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Modelling and Estimation of Maintenance Parameters

ABSTRACT

In modern industrial environments equipment and systems are inherently complex and, therefore effective asset maintenance is crucial for ensuring optimum system availability and reliability to ensure economic operations. This research attempts to focus on the modelling and estimation of some key maintenance parameters particularly, the yearly maintenance budget (YMB), preventive maintenance (PM) strategies under imperfect maintenance conditions, and maintainability performance under variable contextual conditions.

Asset maintenance is driven by allocation of yearly maintenance budget (YMB). Lack of scientific methods for its evaluation has led to budget allocations that are subjective and erroneous causing wastage or lack of resources. To address this, a graph-theory and matrix-based modelling framework, supported by Monte Carlo simulations, is developed to estimate YMB. The method allows for stochastic variation of budget-influencing parameters and provides statistical estimates (mean and variance), which are validated against best-practice benchmarks.

Preventive maintenance in complex systems, suffers from uncertainty due to imperfect maintenance actions arising from contextual constraints. This study studies three failure rate-based preventive maintenance models that incorporate maintenance quality as a probabilistic variable. Weibull distribution is used to represent failure time behaviour

more realistically than traditional models, leading to improved optimization of maintenance intervals and life cycle cost estimates.

Furthermore, this work addresses limitations in traditional maintainability assessment methods, which often ignore the impact of operational covariates. A revised modelling approach is proposed that integrates both time-independent and time-dependent covariates affecting maintainability. The study reveals that ignoring time-dependent factors, such as spare part delays or equipment aging, can significantly bias the maintainability estimates. By incorporating these variables, the proposed maintainability function is expected to provide a more accurate and comprehensive assessment of maintainability performance.

Collectively, this research advances the scientific understanding and estimation of critical maintenance parameters. The proposed models serve as decision-support tools for maintenance managers, enabling more rational budget planning, optimized preventive maintenance strategies, and accurate maintainability evaluation in real-world industrial contexts.

Keywords: *Asset management, Yearly maintenance budget, Monte Carlo Simulation, Graph Theory, Decision making, Maintenance Management, Preventive Maintenance, Imperfect Maintenance, Maintenance Modelling, Maintenance Quality, Failure rate model, Cost-based optimization, Maintainability performance, Maintainability function, Covariates, Regression model, Operating conditions, Time to Repair.*

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CHAPTER 1

INTRODUCTION

1.1 Introduction and Motivation

In modern industrial environments, the inherent complexity of equipment and systems has made asset maintenance a critical function for ensuring system reliability, availability, safety, and cost-effectiveness. Maintenance signifies upkeep of the assets through the use of proactive, reactive and other maintenance strategies [1]. It also ensures quick equipment turnaround to minimize downtime and enhance the availability of physical plant assets. Maintenance is a challenging task, many a times needing out of the box thinking and deployment of optimization algorithms using software modules, for example: inventory management, manpower allocation, maintenance scheduling, etc. Manufacturing firms have realized the importance of this profit enhancing activity, as plant and equipment are becoming increasingly complex and timely maintenance interventions can prevent catastrophic failures.

A key enabler of effective maintenance is the maintenance budget (MB), which directly impacts the organization's ability to plan and execute maintenance activities. It signifies the availability of optimum financial resources to perform the maintenance activities. Budget for maintenance is needed for arranging maintenance workforce, spares, consumables, tools, special equipment, etc. Lack of appropriate maintenance budget can throttle maintenance works. On one hand availability of excess budget can lead to wasteful expenditure, while on the other hand, lack of financial resources can create situations, wherein there are production outages and unsafe plant practices. Therefore, estimation of appropriate maintenance budget is essential for organizations to ensure

that they maintain their competitiveness. The existing methodology for estimating maintenance budget is one of setting MB as some percentage of the asset cost; using historical costs; and, using expert judgment. All of these methods are non-scientific and might lead to wastage of resources. The reasons for wide variations in budget are many, such as, variety of plant assets, safety levels, age and complexities of the plant, operating conditions, etc. Maintenance budget is generally evaluated as % of Asset Replacement Value (ARV) [2][3]. ARV is the current replacement cost of asset including the cost of removing the old assets and the cost of installing new one. High YMB as a % of ARV that leads to large maintenance expenditures do compel the organizations to revisit their maintenance management systems and strike a balance between the need to maintain vis-à-vis asset replacement.

Gupta et al., 2014 [3] proposed a methodology based on graph theory and matrix approach for the estimation of YMB of a plant system under its contextual conditions. They considered eleven budget influencing parameters. These parameters had numerous sub- parameters too. This work is an extension of the previous work. In this, an attempt has been made to allocate a range to each budget influencing parameters as against a point value allotted to each parameter in the earlier work. Monte Carlo simulation is run wherein each budget parameters can pick any random value within the specified ranges to arrive at YMB corresponding to the random data set of the parameters. Such a scenario is more realistic in a dynamic and probabilistic context as the YMB parameter may not have fixed values spread over the entire budgeting year.

However, having an appropriate maintenance budget alone does not guarantee effective maintenance. The quality of preventive maintenance (PM) actions plays a crucial role

in sustaining the operational performance of industrial systems. Unfortunately, maintenance quality is often difficult to quantify and control due to factors such as the skill level of maintenance personnel, inconsistency in resource availability, and the unpredictability of maintenance environments. These challenges contribute to maintenance imperfectness, wherein even scheduled preventive maintenance actions may fail to restore the system to its intended operational condition. Consequently, systems remain vulnerable to unexpected failures or degradation, diminishing overall efficiency and reliability. Maintenance activities are generally categorized into two types: preventative maintenance (PM) plan and corrective maintenance (CM) strategy [4]. Corrective maintenance is also called breakdown or reactive maintenance. In CM strategies maintenance activity is carried out post-failure [5] [6]. PM may be divided into Condition-based and time-based or routine preventive maintenance [7]. Condition-based maintenance uses real-time data, sensor technologies, and predictive analytics to detect early symptoms of equipment deterioration and therefore, maintenance can be planned based on the real time equipment condition [8]. Therefore, condition-based preventative maintenance optimizes maintenance programs using sophisticated technology [9]. The high capital cost and sophisticated technological requirements have so far prevented the broader adoption of this technique [10]. In time-based preventive maintenance (PM) maintenance is performed before breakdown occurrences and therefore, aims to improve the reliability of repairable equipment, extend its operational lifespan, reduce accidents, and reduce manufacturing losses. Preventive maintenance is gaining increasing importance in today's multifaceted industrial equipment as it contributes to maintaining uninterrupted system operations, maximizing their availability, and reducing overall maintenance costs. However, performing routine

maintenance too often leads to excessive and unnecessary maintenance costs and chances of introducing unintended human errors. Due to this, the evaluation of optimal preventive maintenance (PM) policy, including the frequency of PM actions and maintenance intervals, is a crucial decision-making problem in finding a balance between the advantages and costs of PM. The effectiveness and excellence of preventive maintenance action in both CM and PM are compelling subjects in the context of ensuring reliability and play a crucial role when maintenance plans are formulated in real life applications. Consequently, in addition to the widely recognized maintenance classification, a new categorization has emerged in relation to the extent of system restoration following maintenance intervention that deals with a range of possible imperfections and quality in maintenance. These include imperfect maintenance, perfect maintenance, minimal repair, and replacement [11]. In practical applications, the implementation of preventive maintenance measures can enhance the equipment's performance to a specific level. The restoration of the equipment to its original, pristine condition is typically unattainable through repair alone; only a complete replacement can bring it back to its original (new) state. Hence, suboptimal preventive maintenance procedures are predominantly employed for most serviceable and costly equipment. The categories of imperfect maintenance models consist of the following: hybrid hazard rate model, shock model, (p, q) rule concept and its variants, virtual age model, improvement factor method, and (α, β) rule method. Earlier models have operated under the assumption that a unit following any PM procedure will be restored to a condition equivalent to that of a new unit. However, in the recent models it is recognised that this assumption may not hold true and imperfect maintenance outcomes are possible during maintenance intervention. The variability in the quality

of a maintenance action is influenced by factors such as the repairman's expertise, the tools and equipment utilized for maintenance, the thoroughness of inspections, budget constraints, time limitations, and similar other considerations [12]. Consequently, the variability in quality is considered a variable characterized by some probability. This leads to random deviations in the actual preventive maintenance interval and costs from the intended values. The quality of a PM action can be obtained from input provided by experts in the field [13] or actual data gathered from practical observations [14].

This research aims to expand the analysis of imperfect maintenance policies given by [15[16]. The authors, considered fluctuating quality of PM, and gamma distribution of equipment operational life. By incorporating the Weibull distribution to model the operational life of equipment provides a more realistic estimation of PM interval and maintenance cost rates under imperfect maintenance conditions and brings the model closer to real-world scenarios.

In addition to budgetary and quality considerations, the concept of maintainability serves as a pivotal maintenance parameter in effective maintenance planning. Maintainability, as defined by BS 4778, is “The probability of successfully repairing or restoring a failed component or system to a defined state within a given timeframe, through maintenance conducted according to prescribed procedures [17]. Defining the operational state of the system and the circumstances in which maintenance will be conducted is essential for specifying the maintainability of an item [18]. The conditions in which maintenance is to be carried out may include the environmental conditions, maintenance infrastructure, human factors, geographical location, equipment accessibility etc. Environmental factors such as temperature, humidity and seasonal

variations can influence maintenance tasks. Maintenance infrastructure refers to the availability of necessary support systems, facilities, and resources required for maintenance operations. Human factors include factors such as the skills, experience of maintenance personnel and ergonomic considerations. Geographical location may require careful planning for logistics, transportation of equipment and personnel, and access to spare parts and technical support. Equipment accessibility is also a significant parameter, which ensures convenient access to the maintenance points. Standard benchmarks for assessing maintainability consist of repair rate and mean time to repair. Numerous methodologies and models have been developed for predicting maintainability performance [19][20]. These methodologies incorporate a wide range of techniques, from statistical analysis and mathematical models to advanced computational algorithms. However, many existing models and methods fail to account for the impact of "maintenance conditions" on maintainability performance, leading to imprecise and inaccurate predictions, and consequently, incorrect availability performance forecasts. The time required for maintenance work can be significantly influenced by factors such as the availability of resources, the proficiency of maintenance personnel, access to spare parts, accessibility to maintenance points, and local operating conditions when adequate planning is not in place. Factors that have an influence on maintainability characteristics are termed as explanatory variables or covariates. As a result, it is imperative that the methodology utilized in maintainability performance analysis must have the capacity to precisely quantify the correlation between maintenance variables vis-a-vis maintainability. A common challenge that is encountered when analysing the historical database for the time between failures (TBF) and the time to repair (TTR) is to ensure that the contextual conditions under which the

data-sets were collected is not uniform or similar. The frequently utilized models for evaluating system reliability include proportional hazards model (PHM), the proportional covariate model (PCM), and the acceleration model, as highlighted in references [21–23]. The proportional repair model (PRM), proposed by Gao et al. [23] builds upon the framework of proportional hazards model (PHM). In this model, it is assumed that the explanatory variables are time-independent. But this assumption may be invalid in certain instances. At extremely low temperatures, the maintenance intervention is significantly affected by the extreme operating environment. Therefore, the environmental conditions significantly influence the TTR for repairing the component in different seasons. For example, while repairs may take shorter time in favourable conditions to the human body as against more time-consuming in challenging weather conditions. This time dependence of maintainability performance highlights the importance of considering environmental factors when planning maintenance activities. Long downtimes in manufacturing industries and power generation units have a detrimental impact on operational efficiency, resulting in significant decreases in output and huge economic costs. Therefore, it is necessary to develop an improved model for accurate prediction of maintenance time and characteristics of repair time distributions. For accurate prediction of maintenance time distribution, it is crucial to understand how explanatory variables alter the repair time. This involves developing an appropriate statistical method for quantifying the influence of these explanatory variables on maintainability performance.

This thesis integrates these three interconnected aspects; budget estimation, preventive maintenance quality, and maintainability modelling into a cohesive framework for improving maintenance decision-making. It contributes to the literature by (i)

developing a probabilistic budget estimation model using Monte Carlo simulation; (ii) analysing imperfect preventive maintenance and its impact on maintenance interval and cost optimization; and (iii) examining maintainability under varying operational conditions. Through these contributions, the research provides a more realistic approach to asset maintenance management in complex industrial environment.

1.2 Literature Review

A plethora of information is available in academic journals, electronic databases, and books, explaining maintenance budget estimation, imperfect maintenance modelling, maintainability analysis influenced by covariates, and proposing effective strategies to improve maintenance planning, reduce system downtime, and enhance system maintainability. The literature review is organized into four distinct domains. The first domain encompasses research papers and articles that delve into studies on maintenance budget estimation. The second domain offers exhaustive details sourced from research papers, articles, and other archival literature concerning maintenance modelling under imperfect maintenance conditions. The third domain reviews archival literature focused on maintainability analysis under the influence of static and dynamic covariates. With in this framework, subsequent subsection discusses the literature pertinent to Monte Carlo Simulation.

1.2.1 Maintenance budget

Maintenance budget is the money allocated for maintenance before these costs are actually incurred. The budget allocated for maintenance directly influences maintenance activities and plays a more pivotal role than maintenance costs [25]. If the maintenance budget is sub optimally evaluated and allocated then it may result in

inaccurate allocations to resources; such as, maintenance crew hours, maintenance spares, etc. Excessive allocation can lead to unnecessary maintenance of equipment, which is prone to introduction of human errors as discussed earlier. Therefore, the Annual Maintenance Budget (AMB) must be evaluated precisely to ensure effective resource use. In modern manufacturing, mmaintenance-costs can form a substantial share of total manufacturing costs, which can range from 15–30% [26], therefore, accurate budgeting becomes even more critical. Sarker and Faiz [27] highlighted that total maintenance costs are influenced by the number of age groups and age threshold sets of components. In capital-intensive sectors such as mining, maintenance can account for 40–50% of total operating costs [28]. Similarly, the UK manufacturing industry has reported allocating 12–23% of total factory operating costs to maintenance activities [29]. Over time, the deployment of automated production systems has reduced production manpower, while increasing the reliance on skilled maintenance personnel thereby contributing to a rise in the proportion of maintenance-related expenditures. Alsyouf [30] investigated maintenance practices in Swedish industry through a survey of firms with at least 100 employees. The study revealed that maintenance is often undervalued in strategic decision-making. On an average, the maintenance budget constituted about 4% of the companies' total turnover, with only 0–20% of that budget allocated to training and education, averaging around 4%. These findings further reinforce the need for rational budgeting that effectively supports maintenance execution.

A critical review of the literature reveals that while maintenance costs represent a significant portion of operational expenses across industries, there is a notable paucity of comprehensive and consistent data related to maintenance budgeting. The figures

reported vary significantly depending on context-specific factors such as operating conditions, equipment criticality and age, workforce skills, the efficacy of maintenance planning, and the economic trade-offs between reliability, safety, and incurred costs [3].

Despite this variability, several attempts have been made to define benchmarks. For instance, Gulati and Smith [2] report that world-class organizations typically allocate 2.5% to 3.5% of their Asset Replacement Value (ARV) to maintenance, while the broader benchmark range under best practices lies between 3% to 9%. These estimates underscore the importance of contextualized maintenance budgeting.

Multiple studies have shown that an optimized maintenance policy is instrumental in reducing overall maintenance cost or budget. Topal and Ramazan [31] demonstrated that the optimal scheduling of mining equipment using a mixed-integer programming approach could reduce maintenance costs by 16% over ten years. Bahr and Lennerts [32] proposed a Practical Adaptive Budgeting model designed to address limitations in traditional methods, revealing that maintenance costs tend to peak after 30 years of asset life. Similarly, Ottoman et al. [33] provided a comparative evaluation of 18 budgeting models based on asset cost, lifecycle parameters, and physical condition.

Environmental and infrastructural variables also significantly impact maintenance costs. For instance, Kayrbekova et al. [34] emphasized that sub-zero temperatures and insufficient infrastructure notably degrade maintainability and reliability, thereby increasing maintenance costs. Komonen [35] introduced a hierarchical cost model for industrial maintenance aimed at profitability and benchmarking. Using data from 436 production units, he established causal relationships among key variables and

developed a tool for assessing the impact of maintenance costs on plant competitiveness.

Further optimization efforts include the work by Dordevic et al. [36], who developed a reliability-constrained budget minimization model, and Ferreira et al. [37], who formulated a non-linear programming approach to minimize the sum of stochastic and deterministic maintenance costs in multi-component systems. Likewise, Nahas et al. [38] and Pham & Wang [39] examined maintenance sequencing and policy optimization to achieve minimum costs while ensuring system reliability and safety.

To support practical implementation, Gupta et al. [3] introduced a Plant Maintenance Budget Digraph (PMBD) model that enables context-specific evaluation of the YMB as a percentage of ARV. This method accounts for parameters unique to each plant and does not rely on expert judgment. An important threshold emerged from this body of work: if the YMB exceeds 20% of ARV, it is generally considered economically unjustifiable to continue maintaining the asset, and replacement should be considered. The model was further validated through several case studies and further applied to specific systems such as particle accelerators [40] and air conditioning plants [41]. The latter study employed an incidence matrix approach to evaluate influencing parameters under contextual constraints.

However, limitations remain. The original PMBD model was not well-equipped to evaluate stochastic failures or to capture statistical characteristics such as the mean, variance, or distribution of YMB under real-world variability. To address this shortcoming, a Monte Carlo Simulation (MCS) approach was integrated, allowing for random allocation of parameter values to better mirror practical conditions.

A review of the application and benefits of the Monte Carlo Simulation technique in maintenance budgeting is presented in the following subsection.

1.2.2 Monte Carlo Simulation

The estimation of the Yearly Maintenance Budget (YMB) has traditionally relied on fixed or point estimates of budget-influencing parameters under specific contextual conditions. While models such as those developed by Gupta et al. [3] provided a methodology for evaluating YMB, they lacked the ability to incorporate stochastic variations, which are inherent in real-world maintenance environments. Recognizing this limitation, the present work adopted the Monte Carlo Simulation (MCS) technique to overcome the constraints of deterministic modelling. MCS enables the assignment of values to maintenance-related parameters over specified ranges instead of fixed values, thereby facilitating a more realistic and probabilistic estimation of the YMB, including its mean, variance, and distribution for various maintenance scenarios.

MCS is a computational technique that uses pseudo-random number generation to simulate and evaluate system behaviours under uncertainty. It works by converting uniformly distributed random numbers into real-world outcomes using inverse transformation techniques under known probability distributions and mathematical models. MCS is particularly valuable in scenarios where analytical solutions are either difficult or infeasible due to system complexity or the stochastic nature of variables.

Several studies have demonstrated the applicability of MCS in the domain of maintenance and reliability. For instance, Cantoni et al. [42] integrated MCS with

Genetic Algorithms (GA) for optimal plant design under economic and safety constraints, while Borgonovo et al. [43] utilized it to model plant availability considering maintenance, aging, and obsolescence. Similarly, Marseguerra and Zio [44] employed a stochastic model combining GA and MCS to optimize maintenance and repair strategies. In another application, Hall and Strutt [45] used MCS to estimate the characteristic life and shape factor of components, demonstrating its capability in life data analysis.

Further extending its utility, Marquez et al. [46] applied MCS to assess the reliability and availability of complex systems using a continuous-time modelling framework. Ramakrishnan [47] utilized MCS to evaluate unavailability and mean time to failure (MTTF) for fast reactor systems, while Cadini and Gioletta [48] introduced Bayesian MCS to estimate small failure probabilities. Recognizing the dependence of modern systems on highly complex and interconnected components, Crespo and Iung [49] developed an MCS-based model to assess system reliability and availability under varying environmental, sustainability, and system behaviour conditions.

Despite its robustness, one of the known limitations of MCS is its computational cost. To address this, Janssen [50] investigated sampling efficiency and convergence strategies to improve simulation performance and reduce computational load.

Drawing from this literature, the current work leverages MCS on the existing Graph Theory and Matrix Approach (GTMA)-based model developed earlier, enabling the estimation of YMB under different levels of maintenance performance. This probabilistic modelling approach not only captures the inherent variability in

maintenance systems but also generates more informed estimates of YMB, aiding maintenance managers in strategic budgeting and resource allocation. The use of MCS thus bridges the gap between deterministic budget models and the stochastic budget model.

While budget estimation is a critical component of maintenance planning, the quality of maintenance execution plays an important role in determining the actual effectiveness of budget utilization. Even a well-allocated maintenance budget can fail to yield optimal results if maintenance activities are performed imperfectly, due to factors such as insufficient time, lack of skilled manpower, poor diagnostics, or use of substandard parts. Imperfect maintenance leads to increased system downtime, higher failure frequency, and ultimately greater unplanned maintenance expenditure, which in turn disrupt the originally planned yearly maintenance budget (YMB). Therefore, it is essential to explore the interaction between maintenance budget planning and the imperfect nature of real-world maintenance execution.

1.2.3 Imperfect Maintenance

Imperfect maintenance, minimal repair, and repair are interrelated concepts within the field of maintenance and reliability engineering. They represent different approaches to managing the maintenance. Imperfect maintenance refers to maintenance actions that do not completely restore a system to its original, as-good-as-new level of performance. Repair or normal repair implies restoring the system to a condition within a spectrum ranging from an as-good-as-new level of performance to a certain acceptable repair condition. Minimal repair, on the other hand, focuses on performing the least amount of maintenance necessary to return a system to a minimally acceptable operational state.

