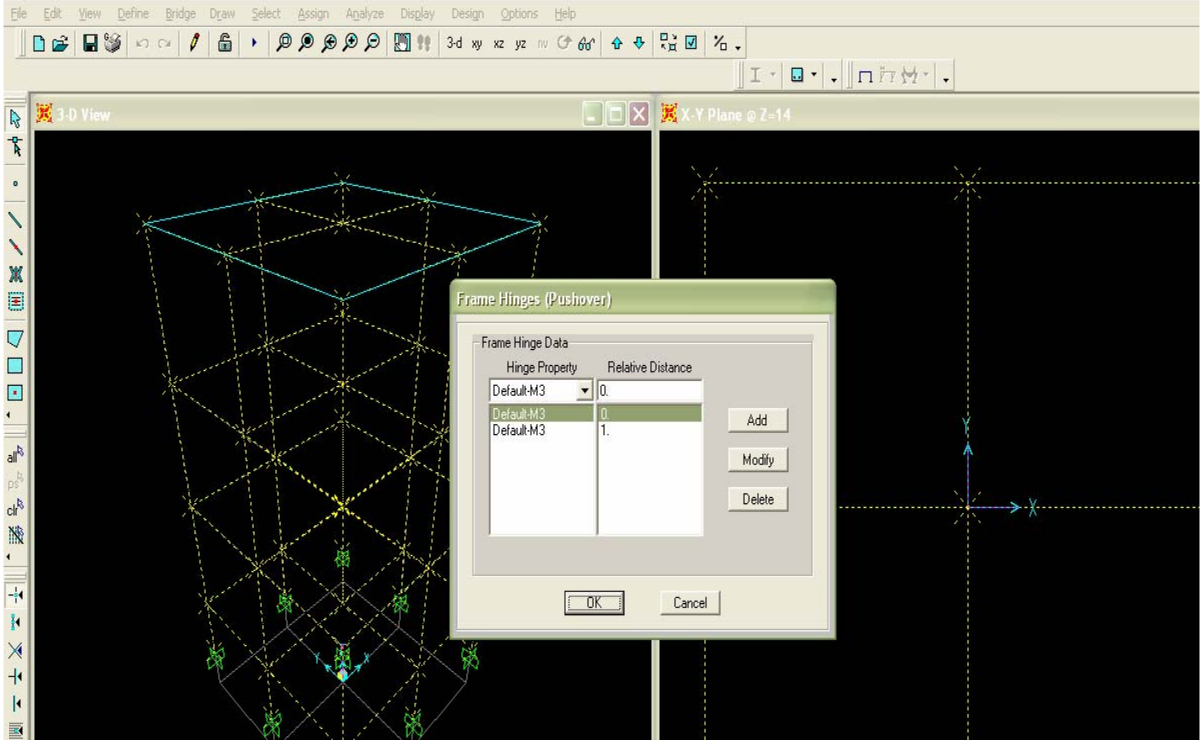
Figure 33. *Lateral Load*



Loads

Load Name	Type	Self Weight Multiplier	Auto Lateral Load
DEAD	DEAD	1	
DEAD	DEAD	1	
LATERAL	QUAKE	0	None

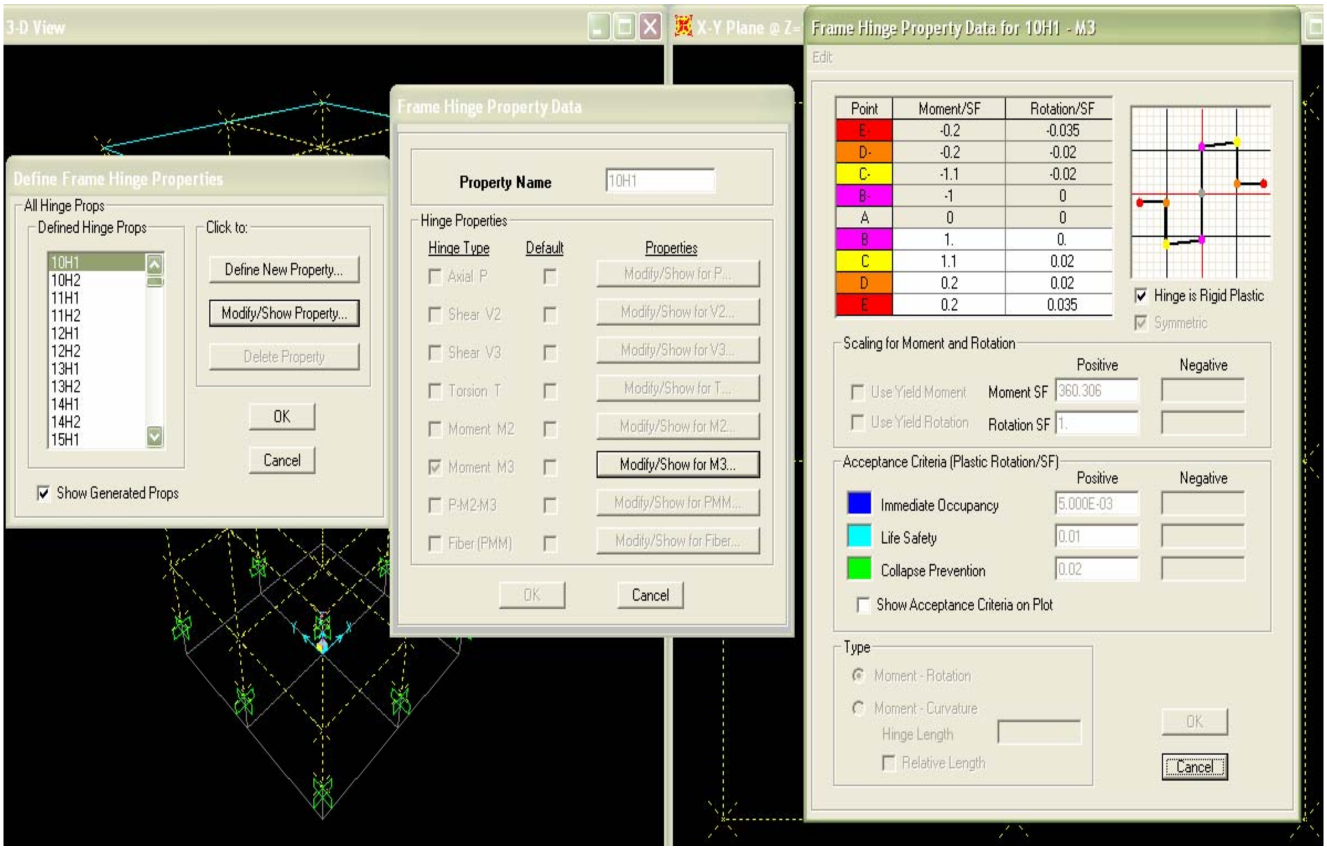
Figure 34. *Load Cases*



Frame Hinge Data

Hinge Property	Relative Distance
Default-M3	0.
Default-M3	0.
Default-M3	1.

Figure 35. *Defining Hinges*



☑ Show Generated Props

Property Name 10H1

Hinge Type	Default
☐ Axial P	☐
☐ Shear V2	☐
☐ Shear V3	☐
☐ Torsion T	☐
☐ Moment M2	☐
☑ Moment M3	☐
☐ P-M2-M3	☐
☐ Fiber (PMM)	☐

Point	Moment/SF	Rotation/SF
E-	-0.2	-0.035
D-	-0.2	-0.02
C-	-1.1	-0.02
B-	-1	0
A	0	0
B	1.	0.
C	1.1	0.02
D	0.2	0.02
E	0.2	0.035

☑ Hinge is Rigid Plastic
☑ Symmetric

Scaling for Moment and Rotation

		Positive	Negative
☐ Use Yield Moment	Moment SF	360.306	
☐ Use Yield Rotation	Rotation SF	1.	

Acceptance Criteria (Plastic Rotation/SF)

	Positive	Negative
🟦 Immediate Occupancy	5.000E-03	
🟦 Life Safety	0.01	
🟩 Collapse Prevention	0.02	

