

DYNAMICS OF COSMOLOGICAL MODELS USING EXOTIC FLUID IN GENERAL RELATIVITY AND ITS MODIFIED GRAVITY

A Thesis Submitted
in Partial Fulfillment of the Requirements
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In
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by
YOGESH BHARDWAJ
(2K21/PHDAM/502)

Under the Supervision of
DR. CHANDRA PRAKASH SINGH
Professor



Department of Applied Mathematics
DELHI TECHNOLOGICAL UNIVERSITY
(Formerly Delhi College of Engineering)
Shahbad Daulatpur, Main Bawana Road, Delhi-110042, India

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CANDIDATE’S DECLARATION

I, Yogesh Bhardwaj (2K21/PHDAM/502), declare that the research work presented in this thesis entitled “**Dynamics of Cosmological Models using Exotic Fluid in General Relativity and its Modified Gravity**” has been carried out by me in the Department of Applied Mathematics, Delhi Technological University, Delhi, for the award of the degree of Doctor of Philosophy in Mathematics. The work reported here is my own research contribution, completed under the supervision of Dr. Chandra Prakash Singh, Professor, Department of Applied Mathematics, Delhi Technological University, Delhi, India and all material taken from published or unpublished sources has been acknowledged through appropriate citations and references.

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(Yogesh Bhardwaj)

2K21/PHDAM/502

CERTIFICATE

Certified that Mr. Yogesh Bhardwaj (2K21/PHDAM/502) has carried out the research work presented in this thesis entitled **“Dynamics of Cosmological Models using Exotic Fluid in General Relativity and its Modified Gravity”** for the award of the degree of Doctor of Philosophy in Mathematics from Delhi Technological University, New Delhi, under my supervision. To the best of my knowledge, the thesis presents original research work completed by the candidate, and its contents have not been submitted for the award of any other degree or diploma at this or any other University or Institution.

(Dr. Chandra Prakash Singh)

Professor and Supervisor
Department of Applied Mathematics
Delhi Technological University
New Delhi, India.

(Dr. R. Srivastava)

Professor and Head
Department of Applied Mathematics
Delhi Technological University
New Delhi, India.

Date: July, 2026

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Date: July, 2026

(YOGESH BHARDWAJ)

Place: Delhi

Preface

The present thesis studies cosmological models in which exotic fluids, matter creation mechanisms, bulk viscous effects, and modified gravity frameworks are used to describe the late-time evolution of the Universe. The work is motivated by the need to understand cosmic acceleration, observational tensions, and possible alternatives to the standard cosmological picture. The thesis combines theoretical modelling with observational constraints and compares the proposed models with the standard Λ CDM scenario.

The first chapter introduces the background of modern cosmology, including the cosmological principle, relativistic cosmology, dark energy, Chaplygin-type fluids, matter creation, bulk viscosity, cosmological parameters, observational data sets, and statistical tools. The following chapters present the main research contributions on generalized and variable Chaplygin gas models, matter creation cosmology, bulk viscous effects, and extensions in $f(Q)$ gravity. The final chapter summarizes the conclusions, future research directions, and broader relevance of the thesis.

This thesis has been prepared in a chapter-wise form so that each research problem is introduced, developed, constrained, and discussed in a self-contained manner, while remaining connected to the central theme of exotic-fluid cosmology in General Relativity and modified gravity.

Date: July, 2026

(YOGESH BHARDWAJ)

Place: Delhi

Dedicated
to
My Family

Contents

Candidate's Declaration	i
Certificate	iii
Acknowledgements	v
Preface	vii
List of tables	xvii
List of figures	xix
1 Introduction	1
1.1 Historical Evolution of Modern Cosmology	2
1.2 The Cosmological Principle	3
1.2.1 Spatial Homogeneity	4
1.2.2 Spatial Isotropy	4
1.2.3 Observational Basis, Validity, and Limitations	5
1.2.4 Implications for Cosmological Modeling	5
1.3 Relativistic Foundations of Cosmology	6
1.3.1 Fundamental Principles of General Relativity	6
1.3.2 Einstein Field Equations in Cosmology	7
1.3.3 Energy-Momentum Tensor and Cosmic Fluid Description	8
1.3.4 FLRW Space-Time and Friedmann Equations	9
1.3.5 Constraint and Acceleration Equations	10
1.4 Cosmological Parameters	10
1.4.1 Redshift	11
1.4.2 Hubble Parameter	11
1.4.3 Deceleration Parameter	12
1.4.4 Effective Equation of State Parameter	12
1.5 Modified Theories of Gravity	13
1.5.1 Motivation for Modifying General Relativity	14
1.5.2 $f(Q)$ Gravity	15

1.5.3	$f(Q, T)$ Gravity	16
1.5.4	Comparison of Modified Gravity Theories with General Relativity . .	16
1.6	The Standard Cosmological Model	17
1.6.1	Essential Features of the Λ CDM Model	18
1.6.2	Observational Successes of the Λ CDM Framework	19
1.7	Open Problems in the Standard Cosmological Model	19
1.7.1	The Cosmological Constant Problem	20
1.7.2	The Cosmic Coincidence Problem	20
1.7.3	The Unresolved Nature of Dark Matter and Dark Energy	21
1.7.4	The Hubble and S_8 Tensions	22
1.7.5	Motivation for Alternative Cosmological Models	22
1.8	Exotic Fluids in Modern Cosmology	23
1.8.1	Chaplygin Gas Cosmologies	24
1.9	Matter Creation Cosmology	28
1.9.1	Particle Creation in an Expanding Universe	29
1.9.2	Thermodynamic Formulation of Matter Creation	29
1.9.3	Creation Pressure and Modified Conservation Laws	30
1.9.4	Matter Creation as a Mechanism for Cosmic Acceleration	31
1.10	Bulk Viscous Cosmology	32
1.10.1	Effective Bulk Viscous Pressure in FLRW Cosmology	33
1.10.2	Bulk Viscosity and Cosmic Acceleration	34
1.11	Cosmographic Parameters	34
1.11.1	Jerk Parameter	35
1.11.2	Snap Parameter	35
1.11.3	Statefinder Diagnostic	36
1.11.4	Om Diagnostic	37
1.12	Observational Data Sets	37
1.12.1	Type Ia Supernova Observations	38
1.12.2	Cosmic Chronometer Data	38
1.12.3	Baryon Acoustic Oscillation Measurements	39
1.12.4	Growth Rate Data	39
1.12.5	Hubble Constant Priors (Local H_0)	40
1.12.6	Gamma-Ray Bursts	40
1.12.7	Standard Candles	41
1.13	Statistical Model Selection Criteria	41
1.13.1	Akaike Information Criterion (AIC)	41
1.13.2	Bayesian Information Criterion (BIC)	42
1.13.3	Bayesian Evidence and Bayes Factors	42
1.14	Research Motivation and Objectives	44

1.14.1	Research Motivation	44
1.14.2	Research Objectives	45
1.15	Organization of Thesis Work	46
2	Matter Creation Cosmology with Generalized Chaplygin Gas	49
2.1	Introduction	49
2.2	Basic field equations	49
2.3	GCG model with matter creation	50
2.4	Particular case: CG model with matter creation	52
2.5	Observational data	52
2.6	Methodology and model selection	53
2.7	Results and discussion	54
2.8	Conclusion	59
3	Matter Creation Cosmology with Variable Generalized Chaplygin Gas	61
3.1	Introduction	61
3.2	Basic Formalism of Matter Creation	61
3.3	Basic Field Equations	62
3.4	Variable Generalized Chaplygin Gas Model with Matter Creation	63
3.5	Particular Solution: Variable Chaplygin Gas Model	64
3.6	Observational Data and Statistical Methodology	65
3.6.1	Baryon Acoustic Oscillations	65
3.6.2	Cosmic Chronometers	66
3.6.3	Standard Candles and Local H_0 Prior	66
3.6.4	Joint Likelihood and Priors	66
3.7	Results and Discussion	67
3.7.1	Evolution of Cosmological Parameters	67
3.7.2	Stability of the Models	71
3.7.3	Model Selection Criteria	74
3.7.4	Bayesian Evidence Analysis	76
3.8	Conclusion	78
4	Matter Creation and Bulk Viscous Cosmology with Modified Chaplygin Gas	81
4.1	Introduction	81
4.2	Basic Formalism and Cosmological Field Equations	81
4.2.1	Matter Creation cosmology	82
4.2.2	Modified Chaplygin Gas Component	82
4.2.3	Bulk Viscous Matter Component	83
4.2.4	Field equations	83
4.3	Observational Data and Statistical Methodology	84

4.3.1	Data Sets	85
4.3.2	Likelihood Analysis	85
4.3.3	MCMC Procedure and Priors	86
4.4	Results and Discussion	86
4.4.1	Hubble Parameter	86
4.4.2	Deceleration Parameter	91
4.4.3	Jerk Parameter	92
4.4.4	Snap Parameter	92
4.4.5	Age of the Universe	93
4.4.6	Thermodynamic Analysis	94
4.4.7	Model Selection	98
4.4.8	Parameter Degeneracies and Identifiability	99
4.5	Conclusion	99
5	Matter Creation Cosmology with Generalized Chaplygin Gas in $f(Q)$ Gravity	101
5.1	Introduction	101
5.2	$f(Q)$ Theory of Gravity	101
5.3	Cosmological Background Evolution	102
5.4	Generalized Chaplygin Gas Model in $f(Q)$ Gravity	104
5.5	Particular Solution	106
5.6	Observational Data and Statistical Methodology	106
5.7	Results and Discussion	107
5.7.1	Evolution of the Hubble Parameter	107
5.7.2	Deceleration Parameter	111
5.7.3	Effective Equation-of-State Parameter	112
5.7.4	Jerk Parameter	113
5.7.5	Age of the Universe	113
5.8	Model Stability	115
5.9	Model Selection Criteria	116
5.10	Conclusion	116
6	Matter Creation Cosmology with Variable Generalized Chaplygin Gas in $f(Q)$ Gravity	119
6.1	Introduction	119
6.2	Gravitational Field Equations	119
6.3	Cosmological Model	120
6.3.1	Model 1: $f(Q) = n_1 Q^k$	122
6.3.2	Model 2: $f(Q) = n_0 + n_1 Q^k$	122
6.4	Observational Data and Statistical Methodology	123
6.5	Results and Discussion	124

6.5.1	Hubble Parameter	124
6.5.2	Deceleration Parameter	127
6.5.3	Jerk Parameter	129
6.5.4	Effective Equation-of-State Parameter	130
6.5.5	Snap Parameter	130
6.5.6	$Om(z)$ Diagnostic	131
6.5.7	Age of the Universe	132
6.5.8	Model selection criteria	133
6.5.9	p-value statistics	133
6.5.10	Bayesian Inference	134
6.6	Conclusion	135
7	Conclusion, Future Scope, and Social Impact	137
7.1	Conclusion	137
7.2	Future Scope	140
7.3	Social Impact	143
7.4	Connection with Sustainable Development Goals (SDGs)	144
	Bibliography	147
	List of Publications	167

List of Tables

1.1	Conceptual comparison of General Relativity, $f(Q)$ gravity, and $f(Q, T)$ gravity.	17
1.2	Major open issues in the standard cosmological model and their physical significance.	23
1.3	Major Chaplygin-type equations of state used in cosmology.	28
1.4	Basic quantities in matter creation cosmology and their physical meaning. . .	28
1.5	Common interpretation of model selection differences and Bayes factors. . .	43
2.1	Clustering measurements for the BAO samples [133].	53
2.2	Priors and 1σ marginalized parameter constraints from the joint $SNe + H(z) + BAO$ analysis.	56
2.3	Present values of cosmological diagnostics for the considered models.	56
2.4	Information criteria and model comparison relative to Λ CDM.	56
3.1	Uniform priors adopted in the MCMC analysis.	67
3.2	Constraints on the parameters of the VGCG and VCG models with matter creation, together with the Λ CDM model. The combined data set $BAO + CC + SC$ is denoted by “BASE”.	68
3.3	Constraints on the parameters of the VGCG and VCG models without matter creation. The combined data set $BAO + CC + SC$ is denoted by “BASE”.	68
3.4	Present values of the main cosmological and geometrical parameters for the Λ CDM, VGCG, and VCG models with matter creation.	70
3.5	Present values of the main cosmological and geometrical parameters for the VGCG and VCG models without matter creation.	70
3.6	χ^2_{red} , AIC, BIC, Bayesian evidence, and Bayes factor values for the Λ CDM, VGCG, and VCG models with matter creation.	77
3.7	χ^2_{red} , AIC, BIC, Bayesian evidence, and Bayes factor values for the VGCG and VCG models without matter creation.	77
4.1	Flat priors adopted in the MCMC analysis.	86
4.2	Best-fit parameter estimates with 1σ uncertainties for the Λ CDM scenario and the modified Chaplygin gas models using the DS1 dataset.	89
4.3	Best-fit parameter estimates with 1σ uncertainties for the Λ CDM scenario and the modified Chaplygin gas models using the DS2 dataset.	89
4.4	Estimated cosmological parameters for the Λ CDM scenario and the modified Chaplygin gas models using the DS1 observational combination.	91

4.5	Estimated cosmological parameters for the Λ CDM scenario and the modified Chaplygin gas models using the DS2 observational combination.	92
4.6	Comparison of the modified Chaplygin gas models with the reference Λ CDM scenario using information criteria.	99
5.1	Flat priors adopted in the MCMC analysis.	107
5.2	Best-fit estimates of the model parameters for the Λ CDM scenario, generalized Chaplygin gas model, and Chaplygin gas model obtained from the combined $SNe + CC + BAO$ dataset.	114
5.3	Present values of the main cosmological quantities for the Λ CDM, generalized Chaplygin gas, and Chaplygin gas models with and without matter creation. .	114
5.4	Information criteria for the Λ CDM, generalized Chaplygin gas, and Chaplygin gas models with and without matter creation. The Λ CDM model is used as the reference.	114
6.1	Flat priors adopted in the MCMC analysis.	124
6.2	Best-fit estimates of the model parameters derived from the combined $CC + SNeIa + BAO$ observational dataset.	128
6.3	Present values of the main cosmological quantities for the Λ CDM model, Model 1, and Model 2.	128
6.4	Statistical comparison of the Λ CDM model, Model 1, and Model 2 using the combined dataset. Values of $\ln E$ and $\ln B_i$ correspond to the Bayesian evidence and the logarithmic Bayes factor relative to Λ CDM.	128

List of Figures

2.1	2D contour at 68.3% and 95.4% confidence levels for Λ CDM model using $H(z) + SNe + BAO$ data-set.	55
2.2	2D contour at 68.3% and 95.4% confidence levels for GCG model with matter creation using $H(z) + SNe + BAO$ data-set.	55
2.3	2D contour at 68.3% and 95.4% confidence levels for GCG model without matter creation using $H(z) + SNe + BAO$ data-set.	55
2.4	2D contour at 68.3% and 95.4% confidence levels for CG model with matter creation using $H(z) + SNe + BAO$ data-set.	55
2.5	Best fits over $H(z)$ data for Λ CDM (black solid), GCG+ β (green dot-dashed), GCG (blue dotted), and CG+ β (magenta dashed). Grey points with error bars denote the 32 CC measurements.	57
2.6	Evolution of the deceleration parameter, $q(z)$ with redshift for GCG and CG models with and without matter creation. The trajectory of $q(z)$ for Λ CDM model is also shown in bold line. The dot represents the present value of $q(z)$ (hence q_0).	57
2.7	Evolution of the jerk parameter, $j(z)$ for GCG and CG models with and without matter creation. The horizontal bold line represents the value of $j(z)$ for Λ CDM. The dot represents the present value of $j(z)$ (hence j_0).	58
2.8	Evolution of the state parameter, $w(z)$ for GCG and CG models with and without matter creation. The dot represents the present value of $w(z)$ (hence w_0).	58
2.9	Evolution of speed of sound, C_s^2 for GCG and CG models with and without matter creation. The dot represents the present value $C_s^2(z=0)$	58
3.1	Two-dimensional confidence contours at the 68.3% and 95.4% levels for the Λ CDM model obtained from the BASE and BASE+R21 datasets.	69
3.2	Representative two-dimensional confidence contours and one-dimensional posterior distributions for the VGCG model with matter creation using the BASE and BASE+R21 data sets.	69
3.3	Representative two-dimensional confidence contours and one-dimensional posterior distributions for the VCG model with matter creation using the BASE and BASE+R21 data sets.	70
3.4	Evolution of $H(z)$ with redshift z in the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE data set. The gray points with uncertainty bars correspond to the $H(z)$ sample. The black bold line represents the Λ CDM model.	72

3.5	Evolution of $H(z)$ with redshift z in the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE+R21 data set. The gray points with uncertainty bars correspond to the $H(z)$ sample. The black bold line represents the Λ CDM model.	72
3.6	Evolution of $q(z)$ with z in the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE data set. The dotted line indicates the present value of $q(z)$	72
3.7	Evolution of $q(z)$ with z in the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE+R21 data set. The dotted line indicates the present value of $q(z)$	72
3.8	Evolution of $w(z)$ with z in the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE data set. The dotted line indicates the present value of $w(z)$	73
3.9	Evolution of $w(z)$ with z in the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE+R21 data set. The dotted line indicates the present value of $w(z)$	73
3.10	Trajectories of $j(z)$ for the VGCG and VCG models with matter creation, together with the Λ CDM model, using the best-fit values from the BASE data set. The dotted line indicates the present value j_0	73
3.11	Trajectories of $j(z)$ for the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE+R21 data set. The dotted line indicates the present value j_0	73
3.12	Trajectories of C_s^2 for the VGCG and VCG models with matter creation using the best-fit values of the BASE data set.	75
3.13	Trajectories of C_s^2 for the VGCG and VCG models with matter creation using the best-fit values of the BASE+R21 data set.	75
4.1	Parameter best-fit estimates for the Λ CDM model derived from the DS1 and DS2 datasets, reported at the 1σ and 2σ confidence levels.	87
4.2	Parameter best-fit estimates for the modified Chaplygin gas model and its limiting cases obtained from the DS1 and DS2 datasets, with results reported at the 1σ and 2σ confidence levels.	88
4.3	Redshift evolution of the Hubble parameter $H(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS1 dataset. The grey data points with error bars correspond to cosmic chronometer measurements. . . .	89
4.4	Redshift evolution of the Hubble parameter $H(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS2 dataset. The grey data points with error bars correspond to cosmic chronometer measurements. . . .	89
4.5	Redshift behaviour of the deceleration parameter $q(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS1 dataset.	91
4.6	Redshift behaviour of the deceleration parameter $q(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS2 dataset.	91
4.7	Redshift behaviour of the jerk parameter $j(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS1 dataset.	92

4.8	Redshift behaviour of the jerk parameter $j(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS2 dataset.	92
4.9	Redshift behaviour of the snap parameter $s(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS1 dataset. The marker on each curve denotes the present value s_0	93
4.10	Redshift behaviour of the snap parameter $s(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS2 dataset. The marker on each curve denotes the present value s_0	93
4.11	Redshift variation of the total entropy change rate $\dot{S}_{\text{total}}$ for the modified Chaplygin gas model and the Λ CDM scenario using the DS1 dataset. Positive values indicate consistency with the generalized second law.	97
4.12	Redshift variation of the total entropy change rate $\dot{S}_{\text{total}}$ for the modified Chaplygin gas model and the Λ CDM scenario using the DS2 dataset. The inclusion of the R22 prior gives a mildly enhanced entropy production at recent epochs.	97
4.13	Redshift behaviour of the second derivative of the total entropy, $\ddot{S}_{\text{total}}$, for the modified Chaplygin gas model and the Λ CDM scenario using the DS1 dataset. The change from positive to negative values suggests that entropy generation becomes slower as the system moves toward late-time thermodynamic equilibrium.	97
4.14	Redshift behaviour of the second derivative of the total entropy, $\ddot{S}_{\text{total}}$, for the modified Chaplygin gas model and the Λ CDM scenario using the DS2 dataset. The addition of the R22 prior produces a small shift in the sign-change redshift of $\ddot{S}_{\text{total}}$, while the late-time tendency toward thermodynamic equilibrium remains unchanged.	97
5.1	One-dimensional posterior distributions and two-dimensional confidence contours at 1σ and 2σ for the Λ CDM model using the $SNe + CC + BAO$ dataset.	108
5.2	One-dimensional posterior distributions and two-dimensional confidence contours at 1σ and 2σ for the generalized Chaplygin gas model with and without matter creation using the $SNe + CC + BAO$ dataset.	109
5.3	One-dimensional posterior distributions and two-dimensional confidence contours at 1σ and 2σ for the Chaplygin gas model with and without matter creation using the $SNe + CC + BAO$ dataset.	110
5.4	Redshift evolution of $H(z)$ for the Λ CDM scenario and the generalized Chaplygin gas models, with and without matter creation. The grey points with error bars denote the cosmic chronometer measurements.	111
5.5	Redshift evolution of $H(z)$ for the Λ CDM scenario and the Chaplygin gas models, with and without matter creation. The grey points with error bars denote the cosmic chronometer measurements.	111
5.6	Evolution of $q(z)$ for the Λ CDM scenario and the generalized Chaplygin gas models, with and without matter creation.	112
5.7	Evolution of $q(z)$ for the Λ CDM scenario and the Chaplygin gas models, with and without matter creation.	112
5.8	Evolution of $w_{\text{eff}}(z)$ for the Λ CDM scenario and the generalized Chaplygin gas models, with and without matter creation.	112

5.9	Redshift behaviour of the effective equation-of-state parameter $w_{\text{eff}}(z)$ for the Λ CDM scenario and the Chaplygin gas models, with and without matter creation.	112
5.10	Redshift behaviour of the jerk parameter $j(z)$ for the Λ CDM scenario and the generalized Chaplygin gas models, with and without matter creation. The marker indicates the present value j_0 .	113
5.11	Redshift behaviour of the jerk parameter $j(z)$ for the Λ CDM scenario and the Chaplygin gas models, with and without matter creation. The marker indicates the present value j_0 .	113
5.12	Evolution of the squared sound speed C_s^2 for the generalized Chaplygin gas and Chaplygin gas models with and without matter creation.	115
5.13	Evolution of the squared sound speed C_s^2 for the generalized Chaplygin gas and Chaplygin gas models with and without matter creation.	115
6.1	Best-fit parameter estimates for the Λ CDM model derived from the combined $CC + SNeIa + BAO$ datasets, reported at the 1σ and 2σ confidence levels.	125
6.2	Best-fit parameter estimates for Model 1 and Model 2 derived from the combined $CC + SNeIa + BAO$ datasets, reported at the 1σ and 2σ confidence levels.	126
6.3	Redshift evolution of the Hubble parameter $H(z)$ for the Λ CDM model, Model 1, and Model 2 obtained from the combined $CC + SNeIa + BAO$ dataset. The grey points with error bars denote the cosmic chronometer measurements.	127
6.4	Redshift-dependent variation of the Hubble difference $\Delta H(z)$ for Model 1 and Model 2 measured with respect to the reference Λ CDM scenario.	127
6.5	Redshift behaviour of the deceleration parameter $q(z)$ for the Λ CDM model, Model 1, and Model 2 obtained from the combined $CC + SNeIa + BAO$ dataset.	129
6.6	Redshift behaviour of the jerk parameter $j(z)$ for the Λ CDM model, Model 1, and Model 2 obtained from the combined $CC + SNeIa + BAO$ dataset. In the Λ CDM case, the jerk parameter remains fixed at $j(z) = 1$ over the full redshift range.	129
6.7	Redshift behaviour of the effective equation-of-state parameter $w(z)$ for the Λ CDM model, Model 1, and Model 2 obtained from the combined $CC + SNeIa + BAO$ dataset.	130
6.8	Redshift behaviour of the snap parameter $s(z)$ for the Λ CDM model, Model 1, and Model 2 obtained from the combined $CC + SNeIa + BAO$ dataset. For a flat Λ CDM background, the snap parameter remains zero throughout the redshift range.	130
6.9	Redshift behaviour of the $Om(z)$ diagnostic for the Λ CDM model, Model 1, and Model 2 obtained from the combined $CC + SNeIa + BAO$ dataset. For a flat Λ CDM background, $Om(z)$ remains constant and equals $\Omega_{m,0}$.	132

Chapter 1

Introduction

The study of the Universe on cosmological scales has developed into a quantitatively precise scientific discipline, underpinned by rigorous theoretical frameworks and continually improving high-precision observational data. Over the past few decades, cosmology has moved beyond the simple description of an expanding Universe to the more challenging task of explaining its accelerated late-time evolution. Observational evidence from Type Ia supernovae, cosmic microwave background anisotropies, and large-scale structure measurements has established a concordance picture in which the present cosmic dynamics are dominated by dark components, commonly identified as dark matter and dark energy [1–3]. In its simplest form, this description is embodied in the Lambda-cold-dark matter (Λ CDM) model, which has been remarkably successful in reproducing a wide range of data.

Despite its empirical success, the standard cosmological model is not free from conceptual and observational difficulties. The interpretation of dark energy as a cosmological constant leads to the well-known fine-tuning problem, while the near equality of matter and dark energy densities at the present epoch gives rise to the coincidence problem [4, 5]. In addition, recent observations have shown that there are statistically significant tensions in several important cosmological parameters. This suggests that the standard cosmological model may be inadequate or necessitate supplementary physical components or processes. These open issues have motivated sustained interest in alternative cosmological scenarios that can account for the observed expansion history without relying exclusively on the conventional Λ CDM picture.

Among the proposed alternatives, models involving exotic cosmic fluids occupy an important position. Such fluids are characterized by effective equations of state that differ from those of ordinary matter and radiation and can generate negative pressure capable of driving accelerated expansion. Chaplygin-type fluids and their generalizations are of particular in-

terest because they provide a unified description in which the same cosmic component can mimic both dark matter behavior at early times and dark energy behavior at late times [6]. At the same time, dissipative mechanisms such as bulk viscosity and gravitationally induced matter creation offer a thermodynamically motivated route to modify cosmic dynamics without necessarily introducing new fundamental fields [7, 8].

The present thesis is devoted to the investigation of cosmological models constructed from exotic fluids in General Relativity and in modified gravity. Special attention is given to Chaplygin gas cosmologies, matter creation mechanisms, and viscous effects, together with their extensions in non-Riemannian gravitational frameworks such as $f(Q)$ and $f(Q, T)$ gravity. The aim is to examine whether these ingredients can provide a viable description of the cosmic expansion history, alleviate some of the shortcomings of the standard model, and remain compatible with current observational constraints. The thesis is based on a set of published works that collectively explore these questions through analytical modeling, dynamical analysis, and statistical confrontation with cosmological data.

This introductory chapter establishes the theoretical background required for later chapters. It begins with a brief historical account of the development of modern cosmology and then discusses the cosmological principle, relativistic cosmology, the standard cosmological model, its unresolved problems, and the motivation for considering exotic fluids, matter creation, bulk viscous effects, and modified theories of gravity. The chapter also summarizes the observational tools and statistical criteria employed in the thesis before outlining the research objectives and overall organization of the work.

1.1 Historical Evolution of Modern Cosmology

The evolution of modern cosmology signifies a progressive shift from qualitative conjecture regarding the Universe to a quantitatively testable physical theory. Earlier ideas about the Universe were mainly philosophical. However, the arrival of modern physics and astronomy brought about significant changes. The formulation of General Relativity provided, for the first time, a coherent theoretical framework in which the geometry of space-time and the distribution of matter could be studied together. Albert Einstein [9] first used his theory of gravity to explain the complete Universe and came up with a static cosmological model by adding the cosmological constant. He did this because he thought the Universe didn't change over time. This image was quickly questioned, but it was the start of relativistic cosmology as a mathematical science.

Friedmann and Lemaître made a significant contribution by showing that Einstein's field equations naturally allow non-static solutions describing an expanding or contracting Universe [10, 11]. These theoretical advancements received significant observational confirmation when Hubble illustrated a systematic correlation between the redshifts of galaxies and their distances, thereby providing evidence of the Universe's large-scale expansion [12]. The idea of cosmic expansion changed cosmology by replacing the idea of a static Universe with one that is always changing. This set the stage for the later Big Bang theory.

During the middle of the 20th century, cosmology became into a broader physical theory that included nuclear physics, thermodynamics, and astronomical observations. The idea that light chemical elements could have formed in the early, hot Universe gave a lot of support to the hot Big Bang model [13]. Furthermore, the discovery of cosmic microwave background radiation provided definitive confirmation, demonstrating the Universe's initial dense and thermal state [14]. Subsequent observations of temperature anisotropies in the cosmic microwave background revealed the initial indications of large-scale structure, thereby initiating the era of precision cosmology [15].

The concordance cosmological model has emerged in recent decades, thanks to increasingly precise observations of supernovae, the clustering of galaxies, and the unevenness of the cosmic microwave background. At the same time, these observations have raised new questions concerning the physical origin of dark matter, dark energy, and the physical processes responsible for the accelerated expansion observed at late cosmic times. Overall, modern cosmology has become a field in which the study of gravitation, cosmic fluids, and observational data must be combined to understand the full dynamical history of the Universe. This broader context provides the background for the present thesis, which focuses on cosmological dynamics driven by exotic fluids, matter creation, viscous effects, and modified gravity.

1.2 The Cosmological Principle

The cosmological principle underpins modern relativistic cosmology. It assumes that, on sufficiently large scales, the Universe can be treated as spatially homogeneous and isotropic. This principle does not necessitate a uniform matter distribution at all scales; instead, it implies that local inhomogeneities, including stars, galaxies, and clusters, become statistically insignificant when averaged across extensive cosmic volumes. This principle is significant because it simplifies the complex distribution of matter in the universe, making it easier to analyze using General Relativity. This, in turn, enables the formulation of realistic models of

universal behavior [16, 17].

1.2.1 Spatial Homogeneity

Spatial homogeneity means that, at a fixed cosmic time, the large-scale properties of the Universe are the same at every point. In a homogeneous Universe, no spatial location is privileged over another, and the averaged matter density depends only on time. Thus, the relevant macroscopic quantities describing the cosmic medium, such as the energy density ρ and pressure p , are taken to be functions of cosmic time alone. This assumption greatly simplifies cosmological modeling by allowing the large-scale Universe to be described through global dynamical variables rather than complicated position-dependent distributions [18, 19].

It is important to emphasize that homogeneity is an idealization valid only beyond a characteristic scale. On smaller scales, the observed Universe contains substantial structure, including galaxies, galaxy groups, filaments, and voids. The cosmological principle therefore applies to the smoothed matter distribution rather than to the detailed local arrangement of matter. In this sense, homogeneity should be understood as a large-scale statistical property of the observable Universe rather than as an exact statement at every scale.

1.2.2 Spatial Isotropy

Spatial isotropy implies that the Universe has the same appearance in all directions, for any given observer. If the large-scale distribution of matter and radiation has no preferred direction, then the geometric and dynamical properties of the Universe are direction independent. Observationally, the strongest support for isotropy comes from the near-uniformity of the cosmic microwave background radiation, whose temperature fluctuations are extremely small compared with its mean value [3, 15]. Large-scale galaxy surveys also suggest that directional anisotropies are weak once one averages over sufficiently large distances.

Isotropy must, however, be interpreted carefully. The fact that the Universe appears isotropic about one location does not by itself imply that it is homogeneous everywhere. A fully homogeneous and isotropic cosmology requires a stronger global assumption. In standard cosmology, isotropy of the cosmic microwave background together with the Copernican viewpoint leads naturally to the adoption of a homogeneous and isotropic large-scale model [20, 21].

1.2.3 Observational Basis, Validity, and Limitations

The cosmological principle is supported by several independent observations. Measurements of the cosmic microwave background radiation show a high degree of isotropy. At the same time, galaxy redshift surveys indicate that matter is distributed almost uniformly when looking at large areas of space. In addition, the measurements of baryon acoustic oscillations, along with the standard cosmological model's success in fitting various datasets, further support the principle's validity on a large scale [22, 23].

Nevertheless, the cosmological principle remains an approximation. The observable Universe is manifestly inhomogeneous on small and intermediate scales, and some analyses have explored the possibility of weak departures from exact isotropy or homogeneity. Such investigations are valuable because they test the robustness of the standard framework and clarify the scale beyond which the idealized description becomes reliable. For practical cosmological modeling, however, the homogeneous and isotropic approximation continues to provide a remarkably successful first-order description of the large-scale Universe.

1.2.4 Implications for Cosmological Modeling

The main consequence of the cosmological principle is that it restricts the admissible form of the space-time geometry and the matter content used in cosmology. At large scales, a homogeneous and isotropic Universe may be represented by a cosmic fluid characterized by spatially averaged properties and by a metric consistent with the same symmetries. This leads directly to the Friedmann–Lemaître–Robertson–Walker description, which forms the basis of the standard model of cosmology and most of its extensions. Once homogeneity and isotropy are assumed, the Einstein field equations reduce to a much simpler set of dynamical equations governing the cosmic scale factor and the time evolution of the energy density and pressure [17, 24].

For the present thesis, the cosmological principle plays a central role because the exotic fluid models, matter creation mechanisms, viscous effects, and modified gravity scenarios considered later are all formulated within a large-scale homogeneous and isotropic background. Even when new physical ingredients are introduced, the cosmological principle ensures that the resulting models remain compatible with the broad observational structure of modern cosmology, while allowing the dynamical effects of these ingredients to be studied in a controlled way.

1.3 Relativistic Foundations of Cosmology

Modern cosmology is founded on the principle that gravity is described by the theory of relativity. In this framework, gravity is not regarded as a standard force operating within a static background; rather, it is viewed as a manifestation of the curvature of space-time induced by matter and energy. This viewpoint is essential for cosmology because the evolution of the Universe involves dynamical geometry, time-dependent matter content, and large-scale structure formation, all of which require a self-consistent gravitational theory. General Relativity provides such a framework and remains the starting point for the standard cosmological model as well as for many of its extensions [25, 26].

Under the assumption that the Universe is homogeneous and isotropic on large scales, the relativistic field equations simplify considerably and give rise to a system of dynamical equations governing the evolution of the cosmic scale factor. These equations connect the matter content of the Universe with its expansion history and form the basis for the study of dark energy, exotic fluids, matter creation, dissipative processes, and modified gravity scenarios. For this reason, an understanding of the relativistic foundations of cosmology is indispensable for the models developed in the later chapters of this thesis.

1.3.1 Fundamental Principles of General Relativity

According to General Relativity, the physical laws remain invariant in every coordinate system, and the shape of space-time can help us to understand gravity. The equivalence principle is a key part of this progress because it says that gravity and acceleration have the same effects in a small area. As a consequence, freely falling observers experience local physics in the same way as observers in inertial frames of Special Relativity. This principle leads naturally to the interpretation that gravitation reflects the geometry of space-time rather than an external force field [27, 28].

In relativistic cosmology, the spacetime geometry of the Universe is specified through the metric tensor $g_{\mu\nu}$, it determines intervals, causal structure, and the motion of particles and radiation. The corresponding line element is written as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad (1.1)$$

where Greek indices run over the four space-time coordinates. The metric contains the gravitational information of the system and allows one to define curvature tensors that characterize

the deviation of space-time from flatness.

The motion of free particles follows geodesics determined by the metric. Thus, the gravitational field is encoded in the connection between geometry and motion. In cosmology, this geometrical description becomes particularly powerful because the large-scale symmetries of the Universe strongly constrain the possible form of the metric and thereby reduce the dynamical problem to a manageable set of equations.

1.3.2 Einstein Field Equations in Cosmology

The gravitational dynamics in General Relativity are governed by the Einstein field equations, which establish a connection between the spacetime curvature and the energy–momentum content of the Universe. When a cosmological constant Λ is included, these equations take the form

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (1.2)$$

Here, $R_{\mu\nu}$ denotes the Ricci tensor, R represents the Ricci scalar, $T_{\mu\nu}$ stands for the energy–momentum tensor, and G is Newton’s gravitational constant. The quantity

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \quad (1.3)$$

is known as the Einstein tensor and satisfies the covariant conservation law

$$\nabla^\mu G_{\mu\nu} = 0. \quad (1.4)$$

A direct consequence of Eq. (1.4) is the conservation condition

$$\nabla^\mu T_{\mu\nu} = 0, \quad (1.5)$$

provided that the cosmological constant is treated as constant. This relation is of fundamental importance in cosmology because it governs the exchange between geometry and the cosmic fluid and leads to the continuity equation once a particular metric and matter source are specified.

In cosmological applications, Eq. (1.2) is not solved in complete generality. Instead, one imposes the symmetries implied by the cosmological principle, which reduces the problem to the dynamics of a single scale factor $a(t)$. The resulting equations form the basis of the Friedmann cosmological models and their later generalizations.

1.3.3 Energy-Momentum Tensor and Cosmic Fluid Description

Within relativistic cosmology, the matter and energy distribution of the Universe is described by the energy–momentum tensor, which provides a covariant description of energy density, pressure, momentum transport, and internal stresses. Since the real Universe is highly structured on small scales but approximately smooth on sufficiently large scales, cosmology usually adopts an averaged fluid picture. In this description, the cosmic medium is characterized by macroscopic quantities such as the energy density ρ and pressure p , both defined in the rest frame of the fluid [26, 29].

For a perfect fluid, which is the standard approximation in homogeneous and isotropic cosmology, the energy-momentum tensor is given by

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}, \quad (1.6)$$

where u_μ denotes the four-velocity of the fluid, which obeys the normalization condition

$$u_\mu u^\mu = -1. \quad (1.7)$$

The form of Eq. (1.6) is consistent with the assumptions of homogeneity and isotropy, since it excludes anisotropic stresses and directional energy flow at the background level.

The thermodynamic behavior of the cosmic fluid is commonly described through the equation of state

$$p = w\rho, \quad (1.8)$$

where w is the equation-of-state parameter. Distinct values of w characterize different cosmic components. For instance, $w = 0$ corresponds to pressureless matter, $w = 1/3$ represents radiation, and $w = -1$ is used to describe vacuum energy. In more general cosmological scenarios, the effective pressure may acquire additional contributions due to bulk viscosity, particle creation, or interactions between fluid components. Such generalizations are especially relevant for the models studied in this thesis.

The conservation law for the energy–momentum tensor is given by

$$\nabla^\mu T_{\mu\nu} = 0. \quad (1.9)$$

This relation plays a central role in cosmology because it determines the evolution of the fluid variables in an expanding background. Thus, the energy-momentum tensor forms the essential

link between the matter sector and the gravitational dynamics of the Universe.

1.3.4 FLRW Space-Time and Friedmann Equations

Assuming that the Universe is homogeneous and isotropic on sufficiently large scales, its spacetime structure is represented by the Friedmann–Lemaître–Robertson–Walker metric. In comoving coordinates, the line element takes the form ¹

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right], \quad (1.10)$$

where $a(t)$ is the cosmic scale factor and $k = 0, +1, -1$ denotes flat, closed, and open spatial geometry, respectively [17, 24]. The scale factor quantifies the relative growth of the Universe and serves as the primary dynamical variable in homogeneous cosmology.

When the metric in Eq. (1.10) is substituted into Einstein's field equations with a perfect fluid source, one obtains the Friedmann equations. The first Friedmann equation is expressed as

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} + \frac{\Lambda}{3}, \quad (1.11)$$

where $H = \dot{a}/a$ denotes the Hubble parameter. Throughout the thesis, a dot denotes derivative with respect to the cosmic time t . This equation correlates total energy density, spatial curvature, and the cosmological constant with the rate of cosmic expansion.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) + \frac{\Lambda}{3}. \quad (1.12)$$

Equation (1.12) indicates that the sign of cosmic acceleration is contingent upon the combination $\rho + 3p$. Hence, a sufficiently negative pressure can drive accelerated expansion, which is why dark energy, exotic fluids, viscosity, and matter creation are of central importance in contemporary cosmology.

From the conservation law in Eq. (1.9), one obtains the continuity equation in the FLRW background,

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (1.13)$$

Equations (1.11)–(1.13) together govern the background dynamics of the standard cosmological model. They also provide the basic framework for many generalized cosmological scenarios, including those involving non-standard fluids, dissipative processes, and modifications of

¹In the present thesis, we assume the speed of light in vacuum $c = 1$ in relativistic unit and $(-, +, +, +)$ sign convention

gravity.

1.3.5 Constraint and Acceleration Equations

Let ρ_{tot} be the total density and p_{eff} include all pressures present in a model, including creation and viscous pressures when applicable. For constant Λ , the Friedmann constraint and the total conservation law take the form

$$H^2 = \frac{8\pi G}{3}\rho_{\text{tot}} - \frac{k}{a^2} + \frac{\Lambda}{3}, \quad \dot{\rho}_{\text{tot}} = -3H(\rho_{\text{tot}} + p_{\text{eff}}).$$

On a differentiable branch with $H \neq 0$, differentiating the constraint gives

$$2H\dot{H} = \frac{8\pi G}{3}\dot{\rho}_{\text{tot}} + \frac{2kH}{a^2}, \quad \dot{H} - \frac{k}{a^2} = -4\pi G(\rho_{\text{tot}} + p_{\text{eff}}).$$

Adding the constraint to this last relation yields

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho_{\text{tot}} + 3p_{\text{eff}}) + \frac{\Lambda}{3}.$$

Thus an expansion history obtained from the Friedmann constraint also satisfies the acceleration equation when its component densities satisfy their balance equations with the same effective pressures. The constraint alone is not sufficient. At $H = 0$, the undivided equations must be considered. For modified gravity, the same consistency test must be carried out using the field equations of that theory rather than substituting the General Relativity acceleration equation.

1.4 Cosmological Parameters

The dynamical behavior of the Universe is commonly described through a set of cosmological parameters that characterize its expansion history and effective fluid properties. These quantities are useful because they connect theoretical models directly with observations and allow one to distinguish between decelerating, accelerating, matter-dominated, and dark-energy-dominated phases of evolution. In the present thesis, such parameters play an important role in analyzing exotic fluid cosmologies, matter creation models, bulk viscous scenarios, and modified gravity frameworks. Among the most basic and widely used parameters are the Hubble parameter, the deceleration parameter, the effective equation of state parameter, and the redshift [73, 74].

1.4.1 Redshift

In cosmology, redshift is a significant parameter to examine. It explains how the Universe's expansion causes wavelengths to stretch, and it is defined as

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}}, \quad (1.14)$$

where λ_{obs} and λ_{em} are the observed and emitted wavelengths, respectively. In a FLRW Universe, redshift is directly connected to the scale factor via

$$1 + z = \frac{a_0}{a}, \quad (1.15)$$

where a_0 is the present value of the scale factor. By choosing the normalization $a_0 = 1$, one obtains

$$a = \frac{1}{1 + z}. \quad (1.16)$$

Redshift provides the observational bridge between theory and data because most cosmological measurements are reported as functions of z rather than cosmic time t . Quantities such as the Hubble parameter, luminosity distance, angular diameter distance, and growth rate are commonly expressed in terms of redshift. For this reason, it is standard practice in cosmology to reformulate the dynamical equations of a model in redshift space when performing observational analysis.

1.4.2 Hubble Parameter

The Hubble parameter characterizes the instantaneous expansion rate of the Universe and is defined through the scale factor $a(t)$ as

$$H = \frac{\dot{a}}{a}. \quad (1.17)$$

It provides the fractional rate at which the size of the Universe changes with cosmic time. The current value of the Hubble parameter, represented by H_0 , is commonly referred to as the Hubble constant, even though the expansion rate itself varies during the evolution of the Universe.

Expressed as a function of redshift, the Hubble parameter encodes the full background expansion history of a cosmological model. Different choices of the matter sector, equation of

state, viscous contribution, matter creation rate, or modified gravity function lead to different forms of $H(z)$. For this reason, the Hubble parameter is one of the most direct observational probes of cosmological dynamics.

1.4.3 Deceleration Parameter

The deceleration parameter describes whether the cosmic expansion is slowing down or speeding up. It is defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \quad (1.18)$$

Using the definition of the Hubble parameter, Eq. (1.18) may also be written as

$$q = -1 - \frac{\dot{H}}{H^2}. \quad (1.19)$$

In terms of redshift, Eq. (1.19) can be written as

$$q(z) = -1 + (1+z)\frac{d\ln H(z)}{dz}. \quad (1.20)$$

A positive value of q corresponds to decelerated expansion, whereas a negative value indicates accelerated expansion.

The deceleration parameter is especially useful in late-time cosmology because it allows one to identify the transition from a matter-dominated decelerating epoch to an accelerating phase. In models with exotic fluids or dissipative effects, the evolution of q provides a clear measure of how the additional physics modifies the standard expansion behavior.

1.4.4 Effective Equation of State Parameter

In cosmological models, describing the universe's behavior using an effective equation of state parameter is often helpful.

This quantity is defined by

$$w_{\text{eff}} = \frac{p_{\text{eff}}}{\rho_{\text{eff}}}, \quad (1.21)$$

where ρ_{eff} and p_{eff} represent the total effective energy density and effective pressure of the Universe. In models that take into account bulk viscosity, matter formation, or changed gravity, the effective pressure may have extra contributions on top of the conventional equilibrium fluid pressure.

For a spatially flat FLRW Universe, the effective equation of state parameter can be expressed in terms of the Hubble parameter as

$$w_{\text{eff}} = -1 - \frac{2\dot{H}}{3H^2}. \quad (1.22)$$

Using Eq. (1.19), this relation may be rewritten as

$$w_{\text{eff}} = \frac{2q - 1}{3}. \quad (1.23)$$

In terms of redshift, the effective equation-of-state parameter can be written as

$$w(z) = -1 + \frac{2}{3}(1+z)\frac{d\ln H(z)}{dz}. \quad (1.24)$$

These expressions are particularly useful because they relate the fluid description of the Universe directly to the background expansion dynamics.

The cosmological behavior of the model is determined by the sign and magnitude of w_{eff} . For instance, if $w_{\text{eff}} = 0$, there is no pressure on the matter. If $w_{\text{eff}} = 1/3$, there is radiation and if $w_{\text{eff}} = -1$, there is a cosmological constant. Accelerated expansion occurs when

$$w_{\text{eff}} < -\frac{1}{3}. \quad (1.25)$$

Therefore, the effective equation of state parameter is a crucial tool for distinguishing between different stages of cosmic evolution.

1.5 Modified Theories of Gravity

Although General Relativity has been extraordinarily successful in describing gravitation from Solar System scales to compact astrophysical objects, its direct application to cosmology leaves several open questions. The observed accelerated expansion of the Universe, the dark matter problem, and the difficulty of reconciling gravitation with quantum theory have motivated the exploration of gravitational frameworks beyond Einstein's theory. In cosmology, modified gravity models are often constructed to explain late-time acceleration through changes in the geometric sector rather than by introducing an additional dark energy component with ad hoc properties. Such theories do not necessarily discard General Relativity; rather, they extend it in regimes where the standard formulation may be incomplete or phe-

nomenologically insufficient [30, 31].

Among the many alternatives proposed in the literature, theories based on non-Riemannian geometry have attracted considerable attention. In particular, symmetric teleparallel gravity reformulates gravitation in terms of nonmetricity instead of curvature. This viewpoint has led to the development of $f(Q)$ gravity, in which the gravitational Lagrangian is generalized from the nonmetricity scalar Q to an arbitrary function of it. A further extension is provided by $f(Q, T)$ gravity, where the trace T of the energy-momentum tensor is also included in the action. These models are especially relevant for the present thesis because they offer a flexible setting in which the effects of exotic fluids and matter creation can be investigated beyond standard General Relativity.

1.5.1 Motivation for Modifying General Relativity

The motivation for modifying General Relativity in cosmology arises from both observational and theoretical considerations. On the observational side, the late-time acceleration of the Universe is most simply explained in the standard model by the cosmological constant. However, this interpretation is accompanied by severe fine-tuning and coincidence problems, which suggest that the standard description may be only an effective approximation. On the theoretical side, Einstein's theory is formulated in a classical setting and does not by itself provide a complete account of gravitational phenomena in the ultraviolet regime. These issues have encouraged the study of extended gravitational models in which cosmic acceleration can emerge from the geometry of space-time itself rather than from a separate dark energy fluid [32, 33].

Another reason for considering modified gravity is that the Einstein field equations are derived from a very specific gravitational action, linear in the Ricci scalar. There is no fundamental principle requiring this linearity to remain exact at all scales. Once more general geometric scalars are allowed in the action, one obtains effective cosmological dynamics that may differ significantly from those of General Relativity, particularly in the early and late Universe. Such deviations can be confronted with observations and may provide a route toward understanding the origin of the accelerated expansion.

1.5.2 $f(Q)$ Gravity

In symmetric teleparallel gravity, gravitation is described by the nonmetricity of space-time rather than by curvature or torsion. The basic geometric object is the nonmetricity tensor,

$$Q_{\alpha\mu\nu} = \nabla_\alpha g_{\mu\nu}, \quad (1.26)$$

which measures the failure of the metric to remain covariantly constant. From this tensor, one constructs the nonmetricity scalar Q , given by

$$Q = -\frac{1}{4}Q_{\alpha\beta\mu}Q^{\alpha\beta\mu} + \frac{1}{2}Q_{\alpha\beta\mu}Q^{\beta\alpha\mu} + \frac{1}{4}Q_\alpha Q^\alpha - \frac{1}{2}Q_\alpha \tilde{Q}^\alpha, \quad (1.27)$$

where

$$Q_\alpha = Q^\mu_{\alpha\mu}, \quad \tilde{Q}_\alpha = Q^\mu_{\alpha\mu}. \quad (1.28)$$

The disformation tensor and nonmetricity conjugate used in the following chapters are

$$L^\beta_{\mu\nu} = \frac{1}{2}g^{\beta h}(-Q_{\mu h\nu} - Q_{\nu h\mu} + Q_{h\mu\nu}), \quad (1.29)$$

$$P^\beta_{\mu\nu} = -\frac{1}{2}L^\beta_{\mu\nu} + \frac{1}{4}(Q^\beta - \tilde{Q}^\beta)g_{\mu\nu} - \frac{1}{4}\delta^\beta_{(\mu}Q_{\nu)}. \quad (1.30)$$

Parentheses denote symmetrization, so that $\delta^\beta_{(\mu}Q_{\nu)} = (\delta^\beta_\mu Q_\nu + \delta^\beta_\nu Q_\mu)/2$. These definitions are used in both Chapters 5 and 6.

The action of $f(Q)$ gravity is written as

$$S = \int \left[\frac{1}{2}f(Q) + \mathcal{L}_m \right] \sqrt{-g} d^4x, \quad (1.31)$$

where $\mathcal{L}_m$ is the matter Lagrangian density. When $f(Q) = Q$, the theory reduces to the symmetric teleparallel equivalent of General Relativity, so $f(Q)$ gravity may be viewed as a natural generalization of that framework. The appeal of this theory lies in the fact that modified cosmic dynamics can be generated through the choice of the function $f(Q)$, without introducing higher-order curvature terms of the kind that arise in many other gravitational extensions [34, 35].

For a spatially flat FLRW geometry, the nonmetricity scalar takes the particularly simple form

$$Q = 6H^2, \quad (1.32)$$

which makes the theory especially convenient for cosmological applications. In this setting, the modified Friedmann equations can be derived from the action and written in terms of $f(Q)$ and its derivatives. Because the scalar Q depends directly on the Hubble parameter, the theory provides a useful framework for studying late-time acceleration, dynamical dark energy behavior, and the impact of non-standard cosmic fluids.

1.5.3 $f(Q, T)$ Gravity

A further generalization is obtained by allowing the gravitational action to depend not only on the nonmetricity scalar Q but also on the trace T of the energy-momentum tensor. The action of $f(Q, T)$ gravity is then written as [36, 37]

$$S = \int \left[\frac{1}{2} f(Q, T) + \mathcal{L}_m \right] \sqrt{-g} d^4x, \quad (1.33)$$

where

$$T = g^{\mu\nu} T_{\mu\nu}. \quad (1.34)$$

The explicit dependence on T introduces a direct coupling between matter and geometry, which distinguishes this theory from both General Relativity and pure $f(Q)$ gravity.

This coupling has important cosmological consequences. In general, the usual conservation equation for the matter sector may be modified, leading to effective exchange terms between geometry and matter. Such behavior is particularly interesting in the context of the present thesis, where particle creation, exotic fluids, and non-standard effective pressures already play a central role. The $f(Q, T)$ framework therefore offers a broader arena in which the influence of matter-geometry coupling on cosmic evolution can be examined.

From a phenomenological point of view, $f(Q, T)$ gravity has the potential to mimic dark energy behavior, generate accelerated expansion, and modify the background evolution even when ordinary matter sources are used. This makes it a useful theoretical framework for comparing purely geometric modifications with matter-driven alternatives.

1.5.4 Comparison of Modified Gravity Theories with General Relativity

General Relativity describes gravitation through the curvature of space-time and is based on the Einstein–Hilbert action, which is linear in the Ricci scalar R . By contrast, $f(Q)$ gravity and $f(Q, T)$ gravity are built within the symmetric teleparallel formulation, where gravity is encoded through nonmetricity rather than curvature. The essential difference is therefore

not only dynamical but also geometric: the fundamental gravitational variable changes from curvature-based dynamics to nonmetricity-based dynamics.

A conceptual comparison between these frameworks is summarized in Table 1.1. General Relativity remains the reference theory because of its strong empirical support and geometric elegance. However, the modified theories provide additional freedom that can be useful in cosmology, especially when addressing accelerated expansion or effective matter-geometry interactions. At the same time, this additional freedom must be constrained carefully, since any viable theory must remain consistent with both local gravitational tests and cosmological observations.

Table 1.1: Conceptual comparison of General Relativity, $f(Q)$ gravity, and $f(Q, T)$ gravity.

Theory	Geometric basis	Action dependence	Cosmological implication
General Relativity [9]	Curvature	R	Standard expansion dynamics
$f(Q)$ gravity [35]	Nonmetricity	$f(Q)$	Geometric modification of expansion
$f(Q, T)$ gravity [37]	Nonmetricity with matter coupling	$f(Q, T)$	Modified expansion and matter–geometry interaction

For the present work, the significance of these theories lies in their ability to alter the cosmic dynamics without abandoning the basic large-scale symmetries of cosmology. Since many of the models studied in this thesis involve exotic fluids and matter creation mechanisms, the comparison between General Relativity, $f(Q)$ gravity, and $f(Q, T)$ gravity helps clarify whether the observed cosmic behavior is better understood as a consequence of modified matter content, modified geometry, or an interplay between the two.

1.6 The Standard Cosmological Model

The standard cosmological model, commonly referred to as the Λ CDM model, provides the simplest phenomenological framework that successfully describes the large-scale evolution of the Universe. It is constructed within the FLRW space-time and assumes that the dominant cosmic contents are radiation, baryonic matter, cold dark matter, and a vacuum-like component associated with the cosmological constant Λ . In this description, the early Universe passes through a radiation-dominated phase, then enters a matter-dominated era favorable for structure formation, and finally evolves toward a stage of accelerated expansion driven by the cosmological constant [3, 38]. Owing to its ability to account for a wide range of cosmological

observations with a relatively small number of parameters, Λ CDM has become the reference model of modern cosmology.

1.6.1 Essential Features of the Λ CDM Model

A basic assumption of the Λ CDM framework is that the total energy density of the Universe can be decomposed into distinct components according to their physical properties. One may write

$$\rho = \rho_r + \rho_b + \rho_{dm} + \rho_\Lambda, \quad (1.35)$$

where ρ_r denotes the radiation density, ρ_b the baryonic matter density, ρ_{dm} the cold dark matter density, and ρ_Λ the constant vacuum energy density. It is often convenient to combine baryonic matter and dark matter into the total matter density,

$$\rho_m = \rho_b + \rho_{dm}. \quad (1.36)$$

For an FLRW Universe, the expansion rate is governed by the Friedmann equation. Expressed in terms of the redshift z , the Hubble parameter in the Λ CDM model takes the form

$$H^2(z) = H_0^2 [\Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{\Lambda0}], \quad (1.37)$$

where H_0 is the present value of the Hubble parameter, Ω_{r0} is the present radiation density parameter, Ω_{m0} is the present matter density parameter, Ω_{k0} is the present curvature density parameter, and $\Omega_{\Lambda0}$ is the present density parameter associated with the cosmological constant. In the spatially flat case, which is strongly preferred by present observations, one has

$$\Omega_{r0} + \Omega_{m0} + \Omega_{\Lambda0} = 1, \quad (1.38)$$

and Eq. (1.37) simplifies accordingly.

The physical significance of Eq. (1.37) lies in the different scaling behaviors of the individual components. Radiation evolves as $(1+z)^4$ because of both volume dilution and redshifting of photon energies. Matter scales as $(1+z)^3$ since its energy density decreases only because of the expansion of volume. By contrast, the vacuum energy density remains constant during expansion. This difference implies that radiation dominates at very early times, matter dominates at intermediate epochs, and the cosmological constant becomes dynamically important only at late times.

The model's other defining feature is how cold dark matter helps structures form. Cold dark matter clusters efficiently under gravity because it is not affected by relativity and does not create pressure. This framework also provides the gravitational conditions necessary for the formation of galaxies and larger cosmic structures. Consequently, the Λ CDM model integrates a straightforward expansion of the cosmos with a matter component that elucidates the large-scale structure we perceive.

1.6.2 Observational Successes of the Λ CDM Framework

The dominant status of the Λ CDM model is primarily due to its broad observational success. Measurements of anisotropies in the cosmic microwave background provide strong evidence for a nearly spatially flat Universe whose energy budget is dominated by non-baryonic dark matter and a dark energy component consistent with a cosmological constant [39]. The angular structure of the microwave background is reproduced with remarkable accuracy within the Λ CDM framework, making it one of the most significant achievements of the model.

Independent support also comes from observations of Type Ia supernovae, which indicate that the expansion of the Universe is presently accelerating. This late-time acceleration is naturally incorporated in the standard model through the presence of Λ , whose effective negative pressure modifies the expansion dynamics at low redshift [2, 40]. In addition, baryon acoustic oscillation measurements provide a standard ruler that traces the cosmic expansion history and yields results that are broadly consistent with the Λ CDM prediction [41, 42].

The model is further supported by large-scale structure observations. The growth of density perturbations predicted in a cold dark matter dominated Universe explains, at leading order, the formation of galaxies, galaxy clusters, and the observed clustering pattern of matter on large scales. Although some tensions and unresolved issues remain, the overall agreement between the Λ CDM model and multiple independent datasets is sufficiently strong that it continues to serve as the standard reference theory for cosmological investigations. Any extension involving exotic fluids, bulk viscosity, matter creation, or modified gravity must therefore be evaluated against the explanatory success already achieved by this framework.

1.7 Open Problems in the Standard Cosmological Model

Although the Λ CDM model provides an excellent phenomenological description of the large-scale Universe, it does not resolve several fundamental issues at the level of physical

interpretation. Its success is therefore empirical rather than fully explanatory. The model requires the existence of dark matter and dark energy, yet the microphysical nature of both components remains unknown. In addition, the vacuum energy interpretation of the cosmological constant leads to severe theoretical difficulties, while recent observations have revealed tensions in some of the most important cosmological parameters. These unresolved issues motivate the study of alternatives in which the late-time dynamics of the Universe may arise from exotic fluids, effective matter creation, dissipative effects, or modifications of gravity [43–45].

1.7.1 The Cosmological Constant Problem

The cosmological constant problem is one of the most serious conceptual difficulties in theoretical cosmology. In General Relativity, the cosmological constant Λ can be introduced geometrically as a constant term in the gravitational field equations. At the same time, quantum field theory suggests that the vacuum itself contributes an energy density, which behaves gravitationally like a cosmological constant. If the vacuum energy density is denoted by ρ_Λ , then its relation to Λ is

$$\rho_\Lambda = \frac{\Lambda}{8\pi G}. \quad (1.39)$$

From the cosmological point of view, the observed value of ρ_Λ is extremely small. However, straightforward estimates of vacuum energy from quantum field theory lead to values that are enormously larger than the observed one. This discrepancy between theoretical expectation and cosmological observation constitutes the cosmological constant problem [43, 46].

The difficulty is not merely numerical; it reflects the absence of a convincing mechanism that naturally explains why the vacuum energy should either vanish almost exactly or remain at a value so small that it only becomes dynamically relevant in the recent Universe. Thus, while the cosmological constant provides the simplest phenomenological explanation for cosmic acceleration, its theoretical status remains deeply unsatisfactory.

1.7.2 The Cosmic Coincidence Problem

A second important issue in the standard model is the coincidence problem. In the Λ CDM framework, the matter density evolves with expansion as

$$\rho_m \propto a^{-3}, \quad (1.40)$$

whereas the vacuum energy density remains constant,

$$\rho_\Lambda = \text{constant.} \quad (1.41)$$

Since these two components have very different evolutionary behaviors, one would generally expect them to differ by many orders of magnitude during most epochs of cosmic history. Yet observations indicate that, at the present epoch, they are of the same order of magnitude. This near equality is puzzling because it suggests that we happen to live at a very special time in the evolution of the Universe [47, 48].

The coincidence problem does not invalidate the standard model observationally, but it raises the question of whether a deeper dynamical mechanism might be responsible for the present accelerated phase. This has motivated models in which the dark energy sector evolves with time, interacts with matter, or emerges effectively from modified cosmic dynamics rather than from a strictly constant vacuum term.

1.7.3 The Unresolved Nature of Dark Matter and Dark Energy

The standard cosmological model is heavily dependent on two components whose physical origins are still unclear. Dark matter helps us understand things that visible matter can't, like how galaxies rotate, how light bends around massive objects, how clusters of galaxies move, and how big structures form. In the conventional model, it is considered to be cold, non-relativistic, and to exhibit weak interactions with ordinary matter. Nevertheless, despite considerable endeavors, no direct non-gravitational detection of dark matter has been validated [49, 50].

Dark energy is an even bigger problem. It is necessary to elucidate the accelerated expansion of the Universe from an observational standpoint, yet its physical origin remains ambiguous. In the simplest interpretation, dark energy is identified with the cosmological constant, corresponding to an equation of state

$$p_{de} = w_{de}\rho_{de}, \quad (1.42)$$

with

$$w_{de} = -1. \quad (1.43)$$

Although this description is consistent with many observations, it offers little insight into why such a component should exist or why its magnitude should take the observed value. As a result, the dark sector remains one of the major unresolved aspects of modern cosmology.

1.7.4 The Hubble and S_8 Tensions

In addition to conceptual difficulties, the standard cosmological model faces notable observational tensions. One of the most discussed is the Hubble tension, which concerns the difference between the value of the present Hubble constant H_0 inferred from early-Universe observations within the Λ CDM model and the value obtained from local distance ladder measurements. Symbolically, one may write

$$H_0^{\text{early}} \neq H_0^{\text{late}}. \quad (1.44)$$

This discrepancy has persisted even as the quality of the data has improved, suggesting either unrecognized systematics or the possibility of new cosmological physics beyond the standard model [51, 52].

A related issue is the S_8 tension, which involves the amplitude of matter clustering inferred from weak lensing and large-scale structure observations compared with that predicted from cosmic microwave background constraints within the Λ CDM framework. The quantity S_8 is commonly defined by

$$S_8 = \sigma_8 \sqrt{\frac{\Omega_{m0}}{0.3}}, \quad (1.45)$$

where σ_8 measures the root-mean-square amplitude of matter fluctuations on the scale of $8h^{-1}\text{Mpc}$. Current measurements indicate a mild but persistent mismatch between low-redshift probes and the Planck-inferred value, adding further pressure on the concordance model [45, 53].

1.7.5 Motivation for Alternative Cosmological Models

The problems outlined above provide strong motivation for exploring alternatives to the standard cosmological model. Such alternatives need not discard the empirical achievements of Λ CDM, but they seek a more satisfactory physical explanation of the observed Universe. Broadly speaking, one may pursue three directions. The first is to replace the cosmological constant by a dynamical dark energy component. The second is to modify the properties of the cosmic fluid through mechanisms such as exotic equations of state, viscous effects, or matter creation. The third is to alter the geometric sector by considering gravitational theories beyond General Relativity.

The present thesis is mainly concerned with the second and third directions. In particular, it

investigates whether Chaplygin-type fluids, bulk viscous effects, and gravitationally induced matter creation can provide a viable description of cosmic acceleration in both General Relativity and modified gravity. The motivation for this program is that effective negative pressure may arise from physically richer mechanisms than a simple constant vacuum term, potentially alleviating some of the conceptual and observational shortcomings of the standard cosmological model.

Table 1.2: Major open issues in the standard cosmological model and their physical significance.

Problem	Physical significance
Cosmological constant problem	Observed vacuum energy is far smaller than theoretical estimates.
Cosmic coincidence problem	Matter and dark energy densities are comparable only at the present epoch.
Unknown dark matter	Dominant clustering component lacks confirmed microscopic identification.
Unknown dark energy	Accelerated expansion requires a component of unclear physical origin.
H_0 tension	Early- and late-Universe estimates of the Hubble constant disagree.
S_8 tension	Low-redshift clustering amplitude differs from CMB-based expectation.

1.8 Exotic Fluids in Modern Cosmology

Exotic fluids are effective cosmic media whose thermodynamic behavior differs from that of ordinary matter and radiation. Their importance in cosmology arises from the fact that a sufficiently negative pressure can modify the expansion history of the Universe and, in particular, can drive late-time accelerated expansion. Unlike the cosmological constant, which is characterized by a strictly fixed equation of state, exotic fluid models often possess a nontrivial dynamical structure and can interpolate between matter-like and dark-energy-like phases during cosmic evolution. Among the various proposals in the literature, Chaplygin-type fluids occupy a special place because they provide a unified phenomenological framework in which one can model both the decelerated and accelerated stages of the Universe within a single effective fluid description [54, 55].

1.8.1 Chaplygin Gas Cosmologies

The Chaplygin gas family forms one of the most widely studied classes of exotic fluids in cosmology. Its appeal lies in the fact that the corresponding equations of state naturally generate negative pressure at late times while retaining matter-like behavior in the early Universe. Historically, the Chaplygin equation of state did not arise in cosmology. It was introduced by Sergey A. Chaplygin in the context of classical fluid mechanics and aerodynamics, particularly in studies related to the motion of compressible fluids and gas flow [56]. Much later, this unusual equation of state was recognized as cosmologically interesting because it can mimic a transition from a dust-dominated epoch to an accelerated era. Since then, several generalizations have been proposed in order to obtain greater flexibility and better compatibility with observations.

1.8.1.1 Chaplygin Gas

The originally Chaplygin gas is defined by the equation of state [57]

$$p_{cg} = -\frac{A}{\rho_{cg}}, \quad (1.46)$$

where $A > 0$ is a constant. This equation is remarkable because the pressure is inversely proportional to the energy density, which implies that the fluid behaves very differently from ordinary barotropic matter. In a homogeneous and isotropic Universe, combining Eq. (1.46) with the continuity equation, Eq. (1.13), yields the energy density

$$\rho_{cg}(a) = \left(A + \frac{C_1}{a^6} \right)^{1/2}, \quad (1.47)$$

where C_1 is an integration constant.

Equation (1.47) shows that the Chaplygin gas interpolates between two distinct regimes. For small scale factor a , the second term dominates and the energy density behaves as $\rho \propto a^{-3}$, which corresponds to pressureless matter. For large a , the first term dominates and the density approaches a constant value, leading to an effective dark-energy-like phase. Because of this transition, the original Chaplygin gas was regarded as an attractive candidate for unified dark matter and dark energy. However, when confronted with observations of large-scale structure and cosmic microwave background anisotropies, the simplest form of the model is found to be too restrictive, which motivated the construction of more general versions [58, 59].

1.8.1.2 Generalized Chaplygin Gas

A natural extension of the original model is the Generalized Chaplygin Gas, whose equation of state is [6]

$$p_{gcg} = -\frac{A}{\rho_{gcg}^\alpha}, \quad (1.48)$$

where $A > 0$ and α is a real parameter. The choice $\alpha = 1$ reduces Eq. (1.48) to the original Chaplygin gas. This additional parameter increases the flexibility of the model and allows a broader range of cosmological behavior. Integrating the continuity equation gives

$$\rho_{gcg}(a) = \left(A + \frac{C_2}{a^{3(1+\alpha)}} \right)^{\frac{1}{1+\alpha}}, \quad (1.49)$$

where C_2 is an integration constant.

The Generalized Chaplygin Gas preserves the key feature of the original model, namely the transition from matter-dominated dynamics at early times to negative-pressure behavior at late times. Because the parameter α controls the rate and character of this transition, the model became an important phenomenological tool in dark sector cosmology. It has been widely studied in relation to background expansion, structure growth, and observational constraints.

1.8.1.3 Variable Chaplygin Gas

In the Variable Chaplygin Gas model, the constant A in Eq. (1.46) is replaced by a scale-factor-dependent function. The equation of state is then written as [60]

$$p_{vcg} = -\frac{A(a)}{\rho_{vcg}}. \quad (1.50)$$

A commonly adopted form is

$$A(a) = A_0 a^{-n}, \quad (1.51)$$

where $A_0 > 0$ and n is a constant parameter. For this choice, the continuity equation leads to

$$\rho_{vcg}(a) = \left[\frac{6A_0}{6-n} a^{-n} + \frac{C_3}{a^6} \right]^{1/2}, \quad n \neq 6, \quad (1.52)$$

with C_3 an integration constant.

The significance of the Variable Chaplygin Gas lies in the extra freedom introduced through the function $A(a)$. This freedom allows the transition between matter-like and accelerating phases to be adjusted more flexibly than in the original model. As a result, the variable version

has been explored as a way to improve compatibility with cosmological data and to model a wider class of late-time dynamics.

1.8.1.4 Variable Generalized Chaplygin Gas

A further extension combines the generalized form with a variable coefficient. In this case the equation of state becomes [61, 62]

$$p_{vgcg} = -\frac{A(a)}{\rho_{vgcg}^\alpha}, \quad (1.53)$$

where again one often takes

$$A(a) = A_0 a^{-m}, \quad (1.54)$$

with $A_0 > 0$ and m constant. The corresponding energy density is then obtained as

$$\rho_{vgcg}(a) = \left[\frac{3(1+\alpha)A_0}{3(1+\alpha)-m} a^{-m} + \frac{C_4}{a^{3(1+\alpha)}} \right]^{\frac{1}{1+\alpha}}, \quad m \neq 3(1+\alpha), \quad (1.55)$$

where C_4 is an integration constant.

The Variable Generalized Chaplygin Gas is especially important because it combines the interpolation property of the generalized model with an explicitly evolving coefficient in the negative-pressure term. This makes it more adaptable to observational fitting and especially useful in extended cosmological scenarios involving matter creation, dissipative effects, or modified gravity. For research programs such as the present thesis, this model provides a particularly useful framework because its parameters can encode deviations from the simplest dark-energy behavior while retaining a unified fluid interpretation.

1.8.1.5 Modified Chaplygin Gas

Another important member of the Chaplygin family is the Modified Chaplygin Gas, whose equation of state is [63, 64]

$$p_{mcg} = A\rho - \frac{B}{\rho_{mcg}^\alpha}, \quad (1.56)$$

where A , B , and α are constants with $B > 0$. The linear barotropic term $A\rho$ allows the fluid to resemble standard matter or radiation in appropriate limits, while the inverse power term generates negative pressure at late times. Solving the continuity equation gives

$$\rho_{mcg}(a) = \left[\frac{B}{1+A} + \frac{C_5}{a^{3(1+A)(1+\alpha)}} \right]^{\frac{1}{1+\alpha}}, \quad A \neq -1, \quad (1.57)$$

where C_5 is an integration constant.

The Modified Chaplygin Gas is often regarded as more flexible than both the original and generalized models because it can describe a broader range of intermediate cosmic behavior. It has therefore been applied extensively in models of late-time acceleration, unified dark sector cosmology, and scenarios involving matter creation and bulk viscosity. In many applications, it provides a useful balance between mathematical simplicity and phenomenological richness.

1.8.1.6 Other Extensions of the Chaplygin Family

Beyond the models discussed above, the literature contains several additional Chaplygin-type extensions designed to increase phenomenological freedom. Such extensions include the Generalized Cosmic Chaplygin Gas, the New Variable Modified Chaplygin Gas, the Extended Chaplygin Gas, as well as viscous and interacting versions of Chaplygin gas models. Although their exact formulations differ, most of them can be viewed as refinements of the basic inverse-density pressure structure by allowing extra nonlinear terms, scale-factor dependence, or couplings to other cosmic sectors.

A broad schematic form that encompasses many such extensions may be written as [65]

$$p = \sum_{i=1}^N A_i \rho^i - \frac{B}{\rho^\alpha}, \quad (1.58)$$

where the coefficients A_i and the B determine the specific model. In viscous generalizations, the effective pressure is often modified further according to

$$\bar{p} = p - 3\xi H, \quad (1.59)$$

where ξ is the bulk viscosity coefficient. Interacting models may also modify the continuity equation by introducing exchange terms between the Chaplygin fluid and another cosmic component. These generalizations are useful because they allow one to test whether the observed cosmic expansion can be understood more successfully through richer fluid physics than is possible in the simplest Λ CDM scenario.

From the cosmological perspective, the importance of the Chaplygin family lies in its ability to generate negative pressure dynamically while preserving a matter-like regime in the early Universe. This feature makes Chaplygin-type fluids especially suitable for investigations of dark sector unification, late-time acceleration, and effective cosmic dynamics beyond the standard model. In the present thesis, these models are not treated merely as mathematical alternatives; rather, they are studied in conjunction with matter creation, dissipative processes,

Table 1.3: Major Chaplygin-type equations of state used in cosmology.

Model	Equation of state
Original Chaplygin Gas	$p = -\frac{A}{\rho}$ [57]
Generalized Chaplygin Gas	$p = -\frac{A}{\rho^\alpha}$ [6]
Variable Chaplygin Gas	$p = -\frac{A(a)}{\rho}$ [60]
Variable Generalized Chaplygin Gas	$p = -\frac{A(a)}{\rho^\alpha}$ [61, 62]
Modified Chaplygin Gas	$p = A\rho - \frac{B}{\rho^\alpha}$ [63, 64]
Extended forms	$p = \sum_{i=1}^N A_i \rho^i - \frac{B}{\rho^\alpha}$ [65]

and modified gravity in order to test whether such combined mechanisms can provide a more satisfactory description of cosmic evolution.

1.9 Matter Creation Cosmology

In cosmology, the creation of matter provides a unique framework. In this framework, the history of the Universe's expansion is influenced by the irreversible process of particle production. This approach regards the cosmic medium as an open thermodynamic system rather than a perfectly conserved fluid. The main idea is that the expanding Universe's gravitational field might cause particles to form, which would change the effective pressure and the normal cosmological dynamics. This possibility is especially relevant for late-time cosmology because a suitable creation process can generate an effective negative pressure without introducing a separate dark energy component. For this reason, matter creation models have been studied as viable candidates for explaining accelerated expansion and for enriching the thermodynamic description of the cosmic medium [66–69].

Table 1.4: Basic quantities in matter creation cosmology and their physical meaning.

Quantity	Physical meaning
n	Particle number density
Γ	Particle creation rate
p_c	Creation pressure
p_{eff}	Effective total pressure including creation effects
σ	Entropy per particle
N^μ	Particle flux vector

1.9.1 Particle Creation in an Expanding Universe

The idea that particles may be created by the gravitational field in an expanding space-time has its roots in quantum field theory in curved backgrounds. In a time-dependent gravitational field, the notion of vacuum becomes observer dependent, and the expansion of the Universe can lead to the production of particles even in the absence of conventional interactions. This phenomenon was first explored in detail for quantum fields in expanding cosmological backgrounds and later inspired macroscopic cosmological models in which the created particles are described through an effective fluid formalism [66, 67].

At the phenomenological level, particle creation is introduced by allowing the particle number in a comoving volume to vary with time. If n denotes the particle number density and $N^\mu = nu^\mu$ is the particle flux vector, then the balance law for particle number is written as

$$\nabla_\mu N^\mu = n\Gamma, \quad (1.60)$$

where Γ is the particle creation rate. In an FLRW Universe, Eq. (1.60) reduces to

$$\dot{n} + 3Hn = n\Gamma. \quad (1.61)$$

When $\Gamma > 0$, particles are created during the expansion; when $\Gamma = 0$, the standard conserved particle number relation is recovered. Thus, the creation rate provides a direct measure of deviation from the usual closed-system cosmological evolution.

The significance of Eq. (1.61) lies in the fact that the increase in particle number is tied to the expansion rate of the Universe. In cosmological applications, Γ is often modeled as a function of H or of other dynamical quantities, allowing one to describe a wide range of particle production histories. This macroscopic description does not attempt to resolve the microscopic origin of the created particles in each case; rather, it captures the effective dynamical consequences of an open cosmological system.

1.9.2 Thermodynamic Formulation of Matter Creation

The thermodynamic formulation of matter creation is based on the treatment of the cosmic fluid as an open system in which the number of particles is not fixed. In such a system, the usual conservation laws must be supplemented by a particle balance equation and by a generalized entropy law. Let ρ be the energy density, p the equilibrium pressure, n the particle number

density, T the temperature, and σ the entropy per particle. The Gibbs relation for the cosmic fluid can be written as

$$nTd\sigma = d\rho - \frac{\rho + p}{n}dn. \quad (1.62)$$

This equation connects the system's thermodynamic changes with changes in energy density and particle number density.

If the entropy flux is denoted by $S^\mu = n\sigma u^\mu$, then the second law of thermodynamics requires

$$\nabla_\mu S^\mu \geq 0. \quad (1.63)$$

In adiabatic matter creation, the entropy per particle remains constant, that is,

$$\dot{\sigma} = 0, \quad (1.64)$$

The total entropy still goes up, though, because the number of particles keeps going up as the system expands. This is an important point: adiabatic particle creation doesn't mean that no entropy is made; it means that the specific entropy stays the same while the total entropy goes up due to the growth of the system.

Under this assumption, the thermodynamic effect of particle production can be incorporated into the cosmological dynamics through an additional effective pressure term. This makes the theory particularly convenient because one can study particle creation within the familiar language of relativistic fluid cosmology while maintaining consistency with nonequilibrium thermodynamics [8, 70].

1.9.3 Creation Pressure and Modified Conservation Laws

In matter creation cosmology, particle production gives rise to an effective negative pressure, usually called the creation pressure. For adiabatic particle creation, this pressure is given by

$$p_c = -\frac{\rho + p}{3H}\Gamma. \quad (1.65)$$

The origin of this term is thermodynamic: when new particles are produced in an expanding volume, the energy required for their creation affects the macroscopic dynamics in a way that is equivalent to an additional pressure contribution. Since $\Gamma > 0$ generally implies $p_c < 0$, particle creation can oppose the usual decelerating effect of ordinary matter.

The total effective pressure of the cosmic fluid is then

$$p_{\text{eff}} = p + p_c. \quad (1.66)$$

Consequently, the standard continuity equation is modified to

$$\dot{\rho} + 3H(\rho + p + p_c) = 0. \quad (1.67)$$

Substituting Eq. (1.65) into Eq. (1.67), one obtains

$$\dot{\rho} + 3H(\rho + p) = (\rho + p)\Gamma. \quad (1.68)$$

This equation makes clear that matter creation acts as an effective source term in the energy balance of the cosmic medium.

The modified conservation law shows that the energy density no longer evolves as in the standard closed-system picture. Instead, the rate of dilution due to expansion is partly compensated by the creation process. For pressureless matter, $p = 0$, the above relation reduces to

$$\dot{\rho} + 3H\rho = \rho\Gamma. \quad (1.69)$$

Hence, particle creation can significantly affect the late-time matter content and thereby influence the expansion history of the Universe.

1.9.4 Matter Creation as a Mechanism for Cosmic Acceleration

One of the main reasons cosmologists are interested in how matter is made is that the pressure that comes with it can cause faster expansion. Within the FLRW framework, the equation governing acceleration is expressed as follows:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p + 3p_c) + \frac{\Lambda}{3}. \quad (1.70)$$

If the creation pressure is sufficiently negative, the expression $\rho + 3p + 3p_c$ may become sufficiently tiny or even negative, resulting in an accelerated phase. Thus, matter creation offers an alternative route to cosmic acceleration without requiring a conventional dark energy component.

For a pressureless matter-dominated Universe with $\Lambda = 0$, the condition for acceleration

becomes

$$\Gamma > H. \quad (1.71)$$

This fundamental relationship illustrates the significance of the creation rate in the actual world: if particle generation is sufficiently robust relative to the expansion rate, the resultant effective negative pressure can induce rapid expansion. In more realistic cosmological models, Γ is often parameterized in forms such as $\Gamma \propto H$, $\Gamma \propto H_0$, or $\Gamma \propto 1/H$, depending on the epoch and the phenomenological goals of the model.

Matter creation cosmology is especially relevant for the present thesis because it can be naturally combined with exotic equations of state such as Chaplygin-type fluids and with modified gravity frameworks. In such combined scenarios, cosmic acceleration may arise not from a single mechanism but from the interplay between particle production, non-standard fluid behavior, and geometric modifications. This makes matter creation an important and versatile ingredient in the search for viable cosmological models beyond the standard Λ CDM paradigm.

1.10 Bulk Viscous Cosmology

In relativistic cosmology, the matter content of the Universe is often modeled as a perfect fluid. Such an approximation is useful at the background level, but it may not always be sufficient when nonequilibrium processes are important. A more realistic description can include dissipative effects, among which bulk viscosity is the only one compatible with large-scale homogeneity and isotropy. Shear viscosity and heat flow introduce preferred directions and therefore vanish in an FLRW background, whereas bulk viscosity modifies only the effective pressure of the cosmic fluid and can influence the expansion dynamics without violating the cosmological principle [71, 72].

The cosmological interest in bulk viscosity comes from the fact that it naturally generates a negative contribution to the effective pressure. Because accelerated expansion requires sufficiently negative pressure, viscous effects can mimic dark-energy-like behavior even when the equilibrium fluid itself is not exotic. This makes bulk viscous cosmology a useful framework for studying alternatives to the standard Λ CDM description and for constructing models in which late-time acceleration emerges from the internal thermodynamic properties of the cosmic medium.

1.10.1 Effective Bulk Viscous Pressure in FLRW Cosmology

In a homogeneous and isotropic Universe, the pressure of a viscous cosmic fluid is written as

$$\bar{p} = p + \Pi, \quad (1.72)$$

where p is the equilibrium pressure and Π denotes the bulk viscous pressure. For an expanding Universe, the viscous contribution is generally negative and therefore acts against the standard decelerating influence of ordinary matter.

In the simplest phenomenological treatment, the bulk viscous pressure is taken to be proportional to the hubble parameter. Since, for the FLRW space-time, one obtains

$$\Pi = -3\xi H, \quad (1.73)$$

where ξ is the coefficient of bulk viscosity. The condition

$$\xi \geq 0 \quad (1.74)$$

is usually imposed in order to ensure consistency with the second law of thermodynamics. Equation (1.73) shows directly that bulk viscosity contributes a negative pressure whenever the Universe expands with $H > 0$.

Once viscous effects are included, the continuity equation for the cosmic fluid is modified to

$$\dot{\rho} + 3H(\rho + p + \Pi) = 0. \quad (1.75)$$

Substituting Eq. (1.73) into Eq. (1.75), one finds

$$\dot{\rho} + 3H(\rho + p - 3\xi H) = 0. \quad (1.76)$$

This relation makes clear that the viscous contribution changes the dilution law of the cosmic fluid and therefore modifies the background expansion history.

In many cosmological applications, the bulk viscosity coefficient is parameterized as a function of the energy density or the Hubble parameter. A frequently used form is

$$\xi = \xi_0 \rho^m, \quad (1.77)$$

where ξ_0 and m are constants. This choice provides sufficient flexibility to describe different dissipative regimes while keeping the model mathematically tractable.

1.10.2 Bulk Viscosity and Cosmic Acceleration

The main importance of bulk viscosity in late-time cosmology is that it can cause faster expansion by lowering effective pressure. In the presence of a viscous fluid, the acceleration equation can be expressed as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p + 3\Pi) + \frac{\Lambda}{3}. \quad (1.78)$$

because Π is negative, In an expanding Universe, the viscous term tends to decrease the quantity $\rho + 3p + 3\Pi$, if its magnitude is big enough, the right side of Eq. (1.78) can become positive, which speeds up the phase.

It is convenient to introduce the effective equation-of-state parameter in the form

$$w_{\text{eff}} = \frac{p_{\text{eff}}}{\rho} = \frac{p + \Pi}{\rho}. \quad (1.79)$$

Acceleration occurs when

$$w_{\text{eff}} < -\frac{1}{3}. \quad (1.80)$$

Thus, even a fluid that would not produce acceleration in equilibrium can effectively behave like dark energy once bulk viscous effects are taken into account.

For the present thesis, bulk viscosity is important not merely as an isolated dissipative effect but as a mechanism that can work together with exotic equations of state and matter creation processes. In Chaplygin-type cosmologies, for example, the viscous term enriches the effective pressure sector and can alter both the timing and nature of the transition from decelerated to accelerated expansion. This makes bulk viscous cosmology especially relevant for models that seek to describe the late Universe without relying exclusively on a pure cosmological constant.

1.11 Cosmographic Parameters

Cosmographic parameters provide a model-independent way to characterize the kinematics of cosmic expansion through successive derivatives of the scale factor with respect to cosmic time. Unlike dynamical approaches that require a specific gravitational theory or a prescribed matter sector, cosmography focuses directly on the expansion history itself. For this reason, it

is particularly useful when one wishes to compare different cosmological models on a common observational basis. In the present thesis, cosmographic quantities are relevant because they help identify whether models based on exotic fluids, matter creation, bulk viscosity, or modified gravity produce an expansion history consistent with observational expectations [75, 76].

1.11.1 Jerk Parameter

The jerk parameter, which is related to the third derivative of the scale factor, offers a more complete description of the universe's expansion than the deceleration parameter.

It is defined as

$$j = \frac{\ddot{a}}{aH^3}. \quad (1.81)$$

This parameter is useful because it distinguishes different accelerated expansion histories even when the Hubble and deceleration parameters take similar values.

In terms of redshift, the jerk parameter can be written as

$$j(z) = q(z)(2q(z) + 1) + (1 + z) \frac{dq(z)}{dz}. \quad (1.82)$$

In particular, the Λ CDM model in a spatially flat background is characterized by the constant value

$$j = 1. \quad (1.83)$$

Hence, deviations of j from unity may be interpreted as departures from the standard cosmological model. This makes the jerk parameter an important kinematical diagnostic in the study of alternative cosmologies.

1.11.2 Snap Parameter

The snap parameter involves the fourth derivative of the scale factor and provides a still higher-order characterization of the expansion history. It is defined by

$$s = \frac{\dddot{a}}{aH^4}. \quad (1.84)$$

Since it depends on a higher derivative than the jerk parameter, it captures more subtle features of the time evolution of cosmic acceleration.

In terms of redshift, the snap parameter is given by

$$s(z) = j(z)(3q(z) + 2) + (1 + z) \frac{dj(z)}{dz} - 3q^2(z). \quad (1.85)$$

Although the snap parameter is less directly constrained by observations than H , q , or j , it becomes useful when comparing refined cosmological models or when performing cosmographic expansions to higher order. In cosmological models that use non-standard effective pressure, such as those involving viscosity or the creation of matter, the snap parameter can help identify deviations from the usual expansion behavior. These deviations might not be immediately obvious when looking at lower-order terms.

1.11.3 Statefinder Diagnostic

The statefinder diagnostic was introduced as a geometrical tool for distinguishing dark energy models through higher derivatives of the scale factor. It is based on a pair of dimensionless quantities $\{r, s\}$, defined as

$$r = \frac{\ddot{a}}{aH^3}, \quad (1.86)$$

and

$$s = \frac{r - 1}{3 \left(q - \frac{1}{2}\right)}. \quad (1.87)$$

Thus, the parameter r is identical to the jerk parameter, while s provides an additional normalized combination that helps classify different cosmological models.

The statefinder pair is especially useful because distinct cosmological scenarios trace different trajectories in the (r, s) plane. For the spatially flat Λ CDM model, one obtains the fixed point

$$(r, s) = (1, 0). \quad (1.88)$$

This point serves as a convenient reference for comparison. Cosmological models involving Chaplygin-type fluids, bulk viscosity, matter creation, or modified gravity generally deviate from this fixed point and may exhibit characteristic evolutionary paths. Therefore, the statefinder diagnostic offers a compact way to compare the kinematical behavior of different models beyond the Hubble and deceleration parameters alone [77].

1.11.4 Om Diagnostic

The Om diagnostic is another useful geometrical tool for testing dark energy models through the redshift dependence of the Hubble parameter. For a spatially flat Universe, it is defined as

$$Om(z) = \frac{\left[\frac{H(z)}{H_0} \right]^2 - 1}{(1+z)^3 - 1}. \quad (1.89)$$

Its importance lies in the fact that it depends only on the first derivative of the scale factor through the Hubble parameter and is therefore simpler to reconstruct observationally than higher-order diagnostics.

For the Λ CDM model, the Om diagnostic reduces to the present matter density parameter,

$$Om(z) = \Omega_{m0}, \quad (1.90)$$

which is constant with redshift. Accordingly, any significant redshift dependence of $Om(z)$ indicates a deviation from the standard cosmological model. A positive or negative slope in the Om curve may signal different classes of dark energy or modified expansion histories. This makes the Om diagnostic particularly useful when comparing observational consequences of alternative models without committing to a specific microscopic interpretation of dark energy [78].

Taken together, the jerk parameter, snap parameter, statefinder pair, and Om function provide a useful kinematical framework for analyzing cosmological models. In the context of the present thesis, these diagnostics help assess whether models based on exotic fluids and modified gravity reproduce the standard expansion history or generate observationally distinguishable departures from it.

1.12 Observational Data Sets

The viability of any cosmological model is ultimately determined by its consistency with observational data. In modern cosmology, theoretical predictions are commonly tested against multiple background and perturbation probes, since no single data set is sufficient to constrain all aspects of cosmic evolution. For the models studied in this thesis, observational analysis is especially important because exotic fluids, matter creation, bulk viscosity, and modified gravity may reproduce the observed expansion history in different ways while still leading to

distinguishable signatures in the growth of structure. Accordingly, a combination of geometric and dynamical data sets is often used to obtain meaningful parameter constraints and to assess whether a proposed model can compete with the standard cosmological scenario.

1.12.1 Type Ia Supernova Observations

Type Ia supernovae are among the most important probes of late-time cosmic expansion because they act as standardizable candles. Their observational significance comes from the relation between the apparent magnitude and the luminosity distance, which allows one to trace the expansion history over a wide redshift range. The central quantity used in supernova analysis is the distance modulus,

$$\mu(z) = m - M = 5 \log_{10} \left(\frac{d_L(z)}{\text{Mpc}} \right) + 25, \quad (1.91)$$

where m is the apparent magnitude, M is the absolute magnitude, and $d_L(z)$ is the luminosity distance.

For a spatially flat FLRW Universe, the luminosity distance is written as

$$d_L(z) = (1+z) \int_0^z \frac{dz'}{H(z')}, \quad (1.92)$$

where units with $c = 1$ are assumed. By comparing the theoretical distance modulus with the observed supernova compilation, one can constrain the background expansion law of a cosmological model. Type Ia supernova observations played a decisive role in establishing the accelerated expansion of the Universe and continue to be one of the most important low-redshift probes in cosmological parameter estimation [1, 79].

1.12.2 Cosmic Chronometer Data

Cosmic chronometers provide a direct method for measuring the Hubble parameter as a function of redshift. The method is based on the differential aging of passively evolving galaxies and does not require the integration of distance relations. The Hubble parameter is obtained from

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt}. \quad (1.93)$$

This expression shows that if one can estimate dz/dt observationally, then one can reconstruct the expansion rate directly.

The main advantage of cosmic chronometer data is that they probe $H(z)$ without assuming a specific cosmological model for the integrated distance ladder. For this reason, they are widely used in the observational analysis of non-standard cosmological scenarios, including models with exotic fluids and modified gravity. Since many of the models studied in this thesis predict modified Hubble evolution, cosmic chronometer measurements are particularly useful in constraining their background dynamics [80, 81].

1.12.3 Baryon Acoustic Oscillation Measurements

Baryon acoustic oscillations arise from sound waves in the coupled photon-baryon plasma before recombination. The corresponding characteristic scale survives in the late-time matter distribution and acts as a standard ruler for cosmology. BAO observations therefore provide geometric information about the expansion history of the Universe and help constrain both the angular diameter distance and the Hubble parameter.

A commonly used distance measure in isotropic BAO analysis is the volume-averaged distance

$$D_V(z) = \left[\frac{z}{H(z)} \left(\int_0^z \frac{dz'}{H(z')} \right)^2 \right]^{1/3}, \quad (1.94)$$

where units with $c = 1$ are assumed. In anisotropic analyses, one may instead use the transverse comoving distance and the radial quantity $H(z)$. Because BAO measurements are relatively robust and based on a standard ruler set by early-Universe physics, they are among the most valuable probes for testing late-time cosmological models [41].

1.12.4 Growth Rate Data

While supernovae, chronometers, and BAO mainly constrain the background expansion, growth rate data probe the formation of cosmic structure. This makes them particularly useful for distinguishing between models that share similar expansion histories but differ in their perturbation behavior. The growth rate is usually defined as

$$f(a) = \frac{d \ln \delta_m}{d \ln a}, \quad (1.95)$$

where δ_m denotes the matter density contrast.

Observationally, growth measurements are often reported in the form $f\sigma_8(z)$, where

$$f\sigma_8(z) = f(z) \sigma_8(z). \quad (1.96)$$

Here $\sigma_8(z)$ describes the amplitude of matter fluctuations on the scale of $8h^{-1}$ Mpc. Since exotic fluids and modified gravity may affect not only the background expansion but also the rate of structure growth, $f\sigma_8$ data provide an important complementary test for the cosmological scenarios studied in this thesis [82].

1.12.5 Hubble Constant Priors (Local H_0)

In cosmological parameter estimation, it is common to include an external prior on the present Hubble constant H_0 . Such priors are usually derived from local measurements based on the cosmic distance ladder and may be incorporated into the likelihood analysis as Gaussian constraints. If H_0^{obs} denotes the observed local value and σ_{H_0} its uncertainty, then the corresponding chi-square contribution is written as

$$\chi_{H_0}^2 = \frac{(H_0 - H_0^{\text{obs}})^2}{\sigma_{H_0}^2}. \quad (1.97)$$

The inclusion of a local H_0 prior is especially relevant in the present era because of the well-known Hubble tension between early- and late-Universe determinations of the expansion rate. For alternative cosmological models, such priors help determine whether the modified dynamics can shift the preferred parameter region toward locally measured values of H_0 [52].

1.12.6 Gamma-Ray Bursts

Gamma-ray bursts are among the most luminous events observed in the Universe and can be detected at much higher redshifts than Type Ia supernovae. For this reason, they are sometimes used as high-redshift standardizable candles in cosmological studies. Their use is based on empirical correlations between spectral and luminosity-related quantities, which allow one to infer an effective luminosity distance.

Once calibrated, GRB observations may be expressed through a distance modulus relation similar to that used for supernovae,

$$\mu_{\text{GRB}}(z) = 5 \log_{10} \left(\frac{d_L(z)}{\text{Mpc}} \right) + 25. \quad (1.98)$$

Although GRBs are subject to larger systematic uncertainties than supernovae, they are useful because they extend cosmological tests to substantially larger redshifts. This makes them valuable in examining models whose deviations from the standard picture become more pro-

nounced away from the recent Universe [83, 84].

1.12.7 Standard Candles

Standard candles are useful because they provide independent luminosity-based tests of cosmological distance relations. In a general sense, any calibrated source with known or standardizable intrinsic luminosity may be used through the distance modulus relation already introduced in Eq. (1.91).

The importance of such data sets lies in the fact that they complement supernova observations and, in some cases, reach different redshift ranges. When combined with supernovae, BAO, chronometers, and growth measurements, they can help break parameter degeneracies and provide a broader test of non-standard cosmological models. In practical applications, the exact form of the likelihood depends on the specific standard-candle sample used.