In the literature, multiple research studies have focused on the development of maintenance policies for various systems, considering the inherent imperfections associated with preventive maintenance practices. The optimal goal of maintenance managers is to minimize maintenance costs and maximize system performance [51]. This goal can be achieved through the development of a maintenance plan that optimizes the preventive maintenance cost and intervals. Weiss [52] made the initial assumption that the inspection process aimed at detecting failures might be imperfect. Nakagawa [15] introduced an imperfect preventive maintenance model, considering the case in which there is a probability that the unit, following preventive maintenance (PM), does not return to a new-like condition. The study discussed optimal PM strategies aimed at maximizing system availability or minimizing expected costs, taking into account a system lifetime characterized by a gamma distribution. Brown and Proschan [53], determined that a failed unit, post-repair, could be considered as good as the new level of performance with a specific probability. They then proceeded to explore and examine different characteristics of the failure distribution associated with this assumption. Doyen and Gaudoin [54] generalized imperfect maintenance models that were similar to those described in [53]. The investigation of Wu and Clements-Croome [55] concentrated on the uncertainty of preventative maintenance quality by considering maintenance quality as a random variable with a predefined probability distribution. They applied two thoroughly studied preventive maintenance model: age reduction model and the failure rate model. Khatab and Aghezzaf [56] addressed the optimization of imperfect preventive maintenance in a complex multiple-component system, incorporating stochastic effectiveness in maintenance actions. The repairman's expertise and tool usage contribute to the effectiveness of the repair

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process. The effectiveness of the repair is treated as a random variable, and this variable is characterized by a predefined probability distribution. The study was focused on managing uncertainties in maintenance outcomes. A sequential model for imperfect PM was developed by Lin et al., [57]. It explored two failure modes: non-maintainable and maintainable. Their work aims to improve the effectiveness of preventive maintenance in both repairable and irreparable circumstances. Periodic, imperfect preventive maintenance strategies were examined by Zequeira and Bérenguer [58]. The optimal strategy was designed for a system that encompasses both repairable and irreparable failure states. The proposed model facilitates the improvement in the state of the equipment as a function of the period during which preventative maintenance was performed. A more generalized version of a periodic model for imperfect preventive maintenance was introduced by Sheu and Chang [59]. The model incorporated age-dependent failure more specifically minor and catastrophic failures. They assumed that, in each preventive maintenance cycle, the system is preventively maintained either upon the occurrence of a catastrophic failure or upon reaching a predetermined maintenance interval, whichever occurs first. After a specified number of such preventive maintenance actions, the system is replaced. Finally, they determined the optimal replacement and preventive maintenance schedule, aiming to minimize the expected cost rate over an infinite time horizon. An optimisation problem based on the virtual age model introduced by Kijima was formulated by Cha and Finkelstein [60]. The authors considered the periodic, imperfect preventive maintenance of components in between a growing deteriorating condition over time and minimal maintenance interventions. The deterministic asymptotic value of the virtual age, obtained through analysis, defines the initial age of the component at the start of each optimization cycle.

They applied similar methodologies to age-based preventive maintenance. The dynamics of the saturation effect in preventive maintenance (PM) actions were explicitly highlighted by Sun et al. [61]. They successfully introduced two innovative preventive maintenance models. The determination of the optimal preventive maintenance schedule and replacement cycle was achieved through the application of the proposed algorithm. Two imperfect PM policies for infinite and finite time frames were developed by Wang et al. [62]; they incorporated delays in the execution of scheduled PM activities. Maintenance unpunctuality is assumed to follow a predetermined probability distribution. After modelling and optimizing the two untimely performed PM policies, they then examined how delays in the execution of scheduled PM activities affected the optimal PM decisions and maintenance costs. An optimal failure management strategy for units subjected to periodic inspections with imperfect maintenance was investigated by Ahmadi et al. [63]. Mercier and Castro [64] compared two imperfect maintenance models designed for a deteriorating system, wherein the degradation level was modelled as a non-homogeneous gamma process. In addition, Li et al. [65] centred their investigation on optimizing maintenance practices through the application of the hybrid hazard model, utilizing the meta unit as the focal point of their research. The paper explored the domain of imperfect preventive maintenance, constructing an optimization model specifically designed for uniform intervals of preventive maintenance. Finally, the authors conducted numerical experiments to analyse the impact of varying corrective and preventive maintenance costs on the optimal maintenance plan. Pedersen et al., [66] developed a condition-based maintenance (CBM) policy for a two-component system with catastrophic failure along with continuous wear and tear (deterioration). The policy optimizes minimal

repair threshold and preventive replacement times to minimize long run cost rates. The modelling framework accommodates delay in maintenance and opportunistic maintenance and demonstrated the strategy through a case study of cooling system used in the process industry. Wang and Chen [67] constructed an imperfect maintenance model under CBM approaches using piecewise deterministic Markov process. They considered natural degradation and random shocks in modelling, with two maintenance options; imperfect and corrective. Optimal maintenance strategies are determined using impulse optimal control theory. Numerical study on component coating maintenance is presented, along with sensitivity analysis on various factors.

Further, imperfect maintenance for multi-component systems was investigated by Laggoune and Chateauneuf [68]. The technique involved the development of an integrated model that takes into account proportional and geometric adjustment of failure intensity. Furthermore, the model includes proportional age adjustment. Bayesian network is used to evaluate the imperfect maintenance model parameters. Liu et al. [69] examined how diversity affects breakdowns of maintainable systems when subjected to imperfect maintenance. The study employed constant repair efficiency and Kijima Type II virtual aging. The failure characteristics of the system were modelled using the Weibull baseline distribution. Multi-state system maintenance activity, using component randomness and stochasticity in imperfect maintenance activities were optimised by Shahraki et al. [70]. Wang et al. [71] proposed a dynamic model based on general renewal process for imperfect preventative maintenance decisions. Virtual age factor and failure intensity update factor are used to explain impact of imperfect maintenance. An optimal maintenance approach for a multi-state system without observation with a limited number of imperfect preventive maintenance operations

between replacements were established by Fan et al. [72]. A numerical example was provided to illustrate the optimal policy. Chen et al. [73] designed a novel approach that combines age-based and random replacement policies for an aging, multi-state system under imperfect maintenance during failure. They analytically derived and numerically discussed the optimal replacement schedule aiming to minimize the mean cost rate. Zhao et al. [74] formulated optimal imperfect preventive maintenance strategies for a previously utilized system. Optimal policies aimed at minimizing cost rates are analytically derived and subsequently numerically discussed. Li et al. [75] introduced an optimization modal for condition- based and scheduled maintenance at the unit level, aiming to overcome the limitations of existing opportunistic maintenance strategies. They established a preventive opportunistic maintenance framework considering imperfect preventive maintenance using Monte Carlo simulation-based optimization algorithm. Results from implementing this model to computer numerical control (CNC) grinding machines revealed substantial cost saving potential relative to other strategies, offering valuable guidance for maintenance strategy development in relevant industries. In their research Mai et al. [76] suggested a reliability model for systems facing multiple dependent failure processes within dynamic environment, emphasizing trade-off between replacement action and imperfect maintenance. They incorporated continuous time Markov processes and gamma process to represent system dynamics and natural degradation. They demonstrated the model through Micro- electrochemical system case study, aiming to minimize long- term expected cost. A novel maintenance strategy for multi- state system in fluctuating environments, where environment and system mutually influence each other was developed by Gan et al. [77]. They considered three decision making actions, namely replacement, imperfect maintenance and do nothing,

with maintenance action influenced by system and environment. Markov decision process optimized the strategy using policy iteration algorithm. Numerical illustration and sensitivity analysis validated the effectiveness of the model. In the paper by Li et al. [78], they designed a CBM model for manufacturing systems, integrating it with quality considerations and working schedule. Perfect and imperfect maintenance actions were used to mimic real-life scenarios. The problem was modelled by Markov decision processes algorithm. Numerical illustration using sensor data from commercial vehicle manufacturing industry demonstrated the models' effectiveness. Zhang et al. [79] developed a maintenance cost model for a multi-component repairable system using a hybrid condition-based opportunistic maintenance strategy. They also considered the effect of imperfect maintenance of equipment. Numerical illustrations are also presented for validation of the proposed model. In their study, Cheng and Zhao [80] presented a maintenance optimization technique for dependent two- component systems, incorporating imperfect maintenance and degradation. They used Markov decision process framework for infinite horizon maintenance, yielding optimal solution through value iteration. The research highlights the substantial impact of imperfect maintenance characteristics on optimal policies, prioritizing the mean effect over variability of imperfect maintenance. Alaswad and Salman [81] designed an optimal preventive maintenance model by incorporating steady- state and transient availability for military systems. They assumed imperfection in preventive maintenance, The result showed a 5% saving in maintenance cost.

1.2.4 Maintainability Performance

Maintainability is a key system characteristic that significantly influences the total life-cycle cost of a machinery. It refers to the probability that a system can be restored to its

fully operational condition after a failure has occurred. The maintainability of a system is interrelated with various factors and is influenced by the operational environment, degree of automation, maintenance process characteristics, and Maintenance management approach.

Several methodologies and models exist for predicting maintainability performance. Agrawal et al. [19] developed a Markov-based analysis demonstrating that system maintainability is highly dependent on its subsystems. The study further highlighted that proper planning significantly enhances maintainability performance. Choudhary and Tripathi [20] analysed the maintainability of a cement plant by testing the Time Between Failures (TBF) and Time to Repair (TTR) data for independence and identical distribution, identifying best-fit distributions, and simulating the real and proposed systems to evaluate the effectiveness of preventive maintenance. They observed that management issues such as poor planning, delayed decisions, and inadequate resources can indirectly reduce maintainability by increasing repair times. Accurate maintainability prediction enables more effective allocation of repair resources, such as spare parts and maintenance training. Repair data often contain heterogeneity arising from varying operational conditions, influenced by both observed and unobserved covariates. Ignoring these covariates can lead to erroneous model selection and inaccurate conclusions. To overcome this challenge, the Mixture Frailty Model (MFM) provides an effective approach by incorporating both observed and unobserved covariates, thereby improving the precision of maintainability performance.

A common challenge when analysing historical databases, such as Time Between Failures (TBF) and Time to Repair (TTR), is ensuring that the conditions under which

the data were collected are uniform or comparable. Repair data are often gathered under varying operational conditions, which can introduce heterogeneity into the dataset. Commonly used models for evaluating system reliability include the Proportional Hazards Model (PHM), the Proportional Covariate Model (PCM), and the Acceleration Model. Ghodrati and Kumar [21] developed Proportional Hazards Model to determine the number of required spare parts by considering the influence of external factors, excluding time, on component reliability. The study highlights the significant impact of the operating environment on system performance and emphasizes that forecasting spare parts needs by incorporating both technical characteristics and environmental conditions is an effective approach to prevent unplanned stoppages. Ansell and Phillips [22] reviews the practical applications of Cox's Proportional Hazards Model (PHM) in lifetime data analysis, emphasizing its semi-parametric nature and minimal assumptions about the hazard function. The study discusses medical advancements that have extended the model's applicability to repairable systems. It highlights the use of diagnostic statistics and graphical tools for model assessment, supported by field data from the semiconductor and electrical industries. Repairable system applications are further illustrated using data from the hydrocarbon industry. Gao et al. [23] developed a Proportional Repair Model (PRM), based on the Proportional Hazards Model (PHM), to predict repair rates and evaluate the influence of operating conditions on maintainability performance in Arctic conditions. Their findings showed that Arctic-specific factors such as harsh climate, adverse maintenance design, and equipment quality can substantially increase repair times, leading to reduced maintainability. The proposed model accounts for these influencing factors, overcoming the limitations of using data from systems in normal climates. The approach was validated through a case

study, emphasizing the critical role of the operating environment in maintainability analysis. The results offer practical guidance for improving maintainability. The Proportional Repair Model (PRM), proposed in [23], builds upon the framework of the Proportional Hazards Model (PHM) and assumes that explanatory variables are time-independent; however, this assumption may not hold true in all situations. Factors such as equipment ageing, spare part availability, and environmental conditions can change over time and significantly affect the Time to Repair (TTR) for components. Ageing equipment may require more complex and time-consuming repairs. Similarly, limited spare part availability caused by supply chain delays, inadequate inventory, can result in extended downtime. In addition, while repairs may be completed more quickly in summer under favourable weather conditions, extreme winter environments can greatly hinder maintenance activities. These time-dependent variations in maintainability performance are particularly critical in industries such as manufacturing and power generation, where prolonged downtimes can severely reduce throughput. Barabadi et al. [82] incorporated time-dependent covariates into maintainability modelling but focused solely on environmental factors such as temperature, working shift, and location etc. This leaves a gap in modelling operationally significant covariates such as spare-parts availability, equipment ageing, and maintenance-crew workload etc. which also have a substantial effect on maintainability performance. Accurate prediction of repair time distributions requires statistical methods that quantify the effects of both time-dependent (operational and environmental) and time-independent covariates. The aim of this study is to investigate the influence of both environmental and operational time-dependent covariates on maintainability performance analysis.

1.3 Findings from Literature Review

1.3.1 Key points

- Maintenance is critical for minimizing downtime and maximizing asset availability.
- Existing methods for maintenance budgeting rely heavily on non-scientific approaches such as rules of thumb, historical data, or expert judgment, which can lead to inefficiencies.
- Maintenance costs and budgets are highly context-dependent and vary widely based on asset type, age, reliability, operational conditions, workforce skills, etc.
- Some scientific models (e.g Gupta et al. [3] digraph model) exist for maintenance budgeting, but do not account for stochastic values of variables.
- While prior studies acknowledge the importance of preventive maintenance quality, there is a significant research gap in quantitatively modelling maintenance imperfectness and integrating it with maintenance planning frameworks.
- Extensive literature exists on Maintainability, but time-dependent and independent maintainability has not been explored in depth.
- Monte Carlo Simulation (MCS) is a promising technique to validate complex maintenance models.
- There is a need to incorporate time-dependent and stochastic factors into budget and maintainability models.

1.3.2 Research Gaps

- Absence of budget estimation models that incorporate the random values of budget-influencing variables and evaluation of budget variability within confidence intervals.
- While prior studies acknowledge the importance of PM quality, there is a significant research gap in quantitatively modelling maintenance imperfectness and integrating it with maintenance planning frameworks**.**
- While Barabadi et al. [82] incorporated time-dependent and time-independent covariates into maintainability modelling, their focus on environmental factors like temperature, working shift, and location has limited applicability (particularly in offshore or Arctic conditions). These variables influence repair conditions directly and are often not within the control purview of the maintenance team. There are still gap in modelling maintainability considering direct operationally relevant covariates such as spare parts availability, component ageing, and maintenance crew workload etc., which have a significant impact on maintainability.

A critical review of the literature reveals no comprehensive framework exists that simultaneously incorporates maintenance budgeting, maintainability analysis, and the effects of imperfect maintenance. For instance, maintenance budgeting research often focuses on optimising budget allocation but generally overlooks how repair times or imperfect maintenance actions influence system availability; while some budget estimation studies consider maintainability to a limited extent as one of several covariates this treatment does not capture the dynamic effects of maintenance quality. Maintainability analysis, incorporates time-dependent and time-independent covariates, but imperfect maintenance quality is rarely modelled explicitly as a

covariate, and maintenance budget constraints are usually addressed only indirectly, through their influence on factors such as maintenance crew skill, ageing of equipment or spare parts availability. Studies on imperfect maintenance evaluates maintenance quality, but rarely links it to budget planning or maintainability metrics. While individual studies have addressed these aspects separately, their integration into a single, unified model remains largely unexplored.

1.3.3 Objectives of Research Work

- Develop a maintenance budget function distribution and evaluate its mean and variance using MCS.
- Extend an existing imperfect-maintenance cost model by generalizing the time-to-failure distribution to Weibull, and evaluate its effect on the average cost rate and optimal PM interval through numerical experiments, yielding results that are more realistic than those reported in the literature.
- Develop a maintainability function that integrates both operational and environmental covariates, some of which may be stochastic in nature. The analysis considers factors such as maintenance crew workload, spare-parts availability, component ageing, and environment conditions, thereby providing a more realistic assessment of industrial system maintainability.

1.4 Organization of the thesis

This thesis is organized into five chapters, based on three integrated studies related to maintenance budgeting, Optimized preventive maintenance strategy under imperfect maintenance conditions, and maintainability analysis.

Chapter 1 introduces the research background, motivation, and literature review, along with key findings that lead to the identification of research gaps. It also presents the objectives of the research work and outlines the overall organization of the thesis.

Chapter 2 presents a methodology to estimate the Yearly Maintenance Budget (YMB) using graph theory and matrix modelling, and Monte Carlo simulation. It addresses the limitations of traditional subjective approaches and provides a validated framework for rational budget estimation.

In chapter 3, three preventive maintenance models based on failure rate PM approaches are developed. It considers varying levels of maintenance quality, applies Weibull distribution to model failure behaviour, and provides optimized preventive maintenance strategies through algorithmic implementation and numerical analysis.

The influence of both time-dependent and time-independent variables on maintainability effectiveness are examined under chapter 4. Also, a maintainability function incorporating stochastic elements is developed and demonstrates how such covariates impact system recovery performance.

Chapter 5 summarizes the key findings and contributions of the thesis. It highlights practical implications for maintenance management and provides directions for future research in maintenance optimization.

The next chapter presents a graph theory and matrix- based framework for maintenance budget estimation, incorporating stochasticity in contextual maintenance parameters through Monte Carlo simulation.

CHAPTER-2

STOCHASTIC MAINTENANCE BUDGET MODELLING USING GRAPH THEORY AND MATRIX METHOD THROUGH MONTE CARLO SIMULATION

2.1 Introduction

Accurate estimation of the Yearly Maintenance Budget (YMB) provides strategic direction to maintenance operations, ensuring that physical assets remain in a reliable state while enabling judicious utilization of resources through appropriate benchmarking, thereby enhancing overall maintenance effectiveness**.** Traditionally, the estimation of YMB has relied on historical data trends, expert opinions, or rule-of-thumb. However, such subjective approaches often result in erroneous budget allocations, leading to either excessive spending or underfunded maintenance, both of which can compromise plant performance. This chapter presents a more realistic methodology that overcomes the limitations of conventional techniques by removing the subjectivity associated with budget-influencing parameters. The proposed approach combines Graph Theory and Matrix Approach (GTMA) with Monte Carlo Simulation (MCS) to estimate YMB across different maintenance performance levels. Randomized values are assigned to key maintenance parameters within pre-defined ranges, and statistical inference is used to identify the distribution, mean, variance, and interval estimates of the YMB. The simulation results reveal critical insights: for world-class maintenance practices, YMB lies between 1.62% and 1.82% of the Asset Replacement Value (ARV), while for standard maintenance levels, the budget may range as high as 6.51% to 7.23% of ARV. These findings suggest that budget-influencing parameters, when poorly managed or falling below certain thresholds, can lead to significant

increases in maintenance expenditure. The model thus acts as a valuable decision-support tool for maintenance managers, providing guidance for optimizing budget allocation and improving overall system efficiency.

2.2 Parameters influencing YMB

The Yearly Maintenance Budget (YMB) is influenced by multiple factors, as illustrated in figure 2.1.

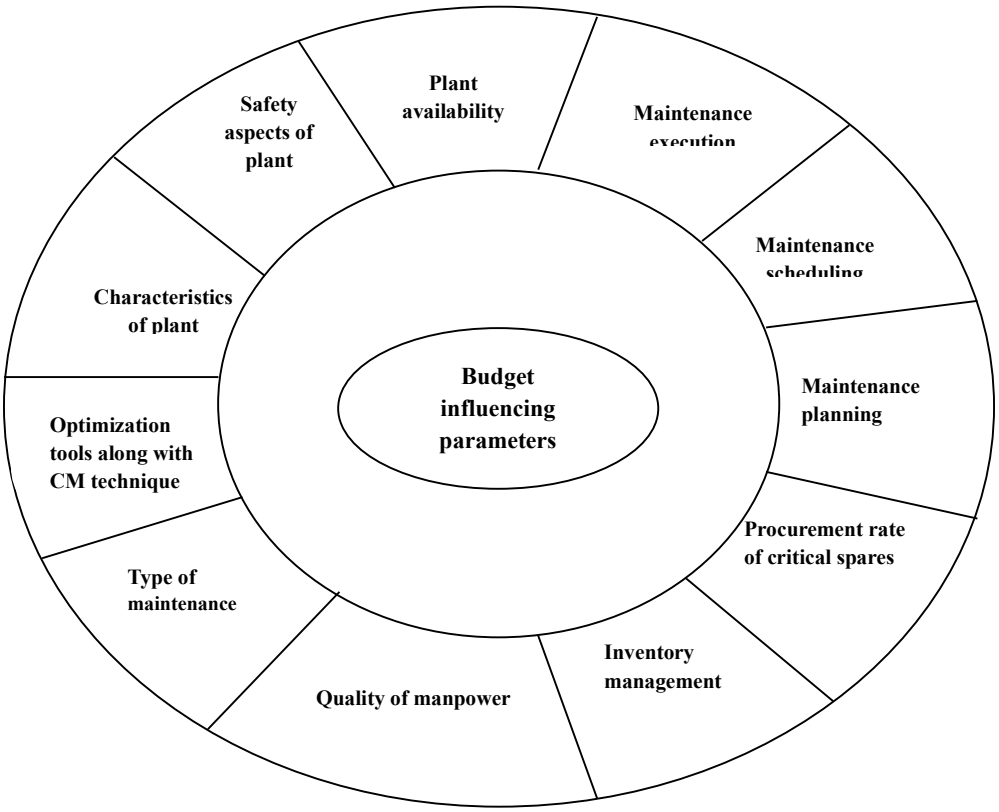


Figure 2.1: Budget influencing parameters

These parameters define the operational context of the plant and significantly impact the estimation of maintenance budget. One of the key parameters is plant availability, defined as the probability that a system performs its required function at a specific point in time under stated operating conditions. Opportunistic maintenance significantly contributes to improving plant availability by utilizing scheduled plant shutdowns to

replace or upgrade partially degraded subsystems or units [83]. A plant availability of over 90% is considered indicative of world-class performance. Due to inherently hazardous nature of industrial operations, safety requirements impose another important constraint.