☐ Show Acceptance Criteria on Plot

Type
● Moment - Rotation
○ Moment - Curvature
Hinge Length
☐ Relative Length

Figure 36. *Hinge Property*

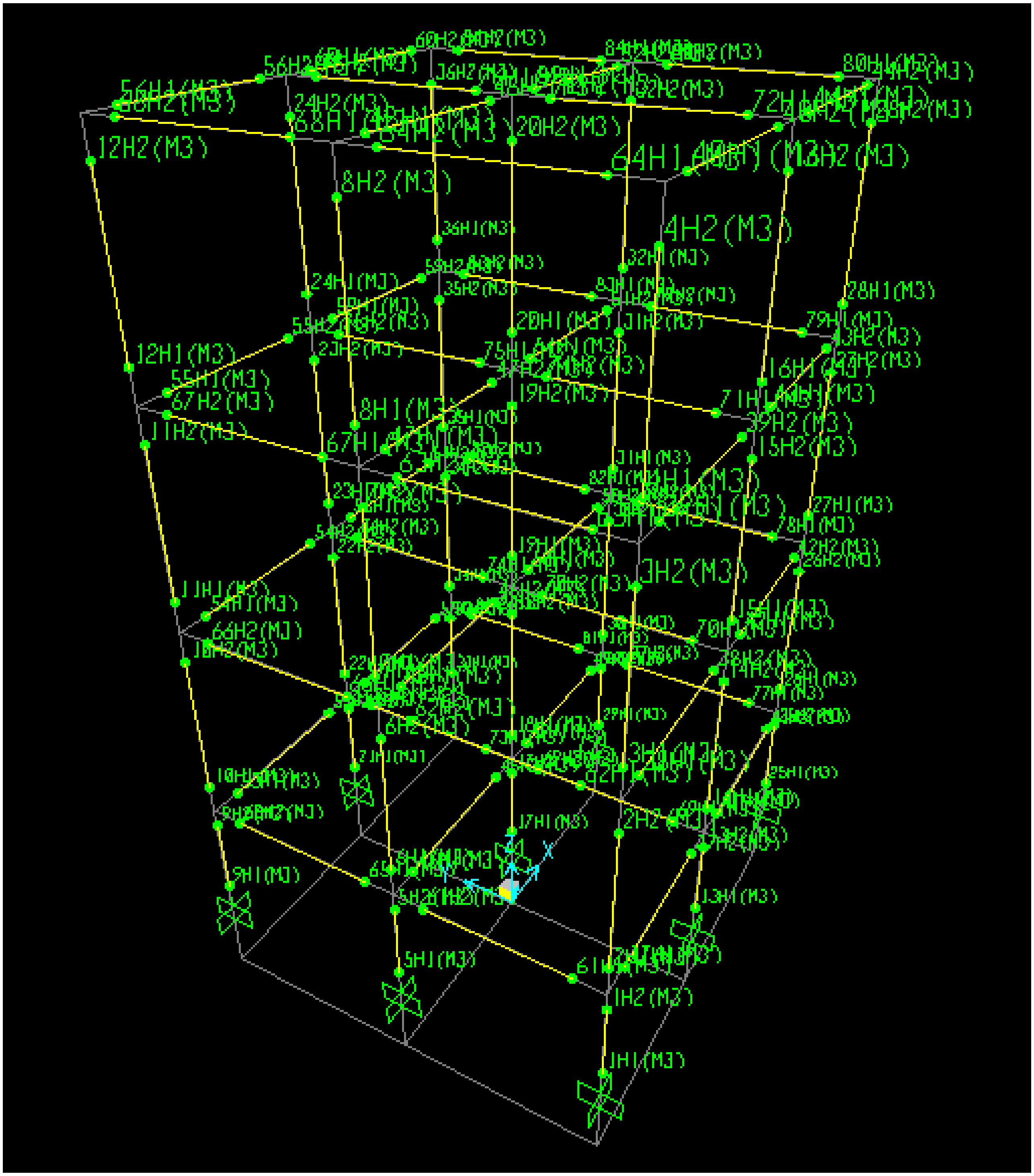


__Figure 37.__ Defined Hinge (3 D)

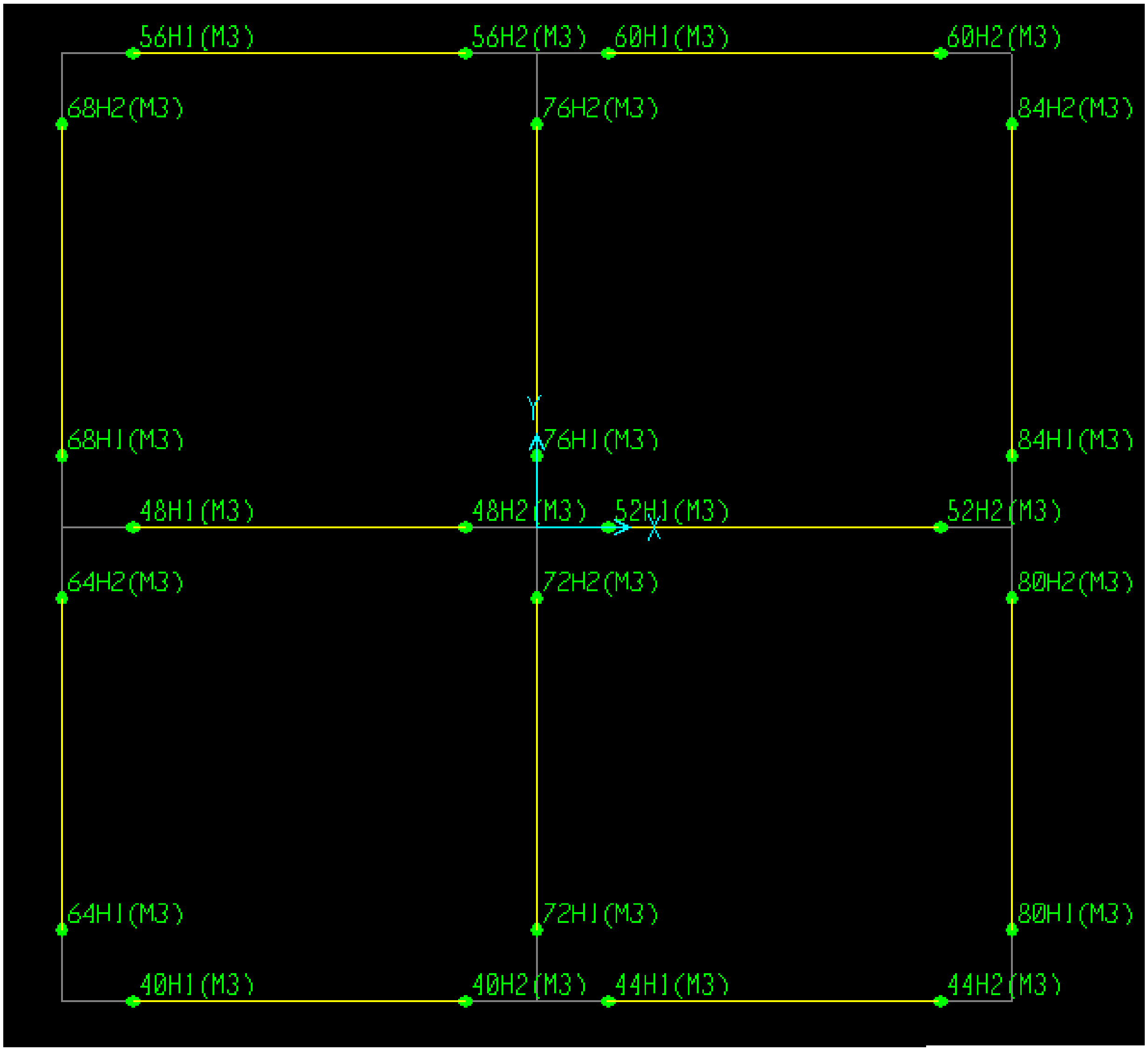
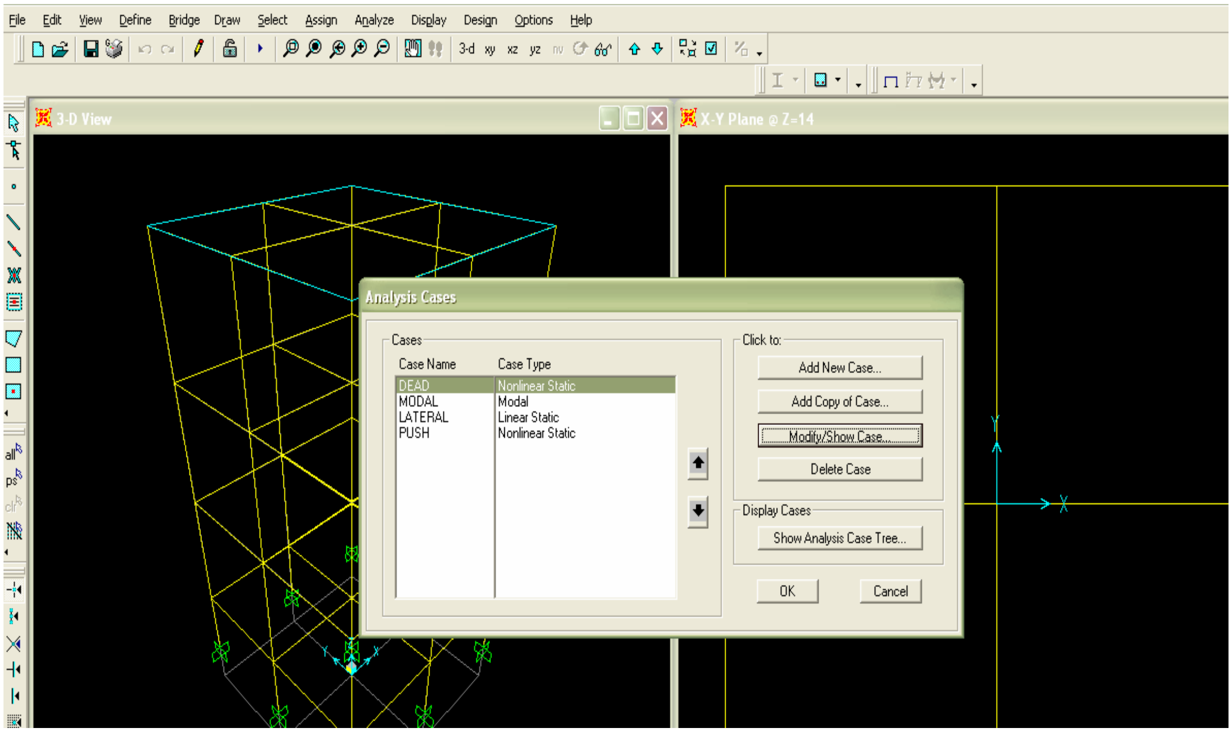


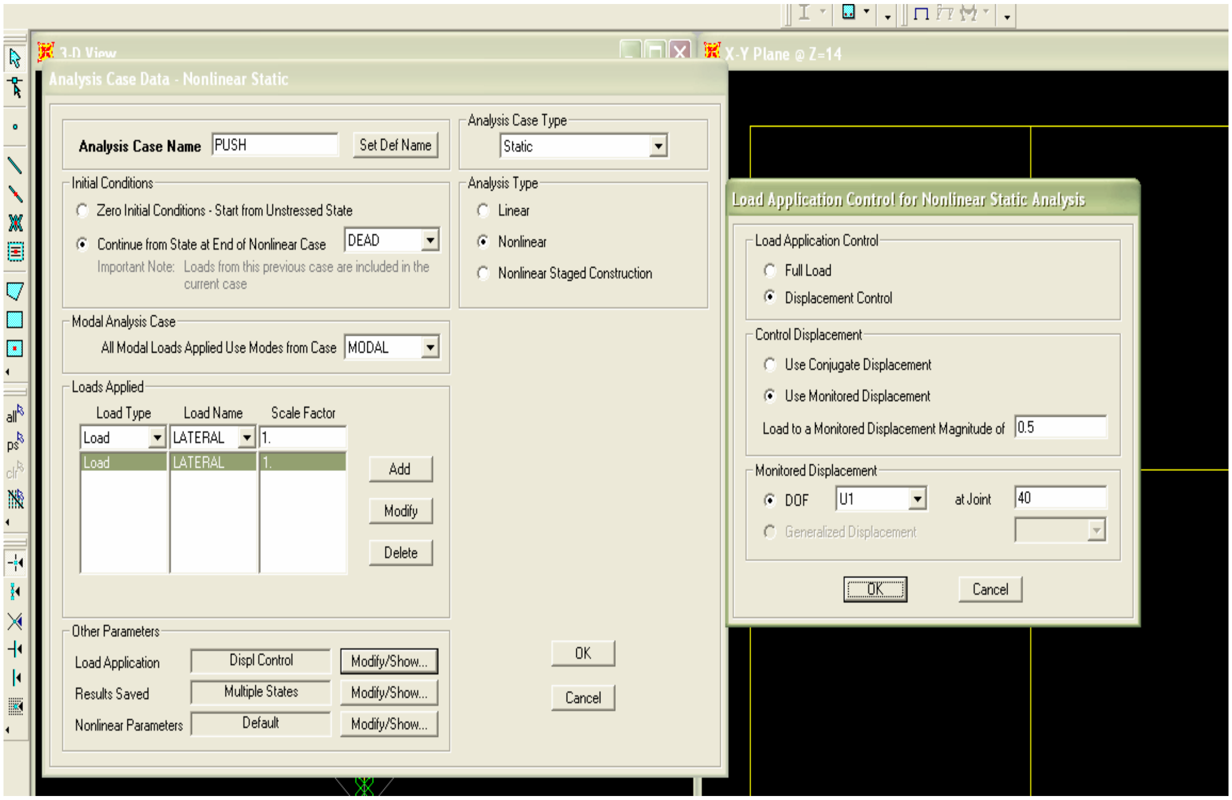
Figure 38. *Defined Hinges (Plan)*



Cases

Case Name	Case Type
DEAD	Nonlinear Static
MODAL	Modal
LATERAL	Linear Static
PUSH	Nonlinear Static

Figure 39. *Analysis Cases*



Analysis Case Name PUSH

Analysis Case Type: Static

Initial Conditions
- ○ Zero Initial Conditions - Start from Unstressed State
- ● Continue from State at End of Nonlinear Case DEAD

Important Note: Loads from this previous case are included in the current case

Analysis Type
- ○ Linear
- ● Nonlinear
- ○ Nonlinear Staged Construction

Modal Analysis Case: All Modal Loads Applied Use Modes from Case MODAL

Loads Applied

Load Type	Load Name	Scale Factor
Load	LATERAL	1.