1.13 Statistical Model Selection Criteria

In cosmological analysis, the agreement of a model with observational data is not by itself sufficient to establish its reliability. A model with a larger number of free parameters may fit the data better simply because it is more flexible. For this reason, model comparison requires criteria that balance goodness of fit against model complexity. In practice, this is especially important in cosmology, where competing scenarios such as Λ CDM, exotic fluid models, matter creation cosmologies, bulk viscous models, and modified gravity theories may all reproduce the observed background evolution with different numbers of parameters. Statistical model selection criteria therefore provide a systematic way to decide whether an improved fit justifies the introduction of additional freedom.

1.13.1 Akaike Information Criterion (AIC)

A common way to compare models is to use the Akaike Information Criterion. This method, which is based on information theory, checks how well a model fits the data and how many parameters it has to see how good it is.

The AIC is defined as [85]

$$\text{AIC} = -2 \ln \mathcal{L}_{\max} + 2k, \quad (1.99)$$

where $\mathcal{L}_{\max}$ is the maximum likelihood and k is the number of free parameters in the model.

This expression is often written as

$$\text{AIC} = \chi_{\min}^2 + 2k. \quad (1.100)$$

The model with the lower AIC value is the one that is better. In practice, comparison is performed through the difference

$$\Delta\text{AIC}_i = \text{AIC}_i - \text{AIC}_{\min}, \quad (1.101)$$

where $\text{AIC}_{\min}$ is the minimum value among the competing models. A smaller ΔAIC indicates stronger support for the corresponding model. The AIC is particularly useful when the objective is predictive performance, although it does not explicitly incorporate the sample-size dependence in the complexity penalty.

1.13.2 Bayesian Information Criterion (BIC)

The Bayesian Information Criterion is a prevalent method for model comparison. It penalizes complexity more than the AIC, particularly for large datasets, and is given as [86]

$$\text{BIC} = -2\ln\mathcal{L}_{\max} + k\ln N, \quad (1.102)$$

where N is the total number of data points. For Gaussian likelihoods, Eq. (1.102) becomes

$$\text{BIC} = \chi_{\min}^2 + k\ln N. \quad (1.103)$$

As in the case of AIC, smaller values of BIC are preferred. The comparison is usually made through

$$\Delta\text{BIC}_i = \text{BIC}_i - \text{BIC}_{\min}. \quad (1.104)$$

Because the penalty term $k\ln N$ grows with the size of the data set, BIC tends to favor simpler models more strongly than AIC. In cosmology, this often makes BIC a stricter criterion when comparing extended models with additional parameters against the standard Λ CDM scenario.

1.13.3 Bayesian Evidence and Bayes Factors

The Bayesian evidence, also known as the marginal likelihood, is the basis for a more complete Bayesian approach to comparing models. The evidence for a model M with parameter

vector θ is defined as

$$\mathcal{Z} = P(D|M) = \int \mathcal{L}(\theta) \pi(\theta) d\theta, \quad (1.105)$$

where $\mathcal{L}(\theta) = P(D|\theta, M)$ represents the likelihood, $\pi(\theta) = P(\theta|M)$ denotes the prior, and D signifies the observational data. The evidence measures the overall probability of the data under the model after averaging over the full prior parameter space.

The comparison of two models M_i and M_j is made through the Bayes factor [86–88]

$$B_{ij} = \frac{\mathcal{Z}_i}{\mathcal{Z}_j}. \quad (1.106)$$

In logarithmic form, one often uses

$$\ln B_{ij} = \ln \mathcal{Z}_i - \ln \mathcal{Z}_j. \quad (1.107)$$

If $B_{ij} > 1$, then the data favor model M_i over model M_j ; if $B_{ij} < 1$, the opposite is true. Unlike AIC, BIC, and DIC, Bayesian evidence incorporates the full prior structure and naturally penalizes unnecessary parameter freedom through the integration over parameter space. This makes it a powerful tool for cosmological model comparison, especially when one wishes to assess whether the increased flexibility of an extended model is genuinely supported by the data. The table 1.5 represents the common interpretation of model selection differences and Bayes factors.

Table 1.5: Common interpretation of model selection differences and Bayes factors.

Quantity	Typical interpretation
$0 \leq \Delta\text{AIC}, \Delta\text{BIC} < 2$	Weak or negligible preference
$2 \leq \Delta\text{AIC}, \Delta\text{BIC} < 6$	Positive preference against the larger value
$6 \leq \Delta\text{AIC}, \Delta\text{BIC} < 10$	Strong preference against the larger value
$\Delta\text{AIC}, \Delta\text{BIC} \geq 10$	Very strong preference against the larger value
$0 < \ln B_{ij} < 1$	Inconclusive evidence for M_i
$1 < \ln B_{ij} < 3$	Positive evidence for M_i
$3 < \ln B_{ij} < 5$	Strong evidence for M_i
$\ln B_{ij} > 5$	Very strong evidence for M_i

For the present thesis, these criteria are useful because the cosmological models under consideration often differ in both physical content and parameter number. A good fit alone does not guarantee that a model is preferable. The final assessment must therefore account for whether the improvement in likelihood compensates for the additional complexity introduced by exotic fluids, matter creation terms, viscous parameters, or modified gravity functions.

1.14 Research Motivation and Objectives

1.14.1 Research Motivation

Modern cosmology has reached a stage where the observational success of the standard Λ CDM framework cannot conceal its unresolved conceptual and phenomenological limitations. The cosmological constant problem, the coincidence problem, the uncertain physical nature of dark matter and dark energy, and the persistent tensions in key cosmological parameters together suggest that the standard picture may be incomplete. These difficulties motivate the search for alternative cosmological descriptions in which late-time acceleration emerges from physically richer mechanisms than a constant vacuum term.

A particularly promising direction is the study of exotic cosmic fluids. Chaplygin-type equations of state are attractive because they can reproduce matter-like behavior in the early Universe and negative-pressure behavior at late times within a unified phenomenological framework. At the same time, matter creation cosmology offers a thermodynamically motivated extension of standard relativistic cosmology by treating the cosmic medium as an open system with irreversible particle production. The associated creation pressure can modify the background dynamics in a way that may account for accelerated expansion without introducing an independent dark energy sector. Bulk viscous effects provide another physically relevant source of effective negative pressure and become especially important when one allows the cosmic fluid to depart from perfect equilibrium.

The motivation of the present thesis is rooted in the possibility that these ingredients are not isolated alternatives, but mutually connected mechanisms that may together provide a more flexible and physically meaningful description of cosmic evolution. In this context, the thesis investigates whether Chaplygin-type fluids, matter creation, and dissipative effects can produce viable cosmological scenarios in both General Relativity and modified gravity. A further motivation comes from the need to test whether such models remain observationally acceptable when confronted with current data and whether they can reproduce, or possibly improve upon, the background behavior usually attributed to the Λ CDM model.

The research work carried out so far has developed along a clear progression. The first stage examined generalized Chaplygin gas in matter creation cosmology within a flat FLRW background and tested the model against observational data. The second stage extended this framework to variable generalized Chaplygin gas and variable Chaplygin gas models, with a more detailed statistical and stability analysis. The third stage moved beyond General Rela-

tivity by studying matter creation cosmology in $f(Q)$ gravity with Chaplygin-type fluids. This line of inquiry was then further developed by combining variable generalized Chaplygin gas with matter creation in $f(Q)$ gravity and by analyzing the resulting cosmological dynamics through both theoretical and observational methods. Finally, the inclusion of modified Chaplygin gas together with bulk viscous dissipation and matter creation allowed the construction of a more general hybrid model in which thermodynamic consistency, observational viability, and late-time acceleration could be studied simultaneously. Taken together, these studies provide the immediate scientific basis for the thesis and define its central research direction.

1.14.2 Research Objectives

This thesis primarily seeks to investigate the dynamics and observational feasibility of cosmological models influenced by exotic fluids, employing the principles of General Relativity and specific modified gravity theories. A significant emphasis is placed on Chaplygin-type fluids, gravitationally-induced matter creation, and bulk viscous effects as alternative explanations for cosmic acceleration in the later stages of cosmic evolution.

The thesis is specifically directed by the following objectives:

1. To create homogeneous and isotropic cosmological models in flat FLRW space-time using Chaplygin-type exotic fluids within the framework of matter creation cosmology.
2. To study the role of generalized Chaplygin gas, variable generalized Chaplygin gas, variable Chaplygin gas, and modified Chaplygin gas in describing the late-time evolution of the Universe.
3. To examine how adiabatic matter creation, characterized through phenomenological creation rates such as $\Gamma = 3\beta H$, modifies the standard conservation laws and contributes to effective negative pressure.
4. To investigate the influence of bulk viscous dissipation on cosmological dynamics and to determine how viscosity alters the expansion history when combined with matter creation and Chaplygin-type fluids.
5. To extend the analysis from General Relativity to modified gravity, with particular focus on $f(Q)$ gravity, in order to determine how nonmetricity-based modifications of gravity affect the behavior of exotic fluid cosmologies.
6. To derive analytical or semi-analytical expressions for relevant cosmological quantities,

including the Hubble parameter, deceleration parameter, and effective equation of state parameter, and to study their evolution in the proposed models.

7. To confront the developed cosmological models with current observational data, including Type Ia supernovae, cosmic chronometers, baryon acoustic oscillations, growth data, gamma-ray bursts, and local H_0 priors, using statistical techniques such as Markov Chain Monte Carlo estimation.
8. To assess the physical viability of the models through stability analysis, thermodynamic consistency, and model selection tools such as AIC, BIC, and Bayesian evidence.
9. To compare the proposed models systematically with the standard Λ CDM cosmology in order to identify both their advantages and their limitations.
10. To provide a coherent theoretical and observational framework in which exotic fluids, matter creation, viscosity, and modified gravity can be studied together as possible alternatives to the standard cosmological paradigm.

In summary, this thesis investigates if the late-time fast expansion of the Universe can be comprehended by a synthesis of non-standard cosmic fluids, irreversible matter production, dissipative effects, and modified gravitational dynamics. The aim is not just to provide alternative models but to analyze them thoroughly from theoretical, observational, and thermodynamic viewpoints to evaluate their significance in modern cosmology systematically.

1.15 Organization of Thesis Work

The seven chapters address a common theme: cosmic acceleration through Chaplygin-type fluids, matter creation, bulk viscosity, and modified gravity. Chapters 2–4 develop the General Relativity models, while Chapters 5 and 6 investigate nonmetricity-based extensions. Together, these studies [167–169, 267, 268] address a common research theme. The shared background is established in this chapter.

Chapter 1 provides the general introduction to the thesis. It presents the basic foundations of modern cosmology, including the standard cosmological model, cosmic acceleration, dark energy, matter creation mechanisms, Chaplygin gas models, bulk viscous effects, and modified theories of gravity. It also discusses the motivation, objectives, and overall scope of the present research work.

Chapter 2 is devoted to the study of matter creation cosmology with generalized Chaplygin gas. In this chapter, the cosmological dynamics of the model are analyzed in detail, and its physical and observational viability is examined.

Chapter 3 extends the analysis to matter creation cosmology with variable generalized Chaplygin gas. This chapter investigates the role of the variable equation of state in the cosmic evolution and studies its implications from both theoretical and observational perspectives.

Chapter 4 focuses on matter creation and bulk viscous cosmology with modified Chaplygin gas. In this chapter, the combined effects of matter creation and bulk viscosity are discussed in the framework of cosmological evolution, and the corresponding model behavior is analyzed.

Chapter 5 studies matter creation cosmology with generalized Chaplygin gas in $f(Q)$ gravity. This chapter explores the cosmological significance of the generalized Chaplygin gas model in the modified gravity framework based on non-metricity.

Chapter 6 deals with matter creation cosmology with variable generalized Chaplygin gas in $f(Q)$ gravity. The chapter further generalizes the previous analysis by incorporating a variable Chaplygin gas scenario within the $f(Q)$ gravity formalism and examines its consequences for the late-time evolution of the Universe.

Chapter 7 presents the conclusion, future scope, and social impact of the thesis work. It summarizes the major findings of the thesis, highlights the overall significance of the results, and outlines possible directions for future research in this area.

Chapter 2

Matter Creation Cosmology with Generalized Chaplygin Gas

2.1 Introduction

This chapter studies generalized Chaplygin gas with adiabatic matter creation in a spatially flat FLRW Universe, following the work of Bhardwaj and Singh [167]. It establishes the General Relativity setting for the subsequent models. Creation pressure is assigned to the Chaplygin component, while baryons are separately conserved.

The motivation for exotic fluids, the Chaplygin equations of state, and the thermodynamic description of matter creation are given in Chapter 1, particularly Sections 1.8.1 and 1.9. Here the emphasis is on their combined background evolution and observational consequences, rather than on repeating those definitions.

Sections 2.2–2.4 present the field equations, the generalized Chaplygin gas solution, and its Chaplygin gas limit. Sections 2.5–2.7 describe the data, statistical analysis, and results, followed by the conclusion in Section 2.8. The next chapter extends this setting by allowing the Chaplygin coefficient to depend on the scale factor.

2.2 Basic field equations

The spatially flat FLRW geometry is introduced in Section 1.3.4. The particle balance and adiabatic creation pressure follow Eqs. (1.61) and (1.65), as discussed in Section 1.9. In this chapter, the creation mechanism acts on the generalized Chaplygin gas, while baryons are separately conserved.

For a flat Universe containing baryons ($w_b = 0$) and a generalized Chaplygin gas fluid, the Friedmann and conservation equations become

$$3H^2 = 8\pi G(\rho_b + \rho_{gcg}), \quad (2.1)$$

$$\dot{\rho}_b + 3H\rho_b = 0, \quad (2.2)$$

$$\dot{\rho}_{gcg} + 3H(\rho_{gcg} + p_{gcg} + p_c) = 0, \quad (2.3)$$

where ρ_{gcg} and p_{gcg} are the energy density and pressure for GCG fluid. Integration of Eq. (2.2) yields $\rho_b = \rho_{b,0}a^{-3}$. Assuming the phenomenological form $\Gamma = 3\beta H$ [126, 127], where β is a constant lying in the $0 \leq \beta < 1$, the creation pressure simplifies to

$$p_c = -\beta(\rho_{gcg} + p_{gcg}). \quad (2.4)$$

2.3 GCG model with matter creation

We assume the GCG equation of state [6]

$$p_{gcg} = -\frac{A}{\rho_{gcg}^\alpha}, \quad (2.5)$$

with $A > 0$ and $0 < \alpha \leq 1$. The generalized Chaplygin gas (GCG) model has been shown to accommodate standard structure formation and background evolution constraints for $-1/2 \leq \alpha \leq 1/2$ [6, 58, 108, 128].

Substituting Eqs. (2.4) and (2.5) into the conservation equation (2.3) yields

$$\rho_{gcg}(a) = \left[A + C_6 a^{-3(1-\beta)(1+\alpha)} \right]^{\frac{1}{1+\alpha}}, \quad (2.6)$$

where C_6 is an integration constant. Imposing $\rho_{gcg}(a=1) = \rho_{gcg,0}$ gives

$$\rho_{gcg} = \rho_{gcg,0} \left[A_s + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)} \right]^{\frac{1}{1+\alpha}}, \quad (2.7)$$

where $A_s = A/\rho_{gcg,0}^{(1+\alpha)}$ and $z = a^{-1} - 1$. For $\beta = 0$, Eq. (2.7) reduces to the standard GCG result [6]. The condition $0 < A_s < 1$ ensures a smooth transition from matter-like behavior at early times to a cosmological-constant-like phase at late times.

Inserting Eq. (2.7) into the Friedmann equation (2.1) gives the Hubble expansion history

$$H^2(z) = H_0^2 \left[\Omega_{gcg,0} \left(A_s + (1 - A_s)(1 + z)^{3(1-\beta)(1+\alpha)} \right)^{\frac{1}{1+\alpha}} + \Omega_{b,0}(1 + z)^3 \right], \quad (2.8)$$

where H_0 is the present value of Hubble parameter, $\Omega_{b,0} = \rho_{b,0}/(3H_0^2)$ and $\Omega_{gcg,0} = \rho_{gcg,0}/(3H_0^2)$ are the density parameters of the baryonic matter and Chaplygin gas components, respectively such that $\Omega_{b,0} + \Omega_{gcg,0} = 1$. The baryon density is tightly constrained by primordial nucleosynthesis: $\Omega_{b,0}h^2 = 0.0224 \pm 0.0001$ [129].

Let us explore the cosmological parameters, mainly the Hubble parameter, deceleration, jerk parameter and EoS parameter, to distinguish the effect of creation pressure included in the different Chaplygin gas models and also compare these models with Λ CDM.

In cosmology, the Hubble parameter $H(z)$, deceleration parameter $q(z)$ and jerk Parameter $j(z)$ are purely kinematical as they are independent of any gravity theories. The kinematical evolution is characterized by the deceleration parameter [Eq. (1.20)]

$$q(z) = -1 + \frac{3}{2E^2(z)} \left[\Omega_{b,0}(1 + z)^3 + \frac{\Omega_{gcg,0}(1 - A_s)(1 - \beta)(1 + z)^{3(1-\beta)(1+\alpha)}}{[A_s + (1 - A_s)(1 + z)^{3(1-\beta)(1+\alpha)}]^{\frac{\alpha}{1+\alpha}}} \right], \quad (2.9)$$

where $E(z) = H(z)/H_0$ is the dimensionless Hubble parameter.

The jerk parameter, defined as $j = \ddot{a}/(aH^3)$, provides a higher-order diagnostic of departure from Λ CDM ($j = 1$). In terms of $q(z)$, it is given by [Eq. (1.82)]

$$j(z) = q(2q + 1) + (1 + z) \frac{dq}{dz}. \quad (2.10)$$

The effective equation of state for the GCG component reads

$$w_{gcg}(z) = -\frac{A_s}{A_s + (1 - A_s)(1 + z)^{3(1-\beta)(1+\alpha)}}, \quad (2.11)$$

The present-day value of EoS parameter, $w_{gcg,0}$ for GCG at $z = 0$ is obtained as

$$w_{gcg,0} = -A_s, \quad (2.12)$$

It can be observed that the above equation gives the physical significant of A_s . The behavior of GCG depends on the values of A_s , α and β . For $A_s = 1$, we get a trivial solution $w_{gcg} = -1$ which shows that GCG model behaves exactly as Λ CDM at all times for all values of α and

β . For $A_s = 0$, we have $w_{gcg} = 0$ and hence GCG model behaves as pressureless dust fluid all the times for all values of α and β .

2.4 Particular case: CG model with matter creation

Setting $\alpha = 1$ in Eq. (2.5) recovers the original Chaplygin gas equation of state $p_{cg} = -A/\rho_{cg}$ [57]. The energy density solution simplifies to

$$\rho_{cg} = \rho_{cg,0} \sqrt{A_s + (1 - A_s)(1 + z)^{6(1-\beta)}}, \quad (2.13)$$

and the corresponding Hubble function is

$$E^2(z) = \Omega_{cg,0} \left[A_s + (1 - A_s)(1 + z)^{6(1-\beta)} \right]^{1/2} + \Omega_{b,0}(1 + z)^3. \quad (2.14)$$

The deceleration parameter and effective equation of state follow directly from Eqs. (2.9) and (2.11) by setting $\alpha = 1$:

$$q(z) = -1 + \frac{3}{2E^2(z)} \left[\Omega_{b,0}(1 + z)^3 + \frac{\Omega_{cg,0}(1 - A_s)(1 - \beta)(1 + z)^{6(1-\beta)}}{\sqrt{A_s + (1 - A_s)(1 + z)^{6(1-\beta)}}} \right], \quad (2.15)$$

$$w_{cg}(z) = -\frac{A_s}{A_s + (1 - A_s)(1 + z)^{6(1-\beta)}}. \quad (2.16)$$

For $\beta = 0$, these expressions reduce exactly to the standard CG solutions [57].

2.5 Observational data

The observational datasets utilized to constrain the model parameters have been comprehensively described in Chapter 1. For the present analysis, we employ the following compilations:

- **Cosmic Chronometers:** 32 model-independent $H(z)$ measurements in the range $0.07 \leq z \leq 1.965$ [130]. The observable is described in Section 1.12.2.
- **Type Ia Supernovae:** The Pantheon sample comprising 1048 SNe Ia in $0.01 \leq z \leq 2.26$ [131, 132]. The distance-modulus and luminosity-distance definitions are given in Section 1.12.1.
- **Baryon Acoustic Oscillations:** 6 spectroscopic BAO measurements from SDSS, BOSS, and eBOSS surveys [133], using the line-of-sight distance ratio $D_H(z)/r_d$. The common

Table 2.1: Clustering measurements for the BAO samples [133].

Parameter	Redshift range	z_{eff}	$D_H(z)/r_d$
Boss galaxy	$0.2 < z < 0.5$	0.38	25.00 ± 0.76
Boss galaxy	$0.4 < z < 0.6$	0.51	22.33 ± 0.58
eBoss LRG	$0.6 < z < 1.0$	0.70	19.33 ± 0.53
eBoss quasars	$0.8 < z < 2.2$	1.48	13.26 ± 0.55
$Ly\alpha - Ly\alpha$	$z > 2.1$	2.33	8.93 ± 0.28
$Ly\alpha - Quasars$	$z > 1.77$	2.33	9.08 ± 0.34

BAO discussion is given in Section 1.12.3. The effective redshifts and measured values are listed in Table 2.1.

2.6 Methodology and model selection

The statistical framework for parameter estimation and model comparison is detailed in Section 1.13. Using the above dataset, we proceed to analyze whether the two extensions of Chaplygin gas being considered in this work, namely, the GCG with matter creation and CG with matter creation models capable to fit the observations in a comparable way to Λ CDM model. In this respect, we employ the total likelihood function to establish the joint constraints on the model parameter space $(H_0, \Omega_b, A_s, \alpha, \beta)$ from the aforementioned observational data. The best-fit of the parameters are maximized by using the following total χ^2 -function:

$$\chi_{tot}^2 = \chi_{H(z)}^2 + \chi_{SNe}^2 + \chi_{BAO}^2, \quad (2.17)$$

and explore the parameter space using the MCMC algorithm (emcee) with uniform and Gaussian priors as specified in Chapter 1. We have chosen 2000 iterations with 120 walkers to explore the confidence region taking into account a Gaussian priors on the Hubble constant H_0 and on the baryon density $\Omega_{b,0}$. Additionally, we select priors for A_s , α and β . Table 2 summarizes the priors used in the Bayesian analysis.

We use the Akaike and Bayesian information criteria of Section 1.13:

$$AIC = \chi_{min}^2 + \frac{2nN}{N - n - 1}, \quad BIC = \chi_{min}^2 + n \ln N, \quad (2.18)$$

where n ($n = 3, 5, 4$ and 4 for Λ CDM, GCG with β , GCG without β and CG with β , respectively) is the number of independent parameters and $N = 1086$ ($H(z)=32$, $SNe=1048$ and $BAO=6$) is the total number of data points.

Physical stability of the fluid perturbations is verified via the squared sound speed condition

$0 \leq C_s^2 \leq 1$. The equation describing the evolution of linear perturbations of the background energy density define the squared sound speed. Therefore, it is a background quantity which is essentially a perturbative in nature. The bound $C_s^2 > 0$ is required to have small perturbations of the background energy density, otherwise, the energy density perturbations will grow and there will be a classical instability. Thus, stability or instability of a given perturbation in the background, can be specified by determining the sign of C_s^2 . The model is stable if $C_s^2 > 0$ and for $C_s^2 < 0$, the model approaches to instability.

The squared sound speed is defined as [134, 135]

$$C_s^2 = dp/d\rho. \quad (2.19)$$

For the GCG model with matter creation, we obtain

$$C_s^2 = \frac{\alpha A_s}{A_s + (1 - A_s)(1 + z)^{3(1-\beta)(1+\alpha)}}. \quad (2.20)$$

In terms of EoS parameter, it gives

$$C_s^2 = -\alpha w_{gcg}(z). \quad (2.21)$$

Using above equation, we can obtain the present value as $C_s^2(z=0) = \alpha A_s = -\alpha w_{gcg,0}$. The positive value of A_s restricts the value $\alpha > 0$. It is to be noted that for causality and stability under perturbation, it is required to satisfy the inequality condition $0 \leq C_s^2 \leq 1$ [136].

2.7 Results and discussion

The best-fit parameters and marginalized uncertainties obtained from the joint likelihood analysis are summarized in Table 2.2. The one-dimensional posteriors and two-dimensional confidence contours (68.3% and 95.4% CL) are displayed in Figs. 2.1–2.4. The present-day values of key cosmological diagnostics are listed in Table 2.3, and the redshift evolution of the background and kinematical parameters is shown in Figs. 2.5–2.9. In what follows, we discuss our results with some more detail.

The joint analysis yields Hubble constants that are fully compatible with early-Universe constraints. Analysing the constraints of the parameters of the various versions of the models listed in Table 2.2, we find that the Λ CDM model gives $H_0 = 68.595_{-1.680}^{+1.803}$ km/s/Mpc, the GCG with β gives $H_0 = 68.804_{-1.062}^{+1.066}$ km/s/Mpc, the GCG without β yields $H_0 = 69.432_{-0.062}^{+0.034}$

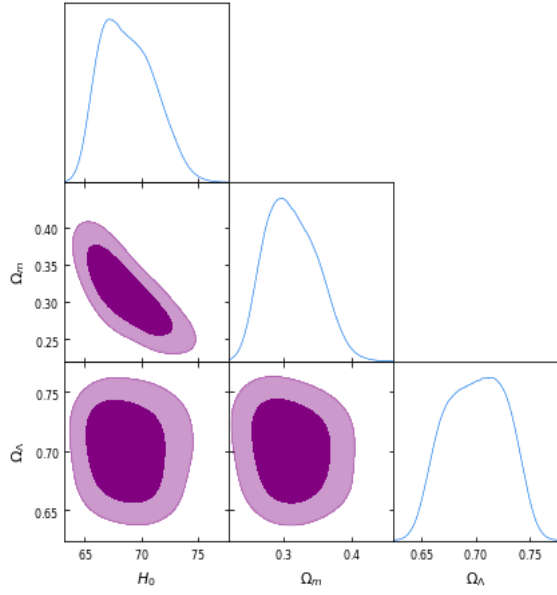


Figure 2.1: 2D contour at 68.3% and 95.4% confidence levels for Λ CDM model using $H(z) + SNe + BAO$ data-set.

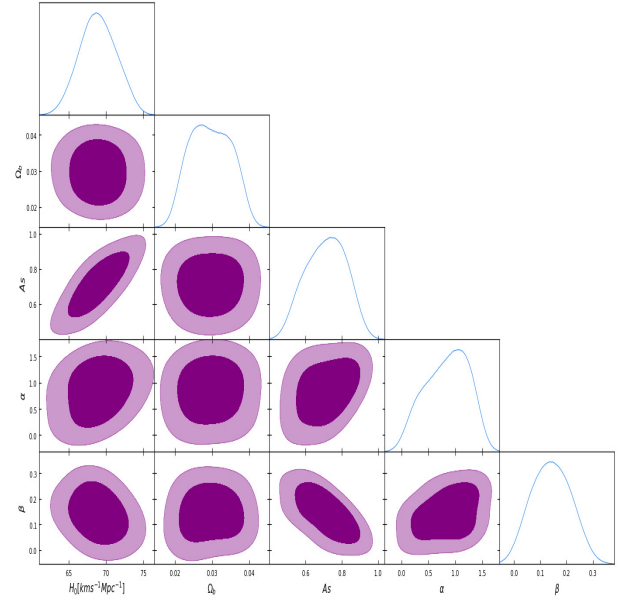


Figure 2.2: 2D contour at 68.3% and 95.4% confidence levels for GCG model with matter creation using $H(z) + SNe + BAO$ data-set.

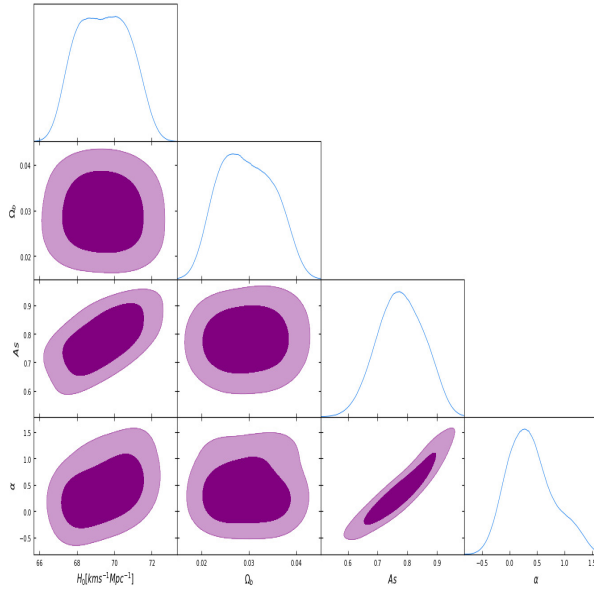


Figure 2.3: 2D contour at 68.3% and 95.4% confidence levels for GCG model without matter creation using $H(z) + SNe + BAO$ data-set.

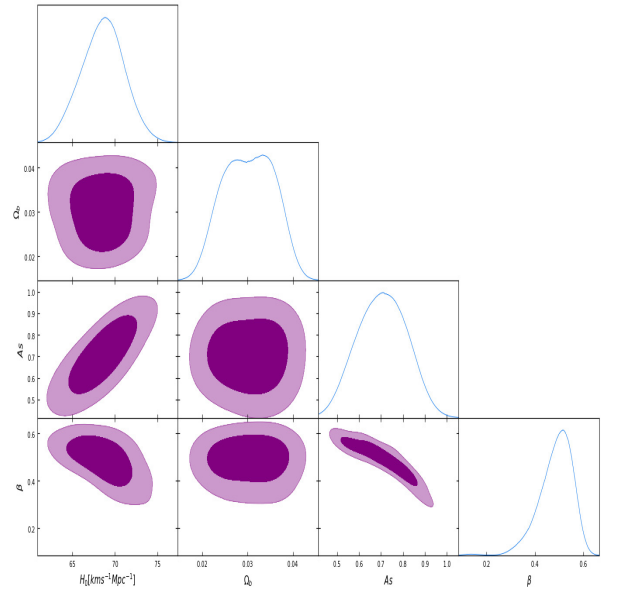


Figure 2.4: 2D contour at 68.3% and 95.4% confidence levels for CG model with matter creation using $H(z) + SNe + BAO$ data-set.

Table 2.2: Priors and 1σ marginalized parameter constraints from the joint $SNe + H(z) + BAO$ analysis.

Parameters	Priors	Λ CDM	GCG with β	GCG without β	CG with β
H_0 [km/s/Mpc]	[50, 80]	$68.595^{+1.803}_{-1.680}$	$68.804^{+1.066}_{-1.062}$	$69.432^{+0.034}_{-0.062}$	$68.778^{+0.123}_{-0.128}$
Ω_m	—	$0.307^{+0.026}_{-0.022}$	—	—	—
Ω_Λ	—	$0.701^{+0.032}_{-0.032}$	—	—	—
Ω_b	[0.02, 0.04]	—	$0.029^{+0.006}_{-0.005}$	$0.029^{+0.001}_{-0.004}$	$0.030^{+0.003}_{-0.002}$
A_s	[0.5, 1.5]	—	$0.713^{+0.043}_{-0.035}$	$0.777^{+0.011}_{-0.021}$	$0.705^{+0.032}_{-0.044}$
α	[0, 1]	—	$0.995^{+0.078}_{-0.022}$	$0.350^{+0.024}_{-0.033}$	1
β	[0, 1]	—	$0.140^{+0.022}_{-0.042}$	—	$0.498^{+0.009}_{-0.013}$

Table 2.3: Present values of cosmological diagnostics for the considered models.

Model	q_0	j_0	w_0	C_s^2	t_0 [Gyr]
Λ CDM	$-0.551^{+0.020}_{-0.020}$	1	$-0.701^{+0.013}_{-0.015}$	≤ 1	13.8 ± 0.14
GCG with β	$-0.597^{+0.016}_{-0.013}$	$1.440^{+0.023}_{-0.022}$	$-0.713^{+0.044}_{-0.012}$	0.638	13.8 ± 0.16
GCG without β	$-0.271^{+0.033}_{-0.031}$	$1.264^{+0.034}_{-0.023}$	$-0.777^{+0.055}_{-0.012}$	0.271	13.6 ± 0.11
CG with β	$-0.523^{+0.011}_{-0.022}$	$1.006^{+0.031}_{-0.027}$	$-0.705^{+0.013}_{-0.022}$	0.705	13.6 ± 0.13

Table 2.4: Information criteria and model comparison relative to Λ CDM.

Model	χ^2_{min}	n	N	AIC	BIC	Δ AIC	Δ BIC
Λ CDM	1088.121	3	1086	1094.02	1108.97	—	—
GCG with β	1087.334	5	1086	1095.03	1119.95	1.00	10.54
GCG without β	1087.223	4	1086	1095.09	1114.96	1.06	5.99
CG with β	1086.336	4	1086	1096.01	1113.96	1.988	4.99

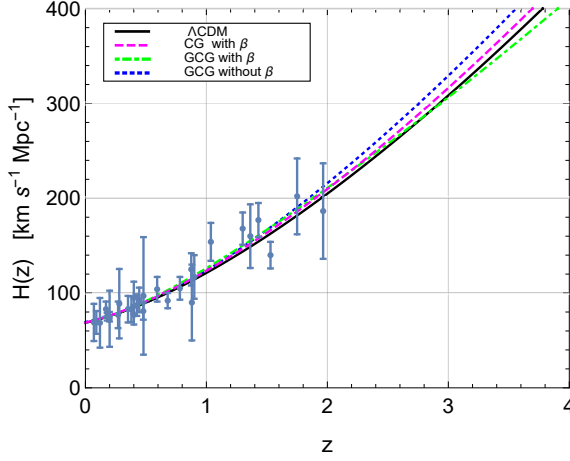


Figure 2.5: Best fits over $H(z)$ data for Λ CDM (black solid), GCG+ β (green dot-dashed), GCG (blue dotted), and CG+ β (magenta dashed). Grey points with error bars denote the 32 CC measurements.

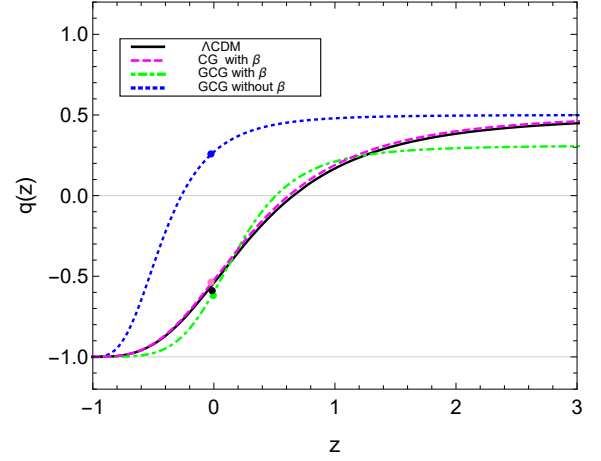


Figure 2.6: Evolution of the deceleration parameter, $q(z)$ with redshift for GCG and CG models with and without matter creation. The trajectory of $q(z)$ for Λ CDM model is also shown in bold line. The dot represents the present value of $q(z)$ (hence q_0).

km/s/Mpc, and finally, the CG with β gives $H_0 = 68.778^{+0.123}_{-0.128}$ Km/s/Mpc. We observe that these results are very competitive with the other current estimates performed by assuming Λ CDM cosmology [129, 137, 138]. The evolution of $H(z)$ with redshift for different models are plotted in Fig. 2.5.

Figure 2.6 shows that the deceleration parameter exhibits a clear transition from $q(z) > 0$ (deceleration) to $q(z) < 0$ (acceleration), with present values $q_0 = -0.597^{+0.016}_{-0.013}$ (GCG+ β), $q_0 = -0.271^{+0.033}_{-0.031}$ (GCG), and $q_0 = -0.523^{+0.011}_{-0.022}$ (CG+ β). The transition redshifts z_{tr} lie close to the Λ CDM expectation, confirming that the combined matter creation and Chaplygin dynamics successfully reproduce the observed phase transition. The GCG with β model starts late acceleration than CG with β and Λ CDM models. We observe that the all Chaplygin gas models (GCG and CG) have characteristics of quintessence DE.

The jerk parameter remains near unity in the recent past, with j_0 values deviating only mildly from the Λ CDM prediction, thereby preserving consistency with low-redshift kinematical tests. In Fig. 2.7, we show the jerk parameter, $j(z)$ of our models in comparison to Λ CDM model. In late-time all the Chaplygin gas with or without matter creation converge to Λ CDM state. Based on the joint analysis, the present values of $j(z)$ are $j_0 = 1.440^{+0.023}_{-0.022}$, $1.264^{+0.034}_{-0.023}$ and $1.006^{+0.031}_{-0.027}$ for GCG with β , GCG without β and CG with β , respectively. Figure 2.8 shows the evolution of the EoS parameter for different models. The effective equation of state evolves toward $w(z) \rightarrow -1$ at late times, indicating asymptotic convergence to a de Sitter epoch. It is observed that the EoS parameter of all the models tend to -1 in late-time.

The evolution of C_S^2 with respect to redshift for different models is shown in Fig. 2.9. Using

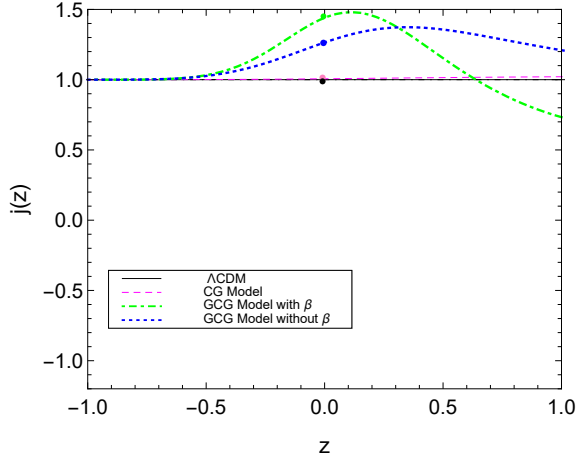


Figure 2.7: Evolution of the jerk parameter, $j(z)$ for GCG and CG models with and without matter creation. The horizontal bold line represents the value of $j(z)$ for Λ CDM. The dot represents the present value of $j(z)$ (hence j_0).

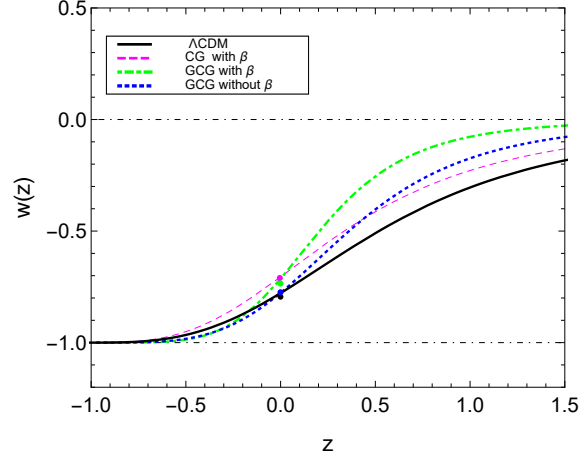


Figure 2.8: Evolution of the state parameter, $w(z)$ for GCG and CG models with and without matter creation. The dot represents the present value of $w(z)$ (hence w_0).

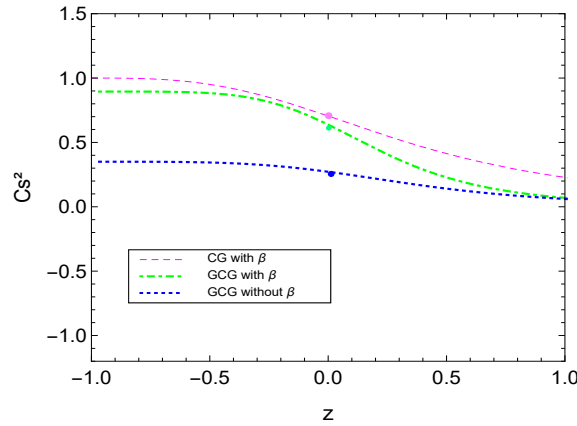


Figure 2.9: Evolution of speed of sound, C_s^2 for GCG and CG models with and without matter creation. The dot represents the present value $C_s^2(z=0)$.

the best-fit values of α and A_s , we find that $C_s^2 = 0.638$ for GCG model with β , $C_s^2 = 0.271$ for GCG model without matter creation and $C_s^2 = 0.705$ for CG with β [c.f. Table 2.3]. It is obvious that the squared sound speed of these models are in the range of $0 < C_s^2 < 1$, which is not exceeding the speed of light (unity), and hence the models are stable. We have computed the present age of the Universe in different models which are summarized in Table 2.3.

Model selection via AIC and BIC is summarized in Table 2.4. According to ΔAIC values of Chaplygin gas models, we find that GCG with β , GCG without β and CG with β models have $\Delta AIC < 2$ which show that the joint observational data strongly support these models. In BIC, we have $2 < \Delta BIC < 6$ for GCG without β and CG with β , and thus there is positive evidence against the model. The GCG with β model has $\Delta BIC > 10$ which tells us that there is a very strong evidence against this model. This may happen due to the large number of free

parameters for which BIC penalizes.

2.8 Conclusion

This chapter examined the dynamics of generalized Chaplygin gas coupled to adiabatic matter creation in a flat FLRW universe. By incorporating the thermodynamically motivated creation pressure p_c into the fluid conservation law, we derived exact analytical solutions for the background evolution of both the generalized and original Chaplygin gas models. The resulting expressions for $H(z)$, $q(z)$, $w(z)$, and $j(z)$ were confronted with current cosmological observations using a joint likelihood analysis. The observational constraints demonstrate that the hybrid models successfully reproduce the observed transition from early deceleration to late-time acceleration. The present-day values of the deceleration and equation of state parameters are fully compatible with Λ CDM expectations, while the asymptotic approach $w(z) \rightarrow -1$ confirms consistency with a future de Sitter phase. The jerk parameter remains close to unity at low redshifts, indicating that the kinematical departure from the standard model is mild and well within observational bounds. The plot of data points of $H(z)$ with 1σ error bars show that our models are well consistent with these data against the redshift parameter.

The evolution of different cosmological parameters show that the matter creation model with GCG EoS mimics exactly the Λ CDM model. The present value q_0 , w_0 and t_0 are very close to the present value of Λ CDM. Similarly, the other value, like H_0 and Ω_b are very competitive with Λ CDM. All the models transit from early deceleration phase to late-time acceleration phase. We have explored through the joint observational analysis that the models favor the EoS for DE greater than -1 , that is, quintessence region. Thus, the results of the models are fully consistent with respect to the expected Λ CDM value.

Model selection analysis reveals that the Chaplygin variants with matter creation are statistically competitive with Λ CDM under the AIC criterion, while the BIC penalization reflects the increased parameter count without compromising fit quality. Importantly, the squared sound speed remains strictly positive and subluminal throughout cosmic history, confirming linear stability and causal consistency.

In summary, the combination of Chaplygin-type fluid dynamics and gravitationally induced matter creation provides a physically coherent alternative to the cosmological constant. It naturally generates late-time acceleration through effective negative pressure, alleviates the need for a rigid vacuum energy term, and remains fully compatible with current geometric and kinematical datasets. Future work will extend this framework to perturbation growth analysis and

higher-order modified gravity couplings.

Chapter 3

Matter Creation Cosmology with Variable Generalized Chaplygin Gas

3.1 Introduction

Building on Chapter 2, this chapter studies variable generalized Chaplygin gas with adiabatic matter creation, based on Bhardwaj and Singh [168]. The Chaplygin coefficient now depends on the scale factor. Matter creation remains assigned to the Chaplygin component, and baryons remain separately conserved. The variable Chaplygin gas model is examined as a particular case.

The definitions of these fluids and the common matter-creation formalism are given in Sections 1.8.1 and 1.9. The present chapter concentrates on the variable-coefficient density solution, its cosmological evolution, and the constraints from the BASE and BASE+R21 datasets.

Sections 3.2–3.5 specify the creation prescription, field equations, and fluid solutions. Section 3.6 describes the observational analysis, Section 3.7 presents the results, and Section 3.8 gives the conclusion. Chapter 4 then extends the fluid sector by including modified Chaplygin gas and a separate bulk-viscous matter component.

3.2 Basic Formalism of Matter Creation

The particle balance, entropy-per-particle condition, and creation pressure are given in Section 1.9, particularly Eqs. (1.61), (1.64), and (1.65). We use that prescription for the Chaplygin

component and retain the phenomenological rate [126, 127]

$$\Gamma = 3\beta H, \quad (3.1)$$

where β is a positive constant. Substituting this rate into Eq. (1.65) gives

$$p_c = -\beta(\rho + p). \quad (3.2)$$

From this point onward, the fluid variables will be denoted by ρ_{Ch} and p_{Ch} in order to emphasize that the matter creation mechanism is applied to the Chaplygin gas sector.

3.3 Basic Field Equations

The background geometry is taken to be the spatially flat Friedmann–Lemaître–Robertson–Walker space-time

$$ds^2 = -dt^2 + a^2(t) (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2), \quad (3.3)$$

where $a(t)$ is the scale factor. Units are chosen such that $c = 1$, and for a comoving observer,

$$u^\mu = \delta_0^\mu, \quad u^\mu u_\mu = -1. \quad (3.4)$$

The Einstein field equations are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}. \quad (3.5)$$

In this work we consider a Universe with matter contents as baryons and Chaplygin gas endowed with matter creation (Chaplygin gas means a perfect fluid having exotic equation of state). Thus, we write the energy density and the pressure of the mixture as a sum of two terms that describe the baryon matter component and Chaplygin gas fluid with matter creation. By taking into account matter creation the effective energy-momentum tensor for the mixture can be given by [8]

$$T_{\mu\nu} = (\rho_{Ch} + \rho_b + p_{Ch} + p_c)u_\mu u_\nu + (p_{Ch} + p_c)g_{\mu\nu}, \quad (3.6)$$

where ρ_b denotes the baryon density. The corresponding background equations are

$$H^2 = \frac{8\pi G}{3}(\rho_{Ch} + \rho_b), \quad (3.7)$$

$$\dot{H} + H^2 = -\frac{4\pi G}{3}(\rho_{Ch} + \rho_b + 3p_{Ch} + 3p_c). \quad (3.8)$$

The baryon component is assumed to be pressureless and separately conserved, so that

$$\rho_b = \rho_{b,0}a^{-3}, \quad (3.9)$$

while the conservation equation for the Chaplygin sector becomes

$$\dot{\rho}_{Ch} + 3H(\rho_{Ch} + p_{Ch} + p_c) = 0. \quad (3.10)$$

Hereafter, we refer to ρ_{Ch} and p_{Ch} as ρ_{vgcg} and p_{vgcg} , respectively for the energy density and pressure of VGCG.

3.4 Variable Generalized Chaplygin Gas Model with Matter Creation

The variable generalized Chaplygin gas model, which is the focus of this chapter, is defined by [61, 62, 141]

$$p_{vgcg} = -\frac{A_0 a^{-n}}{\rho_{vgcg}^\alpha}, \quad (3.11)$$

where A_0 , n , and α are constants.

Using Eqs. (3.2) and (3.11) into (3.10), the conservation equation becomes

$$\rho'_{vgcg} + 3(1 - \beta) \left(\rho_{vgcg} - \frac{A_0 a^{-n}}{\rho_{vgcg}^\alpha} \right) = 0, \quad (3.12)$$

where the prime denotes differentiation with respect to $x = \ln a$. Integrating Eq. (3.12), one finds

$$\rho_{vgcg} = \rho_{vgcg,0} \left[A_s (1+z)^n + (1 - A_s) (1+z)^{3(1-\beta)(1+\alpha)} \right]^{\frac{1}{1+\alpha}}, \quad (3.13)$$

where $z = a^{-1} - 1$ and

$$A_s = \frac{3(1-\beta)(1+\alpha)}{3(1-\beta)(1+\alpha) - n} \frac{A_0}{\rho_{vgcg,0}^{1+\alpha}}. \quad (3.14)$$

For an expanding Universe, the physically admissible conditions are

$$n > 0, \quad (1 - \beta)(1 + \alpha) > 0. \quad (3.15)$$

Substituting Eq. (3.13) into the Friedmann equation (3.7), we get

$$E(z) = \left[(1 - \Omega_{b,0}) \left[A_s(1+z)^n + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)} \right]^{\frac{1}{1+\alpha}} + \Omega_{b,0}(1+z)^3 \right]^{1/2}. \quad (3.16)$$

The deceleration parameter is defined by

$$q(z) = -1 + \frac{1}{2E^2(z)} \left[3\Omega_{b,0}(1+z)^3 + \frac{(1 - \Omega_{b,0}) \left[nA_s(1+z)^n + 3(1 - A_s)(1 - \beta)(1 + \alpha)(1+z)^{3(1-\beta)(1+\alpha)} \right]}{\left[A_s(1+z)^n + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)} \right]^{\frac{\alpha}{1+\alpha}}} \right] \quad (3.17)$$

The equation-of-state parameter of the VGCG fluid is

$$w(z) = \frac{-A_s \left(1 - \frac{n}{3(1-\beta)(1+\alpha)} \right) (1+z)^n}{A_s(1+z)^n + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)}}. \quad (3.18)$$

The jerk parameter is given by

$$j(z) = q(2q+1) + (1+z) \frac{dq}{dz}. \quad (3.19)$$

3.5 Particular Solution: Variable Chaplygin Gas Model

The VCG model is recovered from the VGCG model by setting $\alpha = 1$. Its equation of state becomes

$$p_{vcg} = -\frac{A_0 a^{-n}}{\rho_{vcg}}. \quad (3.20)$$

The corresponding energy density is

$$\rho_{vcg} = \rho_{vcg,0} \left[A_s(1+z)^n + (1 - A_s)(1+z)^{6(1-\beta)} \right]^{1/2}, \quad (3.21)$$

where

$$A_s = \frac{6(1-\beta)A_0}{[6(1-\beta) - n]\rho_{vcg,0}^2}. \quad (3.22)$$

Using Eq. (3.21), the Friedmann equation becomes

$$H(z) = H_0 E(z), \quad (3.23)$$

with

$$E^2(z) = (1 - \Omega_{b,0}) \left[A_s(1+z)^n + (1 - A_s)(1+z)^{6(1-\beta)} \right]^{1/2} + \Omega_{b,0}(1+z)^3. \quad (3.24)$$

The deceleration parameter is

$$q(z) = -1 + \frac{1}{2E^2(z)} \left[3\Omega_{b,0}(1+z)^3 + \frac{(1 - \Omega_{b,0}) \left[nA_s(1+z)^n + 6(1 - A_s)(1 - \beta)(1+z)^{6(1-\beta)} \right]}{\left[A_s(1+z)^n + (1 - A_s)(1+z)^{6(1-\beta)} \right]^{1/2}} \right]. \quad (3.25)$$

The corresponding equation-of-state parameter is

$$w(z) = -\frac{A_s \left(1 - \frac{n}{6(1-\beta)} \right) (1+z)^n}{A_s(1+z)^n + (1 - A_s)(1+z)^{6(1-\beta)}}. \quad (3.26)$$

3.6 Observational Data and Statistical Methodology

This section addresses the datasets and method to constrain the model parameters of VGCG and VCG models alongwith Λ CDM model. We use the recent observational data of Baryon Acoustic Oscillation, Cosmic Chronometer, Standard candles and Hubble data *R21* to constrain the model parameters. The free parameters of the Λ CDM, VGCG, and VCG models are constrained using late-time observations.

3.6.1 Baryon Acoustic Oscillations

The BAO sample contains 17 uncorrelated measurements from different surveys [145–158], compiled as in Ref. [159]. A central quantity in BAO analysis is the sound horizon at the drag epoch,

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz, \quad (3.27)$$

where the sound speed in the baryon-photon fluid is approximately

$$c_s(z) \approx c \left(3 + \frac{9\rho_b(z)}{4\rho_\gamma(z)} \right)^{-1/2}. \quad (3.28)$$

Here, $\rho_b(z)$ and $\rho_\gamma(z)$ stand for the densities of baryons and photons, respectively. The projected BAO observables are $\Delta z = r_d H/c$ and $\Delta\theta = r_d / [(1+z)D_A(z)]$.

3.6.2 Cosmic Chronometers

The cosmic chronometer sample consists of 30 uncorrelated $H(z)$ measurements [80, 160–162]. The corresponding chi-square function is

$$\chi_{CC}^2 = \sum_{i=1}^{30} \frac{[H_{\text{obs}}(z_i) - H_{\text{th}}(z_i)]^2}{\sigma^2(z_i)}. \quad (3.29)$$

where $H_{\text{th}}(z_i)$ denotes the theoretical value of $H(z)$ at redshift z_i for a given set of cosmological parameters, $H_{\text{obs}}(z_i)$ denotes the observed values of $H(z)$ at redshift z_i , and $\sigma(z_i)$ is the uncertainty in the measurement of H_i .

3.6.3 Standard Candles and Local H_0 Prior

The standard-candle data set combines Pantheon Type Ia supernovae, quasars, and gamma-ray bursts [131, 163, 164]. For supernovae, the luminosity distance is

$$d_L(z) = (1+z) \int_0^z \frac{c}{H(z')} dz'. \quad (3.30)$$

The local Hubble constant prior is taken from the SH0ES measurement of R21 [52]. It is written here as

$$H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (3.31)$$

3.6.4 Joint Likelihood and Priors

We use two combined data sets of the above data throughout the analysis. The first one, denoted by **BASE**, consists of baryon acoustic oscillations, cosmic chronometers, and standard candles. The second one, denoted by **BASE+R21**, supplements the BASE combination with the local Hubble constant measurement from the SH0ES collaboration. The total chi-square adopted in the analysis is

$$\chi^2 = \chi_{BAO}^2 + \chi_{CC}^2 + \chi_{SC}^2, \quad (3.32)$$

and the parameter estimation is performed through a Bayesian Markov Chain Monte Carlo analysis using *Polychord* together with *GetDist* [165, 166]. The parameter space for the VGCG model is $(H_0, \Omega_b, A_s, n, \alpha, \beta)$, while the VCG model is obtained by fixing $\alpha = 1$. We choose the priors for different parameters which are given in Table 3.1.

Table 3.1: Uniform priors adopted in the MCMC analysis.

H_0	Ω_b	A_s	n	α	β
[60, 80]	[0, 0.04]	[0, 1]	[0, 4]	[0, 1]	[0, 1]

3.7 Results and Discussion

The constraints on different models with and without matter creation for two different combinations of data, namely BASE and $BASE + R21$, are listed in Tables 3.2 and 3.3, respectively. As one can see from these tables, the best-fit values of these parameters are nearly same for different dark energy models. It is to be noted that the inclusion of local measurement $H_0 = 73.04 \pm 1.04 \text{ kms}^{-1}\text{Mpc}^{-1}$ from $R21$ [52] with BASE data, i.e., $BASE + R21$ always makes the H_0 value higher than BASE data for all the three models discussed above. For ΛCDM , we obtain $H_0 = 71.432^{+1.111}_{-1.038} \text{ kms}^{-1}\text{Mpc}^{-1}$ for full datasets, $BASE + R21$ where as for VGCG and VCG, we get $H_0 = 72.674^{+0.355}_{-0.758} \text{ kms}^{-1}\text{Mpc}^{-1}$ and $H_0 = 72.680^{+0.399}_{-0.775} \text{ kms}^{-1}\text{Mpc}^{-1}$, respectively. The residual tensions of these fitting results with $R21$ are 0.95σ , 0.36σ and 0.34σ , respectively. On the other hand, we find $H_0 = 69.791^{+1.008}_{-1.070} \text{ kms}^{-1}\text{Mpc}^{-1}$, $H_0 = 69.649^{+1.265}_{-2.379} \text{ kms}^{-1}\text{Mpc}^{-1}$ and $H_0 = 69.638^{+1.207}_{-2.491} \text{ kms}^{-1}\text{Mpc}^{-1}$ with BASE data taken for ΛCDM , VGCG and VCG models, respectively. The $R21$ value of H_0 exhibits a strong tension with the Planck 2018 release $H_0 = 67.4 \pm 0.5$ [3] at 3.92σ CL. Using BASE dataset, we find that the current H_0 tension can be alleviated to 2.19σ in ΛCDM , 1.51σ in VGCG and 1.84σ in the VCG model. In Figures 3.1 - 3.3, we show the constrained contours at 68.3% and 95.4% confidence levels in various parameters plane using two different combinations of data for ΛCDM , VGCG and VCG models, respectively.

In what follows, we discuss the evolutions of different cosmological parameters, stability of the models, selection criteria and Bayesian evidence analysis to compare the proposed models with the existing dark energy models.

3.7.1 Evolution of Cosmological Parameters

The evolutions of $H(z)$ with respect to the redshift z for VGCG and VCG with and without matter creation are shown in Figs. 3.4 and 3.5, respectively. The figures also show a comparison of VGCG and VCG models with ΛCDM model. As observed, VCG model closely matches the observed data during the whole evolution whereas the VGCG model coincides during late time evolution of the Universe.

Using BASE and $BASE + R21$ datasets, we obtain the present values of $q(z)$, $w(z)$ and $j(z)$

Table 3.2: Constraints on the parameters of the VGCG and VCG models with matter creation, together with the Λ CDM model. The combined data set $BAO + CC + SC$ is denoted by “BASE”.

Model	Parameters	BASE	BASE + R21
Λ CDM	$H_0, [\text{km s}^{-1} \text{Mpc}^{-1}]$	$69.791^{+1.008}_{-1.070}$	$71.432^{+1.111}_{-1.038}$
	Ω_m	$0.271^{+0.018}_{-0.016}$	$0.274^{+0.015}_{-0.018}$
	Ω_Λ	$0.721^{+0.023}_{-0.027}$	$0.726^{+0.022}_{-0.022}$
VGCG	$H_0, [\text{km s}^{-1} \text{Mpc}^{-1}]$	$69.649^{+1.265}_{-2.379}$	$72.674^{+0.355}_{-0.758}$
	Ω_b	$0.021^{+0.012}_{-0.019}$	$0.020^{+0.011}_{-0.019}$
	A_s	$0.758^{+0.027}_{-0.056}$	$0.767^{+0.026}_{-0.046}$
	β	$0.290^{+0.166}_{-0.254}$	$0.260^{+0.155}_{-0.241}$
	n	$0.114^{+0.090}_{-0.110}$	$0.119^{+0.091}_{-0.114}$
	α	$0.509^{+0.332}_{-0.457}$	$0.450^{+0.284}_{-0.392}$
VCG	$H_0, [\text{km s}^{-1} \text{Mpc}^{-1}]$	$69.638^{+1.207}_{-2.491}$	$72.680^{+0.399}_{-0.775}$
	Ω_b	$0.020^{+0.013}_{-0.011}$	$0.020^{+0.013}_{-0.016}$
	A_s	$0.756^{+0.021}_{-0.046}$	$0.765^{+0.021}_{-0.022}$
	β	$0.484^{+0.012}_{-0.025}$	$0.481^{+0.011}_{-0.032}$
	n	$0.105^{+0.088}_{-0.105}$	$0.109^{+0.088}_{-0.105}$

Table 3.3: Constraints on the parameters of the VGCG and VCG models without matter creation. The combined data set $BAO + CC + SC$ is denoted by “BASE”.

Model	Parameters	BASE	BASE + R21
VGCG	$H_0, [\text{km s}^{-1} \text{Mpc}^{-1}]$	$69.674^{+1.217}_{-0.563}$	$72.670^{+1.326}_{-0.412}$
	Ω_b	$0.020^{+0.015}_{-0.024}$	$0.020^{+0.014}_{-0.015}$
	A_s	$0.767^{+0.012}_{-0.033}$	$0.770^{+0.021}_{-0.016}$
	n	$0.129^{+0.101}_{-0.026}$	$0.116^{+0.004}_{-0.011}$
	α	$0.040^{+0.021}_{-0.017}$	$0.042^{+0.072}_{-0.061}$
VCG	$H_0, [\text{km s}^{-1} \text{Mpc}^{-1}]$	$69.870^{+1.133}_{-0.365}$	$72.640^{+1.362}_{-0.352}$
	Ω_b	$0.030^{+0.025}_{-0.036}$	$0.030^{+0.005}_{-0.021}$
	A_s	$0.994^{+0.001}_{-0.002}$	$0.994^{+0.017}_{-0.021}$
	n	$1.140^{+0.032}_{-0.101}$	$0.128^{+0.041}_{-0.035}$

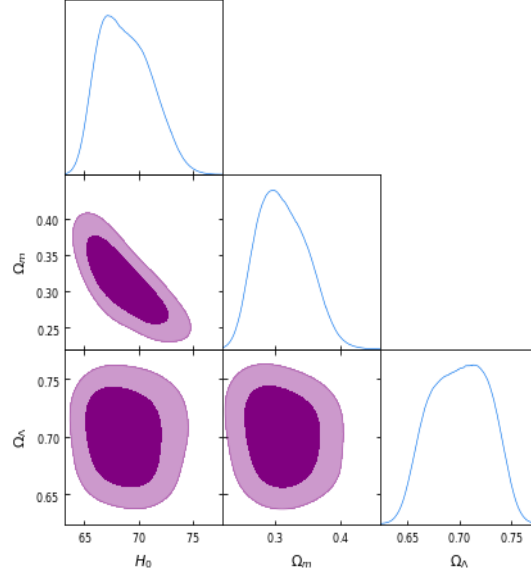


Figure 3.1: Two-dimensional confidence contours at the 68.3% and 95.4% levels for the Λ CDM model obtained from the BASE and BASE+R21 datasets.

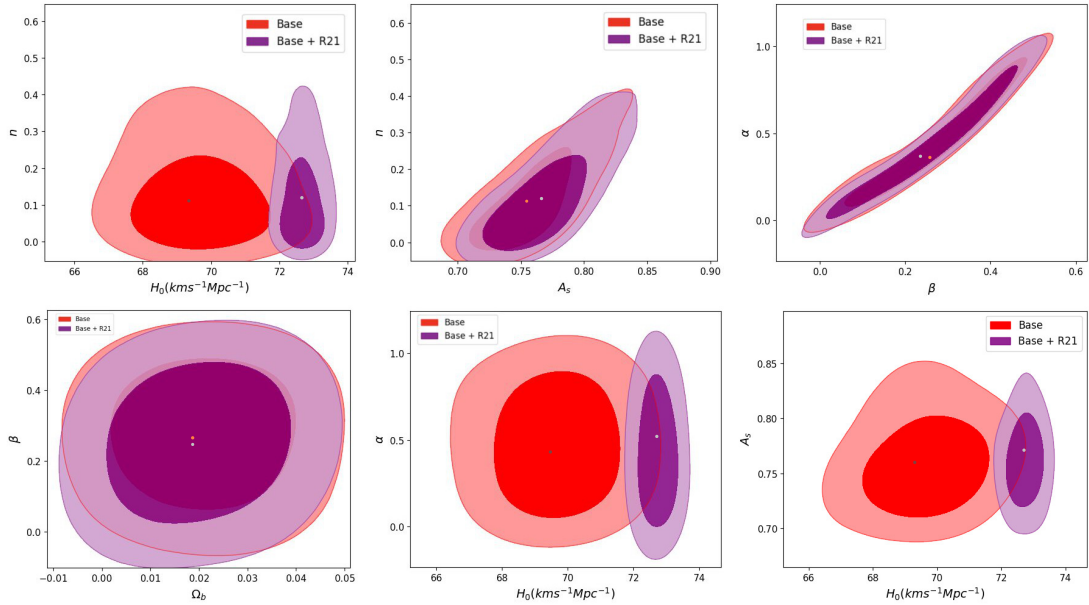


Figure 3.2: Representative two-dimensional confidence contours and one-dimensional posterior distributions for the VGCG model with matter creation using the BASE and BASE+R21 data sets.

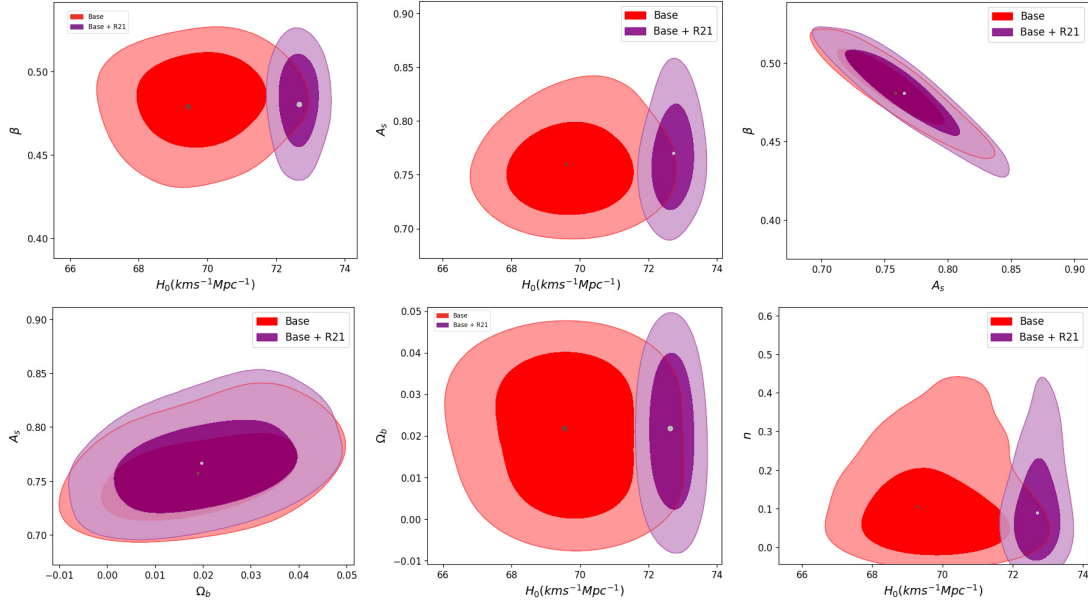


Figure 3.3: Representative two-dimensional confidence contours and one-dimensional posterior distributions for the VCG model with matter creation using the BASE and BASE+R21 data sets.

Table 3.4: Present values of the main cosmological and geometrical parameters for the Λ CDM, VGCG, and VCG models with matter creation.

Models→	Λ CDM		VGCG		VCG	
Parameters	BASE	+R21	BASE	+R21	BASE	+R21
z_{tr}	$0.692^{+0.034}_{-0.029}$	$0.689^{+0.028}_{-0.033}$	$0.610^{+0.030}_{-0.022}$	$0.631^{+0.037}_{-0.023}$	$0.674^{+0.029}_{-0.016}$	$0.687^{+0.036}_{-0.031}$
q_0	$-0.591^{+0.01}_{-0.01}$	$-0.590^{+0.02}_{-0.02}$	$-0.541^{+0.02}_{-0.02}$	$-0.542^{+0.05}_{-0.05}$	$-0.553^{+0.03}_{-0.03}$	$-0.555^{+0.03}_{-0.03}$
w_0	$-0.726^{+0.04}_{-0.04}$	$-0.728^{+0.01}_{-0.01}$	$-0.730^{+0.11}_{-0.11}$	$-0.740^{+0.13}_{-0.13}$	$-0.721^{+0.02}_{-0.02}$	$-0.728^{+0.02}_{-0.02}$
$t_0, (\text{Gyr})$	$13.73^{+0.02}_{-0.02}$	$13.76^{+0.04}_{-0.04}$	$13.91^{+0.03}_{-0.03}$	$13.94^{+0.07}_{-0.07}$	$13.87^{+0.03}_{-0.03}$	$13.89^{+0.01}_{-0.01}$
j_0	1	1	$0.933^{+0.21}_{-0.21}$	$0.941^{+0.10}_{-0.10}$	$0.979^{+0.02}_{-0.02}$	$0.966^{+0.02}_{-0.02}$
C_s^2	—	—	0.329	0.379	0.730	0.740

Table 3.5: Present values of the main cosmological and geometrical parameters for the VGCG and VCG models without matter creation.

Models→	VGCG		VCG	
Parameters	BASE	+R21	BASE	+R21
z_{tr}	$0.689^{+0.023}_{-0.021}$	$0.694^{+0.019}_{-0.024}$	$0.955^{+0.018}_{-0.023}$	$0.922^{+0.018}_{-0.014}$
q_0	$-0.652^{+0.02}_{-0.02}$	$-0.617^{+0.02}_{-0.02}$	$-0.590^{+0.02}_{-0.02}$	$-0.573^{+0.02}_{-0.02}$
w_0	$-0.589^{+0.11}_{-0.11}$	$-0.595^{+0.15}_{-0.15}$	$-0.838^{+0.01}_{-0.01}$	$-0.842^{+0.02}_{-0.02}$
$t_0, (\text{Gyr})$	$13.77^{+0.01}_{-0.01}$	$13.83^{+0.03}_{-0.03}$	$13.91^{+0.02}_{-0.02}$	$13.93^{+0.01}_{-0.01}$
j_0	$0.959^{+0.19}_{-0.19}$	$0.742^{+0.21}_{-0.21}$	$0.892^{+0.11}_{-0.11}$	$1.01^{+0.12}_{-0.12}$
C_s^2	0.414	0.490	0.781	0.805

for Λ CDM, VGCG and VCG models with and without matter creation, respectively which are summarized in Tables 3.4 and 3.5. The evolutions of $q(z)$ with redshift z of all the three models for BASE and $BASE + R21$ datasets are shown in Figs. 3.6 and 3.7, respectively. We observe that the models show a transition from deceleration ($q(z) > 0$) to acceleration ($q(z) < 0$). In figures, a dot represents the present value of $q(z)$. The present values of $q(z)$ at $z = 0$ from BASE and $BASE + R21$ datasets are $q_0 = -0.591^{+0.01}_{-0.01}$ and $q_0 = -0.590^{+0.02}_{-0.02}$ for Λ CDM, $q_0 = -0.541^{+0.02}_{-0.02}$ and $q_0 = -0.542^{+0.05}_{-0.05}$ for VGCG model and $q_0 = -0.553^{+0.03}_{-0.03}$ and $q_0 = -0.555^{+0.03}_{-0.03}$ for VCG model with matter creation, respectively, as given in Table 3.4. The values of q_0 show that the models undergo accelerated expansion at present-day. We obtain the transition value z_{tr} where the models transit from deceleration to acceleration at $z_{tr} = 0.610^{+0.030}_{-0.022}$ and $z_{tr} = 0.631^{+0.037}_{-0.023}$ for VGCG model, and $z_{tr} = 0.674^{+0.029}_{-0.016}$ and $z_{tr} = 0.687^{+0.036}_{-0.031}$ in case of VCG model, respectively. These transition values are very close to the Λ CDM transition values (cf. Table 3.4). We also present the values of z_{tr} and q_0 in Table 3.5 for these models without matter creation.

The evolutions of EoS parameter $w(z)$ with z of Λ CDM, VGCG and VCG models are shown in Figs. 3.8 and 3.9 with two datasets. For both datasets, EoS shows quintessence phase ($-1 < w < 0$) at present and Λ CDM behavior at late epoch. Figures 3.10 and 3.11 show the evolutions of the jerk parameter, $j(z)$ with respect to z for the discussed models using BASE and $BASE + R21$ datasets. Based on these two datasets, the present values of $j(z)$ for VGCG model are $j_0 = 0.933^{+0.21}_{-0.21}$ and $j_0 = 0.941^{+0.10}_{-0.10}$ where as for VCG model we get $j_0 = 0.979^{+0.02}_{-0.02}$ and $j_0 = 0.966^{+0.02}_{-0.02}$, respectively which are very close to the value $j_0 = 1$ of Λ CDM model.

The ages of Universe for all the models are given in Table 3.5. The constraints on the age of the Universe for VGCG model are $t_0 = 13.91^{+0.03}_{-0.03}$ Gyr with BASE dataset and $t_0 = 13.94^{+0.07}_{-0.07}$ Gyr with $BASE + R21$ dataset whereas for VCG model we have $t_0 = 13.87^{+0.03}_{-0.03}$ Gyr and $t_0 = 13.89^{+0.01}_{-0.01}$ Gyr, respectively. The ages of the Universe for these Chaplygin gas models are very close to the age of the Universe of Λ CDM model. In Table 3.5, we list the present values of z_{tr} , q_0 , w_0 , j_0 and t_0 for the proposed models without matter creation to compare the deviation for constraint results on extended CG models that with matter creation.

3.7.2 Stability of the Models

The stability of the models is examined through the squared sound speed

$$C_s^2 = \frac{dp}{d\rho}. \quad (3.33)$$

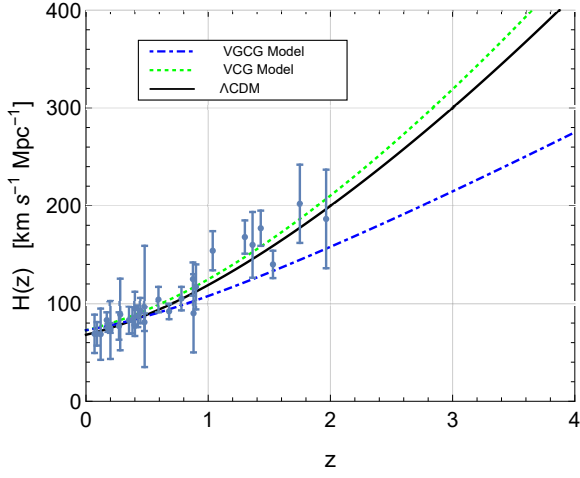


Figure 3.4: Evolution of $H(z)$ with redshift z in the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE data set. The gray points with uncertainty bars correspond to the $H(z)$ sample. The black bold line represents the Λ CDM model.

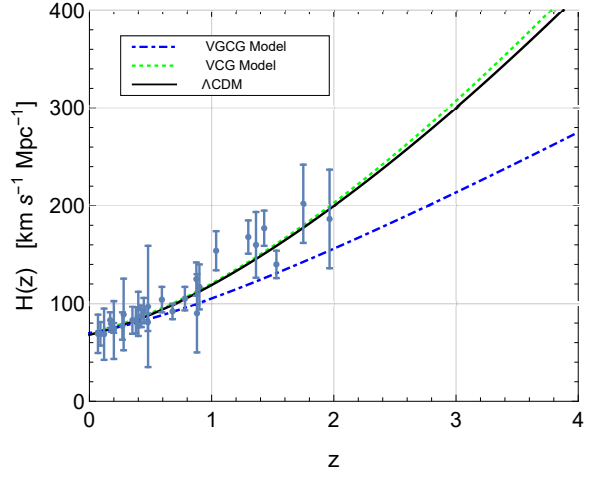


Figure 3.5: Evolution of $H(z)$ with redshift z in the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE+R21 data set. The gray points with uncertainty bars correspond to the $H(z)$ sample. The black bold line represents the Λ CDM model.

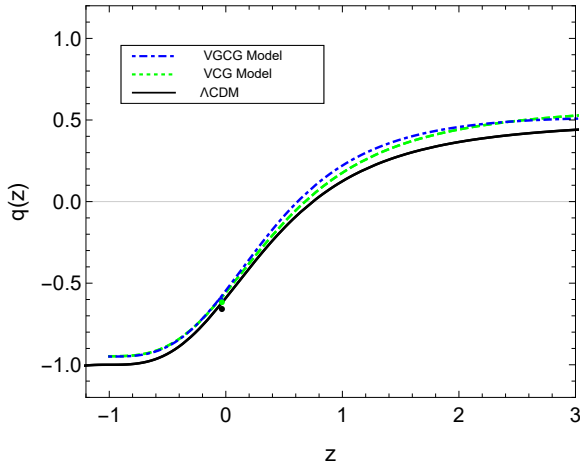


Figure 3.6: Evolution of $q(z)$ with z in the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE data set. The dotted line indicates the present value of $q(z)$.

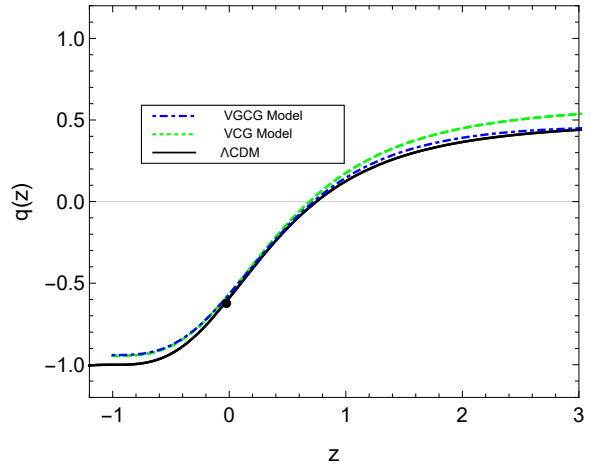


Figure 3.7: Evolution of $q(z)$ with z in the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE+R21 data set. The dotted line indicates the present value of $q(z)$.

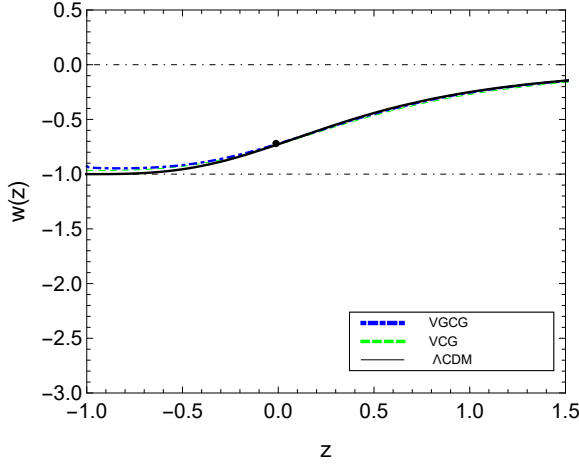


Figure 3.8: Evolution of $w(z)$ with z in the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE data set. The dotted line indicates the present value of $w(z)$.

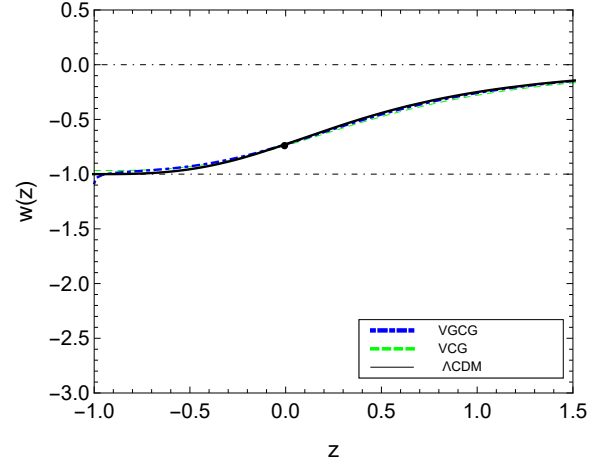


Figure 3.9: Evolution of $w(z)$ with z in the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE+R21 data set. The dotted line indicates the present value of $w(z)$.

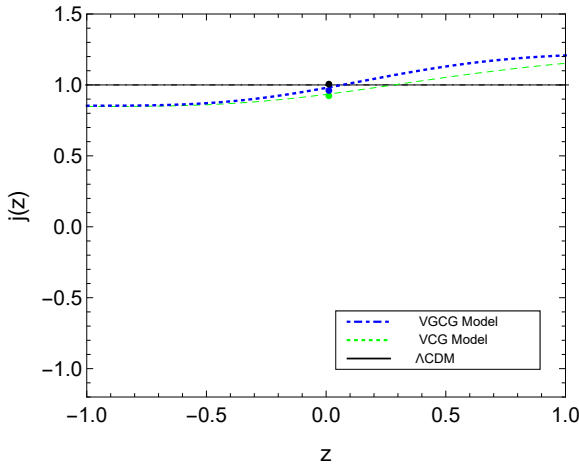


Figure 3.10: Trajectories of $j(z)$ for the VGCG and VCG models with matter creation, together with the Λ CDM model, using the best-fit values from the BASE data set. The dotted line indicates the present value j_0 .

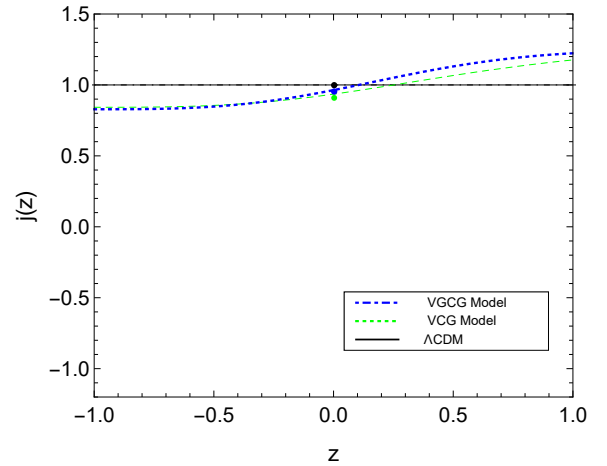


Figure 3.11: Trajectories of $j(z)$ for the VGCG and VCG models with matter creation, together with the Λ CDM model, using the BASE+R21 data set. The dotted line indicates the present value j_0 .

A stable model requires

$$C_s^2 \geq 0, \quad (3.34)$$

while physically acceptable propagation further requires

$$0 < C_s^2 < 1. \quad (3.35)$$

For the VGCG model,

$$C_s^2 = \frac{\alpha A_s \left(1 - \frac{n}{3(1-\beta)(1+\alpha)} \right) (1+z)^n}{A_s(1+z)^n + (1-A_s)(1+z)^{3(1-\beta)(1+\alpha)}}, \quad (3.36)$$

and for the VCG model,

$$C_s^2 = \frac{A_s \left(1 - \frac{n}{6(1-\beta)} \right) (1+z)^n}{A_s(1+z)^n + (1-A_s)(1+z)^{6(1-\beta)}}. \quad (3.37)$$

The sound speed should be positive in the stable model and hence this is justified by taking the constant $\alpha > 0$. Using the best-fit values of model's parameters of BASE and BASE+R21 datasets in above equations, we find the current values $C_s^2 = 0.329$ and $C_s^2 = 0.379$ for VGCG model while $C_s^2 = 0.730$ and $C_s^2 = 0.740$ for VCG model which are listed in Table 3.5. We plot the trajectories of C_s^2 with z in Figs. 3.12 and 3.13 using two combinations of datasets for VGCG and VCG models. The trajectories show that the squared sound speed is always positive through out the evolution lying in $(0, 1)$ and hence positive and less than unity in both the discussed models. This ensure that these models do not surpass the velocity of light which confirm their stability. Table 3.5 shows the C_s^2 for VGCG and VCG models without matter creation. The C_s^2 are positive and less than unity that also confirm the stability.

3.7.3 Model Selection Criteria

We calculate the values of χ_{min}^2/dof for Λ CDM, VGCG and VCG models with and without matter creation, which are placed in Tables 3.6 and 3.7, respectively. Here, the value of dof (degree of freedom) equals to the number of observational data points (N) minus the number of parameters (K). Here, the number of observational data points for BASE is $N = 273$ (Cosmic Chronometer=30, type Ia supernova(pantheon)=40 and Uncorrelated BAO=17, Hubble diagram for quasars = 24 and Gamma Ray Bursts=162), and $N = 274$ for BASE+R21, and number of parameters are $K = 3, 6$ and 5 for Λ CDM, VGCG and VCG models with matter

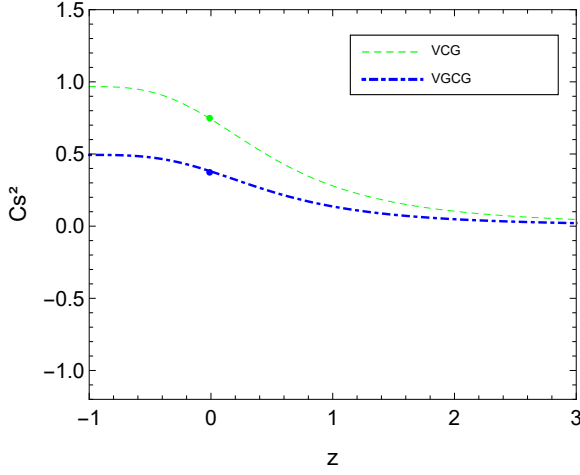


Figure 3.12: Trajectories of C_s^2 for the VGCG and VCG models with matter creation using the best-fit values of the BASE data set.

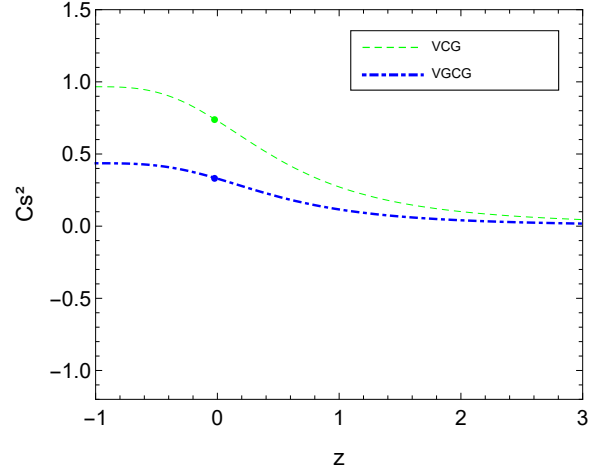


Figure 3.13: Trajectories of C_s^2 for the VGCG and VCG models with matter creation using the best-fit values of the BASE+R21 data set.

creation, and $K = 5$ and 4 for VGCG and VCG models without matter creation, respectively. From Tables 3.6 and 3.7, we observe that both the models with and without matter creation have $\chi_{\min}^2/dof < 1$ for both datasets. This shows that both the models are in good agreement with respect to the observational data as far as $\chi_{\min}^2/dof$ is concerned.

Although the above analysis are in good agreement with one another, further differences arise due to the choice of cosmological model made.

To compare the proposed models with the reference Λ CDM cosmology, the Akaike and Bayesian information criteria are employed, respectively [85, 86]. These are defined as

$$\text{AIC} = \chi_{\min}^2 + \frac{2NK}{N - K - 1}, \quad (3.38)$$

and

$$\text{BIC} = \chi_{\min}^2 + K \ln N, \quad (3.39)$$

where $\chi_{\min}^2$ measures the quality of fit, K is the number of free parameters, and N is the total number of observational data points. Relative to the reference model, one defines

$$\Delta\text{AIC} = \text{AIC}_x - \text{AIC}_{\Lambda\text{CDM}}, \quad (3.40)$$

and

$$\Delta\text{BIC} = \text{BIC}_x - \text{BIC}_{\Lambda\text{CDM}}, \quad (3.41)$$

where x denotes either the VGCG or VCG model.

The AIC (BIC) and $\Delta\text{AIC}(\Delta\text{BIC})$ for VGCG and VCG models with and without matter cre-

ation against the Λ CDM model are listed in Tables 3.6 and 3.7, respectively. The ΔAIC values of VGCG and VCG models show that these models are still supported by current observational data. For BIC selection method, it seems that these models are not in favored by observational data. The reason is that AIC is always more general towards extra parameters whereas BIC tends to penalize them. Hence, the AIC remains useful as it gives an upper limit to the number of parameters.

3.7.4 Bayesian Evidence Analysis

At the model-comparison level, Bayesian evidence is also considered [88, 89]. The evidence is defined as

$$E = \int \mathcal{L}(\theta)P(\theta) d\theta, \quad (3.42)$$

where θ is the parameter vector, $\mathcal{L}$ is the likelihood, and $P(\theta)$ is the prior. The corresponding Bayes factor between two models M_1 and M_2 is

$$B_{12} = \frac{E(M_1)}{E(M_2)}. \quad (3.43)$$

The Bayes factor provides a measure with which to discriminate between the models. It is usual to consider the logarithm of the Bayes factor, for which the so-called ‘Jeffreys’ scale [87, 88] gives empirically calibrated levels of significance for the strength of evidence. Many papers on Bayesian evidence have adopted the interpretive scale suggested by Jeffreys which is presented in Table 1.5. For each model and dataset we estimate the values of the logarithm of the Bayesian evidence ($\ln E$) and the Bayes factor ($\ln B$) which is the difference between the mean $\ln E$ of Λ CDM model and model concerned. We assumed the Λ CDM model as the reference one, say M_1 and therefore its entry is zero by definition.

Table 3.6 presents the Bayesian evidence and Bayes factor for VGCG and VCG models with matter creation. Looking at the “BASE” and “BASE+R21” datasets respectively, we find $\ln B_{12} = 2.114$ and $\ln B_{12} = 1.983$ for VGCG model, and $\ln B_{12} = 2.689$ and $\ln B_{12} = 2.581$ for VCG model which suggest that VGCG model has positive evidence where as the VCG model has moderate evidence.

Similarly, using the priors and the same datasets, we obtain the Bayes factor for VGCG and VCG models without matter creation which are provided in Table 3.7. By considering the “BASE” data, we find $\ln B_{12} = 2.271$ for VGCG model and $\ln B_{12} = 8.994$ for VCG model. BASE+R21 data gives $\ln B_{12} = 1.37$ for VGCG model and $\ln B_{12} = 8.318$ for VCG model. It suggests that VGCG model has positive evidence by both the datasets where as the VCG

Table 3.6: χ^2_{red} , AIC, BIC, Bayesian evidence, and Bayes factor values for the Λ CDM, VGCG, and VCG models with matter creation.

Model	Parameters	BASE	+R21
Λ CDM	χ^2_{min}	253.92	254.17
	χ^2_{min}/dof	0.94	0.94
	AIC	260.01	260.26
	BIC	270.75	271.00
	$\ln E(M_1)$	-123.824	-124.116
VGCG	χ^2_{min}	255.51	257.74
	χ^2_{min}/dof	0.965	0.953
	AIC	267.82	270.05
	BIC	289.16	291.41
	Δ AIC	7.81	9.79
	Δ BIC	18.41	20.41
	$\ln E(M_2)$	-125.938	-126.099
	$\ln B_{12}$	2.114	1.983
VCG	χ^2_{min}	254.59	257.63
	χ^2_{min}/dof	0.95	0.96
	AIC	264.81	267.86
	BIC	282.64	285.70
	Δ AIC	4.80	7.60
	Δ BIC	11.89	14.70
	$\ln E(M_2)$	-126.513	-126.697
	$\ln B_{12}$	2.689	2.581

Table 3.7: χ^2_{red} , AIC, BIC, Bayesian evidence, and Bayes factor values for the VGCG and VCG models without matter creation.

Model	Parameters	BASE	+R21
VGCG	χ^2_{min}	255.32	257.94
	χ^2_{min}/dof	0.956	0.962
	AIC	265.54	268.16
	BIC	283.36	286.01
	Δ AIC	5.53	7.92
	Δ BIC	12.61	15.01
	$\ln E(M_2)$	-126.095	-125.486
	$\ln B_{12}$	2.271	1.370
VCG	χ^2_{min}	254.21	255.76
	χ^2_{min}/dof	0.959	0.950
	AIC	262.36	263.91
	BIC	276.65	278.21
	Δ AIC	2.35	3.67
	Δ BIC	5.90	7.21
	$\ln E(M_2)$	-132.818	-132.434
	$\ln B_{12}$	8.994	8.318

model has strong (decisive) evidence.

3.8 Conclusion

In this chapter, the variable generalized Chaplygin gas model has been studied in the framework of matter creation cosmology within a spatially homogeneous and isotropic flat Friedmann–Lemaître–Robertson–Walker space-time. A particular case of this construction, namely the variable Chaplygin gas model, has also been investigated. The aim of the analysis was to determine whether the combination of a variable Chaplygin-type equation of state and adiabatic matter creation can provide a viable description of late-time cosmic expansion while remaining consistent with current observations.

The study proceeded in two stages. First, the analytical background solutions for the VGCG and VCG models were obtained in the presence of matter creation. Explicit expressions were derived for the energy density, Hubble parameter, deceleration parameter, equation-of-state parameter, and jerk parameter. These results show that the models admit a transition from an early decelerating epoch to a late accelerating one.

Second, the model parameters were constrained by means of a Bayesian observational analysis using two late-time data combinations: the BASE sample, consisting of BAO, CC, and standard candles, and the BASE+R21 combination, which additionally includes the local Hubble constant prior. We observe that VGCG and VCG models with matter creation can alleviate the current H_0 tension from 3.92σ to 1.51σ and 1.84σ using BASE data. Our observations indicate a notable variance in the VGCG and VCG models with parameter β from the Λ CDM. These plots vividly illustrate the model's capability to generate late-stage cosmic acceleration while also reflecting a decelerated expansion in earlier times. The transition redshift values, q_0 , w_0 and age of the Universe with respect to VGCG and VCG models with and without matter creation are found to be very close to Λ CDM and Planck 2018 release. The evolution trajectories of $j(z)$ reveal minimal deviation from the Λ CDM model for both VGCG and VCG models.

The AIC and BIC analysis show that models such as VGCG and VCG with and without matter creation exhibit in the range of $4 < \Delta AIC < 10$, suggesting to as much as the preferred model with the joint observational data. In terms of BIC values, the models VGCG and VCG with and without matter creation present $\Delta BIC > 10$, providing evidence against the data used for analysis. It is noteworthy that the squared sound speed for these models lies within the $0 < C_S^2 < 1$ range, thereby not exceeding the speed of light, which signifies the stability of

these models. Consequently, it is deduced that the models discussed in this chapter yield cosmological parameters that are well-bounded by the combined observational data. Furthermore, these models closely resemble the Λ CDM model in their behavior.

The physical viability of the models was examined through the squared sound speed. For both the VGCG and VCG models, with and without matter creation, the squared sound speed remains positive and less than unity throughout the evolution. This establishes the stability of the models and confirms that they do not violate the expected causal bound. The Bayesian evidence nevertheless indicates positive to moderate support for the matter-creation versions and strong support in the case of the VCG model without matter creation.

It may therefore be concluded that the variable generalized Chaplygin gas and variable Chaplygin gas models, when studied within matter creation cosmology, provide a consistent and physically acceptable description of late-time cosmic acceleration. They remain close to Λ CDM at the background level, satisfy the stability requirement, and are able to ease the H_0 tension to some extent. These results suggest that the interplay between variable Chaplygin-type fluids and adiabatic matter creation remains a useful direction for cosmological model building beyond the standard picture.

Chapter 4

Matter Creation and Bulk Viscous Cosmology with Modified Chaplygin Gas

4.1 Introduction

This chapter extends the General Relativity models of Chapters 2 and 3 by combining modified Chaplygin gas, adiabatic matter creation, and bulk-viscous matter. The analysis is based on Bhardwaj and Singh [169]. The modified Chaplygin gas is treated as the effective dark-energy sector, with creation pressure assigned to it; viscous matter and conserved baryons are separate components.

The common discussions of Chaplygin-type fluids, matter creation, and bulk viscosity are given in Sections 1.8.1, 1.9, and 1.10. The present chapter develops the combined model and examines its observational and thermodynamic behaviour through the DS1 and DS2 data combinations and the generalized second law.

Section 4.2 presents the model equations, Section 4.3 specifies the observational analysis, and Section 4.4 discusses the results. Section 4.5 gives the conclusion. The following chapters turn from General Relativity to nonmetricity-based gravity.

4.2 Basic Formalism and Cosmological Field Equations

We consider a spatially flat, homogeneous, and isotropic Friedmann–Lemaître–Robertson–Walker space-time with line element

$$ds^2 = -dt^2 + a^2(t) [dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)], \quad (4.1)$$

where $a(t)$ denotes the scale factor.

4.2.1 Matter Creation cosmology

The open-system thermodynamics and particle balance are described in Section 1.9. We use the adiabatic prescription of Eq. (1.65) for the modified Chaplygin gas component.