18 Plants operating under higher risk conditions require more frequent maintenance intervention to avoid accidents and ensure safe operation [84][85]. Consequently, such facilities require a larger maintenance budget. Plant-specific factors such as physical environment, system complexity, redundancy, technological sophistication, plant duty cycles, layout, and geographic location also shape the maintenance system's requirements and influence the budget accordingly. Mathematical models and condition monitoring techniques have been shown to optimize maintenance expenditure by guiding the selection of appropriate preventive and condition-based maintenance intervals [86] [87].

It is well established that, the maintenance expenditure can be optimized using mathematical models that includes condition monitoring techniques. Plant availability is enhanced by carrying out maintenance at optimum interval of preventive and condition-based maintenance [86][87].

The type of maintenance work is a crucial factor influencing the maintenance budget. Proactive maintenance, as opposed to reactive maintenance, significantly enhances plant availability. Studies have shown that proactive maintenance can reduce maintenance costs by 15–30% and downtime by 20–40%, while simultaneously increasing production output by 15–25%. It is therefore recommended that over 80% of maintenance activities should be proactive [2].

Additionally, Kumar and Gandhi [88] reported that a majority of equipment failures in plants are caused by human errors. This highlights the importance of skilled manpower, which plays a crucial role in maintenance activities and significantly impacts the Yearly Maintenance Budget (YMB).

Another key factor influencing maintenance budget is inventory and spare parts management. An efficient inventory management system, supported by strong supplier relationships, minimizes waiting time for spare parts during maintenance operations, thereby reducing overall maintenance costs. Gulati and Smith [2] noted that maintenance personnel typically spend up to 30% of their time locating spare parts. To address this inefficiency, organizations are encouraged to implement effective materials management software to improve stocking decisions and avoid unnecessary delays. Additionally, CBM strategies help reduce the need for large in-stock inventories, as part procurement can be initiated based on early detection of potential failures [89]. Maintenance planning based on the work content, methodology, technical documentation, start date, required by date, man/material resources, safety requirements and special requirements, etc. minimizes the probability of system failure and its consequences related to safety, failure train and environment. It helps management in taking decisions concerning investment in maintenance. This will result in better plant utilization. It is well known that simultaneous planning of production and maintenance does result in reducing the maintenance budget. Scheduling intensity, scheduling quality and schedule realization rate are the key indicators to quantify the maintenance scheduling variable [25]. After planning and scheduling, maintenance tasks must be executed in alignment with operational needs. The effectiveness of execution is assessed using key indicators such as maintenance precision, rate of

compliance, and maintenance crew utilization rate. Maintenance precision indicates the extent to which maintenance work is completed without requiring rework, rate of compliance refers to the percentage of maintenance tasks completed within the scheduled time, while maintenance crew utilization reflects how efficiently the available workforce is used.

2.3 Methodology

This section briefly describes the YMB model proposed by [3]. This section also describes the computer algorithm developed for running MCS on the developed model by removing the constraint of contextual and point values of budget influencing parameters.

The YMB system was modelled by identifying the parameters influencing it. The values of the parameters were assigned in percentage based on tables developed for their quantification by the authors. A higher value of the variable implies reduction in the maintenance budget. The value 100 represents the ideal condition of parameters.

In this work, the constraint on variable quantification is removed to avoid subjectivity involved in the quantification methodology proposed in the earlier work. In this work Monte Carlo simulations are run by assigning any random value to the budget parameters within the chosen ranges. Based on the range of parameters, the maintenance levels have been described as: ‘world class practices’ and six other levels namely L_1, L_2, L_3, L_4, L_5 and L_6 in ascending order of expenditure: L_1 being the level that consumes the least and L_6consuming the most. The ranges for the YMB parameters corresponding to these levels are shown in Table2.1

Table 2.1: Maintenance Budget parameters ranges for various maintenance scenarios

S. No.	Maintenance Budget influencing parameters	World class practices (typical range)	World class practices (minimum threshold)	Maintenance budget parameters ranges					
		L_{WC}	L_{WCmin}	L_1	L_2	L_3	L_4	L_5	L_6
1.	Plant availability	90-100	90	90-100	80-100	70-100	80-90	70-80	60-70
2.	Safety aspects of plant	100	100	90-100	80-100	70-100	80-90	70-80	60-70
3.	Characteristics of plant	93-100	93	90-100	80-100	70-100	80-90	70-80	60-70
4.	Optimization tools along with condition monitoring technique	100	100	90-100	80-100	70-100	80-90	70-80	60-70
5.	Type of maintenance	80-100	80	90-100	80-100	70-100	80-90	70-80	60-70
6.	Quality of manpower	95-100	95	90-100	80-100	70-100	80-90	70-80	60-70
7.	Inventory management	98-100	98	90-100	80-100	70-100	80-90	70-80	60-70
8.	Procurement rate of critical spares	99-100	99	90-100	80-100	70-100	80-90	70-80	60-70
9.	Maintenance planning	95-100	95	90-100	80-100	70-100	80-90	70-80	60-70
10.	Maintenance scheduling	95-100	95	90-100	80-100	70-100	80-90	70-80	60-70
11.	Maintenance execution	90-100	90	90-100	80-100	70-100	80-90	70-80	60-70

2.3.1 Model description

The proposed work is the extension of the budget model suggested by [3] by removing the constraint of assigning point values to budget parameters.

The steps followed in formulating the model are briefly discussed as below:

- As a first step, the budget influencing parameters were identified.

- Next, the budget matrix was developed considering the identified parameters and their degree of mutual influences.
- The budget matrix of size n x n (n= number of budget influencing parameters) had diagonal elements, which were logarithm of the percentage value of budget parameters and the values of the off-diagonal elements were the degree of influences among the parameters. The degree of influence was quantified on a scale of 0 to 3, with 0 signifying no influence while 1, 2, and 3 signifying a weak, medium and strong influence.
- Finally, the Permanent Function [90] for the matrix was obtained and was named as maintenance budget function (MBF). Use of Permanent Function ensured that no information was lost.
- The YMB is then evaluated from the equations (2.1) and (2.2).

$$MaintenanceBudgetIndex(MBI) = \frac{MBF_{Ideal}}{MBF_{actual}} \tag{2.1}$$

$$YMB = (MBI \times 100 - 100)\,\%\,ofARV \tag{2.2}$$

The authors Gupta et al. [3] in their model evaluated the values of diagonal elements by quantifying them using tables developed for the purpose. In this work, the YMB, however, is evaluated without resorting to the quantification tables and by assigning random values to the budget parameters within specified ranges and deploying MCS simulations to arrive at steady state means of YMB. The algorithm for the simulation is described in the next sub-sections. A MATLAB code is developed for its implementation.

2.3.2 Algorithm

An algorithm is developed for YMB estimation based on MCS and various steps are detailed below.

Step (i)	:	X_1, X_2, X_3……..X_n are the n budget influencing parameters. These parameters are diagonal elements of the matrix and input to the model.
Step (ii)	:	Off-diagonal elements of matrix for each variable are calculated depending upon the intensity of relation with other parameters.
Step (iii)	:	Allocate the value log100% = 2 to all budget influencing parameters under ideal condition
Step (iv)	:	Generate the ideal maintenance budget matrix of size n x n.
Step (v)	:	Evaluate Permanent of matrix under ideal condition
Step (vi)	:	Generate the random numbers between the chosen range of parameters and allocate the random value to each variable
Step (vii)	:	Generate the maintenance budget matrix for a chosen maintenance level corresponding to levels detailed in Table 2.1.
Step (viii)	:	Evaluate Permanent of matrix arrived at in step (vii) above
Step (ix)	:	Calculate the MBI from equation (2.1) for the chosen maintenance level.
Step (x)	:	Evaluate the YMB from equation (2.2) for the chosen maintenance level.
Step (xi)	:	Repeat the step (vi) through step (x) for N number of cycles till the YMB value converges.
Step (xii)	:	Calculate mean and variance of YMB after each run.
Step(xiii)	:	Simulate the mean for N numbers of cycles using MCS.
Step (xiv)	:	Perform statistical analysis on model output

Based on the algorithm suggested in this sub-section, a MATLAB code is developed for evaluation of YMB.

2.4 Estimation of Yearly Maintenance budget

Following the discussion in Sections 2.3 and 2.4 the mean and the variance of YMB as a percentage of asset replacement value (ARV) are evaluated in this section. Mean and variance of YMB are evaluated for different maintenance levels operative in the plant systems. The classifications of different maintenance levels based on the input data range of parameters are discussed in section 3 and given in Table 2.1. To implement the suggested methodology, the MATLAB code was deployed. The simulations were run for 10^5 trials for convergence of the results. These are tabulated in Table 2.1. Result for the maintenance level 'world class practices' and six other levels namely L_1, L_2, L_3, L_4, L_5 and L_6 are shown graphically also in figures (2.2) to (2.8).

Table 2.2: Mean and variance of YMB as % of ARV for different plant category

	Range of maintenance parameters for various maintenance levels	Mean of YMB as % of ARV	Variance of YMB as % of ARV	Standard deviation as % of ARV
World class practices	Range of parameters as per Table 1	1.71	0.28	0.53
World class practices corresponding to point values of budget parameters at minimum threshold	Values of parameters as per Table 1	3.5	-	
Maintenance levels	L_1	3.20	0.70	0.84
	L_2	6.82	3.40	1.84
	L_3	7.50	10.76	3.28
	L_4	10.52	1.04	1.02
	L_5	19.49	1.63	1.28
	L_6	30.83	2.74	1.65

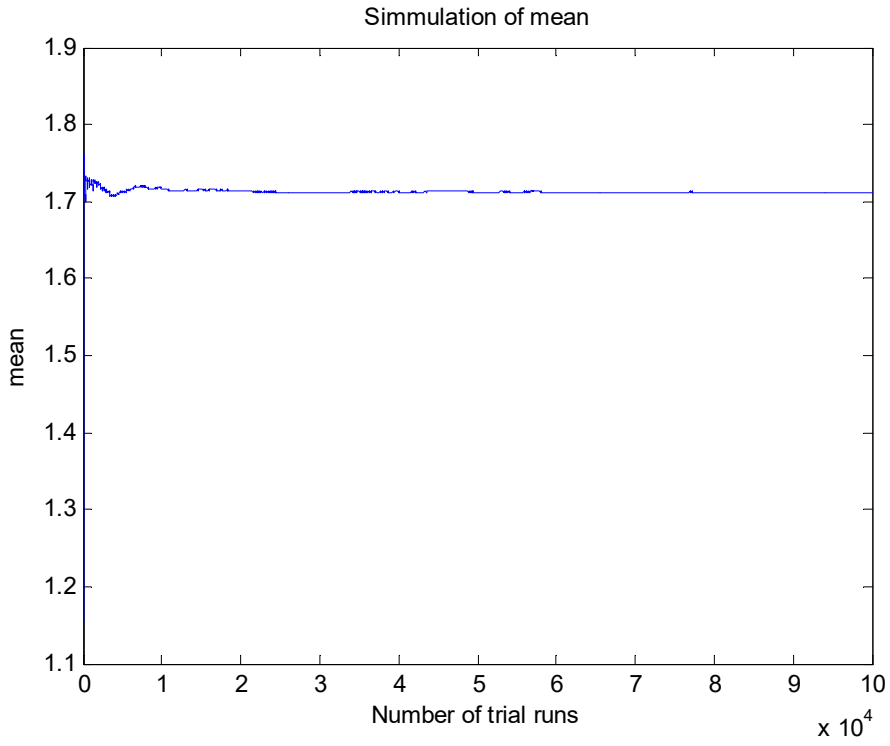


Figure 2.2: Mean of YMB Vs. number of simulations for World class level of maintenance