Other Parameters

Load Application	Displ Control
Results Saved	Multiple States
Nonlinear Parameters	Default

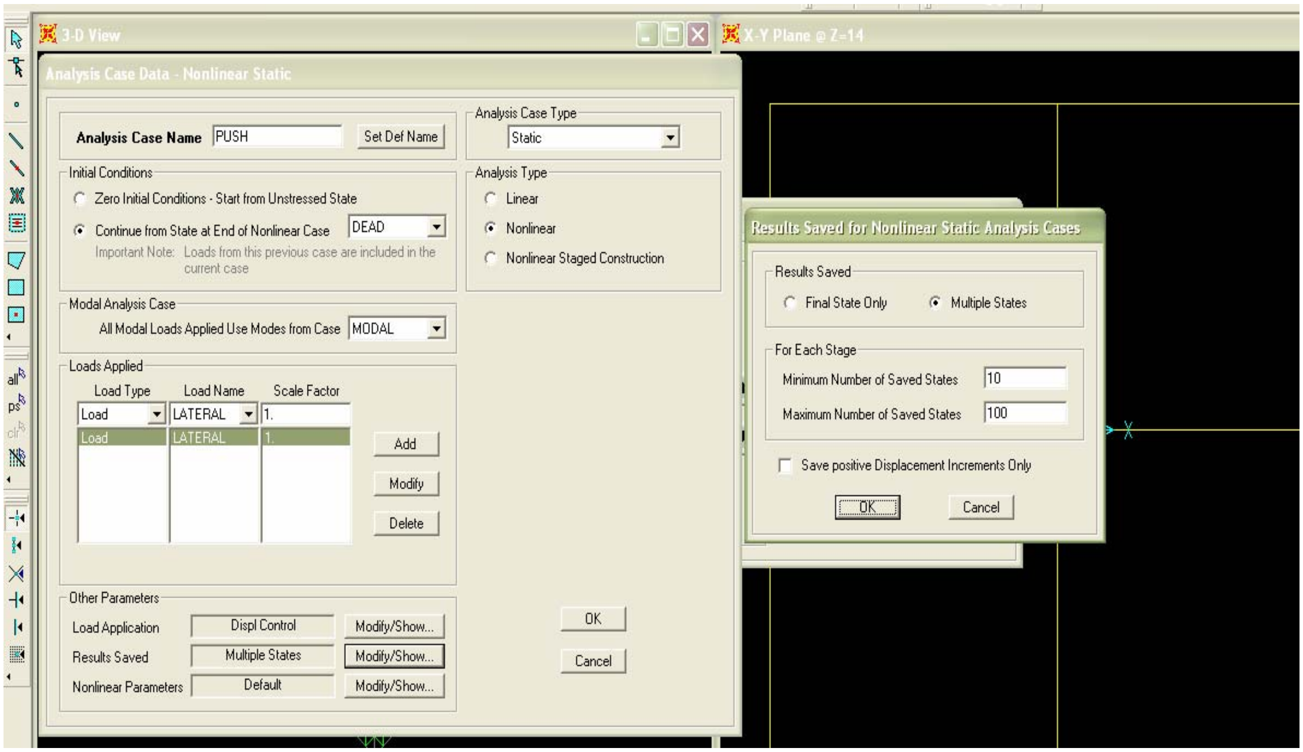
Load Application Control
- ○ Full Load
- ● Displacement Control

Control Displacement
- ○ Use Conjugate Displacement
- ● Use Monitored Displacement

Load to a Monitored Displacement Magnitude of 0.5

Monitored Displacement
- ● DOF U1 at Joint 40
- ○ Generalized Displacement

Figure 40. *Pushover Analysis Case in which top storey drift is assigned as 0.5 m*



Analysis Case Data - Nonlinear Static

Analysis Case Name PUSH

Analysis Case Type: Static

Initial Conditions

○ Zero Initial Conditions - Start from Unstressed State
● Continue from State at End of Nonlinear Case DEAD

Important Note: Loads from this previous case are included in the current case

Analysis Type

○ Linear
● Nonlinear
○ Nonlinear Staged Construction

Modal Analysis Case

All Modal Loads Applied Use Modes from Case MODAL

Loads Applied

Load Type	Load Name	Scale Factor
Load	LATERAL	1.

Other Parameters

Load Application: Displ Control
Results Saved: Multiple States
Nonlinear Parameters: Default

Results Saved for Nonlinear Static Analysis Cases

Results Saved

○ Final State Only ● Multiple States

For Each Stage

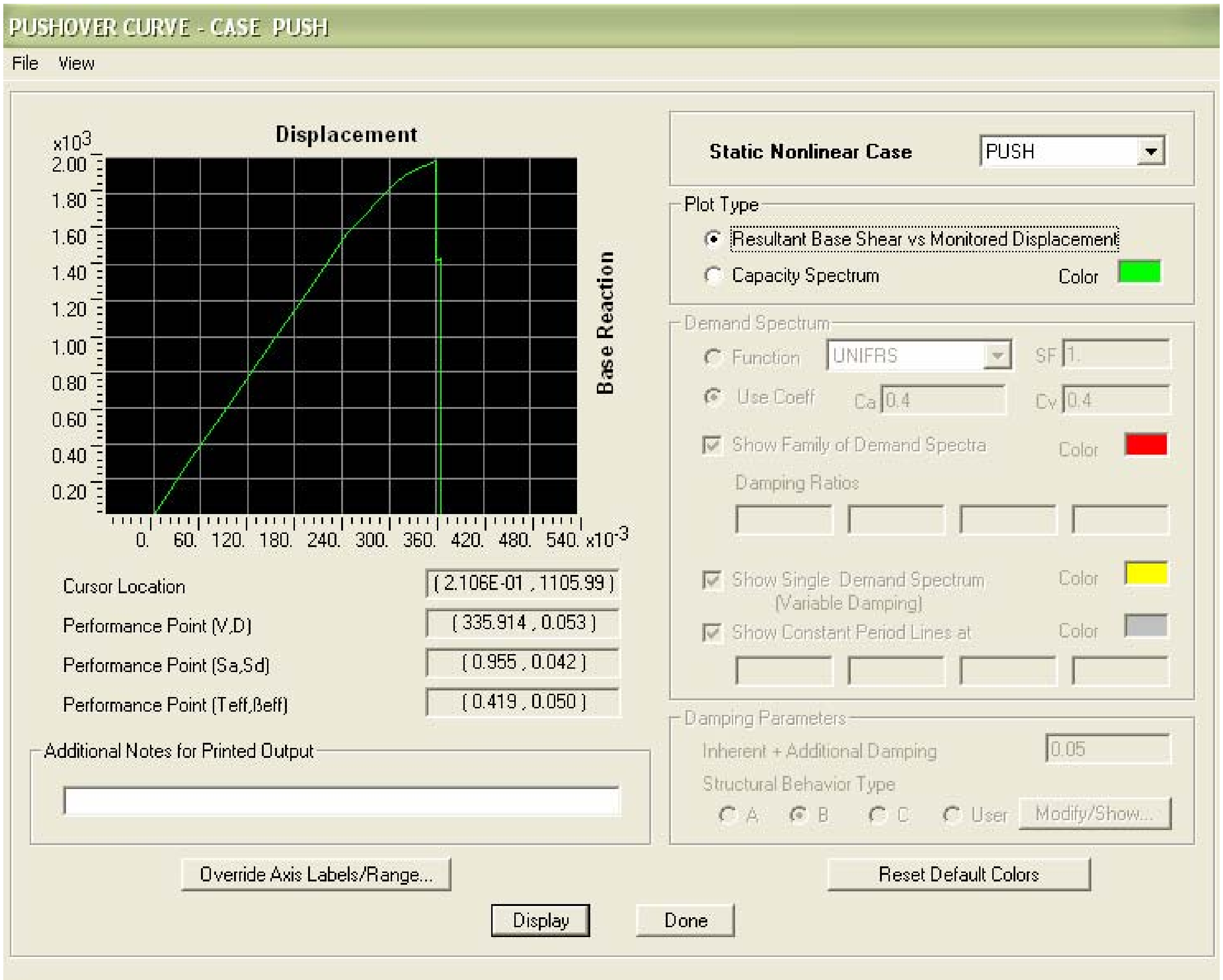
Minimum Number of Saved States: 10
Maximum Number of Saved States: 100

☐ Save positive Displacement Increments Only

Figure 41. *Pushover Analysis Case in which no. of step assigned is 100.*

B. Base Shear Vs Monitored Displacement and Capacity Spectrum

After having modeled and analyzed by Pushover Analysis the Base Shear Vs Monitored Displacement graph is shown in figures from (42 - 47).



