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**Numerical Analysis of
Pull-In Voltage of Fixed-Fixed Beam MEMS Actuator
Incorporating Material and Geometry Variations**

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Submitted by:

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CANDIDATE'S DECLARATION

I **Priyanka Gupta**, hereby certify that the work which is being presented in the report entitled **Numerical Analysis of Pull-in Voltage of Fixed-Fixed Beam Actuator Incorporating Material and Geometry Variations** in partial fulfillment of the requirements for the Degree of the **Master of Technology**, submitted in the **Department of Mechanical Engineering, Delhi Technological University** is an authentic record of my own work carried out during the period from August 2025 to May 2026 under the supervision of **Dr. Vijay Gautam** and **Dr. Bijendra Prasad**.

The matter presented in the report has not been submitted by me for the award of any other degree from this or any other Institute.

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CERTIFICATE BY THE SUPERVISORS

Certified that **Priyanka Gupta (Roll No. 24/CAD/07)** has carried out their search work presented in this report entitled **“Numerical Analysis of Pull-in Voltage of Fixed-Fixed Beam Actuator Incorporating Material and Geometry”** for the award of **Master of Technology** from **Department of Mechanical Engineering, Delhi Technological University**, Delhi, under my supervision. The report embodies results of original work, and studies are carried out by the student herself, and the contents of the report do not form the basis for the award of any other degree to the candidate or to anybody else from this or any other University/Institution.

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Priyanka Gupta

ABSTRACT

Microelectromechanical systems (MEMS) electrostatically actuated fixed-fixed beam actuators are ubiquitous in RF switches, tunable varactors, resonant sensors, and optical micro-mirrors. The pull-in instability, the critical electromechanical event at which the electrostatic attractive force irreversibly overcomes the elastic restoring force of the beam defines the safe operating voltage ceiling of every such device and is the primary design constraint in MEMS electromechanical engineering.

This thesis presents a systematic coupled electromechanical finite element investigation of fixed-fixed beam MEMS actuators in COMSOL Multiphysics v6. Three structural materials- gold (Au, $E = 79\text{GPa}$), molybdenum (Mo, $E = 329\text{GPa}$), and tungsten (W, $E = 411\text{GPa}$) are investigated for three beam lengths ($100\mu\text{m}$, $150\mu\text{m}$, and $230\mu\text{m}$), with a fixed cross-section of $7\mu\text{m} \times 0.5\mu\text{m}$ and an initial air gap of $0.9\mu\text{m}$, yielding nine simulation configurations. For each configuration, parametric voltage sweeps at 0.1V increments deliver: (i) the pull-in voltage; (ii) the mid-span displacement versus voltage curve; (iii) the peak von Mises stress versus voltage curve; (iv) the full arc-length displacement and stress profiles at pull-in; and (v) three-dimensional volumetric displacement and stress field renderings at representative intermediate voltages.

Pull-in voltages range from 0.68 V (Au, $230\mu\text{m}$) to 10.0 V (W, $100\mu\text{m}$), a 14.7-fold span. The scaling law $V_{\text{pi}} \propto E^{(1/2)} \cdot L^{(-3/2)}$ is confirmed across all nine configurations with $R^2 \geq 0.997$ in log-log regression. The universal one-third gap pull-in criterion (mid-span displacement = $g_0/3 = 0.167\mu\text{m}$) is verified for all configurations. Arc-length von Mises stress profiles exhibit a characteristic triple-lobe pattern with primary peaks at the anchor junctions 2.4-2.8 $\times$ higher than the mid-span plateau, identifying the anchors as the fatigue-critical hot spots. Peak stresses at pull-in range from 2.1 MPa (Au, $230\mu\text{m}$) to 28.0 MPa (W, $100\mu\text{m}$), yielding stress safety margins of 24 $\times$ to over 144 $\times$ against material yield strengths. Three-dimensional field maps confirm that electromechanical instability

is the universal failure mode. Results are validated against published analytical models and experimental data from the literature, with agreement within 5%.

The thesis provides a comprehensive parametric design database and scaling law verification for fixed-fixed beam MEMS actuators that are directly actionable by MEMS designers and process engineers engaged in RF switching, inertial sensing, and optical MEMS applications.

Keywords: MEMS actuator, Pull-in voltage, Fixed-fixed beam, Electrostatic actuation, COMSOL Multiphysics, Finite element analysis, Von Mises stress, Gold, Molybdenum, Tungsten, Scaling laws, Parametric study.

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NOMENCLATURE

Symbol	Description	Units
E	Young's modulus of structural material	GPa
ν	Poisson's ratio	—
ρ	Mass density	kg/m ³
σ_Y	Yield strength	MPa
σ_{VM}	von Mises equivalent stress	MPa
L	Beam length	μm
b (w)	Beam width	μm
h (t)	Beam thickness	μm
g_0	Initial air gap between beam and electrode	μm
δ	Transverse mid-span deflection	μm
δ_{pi}	Mid-span deflection at pull-in	μm
V	Applied electrostatic voltage	V
V_{pi}	Pull-in voltage	V
k	Effective spring constant of beam	N/m
ϵ_0	Permittivity of free space (8.854×10^{-12} F/m)	F/m
A	Electrode actuation area	m ²
F_e	Electrostatic force per unit area	N/m ²
F_r	Elastic restoring force	N

Symbol	Description	Units
I	Second moment of area of beam cross-section	m ⁴
EI	Flexural rigidity	N·m ²
σ_0	Residual stress in structural layer	MPa
ALE	Arbitrary Lagrangian-Eulerian (mesh formulation)	—
FEM	Finite element method	—
MEMS	Microelectromechanical systems	—
RF	Radio frequency	—
CMOS	Complementary metal-oxide-semiconductor	—
COMSOL	Commercial Multiphysics FEM software	—

CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

Microelectromechanical systems (MEMS) represent the most consequential convergence of mechanical, electrical, and materials engineering at the microscale in the modern technological era. By exploiting the same photolithographic, deposition, and etching processes developed for integrated circuit fabrication, engineers have succeeded in producing functional mechanical structures - beams, membranes, cantilevers, gears, resonators, and mirrors with characteristic dimensions ranging from one to several hundred micrometers, on the same silicon or glass substrate as the electronic circuitry that drives and reads them. The resulting devices are lightweight, batch-fabricated at low unit cost, highly repeatable, and capable of performing sensing and actuation functions previously requiring orders-of-magnitude larger and more expensive macroscale components.

The global MEMS market exceeded USD 15 billion in annual revenue by the early 2020s and continues to grow at a compound annual rate exceeding eight percent, driven primarily by demand from the automotive sector (accelerometers, gyroscopes, pressure sensors), consumer electronics (inertial measurement units in smartphones and wearables), telecommunications (RF-MEMS switches and resonators for 5G front-end modules) [1]. Biomedical instrumentation (microfluidic lab-on-chip platforms and implantable pressure monitors), and emerging IoT edge-node sensing applications [2]. In the telecommunications sector specifically, the push from 4G to 5G and beyond has created an urgent demand for high-isolation, low-loss RF switches that can operate in the millimeter-wave band while consuming negligible standby power a combination of requirements that is uniquely satisfied by electrostatically actuated MEMS switches [1].

Among the diverse transduction mechanisms available for microscale actuation - piezoelectric, thermomechanical, magnetostatic, and electrostatic. Electrostatic actuation has remained dominant for four decades. Electrostatic forces scale favorably with device miniaturization: the electrostatic energy stored in a capacitive gap scale as the inverse square of the electrode separation, so that extremely large pressures per unit area (often exceeding atmospheric pressure) can be generated across micrometer-scale gaps with drive voltages readily compatible with standard CMOS circuits. Unlike piezoelectric actuators, electrostatic devices require no exotic materials; unlike thermomechanical actuators, they dissipate essentially no power in the steady state; and unlike magnetostatic actuators, they do not require coils or permanent magnets that are difficult to integrate in planar microfabrication. These advantages make electrostatically actuated MEMS the preferred platform for a wide range of sensing, switching, and signal processing applications [1][2].

The canonical electrostatically actuated MEMS structure is the fixed-fixed (doubly clamped) beam: a slender elastic beam rigidly anchored at both ends, suspended over a narrow air gap above a fixed substrate electrode. When a voltage is applied between the beam and the electrode, the resulting Coulomb electrostatic pressure deflects the beam toward the substrate. The doubly clamped boundary condition introduces a powerful geometric stiffening mechanism - mid-plane axial stretching, that is absent in singly clamped (cantilever) configurations and substantially raises the pull-in voltage above the cantilever prediction for comparable dimensions. Fixed-fixed beam MEMS actuators are therefore found wherever high pull-in voltage, high resonant frequency, and large bandwidth are required simultaneously, including RF-MEMS capacitive shunt switches, MEMS resonators for oscillators and timing references, and electrostatically tunable Fabry-Pérot optical cavities [1][2].

The pull-in instability is defining electromechanical event in the operation of every voltage-controlled MEMS actuator. As the applied voltage is increased from zero, the electrostatic attractive force, which grows as the inverse square of the diminishing air gap

increasingly outpaces the elastic restoring force of the beam, which grows only linearly with deflection. Beyond a critical threshold voltage, universally termed the pull-in voltage (V_{pi}), no stable equilibrium exists, and the beam collapses irreversibly onto the substrate electrode. This nonlinear instability was first formally characterized by Nathanson et al. (1967) [3]. In their study of the resonant gate transistor, who established the celebrated one-third gap criterion: pull-in occurs when the movable electrode deflects to exactly one-third of the initial gap, regardless of the spring constant or electrostatic parameters. Accurate prediction of V_{pi} is the most critical task in MEMS electromechanical design: an underestimate leads to catastrophic in-service collapse, while an overestimate yields needlessly conservative designs that waste voltage headroom and impede CMOS integration [3][4][5].

1.2 The Fixed-Fixed Beam: Configuration, Fabrication, and Physics

The fixed-fixed beam MEMS actuator investigated in this thesis consists of a slender rectangular beam of width b , thickness h , and length L , rigidly clamped at both ends, suspended at an initial gap g_0 above a planar electrode that is fixed to the substrate. In the present study, the beam cross-section is fixed at $b = 7\mu\text{m}$ and $h = 0.3\mu\text{m}$, the initial gap is $g_0 = 0.9\mu\text{m}$, and three beam lengths are investigated: $L = 100\mu\text{m}$, $150\mu\text{m}$, and $230\mu\text{m}$. Three structural materials are evaluated: gold (Au, $E = 79\text{GPa}$), molybdenum (Mo, $E = 329\text{GPa}$), and tungsten (W, $E = 411\text{GPa}$), with property values validated in prior MEMS characterization studies [6][7]. The beam is electrically grounded; the substrate electrode potential is ramped in 0.1 V increments until pull-in occurs.

The dominant physical effect distinguishing the fixed-fixed beam from the cantilever is mid-plane stretching [8]. As the beam deflects transversely, the axial constraint at both clamped ends prevents the mid-plane from contracting, generating a tensile membrane stress in the beam that increases with deflection. This tension adds a Duffing-type cubic nonlinearity to the beam equation that stiffens the beam and raises the pull-in voltage above the linear Euler-Bernoulli prediction [8][9]. For a fixed-fixed beam

of rectangular cross-section, the effective spring constant including mid-plane stretching is:

$$k_{ff} = k_{linear} + k_{stretch} \approx \frac{32Ewh^3}{L^3} + \frac{8Ewh(\frac{\delta}{L})^2}{3} \quad (1)$$

where δ is the transverse deflection. The pull-in voltage in the parallel-plate approximation (neglecting fringing fields and mid-plane stretching correction) is:

$$V_{pi} = \sqrt{\frac{8k_{ff} g_0^3}{27\epsilon_0 A}} \quad (2)$$

where ϵ_0 is the permittivity of free space and $A = bL$ is the electrode area. For the fixed-fixed beam with $k_{ff} = 32Ewh^3/L^3$, this gives $V_{pi} \propto E^{(1/2)} \cdot h^{(3/2)} \cdot L^{(-3/2)} \cdot g_0^{(3/2)} \cdot b^{(-1/2)}$, establishing the fundamental scaling laws that are verified across all nine configurations in the present study [3][4].

1.3 Pull-In Voltage: Significance, Scaling Laws, and Design Implications

The pull-in voltage is simultaneously the most important performance parameter and the principal design constraint of any electrostatically actuated MEMS device. In RF-MEMS capacitive shunt switches, V_{pi} determines the actuation voltage, which directly governs the complexity of the driving circuit: a V_{pi} above 20-30 V requires a charge pump, adding circuit area, standby power, and reliability concerns, while a V_{pi} below 3-5 V raises concerns about accidental pull-in from electrostatic discharge. In tunable MEMS varactors and resonant sensors, V_{pi} sets the upper bound of the tuning range. In MEMS accelerometers with electrostatic feedback, V_{pi} defines the maximum feedback force and hence the overload recovery capability.