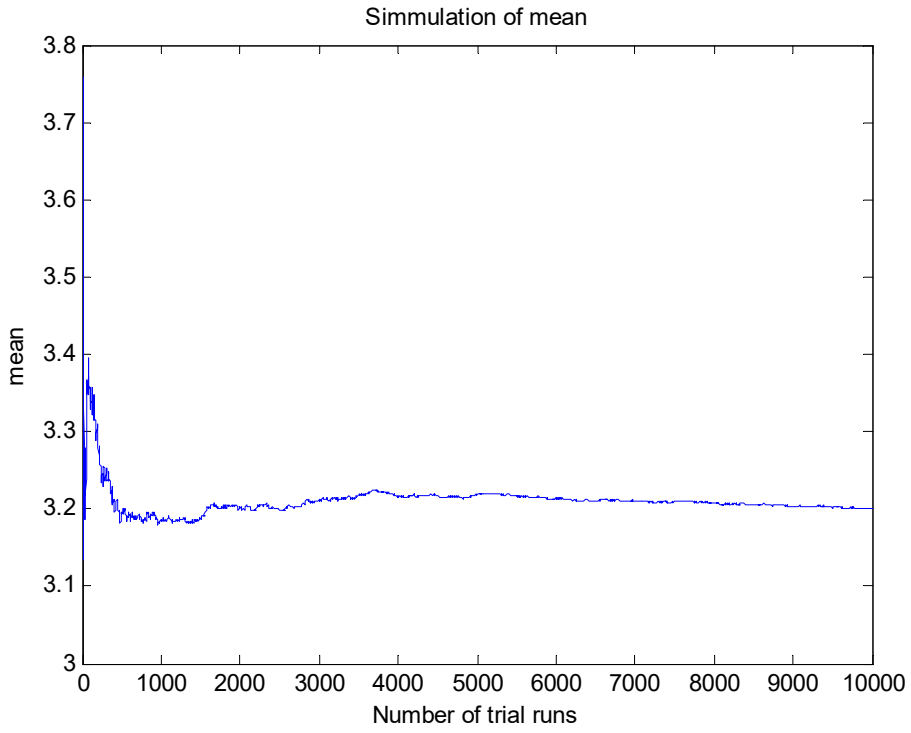


Figure 2.3: Mean of YMB Vs. number of simulations for L_1 level of maintenance

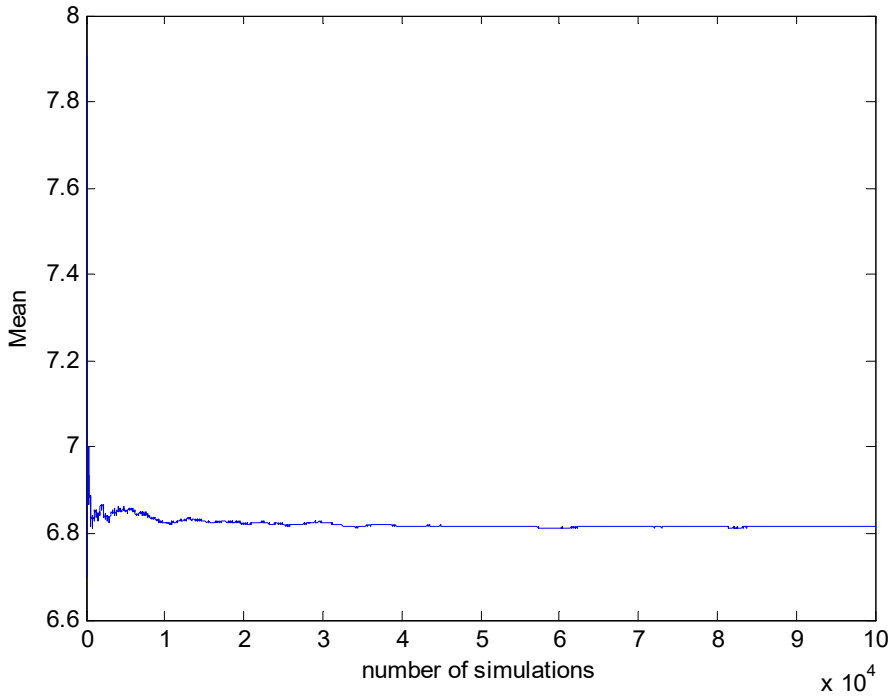


Figure 2.4: Mean of YMB Vs. number of simulations for L_2 level of maintenance

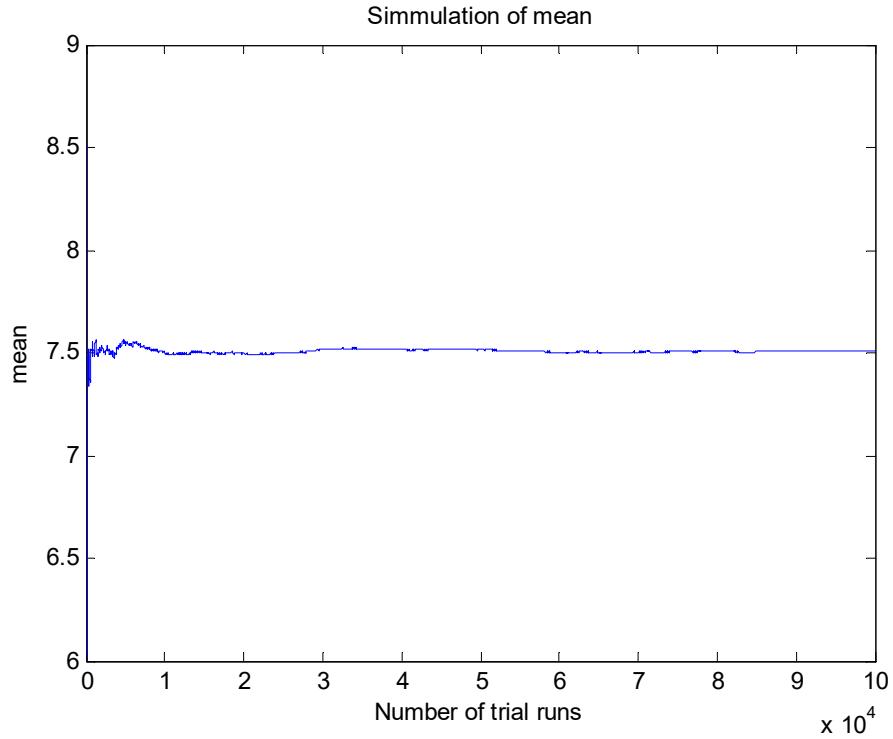


Figure 2.5: Mean of YMB Vs. number of simulations for L_3 level of maintenance

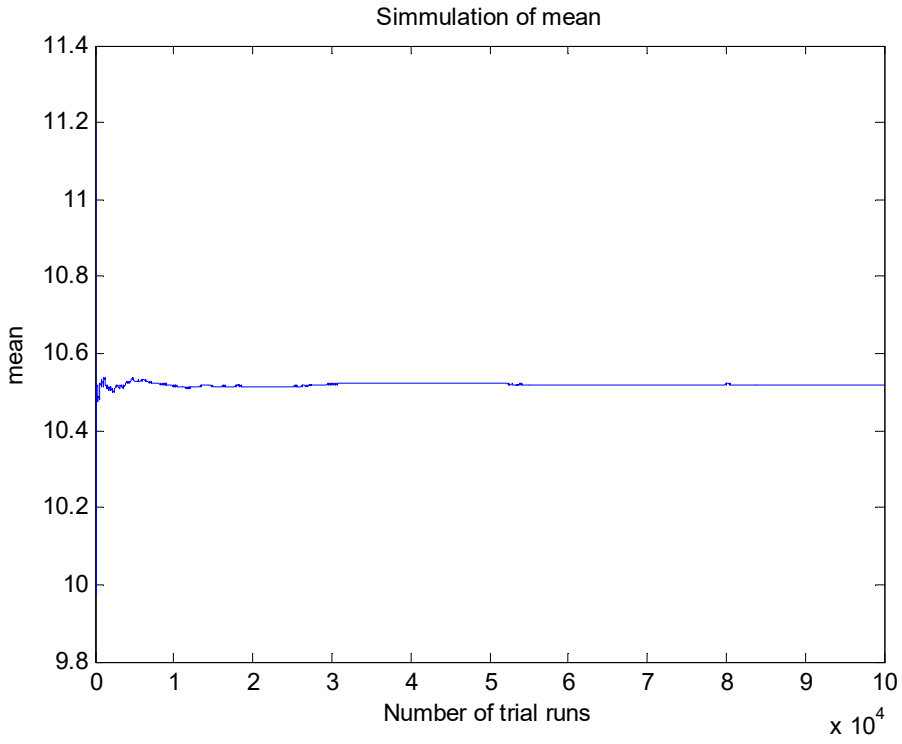


Figure 2.6: Mean of YMB Vs. number of simulations For L_4 level of maintenance

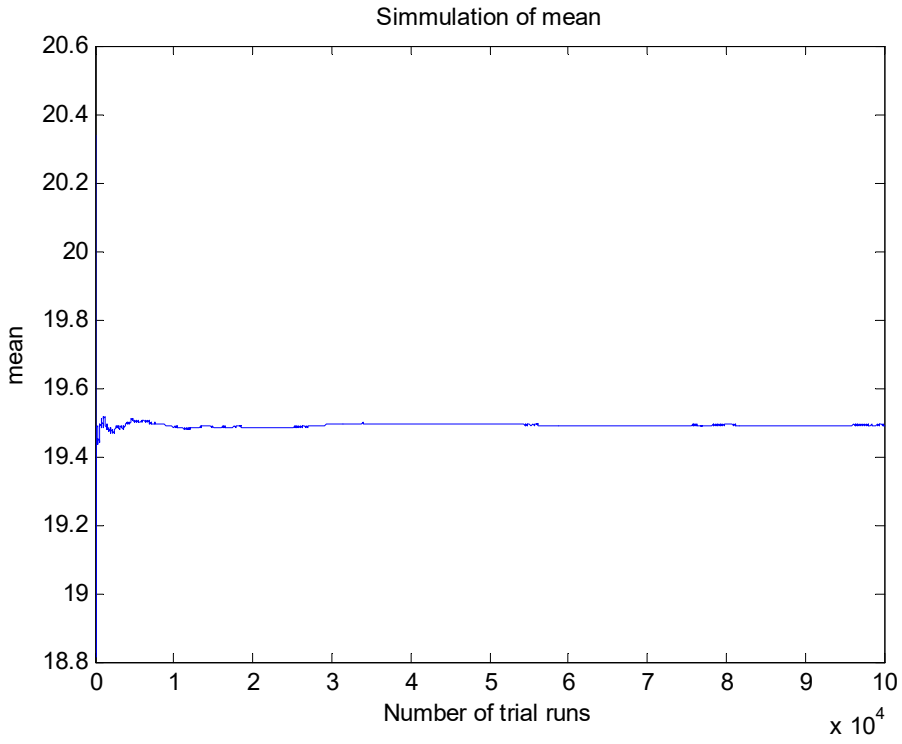


Figure 2.7: Mean of YMB Vs. number of simulationsfor L_5 level of maintenance

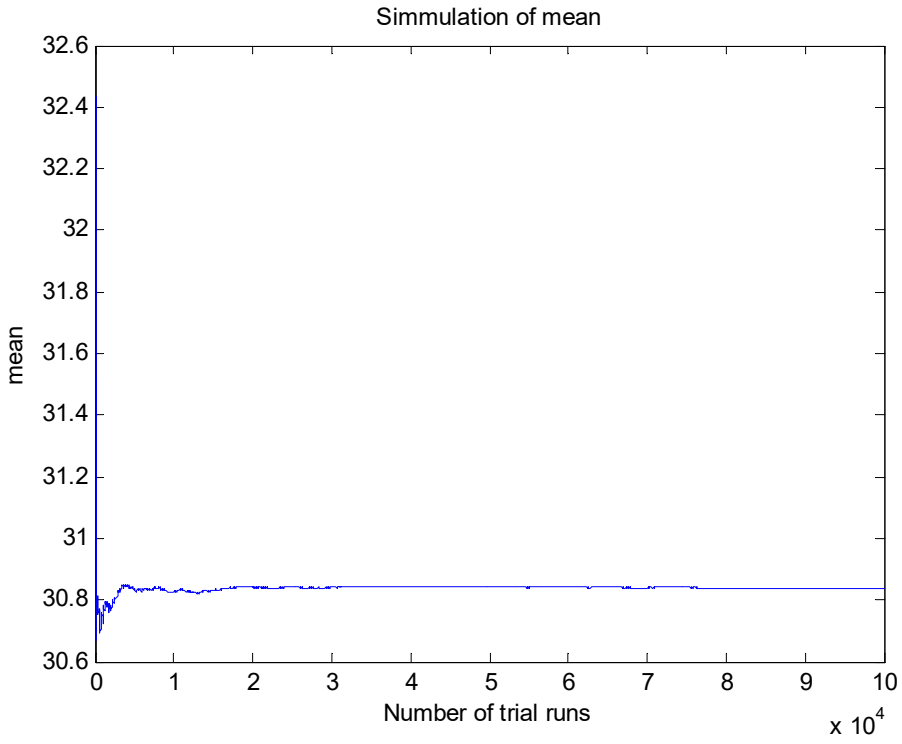


Figure 2.8: Mean of YMB Vs. number of simulations for L_6 level of maintenance

A closer look at the results shown in Table 2.2 indicates that YMB mean of 1.71 % of ARV for maintenance budget parameters having world class values as per the ranges specified in the literature [3]. This is in contradiction to the range of 2.5 to 3.5 % as reported in the literature [2]. The authors are not aware of the mechanism that Gulati and Smith had used to indicate the budget influencing variable range of world class practices. Also, authors apprehend that Gulati and Smith may have used point values of parameters on the lower side of world class bound or else they may have arrived at these figures through an extensive survey of the industries following world class practices. Further, the YMB for point values of budget influencing parameters at their minimum threshold of world class values is evaluated as 3.5% of ARV, which matches with the upper bound of the range specified in the literature. Further, it is relevant to note that the mean of YMB is evaluated as 6.82 with a standard deviation of 1.84, if the

values of budget parameters lie between 80 to 100%. The comparable values reported in the literature are between 3 to 9%. Moreover, for L_4 level of maintenance that corresponds to values of budget parameters lying between 80 to 90%, the mean of YMB is evaluated as 10.52% with a standard deviation of 1.02% of ARV. Experience shows that most well maintained enterprises having modern maintenance management outlook do resort to maintenance budgets of this order. Briefly, the results of Table 2.2 indicate that management must resort to corrective actions, if the values of budget influencing parameters drop below 80%.

The next section discusses the distribution followed by YMB function and the confidence interval (CI) estimation of mean and variance of YMB using sample data generated by MCS through inferential statistics.

2.5 Distribution of YMB function

Inferential statistics was applied to find the appropriate distribution and evaluate the interval estimate of maintenance budget parameters.

2.5.1 Budget function distribution

In this sub-section YMB distribution is identified by generating a random data set of YMB using MCS.

2.5.2 Identification of distribution

The best distribution was identified by fitting theoretical to the generated data set. This was based on statistically significant sample size of 100 with margin of error in the range of 9-10%, which is a norm in industry. For illustration purposes reference may be made to Table 2.3.

Table 2.3: YMB data for maintenance level 'L_2'

S. No.	YMB	S. No.	YMB	S. No.	YMB	S. No.	YMB	S. No.	YMB
1.	8.02	21.	7.55	41.	5.09	61.	4.86	81.	6.01
2.	4.89	22.	7.39	42.	7.73	62.	9.46	82.	8.36
3.	5.082	23.	3.83	43.	7.59	63.	4.38	83.	4.99
4.	5.282	24.	5.18	44.	5.40	64.	7.41	84.	8.88
5.	6.42	25.	7.76	45.	8.53	65.	8.02	85.	7.55
6.	7.05	26.	7.46	46.	5.28	66.	4.89	86.	7.39
7.	8.63	27.	5.49	47.	5.65	67.	5.08	87.	3.83
8.	6.47	28.	9.37	48.	8.67	68.	5.28	88.	5.18
9.	9.35	29.	4.89	49.	6.10	69.	6.44	89.	7.76
10.	5.95	30.	7.49	50.	9.33	70.	7.05	90.	7.46
11.	7.41	31.	10.04	51.	6.76	71.	8.63	91.	5.46
12.	4.94	32.	4.62	52.	6.27	72.	6.47	92.	9.37
13.	9.02	33.	6.25	53.	5.56	73.	9.35	93.	4.89
14.	11.4	34.	7.20	54.	7.26	74.	5.95	94.	7.49
15.	7.68	35.	7.24	55.	8.09	75.	7.41	95.	10.04
16.	5.9	36.	6.34	56.	3.92	76.	4.94	96.	4.62
17.	6.01	37.	9.51	57.	6.02	77.	9.02	97.	6.25
18.	8.36	38.	6.59	58.	3.80	78.	11.4	98.	7.20
19.	4.50	39.	7.13	59.	7.16	79.	7.68	99.	7.24
20.	8.88	40.	3.92	60.	9.51	80.	5.93	100.	6.34

Exponential, Weibull, lognormal and normal distributions were tested for the best fit with the generated data set. Index of fit values of 'r', were found to be 0.990, 0.974, 0.989 and 0.936 for normal, Weibull, lognormal and exponential distributions respectively implying that Normal distribution is the best fit. Illustrative plots for this distribution are only shown in figures 2.9 to 2.12.

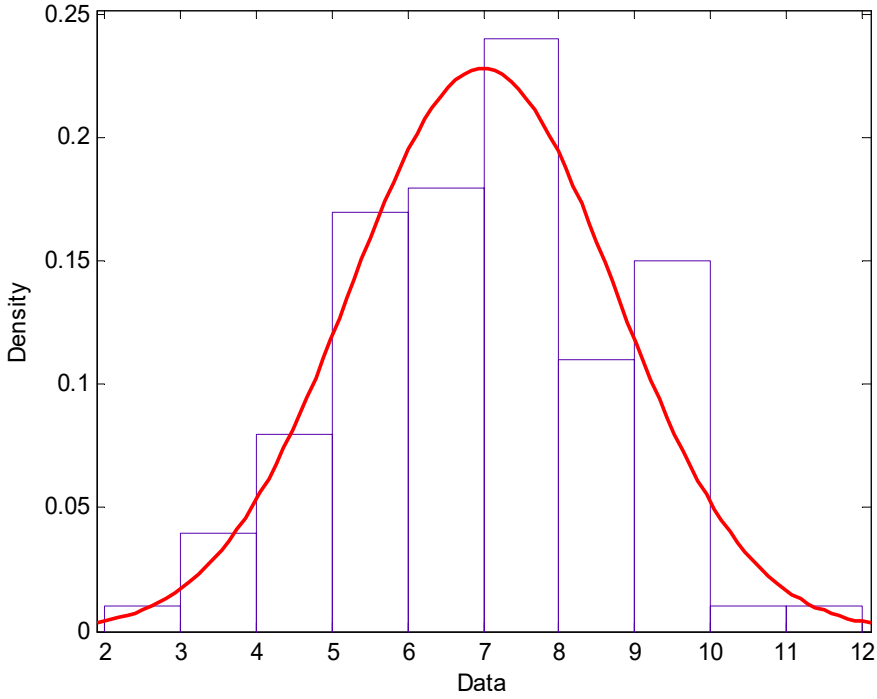


Figure 2.9: Probability density function (Normal) for L_2 level of maintenance

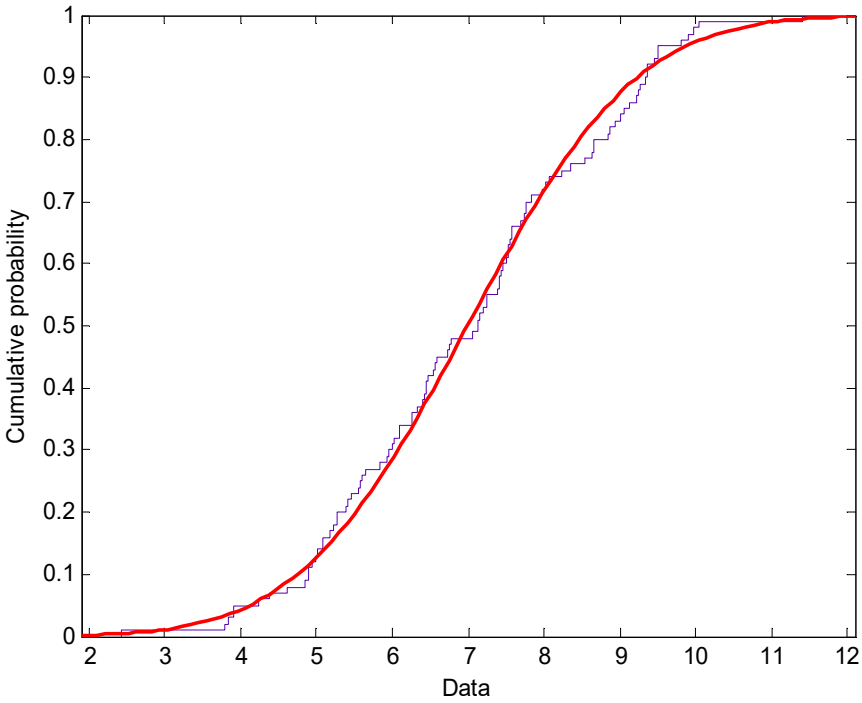


Figure 2.10: Cumulative density function (Normal) for L_2 level of maintenance

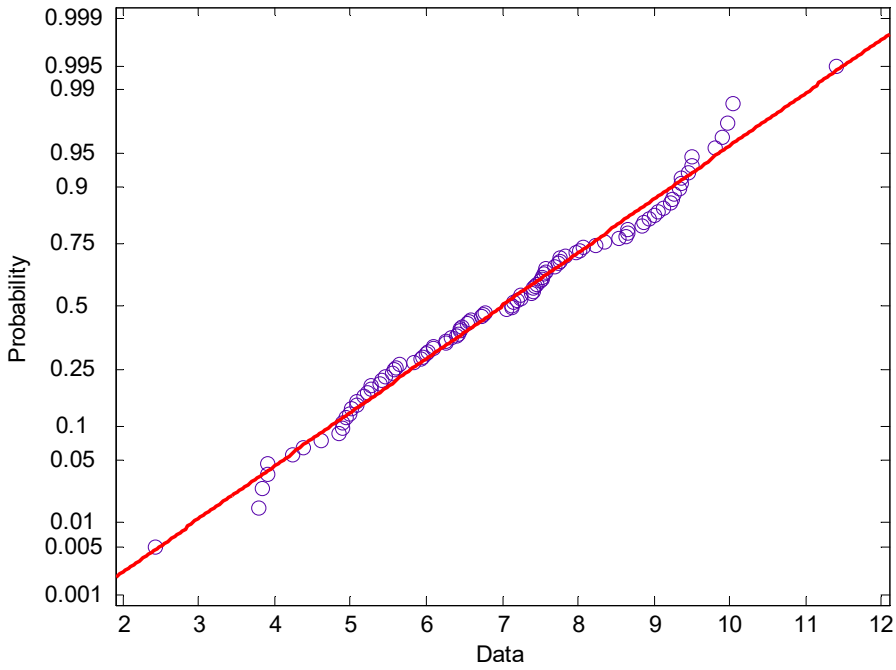


Figure 2.11: Probability plot (Normal) for L_2 level of maintenance

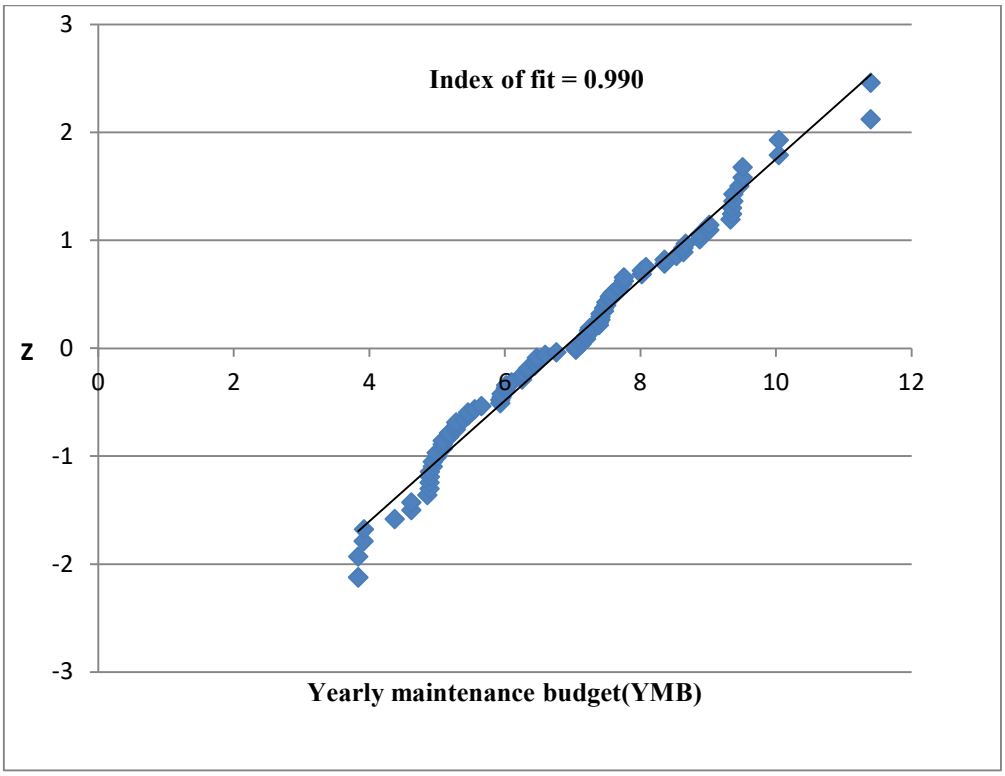


Figure 2.12: Least- square curve-fitting plot (Normal) for L_2 level of maintenance

Similarly, the best fit distributions for other data sets were evaluated for other ranges of budget influencing variable and the results are shown in Table 2.4

Table 2.4: Distribution and goodness-of-fit test

	Range of maintenance parameters for various maintenance levels	Best fit distribution (based on plots and index of fit values)	Result of goodness of fit test (Y for null hypothesis) (N for alternate hypothesis)
World class practices	Range of parameters as per Table 2.1	Normal	Y
Maintenance levels	L_1	Normal	Y
	L_2	Normal	Y
	L_3	Normal	Y
	L_4	Normal	Y
	L_5	Normal	Y
	L_6	Not fitting into any of the four chosen distributions	N

Goodness-of-fit test was carried out for validating the best fit, which is described in the next sub-section.

2.6 Goodness -of -fit test

This sub-section deals with testing of best fit distribution identified in section 2.5.2 using inferential statistics, i.e. Goodness-of-fit test. Ross, [91] described the Chi-square goodness-of-fit test, which is a process of identification of theoretical distribution. The test is carried on data generated for values of maintenance budget parameters in the range of 80-100% and the test procedure is as follows.

The hypothesis is designed as:

H_0: The maintenance budget data follows the normal distribution

H_1: The maintenance budget data does not follow normal distribution

To carry out the chi-square test a significance level of 0.05 is assumed. The test statistic value is calculated using the sample data of Table 2.3 and its value is 6.47. The critical value of Chi-square from the statistical table is noted as 11.07 [92]. It is concluded that mean of YMB follows normal distribution. On similar lines, sample data of YMB for other ranges of YMB parameters was also tested for best fit distribution and the results are summarized in Table 2.4.

Further, further confidence interval estimates for mean and variance of YMB were evaluated and are shown in Table 2.5 for different maintenance levels defined earlier in Table 2.2. Also, Table 2.6 shows the results of hypothesis testing of mean and variance of YMB based on significance level of 0.05.

Table 2.5: Confidence interval estimates for mean and variance

	Range of maintenance parameters for various maintenance levels	**95% CI estimate for Mean**	**95% CI estimate for Variance**
World class practices1	Range of parameters as per Table 2.1	(1.62, 1.82)	(0.22, 0.38)
Maintenance levels	L_1	(2.99, 3.31)	(0.42, 0.73)
	L_2	(6.51, 7.23)	(2.31, 4.04)
	L_3	(6.36, 7.64)	(6.44,11.27)
Maintenance levels	L_4	(10.45,10.85)	(0.93, 1.63)
	L_5	(19.28,19.78)	(1.27,2.23)
	L_6	(30.67, 31.33)	(1.91,3.35)

Table 2.6: Hypothesis test for population parameters

	Range of maintenance parameters for various maintenance levels	Test for Mean at 5% significance level	Test for Variance at 5% significance level
World class practices	Range of parameters as per Table 2.1	H_0 accepted	H_0 accepted
Maintenance levels	L_1	H_0 accepted	H_0 accepted
	L_2	H_0 accepted	H_0 accepted
	L_3	H_0 accepted	H_0 accepted
	L_4	H_0 accepted	H_0 accepted
	L_5	H_0 accepted	H_0 accepted
	L_6	H_0 rejected	H_0 rejected

2.7 Practical implications

The results discussed in Section 2.5 and 2.6 identify the requirement of budget for different maintenance level. For worldclass practices and L_1 to L_3 level of maintenance, the budget value falls between 2 and 9%. This indicates that the maintenance department is following optimum processes of maintenance. If the maintenance budget ranges from 10 to20% as in the case of L_4 and L_5 level, it shows that unnecessary maintenance expenditure reduces the profitability of the organization. To face the above challenges, organizations must constantly pursue maintenance excellence through highly structured maintenance programs. Over 20% of the budget value indicates that yearly maintenance expenditure is so high at L_6 maintenance level and most of the systems in plant should be replaced by a completely new one at regular intervals. Maintenance engineer shall decide the value of maintenance parameters based on plant-specific conditions and may use the suggested methodology to estimate the yearly maintenance budget. Corrective actions must be taken by maintenance engineers if budget influencing parameter values fall below 80%. The methodology recommended in this work will be a valuable guideline to maintenance engineer in deciding YMB

required for plant maintenance. It also provides indications for maintenance system rectifications and avoids expert recommendations on maintenance budget allocation.

2.8 Summary

This chapter addressed a significant gap in the estimation of the Yearly Maintenance Budget (YMB) by introducing a scientifically grounded approach that overcomes the limitations of conventional, subjective practices. Traditional methods often rely on historical data or expert `modern plant systems. While some scientific modelling attempts, such as the digraph-based methodology by Gupta et al. [3] have laid foundational work in this area, they often do not consider the stochastic nature of budget variable, limiting their practical applicability. The present work overcomes this by integrating stochastic simulation, thereby improving the accuracy of maintenance budget. In this study, a novel methodology combining the Graph Theory and Matrix Approach (GTMA) with Monte Carlo Simulation (MCS) was employed to evaluate YMB by assigning probabilistic values to influencing parameters over a defined range. The simulation-based analysis provided interval estimates for YMB as a percentage of Asset Replacement Value (ARV) across various maintenance levels. These results offer maintenance managers clear benchmarks, particularly when maintenance parameters fall below a specific threshold, leading to a substantial increase in YMB and compromising operational efficiency.