Static Nonlinear Case: PUSH

Plot Type
● Resultant Base Shear vs Monitored Displacement
○ Capacity Spectrum
Color 🟩

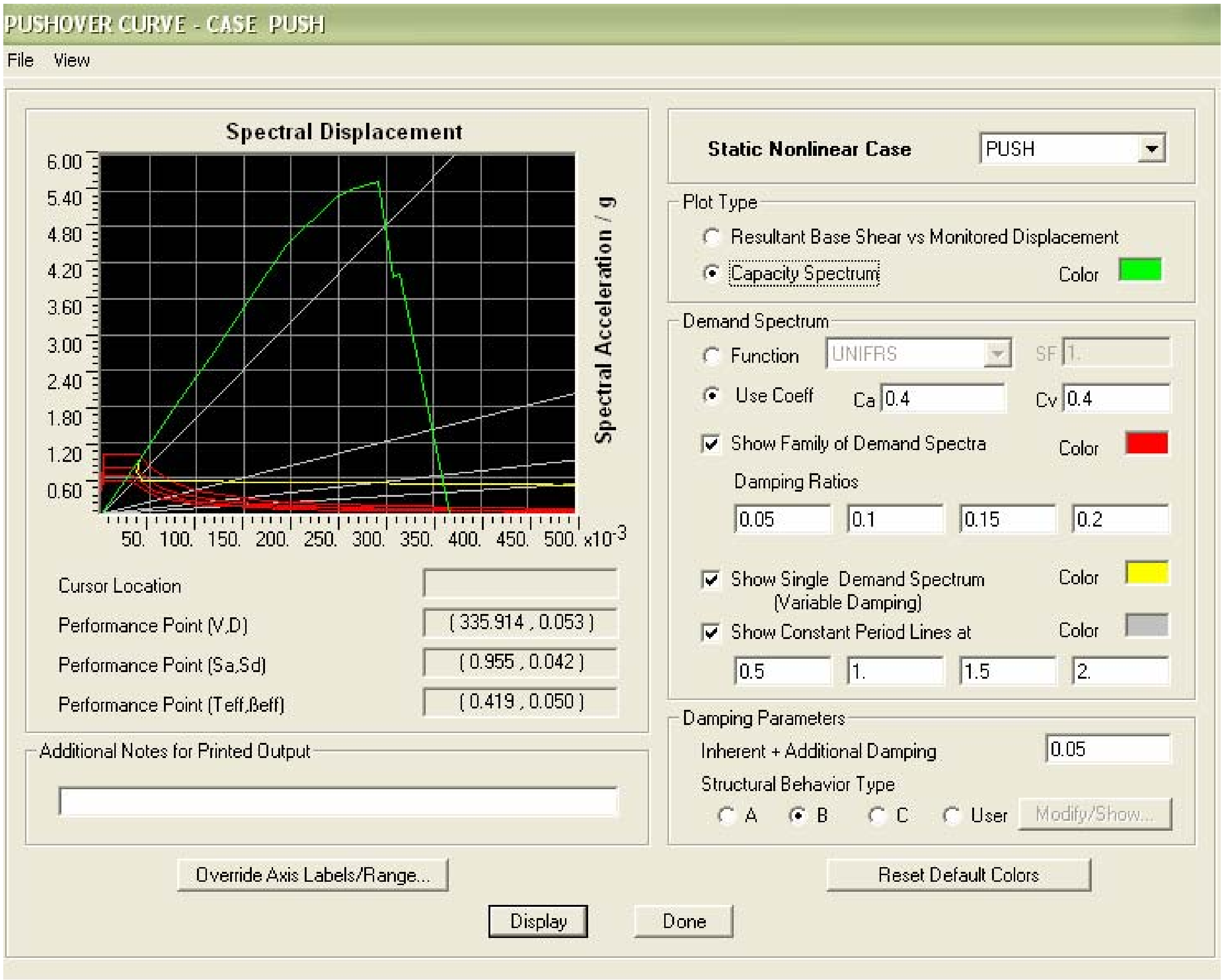
Demand Spectrum
○ Function UNIFRS SF 1.
● Use Coeff Ca 0.4 Cv 0.4
☑ Show Family of Demand Spectra Color 🟥
Damping Ratios
☑ Show Single Demand Spectrum (Variable Damping) Color 🟨
☑ Show Constant Period Lines at Color ▨

Damping Parameters
Inherent + Additional Damping 0.05
Structural Behavior Type
○ A ● B ○ C ○ User

Cursor Location (2.106E-01 , 1105.99)
Performance Point (V,D) (335.914 , 0.053)
Performance Point (Sa,Sd) (0.955 , 0.042)
Performance Point (Teff,βeff) (0.419 , 0.050)

Additional Notes for Printed Output

Figure 42. *Base Shear Vs Monitored Displacement for building designed for Earthquake zone 3*



Cursor Location

Performance Point (V,D) (335.914 , 0.053)

Performance Point (Sa,Sd) (0.955 , 0.042)

Performance Point (Teff,ßeff) (0.419 , 0.050)

Additional Notes for Printed Output

Static Nonlinear Case: PUSH

Plot Type

○ Resultant Base Shear vs Monitored Displacement

● Capacity Spectrum Color 🟩

Demand Spectrum

○ Function UNIFRS SF 1.

● Use Coeff Ca 0.4 Cv 0.4

☑ Show Family of Demand Spectra Color 🟥

Damping Ratios 0.05 0.1 0.15 0.2

☑ Show Single Demand Spectrum (Variable Damping) Color 🟨

☑ Show Constant Period Lines at Color ▨

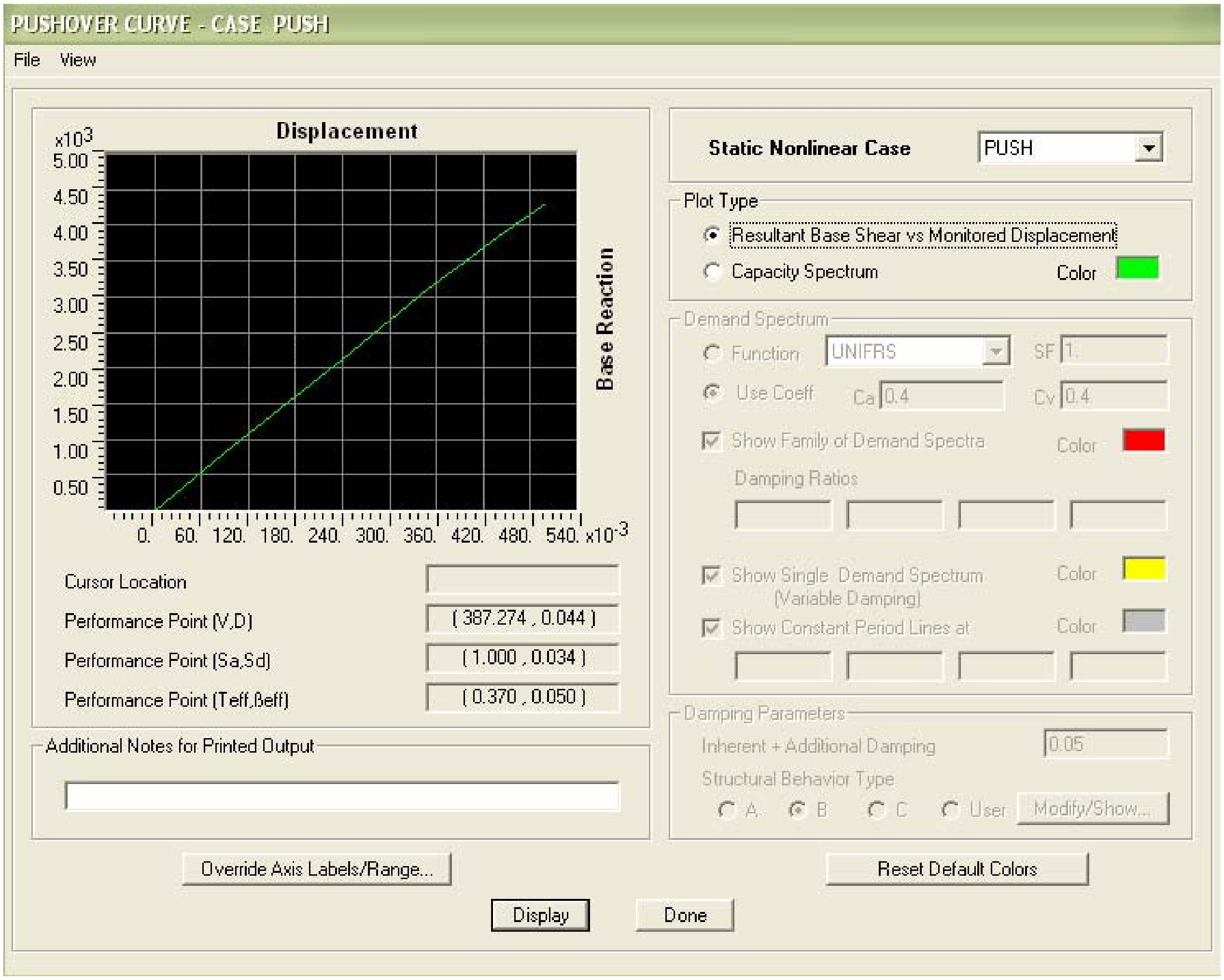
0.5 1. 1.5 2.

Damping Parameters

Inherent + Additional Damping 0.05

Structural Behavior Type ○ A ● B ○ C ○ User

Figure 43. *Capacity spectrum for building designed for Earthquake Zone 3*



Static Nonlinear Case PUSH

Plot Type
● Resultant Base Shear vs Monitored Displacement
○ Capacity Spectrum Color 🟩