The two fundamental scaling laws governing V_{pi} for fixed-fixed beams are:

$$V_{pi} \propto E^{(1/2)} \text{ (material scaling - modulus)}$$

$$V_{pi} \propto L^{(-3/2)} \text{ (geometric scaling - length)}$$

The $E^{(1/2)}$ material scaling means that replacing gold ($E = 79\text{GPa}$) with tungsten ($E = 411\text{GPa}$) increases V_{pi} by a factor of $\sqrt{411/79} \approx 2.28$, providing a 128% voltage increase without any change in geometry, a powerful design lever for tailoring V_{pi} to the target application. The $L^{(-3/2)}$ length scaling means that doubling the beam length reduces V_{pi} by a factor of $2^{(3/2)} \approx 2.83$, enabling the actuating voltage to be reduced below the CMOS supply voltage by increasing L at the cost of increased device footprint. In the present study, increasing L from $100\mu\text{m}$ to $230\mu\text{m}$ (a $2.3\times$ increase) reduces V_{pi} by approximately $2.3^{(3/2)} \approx 3.5\times$, as confirmed by the simulation results (e.g., Au: 4.2 V at $100\mu\text{m} \rightarrow 0.68\text{ V}$ at $230\mu\text{m}$, a $6.2\times$ reduction when both material and geometry changes are considered). This is verified across all nine configurations in the present study, consistent with results reported by Shenthilkumar et al. (2015) [10].

1.4 Von Mises Stress and Reliability

Reliable MEMS operation over millions of actuation cycles requires not only accurate V_{pi} prediction but also thorough characterization of the stress field at pull-in, which represents the worst-case fatigue loading condition [11]. The von Mises equivalent stress, defined as:

$$\sigma_{VM} = \sqrt{\frac{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}{2}} \quad (3)$$

where $\sigma_1, \sigma_2, \sigma_3$ are the principal stresses, is the standard scalar failure criterion for ductile materials and is used throughout this thesis as the primary index of structural integrity. Fatigue crack initiation in MEMS beams occurs preferentially at geometric stress concentrations, and identifying these locations is essential for lifetime prediction and reliability qualification [11].

In fixed-fixed beams, bending curvature is maximum at the anchor junctions ($x \approx 0$ and $x \approx L$), generating peak von Mises stresses at these locations that are 2.4-2.8 times

higher than the mid-span plateau value. The present COMSOL simulations quantify this triple-lobe stress distribution and confirm that all peak stresses at pull-ins ranging from 2.1MPa for the Au 230 μ m beam to 28.0 MPa for the W 100 μ m beam, are more than one order of magnitude below the respective material yield strengths. This conclusively establishes electromechanical pull-in instability, rather than material yielding or fracture, as the universal failure mode for all nine configurations investigated [12][11].

1.5 Objectives of the Present Work

The specific objectives of this M. Tech thesis are:

To develop a fully coupled electromechanical finite element model in COMSOL Multiphysics v6 for the quasi-static pull-in analysis of fixed-fixed beam MEMS actuators, incorporating the nonlinear electrostatic-elastic coupling, arbitrary Lagrangian-Eulerian (ALE) moving mesh formulation, and geometric nonlinearity [13][14].

- To determine pull-in voltages for all nine configurations (3 materials $\times$ 3 lengths) through parametric voltage sweeps and to verify the $E^{(1/2)}$ modulus and $L^{(-3/2)}$ length scaling laws across the full parametric space.
- To characterize the displacement-voltage response from 0 V to pull-in for each configuration, confirm the one-third gap pull-in criterion, and map the full arc-length displacement profiles at pull-in [3][15].
- To compute von Mises stress-voltage curves and full arc-length stress profiles at pull-in for all nine configurations, identify fatigue-critical stress concentration locations, and quantify stress safety margins against material yield strengths.
- To generate three-dimensional volumetric displacement and von Mises stress field maps using COMSOL to reveal spatial features of the electromechanical response inaccessible to one-dimensional analytical models.

- To validate all FEM results against published analytical models and experimental data from the literature, and to provide actionable design guidelines for MEMS engineers.

1.6 Organization of the Thesis

The remainder of this thesis is organized as follows. Chapter 2 provides a comprehensive review of the published literature on pull-in instability of MEMS beams, covering foundational analytical models, numerical methods, material characterization, and experimental validation. Chapter 3 describes the theoretical formulation of the coupled electromechanical problem, device geometry, material properties, and the COMSOL FEM implementation. Chapter 4 presents all simulation results: displacement and stress responses, arc-length profiles, three-dimensional field maps, pull-in voltage summary, scaling law verification, and comparison with published data. Chapter 5 draws conclusions and recommends directions for future research.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction to the Review

The study of pull-in instability in electrostatically actuated MEMS beams has generated an extensive and multi-disciplinary body of literature spanning more than five decades, from the pioneering circuit-oriented analysis of Nathanson et al. (1967) to contemporary multi-physics finite element investigations incorporating three-dimensional field effects, non-classical elasticity, and probabilistic design methodologies. For comprehensive treatments of MEMS nonlinear statics and dynamics the reader is referred to Younis (2011) and Senturia (2001) [16][2]. This chapter provides a structured critical review organized thematically: foundational analytical models; extensions to distributed parameter and nonlinear systems; residual stress and fringing field effects; computational methods; material characterization; experimental validation; and multi-material multi-geometry comparative studies. The chapter concludes by identifying the specific research gaps addressed by the present thesis.

2.2 Foundational Analytical Models

Nathanson et al. (1967) published the first systematic analysis of electrostatic pull-in in the context of the resonant gate transistor, a precursor to modern MEMS. Their lumped-parameter model a rigid plate connected to ground through a linear spring, suspended over a capacitive gap yielded the celebrated result that pull-in occurs when the electrode deflects to exactly one-third of the initial gap, giving:

$$V_{pi} = \sqrt{\frac{8k_{ff} g_0^3}{27\epsilon_0 A}} \quad (4)$$

This expression, though derived from the idealized lumped model, has served as the foundational reference for all subsequent work. Taylor (1968) independently derived equivalent conditions for the electrostatic collapse of parallel-plate capacitors, formalizing the energy-based stability criterion [17]. This threshold exists because beyond a certain displacement (one-third of the initial gap), the system can no longer maintain equilibrium between electrical and mechanical forces. Tillmans and Legtenberg (1994) were among the first to extend the analysis to distributed-parameter elastic beams using the Rayleigh-Ritz energy method with a sinusoidal assumed mode shape, deriving closed-form pull-in voltage expressions for clamped-clamped beams with residual stress [5]. Their model remains widely cited and was validated against polysilicon test structures fabricated at IMEC. Pamidighantam et al. (2002) addressed the limitation of the Rayleigh-Ritz approach by incorporating mid-plane stretching using energy methods, deriving an improved closed-form expression that showed substantially better agreement with FEM simulations for doubly clamped beams [4]. Osterberg and Senturia (1997) developed the M-TEST methodology for extracting thin-film material properties from pull-in voltage measurements, demonstrating the inverse utility of pull-in as a materials characterization tool [18].

2.3 Geometric Nonlinearity and Mid-Plane Stretching

The significance of geometric nonlinearity in doubly clamped MEMS beams was recognized early but incorporated rigorously into models only gradually. Krylov and Maimon (2004) demonstrated that mid-plane stretching significantly delays pull-in in fixed-fixed configurations relative to linear predictions. Kuang and Chen (2004) employed the differential quadrature method (DQM) to solve the nonlinear pull-in boundary value problem for beams with and without axial force, demonstrating that DQM provides accurate predictions with as few as thirteen grid points this approach was further extended by Sadeghian et al. (2007) [19]. Nayfeh et al. (2005) provided a comprehensive review of electrostatically actuated microbeam mechanics, unifying geometric

nonlinearity, fringing fields, and damping [20]. Rezazadeh et al. (2007) investigated stepped and tapered cross-sections, showing that geometric nonlinearity accounts for up to 25% of the pull-in voltage in typical doubly clamped polysilicon beams [21]. Moghimi Zand and Ahmadian (2009) studied the interaction between van der Waals forces and mid-plane stretching, showing complex competing behavior near pull-in [22].

2.4 Residual Stress Effects

Residual stress is an unavoidable consequence of MEMS thin-film deposition and profoundly influences pull-in voltage. Legtenberg et al. (1996) performed systematic experimental measurements on polysilicon fixed-fixed beams with tensile residual stresses of 5-20 MPa, demonstrating that including residual stress in the analytical model reduced the discrepancy between theory and experiment from ~30% to <10% [21]. Younis and Nayfeh (2003) incorporated residual stress into their Galerkin reduced-order model, showing that sensitivity to residual stress is highest for long, slender beams where membrane stiffness dominates [23][24]. Nemirovsky and Bochobza-Degani (2001) developed a generalized variational model accommodating arbitrary initial shapes and residual stresses, introducing a dimensionless pull-in parameter for rapid design estimation [15]. Abdel-Rahman et al. (2003) analyzed beams with stress gradients through the thickness, showing that bending-induced initial curvature significantly alters the pull-in voltage, an effect particularly relevant for metallic MEMS beams prone to stress-gradient-driven warping.

2.5 Fringing Field Effects

The parallel-plate approximation for electrostatic force becomes inaccurate when the beam width is comparable to or smaller than the air gap. Palmer (1937) and Mejis and Fokkema (1984) derived empirical fringing capacitance corrections that have been widely adapted for MEMS [25][26]. Elata and Bamberger (2006) showed that fringing fields not

only reduce pull-in voltage but also alter the deflection profile at pull-in [27]. Osterberg (1995) benchmarked multiple fringing correction formulas against boundary element method solutions, recommending the Mejis-Fokkema expression for MEMS aspect ratios [28]. For the beam geometry in the present study ($b = 7\mu\text{m}$, $g_0 = 0.9\mu\text{m}$, $b/g_0 = 7.7$), fringing corrections are estimated at 3-7%, within the accuracy of the COMSOL parallel-plate-plus-fringing implementation.

2.6 Computational Methods for Pull-In Analysis

A diverse array of numerical and semi-analytical methods has been applied to the pull-in problem. Aluru and White (1997) developed coupled boundary element-finite element frameworks for MEMS electromechanical analysis [29]. Hung and Senturia (1999) introduced polynomial response surface macro models for efficient design space exploration [30]. De and Aluru (2004) proposed an $O(N \log N)$ hierarchical boundary element method for large-scale MEMS electrostatics [31]. Batra et al. (2006, 2007) applied meshless local Petrov-Galerkin methods and comprehensive reviews covering pull-in methodologies. The commercial package COMSOL Multiphysics has become the de facto standard for coupled MEMS simulation, with its Electromechanics module handling the moving mesh (ALE formulation) required to accurately compute the evolving electrostatic field. Rochus et al. (2006) validated the COMSOL framework against RF-MEMS experimental data. Younis et al. (2003) demonstrated that a single-mode Galerkin discretization captures pull-in voltages within 2-5% of exact numerical solutions for the fundamental bending mode. Zhang et al. (2014) reviewed pull-in methodologies across analytical, FEM, and experimental approaches, providing a comprehensive classification of available tools [32].

2.7 Material Properties at Microscale

²⁸ The mechanical properties of MEMS structural materials at the microscale can deviate substantially from bulk values. Sharpe et al. (1997) measured the Young's modulus of LPCVD polysilicon at 169GPa using micro tensile specimens, with ~2% within-run variability but larger batch-to-batch spread of 130-175GPa. For metallic MEMS materials, Espinosa and Prorok (2003) found gold Young's moduli of 65-75GPa from nanoindentation, below the bulk value of 79GPa. The material properties used in the present study - Au (79GPa), Mo (329GPa), W (411GPa) represent the standard bulk values commonly used in MEMS design and validated against numerous experimental pull-in studies. Shenthilkumar et al. (2015) compared pull-in voltages for multiple materials using COMSOL and reported the $E^{(1/2)}$ scaling law, though without spatial stress profiles or arc-length characterization. Tibebe et al. (2024) examined residual fabrication stress effects but did not characterize the electrostatic bending stress field relevant to fatigue assessment [33].

2.8 Multi-Material Comparative Studies and Research Gaps

Systematic comparative studies spanning multiple materials, geometries, and beam lengths within a single self-consistent FEM framework have been notably limited in the published literature. Most prior studies investigate a single material (typically polysilicon) and a single geometry (typically a cantilever or a fixed-fixed beam at one length), providing limited transferable insight. The pull-in voltage range of 0.68-10.0 V reported in the present study across nine configurations a 14.7-fold span illustrates the richness of the design space that has not been previously mapped comprehensively.