Once the scientific evaluation of annual maintenance budget is done based on contextual plant parameters, it becomes imperative for the maintenance managers to make a informed decision on the maintenance approach to be adopted for optimal and economic operations. The next chapter-3 deals with this aspect.

CHAPTER 3

COST-BASED MODELLING AND OPTIMIZATION UNDER IMPERFECT MAINTENANCE QUALITY

3.1 Introduction

In industrial systems, maintenance actions are critical for ensuring equipment reliability, availability, and cost-efficiency. However, in practice, maintenance is often imperfect—meaning it does not fully restore a system to its original or 'as-good-as-new' condition. Such imperfect maintenance introduces variability in maintenance outcomes due to factors such as limited technician skill, resource constraints, inappropriate procedures, or substandard replacement parts. This unpredictability in maintenance quality directly impacts system performance and complicates cost forecasting.

Traditional maintenance optimization models often assume perfect or minimal repair, overlooking the reality that most industrial maintenance actions lie somewhere between these two extremes. This oversight can lead to suboptimal maintenance schedules and inaccurate cost estimations. Therefore, it is essential to integrate maintenance quality variability into cost-based maintenance models to reflect realistic system behaviour.

This chapter presents a modelling framework that incorporates the imperfect nature of maintenance actions into the cost evaluation and optimization process. The proposed approach models the preventive and corrective maintenance strategies under imperfect conditions and quantifies their impact on long-term maintenance cost. Preventive

maintenance intervals are optimized based on maintenance quality levels, cost rates, and failure distributions.

By accounting for imperfect maintenance quality, the model enables more accurate estimation of total maintenance costs and supports the development of economically sound maintenance policies. The methodology offers valuable insights for maintenance managers to make informed decisions in resource-constrained, quality-variable environments.

3.2 Materials and Methods

This chapter is a significant extension of imperfect maintenance model proposed by Nakagawa [15][16]. Model *X* is based on an imperfect preventive maintenance policy with repair at failure. The Model Y is designed with a focus on preventive maintenance with minimal repair after failure. The foundation of Model Z is based on the notion that perfect preventive intervention is the only way to detect a failed unit. Expected costs per unit of time and maintenance time interval are evaluated for all these three models, and optimal policies are subsequently discussed. Finally, for better understanding a numerical illustration is provided.

3.3 Model X: Imperfect Maintenance Policy

In the following subsections, the imperfect maintenance policy is described in detail, including the underlying assumptions, the method for determining the optimal preventive maintenance interval, and the calculation of associated maintenance costs.

3.3.1 Imperfect Maintenance Policy

In real situations, equipment restoration after preventive maintenance is not perfect (like- new condition). Imperfections in restoration, or imperfect maintenance, more accurately reflect practical scenarios. Sometimes, equipment post-PM may degrade due to faulty procedures incorrect adjustments, substandard spare parts, and damages that occurred during the maintenance process resulting in a condition worse than pre-PM. In order to incorporate this possibility, the assumption is made that the rate of failure post-PM remains the same as it was pre-PM with a predetermined probability. The anticipated maintenance cost per unit time (cost rate) of the model undergoing imperfect preventative maintenance is determined in this section. An optimum strategy aimed at minimizing it is presented.

3.3.2 Assumptions involved in Modeling

Considering the imperfect PM policy designed for a system with a single unit intended to operate indefinitely.

The modeling and analysis process can be facilitated by introducing the following specific assumptions.

I. The unit is either repaired in the case of a breakdown or undergoes scheduled preventive maintenance at a specified time T, depending on whichever event takes place first, following its installation or the previous preventive maintenance, where T is the time interval between two consecutive instances of performing preventative maintenance.

II. The repair process has the potential to restore the unit to a like-new condition.

III. There is no change in the rate of failure after PM with some specific probability p, and q, is the probability that the PM will be successful in restoring the unit to a condition as good as new. Where $q = 1 - p$.

IV. The time durations associated with PM and repair activities are considered insignificant and, therefore, ignored.

V. The costs involved on a repair (corrective maintenance), and PM are c_r, and c_p, respectively.

The long-term average maintenance cost rate is derived based on the assumptions provided above. It is determined by combining the costs associated with equipment repair (corrective maintenance) and PM.

The rate of long-term average cost is expressed through the equation (3.1)

$$C_X(T;p) = \frac{c_r\left[1 - q\sum_{k=1}^{\infty} p^{k-1}\bar{F}(kT)\right] + c_p\sum_{k=1}^{\infty} p^{k-1}\bar{F}(kT)}{\sum_{k=1}^{\infty} p^{k-1}\int_{(k-1)T}^{kT} \bar{F}(t)\mathrm{d}t} \quad (3.1)$$

At the beginning of the maintenance planning horizon (T=0), the average maintenance cost is determined by equation (3.2)

$$C_X(0;p) \equiv \lim_{T\to 0} C_X(T;p) = \infty \quad (3.2)$$

This indicates an unbounded expected cost associated with the scenario where no preventive maintenance (PM) is performed, and the unit is only repaired when it fails.

Conversely, when T= ∞, the average maintenance cost is computed as per equation (3.3)

$$C_X(\infty; p) \equiv \lim_{T\to\infty} C_X(T; p) = \frac{c_r}{\mu} \tag{3.3}$$

The value of $C_X(T; p)$ in this case converges to finite value specifically $\frac{c_r}{\mu}$. This value signifies the anticipated cost for the situation in which no preventive maintenance is performed on a unit or system. In this case, the unit is only repaired when it fails to function, with no proactive maintenance measures taken to prevent failures. The comparison of these two extreme cases provides insights into the cost implications associated with different maintenance strategies over time.

3.3.3 Determination of optimum Preventive Maintenance interval T_0

To determine the optimal condition of variable T that results in the minimization of the function $C_X(T; p)$ in equation (3.1), the following optimization problem need to be solved

$$C_X(T_0, p) = \min_T C_X(T; p) \tag{3.4}$$

At this point the term average maintenance cost rate $C_X(T; p)$ represents a two-variable function of T and p.

The cumulative hazard function is defined according to equation (3.5)

$$\Lambda(t; p) \equiv \sum_{k=1}^{\infty} p^{k-1} k f(kt) / \sum_{k=1}^{\infty} p^{k-1} k \tilde{F}(kt) \tag{3.5}$$

An assumption made regarding the cumulative hazard function is that it exhibits an explicitly increasing behaviour with time t. This characteristic holds true for any value of p.

Table 3.1 Steps to find out the optimal value T_0 for Model X

Step I

Inputs

$F(t)$: The cumulative distribution function of failure time;

$\Lambda(t)$: Cumulative hazard function;

c_r : The repair cost;

c_p : The cost associated with PM;

c_{mr} : The cost related to each and every minimal repair;

c_{re} The cost incurred due to the time passed between the occurrence of a fault and its detection

p : Probability that preventive maintenance is imperfect

Step II

Outputs

T_0 : Optimal PM time interval;

$C_X(T_0, p)$: Optimal maintenance cost rate;

Algorithm

i. : For T value ranging from 1 to ∞ do;
ii. : Obtain T by ensuring that the resulting T satisfies the equation (3.6);
iii. : And if inequality (3.7) and (3.8) is satisfied;
iv. : Calculate $C_X(T_0, p)$ from equation (3.9);
v. : $T_0 \Leftarrow T$;
vi. : Break;
vii. : End
viii. : End

Next, find the derivative of function $C_X(T; p)$with respect to T, Set the derivative equal to zero, and solve for T

i. e $\frac{\partial C_X(T;p)}{\partial T} = 0$,

subsequently, equation (3.6) can be obtained, and then an optimal value T_0 exists, satisfying equation (3.6), and meeting the condition of the optimum policy

$$\frac{(c_r q - c_p)}{c_p}\left[\Lambda(t;p)\sum_{k=1}^{\infty} p^{k-1}\int_{(k-1)T}^{kT}\bar{F}(t)\mathrm{d}t - q\sum_{k=1}^{\infty} p^{k-1}F(kT)\right] = 0 \qquad (3.6)$$

$$\frac{c_r q}{(c_r q - c_p)} < \mu\,\Lambda(\infty;p) \qquad (3.7)$$

$$\frac{c_r q}{c_p} > 1 \qquad (3.8)$$

3.3.4 Determination of optimum maintenance cost rate $C_X(T_0, p)$

The optimum Maintenance cost rate is computed by the equation (3.9)

$$C_X(T_0,p) = \Lambda(T_0;p)\left(c_r - \frac{c_p}{q}\right) \qquad (3.9)$$

In the scenario where no preventive maintenance (PM) is performed, and maintenance activities are carried out reactively (run-to-failure), the expected maintenance cost rate is evaluated by equation (3.3)

3.4 Model Y: Imperfect Preventive Maintenance with Minimal Repair at Failure

This section introduces Model Y, which considers imperfect preventive maintenance with minimal repair at failure, outlining the maintenance policy, modelling assumptions, the procedure to determine the optimal preventive maintenance interval, and the evaluation of maintenance costs

3.4.1 Maintenance Policy Description

In this model, an imperfect PM, with minimal maintenance or repair is performed after failures. Minimal repair is a repair process that requires limited effort for maintenance, aiming to restore the device to its functional state as it was prior to the occurrence of

the failure. Moreover, it is reasonable to consider that if a component in a system fails, the act of replacing that failed component with a new one is considered as a minimal repair to the entire system.

3.4.2 Assumptions involved in Modelling

I. PM activities are executed at a specific time, denoted as xT for $x = 1,2 \ldots, N$,

II. The equipment receives minimum repairs only when breakdowns occurs between scheduled PMs.

III. The rate of failure $r(t)$ is monotonically increasing.

IV. After preventive maintenance, the failure rate of the unit remains unchanged. That is, the unit is expected to maintain the same level of reliability and performance as it had before the preventive maintenance action with predefined probability p, or it achieves a state of perfect condition with a specific probability q

V. The minimal repair cost is c_{mr} and the PM cost is c_{p}.

VI. The time required for minimal repairs and PM is considered to be negligible.

VII. The rate of failure $r(t)$ remains unaltered by minimal repair.

Considering the above assumptions, the long-term maintenance cost rate can be expressed by equation (3.10)

$$C_Y(T;p) = \frac{1}{T}\left[c_{\mathrm{mr}} q^2 \sum_{k=1}^{\infty} p^{j-1} \int_0^{kT} h(t)\mathrm{d}t + c_{\mathrm{p}}\right] \qquad (3.10)$$

3.4.3 Determination of Optimum Preventive Maintenance Interval T_0

To determine the optimal condition of variable T that results in the minimization of the function $C_Y(T;p)$ in equation (3.10), the optimization problem is solved using equation (3.11)

$$C_Y(T_0,p) = \min_T C_X(T;p) \tag{3.11}$$

Table 3.2 Steps to find out the optimal value T_0 for Model Y

The inputs and outputs are similar to those of the Algorithm shown in Table 3.1. The calculation steps remain identical to those outlined in Algorithm after implementing the following interchanges: Replace equation (3.6) with equation (3.12), equation (3.7) and equation (3.8) with equation (3.13) and equation (3.9) with equation (3.14) respectively.

Next, find the derivative of function $C_Y(T;p)$ with respect to T, Set the derivative equal to zero, and solve for T

i. e $\frac{\partial C_Y(T;p)}{\partial T} = 0$,

Subsequently, equation (3.12) can be obtained, and then an optimal value T_0 exists, satisfying equation (3.12), and meeting the condition of the optimum policy, equation (3.13)

$$\sum_{k=1}^{\infty} p^{k-1} \int_0^{kT} t\, dh(t) = \frac{c_p}{c_{mr} q^2} \tag{3.12}$$

$$\int_0^{\infty} t\, dh(t) > \frac{c_p}{(c_{mr} q^2)} \tag{3.13}$$

3.4.4 Determination of Optimum Maintenance Cost Rate $C_Y(T_0,p)$

The optimum Maintenance cost rate is computed by the equation (3.14).

$$C_Y(T_0;p) = c_{\text{mr}} q^2 \sum_{k=1}^{\infty} p^{j-1} kh(jT_0) \quad (3.14)$$

3.5 Model Z: Failure Detection only through Perfect Preventive Maintenance

Here, we present Model Z, explaining the maintenance approach in which system failures are detected only through perfect preventive maintenance, the assumptions made, the algorithm to find the optimal maintenance interval, and the method to compute maintenance costs.

3.5.1 Maintenance Policy Description

In this model, it is assumed that the failure of the equipment or system can only be identified when a perfect PM is conducted. Therefore, the failure persists for the time interval between the occurrence of the failure and the subsequent perfect Preventive Maintenance activity.

3.5.2 Assumptions involved in Modelling

I. PM actions are regularly scheduled at specific times xT for $x = 1, \dots, N$, and in case of any failures between these scheduled maintenance events, only minimal repairs are conducted to keep the equipment operational until the next planned maintenance period.

II. There is a continuous and consistent rise in failure rate as time advances.

III. The PM may not fully restore the system to a new-like condition with a predefined probability of p and there is a probability q that the system will become as good as new after maintenance. where $q = 1 - p$.

IV. The time required for both repair and PM activities is very small or insignificant.

Considering the above assumptions, the long-term maintenance cost rate is given by equation (3.15)

$$C_Z(T,p) = \left[c_{\mathrm{p}} + c_{\mathrm{pr}}\tilde{p}^2\Sigma_j p^{k-1}\int_0^{kT} F(t)dt\right]/T \qquad (3.15)$$

3.5.3 Determination of Optimum Preventive Maintenance Interval

To determine the optimal condition of variable T that results in the minimization of the function $C_Z(T_0,p)$ in equation (3.15), the optimization problem can be solve, given by equation (3.16)

$$C_Z(T_0,p) = \min_T C_Z(T;p) \qquad (3.16)$$

Table 3.3 Steps to find out the optimal value T_0 for Model Z

The inputs and outputs are similar to those of the Algorithm shown in Table 3.1. The calculation steps remain identical to those outlined in Algorithm after implementing the following interchanges: equation (3.6) with equation (3.18); equation (3.7) and equation (3.8) with equation (3.19); equation (3.9) with equation (3.20), respectively.

Next, find the derivative of function $C_Z(T;p)$with respect to T, Set the derivative equal to zero, and solve for T

$$\text{i. e } \frac{\partial C_Z(T;p)}{\partial T} = 0, \qquad (3.17)$$

subsequently, equation (3.18) can be obtained, and then an optimal value T_0 exists, satisfying equation (3.18), and meeting the condition of the optimum policy given by equation (3.19)

$$\sum_{k=1}^{\infty} p^{k-1} \int_0^{kT} t \, \mathrm{d}F(t) = \frac{c_{\mathrm{p}}}{c_{\mathrm{pr}} q^2} \tag{3.18}$$

$$\mu > \frac{c_{\mathrm{p}}}{(c_{\mathrm{pr}} q^2)} \tag{3.19}$$

3.5.4 Determination of Optimum Maintenance cost rate $C_Z(T_0; p)$

The optimum Maintenance cost rate is computed by the equation (3.20)

$$C_Z(T_0; p) = c_{\mathrm{mr}} q^2 \sum_{k=1}^{\infty} p^{k-1} kF(\mathrm{k}T_0) \tag{3.20}$$

3.6 Numerical Analysis

In this section, the numerical analysis and results for Models X, Y, and Z are presented, highlighting the application of each maintenance policy, determination of optimal preventive maintenance intervals, and evaluation of the resulting maintenance costs and system performance.

3.6.1 Model X: Numerical Analysis and Results

This section presents a numerical analysis illustrating the implementation of the imperfect PM policy over an infinite time frame. The aim is to explore as to how the execution of imperfect PM policies influences optimal maintenance decisions. The method of solution is displayed in Table 3.1. By implementing the algorithm outlined in Table 3.1 for the optimal maintenance strategy: the most effective maintenance interval plan, and the corresponding optimal maintenance cost rate, $C_X(T_0, p)$ can be

obtained. The required key parameters settings for the model are minimal repair cost c_{mr} , preventive maintenance cost c_p, and Repair cost c_r.

A widely used failure distribution in reliability analysis is the Weibull distribution [93-95]. The Weibull reliability analysis is effective in characterizing the equipment time-to-failure data and designing appropriate maintenance strategies. Here two-parameter Weibull distributions is used.

For the Weibull distribution, the probability density function is

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}} (\beta, \theta > 0) \quad (3.21)$$

Where β is the shape parameter

and θ is the scale parameter or characteristic life

β<1 is the case of decreasing failure rate

β= 1 is the condition for a constant failure rate

β>1 shows the increasing failure rate

Similarly, as θ increases reliability increases

the failure time cumulative distribution is set as

$$F(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^{\beta}} (\beta, \theta > 0) \quad (3.22)$$

Then the reliability function or survivability function can be represented as

$$\bar{F}(t) = 1 - F(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}} (\beta, \theta > 0) \quad (3.23)$$

The failure rate function is determined as

$$r(t) = \frac{f(t)}{\bar{F}(t)} = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \quad (\beta, \theta > 0) \tag{3.24}$$

the cumulative hazard functions, $\Lambda(t) = \int_0^u r(u)du = (t/\theta)^{\beta}$ (3.25)

To illustrate the suggested model, a few parameters are randomly chosen. Following are the specific model parameters that have been set. Repair cost $c_r = 5$ and the preventive maintenance cost $c_p = 1$

After setting the value of the scale parameter to 1, the failure rate function can be represented as follows

$$r(t) = \beta t^{\beta-1} \tag{3.26}$$

The shape parameter must be restricted to β>1, as it is widely recognized that performing preventive maintenance activities becomes economically impractical when β is equal to or less than 1 and the hazard rate function displays a trend that is either decreasing or remains constant. In this context, the analysis focuses on exploring and comparing two typical patterns observed in hazard rate functions i.e. when the parameter β, falling between 1 and 2, corresponds to a concave or moderately increasing hazard rate and for values of β greater than 2, the hazard rate exhibits a convex shape, indicating a more significant surge. Two distinct shape parameters, β=1.6 indicating concave hazard rates, and β=3.5 indicating convex hazard rates, are used.

Initially, the imperfect preventive maintenance for the infinite time frame is examined. Table 3.4 and figure 3.1 present a comprehensive illustration of optimal decisions for preventive maintenance interval, along with associated average maintenance cost rates for various imperfect preventive maintenance probability values.

Table 3.4: Variation in the optimal solution T_0 and associated Cost with the change of Imperfect PM probability

	$(\beta = 1.6)$		$(\beta = 3.5)$	
p	$C_X(T_0, p)$	T_0	$C_X(T_0, p)$	T_0
0.00	4.75	0.60	2.72	0.52
0.01	4.77	0.61	2.76	0.52
0.05	4.84	0.62	2.95	0.51
0.10	4.93	0.64	3.17	0.49
0.15	5.02	0.66	3.39	0.48
0.20	5.10	0.69	3.61	0.47
0.25	5.19	0.72	3.84	0.46
0.30	5.27	0.77	4.06	0.47
0.35	5.34	0.83	4.28	0.48
0.40	5.40	0.92	4.50	0.50
0.50	5.56	1.19	4.93	0.58
0.60	5.57	1.72	5.24	0.80

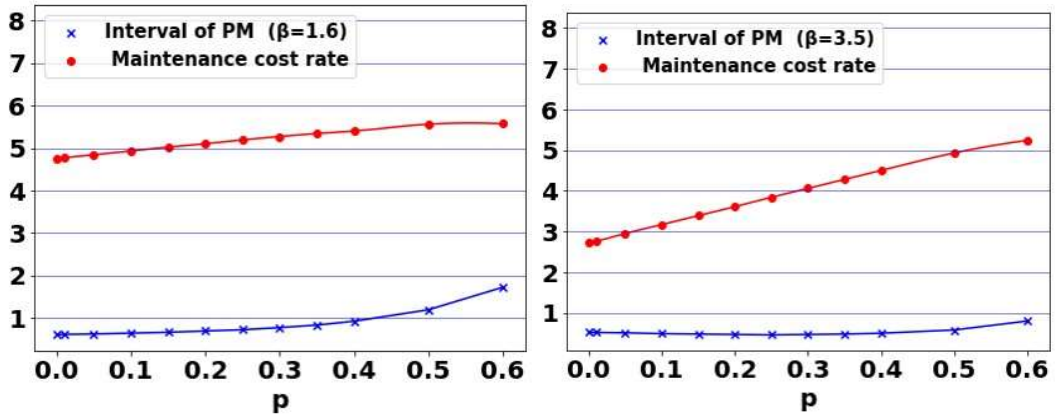


Figure 3.1 Variation in the optimal solution T_0 and $C_X(T_0, p)$ with the change of p.

It is observed that as the imperfect maintenance probability increases, there is an increasing trend in the optimal preventive maintenance time interval and maintenance cost rate. This means that, when the probability of imperfect preventive maintenance is high, it is more practical to intervene with corrective maintenance, repairing a failed

unit promptly when problems arise, rather than performing preventive maintenance frequently. This observation is for concave hazard rates.

In the case of the convex hazard rate, as the probability of imperfection increases from 0 to 25%, there is a corresponding decrease in the maintenance time interval, dropping from 0.52 to 0.47. Therefore, it is recommended to execute preventative maintenance more frequently. As the level of imperfectness exceeds 25%, it becomes more practical to opt for corrective maintenance rather than adhering to a frequent schedule of preventive maintenance. The results obtained by the author are more realistic and closely aligned with real-world observations compared to those reported in existing research (Nakagawa, 1979[16]). The results are further improved when a higher value is assigned to the Weibull shape parameter.