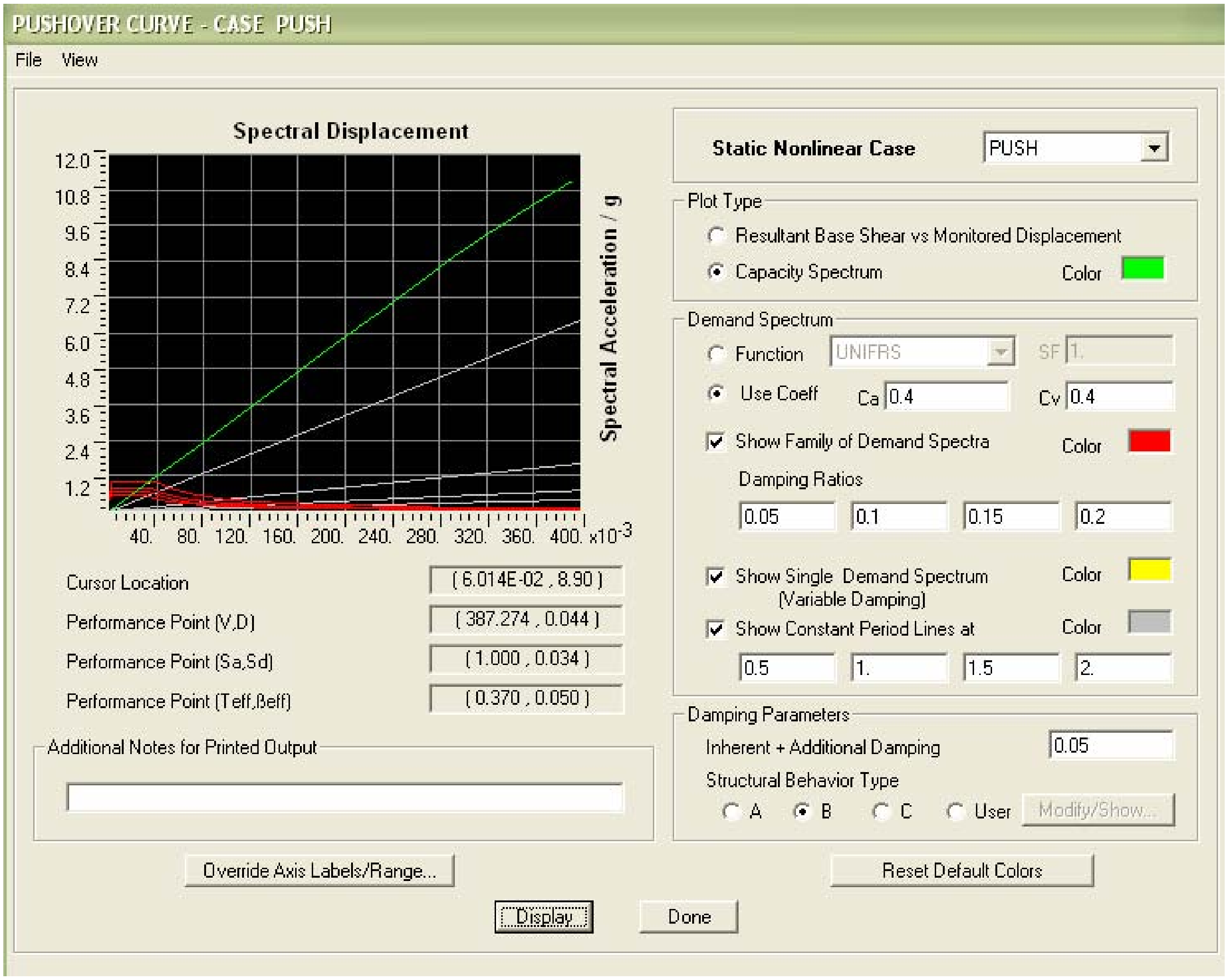
Demand Spectrum
○ Function UNIFRS SF 1.
● Use Coeff Ca 0.4 Cv 0.4
☑ Show Family of Demand Spectra Color 🟥
Damping Ratios
☑ Show Single Demand Spectrum (Variable Damping) Color 🟨
☑ Show Constant Period Lines at Color ⬜

Damping Parameters
Inherent + Additional Damping 0.05
Structural Behavior Type
○ A ● B ○ C ○ User

Cursor Location
Performance Point (V,D) (387.274 , 0.044)
Performance Point (Sa,Sd) (1.000 , 0.034)
Performance Point (Teff,Beff) (0.370 , 0.050)

Additional Notes for Printed Output

Figure 44. *Base Shear Vs Monitored Displacement for building designed for Earthquake zone 4*



Cursor Location (6.014E-02 , 8.90)

Performance Point (V,D) (387.274 , 0.044)

Performance Point (Sa,Sd) (1.000 , 0.034)

Performance Point (Teff,ßeff) (0.370 , 0.050)

Additional Notes for Printed Output

Static Nonlinear Case: PUSH

Plot Type

○ Resultant Base Shear vs Monitored Displacement

● Capacity Spectrum Color 🟩

Demand Spectrum

○ Function UNIFRS SF 1.

● Use Coeff Ca 0.4 Cv 0.4

☑ Show Family of Demand Spectra Color 🟥

Damping Ratios 0.05 0.1 0.15 0.2

☑ Show Single Demand Spectrum (Variable Damping) Color 🟨

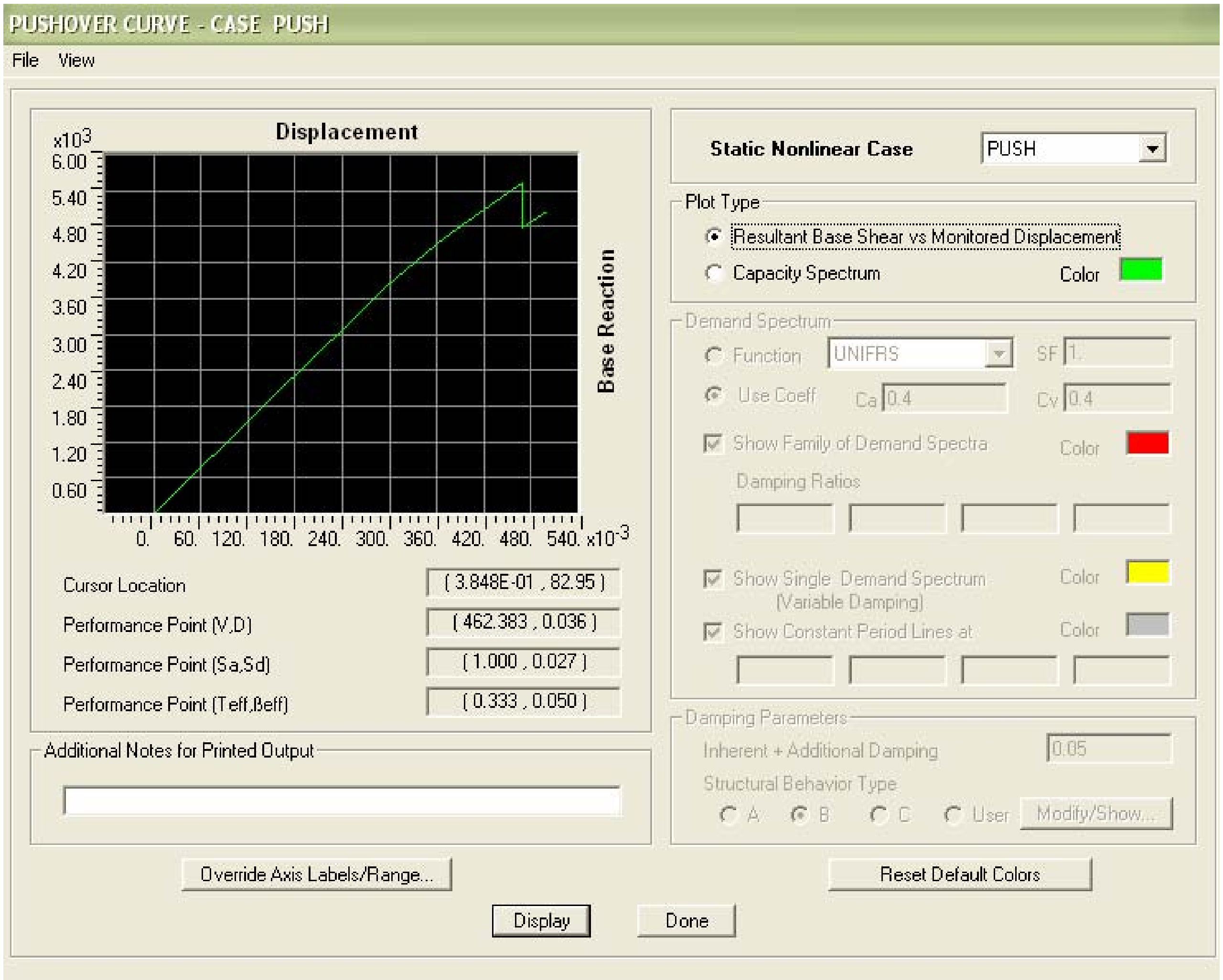
☑ Show Constant Period Lines at 0.5 1. 1.5 2. Color ⬜

Damping Parameters

Inherent + Additional Damping 0.05

Structural Behavior Type ○ A ● B ○ C ○ User

Figure 45. *Capacity spectrum for building designed for Earthquake Zone 4*



Cursor Location (3.848E-01 , 82.95)

Performance Point (V,D) (462.383 , 0.036)

Performance Point (Sa,Sd) (1.000 , 0.027)

Performance Point (Teff,ßeff) (0.333 , 0.050)

Static Nonlinear Case PUSH

Plot Type

● Resultant Base Shear vs Monitored Displacement

○ Capacity Spectrum Color 🟩

Demand Spectrum

○ Function UNIFRS SF 1.