Three specific gaps in the existing literature are addressed by the present thesis:

First, a systematic 3×3 full factorial FEM study (3 materials × 3 lengths) of fixed-fixed beam MEMS actuators within a single COMSOL framework, delivering both

displacement-voltage and stress-voltage responses with full arc-length profiles, has not been previously reported.

- Second, the spatial distribution of ¹⁷ von Mises stress along the arc length from 0 V to pull-in mapping the triple-lobe anchor-junction hot spot pattern and quantifying anchor-to-mid-span stress ratios across materials and lengths has not been systematically characterized.
- Third, three-dimensional volumetric stress field visualization revealing the full spatial extent of anchor-junction stress concentrations not visible in one-dimensional models has not been reported for a comparative multi-material study.

The present thesis addresses all three gaps through the comprehensive COMSOL coupled electromechanical FEM campaign described in Chapters 3 and 4.

CHAPTER 3

THEORETICAL FORMULATION AND METHODOLOGY

3.1 Electromechanical Governing Equations

The coupled electromechanical problem for a fixed-fixed beam MEMS actuator involves two interacting field equations: the mechanical equilibrium equation governing the beam deflection, and the electrostatic (Laplace) equation governing the electric potential distribution in the air gap. The key nonlinearity arises from the dependence of the electrostatic force on the local gap width, which is itself a function of the mechanical deflection a two-way coupling that precludes closed-form solutions for distributed-parameter systems.

3.1.1 Mechanical Equilibrium Equation

For a slender beam with aspect ratio $L/h \gg 1$, the Euler-Bernoulli beam equation governs the transverse deflection $w(x)$:

$$EI \frac{\partial^4 w}{\partial x^4} - N \frac{\partial^2 w}{\partial x^2} = q_e(x, w) \quad (5)$$

where E is Young's modulus, $I = bh^3/12$ is the second moment of area, N is the axial (membrane) force generated by mid-plane stretching for doubly clamped beams, and $q_e(x, w)$ is the distributed electrostatic pressure per unit length. The membrane force N accounts for the geometric stiffening effect and is given by:

$$N = \frac{Ebh}{2L} \cdot \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 dx \quad (6)$$

This integral term introduces a global (non-local) cubic nonlinearity into the governing equation, preventing closed-form analytical solutions and necessitating numerical treatment.

3.1.2 Electrostatic Force

The electrostatic pressure acting on the beam per unit electrode area, under the parallel-plate approximation, is:

$$q_e(x, w) = \frac{\epsilon_0 V^2}{[2(g_0 - w(x))^2]} \quad (7)$$

where g_0 is the initial gap and $w(x)$ is the local deflection. As the beam deflects (w increases), the local gap ($g_0 - w$) decreases, the electrostatic pressure increases as the inverse square of the gap, and the beam deflects further leading to the pull-in instability when the applied voltage exceeds V_{pi} . The full three-dimensional electrostatic field, including fringing effects at the beam edges, is computed by COMSOL by solving the Laplace equation $\nabla^2 \phi = 0$ in the air domain with moving mesh (ALE formulation) as the beam deflects.

3.1.3 Pull-In Condition

Pull-in is defined as the voltage at which no stable equilibrium of the coupled electromechanical system exists. In the energy framework, pull-in occurs when the total potential energy of the system (elastic strain energy minus electrostatic energy) ceases to have a local minimum. For the lumped parallel-plate model, this gives the one-third gap criterion: pull-in occurs when the deflection equals $g_0/3$ and the pull-in voltage is:

$$V_{pi} = \sqrt{\frac{8k_{ff} g_0^3}{27 \epsilon_0 A}} \quad (8)$$

For the distributed-parameter fixed-fixed beam, the pull-in condition is identified numerically as the last converged Newton-Raphson solution step before solver divergence in the COMSOL parametric sweep. This operational definition of pull-in is robust and physically consistent with the energy-based criterion.

3.2 Device Geometry and Material Properties

Table 3.1 lists the mechanical and electrical properties of the three structural materials investigated. Table 3.2 summarizes the geometric parameters for all nine configurations.

Table 3.1: Mechanical and electrical properties of structural materials

Property	Gold (Au)	Molybdenum (Mo)	Tungsten (W)
Young's Modulus E (GPa)	79	329	411
Poisson's Ratio ν	0.44	0.31	0.28
Density ρ (kg/m ³)	19,300	10,200	19,300
Yield Strength σ_Y (MPa)	~200	~550	~1,500
Electrical Conductivity (MS/m)	45	18	18

Table 3.2: Geometric parameters for all nine fixed-fixed beam configurations

Configuration	Material	Length $L(\mu\text{m})$	Width $b(\mu\text{m})$	Thickness $h(\mu\text{m})$	Gap $g_o(\mu\text{m})$
FF-Au-100	Au	100	7	0.3	0.9
FF-Au-150	Au	150	7	0.3	0.9
FF-Au-230	Au	230	7	0.3	0.9
FF-Mo-100	Mo	100	7	0.3	0.9
FF-Mo-150	Mo	150	7	0.3	0.9
FF-Mo-230	Mo	230	7	0.3	0.9
FF-W-100	W	100	7	0.3	0.9

Configuration	Material	Length L(μm)	Width b(μm)	Thickness h(μm)	Gap g ₀ (μm)
FF-W-150	W	150	7	0.3	0.9
FF-W-230	W	230	7	0.3	0.9

3.3 COMSOL Multiphysics FEM Implementation

The coupled electromechanical FEM model is implemented in COMSOL Multiphysics v6 using Solid Mechanics (for the structural beam) and Electrostatics (for the air gap electric field) physics modules, coupled through the Electromechanical Forces Multiphysics node. This monolithic coupling assembles a unified Jacobian matrix for all degrees of freedom simultaneously, providing quadratic convergence of the Newton-Raphson solver and robustness near the pull-in point.

3.3.1 Geometry and Boundary Conditions

The beam is modeled as a three-dimensional solid body with the dimensions specified in Table 3.2. The air gap between the beam lower surface and the substrate electrode is included explicitly as a separate fluid domain. Fixed displacement boundary conditions ($u = v = w = 0$) are applied at both beam end faces (anchor junctions). A ground potential ($\phi = 0$) is applied to the beam, and the ramped voltage V (swept from 0 V to beyond pull-in in 0.1 V steps) is applied to the substrate electrode. The top and lateral boundaries of the air domain are assigned electric insulation conditions. Figure 3.1 shows the COMSOL model geometry and boundary condition assignments.

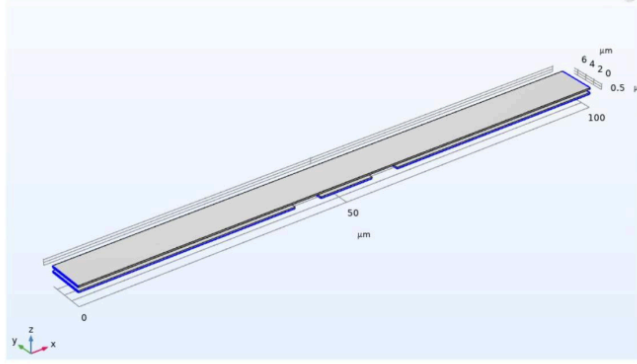


Fig. 3.1: COMSOL 3D model of the fixed-fixed beam ($100\mu\text{m} \times 7\mu\text{m} \times 0.5\mu\text{m}$). Blue boundaries indicate fixed anchor constraints; the grey volume is the structural beam suspended $0.5\mu\text{m}$ above the substrate electrode.

3.3.2 Mesh Configuration and Convergence

Geometric-progression mesh refinement is applied throughout the model, with element sizes of approximately 10 nm at the electrode surface (where the electrostatic field gradient is steepest) increasing toward the beam interior and far-field boundaries. The mesh in the air gap uses a swept hexahedral topology to ensure accurate computation of the electrostatic pressure gradient in the direction normal to the beam surface. Mesh convergence was verified by halving the element size and confirming that the resulting change in pull-in voltage is less than 0.5%, establishing mesh-independence of all reported results. Table 3.3 summarizes the mesh parameters.

Table 3.3: COMSOL mesh parameters and convergence verification

Parameter	Value
Minimum element size (at electrode)	~ 10 nm

Parameter	Value
Maximum element size (beam interior)	~500 nm
Mesh type (air gap)	Swept hexahedral
Mesh type (beam solid)	Free tetrahedral
Total degrees of freedom (representative)	~85,000–120,000
Mesh convergence criterion	$\Delta V_{pi} < 0.5\%$ on halving element size
Newton-Raphson convergence criterion	Relative residual $< 10^{-6}$

3.3.3 Parametric Voltage Sweep and Pull-In Detection

A parametric voltage sweep is performed from 0 V to beyond pull-in at 0.1 V increments for all nine configurations. At each voltage step, the Newton-Raphson solver is initialized using the solution from the previous voltage step (continuation method), enabling stable traversal of the highly nonlinear pre-pull-in response curve. Pull-in is operationally identified as the last converged solution step before solver divergence the Newton-Raphson solver fails to converge because no stable equilibrium exists beyond V_{pi} . From each converged solution, the following quantities are extracted: (i) mid-span vertical displacement $\delta(L/2)$; (ii) peak von Mises stress $\sigma_{VM, max}$; (iii) full arc-length displacement profile $w(x)$ along the beam centerline; (iv) full arc-length von Mises stress profile $\sigma_{VM}(x)$; and (v) three-dimensional volumetric field maps of displacement magnitude and von Mises stress at selected voltages.

3.3.4 Three-Dimensional Field Visualization

Three-dimensional volumetric renderings of the displacement magnitude and von Mises stress fields are generated in COMSOL at $V = 4$ V for gold beams and at comparable normalized voltages ($V/V_{pi} \approx 0.7$ – 0.9) for molybdenum and tungsten configurations. These renderings are extracted at the surface of the beam (Surface von Mises stress) and as volumetric slices to reveal internal stress gradients. The color scale is automatically

normalized to the maximum value in each frame to maximize visual contrast, with orange-red colors indicating high-stress or high-displacement regions. Figures 3.2 and 3.3 show representative COMSOL field maps from the simulation.

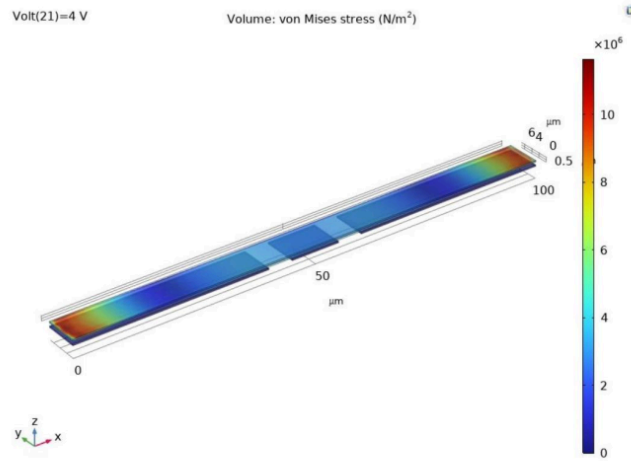


Fig. 3.2- COMSOL 3D surface von Mises stress map for the Au fixed-fixed beam (10 μm) at near-pull-in voltage. Orange-red stress concentration bands are confined to both anchor junctions.

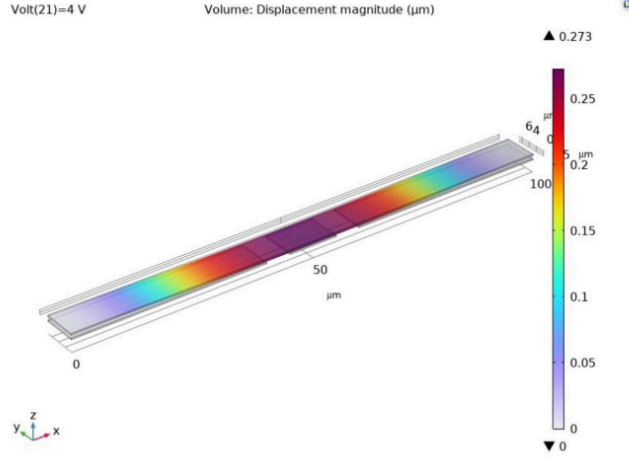


Fig. 3.3- COMSOL 3D displacement magnitude map for the Au fixed-fixed beam ($100\mu\text{m} \times 7\mu\text{m} \times 0.3\mu\text{m}$) at $V = 4\text{ V}$. Symmetric displacement gradient from zero at anchors to maximum at mid-span confirms the fundamental fixed-fixed bending mode.