Next, the imperfect preventive maintenance policy with maintenance imperfections rates of 10% and 30% was examined. Tables 3.5 and 3.6 and figures 3.2 and 3.3 provide a summary of the optimal preventive maintenance decisions and their associated average maintenance cost rates for various c_p values. This observation reveals that when the cost of preventive maintenance rises, there is a corresponding increase in the optimal frequency of preventive maintenance and the rate of maintenance cost. This result applies to hazard rates with concave and convex structure, as well as maintenance imperfectness rates of 10% and 30%.

Table 3.5: Effect of c_p changes on optimal maintenance strategy for 10% Maintenance Imperfectness

c_p	$(\beta = 1.6)$		$(\beta = 3.5)$	
	$C_X(T_0, p)$	T_0	$C_X(T_0, p)$	T_0
0.1	2.34	0.11	0.69	0.20

0.5	4.08	0.35	2.07	0.36
1.0 1.5	4.93 5.34	0.64 0.98	3.17 3.99	0.49 0.62
2.0	5.51	1.42	4.56	0.73
2.5 3.0	5.56 5.58	2.11 3.42	5.00 5.27	0.84 0.96

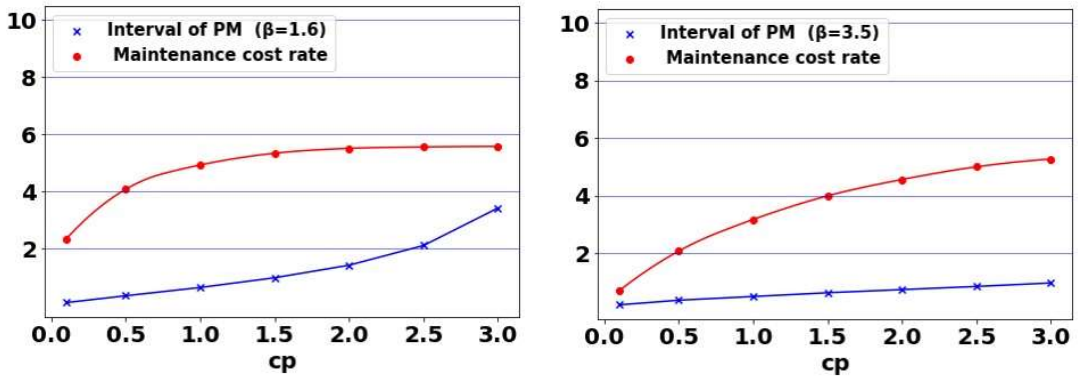


Figure 3.2: Variation in the optimal solution T_0 and $C_X(T_0, p)$ with the change of c_p with a 10% Maintenance Imperfectness

Table 3.6: Effect of $c_p$changes on optimal maintenance strategy for 30% Maintenance Imperfectness

c_p	$(\beta = 1.6)$		$(\beta = 3.5)$	
	$C_X(T_0, p)$	T_0	$C_X(T_0, p)$	T_0
0.1	2.7	0.10	0.99	0.15
0.5	4.54	0.36	2.8	0.29
1.0 1.5	5.27 5.51	0.77 1.34	4.05 4.76	0.47 0.71
2.0	5.57	2.25	5.17	0.86
2.5 3.0	5.58 5.93	4.42 6.40	5.44 5.59	1.035 1.38

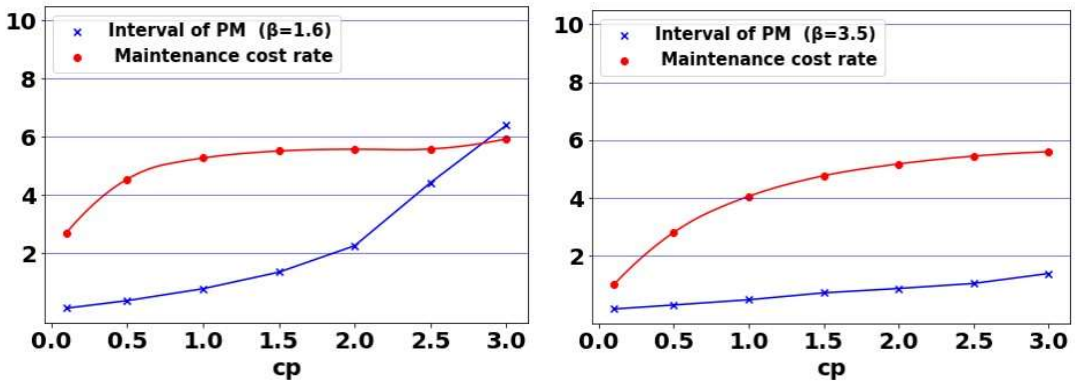


Figure 3.3: Variation in the optimal solution T_0 and $C_X(T_0, p)$ with the change of c_p with a 30% Maintenance Imperfectness

The results of the model are very similar to those found by [62], showing similar patterns in optimum maintenance interval and maintenance cost rate variations in response to changes in preventive maintenance imperfections. Moreover, Studies show a consistent correlation between preventive maintenance costs and optimal maintenance intervals. This alignment with the existing literature enhances the credibility of the approach adopted in this study.

3.6.2 Model Y: Numerical illustration and Results

In this part, numerical experiments are carried out to demonstrate imperfect PM models with minimal maintenance at failures for an infinite time frame. The objective is twofold: analyse how imperfect preventive maintenance executions affect optimal PM decisions and explore the effect of variation in maintenance cost on the optimal maintenance decision. The method of solution is shown in Table 3.2. By utilizing the algorithm outlined in Table 3.2 for the optimal maintenance strategy, the most effective maintenance plan, and the corresponding optimal maintenance cost rate, $C_Y(T_0, p)$ are calculated. The Parameter settings are as follows. Minimal Repair cost c_{mr}=3 and the

preventive maintenance cost $c_p = 1$. $\beta = 1.6$ for concave hazard rates, while for convex hazard rates, $\beta = 3.5$.

We first look at the imperfect preventive maintenance with minimal repair at failure for the infinite time horizon. Table 3.7 and figure 3.4 provides an overview of the optimal PM decisions and associated average maintenance cost rates for various values of imperfect preventive maintenance probability.

Table 3.7: Variation in the optimal solution T_0 and associated Cost rate with the change of Imperfect PM probability (Minimal repair)

p	$(\beta = 1.6)$		$(\beta = 3.5)$	
	$C_Y(T_0, p)$	T_0	$C_Y(T_0, p)$	T_0
0.00	3.85	0.69	2.46	0.56
0.01	3.88	0.68	2.55	0.55
0.05	3.98	0.67	2.81	0.50
0.10	4.11	0.65	3.13	0.45
0.15	4.23	0.63	3.44	0.40
0.20	4.40	0.60	3.58	0.37
0.25	4.57	0.58	4.14	0.34
0.30	4.75	0.56	4.55	0.31
0.35	4.94	0.54	4.96	0.28
0.40	5.15	0.52	5.09	0.25
0.50	5.63	0.47	6.42	0.21
0.60	6.26	0.43	8.09	0.17

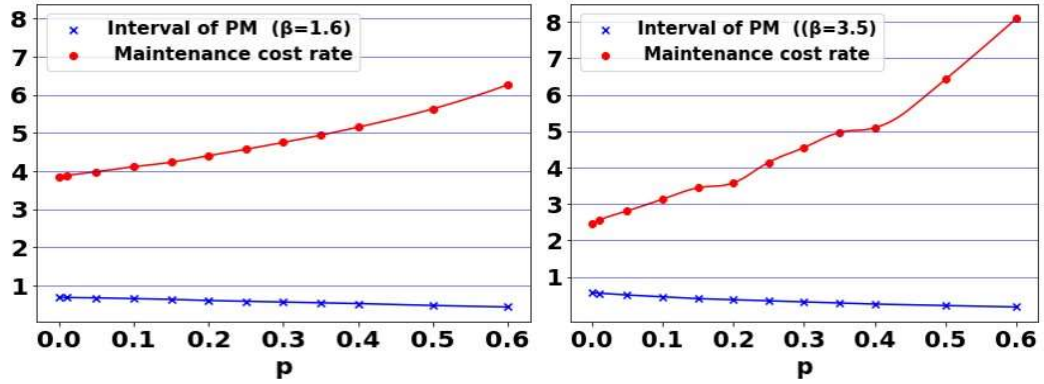


Figure 3.4: Variation in the optimal solution T_0 and $C_Y(T_0, p)$ with the change of p for Minimal repair

As the likelihood of imperfect maintenance rises, there is a noticeable decline in the optimal preventive maintenance time interval and a corresponding increase in the maintenance cost rate. This implies that, with a higher probability of imperfect preventive maintenance, it is advisable to carry out preventative maintenance at more frequent intervals. This observation holds true for both convex and concave hazard rates. The author's results are more authentic and aligned with real-world observations in contrast to the findings presented in prior research [15].

Next, the imperfect preventive maintenance policy, considering maintenance imperfection rates of 10% and 30% were analysed. Tables 3.8, 3.9 and figures 3.5, 3.6 summarize the optimal preventive maintenance decisions and their respective average maintenance cost rates for different cp values. This observation indicates that with an increase in the cost of preventive maintenance, there is a corresponding increase in both the optimal preventive maintenance frequency and maintenance cost rate. This pattern holds true for hazard rates exhibiting both concave and convex patterns, as well as maintenance imperfection rates of 10% and 30%. The findings in this study closely align with the outcomes reported by [62]. This alignment with the existing literature enhances the credibility of the approach adopted in this study.

Table 3.8: Optimal PM Decisions with the change of c_p Under 10% Maintenance Imperfectness rate (Minimal repair)

c_p	$(\beta = 1.6)$		$(\beta = 3.5)$	
	$C_Y(T_0, p)$	T_0	$C_Y(T_0, p)$	T_0
0.1	1.72	0.15	0.60	0.23
0.5	3.16	0.42	1.9	0.37
1.0	4.11	0.65	3.19	0.45
1.5	4.79	0.84	4.17	0.50
2.0	5.33	0.99	5.13	0.54
2.5	5.80	1.15	6.01	0.58
3.0	6.12	1.30	6.87	0.61

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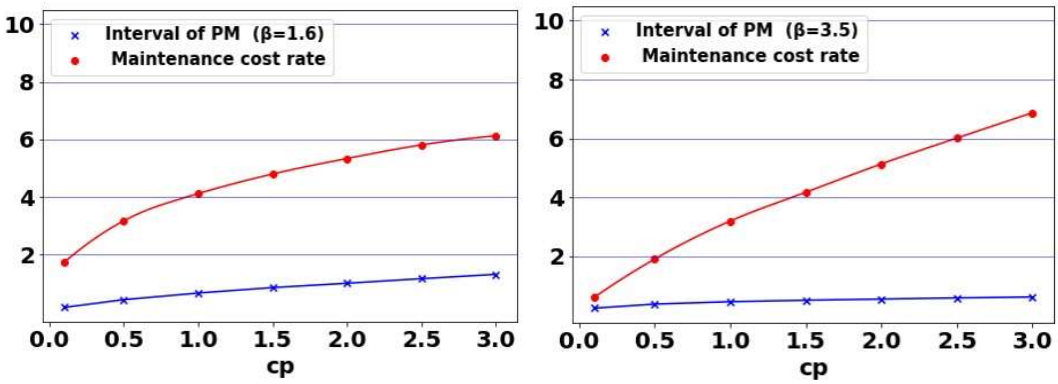


Figure 3.5: Variation in the optimal solution T_0 and $C_Y(T_0, p)$ with the change of c_p with a 10% Maintenance Imperfectness for Minimal repair

Table 3.9: Optimal PM decisions with the change of c_p Under 30% Maintenance Imperfectness rate (Minimal repair)

c_p	$(\beta = 1.6)$		$(\beta = 3.5)$	
	$C_Y(T_0, p)$	T_0	$C_Y(T_0, p)$	T_0
0.1	1.81	0.11	0.87	0.16
0.5	3.66	0.36	2.76	0.25
1.0	4.75	0.56	4.63	0.31
1.5	5.52	0.72	6.05	0.34
2.0	6.18	0.87	7.46	0.38
2.5	6.67	0.99	8.77	0.40
3.0	7.19	1.12	9.96	0.42

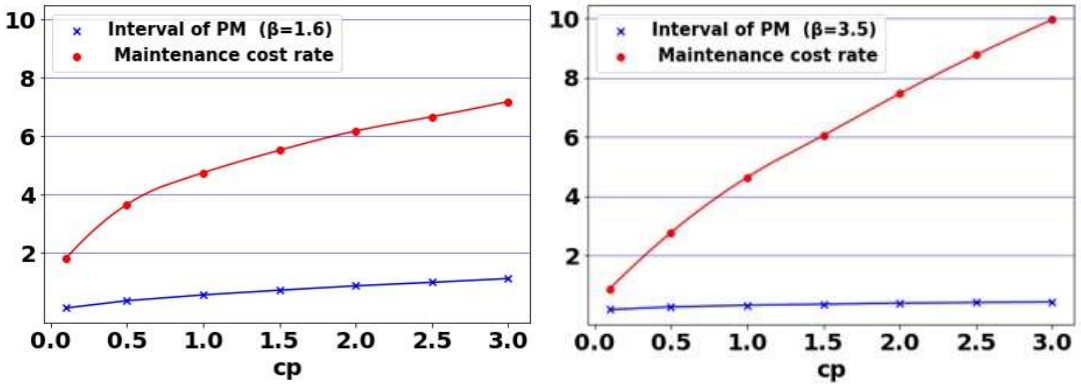


Figure 3.6: Variation in the optimal solutionT_0 and $C_Y(T_0, p)$ with the change of c_p with a 30% Maintenance Imperfectness for Minimal repair

Now according to the findings of [96], the costs associated with minimal repairs are 40 times greater than the costs of preventive maintenance, indicating a significant difference in expenses between minimal repairs and preventive maintenance. We,

therefore, once again set the minimal repair cost c_{mr} to 40 and the preventive maintenance cost c_p to 1 to validate the model. The rest of the parameter settings remain unchanged. The results are tabulated in tables 3.10 to 3.12 and figures 3.7 to 3.9. The trends of the results are almost the same as those of the originally assumed parameters, thereby validating our model.

Table 3.10: Variation in the optimal solution T_0 and associated Cost with the change of Imperfect PM probability (Minimal repair) c_{mr}=40 and $c_p = 1$

P	$(\beta = 1.6)$		$(\beta = 3.5)$	
	$C_Y(T_0, p)$	T_0	$C_Y(T_0, p)$	T_0
0.00	19.41	0.137	5.20	0.27
0.01	19.53	0.136	5.34	0.26
0.05	20.00	0.13	5.87	0.24
0.10	20.18	0.13	6.56	0.21
0.15	21.47	0.12	7.24	0.19
0.20	22.16	0.12	7.87	0.16
0.25	23.01	0.11	8.64	0.16
0.30	23.94	0.11	9.40	0.15
0.35	24.80	0.10	10.39	0.13
0.40	25.90	0.09	11.30	0.12
0.50	28.35	0.08	13.74	0.10
0.60	31.57	0.07	16.66	0.08

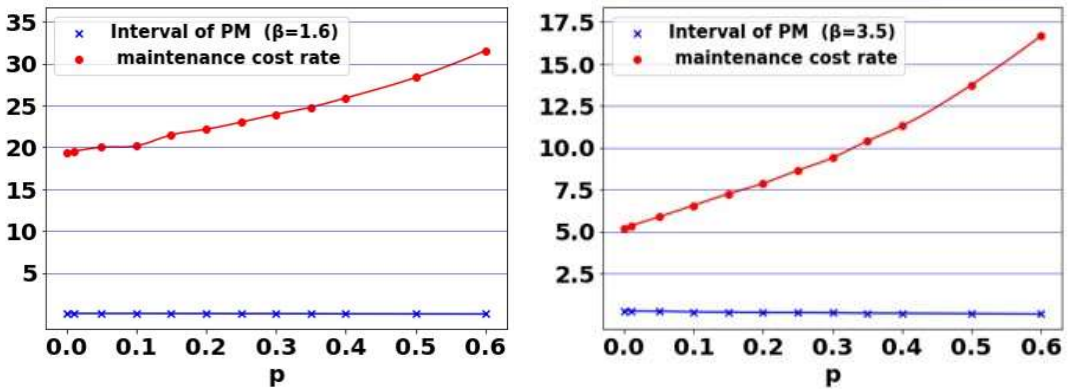


Figure 3.7: Variation in the optimal solutionT_0 and $C_Y(T_0, p)$ with the change of p for Minimal repair c_{mr}=40 and $c_p = 1$.

Table 3.11: Optimal PM decisions with the change of c_p Under 10% Maintenance Imperfectness rate (Minimal repair) c_{mr}=40 and $c_p = 1$.

c_p	$(\beta = 1.6)$		$(\beta = 3.5)$	
	$C_Y(T_0, p)$	T_0	$C_Y(T_0, p)$	T_0
0.1	8.67	0.03	1.25	0.11
0.5	15.63	0.08	3.95	0.17
1.0	20.23	0.12	6.56	0.21
1.5	23.68	0.16	8.74	0.24
2.0	26.91	0.20	10.69	0.26
2.5	29.44	0.23	12.65	0.28
3.0	31.33	0.26	14.42	0.30

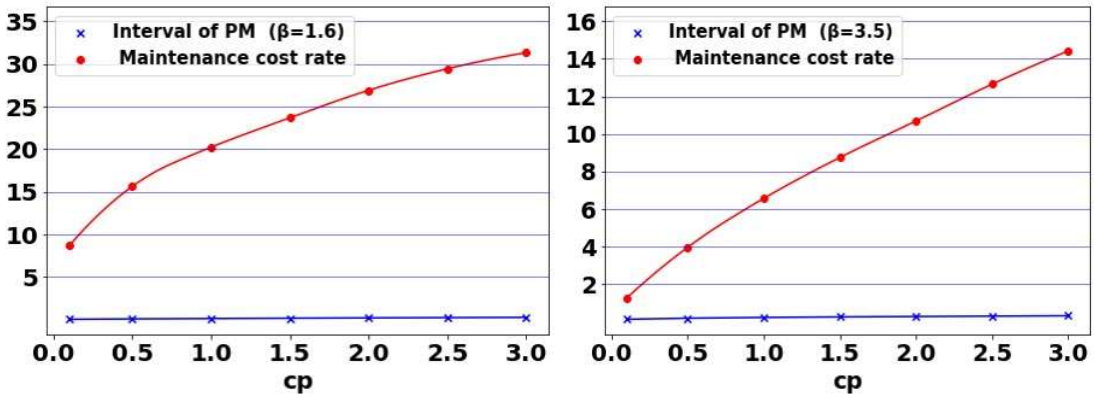


Figure 3.8: Variation in the optimal solution T_0 and $C_Y(T_0, p)$ with the change of c_p with a 10% Maintenance Imperfectness for Minimal repair c_{mr}=40 and $c_p = 1$

Table 3.12: Optimal PM decisions with the change of c_p Under 30% Maintenance Imperfectness rate (Minimal repair) c_{mr}=40 and $c_p = 1$

c_p	$(\beta = 1.6)$		$(\beta = 3.5)$	
	$C_Y(T_0, p)$	T_0	$C_Y(T_0, p)$	T_0
0.1	10.02	0.03	1.84	0.08
0.5	18.15	0.07	5.76	0.12
1.0	23.96	0.11	9.25	0.15
1.5	27.87	0.14	12.58	0.16
2.0	31.13	0.17	15.66	0.18
2.5	33.77	0.20	18.17	0.19
3.0	36.19	0.22	20.66	0.20

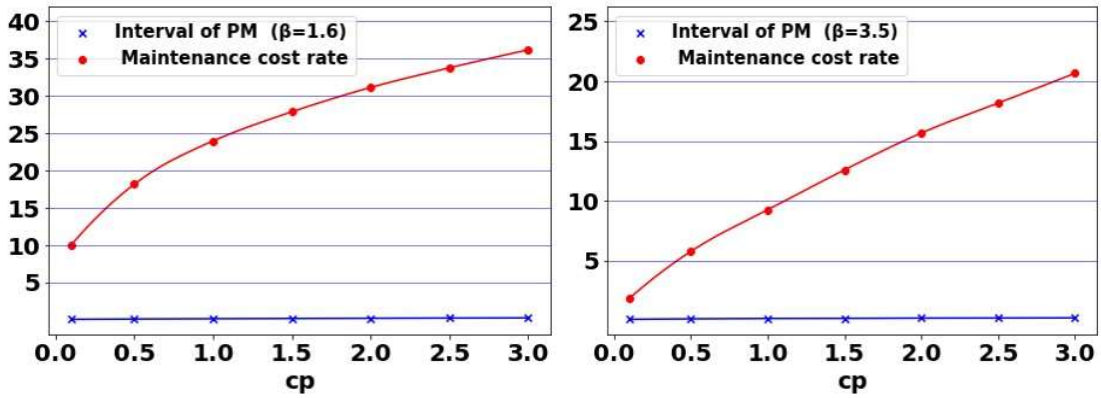


Figure 3.9: Variation in the optimal solutionT_0 and $C_Y(T_0, p)$ with the change of c_p with a 30% Maintenance Imperfectness for Minimal repair c_{mr}=40 and $c_p = 1$

3.6.3 Model Z: Numerical Results and Implications

The solution methodology is presented in Table 3.3. By applying the algorithm specified in Table 3.3, for the optimum maintenance strategy, the most effective maintenance plan, and the corresponding optimum maintenance cost rate, $C_Z(T_0, p)$ can be achieved.

The specified parameter settings are as follows:

The cost of time passes between failure occurrence and its detection $c_{re} = 5$, preventive maintenance cost $c_p = 1$. For concave hazard rates, β=1.6 is considered, while for convex hazard rates, β=3.5 is employed.