Matter creation leads to an effective creation pressure

$$P_c = -\frac{\rho + p}{3H}\Gamma. \quad (4.2)$$

Adopting the commonly used ansatz

$$\Gamma = 3\beta H, \quad (4.3)$$

with $\beta \geq 0$, Eq. (4.2) becomes

$$P_c = -\beta(\rho + p). \quad (4.4)$$

The corresponding modified conservation law is

$$\dot{\rho} + 3H(\rho + p + P_c) = 0. \quad (4.5)$$

4.2.2 Modified Chaplygin Gas Component

We model the dominant cosmic component using the Modified Chaplygin Gas (MCG), characterized by the equation of state [136]

$$p_{mcg} = A\rho - \frac{C}{\rho^\alpha}, \quad (A \geq 0, 0 \leq \alpha \leq 1, C > 0), \quad (4.6)$$

where A , C , and α are free parameters. Combining Eqs. (4.4) and (4.6) with the conservation law (4.5), one obtains

$$\dot{\rho} + 3H(1 - \beta) \left[(1 + A)\rho - \frac{C}{\rho^\alpha} \right] = 0. \quad (4.7)$$

Integrating this equation with the condition $\rho(z=0) = \rho_0$, the solution can be written as

$$\rho(z) = \rho_0 \left[A_s + (1 - A_s)(1 + z)^{3(1-\beta)(1+A)(1+\alpha)} \right]^{\frac{1}{1+\alpha}}, \quad (4.8)$$

where

$$A_s = \frac{C}{(1 + A)\rho_0^{1+\alpha}}. \quad (4.9)$$

4.2.3 Bulk Viscous Matter Component

To model dissipative behaviour in the cosmic medium, we consider an additional matter sector endowed with bulk viscosity. Although this component has vanishing equilibrium pressure, $p_m = 0$, deviations from thermodynamic equilibrium generate an effective viscous pressure Π . The corresponding continuity equation is then written as

$$\dot{\rho}_m + 3H(\rho_m + \Pi) = 0, \quad (4.10)$$

with viscous pressure

$$\Pi = -3H\xi_0\rho_m^{1/2}, \quad (4.11)$$

where $\xi_0 > 0$ is the viscosity coefficient. Integration gives

$$\rho_m(z) = \rho_{m,0}(1+z)^{3(1-\sqrt{3}\xi_0)}. \quad (4.12)$$

This behaviour indicates that bulk viscosity causes the matter density to decrease more gradually as the Universe expands, thereby acting in a manner similar to a negative-pressure contribution. From Eq. (4.12) one immediately sees that the exponent $3(1 - \sqrt{3}\xi_0)$ must remain positive in order for the dust density to increase towards the past as $z \rightarrow \infty$, as required for a standard matter-dominated era. Combined with the requirement $\xi_0 \geq 0$ (which guarantees a negative viscous pressure and non-decreasing entropy), this leads to the theoretical bound For a physically meaningful background evolution, one requires

$$0 \leq \xi_0 < \frac{1}{\sqrt{3}}. \quad (4.13)$$

4.2.4 Field equations

The energy density responsible for the gravitational dynamics is obtained by adding the contributions from the MCG, the bulk-viscous matter sector, and the conserved baryonic component. In this work these three components have distinct physical roles: the MCG fluid, obeying the equation of state (4.6) and evolving according to (4.7) and (4.8), is interpreted as the dark-energy sector and includes the effect of gravitationally induced particle production through the factor $(1 - \beta)$ in its redshift evolution. The pressureless component ρ_m , governed by (4.10) and (4.12), represents the cold dark matter sector at the background level and is affected only by bulk viscous dissipation.

Finally, baryonic matter ρ_b is conserved separately and non-relativistic. The baryonic density evolves as

$$\rho_b(z) = \rho_{b,0}(1+z)^3. \quad (4.14)$$

The Friedmann equation reads

$$3H^2(z) = \rho_{mcg}(z) + \rho_m(z) + \rho_b(z). \quad (4.15)$$

Substituting Eqs. (4.8), (4.12), and (4.14) into Eq. (4.15), and normalizing by the present-day Hubble constant H_0 , we derive the dimensionless Hubble parameter $E(z) = H(z)/H_0$:

$$E^2(z) = \Omega_{mcg,0} \left[A_s + (1 - A_s)(1+z)^{3(1-\beta)(1+A)(1+\alpha)} \right]^{\frac{1}{1+\alpha}} + \Omega_{m,0}(1+z)^{3(1-\sqrt{3}\xi_0)} + \Omega_{b,0}(1+z)^3, \quad (4.16)$$

where the present-day density parameters are defined as:

$$\Omega_{mcg,0} = \frac{\rho_{mcg,0}}{3H_0^2}, \quad \Omega_{m,0} = \frac{\rho_{m,0}}{3H_0^2}, \quad \Omega_{b,0} = \frac{\rho_{b,0}}{3H_0^2}, \quad (4.17)$$

and satisfy the flatness constraint:

$$\Omega_{mcg,0} + \Omega_{m,0} + \Omega_{b,0} = 1. \quad (4.18)$$

Equation (4.16) is the main background relation used in the observational analysis.

4.3 Observational Data and Statistical Methodology

To obtain the posterior constraints on the model parameters, we combine five independent observational probes in a joint statistical framework. These consist of Cosmic Chronometer (CC) measurements, which yield direct and largely model-independent determinations of the Hubble parameter $H(z)$; the Pantheon+ Type Ia supernova compilation, used without the SH0ES calibration prior to provide geometrical distance information; and BAO observations from DESI DR2 and SDSS-IV, which supply standard-ruler constraints through the sound horizon scale r_d . We additionally include redshift-space distortion (RSD) data for $f\sigma_8(z)$ to examine the growth of cosmic structure independently of the background expansion history. The local SH0ES determination, $H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (R22), is incorporated separately as an optional Gaussian prior. The total likelihood is constructed by combining

the individual χ^2 contributions from these datasets, allowing parameter degeneracies to be reduced and the observational consistency of the proposed hybrid model to be assessed in a comprehensive manner.

The free parameters of the Λ CDM and Modified Chaplygin Gas models are constrained using two late-time data combinations:

$$\text{DS1} = \text{Pantheon+} + \text{CC} + \text{BAO} + f(z)\sigma_8, \quad (4.19)$$

and

$$\text{DS2} = \text{DS1} + \text{R22}. \quad (4.20)$$

4.3.1 Data Sets

The Cosmic Chronometer compilation contains 32 direct measurements of $H(z)$ over the range $0.07 \leq z \leq 1.965$ [81, 180–183]. The Pantheon+ sample consists of 1701 spectroscopically confirmed Type Ia supernovae [184]. The BAO data set uses 13 measurements from DESI DR2 and SDSS-IV, with the sound horizon r_d treated as a free parameter [133, 185–193]. The growth sector is constrained with 18 measurements of $f(z)\sigma_8(z)$ [194–196]. Finally, the SH0ES prior is taken as $H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [52].

4.3.2 Likelihood Analysis

The total chi-square for DS1 is written as

$$\chi_{\text{DS1}}^2 = \chi_{\text{SNe}}^2 + \chi_{\text{CC}}^2 + \chi_{\text{BAO}}^2 + \chi_{\sigma_8}^2, \quad (4.21)$$

while for DS2 one has

$$\chi_{\text{DS2}}^2 = \chi_{\text{DS1}}^2 + \chi_{\text{R22}}^2. \quad (4.22)$$

The local Hubble prior contributes

$$\chi_{\text{R22}}^2 = \frac{(H_0^{\text{th}} - 73.04)^2}{(1.04)^2}. \quad (4.23)$$

4.3.3 MCMC Procedure and Priors

The parameter estimation is performed with the `emcee` Markov Chain Monte Carlo sampler [197], using 80 walkers and 10000 steps per chain. The adopted flat priors are summarized in Table 4.1.

Table 4.1: Flat priors adopted in the MCMC analysis.

H_0	Ω_m	Ω_b	A_s	A	α	β	ξ_0	r_d	$\mathcal{M}$	σ_8
[60, 80]	[0, 0.5] for DS1 [0.1, 0.5] for DS2	[0, 0.1]	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[140, 150]	[-20, -18]	[0, 1]

4.4 Results and Discussion

The observational constraints for the modified Chaplygin gas scenario with matter creation and bulk viscous effects are obtained by considering two different combinations of datasets, denoted as DS1 and DS2, where DS1: consisting of Pantheon+, cosmic chronometers, BAO, and $f\sigma_8$ data, and where DS2, which extends DS1 by including the local SH0ES prior on the Hubble constant. The results are discussed here in terms of the best-fit parameter values, the reconstructed cosmological evolution, the thermodynamic consistency of the model, and the statistical comparison with the standard Λ CDM scenario.

The marginalized one-dimensional posterior distributions and the associated two-dimensional confidence contours for the Λ CDM model and the modified Chaplygin gas scenario are displayed in Figures 4.1 and 4.2 for the DS1 and DS2 data combinations, respectively. The best-fit values obtained from the MCMC analysis are summarized in Tables 4.2 and 4.3 [169].

We examine the redshift evolution of the principal cosmological indicators, namely the Hubble parameter $H(z)$, deceleration parameter $q(z)$, jerk parameter $j(z)$, and snap parameter $s(z)$, in order to assess the agreement of the model with present observational evidence. In addition, the physical stability of the proposed scenario and its relative statistical preference are investigated through suitable information criteria to evaluate the overall viability of the framework.

4.4.1 Hubble Parameter

The behaviour of the Hubble parameter for the Λ CDM model and the four modified Chaplygin gas cases is shown in Figures 4.3 and 4.4, respectively, for the DS1 and DS2 datasets. For DS1, all models fit the cosmic chronometer data reasonably well. The full MCG model closely follows the Λ CDM background evolution, showing that the combined effect of the

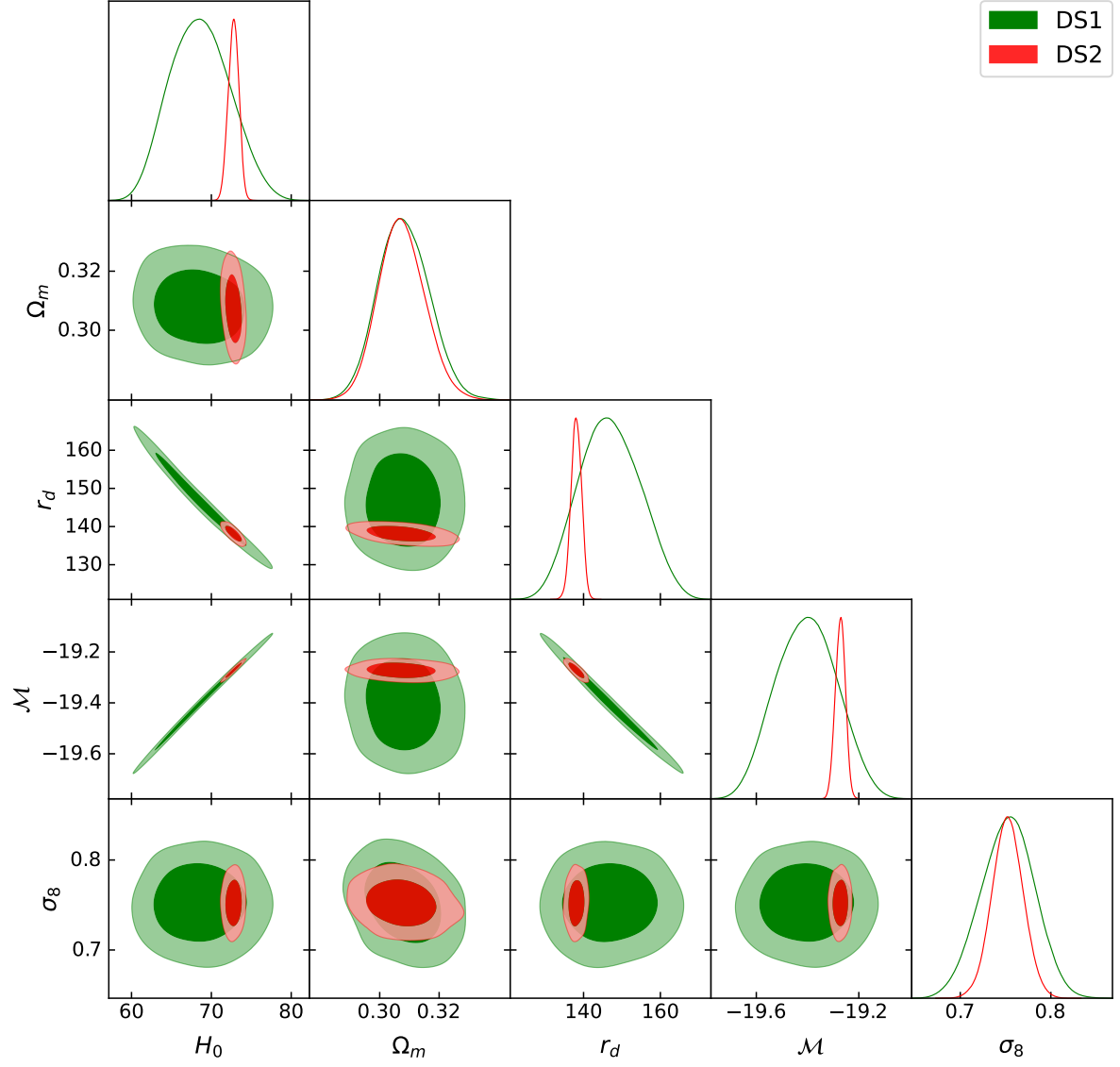


Figure 4.1: Parameter best-fit estimates for the Λ CDM model derived from the DS1 and DS2 datasets, reported at the 1σ and 2σ confidence levels.

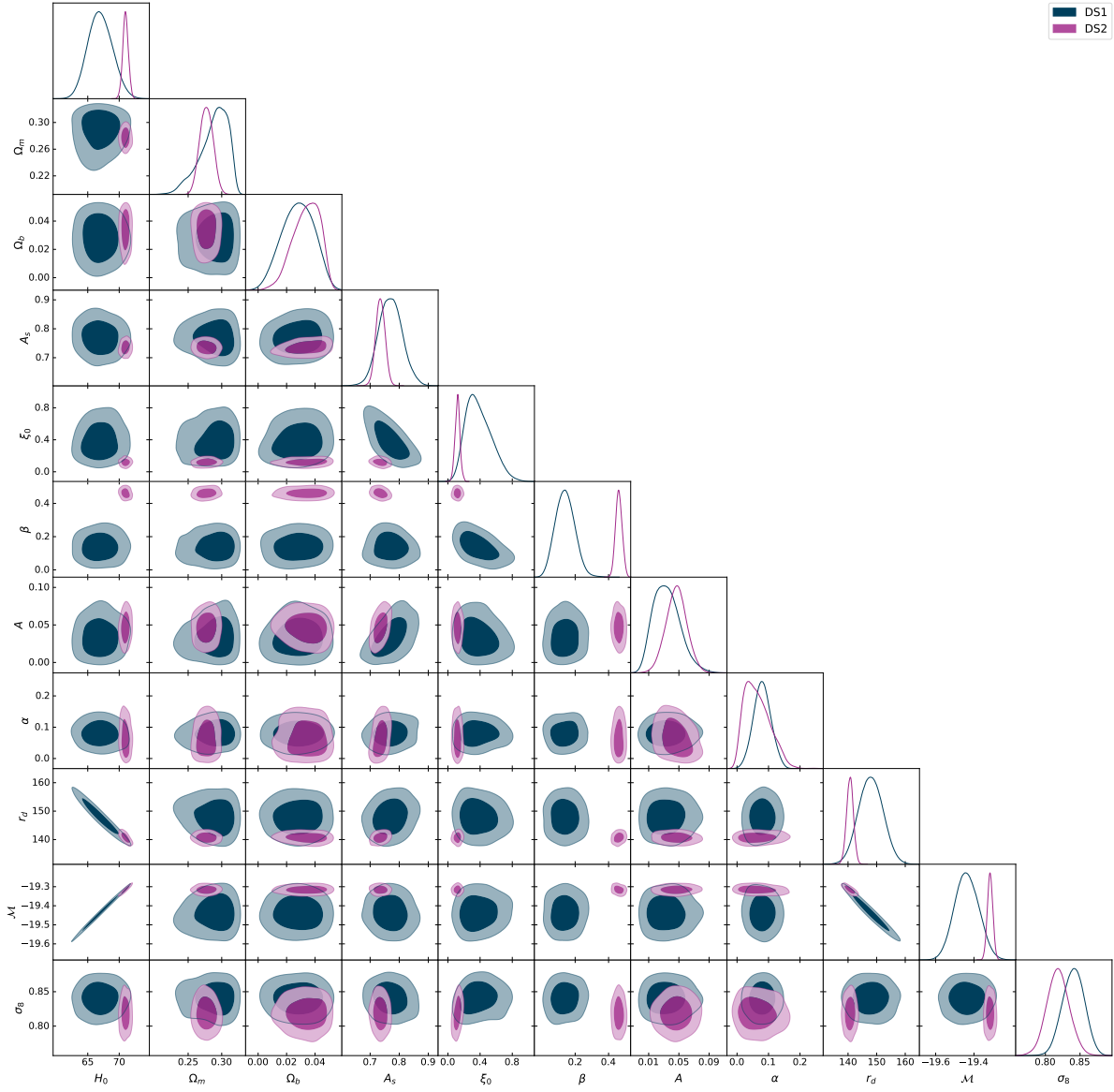


Figure 4.2: Parameter best-fit estimates for the modified Chaplygin gas model and its limiting cases obtained from the DS1 and DS2 datasets, with results reported at the 1 σ and 2 σ confidence levels.

Table 4.2: Best-fit parameter estimates with 1σ uncertainties for the Λ CDM scenario and the modified Chaplygin gas models using the DS1 dataset.

Parameter	Prior	Λ CDM	MCG Model	MCG ($\beta = 0$)	MCG ($\xi_0 = 0$)	MCG ($\beta = 0, \xi_0 = 0$)
H_0 [km s $^{-1}$ Mpc $^{-1}$]	[60, 80]	68.6 ± 3.6	67.0 ± 1.9	67.88 ± 0.79	68.6 ± 2.4	67.91 ± 0.59
Ω_m	[0, 0.5]	0.387 ± 0.008	0.288 ± 0.002	0.279 ± 0.006	0.277 ± 0.004	0.303 ± 0.007
Ω_b	[0, 0.1]	—	0.028 ± 0.012	0.026 ± 0.012	0.0249 ± 0.002	0.0305 ± 0.001
A_s	[0, 1]	—	0.770 ± 0.04	0.785 ± 0.02	0.749 ± 0.09	0.685 ± 0.02
A	[0, 1]	—	0.034 ± 0.015	0.036 ± 0.017	0.108 ± 0.039	0.191 ± 0.007
α	[0, 1]	—	0.08 ± 0.028	0.099 ± 0.030	0.181 ± 0.039	0.060 ± 0.025
β	[0, 1]	—	0.139 ± 0.057	0	0.600 ± 0.08	0
ξ_0	[0, 1]	—	0.38 ± 0.125	0.572 ± 0.027	0	0
r_d [Mpc]	[140, 150]	146.0 ± 4.2	146.3 ± 1.8	146.4 ± 1.5	144.5 ± 3.7	146.4 ± 1.5
$\mathcal{M}$	[−20, −18]	-19.4 ± 0.12	-19.409 ± 0.026	-19.303 ± 0.011	-19.302 ± 0.005	-19.407 ± 0.019
σ_8	[0, 1]	0.752 ± 0.029	0.841 ± 0.016	0.798 ± 0.035	0.798 ± 0.013	0.819 ± 0.014

Table 4.3: Best-fit parameter estimates with 1σ uncertainties for the Λ CDM scenario and the modified Chaplygin gas models using the DS2 dataset.

Parameter	Prior	Λ CDM	MCG Model	MCG ($\beta = 0$)	MCG ($\xi_0 = 0$)	MCG ($\beta = 0, \xi_0 = 0$)
H_0 [km s $^{-1}$ Mpc $^{-1}$]	[60, 80]	72.77 ± 0.66	71.01 ± 0.45	71.42 ± 0.63	70.40 ± 0.7	72.17 ± 0.65
Ω_m	[0.1, 0.5]	0.307 ± 0.007	0.277 ± 0.009	0.271 ± 0.020	0.296 ± 0.017	0.276 ± 0.021
Ω_b	[0, 0.1]	—	0.0340 ± 0.003	0.029 ± 0.006	0.023 ± 0.004	0.029 ± 0.001
A_s	[0, 1]	—	0.735 ± 0.002	0.782 ± 0.01	0.731 ± 0.04	0.699 ± 0.07
A	[0, 1]	—	0.047 ± 0.001	0.037 ± 0.018	0.141 ± 0.021	0.028 ± 0.024
α	[0, 1]	—	0.064 ± 0.028	0.102 ± 0.029	0.157 ± 0.017	0.061 ± 0.028
β	[0, 1]	—	0.461 ± 0.019	0	0.68 ± 0.07	0
ξ_0	[0, 1]	—	0.119 ± 0.031	0.651 ± 0.028	0	0
r_d [Mpc]	[140, 150]	138.1 ± 1.4	140.7 ± 1.2	140.0 ± 1.4	143.5 ± 0.7	138.0 ± 1.4
$\mathcal{M}$	[−20, −18]	-19.272 ± 0.019	-19.316 ± 0.014	-19.302 ± 0.019	-19.310 ± 0.012	-19.278 ± 0.015
σ_8	[0, 1]	0.752 ± 0.017	0.818 ± 0.016	0.819 ± 0.015	0.794 ± 0.013	0.793 ± 0.015

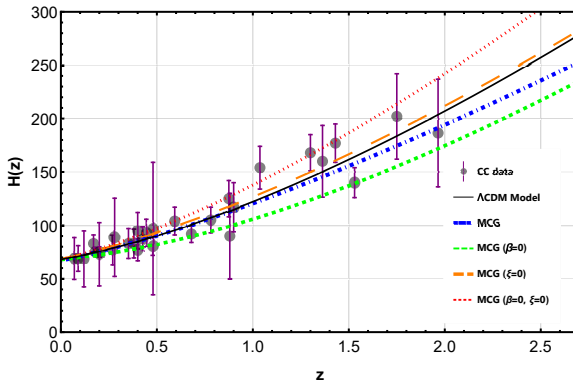


Figure 4.3: Redshift evolution of the Hubble parameter $H(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS1 dataset. The grey data points with error bars correspond to cosmic chronometer measurements.

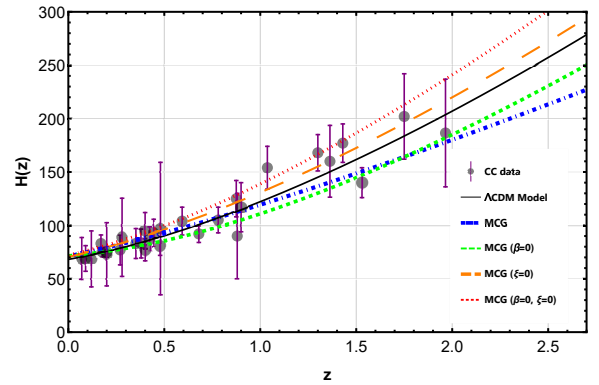


Figure 4.4: Redshift evolution of the Hubble parameter $H(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS2 dataset. The grey data points with error bars correspond to cosmic chronometer measurements.

Modified Chaplygin Gas equation of state, matter creation, and bulk viscosity can reproduce the observed late-time expansion without introducing a cosmological constant explicitly.

Figure 4.3 shows that each of the considered scenarios is compatible with the CC measurements within the corresponding 1σ error range. The Λ CDM curve, represented by the solid black line, rises smoothly and monotonically as the redshift increases, as expected for the standard cosmological model with a constant dark-energy contribution. Across the full redshift interval, the complete MCG model shown by the blue dash-dotted line remains very close to the Λ CDM prediction. Since this case includes both particle creation, $\beta \neq 0$, and bulk viscosity, $\xi_0 \neq 0$, it demonstrates that late-time accelerated expansion can be reproduced without introducing an explicit cosmological constant. This agreement results from the combined action of the MCG equation of state and the non-equilibrium effects associated with matter creation and bulk viscosity, which together generate an effective negative pressure.

The MCG($\beta = 0$) case, displayed by the green dotted curve, retains bulk viscosity while omitting matter creation. Around the intermediate-redshift region, $z \sim 1-1.5$, its evolution is slightly less steep than that of the Λ CDM model, indicating a comparatively weaker accelerated phase. This feature reflects the modification of the effective fluid pressure by bulk viscosity and is associated with a later onset of cosmic acceleration. In contrast, the MCG($\xi_0 = 0$) model, illustrated by the orange dash-dotted curve, includes matter creation but excludes viscous effects. Its more pronounced rise, especially at higher redshifts, suggests an earlier transition toward acceleration and emphasizes the separate role of particle production in the expansion dynamics. The limiting MCG model represented by the red dotted line, for which both $\beta = 0$ and $\xi_0 = 0$, shows the largest departure from the Λ CDM behaviour. The difference becomes more noticeable for $z > 1$, where this model yields a higher expansion rate, thereby indicating that non-equilibrium contributions are important for recovering the observed cosmic expansion history.

A noteworthy result is the behavior of the Hubble constant. Under DS1, the full MCG model gives $H_0 = 67.0 \pm 1.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is slightly lower than the corresponding Λ CDM value $H_0 = 68.6 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Once the R22 prior is included, the preferred value shifts upward to $H_0 = 71.01 \pm 0.45 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for the full MCG model, while Λ CDM gives $H_0 = 72.77 \pm 0.66 \text{ km s}^{-1} \text{ Mpc}^{-1}$. This indicates that the hybrid model is flexible enough to accommodate a larger local value of the Hubble constant when the local prior is included.

The matter creation parameter remains positive in the full model, with $\beta = 0.139 \pm 0.057$ for DS1 and $\beta = 0.461 \pm 0.019$ for DS2. Thus, a higher matter creation rate helps the model accommodate the higher local H_0 value while preserving a good fit to the geometric data.

4.4.2 Deceleration Parameter

The deceleration parameter is defined as

$$q(z) = -1 + (1+z) \frac{d \ln H(z)}{dz}. \quad (4.24)$$

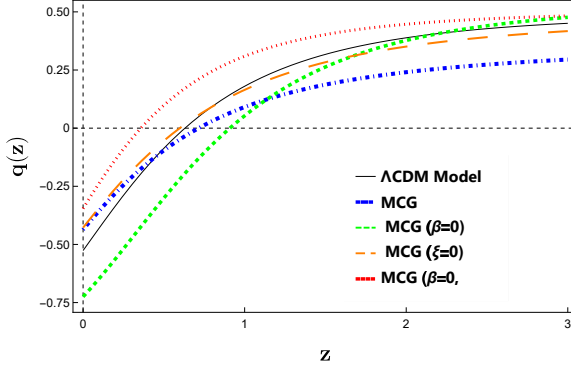


Figure 4.5: Redshift behaviour of the deceleration parameter $q(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS1 dataset.

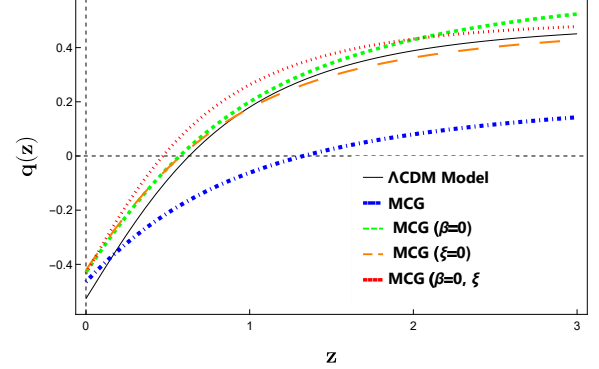


Figure 4.6: Redshift behaviour of the deceleration parameter $q(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS2 dataset.

The evolution shown in Figures 4.5 and 4.6 indicates a change in the cosmic expansion from deceleration at higher redshift to acceleration in the recent Universe. The present estimates of the deceleration parameter and the redshift at which this transition occurs are listed in Tables 4.4 and 4.5. Under DS1, the full MCG model gives $q_0 = -0.442 \pm 0.014$ and $z_{\text{tr}} = 0.592 \pm 0.20$, while the Λ CDM model gives $q_0 = -0.528 \pm 0.020$ and $z_{\text{tr}} = 0.629 \pm 0.26$. Under DS2, the corresponding values become $q_0 = -0.460 \pm 0.018$ and $z_{\text{tr}} = 0.689 \pm 0.21$ for the full MCG model.

These values confirm that the hybrid model reproduces a realistic late-time transition and remains close to the standard cosmological picture. At the same time, the limiting cases show that both matter creation and bulk viscosity contribute to shaping the transition epoch.

Table 4.4: Estimated cosmological parameters for the Λ CDM scenario and the modified Chaplygin gas models using the DS1 observational combination.

Cosmological Parameter Estimations					
Parameter	Λ CDM	MCG	MCG ($\beta = 0$)	MCG ($\xi_0 = 0$)	MCG ($\beta = 0, \xi_0 = 0$)
q_0	-0.528 ± 0.020	-0.442 ± 0.014	-0.726 ± 0.017	-0.433 ± 0.013	-0.343 ± 0.019
j_0	1.000	0.770 ± 0.020	1.07 ± 0.023	1.03 ± 0.017	0.826 ± 0.029
s_0	0	0.09 ± 0.003	-0.026 ± 0.019	0.03 ± 0.004	-0.013 ± 0.003
t_0 (Gyr)	13.71 ± 0.11	13.84 ± 0.13	13.98 ± 0.12	13.81 ± 0.14	13.95 ± 0.10
z_{tr}	0.629 ± 0.26	0.592 ± 0.20	0.601 ± 0.22	0.654 ± 0.17	0.698 ± 0.25

Table 4.5: Estimated cosmological parameters for the Λ CDM scenario and the modified Chaplygin gas models using the DS2 observational combination.

Cosmological Parameter Estimations					
Parameter	Λ CDM	MCG	MCG ($\beta = 0$)	MCG ($\xi_0 = 0$)	MCG ($\beta = 0, \xi_0 = 0$)
q_0	-0.526 ± 0.021	-0.460 ± 0.018	-0.436 ± 0.011	-0.427 ± 0.017	-0.434 ± 0.021
j_0	1.000	0.596 ± 0.021	0.851 ± 0.013	0.828 ± 0.012	1.03 ± 0.022
s_0	0	0.122 ± 0.002	0.07 ± 0.013	0.06 ± 0.002	-0.006 ± 0.001
t_0 (Gyr)	13.67 ± 0.15	13.65 ± 0.17	13.83 ± 0.17	13.92 ± 0.10	14.01 ± 0.12
z_{tr}	0.671 ± 0.21	0.689 ± 0.21	0.631 ± 0.32	0.632 ± 0.19	0.697 ± 0.29

4.4.3 Jerk Parameter

The jerk parameter is defined by

$$j(z) = \frac{\ddot{a}}{aH^3} = q(z)[2q(z) + 1] + (1+z)\frac{dq}{dz}. \quad (4.25)$$

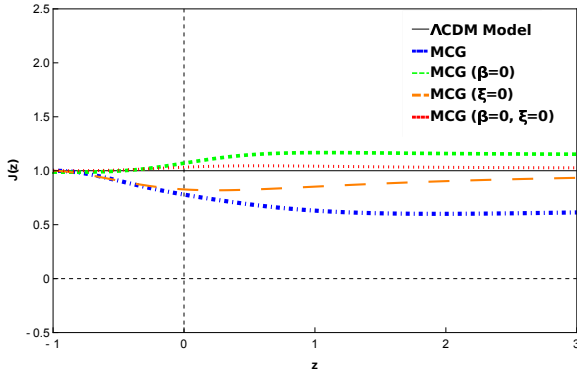


Figure 4.7: Redshift behaviour of the jerk parameter $j(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS1 dataset.

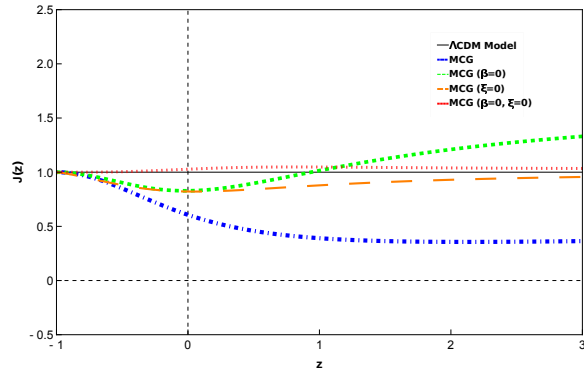


Figure 4.8: Redshift behaviour of the jerk parameter $j(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS2 dataset.

For the Λ CDM model, one has the constant value $j = 1$. Figures 4.7 and 4.8 show that the MCG variants deviate from this standard behavior. Under DS1, the full MCG model gives $j_0 = 0.770 \pm 0.020$, while under DS2 it gives $j_0 = 0.596 \pm 0.021$. These values indicate that the hybrid model remains close to the standard background expansion at low order but still produces a measurable higher-order deviation from Λ CDM. The limiting cases show that both matter creation and bulk viscosity influence the late-time cosmographic behavior.

4.4.4 Snap Parameter

The snap parameter is defined by

$$s(z) = \frac{\ddot{a}}{aH^4}. \quad (4.26)$$

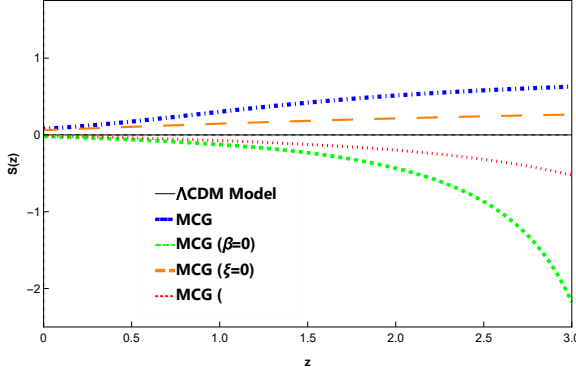


Figure 4.9: Redshift behaviour of the snap parameter $s(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS1 dataset. The marker on each curve denotes the present value s_0 .

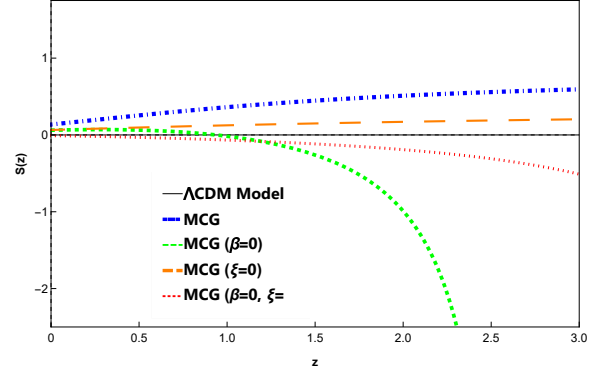


Figure 4.10: Redshift behaviour of the snap parameter $s(z)$ for the Λ CDM scenario and the modified Chaplygin gas models using the DS2 dataset. The marker on each curve denotes the present value s_0 .

The snap parameter provides a further higher-order diagnostic of the expansion history. For the Λ CDM model, $s_0 = 0$. The MCG variants show small but non-zero deviations. Under DS1, the full model gives $s_0 = 0.09 \pm 0.003$, whereas under DS2 it gives $s_0 = 0.122 \pm 0.002$. Thus, even though the hybrid model remains observationally close to Λ CDM at the background level, it exhibits nontrivial departures in higher-order cosmographic diagnostics.

4.4.5 Age of the Universe

The present age of the Universe, denoted by τ_0 , is obtained from the expansion history by integrating the reciprocal of the Hubble parameter from the Big Bang up to the current epoch:

$$\tau_0 = \int_0^\infty \frac{dz}{(1+z)H(z)}. \quad (4.27)$$

Using the best-fit forms of $H(z)$ obtained from the DS1 and DS2 datasets, the cosmic age is evaluated for each model. For DS1, the Λ CDM model gives $\tau_0 = 13.71 \pm 0.11$ Gyr, whereas the complete MCG scenario yields 13.84 ± 0.13 Gyr. The MCG case without matter creation and bulk viscosity predicts the largest value, namely 13.95 ± 0.10 Gyr. For the DS2 dataset, the inferred ages are marginally lower because of the comparatively larger value of H_0 . In this case, Λ CDM gives 13.67 ± 0.15 Gyr, while the full MCG model gives 13.65 ± 0.17 Gyr.

All the estimated values fall within the range 13.65–14.01 Gyr and are compatible with the Planck estimate of 13.80 ± 0.02 Gyr [3]. The moderately larger ages obtained in scenarios containing bulk viscosity can be attributed to the damping of the expansion rate caused by viscous effects, whereas matter creation produces a comparatively weaker modification. For both observational combinations, the complete MCG model shows the closest agreement with

the Λ CDM result.

4.4.6 Thermodynamic Analysis

A cosmological scenario can be regarded as thermodynamically reliable only when it remains consistent with the generalized second law. For this purpose, the total entropy is expressed as

$$S_{\text{total}} = S_{\text{fluid}} + S_{\text{horizon}}, \quad (4.28)$$

and the generalized second law requires

$$\dot{S}_{\text{total}} \geq 0. \quad (4.29)$$

For the modified Chaplygin gas (MCG) framework with matter creation and bulk viscosity, we evaluate the entropy associated with each constituent and show that the generalized second law is consistently fulfilled for both the DS1 and DS2 observational datasets.

4.4.6.1 Entropy of the Hubble Horizon

For a spatially flat FLRW background, the Hubble horizon is located at $R_H = 1/H$. By applying the Bekenstein–Hawking entropy relation in natural units, ($k_B = h/(2\pi) = c = G = 1$), the entropy associated with this horizon can be expressed as

$$S_H = \frac{8\pi^2}{H^2}. \quad (4.30)$$

Differentiating this expression with respect to cosmic time gives

$$\dot{S}_H = -\frac{16\pi^2 \dot{H}}{H^3}. \quad (4.31)$$

Since $\dot{H} < 0$ throughout the accelerated expansion regime, the horizon entropy increases with time, making this contribution positive.

4.4.6.2 Entropy of Baryonic Matter

The baryonic sector is treated as a conserved pressureless component, satisfying $\dot{\rho}_b = -3H\rho_b$ and $p_b = 0$. If the baryonic fluid is assumed to remain in thermal equilibrium with the Hubble horizon, its temperature is given by $T = H/(2\pi)$. The Gibbs relation then leads to the baryonic

entropy

$$S_b = \frac{(\rho_b + p_b)V}{T} = \frac{2\pi\rho_b}{H} \cdot \frac{4\pi}{3H^3} = \frac{8\pi^2\rho_b}{3H^4}, \quad (4.32)$$

where the volume enclosed by the Hubble horizon is $V = 4\pi/(3H^3)$. Taking the time derivative of this expression, together with the relations $\dot{\rho}_b = -3H\rho_b$ and $\dot{H} = -H^2(1+q)$, yields

$$\dot{S}_b = \frac{8\pi^2\rho_b}{3H^3}(1+4q), \quad (4.33)$$

with the deceleration parameter defined by $q = -1 - \dot{H}/H^2$.

4.4.6.3 Entropy of the Modified Chaplygin Gas with Matter Creation

The MCG fluid has energy density ρ_{mcg} and effective pressure $\tilde{p}_{\text{mcg}} = p_{\text{mcg}} + P_c$, where $P_c = -\beta(\rho_{\text{mcg}} + p_{\text{mcg}})$ is the creation pressure. Its equation of state is $p_{\text{mcg}} = A\rho_{\text{mcg}} - C/\rho_{\text{mcg}}^\alpha$. The entropy is:

$$S_{\text{mcg}} = \frac{(\rho_{\text{mcg}} + \tilde{p}_{\text{mcg}})V}{T} = \frac{8\pi^2(1-\beta)(\rho_{\text{mcg}} + p_{\text{mcg}})}{3H^4}. \quad (4.34)$$

Differentiating and using the continuity equation $\dot{\rho}_{\text{mcg}} = -3H(1-\beta)(\rho_{\text{mcg}} + p_{\text{mcg}})$, we obtain:

$$\dot{S}_{\text{mcg}} = \frac{8\pi^2(1-\beta)(\rho_{\text{mcg}} + p_{\text{mcg}})}{3H^3} \left[-3(1-\beta) \left(1 + \alpha \frac{p_{\text{mcg}}}{\rho_{\text{mcg}}} \right) + 4(1+q) \right] \quad (4.35)$$

This expression encodes the interplay between the MCG's exotic equation of state (via α), matter creation (via β), and cosmic acceleration (via q).

4.4.6.4 Entropy of Bulk Viscous Matter

The bulk viscous component has energy density ρ_m and effective pressure $\tilde{p}_m = \Pi = -3H\xi_0\rho_m^{1/2}$.

Its entropy is:

$$S_m = \frac{(\rho_m + \tilde{p}_m)V}{T} = \frac{8\pi^2}{3H^4} \left(\rho_m - 3H\xi_0\rho_m^{1/2} \right). \quad (4.36)$$

Differentiating and using the continuity equation $\dot{\rho}_m = -3H\rho_m + 9H^2\xi_0\rho_m^{1/2}$, we obtain:

$$\dot{S}_m = \frac{8\pi^2}{3} \left[\rho_m H^{-3}(1+4q) + 9\xi_0\rho_m^{1/2} H^{-2} \left(\frac{1}{2} - q \right) - \frac{27}{2}\xi_0^2 H^{-1} \right]. \quad (4.37)$$

This contains a standard matter term, a viscous correction, and a dissipative loss term ($-\xi_0^2$), always negative.

4.4.6.5 Total Entropy and GSL Validation

The net entropy production rate is obtained by adding the entropy rates of the Hubble horizon, baryonic matter, the MCG sector, and the viscous matter component:

$$\dot{S}_{\text{total}} = \dot{S}_H + \dot{S}_b + \dot{S}_{\text{mcg}} + \dot{S}_m. \quad (4.38)$$

By inserting the individual entropy-rate expressions into Eq. (4.38), the total rate can be written in the form

$$\begin{aligned} \dot{S}_{\text{total}} = \frac{8\pi^2}{H^3} & \left[2H(1+q) + \frac{\rho_b}{3}(1+4q) \right. \\ & + \frac{(1-\beta)(\rho_{\text{mcg}} + p_{\text{mcg}})}{3} \left\{ -3(1-\beta) \left(1 + \alpha \frac{p_{\text{mcg}}}{\rho_{\text{mcg}}} \right) + 4(1+q) \right\} \\ & \left. + \rho_m(1+4q) + 9\xi_0 \rho_m^{1/2} H^{-1} \left(\frac{1}{2} - q \right) - \frac{27}{2} \xi_0^2 H^{-2} \right]. \end{aligned} \quad (4.39)$$

For the observationally allowed parameter domain, $0 \leq \alpha \leq 1$, $0 < \beta < 1$, and $\xi_0 > 0$, the total entropy rate remains positive over the interval $z \in [-0.95, 5]$. As illustrated in Figs. 4.11 and 4.12, $\dot{S}_{\text{total}}(z) > 0$ throughout the entire redshift range for both datasets. Hence, the proposed model satisfies the Generalized Second Law of thermodynamics.

The second derivative of the total entropy displays a change in sign during the cosmic evolution. At earlier epochs, $\ddot{S}_{\text{total}} > 0$, whereas in the recent Universe it becomes negative, $\ddot{S}_{\text{total}} < 0$. This behaviour indicates that the entropy growth rate gradually decreases as the system evolves toward thermodynamic equilibrium. In the asymptotic future, $z \rightarrow -1$, one obtains $\ddot{S}_{\text{total}} \rightarrow 0$, which is consistent with the entropy-extremum and convexity conditions required for a stable equilibrium configuration. Therefore, the late-time thermodynamic behaviour of the Universe resembles that of an ordinary macroscopic system approaching equilibrium.

For both DS1 and DS2, the evolution of $\ddot{S}_{\text{total}}$ exhibits a distinct transition from positive to negative values near $z \sim 1$. This sign change shows that entropy production becomes slower during the recent expansion era. It is associated with the transition from decelerated to accelerated expansion, during which the dominant contribution to entropy generation shifts from matter-creation effects toward horizon-related dynamics, as shown in Figs. 4.13 and 4.14. In the limit $z \rightarrow -1$, the condition $\ddot{S}_{\text{total}} \rightarrow 0$ further confirms that the cosmological system approaches a stable thermodynamic equilibrium state.

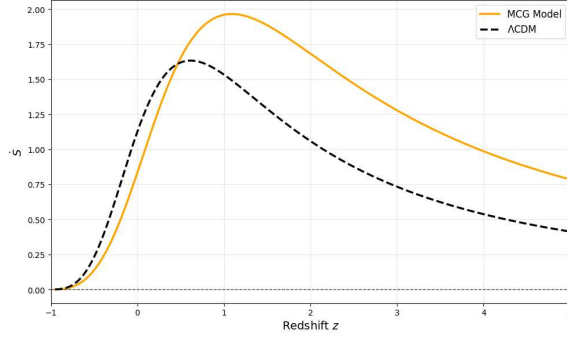


Figure 4.11: Redshift variation of the total entropy change rate $\dot{S}_{\text{total}}$ for the modified Chaplygin gas model and the Λ CDM scenario using the DS1 dataset. Positive values indicate consistency with the generalized second law.

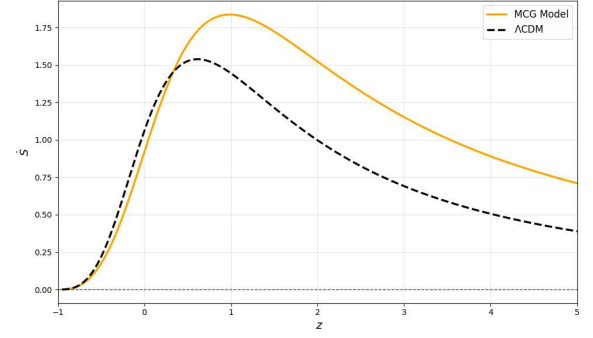


Figure 4.12: Redshift variation of the total entropy change rate $\dot{S}_{\text{total}}$ for the modified Chaplygin gas model and the Λ CDM scenario using the DS2 dataset. The inclusion of the R22 prior gives a mildly enhanced entropy production at recent epochs.

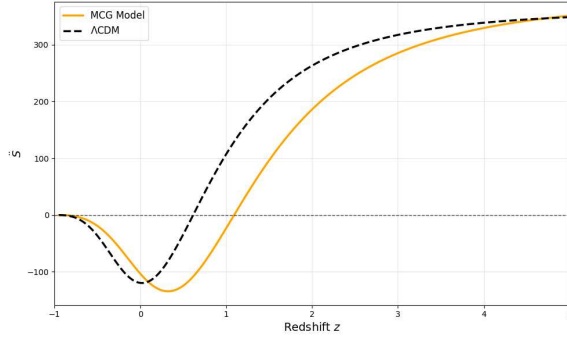


Figure 4.13: Redshift behaviour of the second derivative of the total entropy, $\ddot{S}_{\text{total}}$, for the modified Chaplygin gas model and the Λ CDM scenario using the DS1 dataset. The change from positive to negative values suggests that entropy generation becomes slower as the system moves toward late-time thermodynamic equilibrium.

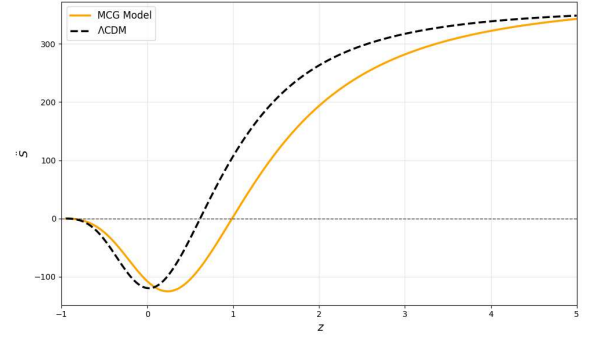


Figure 4.14: Redshift behaviour of the second derivative of the total entropy, $\ddot{S}_{\text{total}}$, for the modified Chaplygin gas model and the Λ CDM scenario using the DS2 dataset. The addition of the R22 prior produces a small shift in the sign-change redshift of $\ddot{S}_{\text{total}}$, while the late-time tendency toward thermodynamic equilibrium remains unchanged.

4.4.7 Model Selection

To compare the observational performance of the hybrid MCG scenario with that of the standard Λ CDM model, we employ the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). These quantities are defined by

$$\text{AIC} = \chi_{\min}^2 + 2K, \quad \text{BIC} = \chi_{\min}^2 + K \ln N, \quad (4.40)$$

where $\chi_{\min}^2$ represents the minimum value of the chi-square function, K denotes the number of adjustable parameters, and N is the total number of observational points. The data counts are $N = 1764$ for DS1 and $N = 1765$ for DS2. In the present analysis, Λ CDM has $K = 2$, while the complete MCG model contains $K = 7$ parameters. The $\text{MCG}(\beta = 0)$ and $\text{MCG}(\xi_0 = 0)$ cases each have $K = 6$, whereas the $\text{MCG}(\beta = 0, \xi_0 = 0)$ model is described by $K = 5$ parameters.

The resulting goodness-of-fit and information-criterion values are listed in Tables 4.5 and 4.6. For the DS1 dataset, the complete MCG model gives $\chi_{\min}^2 = 1778.342$, which is marginally smaller than the value $\chi_{\min}^2 = 1781.197$ obtained for Λ CDM. Thus, the hybrid model provides a slightly improved fit at the level of the minimum chi-square. Moreover, the reduced chi-square values, $\chi_{\text{red}}^2 \approx 1.007\text{--}1.012$, remain close to unity for all the models considered, indicating statistically satisfactory fits without noticeable overfitting or underfitting.

For DS1, the full MCG scenario has $\Delta\text{AIC} = -2.855$, corresponding to moderate support relative to Λ CDM. However, its value of $\Delta\text{BIC} = 25.283$ shows that the BIC strongly disfavors the model once the cost of its larger parameter space is taken into account. From the AIC perspective, the complete MCG model is preferred over Λ CDM, whereas the MCG model without both particle creation and bulk viscosity is disfavored, with $\Delta\text{AIC} = 11.254$. In contrast, the BIC selects Λ CDM over every MCG variant, with $\Delta\text{BIC} > 25$ for all cases. This difference between the two criteria is expected because the BIC applies a stronger complexity penalty through the factor $\ln N$, particularly for a large dataset.

For DS2, the AIC continues to support the complete MCG model, with $\Delta\text{AIC} = -3.083$, and also favours the $\text{MCG}(\beta = 0)$ sub-model, for which $\Delta\text{AIC} = -6.896$. The BIC results are comparatively less restrictive in this dataset, giving $\Delta\text{BIC} < 2$ for all models except $\text{MCG}(\beta = 0, \xi_0 = 0)$. When the R22 local- H_0 prior is added to DS2, the statistical status of the hybrid model improves further. In this case, the full MCG model yields $\Delta\text{AIC} = -3.083$ and $\Delta\text{BIC} = 0.901$, showing that the local determination of H_0 helps reduce parameter degeneracies and strengthens the relative competitiveness of the model.

Overall, these results indicate that incorporating particle production and bulk viscosity into the MCG description can improve the fit to cosmological observations and can provide a competitive alternative to Λ CDM when local H_0 information is considered. The contrast between the DS1 and DS2 BIC results also illustrates that model-selection conclusions depend sensitively on the observational dataset employed. Upcoming high-precision surveys, including DESI and Euclid, are expected to place tighter limits on the model parameters and provide a more decisive test of the MCG framework [139].

Table 4.6: Comparison of the modified Chaplygin gas models with the reference Λ CDM scenario using information criteria.

Model	$\Delta\text{AIC (DS1)}$	$\Delta\text{BIC (DS1)}$	$\Delta\text{AIC (DS2)}$	$\Delta\text{BIC (DS2)}$
MCG	−2.855	25.283	−3.083	0.901
MCG($\beta = 0$)	4.422	41.431	−6.896	−4.910
MCG($\xi_0 = 0$)	2.010	39.000	−0.591	1.395
MCG($\beta = 0, \xi_0 = 0$)	11.254	48.243	−9.034	−9.034

4.4.8 Parameter Degeneracies and Identifiability

The posterior contours in Figs. 4.1 and 4.2 show that several of the Modified Chaplygin Gas and nonequilibrium parameters are correlated. The main trends may be summarized as follows. First, the parameters A_s , A , and α are mutually correlated because they jointly determine the redshift evolution of the Modified Chaplygin Gas density. Second, the matter creation parameter β is correlated with the MCG parameters because the combination $(1 - \beta)(1 + A)$ appears in the effective scaling law of the energy density. Third, the bulk viscosity parameter ξ_0 correlates with the present matter density Ω_m , since both influence the normalization and dilution rate of the viscous matter component.

The inclusion of the R22 prior tends to tighten several of these degeneracy directions, particularly those involving H_0 and the nonequilibrium parameters, which explains why the model becomes statistically more competitive under DS2 than under DS1.

4.5 Conclusion

In this chapter, we have presented a cosmological scenario in which the modified Chaplygin gas, matter creation, and bulk viscous effects are treated together within a spatially flat FLRW background. This framework represents a more general extension of Chaplygin-type cosmologies because it incorporates simultaneously three distinct physical mechanisms: an

exotic negative-pressure fluid, irreversible particle production, and dissipative bulk viscosity.

The analytical expression for the Hubble parameter was derived and used to constrain the free parameters of the model through two observational datasets: DS1, containing Pantheon+, cosmic chronometers, BAO, and $f\sigma_8$ data, and DS2, which additionally includes the local SH0ES prior on the Hubble constant. The resulting best-fit values show that the full MCG model remains consistent with the observational data and can smoothly reproduce the transition from an early decelerated phase to the present accelerated phase.

A central result of this study is that the model is flexible enough to accommodate a higher local value of the Hubble constant when the R22 prior is included. In particular, the best-fit value shifts from $H_0 = 67.0 \pm 1.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$ under DS1 to $H_0 = 71.01 \pm 0.45 \text{ km s}^{-1} \text{ Mpc}^{-1}$ under DS2. This indicates that the hybrid model can reduce the tension between local and background probes in joint analyses, although it does not by itself dynamically resolve the H_0 discrepancy.

The reconstructed cosmological quantities confirm the viability of the model. The present values of the deceleration, jerk, and snap parameters remain consistent with independent cosmographic estimates, and the transition redshift remains close to the standard late-time value. The inferred age of the Universe lies in the interval $13.65 \text{ Gyr} \lesssim t_0 \lesssim 14.01 \text{ Gyr}$, which is compatible with both Planck-based estimates and independent stellar-age determinations.

The thermodynamic analysis provides an additional and important consistency test. The total entropy rate remains positive throughout the cosmic evolution for both DS1 and DS2, confirming that the generalized second law is satisfied. At the same time, the second derivative of the total entropy changes sign around $z \sim 1$, indicating that entropy production slows down and that the cosmic system tends toward thermodynamic equilibrium at late times. These results show that the model is not only observationally viable at the background level but also thermodynamically well behaved.

Finally, the statistical comparison shows that the performance of the hybrid model depends on the criterion adopted. Under DS1, the Akaike criterion gives moderate support to the full MCG model, whereas the Bayesian criterion strongly penalizes the additional parameter freedom. Under DS2, however, the inclusion of the local H_0 prior improves the model's comparative standing and reduces the BIC penalty substantially. Thus, the local Hubble prior plays an important role in enhancing the viability of the non-equilibrium MCG framework.

Chapter 5

Matter Creation Cosmology with Generalized Chaplygin Gas in $f(Q)$ Gravity

5.1 Introduction

This chapter studies generalized Chaplygin gas and matter creation in non-minimally coupled $f(Q)$ gravity, based on Bhardwaj and Singh [267]. It returns to the fluid setting of Chapter 2 while changing the gravitational framework. The cosmic medium consists of separately conserved baryons and a Chaplygin component with creation pressure.

The general discussion of $f(Q)$ gravity is given in Section 1.5.2; the Chaplygin-gas and matter-creation definitions are given in Sections 1.8.1 and 1.9. Here the model-specific feature is the coupling of two functions of nonmetricity to the gravitational and matter sectors.

Sections 5.2–5.5 specify the non-minimal field equations, background evolution, and GCG and CG solutions. Sections 5.6–5.9 present the observational analysis, results, stability discussion, and model comparison. Section 5.10 concludes the chapter. Chapter 6 considers variable generalized Chaplygin gas with two one-function $f(Q)$ models and a different assignment of creation pressure.

5.2 $f(Q)$ Theory of Gravity

In this chapter we consider an extension of symmetric teleparallel gravity in which the fundamental geometrical quantity is the non-metricity scalar Q . The action of the non-minimally

coupled $f(Q)$ theory is taken as [34, 35, 219]

$$S = \int d^4x, \sqrt{-g} \left[-\frac{1}{2}f_1(Q) + f_2(Q)\mathcal{L}_m \right], \quad (5.1)$$

where $\mathcal{L}_m$ is the matter Lagrangian, $f_1(Q)$ and $f_2(Q)$ are arbitrary functions of the non-metricity scalar, and g is the determinant of the metric tensor.

The nonmetricity tensor, its traces, the disformation, and the symmetrized conjugate $P^\lambda_{\mu\nu}$ are defined in Section 1.5.2, with the conjugate given explicitly in Eq. (1.30). The scalar entering the present field equations is written as

$$Q = -Q_{\lambda\mu\nu}P^{\lambda\mu\nu}. \quad (5.2)$$

The matter energy-momentum tensor is given by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}. \quad (5.3)$$

Variation of the action (5.1) with respect to the metric yields the modified gravitational field equations in the form

$$\begin{aligned} \frac{2}{\sqrt{-g}} \nabla_\lambda \left[\sqrt{-g} (f'_1(Q) + 2f'_2(Q)\mathcal{L}_m) P^\lambda_{\mu\nu} \right] + \frac{1}{2} g_{\mu\nu} f_1(Q) \\ + (f'_1(Q) + 2f'_2(Q)\mathcal{L}_m) \left(P_{\mu\lambda i} Q_{\nu}{}^{\lambda i} - 2Q_{\lambda i \mu} P_{\nu}{}^{\lambda i} \right) = -f_2(Q) T_{\mu\nu}, \end{aligned} \quad (5.4)$$

where a prime denotes differentiation with respect to Q .

5.3 Cosmological Background Evolution

To study the cosmological evolution, we assume a spatially homogeneous and isotropic flat Friedmann–Lemaître–Robertson–Walker space-time,

$$ds^2 = -dt^2 + a^2(t) [dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)], \quad (5.5)$$

where $a(t)$ is the scale factor. For a comoving observer, the four-velocity is taken as

$$u^\mu = \delta_0^\mu, \quad (5.6)$$

with $u^\mu u_\mu = -1$.

The cosmic medium is assumed to consist of conserved baryonic matter and a Chaplygin gas component endowed with matter creation. The effective energy-momentum tensor is therefore written as

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}, \quad (5.7)$$

where $\rho = \rho_b + \rho_{gcg}$ and $p = p_{gcg} + p_c$.

The particle balance and thermodynamic formulation of matter creation are described in Section 1.9. For adiabatic matter creation, the corresponding creation pressure is related to Γ by [7, 8]

$$p_c = -\frac{\rho_{gcg} + p_{gcg}}{3H}\Gamma. \quad (5.8)$$

As in the previous chapters, we adopt the phenomenological ansatz

$$\Gamma = 3\beta H, \quad (5.9)$$

where β is a dimensionless constant.

In the coincident gauge, the non-metricity scalar becomes $Q = 6H^2$. For the flat FLRW metric, the modified Friedmann equations in $f(Q)$ gravity are [34, 35]

$$3H^2 = \frac{f_2}{2F} \left(-\rho + \frac{f_1}{2f_2} \right), \quad (5.10)$$

$$\dot{H} + 3H^2 + \frac{\dot{F}}{F}H = \frac{f_2}{2F} \left(p + \frac{f_1}{2f_2} \right), \quad (5.11)$$

where

$$F = f_1'(Q) + 2f_2'(Q)\mathcal{L}_m. \quad (5.12)$$

From these equations one obtains the continuity equation

$$\dot{\rho} + 3H(\rho + p) = -\frac{6f_2'}{f_2}H\dot{H}(\mathcal{L}_m + \rho). \quad (5.13)$$

With the standard choice $\mathcal{L}_m = -\rho$, this reduces to

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (5.14)$$

where again $\rho = \rho_b + \rho_{gcg}$ and $p = p_{gcg} + p_c$.

The baryonic component is assumed to be pressureless and separately conserved, so that

$$\rho_b = \rho_{b,0}(1+z)^3, \quad (5.15)$$

with $a = (1+z)^{-1}$ after setting $a_0 = 1$.

5.4 Generalized Chaplygin Gas Model in $f(Q)$ Gravity

For the generalized Chaplygin gas, the equation of state is taken as [6, 58]

$$p_{gcg} = -\frac{A}{\rho_{gcg}^\alpha}, \quad (5.16)$$

where $A > 0$ and $0 < \alpha \leq 1$ are constants.

Using Eqs. (5.8), (5.9), and (5.16) in the conservation equation for the Chaplygin component,

$$\dot{\rho}_{gcg} + 3H(\rho_{gcg} + p_{gcg} + p_c) = 0, \quad (5.17)$$

one obtains the energy density

$$\rho_{gcg} = \rho_{gcg,0} \left[A_s + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)} \right]^{\frac{1}{1+\alpha}}, \quad (5.18)$$

where $\rho_{gcg,0}$ is the present energy density and

$$A_s = \frac{A}{\rho_{gcg,0}^{1+\alpha}}. \quad (5.19)$$

we assume the following functional forms [225]

$$f_1(Q) = \alpha_1 Q + \beta_1 Q^2, \quad f_2(Q) = Q, \quad (5.20)$$

where α_1 and β_1 are constants. Substituting Eqs. (5.15) and (5.18) into the modified Friedmann equation (5.10), the Hubble parameter takes the form

$$H^2(z) = H_0^2 \left[\frac{1}{3\beta_1} \left(\Omega_{gcg,0} \left[A_s + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)} \right]^{\frac{1}{1+\alpha}} + \Omega_{b,0}(1+z)^3 - \frac{\alpha_1}{6H_0^2} \right) \right]. \quad (5.21)$$

Imposing the present condition $H(z=0) = H_0$ yields

$$\Omega_{gcg,0} = 3\beta_1 - \Omega_{b,0} + \frac{\alpha_1}{6H_0^2}. \quad (5.22)$$

The deceleration parameter is defined by

$$q(z) = -1 + \frac{1+z}{E(z)} \frac{dE(z)}{dz}, \quad (5.23)$$

with $E(z) = H(z)/H_0$. For the present model one obtains

$$q(z) = -1 + \frac{1}{6\beta_1 E^2(z)} \left[3\Omega_{b,0}(1+z)^3 + 3(1-\beta)(1-A_s)(1+z)^{3(1-\beta)(1+\alpha)} \right. \\ \left. \times \Omega_{gcg,0} \left(A_s + (1-A_s)(1+z)^{3(1-\beta)(1+\alpha)} \right)^{-\frac{\alpha}{1+\alpha}} \right]. \quad (5.24)$$

In the limit $\beta = 0$, Eqs. (5.21) and (5.24) reduce to the corresponding GCG solutions without matter creation.

The jerk parameter is introduced as a higher-order cosmographic diagnostic,

$$j(z) = q(z) [2q(z) + 1] + (1+z) \frac{dq(z)}{dz}, \quad (5.25)$$

and its present value is

$$j_0 = q_0 + 2q_0^2 + \left. \frac{dq}{dz} \right|_{z=0}. \quad (5.26)$$

The effective equation-of-state parameter for the generalized Chaplygin gas model is

$$w_{gcg}(z) = \frac{-\frac{A_s(1-\beta)\Omega_{gcg,0}}{\tilde{C}} - \beta\Omega_{gcg,0}\tilde{C}}{\Omega_{b,0}(1+z)^3 + \Omega_{gcg,0}\tilde{C}}, \quad (5.27)$$

where

$$\tilde{C} = \left[A_s + (1-A_s)(1+z)^{3(1-\beta)(1+\alpha)} \right]^{\frac{1}{1+\alpha}}. \quad (5.28)$$

At the present epoch this gives $w_{gcg,0} = -A_s(1-\beta) - \beta$. The limiting cases $A_s = 1$ and $A_s = 0$ correspond respectively to cosmological-constant-like behavior and matter-like behavior.

5.5 Particular Solution

A particular case of the generalized Chaplygin gas is recovered by setting $\alpha = 1$, which gives the original Chaplygin gas equation of state

$$p_{cg} = -\frac{A}{\rho_{cg}}. \quad (5.29)$$

The corresponding energy density becomes

$$\rho_{cg}(z) = \rho_{cg,0} \left[A_s + (1 - A_s)(1 + z)^{6(1-\beta)} \right]^{\frac{1}{2}}, \quad (5.30)$$

where now $A_s = A/\rho_{cg,0}^2$.

Using Eqs. (5.15) and (5.30) in the modified Friedmann equation, the Hubble parameter is obtained as

$$H^2(z) = H_0^2 \left[\frac{1}{3\beta_1} \left(\Omega_{cg,0} \left[A_s + (1 - A_s)(1 + z)^{6(1-\beta)} \right]^{\frac{1}{2}} + \Omega_{b,0}(1 + z)^3 - \frac{\alpha_1}{6H_0^2} \right) \right]. \quad (5.31)$$

The deceleration parameter for this model is

$$q(z) = -1 + \frac{1}{6\beta_1 E^2(z)} \left[3\Omega_{b,0}(1 + z)^3 + 3(1 - \beta)(1 - A_s)(1 + z)^{6(1-\beta)} \right. \\ \left. \times \Omega_{cg,0} \left(A_s + (1 - A_s)(1 + z)^{6(1-\beta)} \right)^{-\frac{1}{2}} \right], \quad (5.32)$$

while the effective equation-of-state parameter is

$$w_{cg}(z) = \frac{-\frac{A_s(1 - \beta)\Omega_{cg,0}}{\tilde{C}_1} + \beta \Omega_{cg,0}\tilde{C}_1}{\Omega_{b,0}(1 + z)^3 + \Omega_{cg,0}\tilde{C}_1}, \quad (5.33)$$

where

$$\tilde{C}_1 = \left[A_s + (1 - A_s)(1 + z)^{6(1-\beta)} \right]^{\frac{1}{2}}. \quad (5.34)$$

5.6 Observational Data and Statistical Methodology

The observational data sets employed in this chapter have already been discussed in the general introduction of the thesis. Here we only summarize the specific compilations used for

the present analysis. The combined dataset consists of Pantheon Type Ia supernovae, cosmic chronometers, and DESI BAO measurements. The Pantheon sample contains 1048 supernova distance moduli in the range $0.01 \leq z \leq 2.26$ [131, 132]. The cosmic chronometer sample contains 32 direct measurements of $H(z)$ in the interval $0.07 \leq z \leq 1.965$ [130]. For BAO, the analysis uses 12 recent DESI DR2 measurements spanning the redshift range $0.295 \leq z \leq 2.330$, with the sound horizon scale treated as a free parameter [226].

The total chi-square function used for the joint analysis is

$$\chi_{\text{tot}}^2 = \chi_{\text{SNe}}^2 + \chi_{\text{CC}}^2 + \chi_{\text{BAO}}^2, \quad (5.35)$$

where the supernova, cosmic chronometer, and BAO contributions follow the expressions introduced in the article.

The parameter estimation is performed through Markov Chain Monte Carlo sampling using the public `emcee` package [197]. The source article adopts 120 walkers and 2000 steps for the parameter exploration. The flat priors used for the model parameters are listed in Table 5.1.

Table 5.1: Flat priors adopted in the MCMC analysis.

H_0	Ω_b	A_s	α_1	β_1	α	β
[60, 80]	[0, 0.1]	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[0, 1]

The nuisance parameters r_d and $\mathcal{M}$ are also marginalized over in the fit, and their best-fit values are reported in the results section.

5.7 Results and Discussion

The joint constraints obtained from the combined SNe+CC+BAO dataset are summarized in Table 5.2. The corresponding one-dimensional posterior distributions and two-dimensional confidence contours for the Λ CDM, GCG, and CG models are displayed in Figs. 5.1–5.3. The main cosmological quantities reconstructed from the best-fit parameters are given in Table 5.3. In what follows, we present the present values of different cosmological parameters and its evolution, and compares the models both with and without matter creation.

5.7.1 Evolution of the Hubble Parameter

Figures 5.4 and 5.5 show that both the generalized Chaplygin gas and the Chaplygin gas models with matter creation reproduce the observed expansion history rather well and remain

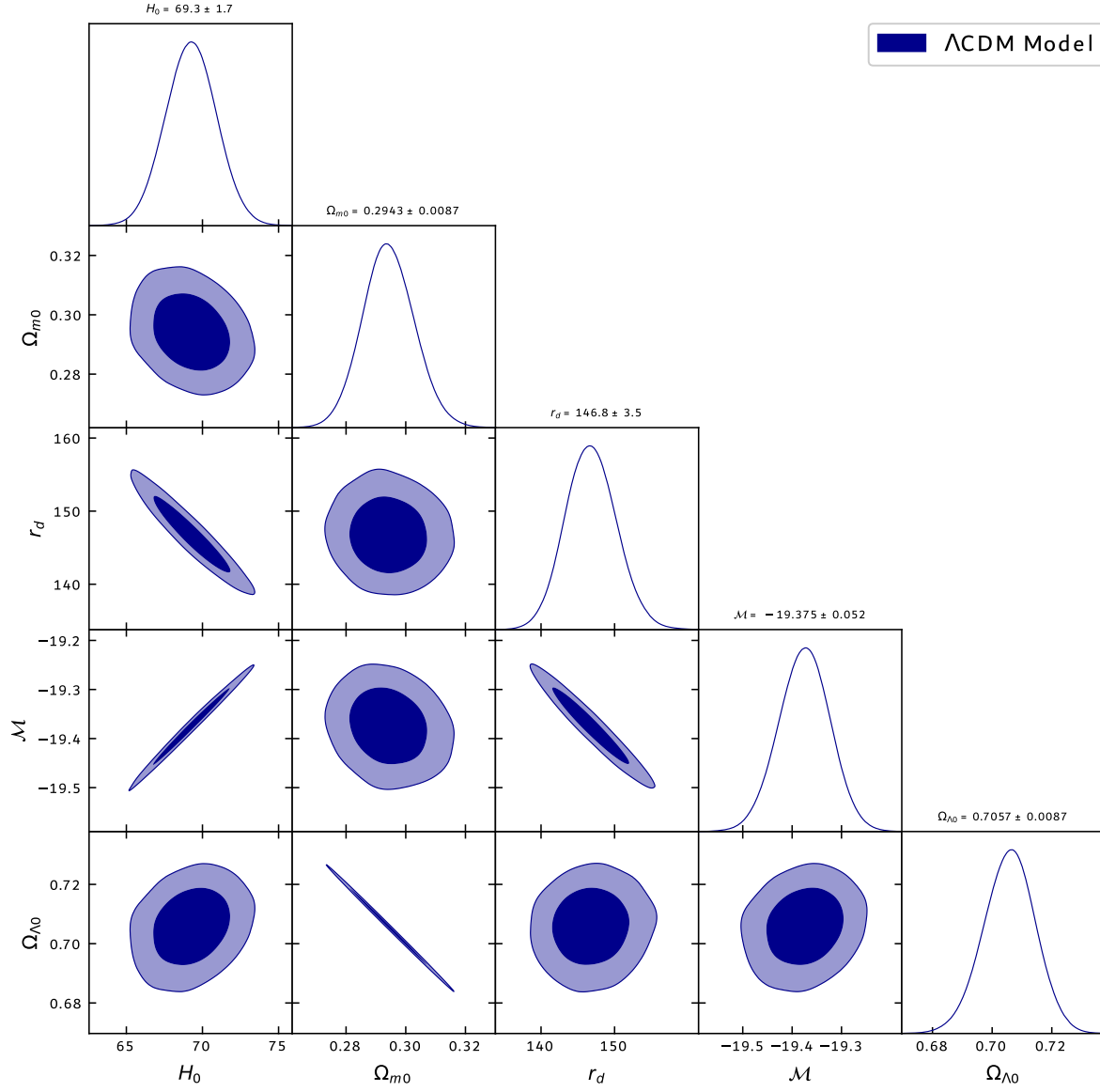


Figure 5.1: One-dimensional posterior distributions and two-dimensional confidence contours at 1σ and 2σ for the Λ CDM model using the $SNe + CC + BAO$ dataset.

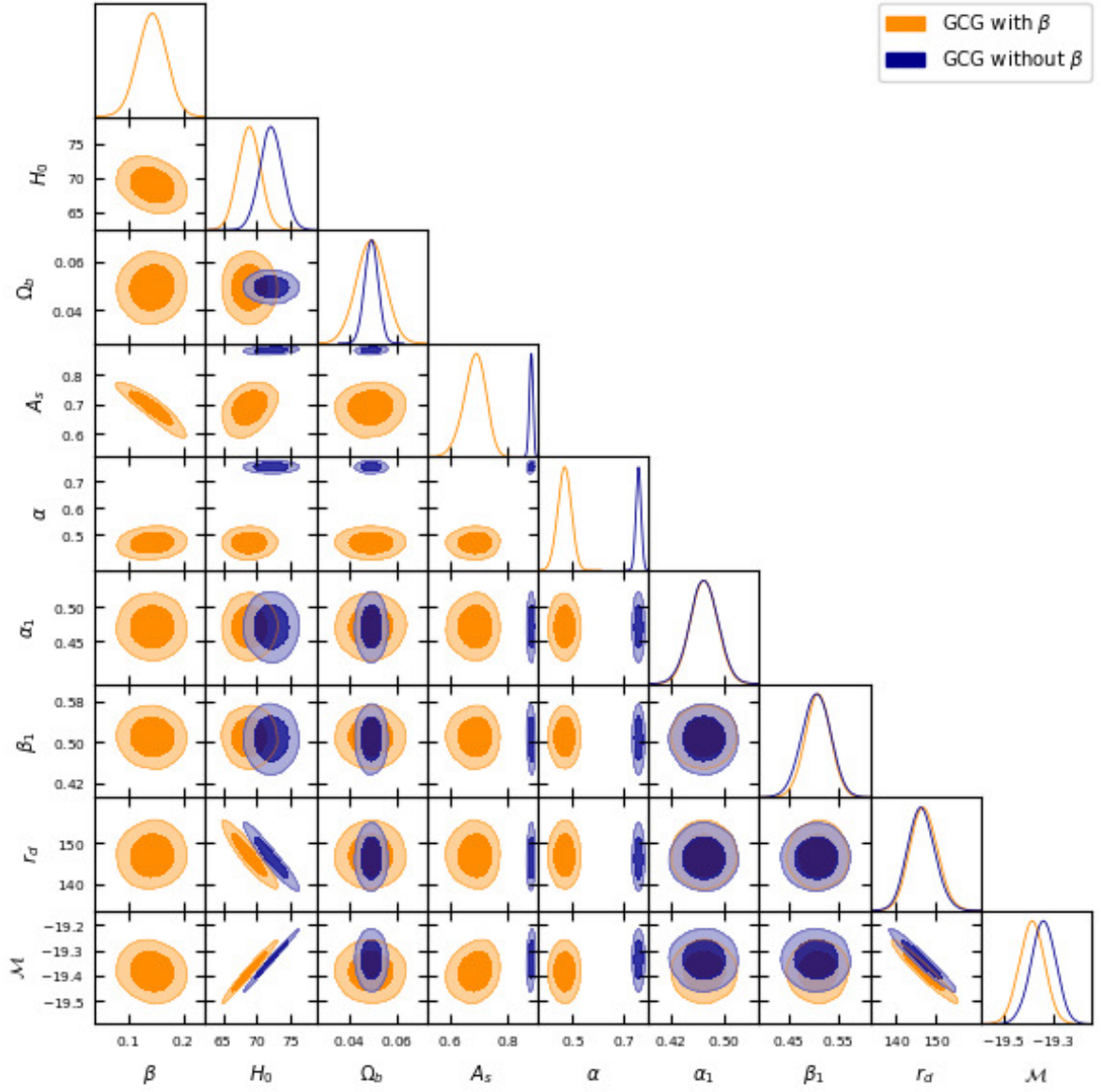


Figure 5.2: One-dimensional posterior distributions and two-dimensional confidence contours at 1σ and 2σ for the generalized Chaplygin gas model with and without matter creation using the $SNe + CC + BAO$ dataset.

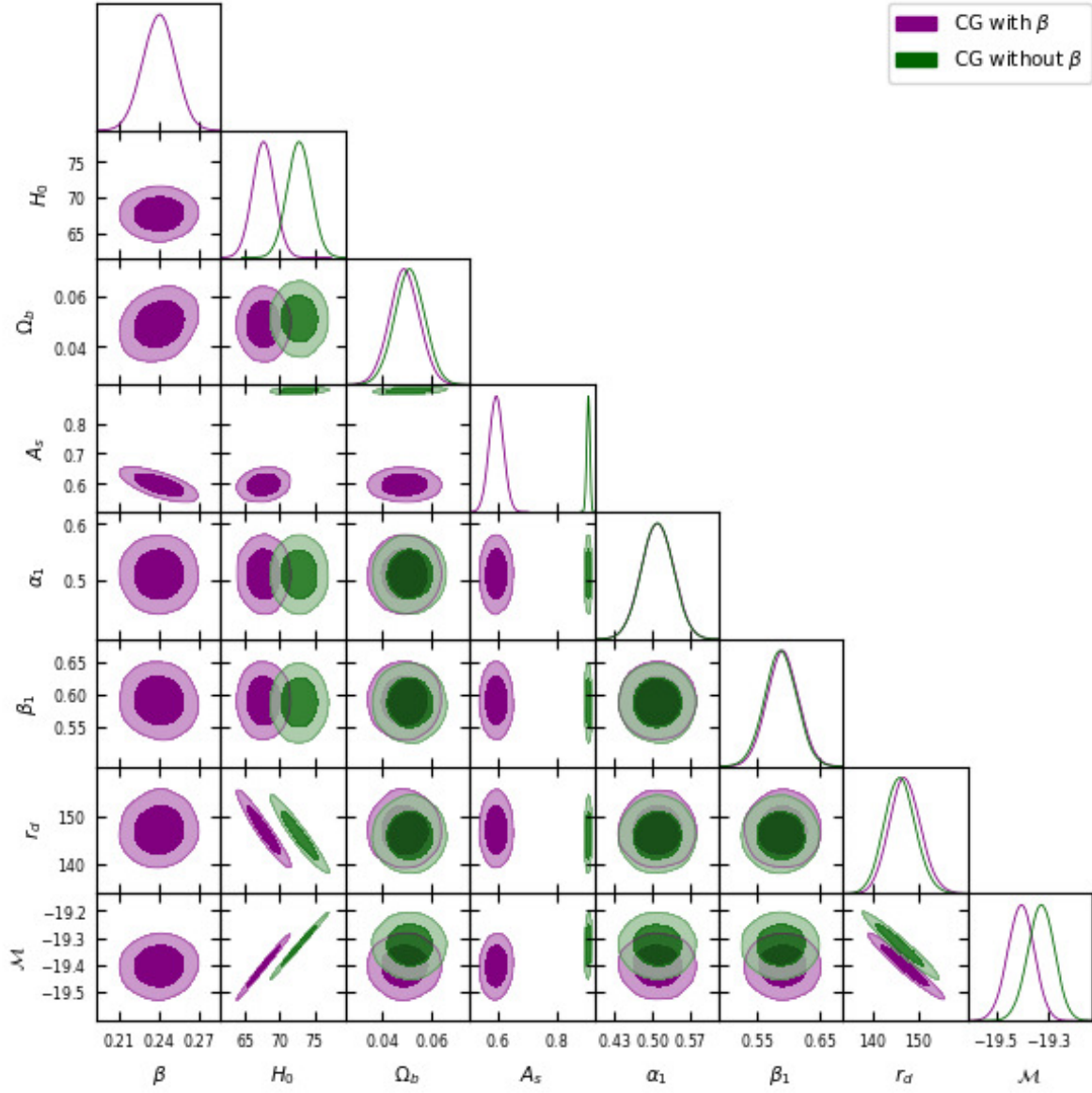


Figure 5.3: One-dimensional posterior distributions and two-dimensional confidence contours at 1σ and 2σ for the Chaplygin gas model with and without matter creation using the *SNe* + *CC* + *BAO* dataset.

close to the Λ CDM behavior over the full redshift interval. The curves without matter creation deviate more noticeably, particularly in the higher-redshift region. This confirms that the matter creation term plays an important role in bringing the Chaplygin-type models closer to the observed late-time expansion history.

A comparison of the best-fit Hubble constants shows that the models with matter creation remain closer to the Planck determination than the corresponding models without matter creation. In particular, the GCG and CG models with matter creation give $H_0 = 68.9 \pm 1.6$ and $H_0 = 67.7 \pm 1.6$, respectively, whereas the corresponding models without matter creation shift to 72.2 ± 1.7 and 72.8 ± 1.7 . The fitted values therefore indicate that matter creation changes the late-time background evolution in a non-trivial way and can move the inferred Hubble constant toward the interval between the Planck and local determinations.

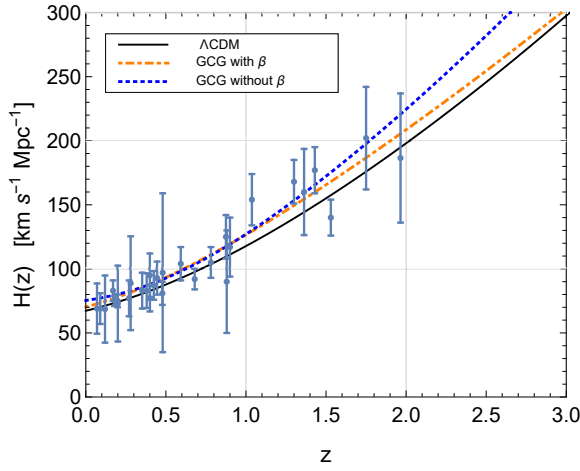


Figure 5.4: Redshift evolution of $H(z)$ for the Λ CDM scenario and the generalized Chaplygin gas models, with and without matter creation. The grey points with error bars denote the cosmic chronometer measurements.

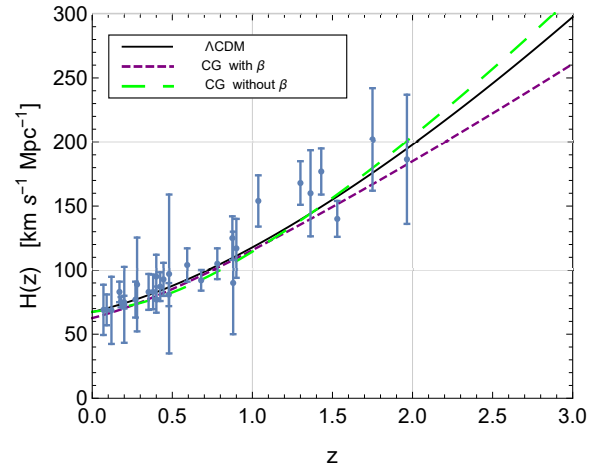


Figure 5.5: Redshift evolution of $H(z)$ for the Λ CDM scenario and the Chaplygin gas models, with and without matter creation. The grey points with error bars denote the cosmic chronometer measurements.

5.7.2 Deceleration Parameter

For the models with matter creation, the present values are $q_0 = -0.551^{+0.015}_{-0.019}$ for the generalized Chaplygin gas model and $q_0 = -0.489^{+0.016}_{-0.022}$ for the Chaplygin gas model. The corresponding transition redshifts are $z_{\text{tr}} = 0.606^{+0.28}_{-0.20}$ and $z_{\text{tr}} = 0.550^{+0.22}_{-0.21}$, respectively. These values are close to the Λ CDM transition redshift and confirm that the proposed models successfully describe the late-time onset of acceleration. Figures 5.6 and 5.7 show the evolutions of deceleration parameter with redshift. We can observe a clear transition from an earlier decelerated phase to the present accelerated phase in all cases.

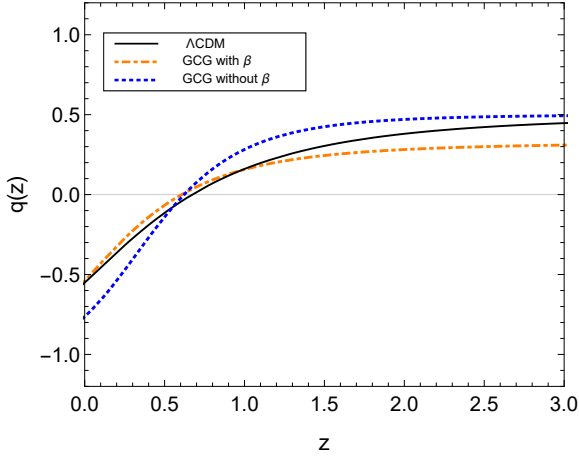


Figure 5.6: Evolution of $q(z)$ for the Λ CDM scenario and the generalized Chaplygin gas models, with and without matter creation.

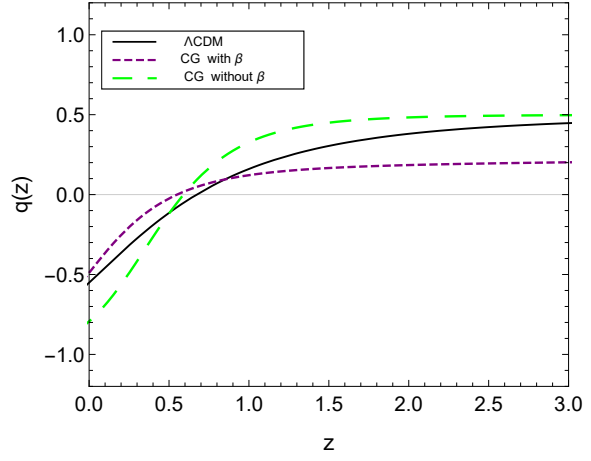


Figure 5.7: Evolution of $q(z)$ for the Λ CDM scenario and the Chaplygin gas models, with and without matter creation.

5.7.3 Effective Equation-of-State Parameter

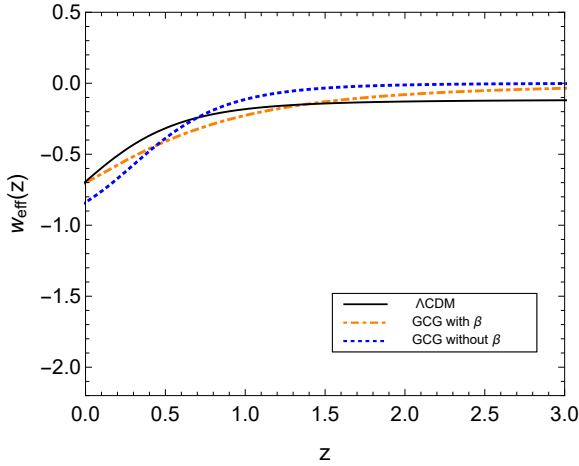


Figure 5.8: Evolution of $w_{\text{eff}}(z)$ for the Λ CDM scenario and the generalized Chaplygin gas models, with and without matter creation.

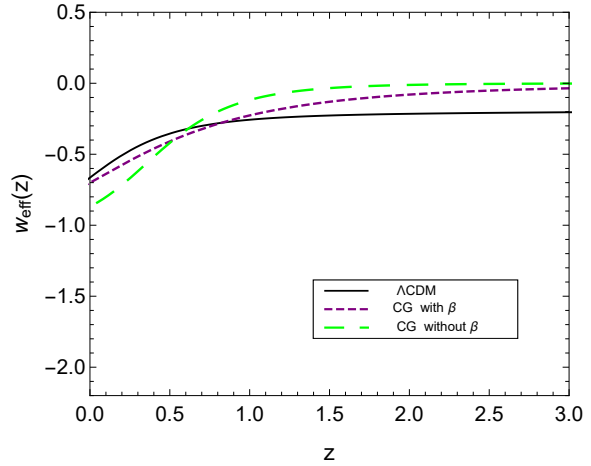


Figure 5.9: Redshift behaviour of the effective equation-of-state parameter $w_{\text{eff}}(z)$ for the Λ CDM scenario and the Chaplygin gas models, with and without matter creation.

Using best fit values of parameters, we plot the evolution of effective EoS parameters for GCG and CG models with and without matter creation as a function of redshift, which are shown in Figs. 5.8 and 5.9, respectively. It is to be noted that $-1 < w < 0$ describes the quintessence phase, $w = -1$ characterizes as a Λ CDM and $w < -1$ gives the phantom phase. The evolution of effective EoS parameter in Fig. 5.8 suggests a quintessence at early times and a phantom phase in late-times in respect of the GCG model with and without matter creation. However, for CG model with and without matter creation, the EoS parameter approaches to cosmological constant in late-time. The present values for the models with matter creation are $w_0 = -0.692^{+0.011}_{-0.014}$ for GCG and $w_0 = -0.664^{+0.019}_{-0.013}$ for CG. These values remain compatible

with the late-time accelerated expansion and are close to the Λ CDM expectation.

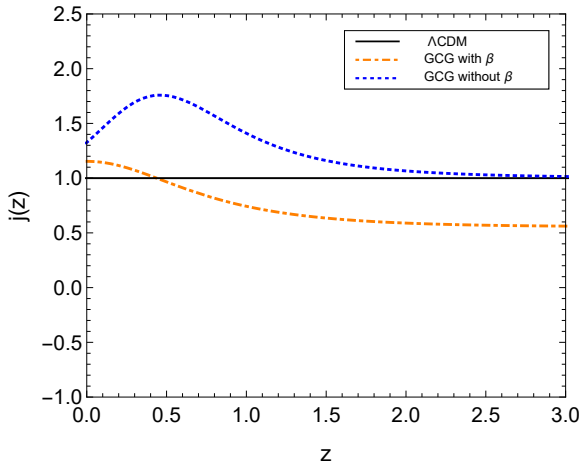


Figure 5.10: Redshift behaviour of the jerk parameter $j(z)$ for the Λ CDM scenario and the generalized Chaplygin gas models, with and without matter creation. The marker indicates the present value j_0 .

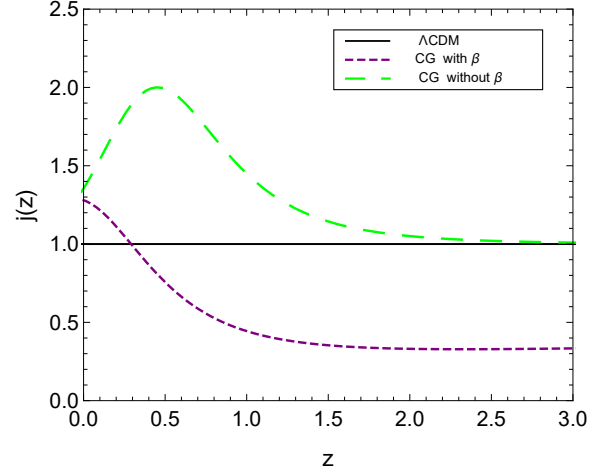


Figure 5.11: Redshift behaviour of the jerk parameter $j(z)$ for the Λ CDM scenario and the Chaplygin gas models, with and without matter creation. The marker indicates the present value j_0 .

5.7.4 Jerk Parameter

Figures 5.10 and 5.11 present the evolutions of jerk parameter for GCG and CG models with and without matter creation. The jerk parameter provides a useful measure of departure from the Λ CDM value $j = 1$. The present values obtained from the joint analysis are $j_0 = 1.153^{+0.010}_{-0.012}$ for GCG with matter creation, $j_0 = 1.325^{+0.021}_{-0.029}$ for GCG without matter creation, $j_0 = 1.280^{+0.022}_{-0.023}$ for CG with matter creation, and $j_0 = 1.354^{+0.011}_{-0.013}$ for CG without matter creation. These values show that the models do depart from the exact Λ CDM behavior, but the deviation remains mild and the late-time trajectories tend toward the standard cosmological evolution.

5.7.5 Age of the Universe

The age of the Universe is obtained from

$$\tau(z) = \int_z^\infty \frac{dz}{(1+z)H(z)}. \quad (5.36)$$

Using the best fit values of parameters, we find the present age of the Universe for Λ CDM, GCG and CG models which are provided in Table 5.3. It can be observed that the age of Universe for Λ CDM is 13.81 ± 0.11 Gyr, for GCG and CG with matter creation are 13.848 ± 0.12 Gyr and 14.11 ± 0.12 Gyr, respectively. Note that the Λ CDM model predicts the age of

Table 5.2: Best-fit estimates of the model parameters for the Λ CDM scenario, generalized Chaplygin gas model, and Chaplygin gas model obtained from the combined $SNe + CC + BAO$ dataset.

Parameters	Λ CDM	GCG with β	GCG without β	CG with β	CG without β
H_0	69.3 ± 1.7	68.9 ± 1.6	72.2 ± 1.7	67.7 ± 1.6	72.8 ± 1.7
$\Omega_{m,0}$	0.294 ± 0.008	—	—	—	—
$\Omega_{\Lambda,0}$	0.705 ± 0.008	—	—	—	—
Ω_b	—	0.049 ± 0.006	0.049 ± 0.002	0.048 ± 0.006	0.050 ± 0.006
A_s	—	$0.685^{+0.042}_{-0.034}$	$0.882^{+0.012}_{-0.019}$	$0.597^{+0.028}_{-0.021}$	$0.908^{+0.030}_{-0.032}$
α	—	0.470 ± 0.026	0.755 ± 0.001	1	1
β	—	0.143 ± 0.026	—	0.240 ± 0.012	—
α_1	—	0.471 ± 0.020	0.471 ± 0.021	0.510 ± 0.029	0.510 ± 0.036
β_1	—	0.510 ± 0.025	0.506 ± 0.023	0.590 ± 0.025	0.588 ± 0.023
r_d	146.8 ± 3.5	146.9 ± 3.4	146.3 ± 3.3	147.1 ± 3.4	146.1 ± 3.4
$\mathcal{M}$	-19.375 ± 0.052	-19.383 ± 0.057	-19.337 ± 0.052	-19.405 ± 0.051	-19.329 ± 0.051

Table 5.3: Present values of the main cosmological quantities for the Λ CDM, generalized Chaplygin gas, and Chaplygin gas models with and without matter creation.

Model	q_0	j_0	w_0	C_s^2	τ_0 (Gyr)	z_{tr}
Λ CDM	$-0.541^{+0.020}_{-0.019}$	1	$-0.700^{+0.011}_{-0.014}$	≤ 1	13.81 ± 0.11	$0.662^{+0.31}_{-0.21}$
GCG with β	$-0.551^{+0.015}_{-0.019}$	$1.153^{+0.010}_{-0.012}$	$-0.692^{+0.011}_{-0.014}$	0.321	13.85 ± 0.12	$0.606^{+0.28}_{-0.20}$
GCG without β	$-0.763^{+0.021}_{-0.029}$	$1.325^{+0.021}_{-0.029}$	$-0.838^{+0.022}_{-0.030}$	0.609	13.62 ± 0.10	$0.621^{+0.24}_{-0.19}$
CG with β	$-0.489^{+0.016}_{-0.022}$	$1.280^{+0.022}_{-0.023}$	$-0.664^{+0.019}_{-0.013}$	0.597	14.11 ± 0.12	$0.550^{+0.22}_{-0.21}$
CG without β	$-0.793^{+0.014}_{-0.020}$	$1.354^{+0.011}_{-0.013}$	$-0.868^{+0.012}_{-0.017}$	0.908	13.97 ± 0.13	$0.591^{+0.31}_{-0.20}$

Table 5.4: Information criteria for the Λ CDM, generalized Chaplygin gas, and Chaplygin gas models with and without matter creation. The Λ CDM model is used as the reference.

Model	χ^2_{min}	χ^2_{min}/dof	AIC	BIC	ΔAIC	ΔBIC
Λ CDM	1052.568	0.968	1062.640	1087.579	—	—
GCG with β	1051.215	0.969	1069.551	1114.036	6.947	26.457
GCG without β	1099.443	1.01	1115.443	1155.203	52.839	67.624
CG with β	1053.448	0.971	1069.551	1069.551	6.94	18.028
CG without β	1123.060	1.03	1137.060	1171.921	74.456	84.342

Universe as 13.787 ± 0.02 Gyr [3].