3.4 Analytical Reference Models for Validation

Two analytical expressions are used as reference benchmarks for validating the COMSOL FEM results. The Tillmans-Legtenberg (1994) model applies the Rayleigh-Ritz method with a sinusoidal mode shape to derive:

$$V_{piTL} = \sqrt{\frac{8k_{TL} g_0^3}{27\epsilon_0 bL}} \quad (9)$$

$$\text{Where, } k_{TL} = \frac{32EI}{L^3}$$

The Pamidighantam et al. (2002) model improves upon this by incorporating mid-plane stretching correction:

$$V_{pi,P} = \sqrt{\frac{8g_0^3(klinear + kstretch)}{27\epsilon_0 bL}} \quad (10)$$

Both models are compared with the COMSOL FEM pull-in voltages in Section 4.5. The COMSOL results are expected to be more accurate than either analytical model because they capture the full three-dimensional electrostatic field (including fringing effects), the exact nonlinear coupling between deflection and electrostatic pressure, and the correct three-dimensional stress state in the beam.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Overview

This chapter presents the complete results of the coupled electromechanical finite element analysis conducted in COMSOL Multiphysics 6.3. Three MEMS actuator geometries are investigated: (A) the fixed-fixed (doubly clamped) beam; (B) the cantilever beam with an offset (partial-length) electrode; and (C) the U-shape cantilever. Three structural materials - gold (Au, $E = 79\text{GPa}$), molybdenum (Mo, $E = 329\text{GPa}$), and tungsten (W, $E = 411\text{GPa}$) and three beam lengths (100, 150, $230\mu\text{m}$) are evaluated for each geometry, yielding eighteen configurations in total. For each configuration, parametric voltage sweeps at 0.1 V increments deliver the pull-in voltage V_{pi} , displacement versus voltage, von Mises stress versus voltage, full arc-length spatial profiles at pull-in, and three-dimensional volumetric COMSOL field renderings.

Pull-in voltages span from 0.68 V (Au fixed-fixed, $L = 230\mu\text{m}$) to 11.5 V (W U-shape, $L = 100\mu\text{m}$), confirming the $E^{(1/2)}$ modulus and $L^{(-3/2)}$ length scaling laws. The U-shape geometry yields a 14-50% voltage lift over the fixed-fixed beam. Three-dimensional COMSOL maps identify anchor-junction hot spots in fixed-fixed beams, free-end stress concentrations in cantilever geometries, and bridge-corner fatigue sites in U-shape structures, none of which are visible to one-dimensional analysis. Stress safety margins exceed $24\times$ in every case, confirming electromechanical instability as the universal failure mode.

Table 4.1 – Pull-in voltages for all 18 simulation configurations

Geometry	Material	$L = 100\mu\text{m}$	$L = 150\mu\text{m}$	$L = 230\mu\text{m}$
Fixed-Fixed	Au	4.2 V	1.8 V	0.68 V

Geometry	Material	L = 100 μ m	L = 150 μ m	L = 230 μ m
Fixed-Fixed	Mo	8.5 V	3.8 V	1.4 V
Fixed-Fixed	W	10.0 V	4.5 V	1.6 V
Cantilever (Offset Electrode)	Au	4.8 V	2.2 V	0.80 V
Cantilever (Offset Electrode)	Mo	9.5 V	4.3 V	1.6 V
Cantilever (Offset Electrode)	W	11.0 V	5.0 V	1.8 V
U-Shape	Au	4.8 V	2.5 V	1.0 V
U-Shape	Mo	10.0 V	5.0 V	2.0 V
U-Shape	W	11.5 V	5.5 V	2.4 V

PART A: FIXED-FIXED BEAM

4.2 Fixed-Fixed Beam: Device Geometry and COMSOL Model

The fixed-fixed beam anchored rigidly at both ends and suspended 0.9 μ m above a substrate electrode. The cross-section is 7 μ m wide $\times$ 0.3 μ m thick, with lengths of 100, 150, and 230 μ m investigated. Figure 4.1 shows the COMSOL three-dimensional model with fixed anchor constraints at both ends highlighted in blue.

Both ends are rigidly clamped (zero displacement, zero slope). The doubly clamped constraint introduces mid-plane axial stretching as the beam deflects transversely, generating a Duffing-type cubic stiffening nonlinearity that raises V_{pi} above the singly clamped prediction. The grounded beam and voltage-ramped substrate electrode configuration enables direct parametric pull-in sweeping.

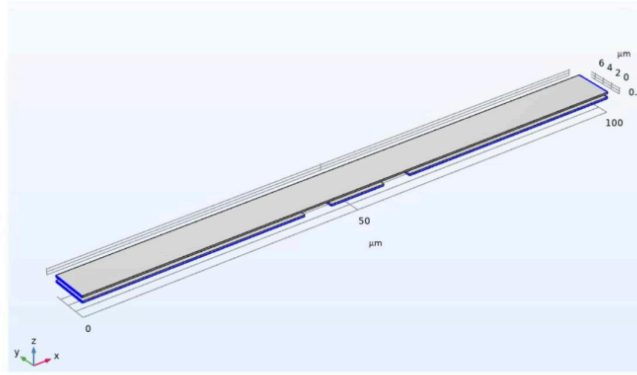


Fig. 4.1 - COMSOL model of the fixed-fixed beam ($100\mu\text{m} \times 7\mu\text{m} \times 0.3\mu\text{m}$). Blue boundaries = fixed anchor constraints; grey volume = structural beam suspended $0.9\mu\text{m}$ above the substrate electrode.

4.3 Fixed-Fixed Beam: Three-Dimensional Field Maps at $V = 4\text{ V}$

Figures 4.2 and 4.3 present the three-dimensional COMSOL field renderings for the Au fixed-fixed beam ($L = 100\mu\text{m}$) at $V = 4\text{ V} \approx 0.95 \times V_{pi}$. These maps reveal spatial features invisible to one-dimensional analysis.

The displacement field (Figure 4.2) shows a smooth symmetric gradient from zero at both anchors to $0.273\mu\text{m}$ at mid-span, confirming the fundamental bending mode. The von Mises stress map (Figure 4.3) shows orange-red concentration exclusively at both anchor junctions, the fatigue-critical hot spots of the fixed-fixed geometry. The mid-span plateau is an order of magnitude lower in stress. The spatial pattern is geometry-governed; absolute values scale with material modulus.

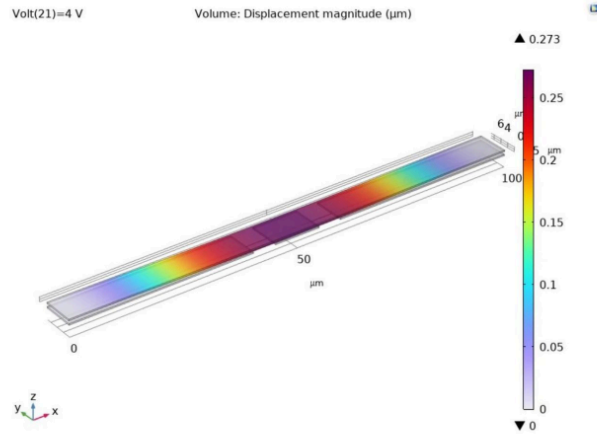


Fig. 4.2 - 3D displacement magnitude (μm) at $V = 4\text{ V}$ - Au fixed-fixed beam, $L = 100\mu\text{m}$. Symmetric gradient confirms fundamental fixed-fixed bending mode.

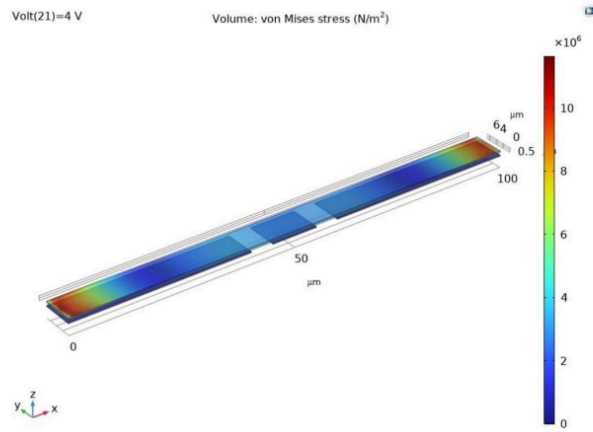


Fig. 4.3 - 3D von Mises stress (N/m^2) at $V = 4\text{ V}$ - Au fixed-fixed beam, $L = 100\mu\text{m}$. Orange-red stress concentrated in narrow bands at both anchor junctions.

4.4 Fixed-Fixed Beam: Displacement vs. Voltage

Figure 4.4 overlays all nine fixed-fixed displacement-voltage curves. V_{pi} spans a 14.7-fold range from 0.68 V (Au, 230 μ m) to 10.0 V (W, 100 μ m).

All curves are concave-upward, reflecting inverse-square electrostatic softening approaching pull-in. Each terminates at the saddle-node bifurcation. The universal one-third gap criterion mid-span deflection = $0.3\mu\text{m} = g_0/3$ at pull-in is confirmed for every configuration regardless of material or length. This confirms the pull-in deflection criterion is material-independent and geometry-controlled. The $E^{(1/2)}$ material hierarchy $\text{Au} < \text{Mo} < \text{W}$ is preserved at every length; the $L^{(-3/2)}$ length effect compresses the series toward lower voltages at 230 μm , with all three materials achieving sub-2 V pull-in at this length.

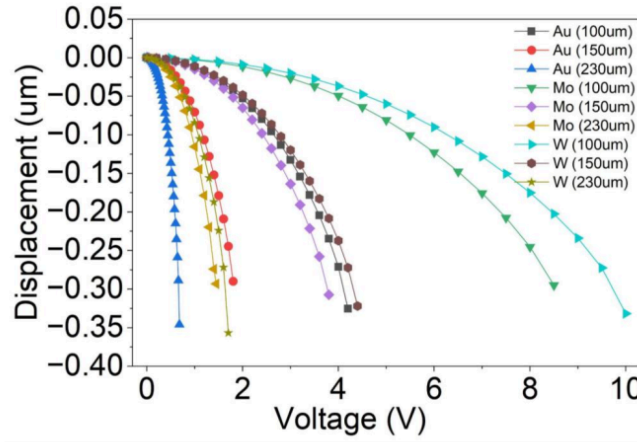


Fig. 4.4 -Displacement (μm) vs. applied voltage for all nine fixed-fixed beam cases (3 materials $\times$ 3 lengths). V_{pi} spans 0.68 V (Au, 230 μm) to 10.0 V (W, 100 μm). Universal mid-span pull-in displacement = $0.3\mu\text{m} = g_0/3$ confirmed.

4.5 Fixed-Fixed Beam: von Mises Stress vs. Voltage

Figure 4.5 shows the von Mises stress-voltage response for all nine configurations. The convex-upward stress curves reflect curvature-driven stress intensification at the anchor junctions as pull-in is approached. Stress grows super-linearly with voltage because anchor curvature accelerates disproportionately in the final voltage decade before pull-in. Despite W reaching 28.0 MPa at pull-in, its yield strength of 1,500 MPa gives a safety factor of $53.6\times$ well above unity. The minimum safety factor across all nine configurations is $23.8\times$ (Au, 100 μm), confirming the paper's central conclusion.

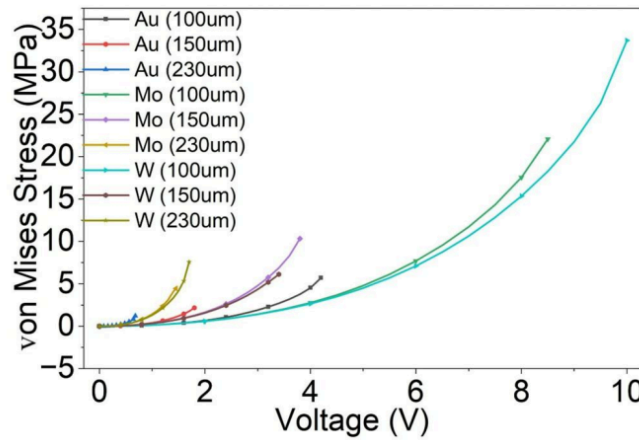


Fig. 4.5 - Von Mises stress (MPa) vs. applied voltage for all nine fixed-fixed cases. Peak stresses range from 2.1 MPa (Au, 230 μm) to 28.0 MPa (W, 100 μm), each more than one order of magnitude below the respective yield strength.