Table 3.13 and figure 3.10 present a summary of the optimum PM decisions and corresponding average maintenance cost rates for different values of imperfect preventive maintenance probability.

Table 3.13: Variation in the optimal solution T_0 and associated Cost with the change of Imperfect PM probability (only perfect PM can detect the defective unit)

p	$(\beta = 1.6)$		$(\beta = 3.5)$	
	$C_Z(T_0, p)$	T_0	$C_Z(T_0, p)$	T_0
0.00	2.12	0.84	1.62	0.83
0.01	2.13	0.83	1.62	0.82
0.05	2.23	0.83	1.75	0.80
0.10	2.34	0.81	1.91	0.79
0.15	2.45	0.80	2.10	0.77
0.20	2.56	0.79	2.21	0.75
0.25	2.67	0.78	2.36	0.73
0.30	2.82	0.78	2.51	0.71
0.35	2.95	0.77	2.66	0.69
0.40	3.09	0.77	2.84	0.69
0.50	3.38	0.81	3.21	0.70
0.60	3.69	0.94	3.59	0.80

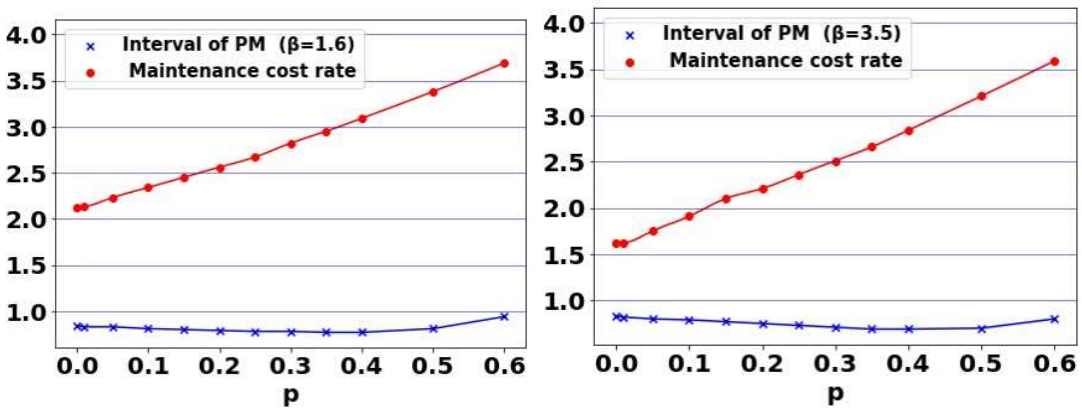


Figure 3.10: Variation in the optimal solution T_0 and $C_Z(T_0, p)$ with the change of p when only a perfect PM can detect a defective unit

The information derived from the Table 3.13 and Figure 3.10 yields the following observations:

- In both concave and convex hazard rate conditions, as the probability of imperfection increases from 0 to 40%, there is a corresponding decrease in the

maintenance time interval. Specifically, it drops from 0.84 to 0.77 for concave hazard rate and from 0.83 to 0.69 for convex hazard rate. This highlights the importance of more frequent implementation of preventive maintenance under lower to moderate levels of imperfection to ensure system reliability.

- Once the imperfection level goes beyond 40% in both hazard rate cases, there is a notable increase in the maintenance time interval. As a result, opting for corrective maintenance becomes a more practical approach than maintaining a frequent preventive maintenance schedule.
- The maintenance cost rate increases in both cases as imperfections in maintenance rise.

These findings offer helpful guidance for making smart maintenance decisions. Maintenance professionals can use this information to figure out the best way to handle maintenance tasks based on the current state of imperfections.

Tables 3.14 and 3.15 and figures 3.11 and 3.12 provide a summary of optimal preventive maintenance decisions and corresponding average maintenance cost rates for various cp values. The observation indicates a consistent trend, with a simultaneous increase in both the optimal preventive maintenance frequency and maintenance cost rate as the cost of preventive maintenance rises. This trend remains same for both hazard rates and maintenance imperfection rates of 10% and 30%. The results closely match with the observations presented by [62], and strengthening the validity of the results.

Table 3.14: Optimal PM Decisions with the change of c_p Under 10% Maintenance Imperfectness rate (Only perfect PM can detect the defective unit)

c_p	$(\beta = 1.6)$		$(\beta = 3.5)$	
	$C_Z(T_0, p)$	T_0	$C_Z(T_0, p)$	T_0
0.1	0.64	0.27	0.96	0.37
0.5	1.16	0.56	1.74	0.60
1.0	2.34	0.81	2.29	0.79
1.5	2.88	1.05	2.60	0.92
2.0	3.30	1.31	2.86	1.04
2.5	3.64	1.64	3.09	1.17
3.0	3.92	2.28	3.37	1.37

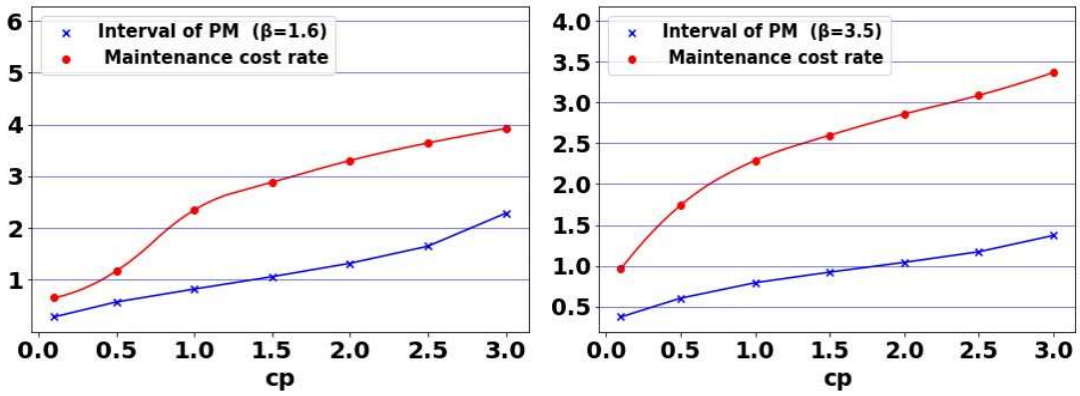


Figure 3.11: Variation in the optimal solution T_0 , $C_Z(T_0, p)$ with change of c_p with a 10%Maintenance Imperfectness when only perfect PM can detect the defective unit

Table 3.15:. Optimal PM decisions with the change of c_p under 30% Maintenance Imperfectness rate (Only perfect PM can detect the defective unit)

c_p	$(\beta = 1.6)$		$(\beta = 3.5)$	
	$C_Z(T_0, p)$	T_0	$C_Z(T_0, p)$	T_0
0.1	0.81	0.21	0.52	0.25
0.5	1.99	0.48	1.63	0.47
1.0	2.82	0.77	2.51	0.71
1.5	3.35	1.10	3.12	0.94
2.0	3.74	1.56	3.93	1.41
2.5	3.99	3.22	3.99	1.59
3.0	4.03	4.3	4.21	2.10

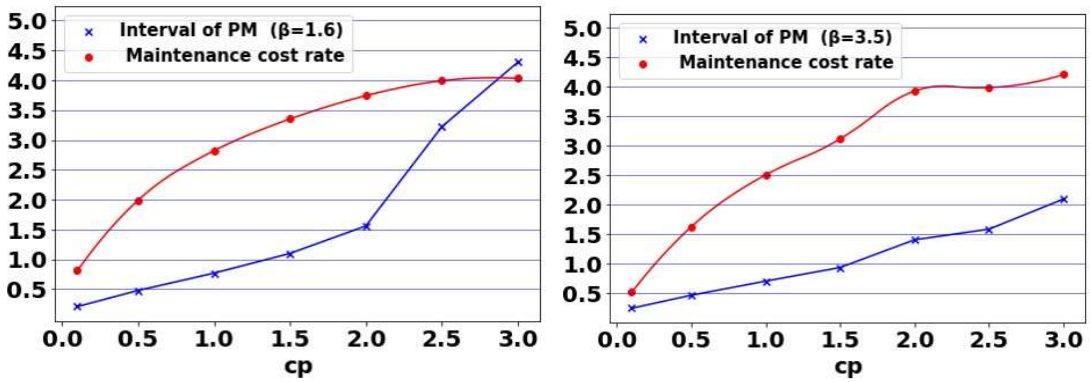


Figure 3.12: Variation in the optimal solution T_0 , $C_Z(T_0, p)$ with change of c_p with a 30% Maintenance Imperfectness when only perfect PM can detect the defective unit

3.7 Summary

This chapter explored a cost-based modelling framework that addresses the challenges posed by imperfect maintenance quality in industrial systems. Recognizing that maintenance actions rarely restore equipment to an ideal state, the chapter presented models that incorporate varying levels of maintenance effectiveness into preventive and corrective strategies. By treating maintenance quality as a probabilistic variable, the analysis enabled realistic estimation of maintenance cost under uncertainty.

Three imperfect maintenance scenarios were modelled, and the optimum preventive maintenance intervals and associated cost rates were derived. The models accounted for the impact of imperfect interventions on system reliability and downtime, enabling cost optimization that balances maintenance frequency and long-term expenses. Numerical analysis demonstrated how declining maintenance quality increases maintenance cost and necessitates shorter maintenance intervals.

The findings from this chapter provide critical inputs for maintenance managers and decision-makers, supporting the development of economically efficient maintenance

policies under real-world constraints of imperfect maintenance execution. This, in turn, enhances the economic sustainability of asset management practices under uncertainty.

This chapter evaluated the life cycle maintenance cost-based approach in deciding the optimal maintenance strategy. Consequently, a frame work needs to be figured out to evaluate maintainability that incorporates time dependent and time independent variables that influence maintainability of an asset system. The next chapter discusses this.

CHAPTER 4

INVESTIGATION OF THE EFFECT OF TIME-DEPENDENT COVARIATES ON MAINTAINABILITY ANALYSIS

4.1 Introduction

The following subsection discusses the historical and technological progression of maintainability analysis.

4.1.1 Historical and Technological Development of Maintainability Analysis

Maintainability has long been recognized as a key component of system performance, mainly in industries where system availability is critical. Traditionally, maintainability assessments primarily focused on "time to repair" as the sole indicator of performance. Early models, such as the Proportional Repair Model (PRM), were rooted in the framework of the Proportional Hazards Model (PHM) and assumed that the explanatory variables (covariates) are time-independent. Over the years, this assumption has proven inadequate in dynamic industrial environments. Despite the development of a wide range of methods, from basic statistical techniques to advanced computational models, many of these approaches still fail to account for fluctuating operating and environmental conditions, which are inherently time-dependent. This gap necessitates a shift toward models capable of capturing the influence of both static (time-independent) and dynamic (time-dependent) covariates on maintainability. For example, factors such as spare parts availability, equipment aging, weather conditions, and maintenance personnel skill levels can all change over time, significantly affecting maintenance durations.

4.1.2 Role of Covariates in Maintainability Modelling

One of the major limitations of conventional maintainability models is their tendency to ignore "maintenance conditions" such as environmental, operational, and human factors that significantly affect maintenance performance. These conditions are often embedded as covariates, which, if not accurately accounted for, can lead to biased and imprecise estimations of maintainability. For instance, seasonal variations like extreme cold can prolong repair times, while extreme summer conditions may cause workmen fatigue. Similarly, logistical issues due to geographic remoteness, unavailability of spare parts, or workforce limitations introduces variability in time to repair. In addition, the ageing of equipment also plays a critical role in determining maintainability performance, as older assets are generally more susceptible to failures and require longer and more complex repair interventions. Existing historical data sets such as Time to Repair (TTR) often represent maintenance events under diverse, non-uniform conditions. Without acknowledging this heterogeneity using covariates, any predictive modelling becomes less reliable. Therefore, there is a strong need to build and apply maintainability assessment models that can realistically incorporate both time-independent and time-dependent influencing factors. This chapter proposes a refined statistical framework based on Cox regression and its extended version to accurately analyse maintainability in complex industrial environments.

4.1.3 Importance of Maintainability

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Maintainability plays a vital role in ensuring the availability, maintaining reliability, and overall performance of industrial systems. In sectors such as manufacturing, power generation, and transportation, long downtimes can lead to significant economic losses

and safety concerns. Maintenance not only restores the system to its operational state but also directly influences long-term performance by reducing downtime, improving efficiency, and extending asset life. The British Standard BS 4778 defines maintainability as "the probability of successfully repairing or restoring a failed component or system to a defined state within a given timeframe, through maintenance conducted according to prescribed procedures." However, this definition implicitly assumes standard maintenance conditions. In reality, maintainability is subject to many influencing factors including environmental variables, infrastructure, human capabilities, and equipment accessibility etc., that fluctuate over time. Incorporating these real-world complexities into the maintainability framework is essential to produce more realistic results. This chapter addresses the need to modify traditional models by recognizing and integrating these variables, ultimately aiming to enhance the predictive accuracy and operational relevance of maintainability assessments.

4.2 Maintainability performance in Engineering Design

Maintainability involves the strategies implemented throughout the development, design, and installation phases of a manufactured product to minimize the necessary maintenance such as., manhours, tool usage, logistical expenses, skill requirements, and facility demands, etc. The objective is to ensure that the product fulfils the requirements of its designated purpose [111]. Accurate prediction of maintainability during the design and development stages is crucial as improvements often necessitate significant changes in the equipment/ system's layout [97].

Maintainability analysis involves modelling the probability of a successful repair of the equipment within a specified time frame, by using prescribed procedures and available

resources. Time to Repair (TTR) is a stochastic variable. The factors contributing to the overall TTR include: time to recognize that a failure has occurred, time to pinpoint the location of the failure, time for diagnosis, time for corrective maintenance, and time spent in verifying that the repair was successful [24],[98]. To enhance the accuracy of maintainability characteristics estimation, the use of regression models is recommended due to their ability to incorporate covariates. Quantitative models and those predominantly belonging to the proportional hazard's family are the two main categories of regression models in maintainability modelling.

4.3 Quantitative approaches in maintainability Modelling

Quantitative methods in maintainability analysis encompasses the use of mathematical models and statistical techniques to assess and predict the maintainability of a system. The methods assume that the data follows a particular probability distribution. However, a major challenge in quantitative techniques is that if the assumptions are violated, the results may be grossly inaccurate or misleading. Common distributions in quantitative approach include the normal (Gaussian), lognormal, Weibull, exponential, Gamma, and others. Each probability distribution is characterized by parameters. The goal of quantitative methods is to estimate these parameters from the time to repair data. These Parameters can be determined through either least-squares fit or maximum likelihood estimation and goodness of fit analysis, such as the Kolmogorov-Smirnov test [99], Anderson-Darling test [100], and Pearson Chi-squared tests [101], which can serve as a tool for determining the optimal probability distribution. To quantify the duration of the repair process, let T be a continuous random variable representing the time taken to repair a failed unit, with a probability density function denoted by $h(t)$. Then, the cumulative distribution function can be expressed as in equation (4.1) [18]

$$\Pr\{T \leq t\} = H(t) = \int_0^t h(t')dt' \quad (4.1)$$

equation (4.1) represents the probability of completing a repair within a specified time frame, "t" . The mean time to repair (MTTR), which is a measure of the performance of maintainability is calculated by equation (4.2) and repair rate may be computed by equation (4.3)

$$MTTR = \int_0^\infty th(t)dt = \int_0^\infty \big(1 - H(t)\big)dt \quad (4.2)$$

$$\lambda(t) = -\frac{h(t)}{1-H(t)} \quad (4.3)$$

In quantitative maintainability modelling, it is established that identical maintenance intervention are tested under identical environmental settings, which simplifies the analysis and facilitates the application of statistical methods. However, this assumption may not always hold true when dealing with real word operational data. For example, the maintenance circumstances (variability in environmental condition, skill and experience of maintenance personnel) or equipment age and condition can vary over time. This indicates that repair rate exhibits stochastic nature, influenced by the specific conditions of the environment. Therefore, including covariates in the modelling enhances the effectiveness of the statistical analysis and eliminates influence of variables. The characteristics of maintainability data suggests that non-parametric methods are well-suited for exploring as to how covariates influence maintainability performance.

4.4 Proportional repair model (PRM)

The repair rate in PRM is determined by multiplying a baseline repair rate, $\lambda_0(t)$, with a function $\varphi(z;\beta)$ that characterizes the variation in the repair rate based on

influencing covariates.

$$\lambda(t;z) = \lambda_0(t)\varphi(z;\beta) \tag{4.4}$$

$\lambda_0(t)$ is time-dependent only, and a positive functional term $\varphi(z;\beta)$, that is time - invariant, considering the influence of various covariates.

In the given model, the row vector Z represents covariates associated with the system, while β is a column vector comprising the unknown regression parameters. β serves as a parameter in the model, describing the impact of covariates on the system. The baseline repair rate refers to the rate at which repairs are conducted under ideal environment, unaffected by any covariates influencing the pattern of failures or repair rate. Under these circumstances, the baseline hazard rate is considered consistent and equivalent to the overall hazard rate. Explanatory variables have the potential to influence the hazard rate, resulting in deviations from the baseline hazard rate. Factors like suboptimal maintenance or the introduction of new and improved components can cause the hazard rate to deviate from the baseline, either increasing or decreasing. The baseline repair rate pattern, along with the regression coefficients associated with explanatory variables can be determined through two main approaches: by deriving information from reported data in literature or by input provided by subject experts.

Various function form of $\varphi(z;\beta)$ can be used in repair rate equation to achieve specific objectives including the log linear form, $\varphi(z;\beta) = e^{\beta^t z}$; the linear form, $\varphi(z;\beta) = 1 + \beta^t z$ and the log form, $\varphi(z;\beta) = \log(1 + \beta^t z)$. The repair rate, when modelled in exponential form, it is defined in equation (4.5)

$$\lambda(t;z) = \lambda_0(t)\exp(z\beta) = \lambda_0(t)\exp\left(\sum_{j=1}^{n} \beta_j z_j\right) \tag{4.5}$$

The following equation represents the maintainability function.

$$H(t;z) = 1 - \exp\left[-\int_0^t \lambda(t;z)dt\right] = 1 - (1 - H_0(t))^{\exp\left(\sum_{j=1}^{n} \alpha_j z_j\right)} \quad (4.6)$$

The expression for the baseline maintainability function, $H_0(t)$, is given in equation (4.7)

$$H_0(t) = 1 - \exp\left[-\int_0^t \lambda(t)dt\right] \quad (4.7)$$

The PRM is generally considered a non-parametric regression technique. However, in reality, it is not a purely non-parametric regression method, as it incorporates parametric modelling to establish the relationship between rate of failure and designated covariate [102]. Two different methodologies can be employed to model the baseline repair rate: either by considering parametric distribution [103],[104] (e.g., failure rate follows a Weibull or lognormal distribution) or without making assumptions about the underlying distribution. In fact, when there is no strong theoretical justification for assuming a specific mathematical form for the repair rate (such as a Weibull or Exponential distribution), parametric models are replaced by the Proportional Repair Model (PRM) regression.

The Proportional Repair Model (PRM) operates under the following assumptions

- It models covariates as time-independent.
- It conforms to the proportionality requirement.

$$\lambda_i(t) = e^{\left(\beta_1(z_{i1}-z_{j1})+\cdots+\beta_k(z_{kk}-z_{jk})\right)} * \lambda_j(t) \quad (4.8)$$

Equation (4.8) represents the proportionality assumption of the Proportional Repair Model (PRM). It indicates that repair ratio between operational conditions, i and j, with differing z-values that are constant over time.

In the analysis of TTR data sets using the PRM, the examination of the proportionality assumption plays a crucial role. This assumption can be verified through various graphical and numerical techniques. The graphical approach is extensively used [105]. In graphical methods, TTR data is Partitioned into arbitrary time intervals and/or stratified in covariate levels.

- In the former case, if the shapes of the repair rate curves remain consistent across these intervals, it supports the proportionality assumption of the PRM.
- In the latter case, if parallel trends are observed in graphical plots, the PRM assumption is valid.

However, if significant deviations occur, a time-dependent covariate model may be required instead. Overall, it is necessary that if proportionality holds, the shape of the baseline repair functions remains unchanged across different time intervals else stratified. Log-Minus-Log (LML) survival plots and log cumulative repair plots versus time serve as tools for checking proportionality condition. The assumption will be valid when plots corresponding to various strata display parallel trend. It is important to verify that each stratum must contain enough repair events to ensure reliable graphical trends. If a stratum has few events, the results will be statistically insignificant.

4.4.1 Computation of Regression coefficient (β)

The baseline repair rate is not constrained to a specific functional form when computing the regression coefficient, and is determined by maximizing various likelihood

functions, such as marginal likelihood, partial likelihood, or maximum likelihood. The marginal likelihood approach is used, and the distribution is derived from the rank statistics of failure times while, partial likelihood method is derived from the conditional probability of failure. The regression vector β is estimated by maximizing the likelihood function using the maximum likelihood approach. The highest probability of observing the actual failure times is ensured in this approach. The estimates from partial and marginal likelihood methods produce identical results. When time-dependent covariates are included in the modelling process, the partial likelihood method proves to be effective. Thus, the marginal or partial likelihood approach is typically favored for estimating the regression coefficient [105].

4.4.2 Computation of Maintainability and base-line cumulative repair rate (CRR)

Following the evaluation of regression vector and validation of the proportionality assumption of repair rates under different covariates, the maintainability and cumulative repair rate functions need to be computed. Three methods for estimating these functions are described in literature [82].

These are:

Breslow estimator - This approach, introduced by Breslow, is simple to compute but yields a good approximation only when the proportion of tied failure data remains relatively low in comparison to the total failure count.