● Use Coeff Ca 0.4 Cv 0.4

☑ Show Family of Demand Spectra Color 🟥

Damping Ratios

☑ Show Single Demand Spectrum (Variable Damping) Color 🟨

☑ Show Constant Period Lines at Color ⬜

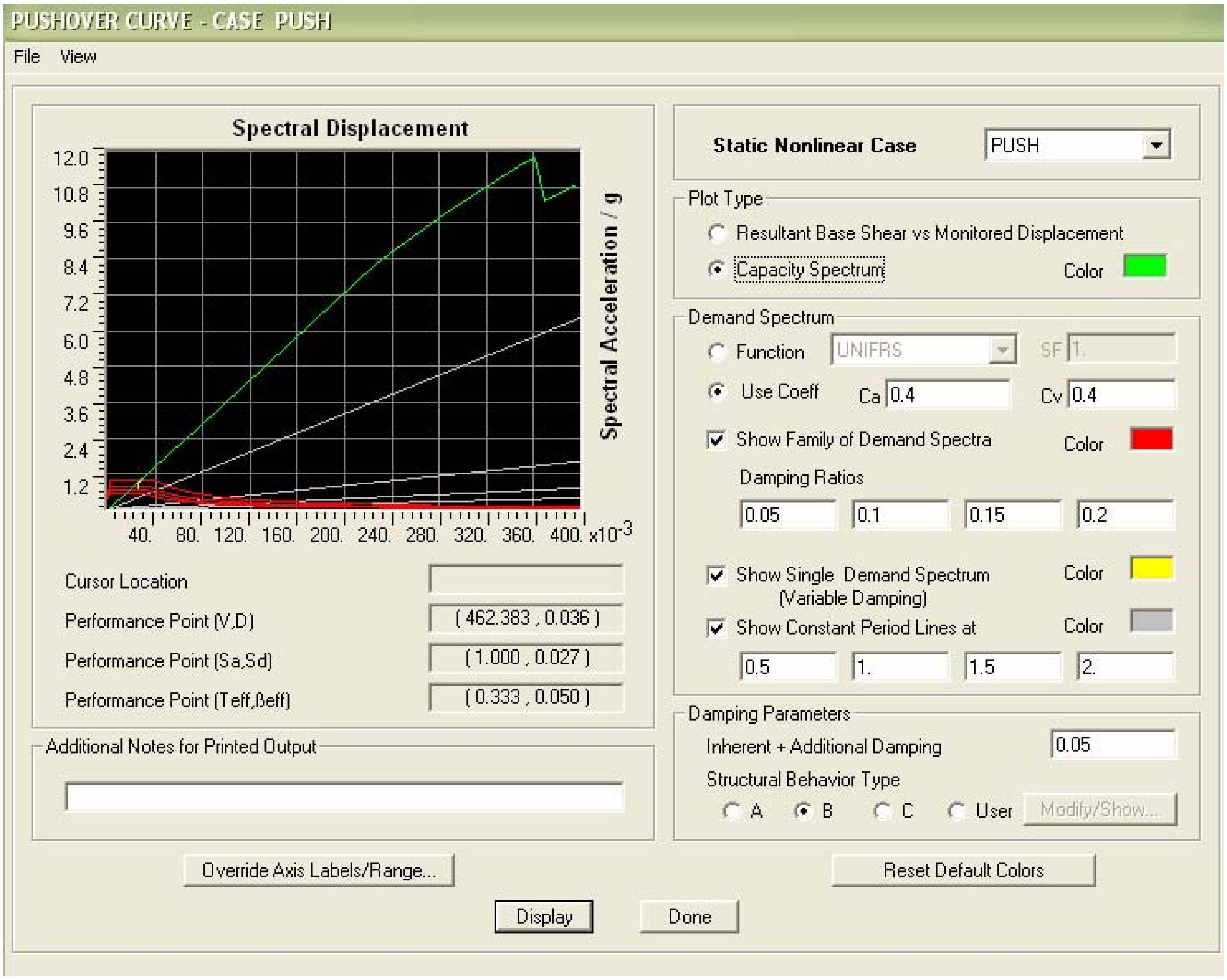
Damping Parameters

Inherent + Additional Damping 0.05

Structural Behavior Type

○ A ● B ○ C ○ User

__Figure 46.__ Base Shear Vs Monitored Displacement for building designed for Earthquake zone 5



Cursor Location

Performance Point (V,D) (462.383 , 0.036)

Performance Point (Sa,Sd) (1.000 , 0.027)

Performance Point (Teff,ßeff) (0.333 , 0.050)

Additional Notes for Printed Output

Static Nonlinear Case PUSH

Plot Type

○ Resultant Base Shear vs Monitored Displacement

● Capacity Spectrum Color 🟩

Demand Spectrum

○ Function UNIFRS SF 1.

● Use Coeff Ca 0.4 Cv 0.4

☑ Show Family of Demand Spectra Color 🟥

Damping Ratios 0.05 0.1 0.15 0.2

☑ Show Single Demand Spectrum (Variable Damping) Color 🟨

☑ Show Constant Period Lines at Color ⬜

0.5 1. 1.5 2.

Damping Parameters

Inherent + Additional Damping 0.05

Structural Behavior Type

○ A ● B ○ C ○ User

Figure 47. *Capacity spectrum for building designed for Earthquake Zone 5*

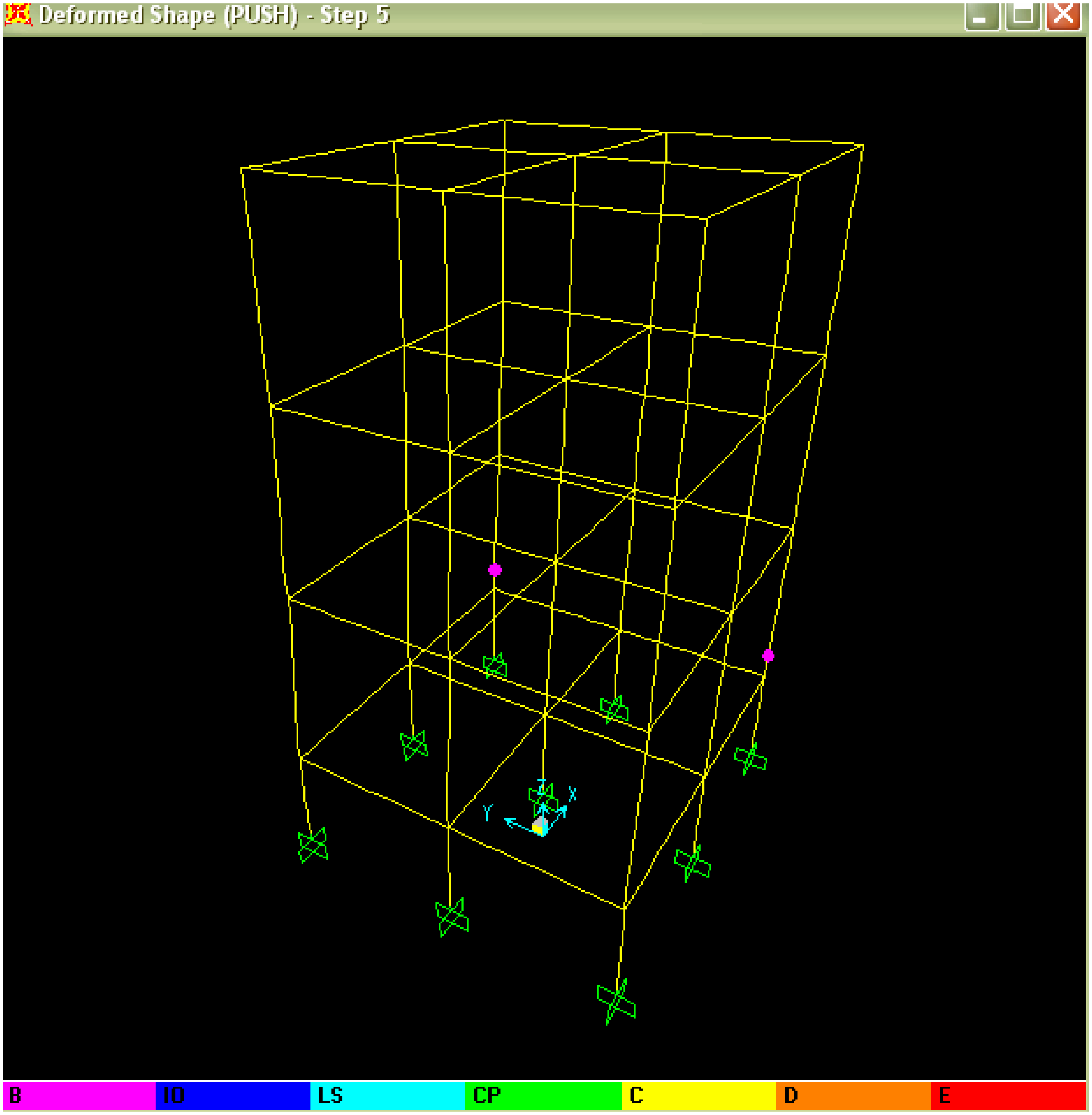


Figure 48. *Location of first occurrence of Plastic hinge for building designed for Zone 3*

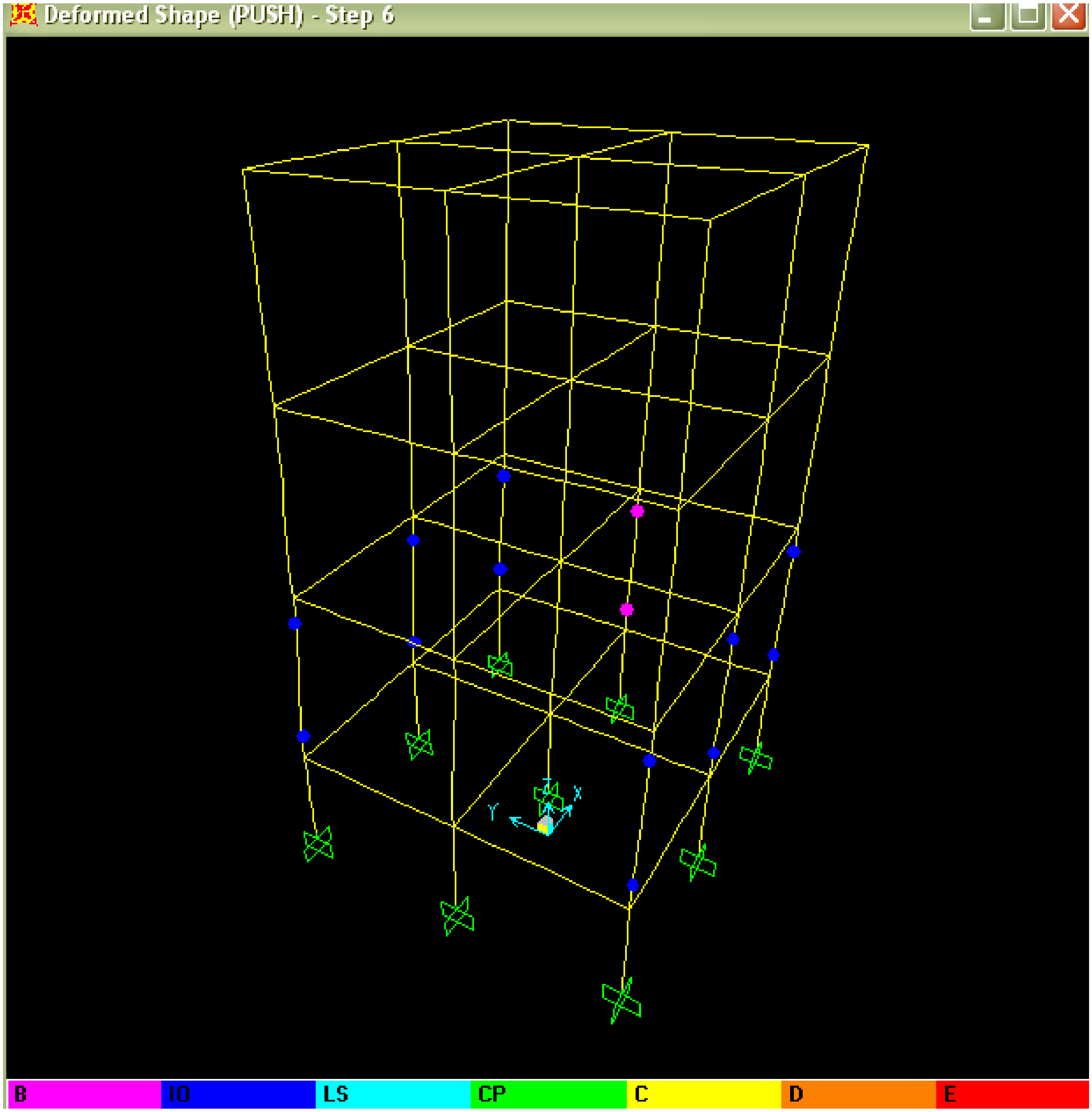


Figure 49. *Occurrence of Plastic hinges in succession of previous figure for building designed for Zone 3*