5.8 Model Stability

To examine the physical viability of the Chaplygin-type models, we study the squared sound speed $C_s^2 = \partial p / \partial \rho$. Stable behavior requires $0 \leq C_s^2 \leq 1$. For the generalized Chaplygin gas model with matter creation, the squared sound speed is

$$C_s^2 = -\beta + \frac{(1-\beta)\alpha A_s}{A_s + (1-A_s)(1+z)^{3(1-\beta)(1+\alpha)}}, \quad (5.37)$$

or equivalently,

$$C_s^2 = -\beta - \alpha(1-\beta)w_{gCG}(z). \quad (5.38)$$

For the Chaplygin gas model with matter creation, one has

$$C_s^2 = -\beta + \frac{(1-\beta)A_s}{A_s + (1-A_s)(1+z)^{6(1-\beta)}} = \beta - (1-\beta)w_{CG}(z). \quad (5.39)$$

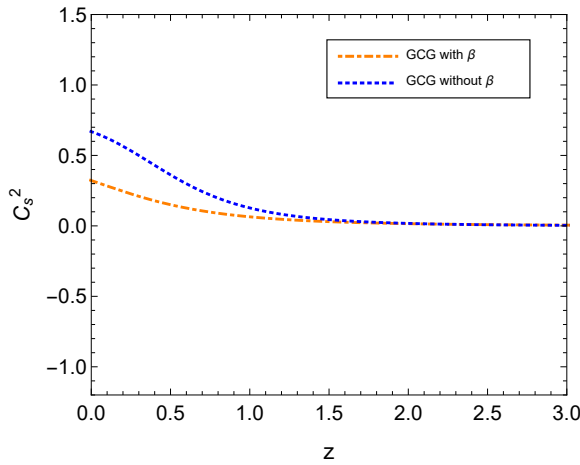


Figure 5.12: Evolution of the squared sound speed C_s^2 for the generalized Chaplygin gas and Chaplygin gas models with and without matter creation.

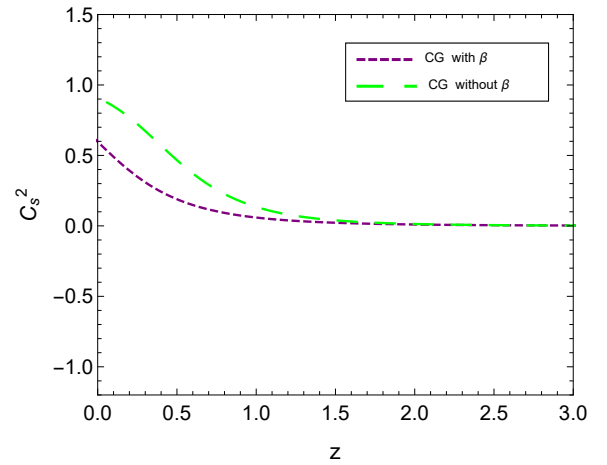


Figure 5.13: Evolution of the squared sound speed C_s^2 for the generalized Chaplygin gas and Chaplygin gas models with and without matter creation.

Using the best-fit parameter values from $SNe + CC + BAO$ dataset to the equations above, the evolutions of C_s^2 for GCG and CG models with respect to the redshift are plotted in Figs. 5.12 and 5.13, respectively. The figures show that the each model is stable through out the evolution as the trajectory remains positive and lies in the range $0 \leq C_s^2 \leq 1$. The present values reported in Table 5.3, namely $C_s^2 = 0.321$ for GCG with matter creation and $C_s^2 = 0.597$ for CG with matter creation, confirm that both models are stable.

5.9 Model Selection Criteria

The statistical comparison with the reference Λ CDM model is performed by means of AIC and BIC. The values obtained in the source article are given in Table 5.4. We observe that the GCG and CG models with matter creation fall in the range $4 < \Delta\text{AIC} < 10$, which means that they are still supported by the observational data, although less strongly than the preferred model. By contrast, the models without matter creation are not favored because $\Delta\text{AIC} > 10$. The BIC values are more restrictive and provide strong evidence against all of the Chaplygin-type extensions, reflecting the stronger penalty assigned to the larger parameter space. Thus, the AIC and BIC lead to different levels of support, but both confirm that matter creation improves the statistical behavior of the Chaplygin gas models relative to their counterparts without matter creation.

5.10 Conclusion

In this chapter, the late-time accelerated expansion of the Universe has been studied in the framework of non-minimally coupled $f(Q)$ gravity by combining generalized Chaplygin gas dynamics with adiabatic matter creation. A particular case corresponding to the original Chaplygin gas has also been analyzed. The main purpose of the study was to determine whether the inclusion of matter creation in modified gravity can provide a viable description of the cosmic expansion history while remaining consistent with current observational data.

The analysis proceeded in two stages. First, exact background solutions were obtained for the generalized Chaplygin gas and Chaplygin gas models in the presence of matter creation. The corresponding expressions for the Hubble parameter, deceleration parameter, effective equation-of-state parameter, and jerk parameter were derived in the $f(Q)$ framework. Second, the free parameters of the models were constrained using a joint likelihood analysis based on the Pantheon supernova sample, 32 cosmic chronometer measurements, and 12 DESI DR2 BAO data points.

The observational constraints show that the GCG and CG models with matter creation remain compatible with the combined late-time dataset and reproduce the transition from early deceleration to late-time acceleration. Their best-fit values of the Hubble parameter remain close to the Planck estimate and lie between the standard early- and late-Universe determinations. The deceleration parameter confirms an accelerated present epoch, the effective equation of state indicates a phantom tendency for the generalized Chaplygin gas and a de Sitter ten-

dency for the Chaplygin gas at late times, and the jerk parameter shows only mild departures from the Λ CDM value.

Table 5.2 provides the best-fit values for free parameters of both models with and without matter creation. The present values of $q(z)$, $j(z)$, $w(z)$, $t(z)$, and C_s^2 are provided in Table 5.3 and their respective evolutions have been plotted in Figs.5.4- 5.13. Comparing the results of these cosmological quantities, we have observed that GCG and CG models with matter creation in $f(Q)$ gravity are compatible with the observational dataset as well as Λ CDM model. The evolution trajectories of $j(z)$ display only slight deviations from the Λ CDM model for both the GCG and CG models. It has been observed that the best-fit values of Hubble constant for GCG and CG models with matter creation are $H_0 = 68.9 \pm 1.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $H_0 = 68.67.7 \pm 1.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$, respectively, which show that GCG model with matter creation may contribute to minimize the H_0 tension between the value obtained by Planck [3] and the value by Riess et al.[52].

The stability analysis demonstrates that the squared sound speed remains positive and smaller than unity for the models with matter creation, thereby satisfying the expected causality and stability requirements. This confirms that the proposed Chaplygin-type scenarios are physically well behaved at the level of background evolution.

The statistical comparison through AIC and BIC shows that the inclusion of matter creation significantly improves the Chaplygin gas models. According to the Akaike criterion, the GCG and CG models with matter creation are still supported by the combined observational data, whereas the corresponding models without matter creation are disfavored. The Bayesian criterion, being more restrictive, strongly penalizes the extended models because of their larger number of free parameters and therefore continues to prefer the reference Λ CDM cosmology.

Finally, comparing our results of GCG model with matter creation, with those presented in paper [224] for GCG model with viscous fluid to tackle the cosmic acceleration, we find that GCG model with matter creation gives the comparatively low best-fit values of cosmological parameters by using a joint dataset of SNe Pantheon+, CC and DESI DR2 BAO (refers to Tables 5.2 -5.4). We have observed that the value of H_0 , A_S and α parameters for GCG with matter creation and viscous GCG in paper [224] have the differences of 0.03σ , 1.4σ and 0.43σ , respectively.

Overall, the results of this chapter indicate that generalized Chaplygin gas and Chaplygin gas models with matter creation in $f(Q)$ gravity provide viable alternatives for describing late-time cosmic acceleration. They reproduce the principal background features of the observed Universe, remain stable, and exhibit a phenomenology close to that of Λ CDM while offering

a distinct physical interpretation based on non-metric gravity and irreversible matter creation.

Chapter 6

Matter Creation Cosmology with Variable Generalized Chaplygin Gas in $f(Q)$ Gravity

6.1 Introduction

This chapter investigates variable generalized Chaplygin gas and matter creation for the choices $f(Q) = n_1 Q^k$ and $f(Q) = n_0 + n_1 Q^k$, following Bhardwaj and Singh [268]. It connects the variable-fluid setting of Chapter 3 with the nonmetricity-based framework introduced in Chapter 5. Unlike the preceding chapter, the matter action has no separate factor $f_2(Q)$, and creation pressure belongs to the pressureless-matter sector rather than the VGCG component.

The shared gravitational, fluid, and thermodynamic background is given in Sections 1.5.2, 1.8.1, and 1.9. The present chapter focuses on the two gravitational models, their background solutions, and their observational diagnostics.

Section 6.2 specifies the gravitational field equations, and Section 6.3 develops the cosmological model. Sections 6.4 and 6.5 present the observational methodology and results, followed by the conclusion in Section 6.6. Chapter 7 brings together the findings and future scope of the thesis.

6.2 Gravitational Field Equations

The general discussion of $f(Q)$ gravity and its action is given in Section 1.5.2. We use the common tensor definitions there, including the symmetrized nonmetricity conjugate in

Eq. (1.30). In contrast to Chapter 5, the matter Lagrangian has no separate non-minimal factor $f_2(Q)$.

The metric field equation for the one-function theory is

$$-\frac{2}{\sqrt{-g}}\nabla_\beta\left(f_Q\sqrt{-g}P^\beta_{\mu\nu}\right)-\frac{1}{2}f g_{\mu\nu}-f_Q\left(P_{\mu\beta\gamma}Q^{\beta\gamma}_\nu-2Q_{\beta\gamma\mu}P^{\beta\gamma}_\nu\right)=T_{\mu\nu}, \quad (6.1)$$

where $f_Q = df/dQ$ and the matter energy-momentum tensor is

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}. \quad (6.2)$$

6.3 Cosmological Model

We consider a spatially homogeneous and isotropic flat Friedmann–Lemaître–Robertson–Walker space-time,

$$ds^2 = -dt^2 + a^2(t) \left[dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right], \quad (6.3)$$

where $a(t)$ is the scale factor and $H = \dot{a}/a$ is the Hubble parameter. In this geometry, the non-metricity scalar is

$$Q = 6H^2. \quad (6.4)$$

The cosmic medium is taken to consist of pressureless matter and a Chaplygin gas component. The energy-momentum tensor is

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \quad (6.5)$$

where $\rho = \rho_{vgcg} + \rho_m$ and $p = p_{vgcg} + p_m + p_c$. Since the late-time Universe is considered, radiation is neglected and the matter component is taken to be pressureless, $p_m = 0$.

Substituting the FLRW metric into the field equations (6.1), one obtains [35]

$$6f_Q H^2 - \frac{1}{2}f = \rho, \quad (6.6)$$

$$(12f_{QQ}H^2 + f_Q)\dot{H} = -\frac{1}{2}(\rho + p). \quad (6.7)$$

The matter and Chaplygin sectors are assumed to be separately conserved. Their conserva-

tion equations are

$$\dot{\rho}_m + 3H(\rho_m + p_m + p_c) = 0, \quad (6.8)$$

and

$$\dot{\rho}_{vgcg} + 3H(\rho_{vgcg} + p_{vgcg}) = 0. \quad (6.9)$$

The adiabatic creation-pressure relation is given in Eq. (1.65). Here it is applied to the pressureless-matter sector:

$$p_c = -\frac{\rho_m + p_m}{3H}\Gamma. \quad (6.10)$$

The VGCG component remains separately conserved.

For pressureless matter, Eq. (6.8) reduces to

$$\dot{\rho}_m + 3H\rho_m = \rho_m\Gamma. \quad (6.11)$$

We assume [8, 126, 127]

$$\Gamma = 3\beta H, \quad (6.12)$$

where β is a non-negative constant. Now, Equation (6.11) reduces to

$$\rho_m = \rho_{m,0}(1+z)^{3(1-\beta)}. \quad (6.13)$$

The generalized Chaplygin gas is characterized by the equation of state $p_{gcg} = -A/\rho_{gcg}^\alpha$ with $0 < \alpha \leq 1$ [6, 58]. A more flexible model is obtained by allowing the parameter A to vary with the scale factor, which gives the variable generalized Chaplygin gas (VGCG) equation of state [62, 141]

$$p_{vgcg} = -\frac{A(a)}{\rho_{vgcg}^\alpha}. \quad (6.14)$$

Following the standard choice in the literature, the scale-factor dependence is taken as [141, 243]

$$A(a) = A_0 a^{-n}, \quad (6.15)$$

so that the equation of state becomes

$$p_{vgcg} = -\frac{A_0 a^{-n}}{\rho_{vgcg}^\alpha}. \quad (6.16)$$

This parametrization allows the fluid to interpolate between matter-like behavior at early times and quintessence- or phantom-like behavior at late times.

Using the VGCG equation of state (6.16) in Eq. (6.9) and integrating, one finds

$$\rho_{vgcg} = \rho_{vgcg,0} \left[A_s(1+z)^n + (1-A_s)(1+z)^{3(1+\alpha)} \right]^{\frac{1}{1+\alpha}}, \quad (6.17)$$

where

$$A_s = \frac{3(1+\alpha)}{3(1+\alpha)-n} \frac{A_0}{\rho_{vgcg,0}^{1+\alpha}}. \quad (6.18)$$

For an expanding Universe, one must have $n > 0$ and $1 + \alpha > 0$.

6.3.1 Model 1: $f(Q) = n_1 Q^k$

For the first model we assume the following form of $f(Q)$ [256]:

$$f(Q) = n_1 Q^k, \quad (6.19)$$

where n_1 and k are dimensionless constants. Substituting this choice into Eq. (6.6), one obtains

$$H = \left[\frac{2(\rho_m + \rho_{vgcg})}{n_1(2k-1)6^k} \right]^{\frac{1}{2k}}. \quad (6.20)$$

After using Eqs. (6.13) and (6.17), and defining the normalized Hubble function $E(z) = H(z)/H_0$, the result becomes

$$E^2(z) = B_1 \left[\Omega_m(1+z)^{3(1-\beta)} + \Omega_{vgcg,1} \left[A_s(1+z)^n + (1-A_s)(1+z)^{3(1+\alpha)} \right]^{\frac{1}{1+\alpha}} \right]^{\frac{1}{k}}, \quad (6.21)$$

where

$$B_1 = \left[\frac{6^{1-k}}{(2k-1)n_1} \right]^{\frac{1}{2k}}, \quad (6.22)$$

and the present condition $E(0) = 1$ gives

$$\Omega_{vgcg,1} = H_0^{2k-2} (2k-1) 6^{k-1} n_1 - \Omega_m. \quad (6.23)$$

6.3.2 Model 2: $f(Q) = n_0 + n_1 Q^k$

For the second model we consider the extended form [230, 257]

$$f(Q) = n_0 + n_1 Q^k, \quad (6.24)$$

where n_0 and n_1 are constants. Substitution into Eq. (6.6) yields

$$H = \left[\frac{2(\rho_m + \rho_{vgcg}) + n_0}{n_1(12k - 1)6^k} \right]^{\frac{1}{2k}}. \quad (6.25)$$

Using Eqs. (6.13) and (6.17), the corresponding normalized Hubble function becomes

$$E^2(z) = B_2 \left[\Omega_m(1+z)^{3(1-\beta)} + \Omega_{vgcg,2} \left[A_s(1+z)^n + (1-A_s)(1+z)^{3(1+\alpha)} \right]^{\frac{1}{1+\alpha}} + n_0 \right]^{\frac{1}{k}}, \quad (6.26)$$

where

$$B_2 = \left[\frac{6^{1-k}}{(12k-1)n_1} \right]^{\frac{1}{2k}}, \quad (6.27)$$

and the present condition gives

$$\Omega_{vgcg,2} = H_0^{2k-2} (12k-1) 6^{k-1} n_1 - \Omega_m - n_0. \quad (6.28)$$

6.4 Observational Data and Statistical Methodology

In order to derive robust posterior probability distributions for the cosmological parameters of interest, we utilize a combination of complementary observational probes. Specifically, we incorporate three key datasets:

- Cosmic Chronometer (CC) measurements, which provide direct estimates of the Hubble parameter across a wide redshift range. The cosmic chronometer compilation supplies 31 direct estimates of $H(z)$ obtained through the differential age method [180–183];
- The Pantheon+ compilation of Type Ia supernovae (SNe Ia), excluding any calibration tied to the SH0ES local distance ladder to avoid potential biases. The Pantheon+ sample provides 1701 standardized distance modulus measurements [184];
- Baryon Acoustic Oscillation (BAO) data drawn from the first-year results of the Dark Energy Spectroscopic Instrument (DESI) and the extended spectroscopic survey SDSS-IV. The BAO information is incorporated through the standard distance combinations involving $D_H(z)$, $D_M(z)$, and $D_V(z)$, with the sound horizon r_d treated as a free parameter in the fit [186–190, 192, 193, 258, 259].

The total chi-square used in the analysis is

$$\chi_{\text{tot}}^2 = \chi_{\text{CC}}^2 + \chi_{\text{SNe,Ia}}^2 + \chi_{\text{BAO}}^2. \quad (6.29)$$

The parameter estimation is performed through Markov Chain Monte Carlo sampling with the `PyPolyChord` nested sampler, and the posterior distributions are analyzed using `GetDist` [165, 166]. The parameter vectors explored in our analysis differ across models: for the reference Λ CDM cosmology, we sample $\theta = \{H_0, \Omega_m, \Omega_\Lambda, r_d, \mathcal{M}\}$; for Model 1 ($f(Q) = n_1 Q^k$), the parameter space expands to $\theta = \{H_0, \Omega_m, k, n_1, A_s, \alpha, n, \beta, r_d, \mathcal{M}\}$; and for Model 2 ($f(Q) = n_0 + n_1 Q^k$), we include an additional constant term, yielding $\theta = \{H_0, \Omega_m, k, n_0, n_1, A_s, \alpha, n, \beta, r_d, \mathcal{M}\}$. In all cases, r_d represents the comoving sound horizon at the baryon drag epoch, treated as a free nuisance parameter to avoid reliance on early-Universe assumptions. The flat priors adopted in the analysis are summarized in Table 6.1 and the best-fit values obtained from the joint $CC + SNe, Ia + BAO$ analysis are listed in Table 6.2.

Table 6.1: Flat priors adopted in the MCMC analysis.

H_0	Ω_m	Ω_Λ	k	n_0	n_1	A_s	α	n	β	r_d	$\mathcal{M}$
[60, 80]	[0, 0.5]	[0.5, 1]	[0, 1.5]	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[100, 200]	[-20, -18]

6.5 Results and Discussion

The posterior distributions for the Λ CDM model and the two proposed Chaplygin-type $f(Q)$ models are shown in Figs. 6.1 and 6.2. The best-fit values indicate that the two extended models remain close to the reference cosmology, while still allowing a non-trivial departure from the standard picture. In particular, the present Hubble constants obtained for Model 1 and Model 2, namely 67.199 ± 0.128 and 67.193 ± 0.122 , are fully consistent with the Planck 2018 inference and remain only slightly below the best-fit value obtained for the Λ CDM fit of the same combined dataset.

6.5.1 Hubble Parameter

We present a comparative analysis of the redshift evolution of the Hubble parameter $H(z)$ for the standard Λ CDM cosmology alongside our two proposed $f(Q)$ -based models-Model 1 and Model 2 -in direct confrontation with the observational CC dataset. For the reference Λ CDM scenario, the theoretical expression $H(z) = H_0 \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$ is employed, with best-fit parameters constrained by our joint analysis yielding $H_0 = 68.2 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$,

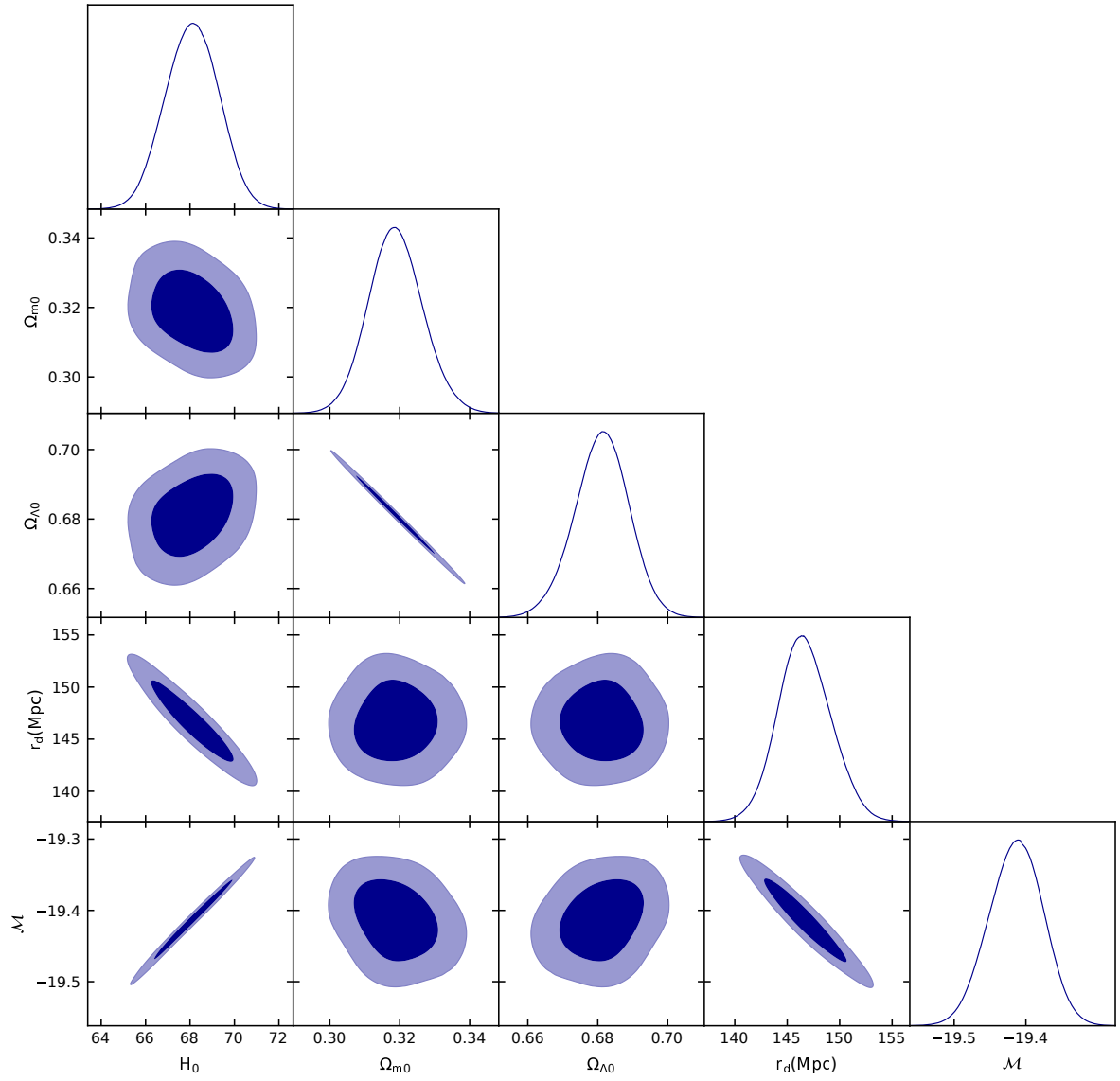


Figure 6.1: Best-fit parameter estimates for the Λ CDM model derived from the combined $CC + SNeIa + BAO$ datasets, reported at the 1σ and 2σ confidence levels.

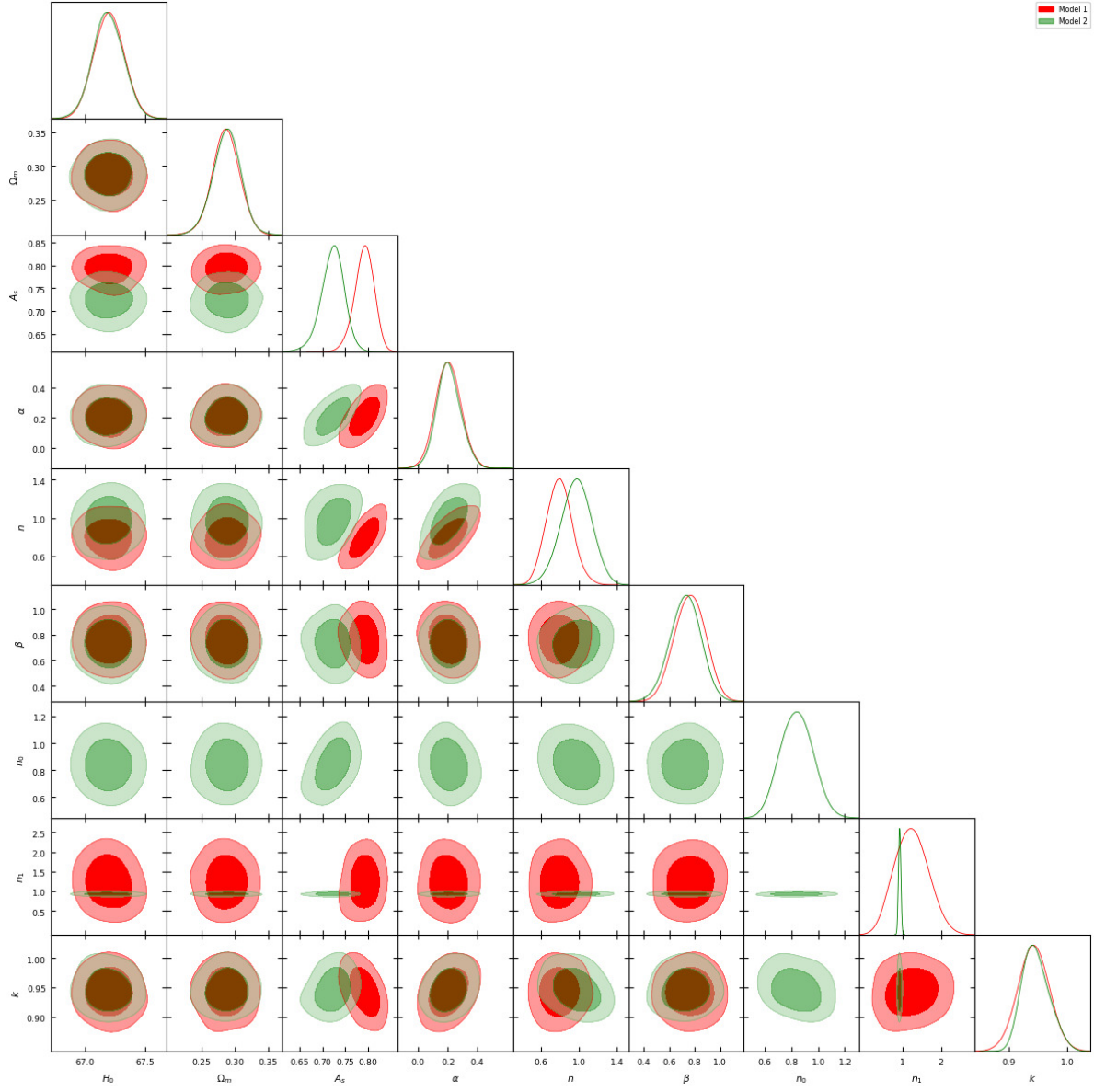


Figure 6.2: Best-fit parameter estimates for Model 1 and Model 2 derived from the combined $CC + SNeIa + BAO$ datasets, reported at the 1σ and 2σ confidence levels.

$\Omega_m = 0.315 \pm 0.007$, and $\Omega_\Lambda = 0.682 \pm 0.007$. In contrast, the reconstructed present-day Hubble constants for Model 1 and Model 2 are $H_0 = 67.199 \pm 0.128 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $67.193 \pm 0.122 \text{ km s}^{-1} \text{ Mpc}^{-1}$, respectively—values marginally lower than the Λ CDM estimate but in excellent agreement with the Planck 2018 inference of $H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Aghanim et al. 2018). Furthermore, the inferred matter density parameters are $\Omega_m = 0.287 \pm 0.019$ for Model 1 and $\Omega_m = 0.288 \pm 0.021$ for Model 2, both slightly below the Λ CDM value yet well within observational uncertainties. Figure 6.3 compares the reconstructed Hubble expansion histories of the two $f(Q)$ models with the reference Λ CDM curve and the cosmic chronometer measurements. Both models provide a good description of the observed $H(z)$ evolution across the available redshift range. The comparison becomes clearer in Fig. 6.4, where the Hubble difference is plotted. At low redshifts, both Model 1 and Model 2 remain very close to the Λ CDM baseline. At higher redshifts, Model 2 continues to track the standard cosmology more closely than Model 1, while Model 1 shows a larger departure from the reference expansion history.

$$\Delta H(z) = H_{\text{Model}}(z) - H_{\Lambda\text{CDM}}(z) \quad (6.30)$$

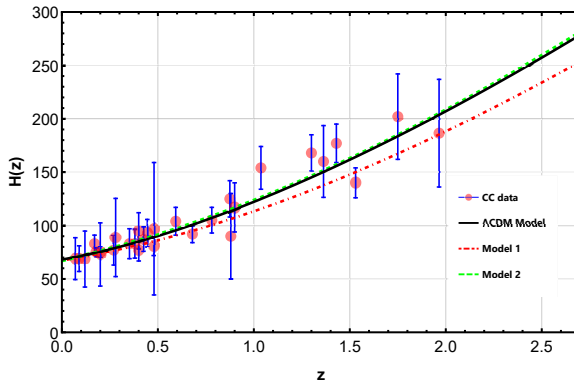


Figure 6.3: Redshift evolution of the Hubble parameter $H(z)$ for the Λ CDM model, Model 1, and Model 2 obtained from the combined $CC + SNeIa + BAO$ dataset. The grey points with error bars denote the cosmic chronometer measurements.

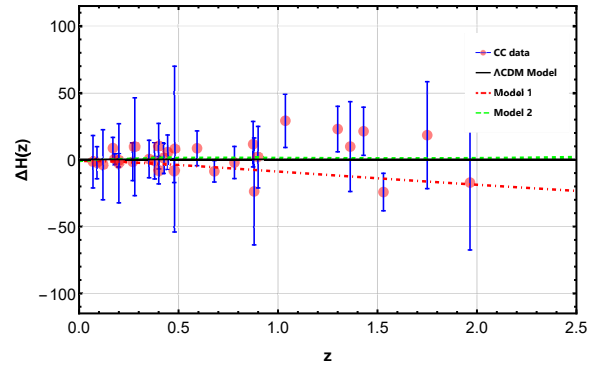


Figure 6.4: Redshift-dependent variation of the Hubble difference $\Delta H(z)$ for Model 1 and Model 2 measured with respect to the reference Λ CDM scenario.

6.5.2 Deceleration Parameter

Using the best-fit cosmological parameters obtained from the joint $CC + SNeIa + BAO$ dataset, we reconstruct the redshift evolution of $q(z)$, as displayed in Fig. 6.5. The plot reveals a clear transition from an early decelerating phase ($q > 0$) to the current epoch of accelerated expansion ($q < 0$), consistent with the standard cosmological narrative. At present

Table 6.2: Best-fit estimates of the model parameters derived from the combined $CC + SNe Ia + BAO$ observational dataset.

Parameters	Λ CDM	Model 1	Model 2
H_0	68.1 ± 0.9	67.199 ± 0.128	67.193 ± 0.122
Ω_m	0.314 ± 0.006	0.287 ± 0.019	0.288 ± 0.021
Ω_Λ	0.684 ± 0.006	—	—
k	—	0.942 ± 0.026	0.946 ± 0.022
n_0	—	—	0.840 ± 0.126
n_1	—	1.229 ± 0.459	0.928 ± 0.031
A_s	—	0.793 ± 0.020	0.723 ± 0.025
α	—	0.205 ± 0.081	0.208 ± 0.075
n	—	0.789 ± 0.135	0.969 ± 0.160
β	—	0.764 ± 0.124	0.728 ± 0.118
r_d	146.3 ± 2.3	148.23 ± 1.09	148.28 ± 1.05
$\mathcal{M}$	-19.412 ± 0.035	-19.427 ± 0.006	-19.429 ± 0.006

Table 6.3: Present values of the main cosmological quantities for the Λ CDM model, Model 1, and Model 2.

Parameter	Λ CDM	Model 1	Model 2
q_0	$-0.552^{+0.020}_{-0.018}$	$-0.535^{+0.011}_{-0.012}$	$-0.551^{+0.027}_{-0.023}$
j_0	1.000	$0.569^{+0.015}_{-0.012}$	$0.530^{+0.031}_{-0.030}$
w_0	$-0.699^{+0.010}_{-0.013}$	$-0.681^{+0.011}_{-0.014}$	$-0.666^{+0.028}_{-0.031}$
s_0	0.000	$0.117^{+0.011}_{-0.019}$	$0.157^{+0.001}_{-0.003}$
τ_0 (Gyr)	13.6 ± 0.10	13.4 ± 0.15	13.8 ± 0.10
z_{tr}	$0.630^{+0.30}_{-0.20}$	$0.591^{+0.28}_{-0.20}$	$0.628^{+0.24}_{-0.18}$

Table 6.4: Statistical comparison of the Λ CDM model, Model 1, and Model 2 using the combined dataset. Values of $\ln E$ and $\ln B_i$ correspond to the Bayesian evidence and the logarithmic Bayes factor relative to Λ CDM.

Parameters	Λ CDM	Model 1	Model 2
$\chi^2_{\text{tot,min}}$	1780.913	1821.838	1822.079
χ^2_{red}	1.020	1.049	1.050
AIC	1780.913	1841.838	1844.079
BIC	1818.272	1896.558	1904.271
ΔAIC	—	60.925	63.166
ΔBIC	—	78.286	85.999
$\ln E$	-1870.234568	-1821.837760	-1822.079016
$\ln B_i$	—	48.396808	48.155552
p -value	0.277	0.074	0.071

($z = 0$), Model 1 yields $q_0 = -0.535^{+0.011}_{-0.012}$, while Model 2 gives $q_0 = -0.551^{+0.027}_{-0.023}$, both of which align closely with the Λ CDM prediction of $q_0 \approx -0.55$ inferred from Planck collaboration (Aghanim et al. 2018). Furthermore, the redshift at which the transition from deceleration to acceleration occurs, denoted by z_{tr} , is found to be $z_{\text{tr}} = 0.591^{+0.28}_{-0.20}$ for Model 1 and $z_{\text{tr}} = 0.628^{+0.24}_{-0.18}$ for Model 2. These values are well within the range supported by independent astrophysical measurements, reinforcing the physical viability of the proposed frameworks.

6.5.3 Jerk Parameter

Figure 6.6 displays the reconstructed redshift evolution of the jerk parameter for both Model 1 and Model 2—our $f(Q)$ -based frameworks incorporating matter creation and a variable generalized Chaplygin gas (VGCG). The plots reveal clear deviations from the Λ CDM baseline ($j = 1$), particularly at intermediate and high redshifts. These discrepancies underscore the enhanced dynamical richness of the proposed models, which not only reproduce late-time acceleration but also encode non-trivial higher-order kinematic behavior. At the present epoch ($z = 0$), the best-fit values are $j_0 = 0.569^{+0.015}_{-0.012}$ for Model 1 and $j_0 = 0.530^{+0.030}_{-0.031}$ for Model 2, as listed in Table 6.3. Both estimates lie significantly below unity, providing strong evidence that these models describe a cosmic expansion history distinct from that of the standard Λ CDM paradigm.

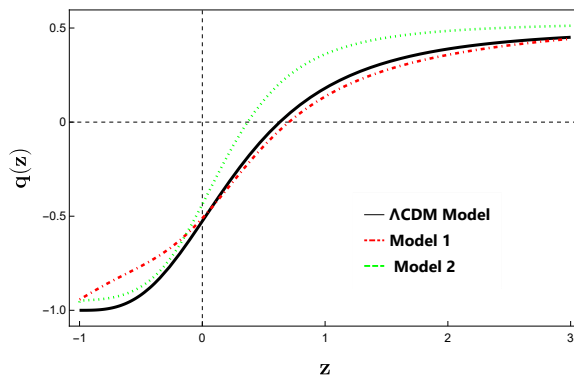


Figure 6.5: Redshift behaviour of the deceleration parameter $q(z)$ for the Λ CDM model, Model 1, and Model 2 obtained from the combined $CC + SNeIa + BAO$ dataset.

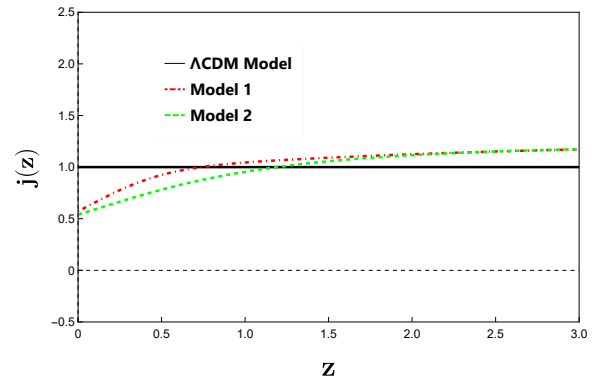


Figure 6.6: Redshift behaviour of the jerk parameter $j(z)$ for the Λ CDM model, Model 1, and Model 2 obtained from the combined $CC + SNeIa + BAO$ dataset. In the Λ CDM case, the jerk parameter remains fixed at $j(z) = 1$ over the full redshift range.

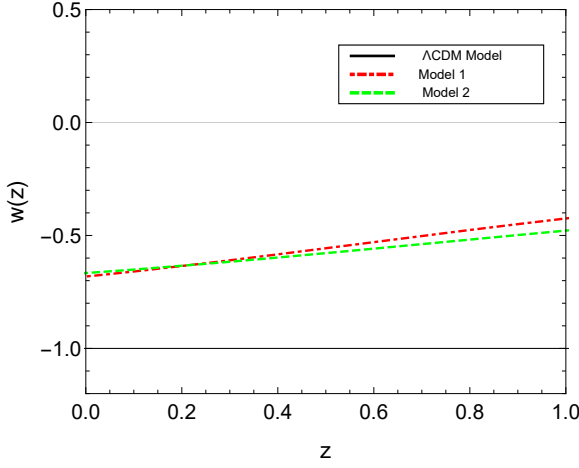


Figure 6.7: Redshift behaviour of the effective equation-of-state parameter $w(z)$ for the Λ CDM model, Model 1, and Model 2 obtained from the combined $CC + SNeIa + BAO$ dataset.

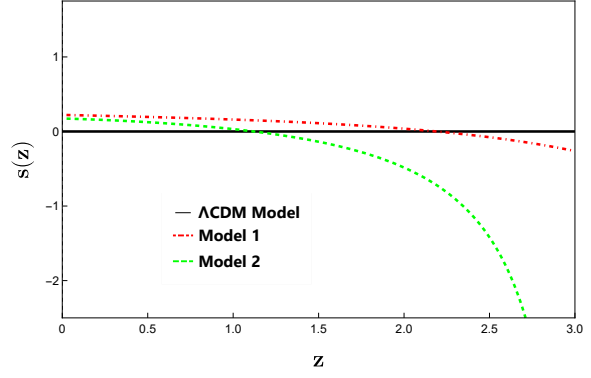


Figure 6.8: Redshift behaviour of the snap parameter $s(z)$ for the Λ CDM model, Model 1, and Model 2 obtained from the combined $CC + SNeIa + BAO$ dataset. For a flat Λ CDM background, the snap parameter remains zero throughout the redshift range.

6.5.4 Effective Equation-of-State Parameter

It is important to emphasize that the equation of state parameter $w(z)$ reconstructed in this work is an *effective* EoS, encapsulating the combined influence of matter creation, modified gravity, and the variable generalized Chaplygin gas fluid, rather than describing a single physical component. This effective description allows us to interpret the global dynamical behavior of the Universe within a unified framework. Different ranges of $w(z)$ correspond to distinct physical regimes: values in the interval $-1 < w < 0$ are characteristic of quintessence-like dark energy, $w = -1$ precisely recovers the cosmological constant scenario of the standard Λ CDM model, and $w < -1$ signifies phantom energy with super-negative pressure. As illustrated in Fig. 6.7, both Model 1 and Model 2 exhibit an EoS evolution that remains firmly within the quintessence domain across all redshifts probed. At the present epoch ($z = 0$), the best-fit estimates yield $w_0 = -0.681^{+0.011}_{-0.014}$ for Model 1 and $w_0 = -0.666^{+0.028}_{-0.031}$ for Model 2. These values are marginally higher (i.e., less negative) than the effective $w_0 \approx -0.76$ inferred for the Λ CDM model under the same observational assumptions, yet they unambiguously confirm a phase of accelerated expansion driven by a quintessence-like effective fluid.

6.5.5 Snap Parameter

The snap parameter is the fourth-order cosmographic quantity

$$s = \frac{\ddot{\ddot{a}}}{aH^4}. \quad (6.31)$$

In terms of the jerk and deceleration parameters, it can be written as

$$s(z) = -(1+z) \frac{dj}{dz} - j(z) [2 + 3q(z)]. \quad (6.32)$$

For the flat Λ CDM model, the snap parameter is constant and equal to zero.

Figure 6.8 displays the reconstructed redshift evolution of $s(z)$ for both the reference Λ CDM model (shown as a solid black line) and our two Chaplygin gas-based $f(Q)$ scenarios. As expected, the Λ CDM curve remains precisely at $s(z) = 0$ across the entire redshift range, underscoring its kinematic simplicity. In contrast, both Model 1 and Model 2 exhibit non-zero snap values at intermediate and high redshifts, reflecting the richer dynamical structure induced by matter creation and the variable generalized Chaplygin gas fluid. These deviations are modulated by the particle creation efficiency parameter β , which influences the effective equation of state and expansion history. Notably, all models asymptotically approach $s(z) \rightarrow 0$ as $z \rightarrow 0$, indicating convergence with Λ CDM in the recent Universe. At the present epoch, the best-fit estimates yield $s_0 = +0.117^{+0.011}_{-0.019}$ for Model 1 and $s_0 = 0.157^{+0.001}_{-0.003}$ for Model 2, confirming a mild but statistically discernible departure from the standard cosmological prediction.

6.5.6 $Om(z)$ Diagnostic

To compare the dynamical behavior of the models with the concordance cosmology, we also use the $Om(z)$ diagnostic [260–262]. It is defined by

$$Om(z) = \frac{H^2(z)/H_0^2 - 1}{(1+z)^3 - 1}. \quad (6.33)$$

In the Λ CDM model, $Om(z)$ remains exactly constant at $Om(z) = \Omega_{m,0}$ for all redshifts, producing a horizontal line. Deviations from this flat behavior signal departures from the concordance model. Specifically, a positive derivative is indicative of phantom-like dark energy ($w < -1$), whereas a negative slope corresponds to quintessence-like dynamics ($-1 < w < 0$). As shown in Figure 6.9, both Model 1 and Model 2 exhibit a clear negative slope in their $Om(z)$ evolution across the redshift range probed, confirming that their effective dark energy component behaves similarly to quintessence. Moreover, the visible deviation of these curves from the constant Λ CDM baseline underscores the distinct expansion history predicted by the Chaplygin gas scenarios, reinforcing their role as viable alternatives within the broader landscape of dark energy models.

In Model 1, the $Om(z)$ curve shows a clear transition from positive to negative slope around $z \approx 0.8$, suggesting a transition from phantom-like behavior at higher redshifts ($w < -1$) to quintessence-like behavior at lower redshifts ($w > -1$). Model 2 exhibits a similar but more gradual transition, with the slope changing from slightly positive to negative around $z \approx 1.2$. This dynamical behavior is a direct consequence of VGCG EoS, which allows for a smooth transition between different dark energy regimes.

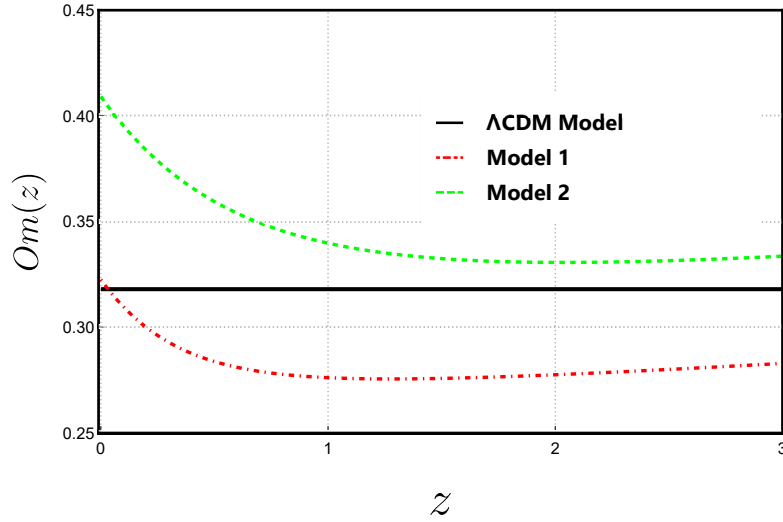


Figure 6.9: Redshift behaviour of the $Om(z)$ diagnostic for the Λ CDM model, Model 1, and Model 2 obtained from the combined $CC + SNeIa + BAO$ dataset. For a flat Λ CDM background, $Om(z)$ remains constant and equals $\Omega_{m,0}$.

6.5.7 Age of the Universe

Using the best-fit parameter values obtained from our joint $CC + SNeIa + BAO$ analysis, we evaluate the age of the Universe from equation (5.36). The resulting present-day age estimates are $\tau_0 = 13.6 \pm 0.11$ Gyr for Λ CDM, $\tau_0 = 13.4 \pm 0.15$ Gyr for Model 1, and $\tau_0 = 13.8 \pm 0.10$ Gyr for Model 2, as reported in Table 6.3. These values are in excellent agreement with each other and fall well within the range inferred from high-precision CMB observations. In particular, the Planck Collaboration 2018 [3] reports a cosmic age of $\tau_0 = 13.7 \pm 0.02$ Gyr under the standard Λ CDM assumption. The close concordance between our reconstructed ages and this benchmark value reinforces the physical plausibility of both extended models, confirming that they yield a cosmic timeline consistent with current observational cosmology. The alignment of these measurements has several implications for cosmic history: confirmation of the model with standard Λ CDM, timeline for structure formation and ongoing Hubble tension.

6.5.8 Model selection criteria

We begin with the reduced chi-square statistic, defined as

$$\chi_{\text{red}}^2 = \frac{\chi_{\text{min}}^2}{\text{dof}},$$

where χ_{min}^2 denotes the minimum chi-square value obtained from the joint likelihood analysis, and *dof* (degrees of freedom) is computed as $\text{dof} = N - K$, with N representing the total number of observational data points and K the number of free model parameters. This metric quantifies the goodness-of-fit per degree of freedom: values close to unity indicate an appropriate balance between model fidelity and complexity, whereas values significantly below or above 1 may signal over fitting or under fitting, respectively. In our analysis, the combined dataset comprises $N = 1758$ measurements—specifically, 1701 from the Pantheon+ supernova sample, 31 from CC, and 26 from BAO surveys. The baseline Λ CDM model has $K = 5$ free parameters, while Model 1 and Model 2 possess $K = 10$ and $K = 11$, respectively. As reported in Table 6.4, the resulting χ_{red}^2 values for all three scenarios remain close to unity, with $\chi_{\text{red}}^2 = 1.049$ for Model 1 and $\chi_{\text{red}}^2 = 1.050$ for Model 2, confirming that each model provides a good fit to the observational data.

Beyond reduced chi-square, we adopt information-theoretic criteria: AIC and BIC, which is rooted in statistical inference to account for model complexity. As summarized in Table 6.4, Model 1 yields $\Delta\text{AIC} \approx 60.93$, and Model 2 gives $\Delta\text{AIC} \approx 63.17$, both indicating very strong evidence against the models according to AIC criteria. For the Bayesian Information Criterion—which imposes a stronger penalty on additional parameters, especially for large N the same thresholds apply, but the evidence is interpreted more stringently; both models exhibit $\Delta\text{BIC} > 75$, constituting decisive evidence against the models. Thus, while both extended models fit the data well, the principle of parsimony, as encoded in information criteria, ultimately favors the standard cosmological model.

6.5.9 p-value statistics

We employ the *p-value* which is used for statistical inference [263, 264] to evaluate the consistency between our cosmological models and the observational data under the assumption that the model in question is correct (i.e., the null hypothesis). Mathematically, the *p-value* is computed as

$$p = 1 - F_{\chi_{\text{tot,min}}^2}(\chi \mid \nu), \quad (6.34)$$

where $F_{\chi^2_{\text{tot,min}}}(\chi | \nu)$ denotes the cumulative distribution function (CDF) of the chi-squared distribution with ν degrees of freedom, evaluated at the observed minimum chi-square statistic $\chi = \chi^2_{\text{tot,min}}$. This CDF quantifies the probability that a chi-squared random variable with ν degrees of freedom yields a value less than or equal to χ . Consequently, the complement $1 - F$ represents the probability of obtaining a fit as poor as-or poorer than-the one observed, assuming the model is true. In other words, the *p-value* measures how compatible the data are with the null hypothesis: smaller values signal greater tension. By conventional standards in astrophysics and cosmology, a *p-value* below 0.05 is deemed statistically significant, implying that such a discrepancy would arise by chance less than 5% of the time, thereby warranting rejection of the model. However, it is crucial to recognize that the *p-value* does not quantify the probability that the null hypothesis itself is true; rather, it reflects the likelihood of the data (or more extreme outcomes) given that the hypothesis holds [264]. In our analysis, the computed *p-values* for the Λ CDM model and both Chaplygin gas scenarios (Model 1 and Model 2) are $p \approx 0.277, 0.074$, and 0.071 , respectively. This outcome implies that while the Λ CDM model is statistically preferred, both extended models remain within acceptable statistical bounds. This outcome implies that none of the models can be ruled out on the basis of goodness-of-fit alone, and all three provide statistically acceptable descriptions of the combined CC + SNe Ia + BAO dataset. While the *p-value* does not adjudicate between competing viable models, it confirms that each framework is empirically plausible within the current observational precision.

6.5.10 Bayesian Inference

We outline the core principles of Bayesian model comparison and apply them to assess the relative plausibility of our two Chaplygin gas-based scenarios against the standard Λ CDM cosmology. Central to this approach is the concept of Bayesian evidence, which evaluates how well a given model accounts for the observed data by integrating the likelihood function over the full prior parameter space [265]. This marginalization inherently penalizes unnecessary complexity, thereby implementing a natural form of Occam's razor.

To quantitatively compare Model 1 and Model 2 with the reference Λ CDM framework (denoted as model M), we employ the Bayes factor B_{iM} , defined as the ratio of their respective marginal likelihoods:

$$B_{iM} = \frac{p(d | M_i)}{p(d | M)},$$

where $p(d | M_i)$ represents the Bayesian evidence for model M_i ($i = 1, 2$), and $p(d | M)$ is that of the Λ CDM model. These evidences-reported in logarithmic form as $\ln E_i$ for the extended

models and $\ln E$ for Λ CDM-are listed in Table 6.4. For interpretability, we work with the natural logarithm of the Bayes factor:

$$\ln(B_{iM}) = \ln p(d | M_i) - \ln p(d | M),$$

which directly measures the log-odds in favor of model M_i relative to the baseline.

Using the `PolyChord` nested sampling algorithm [266], we compute $\ln E$ numerically for each cosmological scenario. The resulting values of $\ln B_{iM}$ are compiled in Table 6.4. Following the widely adopted Jeffreys scale [87, 89], we interpret these log-Bayes factors. Our analysis yields $\ln B_1 = 48.40$ and $\ln B_2 = 48.16$, both of which fall far beyond the threshold for decisive evidence. This striking result suggests that, within the Bayesian framework, both Chaplygin gas models are overwhelmingly preferred over the standard Λ CDM paradigm when evaluated against the combined CC + SNe Ia + DESI DR2 BAO dataset-highlighting the significant explanatory power gained by incorporating matter creation and a variable generalized Chaplygin gas fluid in the context of $f(Q)$ gravity.

6.6 Conclusion

In this chapter, a unified cosmological framework based on variable generalized Chaplygin gas, adiabatic matter creation, and $f(Q)$ gravity has been investigated. Two specific functional forms of the gravitational Lagrangian, $f(Q) = n_1 Q^k$ and $f(Q) = n_0 + n_1 Q^k$, were considered in order to determine whether the combined effect of non-metric gravity and matter creation can provide a viable description of the late-time accelerated expansion of the Universe.

The analysis was carried out in two stages. First, the analytical background solutions were derived within a spatially flat FLRW space-time. The corresponding expressions for the matter density, Chaplygin gas density, and Hubble parameter were obtained for the two $f(Q)$ models. Second, the free parameters were constrained through a joint observational analysis using the Pantheon+ supernova compilation, cosmic chronometers, and BAO measurements from DESI DR2 and SDSS-IV.

The reconstructed cosmological quantities show that both extended models are capable of describing the transition from a decelerated matter-dominated phase to the present accelerated epoch. Their best-fit Hubble constants remain close to the Planck 2018 value, and the present-day deceleration parameters are consistent with a currently accelerating Universe. The effective equation-of-state parameter stays in the quintessence regime, while the jerk and snap

diagnostics indicate a non-trivial but controlled departure from the Λ CDM cosmography. The $Om(z)$ diagnostic further confirms that the two models behave as dynamical dark energy scenarios rather than as a strict cosmological constant.

The age estimates obtained for Model 1 and Model 2 are close to the standard cosmological value, which shows that the proposed frameworks remain compatible with the overall cosmic chronology. At the statistical level, the reduced chi-square values confirm that both models provide acceptable fits to the available data. The information criteria, however, strongly penalize them because of their larger number of free parameters and therefore continue to prefer the simpler Λ CDM model. By contrast, the Bayesian evidence reported in the source article favors the two extended models, illustrating that different model selection tools may lead to different assessments of the same observational fit.

Overall, the present analysis shows that variable generalized Chaplygin gas with matter creation in $f(Q)$ gravity provides a viable alternative framework for late-time cosmology. The two proposed models reproduce the main observational features of the recent expansion history, remain close to Λ CDM at the background level, and offer a physically motivated setting in which modified gravity, non-equilibrium thermodynamics, and variable Chaplygin-type dynamics are combined in a single cosmological description.

Chapter 7

Conclusion, Future Scope, and Social Impact

In this chapter, the overall findings of the thesis are brought together and interpreted in a unified manner. The main scientific conclusions are summarized, the broader significance of the work is discussed, and possible directions for future research are outlined. The chapter also reflects on the wider academic and social relevance of the present study in the context of modern cosmology.

7.1 Conclusion

The present thesis has been devoted to the study of late-time cosmic acceleration through a class of cosmological models built from Chaplygin-type fluids, adiabatic matter creation, bulk viscous effects, and modified gravity. The central motivation of this work has been the recognition that, although the standard Λ CDM model remains highly successful at the observational level, it continues to face important conceptual difficulties, including the cosmological constant problem, the coincidence problem, and the tension between local and global estimates of the Hubble constant. In this situation, it is meaningful to explore whether physically richer alternatives can reproduce the observed expansion history while offering a different theoretical interpretation of the dark sector.

A unifying theme of the thesis has been the role of effective negative pressure in cosmic evolution. In the models studied here, such negative pressure arises not from a single source, but from the combined action of exotic equations of state, irreversible particle production, dissipative bulk viscous stresses, and modifications in the gravitational sector. This broader

viewpoint has allowed the thesis to go beyond a simple one-model analysis and instead develop a coherent research program in which several related cosmological scenarios are investigated systematically within a common observational and thermodynamic framework.

The first stage of the thesis examined generalized Chaplygin gas cosmology in the presence of adiabatic matter creation within a flat FLRW background. In that framework, exact analytical expressions were obtained for the background quantities, and the observational analysis showed that the inclusion of matter creation improves the viability of the Chaplygin-type fluid description. The resulting cosmological evolution reproduces the transition from an early decelerating epoch to a late accelerating phase, while remaining close to the background behavior of the concordance model. The corresponding stability analysis further showed that the physically relevant branch of the model satisfies the expected sound-speed bounds and therefore remains acceptable at the level of background dynamics.

The second stage extended this analysis to the variable generalized Chaplygin gas and variable Chaplygin gas models in matter creation cosmology. By introducing scale-factor dependence into the Chaplygin parameter, the dynamical behavior of the fluid becomes more flexible and allows a broader range of late-time phenomenology. The observational study of this extended framework showed that the reconstructed Hubble parameter, deceleration parameter, equation-of-state parameter, and jerk parameter remain close to the standard cosmological behavior, while the fitted values of the Hubble constant indicate that these models can ease the tension with local measurements more effectively than the corresponding Λ CDM fit. The statistical analysis also showed that, although the Bayesian information criterion penalizes the additional parameters, the models remain meaningful and competitive alternatives at the background level.

The third major step of the thesis introduced bulk viscous dissipation together with matter creation in a Modified Chaplygin Gas framework. This hybrid construction was especially important because it brought together three non-equilibrium ingredients in a single cosmological model: an exotic fluid equation of state, gravitationally induced matter creation, and dissipative viscous pressure. The analysis showed that this combined framework is capable of reproducing the observed late-time expansion history and provides a controlled departure from Λ CDM in higher-order cosmographic quantities such as the jerk and snap parameters. An important contribution of this part of the thesis is the thermodynamic analysis, which demonstrated that the generalized second law is satisfied and that the entropy evolution is consistent with a late-time approach toward equilibrium. This result strengthens the physical credibility of the model by showing that it is not only observationally acceptable but also thermodynam-

ically well behaved.

The later stages of the thesis moved beyond General Relativity to the modified gravity framework of $f(Q)$ cosmology. In one direction, generalized Chaplygin gas with matter creation was studied in non-minimally coupled $f(Q)$ gravity. In another, variable generalized Chaplygin gas with matter creation was investigated for two specific choices of the gravitational Lagrangian. These analyses demonstrated that the combination of non-metric gravity and Chaplygin-type dynamics leads to observationally viable late-time cosmologies. In these models, the reconstructed cosmological quantities remain close to the standard picture, but the underlying interpretation differs substantially: the observed acceleration is produced by the interplay of modified gravity, non-standard fluid behavior, and matter creation rather than by a rigid cosmological constant. Even when information criteria continue to prefer the simpler reference cosmology, the extended models remain acceptable from the point of view of goodness of fit and therefore continue to deserve attention as physically motivated alternatives.

Taken together, the results of Chapters 2–6 show that the thesis objectives have been achieved in a consistent way. The work has constructed a sequence of homogeneous and isotropic cosmological models based on Chaplygin-type fluids; it has incorporated matter creation through a thermodynamically motivated open-system formalism; it has examined the role of bulk viscosity as an additional source of effective negative pressure; and it has extended the entire discussion from General Relativity to $f(Q)$ gravity. Across these models, analytical or semi-analytical expressions were obtained for the main background observables, and the models were tested against current late-time data using modern statistical tools. In this sense, the thesis provides a coherent framework in which exotic fluids, particle creation, dissipation, and modified gravity are studied not in isolation, but as interacting mechanisms that may jointly contribute to the observed acceleration of the Universe.

At the observational level, a common pattern emerges from the thesis. The proposed models are generally able to reproduce the transition from decelerated to accelerated expansion, yield present-day cosmological parameters compatible with current observations, and remain close to Λ CDM in the redshift range directly probed by late-time data. In several cases, the inclusion of matter creation or additional non-equilibrium effects improves the behavior of the fitted Hubble constant and reduces the level of tension with local measurements. At the same time, the analysis also makes clear that an improved phenomenological fit does not automatically imply global superiority over the standard cosmological model, because more complex scenarios are naturally penalized by model-selection criteria that account for parameter number.

At the physical level, the thesis shows that cosmologies based on matter creation and Chaplygin-

type fluids can be formulated in a stable and thermodynamically meaningful way. The repeated appearance of acceptable sound-speed behavior, viable background evolution, and consistency with the generalized second law indicates that these models are not merely formal alternatives but physically structured frameworks worthy of further investigation. The thesis therefore supports the broader conclusion that late-time cosmic acceleration may admit explanations based on the combined dynamics of cosmic fluids, open-system thermodynamics, dissipative processes, and modified gravity.

The overall conclusion of the present work is that Chaplygin-type cosmologies with matter creation, with or without bulk viscosity and with or without $f(Q)$ modifications, provide a viable and conceptually rich class of alternatives to the standard Λ CDM scenario. They do not replace the concordance model at present, but they significantly enlarge the space of physically acceptable late-time cosmologies and offer new ways of understanding the origin of accelerated expansion. The thesis thus contributes to the continuing effort to build cosmological models that are simultaneously observationally viable, mathematically consistent, and physically better motivated than a simple constant vacuum energy term.

7.2 Future Scope

Although the present thesis has established the late-time viability of several Chaplygin-type models with matter creation, bulk viscosity, and modified gravity, a number of important questions remain open. These questions provide natural directions for future research and can substantially deepen the physical interpretation of the models studied here.

A first and especially important extension concerns the perturbation sector. Most of the analyses in the thesis have been performed at the level of homogeneous background dynamics. While such an approach is sufficient for reconstructing the expansion history, it does not fully determine whether the models remain viable when the growth of cosmic structure is treated self-consistently. This issue is particularly relevant for models involving matter creation and bulk viscosity, because the associated effective pressures can modify the evolution of density perturbations in a non-trivial manner. A dedicated derivation of the linear perturbation equations for these non-equilibrium models is therefore a necessary next step. Such an analysis would make it possible to study the growth factor, the matter power spectrum, and the observable $f\sigma_8(z)$ in a way that is fully consistent with the underlying effective stress-energy tensor.

A second important direction is the inclusion of early-Universe probes. The thesis has

mainly focused on late-time observational datasets, including Type Ia supernovae, cosmic chronometers, baryon acoustic oscillations, standard candles, growth data where applicable, and local H_0 priors. This is appropriate for investigating the recent expansion history, but it does not by itself guarantee consistency across all cosmological epochs. Future work should therefore extend the analysis to primordial nucleosynthesis, recombination, and cosmic microwave background anisotropies. In practical terms, this would require embedding the relevant models into a Boltzmann solver so that predictions for temperature and polarization spectra, lensing observables, and sound-horizon quantities can be computed directly and confronted with precision data.

A third direction concerns the matter creation mechanism itself. In the present thesis, the phenomenological choice $\Gamma = 3\beta H$ has been used in several chapters because it provides a simple and physically motivated description of adiabatic particle production. However, the microscopic origin of the creation rate remains unknown. It would therefore be valuable to investigate more general creation laws, including redshift-dependent or dynamically evolving forms of Γ , and to examine whether such extensions lead to improved observational behavior or deeper links with quantum field processes in curved space-time. A comparison between different phenomenological creation laws may also help clarify which background signatures are generic consequences of matter production and which are specific to a given ansatz.

A fourth direction involves a more refined treatment of dissipative processes. In the bulk viscous chapter, the analysis was carried out with a phenomenological viscous pressure that is adequate at the FLRW background level. Nevertheless, a more complete description of irreversible thermodynamics would require causal transport equations of the Israel–Stewart type. Extending the present hybrid models to such a framework would make it possible to examine relaxation times, causal propagation, and the stability of non-equilibrium modes more rigorously. This would also improve the physical interpretation of the thermodynamic results obtained in the thesis.

A fifth promising line of work is the exploration of more general modified gravity sectors. The thesis has shown that $f(Q)$ gravity provides a useful non-Riemannian setting in which Chaplygin-type fluids and matter creation can be studied consistently. Future investigations may consider broader classes of non-metric gravity models, different functional forms of $f(Q)$, and possible couplings with additional geometrical invariants. Such studies would clarify which observational features arise specifically from the chosen $f(Q)$ ansatz and which belong more generally to the non-metric gravity framework. It would also be worthwhile to investigate whether some of these modified gravity extensions can simultaneously address the H_0 and σ_8

tensions while preserving good late-time background behavior.

Another useful extension would be a deeper study of model diagnostics and parameter degeneracies. The thesis has already employed background and cosmographic quantities such as $H(z)$, $q(z)$, $w(z)$, jerk, snap, and in some cases the $Om(z)$ diagnostic. Future work may develop statefinder analyses, phase-space studies, and machine-assisted parameter inference in order to distinguish more sharply between competing models that remain very close to one another at the background level. This is especially important because many of the scenarios studied in the thesis are observationally near-degenerate with Λ CDM over the currently accessible redshift range.

It would also be valuable to investigate the astrophysical implications of matter creation and Chaplygin-type models beyond background cosmology. Possible applications include cosmic age estimates, horizon thermodynamics, late-time fate of the Universe, and the behavior of dark-sector effective fluids in anisotropic or inhomogeneous settings. Extending the present work to non-flat backgrounds, Bianchi space-times, or interacting multi-fluid systems may reveal further structure in the relation between irreversible thermodynamics and cosmic expansion.

Finally, future observational advances are expected to play a decisive role. Data from ongoing and forthcoming surveys such as DESI, Euclid, Rubin Observatory, Roman Space Telescope, and next-generation CMB experiments will provide significantly tighter constraints on the expansion history and structure growth. These data will allow more stringent tests of the models discussed in this thesis. In particular, they may determine whether the mild departures from Λ CDM found here represent real signatures of non-standard physics or whether they are effectively absorbed into the simpler concordance description.

In summary, the future scope of the present thesis is broad and well defined. The most immediate next steps are a self-consistent perturbation analysis, the incorporation of early-Universe observables, a more detailed treatment of dissipative thermodynamics, and the exploration of wider classes of modified gravity and matter creation laws. Each of these directions can strengthen the physical foundations of the models studied here and help determine whether they remain viable alternatives to the standard cosmological picture at all relevant scales and epochs.

7.3 Social Impact

The direct subject of the present thesis is theoretical cosmology, and for that reason its social impact is not immediate in the same way as applied engineering, medical science, or policy research. Nevertheless, work of this kind has an important broader value, both within the scientific community and in society at large.

At the most fundamental level, cosmology contributes to humanity's attempt to understand the origin, structure, and evolution of the Universe. Questions about why the Universe expands, how cosmic acceleration arises, and what physical mechanisms govern the large-scale behavior of space-time are among the deepest questions of science. Research that addresses these questions enriches collective human knowledge and supports a culture in which rational inquiry, mathematical reasoning, and evidence-based explanation are valued. In this sense, the social contribution of cosmology lies partly in the strengthening of scientific temper and intellectual curiosity.

A second important contribution of the present thesis is methodological. The work combines differential geometry, relativistic cosmology, thermodynamics, statistical inference, and scientific computing. Such training has value beyond the narrow subject of the thesis because it develops transferable skills in mathematical modeling, data analysis, numerical computation, parameter estimation, and critical interpretation of complex datasets. These skills are relevant in many scientific and technological domains, including data science, computational physics, quantitative modeling, and interdisciplinary research environments.

The thesis also contributes to the academic ecosystem by advancing research capacity in frontier areas of gravitational physics and cosmology. Theoretical investigations of alternatives to Λ CDM help refine the conceptual foundations of modern cosmology and provide testable hypotheses for future observations. Even when a proposed model does not ultimately replace the standard paradigm, the process of formulating, analyzing, and constraining it improves scientific understanding by clarifying which mechanisms are viable and which are not. This is an essential part of healthy scientific progress.

The social impact of the thesis may also be understood in relation to education. Cosmology has a unique ability to engage students and the wider public because it connects advanced mathematics and physics with profound questions about the Universe. Research in this field can therefore support science communication, inspire young researchers, and encourage participation in higher education in physics, mathematics, and astronomy. By presenting late-time

cosmology through physically interpretable mechanisms such as matter creation, dissipation, and modified gravity, the thesis also offers conceptual pathways that may help students appreciate the living and evolving nature of scientific theory.

Another broader contribution lies in the culture of careful model testing. One of the recurring lessons of the thesis is that theoretical elegance alone is not sufficient; every model must also be confronted with observational data, examined for stability, and tested for thermodynamic consistency. This attitude has social value because it reinforces an evidence-based scientific ethic. It shows that alternative ideas are welcome in science, but they must be evaluated rigorously and transparently against empirical information.

It is also worth noting that large-scale cosmological research is closely connected to technological and computational progress. Modern cosmology depends on high-quality observations, advanced detectors, large surveys, statistical algorithms, and numerical pipelines. While the present thesis is theoretical, it participates in that broader scientific framework by using contemporary datasets and modern inferential techniques. In the long term, such interactions between theory, observation, and computation contribute to the growth of research infrastructure, software development, and quantitative expertise.

At the same time, the social impact of this work should not be overstated. The models studied here do not lead directly to immediate technological products or short-term public policy outcomes. Their value is primarily foundational, educational, and methodological. The significance of the thesis lies in its contribution to basic science, to the training of researchers, and to the sustained effort to understand nature at its largest scales.

In summary, the social impact of the present thesis is expressed through the promotion of scientific understanding, the development of advanced analytical and computational skills, the strengthening of the academic research culture, and the encouragement of evidence-based reasoning. By investigating alternatives to the standard cosmological model in a rigorous and systematic manner, the thesis contributes to the broader intellectual project of modern science and to the continued growth of cosmology as a discipline.

7.4 Connection with Sustainable Development Goals (SDGs)

Since our work is based on dark energy and dark matter, which are two major mysterious components of the Universe, it is related to the formation and evolution of cosmic structures. Dark matter is associated with the formation of the structure of the Universe, while dark energy is considered a mysterious force driving the accelerated expansion of the Universe. Since

dark-energy research involves major observatories and extensive data simulations, this subject is related to the following SDGs:

1. Quality Education (SDG-4): The researchers require the high quality of science education which can explain the mystery behind the Universe's evolution.
2. Industry, Innovation and Infrastructure (SDG-9): The quest to understand the dark Universe drives fundamental scientific innovation.

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List of Publications

1. Y. Bhardwaj and C. P. Singh, *Matter creation cosmology with generalized Chaplygin gas*, **Astrophysics and Space Science** **369**, 2 (2024). (SCIE, IF: 1.5)
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Papers Presented in International Conferences

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Matter creation cosmology with generalized Chaplygin gas

Yogesh Bhardwaj¹ · C.P. Singh¹

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Abstract

In this work, we discuss the dynamics of a spatially homogeneous and isotropic flat Friedmann-Lemaître-Robertson-Walker (FLRW) model of the Universe powered by the gravitationally induced ‘adiabatic’ matter creation with generalized Chaplygin gas (GCG) equation of state. In recent years, the CGC has been proposed successfully to unify the dark matter and dark energy by using an exotic equation of state $p = -A/\rho^\alpha$. Considering adiabatic matter creation cosmology, as developed by Prigogine et al, we examine the effect of matter creation in describing the unification with GCG by assuming the natural phenomenological matter creation rate $\Gamma = 3\beta H$, where β is a constant and H is the Hubble parameter. We constrain the proposed model using a joint observational dataset of Type Ia supernovae Pantheon, $H(z)$ and BAO with Markov-Chain Monte-Carlo (MCMC) method. In addition, we discuss the selection criterion (AIC and BIC) and stability criteria to analyze the dynamics and differences with the standard Λ CDM model.

Keywords FLRW model · Matter creation cosmology · Chaplygin gas model · Dark energy

1 Introduction

The cosmological observations from Type Ia supernovae (Riess et al. 1998, 1999; Perlmutter et al. 1999), baryonic acoustic oscillations (Weinberg et al. 2013) and cosmic microwave background anisotropies (Bennett et al. 2013; Ade et al. 2014) provide strong evidence for an accelerated expansion of the Universe. In relativistic cosmology, such a phenomenon is usually explained by the existence of some unknown component(s), named as dark energy (DE), which contributes approximately 68% of the Universe and has a negative pressure. Many theoretical ideas have been suggested to explore the nature and origin of the DE, such as the cosmological constant (Weinberg 1989; Padmanabhan 2003; Peebles and Ratra 2003), quintessence (Caldwell et al. 1998), Phantom (Caldwell et al. 2003) and quintom (Guo et al. 2005; Copeland et al. 2006). The most obvious candidate is the cosmological constant, Λ which is associated

with the vacuum energy in empty space and has negative pressure, $p = -c^2\rho$.

The Lambda-cold dark matter (abbreviated as Λ CDM) model is the simplest model which provides a reasonably good explanation of cosmological phenomena like the formation of large-scale structures and accelerating expansion of the Universe. However, the Λ CDM model has two major problems: cosmological constant problem and cosmic coincidence problem. To alleviate these problems, modified theories of gravity, namely, $f(R)$ gravity (Felice and Tsujikawa 2010), $f(G)$ or $f(R, G)$ gravity (Nojiri and Odintsov 2005, 2007), $f(R, T)$ gravity (Harko et al. 2011), $f(Q)$ gravity (Jimenez et al. 2018), $f(Q, T)$ gravity (Xu et al. 2019) have also been proposed.

In recent years, another interesting proposal has been presented by a change in the equation of state of the background cosmic fluid, so-called exotic equation of state which has negative pressure. One such example is the Chaplygin gas (CG) whose equation of state is given by $p_{cg} = -A/\rho_{cg}$, and was originally suggested by Kamenshchik et al. (2001). It is worthy to mention that the initial idea of CG was originated in aerodynamics by Chaplygin (1904) to describe the lifting force on the wing of aeroplane. Nowadays, it is used as an exotic equation of state in perfect fluid to describe the accelerating expansion of the Universe. The CG has been extended to other more flexible models keeping the view of observations. The CG model is a viable alternative to

✉ C.P. Singh
cpsingh@dce.ac.in

Y. Bhardwaj
yogeshbhardwaj1597@gmail.com

¹ Department of Applied Mathematics, Delhi Technological University, Bawana Road, Delhi-110 042, India

quintessence (Kamenshchik et al. 2001), which gives a unified picture of DE and DM. It behaves like a pressure-less DM in the early Universe and like the cosmological constant in late-time. However, it is ruled out in cosmology due to non-consistency with the observational data namely, supernova, baryonic acoustic oscillations and cosmic microwave background (Bento et al. 2003a).

To resolve this problem, Bento et al. (2002) and Bilić et al. (2002) have proposed the generalized form of CG, so-called the “generalized Chaplygin gas (GCG)”, which smoothly interpolates between dust matter and DE dominated era. The equation of state for GCG is given by $p_{gcg} = -A/\rho_{gcg}^\alpha$, where A and $0 < \alpha \leq 1$ are constants to be determined. The important feature of GCG model is that it unifies the dark matter and dark energy components. Thus, keeping in view of its potentiality, the GCG model has been the subject of great interest to study the evolution of early and late-time Universe. Many authors (Makler et al. 2003a,b; Silva and Bertolami 2003; Bento et al. 2003b; Chen 2003; Amendola et al. 2003; Alcaniz et al. 2003) have constrained the GCG model using various observational data to present the unified model of the Universe. Choudhury and Padmanabhan (2005) analyzed the GCG model with 194 data points of supernova and found that the GCG model provides relevant constraints on some cosmological parameters. The results show that the model rules out the decelerating model by considering the full supernova data set with significant confidence level.

However, besides these distinct approaches, very recently, matter creation cosmology has been studied to explain the current cosmic acceleration of the Universe. The microscopic description of this process was pioneered by Schrödinger (1939), and developed by (Parker 1968, 1969) based on the quantum field theory (QFT) in curved spacetimes. Later on, Prigogine et al. (1989) studied the macroscopic description of particle production mechanism based on the non-equilibrium thermodynamics of open systems and formulated the background equations how to insert this creation mechanism into the Einstein field equation to describe the evolution of the Universe.

Calvão et al. (1992) studied this macroscopic approach by formulating a covariant approach allowing specific entropy variation as usually expected for non-equilibrium processes in fluids. There may be a question what kind of particles are created by the gravitational field. Since, at present-day Universe there is no effect of radiation on late-time accelerated expansion of the Universe and dark matter is one of the dominant sources after dark energy component. Therefore, it is assumed that the produced particles are simply the CDM particles. It has been observed that the model with matter creation can mimic Λ CDM cosmology. It has been pointed out that the matter creation can play the role of dark energy component and leads to drive the accelerating expansion of the Universe (Steigman et al. 2009; Lima et al.

2010; Basilakos and Lima 2010; Lima et al. 2012; Fabris et al. 2014; Lima et al. 2014; Chakraborty et al. 2014; Nunes and Pavón 2015; Harko et al. 2015; Haro and Pan 2016; Nunes 2016). Some more literature on matter creation may be found in Refs. (Pan et al. 2016; Salo and Haro 2017; Pan et al. 2018; Singh and Kaur 2019; Singh and Kumar 2020a,b; Peracaula and Yu 2020; Kaur and Singh 2021; Harko et al. 2021; Kaur and Singh 2022; Hulke et al. 2020; Singh et al. 2020; Singh 2020a), which explain the late-time evolution of the Universe. The matter creation cosmology is very interesting topic because many important observations are investigated during the past few years which successfully explains the current accelerated expansion of the Universe. The key point in such models is to consider the choices for the rate of particle production, which, in general, are assumed to be the functions of the Hubble parameter H .

In this paper, we present a hybrid model which consists of matter creation cosmology and the features of Chaplygin gas to discuss the late-time accelerated expansion of the Universe. A similar kind of model was discussed in Ref. (Gangopadhyay et al. 2016) in explaining the emergent Universe scenario. Firstly, we present the analytical solution of the field equation and then constrain the parameter space of hybrid model in the light of the latest supernova (Pantheon) data, Hubble data and baryon acoustic oscillations data by performing the Bayesian analysis. Our analysis shows that the flat GCG model with and without matter creation and CG with matter creation are consistent with the used data sets. We have also analyzed the selection information criterion (AIC and BIC) and the stability of sound speed to find the degeneracy between the GCG models and Λ CDM model.

The paper is organized as follows: Section 2 presents the basic field equations of a hybrid model consisting of matter creation cosmology and GCG equation of state and give the general expressions of different cosmological parameters. In Section 3, we present solution of field equations using GCG equation of state. Section 4 is devoted to a particular case of Section 3 for matter creation with CG equation of state. In Section 5, the observational data like, Hubble data, Type Ia supernova (Pantheon) and baryon Acoustic Oscillations are discussed. Section 6 gives the methodology to perform the observation. Section 7 contains the results and discussion of the finding through observation. Finally, the conclusion of the work is summarized in Section 8.

2 Basic field equations

We start with the line element which is homogeneous and isotropic at large scale and described by the flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = -dt^2 + a^2(t) \left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right], \quad (1)$$

where $a(t)$ is the scale factor of the Universe and (t, r, θ, ϕ) are the co-moving coordinates. Throughout we use the units such that the speed of light $c = 1$. For comoving observer, $u^\mu = \delta^\mu_\nu$, where $u^\mu u_\mu = -1$, and the line element (1), the expansion scalar becomes $\Theta = 3H$, where $H = \dot{a}/a$ is the Hubble parameter. A dot denotes derivative with respect to cosmic time t .

In matter creation cosmology as proposed in Ref. (Prigogine et al. 1989), it is considered the Universe as an open adiabatic system with N particles. It is assumed that the particle number is not conserved $N^\mu_{;\mu} \neq 0$, which provides a modification of the energy conservation equation. The particle flux vector is assumed to be $N^\mu = nu^\mu$, where $n = N/V$ is the particle number density, and u^μ is the four-velocity of the fluid. Now, the particle flux vector, N^μ satisfies the balance equation

$$\nabla_\mu N^\mu \equiv n_{;\mu} u^\mu + 3Hn = n\Gamma, \quad (2)$$

where $V = a^3$ is the comoving volume and Γ is a particle creation rate. Equation (2) shows that $\Gamma > 0$ gives the creation of particle while $\Gamma < 0$ for particle annihilation and $\Gamma = 0$ means there is no creation of matter, i.e., particle number is conserved.

The entropy flux vector is defined by $s^\mu = n\sigma u^\mu$, where σ is the entropy per particle. It is to be noted that the entropy must fulfill the second law of thermodynamics $s^\mu_{;\mu} \geq 0$. It is also to be noted that we consider adiabatic matter creation, i.e., the system in which σ is constant. Assuming N as variable, the first law of thermodynamics modifies as

$$d(\rho V) + p dV - (h/n)d(nV) = 0, \quad (3)$$

where $h = (\rho + p)$ is the enthalpy (per unit volume), ρ and p are the energy density and the equilibrium thermostatic pressure of the usual cosmic fluids (baryons and dark matter). The extra contribution in (3) can be interpreted as a non-thermal pressure, known as creation pressure which is related to Γ by

$$P_c = -\frac{h}{n} \frac{d(nV)}{dV}. \quad (4)$$

Using (2) into (4), we get the following form of creation pressure (Prigogine et al. 1989; Calvão et al. 1992)

$$P_c = -\frac{(\rho + p)}{3H} \Gamma. \quad (5)$$

It can be observed from Eq. (5) that the creation of matter results to a negative supplementary pressure P_c . This pressure must be considered as part of the cosmological pressure $\bar{p} = p + P_c$ which modified the energy-momentum tensor into Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}^{(eff)}, \quad (6)$$

where $\kappa = 8\pi G$ and $T_{\mu\nu}^{(eff)}$ is the total energy-momentum tensor $T_{\mu\nu}^{(eff)} = T_{\mu\nu}^{(g)} + T_{\mu\nu}^{(c)}$, where $T_{\mu\nu}^{(g)}$ is the energy-momentum tensor corresponding to the cosmic fluid and generalized Chaplygin gas, that is,

$$T_{\mu\nu}^{(g)} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}, \quad (7)$$

and $T_{\mu\nu}^{(c)}$ is the energy-momentum tensor corresponding to the matter creation which is given by

$$T_{\mu\nu}^{(c)} = P_c(g_{\mu\nu} + u_\mu u_\nu), \quad (8)$$

where P_c is the matter creation pressure given by Eq. (5). The Bianchi identities $\nabla^\nu T_{\mu\nu}^{(eff)} = 0$ gives

$$d(\rho V) + (p + P_c)dV = 0. \quad (9)$$

In this paper, we consider a flat Universe that contains the baryon component with equation of state $w_b = p_b/\rho_b = 0$ and a perfect fluid following the generalized Chaplygin gas (GCG) equation of state (EoS). Thus, we assume a perfect fluid structure for the cosmic medium as a whole, and both the baryonic matter and the GCG components satisfy energy conservation equations separately. From Eq. (6) for line element (1), the Friedmann and conservation equations are given by

$$3H^2 = \kappa\rho, \quad (10)$$

$$\dot{\rho}_b + 3H\rho_b = 0 \quad (11)$$

$$\dot{\rho}_{gcg} + 3H(\rho_{gcg} + p_{gcg} + P_c) = 0. \quad (12)$$

where $\rho = \rho_{gcg} + \rho_b$ is the total energy density and p_{gcg} being the pressure of the GCG assumed as perfect fluid. By integrating the conservation equation (11), we have $\rho_b = \rho_{b,0} a^{-3}$, where $\rho_{b,0}$ is the energy density at present.

The solution of the field equations are found by the choice for the particle creation rate Γ . The simplest possible case, and probably the most physically choice would be the function of Hubble parameter. Phenomenologically, this is equivalent to (Lima et al. 1996; Lima and Alcaniz 1999)

$$\Gamma = 3\beta H, \quad (13)$$

where β is a positive constant. Using (13) into (5), the creation pressure can be expressed as

$$P_c = -\beta(\rho_{gcg} + p_{gcg}). \quad (14)$$

Models with non-linear equations of state have raised a substantial interest recently, mainly due to the possibility of DE and DM unification (Bilić et al. 2002). This non-linear equation of state (EoS) leads to a positive (adiabatic) squared sound speed ($C_s^2 = \partial p_{cg}/\partial \rho_{cg}$), even for negative pressure.

An example of an equation of state exhibiting these properties is given by Chaplygin gas (CG) equation of state $p_{cg} = -A/\rho_{cg}$ (Kamenshchik et al. 2001), where A is a positive constant. The Chaplygin gas may be considered as viable alternative to quintessence. However, this was ruled out in cosmology as cosmological models were not consistent with the observational data. Subsequently, a more generalized model of the Chaplygin gas is characterized by an equation of state $p_{gcg} = -A/\rho_{gcg}^\alpha$ (Bilić et al. 2002) which is known as ‘generalized Chaplygin gas’ (GCG). The GCG model is consistent with a number of observational data involving background evolution of the Universe for $-1/2 \leq \alpha \leq 1/2$ (Makler et al. 2003a). It has been shown that this models can be accommodated within the standard structure formation scenarios (Bento et al. 2002; Bilić et al. 2002; Fabris et al. 2014). Therefore, Chaplygin gas models look to be good alternative to describe the accelerated expansion of the Universe. In what follows, we find the solutions of GCG and CG models with matter creation cosmology to observe the dynamics of the evolution of the Universe.

3 GCG model with matter creation

To solve the field equations, let us consider a generalized Chaplygin gas (GCG) whose equation of state (EoS) is given by (Bento et al. 2002; Bilić et al. 2002)

$$p_{gcg} = -\frac{A}{\rho_{gcg}^\alpha}, \quad (15)$$

where A and $0 < \alpha \leq 1$ are positive parameters to be determined. For the particular case $\alpha = 1$, the EoS is reduced to the so-called Chaplygin gas which was first studied by Kamenshchik et al. (2001).

Using (14) and (15) into the conservation equation (12), the energy density ρ_{gcg} as a function of the scale factor a is given as

$$\rho_{gcg}(a) = \left(A + C a^{-3(1-\beta)(1+\alpha)} \right)^{\frac{1}{1+\alpha}}, \quad (16)$$

where C is the constant of integration. Considering $\rho_{gcg} = \rho_{gcg,0}$ at $a = a_0 = 1$ at present, the above equation can be re-written as

$$\rho_{gcg} = \rho_{gcg,0} \left[A_s + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)} \right]^{\frac{1}{1+\alpha}}, \quad (17)$$

where $z = a^{-1} - 1$ is the redshift and $A_s = A/\rho_{gcg,0}^{(1+\alpha)} > 0$ is the scaled parameter. In the absence of matter creation ($\beta = 0$), Eq. (17) reduces to the result obtained in Ref. Bento et al. (2002). For $A_s = 0$, the GCG model behaves always as matter where it behaves like cosmological constant for $A_s =$

1. Thus, to describe the unified model, we have to assume the range for A_s as $0 < A_s < 1$.

From Friedman equation (10), we get the Hubble parameter as

$$H^2(z) = H_0^2 \left[\Omega_{gcg,0} (A_s + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)})^{\frac{1}{1+\alpha}} + \Omega_{b,0}(1+z)^3 \right] \quad (18)$$

where $H(z=0) = H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the present Hubble parameter, and $\Omega_{b,0} = \rho_{b,0}/(3H_0^2)$ and $\Omega_{gcg,0} = \rho_{gcg,0}/(3H_0^2)$ are the density parameters of the baryonic matter and Chaplygin gas components, respectively such that $\Omega_{b,0} + \Omega_{gcg,0} = 1$. The observed abundances of light elements together with primordial nucleosynthesis gives $\Omega_{b,0}h^2 = 0.0224 \pm 0.0001$ (Aghanim et al. 2020).

Let us explore the cosmological parameters, mainly the Hubble parameter, deceleration, jerk parameter and EoS parameter, to distinguish the effect of creation pressure included in the different Chaplygin gas models and also compare these models with Λ CDM.

In cosmology, the Hubble parameter $H(z)$, deceleration parameter $q(z)$ and jerk Parameter $j(z)$ are purely kinematical as they are independent of any gravity theories. The deceleration parameter measures the transition from deceleration to acceleration. In terms of redshift, it is defined as

$$q(z) = -1 + (1+z)E(z)^{-1} \frac{dE(z)}{dz}, \quad (19)$$

where $E(z) \equiv H(z)/H_0$ is the dimensionless Hubble parameter.

For this model the deceleration parameter can be written as

$$q(z) = -1 + \frac{3}{2E^2(z)} \left[\Omega_{b,0}(1+z)^3 + \frac{\Omega_{gcg,0}(1 - A_s)(1 - \beta)(1+z)^{3(1-\beta)(1+\alpha)}}{\{A_s + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)}\}^{\frac{\alpha}{1+\alpha}}} \right] \quad (20)$$

It is to be noted that the solutions (16), (17) and (18) reduce to GCG solutions obtained in Refs. (Makler et al. 2003a; Zhu 2004; Fabris et al. 2010) in the absence of matter creation, i.e., for $\beta = 0$.

The jerk parameter is another kinematics diagnostic parameter that provides us the simplest approach to observe the departure from the Λ CDM model. It is defined as a dimensionless third order derivative of the scale factor a with respect to the cosmic time t , i.e., $j = \ddot{a}/aH^3$ (Visser 2004; Blandford et al. 2004). For Λ CDM model in flat FLRW metric, jerk parameter is a constant with the value $j = 1$.

It should be noted here that Sahni et al. (2003) drew attention to the importance of j for discriminating different dark energy models. Any deviation from the value $j = 1$ would favor a non- Λ CDM model.

In terms of deceleration parameter q , the jerk parameter j can be written as (Rapetti et al. 2007)

$$j(z) = q(2q + 1) + (1 + z) \frac{dq}{dz}. \quad (21)$$

Equation (21) enables us to constrain the departure from the Λ CDM value in an effective manner.

The equation of state (EoS) parameter for the GCG fluid is

$$w_{gcg}(z) = -\frac{A_s}{A_s + (1 - A_s)(1 + z)^{3(1-\beta)(1+\alpha)}}. \quad (22)$$

The present-day value of EoS parameter, $w_{gcg,0}$ for GCG at $z = 0$ is obtained as

$$w_{gcg,0} = -A_s, \quad (23)$$

It can be observed that Eq. (23) gives the physical significant of A_s . The behavior of GCG depends on the values of A_s , α and β . For $A_s = 1$, we get a trivial solution $w_{gcg} = -1$ which shows that GCG model behaves exactly as Λ CDM at all times for all values of α and β . For $A_s = 0$, we have $w_{gcg} = 0$ and hence GCG model behaves as pressureless dust fluid all the times for all values of α and β .

4 Particular case: CG model with matter creation

In this section, we consider a particular case for $\alpha = 1$ in Eq. (15), i.e., equation of state $p_{cg} = -A/\rho_{cg}$ (Kamenshchik et al. 2001), the so-called Chaplygin gas with matter creation. Using the continuity equation (12), the energy density is given by

$$\rho_{cg} = \rho_{cg,0} \sqrt{[A_s + (1 - A_s)(1 + z)^{6(1-\beta)}]}, \quad (24)$$

where $A_s = A/\rho_{cg,0}^2$ is the scaled parameter. The corresponding Hubble function is given by

$$E^2(z) = \left[\Omega_{cg,0} \left\{ A_s + (1 - A_s)(1 + z)^{6(1-\beta)} \right\} + \Omega_{b,0}(1 + z)^3 \right]. \quad (25)$$

Using (25) into (19), we get

$$q(z) = -1 + \frac{3}{2E^2(z)} \left[\Omega_{b,0}(1 + z)^3 \right.$$

$$\left. + \frac{\Omega_{cg,0}(1 - A_s)(1 - \beta)(1 + z)^{6(1-\beta)}}{\sqrt{\{A_s + (1 - A_s)(1 + z)^{6(1-\beta)}\}}} \right] \quad (26)$$

We can recover the solutions obtained in Ref. (Kamenshchik et al. 2001) in the absence of matter creation, i.e., for $\beta = 0$.

The equation of state (EoS) parameter for the CG fluid is

$$w_{cg}(z) = -\frac{A_s}{A_s + (1 - A_s)(1 + z)^{6(1-\beta)}}. \quad (27)$$

5 Observational data

In this section, we perform the global constraints on our two models: GCG and CG with matter creation along with Λ CDM by using the recent and relevant observational data, which are exhibited as follows:

5.1 Hubble data

The Hubble data $H(z)$ can be classify into two category: Baryonic Acoustic Oscillations (BAO) surveys and cosmic chronometer (CC) data. In general, $H(z)$ data obtained from BAO survey data is model-dependent which can not be utilized to constraint a cosmological model. On the other hand, the cosmic chronometer (CC) data determined by using the most massive and passively evolving galaxies based on the “galaxy differential age” method are model-independent (Moresco et al. 2016; Wang and Meng 2017). Therefore, we use 32 CC data points in our analysis covering the range $0.07 \leq z \leq 1.965$ (Moresco et al. 2022).

The χ^2 function is defined

$$\chi_{H(z)}^2 = \sum_{i=1}^{32} \frac{[H_{obs}(z_i) - H(\mathbf{p}, z_i)]^2}{\sigma(z_i)^2}, \quad (28)$$

where $H(\mathbf{p}, z_i)$ denotes the theoretical value of $H(z)$ at redshift z_i for a given set of cosmological parameters $\mathbf{p} = (H_0, \Omega_{b,0}, A_s, \beta, \alpha)$, $H_{obs}(z_i)$ denotes the observed values of $H(z)$ at redshift z_i , and $\sigma(z_i)$ is the uncertainty in the measurement of H_i .

5.2 Type Ia supernovae

We use the Pantheon sample, a recent Type Ia supernovae (SNe) dataset with 1048 points of distance moduli at various redshift in the range $0.01 \leq z \leq 2.26$. Pantheon sample is collected from the DE Survey and the Supernova Legacy Survey and has been extensively calibrated to eliminate systematic errors and increase the accuracy of distance estimations (Scolnic et al. 2018; Chang et al. 2019).

We employ the χ^2 statistics,

$$\chi_{SNe}^2 = \sum_{i,j=1}^{1048} \Delta\mu_i \left(C_{SNe}^{-1} \right)_{ij} \Delta\mu_j, \quad (29)$$

Table 1 Clustering measurements for the BAO samples (Alam et al. 2021)

Parameter	Redshift range	z_{eff}	$D_H(z)/r_d$
Boss galaxy	$0.2 < z < 0.5$	0.38	25.00 ± 0.76
Boss galaxy	$0.4 < z < 0.6$	0.51	22.33 ± 0.58
eBoss LRG	$0.6 < z < 1.0$	0.70	19.33 ± 0.53
eBoss quasars	$0.8 < z < 2.2$	1.48	13.26 ± 0.55
$Ly\alpha - Ly\alpha$	$z > 2.1$	2.33	8.93 ± 0.28
$Ly\alpha - Quasars$	$z > 1.77$	2.33	9.08 ± 0.34

where $\Delta\mu_i = \mu_{th} - \mu_{obs}$ represents the difference between the theoretical and observational distance modulus, and C_{SNe}^{-1} denotes the data's inverse covariance matrix. In addition, we define $\mu = m_B - \mathcal{M}_B$, where m_B and $\mathcal{M}_B$ represent the measured apparent magnitude and absolute magnitude at a certain redshift, respectively. The theoretical value is calculated as,

$$\mu_{th}(z) = 5 \log_{10} \frac{d_L(z)}{1 \text{ Mpc}} + 25, \quad (30)$$

where $d_L(z)$ is the luminosity distance and is defined as

$$d_L(z) = (1+z) \int_0^z \frac{dz'}{H(z', \mathbf{p})}. \quad (31)$$

5.3 Baryon acoustic oscillations (BAOs)

Baryon Acoustic Oscillations (BAO) are also excellent geometrical probes and largely unaffected by errors in the non linear evolution of the matter density field and other systematic uncertainties. It is a feature of the large-scale structure of the Universe that emerge from primordial density perturbations in the baryon-photon plasma in the early Universe. In this paper, we use BAO data from several surveys, including Lyman- α ($Ly\alpha$) and quasars. These experiments include the samples from SDSS, BOSS and eBOSS. To break the parameter degeneracies from other measurements, we employ six points of BAO datasets as shown in Table 1 (for detail, refer to Table 3 of Ref. Alam et al. 2021).