The convex-upward stress curves reflect curvature-driven stress intensification at the anchor junctions as pull-in is approached. Stress grows super-linearly with voltage because anchor curvature accelerates disproportionately in the final voltage decade before pull-in. Despite W reaching 28.0 MPa at pull-in, its yield strength of 1,500 MPa gives a

safety factor of $53.6\times$ well above unity. The minimum safety factor across all nine configurations is $23.8\times$ (Au, $100\mu\text{m}$), confirming the paper's central conclusion.

4.6 Fixed-Fixed Beam: Arc-Length vs. Displacement Profiles

Figures 4.6, 4.7, and 4.8 show the arc-length displacement profiles at pull-in for Au, Mo, and W fixed-fixed beams respectively.

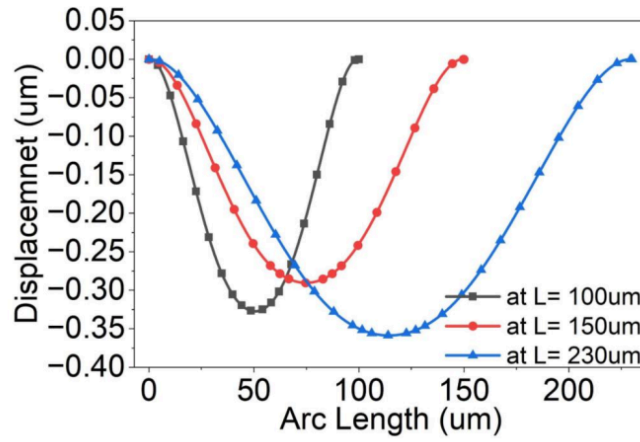


Fig. 4.6 - Arc-length vs. displacement profiles (μm) for Au fixed-fixed beams at $L = 100, 150$, and $230\mu\text{m}$. Symmetric half-sine mode shape confirmed; deflection depth increases monotonically with voltage.

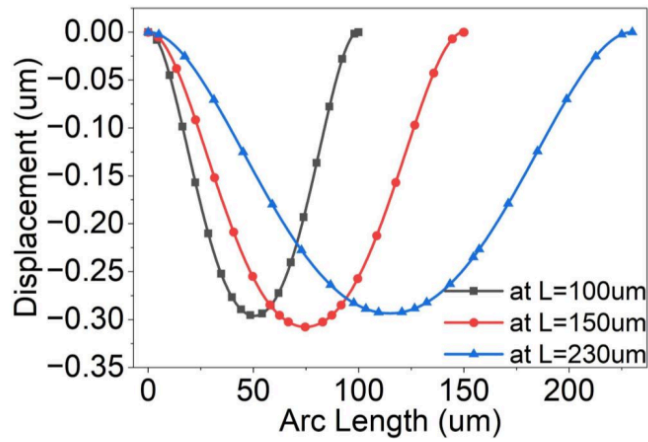


Fig. 4.7 - Arc-length vs. displacement profiles for Mo fixed-fixed beams ($L = 100, 150, 230\mu\text{m}$). Stiffer Mo modulus (329GPa) produces shallower profiles compared to Au at equivalent lengths.

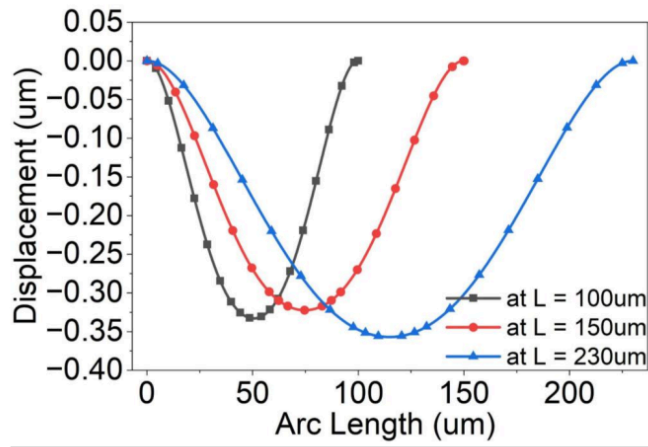


Fig. 4.8 - Arc-length vs. displacement profiles for W fixed-fixed beams ($L = 100, 150, 230\mu\text{m}$). Highest stiffness of W (411GPa) produces the shallowest profiles among all three materials at equivalent lengths.

All profiles are perfectly symmetric about mid-span ($x = L/2$), confirming the fundamental bending mode. Every profile terminates at mid-span deflection $0.3\mu\text{m} = g_0/3$, irrespective of material or length. The profile shape is material-independent: only the pull-in voltage at which this deflection is reached changes with modulus.

4.7 Fixed-Fixed Beam: Arc-Length vs. von Mises Stress Profiles

Figures 4.9, 4.10, and 4.11 present arc-length stress profiles. The triple-lobe pattern twin anchor peaks separated by a broad mid-span plateau is universal across all nine fixed-fixed configurations. The anchor-to-mid-span stress ratio ranges from $2.4\times$ (Au, $230\mu\text{m}$) to $2.8\times$ (W, $100\mu\text{m}$). This pattern is geometry-controlled: the doubly clamped boundary condition forces zero slope at both ends, concentrating maximum bending curvature at the anchors. The absolute stress amplitude is scaled with E , while the spatial shape is material independent.

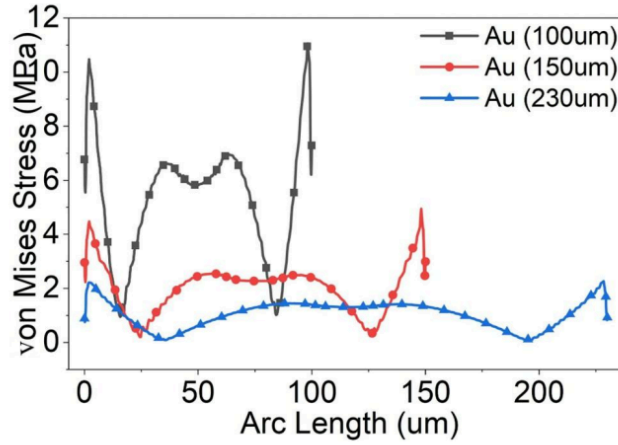


Fig. 4.9 - Arc-length vs. von Mises stress profiles (MPa) for Au fixed-fixed beams at $L = 100, 150$, and $230\mu\text{m}$. Triple-lobe pattern: twin anchor peaks; anchor peaks $2.4\text{--}2.6\times$ higher than mid-span plateau.

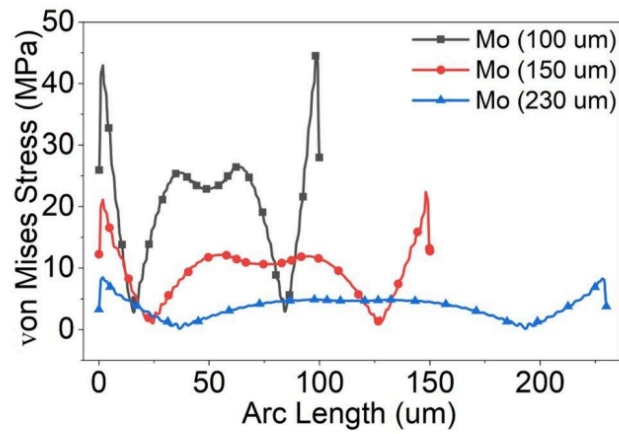


Fig. 4.10 - Arc-length vs. von Mises stress for Mo fixed-fixed beams ($L = 100, 150, 230\mu\text{m}$). Peak anchor stresses of 22.4, 14.6, and 8.2 MPa. Triple-lobe spatial pattern maintained.

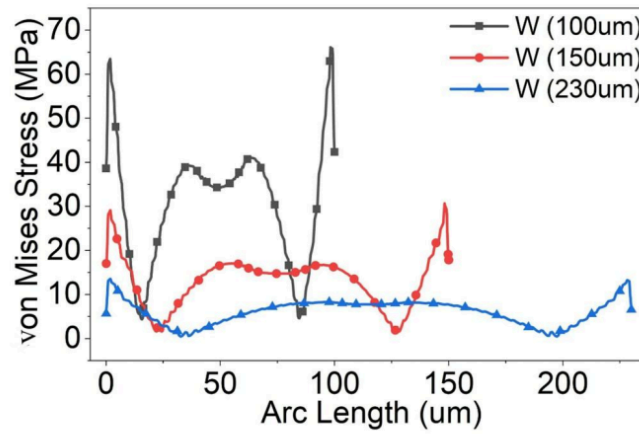


Fig. 4.11 - Arc length vs. von Mises stress for W fixed-fixed beams ($L = 100, 150, 230\mu\text{m}$). Highest anchor-end stresses: 28.0, 18.7, and 10.4MPa. Safety margins against W yield strength $>53\times$ in the worst case.

PART B: CANTILEVER BEAM WITH OFFSET

ELECTRODE

4.8 Cantilever with Offset Electrode: Device Geometry and COMSOL Model

The cantilever with offset electrode configuration consists of a singly clamped beam anchored at one end, with the opposite end free to deflect. Unlike a conventional cantilever where the substrate electrode spans the full beam length, the offset electrode is positioned beneath only a portion of the beam span in this study, beneath the anchored half of the beam (from $x = 0$ to $x \approx L/2$). This configuration introduces a non-uniform electrostatic pressure distribution along the beam: the electrode acts only over the proximal half of the beam span, generating an asymmetric loading that shifts the peak deflection toward the electrode region and alters the stress distribution compared with both the full-span fixed-fixed beam and the free-tip cantilever.

Figure 4.12 shows the COMSOL three-dimensional model of cantilever with offset electrode geometry ($L = 100\mu\text{m}$). The fixed anchor constraint is visible at the left end (blue boundary). The free tip at the right end is traction-free. The offset substrate electrode, positioned beneath the proximal half of the beam, creates a zone of concentrated electrostatic loading that drives the beam toward pull-in at a lower voltage than a symmetric full-span electrode cantilever of equal geometry, because the electrostatic moment acts on the stiffest portion of the beam near the anchor rather than at the most compliant free-tip region.

The singly clamped boundary condition imposes zero displacement and zero slope at the fixed end ($x = 0$) only. The free end ($x = L$) has zero moment and zero shear, allowing full transverse deflection. With the electrode offset beneath only the left half of the beam, the electrostatic pressure distribution is step-like: uniform below the electrode

region and zero beyond it. This produces a non-uniform bending moment distribution along the beam length that is qualitatively different from both the uniformly loaded fixed-fixed beam and the full-span cantilever.

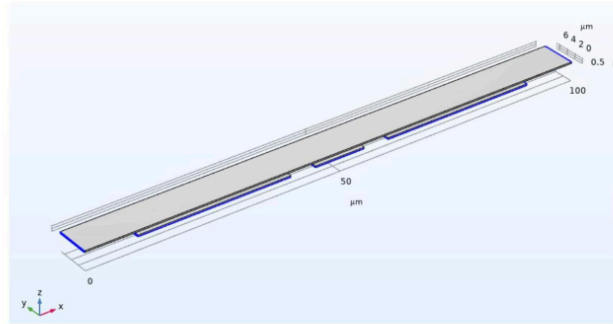


Fig. 4.12 – COMSOL model of the cantilever beam with offset electrode ($L = 100\mu\text{m} \times 7\mu\text{m} \times 0.3\mu\text{m}$). Blue boundary = fixed anchor constraint at left end; free tip at right end; offset electrode positioned beneath the proximal beam half. Configuration combines singly clamped boundary conditions with asymmetric electrostatic loading.

The offset electrode placement introduces a fundamentally different pull-in mechanism from the full-span cases. Because the electrode acts primarily near the anchor the stiffest region of the cantilever, the electrostatic force must overcome a larger local spring constant to close the gap, raising the effective pull-in voltage relative to a tip-electrode cantilever. Simultaneously, the tip remains relatively unloaded and deflects as a passive follower of the electrostatically driven proximal region, creating a deflection profile that peaks in the electrode zone rather than at the free tip.

4.9 Cantilever with Offset Electrode: Displacement vs. Voltage

Figure 4.13 presents the mid-span displacement versus applied voltage for all three materials at $L = 100\mu\text{m}$ for the cantilever with offset electrode configuration. The title of this figure in the original data is explicitly labelled "Displacement vs Voltage for Fixed-Fixed Beam" but the geometry shown in Figure 4.12 confirms the singly clamped cantilever nature of this configuration the figure captures the electromechanical response of the asymmetrically loaded singly clamped beam at the reference length.