Kalbfleisch and Prentice estimator- When the failure data contains a significant number of tied observations, the Kalbfleisch and Prentice estimator is utilized for analysis.

Piecewise exponential estimator- For a dataset with censored observations, the piecewise exponential estimator is used, offering a continuous repair rate estimation, unlike the other two methods.

4.4.3 Time dependent Covariates

A covariate in repair models represents factors such as degradation, wear, or other types of damage that a component experiences while it is in operation. A covariate that varies over time is referred to as a time-dependent. In this case the model ceases to be classified as a Proportional repair model (PRM), because the repair rate ratio will no longer remain proportional over time. In such cases, a non-proportional repair Model is more appropriate, as it allows repair rates to vary with time. This is described in the next section.

4.5 Non-Proportional repair rate models

Covariates may continuously change over time in a stochastic manner (such as the increasing difficulty of disassembling an internal combustion engine with age). In the non-proportional repair rate model, the ratio of repair rate for instances i and j can be computed in equation (4.9)

$$\lambda_i(t) = e^{\left(\beta_1(z_{i1}(t)-z_{j1}(t))+\cdots+\beta_k(z_{kk}(t)-z_{jk}(t))\right)} * \lambda_j(t) \qquad (4.9)$$

The assumption of non-proportional repair is typically addressed using two methods: the Stratified Cox regression method, and the extended Cox regression model, which incorporates time-dependent covariates to handle deviations from proportionality. Although these methods are widely applied in reliability engineering and the medical field [106-108], they can also be utilized in the maintainability domain. However, it is

imperative to note that the human factor in maintenance activities may add complexity to these models.

4.5.1 Maintainability analysis based on the extended Cox- regression model

The extended Cox regression model expands upon the Proportional Hazards Model (PHM) to facilitate the simultaneous analysis of time-dependent and time-independent covariates, specifically in the context of reliability assessment. Given this framework, the Proportional Repair Model (PRM) can be implemented when time-dependent covariates are present, while accounting for the assumption of non-proportional repair rates. equation (4.10) represents the mathematical formulation of the Extended Cox regression model [107]

$$\lambda(t \mid \mathbf{z}(t)) = \lambda_0(t)\exp\left(\beta^T \mathbf{z} + \delta^T \mathbf{z}(t)\right) \tag{4.10}$$

Where $\mathbf{z}(t)$ represents the time-dependent covariates. Presence of time-dependent predictors results in the violation of the proportional repair assumption. Repair ratio explicitly defined as a time-dependent function (equation 4.11)

$$\begin{aligned} \lambda_i(t) &= \frac{\lambda(t|z_i(t))}{\lambda(t|z_j(t))}\lambda_j(t) = \frac{\lambda_0(t)\exp\left(\beta^T z_i(t)\right)}{\lambda_0(t)\exp\left(\beta^T z_j(t)\right)}\lambda_j(t) \\ &= \exp\left[\left(\beta^T\left(z_i{}^* - z_j\right)\right) + \left(\delta^T\left(z_i{}^*(t) - z_j(t)\right)\right)\right] \end{aligned} \tag{4.11}$$

4.5.2 Maintainability analysis based on the Stratified Cox regression model

In the stratified Cox regression method, the model is divided into strata based on a covariate that does not satisfy the proportional repair assumption. This technique can also be systematically applied to model time-dependent predictors within the framework of maintainability analysis. Under this methodology, time-dependent covariates are categorized into different levels, determined either through expert

knowledge or statistical methods. Each level of the covariate is treated as an independent stratum. The regression coefficients remain constant across all strata. Each stratum is characterized by a unique baseline repair rate. For instance, when maintenance activities are carried out under different levels of spare part availability, the repair completion time will vary significantly. When spare parts are readily available, maintenance tasks can be completed quickly without delays. However, if spare parts are out of stock or need to be ordered, the repair process may be prolonged due to waiting times for procurement and delivery. If spare part availability is treated as a covariate, the acquired data can be systematically stratified according to this parameter. For instance, two levels of spare part availability can be defined: one for scenarios where spare parts are immediately available and another for scenarios where spare parts are delayed or unavailable.

In the stratification approach, several key assumptions are typically made:

1. The covariate used for stratification divided into ‘k’ distinct levels.
2. Items that belong to the same stratum demonstrate repair functions that maintain a proportional relationship.
3. The effect of regressor variables remains uniform across all defined strata.

Consider a scenario where multiple Time-to-Repair (TTR) values are classified into ‘p’ distinct strata. These strata are defined based on the levels of different time-dependent covariates. For j-th stratum, the corresponding repair function is expressed in equation (4.12)

$$\lambda_j(t \mid z) = \lambda_{0j}(t)e^{z\beta} \qquad (4.12)$$

Similar to the standard PRM (Proportional Repair Model), the model contains two unknown elements: Regression coefficient β and Baseline repair function λ_{0j} corresponding to every stratum. Baseline repair functions for the p strata are considered independent and adaptable, with no assumed interdependence among them.

$$L(\beta) = \prod_{j=1}^{p} L_j(\beta) \tag{4.13}$$

Baseline repair function is calculated individually for every stratum, after determining the regression coefficients β. A key benefit of this method is its ability to perform analysis without imposing any prior assumptions about how the stratifying predictors interact with time. A significant drawback of the stratification method is that it limits the ability to analyses or evaluate the impact of the stratifying predictors on maintainability. This occurs because, in stratification, the covariates are used to divide the data into distinct groups or strata, and the analysis is conducted separately within each stratum. As a result, the covariates themselves are no longer treated as variables whose effects can be directly measured.

4.6 Case Study: Maintainability analysis in a Complex Modern Industrial setting

Pumps are extensively used in many industrial applications, and they represent one-fifth of the utilization of industrial energy. Pumps consist of various expensive capital parts affected by specific processes of degradation. A proactive maintenance strategy is essential to enhance the reliability and availability of centrifugal pumps while reducing production outages [109][110]. Therefore, in this paper, we have chosen the industrial pump for analysing the maintainability performance index, considering covariates.

In this study, several assumptions have been made: the system is repairable, and it undergoes repair and maintenance throughout its life cycle. At the initial stage, all covariates that could potentially affect the industrial pump maintenance are identified. Subsequently, it is crucial to determine the key covariates affecting the maintainability of the industrial pump and assess their degree of influence on the performance of maintainability, The following covariates have been considered for the possible case study on pumps.

(a) Spare parts availability(z_1): Spare part requirements are influenced by component failures, which occur randomly due to factors such as wear, operational stress, and unforeseen defects. As these failures are unpredictable, the demand for spare parts is also stochastic in nature. Delays in part procurement can extend repair times. This covariate is assigned a value of 0 if parts are readily available and 1 if there is a delay in procurement during the maintenance activity [107].

(b) Ageing of components(z_2): Older equipment tends to have more wear and tear, (degradation) making repairs more complex and time-consuming, whereas newer equipment generally requires less time for maintenance.

(c) Maintenance crew workload(z_3): When the stochastic maintenance work, is optimally distributed among maintenance crew members, more effective maintenance actions are carried out, leading to improved overall maintainability of the system. A higher workload can slow down maintenance activities due to resource constraints and fatigue. This covariate is assigned a value proportional to the crew's workload, where lower values indicate a lighter workload and higher values represent increased workload and possible delays [111].

(d) Environmental conditions(z_4): The environmental conditions, such as temperature and humidity, can fluctuate between winter and summer, potentially influencing the effectiveness of maintenance activities. To account for this variation, the average environmental conditions recorded at the time of maintenance are included as a covariate in the study [82].

Field data can be collected and can be categorized according to the relevant covariates.

The results of the case study can be interpreted by: the parametric method, using proportionality assumption, and the stratification approach for non-proportionality repair assumption.

The steps are enumerated in the following section

4.6.1 Parametric Regression model

Before performing parametric analysis, the collected data must go through sorting (Arranging the data in a meaningful order, such as chronological order or grouping similar failure modes together) and classification (Categorizing the data based on different covariates, such as environmental conditions, maintenance crew work load, or ageing of component etc.). Once the data has been processed, the next step is to verify whether the Time-to-Repair (TTR) data adheres to the condition of being independent and identically distributed (iid). Validation of the iid assumption ensures that the statistical analysis is robust and the conclusions drawn about system maintainability are reliable. The independent and identically distributed (iid) assumption is validated in this study through Trend Test and Serial Correlation Test. These tests may be conducted using either graphical or analytical methods.

Trend Test along with Serial Correlation Test are performed on the Time to Repair (TTR) data to analyse patterns and dependencies within the dataset. If results confirmed that the iid assumption holds true then the best-fit probability distribution for Time to Repair (TTR) data should be determined. To better capture the influence of covariates in the analysis, non-parametric method is essential.

4.6.2 Analysis through Proportional repair model

The Proportional Repair Model (PRM) provides a robust framework for analysing, as to how various covariates influence the maintainability of centrifugal pumps. In the initial stage, the regression coefficient are computed. The regression coefficients indicate the rate at which the system's repair effectiveness improves because of changing covariates. To test whether each coefficient significantly influences the dependent variable e.g., maintainability of the centrifugal pump, the Wald statistic and its corresponding p-value is used to assess this significance. In this study, a 5% significance level (p-value ≤ 0.05) is used. If the p-value ≤ 0.05, it suggests that the corresponding covariate significantly affects the dependent variable (the maintainability of the centrifugal pump). If the p-value > 0.05, it suggests that the covariate does not significantly impact the maintainability and can be excluded from the model. Exponential of regression coefficient $Exp(\beta)$ represents the repair ratio. In the case of categorical covariates, $Exp(\beta)$ tells us how the risk of the terminal event (e.g., repair) changes when the covariate shifts from one category to another. For continuous covariates, $Exp(\beta)$ quantifies how the repair rate changes when the covariate increases by one unit. If $Exp(\beta) < 1.0$, it indicates that the covariate negatively impacts the repair rate, meaning the system is less likely to require repairs

as the covariate increases. This suggests that the covariate contributes to improving system reliability by reducing the frequency of repairs. When the value of $Exp(\beta)$ is equal to one and within the range of the confidence interval of a covariate, it implies that the effect of that covariate on the repair rate is statistically insignificant.

If the covariates; maintenance crew workload (z_3), ageing of component (z_2), and spare part availability (z_1) have a notable impact on the maintainability, the repair rate of the centrifugal pump may be modelled as a function of these covariates represented by equation (4.14)

$$\lambda(t,z) = \lambda_0(t)\exp(\beta_1 z_2 + \beta_2 z_3 + \beta_3 z_1) \quad (4.14)$$

In this equation If β_1, is greater than zero, it means that as components age, the repair rate increases similarly if β_2 is <0, an excessive workload reduces efficiency, leading to slower repairs. Finally, when β_3 is < 0, it suggests that Spare Part Deficiency reduces the repair rate.

To ensure unbiased results in the Proportional Repair Model (PRM) analysis, it is essential to validate the assumption of proportionality. In maintainability analysis, it is crucial to determine whether a covariate's effect changes over time. The time-sensitive behaviour of a covariates are examined by plotting Log Minus Log (LML) function or the repair function against time. In this study, strata (subgroups) are defined based on literature.

If covariate, Spare Part Availability (z_1) graph shows an intersection between the two Log Minus Log (LML) curves corresponding to Stratum 1 (High Availability) and Stratum 2 (Low Availability/delay) and if the LML curves and cumulative repair rate

curves for Stratum 1 and Stratum 2 are not parallel, it indicates that the effect of Spare Part Availability changes over time. As a result, Spare Part availability should be treated as a time-dependent covariate. Similar methodology is applied to other covariates to determine whether they are time-independent or time-dependent.

4.6.3 Stratification model

Maintainability analysis using stratification begins with categorizing Spare Part availability into two stratums. Here, spare part availability is a time-dependent covariate.

- Stratum 1: Adequate Spare Part Availability
- Stratum 2: Spare Part Deficiency

Based on expert experience, Spare Part Availability level (balanced) is chosen as the breakpoint. The data are analyzed after the stratification process. If two covariates—Ageing of components(z_2) and Maintenance crew workload(z_3)—have a strong statistical significance, While, the covariate Environmental condition(z_4) is "marginally significant," then the centrifugal pump repair rate can be computed using the equation (4.15)

$$\lambda(t,z) = \lambda_0(t)\exp(\beta_4 z_2 + \beta_5 z_3) \quad (4.15)$$

The value of β_4, β_5 may be positive or negative based on the pattern of the data. The effect of Spare Part Availability is observed through baseline repair rate i.e. Variations in the repair rate are driven by changes in Spare Part Availability. A graph between the cumulative baseline repair rate versus TTR should be drawn for Stratum 1 and Stratum 2. This graph may be utilized to estimate the baseline repair rate at any specific level of

spare part availability. Subsequently, the actual repair rate will be obtained using equation (4.15). If spare part availability is a time-dependent variables, the stratification approach reveals a significant difference in the maintainability of centrifugal pumps based on spare part availability. For example, the probability of completing the repair within 30 minutes is 50 % when spare part availability is deficient. However, if spare part availability is sufficient, the probability of completing the repair within the same time increases to 80 %. Therefore, considering the impact of spare part availability is essential for optimizing maintenance strategies for centrifugal pumps. A comparative evaluation of maintainability performance should be conducted through stratification, PRM, and parametric methods. Stratification highlights time-dependent covariates, PRM accounts for proportional effects, and parametric models help quantify the exact relationship between variables. Comparing them can clarify as to how covariates influence maintainability. The optimal probability distribution for each sub-sample (Stratum 1 and Stratum 2) should be determined. However, analyzing and formulating the impact of multiple covariates, such as component aging, maintenance crew workload, and environmental conditions, are still highly complex.

The above analysis will be more valuable when actual TTR data for key covariates such as spare part availability, component aging, maintenance crew workload, and environmental conditions, become available from the industry. This can form a significant part of future research.

4.7 Summary

This chapter critically evaluates the limitations of traditional maintainability assessment methods, which often rely solely on repair time as the primary determinant. It highlights the need to incorporate covariates particularly time-dependent and time-independent like

spare part availability and equipment aging, etc for more accurate estimation of maintainability performance. The widely used Proportional Repair Model (PRM), which assumes covariates are time-independent, is challenged for its inadequacy in dynamic industrial environment. The chapter emphasizes the importance of identifying both operational and environmental covariates that influence maintenance efficiency and system availability. To overcome these gaps, a refined modelling approach using Cox regression and its variants is proposed.

This approach is capable of handling both time-dependent and time-independent covariates. To validate the proposed methodology, a possible case study is proposed on pumps, which are extensively used in various industrial applications. Their frequent exposure to varying operating conditions makes them ideal for evaluating the effects of covariates on maintainability performance. The analysis will demonstrate as to how the proposed model improves predictive accuracy in maintainability assessment by incorporating both static and dynamic covariates.

The insights obtained from the analyses presented in the preceding chapters form the basis for the concluding remarks and future research directions outlined next.

CHAPTER 5

CONCLUSIONS AND FUTURE SCOPE

5.1 Conclusions

The primary objective of this research was to model and estimate critical maintenance parameters namely maintenance budget, appropriate maintenance strategy and the role of time dependent and independent variables into maintainability evaluation that influence the maintenance decision of industrial systems. The thesis is structured around three key studies that address different but interconnected aspects of maintenance decision-making. Chapter 2 focused on the estimation of the Yearly Maintenance Budget (YMB) using a probabilistic approach that combines Monte Carlo Simulation (MCS) with the Graph Theory and Matrix Approach (GTMA). Traditional methods of estimating YMB are primarily based on historical data trends, expert opinions, or assumed percentages of capital cost—approaches that are often subjective and lack scientific rigor. This work addressed that gap by providing a structured methodology that removes the subjectivity associated with it.

The study simulated budget-influencing parameters across various predefined maintenance levels and generated statistically meaningful outputs in terms of mean and variance. The findings from this chapter are summarized below:

- For world-class maintenance practices, the 95% confidence interval (CI) for the mean YMB lies between 1.62% and 1.82% of the Asset Replacement Value (ARV).
- For Level 1 maintenance standards, where performance parameters exceed 90%, the estimated YMB ranges between 2.99% and 3.31% of ARV.
- For Level 2 maintenance standards, where performance parameters exceed 80%, the estimated YMB lies between 6.51% and 7.23% of ARV.

The results demonstrate that as maintenance effectiveness declines, the required YMB increases significantly. This underlines the importance for maintenance managers to monitor key indicators and intervene when values fall below acceptable thresholds—particularly below 80%—as such deviations could lead to substantial costs and compromise operational efficiency.

The model's outputs provides a more realistic basis for budget allocation and are useful for maintenance managers in avoiding both overestimation and underfunding. The estimates generated by this model are validated in practical settings by plant professionals and adapted to different operational scenarios.

Chapter 3 addressed the complexities of Imperfect Maintenance Modelling through a cost-based optimization, focusing on variability in maintenance execution quality. This chapter introduced a more practical approach that considers the uncertainty in maintenance quality as a probabilistic variable. This allows for more realistic and applicable decision-making in industrial maintenance environments where perfect execution is often unachievable. The research presented three distinct imperfect maintenance models, (i) repair after failure, (ii) minimal repair after failure, and (iii) detection of failure only through perfect PM. It derived the optimal preventive maintenance intervals and average maintenance cost rates using failure rate-model with Weibull distribution for component failure times.

Key findings from this chapter include:

- In the repair-at-failure model, when the probability of imperfect maintenance is low (0–25%), it is economically and operationally advantageous to perform preventive maintenance more frequently. However, as imperfection increases beyond 25%, corrective maintenance becomes preferable over scheduled PM.

- In the minimal maintenance after failure model, rising levels of imperfection suggest the need for shorter PM intervals to maintain reliability and cost-effectiveness.
- For the model where failure is detectable only through perfect PM, PM frequency should be increased at imperfection levels up to 40%. Beyond that, reliance on corrective maintenance proves more viable.
- Numerical simulations further illustrated how variations in maintenance cost influences the optimal maintenance strategy, providing practical insight into real-world applications. These findings offer a more realistic improvement in the average maintenance cost rate and optimum maintenance time interval compared to earlier research.

The validated outcomes provide strategies for maintenance professionals, enabling them to make decisions in uncertain maintenance environments. The methodology equips managers with the tools to adjust maintenance intervals based on the current level of imperfection.

Chapter 4 focused on the Investigation of the Effect of Time-Dependent Covariates on Maintainability Analysis, addressing the limitations of traditional methods that often overlook the influence of operational factors on maintainability. In complex industrial systems, covariates such as operating conditions, ageing of equipment, spare part availability, and maintenance crew workload etc., significantly impact the repair process. Accurately modelling these time-dependent covariates is essential for realistic maintainability evaluation.

This chapter extended the use of Cox regression and its advanced variants to the domain of maintainability, enabling the estimation of repair rates while accounting for both time-independent and time-dependent variables. The proposed statistical methodology provides a more nuanced approach for understanding how covariates influence system repair time distributions and maintainability performance.

The effectiveness of the proposed framework was suggested to be demonstrated using data sets on centrifugal pumps from industry, which can show that neglecting time-dependent covariates leads to underestimation or overestimation of repair times, potentially compromising maintenance planning and resource allocation. The study confirmed that integrating these dynamic factors improves the predictive accuracy of repair rate models and enhances decision-making in maintenance management.

5.2 Contributions

The key contributions of this research are as follows:

- The first contribution is to develop a scientific methodology that estimates the Yearly Maintenance Budget (YMB) using a combination of graph theory and matrix approach and Monte Carlo simulations. It eliminates subjectivity by assigning randomized values to budget parameters and identifying statistical distributions for interval estimation.
- The second contribution is to incorporate the unpredictability of maintenance quality into imperfect maintenance modelling, resulting in more realistic maintenance planning strategies.
- The third contribution is to assess system maintainability by incorporating time-dependent, time-independent, and stochastic covariates (including maintenance crew workload, spare-parts availability, component ageing, and environmental conditions), resulting in a more realistic evaluation of maintainability performance. Within this framework, the maintenance budget may be treated as a key influencing factor that governs spare-parts availability and workload distribution, while component ageing and environmental stresses may be modelled as determinants of maintenance quality.
- Last but not the least, the study contributes by developing a framework that can be adapted across various industries to improve maintenance planning and budgeting decisions.

5.3 Future direction

This research provides a comprehensive methodology for estimating maintenance budget, imperfect maintenance modelling, and maintainability analysis under the influence of time-dependent covariates.

Future investigations can include multiple case studies on different systems, such as: small and medium manufacturing units, refineries, fertilizer plants, power plants, automobile manufacturing units, aircraft manufacturing, etc. This will further validate the evaluated range of budget data.

While this study focuses on cost optimization and preventive maintenance under imperfect conditions, future work could also incorporate condition-based maintenance strategies, downtime cost implications, and availability metrics for a more accurate assessment. Additionally, exploring the impact of real-time degradation monitoring and integrating predictive analytics tools, such as machine learning models, could enhance the adaptability and responsiveness of the budget estimation framework.

Another important direction would be to account for covariate-driven stochastic behaviour of systems in greater depth, especially in the context of maintainability. Including time-dependent covariates through multiple regression models may improve the accuracy of maintainability prediction. The maintainability analysis presented in this thesis considered a limited number of covariates. Future research could include a more diverse set of covariates. Collecting real-time covariate TTR data from industry will help validate the model more effectively and allow researchers to test whether these covariates exhibit time-dependent or time-independent behaviour, thus strengthening the robustness of maintainability prediction.

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Lastly, the integration of Artificial Intelligence (AI) methods—such as Artificial Neural Networks (ANNs), evolutionary optimization algorithms or hybrid method—could offer

dynamic modelling capabilities and reduce reliance on subjective expert judgment in maintenance decision.

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