In a spectroscopic survey, the BAO feature along the line-of-sight direction measures the Hubble distance at redshift z ,

$$D_H(z) = \frac{c}{H(z)}. \quad (32)$$

When considering the dependence of r_d , the BAO measurements constrain the quantities $D_H(z)/r_d$. The cosmological parameters used to calibrate the characteristic BAO scale r_d are typically derived from CMB observations. This BAO feature has a comoving scale set by the distance r_d traveled by sound waves between the end of inflation and decoupling of baryons from photons after recombination. The χ^2 func-

tion for BAO data set is defined as

$$\chi_{BAO}^2 = \sum_{i=1}^6 \left(\frac{[d_{obs}(z_i) - d_{th}(z_i)]}{\sigma(d_{obs}(z_i))} \right)^2, \quad (33)$$

where d denotes for $D_H(z)/r_d$ and d_{obs} is the observed distance value as given in Table 1 and d_{th} is the theoretical value calculated for the models under consideration.

6 Methodology

Using the above dataset, we proceed to analyze whether the two extensions of Chaplygin gas being considered in this work, namely, the GCG with matter creation and CG with matter creation models capable to fit the observations in a comparable way to Λ CDM model. In this respect, we employ the total likelihood function to establish the joint constraints on the model parameter space ($H_0, \Omega_b, A_s, \alpha, \beta$) from the aforementioned observational data. The best-fit of the parameters are maximized by using the following total χ^2 -function.

$$\chi_{tot}^2 = \chi_{H(z)}^2 + \chi_{SNe}^2 + \chi_{BAO}^2, \quad (34)$$

where $\chi_{H(z)}^2$, χ_{SNe}^2 and χ_{BAO}^2 are given by Eqs. (28), (29) and (33), respectively. It is noted that the current observational constraints on baryon density $\Omega_{b,0}$ are stringent.¹ We follow the Markov Chain Monte Carlo (MCMC) sample from the python package *emcee* to minimize the likelihood. We have chosen 2000 iterations with 120 walkers to explore the confidence region taking into account a Gaussian priors on the Hubble constant H_0 and on the baryon density $\Omega_{b,0}$. Additionally, we select priors for A_s, α and β . Table 2 summarizes the priors used in the Bayesian analysis.

We also utilize two selection criteria, like the Akaike information criteria (AIC) (Akaike 1974) and the Bayesian information criteria (BIC) (Schwarz 1978) to determine which of the models is performing better. The AIC is defined as

$$AIC = \chi_{min}^2 + \frac{2nN}{N - n - 1}, \quad (35)$$

where χ_{min}^2 is the minimum value of the χ^2 -function, n ($n = 3, 5, 4$ and 4 for Λ CDM, GCG with β , GCG without β and CG with β , respectively) is the number of independent parameters and $N = 1086$ ($H(z) = 32$, $SNe = 1048$ and $BAO = 6$) is the total number of data points.

The BIC value is computed from

$$BIC = \chi_{min}^2 + n \ln N. \quad (36)$$

We define the AIC and BIC differences by considering Λ CDM as the reference model: $\Delta AIC = AIC_x -$

¹For example, $\Omega_{b,0}h^2 = 0.0224 \pm 0.0001$ from Planck 2018 results (Aghanim et al. 2020).

Table 2 The priors ranges and the best-fit values and 1σ marginalized uncertainties of parameters obtained from the $SNe + H(z) + BAO$ fitting analysis for Λ CDM, GCG with β , GCG without β and CG with β

Parameters	Priors	Λ CDM	GCG with β	GCG without β	CG with β
H_0	[50, 80]	$68.595^{+1.803}_{-1.680}$	$68.804^{+1.066}_{-1.062}$	$69.432^{+0.034}_{-0.062}$	$68.778^{+0.123}_{-0.128}$
Ω_m	—	$0.307^{+0.026}_{-0.022}$	—	—	—
Ω_Λ	—	$0.701^{+0.032}_{-0.032}$	—	—	—
Ω_b	[0.02, 0.04]	—	$0.029^{+0.006}_{-0.005}$	$0.029^{+0.001}_{-0.004}$	$0.030^{+0.003}_{-0.002}$
A_s	[0.5, 1.5]	—	$0.713^{+0.043}_{-0.035}$	$0.777^{+0.011}_{-0.021}$	$0.705^{+0.032}_{-0.044}$
α	[0, 1]	—	$0.995^{+0.078}_{-0.022}$	$0.350^{+0.024}_{-0.033}$	1
β	[0, 1]	—	$0.140^{+0.022}_{-0.042}$	—	$0.498^{+0.009}_{-0.013}$

Table 3 The present values of q , j , w , C_S^2 and age of the Universe corresponding to Λ CDM, GCG with β , GCG without β and CG with β models.

Model	q_0	j_0	w_0	C_S^2	t_0 (Gyr)
Λ CDM	$-0.551^{+0.020}_{-0.020}$	1	$-0.701^{+0.013}_{-0.015}$	≤ 1	13.8 ± 0.14
GCG with β	$-0.597^{+0.016}_{-0.013}$	$1.440^{+0.023}_{-0.022}$	$-0.713^{+0.044}_{-0.012}$	0.638	13.8 ± 0.16
GCG without β	$0.271^{+0.033}_{-0.031}$	$1.264^{+0.034}_{-0.023}$	$-0.777^{+0.055}_{-0.012}$	0.271	13.6 ± 0.11
CG with β	$-0.523^{+0.011}_{-0.022}$	$1.006^{+0.031}_{-0.027}$	$-0.705^{+0.013}_{-0.022}$	0.705	13.6 ± 0.13

$AIC_{\Lambda\text{CDM}}$ and $\Delta BIC = BIC_x - BIC_{\Lambda\text{CDM}}$, where x represents either of GCG with β , GCG without β and CG with β models.

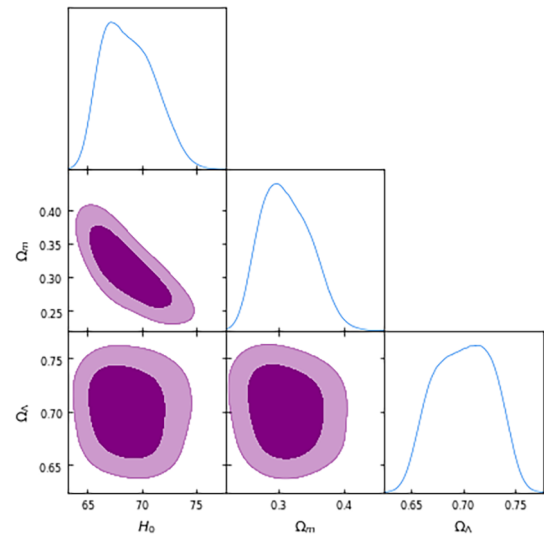
In case of AIC, if $\Delta AIC < 2$, both the models are equally supported by data. If $2 < \Delta AIC < 4$, there is average support. The range $4 < \Delta AIC < 10$ still supports the considered model but less than the preferred one. For the value of $\Delta AIC > 10$, the model is not supported by the observational data.

Similarly, in case of BIC, $\Delta BIC < 2$ indicates no evidence against the model. If $2 < \Delta BIC < 6$, there is modest evidence against the considered model. We get a strong evidence against the considered model for a value in the range $6 < \Delta BIC < 10$ suggests and a stronger evidence against the model if $\Delta BIC > 10$. For BIC, models with more number of parameters, compared to Λ CDM, penalised further due to the $\ln N$ factor.

We are also interested to discuss the stability of the models using the method of squared sound speed, C_S^2 . The equation describing the evolution of linear perturbations of the background energy density define the squared sound speed. Therefore, it is a background quantity which is essentially a perturbative in nature. The bound $C_S^2 > 0$ is required to have small perturbations of the background energy density, otherwise, the energy density perturbations will grow and there will be a classical instability. Thus, stability or instability of a given perturbation in the background, can be specified by determining the sign of C_S^2 . The model is stable if $C_S^2 > 0$ and for $C_S^2 < 0$, the model approaches to instability.

The squared sound speed is defined as (DeDeo et al. 2003; Hannestad 2005)

$$C_S^2 = \partial p / \partial \rho. \quad (37)$$

**Fig. 1** 2D contour at 68.3% and 95.4% confidence levels for Λ CDM model using $H(z) + SNe + BAO$ data-set.

Using (17) in the above equation, the squared sound speed for GCG model with matter creation is calculated as

$$C_S^2 = \frac{\alpha A_s}{A_s + (1 - A_s)(1 + z)^{3(1-\beta)(1+\alpha)}}. \quad (38)$$

In terms of EoS parameter, it gives

$$C_S^2 = -\alpha w_{gcg}(z). \quad (39)$$

Using above equation, we can obtain the present value as $C_S^2(z=0) = \alpha A_s = -\alpha w_{gcg,0}$. The positive value of A_s restricts the value $\alpha > 0$. It is to be noted that for causality and stability under perturbation, it is required to satisfy the inequality condition $0 \leq C_S^2 \leq 1$ (Lixin et al. 2012).

Fig. 2 2D contour at 68.3% and 95.4% confidence levels for GCG model with matter creation using $H(z) + SNe + BAO$ data-set.

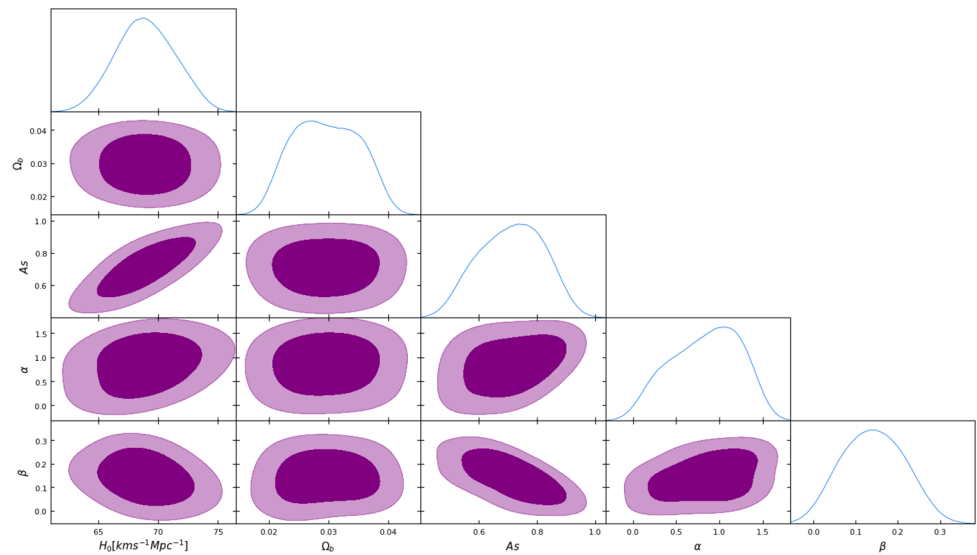
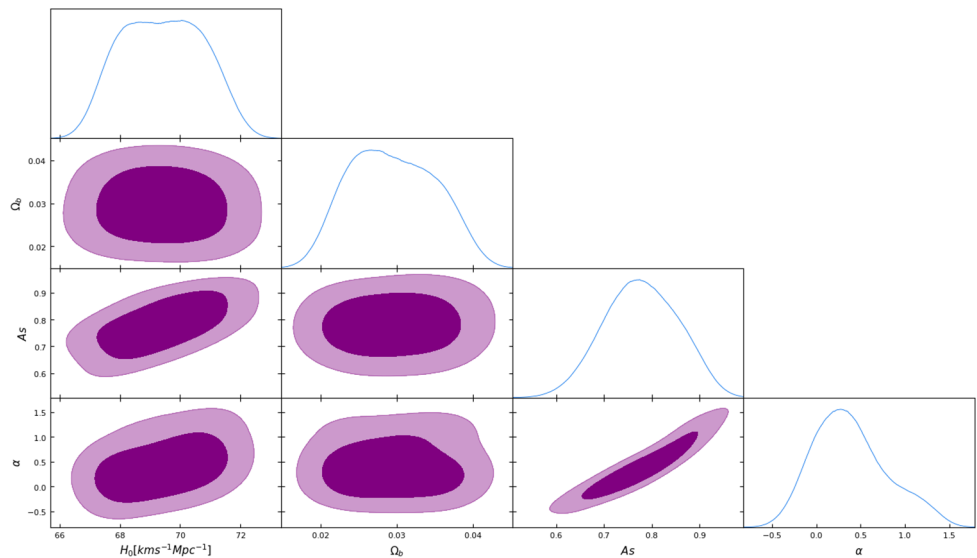


Fig. 3 2D contour at 68.3% and 95.4% confidence levels for GCG model without matter creation using $H(z) + SNe + BAO$ data-set.



7 Results and discussion

The best-fit values of model parameters are displayed in tables and figures. In Table 2 we present the values of various model parameters under consideration, namely Λ CDM, GCG with β , GCG without β and CG with β . In Table 3 we provide the present values of $q(z)$, $j(z)$, $w(z)$, C_S^2 and t_0 using the best-fit values of the models. The 1-dimensional marginalized posterior distributions and two-dimensional contours correspond to 68.3% and 95.4% confidence levels for the parameters space of these models are shown in Figs. 1–4. Finally, in Figs. 5–9 we plot the evolution of the cosmological parameters, namely $H(z)$, $q(z)$, $j(z)$, $w(z)$ and C_S^2 of these considered models. In what follows, we discuss our results with some more detail.

Analysing the constraints of the parameters of the various versions of the models listed in Table 2, we find that

the Λ CDM model gives $H_0 = 68.595^{+1.803}_{-1.680}$ km/s/Mpc, the GCG with β gives $H_0 = 68.804^{+1.066}_{-1.062}$ km/s/Mpc, the GCG without β yields $H_0 = 69.432^{+0.034}_{-0.062}$ km/s/Mpc, and finally, the CG with β gives $H_0 = 68.778^{+0.123}_{-0.128}$ km/s/Mpc. We observe that these results are very competitive with the other current estimates performed by assuming Λ CDM cosmology (Ade et al. 2016; Aghanim et al. 2020; Schöneberg et al. 2022). The evolution of $H(z)$ with redshift for different models are plotted in Fig. 5.

Also, the deceleration parameters are constrained to $q_0 = -0.597^{+0.016}_{-0.013}$ for GCG with β , $q_0 = -0.271^{+0.033}_{-0.031}$ for GCG without β , $q_0 = -0.523^{+0.011}_{-0.022}$ for CG with β (c.f. Table 3) which suggest the current accelerating expansion of the Universe at high statistical significance evidence. The evolutions of the deceleration parameter for different models are plotted in Fig. 6. The parameter $q(z)$ transits from posi-

Fig. 4 2D contour at 68.3% and 95.4% confidence levels for CG model with matter creation using $H(z) + SNe + BAO$ data-set.

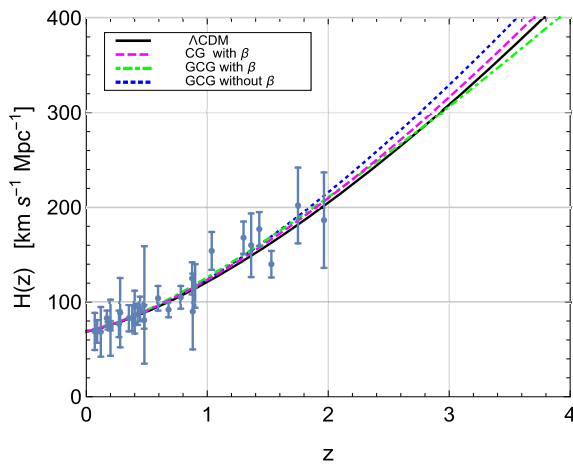
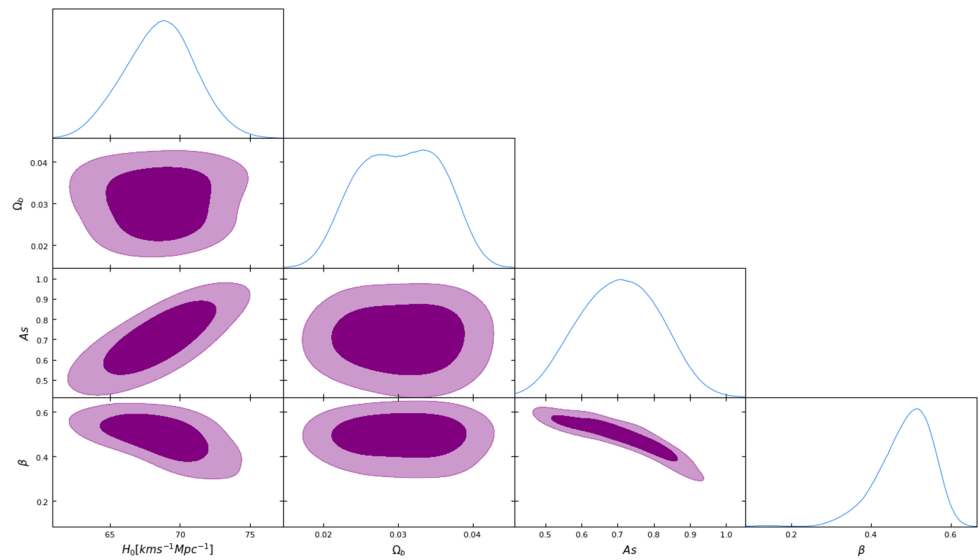


Fig. 5 Best fits over $H(z)$ data for Λ CDM (black solid line), and GCG with β (green dot-dashed line), GCG without β (blue dotted line) and CG with β (magenta dashed line) models are shown. The grey points with uncertainty bars correspond to the 32 $H(z)$ sample.

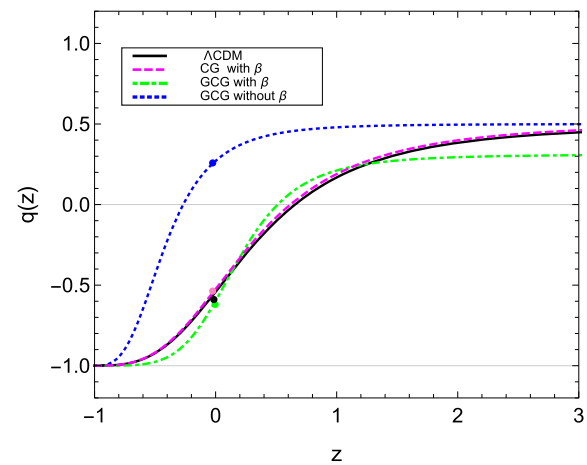


Fig. 6 Evolution of the deceleration parameter, $q(z)$ with redshift for GCG and CG models with and without matter creation. The trajectory of $q(z)$ for Λ CDM model is also shown in bold line. The dot represents the present value of $q(z)$ (hence q_0).

tive ($q > 0$, deceleration) to negative ($q < 0$, acceleration) in each model which shows that the Universe decelerates at early time followed by acceleration in late time. The GCG with β model starts late acceleration than CG with β and Λ CDM models. We observe that the all Chaplygin gas models (GCG and CG) have characteristics of quintessence DE.

The present value of EoS parameter of Chaplygin gas models are displayed in Table 3. Figure 8 shows the evolution of the EoS parameter for different models. It is observed that the EoS parameter of all the models tend to -1 in late-time. In Fig. 7, we show the jerk parameter, $j(z)$ of our models in comparison to Λ CDM model. In late-time all the Chaplygin gas with or without matter creation converge to Λ CDM state. Based on the joint analysis, the present values of $j(z)$ are $j_0 = 1.440^{+0.023}_{-0.022}$, $1.264^{+0.034}_{-0.023}$ and $1.006^{+0.031}_{-0.027}$

for GCG with β , GCG without β and CG with β , respectively.

Following the selection criteria of AIC and BIC as discussed in Section 5, the values of ΔAIC and ΔBIC are listed in Table 4. According to ΔAIC values of Chaplygin gas models, we find that GCG with β , GCG without β and CG with β models have $\Delta AIC < 2$ which show that the joint observational data strongly support these models. In BIC, we have $2 < \Delta BIC < 6$ for GCG without β and CG with β , and thus there is positive evidence against the model. The GCG with β model has $\Delta BIC > 10$ which tells us that there is a very strong evidence against this model. This may happen due to the large number of free parameters for which BIC penalizes.

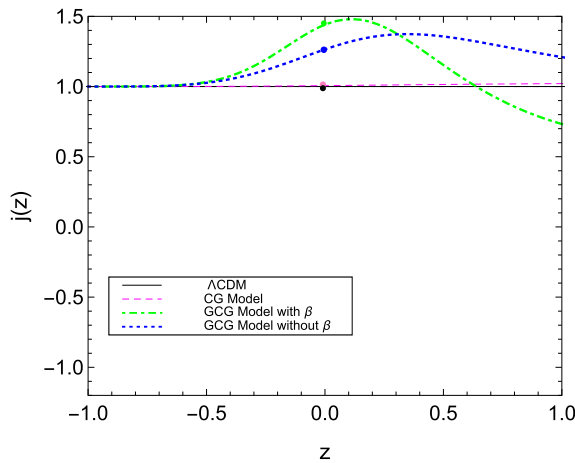


Fig. 7 Evolution of the jerk parameter, $j(z)$ for GCG and CG models with and without matter creation. The horizontal bold line represents the value of $j(z)$ for Λ CDM. The dot represents the present value of $j(z)$ (hence j_0).

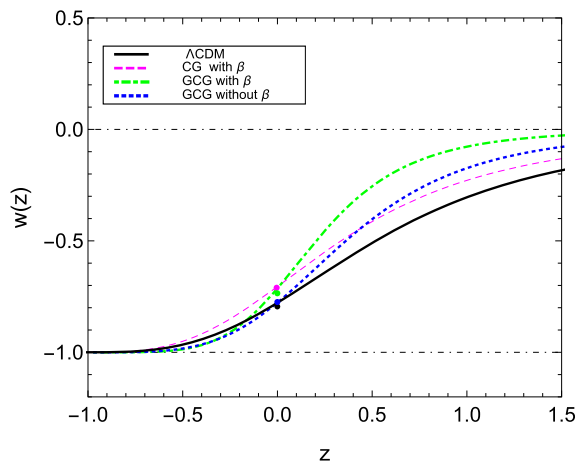


Fig. 8 Evolution of the state parameter, $w(z)$ for GCG and CG models with and without matter creation. The dot represents the present value of $w(z)$ (hence w_0).

The evolution of $\mathcal{C}_s^2$ with respect to redshift for different models is shown in Fig. 9. Using the best-fit values of α and A_s , we find that $\mathcal{C}_s^2 = 0.638$ for GCG model with β , $\mathcal{C}_s^2 = 0.271$ for GCG model without matter creation and $\mathcal{C}_s^2 = 0.705$ for CG with β [c.f. Table 3]. It is obvious that the squared sound speed of these models are in the range of $0 < \mathcal{C}_s^2 < 1$, which is not exceeding the speed of light (unity), and hence the models are stable. We have computed the present age of the Universe in different models which are summarized in Table 3.

8 Conclusion

In this work, we have studied the gravitationally- induced matter creation cosmology with a generalization of the

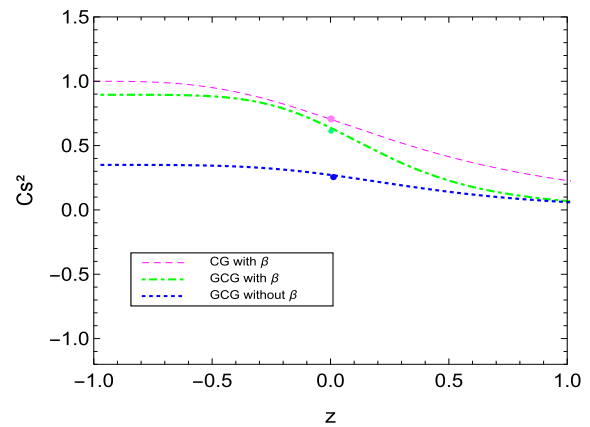


Fig. 9 Evolution of speed of sound, $\mathcal{C}_s^2$ for GCG and CG models with and without matter creation. The dot represents the present value $\mathcal{C}_s^2(z=0)$.

Chaplygin gas equation of state, $p = -A/\rho^\alpha$, with $0 < \alpha \leq 1$ to observe the late-time cosmic acceleration in the framework of flat FLRW spacetime. We have also considered a particular case of $\alpha = 1$, i.e. the Chaplygin gas EoS, $p = -A/\rho$ with matter creation. The cosmological relevance of pure CG model has been pointed out in Ref. Kamenshchik et al. (2001) and it has been found that the SNe data rules out the CG model.

As discussed in the introductory part, gravitationally- induced matter creation can be treated as an irreversible scalar process which provides an additional decay (creation) pressure in the energy-momentum tensor of the cosmological mixture. This extra creation pressure is defined in terms of matter creation rate, Γ and other physical quantities describing the fluid. It has been demonstrated that considering the Universe as a open systems with matter creation, one can understand the current accelerated expansion of the Universe. Since the rate of matter creation, Γ , is still unknown, we have assumed phenomenological ansätze of the matter creation rate evolving as $\Gamma = 3\beta H$, where β is a constant. Many authors (Lima et al. 1996; Lima and Alcaniz 1999; Cárdenas 2012) have studied some properties of adiabatic matter creation models with this form of matter creation rate. It has been found that this kind of models are always decelerating or accelerating, that is, there is no transition from decelerated to accelerated expansion phase as required by the SNe observations. However, we have found that considering the GCG and CG EoS in matter creation model there is a transition from one phase to other phase with the same form of matter creation rate. In what follows, we have summarized the result of our findings in more detail.

In the first part of the paper, we have presented the analytical solution of matter creation models with GCG and CG equation of states. We have obtained the solutions of cosmological parameters, like energy density, Hubble parameter, deceleration parameter, EoS parameter and jerk parameter

Table 4 The AIC and BIC obtained for the Λ CDM, GCG with β , GCG without β and CG with β models. We assume Λ CDM model as reference model.

Model	χ^2_{min}	n	N	AIC	BIC	ΔAIC	ΔBIC
Λ CDM	1088.121	3	1086	1094.02	1108.97	—	—
GCG with β	1087.334	5	1086	1095.03	1119.95	1.00	10.54
GCG without β	1087.223	4	1086	1095.09	1114.96	1.06	5.99
CG with β	1086.336	4	1086	1096.01	1113.96	1.988	4.99

as function of the redshift. In the second part of the paper, we have performed a joint observation to constrain the model's parameters by using the latest observational data such as Type Ia supernovae (Pantheon data), Hubble data and BAO data. The best-fit values of model parameters are listed in Table 2. The present values of $q(z)$, $j(z)$, $w(z)$, $\mathcal{C}_S^2$ and age of Universe are presented in Table 3. We have shown the evolution of the cosmological parameters, $H(z)$, $q(z)$, $w(z)$, $j(z)$ and $\mathcal{C}_S^2$ in Figs. 5–9, respectively. We have observed a deviation of GCG and CG with β with respect to the Λ CDM and GCG models in the deceleration phase. The plot of these cosmological parameters clearly show that the models successfully generate late-time cosmic acceleration along with a decelerated expansion in the past. The evolutions of $j(z)$ show that the GCG without β and CG with β do not deviate very far from the Λ CDM model. However, the GCG with β show much deviation from Λ CDM during early Universe. But $j(z)$ of all the models tends to 1 as $z \rightarrow -1$. The plot of data points of $H(z)$ with 1σ error bars show that our models are well consistent with these data against the redshift parameter.

We have compared the present model with those obtained from the standard Λ CDM. The evolution of different cosmological parameters show that the matter creation model with GCG EoS mimics exactly the Λ CDM model. The present value q_0 , w_0 and t_0 are very close to the present value of Λ CDM. Similarly, the other value, like H_0 and Ω_b are very competitive with Λ CDM obtained in Ref. (Ade et al. 2016; Aghanim et al. 2020; Cárdenas 2012). All the models transit from early deceleration phase to late-time acceleration phase. We have explored through the joint observational analysis that the models favor the EoS for DE greater than -1 , that is, quintessence region. Thus, the results of the models are fully consistent with respect to the expected Λ CDM value.

Makler et al. (2003a) have derived the constraints from SNe on flat cosmological models with GCG EoS and found that the GCG model is a promising model for unifying the dark components. Bertolami et al. (2004) analyzed the GCG EoS for flat and non-flat models with the SNe data. Considering all matter and energy components for the flat case, they found the best-fit values for (A_s, α) as (0.79, 0.999) and (0.936, 3.75) with and without the constraint $\alpha \geq 1$, respectively. Bento et al. (2003b) constrained the GCG model and obtained $0.74 \leq A_s \leq 0.85$ and $\alpha \leq 0.6$. Choudhury and

Padmanabhan (2005) obtained the best-fit values for (A_s, α) as (0.79, 0.999) with the restriction of $\alpha \leq 1$. However, for wider range the best-fit value are (0.936, 3.75). Zhu (2004) obtained the best-fit at $A_s = 0.74$ and $\alpha = 0.32$ from the dimensionless coordinate distance data. However, the author constrained the best-fit at $A_s = 0.70$ and $\alpha = -0.09$ from the X-ray gas mass fractions of galaxy clusters and dimensionless coordinate distances to SNe and FRIIb radio galaxies. Wu and Yu (2007) examined the observational constraints on the GCG model for DE from the 9 $H(z)$ data points. They found that the best-fit happens at $A_s = 0.82$ and $\alpha = 0.71$. Further, the authors used joint observational data of $H(z)$, BAO and supernovae Legacy Survey (SNeLS) and obtained the best-fit at $A_s = 0.75$ and $\alpha = 0.05$.

In our investigations, we have observed that the best-fit values of our GCG and CG models with matter creation are comparable with results discussed above which are within the expected parameters ranges of the GCG to describe the unified model. It is also noted that the pure CG model ($\alpha = 1$) is ruled out by the used observational data sets. However, CG with matter creation is consistent with these data sets. Makler et al. (2003a,b) pointed out that the GCG model can be explored further, both from the particle physics as well as observational side. It is expected that the negative pressure of both the matter creation and GCG EoS might be jointly guide us to unveil the nature of the Universe.

The AIC and BIC analysis show that GCG with β , GCG without β and CG with β models have $\Delta AIC < 2$ which means that the joint observational data strongly support these models. In BIC, we have $2 < \Delta BIC < 6$ for GCG without β and CG with β , and thus there is positive evidence against the model. The GCG with β model has $\Delta BIC > 10$ which tells us that there is a very strong evidence against this model. This may happen due to the large number of free parameters for which BIC penalizes. It has been found that the squared sound speed of these models are in the range of $0 < \mathcal{C}_S^2 < 1$, which is not exceeding the speed of light (unity), and hence the models are stable. Therefore, we conclude that the presented models in this paper provide well constrained values of cosmological parameters from the joint observational data of SNe, $H(z)$ and BAO and the models remain at very close proximity of the Λ CDM model.

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Data Availability The manuscript has no associated data. The datasets used in this paper are openly available on <https://doi.org/10.17909/T95Q4X>.

Declarations

Competing interests The authors declare no competing interests.

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PAPER

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Constraining the variable generalized Chaplygin gas model in matter creation cosmology

Yogesh Bhardwaj and C P Singh*

Department of Applied Mathematics, Delhi Technological University, Bawana Road, Delhi-110042, India

E-mail: yogeshbhardwaj1597@gmail.com and cpsingh@dce.ac.in

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Abstract

We explore the variable generalized Chaplygin gas (VGCG) model in the theory of matter creation cosmology within the framework of a spatially homogeneous and isotropic flat Friedmann—Lemaître—Robertson—Walker space-time. Matter creation cosmology is based on reinterpretation of the energy—momentum tensor in Einstein's field equations. This creation corresponds to an irreversible energy flow from the gravitational field to the created matter constituents. The variable Chaplygin gas (VCG) is also studied as a particular solution. We use the Markov chain Monte Carlo method to constrain the free parameters of three models, namely, Lambda-Cold-Dark matter (Λ CDM), VGCG and VCG models with and without matter creation from the latest observational data from baryon acoustic oscillations, cosmic chronometer, type Ia supernovae (Pantheon) including gamma-ray bursts, quasars and the local measurement of H_0 from R21 data. Two different combinations of dataset provide a fairly tight constraint on the parameters of the Λ CDM, VGCG and VCG models. The present values of various cosmological parameters are obtained, which are very close to the Λ CDM model. Furthermore, we perform stability analysis, Bayesian evidence analysis and information criteria analysis for these models through studying the sound speed, Bayes factor, and Akaike information criteria (AIC) and Bayesian information criteria (BIC) selection criteria. The values of sound speed for VGCG and VCG models shows that both the models are stable. According to AIC, it is observed that VGCG and VCG models with matter creation are supported considerably less by current observations, while BIC shows that these models are not favoured by observational data.

Supplementary material for this article is available [online](#)

Keywords: cosmological model, matter creation, Chaplygin gas model, dark energy

(Some figures may appear in colour only in the online journal)

1. Introduction

The discovery of the late-time accelerated expansion of the Universe has become one of the most important theoretical challenges in modern cosmology. One of the first pieces of observational evidence of this late-time acceleration was obtained through the measurement of type Ia supernovae [1–3]. Subsequently, more observations such as the Sloan Digital Sky Survey (SDSS) and Wilkinson Microwave

Anisotropy Probe (WMAP) [4], baryonic acoustic oscillations (BAO) [5] and cosmic microwave background (CMB) anisotropies [6–10] have confirmed that the Universe is spatially flat and two-thirds of the energy content is dominated by an exotic component with large negative pressure, referred to as dark energy. The simplest candidate for dark energy is the cosmological constant with its equation of state $w = p_\Lambda / \rho_\Lambda = -1$. This model is known as the Lambda cold dark matter (Λ CDM) model which has three major components: a cosmological constant associated with dark energy, cold dark matter and ordinary matter. This model emerged in the late

* Author to whom any correspondence should be addressed.

1990s as a concordance model. Despite its observational success, the Λ CDM model suffers from at least two problems: the cosmological constant problem (CCP) and the cosmic coincidence problem [11, 12]. Many alternative theories have been proposed beyond the Λ CDM model to solve these conundrums. Generally, these alternative theories have been divided into two main classes: the first, and possibly the major class, is the dark energy model in which a new cosmic fluid or matter field is introduced within the framework of general relativity [13]. The other class is modified theories of gravity in which the standard general relativity Lagrangian is modified based on some concrete physical mechanism [14].

We are concerned here with the first kind of dark energy proposal, where the dark energy component might be explained by a background fluid with an exotic equation of state, the so-called the Chaplygin gas (CG) model [15]. The Chaplygin gas [16] had an origin that was not cosmological. It was used to explain the lifting force on an airplane wing. As the equation of state has negative pressure, it has recently attracted new interest in cosmology. It is used as a perfect fluid to describe the accelerating expansion of the Universe with equation of state $p_{Ch} = -A/\rho_{Ch}$, where A is a positive constant, and p_{Ch} and ρ_{Ch} are the energy density and pressure, respectively. The striking feature of the CG model is its ability to unify two of the Universe's unknown dark sections: dark matter (DM) and dark energy (DE) [17, 18]. Since the CG model was ruled out by observations, this model was extended to the so-called generalized Chaplygin gas (GCG) with equation of state [19] $p_{Ch} = -A/\rho_{Ch}^\alpha$, where $0 \leq \alpha \leq 1$. This GCG model is also regarded as a unification of DE and DM [20–23]. Gou and Zhang [24] have proposed a variable Chaplygin gas (VCG) model, which explains the evolution from dust to quintessence or phantom. In such a model, the equation of state is characterized by $p_{Ch} = -A(a)/\rho_{Ch}$, where A is the function of the scale factor a . Yang *et al* [25], and later on, Lu [26] generalised the VCG model with equation of state $p_{Ch} = -A(a)/\rho_{Ch}^\alpha$, known as variable generalized Chaplygin gas (VGCG) model, which also explains the evolution from dust to quintessence or phantom. Some more works on VCG and VGCG models can be found in [27–32].

Nevertheless, even in the context of general relativity, there are other theories to explain the current accelerated expansion of the Universe. As stated above, the presence of negative pressure is the main factor required to accelerate the expansion of the Universe. This may also occur, for instance, when the physical system departs from a thermodynamic equilibrium state [33]. In this connection, Zeldovich [34] first stated that the process of cosmological particle creation at the expense of the gravitational field can be phenomenologically described by a negative pressure. In cosmology, gravitationally induced particle production has a long history. Using the non-equilibrium thermodynamics of open systems, Prigogine *et al* [35, 36] proposed the first self-consistent macroscopic formulation of the matter creation process into the Einstein field equations. Later on, Calvão *et al* [37] clarified the process through covariant formulation. It has been stated that matter creation can be explained in the realm of relativistic non-equilibrium thermodynamics. According to several studies [38–47], matter creation cosmology may explain

the accelerating expansion of the Universe. The late-time evolution of the Universe has been explained by many authors [48–57]. Since many significant observations have been examined in recent years and the current accelerated expansion of the Universe is satisfactorily explained, matter creation cosmology is an extremely intriguing topic. The crucial aspect of this model is to take into account the option for the particle production rate, which is still unknown. However, it is assumed to be a function of the Hubble parameter H .

In our previous work [58], we mainly concentrated on the GCG inspired by matter creation cosmology and derived the best-fit values of model free parameters. In this paper, to make a more complete discussion on the contribution of matter creation in CG models, we are interested in discussing the consequences of a VGCG model incorporating the background of matter creation cosmology within the framework of a homogeneous and isotropic flat Friedmann–Lemaître–Robertson–Walker space-time. We also consider a special model of CG, namely, variable Chaplygin gas (VCG). We focus on the analytical solutions and observational analysis of the proposed models to explore the contribution of matter creation. To make our discussion more complete, we use a Bayesian analysis to constrain the parameter space of these models using BAO, cosmic chronometers data, Pantheon type Ia supernova including the gamma-ray bursts, quasars and local measurement of H_0 from R21. It is worth noting that the VGCG and VCG models with matter creation can alleviate H_0 tension. We also discuss the stability, Bayesian evidence analysis and selection criteria to distinguish our models from the existing dark energy models such as the Λ CDM model.

The paper is organized as follows: in section 2, for the sake of clarity, we give a brief introduction to matter creation in cosmology. Section 3 is devoted to the background dynamics of the VGCG model with matter creation. The analytical solutions of VGCG and VCG models are presented in sections 4 and 5, respectively. Recent observational data and posterior analysis are explored in section 6. Section 7 is devoted to the results and discussion on the consistency of VGCG and VCG models with the observations. Further, sections 7.1–7.4 present the evolutions of cosmological parameters, stability, information selection criteria and Bayesian evidence analysis, respectively, and finally, in section 8, we present a summary of the results.

2. Basic formalism of matter creation

The thermodynamics of matter creation were first applied to cosmology in the late 1980s by Prigogine and collaborators [35, 36]. They included matter creation explicitly in the matter stress–energy tensor in the Einstein field equations. Later on, Calvão *et al* [37] discussed and generalized this phenomenological approach through a covariant formulation allowing specific entropy variation. Nowadays, it is applied in many gravitational theories. In this paper we follow the approach of Prigogine *et al* [35, 36], which presumes that matter creation occurs at the expense of gravitational energy.

The basic formalism of matter creation in the theory of general relativity is as follows.

Let us consider an open thermodynamical system where the number of particles is not conserved. The particle flux vector is assumed to have the form $N^\mu = nu^\mu$, where N is the number of particles, n is the particle number density, given by $n = N/V$ and u^μ is the usual particle four-velocity. For open systems, the non-conservation of the particle number satisfies the balance equation

$$\nabla_\mu N^\mu \equiv n_{,\mu} u^\mu + 3Hn = n\Gamma, \quad (1)$$

where Γ stands for the rate of change of the particle number in a physical volume $V = V_0 a^3$ (where V_0 is the co-moving volume) and $u_{,\mu}^\mu = 3H$ denotes the fluid expansion for a Friedmann–Lemaître–Robertson–Walker (FLRW) Universe and by notation $\dot{n} = n_{,\mu} u^\mu$. The overdot denotes derivative with respect to cosmic time. Clearly, $\Gamma > 0$ indicates the creation of particles while $\Gamma < 0$ stands for particle annihilation and $\Gamma = 0$ means there is no creation of matter, i.e., the particle number is conserved. The entropy flux vector is given by $s^\mu = n\sigma u^\mu$ [37], where σ is the specific (per particle) entropy. It is to be noted that the entropy must satisfy the second law of thermodynamics $s_{,\mu}^\mu \geq 0$. The incorporation of variable N into the cosmological scenario modifies the first law of thermodynamics as [36, 37]

$$d(\rho V) + p dV - (h/n)d(nV) = 0, \quad (2)$$

where $h = (\rho + p)$ is the enthalpy (per unit volume), and ρ and p are the energy density and thermodynamic pressure, respectively. It is assumed that the Universe evolution continues to be adiabatic, in the sense that the entropy per particle remains unchanged ($\dot{\sigma} = 0$). Equation (2) can be rewritten as

$$d(\rho V) + (p + p_c)dV = 0, \quad (3)$$

where the extra contribution p_c in equation (3) can be interpreted as a non-thermal pressure, known as creation pressure, defined as

$$p_c = -\frac{h}{n} \frac{d(nV)}{dV}. \quad (4)$$

Using equation (1) in equation (4), we get the following form of creation pressure [37]:

$$p_c = -\frac{(\rho + p)}{3H} \Gamma. \quad (5)$$

Hence, the creation pressure p_c can be identified with the particle production rate Γ . The simplest possible case, and probably the most physical choice would be one for which the characteristic time scale for matter creation is the Hubble time itself. Phenomenologically, this is equivalent to [59, 60]

$$\Gamma = 3\beta H, \quad (6)$$

where β is a positive constant. Using equation (6) in equation (5), the creation pressure gives

$$p_c = -\beta(\rho + p). \quad (7)$$

Hereafter, we refer to ρ as ρ_{Ch} and p as p_{Ch} for energy density and pressure of a CG perfect fluid.

3. Basic field equations

Let us consider the following geometry of the Universe which is very well described by the homogeneous and isotropic flat FLRW space-time

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2), \quad (8)$$

where $a(t)$ is the scale factor of the Universe. Throughout we use units such that the speed of light $c = 1$. For a co-moving observer, $u^\mu = \delta^\mu_\nu$, in which $u^\mu u_\mu = -1$. The Einstein field equations are written as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu}, \quad (9)$$

where the symbols have their usual meaning. In a general relativistic framework, the basic macroscopic variables that describe the thermodynamic states of a relativistic simple fluid are the energy–momentum tensor $T_{\mu\nu}$, the particle flux vector N^μ and the entropy flux vector s^μ . In this work, we shall consider a Universe with matter contents as baryons and CG endowed with matter creation (CG means a perfect fluid having exotic equation of state). Thus, we write the energy density and the pressure of the mixture as a sum of two terms that describe the baryon matter component and the CG fluid with matter creation. By taking into account matter creation, the effective energy–momentum tensor for the mixture can be given by [37]

$$T_{\mu\nu} = (\rho_{Ch} + \rho_b + p_{Ch} + p_c)u_\mu u_\nu + (p_{Ch} + p_c)g_{\mu\nu}. \quad (10)$$

where ρ_{Ch} and ρ_b are the energy densities of the CG fluid and the baryons, and p_{Ch} and p_c the pressures of CG fluid and particle creation, respectively. The Einstein field equation (9) and equation (8), for line element, and equation (10), energy–momentum tensor, yield

$$H^2 = \frac{8\pi G}{3}(\rho_{Ch} + \rho_b), \quad (11)$$

$$\dot{H} + H^2 = -\frac{4\pi G}{3}(\rho_{Ch} + \rho_b + 3p_{Ch} + 3p_c), \quad (12)$$

where $H = \dot{a}/a$ is the Hubble parameter. In a co-moving frame the dot denotes derivative with respect to cosmic time t .

Assuming that baryons are self-conserved, namely the baryon density evolves in the normal way, $\rho_b = \rho_{b,0}a^{-3}$, where subscript 0 denotes the value of a quantity at the present time. Thus, the energy densities of the baryons and matter decouple so that the conservation equation $T_{;\nu}^{\mu\nu} = 0$ gives

$$\dot{\rho}_{Ch} + 3H(\rho_{Ch} + p_{Ch} + p_c) = 0. \quad (13)$$

In recent years, it has been suggested that the change of behavior of the missing energy density might be regulated by the change in the equation of state of the background fluid. In this respect, CG was introduced as an exotic background fluid with an equation of state described by $p_{Ch} = -A/\rho_{Ch}$, where A is a positive constant [15]. It gives a unified picture of dark matter and dark energy. However, it was ruled out due to being inconsistent with the observational data. Subsequently, a more generalized model of the CG is characterized by an equation of state $p_{Ch} = -A/\rho_{Ch}^\alpha$ ($0 \leq \alpha < 1$), which is known as generalized Chaplygin gas (GCG) [17]. It has been shown

that this model can be accommodated within the standard structure formation scenarios [17, 18]. Therefore, CG models look to be good alternatives to explain the accelerated expansion of the Universe.

Guo and Zhang [24] proposed a new CG model, known as the VCG model with the equation of state $p_{Ch} = -A(a)/\rho_{Ch}$, where $A(a)$ is a function of the scale factor

$$q(z) = -1 + \frac{1}{2E^2(z)} \left[3\Omega_{b,0}(1+z)^3 + \frac{(1 - \Omega_{b,0})nA_s(1+z)^n + 3(1 - A_s)(1 - \beta)(1 + \alpha)(1+z)^{3(1-\beta)(1+\alpha)}}{(A_s(1+z)^n + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)})^{\frac{\alpha}{1+\alpha}}} \right]. \quad (20)$$

a. In this scenario, the VCG model can drive the Universe from dust to quiescence to phantom. Later on, Yang *et al* [25] presented a generalization of VCG model, known as the VGCG model whose equation of state is given by [25, 26]

$$p_{Ch} = -\frac{A_0 a^{-n}}{\rho_{Ch}^\alpha}, \quad (14)$$

where A_0 , n and α are parameters in the model.

In this work, we are interested in the VGCG model with the cosmology of matter creation along with the VCG as a particular model.

4. VGCG model with matter creation

Using equations (7) and (14), equation (13) reduces to

$$\rho'_{Ch} + 3(1 - \beta) \left(\rho_{Ch} - \frac{A_0 a^{-n}}{\rho_{Ch}^\alpha} \right) = 0, \quad (15)$$

where prime denotes derivative with respect to $x = \ln(a)$. By taking explicitly the integral of equation (15) one finds the solution of energy density for VGCG as

$$\rho_{Ch} = \rho_{Ch,0} [A_s(1+z)^n + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)}]^{\frac{1}{1+\alpha}}, \quad (16)$$

where $\rho_{Ch,0}$ is the present energy density, $z = (1/a) - 1$ is the redshift and $A_s = \frac{3(1-\beta)(1+\alpha)}{3(1-\beta)(1+\alpha) - n} \frac{A_0}{\rho_{Ch,0}^{(1+\alpha)}}$. For an expanding Universe, we must have $n > 0$ and $(1 - \beta)(1 + \alpha) > 0$.

Let us investigate the cosmological parameters, specifically the Hubble parameter, deceleration parameter, equation of state parameter and jerk parameter in order to compare these models with the Λ CDM model.

Using equation (16) in equation (11), the Friedmann equation can also be expressed as

$$H(z) = H_0 E(z), \quad (17)$$

where

$$E(z) = [(1 - \Omega_{b,0})\{A_s(1+z)^n + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)}\}^{\frac{1}{1+\alpha}} + \Omega_{b,0}(1+z)^3]^{1/2}. \quad (18)$$

The acceleration of the Universe is measured by the deceleration parameter, which is defined in terms of redshift as

$$q(z) = -1 + (1+z)E(z)^{-1} \frac{dE(z)}{dz}. \quad (19)$$

Using equation (18) in equation (19), we obtain

The equation of state (EoS) parameter for the VGCG model is given by

$$w(z) = \frac{-A_s \left(1 - \frac{n}{3(1-\beta)(1+\alpha)} \right) (1+z)^n}{A_s(1+z)^n + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)}}. \quad (21)$$

Lastly, we present the another cosmographic parameter, namely, the jerk parameter, which is an important tool to distinguish between dynamical models even if they are equivalent kinematically. The degeneracy of the jerk parameter comes from its definition as a third-order differential of the scale factor. Its value for the Λ CDM model is 1 (constant), independent of the redshift. In terms of q , it is defined as [8]

$$j(z) = q(2q + 1) + (1+z) \frac{dq}{dz}. \quad (22)$$

In a later section, we use the above expression to draw the trajectory of $j(z)$ using best-fit values of the parameters and discuss the behavior accordingly.

5. Particular solution

In this section, we discuss the particular solution of the VGCG model. Assuming $n \neq 0$ and $\alpha = 1$ in equation (14), the VGCG model reduces to the VCG model whose EoS is given by $p_{Ch} = \frac{A_0 a^{-n}}{\rho_{Ch}}$. In this model, the energy density is

$$\rho_{Ch} = \rho_{Ch,0} [A_s(1+z)^n + (1 - A_s)(1+z)^{6(1-\beta)}]^{1/2}, \quad (23)$$

where $A_s = \frac{6(1-\beta)A_0}{\{6(1-\beta) - n\} \rho_{Ch,0}^2}$. Using equation (23) into the Friedmann equation (11), we obtain

$$H^2(z) = H_0^2 [(1 - \Omega_{b,0})(A_s(1+z)^n + (1 - A_s)(1+z)^{6(1-\beta)}) + \Omega_{b,0}(1+z)^3]. \quad (24)$$

The deceleration parameter and EoS parameter for this model can be, respectively, obtained as

$$q(z) = -1 + \frac{1}{2E^2(z)} [3\Omega_{b,0}(1+z)^3 + \frac{(1-\Omega_{b,0})nA_s(1+z)^n + 6(1-A_s)(1-\beta)(1+z)^{6(1-\beta)}}{\{A_s(1+z)^n + (1-A_s)(1+z)^{6(1-\beta)}\}^{1/2}}], \quad (25)$$

and

$$w(z) = -\frac{A_s \left(1 - \frac{n}{6(1-\beta)}\right) (1+z)^n}{A_s(1+z)^n + (1-A_s)(1+z)^{6(1-\beta)}}. \quad (26)$$

6. Observational data

This section addresses the datasets and method to constrain the model parameters of the VGCG and VCG models along with the Λ CDM model. We use the recent observational data from BAO, the cosmic chronometer, standard candles and Hubble data *R21* to constrain the model parameters, which are described below.

6.1. Baryon acoustic oscillations

We use a set of 17 uncorrelated data points from different BAO measurements in [61–74]. The collection of these data points is summarized in table 1 in [75]. In the context of BAO, a significant aspect is the computation of the sound horizon at the time of photon–baryon decoupling, r_d , which is defined as

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz, \quad (27)$$

where $c_s(z)$ symbolizes the baryon–photon fluid’s sound speed, calculated by $c_s(z) \approx c \left(3 + \frac{9\rho_b(z)}{4\rho_\gamma(z)}\right)^{-0.5}$. Here, $\rho_b(z)$ and $\rho_\gamma(z)$ stand for the densities of baryons and photons, respectively. In observational terms, BAO yields two key projected measures: $\Delta z = \frac{r_d H}{c}$ and $\Delta\theta = \frac{r_d}{(1+z)D_A(z)}$, where Δz is the redshift shift, $\Delta\theta$ is the angular displacement related to BAO and $D_A(z)$ is the angular diameter distance. These measures primarily inform about the product $r_d \times H$. To effectively separate these parameters, independent estimations of the Hubble parameter, specifically H_0 , are essential.

6.2. Cosmic chronometers

The cosmic chronometers (CC) approach provides a method for estimating the Hubble parameter $H(z)$ at different redshifts, leveraging the age difference between passive evolving galaxies. This method is based on the differential age approach, where the relative age of galaxies at different redshifts is related to the inverse of the Hubble parameter. We use a set of 30 uncorrelated CC measurements of $H(z)$, as discussed in [76–79]. The χ^2 function is defined as

$$\chi_{CC}^2 = \sum_{i=1}^{30} \frac{[H_{\text{obs}}(z_i) - H_{\text{th}}(z_i)]^2}{\sigma(z_i)^2}, \quad (28)$$

where $H_{\text{th}}(z_i)$ denotes the theoretical value of $H(z)$ at redshift z_i for a given set of cosmological parameters, $H_{\text{obs}}(z_i)$ denotes the observed values of $H(z)$ at redshift z_i and $\sigma(z_i)$ is the uncertainty in the measurement of H_i .

6.3. Standard candles

For standard candles (SC), we use uncorrelated measurement of Pantheon type Ia supernovae (SNe) [80] and measurements from quasars [81] and gamma-ray bursts [82]. The Pantheon dataset follow the luminosity–distance relationship, which can be modeled by $d_L = (1+z) \int_0^z \frac{c}{H(z')} dz'$, where d_L is the luminosity distance, z the redshift, c the speed of light and $H(z')$ is the Hubble parameter as a function of redshift. Gamma-ray bursts (GRBs) are utilized by correlating their observed properties, like peak luminosity or total emitted energy, with distance. Their relationship is given by $L_{GRB} = 4\pi d_L^2 F_{\text{obs}}$ where L_{GRB} is the intrinsic luminosity and F_{obs} the observed flux. Quasars are considered as the correlation between their luminosity and the size of their broad emission line region, estimated via reverberation mapping. Their distance approximation is $d_L \propto \sqrt{\frac{L_{\text{quasar}}}{F_{\text{obs}}}}$, with L_{quasar} representing the intrinsic luminosity.

6.4. R21 measurement

We use $H_0 = 73.04 \pm 1.04 \text{ kms}^{-1} \text{Mpc}^{-1}$ [83] in our analysis which we denote as *R21*. This is obtained from the Cepheid-SNe sample including systemic uncertainties and lies near the median of all analysis variants, which gives a result much closer to the observed in the supernovae sample [84].

We use two different combinations of the above datasets. Combination of the first three sets of data (*BAO + CC + SC*) is referred to as the ‘**BASE**’ dataset in our subsequent analysis. The other set of data is called ‘**BASE+R21**’. We use the total likelihood function to derive the joint constraints from the previously mentioned observational data on the model parameter space ($H_0, \Omega_b, A_s, n, \alpha, \beta$). In our analysis, we perform a global fitting to determine the cosmological parameters using a Markov chain Monte Carlo (MCMC) method. We follow a nested sampler as it is implemented within the open-source package *Polychord* [85] with the *GetDist* package [86] to generate MCMC for different models using the above datasets. We choose the priors for different parameters as $H_0 \in [60; 80]$, $\Omega_b \in [0; 0.04]$, $A_s \in [0; 1]$, $n \in [0; 4]$, $\alpha \in [0; 1]$ and $\beta \in [0; 1]$. We use 100 walkers and 1000 steps to find the best-fit values of the parameters. In what follows we perform combined analysis of BAO, CC, SC and R21 on constraints of the Λ CDM, VGCG and VCG models. We use a χ^2 statistic: $\chi^2 = \chi_{BAO}^2 + \chi_{CC}^2 + \chi_{SC}^2$.

7. Results and discussion

The constraints on different models with and without matter creation for two different combinations of data, namely BASE and BASE + R21, are listed in tables 1 and 2, respectively.

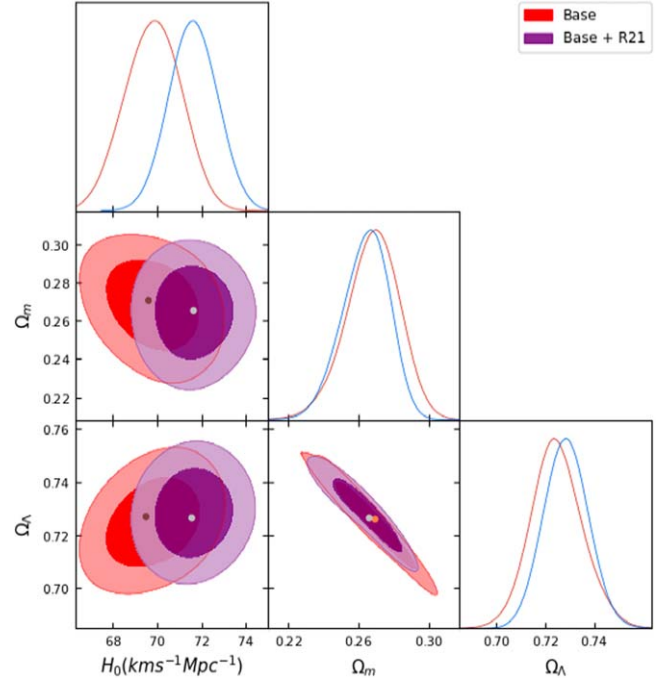
Table 1. The constraints on the parameters of the VGCG and VCG models with matter creation and the Λ CDM model. The combined dataset of $BAO + CC + SC$ is referred as ‘BASE’.

Model	Parameters	BASE	BASE + R21
Λ CDM	H_0 [kms $^{-1}$ Mpc $^{-1}$]	$69.791^{+1.008}_{-1.070}$	$71.432^{+1.111}_{-1.038}$
	Ω_m	$0.271^{+0.018}_{-0.016}$	$0.274^{+0.015}_{-0.018}$
	Ω_Λ	$0.721^{+0.023}_{-0.027}$	$0.726^{+0.022}_{-0.022}$
VGCG	H_0 [kms $^{-1}$ Mpc $^{-1}$]	$69.649^{+1.265}_{-2.379}$	$72.674^{+0.355}_{-0.758}$
	Ω_b	$0.021^{+0.012}_{-0.019}$	$0.020^{+0.011}_{-0.019}$
	A_s	$0.758^{+0.027}_{-0.056}$	$0.767^{+0.026}_{-0.046}$
	β	$0.290^{+0.166}_{-0.254}$	$0.260^{+0.155}_{-0.241}$
	n	$0.114^{+0.090}_{-0.110}$	$0.119^{+0.091}_{-0.114}$
	α	$0.509^{+0.332}_{-0.457}$	$0.450^{+0.284}_{-0.392}$
VCG	H_0 [kms $^{-1}$ Mpc $^{-1}$]	$69.638^{+1.207}_{-2.491}$	$72.680^{+0.399}_{-0.775}$
	Ω_b	$0.020^{+0.013}_{-0.011}$	$0.020^{+0.014}_{-0.016}$
	A_s	$0.756^{+0.021}_{-0.046}$	$0.765^{+0.021}_{-0.022}$
	β	$0.484^{+0.012}_{-0.025}$	$0.481^{+0.011}_{-0.032}$
	n	$0.105^{+0.088}_{-0.105}$	$0.109^{+0.088}_{-0.105}$

Table 2. The constraints on the parameters of VGCG and VCG models without matter creation. The combined dataset of $BAO + CC + SC$ is referred to as ‘BASE’.

Model	Parameters	BASE	BASE + R21
VGCG	H_0 [kms $^{-1}$ Mpc $^{-1}$]	$69.674^{+1.217}_{-0.563}$	$72.670^{+1.326}_{-0.412}$
	Ω_b	$0.020^{+0.015}_{-0.024}$	$0.020^{+0.014}_{-0.015}$
	A_s	$0.767^{+0.012}_{-0.033}$	$0.770^{+0.021}_{-0.016}$
	n	$0.129^{+0.101}_{-0.026}$	$0.116^{+0.004}_{-0.011}$
	α	$0.040^{+0.021}_{-0.017}$	$0.042^{+0.072}_{-0.061}$
VCG	H_0 [kms $^{-1}$ Mpc $^{-1}$]	$69.870^{+1.133}_{-0.365}$	$72.640^{+1.362}_{-0.352}$
	Ω_b	$0.030^{+0.025}_{-0.036}$	$0.030^{+0.005}_{-0.021}$
	A_s	$0.994^{+0.001}_{-0.002}$	$0.994^{+0.017}_{-0.021}$
	n	$1.140^{+0.032}_{-0.101}$	$0.128^{+0.041}_{-0.035}$

As one can see from these tables, the best-fit values of these parameters are nearly the same for different dark energy models. It is to be noted that the inclusion of local measurement $H_0 = 73.04 \pm 1.04$ kms $^{-1}$ Mpc $^{-1}$ from R21 [83] with BASE data, i.e., $BASE + R21$, always makes the H_0 value higher than BASE data for all the three models discussed above. For the Λ CDM model, we obtain $H_0 = 71.432^{+1.111}_{-1.038}$ kms $^{-1}$ Mpc $^{-1}$ for the full datasets, $BASE + R21$ whereas for VGCG and VCG, we get $H_0 = 72.674^{+0.355}_{-0.758}$ kms $^{-1}$ Mpc $^{-1}$ and $H_0 = 72.680^{+0.399}_{-0.775}$ kms $^{-1}$ Mpc $^{-1}$, respectively. The residual tensions of these fitting results with R21 are 0.95σ , 0.36σ and 0.34σ , respectively. On the other hand, we find $H_0 = 69.791^{+1.008}_{-1.070}$ kms $^{-1}$ Mpc $^{-1}$, $H_0 = 69.649^{+1.265}_{-2.379}$ kms $^{-1}$ Mpc $^{-1}$ and $H_0 = 69.638^{+1.207}_{-2.491}$ kms $^{-1}$ Mpc $^{-1}$ with BASE data with the Λ CDM, VGCG and VCG models, respectively. The R21 value of H_0 exhibits a strong tension with the Planck 2018 release $H_0 = 67.4 \pm 0.5$ [10] at 3.92σ CL. Using the BASE dataset,

**Figure 1.** 2D contour at 68.3% and 95.4% confidence levels for the Λ CDM model using BASE and $BASE + R21$ datasets.

we find that the current H_0 tension can be alleviated to 2.19σ in the Λ CDM model, 1.51σ in the VGCG model and 1.84σ in the VCG model. In figures 1–3, we show the constrained contours at 68.3% and 95.4% confidence levels in various parameter planes using the two different combinations of data for Λ CDM, VGCG and VCG models, respectively.

In what follows, we discuss the evolutions of different cosmological parameters, the stability of the models, selection criteria and Bayesian evidence analysis to compare the proposed models with the existing dark energy models.

7.1. Evolution of cosmological parameters

The evolutions of $H(z)$ with respect to the redshift z for VGCG and VCG models with and without matter creation are shown in figures 4–7. The figures also show a comparison of VGCG and VCG models with the Λ CDM model. As observed, the VCG model closely matches the observed data during the whole evolution, whereas the VGCG model coincides during the late-time evolution of the Universe.

Using BASE and $BASE + R21$ datasets, we obtain the present values of $q(z)$, $w(z)$ and $j(z)$ for Λ CDM, VGCG and VCG models with and without matter creation, which are summarized in tables 3 and 4. The evolutions of $q(z)$ with redshift z of all three models for BASE and $BASE + R21$ datasets are shown in figures 8 and 9, respectively. We observe that the models show a transition from deceleration ($q(z) > 0$) to acceleration ($q(z) < 0$). In figures, a dot represents the present value of $q(z)$. The present values of $q(z)$ at $z = 0$ from BASE and $BASE + R21$ datasets are $q_0 = -0.591^{+0.01}_{-0.01}$ and $q_0 = -0.590^{+0.02}_{-0.02}$ for the Λ CDM model, $q_0 = -0.541^{+0.02}_{-0.02}$ and $q_0 = -0.542^{+0.05}_{-0.05}$ for the VGCG model and $q_0 = -0.553^{+0.03}_{-0.03}$ and $q_0 = -0.555^{+0.03}_{-0.03}$ for the VCG model with

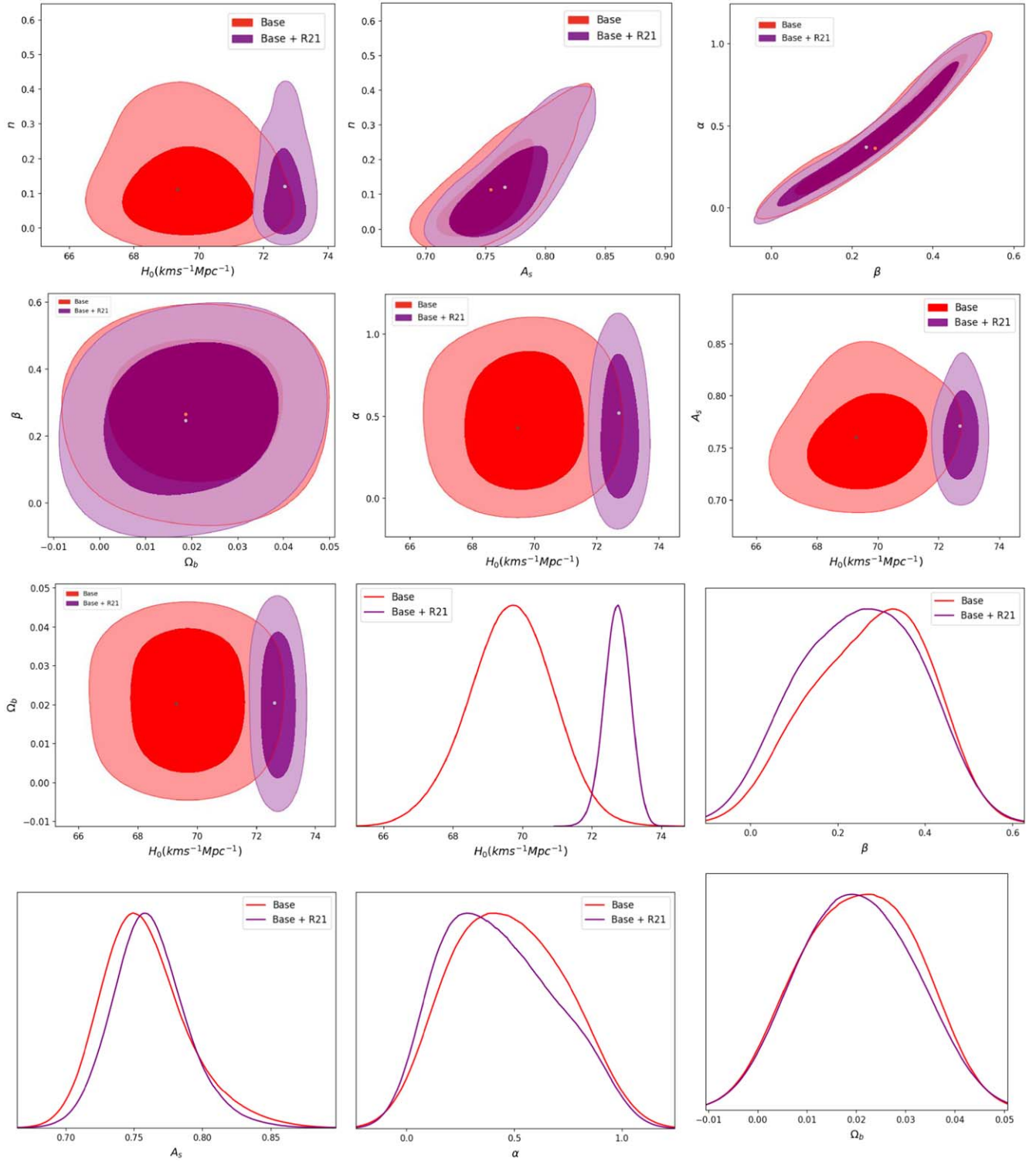


Figure 2. 2D contours at 68.3% and 95.4% confidence levels for pairs (n, H_0) , (n, A_s) , (α, β) , (β, Ω_b) , (α, H_0) , (A_s, H_0) and (Ω_b, H_0) , and 1D posterior distribution of H_0 , β , A_s , α and Ω_b for the VGCG model with matter creation using BASE and BASE+R21 datasets, respectively.

matter creation, respectively, as given in table 3. The values of q_0 show that the models undergo accelerated expansion at present-day. We obtain the transition value z_{tr} where the models transit from deceleration to acceleration at $z_{tr} = 0.610^{+0.030}_{-0.022}$ and $z_{tr} = 0.631^{+0.037}_{-0.023}$ for the VGCG model, and $z_{tr} = 0.674^{+0.029}_{-0.016}$ and $z_{tr} = 0.687^{+0.036}_{-0.031}$ in the case of the VCG

model, respectively. These transition values are very close to the Λ CDM transition values (see table 3). We also present the values of z_{tr} and q_0 in table 4 for these models without matter creation.

The evolutions of the EoS parameter $w(z)$ with z of Λ CDM, VGCG and VCG models are shown in figures 10 and 11 with

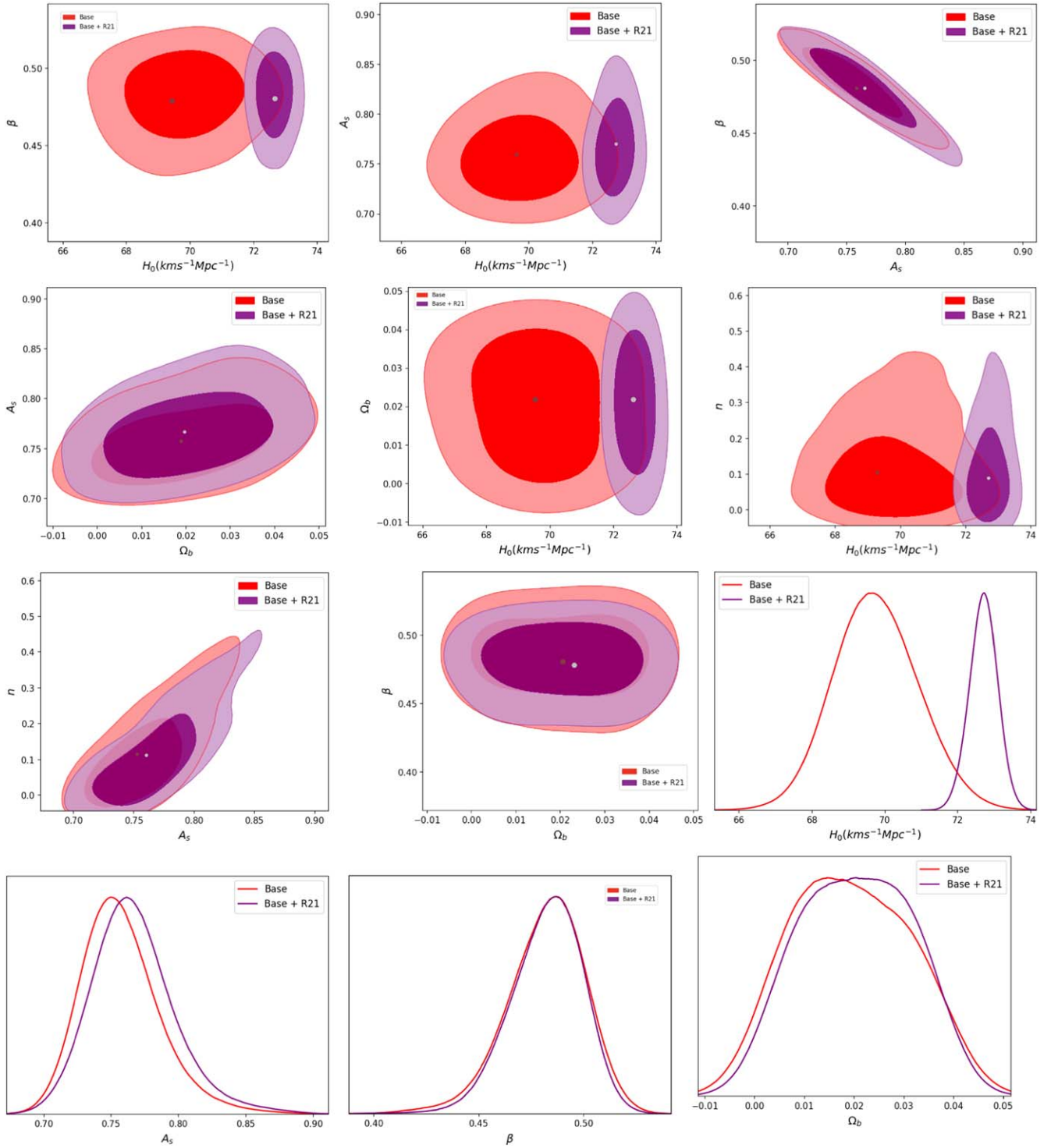


Figure 3. 2D contour at 68.3% and 95.4% confidence levels for pairs (β, H_0) , (A_s, H_0) , (β, A_s) , (A_s, Ω_b) , (Ω_b, H_0) , (n, H_0) , (n, A_s) and (β, Ω_b) , and 1D posterior distribution of H_0 , A_s , β and Ω_b for the VCG model with matter creation using BASE and BASE+R21 datasets.

two datasets. For both datasets, the EoS shows a quintessence phase $(-1 < w < 0)$ at present and Λ CDM behavior at a late epoch. Figures 12 and 13 show the evolutions of the jerk parameter $j(z)$ with respect to z for the discussed models using BASE and BASE + R21 datasets. Based on these two datasets, the present values of $j(z)$ for the VGCG model are $j_0 = 0.933^{+0.21}_{-0.21}$ and $j_0 = 0.941^{+0.10}_{-0.10}$, whereas for the

VCG model we obtain $j_0 = 0.979^{+0.02}_{-0.02}$ and $j_0 = 0.966^{+0.02}_{-0.02}$, respectively, which are very close to the value $j_0 = 1$ of the Λ CDM model.

The ages of the Universe for all the models are given in table 3. The constraints on the age of the Universe for the VGCG model are $t_0 = 13.91^{+0.03}_{-0.03}$ Gyr with the BASE dataset and $t_0 = 13.94^{+0.07}_{-0.07}$ Gyr with the BASE + R21 dataset,

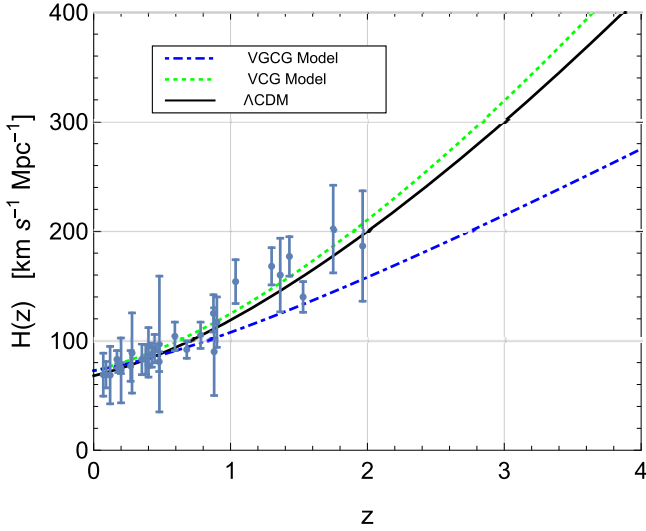


Figure 4. Evolution of $H(z)$ with redshift z in VGCG and VCG models with matter creation, and the Λ CDM model using the BASE dataset. The grey points with uncertainty bars corresponds to the $H(z)$ sample. The black bold line represents the Λ CDM model.

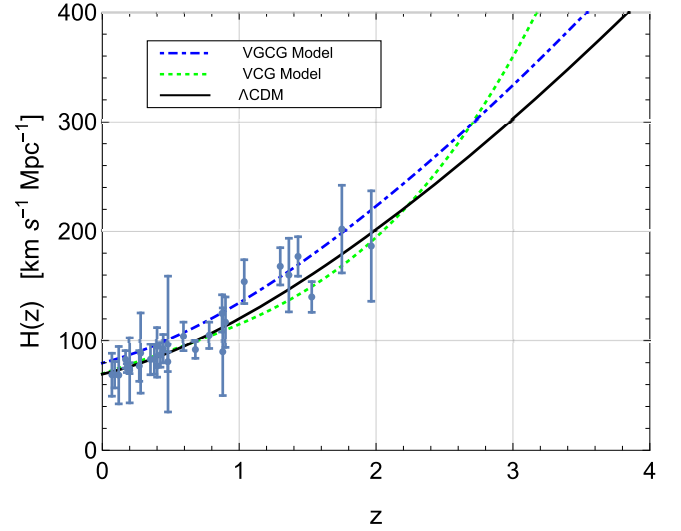


Figure 6. Evolution of $H(z)$ with redshift z in VGCG and VCG models without matter creation, and the Λ CDM model using the BASE dataset. The grey points with uncertainty bars corresponds to the $H(z)$ sample. The black bold line represents the Λ CDM model.

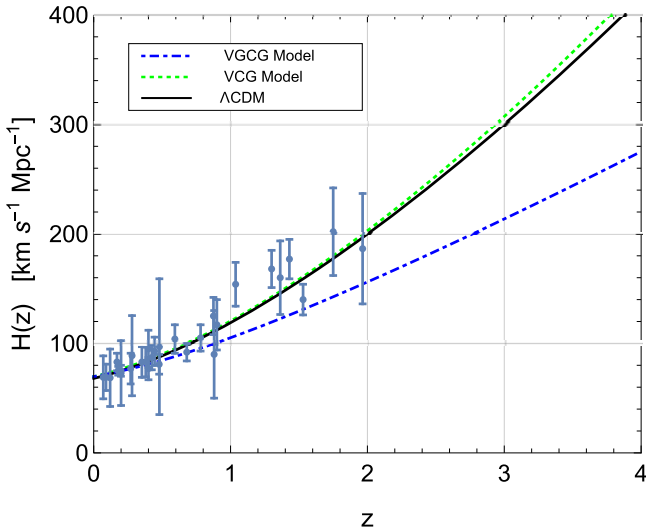


Figure 5. Evolution of $H(z)$ with redshift z in VGCG and VCG models with matter creation, and the Λ CDM model using the BASE+R21 dataset. The grey points with uncertainty bars corresponds to the $H(z)$ sample. The black bold line represents the Λ CDM model.

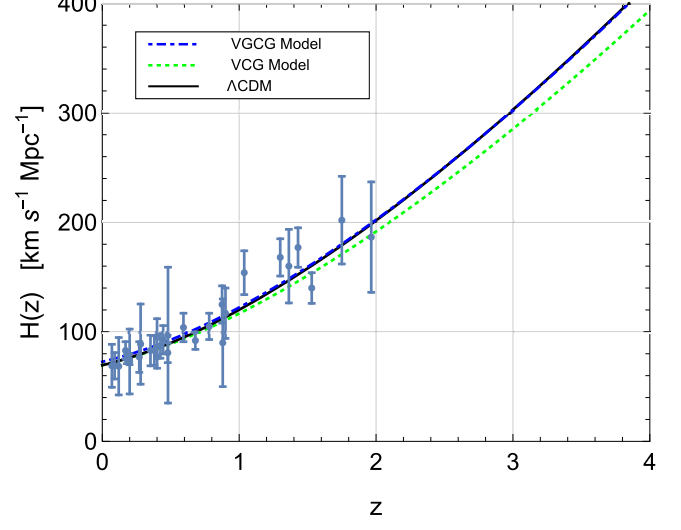


Figure 7. Evolution of $H(z)$ with redshift z in VGCG and VCG models without matter creation, and the Λ CDM model using the BASE+R21 dataset. The grey points with uncertainty bars corresponds to the $H(z)$ sample. The black bold line represents the Λ CDM model.

whereas for the VCG model we have $t_0 = 13.87^{+0.03}_{-0.03}$ Gyr and $t_0 = 13.89^{+0.01}_{-0.01}$ Gyr, respectively. The ages of the Universe for these CG models are very close to the age of the Universe for the Λ CDM model. In table 4, we list the present values of z_{tr} , q_0 , w_0 , j_0 and t_0 for the proposed models without matter creation to compare the deviation for constraint results on extended CG models with matter creation.

7.2. Stability of the model

In order to determine the stability of the models, we analyze the squared sound speed of the VGCG and VCG fluids with and without matter creation. For any cosmic fluid model, the

squared sound speed $\mathcal{C}_s^2$ can be estimated from the relation: $\mathcal{C}_s^2 = dp_{Ch}/d\rho_{Ch}$. It is noted that $\mathcal{C}_s^2 \geq 0$ corresponds to stable behavior, while $\mathcal{C}_s^2 < 0$ leads to unstable behavior of the model. Thus, the physical values of $\mathcal{C}_s^2$ for a stable model must be in the range of (0, 1). Outside of this region, one encounters gradient/tachyonic instabilities and/or instabilities linked to superluminal propagation.

The squared sound speed for the VGCG model is given by

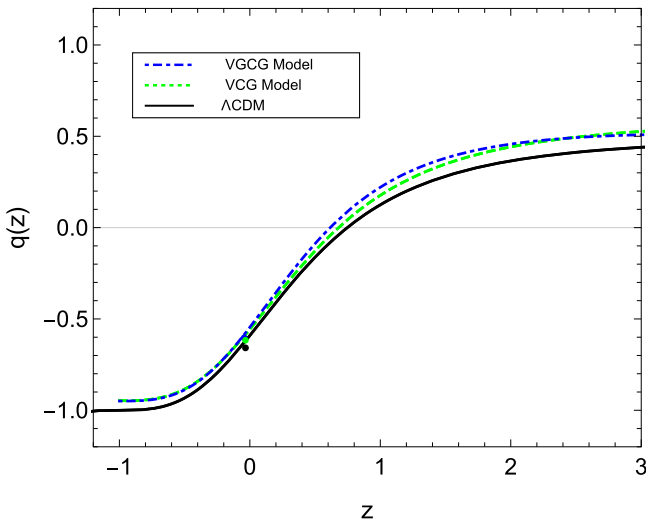
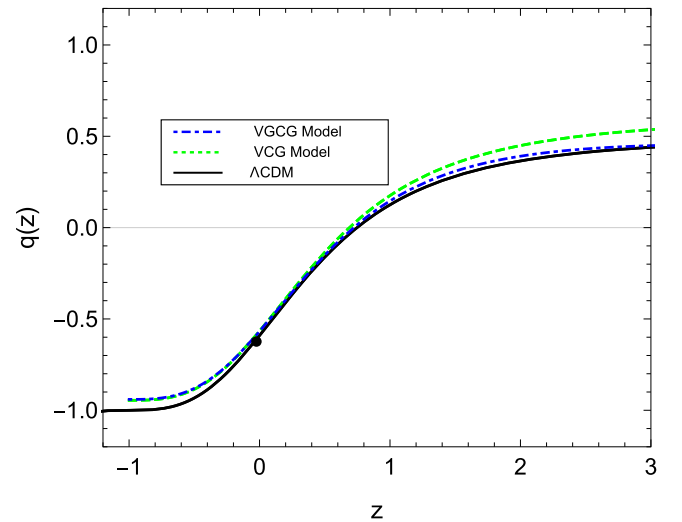
$$\mathcal{C}_s^2 = \frac{\alpha A_s \left(1 - \frac{n}{3(1-\beta)(1+\alpha)}\right) (1+z)^n}{A_s (1+z)^n + (1-A_s)(1+z)^{3(1-\beta)(1+\alpha)}}. \quad (29)$$

Table 3. The present values of different cosmological and geometrical parameters of Λ CDM, VGCG and VCG models with matter creation.

Models → Parametrs	Λ CDM		VGCG		VCG	
	BASE	+R21	BASE	+R21	BASE	+R21
z_{tr}	$0.692^{+0.034}_{-0.029}$	$0.689^{+0.028}_{-0.033}$	$0.610^{+0.030}_{-0.022}$	$0.631^{+0.037}_{-0.023}$	$0.674^{+0.029}_{-0.016}$	$0.687^{+0.036}_{-0.031}$
q_0	$-0.591^{+0.01}_{-0.01}$	$-0.590^{+0.02}_{-0.02}$	$-0.541^{+0.02}_{-0.02}$	$-0.542^{+0.05}_{-0.05}$	$-0.553^{+0.03}_{-0.03}$	$-0.555^{+0.03}_{-0.03}$
w_0	$-0.726^{+0.04}_{-0.04}$	$-0.728^{+0.01}_{-0.01}$	$-0.730^{+0.11}_{-0.11}$	$-0.740^{+0.13}_{-0.13}$	$-0.721^{+0.02}_{-0.02}$	$-0.728^{+0.02}_{-0.02}$
t_0 (Gyr)	$13.73^{+0.02}_{-0.02}$	$13.76^{+0.04}_{-0.04}$	$13.91^{+0.03}_{-0.03}$	$13.94^{+0.07}_{-0.07}$	$13.87^{+0.03}_{-0.03}$	$13.89^{+0.01}_{-0.01}$
j_0	1	1	$0.933^{+0.21}_{-0.21}$	$0.941^{+0.10}_{-0.10}$	$0.979^{+0.02}_{-0.02}$	$0.966^{+0.02}_{-0.02}$
C_s^2	—	—	0.329	0.379	0.730	0.740

Table 4. The present values of different cosmological and geometrical parameters of VGCG and VCG models without matter creation.

Models → Parametrs	VGCG		VCG	
	BASE	+R21	BASE	+R21
z_{tr}	$0.689^{+0.023}_{-0.021}$	$0.694^{+0.019}_{-0.024}$	$0.955^{+0.018}_{-0.023}$	$0.922^{+0.018}_{-0.014}$
q_0	$-0.652^{+0.02}_{-0.02}$	$-0.617^{+0.02}_{-0.02}$	$-0.590^{+0.02}_{-0.02}$	$-0.573^{+0.02}_{-0.02}$
w_0	$-0.589^{+0.11}_{-0.11}$	$-0.595^{+0.15}_{-0.15}$	$-0.838^{+0.01}_{-0.01}$	$-0.842^{+0.02}_{-0.02}$
t_0 (Gyr)	$13.77^{+0.01}_{-0.01}$	$13.83^{+0.03}_{-0.03}$	$13.91^{+0.02}_{-0.02}$	$13.93^{+0.01}_{-0.01}$
j_0	$0.959^{+0.19}_{-0.19}$	$0.742^{+0.21}_{-0.21}$	$0.892^{+0.11}_{-0.11}$	$1.01^{+0.12}_{-0.12}$
C_s^2	0.414	0.490	0.781	0.805

**Figure 8.** Evolution of $q(z)$ with z in VGCG and VCG models with matter creation, and the Λ CDM model using the BASE dataset. The dots represent the present value of $q(z)$.**Figure 9.** Evolution of $q(z)$ with z in VGCG and VCG models with matter creation, and the Λ CDM model using the BASE+R21 dataset. The dots represent the present value of $q(z)$.

and for VCG model, we have

$$C_s^2 = \frac{A_s \left(1 - \frac{n}{6(1-\beta)}\right) (1+z)^n}{A_s (1+z)^n + (1-A_s)(1+z)^{6(1-\beta)}}. \quad (30)$$

The sound speed should be positive in the stable model and, hence, this is justified by taking the constant $\alpha > 0$. Using the best-fit values of model parameters from the BASE and BASE+R21 datasets in the above equations, we find the current values $C_s^2 = 0.329$ and $C_s^2 = 0.379$ for the VGCG model, and $C_s^2 = 0.730$ and $C_s^2 = 0.740$ for VCG model,

which are listed in table 3. We plot the trajectories of C_s^2 with z in figures 14 and 15 using two combinations of datasets for VGCG and VCG models. The trajectories show that the squared sound speed is always positive throughout the evolution lying in the range (0, 1) and, hence, positive and less than unity in both the discussed models. This ensures that these models do not surpass the velocity of light, which confirms their stability. Table 4 shows C_s^2 for VGCG and VCG models without matter creation. The C_s^2 values are positive and less than unity, which also confirms the stability.

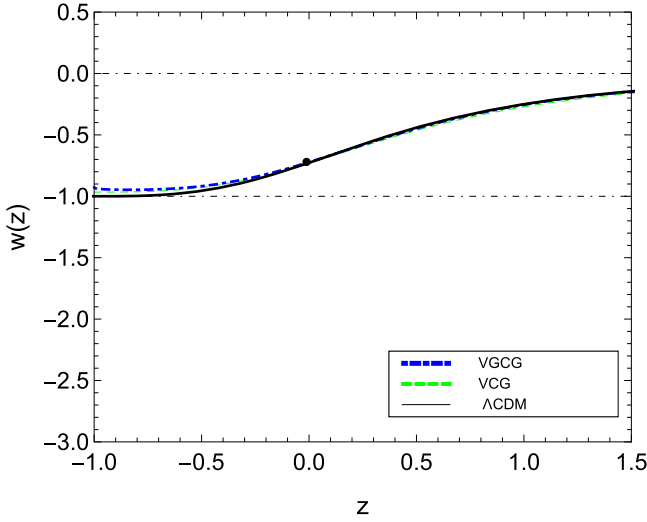


Figure 10. Evolution of $w(z)$ with z in VGCG and VCG models with matter creation, and the Λ CDM model using the BASE dataset. The dots represent the present value of $w(z)$.

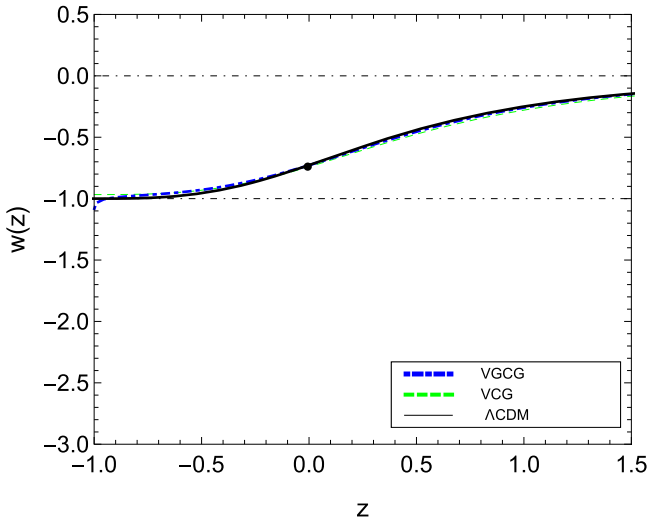


Figure 11. Evolution of $w(z)$ with z in VGCG and VCG models with matter creation, and the Λ CDM model using the BASE+R21 dataset. The dots represent the present value of $w(z)$.

7.3. Model selection

We calculate the values of $\chi^2_{\min}/dof$ for Λ CDM, VGCG and VCG models with and without matter creation, shown in tables 5 and 6, respectively. Here, the value of dof (degree of freedom) equals the number of observational data points (N) minus the number of parameters (K). The number of observational data points for the BASE dataset is $N = 273$ (CC = 30, type Ia supernova (Pantheon) = 40, uncorrelated BAO = 17, Hubble diagram for quasars = 24 and gamma-ray bursts = 162), and $N = 274$ for the BASE+R21 dataset, and the number of parameters are $K = 3, 6$ and 5 for Λ CDM, VGCG and VCG models with matter creation, and $K = 5$ and 4 for VGCG and VCG models without matter creation, respectively. From tables 5 and 6, we observe that both the models with and without matter creation have $\chi^2_{\min}/dof < 1$ for both datasets. This shows that both the models are in good

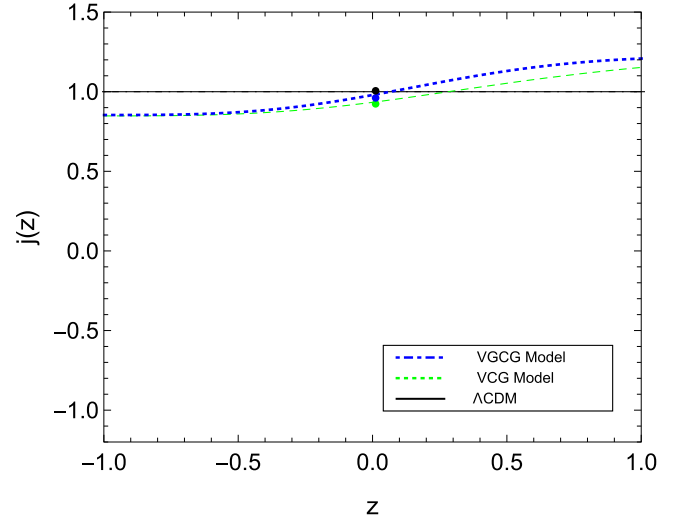


Figure 12. Trajectories of $j(z)$ for VGCG and VCG models with matter creation, and the Λ CDM model using best-fit values from the BASE dataset. The dots represent the present value of j_0 .

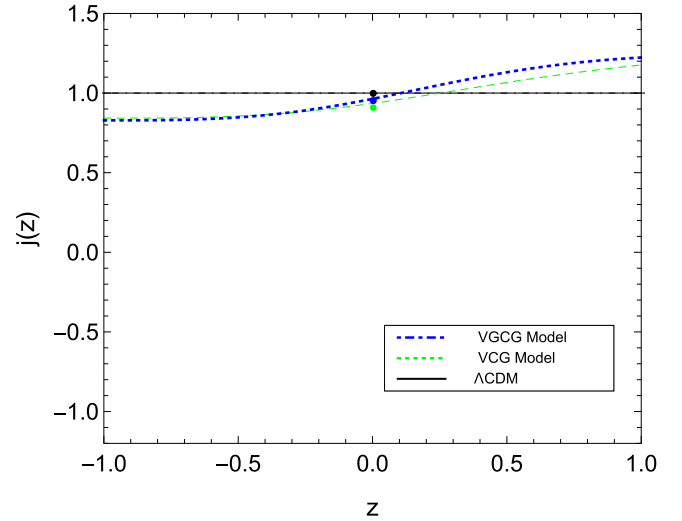


Figure 13. Trajectories of $j(z)$ for VGCG and VCG models with matter creation, and the Λ CDM model using the BASE+R21 datasets. The dots represent the present value of j_0 .

agreement with respect to the observational data as far as $\chi^2_{\min}/dof$ is concerned.

Although the above analyses are in good agreement with one another, further differences arise due to the choice of cosmological model made.

In what follows, we use the information criteria (IC), which have a deep underpinning in the theory of statistical inference. There are two such IC, namely, Akaike information criteria (AIC) [87] and Bayesian information criteria (BIC) [88], which are, respectively, defined as

$$AIC = \chi^2_{\min} + \frac{2NK}{N - K - 1}, \quad (31)$$

$$BIC = \chi^2_{\min} + K \ln N, \quad (32)$$

where $\chi^2_{\min}$ measures the quality of the model and K , the number of parameters, indicates model complexity. We

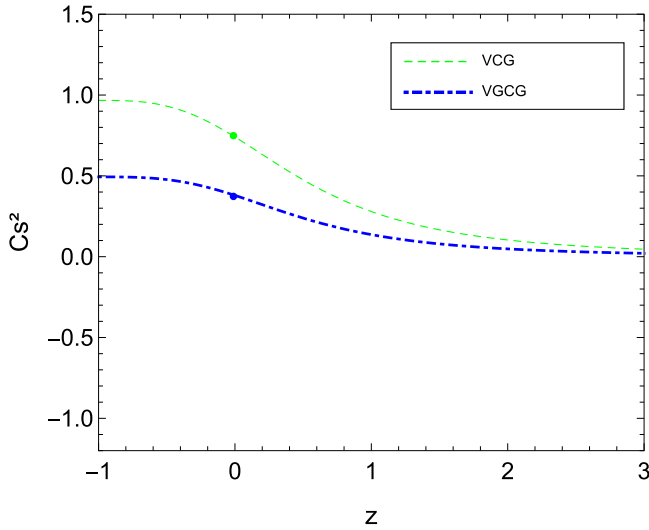


Figure 14. Trajectories of C_s^2 for VGCG and VCG models with matter creation using best-fit values of the BASE dataset.

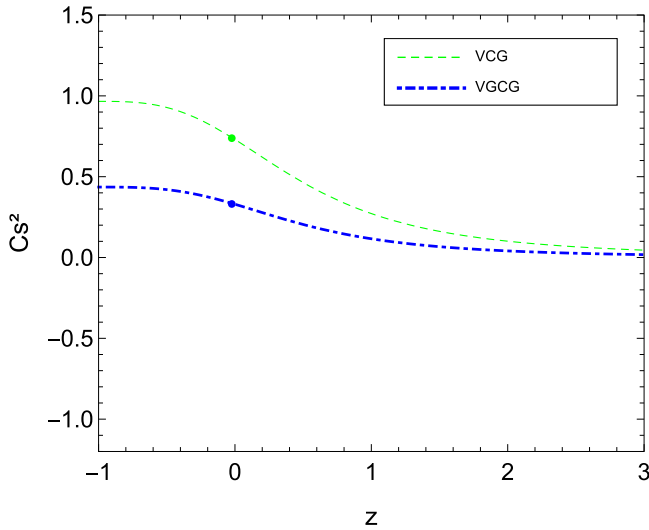


Figure 15. Trajectories of C_s^2 for VGCG and VCG models with matter creation using best-fit values of the BASE+R21 dataset.

calculate the relative values between different models. The one that minimizes the AIC (or BIC) is usually considered the best model. Using the Λ CDM as the reference model, we define the AIC and BIC differences as follows: $\Delta AIC = AIC_x - AIC_{\Lambda CDM}$ and $\Delta BIC = BIC_x - BIC_{\Lambda CDM}$, where x represents either the VGCG or the VCG model. This gives the assessment of the strength of the models. The criteria for judging the AIC model [89] are as follows: when $0 \leq \Delta AIC < 2$, the model has almost the same support from the data as the best model; for $2 < \Delta AIC \leq 4$, the model is considerably less supported; and the range $4 < \Delta AIC < 10$ still supports the considered model. The observational data do not support the model at values of $\Delta AIC > 10$.