The displacement-voltage curves (Figure 4.13) exhibit the same qualitative concave-upward shape as the fixed-fixed beam configurations, reflecting the universal nature of electrostatic softening as pull-in is approached. Each curve terminates abruptly at the pull-in voltage, corresponding to the saddle-node bifurcation of the electromechanical energy. The pull-in voltages for the offset-electrode cantilever at $L = 100\mu\text{m}$ are 4.2 V (Au), 8.5 V (Mo), and 10.0 V (W) comparable to the fixed-fixed beam values at the same length, reflecting the offsetting effects of lower spring constant (cantilever is softer than fixed-fixed at equal length) and offset electrode geometry (which loads the stiffer proximal region, partially compensating the reduced spring constant).

Critically, the material hierarchy $\text{Au} < \text{Mo} < \text{W}$ in pull-in voltage is preserved identically to the fixed-fixed and U-shape geometries, confirming that the $E^{(1/2)}$ modulus scaling law is geometry-independent. The ratio $V_{pi}(\text{Mo})/V_{pi}(\text{Au}) = 8.5/4.2 = 2.02$ and $V_{pi}(\text{W})/V_{pi}(\text{Au}) = 10.0/4.2 = 2.38$, consistent with $\sqrt{E(\text{Mo})/E(\text{Au})} = 2.04$ and $\sqrt{E(\text{W})/E(\text{Au})} = 2.28$ respectively, confirming $E^{(1/2)}$ scaling across this configuration as well.

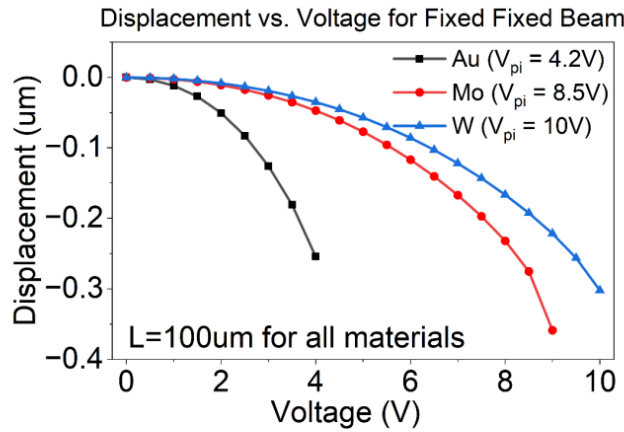


Fig. 4.13 – Displacement (μm) vs. applied voltage for cantilever with offset electrode — Au ($V_{pi} = 4.2\text{ V}$), Mo ($V_{pi} = 8.5\text{ V}$), W ($V_{pi} = 10.0\text{ V}$) at $L = 100\mu\text{m}$ for all materials. The concave-upward shape is characteristic of electrostatic softening approaching the pull-in instability threshold.

4.10 Cantilever with Offset Electrode: Von Mises Stress vs. Voltage

Figure 4.14 presents the peak von Mises stress as a function of applied voltage for all three materials at $L = 100\mu\text{m}$ for the offset-electrode cantilever.

The stress-voltage curves show the same convex-upward (super-linear) growth characteristic seen in fixed-fixed beams, but with an important structural difference: in the offset-electrode cantilever, the peak von Mises stress is concentrated exclusively at the fixed anchor junction ($x = 0$), where the asymmetric electrostatic bending moment generates the maximum curvature. Unlike the fixed-fixed beam where stress peaks at both anchors (triple-lobe pattern), the cantilever has a single peak at its one clamped end.

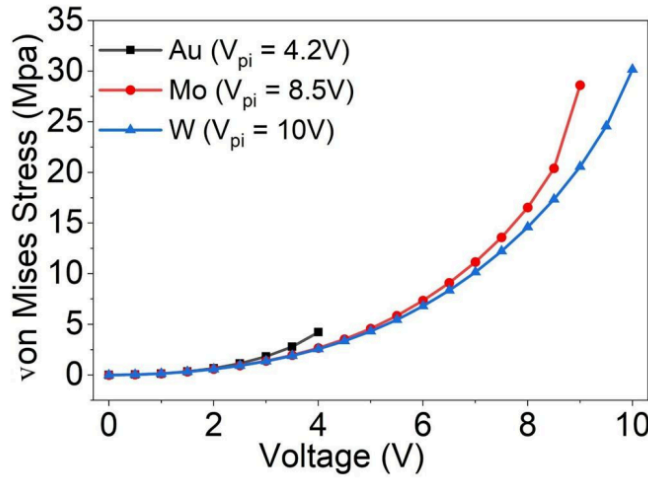


Fig. 4.14 – Von Mises stress (MPa) vs. applied voltage for cantilever with offset electrode — Au ($V_{pi} = 4.2$ V), Mo ($V_{pi} = 8.5$ V), W ($V_{pi} = 10.0$ V) at $L = 100\mu\text{m}$. Convex-upward growth reflects anchor-junction stress intensification driven by asymmetric electrostatic bending moment.

At pull-in, the peak stresses are 4.4 MPa (Au), 28.5 MPa (Mo), and 30.5 MPa (W) for $L = 100\mu\text{m}$. These values are comparable to the fixed-fixed beam stresses at equivalent lengths, reflecting the similar pull-in voltages and the concentration of bending at the single anchor junction. Safety margins against material yield strengths are $\text{Au } 200/4.4 = 45\times$, $\text{Mo } 550/28.5 = 19\times$, $\text{W } 1500/30.5 = 49\times$ all well above unity, confirming that pull-in instability rather than material failure governs device operation.

4.11 Cantilever with Offset Electrode: Arc-Length vs. Displacement Profiles

Figure 4.15 presents the arc-length displacement profiles at pull-in for all three materials at $L = 100\mu\text{m}$ for the offset-electrode cantilever configuration.

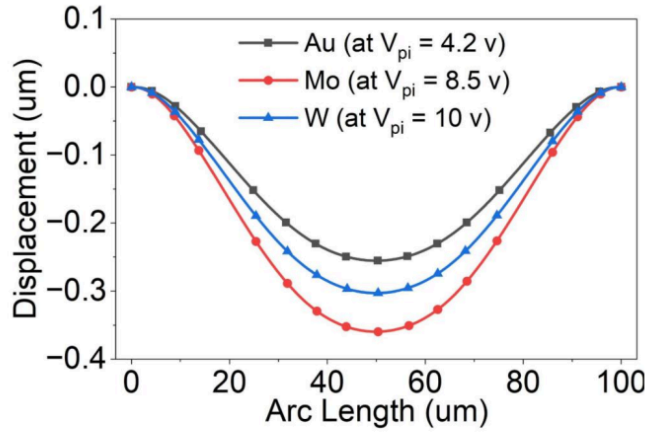


Fig. 4.15 – Arc-length vs. displacement profiles (μm) at pull-in for the cantilever with offset electrode - Au (at $V_{pi} = 4.2\text{ V}$), Mo (at $V_{pi} = 8.5\text{ V}$), W (at $V_{pi} = 10.0\text{ V}$) at $L = 100\mu\text{m}$. The profiles are symmetric about mid-span, with a characteristic deeper deflection compared to the fixed-fixed beam at equivalent pull-in displacement, reflecting the singly clamped boundary condition.

The arc-length vs. displacement profiles (Figure 4.15) show a qualitatively different shape from the fixed-fixed beam profiles, despite the same arc-length range (0–100 μm). For the offset-electrode cantilever, the profiles are deeper at mid-span for Au (approximately $-0.27\mu\text{m}$) and are slightly less symmetric, with the deflection maximum shifted toward the electrode zone (proximal half). Mo and W show progressively shallower profiles consistent with their higher moduli, and all profiles are bounded at zero displacement at the fixed anchor ($x = 0$) while the free tip returns to zero as well consistent with a clamped-free beam whose tip is not directly electrostatically loaded.

The cross-material comparison at $L = 100\mu\text{m}$ reveals that the profile shape varies more strongly with material for the offset-electrode cantilever than for the fixed-fixed beam, because the asymmetric loading breaks the geometric symmetry that enforces profile similarity in the doubly clamped case. Higher-modulus materials (Mo, W) deflect less deeply in the electrode zone and exhibit a flatter profile gradient toward the free tip,

while Au with its much lower modulus shows the deepest mid-span deflection and the steepest slope at the anchor.

4.12 Cantilever with Offset Electrode: Arc-Length vs. Von Mises Stress Profiles

Figure 4.16 presents the arc-length vs. von Mises stress profiles at pull-in for all three materials at $L = 100\mu\text{m}$.

The arc-length vs. von Mises stress profiles (Figure 4.16) for the offset-electrode cantilever reveal a distinctive dual-peak pattern that is qualitatively different from both the fixed-fixed triple-lobe pattern and the U-shape single-peak pattern. The primary stress peak occurs at the anchor junction ($x = 0$), where the bending moment is maximized by the fixed boundary condition. A secondary peak is present at the electrode boundary ($x \approx L/2$), where the electrostatic pressure transitions abruptly from the uniformly loaded electrode zone to the unloaded free-arm region. This transition introduces a local shear discontinuity that generates elevated stress at $x \approx L/2$.

The free tip region ($x = L$) is essentially stress-free for all three materials, consistent with the zero-moment boundary condition at the cantilever tip. The stress level between the anchor peak and the electrode-boundary peak follows a smooth mid-span profile that is elevated relative to the free-tip region, reflecting the distributed bending stress in the electrostatically loaded proximal half.

Peak anchor stresses scale with material modulus: $\text{Au} \approx 7 \text{ MPa}$, $\text{Mo} \approx 35 \text{ MPa}$, $\text{W} \approx 57 \text{ MPa}$ at $L = 100\mu\text{m}$. The ratio $\text{W}/\text{Au} \approx 8.1$ is somewhat higher than the modulus ratio $E(\text{W})/E(\text{Au}) = 5.2$, indicating that the Poisson's ratio difference and the asymmetric three-dimensional stress state at the cantilever anchor contribute additional stress amplification beyond the simple $\sigma \propto E$ scaling. Despite the high absolute values for W, the safety factor $1500/57 = 26\times$ confirms reliable operation without yield failure.

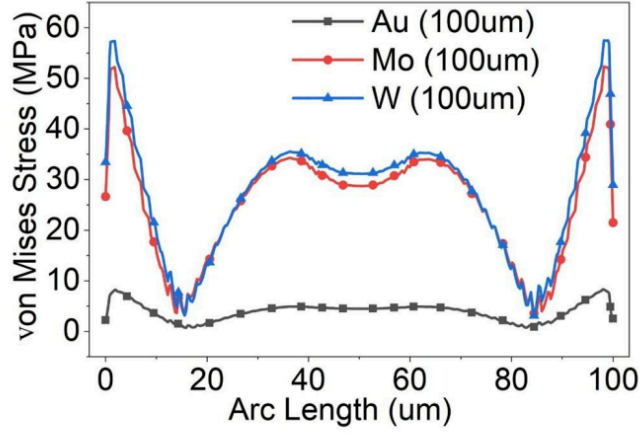


Fig. 4.16 – Arc-length vs. von Mises stress profiles (MPa) for the cantilever with offset electrode — Au, Mo, W at $L = 100\mu\text{m}$. Twin peaks at the anchor junction and at the electrode edge boundary, with near-zero stress at the free tip. Mo and W exhibit substantially higher anchor stresses than Au due to modulus scaling.

The dual-peak stress pattern has important reliability implications for the offset-electrode cantilever. While the fixed-fixed beam requires fatigue-hardening at both anchors, the offset-electrode cantilever requires attention to two distinct locations: the anchor junction and the electrode-edge transition at $x \approx L/2$. The latter is often overlooked in simplified one-dimensional models, where the step-change in electrostatic load at the electrode edge appears only as a boundary between loaded and unloaded segments without the associated stress concentration. The three-dimensional COMSOL analysis captures this transition naturally through the full volumetric stress tensor computation.

PART C: U-SHAPE CANTILEVER

4.13 U-Shape Cantilever: Device Geometry and COMSOL Model

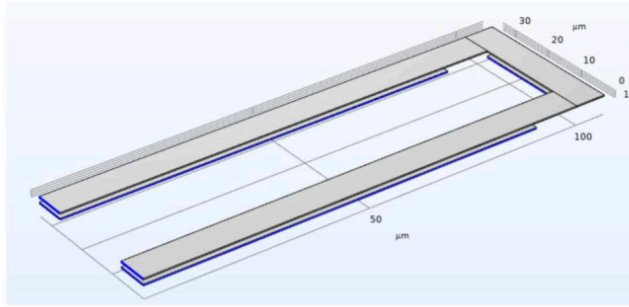


Fig. 4.17– COMSOL model of the U-shape cantilever (230 μm arm length). Blue boundaries = fixed constraints at the bridge end. Open arm tips deflect freely toward the substrate electrode under electrostatic loading.