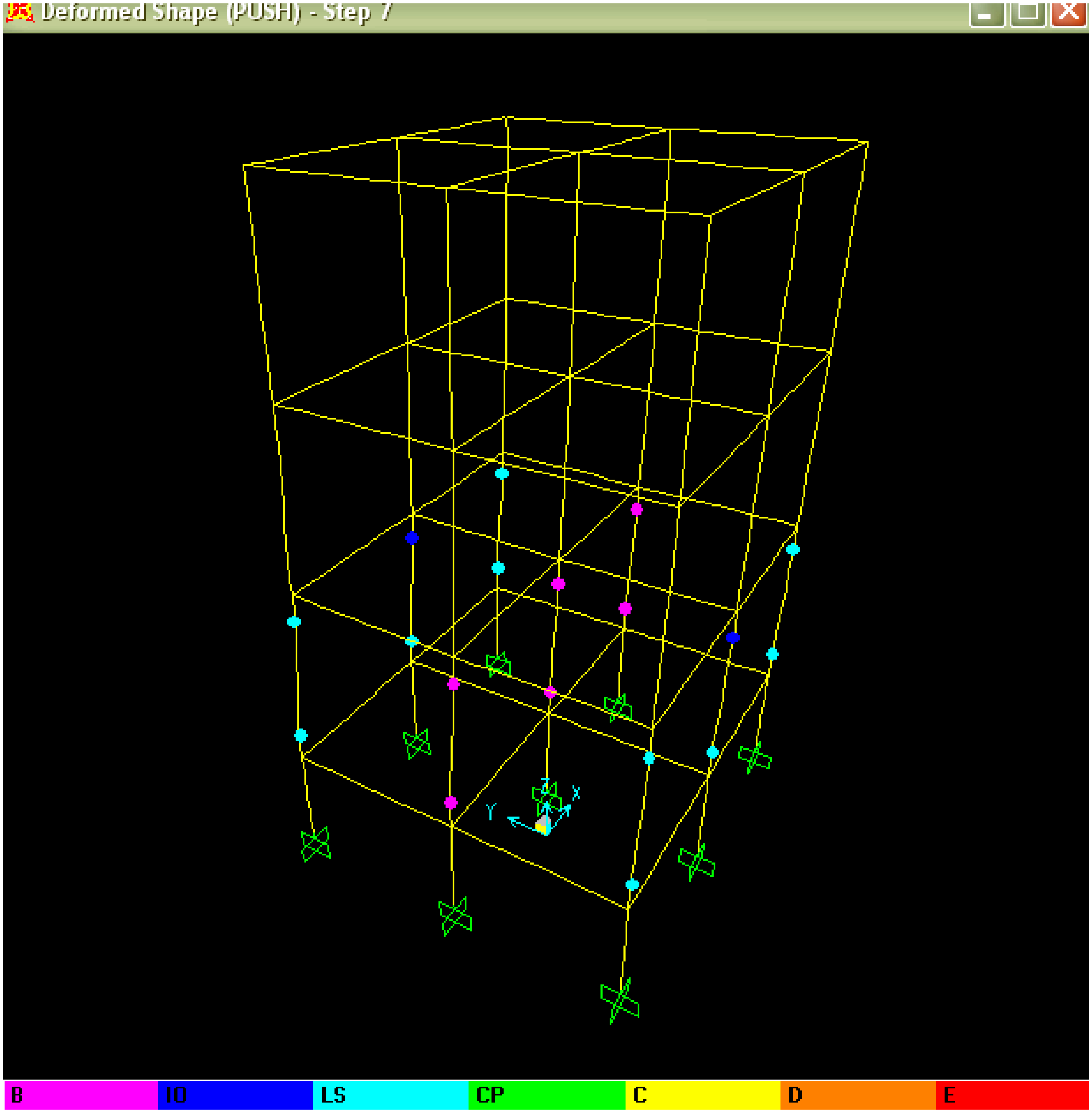


Figure 50. *Occurrence of Plastic hinges in succession of previous figure for building designed for Zone 3*

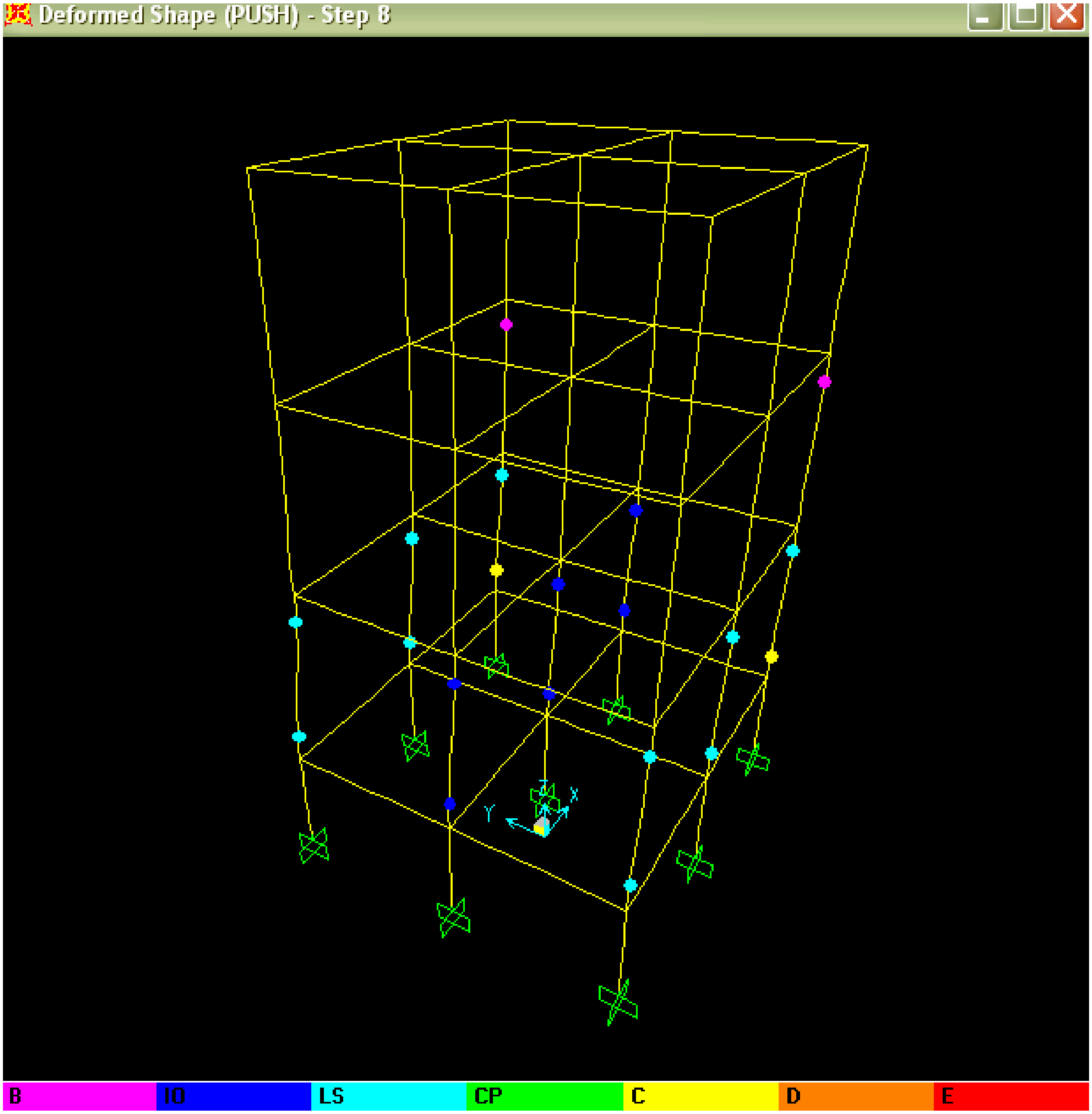


Figure 51. *Occurrence of Plastic hinges in succession of previous figure for building designed for Zone 3*

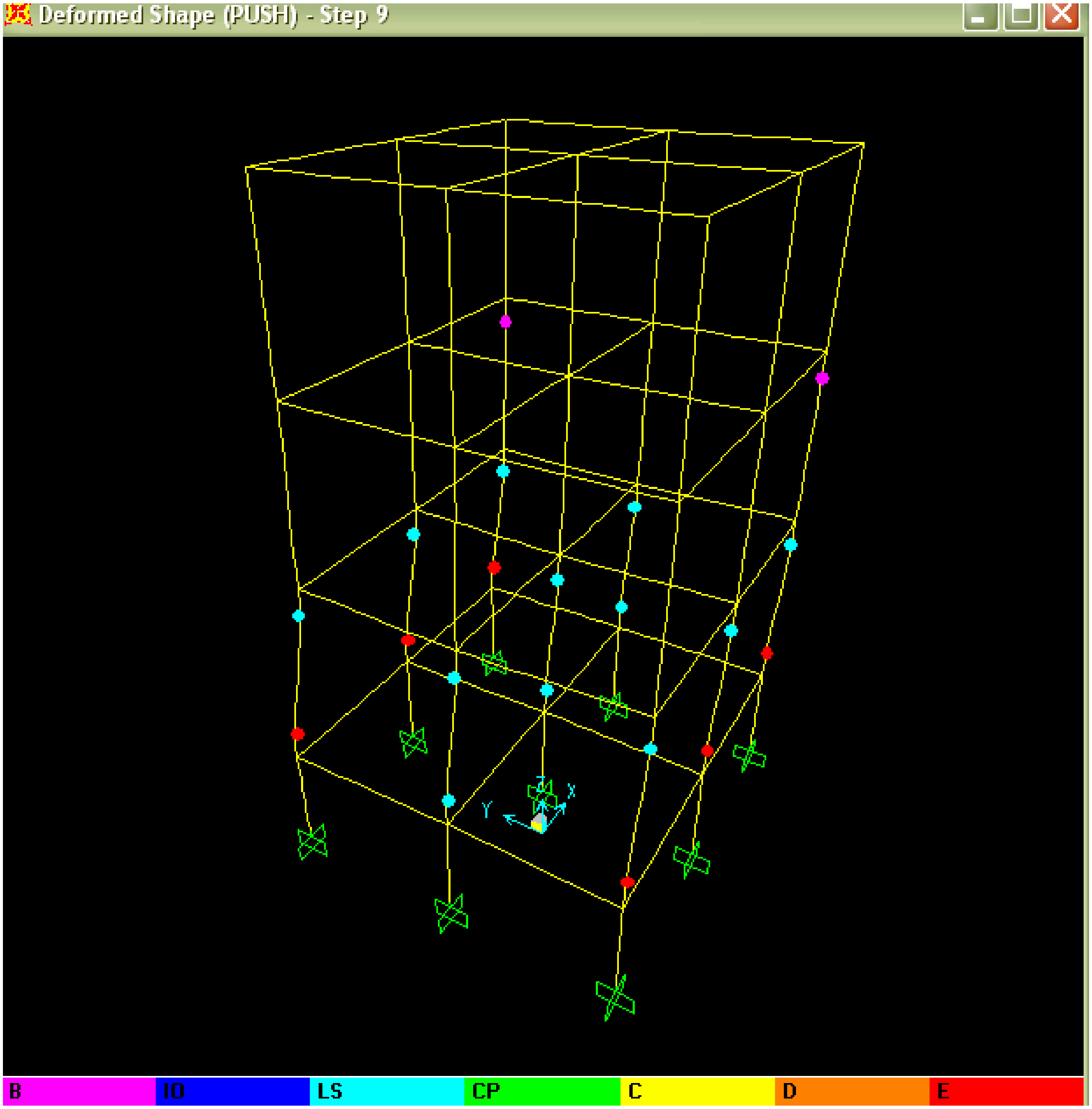


Figure 52. *Occurrence of Plastic hinges in succession of previous figure for building designed for Zone 3*