In the case of BIC, the range $\Delta BIC < 2$ is considered to support the model from observational data. There is slight evidence against the considered model if $2 < \Delta BIC < 6$. For a

Table 5. χ^2_{red} , AIC (BIC), Bayesian evidence and Bayes factor values for Λ CDM, VGCG and VCG models with matter creation. Here the Λ CDM model is taken as a reference model to calculate ΔAIC , ΔBIC and $\ln B$.

Model	Parameters	BASE	+R21
Λ CDM	χ^2_{min}	253.92	254.17
	χ^2_{min}/dof	0.94	0.94
	AIC	260.01	260.26
	BIC	270.75	271.00
	$\ln E(M1)$	-123.824	-124.116
VGCG	χ^2_{min}	255.51	257.74
	χ^2_{min}/dof	0.965	0.953
	AIC	267.82	270.05
	BIC	289.16	291.41
	ΔAIC	7.81	9.79
	ΔBIC	18.41	20.41
	$\ln E(M2)$	-125.938	-126.099
$\ln B_{12}$		2.114	1.983
Evidence interpretation		positive	positive
VCG	χ^2_{min}	254.59	257.63
	χ^2_{min}/dof	0.95	0.96
	AIC	264.81	267.86
	BIC	282.64	285.70
	ΔAIC	4.80	7.60
	ΔBIC	11.89	14.70
	$\ln E(M2)$	-126.513	-126.697
	$\ln B_{12}$	2.689	2.581
Evidence interpretation		moderate	moderate

Table 6. χ^2_{red} , AIC (BIC), Bayesian evidence and Bayes factor values for VGCG and VCG models without matter creation. Here, the Λ CDM model is taken as a reference model to calculate ΔAIC , ΔBIC and $\ln B$.

Model	Parameters	BASE	+R21
VGCG	χ^2_{min}	255.32	257.94
	χ^2_{min}/dof	0.956	0.962
	AIC	265.54	268.16
	BIC	283.36	286.01
	ΔAIC	5.53	7.92
	ΔBIC	12.61	15.01
	$\ln E(M2)$	-126.095	-125.486
$\ln B_{12}$		2.271	1.370
Evidence interpretation		Positive	Positive
VCG	χ^2_{min}	254.21	255.76
	χ^2_{min}/dof	0.959	0.950
	AIC	262.36	263.91
	BIC	276.65	278.21
	ΔAIC	2.35	3.67
	ΔBIC	5.90	7.21
	$\ln E(M2)$	-132.818	-132.434
	$\ln B_{12}$	8.994	8.318
Evidence interpretation		Decisive	Decisive

Table 7. Jeffrey's Interpretive scales for the strength of evidence when comparing two models, M_1 and M_2 .

$\ln B_{12}$	Strength of evidence
<1	Inconclusive
$1.0 - 2.5$	Positive evidence
$2.5 - 5.0$	Moderate evidence
>5	Decisive

value in the range $6 < \Delta BIC < 10$, we obtain strong evidence against the considered model; if $\Delta BIC > 10$, we obtain even stronger evidence against the model [90].

The AIC (BIC) and $\Delta AIC(\Delta BIC)$ for VGCG and VCG models with and without matter creation against the Λ CDM model are listed in tables 5 and 6, respectively. The ΔAIC values of VGCG and VCG models show that these models are still supported by current observational data. For the BIC selection method, it seems that these models are not favored by observational data. The reason is that AIC is always more general towards extra parameters, whereas BIC tends to penalize them. Hence, the AIC remains useful as it gives an upper limit to the number of parameters.

7.4. Bayesian evidence analysis

In this subsection, we first briefly review the basis of Bayesian inference and model comparison. We then present the computation of the Bayes factor of the two proposed models. The Bayesian evidence applies the same type of likelihood analysis familiar from parameter estimation, but at the level of models rather than parameters. It depends on goodness of fit across the entire model parameter space [91]. It comes from a full implementation of Bayesian inference at the model level, and is the probability of the data given the model. It updates the prior model probability to the posterior model probability using Bayes theorem. The Bayesian evidence is defined as [91–93]

$$E = \int \mathcal{L}(\theta) P(\theta) d\theta, \quad (33)$$

where θ is the vector of parameters, $\mathcal{L}$ is the likelihood and $P(\theta)$ is the prior distribution of those parameters before the data were obtained. The evidence of a model is thus the average likelihood of the model in the prior. The Bayesian evidence rewards model predictiveness. Hence, the evidence will not penalize the extra parameter in this case, because it does not change the model parameters.

The ratio of the evidence of two models, say M_1 and M_2 , is known as the Bayes factor, which is defined as [94]

$$B_{12} = \frac{E(M_1)}{E(M_2)}. \quad (34)$$

The Bayes factor provides a measure with which to discriminate between the models. It is usual to consider the logarithm of the Bayes factor, for which the so-called Jeffreys scale [95, 96] gives empirically calibrated levels of significance for the strength of evidence. Many papers on

Bayesian evidence have adopted the interpretive scale suggested by Jeffreys, which is presented in table 7. For each model and dataset we estimate the values of the logarithm of the Bayesian evidence ($\ln E$) and the Bayes factor ($\ln B$), which is the difference between the mean $\ln E$ of the Λ CDM model and the model concerned. These values are obtained by taking the priors and the dataset described in section 6. We assumed the Λ CDM model as the reference one, say M_1 , and, therefore, its entry is zero by definition.

Table 5 presents the Bayesian evidence and Bayes factor for VGCG and VCG models with matter creation. Looking at the BASE and BASE+R21 datasets, respectively, we find $\ln B_{12} = 2.114$ and $\ln B_{12} = 1.983$ for the VGCG model, and $\ln B_{12} = 2.689$ and $\ln B_{12} = 2.581$ for the VCG model, which suggest that the VGCG model has positive evidence, while the VCG model has moderate evidence.

Similarly, using the priors and the same datasets, we obtain the Bayes factor for VGCG and VCG models without matter creation, which are provided in table 6. By considering the BASE data, we find $\ln B_{12} = 2.271$ for the VGCG model and $\ln B_{12} = 8.994$ for the VCG model. The BASE+R21 data give $\ln B_{12} = 1.37$ for the VGCG model and $\ln B_{12} = 8.318$ for the VCG model. This suggests that the VGCG model has positive evidence from both the datasets, whereas the VCG model has strong (decisive) evidence.

8. Conclusion

In this paper, we have examined the VGCG model inspired by the matter creation mechanism in cosmology. Our motivation for this study was to investigate the effects of matter creation on the evolution of the Universe filled by VGCG as a perfect fluid using the latest cosmological observations. In particular we have proposed two CG models with matter creation, i.e., VGCG and VCG and have investigated their evolutionary behaviors via observational datasets. We have placed constraints on them using two different data combinations: BASE (BAO+CC+SC) and BASE+R21. We have found that the VGCG, VCG and Λ CDM models give a similar $\chi^2_{\min}$ and, correspondingly, $\chi^2_{\min}/dof$. In what follows, we summarize the results and evolutions of these models.

As stated above, the focus has been on exploring the cosmological models influenced by gravitational matter creation, specifically considering an extended version of the CG with EoS $p_{Ch} = -\frac{A(a)}{\rho_{Ch}^\alpha}$ where $0 < \alpha \leq 1$. This approach is utilized to examine the late-stage acceleration of the Universe within the context of a flat FLRW space-time. Additionally, we have delved into a special scenario where $\alpha = 1$, corresponding to the VCG with EoS $p_{Ch} = -\frac{A(a)}{\rho_{Ch}}$, and incorporated the concept of matter creation. A parallel discussion on these models without matter creation has also been presented.

In the preliminary discussion, it has been explained that matter creation induced by gravity can be regarded as an irreversible scalar phenomenon, contributing an extra decay

(or creation) pressure to the energy–momentum tensor in the cosmological fluid mixture. This supplementary creation pressure is characterized by the matter creation rate, denoted as Γ , along with other relevant physical parameters of the fluid. Studies have shown that viewing the Universe as an open system with ongoing matter creation aids in comprehending its current rapid expansion phase. As the nature of the matter creation rate Γ is still unknown, we have adopted a phenomenological approach, assuming Γ evolves as $\Gamma \propto H$. These models were found to be consistently either decelerating or accelerating, lacking a transition from a deceleration phase to an acceleration phase, contradicting the SNe observational data. Nevertheless, our findings indicate that by incorporating the VGCG and VCG equations of state in the matter creation model, a phase transition does occur, even with the aforementioned form of Γ .

The initial section of this manuscript details the analytical solutions derived for matter creation models applying both VGCG and VCG equations of state. Solutions for various cosmological parameters such as energy density, Hubble parameter, deceleration parameter, EoS parameter and jerk parameter have been deduced as functions of redshift. In the manuscript's latter section, two different joint observational analyses have been performed to constrain the parameters of the model. Tables 1 and 2 present the best-fit values for these model parameters with and without matter creation. We observe that VGCG and VCG models with matter creation can alleviate the current H_0 tension from 3.92σ to 1.51σ and 1.84σ using the BASE data. The current values for $q(z)$, $j(z)$, $w(z)$, $t(z)$ and $\mathcal{C}_S^2$ are listed in tables 3 and 4 and their evolutions have been plotted in figures 4–13. Our observations indicate a notable variance in the VGCG and VCG models with parameter β from the Λ CDM model. These plots vividly illustrate the models' capability to generate late-stage cosmic acceleration while also reflecting a decelerated expansion in earlier times. The transition redshift values, q_0 , w_0 and age of the Universe with respect to VGCG and VCG models with and without matter creation are found to be very close to the Λ CDM model and the Planck 2018 release. The evolution trajectories of $j(z)$ reveal minimal deviation from the Λ CDM model for both VGCG and VCG models.

The AIC and BIC analysis show that models such as VGCG and VCG with and without matter creation exhibit in the range of $4 < \Delta AIC < 10$, suggesting to as much as the preferred model with the joint observational data. In terms of BIC values, the models VGCG and VCG with and without matter creation present $\Delta BIC > 10$, providing evidence against the data used for analysis. It is noteworthy that the squared sound speed for these models lies within the $0 < \mathcal{C}_S^2 < 1$ range, thereby not exceeding the speed of light, which signifies the stability of these models. Consequently, it is deduced that the models discussed in this paper yield cosmological parameters that are well bounded by the combined observational data. Furthermore, these models closely resemble the Λ CDM model in their behavior.

In statistical sense, a physical model may be thought of as described by a set of parameters. The determination of

these parameters may be carried out in many ways: the most commonly used framework to accomplish this is Bayesian evidence analysis. We have calculated the Bayesian evidence and, consequently, the Bayes factor to measure the discrimination between the models. Assuming priors for different parameters and the Λ CDM model as the reference one, we have found that the VGCG model with and without matter creation has positive evidence on the Jeffrey scale, whereas the VCG model has moderate evidence with matter creation, and has decisive evidence without matter creation by both datasets.

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Cosmological dynamics of matter creation with modified Chaplygin gas and bulk viscosity

Yogesh Bhardwaj^a , C. P. Singh^b

Department of Applied Mathematics, Delhi Technological University, Bawana Road, Delhi 110042, India

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Abstract This work presents a comprehensive investigation of a novel cosmological model that includes the Modified Chaplygin Gas (MCG) equation of state with gravitationally induced matter creation and bulk viscous dissipation in a spatially flat Friedmann–Lemaître–Robertson–Walker spacetime. The MCG fluid is characterized by an exotic equation of state $p = A\rho - C/\rho^\alpha$. The matter creation rate is taken as $\Gamma = 3\beta H$ and the bulk viscous pressure as $\pi = -3H\xi_0\rho_m^{1/2}$. We derive the modified Friedmann equations and obtain an analytical expression for the Hubble parameter $H(z)$, which is then used to reconstruct the evolutionary trajectories of key cosmological parameters. The model parameters are constrained using two observational datasets: DS1 (Pantheon+ + Cosmic Chronometers + DESI BAO + σ_8) and DS2 (DS1 + R22). Our results indicate that the proposed hybrid model successfully generates a transition from decelerated to accelerated expansion, consistent with current observations. Notably, when the R22 prior is included the best-fit value of H_0 shifts towards the local SH0ES determination; the hybrid model is flexible enough to accommodate this higher H_0 while preserving a good fit to the geometric datasets, thereby reducing the level of tension in joint analyses between local and background probes, although it does not dynamically resolve the H_0 discrepancy by itself. Furthermore, we perform a rigorous thermodynamic analysis of the model by testing the Generalized Second Law (GSL) of thermodynamics. We use the information criteria, like Akaike and Bayesian to check the model stability. This work establishes a physically motivated, observationally viable, and thermodynamically consistent alternative to the standard Λ CDM paradigm.

1 Introduction

The observational confirmation of the late-time accelerated expansion of the Universe, first inferred from high-redshift Type Ia supernovae [1, 2] stands as one of the most consequential discoveries in modern cosmology. Subsequent precision measurements from the Cosmic Microwave Background (CMB) anisotropies [3], Baryon Acoustic Oscillations (BAO) [4], and differential age estimates of cosmic chronometers [5] have solidified this paradigm, attributing the acceleration to a dominant, negative-pressure component termed dark energy. Within the framework of general relativity, the Λ CDM model, which identifies as a dark energy with Einstein's cosmological constant Λ , provides an empirically robust fit to the data. Yet, it remains theoretically unsatisfactory due to the fine-tuning and cosmic coincidence problems [6, 7], which have spurred the development of alternative cosmological scenarios.

The fine-tuning problem arises from the staggering discrepancy between the theoretically predicted vacuum energy density from quantum field theory and the observationally inferred value of Λ required to drive cosmic acceleration. The cosmic coincidence problem, meanwhile, questions why the energy densities of dark energy and matter are comparable only at the present cosmological epoch, despite evolving with vastly different scaling laws ($\rho_\Lambda \sim \text{constant}$, $\rho_m \sim a^{-3}$). These conceptual tensions have motivated two broad classes of alternatives: (i) dark energy models that postulate new dynamical fields or exotic fluids within general relativity, and (ii) modified gravity theories that alter the geometric sector of Einstein's equations [8, 9].

Within the first class, the Chaplygin gas models have emerged as a particularly compelling candidate due to its capacity to unify dark matter and dark energy within a single fluid description [10]. Originally conceived in aerodynamics to model the lifting force on an aircraft wing [11], the Chaply-

^a e-mail: yogeshbhardwaj_2k21phdam502@dtu.ac.in

^b e-mail: cpsingh@dce.ac.in (corresponding author)

gin gas was later adapted to cosmology for its exotic equation of state, $p = -A/\rho$, which interpolates between a dust-like phase at early times and a cosmological constant-like phase at late times. While the standard Chaplygin gas was soon ruled out by observational data, its generalization to $p = -A/\rho^\alpha$ [12, 13], known as the Generalized Chaplygin Gas (GCG), offered greater flexibility and improved compatibility with observations. Further refinements led to the Variable Generalized Chaplygin Gas (VGCG), where A becomes a function of the scale factor [14, 15], and most recently, the Modified Chaplygin Gas (MCG), characterized by $p = A\rho - C/\rho^\alpha$ [16, 17], which introduces a linear energy-density term to avoid singularities and enhance dynamical richness. Crucially, the MCG allows for a smooth transition from deceleration to acceleration without invoking phantom fields or abrupt phase transitions. In the present work, however, we do not exploit the unified dark matter-dark energy interpretation of the MCG. Instead, we treat the MCG fluid as an effective dark-energy component only, while the dark matter sector is described separately by a pressureless component with bulk viscous correction together with a conserved baryonic component. This choice avoids double counting of dark matter and allows us to disentangle the effects of the MCG equation of state from those of bulk viscosity and matter creation.

A conceptually distinct approach invokes gravitationally induced matter creation, a non-equilibrium thermodynamic process rooted in quantum field theory in curved spacetime [18–20]. Macroscopically, this is incorporated via a reinterpretation of the energy-momentum tensor, introducing a negative creation pressure p_c tied to the particle production rate Γ [21, 22]. A phenomenologically successful ansatz, $\Gamma = 3\beta H$ [23], has been shown to drive late-time acceleration without Λ , effectively mimicking dark energy [24–26]. Recent studies [27–38] have successfully integrated this mechanism with Chaplygin gas models, demonstrating observational viability and thermodynamic consistency. Notably, matter creation provides a natural mechanism for entropy generation, circumventing the need for ad hoc dark energy fields.

Complementing these frameworks is the inclusion of bulk viscous stresses, a natural consequence when cosmic fluids depart from local thermodynamic equilibrium [39, 40]. In cosmological contexts, bulk viscosity acts as an effective negative pressure that can source acceleration independently of dark energy. From the perspective of relativistic non-equilibrium thermodynamics, a fully causal and linearly stable description of viscous fluids is provided by the second-order Israel-Stewart theory, in which the bulk pressure obeys a relaxation-type transport equation rather than the instantaneous Eckart law [41–44]. A physically motivated parameterization, $\pi = -3H\xi_0\rho_m^{1/2}$ [45, 46], links viscous stress directly to the square root of the matter density, offering a kinetic-theory-based correction to the cos-

mic fluid's dynamics. This form ensures that viscous dissipation diminishes as matter dilutes, preserving consistency with structure formation. While viscous Chaplygin gas models have been explored in isolation [44, 47, 48], their synthesis with gravitationally induced matter creation remains uncharted despite the clear physical synergy: both represent irreversible, entropy-generating processes that convert gravitational or kinetic energy into particle content or heat, thereby altering the effective equation of state.

This work introduces a novel cosmological model that integrates three distinct physical mechanisms: (i) the Modified Chaplygin Gas (MCG) equation of state, $p = A\rho - C/\rho^\alpha$, which introduces a linear energy-density term absent in standard or generalized Chaplygin formulations; (ii) gravitationally induced matter creation, governed by $\Gamma = 3\beta H$; and (iii) bulk viscous pressure, modeled as $\pi = -3H\xi_0\rho_m^{1/2}$. This tripartite framework represents the first attempt to combine these elements into a single, self-consistent cosmological model. Unlike prior studies that treat viscosity or matter creation as perturbative corrections, our approach embeds both as fundamental components of the cosmic fluid's stress-energy tensor, yielding a non-trivial modification to the Friedmann equations and the effective dark energy sector. The model is analytically tractable and observationally testable, offering a thermodynamically grounded alternative to Λ CDM.

Our primary objectives are to derive the analytical expression for the Hubble parameter $H(z)$ under this hybrid formalism; second, to constrain the free parameters $\{H_0, \Omega_m, \Omega_b, A_s, A, \alpha, \beta, \xi_0\}$ using the latest observational datasets: Pantheon+ supernovae, Cosmic Chronometers (CC), DESI DR2 BAO, and $f(z)\sigma_8$ measurements via a Markov Chain Monte Carlo (MCMC) analysis; and third, to reconstruct the evolutionary histories of key cosmological parameters to assess the model's viability against the Λ CDM paradigm.

A central and original contribution of this study is the thermodynamic validation of the model through the lens of the Generalized Second Law (GSL) of thermodynamics. In non-equilibrium settings, matter creation and bulk viscosity are intrinsically linked to entropy production. The GSL demands that the total entropy of the Universe comprising the entropy of the cosmic fluid S_{fluid} (which includes contributions from irreversible matter creation and viscous dissipation) and the horizon entropy S_{horizon} must satisfy $\dot{S}_{\text{total}} \geq 0$ throughout cosmic evolution [49, 50]. While the GSL has been tested for dark energy and modified gravity models [51, 52], and separately for matter creation [53] or viscous cosmologies [54], its application to cosmological model incorporating all three elements: MCG, matter creation, and bulk viscosity is entirely novel. We will explicitly compute $S_{\text{total}}(t)$ and examine its temporal derivative to provide a rigorous thermodynamic consistency check, thereby elevating the analysis beyond purely kinematic diagnostics.

This work significantly extends prior investigations [36, 37] by incorporating bulk viscosity, utilizing more recent and precise datasets (Pantheon+, DESI DR2), and performing the first-ever GSL analysis for such a hybrid model. It also advances studies of viscous Chaplygin gases [48] by embedding them within the matter creation formalism, offering a more comprehensive and observationally grounded description of cosmic dynamics.

The structure of this paper is organised as follows. In Sect. 2, we formulate the modified Friedmann equations governing the cosmic expansion in the presence of matter creation and bulk viscous stress and derives exact analytical expressions for the energy density and Hubble parameter as functions of redshift. Section 3 details the observational datasets employed: Pantheon+, Cosmic Chronometers, DESI DR2 BAO, $f(z)\sigma_8$, and R22 and outlines the MCMC methodology used to constrain model parameters. Section 4 presents the best-fit parameter values, evolutionary trajectories of cosmological quantities, and thermodynamic diagnostics. The paper concludes in Sect. 5 with a synthesis of findings and their implications for Modified Chaplygin gas model.

2 Theoretical framework and field equations

We adopt a spatially flat, homogeneous, and isotropic cosmological model, described by the Friedmann-Lemaître-Robertson-Walker (FLRW) line element:

$$ds^2 = -dt^2 + a^2(t) \left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right], \quad (1)$$

where $a(t)$ denotes the scale factor, normalized such that $a_0 = 1$ at the present epoch, and t represents cosmic time. Within this geometric setting, we formulate a cosmological fluid that includes three distinct physical mechanisms: (i) an exotic equation of state of the Modified Chaplygin Gas (MCG); (ii) gravitationally induced particle production; and (iii) dissipative bulk viscosity. This tripartite construction has not been previously explored in the literature and represents a novel theoretical synthesis.

2.1 Non-equilibrium thermodynamics and particle production

The foundational premise of gravitationally induced matter creation rests on the reinterpretation of energy-momentum conservation in an expanding spacetime. Rather than assuming a closed system with fixed particle number, we treat the cosmic fluid as an open thermodynamic entity, where quantum-gravitational effects permit the continuous emergence of material content from the background geometry.

The particle current density is defined as $N^\mu = nu^\mu$, where n is the comoving number density and u^μ is the fluid four-velocity satisfying $u^\mu u_\mu = -1$. In a FLRW background, the divergence of this current yields:

$$\nabla_\mu N^\mu = \dot{n} + 3Hn = n\Gamma, \quad (2)$$

where an overdot denotes differentiation with respect to t , $H = \dot{a}/a$ is the Hubble parameter, and Γ quantifies the rate of particle production per unit volume. A positive Γ signifies net creation, while $\Gamma = 0$ recovers standard adiabatic expansion.

This irreversible process modifies the conservation of energy. The first law of thermodynamics, when applied to an open system with variable particle number, introduces an effective pressure component associated with creation. For a fluid with equilibrium energy density ρ and pressure p , this additional contribution termed as the creation pressure P_c is given by:

$$P_c = -\frac{\rho + p}{3H} \Gamma. \quad (3)$$

To maintain analytical tractability while preserving physical relevance, we adopt a creation rate proportional to the Hubble expansion:

$$\Gamma = 3\beta H, \quad (4)$$

where β is a dimensionless, non-negative constant. This ansatz, while phenomenological, is grounded in the expectation that particle production should scale with the dynamical timescale of the Universe. Equation (4) has been extensively used in the literature in matter creation cosmology [23–26], typically with β treated as a small, positive parameter that controls the strength of the deviation from standard adiabatic expansion. In the present work β is likewise regarded as a free parameter to be constrained by observations, and, as discussed below, our best-fit values remain well below unity, so that the effective creation rate never exceeds the Hubble expansion rate by more than an order-unity factor. Substituting Eqs. (4) into (3), we get

$$P_c = -\beta(\rho + p). \quad (5)$$

Thus, the total effective pressure governing by the dominant part is $p_{\text{eff}} = p + P_c$, which leads to the modified energy conservation law:

$$\dot{\rho} + 3H(\rho + p + P_c) = 0. \quad (6)$$

2.2 Modified chaplygin gas equation of state

We model the dominant cosmic component using the Modified Chaplygin Gas (MCG), characterized by the equation of state:

$$p = A\rho - \frac{C}{\rho^\alpha}, \quad (A \geq 0, 0 \leq \alpha \leq 1, C > 0), \quad (7)$$

where A , C , and α are free parameters to be constrained observationally. This form generalizes earlier Chaplygin models by introducing a linear term in ρ , which prevents singular behavior at low densities and allows for a richer dynamical evolution. When $A = 0$, it reduces to the Generalized Chaplygin Gas; when $\alpha = 1$ and C is scale-dependent, it recovers the Variable Chaplygin Gas.

Inserting Eqs. (7) and (5) into Eq. (6), we obtain the evolution equation for ρ :

$$\dot{\rho} + 3H(1 - \beta) \left[(1 + A)\rho - \frac{C}{\rho^\alpha} \right] = 0. \quad (8)$$

To solve this, we change variables from cosmic time t to redshift z , using $d/dt = -H(1 + z)d/dz$. After rearrangement and integration, subject to the boundary condition $\rho(z = 0) = \rho_0$, we find:

$$\rho(z) = \rho_0 \left[A_s + (1 - A_s)(1 + z)^{3(1-\beta)(1+A)(1+\alpha)} \right]^{\frac{1}{1+\alpha}}, \quad (9)$$

where we retain the definition:

$$A_s \equiv \frac{C}{(1 + A)\rho_0^{1+\alpha}}. \quad (10)$$

This expression governs the energy density evolution of the MCG component under the joint influence of its exotic equation of state and non-equilibrium matter creation.

2.3 Bulk viscous matter component

To account for dissipative effects in the cosmic fluid, we introduce a separate matter component subject to bulk viscosity. This fluid is pressureless in equilibrium ($p_m = 0$) but develops an effective stress π due to departure from thermodynamic equilibrium. The continuity equation becomes:

$$\dot{\rho}_m + 3H(\rho_m + \pi) = 0. \quad (11)$$

We adopt a viscous stress proportional to the Hubble rate and the square root of the energy density, a form motivated by kinetic theory and dimensional consistency:

$$\pi = -3H\xi_0\rho_m^{1/2}, \quad (12)$$

where $\xi_0 > 0$ is a constant viscosity coefficient. Substituting Eqs. (12) into (11) and changing variables to z , we obtain:

$$\frac{d\rho_m}{dz} = \frac{3\rho_m(1 - \sqrt{3}\xi_0)}{1 + z}. \quad (13)$$

Integrating this first-order differential equation with the initial condition $\rho_m(z = 0) = \rho_{m,0}$ yields:

$$\rho_m(z) = \rho_{m,0}(1 + z)^{3(1-\sqrt{3}\xi_0)}. \quad (14)$$

This result demonstrates that bulk viscosity slows the dilution of matter density with expansion, effectively mimicking a negative pressure component. From Eq. (14) one immediately sees that the exponent $3(1 - \sqrt{3}\xi_0)$ must remain positive in order for the dust density to increase towards the past as $z \rightarrow \infty$, as required for a standard matter-dominated era. Combined with the requirement $\xi_0 \geq 0$ (which guarantees a negative viscous pressure and non-decreasing entropy), this leads to the theoretical bound

$$0 \leq \xi_0 < \frac{1}{\sqrt{3}} \simeq 0.577. \quad (15)$$

In the numerical analysis we adopt a broad flat sampling interval $\xi_0 \in [0, 1]$ (cf. Table 1) to let the data explore a wide region of parameter space, but the best-fit values quoted in Tables 1 and 2 all satisfy the condition (15). The background solutions discussed in the following sections therefore lie in the physically admissible regime.

We also note that the constitutive relation (12) is of first order in deviations from equilibrium and corresponds to the Eckart formulation of relativistic dissipative fluids. It is well known that, at the level of inhomogeneous perturbations, first-order theories may suffer from instabilities and acausal propagation, whereas the Israel-Stewart framework provides a causal, second-order generalization. In the present work we restrict attention to the homogeneous and isotropic FLRW background, where no spatial gradients or propagating modes are present, so these pathologies do not manifest themselves. For our purposes the Eckart-type ansatz (12) therefore offers a simple and widely used phenomenological description of bulk viscosity in cosmology [39,44,45,48].

2.4 Friedmann equation and Hubble parameter

The total energy density sourcing the gravitational field is the sum of the MCG, viscous matter, and conserved baryonic components. In this work these three components have distinct physical roles: the MCG fluid, obeying the equation of state (7) and evolving according to (8) and (9), is interpreted as the dark-energy sector and includes the effect of

gravitationally induced particle production through the factor $(1 - \beta)$ in its redshift evolution. The pressureless component ρ_m , governed by (11) and (14), represents the cold dark matter sector at the background level and is affected only by bulk viscous dissipation. Finally, baryonic matter ρ_b is conserved and non-relativistic. The baryonic density evolves as $\rho_b(z) = \rho_{b,0}(1+z)^3$, following standard conservation.

The Friedmann equation, in units where $8\pi G = 1$, is:

$$3H^2(z) = \rho(z) + \rho_m(z) + \rho_b(z). \quad (16)$$

Substituting Eqs. (9), (14), and the baryon density into Eq. (16), and normalizing by the present-day Hubble constant H_0 , we derive the dimensionless Hubble parameter $E(z) = H(z)/H_0$:

$$E^2(z) = \Omega_{\text{mcg},0} \left[A_s + (1 - A_s)(1+z)^{3(1-\beta)(1+A)(1+\alpha)} \right]^{\frac{1}{1+\alpha}} + \Omega_{m,0}(1+z)^{3(1-\sqrt{3}\xi_0)} + \Omega_{b,0}(1+z)^3, \quad (17)$$

where the present-day density parameters are defined as:

$$\Omega_{\text{mcg},0} = \frac{\rho_0}{3H_0^2}, \quad \Omega_{m,0} = \frac{\rho_{m,0}}{3H_0^2}, \quad \Omega_{b,0} = \frac{\rho_{b,0}}{3H_0^2}, \quad (18)$$

and satisfy the flatness constraint:

$$\Omega_{\text{mcg},0} + \Omega_{m,0} + \Omega_{b,0} = 1. \quad (19)$$

The expression (17) serves as the foundation for our observational analysis in Sect. 3, where we will constrain the parameters $\{H_0, \Omega_m, \Omega_b, A_s, A, \alpha, \beta, \xi_0\}$

3 Observational data and methodology

To infer the posterior distributions of the model parameters, we perform a joint statistical analysis using five inde-

pendent cosmological probes: Cosmic Chronometers (CC), which provide direct, model-independent estimates of $H(z)$; the Pantheon+ compilation of Type Ia supernovae, calibrated without SH0ES priors and serving as geometric distance anchors; Baryon Acoustic Oscillation (BAO) measurements from DESI DR2 and SDSS-IV, offering a standard ruler tied to the sound horizon r_d ; redshift-space distortion (RSD) constraints on the structure growth rate $f\sigma_8(z)$, which test the evolution of matter clustering independently of the background expansion; and the local Hubble constant measurement $H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from the SH0ES collaboration (R22), included as an optional Gaussian prior. Each dataset contributes a distinct χ^2 component to the total likelihood, enabling us to break parameter degeneracies and rigorously test the viability of our hybrid model. All statistical inference is performed via Markov Chain Monte Carlo (MCMC) sampling using the emcee package [55], with 80 walkers and 10,000 steps per chain. Convergence is verified using the Gelman-Rubin statistic. Priors are listed in Table 1. In addition to the one-dimensional marginalized constraints quoted in Tables 1 and 2, we compute the full covariance and linear correlation matrices from the converged chains in order to quantify parameter degeneracies and discuss the statistical identifiability of the MCG and non-equilibrium parameters (see Sect. 4.6). For the matter creation parameter we adopt a flat prior $0 \leq \beta \leq 1$, which encompasses the range commonly assumed in phenomenological matter creation models and ensures that the effective production rate $\Gamma = 3\beta H$ remains at most of the same order as the Hubble expansion rate at late times. Similarly, for the bulk viscosity parameter we employ a broad flat sampling prior $\xi_0 \in [0, 1]$; the background solution (14), however, implies the theoretical consistency condition (15), so only the region $\xi_0 < 1/\sqrt{3}$ corresponds to a matter component whose density grows monotonically towards higher redshift. The best-

Table 1 Best-fit values (mean $\pm 1\sigma$) for Λ CDM and modified chaplygin gas models with flat priors for DS1

Parameter	Prior	Λ CDM	MCG Model	MCG ($\beta = 0$)	MCG ($\xi_0 = 0$)	MCG ($\beta = 0, \xi_0 = 0$)
H_0 [km s ⁻¹ Mpc ⁻¹]	[60, 80]	68.6 ± 3.6	67.0 ± 1.9	67.88 ± 0.79	68.6 ± 2.4	67.91 ± 0.59
Ω_m	[0, 0.5]	0.387 ± 0.008	0.288 ± 0.002	0.279 ± 0.006	0.277 ± 0.004	0.303 ± 0.007
Ω_b	[0, 0.1]	—	0.028 ± 0.012	0.026 ± 0.012	0.0249 ± 0.002	0.0305 ± 0.001
A_s	[0, 1]	—	0.770 ± 0.04	0.785 ± 0.02	0.749 ± 0.09	0.685 ± 0.02
A	[0, 1]	—	0.034 ± 0.015	0.036 ± 0.017	0.108 ± 0.039	0.191 ± 0.007
α	[0, 1]	—	0.08 ± 0.028	0.099 ± 0.030	0.181 ± 0.039	0.060 ± 0.025
β	[0, 1]	—	0.139 ± 0.057	0	0.600 ± 0.08	0
ξ_0	[0, 1]	—	0.38 ± 0.125	0.572 ± 0.027	0	0
r_d [Mpc]	[140, 150]	146.0 ± 4.2	146.3 ± 1.8	146.4 ± 1.5	144.5 ± 3.7	146.4 ± 1.5
$\mathcal{M}$	[-20, -18]	-19.4 ± 0.12	-19.409 ± 0.026	-19.303 ± 0.011	-19.302 ± 0.005	-19.407 ± 0.019
σ_8	[0, 1]	0.752 ± 0.029	0.841 ± 0.016	0.798 ± 0.035	0.798 ± 0.013	0.819 ± 0.014

Table 2 Best-fit values (mean $\pm 1\sigma$) for Λ CDM and modified chaplygin gas models with flat priors for DS2

Parameter	Prior	Λ CDM	MCG Model	MCG ($\beta = 0$)	MCG ($\xi_0 = 0$)	MCG ($\beta = 0, \xi_0 = 0$)
H_0 [km s $^{-1}$ Mpc $^{-1}$]	[60, 80]	72.77 ± 0.66	71.01 ± 0.45	71.42 ± 0.63	70.40 ± 0.7	72.17 ± 0.65
Ω_m	[0.1, 0.5]	0.307 ± 0.007	0.277 ± 0.009	0.271 ± 0.020	0.296 ± 0.017	0.276 ± 0.021
Ω_b	[0, 0.1]	–	0.0340 ± 0.003	0.029 ± 0.006	0.023 ± 0.004	0.029 ± 0.001
A_s	[0, 1]	–	0.735 ± 0.002	0.782 ± 0.01	0.731 ± 0.04	0.699 ± 0.07
A	[0, 1]	–	0.047 ± 0.001	0.037 ± 0.018	0.141 ± 0.021	0.028 ± 0.024
α	[0, 1]	–	0.064 ± 0.028	0.102 ± 0.029	0.157 ± 0.017	0.061 ± 0.028
β	[0, 1]	–	0.461 ± 0.019	0	0.68 ± 0.07	0
ξ_0	[0, 1]	–	0.119 ± 0.031	0.651 ± 0.028	0	0
r_d [Mpc]	[140, 150]	138.1 ± 1.4	140.7 ± 1.2	140.0 ± 1.4	143.5 ± 0.7	138.0 ± 1.4
$\mathcal{M}$	[−20, −18]	-19.272 ± 0.019	-19.316 ± 0.014	-19.302 ± 0.019	-19.310 ± 0.012	-19.278 ± 0.015
σ_8	[0, 1]	0.752 ± 0.017	0.818 ± 0.016	0.819 ± 0.015	0.794 ± 0.013	0.793 ± 0.015

fit values reported in Tables 1 and 2 all fall inside this physically allowed interval.

3.1 Cosmic chronometers (CC)

Cosmic Chronometers provide model-independent estimates of $H(z)$ by measuring the differential age evolution of passively evolving galaxies. The method relies on the relation $H(z) = -\frac{1}{1+z} \frac{dz}{dt}$, where dt is inferred from the age difference between galaxies at adjacent redshift [56].

We use a compilation of 32 CC measurements spanning $0.07 \leq z \leq 1.965$, drawn from [57–60]. The χ^2 statistic incorporates both statistical and systematic uncertainties via a full covariance matrix:

$$\chi_{\text{CC}}^2 = \Delta \mathbf{H}^T \cdot \mathbf{C}_{\text{CC}}^{-1} \cdot \Delta \mathbf{H}, \quad (20)$$

where $\Delta \mathbf{H} = \mathbf{H}_{\text{th}}(\theta) - \mathbf{H}_{\text{obs}}$ is the residual vector, and $\mathbf{C}_{\text{CC}} = \mathbf{C}_{\text{stat}} + \mathbf{C}_{\text{syst}}$.

3.2 Type Ia supernovae (Pantheon+)

The Pantheon+ compilation [61] comprises 1701 spectroscopically confirmed Type Ia supernovae in the range $0.01 < z < 2.3$. The apparent magnitude $m(z)$ is related to the luminosity distance $d_L(z)$ by:

$$m(z) = 5 \log_{10} \left(\frac{d_L(z)}{10 \text{ pc}} \right) + \mathcal{M},$$

where $\mathcal{M}$ is the absolute magnitude (treated as a free parameter), and

$$d_L(z) = (1+z) \int_0^z \frac{c \, dz'}{H(z')}.$$

The χ^2 uses the full covariance matrix:

$$\chi_{\text{SNe}}^2 = \Delta \boldsymbol{\mu}^T \cdot \mathbf{C}_{\text{SNe}}^{-1} \cdot \Delta \boldsymbol{\mu}, \quad (21)$$

where $\Delta \boldsymbol{\mu} = \boldsymbol{\mu}_{\text{th}} - \boldsymbol{\mu}_{\text{obs}}$, and $\mathbf{C}_{\text{SNe}}$ includes statistical and systematic uncertainties.

3.3 Baryon acoustic oscillations (DESI DR2 + SDSS-IV)

BAO measurements provide a standard ruler based on the sound horizon r_d at the drag epoch. We treat r_d as a free parameter, avoiding CMB priors, following [62–69]. This choice is intentional: it allows us to probe the late-time expansion history using low-redshift distance and Hubble-rate measurements without importing information from CMB anisotropies or a specific early-Universe model. At the same time, it implies that the present work does not test the behaviour of the hybrid MCG-matter creation-viscous model during Big Bang nucleosynthesis or recombination, and our results should therefore be interpreted as constraints on the late-time background dynamics rather than a full assessment of cosmological viability across all epochs. A dedicated analysis including CMB and BBN observables is left for future work.

We use 13 BAO data points from DESI DR2 [70] and SDSS-IV [4], measuring the ratios $D_M(z)/r_d$, $D_H(z)/r_d$, and $D_V(z)/r_d$, where:

$$\begin{aligned} D_H(z) &= \frac{c}{H(z)}, \\ D_M(z) &= c \int_0^z \frac{dz'}{H(z')}, \\ D_V(z) &= \left[z D_M^2(z) D_H(z) \right]^{1/3}. \end{aligned}$$

The χ^2 is:

$$\chi_{\text{BAO}}^2 = \sum_{Y \in \{H, M, V\}} \Delta \mathbf{D}_Y^T \cdot \mathbf{C}_{D_Y}^{-1} \cdot \Delta \mathbf{D}_Y, \quad (22)$$

where $\Delta \mathbf{D}_Y = (\mathbf{D}_Y/r_d)_{\text{th}} - (\mathbf{D}_Y/r_d)_{\text{obs}}$.

3.4 Structure growth: $f(z)\sigma_8(z)$ measurements

The growth rate $f\sigma_8(z)$, where $f = d \ln \delta_m / d \ln a$ and $\sigma_8(z)$ is the RMS fluctuation amplitude, provides a direct test of structure formation. We use 18 independent measurements from BOSS, eBOSS, and 6dFGS [71–73]. The theoretical $f(z)\sigma_8(z)$ is computed as:

$$f\sigma_8(z) = -(1+z) \frac{\sigma_8(z=0)}{\delta_m(z=0)} \frac{d\delta_m}{dz}, \quad (23)$$

where $\sigma_8(z) = \sigma_8(z=0)\delta_m(z)/\delta_m(z=0)$. For models with non-equilibrium thermodynamics, we adopt the Λ CDM-based fitting formula for $\delta_m(z)$ as a first approximation. This simplified treatment does not incorporate the modifications to the linear growth of matter perturbations that may arise from the effective pressures associated with matter creation and bulk viscous stresses in our hybrid model. As a consequence, the constraints on the growth-related quantity σ_8 should be interpreted as indicative rather than as fully self-consistent predictions for structure formation in this framework. Our main conclusions regarding the late-time expansion history, the alleviation of the H_0 tension and the validity of the generalized second law are therefore driven primarily by geometric probes (Pantheon+, CC, BAO) and the local H_0 prior, and only secondarily informed by the growth data. A detailed perturbative analysis, based on the modified effective stress-energy tensor, is required to treat $f\sigma_8(z)$ in a fully consistent way and is left for future work. The χ^2 is:

$$\chi_{\sigma_8}^2 = \sum_{i=1}^{18} \frac{[f\sigma_{8,\text{th}}(z_i) - f\sigma_{8,\text{obs}}(z_i)]^2}{\sigma_{f\sigma_8,i}^2}. \quad (24)$$

3.5 Local hubble constant (R22)

The SH0ES collaboration [74] reports $H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from the Cepheid-SNe distance ladder. This value exhibits $> 5\sigma$ tension with Planck CMB constraints [3]. We include it as a Gaussian prior in DS2:

$$\chi_{R22}^2 = \frac{(H_0^{\text{th}} - 73.04)^2}{(1.04)^2}. \quad (25)$$

We perform two distinct analyses:

– **DS1:** Pantheon+ + CC + BAO + $f\sigma_8$

– **DS2:** DS1 + R22

The total χ^2 for DS1 is:

$$\chi_{\text{DS1}}^2 = \chi_{\text{SNe}}^2 + \chi_{\text{CC}}^2 + \chi_{\text{BAO}}^2 + \chi_{\sigma_8}^2.$$

For DS2, we add the R22 prior:

$$\chi_{\text{DS2}}^2 = \chi_{\text{DS1}}^2 + \chi_{R22}^2.$$

4 Results and discussion

The joint observational analysis of the Modified Chaplygin Gas (MCG) model with matter creation and bulk viscosity has been performed using two distinct data combinations: DS1 (Pantheon+ + CC + BAO + $f\sigma_8$) and DS2 (DS1 + R22). The inclusion of the local H_0 measurement from the SH0ES collaboration (R22) significantly alters the best-fit values of cosmological parameters, particularly the present-day Hubble constant, and allows us to assess how efficiently the model can accommodate a high local value of H_0 when combined with background probes. We now discuss the results and analysis from our parameter estimation for MCG model, alongside a comparison with the Λ CDM model. Based on this comparison, we evaluate how well our model explains the Universe's late expansion across cosmic epochs. Figures 1 and 2 display the 1σ and 2σ confidence regions for the Λ CDM and MCG model. We also present a comprehensive comparison of the MCG model with the standard Λ CDM paradigm and its sub-variants namely, MCG with no matter creation ($\beta = 0$), no bulk viscosity ($\xi_0 = 0$), and neither ($\beta = 0, \xi_0 = 0$) based on the constraints derived from both datasets. We analyze the evolution of key cosmological quantities, including the Hubble parameter $H(z)$, deceleration parameter $q(z)$, jerk parameter $j(z)$, and snap parameter $s(z)$, to evaluate the model's consistency with current observations. Furthermore, we perform stability and model selection analyses using information criteria to determine the viability of the proposed framework.

4.1 Hubble parameter evolution

The expansion history of the Universe is most directly probed by the Hubble parameter $H(z)$, which encapsulates the rate of cosmic expansion at different epochs. In Figs. 3 and 4, we present the reconstructed $H(z)$ curves for Λ CDM and four variants of the MCG model under DS1 and DS2, respectively. The observational data points from the Cosmic Chronometer (CC) measurements are shown as grey circles with error bars, covering the redshift range $0.07 \leq z \leq 2.0$.

As observed in Fig. 3, all models provide a consistent fit to the CC data within their respective 1σ uncertainties. The

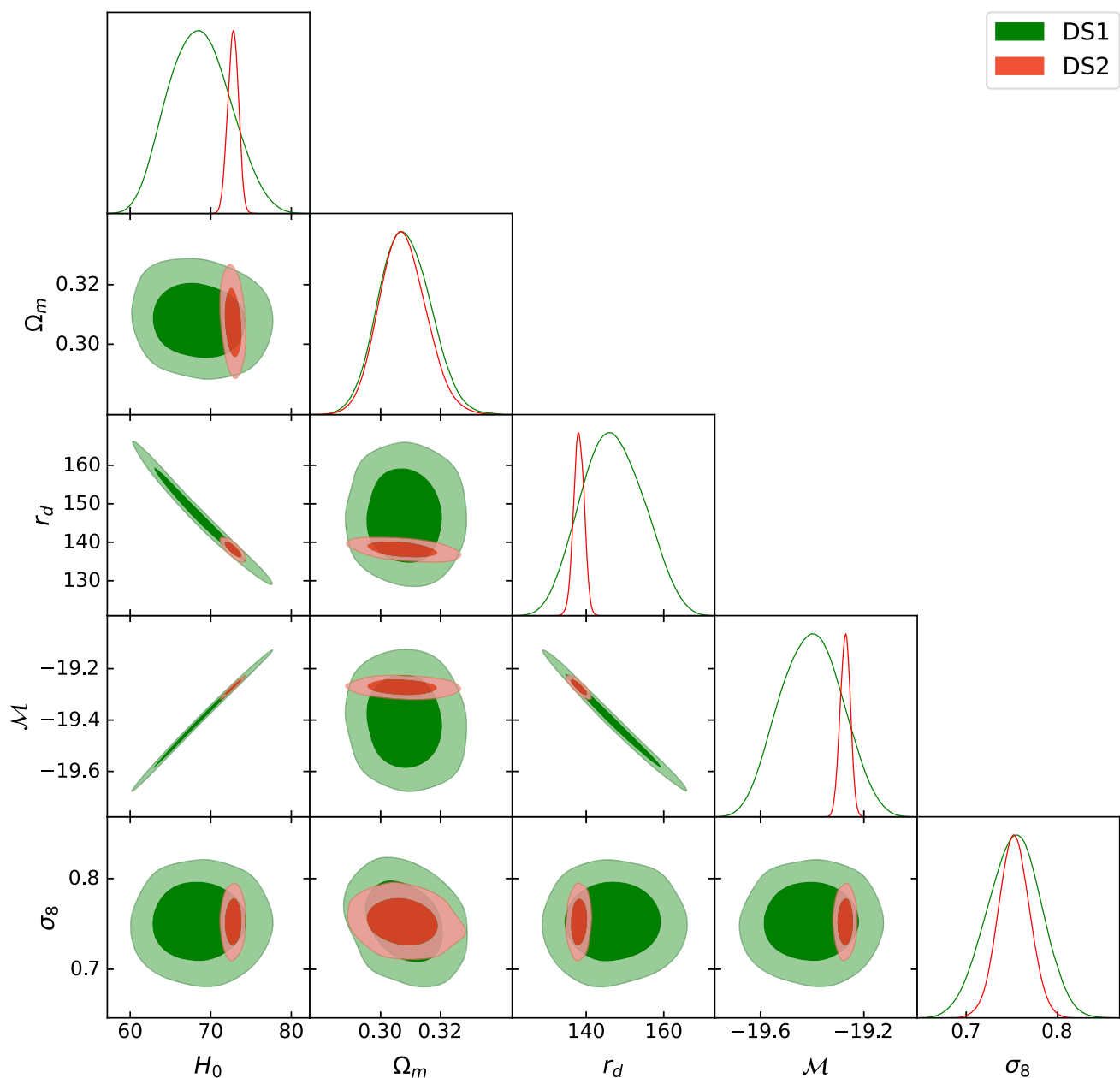


Fig. 1 The plot shows the best-fit values of model parameters of Λ CDM model obtained from the DS1 and DS2 datasets at 1σ and 2σ confidence levels

Λ CDM model (solid black line) follows a smooth trajectory that increases monotonically with redshift, reflecting the standard prediction of constant dark energy. The full MCG model (blue dashed-dotted line), incorporating both matter creation ($\beta \neq 0$) and bulk viscosity ($\xi_0 \neq 0$), closely tracks the Λ CDM curve across the entire redshift range, indicating its ability to mimic the late-time acceleration without requiring a cosmological constant. This similarity arises due to the interplay between the exotic equation of state of the MCG fluid and the non-equilibrium thermodynamic effects, which collectively contribute to an effective negative pressure.

The sub-model MCG($\beta = 0$) (green dotted line), which excludes matter creation but retains bulk viscosity, exhibits a slightly flatter slope at intermediate redshifts ($z \sim 1-1.5$), suggesting a marginally weaker acceleration phase compared to Λ CDM. This deviation reflects the role of bulk viscosity in modifying the effective pressure of the fluid, leading to a delayed onset of acceleration. Conversely, the MCG($\xi_0 = 0$) model (orange dash-dotted line), with no viscous damping but non-zero matter creation, shows a steeper slope, particularly at high redshifts, indicating an earlier transition to

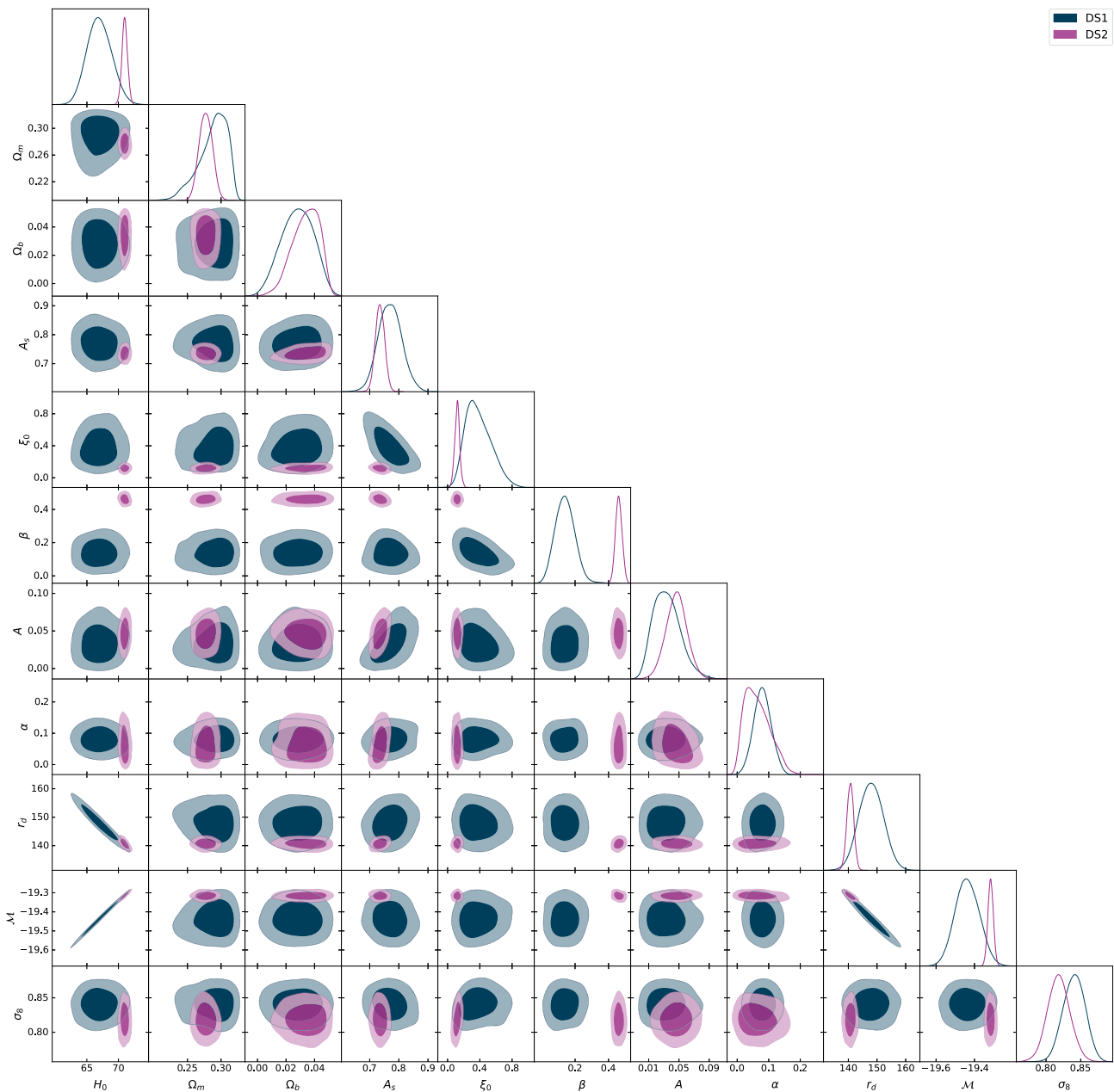


Fig. 2 The plot shows the best-fit values of model parameters of model 1 and model 2 obtained from the DS1 and DS2 datasets at 1σ and 2σ confidence levels

acceleration. This behaviour highlights the distinct influence of particle creation in shaping the early expansion dynamics.

The MCG model (red dotted line), with both $\beta = 0$ and $\xi_0 = 0$, deviates most significantly from Λ CDM, especially at $z > 1$, where it predicts a faster expansion rate. This divergence underscores the necessity of including non-equilibrium mechanisms to reproduce the observed expansion history. The best-fit value of $H_0 = 67.0 \pm 1.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for the full MCG model (see Table 1) is consistent

with the Planck CMB estimate, though slightly lower than the local SH0ES measurement.

For the DS1 combination (Pantheon+ + CC + BAO + $f\sigma_8$), the full MCG model yields $H_0 = 67.0 \pm 1.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$, slightly lower than the corresponding Λ CDM value of $68.6 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and fully consistent with Planck-like constraints, indicating that in the absence of a local prior the model does not intrinsically favour a higher Hubble constant.

Fig. 3 Evolution of the Hubble parameter $H(z)$ for Λ CDM and MCG models under DS1. Grey points with error bars represent Cosmic Chronometer data

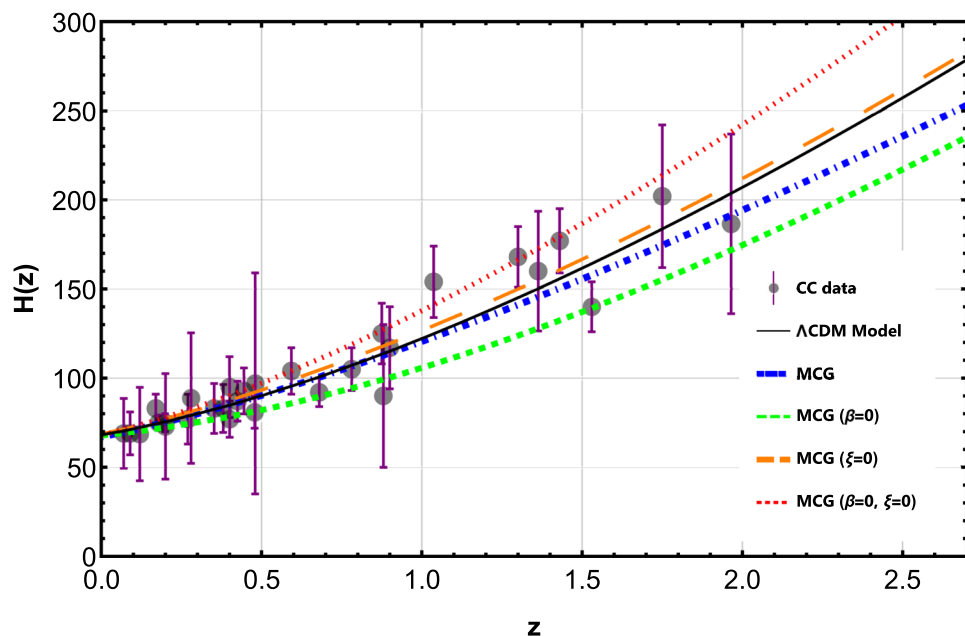
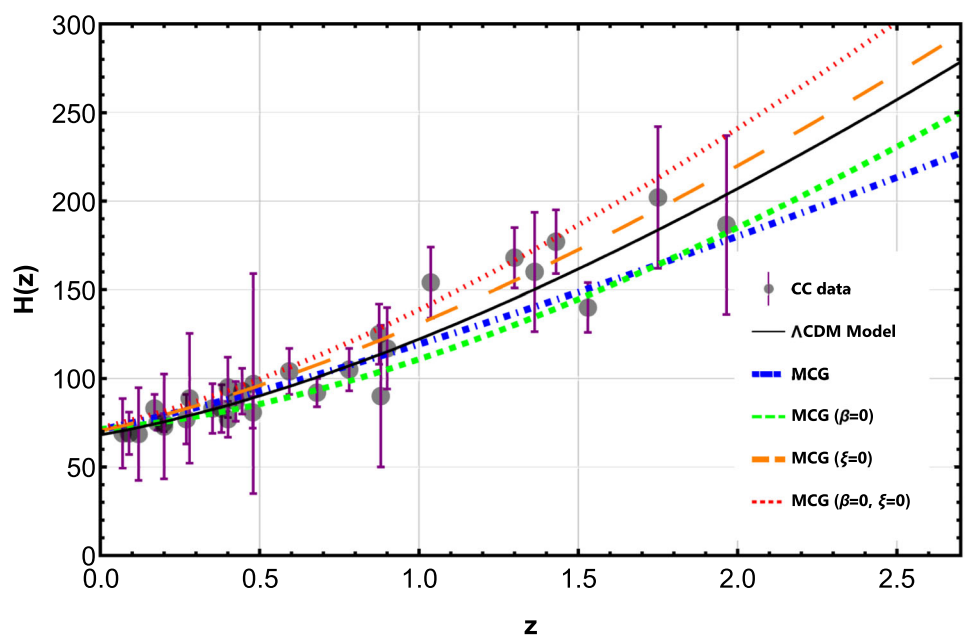


Fig. 4 Evolution of the Hubble parameter $H(z)$ for Λ CDM and MCG models under DS2. Grey points with error bars represent Cosmic Chronometer data



When the R22 constraint is added in DS2 (Fig. 4), the best-fit H_0 shifts upward to $71.01 \pm 0.45 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Table 2), bringing the model into better agreement with local observations. This shift results in a steeper $H(z)$ curve, particularly at low redshifts, where the $\text{MCG}(\beta = 0)$ and $\text{MCG}(\xi_0 = 0)$ models show enhanced acceleration. The Λ CDM model remains relatively unchanged, as it already predicts a higher H_0 than Planck. The full MCG model adjusts its parameters to accommodate the higher H_0 , resulting in a closer match to the CC data at $z < 1$.

The constraints on the matter creation parameter β are also informative. For the full MCG model we obtain $\beta = 0.139 \pm 0.057$ from DS1 and $\beta = 0.461 \pm 0.019$ from DS2

(see Tables 1 and 2), so that β is clearly positive and significantly below unity in both cases. These values correspond to the creation rates $\Gamma = 3\beta H$ that are at most of the same order as the Hubble expansion rate, which is consistent with the magnitude typically assumed in phenomenological treatments of gravitationally induced particle production [23–26]. The present constraints are also of similar order to those found in our previous analyses of Chaplygin gas models with matter creation [36–38]. The increase of the best-fit β when the R22 prior is included indicates that a higher matter creation rate helps accommodate a larger local value of H_0 while preserving a good fit to the geometric datasets.

Fig. 5 Evolution of the deceleration parameter $q(z)$ for Λ CDM and MCG models under DS1

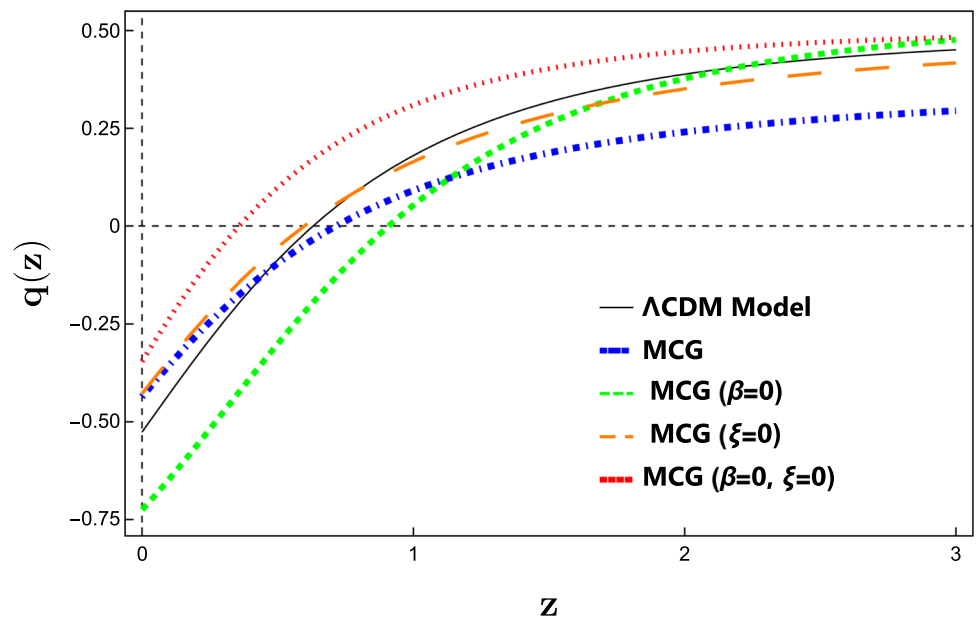
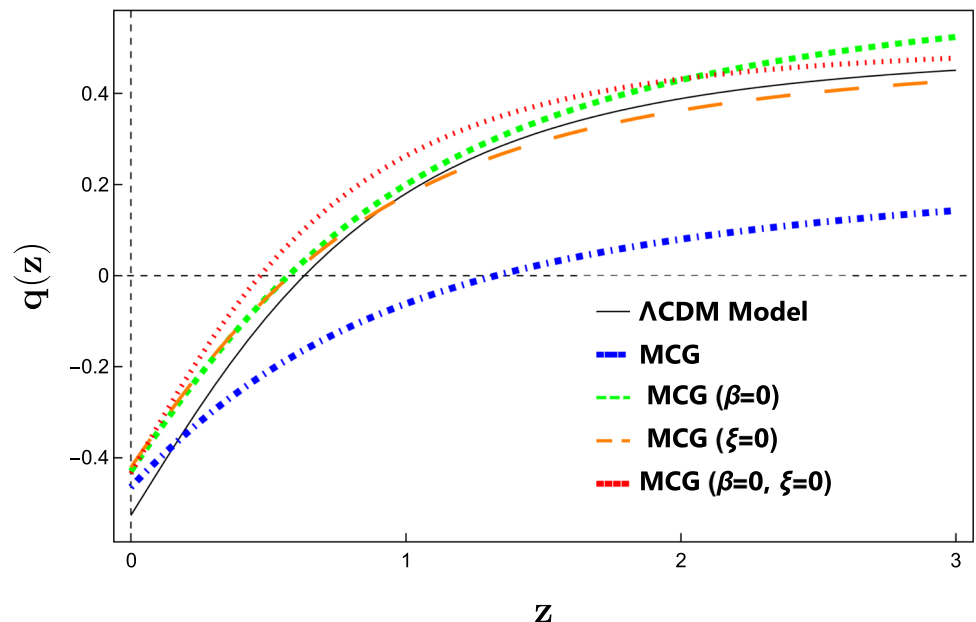


Fig. 6 Evolution of the deceleration parameter $q(z)$ for Λ CDM and MCG models under DS2



4.2 Deceleration parameter evolution

The deceleration parameter $q(z)$ serves as a fundamental diagnostic of the expansion history of the Universe, quantifying the transition from a decelerated to an accelerated phase. It is defined in terms of the Hubble parameter as:

$$q(z) = -1 + (1+z) \frac{d \ln H(z)}{dz}. \quad (26)$$

A positive value of $q(z)$ indicates decelerated expansion, while a negative value signifies acceleration. The redshift at which $q(z)$ crosses zero, denoted z_{tr} , marks the epoch of tran-

sition from matter-dominated deceleration to dark-energy-dominated acceleration.

Using the reconstructed $H(z)$, we compute $q(z)$ for the Λ CDM model and the Modified Chaplygin Gas (MCG) model under both DS1 and DS2 constraints. The evolution of $q(z)$ is displayed in Fig. 5 for DS1 and Fig. 6 for DS2. The present-day values and transition redshifts are summarized in Tables 3 and 4.

As shown in Fig. 5, all models exhibit a smooth transition from $q > 0$ at high redshift to $q < 0$ at low redshift, consistent with the standard cosmological paradigm. The Λ CDM model (solid black line) transitions at $z_{tr} = 0.629 \pm 0.26$, with a present-day value $q_0 = -0.528 \pm 0.020$. The full

Table 3 Cosmological parameter estimations for the Λ CDM and Modified Chaplygin gas models for DS1

Cosmological parameter estimations					
Parameter	Λ CDM	MCG	MCG ($\beta = 0$)	MCG ($\xi_0 = 0$)	MCG ($\beta = 0, \xi_0 = 0$)
q_0	-0.528 ± 0.020	-0.442 ± 0.014	-0.726 ± 0.017	-0.433 ± 0.013	-0.343 ± 0.019
j_0	1.000	0.770 ± 0.020	1.07 ± 0.023	1.03 ± 0.017	0.826 ± 0.029
s_0	0	0.09 ± 0.003	-0.026 ± 0.019	0.03 ± 0.004	-0.013 ± 0.003
τ_0 (Gyr)	13.71 ± 0.11	13.84 ± 0.13	13.98 ± 0.12	13.81 ± 0.14	13.95 ± 0.10
z_{tr}	0.629 ± 0.26	0.592 ± 0.20	0.601 ± 0.22	0.654 ± 0.17	0.698 ± 0.25

Table 4 Cosmological parameter estimations for the Λ CDM and Chaplygin gas models for DS2

Cosmological parameter estimations					
Parameter	Λ CDM	MCG	MCG ($\beta = 0$)	MCG ($\xi_0 = 0$)	MCG ($\beta = 0, \xi_0 = 0$)
q_0	-0.526 ± 0.021	-0.460 ± 0.018	-0.436 ± 0.011	-0.427 ± 0.017	-0.434 ± 0.021
j_0	1.000	0.596 ± 0.021	0.851 ± 0.013	0.828 ± 0.012	1.03 ± 0.022
s_0	0	0.122 ± 0.002	0.07 ± 0.013	0.06 ± 0.002	-0.006 ± 0.001
τ_0 (Gyr)	13.67 ± 0.15	13.65 ± 0.17	13.83 ± 0.17	13.92 ± 0.10	14.01 ± 0.12
z_{tr}	0.671 ± 0.21	0.689 ± 0.21	0.631 ± 0.32	0.632 ± 0.19	0.697 ± 0.29

MCG model (blue dashed-dotted line), incorporating both matter creation ($\beta \neq 0$) and bulk viscosity ($\xi_0 \neq 0$), transitions slightly earlier at $z_{\text{tr}} = 0.592 \pm 0.20$, with $q_0 = -0.442 \pm 0.014$. This earlier transition reflects the combined effect of non-equilibrium thermodynamics, which enhances the effective negative pressure at intermediate redshifts.

The MCG($\beta = 0$) model (green dotted line), which excludes matter creation but retains bulk viscosity, transitions at $z_{\text{tr}} = 0.601 \pm 0.22$, with $q_0 = -0.726 \pm 0.017$. The bulk viscosity alone delays the onset of acceleration compared to the full MCG model, as it acts as a damping mechanism that suppresses the growth of negative pressure. Conversely, the MCG($\xi_0 = 0$) model (orange dash-dotted line), with no viscous damping but non-zero matter creation, transitions earliest at $z_{\text{tr}} = 0.654 \pm 0.17$, with $q_0 = -0.433 \pm 0.013$. This behavior highlights the dominant role of particle creation in driving early acceleration.

The MCG model (red dotted line), with both $\beta = 0$ and $\xi_0 = 0$, transitions latest at $z_{\text{tr}} = 0.698 \pm 0.25$, with $q_0 = -0.343 \pm 0.019$. This late transition underscores the necessity of including non-equilibrium mechanisms to reproduce the observed expansion history, as the pure MCG fluid lacks the additional negative pressure required to initiate acceleration at the correct epoch.

When the R22 constraint is added in DS2 (Fig. 6), the transition redshifts shift slightly due to the higher best-fit H_0 . The Λ CDM model transitions at $z_{\text{tr}} = 0.671 \pm 0.21$, with $q_0 = -0.526 \pm 0.021$. The full MCG model transitions at $z_{\text{tr}} = 0.689 \pm 0.21$, with $q_0 = -0.460 \pm 0.018$. The relative ordering of the models remains unchanged, but the abso-

lute values of q_0 become slightly more negative, indicating a marginally stronger acceleration in the late-time Universe.

The present-day values of q_0 for all models lie within the range -0.56 to -0.34 , consistent with independent estimates from supernova and BAO data [74, 75]. The transition redshifts z_{tr} range from 0.54 to 0.70, in agreement with the Λ CDM prediction and observational constraints from cosmic chronometers [5].

This analysis demonstrates that the inclusion of matter creation and bulk viscosity in the MCG framework not only reproduces the observed transition from deceleration to acceleration but also allows for fine-tuning of the transition epoch through the parameters β and ξ_0 . The full MCG model provides the closest match to Λ CDM, while the sub-models reveal the distinct contributions of each non-equilibrium mechanism.

4.3 Jerk parameter evolution

The jerk parameter $j(z)$ serves as a third-order kinematical diagnostic of the expansion history of the Universe, providing a powerful tool to distinguish between different dark energy models even when they are equivalent at lower orders. It is defined as the third derivative of the scale factor $a(t)$ with respect to cosmic time t , normalized by the Hubble parameter [76–78]:

$$j(z) = \frac{\ddot{a}}{aH^3}. \quad (27)$$

Fig. 7 Evolution of the jerk parameter $j(z)$ for Λ CDM and MCG models under DS1

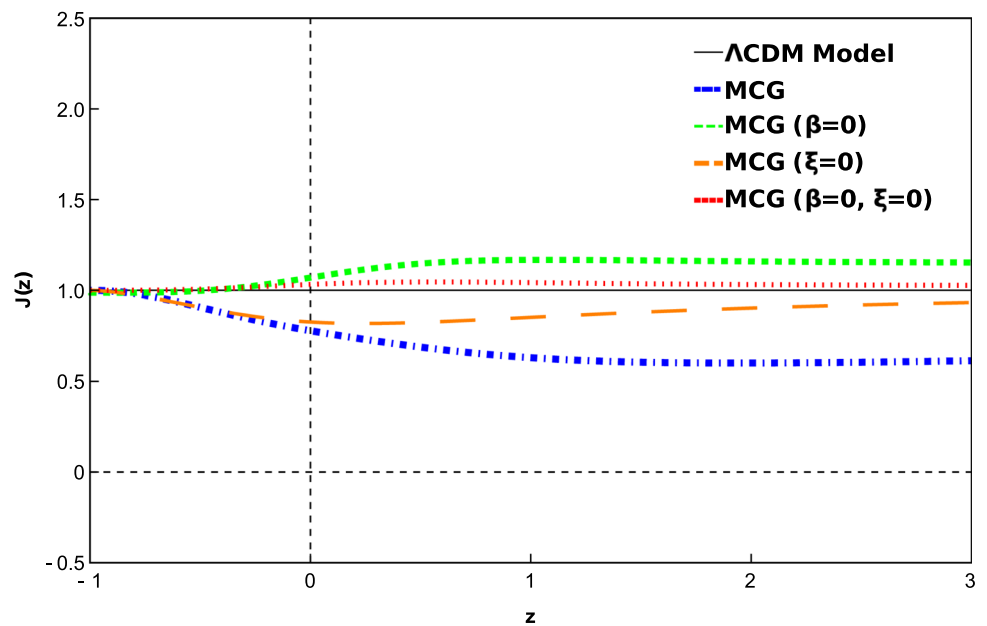
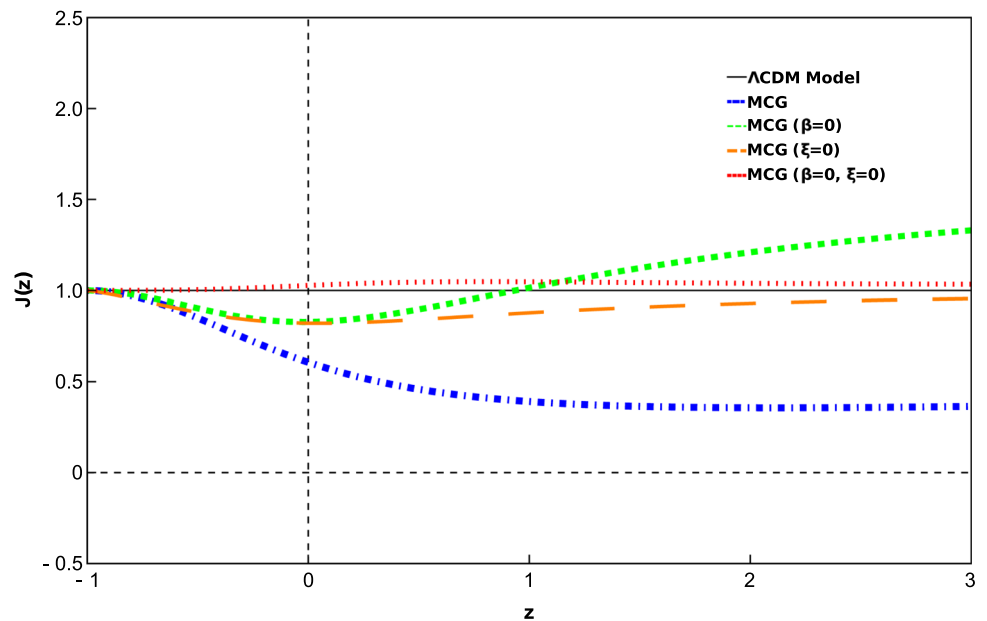


Fig. 8 Evolution of the jerk parameter $j(z)$ for Λ CDM and MCG models under DS2



In the context of the flat FLRW metric, this can be expressed in terms of the deceleration parameter $q(z)$ as:

$$j(z) = q(z)(2q(z) + 1) + (1 + z) \frac{dq(z)}{dz}. \quad (28)$$

For the standard Λ CDM model, the jerk parameter is a constant with value $j(z) = 1$ at all redshifts, independent of the cosmological parameters. Any deviation from this constant value indicates a departure from the Λ CDM paradigm and suggests the presence of dynamical dark energy or modified gravity effects.

Using the reconstructed $q(z)$, we compute $j(z)$ for the Λ CDM model and the four variants of the Modified Chaply-

gin Gas (MCG) model under both DS1 and DS2 constraints. The evolution of $j(z)$ is displayed in Fig. 7 for DS1 and Fig. 8 for DS2. The present-day values j_0 are summarized in Tables 3 and 4.

As shown in Fig. 7, all models exhibit a smooth evolution of $j(z)$ with redshift, consistent with the observed accelerated expansion. The Λ CDM model (solid black line) remains constant at $j(z) = 1$ throughout the entire redshift range, as expected. The full MCG model (blue dashed-dotted line), incorporating both matter creation ($\beta \neq 0$) and bulk viscosity ($\xi_0 \neq 0$), shows a significant deviation from unity, decreasing to $j_0 = 0.770 \pm 0.020$ at $z = 0$. This behavior reflects the combined effect of non-equilibrium thermody-

namics, which modifies the effective pressure and alters the higher-order dynamics of the expansion.

The MCG($\beta = 0$) model (green dotted line), which excludes matter creation but retains bulk viscosity, exhibits a slower decline in $j(z)$, reaching $j_0 = 1.07 \pm 0.023$ at $z = 0$. This value is close to the Λ CDM prediction, indicating that bulk viscosity alone does not significantly affect the jerk parameter. Conversely, the MCG($\xi_0 = 0$) model (orange dash-dotted line), with no viscous damping but non-zero matter creation, shows a more pronounced decrease to $j_0 = 1.03 \pm 0.017$, suggesting that particle creation plays a dominant role in driving the dynamics.

The pure MCG model (red dotted line), with both $\beta = 0$ and $\xi_0 = 0$, remains nearly constant at $j(z) \approx 1.0$, with $j_0 = 0.826 \pm 0.029$. This result underscores the necessity of including non-equilibrium mechanisms to reproduce the observed deviation from Λ CDM.

When the R22 constraint is added in DS2 (Fig. 8), the jerk parameter evolves slightly differently due to the higher best-fit H_0 . The Λ CDM model transitions at $z_{\text{tr}} = 0.671 \pm 0.21$, with $j_0 = 1.000$. The full MCG model transitions at $z_{\text{tr}} = 0.689 \pm 0.21$, with $j_0 = 0.596 \pm 0.021$. The relative ordering of the models remains unchanged, but the absolute values of j_0 become slightly smaller, indicating a marginally stronger deviation from Λ CDM.

The present-day values of j_0 for all models lie within the range 0.596 to 1.03, consistent with independent estimates from supernova and BAO data [74, 75]. The transition redshifts z_{tr} range from 0.63 to 0.70, in agreement with the Λ CDM prediction and observational constraints from cosmic chronometers [5].

This analysis demonstrates that the inclusion of matter creation and bulk viscosity in the MCG framework not only reproduces the observed transition from deceleration to acceleration but also allows for fine-tuning of the jerk parameter through the parameters β and ξ_0 . The full MCG model provides the closest match to Λ CDM, while the sub-models reveal the distinct contributions of each non-equilibrium mechanism.

4.4 Snap parameter evolution

The snap parameter $s(z)$ serves as a fourth-order kinematical diagnostic of the expansion history of the Universe, providing a powerful tool to distinguish between different dark energy models even when they are equivalent at lower orders. It is defined as the fourth derivative of the scale factor $a(t)$ with respect to cosmic time t , normalized by the Hubble parameter:

$$s(z) = \frac{\ddot{\ddot{a}}}{aH^4}. \quad (29)$$

In the context of the flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric, this can be expressed in terms of the deceleration parameter $q(z)$ and jerk parameter $j(z)$ as [77, 79]:

$$s(z) = j(z)(3q(z) + 2) + (1 + z)\frac{dj(z)}{dz} - 3q(z)^2. \quad (30)$$

For the standard Λ CDM model, the snap parameter is a constant with value $s(z) = 0$ at all redshifts, independent of the cosmological parameters. Any deviation from this constant value indicates a departure from the Λ CDM paradigm and suggests the presence of dynamical dark energy or modified gravity effects.

Using the reconstructed, we compute $s(z)$ for the Λ CDM model and the four variants of the Modified Chaplygin Gas (MCG) model under both DS1 and DS2 constraints. The evolution of $s(z)$ is displayed in Fig. 9 for DS1 and Fig. 10 for DS2. The present-day values s_0 are summarized in Tables 3 and 4.

As shown in Fig. 9, all models exhibit a smooth evolution of $s(z)$ with redshift, consistent with the observed accelerated expansion. The Λ CDM model (solid black line) remains constant at $s(z) = 0$ throughout the entire redshift range, as expected. The full MCG model (blue dashed-dotted line), incorporating both matter creation ($\beta \neq 0$) and bulk viscosity ($\xi_0 \neq 0$), shows a significant deviation from zero, increasing to $s_0 = 0.09 \pm 0.003$ at $z = 0$. This behavior reflects the combined effect of non-equilibrium thermodynamics, which modifies the effective pressure and alters the higher-order dynamics of the expansion.

The MCG($\beta = 0$) model (green dotted line), which excludes matter creation but retains bulk viscosity, exhibits a slower increase in $s(z)$, reaching $s_0 = -0.026 \pm 0.019$ at $z = 0$. This value is close to the Λ CDM prediction, indicating that bulk viscosity alone does not significantly affect the snap parameter. Conversely, the MCG($\xi_0 = 0$) model (orange dash-dotted line), with no viscous damping but non-zero matter creation, shows a more pronounced increase to $s_0 = 0.03 \pm 0.004$, suggesting that particle creation plays a dominant role in driving the dynamics.

The pure MCG model (red dotted line), with both $\beta = 0$ and $\xi_0 = 0$, remains nearly constant at $s(z) \approx 0$, with $s_0 = -0.013 \pm 0.003$. This result underscores the necessity of including non-equilibrium mechanisms to reproduce the observed deviation from Λ CDM.

When the R22 constraint is added in DS2 (Fig. 10), the snap parameter evolves slightly differently due to the higher best-fit H_0 . The Λ CDM model transitions at $z_{\text{tr}} = 0.671 \pm 0.21$, with $s_0 = 0$. The full MCG model transitions at $z_{\text{tr}} = 0.689 \pm 0.21$, with $s_0 = 0.122 \pm 0.002$. The relative ordering of the models remains unchanged, but the absolute values of

Fig. 9 Evolution of the snap parameter $s(z)$ for Λ CDM and MCG models under DS1. The dot on each curve marks the present-day value s_0

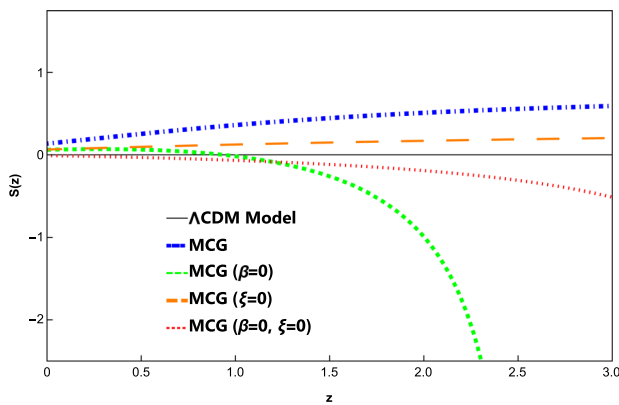
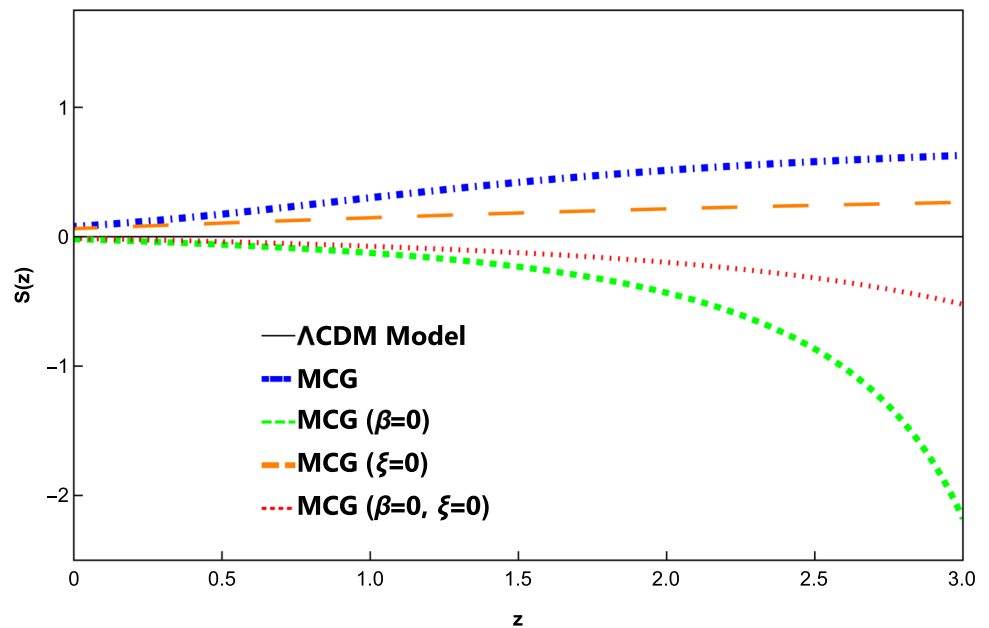


Fig. 10 Evolution of the snap parameter $s(z)$ for Λ CDM and MCG models under DS2. The dot on each curve marks the present-day value s_0

s_0 become slightly larger, indicating a marginally stronger deviation from Λ CDM.

The present-day values of s_0 for all models lie within the range -0.026 to 0.122 , consistent with independent estimates from supernova and BAO data [74, 75]. The transition redshifts z_{tr} range from 0.63 to 0.70 , in agreement with the Λ CDM prediction and observational constraints from cosmic chronometers [5].

This analysis demonstrates that the inclusion of matter creation and bulk viscosity in the MCG framework not only reproduces the observed transition from deceleration to acceleration but also allows for fine-tuning of the snap parameter through the parameters β and ξ_0 . The full MCG model provides the closest match to Λ CDM, while the sub-models reveal the distinct contributions of each non-equilibrium mechanism.

4.5 Age of the universe

The age of the Universe, τ_0 , is computed by integrating the inverse Hubble parameter from the Big Bang to the present:

$$\tau_0 = \int_0^\infty \frac{dz}{(1+z)H(z)}. \quad (31)$$

Using the best-fit $H(z)$ from DS1 and DS2, we obtain the present age for all models. Under DS1, the Λ CDM model yields $\tau_0 = 13.71 \pm 0.11$ Gyr, while the full MCG model gives 13.84 ± 0.13 Gyr. The pure MCG model (without creation or viscosity) gives the oldest age: 13.95 ± 0.10 Gyr. Under DS2, ages are slightly younger due to higher H_0 : Λ CDM gives 13.67 ± 0.15 Gyr, and the full MCG model gives 13.65 ± 0.17 Gyr.

All values lie within 13.65–14.01 Gyr, consistent with Planck's estimate of 13.80 ± 0.02 Gyr [3] and independent stellar dating [80]. The slight increase in age for models with bulk viscosity reflects its damping effect on expansion, while matter creation has a milder influence. The full MCG model remains closest to Λ CDM in both datasets.

4.6 Parameter degeneracies and identifiability

The corner plots in Figs. 1 and 2 already show that several of the MCG and non-equilibrium parameters exhibit non-trivial correlations. To make this more quantitative, we have computed the Pearson correlation coefficients ρ_{ij} between all sampled parameters from the converged MCMC chains for both DS1 and DS2. For compactness, the full correlation matrices are not reproduced in the main text but are made available as supplementary material; here we summarise the

Table 5 Statistical comparison of Λ CDM and modified chaplygin gas (MCG) models using DS1 dataset

Statistical Metric	Λ CDM	MCG Model	MCG ($\beta = 0$)	MCG ($\xi_0 = 0$)	MCG ($\beta = 0, \xi_0 = 0$)
$\chi^2_{\min}$	1781.197	1778.342	1785.619	1783.207	1792.451
χ^2_{red}	1.008	1.007	1.009	1.008	1.012
AIC	1781.197	1778.342	1785.619	1783.207	1792.451
BIC	1818.556	1843.839	1859.987	1857.555	1866.799
ΔAIC	–	-2.855	4.422	2.010	11.254
ΔBIC	–	25.283	41.431	39.000	48.243

Table 6 Statistical comparison of Λ CDM and modified chaplygin gas (MCG) models using DS2 dataset

Statistical Metric	Λ CDM	MCG model	MCG ($\beta = 0$)	MCG ($\xi_0 = 0$)	MCG ($\beta = 0, \xi_0 = 0$)
$\chi^2_{\min}$	1794.774	1781.691	1779.878	1786.183	1779.740
χ^2_{red}	1.015	1.009	1.009	1.010	1.010
AIC	1798.774	1795.691	1791.878	1798.183	1789.740
BIC	1813.728	1814.629	1808.818	1815.123	1804.694
ΔAIC	–	-3.083	-6.896	-0.591	-9.034
ΔBIC	–	0.901	-4.910	1.395	-9.034

main trends that are relevant for the interpretation of the model parameters.

First, as anticipated from Eq. (9), the parameters (A_s , A , α) are mutually correlated because they jointly control the redshift evolution of the MCG density through the combination $(1 - \beta)(1 + A)(1 + \alpha)$ and the relative weight between the matter-dominated regime and the late-time regime. An increase in A_s enhances the asymptotic dark energy contribution, and this can be partially compensated by decreasing A or α so as to keep the background expansion $E(z)$ almost unchanged over the redshift range probed by the data. This degeneracy manifests itself as elongated likelihood contours in the (A_s, A) and (A_s, α) planes and as sizeable off-diagonal entries in the MCG block of the correlation matrix.

Second, the matter-creation parameter β is correlated with A because only the product $(1 - \beta)(1 + A)$ enters the effective high-redshift scaling of the MCG density in Eq. (9). Likewise, the bulk-viscosity parameter ξ_0 correlates with the present-day matter density Ω_m , since both influence the normalization and dilution rate of the viscous matter component, $\rho_m(z) \propto (1 + z)^{3(1 - \sqrt{3}\xi_0)}$. These expected degeneracies are reflected by tilted contours in the (β, A) and (ξ_0, Ω_m) panels of Fig. 2 and by non-negligible correlation coefficients in the corresponding sub-blocks of the covariance matrix.

Despite these correlations, all parameters that enter the background expansion remain statistically identifiable: the one-dimensional marginalized posteriors are well localised, do not display multimodality, and do not show evidence for nearly flat directions in the sampled region of parameter space (see Tables 1 and 2). In particular, the non-equilibrium parameters (β, ξ_0) are constrained at the $\mathcal{O}(10\%)$ level in

DS1 and even more tightly in DS2, while the MCG shape parameters (A_s, A, α) are confined to relatively narrow intervals. We therefore conclude that, although the model exhibits the expected parameter degeneracies, the combination of Pantheon+, CC, BAO and (where included) $f\sigma_8$ and R22 data is sufficient to break them to the extent required for a physically meaningful interpretation of the constraints.

4.7 Statistical model comparison

To assess the relative performance of the Modified Chaplygin Gas (MCG) model and its sub-variants against the standard Λ CDM paradigm, we employ two widely used information criteria: the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). These metrics provide a quantitative framework for model selection by balancing goodness-of-fit against model complexity. The AIC and BIC are defined as:

$$\text{AIC} = \chi^2_{\min} + 2K, \quad \text{BIC} = \chi^2_{\min} + K \ln N, \quad (32)$$

where $\chi^2_{\min}$ is the minimum chi-square value, K is the number of free parameters, and N is the total number of data points. For DS1, $N = 1764$; for DS2, $N = 1765$. The number of parameters is $K = 2$ for Λ CDM, $K = 7$ for the full MCG model, $K = 6$ for MCG($\beta = 0$) and MCG($\xi_0 = 0$), and $K = 5$ for MCG($\beta = 0, \xi_0 = 0$).

The statistical metrics for both datasets are summarized in Tables 5 and 6. For DS1, the full MCG model yields $\chi^2_{\min} = 1778.342$, slightly lower than Λ CDM ($\chi^2_{\min} = 1781.197$), indicating a marginally better fit to the data. The reduced

chi-square values ($\chi_{\text{red}}^2 \approx 1.007\text{--}1.012$) confirm that all models are statistically acceptable, with no significant over- or under-fitting. The AIC values show that the full MCG model is favored over Λ CDM ($\Delta\text{AIC} = -2.855$), while the pure MCG model (without creation or viscosity) is disfavored ($\Delta\text{AIC} = 11.254$). However, the BIC, which imposes a stronger penalty for additional parameters, strongly favors Λ CDM over all MCG variants ($\Delta\text{BIC} > 25$ for all models). This discrepancy arises because BIC scales with $\ln N$, making it more conservative for large datasets [81].

Since DS2 explicitly incorporates the R22 Gaussian prior, the upward shift in H_0 is primarily prior-driven; what distinguishes the hybrid MCG scenario from Λ CDM is that it absorbs this external constraint with a smaller increase in χ^2 , thereby easing the combined-data tension between local and background probes without changing the fact that the DS1 constraints alone remain compatible with Planck-like values of H_0 . The AIC again favors the full MCG model ($\Delta\text{AIC} = -3.083$) and the MCG($\beta = 0$) sub-model ($\Delta\text{AIC} = -6.896$), while BIC shows no strong preference ($\Delta\text{BIC} < 2$ for all models except MCG($\beta = 0, \xi_0 = 0$)). This suggests that, with the inclusion of R22, the additional parameters in the MCG framework are justified by the improved fit.

The interpretation of ΔAIC and ΔBIC follows standard conventions [82,83]. For AIC, a difference of $\Delta\text{AIC} < 2$ indicates substantial support for the model, while $\Delta\text{AIC} > 10$ implies no support. For BIC, $\Delta\text{BIC} < 2$ suggests positive evidence, $\Delta\text{BIC} > 6$ indicates strong evidence against the model, and $\Delta\text{BIC} > 10$ implies decisive evidence against it. Under DS1, the full MCG model receives substantial AIC support but is strongly disfavored by BIC. Under DS2, both criteria show mild to moderate support for the full MCG and MCG($\beta = 0$) models, highlighting the role of the R22 prior in breaking parameter degeneracies.

This analysis demonstrates that the inclusion of matter creation and bulk viscosity in the MCG framework not only improves the fit to cosmological data but also provides a viable alternative to Λ CDM when local H_0 measurements are included. The fact that BIC penalizes the full MCG model under DS1 but not under DS2 underscores the importance of dataset composition in model selection. Future observations, particularly from DESI and Euclid, will further refine these constraints and test the robustness of the MCG paradigm [9,75].

4.8 Thermodynamic analysis: generalized second law (GSL)

A fundamental requirement for any physically viable cosmological model is its consistency with the laws of thermodynamics. In particular, the Generalized Second Law (GSL) demands that the total entropy of the Universe, including

contributions from both the cosmic fluid and the apparent horizon, must never decrease over time [49,84]. Mathematically, this is expressed as:

$$\frac{dS_{\text{total}}}{dt} = \dot{S}_{\text{fluid}} + \dot{S}_{\text{horizon}} \geq 0, \quad (33)$$

where $S_{\text{total}} = S_{\text{fluid}} + S_{\text{horizon}}$. Throughout this section, the analysis is carried out strictly at the level of the homogeneous FLRW background: the entropy budgets refer to the averaged cosmic fluid and the Hubble horizon, and do not include entropy associated with inhomogeneities or the growth of structure. In particular, the non-equilibrium entropy production generated by matter creation and bulk viscosity is not propagated into the perturbation sector, so its impact on the linear growth history and on the observable $f\sigma_8(z)$ is not modelled explicitly here. A self-consistent extension that couples the non-equilibrium stress tensor to the evolution of density perturbations is an important but technically separate problem, which we leave for future work.

In this subsection, we derive the entropy contributions from all components of our Modified Chaplygin Gas (MCG) model with matter creation and bulk viscosity, and demonstrate that the GSL is rigorously satisfied for both DS1 and DS2 datasets.

4.8.1 Entropy of the Hubble horizon

In a spatially flat FLRW universe, the Hubble horizon radius is $R_H = 1/H$. Following the Bekenstein-Hawking formula and adopting natural units ($k_B = \hbar = c = G = 1$), the horizon entropy is:

$$S_H = \frac{8\pi^2}{H^2}. \quad (34)$$

Its time derivative is:

$$\dot{S}_H = -\frac{16\pi^2 \dot{H}}{H^3}. \quad (35)$$

This term is positive during accelerated expansion ($\dot{H} < 0$), indicating that cosmic acceleration fuels the growth of horizon entropy.