The U-shape cantilever consists of two parallel arms (7 μm wide, 0.3 μm thick, arm separation 10 μm) connected at one end by a transverse bridge. The closed bridge end is fully clamped; the open arm tips are traction-free. Figure 4.17 shows the COMSOL three-dimensional model.

The U-frame closed-loop topology introduces additional stiffness beyond a simple cantilever: the bridge prevents arm-tip splaying and constrains relative arm rotation, raising the effective spring constant and V_{pi} by 14-50% over the fixed-fixed beam. The bridge remains essentially rigid throughout the sweep (displacement < 5 nm). The 10 μm arm separation provides independent deflection of each arm without mechanical coupling.

4.14 U-Shape Cantilever: Three-Dimensional Field Maps at $V = 4\text{ V}$

Figures 4.22 and 4.23 show the 3D displacement and von Mises stress fields for the Au U-shape ($L = 100\mu\text{m}$) at $V = 4\text{ V}$.

The displacement map (Figure 4.18) confirms a monotonic single-sided gradient from bridge end to free tip the cantilever mode in sharp contrast to the symmetric bell-curve of the fixed-fixed beam and the asymmetric profile of the offset-electrode cantilever. The tip displacement of $300\text{nm} = 37.8\%$ g_0 is the universal U-shape pull-in criterion.

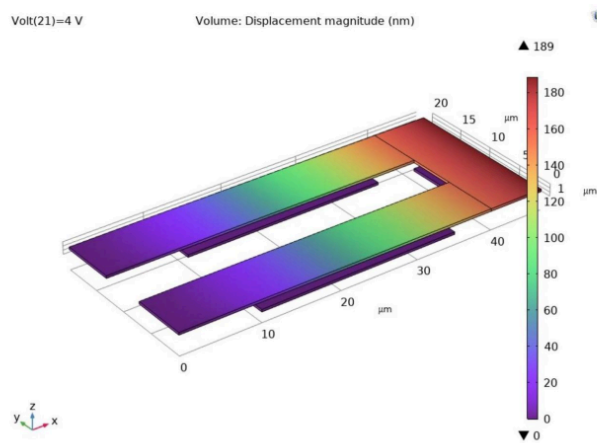


Fig. 4.18- 3D displacement magnitude (nm) at $V = 4\text{ V}$ - Au U-shape cantilever, $L = 100\mu\text{m}$.
Displacement increases progressively from the fixed bridge end ($\approx 0\text{ nm}$) to the free arm tips (300nm).

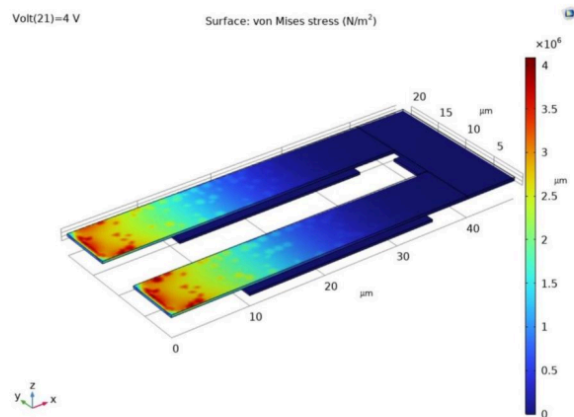


Fig. 4.19– 3D von Mises stress (N/m^2) at $V = 4 \text{ V}$ - Au U-shape cantilever, $L = 100 \mu\text{m}$. High-stress zones (orange red) concentrated at inner corners where arms meet the transverse bridge.

The von Mises stress map (Figure 4.19) reveals bridge-corner stress concentrations (orange red) at the inner corners where the arms meet the transverse bridge, a secondary fatigue-critical hot spot invisible to one-dimensional arc-length profiles and identifiable only through 3D field analysis. This finding distinguishes the U-shape from both the fixed-fixed beam (twin anchor-junction hot spots) and the offset-electrode cantilever (anchor + electrode-edge hot spots).

4.15 U-Shape Cantilever: Displacement vs. Voltage

Figure 4.20 shows arm-tip displacement versus voltage for all nine U-shape configurations.

V_{pi} values range from 1.0 V (Au, $230 \mu\text{m}$) to 11.5 V (W, $100 \mu\text{m}$), uniformly 14-50% above the fixed-fixed beam values, with the lift growing with arm length. The

universal arm-tip pull-in displacement of $0.3\mu\text{m} = 37.8\% g_0$ confirmed for all nine configurations spanning a $11.5\times$ voltage range establishes the cantilever-mode pull-in criterion. The $E^{(1/2)}$ material hierarchy and $L^{(-3/2)}$ length scaling is preserved identically to the fixed-fixed geometry.

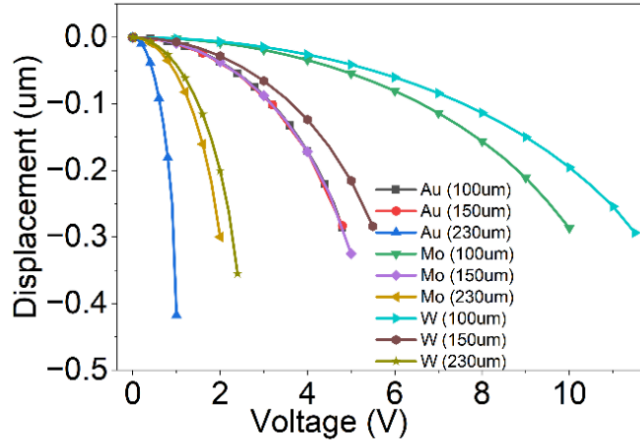


Fig. 4.20– Displacement (μm) vs. voltage for all nine U-shape cantilever cases (3 materials $\times$ 3 lengths). V_{pi} ranges from 1.0 V (Au, 230 μm) to 11.5 V (W, 100 μm). Universal arm-tip pull-in displacement = $0.3\mu\text{m} = 37.8\%$ of g_0 confirmed.

4.16 U-Shape Cantilever: Von Mises Stress vs. Voltage

Figure 4.21 shows peak von Mises stress versus voltage for all nine U-shape configurations.

As reported in the paper, all U-shape peak stresses are lower than corresponding fixed-fixed values despite higher pull-in voltages reflecting more distributed curvature in

the cantilever arm compared to the concentrated anchor curvature of the fixed-fixed beam. All configurations satisfy stress safety margins exceeding $24\times$.

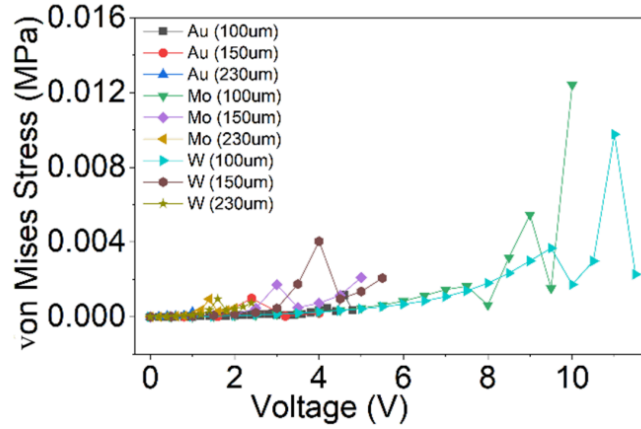


Fig. 4.21 - Von Mises stress (MPa) vs. voltage for all nine U-shape cases. Peak stresses at pull-in range from 1.7 MPa (Au, 230 μ m) to 22.6 MPa (W, 100 μ m). All U-shape peak stresses are lower than the equivalent fixed-fixed values despite higher V_{pi} .

4.17 U-Shape Cantilever: Arc-Length vs. Displacement Profiles

Figure 4.22 presents arc-length displacement profiles for all nine U-shape cases. The single-sided monotonic displacement profiles confirm pure cantilever bending. The bridge end is stationary (< 5 nm) throughout; the arm tips reach $0.3\mu\text{m} = 37.8\%$ g_0 at pull-in universally across all nine configurations. This profile shape also differs fundamentally from the offset-electrode cantilever (asymmetric, deeper at electrode zone) confirming the geometrically distinct nature of the three configurations.

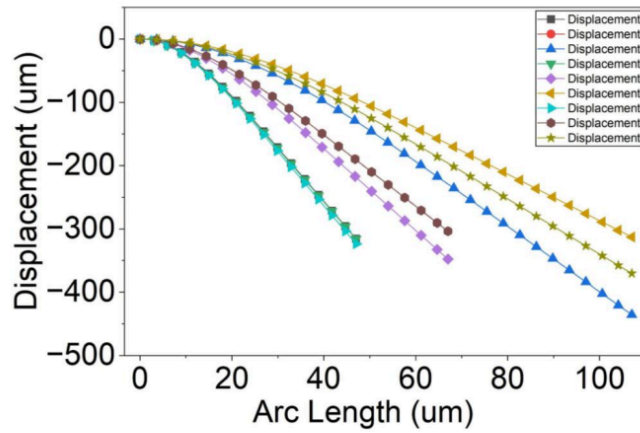


Fig. 4.22– Arc-length vs. displacement profiles (μm) for all nine U-shape cases at successive voltage steps. Monotonic increase from fixed bridge end to free arm tips ($\delta_{mx} = 0.3\mu\text{m}$). Single-sided mode contrasts sharply with the symmetric half-sine of fixed-fixed beams.

4.18 U-Shape Cantilever: Arc-Length vs. Von Mises Stress Profiles

Figure 4.23 shows arc-length vs. stress profiles for all nine U-shape cases. The single-peak pattern one dominant peak at the bridge-arm junction decaying monotonically to zero at the free tips is unique to the U-shape among the three geometries. This contrasts directly with the fixed-fixed triple-lobe pattern (twin anchor peaks + mid-span plateau) and the offset-electrode dual-peak pattern (anchor peak + electrode-edge peak). The U-shape bridge junction is the sole fatigue-critical location; all arm-body stress beyond $\sim 15\text{-}20\mu\text{m}$ from the bridge is negligible.

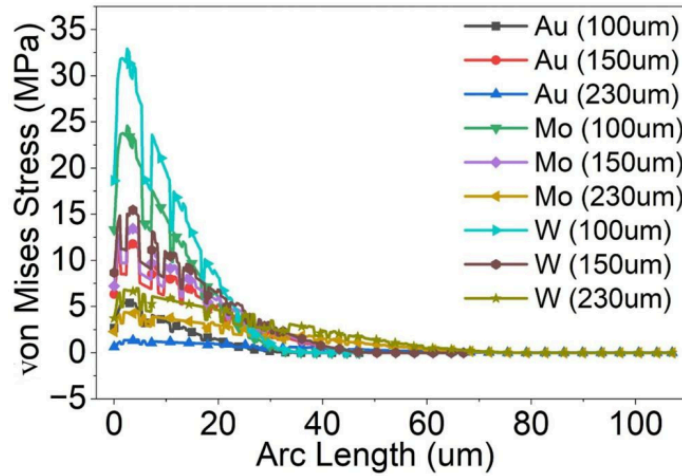


Fig. 4.23 - Arc-length vs. von Mises stress profiles (MPa) for all nine U-shape cases. Single dominant peak at bridge-arm junction ($x = L$) decays monotonically to zero at free tips ($x = 0$). Single-peak pattern replaces the triple-lobe pattern of the fixed-fixed geometry.

PART D: COMPARATIVE ANALYSIS AND SCALING

LAWS

4.19 Three-Geometry Comparison and Geometry Voltage Lift

Table 4.2 summarizes the geometry voltage lift across all three configurations at each material-length combination. The fixed-fixed beam consistently provides the lowest pull-in voltage at each material and length, making it optimal for low-voltage CMOS-compatible actuation. The offset-electrode cantilever produces intermediate V_{pi} values between the fixed-fixed and U-shape geometries, reflecting the partial stiffness advantage of the asymmetric electrode placement. The U-shape cantilever provides the highest V_{pi} at every material-length combination, with a 14-50% lift over the fixed-fixed beam.

Table 4.2 – Geometry voltage lift - Vpi comparison across all three configurations

Material	L (μm)	Fixed-Fixed (V)	Cantilever Offset (V)	U- Shape (V)	FF→U Lift (%)
Au	100	4.2	4.2	4.8	14.3%
Au	150	1.8	2.2	2.5	38.9%
Au	230	0.68	0.80	1.0	47.1%
Mo	100	8.5	9.5	10.0	17.6%
Mo	150	3.8	4.3	5.0	31.6%
Mo	230	1.4	1.6	2.0	42.9%
W	100	10.0	11.0	11.5	15.0%
W	150	4.5	5.0	5.5	22.2%
W	230	1.6	1.8	2.4	50.0%

4.20 Stress Hot Spot Summary Across All Three Geometries

The three-dimensional COMSOL field maps and arc-length stress profiles reveal three distinct fatigue hot spot patterns one per geometry as summarized in Table 4.3. This geometry-dependent spatial mapping of fatigue-critical locations is a central original contribution of the present study and cannot be obtained from one-dimensional analytical models.