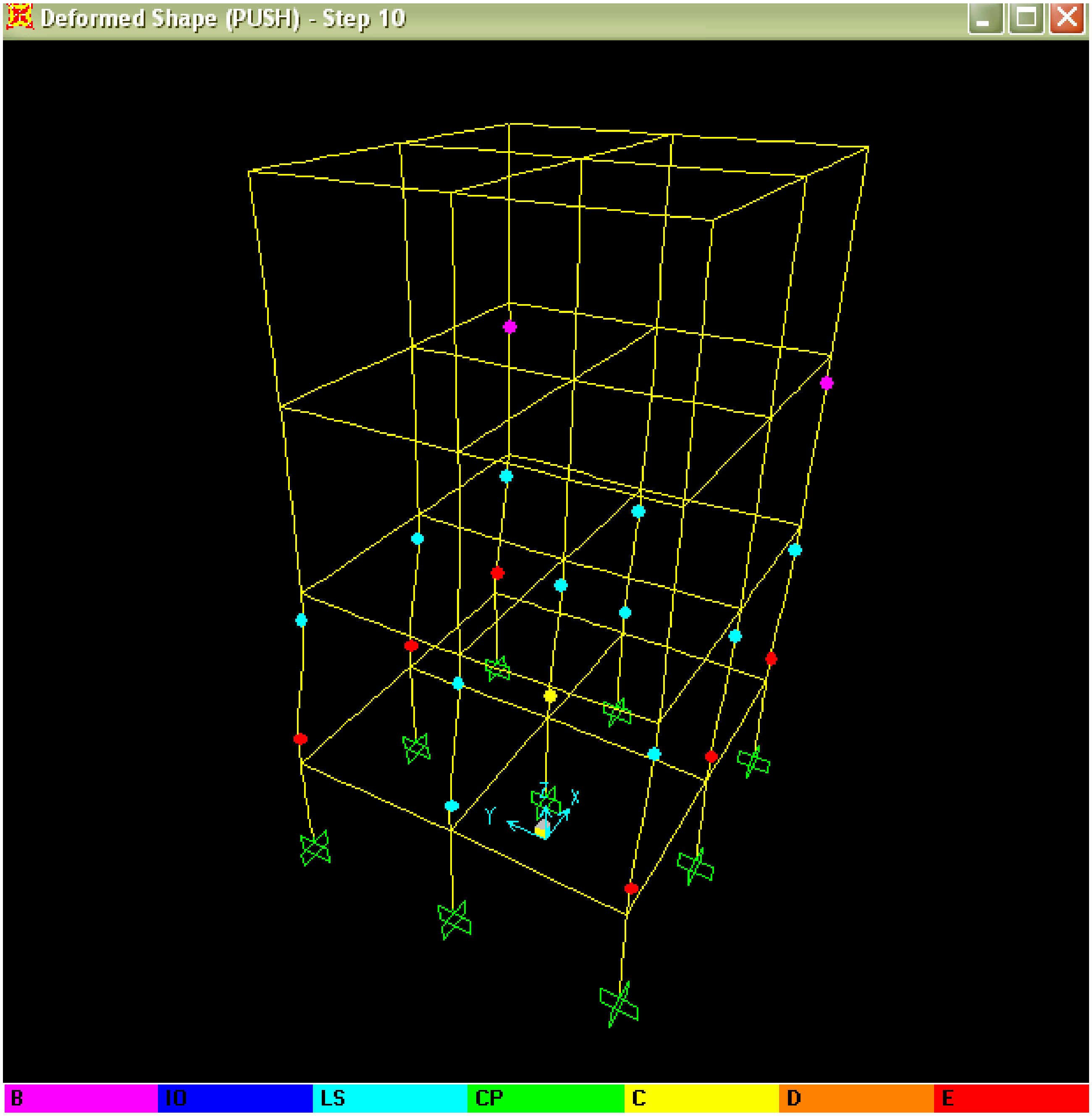


__Figure 53.__ Occurrence of Plastic hinges in succession of previous figure for building designed for Zone 3

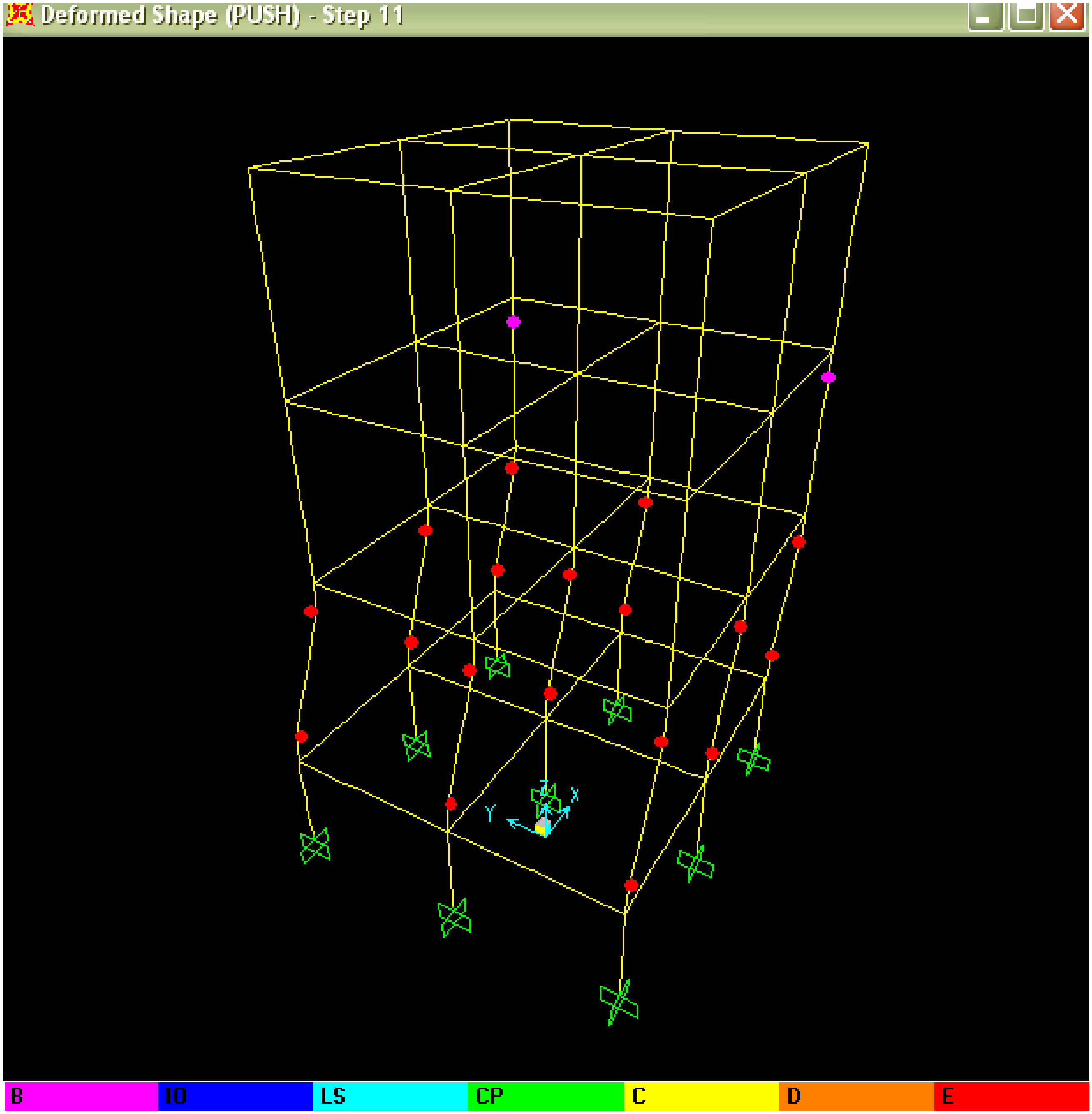


Figure 54. Occurrence of Plastic hinges in succession of previous figure for building designed for Zone 3

7. CONCLUSION

Static pushover analysis is an attempt by the structural engineering profession to evaluate the real strength of the structure and it promises to be useful and effective tool for performance based design.

The non-linear dynamic analysis is the most rigorous method, but it is still too complex for design use. The non-linear static pushover analysis seems to be a more rational method for estimating the lateral strength and the distribution of inelastic deformations.

Performance Point	Zone 3	Zone 4	Zone 5
(V , D)	(333.914 , 0.053)	(387.274, 0.44)	(462.383, 0.036)
(S_a , S_d)	(0.955 , 0.042)	(1 , 0.034)	(1, 0.27)
(T_{eff} , β_{eff})	(0.419 , 0.050)	(0.370 , 0.050)	(0.333 , 0.050)

1. Table shows that shear taking capacity of the building increases as the building is designed for the higher earthquake zone, because the stiffness of the structure is increased in successive zones, to carry the increasing base shear. The roof displacement D also decreases accordingly.

2. The Time period (T_{eff}) is decreasing from Zone 3 to Zone 5, because the structure would be more flexible in Zone 3, and in Zone 5 it has the greatest stiffness.

The retrofit strategies can be devised from Pushover analysis, as it not only shows the point of occurrence of the first plastic hinge (as shown in figure 48) but also the formation of hinge in succession of it (as shown in figures 48-54), which in turn provides the mode of failure, that can’t be determined in case of elastic analysis. This provides the real insight of the behavior of the structure in non-elastic range, when an earthquake of higher intensity occurs.

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