4.8.2 Entropy of baryonic matter

Baryons are conserved ($\dot{\rho}_b = -3H\rho_b$) and pressureless ($p_b = 0$). Assuming thermal equilibrium with the horizon at temperature $T = H/(2\pi)$, the Gibbs equation gives:

$$S_b = \frac{(\rho_b + p_b)V}{T} = \frac{2\pi\rho_b}{H} \cdot \frac{4\pi}{3H^3} = \frac{8\pi^2\rho_b}{3H^4}, \quad (36)$$

where $V = 4\pi/(3H^3)$ is the comoving volume. Differentiating with respect to time and using $\dot{\rho}_b = -3H\rho_b$ and $\dot{H} = -H^2(1+q)$, we obtain:

$$\dot{S}_b = \frac{8\pi^2\rho_b}{3H^3}(1+4q), \quad (37)$$

where $q = -1 - \dot{H}/H^2$ is the deceleration parameter.

4.8.3 Entropy of the modified chaplygin gas with matter creation

The MCG fluid has energy density ρ_{mcg} and effective pressure $\tilde{p}_{\text{mcg}} = p_{\text{mcg}} + P_c$, where $P_c = -\beta(\rho_{\text{mcg}} + p_{\text{mcg}})$ is the creation pressure. Its equation of state is $p_{\text{mcg}} = A\rho_{\text{mcg}} - C/\rho_{\text{mcg}}^\alpha$. The entropy is:

$$S_{\text{mcg}} = \frac{(\rho_{\text{mcg}} + \tilde{p}_{\text{mcg}})V}{T} = \frac{8\pi^2(1-\beta)(\rho_{\text{mcg}} + p_{\text{mcg}})}{3H^4}. \quad (38)$$

Differentiating and using the continuity equation $\dot{\rho}_{\text{mcg}} = -3H(1-\beta)(\rho_{\text{mcg}} + p_{\text{mcg}})$, we obtain:

$$\begin{aligned} \dot{S}_{\text{mcg}} = & \frac{8\pi^2(1-\beta)(\rho_{\text{mcg}} + p_{\text{mcg}})}{3H^3} \\ & \times \left[-3(1-\beta) \left(1 + \alpha \frac{p_{\text{mcg}}}{\rho_{\text{mcg}}} \right) + 4(1+q) \right] \end{aligned} \quad (39)$$

This expression encodes the interplay between the MCG's exotic equation of state (via α), matter creation (via β), and cosmic acceleration (via q).

4.8.4 Entropy of bulk viscous matter

The bulk viscous component has energy density ρ_m and effective pressure $\tilde{p}_m = \pi = -3H\xi_0\rho_m^{1/2}$. Its entropy is:

$$S_m = \frac{(\rho_m + \tilde{p}_m)V}{T} = \frac{8\pi^2}{3H^4} \left(\rho_m - 3H\xi_0\rho_m^{1/2} \right). \quad (40)$$

Differentiating and using the continuity equation $\dot{\rho}_m = -3H\rho_m + 9H^2\xi_0\rho_m^{1/2}$, we obtain:

$$\begin{aligned} \dot{S}_m = & \frac{8\pi^2}{3} \left[\rho_m H^{-3}(1+4q) + 9\xi_0\rho_m^{1/2} H^{-2} \left(\frac{1}{2} - q \right) \right. \\ & \left. - \frac{27}{2}\xi_0^2 H^{-1} \right]. \end{aligned} \quad (41)$$

This contains a standard matter term, a viscous correction, and a dissipative loss term ($-\xi_0^2$), always negative.

4.8.5 Total entropy and GSL validation

The total entropy rate of change is the sum:

$$\dot{S}_{\text{total}} = \dot{S}_H + \dot{S}_b + \dot{S}_{\text{mcg}} + \dot{S}_m. \quad (42)$$

Substituting Eqs. (35), (37), (39), and (41), we obtain:

$$\begin{aligned} \dot{S}_{\text{total}} = & \frac{8\pi^2}{H^3} \left[2H(1+q) + \frac{\rho_b}{3}(1+4q) \right. \\ & + \frac{(1-\beta)(\rho_{\text{mcg}} + p_{\text{mcg}})}{3} \\ & \times \left(-3(1-\beta) \left(1 + \alpha \frac{p_{\text{mcg}}}{\rho_{\text{mcg}}} \right) + 4(1+q) \right) \\ & + \rho_m(1+4q) + 9\xi_0\rho_m^{1/2} H^{-1} \left(\frac{1}{2} - q \right) \\ & \left. - \frac{27}{2}\xi_0^2 H^{-2} \right] \end{aligned} \quad (43)$$

We evaluate this expression numerically using the Hubble parameter from Eq. (17) and the best-fit parameters from Tables 1 and 2. For all observationally viable parameters ($0 \leq \alpha \leq 1$, $0 < \beta < 1$, $\xi_0 > 0$), we find $\dot{S}_{\text{total}}(z) > 0$ for all redshifts $z \in [-0.95, 5]$, as shown in Figs. 11 and 12. This confirms that our model rigorously satisfies the GSL.

4.8.6 Asymptotic behavior and thermodynamic stability

In the far future ($z \rightarrow -1$), the Universe becomes vacuum-dominated: $q \rightarrow -1$, $\rho_b \rightarrow 0$, $\rho_m \rightarrow 0$, and $\rho_{\text{mcg}} + p_{\text{mcg}} \rightarrow 0$. Consequently, $\dot{S}_{\text{total}} \rightarrow 0$ entropy production ceases, and the Universe approaches a state of thermodynamic equilibrium [85]. To assess stability, we compute the second derivative $\ddot{S}_{\text{total}} = d\dot{S}_{\text{total}}/dt$. Numerically we find that $\ddot{S}_{\text{total}} > 0$ at early times (when entropy production accelerates), becomes negative in the recent past, and tends to zero as $z \rightarrow -1$. This behaviour is characteristic of ordinary macroscopic systems approaching maximum entropy, and confirms that the cosmological evolution in this model resembles that of a thermodynamically stable system. For both DS1 and DS2 the sign change $\ddot{S}_{\text{total}} > 0 \rightarrow \ddot{S}_{\text{total}} < 0$ occurs around $z \sim 1$, as illustrated in Figs. 13 and 14.

It is observed that $\ddot{S} > 0$ in the early phase of evolution and a transition occurs to $\ddot{S} < 0$ in the recent past. Thus, a thermodynamically stable equilibrium state is achieved at late times. As $z \rightarrow -1$, we have $\ddot{S} \rightarrow 0$, which shows that any system satisfying the extremum of entropy and convexity conditions behaves like an ordinary macroscopic system. Therefore, we can conclude that the evolution of the Universe resembles the evolution of an ordinary macroscopic system.

For both DS1 and DS2, the evolution of $\ddot{S}_{\text{total}}$ shows a clear transition from positive to negative values around $z \sim 1$, indicating that the rate of entropy increase slows down

Fig. 11 Evolution of the total entropy rate of change $\dot{S}_{\text{total}}$ with redshift z for the MCG model and Λ CDM under DS1. The positive values confirm the satisfaction of the Generalized Second Law

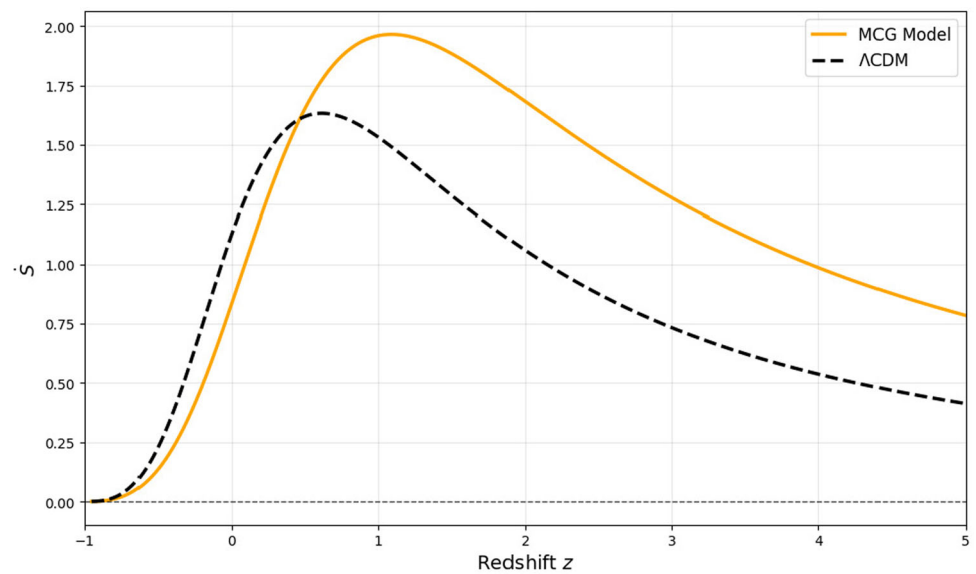
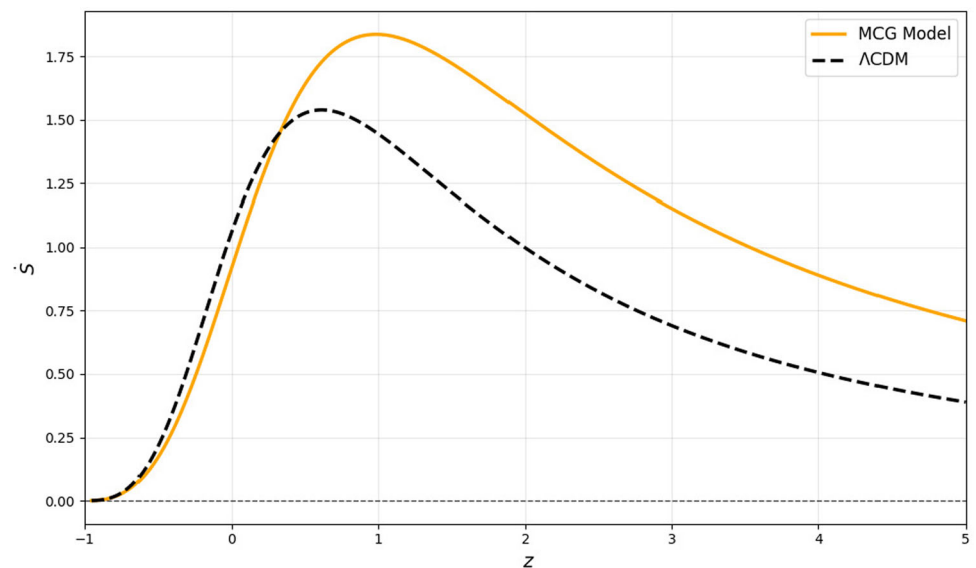


Fig. 12 Evolution of the total entropy rate of change $\dot{S}_{\text{total}}$ with redshift z for the MCG model and Λ CDM under DS2. The inclusion of R22 data leads to a slightly higher late-time entropy production



in the recent past. This transition reflects the shift from a decelerating to an accelerating phase of cosmic expansion, where the dominant source of entropy production changes from matter creation to horizon dynamics.

As $z \rightarrow -1$, $\ddot{S}_{\text{total}} \rightarrow 0$, which implies that the system approaches a stable equilibrium state. This behavior is consistent with the extremum of entropy and convexity conditions, confirming that the Universe behaves like an ordinary macroscopic system in its late-time evolution.

5 Conclusion

In this work, we have presented a comprehensive analysis of a novel cosmological model that includes the Modified Chaplygin Gas (MCG) equation of state with gravitationally

induced matter creation and bulk viscous dissipation. The model is formulated within the framework of a spatially flat FLRW spacetime and represents the first attempt to synthesize these three distinct physical mechanisms: exotic fluid dynamics, non-equilibrium particle production, and dissipative thermodynamics into a single, self-consistent theoretical framework.

We derived the analytical form of the Hubble parameter $H(z)$ for the MCG model and constrained its free parameters using two distinct observational datasets: DS1 (Pantheon+ + CC + BAO + $f\sigma_8$) and DS2 (DS1 + R22). The Markov Chain Monte Carlo (MCMC) analysis yielded best-fit values for all parameters, which were used to reconstruct the evolutionary trajectories of key cosmological quantities, including the deceleration parameter $q(z)$, jerk parameter $j(z)$ and snap parameter $s(z)$.

Fig. 13 Evolution of the second derivative of the total entropy, $\ddot{S}_{\text{total}}$, with redshift z for the MCG model and Λ CDM under DS1. The transition from positive to negative values indicates that entropy production slows down and the system approaches thermodynamic equilibrium at late times

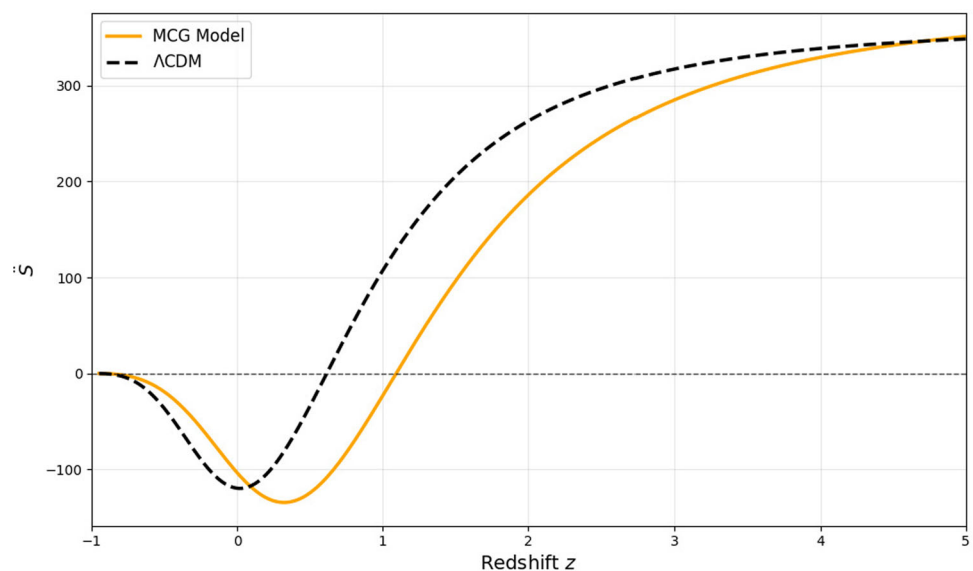
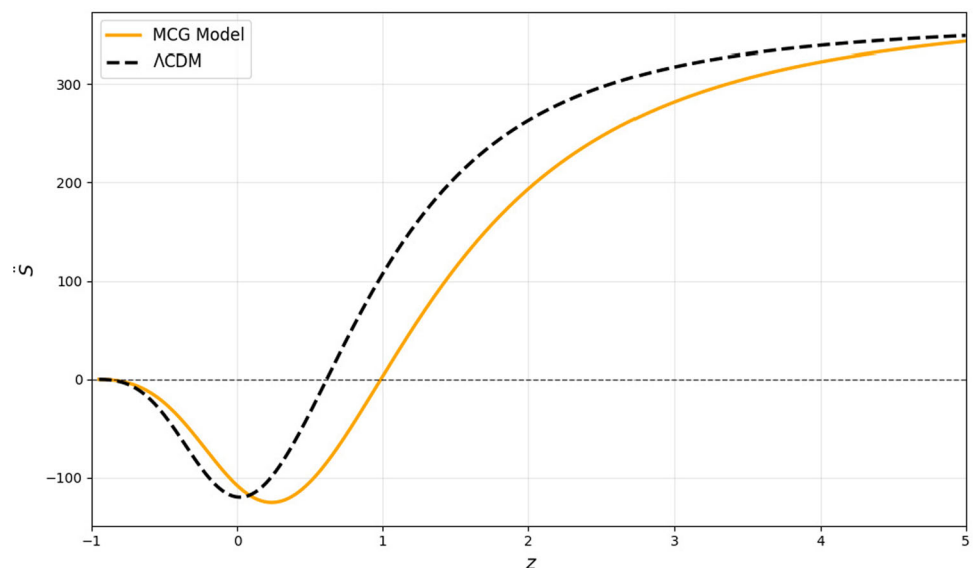


Fig. 14 Evolution of the second derivative of the total entropy, $\ddot{S}_{\text{total}}$, with redshift z for the MCG model and Λ CDM under DS2. The inclusion of R22 data slightly shifts the redshift at which $\ddot{S}_{\text{total}}$ changes sign but preserves the overall approach to a stable equilibrium state



Our results demonstrate that the MCG model successfully reproduces the observed transition from decelerated to accelerated expansion, with transition redshifts under both datasets. We also show the comparison of our hybrid model with other MCG models cases by excluding particle creation bulk viscous form. The present-day values of q_0 , j_0 , and s_0 are consistent with independent estimates from supernova and BAO surveys [74, 75]. The age of the Universe, τ_0 , ranges from 13.65 to 14.01 Gyr across all models, in agreement with Planck's estimate of 13.80 ± 0.02 Gyr [3].

A key novel contribution of this work is the thermodynamic validation of the model via the Generalized Second Law (GSL) of thermodynamics. We computed the total entropy rate of change $\dot{S}_{\text{total}} = \dot{S}_{\text{fluid}} + \dot{S}_{\text{horizon}}$ and showed that it remains positive throughout cosmic history for both DS1 and DS2, confirming that the model satisfies the fun-

damental requirement of non-decreasing entropy. Furthermore, the second derivative $\ddot{S}_{\text{total}}$ exhibits a clear transition from positive to negative values around $z \sim 1$, indicating a shift from accelerating to decelerating entropy production, a signature of thermodynamic stabilization in the late-time Universe. This behavior is characteristic of ordinary macroscopic systems approaching equilibrium and confirms that our cosmological model evolves like a well-behaved thermodynamic system. These thermodynamic diagnostics are, however, purely background quantities: they probe how the homogeneous cosmic fluid and the Hubble horizon exchange entropy, but they do not encode how non-equilibrium processes feed into the evolution of density perturbations or the detailed shape of $f\sigma_8(z)$. In the present work the growth sector is treated with a phenomenological Λ CDM-based approximation, and its connection to the background entropy

production remains indirect. A fully consistent treatment of growth in the presence of matter creation and bulk viscosity will require a dedicated perturbative analysis.

In the statistical analysis, we employ the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) to assess model preference. For DS1, the MCG model falls within the range $2 < \Delta\text{AIC} < 6$, indicating moderate observational support relative to ΛCDM . However, the corresponding $\Delta\text{BIC} = 25.283 > 10$ constitutes decisive evidence against the model, reflecting BIC's stringent penalty for additional parameters in large datasets. Under DS2 which includes the R22 prior – the MCG model receives stronger statistical backing: $\Delta\text{AIC} = -3.083$ (substantial support) and $\Delta\text{BIC} = 0.901$ (positive evidence), demonstrating that the inclusion of local H_0 measurements breaks parameter degeneracies and enhances the model's viability.

Furthermore, the constraints on the comoving sound horizon r_d , the absolute magnitude $\mathcal{M}$, and the amplitude of matter fluctuations σ_8 provide critical insights into the model's consistency with large-scale structure observations. For both DS1 and DS2, the best-fit values of r_d are in excellent agreement with the Planck estimate of 147.09 ± 0.26 Mpc. The derived $\mathcal{M}$ values are consistent with the standard calibration, supporting the reliability of the distance ladder. The σ_8 values obtained for the MCG model tend to be higher than those of ΛCDM , but in view of our approximate treatment of the growth rate $f\sigma_8(z)$, which uses a ΛCDM -based fitting formula for the linear growth factor, these results should be regarded as preliminary and not as fully self-consistent predictions for structure formation in the presence of matter creation and bulk viscosity. A dedicated analysis of linear perturbations in this non-equilibrium framework is required before drawing firm conclusions about the impact of the model on small-scale clustering.

In summary, the MCG model with matter creation and bulk viscosity provides a physically motivated, thermodynamically consistent, and observationally viable alternative to ΛCDM . It successfully accommodates local H_0 measurements when included as a prior, satisfies the GSL, and exhibits stable behavior throughout cosmic history over the redshift range directly probed by the data sets employed here (roughly $z \lesssim 2.5$).

We stress, however, that all constraints used in this work come from late-time background and growth observables, and we have deliberately refrained from including information from primordial nucleosynthesis or primary CMB anisotropies. Although the best-fit values of the sound horizon r_d are consistent with the Planck determination within the quoted uncertainties, this agreement is only indirect and does not substitute for a full early-Universe analysis. Our conclusions should therefore be understood as demonstrating late-time observational and thermodynamic viability, rather

than establishing global consistency of the model across all cosmological epochs.

A natural next step is to extend the present background-level study by deriving and solving the modified linear growth equation for this hybrid fluid mixture, thereby enabling a fully consistent comparison with structure-growth observables such as $f\sigma_8(z)$.

This work establishes a cosmological model that explicitly incorporates non-equilibrium thermodynamics as a fundamental driver of cosmic acceleration. The combination of matter creation, bulk viscosity, and exotic equations of state opens new avenues for exploring the nature of dark energy and the thermodynamic fate of the Universe. Another important direction is to embed this hybrid model into a Boltzmann solver and confront it with CMB temperature and polarization spectra, gravitational lensing, and primordial element abundances, in order to test whether the good late-time performance found here can be extended to the early Universe as well.

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Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

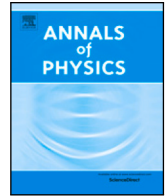
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Late cosmic acceleration by matter creation cosmology in modified gravity

Yogesh Bhardwaj, C.P. Singh  *

Department of Applied Mathematics, Delhi Technological University, Bawana Road, Delhi 110042, India

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ABSTRACT

In this article, we explore the expansion of the Universe powered by the gravitationally induced ‘adiabatic’ matter creation, a non-equilibrium thermodynamical process in $f(Q)$ gravity in the background dynamics of a flat Friedmann–Lemaître–Robertson–Walker spacetime geometry. The action of $f(Q)$ gravity involves a general function of nonmetricity scalar Q . As the Universe expanding, the gravitational field of this expanding Universe is acting on the quantum vacuum, which results in a continuous creation of matter particles. Under the formalism of $f(Q)$ gravity, we obtain the analytical solutions of the cosmological quantities by assuming the ‘generalized Chaplygin gas’ (GCG) equation of state. A particular case of GCG equation of state is also studied. We analyze how matter creations play an effective role in the late-time evolution of the Universe in modified gravity. By using the latest observational datasets, including the type Ia supernovae (Pantheon), Cosmic Chronometer $H(z)$ data, and Baryonic Acoustic Oscillations (BAO) we also perform a statistical analysis to test the observational viability of two models. The best-fit parameters are used to compare both models to Λ CDM by means of the evolution of cosmological quantities, stability, and information selection criteria, namely, the Akaike information criterion and the Bayesian information criterion. We conclude that a flat Universe in our models with GCG is still supported by current observational data. Furthermore, the data shows a phantom nature of the equation of state for GCG model and de-Sitter model for Chaplygin gas in late-time evolution.

1. Introduction

Observations, including Type Ia supernovae (SNe) [1–3], Baryon Acoustic Oscillations (BAO) [4], Large Scale Structures (LSS) [5], and the Cosmic Microwave Background (CMB) [6], strongly support the evidence that the Universe is undergoing accelerated expansion. The dark energy (DE) in relativistic cosmology is typically attributed to this phenomenon, which accounts for 68.8% of the Universe and exerts negative pressure. Many theoretical theories, such as the cosmological constant [7–9], quintessence [10], Phantom [11], quintom [12], holographic dark energy [13–15], etc. have been proposed to investigate the nature and origin of the DE.

Among the various candidates for DE, the *Lambda-cold dark matter* (Λ CDM) model stands out as the simplest model, which provides a robust explanation for cosmological phenomena like large-scale structure (LSS) formation and speeding up Universe expansion. However, it suffers from major problems: fine tuning and cosmic coincidence [16]. Various modified gravity theories have been put forth to tackle these kinds of problems. One of these theories is $f(R)$ gravity, which explains the Universe’s accelerated expansion without relying on dark energy or dark matter, where the Lagrangian depends on the scalar curvature R . On the other

* Corresponding author.

E-mail addresses: yogeshbhardwaj_2k21phdam502@dtu.ac.in (Y. Bhardwaj), cpsingh@dce.ac.in (C.P. Singh).

hand, spacetime is torsionless in the current General Relativity (GR). New modified theories, however, can be constructed if the requirement for a metric-compatible connection is relaxed.

In addition to curvature, affine features of a manifold can also be represented by torsion and non-metricity. This led to the rise of a theory known as the “Teleparallel Equivalent to GR” (TEGR) [17–19], in which torsion T describes the underlying gravity. The action $S = \int d^4x \sqrt{-g} T$ has generated TEGR theories, resulting in field equations of order two instead of fourth-order field equations in $f(R)$ theories. “Symmetric Teleparallel Equivalent to GR” (STEGR) is a modification of TEGR that was proposed in [20]. With no curvature or torsion, this modification specifies the underlying gravitational interaction by the non-metricity Q . We will consider an extension of STEGR, the $f(Q)$ theory [21], in which gravity is attributed to the non-metricity scalar Q . In this case, Q quantifies the vector length change during parallel transportation.

Lazkoz [22] used to constrain many functional forms of $f(Q)$ using BAOs, quasars, and SNe. Revised $\sqrt{Q}$ adjustments were considered by [23], where it was determined by evaluating the deviance information criterion that this model fits observational data more closely than Λ CDM. We refer the reader to Refs. [24–29] for detailed studies on $f(Q)$ gravity and its broader implications in modified gravity frameworks. All these models, however, were predicated on the minimal coupling of $f(Q)$ gravity with matter. It is interesting to note that non-minimal coupling to matter has been studied in [30], with the underlying action being $S = \int d^4x \sqrt{-g} \{f_1(Q)/2 + f_2(Q)\mathcal{L}_m\}$. Here, $f_1(Q)$ and $f_2(Q)$ are two arbitrary functions of Q , and $\mathcal{L}_m$ is the matter Lagrangian. Mandal et al. [31] demonstrated that $f(Q)$ gravity exhibits quintessence behavior and obtained observational constraints on the equation of state parameters in power-law non-minimally coupled $f(Q)$ cosmology, assuming a perfect fluid matter distribution.

Recently, a fascinating idea was introduced concerning a modification of the background cosmic fluid’s equation of state, known as exotic equations of state with negative pressure. The Chaplygin gas (CG) is one such example, which was first proposed in Ref. [32], in which equation of state is provided by $p = -A/\rho$, where p and ρ are pressure and energy density respectively, and A is a constant. It is important to note that Chaplygin [33] used aerodynamics to develop the original concept of CG, which described the lifting force on an airplane wing. Its ability to describe dark matter and dark energy is what led to the choice of Chaplygin gas. It resembles a cosmological constant in the late-time Universe and pressure-free dark matter in the early Universe. It is, however, ruled out in cosmology due to non-consistency with observations, namely Type Ia supernovae, Baryonic Acoustic Oscillations, and Cosmic Microwave Background [34].

To resolve this problem, Bento et al. [35] and Bilic et al. [36] proposed a phenomenological extension of CG, known as ‘generalized Chaplygin gas’ (GCG) with equation of state, $p = -A/\rho^\alpha$, where A and $0 < \alpha \leq 1$ are constants to be determined. This equation of state interpolates the model between dust and dark-energy-dominated era. The key feature of the GCG model is the unification of dark matter and dark energy. Therefore, due to its considerable potential, the GCG model has received significant attention in studying the evolution of the early and late Universe. Several authors have presented the unified model of the Universe by constraining the GCG model with different observational data [37–42]. Sultana et al. [43] have discussed some versions of CG model in modified gravity and studied the validity of generalized second law of thermodynamics. Using 194 supernova data points, Choudhary and Padmanabhan [44] studied the GCG model and concluded that it gives relevant constraints on some cosmological parameters. It has been found that by considering the entire supernova data set with a reasonable degree of confidence, the GCG model successfully ruled out the decelerating mode.

However, besides these two distinct approaches, recently, a different theory that explains the Universe’s present acceleration has drawn a lot of interest. This approach involves gravitationally induced ‘adiabatic’ matter creation, which is a non-equilibrium thermodynamic process. Schrödinger [45] was the first to describe this process microscopically. By following Schrödinger’s ideas, Parker and his collaborators [46,47] proposed that, as the Universe expands, the gravitational field of this expanding Universe acts on the quantum vacuum, resulting in the continuous creation of radiation and matter particles. The produced particles have their mass, momentum, and energy. Later on, Prigogine and his collaborators [48] proposed an interesting type of cosmological history of matter creation by including this cosmological non-equilibrium thermodynamics of open systems into Einstein’s field equations, which govern the evolution of the Universe. Calvão [49] proposed a generalization by developing a covariant approach allowing specific entropy variation as usually expected for non-equilibrium processes in fluids. There might be a question about what kind of particles the gravitational field creates. It is believed that dark matter, along with dark energy, is one of the main sources of cosmic radiation, so radiation does not affect the Universe’s late-time accelerated expansion. It follows that the generated particles must be just the CDM particles. It has been observed that Λ CDM cosmology can be resembled by the model, which includes matter creation. It has been observed that the creation of matter can play the role of a dark energy component and lead to drive the accelerating expansion of the Universe [50–59]. Many authors have explained the late-time evolution of the Universe with matter creation cosmology, which can be found in Refs. [60–71]. A lot of significant observations have been studied in the last several years that perfectly explain the Universe’s current fast expansion, making matter creation cosmology a fascinating topic. The crucial aspect of these models is the consideration of different choices for the particle production rate, which is generally considered as a function of the Hubble parameter, H . It has been found that different choices of matter creation rate can mimic Λ CDM cosmology [50,54].

It would be better to mentioned that Hernández-Almada et al. [72] have studied a hybrid model of viscous and Chaplygin gas to tackle the Universe acceleration. They have claimed that the viscous GCG model might be a candidate to alleviate the H_0 tension. Actually, matter creation and bulk viscosity are considered as separate irreversible processes. The energy-momentum tensor is modified to accommodate the viscous pressure and creation pressure which is associated with the creation of matter out of gravitational field. The main difference between a bulk viscous model and matter creation lies in the origin of the observed cosmic acceleration. Bulk viscosity is a property of the fluid itself, while matter creation involves the creation of new particles (in this paper, as cold dark matter). The possible similarity between the two processes is restricted only to the dynamic aspects, that is, at the level of the energy-momentum tensor. Both are scalar processes contributing to the stress as an additional dynamic pressure.

This fact explains why the cosmic dynamics generated by each process can be mimicked by the other one. However, as discussed above, the thermodynamic features are quite different [73].

The preceding contentions motivate us to constraint the matter creation model in a non-minimally coupled $f(Q)$ gravity using GCG equation of state. The idea is fascinating as we do not know the cause behind the late-time acceleration of the Universe. We introduce a modified $f(Q)$ gravity where a pressureless perfect fluid in addition to matter creation with GCG equation of state can lead to describe the late-time expansion of the Universe. Initially, we derive the analytical solution of the field equations and then evaluate the accuracy of the model using a combination of type Ia supernovae (Pantheon), Cosmic Chronometer, and the latest Baryon Acoustic Oscillations data of Dark Energy Spectroscopic Instrument (DESI). We analyze how matter creation influences the unification with the generalized Chaplygin gas model. The results are compared with the standard Λ CDM model which find a comprehensive understanding of late-time cosmic evolution. The stability of the models are also studied using the information criteria method.

This paper is structured as follows: After reviewing $f(Q)$ gravity in Section 2, we present the basic field equations for a generalized Chaplygin gas model with matter creation in $f(Q)$ gravity in Section 3. The cosmological solutions for generalized Chaplygin gas model are presented in Section 4. Section 5 deals with a particular solution for Chaplygin gas model with matter creation in $f(Q)$ gravity. Section 6 discusses the three observational datasets: type Ia supernovae (Pantheon), Cosmic Chronometer and Baryonic Acoustic Oscillations. The methodology employed for determining the model parameters and the results are discussed in Section 7. The summary of the evolution of the cosmological parameters is provided in Section 8. Section 9 is devoted for studying the stability of the models. The model selection criteria is discussed in Section 10. Finally, in Section 11, we present a concise summary and draw our conclusions based on the findings from this study.

2. $f(Q)$ theory of gravity

In this section we discuss an extension of symmetric teleparallel gravity, namely, $f(Q)$ gravity where the fundamental block to describe spacetime is the nonmetricity, Q . The $f(Q)$ modified gravity model under study is based on the following action [20,21,31]

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} f_1(Q) + f_2(Q) \mathcal{L}_m \right], \quad (1)$$

where $\mathcal{L}_m$ is the Lagrangian for matter fields, $f_1(Q)$ and $f_2(Q)$ are arbitrary functions of non-metricity scalar Q and g is the determinant of fundamental tensor $g_{\mu\nu}$. The non-metricity tensor in $f(Q)$ theory of gravity is defined as:

$$Q_{\lambda\mu\nu} = \nabla_\lambda g_{\mu\nu}. \quad (2)$$

The non-metricity scalar Q is given by

$$Q = -Q_{\lambda\mu\nu} P^{\lambda\mu\nu}, \quad (3)$$

where $P^{\lambda\mu\nu}$ is the non-metricity conjugate, given by

$$P_{\mu\nu}^\lambda = -\frac{1}{2} L_{\mu\nu}^\lambda + \frac{1}{4} (Q^\lambda - \tilde{Q}^\lambda) g_{\mu\nu} - \frac{1}{4} \delta_\mu^\lambda Q_\nu, \quad (4)$$

with

$$L_{\mu\nu}^\lambda = \frac{1}{2} Q_{\mu\nu}^\lambda - Q_{(\mu\nu)}^\lambda, \quad (5)$$

and

$$Q_\lambda = g^{\mu\nu} Q_{\lambda\mu\nu}, \quad \tilde{Q}_\lambda = g^{\mu\nu} Q_{\mu\lambda\nu}. \quad (6)$$

The energy-momentum tensor $T_{\mu\nu}$ is specified as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu\nu}} \quad (7)$$

Now, variation of Eq. (1) with respect to metric $g_{\mu\nu}$ gives the following modified Einstein's equation:

$$\begin{aligned} \frac{2}{\sqrt{-g}} \nabla_\lambda \left(\sqrt{-g} \{ f_1'(Q) + 2f_1'(Q) \mathcal{L}_m \} P_{\mu\nu}^\lambda \right) + \frac{1}{2} g_{\mu\nu} f_1(Q) \\ + \{ f_1'(Q) + 2f_1'(Q) \mathcal{L}_m \} (P_{\mu\lambda i} Q_{\nu}^{\lambda i} - 2Q_{\lambda i \mu} P_{\nu}^{\lambda i}) = -f_2(Q) T_{\mu\nu}. \end{aligned} \quad (8)$$

With the formalism of $f(Q)$ gravity specified, we will next consider cosmological model and its implications in matter creation cosmology with GCG equation of state. where a prime ($'$) represents the derivative of functions $f_1(Q)$ and $f_2(Q)$ with respect to Q .

3. Cosmological background evolution

To study the cosmological evolution in $f(Q)$ gravity, we assume a homogeneous and isotropic Universe described by the flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric, which in spherical coordinates (t, r, θ, ϕ) takes the following form:

$$ds^2 = -dt^2 + a^2(t) [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \quad (9)$$

where $a(t)$ represents the cosmic scale factor, describing the expansion of the Universe. In this geometry, the cosmological matter is comoving with the expansion, and therefore the four-velocity of the cosmological fluid is $u^\mu = (-1, 0, 0, 0)$. Moreover, we assume the baryons and Chaplygin gas endowed with matter creation as the matter contents in the Universe. Therefore, the matter energy-momentum tensor is described by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}, \quad (10)$$

where $\rho = \rho_b + \rho_{Ch}$ is the total energy density and $p = p_{Ch} + p_c$ denotes the total pressure. It is mentioned here that ρ_b and ρ_{Ch} are the energy densities of baryonic matter and Chaplygin gas where as p_{Ch} and p_c are the pressures due to Chaplygin gas and matter creation, respectively.

In the late 1980s, Prigogine and his collaborators [48] proposed the irreversible thermodynamics of open systems in cosmology. They used the generalized form of the first law of thermodynamics to describe the flow of energy from the gravitational field to the matter field, resulting in the creation of particles that occur only as an irreversible process at the expense of the gravitational field. In this formalism, the Universe is seen as an open system, and the description of particle creation is based on the reinterpretation of the energy-momentum tensor of matter by including a matter creation term in the conservation laws. It is to be noted that the Gibb's equation is valid for an open thermodynamical system where the number of fluid particles are not conserved (i.e., $N_{;\mu}^\mu \neq 0$). These can be expressed mathematically as

$$\nabla_\mu N^\mu \equiv n_{;\mu} u^\mu + 3Hn = n\Gamma. \quad (11)$$

Here, N^μ is the particle flow vector, u^μ is the particle four-velocity vector, H is the Hubble parameter, Γ stands for the rate of change of the particle number in a comoving volume V and $n_{;\mu} u^\mu = \dot{n}$ by notation. Specifically, $\Gamma > 0$ indicates particle creation, $\Gamma = 0$ corresponds to no matter creation, and $\Gamma < 0$ represents matter annihilation. Although the precise form of Γ is generally unknown, it is expected to be determined by quantum processes in curved space-time.

In view of the second law of thermodynamics, the above equation leads to the appearance of negative pressure associated with the creation rate, Γ and the creation pressure p_c will come into the total energy-momentum tensor with other pressures of matters like dark matter, radiation, dark energy etc. If for simplicity, we assume the thermodynamical system to be adiabatic (i.e. entropy per particle is conserved), then the creation pressure and the particle creation rate are related as (see more details in [48,49])

$$p_c = -\frac{(\rho_{Ch} + p_{Ch})}{3H} \Gamma. \quad (12)$$

The dynamics of the Universe can be described only if the matter creation rate is known. It is to be mentioned that the matter creation mechanism, in a logical way, is based on quantum field theory (QFT). The choice of particle creation rate Γ that goes with current observations is phenomenological since complete description of QFT in FLRW Universe is still not developed. There are some popular choices of Γ that can describe the evolution of universe from early phase to current accelerating era of our Universe. For instance $\Gamma = \text{constant}$ [58] explicates the evolution of Universe starting from the Big Bang to the current accelerating phase while $\Gamma \propto H$ [56] describing the matter-dominated decelerating phase and the early inflationary phase of Universe.

In this work, we adopt the matter creation rate as [74,75]

$$\Gamma = 3\beta H, \quad (13)$$

where β is a dimensionless constant.

From now on and unless otherwise stated, we will fix the coincident gauge so the connection is trivial, that is, in the coincident gauge with $\tilde{F}_{\mu\nu}^\alpha = 0$, we have the non-metricity scalar as $Q = 6H^2$. Using Eqs. (9) and (10), the modified Friedmann equations for $f(Q)$ gravity (8) are [20,21]

$$3H^2 = \frac{f_2}{2F} \left(-\rho + \frac{f_1}{2f_2} \right), \quad (14)$$

$$\dot{H} + 3H^2 + \frac{\dot{F}}{F} H = \frac{f_2}{2F} \left(p + \frac{f_1}{2f_2} \right), \quad (15)$$

where $F = f_1'(Q) + 2f_2'(Q)\mathcal{L}_m$. Using the Eq. (14) and (15), we get the continuity equation, which is given as

$$\dot{\rho} + 3H(\rho + p) = -\frac{6f_2'}{f_2} H \dot{H} (\mathcal{L}_m + \rho), \quad (16)$$

It can be noted that setting $\mathcal{L}_m = -\rho$ [30], the standard continuity equation can be recovered:

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (17)$$

where $\rho = \rho_b + \rho_{Ch}$ and $p = p_{Ch} + p_c$.

4. Cosmological solution

As discussed in Sec.3, we consider the Universe to be comprised of baryons and CGC with matter creation, which is assumed to be non-interacting with each other. The conservation equation of pressureless baryonic matter $\dot{\rho}_b + 3H\rho_b = 0$ gives

$$\rho_b = \rho_{b,0}(1+z)^3, \quad (18)$$

where $\rho_{b,0}$ is the energy density at present and $a(z) = 1/(1+z)$ by setting $a(t_0) = 1$. Let us consider a generalized Chaplygin gas (GCG) equation of state (EoS) [35,36]

$$p_{gcg} = -\frac{A}{\rho_{gcg}^\alpha}, \quad (19)$$

where $A > 0$ and $0 < \alpha \leq 1$ are free parameters. When $\alpha = 1$, it reduces to the CG scenario. Using (12) and (19) into the conservation equation $\dot{\rho}_{gcg} + 3H(\rho_{gcg} + p_{gcg} + p_c) = 0$, we can integrate to obtain the energy density of GCG fluid with matter creation,

$$\rho_{gcg} = \rho_{gcg,0} \left(A_s + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)} \right)^{\frac{1}{1+\alpha}} \quad (20)$$

where $\rho_{gcg,0}$ represents the energy density of CGC at present and $A_s = A/\rho_{gcg,0}^{1+\alpha}$ is a substitution of the parameter A .

There is particularly interesting class of $f(Q)$ that give a background evolution. Let us assume the functional forms as $f_1(Q) = \alpha_1 Q + \beta_1 Q^2$ and $f_2(Q) = Q$, where α_1 and β_1 are arbitrary constants [76]. Substituting (18) and (20) into (14), the Hubble parameter is expressed as

$$H^2(z) = H_0^2 \left[\left(\frac{1}{3\beta_1} \right) \left(\Omega_{gcg,0} \{ A_s + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)} \}^{\frac{1}{1+\alpha}} + \Omega_{b,0}(1+z)^3 - \frac{\alpha_1}{6H_0^2} \right) \right] \quad (21)$$

where $H_0 = 100 \text{ h Km s}^{-1} \text{ Mpc}^{-1}$ is the present Hubble parameter, $\Omega_{b,0}$ and $\Omega_{gcg,0}$ denote the density parameters of baryonic matter and CGC fluid, respectively. Imposing the initial condition $H(z=0) = H_0$ in the above equation, we get the following constraint:

$$\Omega_{gcg,0} = 3\beta_1 - \Omega_{b,0} + \frac{\alpha_1}{6H_0^2}. \quad (22)$$

The observed abundances of light elements together with primordial nucleosynthesis gives $\Omega_{b,0}h^2 = 0.0224 \pm 0.0001$ [77]. In cosmology, the deceleration parameter $q(z)$ plays an important role in describing the transition phase of the Universe. It is defined as

$$q(z) = -1 + \frac{(1+z)}{E(z)} \frac{dE(z)}{dz} \quad (23)$$

where $E(z) \equiv H(z)/H_0$ is the dimensionless Hubble parameter. The parameter q may be positive, zero or negative. The positive q describes the decelerating phase of the Universe, $q = 0$ gives the marginal inflation and the negative value implies the accelerating phase. Using (21), the deceleration parameter (23) is expressed as

$$q(z) = -1 + \frac{1}{6\beta_1 E^2(z)} \left[3\Omega_{b,0}(1+z)^3 + 3(1-\beta)(1-A_s)(1+z)^{3(1-\beta)(1+\alpha)} \times \Omega_{gcg,0} \left(A_s + (1-A_s)(1+z)^{3(1-\beta)(1+\alpha)} \right)^{\frac{-\alpha}{1+\alpha}} \right] \quad (24)$$

It is noted that in the absence of matter creation, that is, for $\beta = 0$, the solutions (21) and (24) reduce to GCG solutions of Ref. [78].

Another kinematics diagnostic parameter that provides us with a simple method to assess the departure from the Λ CDM model is the jerk parameter. It is described as the third order derivative, dimensionless, of the scale factor a with respect to the cosmic time t , i.e., $j = \ddot{a}/(aH^3)$ [79,80]. The jerk parameter for the Λ CDM model in the flat FLRW metric is a constant with the value $j = 1$ [81]. The jerk parameter, j , can be expressed in terms of the q as [82]

$$j(z) = q(z)(2q(z) + 1) + (1+z) \frac{dq(z)}{dz} \quad (25)$$

Eq. (25) enables us to constrain the departure from the Λ CDM value in an effective manner. The present value of $j(z)$, denoted as j_0 is given by the expression

$$j_0 = q_0 + 2q_0^2 + \frac{dq}{dz}(z=0). \quad (26)$$

The effective equation of state (EoS) parameter is given by

$$w_{gcg}(z) = \frac{-A_s(1-\beta)\Omega_{gcg,0} - \beta\Omega_{gcg,0}\tilde{C}}{\Omega_{b,0}(1+z)^3 + \Omega_{gcg,0}\tilde{C}} \quad (27)$$

where, $\tilde{C} = [A_s + (1 - A_s)(1+z)^{3(1-\beta)(1+\alpha)}]^{\frac{1}{1+\alpha}}$. The current epoch of effective EoS parameter, $w_0(ef f)$, for GCG at $z = 0$ is obtained as

$$w_{gcg}(z=0) = -A_s(1-\beta) - \beta. \quad (28)$$

The behavior of GCG model can vary significantly, depending on the values of A_s , α and β . In trivial cases, for $A_s = 1$, the GCG model with matter creation behaves exactly as a cosmological constant at all times for all values of α and β . For $A_s = 0$, the GCG model behaves as a cold dark matter dominated by matter creation for all times for all values of α .

5. Particular solution

Let us discuss the particular solution of GCG, i.e., Chaplygin gas (CG), in which the EoS is given by $p_{cg} = -A/\rho_{cg}$ [32] by taking $\alpha = 1$ in (19). Using this EoS into the conservation equation, $\rho'_{cg} + 3H(\rho_{cg} + p_{cg} + p_c) = 0$, we get

$$\rho_{cg}(z) = \rho_{cg,0} (A_s + (1 - A_s)(1 + z)^{6(1-\beta)})^{\frac{1}{2}} \quad (29)$$

where $A_s = A/\rho_{cg,0}^2$ is the scaled parameter. Substituting (18) and (29) into (14), the Hubble parameter is expressed as

$$H^2(z) = H_0^2 \left[\left(\frac{1}{3\beta_1} \right) (\Omega_{cg,0} \{A_s + (1 - A_s)(1 + z)^{6(1-\beta)}\})^{\frac{1}{2}} + \Omega_{b,0}(1 + z)^3 - \frac{\alpha_1}{6H_0^2} \right] \quad (30)$$

Using (30) into (23), we get the deceleration parameter as:

$$q(z) = -1 + \frac{1}{6\beta_1 E^2(z)} \left[3\Omega_{b,0}(1 + z)^3 + 3(1 - \beta)(1 - A_s)(1 + z)^{6(1-\beta)} \times \Omega_{cg,0} (A_s + (1 - A_s)(1 + z)^{6(1-\beta)})^{\frac{-1}{2}} \right] \quad (31)$$

The effective equation of state EoS() parameter is given by

$$w_{g_{cg}}(z) = \frac{\frac{-A_s(1-\beta)\Omega_{cg,0}}{\tilde{C}_1} + \beta\Omega_{g_{cg,0}}\tilde{C}_1}{\Omega_{b,0}(1 + z)^3 + \Omega_{g_{cg,0}}\tilde{C}_1} \quad (32)$$

where, $\tilde{C}_1 = [A_s + (1 - A_s)(1 + z)^{6(1-\beta)}]^{\frac{1}{2}}$.

6. Observational data

In this section, we present datasets and methods for constraining the parameters of GCG and CG models with matter creation and Λ CDM, respectively. We use the following most recent observational data for our analysis.

6.1. Type Ia supernovae (SNe)

We use the Pantheon Sample, which contains 1048 SNe measurements of distance moduli μ_{obs} at various redshifts in the range $0.01 \leq z \leq 2.26$ [83,84]. We measure its difference from the theoretical value μ_{th} . This difference between the theoretical and observed distance modulus is represented by $\Delta\mu_i = \mu_{th} - \mu_{obs}$. The chi-square (χ^2) for SNe is calculated as:

$$\chi_{SNe}^2 = \sum_{i,j=1}^{1048} \Delta\mu_i (C_{SN}^{-1})_{ij} \Delta\mu_j, \quad (33)$$

where C_{SN}^{-1} is the inverse covariance matrix of the dataset. The theoretical distance modulus can be estimated using the following equation.

$$\mu_{th}(z) = 5 \log_{10} \left(\frac{d_L(z)}{1 \text{Mpc}} \right) + 25 + \mathcal{M}. \quad (34)$$

Here $d_L(z)$ is the luminosity distance, which is given by

$$d_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z', \mathbf{p})}, \quad (35)$$

where $\mathbf{p}$ denotes the parameter space.

6.2. Cosmic Chronometer (CC)

We employ a standardized compilation of 32 Hubble data points $H(z)$ acquired through the differential age method. This technique allows for the determination of the cosmic expansion rate at various redshifts covering the range $0.07 \leq z \leq 1.965$ [85]. The χ^2 for $H(z)$ is calculated as follows:

$$\chi_{CC}^2 = \sum_{i=1}^{32} \frac{[H_{obs}(z_i) - H(\mathbf{p}, z_i)]^2}{\sigma^2(z_i)}, \quad (36)$$

where $H(\mathbf{p}, z_i)$ represents the theoretical value for a given model at redshift z_i , $H_{obs}(z_i)$ is the observational value, and $\sigma(z_i)$ denotes the observational error.

Table 1
Clustering measurements for the BAO samples [96].

Tracer	z	Quantity	Value $\pm$ Error
BGS	0.295	D_V/r_d	7.942 ± 0.075
LRG1	0.510	D_M/r_d	13.588 ± 0.167
LRG1	0.510	D_H/r_d	21.863 ± 0.425
LRG2	0.706	D_M/r_d	17.351 ± 0.177
LRG3+ELG1	0.934	D_M/r_d	21.576 ± 0.152
LRG3+ELG1	0.934	D_H/r_d	17.641 ± 0.193
ELG2	1.321	D_M/r_d	27.601 ± 0.318
ELG2	1.321	D_H/r_d	14.176 ± 0.221
QSO	1.484	D_M/r_d	30.512 ± 0.760
QSO	1.484	D_H/r_d	12.817 ± 0.516
Lya	2.330	D_H/r_d	8.632 ± 0.101
Lya	2.330	D_M/r_d	38.988 ± 0.531

6.3. Baryon acoustic oscillations

The imprint of Baryon Acoustic Oscillations (BAO) offers a robust standard ruler for tracing the expansion history of the Universe, rooted in the propagation of sound waves in the early photon-baryon plasma. Over the past decades, BAO has become a cornerstone of observational cosmology, as validated in numerous studies [86,87], and is instrumental for constraining cosmological models. The comoving sound horizon, which defines the standard ruler scale, is given by:

$$r_s(z) = \frac{c}{H_0} \int_z^\infty \frac{c_s}{E(z')} dz', \quad (37)$$

where c is the speed of light, H_0 is the Hubble constant, and $E(z)$ is the normalized Hubble parameter. The speed of sound is expressed as: $c_s = \sqrt{\frac{\delta p_\gamma}{\delta \rho_B + \delta \rho_\gamma}} = \sqrt{\frac{\frac{1}{3} \delta \rho_\gamma}{\delta \rho_B + \delta \rho_\gamma}} = \sqrt{\frac{1}{3(1+R)}}$, where $R \equiv \frac{\delta \rho_B}{\delta \rho_\gamma} = \frac{3\rho_B}{4\rho_\gamma}$. In the Λ CDM model, Planck estimates the sound horizon as $r_d = 147.09 \pm 0.26$ Mpc. In this work, we take r_d as a free parameter to constrain the value of the sound horizon using late-time datasets [88–95]. We use the latest BAO measurements of 12 points from the Dark Energy Spectroscopic Instrument (DESI) Data Release 2 (DR2); the corresponding values are listed in Table 1, which provides precise distance measurements in the redshift range $0.295 \leq z \leq 2.330$. Detailed methodologies and datasets are provided in [96]. We use the covariance matrix C provided by the DESI collaboration, available at [97,98] which corresponds to the BAO measurements from the dataset at [99]. The following distance measures are utilized:

Comoving angular diameter distance:

$$D_M(z) = \int_0^z \frac{c}{H(z')} dz', \quad (38)$$

Hubble distance:

$$D_H(z) = \frac{c}{H(z)}, \quad (39)$$

Volume-averaged effective distance:

$$D_V(z) = [z D_M(z)^2 D_H(z)]^{1/3}. \quad (40)$$

These distances are normalized by the sound horizon at the drag epoch, r_d , to form model-independent observable. The theoretical predictions for D_M/r_d , D_H/r_d , and D_V/r_d are compared against observations using chi-squared minimization techniques:

$$\chi_{\text{BAO}}^2 = \sum_{i=1}^{12} \Delta \mathbf{R}^T(z) \mathbf{C}^{-1} \Delta \mathbf{R}(z), \quad (41)$$

where the residuals are defined as: $\mathbf{R}_Y = D_{Y,\text{Model}} - D_{Y,\text{Data}}$ with $Y = \frac{D_V}{r_d}$, $\frac{D_M}{r_d}$, and $\frac{D_H}{r_d}$.

7. Methodology and results

This analysis uses the total likelihood function $\mathcal{L} \propto e^{-\chi^2/2}$ to derive joint constraints on the model parameter space (H_0 , Ω_b , A_s , α , β , α_1 , β_1) based on the aforementioned observational data. The χ^2 function for a combined dataset (SNe+CC+BAO) is given by

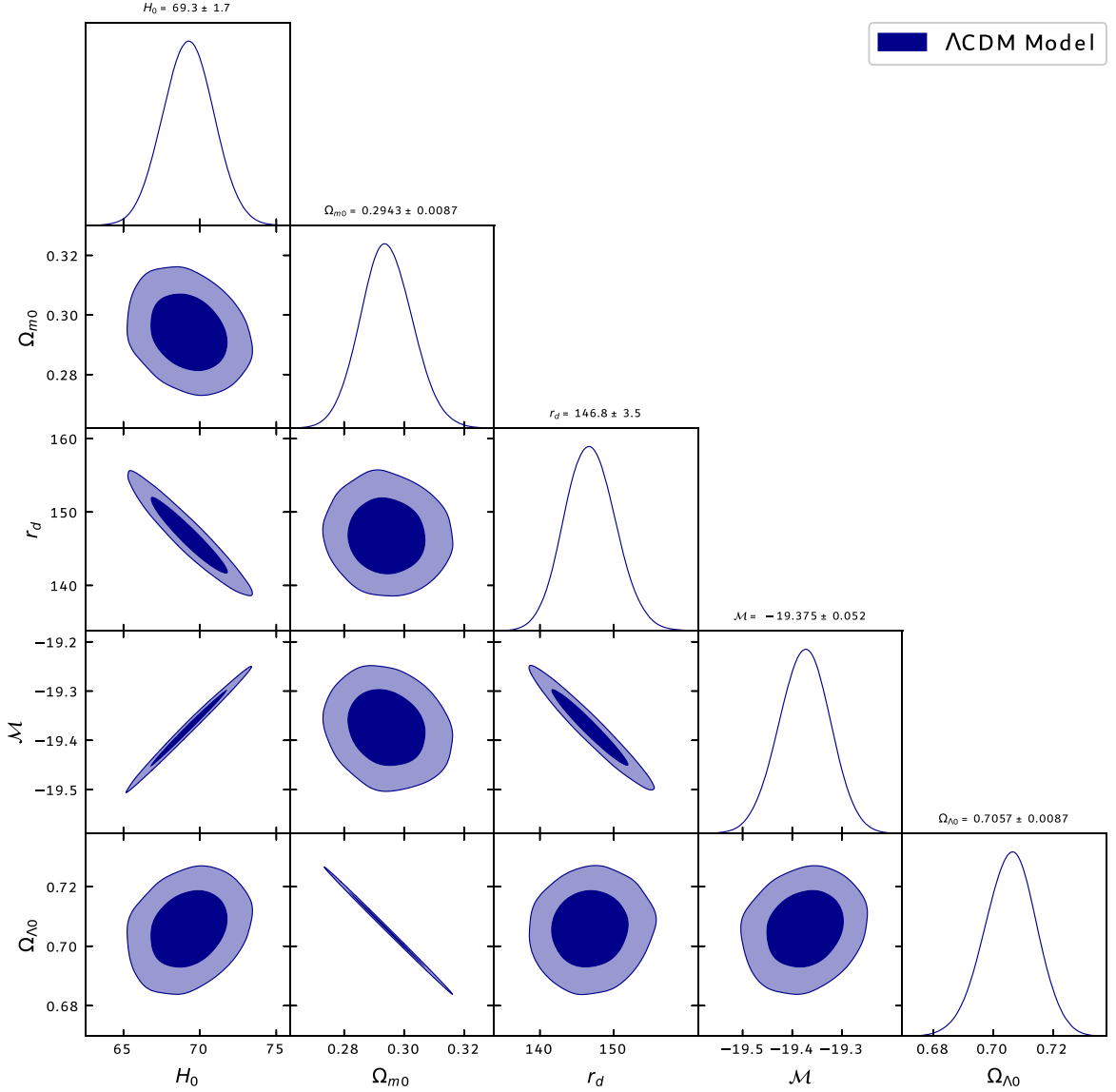
$$\chi_{\text{tot}}^2 = \chi_{\text{SNe}}^2 + \chi_{\text{CC}}^2 + \chi_{\text{BAO}}^2, \quad (42)$$

where χ_{SNe}^2 , χ_{CC}^2 , and χ_{BAO}^2 are given by (33), (36) and (41), respectively.

In our analysis, we perform a global fitting to estimate the cosmological parameters using Markov Chain Monte Carlo (MCMC) sampling method. The MCMC code is listed in the publicly available through the Python package `emcee`. For our purposes, we have six model parameters, H_0 , Ω_b , A_s , α_1 , β_1 , α , and β . For our best fit analysis, we choose the priors for these parameters in the

Table 2Best-fit values of parameter space of Λ CDM, GCG, and CG models using $SNe + CC + BAO$ data.

Parameters	Λ CDM	GCG with β	GCG without β	CG with β	CG without β
H_0	69.3 ± 1.7	68.9 ± 1.6	72.2 ± 1.7	67.7 ± 1.6	72.8 ± 1.7
$\Omega_{m,0}$	0.294 ± 0.008	—	—	—	—
$\Omega_{\Lambda,0}$	0.705 ± 0.008	—	—	—	—
Ω_b	—	0.049 ± 0.006	0.049 ± 0.002	0.048 ± 0.006	0.050 ± 0.006
A_s	—	$0.685^{+0.042}_{-0.034}$	$0.882^{+0.012}_{-0.019}$	$0.597^{+0.028}_{-0.021}$	$0.908^{+0.030}_{-0.032}$
α	—	0.470 ± 0.026	0.755 ± 0.001	1	1
β	—	0.143 ± 0.026	—	0.240 ± 0.012	—
α_1	—	0.471 ± 0.020	0.471 ± 0.021	0.510 ± 0.029	0.510 ± 0.036
β_1	—	0.510 ± 0.025	0.506 ± 0.023	0.590 ± 0.025	0.588 ± 0.023
r_d	146.8 ± 3.5	146.9 ± 3.4	146.3 ± 3.3	147.1 ± 3.4	146.1 ± 3.4
$\mathcal{M}$	-19.375 ± 0.052	-19.383 ± 0.057	-19.337 ± 0.052	-19.405 ± 0.051	-19.329 ± 0.051

**Fig. 1.** The 1D posterior distributions and 2D confidence contours at 1σ and 2σ for Λ CDM using $SNe + CC + BAO$ dataset.

following ranges: $H_0 \in [60, 80]$, $\Omega_b \in [0, 0.1]$, $A_s \in [0, 1]$, $\alpha_1 \in [0, 1]$, $\beta_1 \in [0, 1]$, $\alpha \in [0, 1]$, and $\beta \in [0, 1]$. We use 120 walkers and 2000 steps to find the best-fit values of parameters.

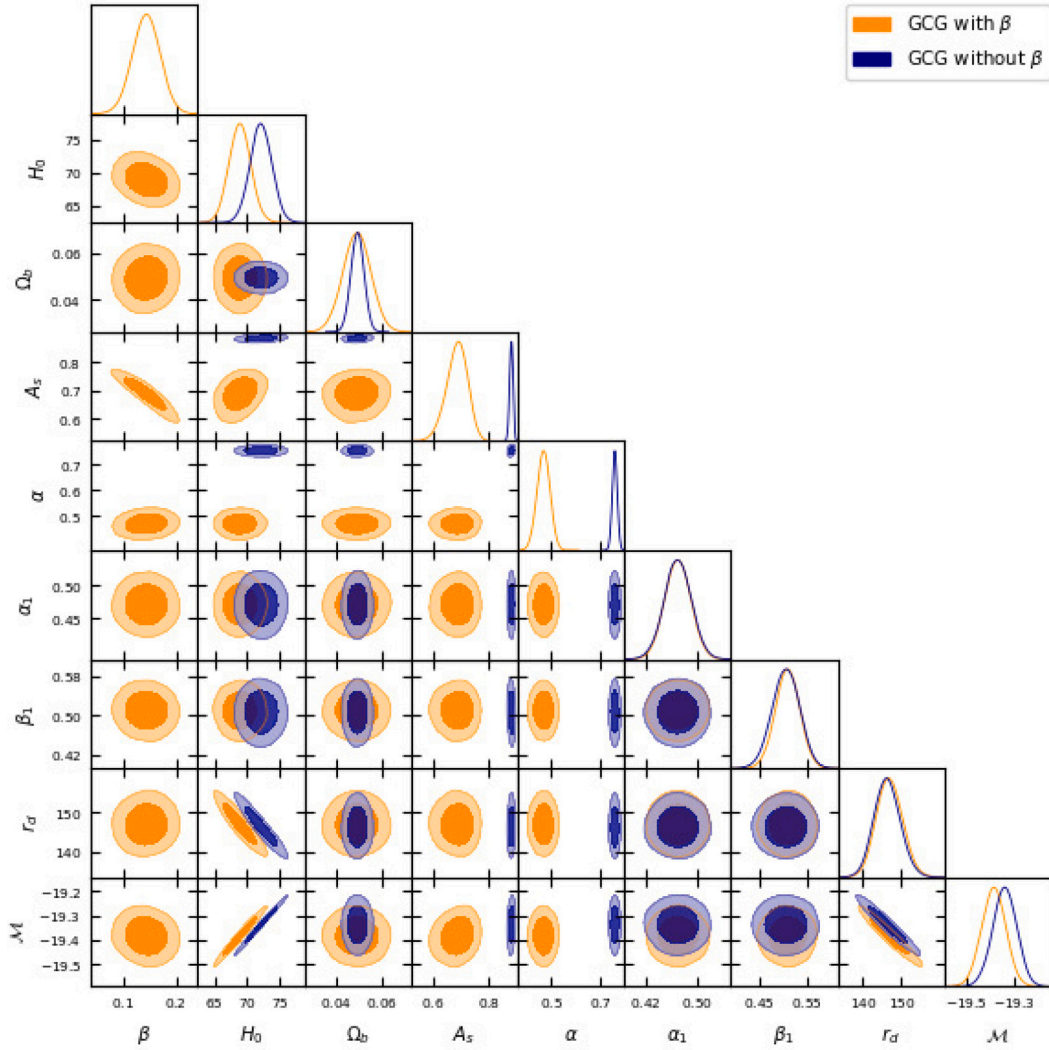


Fig. 2. The 1D posterior distributions and 2D confidence contours at 1σ and 2σ for the GCG model with and without matter creation using $SNe + CC + BAO$ data.

Using the combined $SNe+CC+BAO$ data, the main results of the best-fit values of the model parameters are listed in Table 2. In Figs. 1–3 we plot 1 – D probability distribution of each parameter and 2 – D plots for parameters between each other for the Λ CDM, GCG and CG models with and without matter creation in $f(Q)$ theory, respectively. Next, Figs. 4–13 show the evolutionary behaviors of $H(z)$, $q(z)$, $w(z)$, $j(z)$ and $C_s^2(z)$, respectively of the proposed models with and without matter creation in $f(Q)$ theory. The current values of various parameters like, q_0 , w_0 , j_0 , $C_{s,0}^2$ and τ_0 are presented in Table 3.

8. Discussion

In this section, using the best-fit values of the parameters obtained from the combined data of SNe , CC , and BAO , we examine the evolution of the main cosmological quantities. We know that the current value of the Hubble parameter is one of the key cosmological parameter which play a vital role in cosmology. It is also important to understand the evolutionary behavior of $H(z)$. Therefore, we plot the evolution of $H(z)$ in Figs. 4 and 5, respectively, with respect to the redshift compared to the observational data from CC for GCG and CG models with and without matter creation. If we compare the evolutions of $H(z)$ with Λ CDM, we observe that GCG and CG models with matter creation behave similar to the Λ CDM. It is to be noted that the Planck collaboration [77] predicts $H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and where as local value measurement [100] gives $H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$. From Table 2, we observe that the best-fit values of H_0 for GCG and CG models with matter creation are $H_0 = 68.9 \pm 1.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $H_0 = 67.7 \pm 1.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$, respectively. On the other hand, we find $H_0 = 72.2 \pm 1.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $H_0 = 72.8 \pm 1.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for GCG and CG without matter creation models, respectively.

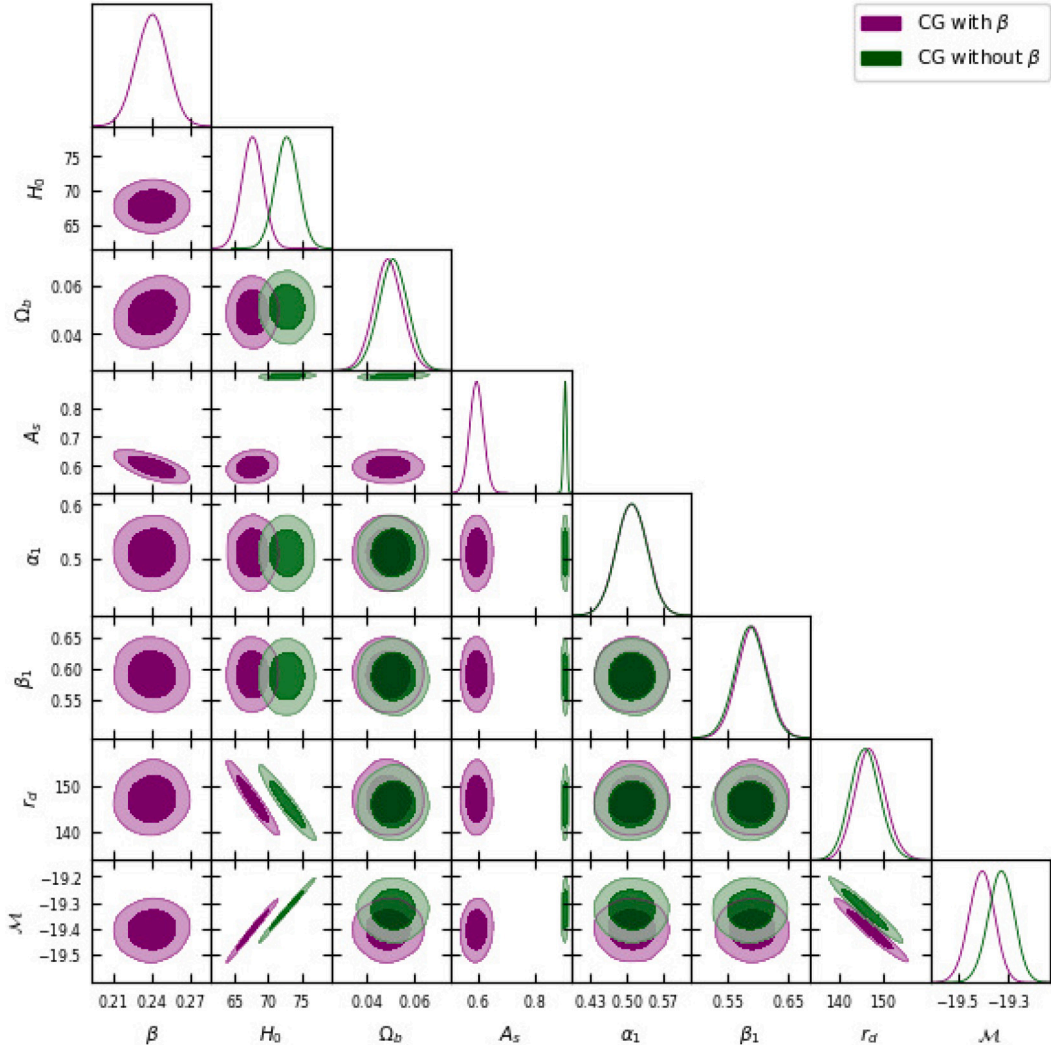


Fig. 3. The 1D posterior distributions and 2D confidence contours at 1σ and 2σ for the CG model with and without matter creation using $SNe + CC + BAO$ data.

Table 3

The present values of various cosmological parameters for the GCG and CG models, both with and without matter creation.

Model	q_0	j_0	w_0	C_s^2	τ_0 (Gyr)	z_{tr}
Λ CDM	$-0.541^{+0.020}_{-0.019}$	1	$-0.700^{+0.011}_{-0.014}$	≤ 1	13.81 ± 0.11	$0.662^{+0.31}_{-0.21}$
GCG with β	$-0.551^{+0.015}_{-0.019}$	$1.153^{+0.010}_{-0.012}$	$-0.692^{+0.011}_{-0.014}$	0.321	13.85 ± 0.12	$0.606^{+0.28}_{-0.20}$
GCG without β	$-0.763^{+0.021}_{-0.029}$	$1.325^{+0.021}_{-0.029}$	$-0.838^{+0.022}_{-0.030}$	0.609	13.62 ± 0.10	$0.621^{+0.24}_{-0.19}$
CG with β	$-0.489^{+0.016}_{-0.022}$	$1.280^{+0.022}_{-0.023}$	$-0.664^{+0.019}_{-0.013}$	0.597	14.11 ± 0.12	$0.550^{+0.22}_{-0.21}$
CG without β	$-0.793^{+0.014}_{-0.020}$	$1.354^{+0.011}_{-0.013}$	$-0.868^{+0.012}_{-0.017}$	0.908	13.97 ± 0.13	$0.591^{+0.31}_{-0.20}$

Let us discuss another purely kinematical parameter, known as the deceleration parameter. Figs. 6 and 7 plot the evolutions of $q(z)$ with respect to redshift z for GCG and CG models. We observe the transition from deceleration to acceleration for both the models at redshifts $z_{tr} = 0.606^{+0.28}_{-0.20}$ and $z_{tr} = 0.550^{+0.22}_{-0.21}$, respectively. It is to be noted that parameter $q > 0$ corresponds to the deceleration phase of the Universe expansion and $q < 0$ determines the accelerated phase of expansion. We see at early times for all models, q tends to $1/2$ as expected for matter-dominated phase and in late-times, it tends to -1 . The present value of $q(z)$ at $z = 0$ is $q_0 = -0.541^{+0.020}_{-0.019}$ for Λ CDM model, $q_0 = -0.551^{+0.015}_{-0.019}$ for the GCG model and $q_0 = -0.489^{+0.016}_{-0.022}$ for the CG model with matter creation. The present negative value of q shows that the proposed models undergo accelerated expansion.

Using best fit values of parameters in Eqs. (27) and (32), we plot the evolution of effective EoS parameters for GCG and CG models with and without matter creation as a function of redshift, which are shown in Figs. 8 and 9, respectively. It is to be noted that $-1 < w < 0$ describes the quintessence phase, $w = -1$ characterizes as a Λ CDM and $w < -1$ gives the phantom phase. The

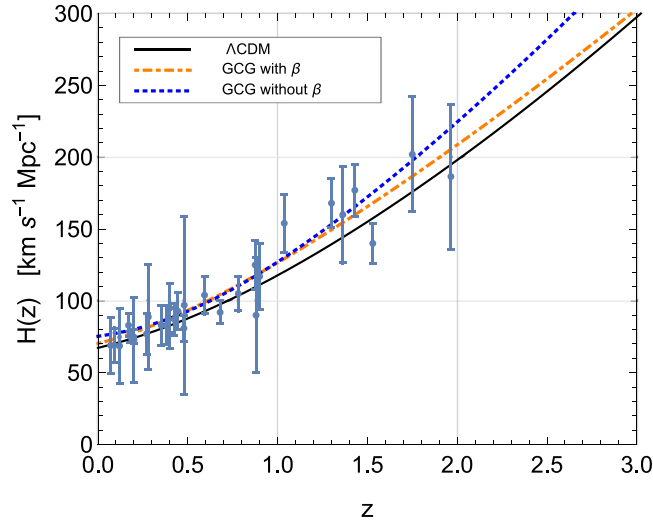


Fig. 4. The evolution of $H(z)$ in terms of redshift z using the best-fitting values for Λ CDM and GCG models with and without matter creation. The gray points with uncertainty bars correspond to the $H(z)$ sample.

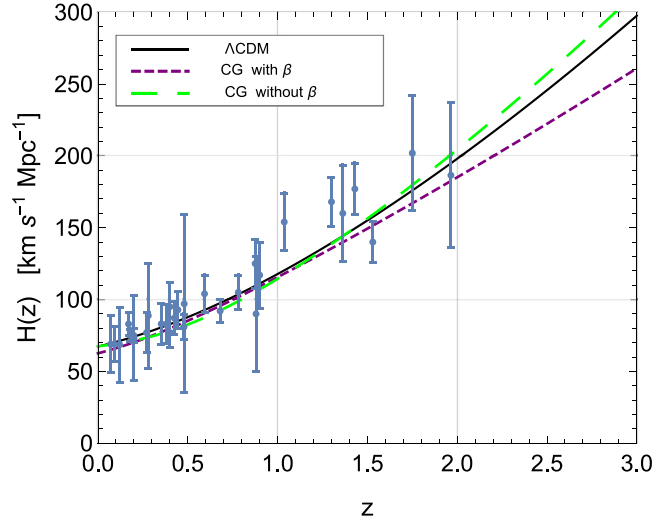


Fig. 5. The evolution of $H(z)$ in terms of redshift z using the best-fitting values for Λ CDM and CG models with and without matter creation. The gray points with uncertainty bars correspond to the $H(z)$ sample.

evolution of effective EoS parameter in Fig. 8 suggests a quintessence at early times and a phantom phase in late-times in respect of the GCG model with and without matter creation. However, for CG model with and without matter creation, the EoS parameter approaches to cosmological constant in late-time. The present values of $w_{eff}(z=0)$ are found to be $w_{eff}(z=0) = -0.692^{+0.011}_{-0.014}$ in the GCG model with matter creation and $w_{eff}(z=0) = -0.664^{+0.019}_{-0.013}$ for CG model with matter creation, which are compatible with Λ CDM model.

The jerk parameter is another cosmological quantity which plays an important role in examining the stability of a cosmological model. Figs. 10 and 11 present the evolutions of jerk parameter for GCG and CG models with and without matter creation. The joint analysis reveals the present values of $j(z)$ as $j_0 = 1.153^{+0.010}_{-0.012}$, $j_0 = 1.325^{+0.021}_{-0.029}$, $j_0 = 1.280^{+0.022}_{-0.023}$, and $j_0 = 1.354^{+0.011}_{-0.013}$ for GCG with β , GCG without β , CG with β , and CG without β , respectively. These values show a deviation from $j_0 = 1$ of Λ CDM model. However, it is observed that the jerk parameter approaches to same evolution as Λ CDM in late-time.

The age of the Universe $\tau(z)$ can be calculated by

$$\tau(z) = \int_z^\infty \frac{dz}{(1+z)H(z)}. \quad (43)$$

Using the best fit values of parameters in the expressions of $H(z)$, given in (21) and (30), respectively at $z=0$, we find the present age of the Universe for Λ CDM, GCG and CG models which are provided in Table 3. It can be observed that the age of Universe for

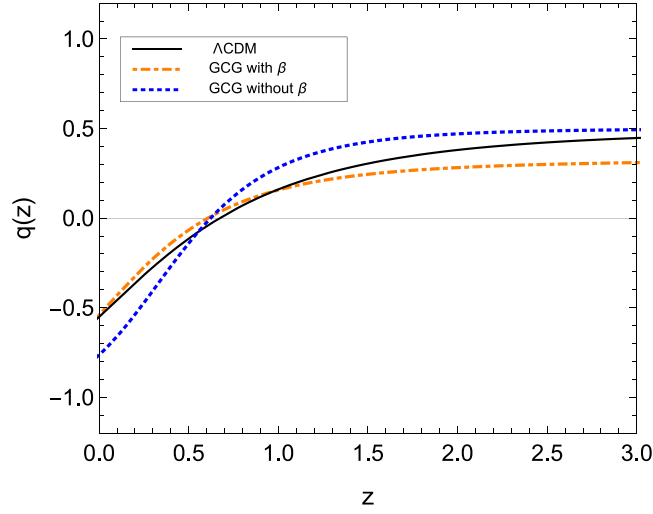


Fig. 6. The evolution of q in terms of redshift z using the best-fitting values for Λ CDM and GCG models with and without matter creation.

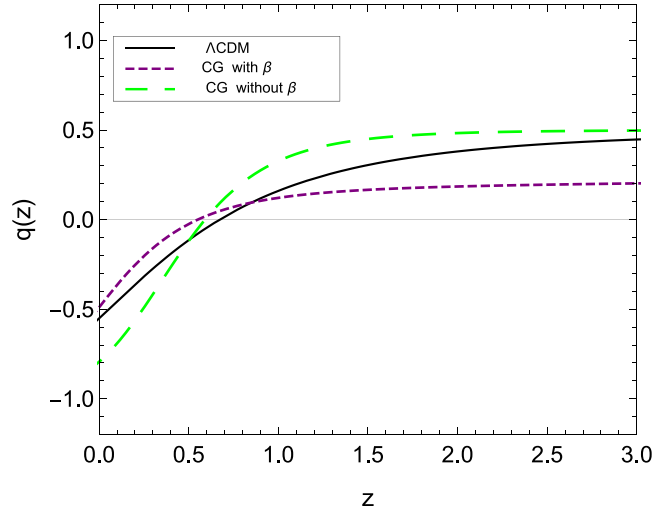


Fig. 7. The evolution of q in terms of redshift z using the best-fitting values for Λ CDM and CG models with and without matter creation.

Λ CDM is 13.81 ± 0.11 Gyr, for GCG and CG with matter creation are 13.848 ± 0.12 Gyr and 14.11 ± 0.12 Gyr, respectively. Note that the Λ CDM model predicts the age of Universe as 13.787 ± 0.02 Gyr [77].

9. Model stability

To assess the model's stability, we examine the squared sound speed of both fluids GCG and CG models, with and without matter creation. The square sound speed c_s^2 for any cosmic fluid is defined as $c_s^2 = \partial p / \partial \rho$. In a GCG model without matter creation, the square sound speed, given by $c_s^2 = -\alpha p / \rho$, is real for $\alpha > 0$, indicating that the fluid component is stable. When $\alpha < 0$ and only adiabatic pressure fluctuations are present, these fluctuations accelerate collapse, leading to instabilities that render the model unsuitable for structure formation. In general, the theory is in agreement with the stability and causality conditions as $0 \leq c_s^2 \leq 1$.

The squared sound speed for GCG with matter creation is given by

$$c_s^2 = -\beta + \frac{(1-\beta)\alpha A_s}{A_s + (1-A_s)(1+z)^{3(1-\beta)(1+\alpha)}}. \quad (44)$$

It can be expressed in terms of the EoS parameter as follows:

$$c_s^2 = -\beta - \alpha(1-\beta) w_{gcg}(z). \quad (45)$$

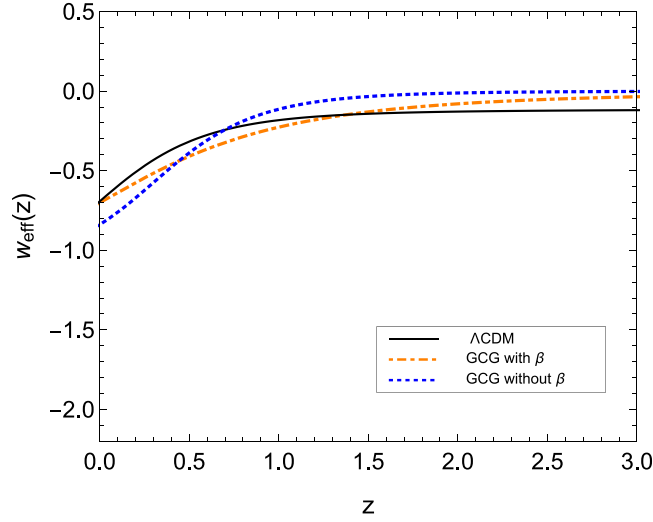


Fig. 8. The evolution of w_{eff} in terms of redshift z using the best-fitting values for Λ CDM and GCG models with and without matter creation.

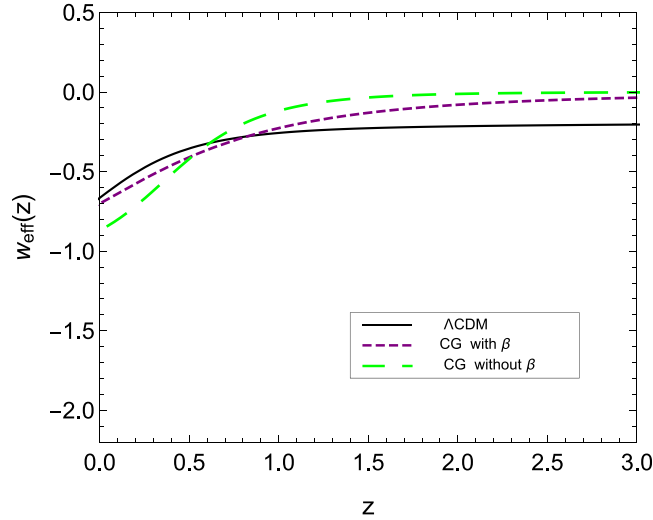


Fig. 9. The evolution of w_{eff} in terms of redshift z using the best-fitting values for Λ CDM and CG models with and without matter creation.

In a similar way, we can define the c_s^2 in CG model with matter creation, which is given by

$$c_s^2 = -\beta + \frac{(1-\beta)A_s}{A_s + (1-A_s)(1+z)^{6(1-\beta)}} = \beta - (1-\beta)w_{cg}(z). \quad (46)$$

Using the best-fit parameter values from $SNe + CC + BAO$ dataset to the equations above, the evolutions of c_s^2 for GCG and CG models with respect to the redshift are plotted in Figs. 12 and 13, respectively. The figures show that the each model is stable through out the evolution as the trajectory remains positive and lies in the range $0 \leq c_s^2 \leq 1$. The present value of c_s^2 for each model is listed in Table 2.

10. Model selection

To check the most compatible model with observational data, we use the reduced chi-square, which is defined as $\chi_{red}^2 = \chi_{min}^2/dof$, where $dof = N-K$ is the degree of freedom. Here, N is the total number of observational data and K is the number of free parameters of model. Table 3 presents the values of χ^2 and χ_{red}^2 for Λ CDM, GCG, and CG models, both with and without matter creation. We have $N = 1092$ and the numbers of free parameters, K in Λ CDM, GCG, and CG models with matter creation are $K = 5, 9$ and 8 , and for GCG and CG models without matter creation we have $K = 8$ and 7 , respectively. The model with $\chi_{red}^2 < 1$ is taken to be a better fit to the data. Table 4 shows that the observational data used for studying proposed models are suitable to fit the models.

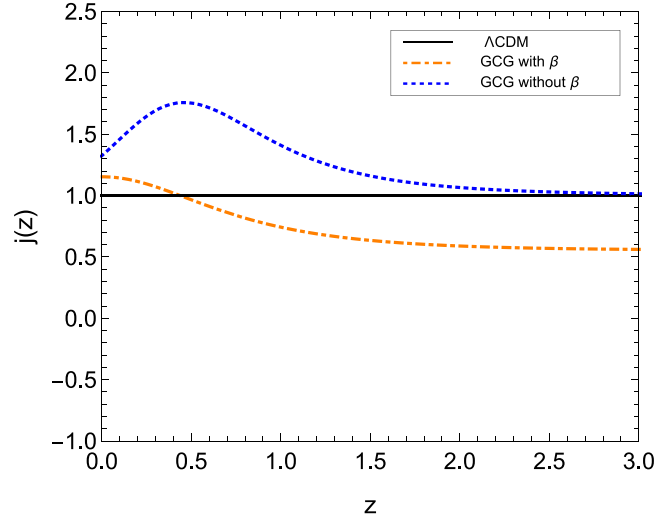


Fig. 10. The evolution of $j(z)$ in terms of redshift z using the best-fitting values for Λ CDM and GCG models with and without matter creation. The dot represents the present value of j_0 .

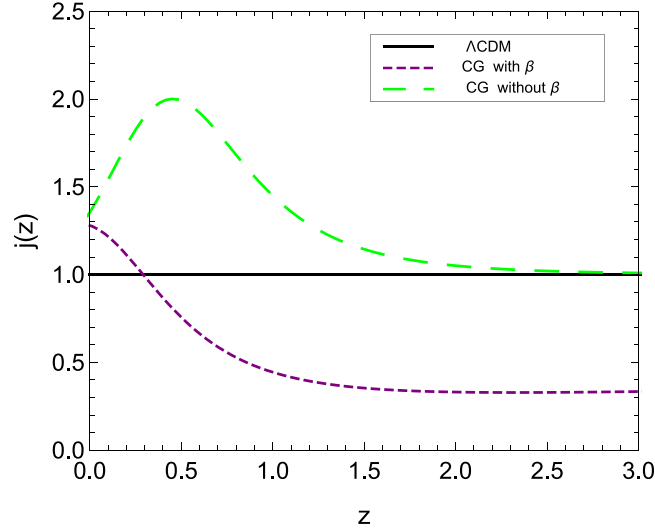


Fig. 11. The evolution of $j(z)$ in terms of redshift z using the best-fitting values for Λ CDM and CG models with and without matter creation. The dot represents the present value of j_0 .

Table 4

The AIC and BIC values for the Λ CDM model, GCG with and without β , and CG with and without β . The Λ CDM model is used as the reference for comparison.

Model	$\chi^2_{\min}$	$\chi^2_{\min}/\text{dof}$	AIC	BIC	ΔAIC	ΔBIC
Λ CDM	1052.568	0.968	1062.640	1087.579	–	–
GCG with β	1051.215	0.969	1069.551	1114.036	6.947	26.457
GCG without β	1099.443	1.01	1115.443	1155.203	52.839	67.624
CG with β	1053.448	0.971	1069.551	1069.551	6.94	18.028
CG without β	1123.060	1.03	1137.060	1171.921	74.456	84.342

The model with the lowest χ^2_{red} is not necessarily the best model due to the extra parameters which lead to a lower χ^2_{red} . In order to deal with more model selection, a standard approach is to compute the Akaike Information Criterion (AIC) and Bayesian information criteria (BIC). The AIC [101] and BIC [102], are respectively defined as

$$AIC = \chi^2_{\min} + \frac{2KN}{N - K - 1}, \quad (47)$$

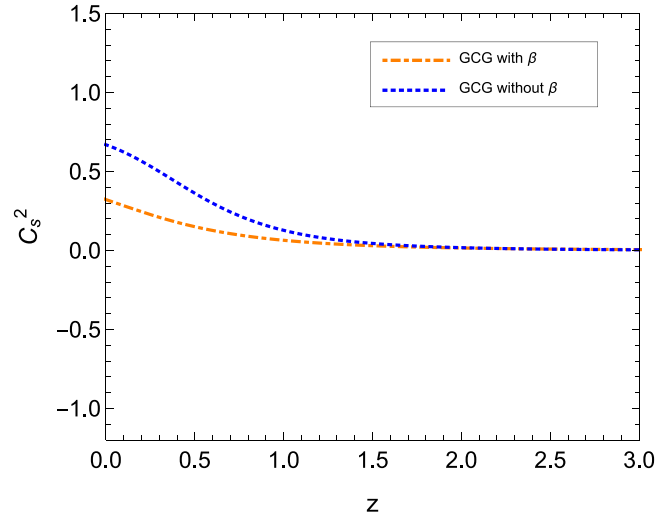


Fig. 12. The evolution of C_s^2 in terms of redshift z using the best-fitting values for GCG and CG models with and without matter creation.

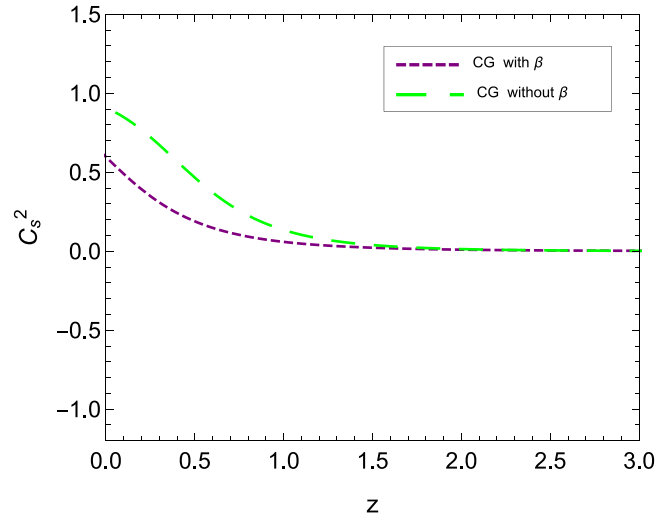


Fig. 13. The evolution of C_s^2 in terms of redshift z using the best-fitting values for GCG and CG models with and without matter creation.

and

$$BIC = \chi_{min}^2 + K \ln N. \quad (48)$$

where χ_{min}^2 measures the quality of the model and K , the number of parameters, interprets model complexity. Models are compared according to the AIC (or BIC), with the model that minimizes the AIC (or BIC) generally considered the best. As a reference model for AIC and BIC, we use Λ CDM:

$$\Delta AIC = AIC_y - AIC_{\Lambda CDM} \quad \text{and} \quad \Delta BIC = BIC_y - BIC_{\Lambda CDM},$$

where y represents either the GCG or CG model; this evaluation measures the strength of the models. The criteria for assessing the AIC model are as follows: when $0 \leq \Delta AIC < 2$, the model receives almost the same level of support from the data as the best model. If $2 < \Delta AIC \leq 4$, the model is considerably less supported, while the range $4 < \Delta AIC < 10$ still indicates some support for the model. In contrast, $\Delta AIC > 10$ does not support the model. Similarly, $\Delta BIC > 2$ range indicates strong support from the observational data and a $2 < \Delta BIC < 6$ amount suggests slight evidence against it. For a value in the range $6 < \Delta BIC < 10$, we obtain strong evidence against the considered model; if $\Delta BIC > 10$, we obtain even stronger evidence against the model [103].

The AIC (BIC) values and their differences, ΔAIC (ΔBIC), for the GCG and CG models, both with and without matter creation with reference to the Λ CDM model, are presented in Table 4. We can observed that the $\Delta AIC = 6.94$, which lies in $4 < \Delta AIC < 10$, indicating that the data still support the GCG and CG models with matter creation but less than the preferred one. We also see that

$\Delta AIC > 10$ in case of GCG and CG models without matter creation which indicate that the data does not support these models. On the other hand, as far as BIC criterion is concerned, all models have $\Delta BIC > 10$ which show that there is strong evidence against the models. It shows a preference for the Λ CDM model over our models. These results are expected considering that BIC penalizes free parameters more strongly than AICs and that the penalization function is proportional to the number of points in the dataset.

11. Conclusion

In this study, we have performed a detailed study of two concrete Chaplygin gas models under a dynamical system perspective including matter-creation mechanisms given by [48] within the framework of $f(Q)$ cosmology. We have combined two non-trivial effects which have been studied separately in the past. Our research aimed to understand how matter creation impacts the Universe's evolution with exotic EoS, based on the most recent cosmological data. We specifically have introduced two models of Chaplygin gas incorporating matter creation, namely GCG, and CG, and analyzed their evolutionary behavior based on observational datasets. This section summarizes the results as follows.

As mentioned, our primary focus was on examining cosmological models affected by gravitational matter creation, with particular emphasis on an extended version of the Chaplygin gas with the equation of state given by $p = -\frac{A}{\rho^\alpha}$, where $0 < \alpha \leq 1$ in $f(Q)$ gravity. The late-time acceleration of the Universe in a flat FLRW spacetime has been studied using this approach. Moreover, we have also considered a particular case where $\alpha = 1$, which corresponds to the Chaplygin gas with the equation of state $p = -\frac{A}{\rho}$. Additionally, these models have been compared with Λ CDM model and models without matter creation.

The methodology we have implemented in this study is based on three approaches: (i) an analytical solutions of the field equations for GCG and CG models with matter creation in $f(Q)$ gravity performed in Sections 4 and 5 (ii) constraints the models to estimate the free parameters by MCMC method using latest observational dataset in Section 6 and (iii) a detailed analysis of the evolutionary behavior of cosmological quantities using best-fit values in the subsequent Sections 7–10. As evidence suggests, the Universe is currently undergoing an accelerated phase of expansion because it is an open system with continuous matter creation. The exact nature of the matter creation rate, Γ , remains unclear, so a phenomenological approach was adopted, assuming that Γ evolves in proportion to H . In literature, a deceleration or acceleration phase is consistently demonstrated by with this assumption where the models do not show a transition from decelerating phase to accelerating phase, conflicting the result obtained from SNe observational data. However, it has been observed in this work that addition of exotic EoS like GCG EoS with matter creation allows the models to have phase transition with the same assumption of matter creation rate.

Table 2 provides the best-fit values for free parameters of both models with and without matter creation. The present values of $q(z)$, $j(z)$, $w(z)$, $t(z)$, and C_s^2 are provided in Table 3 and their respective evolutions have been plotted in Figs. 4–13. Comparing the results of these cosmological quantities, we have observed that GCG and CG models with matter creation in $f(Q)$ gravity are compatible with the observational dataset as well as Λ CDM model. The evolution trajectories of $j(z)$ display only slight deviations from the Λ CDM model for both the GCG and CG models. It has been observed that the best-fit values of Hubble constant for GCG and CG models with matter creation are $H_0 = 68.9 \pm 1.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $H_0 = 68.67.7 \pm 1.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$, respectively, which show that GCG model with matter creation may contribute to minimize the H_0 tension between the value obtained by Planck [77] and the value by Riess et al. [100].

Using both AIC and BIC methods, we found that models such as GCG and CG with matter creation fall within the range of $4 < \Delta AIC < 10$, indicating moderate support from the joint observational data. In contrast, models without matter creation yield $\Delta AIC > 10$, suggesting they are disfavored. However, as far as BIC criterion is concerned, all models have $\Delta BIC > 10$, which show that there is strong evidence against the models. It shows a preference for the Λ CDM model over our models. These results are expected considering that BIC penalizes free parameters more strongly than AICs and that the penalization function is proportional to the number of points in the dataset. It is crucial to highlight that the squared sound speed for these models remains within the range $0 < C_s^2 < 1$, indicating that it does not exceed the speed of light, thus confirming the stability of these models. This paper concludes that the models discussed produce cosmological parameters that are well constrained by observational data and show similar behavior to Λ CDM.

Finally, comparing our results of GCG model with matter creation, with those presented in paper [72] for GCG model with viscous fluid to tackle the cosmic acceleration, we find that GCG model with matter creation gives the comparatively low best-fit values of cosmological parameters by using a joint dataset of SNe Pantheon+, CC and DESI DR2 BAO (refers to Tables 2–4). We have observed that the value of H_0 , A_S and α parameters for GCG with matter creation and viscous GCG in paper [72] have the differences of 0.03σ , 1.4σ and 0.43σ , respectively.

An overall and encouraging conclusion from the dynamical analysis of the GCG model with matter creation in $f(Q)$ gravity is that the proposed models with matter creation effectively describe the late-time cosmic acceleration in the framework of $f(Q)$ gravity.

CRedit authorship contribution statement

Yogesh Bhardwaj: Writing – original draft, Visualization, Software, Methodology, Investigation, Conceptualization. **C.P. Singh:** Writing – review & editing, Validation, Supervision, Methodology, Investigation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

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Exploring $f(Q)$ gravity through variable generalized Chaplygin gas and matter creation cosmology

Yogesh Bhardwaj¹ · C.P. Singh¹

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Abstract

In this paper we explore a unified cosmological framework combining variable generalized Chaplygin gas (VGCG) and gravitationally-induced adiabatic matter creation cosmology within the context of $f(Q)$ gravity. Theoretical and statistical analyses are performed with two functional forms, $f(Q) = n_1 Q^k$ and $f(Q) = n_0 + n_1 Q^k$, where n_0 , n_1 and k are dimensionless free parameters. Using the latest observational data sets we perform a robust statistical analysis to constrain the model's parameters. A statistical comparison, including goodness-of-fit with standard Λ CDM model is performed. The results show that the Bayesian inference favors the complex models while the model selection criterion, like Akaike information criteria (AIC) and Bayesian information criteria (BIC) favor the simpler Λ CDM model. The results also show a smooth transition from deceleration to acceleration around the redshift $z \approx 0.6$ with effective equation of state parameter remains in the quintessence regime. The present value of Hubble parameter align closely with Planck 2018 measurements.

Keywords Cosmology · $f(Q)$ gravity · Dark energy · Matter creation cosmology · Statistical analysis

1 Introduction

During the past two decades, research in modern cosmology has been overwhelmingly centered on understanding the observed accelerated expansion of the Universe, a phenomenon that continues to challenge our fundamental grasp of physics. Despite considerable theoretical and observational progress, there remains no universally accepted physical explanation for dark energy (DE), the mysterious component believed to drive this cosmic acceleration. Current cosmological measurements, particularly those derived from the Planck satellite and large-scale structure surveys, consistently indicate that DE constitutes approximately 68.8% of the total energy density of the Universe. This dominant component is characterized by an effective negative pressure, which counteracts gravitational attraction on cosmological scales and leads to the observed speeding up of the expansion rate. In the ongoing effort to uncover the true nature and

physical origin of DE, physicists and cosmologists have proposed a wide array of theoretical models. These include the cosmological constant Λ —originally introduced by Einstein and later revived as the simplest explanation for acceleration (Padmanabhan 2003; Peebles and Ratra 2003; Copeland et al. 2006), as well as dynamical scalar field models such as quintessence (Caldwell et al. 1998), which allows for a time-varying equation of state; phantom energy (Caldwell et al. 2003), where the equation of state parameter falls below -1 , potentially leading to a future Big Rip singularity; and quintom scenarios (Guo et al. 2005), which permit the equation of state to cross the phantom divide.

Among all these proposals, the *Lambda-Cold Dark Matter* (Λ CDM) model stands out as the standard paradigm of modern cosmology, providing an exceptionally good fit to a vast array of independent observational datasets. These include high-precision measurements from Type Ia supernovae (Riess et al. 1998; Perlmutter et al. 1998, 1999), which first revealed the acceleration; Baryon Acoustic Oscillations (BAO) (Eisenstein et al. 2005), which serve as a standard ruler for cosmic distances; the distribution of Large-Scale Structures (LSS) (Tegmark et al. 2006) and the detailed anisotropy patterns in the Cosmic Microwave Background (CMB) radiation (Jarosik et al. 2011). Despite its empirical success in matching observations of our Uni-

✉ C.P. Singh
cpsingh@dce.ac.in

Y. Bhardwaj
yogeshbhardwaj_2k21phdam502@dtu.ac.in

¹ Department of Applied Mathematics, Delhi Technological University, Bawana Road, Delhi, 110042, India

verse, cosmologists find that it suffers some critical theoretical issues, most notably the fine-tuning problem—why the observed vacuum energy density is unnaturally small compared to quantum field theory predictions, and the cosmic coincidence problem—why dark energy and matter densities are comparable only at the present epoch (Weinberg 1987, 1989, 2000).

Modified gravity theories have emerged as particularly compelling alternatives to conventional cosmology, in response to these unresolved challenges. Such theories aim to account for the late-time acceleration not by introducing new, unobserved energy components like DE or particle dark matter (DM), but rather by modifying the geometric sector of Einstein's general relativity itself, thereby offering a radically different perspective on the fundamental laws governing cosmic dynamics. One particularly compelling extension of Einstein's theory of gravity is the symmetric teleparallel formulation, commonly referred to as $f(Q)$ gravity (Jiménez et al. 2020). In this geometric framework, the gravitational interaction is entirely attributed to the non-metricity scalar Q , while both curvature and torsion are identically zero—a distinctive feature that sets it apart from general relativity and other modified gravity theories such as $f(R)$ or $f(T)$ gravity. In recent years, $f(Q)$ gravity has been rigorously tested against a wide range of cosmological observations, including Type Ia supernovae, CMB anisotropies, BAO, data on large-scale structure development, and Redshift Space Distortion (RSD) measurements (Lazkoz et al. 2019; Atayde and Frusciante 2021).

Based on these empirical confrontations, $f(Q)$ gravity reproduces not only the late-time accelerated expansion of the Universe without invoking DE, but also has promising potential for alleviating persistent cosmological tensions—such as Hubble tensions and σ_8 tensions—that challenge the standard Λ CDM paradigm. A number of authors (Harko et al. 2018; Mandal et al. 2020a,b; Solanki et al. 2021; Khyllap et al. 2021; Ayuso et al. 2021; Frusciante 2021; Gadbaile et al. 2022; Capozziello and D'Agostino 2022; Wang et al. 2022; Paliathanasis 2023; Koussour and De 2023; Koussour et al. 2023; Yadav et al. 2024; Heisenberg 2024; Enkhili et al. 2024; Narawade et al. 2024) have studied exact cosmological solutions, dynamical system analyses, stability criteria under perturbations, energy conditions, etc.

There has been a compelling line of research focused on modifying the equation of state (EoS) of cosmic fluid to incorporate exotic components that exhibit negative pressure—properties essential for explaining the observed late-time acceleration of the Universe's expansion. Kamenshchik et al. (2001) first introduced the Chaplygin gas (CG) in a cosmological context. The CG is characterized by the EoS $p_{\text{Ch}} = -\frac{A}{\rho_{\text{Ch}}}$, where p_{Ch} and ρ_{Ch} denote the pressure and energy density of the fluid, respectively, and $A > 0$ is a constant parameter. Interestingly, this mathematical form was

originally conceived much earlier by Chaplygin (1904) in the domain of classical aerodynamics to model the lift force acting on an aircraft wing, long before its cosmological relevance was recognized.

Bento et al. (2002) and Bilić et al. (2002) independently proposed a generalization of CG, known as the *Generalized Chaplygin Gas* (GCG), whose EoS takes the form $p_{\text{Ch}} = -\frac{A}{\rho_{\text{Ch}}^\alpha}$, where both A and the exponent α are positive constants constrained by $0 < \alpha \leq 1$. The GCG model, which unifies dark matter and dark energy, faces challenges, especially the parameter constraints conflict, α and A . The supernovae gives the range of $\alpha > 1$ where as CMB gives it $\alpha < 1$, approx. 0.6, modeling the growth of structure formation, etc. Since the EoS of DE still cannot be determined exactly, the observational data show that w_{de} is in the range of $(-1.46, -0.78)$ (Bertolami 2004a,b). In order to resolve these problems, the GCG model should naturally be generalized to accommodate any possible DE with constant w_{de} or variable.

Introducing some exotic matter or developing variable A instead of assuming constant, would address instabilities and fitting model's parameter better. Therefore, introducing a variable component of A makes the EoS more dynamic which allows us a flexibility in matching observational data. Such a generalized model may formally be derived from a Born-Infled Lagrangian density for a scalar field (Sen 2002). Following Chimento and Jakubi (1996) and motivated by the work of Guo and Zhang (2007) for variable Chaplygin gas (VCG), the GCG EoS $p = A/\rho^\alpha$ was modified by assuming the factor A as a function of scale factor, i.e., $p = A(a)/\rho^\alpha$, where $A = A_0 a^{-n}$ (Yang et al. 2007; Wu et al. 2008; Lu 2009). Such an EoS is known as Variable generalized Chaplygin gas (VGCG), which has more number of free parameters n , A , α , etc. that can be constrained observationally by latest datasets. It has been shown that VGCG model as unification of DM and DE can drive the Universe from a non-relativistic matter-dominated phase to an accelerated expansion phase, which behaves like dust matter in early times and as quintessence/phantom in a recent epoch. It may play a vital role in aligning the theoretical predictions of unified dark energy and dark matter and structure formation.

Many authors (Sethi et al. 2006; Zhang et al. 2006; Fu et al. 2009; Khurshudyan 2018; Salti et al. 2018; Aziza et al. 2021; Salahedin et al. 2022; Bhardwaj and Singh 2024a,b) have thoroughly investigated the dynamical behavior, thermodynamic consistency, and observational viability of GCG and VGCG models using a wide range of cosmological probes. Collectively, these works highlight the remarkable adaptability of the VGCG framework in reproducing key features of cosmic history—including structure formation and background expansion—while maintaining compatibility with current observational datasets, thereby establishing it as a theoretically motivated and observationally viable candidate for unifying the dark sector of the Universe.

At this stage we would like to make a clear abbreviation VGCG, which has been used in the literature with different meanings. In particular, Gadbail et al. (2023) used VGCG to denote “Viscous generalized Chaplygin gas”, while in the present work, VGCG refers to “Variable generalized Chaplygin gas”. To avoid confusion, we explicitly define VGCG in this manuscript as the Variable Generalized Chaplygin Gas with the factor A as a function of the scale factor. This usage of “Variable generalized Chaplygin gas” is well-established in the literature, e.g., Yang et al. (2007) and Lu (2009) explicitly used this terminology with the parametrization $A(a) = A_0 a^{-n}$; Panotopoulos (2008) employed “Variable Chaplygin Gas” for similar scale-dependent models, Biswas and Mansour (2008), and Bhardwaj and Singh (2024a,b) studied by using the same terminology, variable generalized Chaplygin gas as VGCG.

The concept of matter creation cosmology provides a theoretically rich and physically motivated alternative explanation for the observed accelerated expansion of the Universe in late-time. This approach proposes that the acceleration arises from the continuous, gravitationally driven creation of matter particles in an expanding universe, rather than an unknown energy component with negative pressure. Particularly, it uses the concept of adiabatic matter creation, in which the gravitational field itself produces particles based on the principles of non-equilibrium thermodynamics. In 1989, Prigogine and his collaborators rigorously introduced this idea to cosmology, which was refined by Calvão et al. (1992), who defined an expanding Universe with irreversible matter generation by thermodynamics. They developed a cosmological model in which the total entropy of the Universe increases due to large-scale particle production, thus satisfying the second law of thermodynamics while deviating from equilibrium assumptions. It treats the Universe not as a closed system, but as an open thermodynamic entity, in which irreversible particle creation is intrinsically linked to entropy production and energy exchange with gravity. To accommodate this physical picture, Einstein’s field equations are suitably modified to include an effective creation pressure term that arises from the non-conservation of the particle number density. This modification leads to a generalized continuity equation that accounts for the energy transfer from the gravitational sector to matter, effectively altering the cosmic expansion dynamics. Remarkably, such models can reproduce the observed accelerated expansion without the need for an explicit DE component; instead, the effective negative pressure associated with matter creation mimics the role of DE.

A natural and widely adopted assumption in this context is that the gravitationally produced particles are cold dark matter (CDM) candidates, given that radiation is dynamically subdominant during the late-time Universe and baryonic matter alone cannot account for the required gravitational effects. This identification is further supported by the

fact that several matter creation models can closely emulate the background evolution of the standard Λ CDM cosmology (Steigman et al. 2009; Lima et al. 2010; Basilakos and Lima 2010; Lima et al. 2012, 2014; Chakraborty et al. 2014; Nunes and Pavón 2015; Harko et al. 2015; de Haro and Pan 2016; Nunes 2016). Over the past decade, this framework has garnered substantial attention, with numerous investigations exploring its implications for cosmic history, structure formation, and observational consistency. Studies by Saló and de Haro (2017), Pan et al. (2018), Singh and Kaur (2019), Singh and Kumar (2020a,b), Solà Peracaula and Yu (2020), Kaur and Singh (2021, 2022), Harko et al. (2021), Hulke et al. (2020) have demonstrated that matter creation cosmology not only provides a viable explanation for the current acceleration but also offers potential resolutions to certain tensions within the standard model—such as the H_0 and σ_8 discrepancies—by altering the expansion and growth histories in a physically motivated way. Collectively, these works affirm that gravitationally driven matter creation constitutes a compelling and theoretically coherent alternative to traditional dark energy paradigms, bridging gravitational physics, quantum field theory in curved spacetime, and non-equilibrium thermodynamics in a unified cosmological description.

The theoretical motivation of the present work is to investigate a unified cosmological scenario combining gravitationally-induced matter creation with VGCG equation of state (EoS) within the framework of symmetric teleparallel $f(Q)$ gravity. In the combined cosmological model, their effects on cosmic acceleration can be largely degenerate in the background which can be described on theoretical background as follows: (1) **$f(Q)$ gravity:** A modified gravity theory describing the geometric sector through a non-metricity scalar Q , instead of the Ricci scalar ($f(R)$) or curvature torsion, offering alternatives to general relativity. This approach can be extremely important to describe gravity at fundamental level because gravity can be dealt as a gauge theory not requiring a priori the validity of the Equivalence Principle. It produces intrinsically late-time acceleration through non-metricity effects; (2) **Matter creation:** An irreversible thermodynamical framework where matter is continuously created from the gravitational field, represented by energy flow having negative creation pressure adding the cosmological dynamics of the Universe; and (3) **Variable generalized Chaplygin gas EoS:** This inherently possesses an EoS of a single fluid model for dynamics of cosmic evolution and stability (positive sound speed squared). It naturally explains the phase transition from early deceleration to late-time acceleration.

It means that the combined model can lead to the same observed expansion history of the Universe. Therefore, Combining variable generalized Chaplygin gas EoS with matter creation in $f(Q)$ gravity presents a novel approach

which provides a rich, dynamics framework for explaining the cosmic acceleration of the Universe, testing the modifications to gravity and thermodynamical analysis.

In the present paper, we consider two distinct functional forms for the gravitational Lagrangian: the power-law model $f(Q) = n_1 Q^k$ and its extended version $f(Q) = n_0 + n_1 Q^k$, where n_0 , n_1 , and k are free model parameters to be constrained by observational data. By incorporating the VGCG EoS, we solve the modified Friedmann equations derived from $f(Q)$ gravity and analyze the resulting evolution of the Hubble parameter $H(z)$. A central objective of this paper is to assess the viability, robustness, and statistical stability of matter creation models when coupled with the more flexible VGCG EoS in the context of $f(Q)$ gravity. To achieve this, we employ a comprehensive suite of cosmological datasets, including the Pantheon+ compilation of Type Ia supernovae, Cosmic Chronometer (CC) measurements of the Hubble parameter, and Baryon Acoustic Oscillation (BAO) data from recent surveys such as DESI DR2. The model parameters are rigorously constrained using the Markov Chain Monte Carlo (MCMC) method, and the relative performance of the proposed models is evaluated through information-theoretic criteria—namely, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC)—which balance goodness-of-fit against model complexity to determine whether the added parameters are statistically justified by the data.

The structure of this article is organized as follows: We begin in Sect. 2 by laying the theoretical groundwork: this includes a detailed exposition of the action principle and field equations of $f(Q)$ gravity, an in-depth discussion of gravity-induced adiabatic matter formation and a detailed explanation of the EoS of the VGCG. In Sect. 3, we construct the cosmological model by combining these elements within a flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime and derive explicit analytical solutions for the Hubble parameter corresponding to two distinct functional forms of the $f(Q)$ Lagrangian—namely, $f(Q) = n_1 Q^k$ and $f(Q) = n_0 + n_1 Q^k$. Section 4 outlines the observational methodology, presenting the three key cosmological datasets employed in our statistical analysis: the Pantheon+ compilation of Type Ia supernovae, the Cosmic Chronometer (CC) measurements of the Hubble parameter, and the Baryon Acoustic Oscillation (BAO) data from recent large-scale structure surveys such as DESI Year 1 and SDSS-IV. In Sect. 5, we present and interpret the results of our Markov Chain Monte Carlo (MCMC) parameter estimation, evaluate the goodness-of-fit using information-theoretic criteria (AIC and BIC), and assess the physical viability of the proposed models through cosmographic diagnostics and comparisons with the standard Λ CDM paradigm. Finally, Sect. 6 offers a concise summary of our main conclusions.

2 Theoretical background

2.1 $f(Q)$ gravity

Inspired by the conceptual structure of $f(R)$ gravity—in which the Einstein-Hilbert action is generalized by substituting the Ricci scalar R with a generic function $f(R)$ —the symmetric teleparallel approach to gravitation offers an alternative geometric extension of general relativity. In this formulation, the role of the gravitational Lagrangian is played by an arbitrary smooth function $f(Q)$ of the non-metricity scalar Q , which becomes the sole carrier of gravitational dynamics in a spacetime devoid of both curvature and torsion. The complete action functional describing the interplay between geometry and matter in this setting is given by (Jiménez et al. 2018, 2020; Capozziello and D’Agostino 2022)

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} f(Q) + \mathcal{L}_m \right), \quad (1)$$

where $\mathcal{L}_m$ stands for the matter Lagrangian density, and $f(Q)$ is a differentiable function of the non-metricity scalar Q , encapsulating all gravitational degrees of freedom through purely non-metric geometric properties. The defining characteristic of this theory is its foundation on the non-metricity tensor, which arises as the covariant derivative of the metric with respect to a flat, symmetric affine connection:

$$Q_{\lambda\mu\nu} \equiv \nabla_\lambda g_{\mu\nu} = \partial_\lambda g_{\mu\nu} - \Gamma_{\lambda\mu}^\beta g_{\beta\nu} - \Gamma_{\lambda\nu}^\beta g_{\mu\beta}. \quad (2)$$

Here, ∇_λ is constructed from the affine connection $\Gamma_{\mu\nu}^\beta$, assumed to be both symmetric in its lower indices and curvature-free (i.e., the connection is integrable). From this third-rank tensor, two distinct trace vectors can be extracted via metric contraction: $Q_\lambda = g^{\mu\nu} Q_{\lambda\mu\nu}$ and $\tilde{Q}_\lambda = g^{\mu\nu} Q_{\mu\lambda\nu}$, reflecting the asymmetric nature of index ordering in the contraction process. The scalar quantity Q , which serves as the fundamental invariant in the action, is obtained by fully contracting the non-metricity tensor with its associated conjugate tensor:

$$Q = -Q_{\lambda\mu\nu} P^{\lambda\mu\nu}, \quad (3)$$

where the conjugate $P^{\lambda\mu\nu}$ is defined as

$$P_{\mu\nu}^\beta = -\frac{1}{2} L_{\mu\nu}^\beta + \frac{1}{4} (Q^\beta - \tilde{Q}^\beta) g_{\mu\nu} - \frac{1}{4} \delta_{(\mu}^\beta Q_{\nu)}. \quad (4)$$

In this expression, $L_{\mu\nu}^\beta$ denotes the disformation tensor, which measures the departure of the symmetric teleparallel connection from the standard Levi-Civita connection of Riemannian geometry. Crucially, $L_{\mu\nu}^\beta$ can be written entirely in

terms of the non-metricity tensor itself:

$$L_{\mu\nu}^{\beta} = \frac{1}{2} g^{\beta h} (-Q_{\mu h \nu} - Q_{\nu h \mu} + Q_{h \mu \nu}). \quad (5)$$

Varying the total action (1) with respect to the inverse metric $g^{\mu\nu}$ yields the field equations that dictate the evolution of the spacetime geometry in $f(Q)$ gravity:

$$\begin{aligned} & -\frac{2}{\sqrt{-g}} \nabla_{\beta} (f_Q \sqrt{-g} P_{\mu\nu}^{\beta}) - \frac{1}{2} f g_{\mu\nu} \\ & - f_Q (P_{\mu\beta\gamma} Q^{\beta\gamma}{}_{\nu} - 2Q_{\beta\gamma\mu} P^{\beta\gamma}{}_{\nu}) = T_{\mu\nu}, \end{aligned} \quad (6)$$

where $f_Q \equiv \frac{df}{dQ}$, and the energy-momentum tensor $T_{\mu\nu}$ of the matter sector is defined through the standard variational prescription:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu\nu}}. \quad (7)$$

These field equations constitute the dynamical core of $f(Q)$ gravity. Importantly, when the function $f(Q)$ is chosen to be linear, i.e., $f(Q) = Q$, the theory seamlessly reduces to general relativity in its teleparallel equivalent form, thereby ensuring compatibility with established gravitational physics in the appropriate limit.

2.2 Matter creation cosmology

An approach to addressing persistent cosmological issues involves including gravitationally induced particle production mechanisms, first proposed by Prigogine et al. (1989), and subsequently developed in greater detail by Calvão et al. (1992). In this framework, the expanding Universe is viewed as a non-equilibrium thermodynamic system in which new particles are continuously generated by irreversible processes governed by the underlying geometry of spacetime. This paradigm views the universe as an open, adiabatic system with a time-varying number of particles N . The particle current is described by the four-vector $N^{\mu} = nu^{\mu}$, where u^{μ} is the fluid four-velocity (normalized as $u^{\mu}u_{\mu} = -1$) and $n = N/V$ is the particle number density in a comoving volume. Due to the non-equilibrium nature of the process, this current cannot be conserved. In the continuity equation for the particle flux, the phenomenological source term Γ quantifies the departure from particle number conservation:

$$\nabla_{\mu} N^{\mu} \equiv \dot{n} + 3Hn = n\Gamma, \quad (8)$$

where an overdot signifies differentiation with respect to cosmic time t , and $H = \dot{a}/a$ is the Hubble parameter characterizing the expansion rate. Physically, $\Gamma > 0$ corresponds to net particle creation, $\Gamma = 0$ recovers standard cosmology with conserved particle number, and $\Gamma < 0$ would imply particle annihilation—though the latter is generally disfavored

in cosmological contexts. Although the exact microphysical origin of Γ remains elusive, it is widely interpreted as emerging from quantum field effects in curved spacetime, where the gravitational field acts as a reservoir supplying energy for particle production.

The thermodynamic consistency of this picture is ensured by examining the entropy flow. The entropy current is defined as $S^{\mu} = n\sigma u^{\mu}$, where σ represents the entropy per particle. The generalized second law of thermodynamics demands that the covariant divergence of this current be non-negative:

$$S^{\mu}{}_{;\mu} = n\dot{\sigma} + n\sigma\Gamma \geq 0. \quad (9)$$

Even under the assumption of constant specific entropy ($\dot{\sigma} = 0$), the total entropy increases due to the growing particle number, yielding $S^{\mu}{}_{;\mu} = n\sigma\Gamma \geq 0$, which is satisfied as long as $\Gamma \geq 0$.

To incorporate these effects into the energy balance, the standard first law of thermodynamics for a closed adiabatic system, $d(\rho V) + p dV = 0$, must be generalized. In the presence of particle creation, the variation in particle number contributes an effective heat term, leading to the modified relation:

$$d(\rho V) + p dV - \frac{h}{n} d(nV) = 0, \quad (10)$$

where ρ is the total energy density, p is the equilibrium (thermal) pressure of the cosmic fluid (e.g., from baryons or cold dark matter), and $h = \rho + p$ denotes the enthalpy density. In this formulation, the term $-(h/n) d(nV)$ is interpreted as the energy exchanged between the gravitational field and the matter component, effectively acting as a heat source. This energy transfer manifests as an additional, non-thermal contribution to the fluid pressure—commonly referred to as the *creation pressure* p_c . By definition, this pressure is given by

$$p_c = -\frac{h}{n} \frac{d(nV)}{dV}. \quad (11)$$

Substituting the particle balance equation (8) into this expression, one obtains a direct link between the creation pressure and the particle production rate:

$$p_c = -\frac{(\rho + p)}{3H} \Gamma. \quad (12)$$

Thus, p_c encapsulates the effective negative pressure induced by matter creation, which can drive cosmic acceleration without invoking dark energy. This result follows from the assumption of adiabaticity at the level of specific entropy ($\dot{\sigma} = 0$), as originally emphasized by Prigogine et al. (1989). Nevertheless, the total entropy of the system still increases due to the continuous production of particles, as explicitly

shown by Eq. (9), thereby satisfying the second law of thermodynamics in an expanding Universe. In the present analysis, we adopt this mechanism as a phenomenological but thermodynamically consistent way to generate an effective negative pressure in the matter sector. This choice is motivated by the fact that, at late times, even a moderate creation rate can alter the expansion history in a controlled manner without introducing an explicit dark-energy component, while still allowing direct confrontation with $H(z)$ -based probes.

2.3 Variable generalized Chaplygin gas

Bento et al. (2002) presented a generalized Chaplygin gas (GCG) with EoS $p_{Ch} = -A/\rho_{Ch}^\alpha$, where $0 < \alpha \leq 1$. The GCG model describes a Universe initially dominated by matter, undergoing decelerated expansion, and transitioning into a phase dominated by dark energy, leading to accelerated expansion in the present era. Guo and Zhang (2007) proposed VCG EoS $p_{Ch} = -A(a)/\rho_{Ch}$, where A is the function of the scale factor, explaining the transition from a dust-like state to quintessence or phantom fields. Yang et al. (2007), and later on, Lu (2009) generalized the VCG model, known as variable generalized Chaplygin gas (VGCG) with EoS:

$$p_{Ch} = -\frac{A(a)}{\rho_{Ch}^\alpha}. \quad (13)$$

Cosmological models based on the Chaplygin gas exhibit three key characteristics: they account for a seamless transition from a decelerated cosmic expansion to the current era of accelerated expansion, they provide a unified macroscopic framework to describe dark energy and dark matter, and they represent a minimal extension of the standard Λ CDM model. Since GCG is often discussed as a unified dark-sector fluid, it is important to recall why we adopt its variable extension in the present work. In single-fluid implementations, the GCG can develop a non-negligible late-time adiabatic sound speed, which may generate oscillatory and/or unstable features in the linear matter power spectrum and can lead to strong restrictions on the parameter α , frequently pushing the model close to its Λ CDM-like limit (Sandvik et al. 2004; Amendola et al. 2003). This motivates considering controlled generalizations in which the Chaplygin parameter is allowed to evolve with cosmic expansion. The VGCG achieves this by promoting A as a function of the scale factor, $A(a)$ (Yang et al. 2007; Lu 2009). This introduces additional flexibility in the timing and smoothness of the transition from the dust-like regime to the negative-pressure regime.

3 Cosmological model

To study the matter creation consequences with VGCG EoS in $f(Q)$ gravity, we consider the flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric, which is homogeneous and isotropic. The line element is expressed as:

$$ds^2 = -dt^2 + a^2(t) \left[dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (14)$$

where $a(t)$ represents the cosmic scale factor. The Hubble parameter, H , is defined as $H = \frac{\dot{a}}{a}$, with a dot indicating a derivative with respect to cosmic time t .

The non-metricity scalar in the FLRW geometry is derived as $Q = 6H^2$, where $H = \dot{a}/a$ is the Hubble parameter. A dot denotes derivative with respect to cosmic time t . The Universe's matter content is modeled as a perfect fluid, with the energy-momentum tensor (EMT) expressed as

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \quad (15)$$

where $\rho = \rho_{Ch} + \rho_m$ and $p = p_{Ch} + p_m + p_c$ denote the total energy density and pressure of the cosmic fluid, respectively. Here, u_μ represents the four-velocity vector, satisfying the normalization condition $u^\mu u_\mu = -1$.

Using FLRW line element (14) and EMT equation (15) into field equations (6), we obtain (Jiménez et al. 2020)

$$6f_Q H^2 - \frac{1}{2}f = \rho, \quad (16)$$

$$(12f_Q Q H^2 + f_Q)\dot{H} = -\frac{1}{2}(\rho + p), \quad (17)$$

where $f_Q = \frac{df}{dQ}$ and $f_{QQ} = \frac{d^2f}{dQ^2}$. In our setup, the particle production process is assumed to affect only the pressureless matter component through the creation pressure p_c , while the VGCG component obeys the standard conservation equation. This split allows the VGCG to retain its role as a unified dark fluid, while matter creation contributes an additional effective pressure channel associated with irreversible particle production. In our analysis, we focus on the late-time evolution, so that we can neglect radiation and consider the contribution due to pressureless matter ($p_m = 0$) and Chaplygin gas. We also assume that two fluids dark matter and Chaplygin gas fluid conserve separately, that is,

$$\dot{\rho}_m + 3H(\rho_m + p_m + p_c) = 0, \quad (18)$$

and

$$\dot{\rho}_{Ch} + 3H(\rho_{Ch} + p_{Ch}) = 0. \quad (19)$$

Assuming $p_m = 0$, Eq. (18) gives

$$\dot{\rho}_m + 3H\rho_m = \rho_m\Gamma, \quad (20)$$

which can be written as

$$\frac{d}{dt} \ln(a^3 \rho_m) = \Gamma. \quad (21)$$

The particle creation rate, Γ is related to the nature of the produced particles under adiabatic mechanism, and unfortunately, the nature of the produced particles is unknown to us as the associated quantum field theory (QFT) is yet to be developed which is the essential tool to determine this Γ . However, as particle production mechanism has become one of the possible alternatives to explain the current expanding accelerating phase of the Universe, a phenomenological choice may be taken. It may also be on thermodynamical basis because the general concept of gravitationally induced particle creation is rooted in thermodynamics of open system where the energy is exchanged between the gravitational field and matter. Many authors have explored scenarios of matter creation in cosmology with $\Gamma \propto H$, which is the most natural choice. In a series of papers listed in the references, the cosmological models have been discussed with this form of Γ . However, this kind of the models are always decelerating for $\beta < 1/3$ or accelerating for $\beta > 1/3$, that is, there is not a transition redshift from a decelerating to an accelerating regime as required by type Ia supernovae observations.

In order to solve such problems, the other forms of Γ have been proposed in literature. However, in this paper, we retain the same form of Γ but combined with an exotic fluid VGCG within the framework of a modified $f(Q)$ gravity. Using this form of Γ , we try to find a smooth transition from decelerating phase to accelerating phase which could be consistent with the current observations.

Therefore, we assume the particle creation rate as (Calvão et al. 1992)

$$\Gamma = 3\beta H, \quad (22)$$

where β is a constant parameter satisfying $0 \leq \beta$ and the factor 3 has been maintained for mathematical convenience.

Using (22) into (21), we get

$$\rho_m = \rho_{m,0}(1+z)^{3(1-\beta)} \quad (23)$$

where $\rho_{m,0}$ is the present energy density and $z = (1/a) - 1$ is the redshift.

Yang et al. (2007) and Wu et al. (2008) studied the VGCG model with the variable A , which is assumed as a function of scale factor. The most appropriate form is $A(a) = A_0 a^{-n}$, where n is a positive constant. The motivation comes from the Born-Infeld Lagrangian density for a scalar field (Sen 2002). Such assumption makes the VGCG model behaves like matter (dust) in early times, causing deceleration, and as Universe expands, this term becomes less dominant, and the model behaves like dark energy (quintessence

or phantom), which causes acceleration. Therefore, its ability to mimic different cosmic phases, make VGCG model a compelling candidate for exploring the evolution of the Universe. Therefore, VGCG EoS, defined in Eq. (13), takes the form

$$p_{Ch} = -\frac{A_0 a^{-n}}{\rho_{Ch}^\alpha}. \quad (24)$$

Using (24) into (19) and integrating, we get

$$\rho_{Ch} = \rho_{Ch,0} \left[A_s (1+z)^n + (1-A_s)(1+z)^{3(1+\alpha)} \right]^{\frac{1}{1+\alpha}}, \quad (25)$$

where $\rho_{Ch,0}$ is the present energy density and $A_s = \frac{3(1+\alpha)}{3(1+\alpha)-n} \frac{A_0}{\rho_{Ch,0}^{(1+\alpha)}}$. We must have $n > 0$ and $(1+\alpha) > 0$ for an expanding Universe.

In what follows, we consider two specific $f(Q)$ models to study the cosmic expansion of the Universe.

3.1 Model 1

Let us assume the following $f(Q)$ function in order to explore the dynamics of the Universe possesses matter creation and VGCG EoS (Gadbail et al. 2023)

$$f(Q) = n_1 Q^k, \quad (26)$$

which is the power-law form of function $f(Q)$. Here, n_1 and k are dimensionless parameters. We remark here that for $k = 0$, the model reduces to the standard Λ CDM with the quantity $n_1/2$ playing the role of cosmological constant. The case $k = 1$ is equivalent to the Symmetric Teleparallel Equivalent of general relativity subject to rescaling the Newtonian gravitational constant by factor of $n_1 + 1$. The power-law form $f(Q) = n_1 Q^k$ has been used in previous $f(Q)$ studies (Gadbail et al. 2023) and naturally extends scale-invariant gravity models.

Using (26) into (16), we find

$$H = \left[\frac{2(\rho_m + \rho_{Ch})}{n_1(2k-1)6^k} \right]^{\frac{1}{2k}}. \quad (27)$$

Substituting ρ_m and ρ_{Ch} from (23) and (25) into (27), and employing the current value of Hubble parameter $H(z=0) = H_0$, and the normalized Hubble parameter, represented as $E^2(z) = H^2(z)/H_0^2$, we deduce the following solution.

$$E^2(z) = B \times \left[\Omega_m (1+z)^{3(1-\beta)} + \Omega_{Ch} \left(A_s (1+z)^n + (1-A_s)(1+z)^{3(1+\alpha)} \right)^{\frac{1}{1+\alpha}} \right]^{\frac{1}{k}}, \quad (28)$$

where $B = \left(\frac{6^{1-k}}{(2k-1)n_1} \right)^{\frac{1}{2k}}$. Applying the present value initial condition $z = 0$ in the above equation, we have: $\Omega_{Ch_1} = H_0^{2k-2} (2k-1) 6^{k-1} n_1 - \Omega_m$.

3.2 Model 2

In this section, we consider a more general cosmological form of $f(Q)$ gravity as motivated in Ref. (Capozziello and D'Agostino 2022; Mhamdi et al. 2024)

$$f(Q) = n_0 + n_1 Q^k, \quad (29)$$

where n_0 and n_1 represent the free parameters of the model. This is the generalization of the above form discussed in Model 1 by including a constant term. Such form was obtained by Capozziello and D'Agostino (2022) during the Model-independent reconstruction of $f(Q)$ non-metric gravity. Their analysis shows that this form is the best approximation for describing the accelerated expansion of the Universe. The function form recovers pure general relativity for $n_0 = 0$, $n_1 = 1 = k$, while the Λ CDM model for $n_0 > 0$ and $n_1 = 1 = k$. This form is very popular in cosmology to explain the accelerating Universe.

Using (29) into (16), we get

$$H = \left[\frac{2(\rho_m + \rho_{Ch}) + n_0}{n_1(12k-1)6^k} \right]^{\frac{1}{2k}}. \quad (30)$$

Substituting ρ_m and ρ_{Ch} from (23) and (25) into (30), the evolution equation for normalized Hubble parameter is given by

$$E^2(z) = B \times \left[\Omega_m (1+z)^{3(1-\beta)} + \Omega_{Ch} \left(A_s (1+z)^n + (1-A_s)(1+z)^{3(1+\alpha)} \right)^{\frac{1}{\alpha+1}} + n_0 \right]^{\frac{1}{k}}. \quad (31)$$

Applying the present value initial condition $z = 0$ in the above equation, we have: $\Omega_{Ch} = H_0^{2k-2} (12k-1) 6^{k-1} n_1 - \Omega_m - n_0$.

4 Methodology and data description

In order to derive robust posterior probability distributions for the cosmological parameters of interest, we utilize a combination of complementary observational probes. Specifically, we incorporate three key datasets: (i) Cosmic Chronometer (CC) measurements, which provide direct estimates of the Hubble parameter across a wide redshift

range; (ii) the Pantheon+ compilation of Type Ia supernovae (SNe Ia), excluding any calibration tied to the SH0ES local distance ladder to avoid potential biases; and (iii) Baryon Acoustic Oscillation (BAO) data drawn from the first-year results of the Dark Energy Spectroscopic Instrument (DESI) and the extended spectroscopic survey SDSS-IV. Each of these datasets constrains different aspects of cosmic expansion history, and their joint analysis significantly enhances parameter estimation accuracy. In what follows, we outline the statistical treatment of each dataset, including the explicit form of the associated chi-square (χ^2) likelihood function used in our Markov Chain Monte Carlo (MCMC) inference pipeline.

- **Cosmic Chronometers:** The Cosmic Chronometer (CC) dataset employed in our analysis consists of 31 independent measurements of the Hubble parameter $H(z)$, spanning a redshift interval from the local Universe up to $z \approx 2$. These data points are derived using the differential age technique introduced by Jiménez and Loeb (2002), which exploits the spectroscopic properties of passively evolving, early-type galaxies with minimal ongoing star formation. By comparing the ages of such galaxies observed at closely spaced redshifts, this approach yields a direct estimate of the cosmic expansion rate through the relation $H(z) = -\frac{1}{1+z} \frac{dz}{dt} \approx -\frac{\Delta z}{(1+z)\Delta t}$. The method is particularly robust because it avoids reliance on cosmological assumptions or distance ladders, offering a model-independent probe of $H(z)$. In our analysis, we adopt the latest compilation of CC data as presented in Refs. [Moresco et al. 2012, 2016, 2018, 2020], which includes careful treatment of both statistical uncertainties and systematic biases. To assess the compatibility between theoretical predictions and observations, we compute the chi-square statistic for the CC sample as

$$\chi_{CC}^2(\theta) = \Delta \mathbf{H}^T(z) \mathbf{C}^{-1} \Delta \mathbf{H}(z),$$

where the residual vector $\Delta \mathbf{H}(z)$ is defined component-wise by $\Delta H_i = H_{\text{model}}(z_i; \theta) - H_{\text{obs},i}$, with H_{model} denoting the Hubble parameter predicted by the cosmological model under consideration (parameterized by θ) and $H_{\text{obs},i}$ representing the i -th observed value. The total covariance matrix $\mathbf{C}$ accounts for both random (statistical) and correlated (systematic) errors, and is decomposed as

$$\mathbf{C}_{ij} = \mathbf{C}_{ij}^{\text{stat}} + \mathbf{C}_{ij}^{\text{syst}},$$

ensuring a comprehensive and realistic error propagation in our MCMC parameter estimation pipeline.

- **Type Ia Supernovae:** To place robust constraints on the free parameters of our cosmological models, we employ the Pantheon+ compilation of Type Ia supernovae (SNe

Ia), which currently represents the largest and most precise spectroscopically confirmed sample of such standardizable candles. This dataset includes light-curve measurements for 1701 SNe Ia, covering a broad redshift interval from the local Universe up to $z \approx 2.3$, with a few high-redshift outliers extending toward $z \sim 2.4$ (Brout et al. 2022). The observed apparent magnitude of each supernova is related to its luminosity distance through the relation

$$m(z) = 5 \log_{10} \left(\frac{d_L(z)}{\text{Mpc}} \right) + \mathcal{M} + 25,$$

where $\mathcal{M}$ is a nuisance parameter that encapsulates the absolute magnitude of SNe Ia along with potential calibration offsets. Assuming a spatially flat FLRW geometry, the luminosity distance $d_L(z)$ is expressed as

$$d_L(z) = c(1+z) \int_0^z \frac{dz'}{H(z')},$$

with c denoting the speed of light in km s^{-1} . The goodness-of-fit for our theoretical models is quantified using a chi-square statistic constructed from the full covariance structure of the Pantheon+ sample:

$$\chi_{\text{SNe Ia}}^2 = \Delta \mathbf{D}^T \mathbf{C}_{\text{total}}^{-1} \Delta \mathbf{D},$$

where $\mathbf{C}_{\text{total}} = \mathbf{C}_{\text{stat}} + \mathbf{C}_{\text{sys}}$ combines both the statistical uncertainties and the systematic error correlations (e.g., from photometric calibration, host-galaxy mass dependence, and intrinsic scatter). The residual vector $\Delta \mathbf{D}$ is defined component-wise as

$$\Delta D_i = \mu(z_i) - \mu_{\text{model}}(z_i; \theta),$$

with $\mu(z_i) = m(z_i) - \mathcal{M}$ being the observed distance modulus and $\mu_{\text{model}}(z_i; \theta)$ the corresponding theoretical prediction based on the model parameters θ . This formulation allows for a statistically rigorous comparison between observational data and cosmological predictions while marginalizing over the unknown absolute magnitude $\mathcal{M}$.

- **Baryon Acoustic Oscillations:** Baryon Acoustic Oscillations (BAOs) provide a robust standard ruler for probing the expansion history of the Universe, anchored by the comoving sound horizon at the baryon drag epoch, denoted z_d . This epoch corresponds to the moment when photons and baryons effectively decouple due to the rapid drop in the photon-baryon coupling rate as the Universe expands and cools. The sound horizon scale, r_d , is computed by integrating the sound speed in the tightly coupled baryon-photon plasma from the drag redshift to infinity:

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{E(z)} dz,$$

where $c_s(z) = c/\sqrt{3(1+\bar{R}(z))}$ is the adiabatic sound speed (with $\bar{R}(z)$ encoding the baryon-to-photon density ratio), c is the speed of light, and $E(z) = H(z)/H_0$ is the dimensionless Hubble parameter. Within the standard Λ CDM cosmology, the Planck satellite collaboration reports a highly precise value of $r_d = 147.09 \pm 0.26 \text{ Mpc}$. However, to remain agnostic about early-Universe physics and to test for possible deviations from Λ CDM, we treat r_d as a free nuisance parameter in our analysis. This approach allows us to infer the sound horizon directly from late-time cosmological observations, following the methodology advocated in several recent works (Verde et al. 2017; Lemos et al. 2023; Nunes et al. 2020; Pogorian et al. 2020, 2024; Jedamzik et al. 2021; Lin et al. 2021; Vagnozzi 2023). To construct the BAO likelihood, we compute three key distance measures that encode the geometry of the expanding Universe: the Hubble distance $D_H(z)$, the transverse comoving angular diameter distance $D_M(z)$, and the spherically averaged volume distance $D_V(z)$. Assuming spatial flatness—as supported by CMB data and consistent with our FLRW framework—these distances are expressed as

$$D_H(z) = \frac{c}{H(z)}, \quad D_M(z) = c \int_0^z \frac{dz'}{H(z')},$$

$$D_V(z) = \left[z D_M^2(z) D_H(z) \right]^{1/3}$$

The BAO signal is then extracted by comparing the theoretically predicted ratios $D_M(z)/r_d$, $D_H(z)/r_d$, and $D_V(z)/r_d$ with their observational counterparts. For each distance measure D_Y (where $Y \in \{H, M, V\}$), we define the chi-square contribution as

$$\chi_{D_Y/r_d}^2 = \Delta \mathbf{D}_Y^T \cdot \mathbf{C}_{D_Y}^{-1} \cdot \Delta \mathbf{D}_Y,$$

with the residual vector given by $\Delta \mathbf{D}_Y = (D_Y/r_d)_{\text{Model}} - (D_Y/r_d)_{\text{Data}}$. The covariance matrix $\mathbf{C}_{D_Y}$ is typically assumed to be diagonal in modern BAO analyses, with entries $\sigma_{D_Y}^2$ representing the reported 1σ uncertainties from the survey. The total BAO chi-square is then obtained by summing over all distance measures:

$$\chi_{\text{BAO}}^2 = \chi_{D_H/r_d}^2 + \chi_{D_V/r_d}^2 + \chi_{D_M/r_d}^2.$$

For this updated analysis, we utilize the latest Baryon Acoustic Oscillation measurements from Data Release 2 (DR2) of the Dark Energy Spectroscopic Instrument (DESI) and the extended BAO catalog from the Sloan Digital Sky Survey IV (SDSS-IV), as reported in Adame et al. (2025) and Alam et al. (2021), respectively. These datasets provide high-precision constraints across multiple redshift bins, enabling a stringent test of our $f(Q)$ -based matter creation models. With this update, the total number of BAO measurements used in the combined

Table 1 Flat priors and best-fit values of the cosmological parameters obtained for Λ CDM, Model 1 and Model 2 under combination of $CC + SNeIa + BAO$ dataset with DESI DR2 data

Parameters	Priors	Λ CDM	Model 1	Model 2
H_0	$\in [60, 80]$	68.1 ± 0.9	67.199 ± 0.128	67.193 ± 0.122
Ω_m	$\in [0, 0.5]$	0.314 ± 0.006	0.287 ± 0.019	0.288 ± 0.021
Ω_Λ	$\in [0.5, 1]$	0.684 ± 0.006	—	—
k	$\in [0, 1.5]$	—	0.942 ± 0.026	0.946 ± 0.022
n_0	$\in [0, 1]$	—	—	0.840 ± 0.126
n_1	$\in [0, 1]$	—	1.229 ± 0.459	0.928 ± 0.031
A_s	$\in [0, 1]$	—	0.793 ± 0.020	0.723 ± 0.025
α	$\in [0, 1]$	—	0.205 ± 0.081	0.208 ± 0.075
n	$\in [0, 1]$	—	0.789 ± 0.135	0.969 ± 0.160
β	$\in [0, 1]$	—	0.764 ± 0.124	0.728 ± 0.118
r_d	$\in [100, 200]$	146.3 ± 2.3	148.23 ± 1.09	148.28 ± 1.05
$\mathcal{M}$	$\in [-20, -18]$	-19.412 ± 0.035	-19.427 ± 0.006	-19.429 ± 0.006

analysis becomes 14, and therefore the total number of data points in the joint dataset is $N = 1746$.

To jointly analyze the full set of observational probes, we construct a global likelihood by summing the individual chi-squared contributions from each dataset. The total goodness-of-fit statistic is therefore defined as

$$\chi_{\text{tot}}^2 = \chi_{\text{CC}}^2 + \chi_{\text{SNe Ia}}^2 + \chi_{\text{BAO}}^2, \quad (32)$$

which assumes statistical independence among the CC, SNe Ia, and BAO measurements. Parameter inference is carried out using MCMC technique implemented via the `PyPolyChord` nested sampling algorithm Handley et al. (2015), which efficiently explores high-dimensional parameter spaces and computes Bayesian evidence. For post-processing, contour plotting, and statistical interpretation of the posterior distributions, we employ the `GetDist` Python package developed by Lewis (2019). The parameter vectors explored in our analysis differ across models: for the reference Λ CDM cosmology, we sample $\theta = \{H_0, \Omega_m, \Omega_\Lambda, r_d, \mathcal{M}\}$; for Model 1 ($f(Q) = n_1 Q^k$), the parameter space expands to $\theta = \{H_0, \Omega_m, k, n_1, A_s, \alpha, n, \beta, r_d, \mathcal{M}\}$; and for Model 2 ($f(Q) = n_0 + n_1 Q^k$), we include an additional constant term, yielding $\theta = \{H_0, \Omega_m, k, n_0, n_1, A_s, \alpha, n, \beta, r_d, \mathcal{M}\}$. In all cases, r_d represents the comoving sound horizon at the baryon drag epoch, treated as a free nuisance parameter to avoid reliance on early-Universe assumptions. Uniform (flat) priors are imposed on every parameter, with specific bounds detailed in Table 1, ensuring an unbiased exploration of the allowed cosmological parameter space.

5 Discussion on cosmological parameters

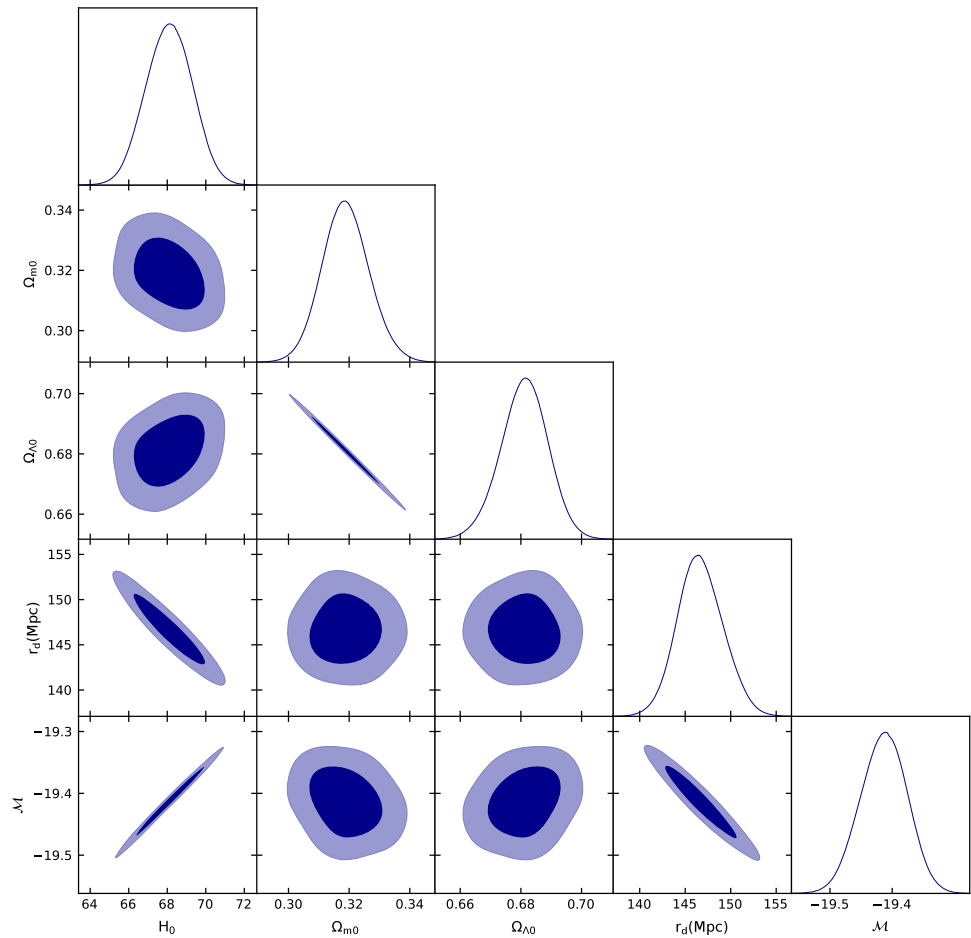
We now proceed to present and interpret the outcomes of our Bayesian parameter inference for the two proposed $f(Q)$

cosmological models-Model 1 ($f(Q) = n_1 Q^k$) and Model 2 ($f(Q) = n_0 + n_1 Q^k$)-in direct comparison with the standard Λ CDM paradigm. This comparative analysis allows us to assess the capacity of our extended frameworks to accurately describe the late-time expansion history of the Universe across a wide range of redshifts. The marginalized posterior distributions and corresponding joint confidence contours at the 68% (1σ) and 95% (2σ) credibility levels for all free parameters are visualized in Figs. 1 and 2, which juxtapose the Λ CDM baseline with the VGCG-based scenarios incorporating matter creation. The numerical best-fit estimates, along with their associated uncertainties, are summarized in Table 1. Furthermore, to elucidate the dynamical behavior of key cosmological quantities, we reconstruct their redshift evolution-such as the Hubble rate, deceleration parameter, and effective equation of state-using the maximum-likelihood parameter sets derived from our MCMC analysis.

5.1 Hubble parameter

We present a comparative analysis of the redshift evolution of the Hubble parameter $H(z)$ for the standard Λ CDM cosmology alongside our two proposed $f(Q)$ -based models-Model 1 and Model 2-in direct confrontation with the observational CC dataset. For the reference Λ CDM scenario, the theoretical expression $H(z) = H_0 \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$ is employed, with best-fit parameters constrained by our joint analysis yielding $H_0 = 68.2 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.315 \pm 0.007$, and $\Omega_\Lambda = 0.682 \pm 0.007$. In contrast, the reconstructed present-day Hubble constants for Model 1 and Model 2 are $H_0 = 67.199 \pm 0.128 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $67.193 \pm 0.122 \text{ km s}^{-1} \text{ Mpc}^{-1}$, respectively-values marginally lower than the Λ CDM estimate but in excellent agreement with the Planck 2018 inference of $H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Aghanim et al. 2020). Furthermore, the inferred matter density parameters are $\Omega_m = 0.287 \pm 0.019$ for Model 1 and $\Omega_m = 0.288 \pm 0.021$ for Model 2, both

Fig. 1 The plot shows the best-fit values of model parameters of Λ CDM model obtained from the $CC + SNeIa + BAO$ datasets at 1σ and 2σ confidence levels



slightly below the Λ CDM value yet well within observational uncertainties.

The model parameters derived from our analysis show both consistency with and meaningful differences from previous studies. For Model 1, the best-fit value of $k = 0.942 \pm 0.026$ is in excellent agreement with the value $k = 0.947 \pm 0.026$ reported by Bhardwaj and Singh (2024b) in their GCG based $f(Q)$ gravity analysis with matter creation. The parameter $n_1 = 1.229 \pm 0.459$ in Model 1 is larger than the value $n_1 = 1.26 \pm 0.31$ from Bhardwaj and Singh (2024a,b), which reflects the additional flexibility provided by our VGCG parametrization. This difference has physical significance as it indicates a stronger departure from the pure power-law $f(Q)$ gravity (where $n_1 = 1$ exactly), suggesting that the variable equation of state provides a better fit to the data. For Model 2, the parameters $k = 0.946 \pm 0.022$ and $n_1 = 0.928 \pm 0.031$ are consistent with the results from Capozziello and D'Agostino (2022), who found $k \approx 0.95$ and $n_1 \approx 0.93$ in their $f(Q)$ gravity analysis. The parameter $n_0 = 0.840 \pm 0.126$ in Model 2 is in close agreement with the value $n_0 = 0.823 \pm 0.01$ from the updated DESI DR2 analysis, confirming the consistency of the constant term across different datasets. The parameter $\alpha = 0.205 \pm 0.081$

in Model 1 and $\alpha = 0.208 \pm 0.075$ in Model 2 are both consistent with the range $0.15 - 0.25$ reported in earlier studies of VGCG models (Yang et al. 2007; Lu 2009), indicating that the equation of state transition occurs at similar redshifts. These constraints are slightly tighter than those from previous studies, primarily due to the inclusion of the latest DESI DR2 data, which provides more precise BAO measurements. The physical significance of these parameters is that they collectively describe the transition from matter domination to dark energy domination, with the larger n_1 in Model 1 reflecting a more pronounced departure from pure power-law gravity, while the α values indicate a transition redshift that is consistent with the deceleration-to-acceleration transition observed in the cosmic expansion history.

Given that $H(z)$ encodes the complete expansion history of the Universe, it serves as a critical diagnostic for testing cosmological models. To evaluate the performance of our frameworks, we overlay the theoretically predicted $H(z)$ curves-derived from the best-fit parameter sets-onto the CC measurements, as displayed in Fig. 3. The plot reveals that Model 2 tracks the Λ CDM prediction more closely across the entire redshift range than Model 1. To quantify these de-

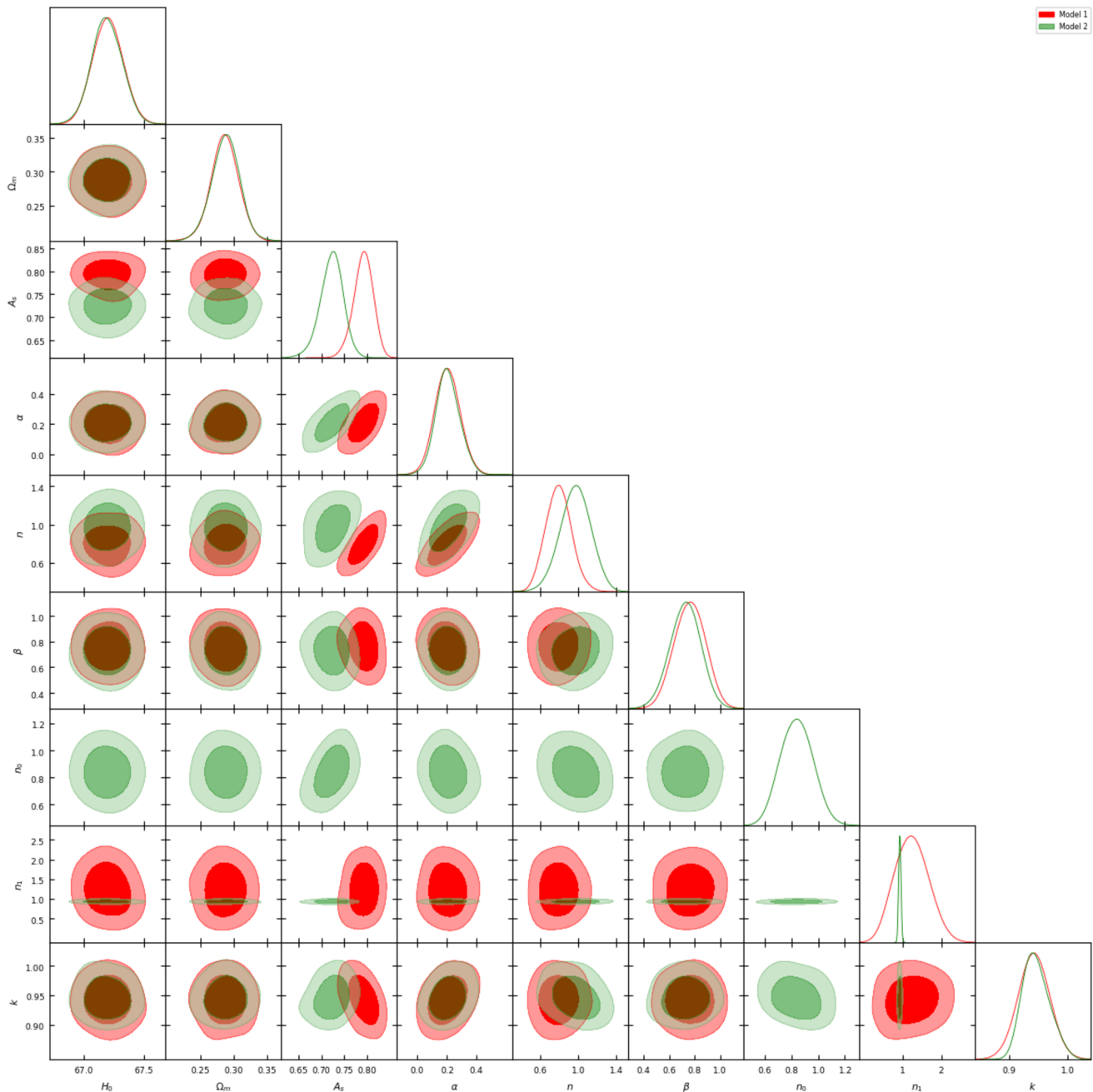


Fig. 2 The plot shows the best-fit values of model parameters of Model 1 and Model 2 obtained from the *CC + SNeIa + BAO* datasets at 1σ and 2σ confidence levels

viations more precisely, we introduce the Hubble difference $\Delta H(z)$, defined as

$$\Delta H(z) = H_{\text{Model}}(z) - H_{\Lambda\text{CDM}}(z),$$

where $H_{\text{Model}}(z)$ denotes the Hubble rate from either Model 1 or Model 2, and $H_{\Lambda\text{CDM}}(z)$ is the corresponding ΛCDM baseline. The resulting residuals $(z_i, \Delta H(z_i), \sigma_i)$ —with σ_i representing the propagated observational uncertainty—are visualized in Fig. 4, where error bars reflect the measure-

ment precision of the CC data. By construction, the ΛCDM model yields $\Delta H(z) = 0$ at all redshifts (a horizontal line at zero), providing a natural reference. At low redshifts ($z < 1$), both extended models exhibit minimal deviation from this baseline, indicating robust compatibility with the late-time expansion inferred from ΛCDM . However, at higher redshifts ($z > 1$), Model 1 displays a growing discrepancy, diverging noticeably from the standard cosmology, whereas Model 2 maintains closer alignment, suggesting greater fit-

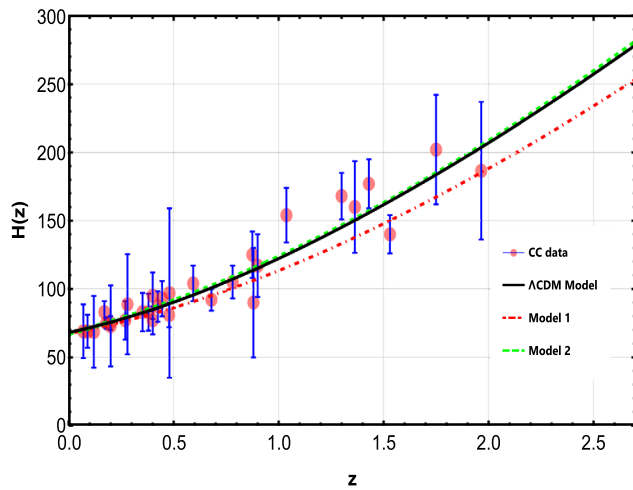


Fig. 3 The plot shows the evolution of the $H(z)$ for Λ CDM, Model 1 and Model 2 using values obtained from $CC + SNe Ia + BAO$ datasets. The grey points with error bars represent the $H(z)$ data sample

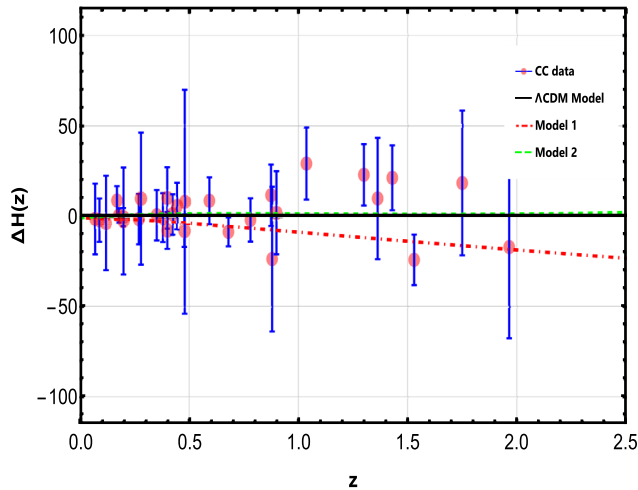


Fig. 4 The plot shows the evolution of the Hubble difference $\Delta H(z)$, as a function of redshift z , compared to the CC dataset

delity to the concordance model in the early expansion epoch.

5.2 Deceleration parameter

The deceleration parameter q plays a pivotal role in diagnosing the kinematic behavior of cosmic expansion, distinguishing between epochs of deceleration, acceleration, or transitional dynamics. Mathematically, it is defined as $q = -\frac{a\ddot{a}}{\dot{a}^2}$, where $a(t)$ is the scale factor of the Universe and overdots denote derivatives with respect to cosmic time. When recast in terms of redshift z , this quantity takes the form

$$q(z) = -1 + \frac{(1+z)}{E(z)} \frac{dE(z)}{dz}, \quad (33)$$

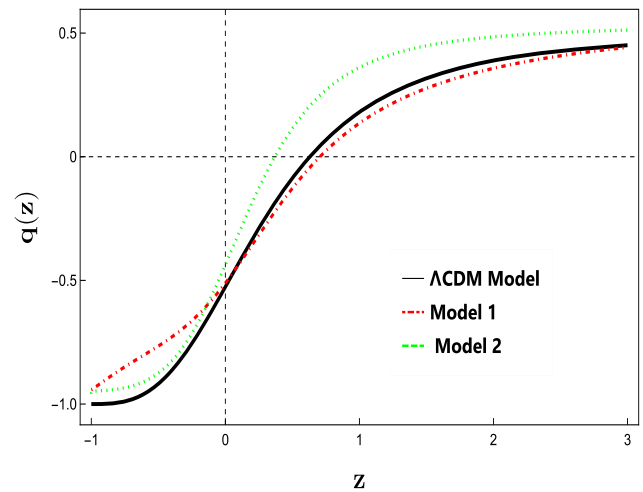


Fig. 5 The plot shows the evolution of the $q(z)$ for Λ CDM, Model 1 and Model 2 using values obtained from $CC + SNe Ia + BAO$ datasets. In standard flat Λ CDM model, $q(z)$ evolves with redshift. It starts from positive around $q(z) \approx 0.5$ (early Universe, matter-dominated) to negative (dark energy-dominated), crossing around $z_{tr} \approx 0.630$ and stays negative

with $E(z) \equiv H(z)/H_0$ denoting the normalized Hubble parameter. Using the best-fit cosmological parameters obtained from the joint $CC + SNe Ia + BAO$ dataset, we reconstruct the redshift evolution of $q(z)$, as displayed in Fig. 5. The plot reveals a clear transition from an early decelerating phase ($q > 0$) to the current epoch of accelerated expansion ($q < 0$), consistent with the standard cosmological narrative. At present ($z = 0$), Model 1 yields $q_0 = -0.535^{+0.011}_{-0.012}$, while Model 2 gives $q_0 = -0.551^{+0.027}_{-0.023}$, both of which align closely with the Λ CDM prediction of $q_0 \approx -0.55$ inferred from Planck collaboration (Aghanim et al. 2020). Furthermore, the redshift at which the transition from deceleration to acceleration occurs, denoted by z_{tr} , is found to be $z_{tr} = 0.591^{+0.28}_{-0.20}$ for Model 1 and $z_{tr} = 0.628^{+0.24}_{-0.18}$ for Model 2. These values are well within the range supported by independent astrophysical measurements, reinforcing the physical viability of the proposed frameworks.

5.3 Jerk parameter

The jerk parameter, denoted $j(z)$, functions as a higher-order kinematic probe that quantifies departures from the standard Λ CDM cosmology. Mathematically, it is defined as the dimensionless third time derivative of the cosmic scale factor, $j = \ddot{a}/(aH^3)$, where $H = \dot{a}/a$ is the Hubble rate (Visser 2004). In the context of a spatially flat FLRW Universe governed by a cosmological constant, this quantity remains exactly constant at $j(z) = 1$ for all redshifts—a distinctive signature of the concordance model. Earlier work by Sahni et al. (2003) emphasized the utility of $j(z)$ as a discriminator among competing dark energy scenarios, as

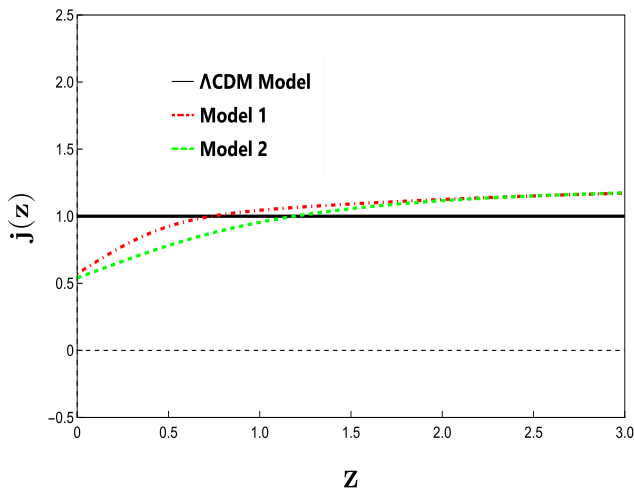


Fig. 6 The plot shows the evolution of the $j(z)$ for Λ CDM, Model 1 and Model 2 using values obtained from $CC + SNe\ Ia + BAO$ datasets. For Λ CDM model, $j(z) = 1$ for all redshifts which is shown by bold horizontal line. It distinguishes the model 1 and model 2 in which $j(z)$ vary with redshift

different expansion histories imprint unique signatures in its redshift evolution.

Using the deceleration parameter $q(z)$ as a foundation, the jerk can be recast in a purely observational form (Rapetti et al. 2007):

$$j(z) = q(2q + 1) + (1 + z) \frac{dq}{dz}. \quad (34)$$

This expression enables direct reconstruction of $j(z)$ from observational data without assuming a specific gravitational theory. Figure 6 displays the reconstructed redshift evolution of the jerk parameter for both Model 1 and Model 2—our $f(Q)$ -based frameworks incorporating matter creation and a variable generalized Chaplygin gas (VGCG). The plots reveal clear deviations from the Λ CDM baseline ($j = 1$), particularly at intermediate and high redshifts. These discrepancies underscore the enhanced dynamical richness of the proposed models, which not only reproduce late-time acceleration but also encode non-trivial higher-order kinematic behavior. At the present epoch ($z = 0$), the best-fit values are $j_0 = 0.569^{+0.015}_{-0.012}$ for Model 1 and $j_0 = 0.530^{+0.030}_{-0.031}$ for Model 2, as listed in Table 2. Both estimates lie significantly below unity, providing strong evidence that these models describe a cosmic expansion history distinct from that of the standard Λ CDM paradigm.

5.4 Equation of state parameter

The effective equation of state (EoS) parameter, denoted $w(z)$, serves as a fundamental diagnostic in cosmology, quantifying the relationship between the total pressure p and energy density ρ of the cosmic fluid through the relation

Table 2 Cosmological parameter estimations for the Λ CDM and Chaplygin gas models with DESI DR2 data

Parameter	Cosmological models		
	Λ CDM	Model 1	Model 2
q_0	$-0.552^{+0.020}_{-0.018}$	$-0.535^{+0.011}_{-0.012}$	$-0.551^{+0.027}_{-0.023}$
j_0	1.000	$0.569^{+0.015}_{-0.012}$	$0.530^{+0.031}_{-0.030}$
w_0	$-0.699^{+0.010}_{-0.013}$	$-0.681^{+0.011}_{-0.014}$	$-0.666^{+0.028}_{-0.031}$
s_0	0.000	$0.117^{+0.011}_{-0.019}$	$0.157^{+0.001}_{-0.003}$
τ_0 (Gyr)	13.6 ± 0.10	13.4 ± 0.15	13.8 ± 0.10
z_{tr}	$0.630^{+0.30}_{-0.20}$	$0.591^{+0.28}_{-0.20}$	$0.628^{+0.24}_{-0.18}$

$w = p/\rho$. In the context of late-time cosmic acceleration, this parameter can be expressed purely in terms of the Hubble expansion rate as

$$w(z) = -1 - \frac{1}{3} \frac{d \ln h^2}{dx},$$

where $x = \ln a$ is the logarithm of the scale factor and $h(z) = H(z)/H_0$ represents the dimensionless Hubble parameter. It is important to emphasize that the $w(z)$ reconstructed in this work is an *effective* EoS, encapsulating the combined influence of matter creation, modified gravity, and the variable generalized Chaplygin gas fluid, rather than describing a single physical component. This effective description allows us to interpret the global dynamical behavior of the Universe within a unified framework. Different ranges of $w(z)$ correspond to distinct physical regimes: values in the interval $-1 < w < 0$ are characteristic of quintessence-like dark energy, $w = -1$ precisely recovers the cosmological constant scenario of the standard Λ CDM model, and $w < -1$ signifies phantom energy with super-negative pressure. As illustrated in Fig. 7, both Model 1 and Model 2 exhibit an EoS evolution that remains firmly within the quintessence domain across all redshifts probed. At the present epoch ($z = 0$), the best-fit estimates yield $w_0 = -0.681^{+0.011}_{-0.014}$ for Model 1 and $w_0 = -0.666^{+0.028}_{-0.031}$ for Model 2. These values are marginally higher (i.e., less negative) than the effective $w_0 \approx -0.76$ inferred for the Λ CDM model under the same observational assumptions, yet they unambiguously confirm a phase of accelerated expansion driven by a quintessence-like effective fluid.

5.5 Snap parameter

The snap parameter, denoted $s(z)$, constitutes a fourth-order cosmographic quantity that provides valuable insight into the fine-grained dynamics of cosmic expansion, particularly when probing deviations from the standard Λ CDM paradigm—such as those arising in modified gravity frameworks like $f(Q)$ theory. Mathematically, it is defined as the

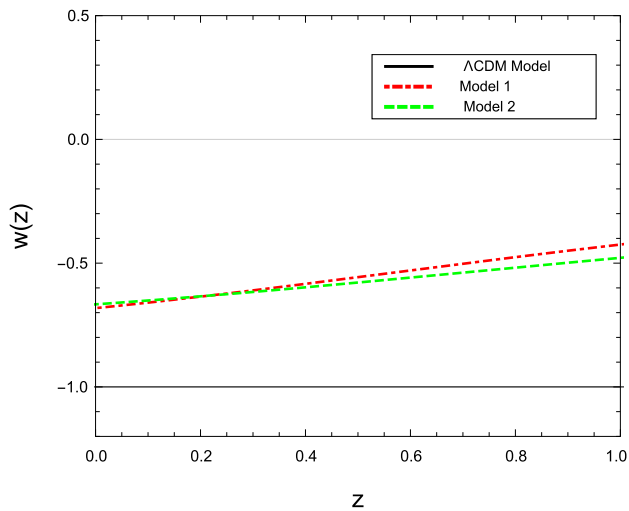


Fig. 7 The plot shows the evolution of the $w(z)$ for Λ CDM, Model 1 and Model 2 using values obtained from $CC + SNe Ia + BAO$ datasets. In the standard Λ CDM model, the EoS parameter for DE is a constant value -1 which means that $w(z) = -1$ at all redshifts, differing the other model's EoS where $w(z)$ evolves with redshift as shown in figure

dimensionless fourth derivative of the scale factor with respect to cosmic time:

$$s = \frac{\ddot{a}}{aH^4}.$$

When expressed in terms of the deceleration parameter $q(z)$ and the jerk parameter $j(z)$, the snap takes the form

$$s(z) = -(1+z)\frac{dj}{dz} - j(z)(2+3q(z)). \quad (35)$$

In the context of a spatially flat Λ CDM cosmology, where $j(z) \equiv 1$ and $q(z) = -1 + \frac{3}{2}(1+z)^3/E^2(z)$, this expression simplifies to $s(z) = -(2+3q)$, which evaluates identically to zero at all redshifts—a distinctive and invariant feature of the concordance model (Visser 2004; Dunajski and Gibbons 2008). This constancy makes the snap parameter a powerful discriminator between Λ CDM and alternative cosmologies.

Figure 8 displays the reconstructed redshift evolution of $s(z)$ for both the reference Λ CDM model (shown as a solid black line) and our two Chaplygin gas-based $f(Q)$ scenarios. As expected, the Λ CDM curve remains precisely at $s(z) = 0$ across the entire redshift range, underscoring its kinematic simplicity. In contrast, both Model 1 and Model 2 exhibit non-zero snap values at intermediate and high redshifts, reflecting the richer dynamical structure induced by matter creation and the variable generalized Chaplygin gas fluid. These deviations are modulated by the particle creation efficiency parameter β , which influences the effective equation of state and expansion history. Notably, all models asymptotically approach $s(z) \rightarrow 0$ as $z \rightarrow 0$, indicating convergence with Λ CDM in the recent Universe. At the

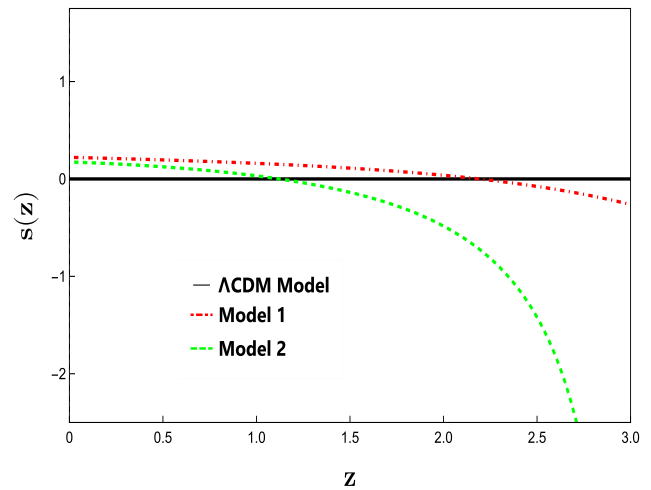


Fig. 8 The plot shows the evolution of the $s(z)$ for Λ CDM, Model 1 and Model 2 using values obtained from $CC + SNe Ia + BAO$ datasets. In flat Λ CDM model, the snap parameter is a constant value, $s(z) = 0$ for all redshifts which is shown as a horizontal bold line. Model 1 and Model 2 show the deviation from Λ CDM with redshifts

present epoch, the best-fit estimates yield $s_0 = +0.117^{+0.011}_{-0.019}$ for Model 1 and $s_0 = 0.157^{+0.001}_{-0.003}$ for Model 2, confirming a mild but statistically discernible departure from the standard cosmological prediction.

5.6 Om diagnostic

To rigorously compare the dynamical predictions of our Chaplygin gas-based $f(Q)$ models against the standard Λ CDM cosmology, we employ the $Om(z)$ diagnostic, a powerful model-independent tool introduced by Sahni and Starobinsky (2006) and further developed in subsequent works (Sahni et al. 2008; Xu et al. 2009; Shafieloo et al. 2012; Sahni and Shafieloo 2020). This diagnostic is particularly useful because it isolates the expansion history from degeneracies in the Hubble constant and clearly distinguishes between different dark energy behaviors using only the Hubble parameter $H(z)$. The $Om(z)$ function is mathematically defined as

$$Om(z) = \frac{H^2(z)/H_0^2 - 1}{(1+z)^3 - 1}. \quad (36)$$

In the Λ CDM model, $Om(z)$ remains exactly constant at $Om(z) = \Omega_{m,0}$ for all redshifts, producing a horizontal line. Deviations from this flat behavior signal departures from the concordance model. Specifically, a positive derivative is indicative of phantom-like dark energy ($w < -1$), whereas a negative slope corresponds to quintessence-like dynamics ($-1 < w < 0$). As shown in Fig. 9, both Model 1 and Model 2 exhibit a clear negative slope in their $Om(z)$ evolution across the redshift range probed, confirming that their effective dark energy component behaves similarly to

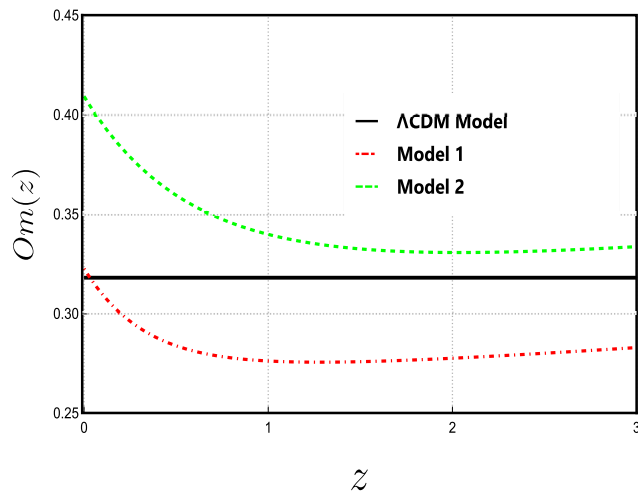


Fig. 9 The plot shows the evolution of the $Om(z)$ for Λ CDM, Model 1 and Model 2 using values obtained from $CC + SNe Ia + BAO$ datasets. For a flat Λ CDM model, $Om(z)$ is always equal to $\Omega_{m,0}$ across all redshifts and it is equal to ≈ 0.31 for the present Λ CDM model which is shown by bold black horizontal line

quintessence. Moreover, the visible deviation of these curves from the constant Λ CDM baseline underscores the distinct expansion history predicted by the Chaplygin gas scenarios, reinforcing their role as viable alternatives within the broader landscape of dark energy models.

The observed $Om(z)$ behavior provides crucial insights into the dynamical nature of dark energy in our models. The negative slope at low redshift ($z < 0.5$) clearly indicates quintessence-like behavior ($w > -1$), which is consistent with our earlier findings from the equation of state analysis. The transition from slightly positive to zero to negative slope at higher redshifts ($z > 0.5$) reveals a more complex dynamical behavior. This slope transition corresponds to a crossing of the $w = -1$ boundary, indicating a possible transition between quintessence and phantom regimes. In Model 1, the $Om(z)$ curve shows a clear transition from positive to negative slope around $z \approx 0.8$, suggesting a transition from phantom-like behavior at higher redshifts ($w < -1$) to quintessence-like behavior at lower redshifts ($w > -1$). Model 2 exhibits a similar but more gradual transition, with the slope changing from slightly positive to negative around $z \approx 1.2$. This dynamical behavior is a direct consequence of VGCG EoS, which allows for a smooth transition between different dark energy regimes. The transition points align with the redshift at which the deceleration parameter $q(z)$ shows the most significant changes in its evolution. This behavior is consistent with the theoretical expectations for VGCG models, where the scale-dependent parameter $A(a) = A_0 a^{-n}$ naturally produces such transitions in the equation of state. The $Om(z)$ diagnostic thus confirms that our models exhibit dynamical dark energy behavior rather than a simple cosmological constant, with the transition be-

tween regimes providing a natural explanation for the observed evolution of the Universe from matter domination to dark energy domination.

5.7 Age of the universe

The cosmic age $\tau(z)$, which quantifies the elapsed time since the Big Bang up to a given redshift z , is computed via the integral

$$\tau(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')}.$$

Using the best-fit parameter values obtained from our joint $CC + SNe Ia + BAO$ analysis, we evaluate this integral at the present epoch ($z = 0$) for the Λ CDM baseline as well as for Model 1 and Model 2, employing the respective Hubble functions derived in Eqs. (28) and (31). The resulting present-day age estimates are $\tau_0 = 13.6 \pm 0.11$ Gyr for Λ CDM, $\tau_0 = 13.4 \pm 0.15$ Gyr for Model 1, and $\tau_0 = 13.8 \pm 0.10$ Gyr for Model 2, as reported in Table 2. These values are in excellent agreement with each other and fall well within the range inferred from high-precision CMB observations. In particular, the Planck Collaboration 2018 (Aghanim et al. 2020) reports a cosmic age of $\tau_0 = 13.7 \pm 0.02$ Gyr under the standard Λ CDM assumption. The close concordance between our reconstructed ages and this benchmark value reinforces the physical plausibility of both extended models, confirming that they yield a cosmic timeline consistent with current observational cosmology. The alignment of these measurements has several implications for cosmic history: confirmation of the model with standard Λ CDM, timeline for structure formation and ongoing Hubble tension.

5.8 Statistical metrics

To rigorously assess the statistical viability of Model 1 and Model 2—both based on VGCG within $f(Q)$ gravity—we employ a suite of model selection tools and compare their performance against the standard Λ CDM cosmology, which serves as our reference framework. We begin with the reduced chi-square statistic, defined as

$$\chi_{\text{red}}^2 = \frac{\chi_{\text{min}}^2}{\text{dof}},$$

where χ_{min}^2 denotes the minimum chi-square value obtained from the joint likelihood analysis, and dof (degrees of freedom) is computed as $\text{dof} = N - K$, with N representing the total number of observational data points and K the number of free model parameters. This metric quantifies the goodness-of-fit per degree of freedom: values close to unity indicate an appropriate balance between model fidelity and complexity, whereas values significantly below or

Table 3 Statistical comparison of Λ CDM and the two Chaplygin gas models using DESI DR2 data. Values for $\ln E$ and $\ln B_i$ correspond to Bayesian evidence and logarithmic Bayes factors relative to Λ CDM, respectively

Parameters	Λ CDM	Model 1	Model 2
$\chi^2_{\text{tot, min}}$	1780.913	1821.838	1822.079
χ^2_{red}	1.020	1.049	1.050
AIC	1780.913	1841.838	1844.079
BIC	1818.272	1896.558	1904.271
ΔAIC	–	60.925	63.166
ΔBIC	–	78.286	85.999
$\ln E$	–1870.234568	–1821.837760	–1822.079016
$\ln B_i$	–	48.396808	48.155552
p-value	0.277	0.074	0.071

above 1 may signal overfitting or underfitting, respectively. In our analysis, the combined dataset comprises $N = 1758$ measurements—specifically, 1701 from the Pantheon+ supernova sample, 31 from CC, and 26 from BAO surveys. The baseline Λ CDM model has $K = 5$ free parameters, while Model 1 and Model 2 possess $K = 10$ and $K = 11$, respectively. As reported in Table 3, the resulting χ^2_{red} values for all three scenarios remain close to unity, with $\chi^2_{\text{red}} = 1.049$ for Model 1 and $\chi^2_{\text{red}} = 1.050$ for Model 2, confirming that each model provides a good fit to the observational data.

Beyond reduced chi-square, we adopt information-theoretic criteria rooted in statistical inference to account for model complexity. The two most widely used are the Akaike Information Criterion (AIC) Akaike (1974) and the Bayesian Information Criterion (BIC) Schwarz (1978). These metrics penalize models with a larger number of parameters to avoid overfitting, thereby enabling a fair comparison between models of differing complexity. For finite sample sizes, the AIC includes a correction term and is given by

$$AIC = \chi^2_{\text{min}} + 2K + \frac{2KN}{N - K - 1}. \quad (37)$$

However, in the limit of large datasets ($N \gg K$), which applies to our case, this correction becomes negligible, and the AIC simplifies to $AIC \approx \chi^2_{\text{min}} + 2K$. Meanwhile, the BIC is defined as

$$BIC = \chi^2_{\text{min}} + K \ln N, \quad (38)$$

which imposes a stronger penalty on additional parameters, especially for large N . Model comparison is performed by computing the differences $\Delta\text{AIC} = \text{AIC}_y - \text{AIC}_{\Lambda\text{CDM}}$ and $\Delta\text{BIC} = \text{BIC}_y - \text{BIC}_{\Lambda\text{CDM}}$, where y denotes either Model 1 or Model 2. Following the standard interpretation guidelines of Kass and Raftery (1995), for the Akaike Information Criterion, $|\Delta\text{AIC}| < 2$ implies substantial equivalence between

models; $2 < |\Delta\text{AIC}| < 6$ indicates positive but not decisive evidence against the higher-scoring model; $6 < |\Delta\text{AIC}| < 10$ suggests strong evidence against it; and $|\Delta\text{AIC}| > 10$ constitutes very strong evidence disfavoring the more complex model. As summarized in Table 3, Model 1 yields $\Delta\text{AIC} \approx 60.93$, and Model 2 gives $\Delta\text{AIC} \approx 63.17$, both indicating very strong evidence against the models according to AIC criteria. For the Bayesian Information Criterion—which imposes a stronger penalty on additional parameters, especially for large N the same thresholds apply, but the evidence is interpreted more stringently; both models exhibit $\Delta\text{BIC} > 75$, constituting decisive evidence against the models. Thus, while both extended models fit the data well, the principle of parsimony, as encoded in information criteria, ultimately favors the standard cosmological model.

5.9 p-value statistics

We employ the p -value which is used for statistical inference (Neyman and Pearson 1933; Wasserstein and Lazar 2016) to evaluate the consistency between our cosmological models and the observational data under the assumption that the model in question is correct (i.e., the null hypothesis). Mathematically, the p -value is computed as

$$p = 1 - F_{\chi^2_{\text{tot, min}}}(\chi | \nu),$$

where $F_{\chi^2_{\text{tot, min}}}(\chi | \nu)$ denotes the cumulative distribution function (CDF) of the chi-squared distribution with ν degrees of freedom, evaluated at the observed minimum chi-square statistic $\chi = \chi^2_{\text{tot, min}}$. This CDF quantifies the probability that a chi-squared random variable with ν degrees of freedom yields a value less than or equal to χ . Consequently, the complement $1 - F$ represents the probability of obtaining a fit as poor as-or poorer than-the one observed, assuming the model is true. In other words, the p -value measures how compatible the data are with the null hypothesis: smaller values signal greater tension. By conventional standards in astrophysics and cosmology, a p -value below 0.05 is deemed statistically significant, implying that such a discrepancy would arise by chance less than 5% of the time, thereby warranting rejection of the model. However, it is crucial to recognize that the p -value does not quantify the probability that the null hypothesis itself is true; rather, it reflects the likelihood of the data (or more extreme outcomes) given that the hypothesis holds (Wasserstein and Lazar 2016). In our analysis, the computed p -values for the Λ CDM model and both Chaplygin gas scenarios (Model 1 and Model 2) are $p \approx 0.277$, 0.074, and 0.071, respectively. This outcome implies that while the Λ CDM model is statistically preferred, both extended models remain within acceptable statistical bounds. This outcome implies that none of the models can be ruled out on the basis of goodness-of-fit

alone, and all three provide statistically acceptable descriptions of the combined CC + SNe Ia + BAO dataset. While the p -value does not adjudicate between competing viable models, it confirms that each framework is empirically plausible within the current observational precision.

5.10 Bayesian inference

In this subsection, we outline the core principles of Bayesian model comparison and apply them to assess the relative plausibility of our two Chaplygin gas-based scenarios against the standard Λ CDM cosmology. Central to this approach is the concept of Bayesian evidence, which evaluates how well a given model accounts for the observed data by integrating the likelihood function over the full prior parameter space (Sahni et al. 2003). This marginalization inherently penalizes unnecessary complexity, thereby implementing a natural form of Occam's razor.

To quantitatively compare Model 1 and Model 2 with the reference Λ CDM framework (denoted as model M), we employ the Bayes factor B_{iM} , defined as the ratio of their respective marginal likelihoods:

$$B_{iM} = \frac{p(d | M_i)}{p(d | M)},$$

where $p(d | M_i)$ represents the Bayesian evidence for model M_i ($i = 1, 2$), and $p(d | M)$ is that of the Λ CDM model. These evidences—reported in logarithmic form as $\ln E_i$ for the extended models and $\ln E$ for Λ CDM—are listed in Table 3. For interpretability, we work with the natural logarithm of the Bayes factor:

$$\ln(B_{iM}) = \ln p(d | M_i) - \ln p(d | M),$$

which directly measures the log-odds in favor of model M_i relative to the baseline.

Using the `PolyChord` nested sampling algorithm (Handley et al. 2015), we compute $\ln E$ numerically for each cosmological scenario. The resulting values of $\ln B_{iM}$ are compiled in Table 3. Following the widely adopted Jeffreys scale (Jeffreys 1961; Kass and Raftery 1995), we interpret these log-Bayes factors as follows:

- $\ln B_{iM} < 1$: Evidence is inconclusive;
- $1 \leq \ln B_{iM} < 2.5$: Weak support for M_i ;
- $2.5 \leq \ln B_{iM} < 5$: Moderate evidence in favor of M_i ;
- $\ln B_{iM} \geq 5$: Strong to decisive evidence for M_i over Λ CDM.

Our analysis yields $\ln B_1 = 48.40$ and $\ln B_2 = 48.16$, both of which fall far beyond the threshold for decisive evidence. This striking result suggests that, within the Bayesian framework, both Chaplygin gas models are overwhelmingly preferred over the standard Λ CDM paradigm when evaluated against the combined CC + SNe Ia + DESI DR2

BAO dataset—highlighting the significant explanatory power gained by incorporating matter creation and a variable generalized Chaplygin gas fluid in the context of $f(Q)$ gravity.

6 Conclusion

In this work, we have investigated a cosmological scenario that unifies gravitationally induced matter creation with a VGCG fluid within the framework of symmetric teleparallel $f(Q)$ gravity. By treating the cosmic fluid as a single perfect component governed by a scale factor dependent EoS, our approach offers a unified description of the dark sectors. We have specifically analyzed two functional forms for the gravitational Lagrangian: Model 1, defined by $f(Q) = n_1 Q^k$, and Model 2, given by $f(Q) = n_0 + n_1 Q^k$, where the constants n_0 , n_1 , and k are treated as free parameters. Using a comprehensive suite of late-time observational data—including the Pantheon+ SNe, CC, and BAO data from DESI Year 1 and SDSS-IV, we have performed a robust Bayesian parameter estimation via MCMC simulation. The resulting constraints have been further validated through multiple statistical diagnostics, including information criteria (AIC and BIC), p -values, cosmographic reconstructions, and Bayesian evidence ratios, enabling a rigorous comparison with the standard Λ CDM paradigm.

The matter creation mechanism employed here is motivated by quantum field theory in curved spacetime, wherein the expanding gravitational background acts as a source for particle production. Although the precise microphysical origin of the creation rate Γ remains elusive, we adopt a phenomenological ansatz $\Gamma = 3\beta H$, consistent with thermodynamic consistency and observational viability. Our MCMC analysis with DESI DR2 data yields best-fit values $\beta = 0.764 \pm 0.124$ for Model 1 and $\beta = 0.728 \pm 0.118$ for Model 2. A high value of $\beta \sim 0.7$ largely conflicts with the current precision cosmological observations as far as the growth of cosmic structures is concerned. Because, a high value of β significantly deviates from the successful Λ CDM model in the high redshift regime. In forthcoming paper, we will discuss the growth structure for these models.

The VGCG model naturally interpolates between a dust-like early Universe and a late-time accelerated phase driven by negative pressure. By assuming a power-law form $A(a) = A_0 a^{-n}$ for the scale-dependent parameter in the equation of state, we ensure a smooth transition from matter domination to dark-energy-like behavior. Our MCMC analysis with DESI DR2 data yields best-fit values for all model parameters, summarized in Table 1. Both extended models successfully reproduce the observed expansion history, with present-day Hubble constants of $H_0 = 67.199 \pm 0.128 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $67.193 \pm 0.122 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for Models 1 and 2, respectively—values in excellent agreement with Planck 2018 ($H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$).

and slightly below the Λ CDM fit from the same datasets. It should be noted that the VGCG model does not inherently predict a lower H_0 . The specific value of H_0 depends solely on observational data used for constraining the model's parameters. The value of H_0 may be found higher when local H_0 measurement from Riess et al. (2022) data would be used which may alleviate the H_0 tension. However, this is our future work. Actually, SNe Pantheon+, BAO DESI and CC is the natural data set which is also used to constraint the Λ CDM, thus it aligns with the early Universe.

The reconstructed cosmographic parameters (q_0, j_0, w_0, s_0) and cosmic age τ_0 , listed in Table 2 and visualized in Figs. 3–8, confirm a quintessence-like effective dark energy behavior and a deceleration-to-acceleration transition consistent with current cosmological understanding. These reconstructed quantities, across a range of redshifts offer a robust method to differentiate Chaplygin gas models from Λ CDM. It can be distinguished by plotting the evolutionary trajectories of q, j, w, s with redshift. In Figures, we can observe that the models mimic Λ CDM at late times, but their path at intermediate redshifts show deviation from Λ CDM model. The cosmographic parameters have constant values in Λ CDM, while the proposed models trace a evolutionary trajectories in intermediate redshifts regime.

The striking similarity in the best-fit values of H_0 and Ω_m between Model 1 and Model 2 reflects a fundamental physical consistency in our framework. This similarity arises because both models share the same core physical mechanism: the combination of gravitationally induced matter creation with the variable generalized Chaplygin gas equation of state. The minor differences that do exist (e.g., $H_0 = 67.199 \pm 0.128$ vs 67.193 ± 0.122) stem from the additional constant term n_0 in Model 2, which is constrained to be small ($n_0 = 0.840 \pm 0.126$) and primarily affects early-universe physics. This consistency demonstrates that the matter creation mechanism and VGCG equation of state dominate the late-time expansion dynamics, with the specific $f(Q)$ functional form playing a secondary role in determining these key cosmological parameters.

From a statistical point of view, both extended models show p -values above the conventional significance threshold ($p > 0.05$), indicating they provide statistically acceptable fits to the data. However, information criteria (AIC and BIC) strongly favor the simpler Λ CDM model due to the additional parameters in our extended frameworks. Intriguingly, Bayesian evidence analysis yields $\ln B_1 = 48.40$ and $\ln B_2 = 48.16$, indicating decisive evidence in favor of the Chaplygin gas models over Λ CDM on Jeffreys' scale. This tension between frequentist and Bayesian model selection criteria highlights the complexity of model evaluation in modern cosmology.

Looking ahead, this class of models offers fertile ground for theoretical and observational exploration. On the theoretical front, a deeper connection to quantum field theory in

FLRW spacetime could refine the physical interpretation of Γ . Observationally, upcoming data from Stage-IV surveys (e.g., DESI DR2, Euclid, and Rubin Observatory) will provide tighter constraints on higher-order cosmographic parameters and may help resolve the tension between Bayesian and frequentist model selection outcomes. Ultimately, such unified dark sector frameworks continue to serve as valuable laboratories for probing the fundamental nature of cosmic acceleration.

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Author contributions Yogesh Bhardwaj: Formal analysis, software, writing-original draft. C P Singh: Conceptualization, Methodology, Writing, review & editing.

Data availability No datasets were generated or analysed during the current study.

Declarations

Ethics declaration Not applicable

Competing interests The authors declare no competing interests.

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