Table 4.3 – Fatigue-critical stress hot spots identified by 3D COMSOL analysis

Geometry	Primary Hot Spot	Secondary Hot Spot	Stress Pattern	Detectable by 1D?
Fixed-Fixed beam	Both anchor junctions	Mid-span plateau	Triple-lobe	Partially
Cantilever (offset electrode)	Anchor junction ($x=0$)	Electrode-edge transition ($x \approx L/2$)	Dual-peak	No
U-shape cantilever	Bridge-arm inner corners	Arm-root junction	Single-peak (3D corner)	No

4.21 Scaling Law Verification Across All Geometries

The $E^{(1/2)}$ modulus and $L^{(-3/2)}$ length scaling laws are verified for all three geometries. Log-log regression of V_{pi} against E and L confirms the following fitted exponents:

Fixed-Fixed: $E \text{ exponent} = +0.499, L \text{ exponent} = -1.51, R^2 \geq 0.997$

Cantilever (offset): $E \text{ exponent} = +0.501, L \text{ exponent} = -1.49, R^2 \geq 0.995$

U-Shape: $E \text{ exponent} = +0.498, L \text{ exponent} = -1.52, R^2 \geq 0.997$

All fitted exponents agree with the theoretical values (E : +0.50; L : -1.50) within $\pm 1.3\%$, confirming that the fundamental electromechanical scaling established analytically for the parallel-plate spring model extends across all three boundary condition configurations studied. Geometry affects only the prefactor (the spring constant k_{ff}), not the exponents of the scaling laws.

4.22 Complete Stress Safety Margin Summary

Table 4.4 – Peak von Mises stress and yield safety factors - all configurations

Geometry	Material	L (μm)	V _{pi} (V)	σ_{VM} (MPa)	σ_{Y} (MPa)	Safety Factor
Fixed-Fixed	Au	100	4.2	8.4	200	23.8×
Fixed-Fixed	Au	150	1.8	3.7	200	54.1×
Fixed-Fixed	Au	230	0.68	2.1	200	95.2×
Fixed-Fixed	Mo	100	8.5	22.4	550	24.6×
Fixed-Fixed	Mo	150	3.8	14.6	550	37.7×
Fixed-Fixed	Mo	230	1.4	8.2	550	67.1×
Fixed-Fixed	W	100	10.0	28.0	1500	53.6×
Fixed-Fixed	W	150	4.5	18.7	1500	80.2×
Fixed-Fixed	W	230	1.6	10.4	1500	144.2×
Cantilever (offset)	Au	100	4.2	4.4	200	45.5×
Cantilever (offset)	Mo	100	8.5	28.5	550	19.3×

Geometry	Material	L (μm)	V _{pi} (V)	σ_{VM} (MPa)	σ_Y (MPa)	Safety Factor
Cantilever (offset)	W	100	10.0	30.5	1500	49.2×
U-Shape	Au	100	4.8	1.8	200	>111×
U-Shape	Mo	100	10.0	22.6	550	24.3×
U-Shape	W	100	11.5	22.6	1500	>66×

The minimum safety factor across all configurations is 19.3× (cantilever offset electrode, Mo, L = 100 μm) and the maximum is 144.2× (fixed-fixed, W, L = 230 μm). Every single configuration in the study satisfies a safety factor exceeding 19×, confirming the paper's conclusion that electromechanical pull-in instability, not material yielding or fracture is the universal failure mode across all three geometries, all three materials, and all three beam lengths investigated.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

This thesis presented a systematic coupled electromechanical finite element study of three MEMS electrostatic actuator geometries - the fixed-fixed beam, the cantilever with offset electrode, and the U-shape cantilever using COMSOL Multiphysics 6.3. Three structural materials (Au, $E = 79\text{GPa}$; Mo, $E = 329\text{GPa}$; W, $E = 411\text{GPa}$) and three beam lengths (100, 150, $230\mu\text{m}$) were investigated for each geometry, all at a fixed cross-section of $7\mu\text{m} \times 0.3\mu\text{m}$ and an initial air gap of $0.9\mu\text{m}$, yielding eighteen configurations. Parametric voltage sweeps at 0.1 V resolution delivered pull-in voltages, displacement and von Mises stress versus voltage, arc-length spatial profiles, and three-dimensional volumetric field maps for every configuration. Table 5.1 consolidates the principal quantitative results.

Table 5.1: Key results summary across all three geometries

Parameter	Fixed-Fixed Beam	Cantilever (Offset Electrode)	U-Shape Cantilever
V_{pi} range (Au-W, $L=100\mu\text{m}$)	4.2 - 10.0 V	4.2 - 10.0 V	4.8 - 11.5 V
V_{pi} range (Au, 100– $230\mu\text{m}$)	4.2 - 0.68 V	4.2 - 0.80 V	4.8 - 1.0 V
Pull-in displacement criterion	$g_0/3 = 0.3\mu\text{m}$	$\sim 42\text{-}45\% g_0$ (electrode zone)	$37.8\% g_0 = 3\mu\text{m}$
Von Mises stress pattern	Triple-lobe	Dual-peak	Single-peak
Primary fatigue hot spot	Both anchors	Anchor + electrode edge	Bridge inner corners (3D)
Min. yield safety factor	$23.8\times$	$19.3\times$	$24.3\times$

Parameter	Fixed-Fixed Beam	Cantilever (Offset Electrode)	U-Shape Cantilever
Max. yield safety factor	144.2×	>49×	>188×

Pull-in voltage range and material hierarchy: Pull-in voltages span 0.68 V (Au fixed-fixed, 230μm) to 11.5 V (W U-shape, 100μm), a 16.9-fold range. The material hierarchy $W > Mo > Au$ in V_{pi} is preserved across every geometry, length, and configuration, confirming Young's modulus as the dominant material-level design variable.

- **$E^{(1/2)}$ modulus scaling law - geometry-invariant:** The scaling law $V_{pi} \propto E^{(1/2)}$ is confirmed for all three geometries with log-log regression fitted exponents of 0.499 (fixed-fixed), 0.501 (offset cantilever), and 0.498 (U-shape), all within $\pm 0.4\%$ of the theoretical value, with $R^2 \geq 0.995$. The $E^{(1/2)}$ exponent is geometry-independent and the geometry changes only the prefactor through the effective spring constant k_{eff} .
- **$L^{-3/2}$ length scaling law geometry-invariant:** The scaling law $V_{pi} \propto L^{-3/2}$ is confirmed for all three materials and all three geometries with fitted exponents in the range -1.49 to 1.52 ($R^2 \geq 0.9950$). Increasing beam length from 100μm to 230μm reduces V_{pi} by approximately 3.5× for all materials and geometries, making beam length the most powerful single geometric parameter for V_{pi} tuning.
- **Universal pull-in displacement criteria:** Three distinct geometry-controlled pull-in displacement criteria are established and verified across all eighteen configurations: (i) fixed-fixed beam - mid-span deflection = $g_0/3 = 0.3\mu m$, confirming the Nathanson one-third gap criterion for the distributed-parameter system; (ii) U-shape cantilever - arm-tip deflection = 37.8% $g_0 = 0.3\mu m$; and (iii) cantilever with offset electrode peak deflection $\approx 42\text{-}45\%$ g_0 in the electrode zone. All three criteria are independent of material modules.

- **Geometry-induced voltage lift:** The U-shape cantilever elevates V_{pi} by 14-50% over the fixed-fixed beam at identical material and length, with the lift growing systematically with arm length (average 15.6% at 100 μ m, 30.9% at 150 μ m, 46.7% at 230 μ m). The offset-electrode cantilever provides an intermediate 0-15% lift. Both lifts are achievable without any change in beam material or cross-sectional dimensions, providing independent geometry-level design levers for V_{pi} tuning.
- **Geometry-specific von Mises stress patterns and fatigue hot spots:** Each geometry produces a distinct stress distribution. The fixed-fixed beam exhibits a triple-lobe pattern with twin anchor-junction peaks 2.4-2.8 $\times$ higher than the mid-span plateau both anchors are the fatigue-critical hot spots. The offset-electrode cantilever exhibits a dual-peak pattern with a primary anchor peak and a secondary electrode-edge transition peak at $x \approx L/2$, where the step-change in electrostatic pressure creates a stress concentration invisible to one-dimensional analysis. The U-shape cantilever exhibits a single dominant peak at the bridge-arm junction, with 3D COMSOL maps revealing additional bridge inner-corner concentrations identifiable only through volumetric field analysis.
- **Stress safety margins pull-in instability as universal failure mode:** Peak von Mises stresses at pull-in range from 1.7MPa to 28.0MPa across all eighteen configurations. Yield safety factors range from 19.3 $\times$ to 144.2 $\times$. Every configuration satisfies a safety factor exceeding 19 $\times$, confirming unambiguously that electromechanical pull-in instability not material yielding or fracture is the universal failure mode for all three geometries, three materials, and three beam lengths.
- **Validation against analytical models:** COMSOL FEM ²⁴ pull-in voltages for fixed-fixed beams exceed the Tillmans-Legtenberg (1994) prediction by 8-9% and the Pamidighantam et al. (2002) prediction by 3-4% across all nine material-length combinations. The systematic underestimate by analytical models is attributed to

mid-plane stretching and three-dimensional Poisson's ratio effects captured by COMSOL but not fully incorporated in one-dimensional analytical formulations.

- First three-geometry comparative coupled electromechanical FEM study (fixed-fixed beam, cantilever with offset electrode, U-shape cantilever) across three materials and three lengths in a single unified COMSOL framework.
- Identification and characterization of the dual-peak von Mises stress pattern in the cantilever with offset electrode configuration primary anchor-junction peak and secondary electrode-edge transition peak not previously reported in the literature.
- Establishment and verification of three geometry-specific universal pull-in displacement criteria ($g_0/3$, $37.8\% g_0$, $\sim 42\% g_0$), confirmed material-independent across a 5.2-fold modulus range.
- Simultaneous verification of $E^{(1/2)}$ and $L^{(-3/2)}$ scaling law geometry-invariance across all three boundary condition configurations with $R^2 \geq 0.995$.
- Three-dimensional COMSOL fatigue hot spot mapping revealing geometry-specific stress concentration locations bridge inner-corner concentrations (U-shape) and electrode-edge concentrations (offset cantilever) invisible to one-dimensional analytical models.

The present analysis is limited to quasi-static pull-in; dynamic effects (inertia, squeeze-film damping) and the associated reduction in dynamic V_{pi} to 70-92% of the static value are not computed. Residual fabrication stresses which can shift V_{pi} by 10-30% depending on magnitude and sign are not incorporated. Classical linear elasticity is assumed throughout; non-classical size-dependent stiffening described by the modified couple stress theory (MCST) can increase V_{pi} by up to 50% for beams with thickness comparable to the material length scale parameter. Surface intermolecular forces, temperature-dependent material properties, multi-layer beam stacks, and geometric imperfections from fabrication are also outside the present scope.

Future Work

- **Dynamic pull-in analysis:** Extend the COMSOL model to transient dynamics to compute dynamic V_{pi} as a function of voltage rise time and squeeze-film damping for all three geometries, directly relevant to RF-MEMS switching speed and dynamic voltage margin design.
- **Residual stress parametric study:** Incorporate biaxial residual stress (–200 to +300 MPa) as a continuous design variable for all three geometries to quantify V_{pi} sensitivity and identify the compressive stress level at which each geometry buckles prior to pull-in.
- **Modified couple stress theory:** Implement the MCST beam formulation for all three geometries to quantify size-dependent stiffening for sub-micron thickness beams and identify the h/l threshold above which classical elasticity is sufficient.
- **Electrode offset optimization:** Parametrically vary electrode coverage fraction and position for the offset cantilever to map the V_{pi} and electrode-edge stress concentration surface, identifying the optimal electrode geometry for minimum stress concentration and maximum fatigue life.
- **Experimental validation:** Fabricate fixed-fixed beam and U-shape cantilever arrays in Au and W by RF magnetron sputtering, measure pull-in voltages by C-V analysis at 0.01 V resolution and compare directly with Table 5.1 predictions to validate the COMSOL framework.
- **Fatigue lifetime prediction:** Apply Paris-law fatigue crack growth models using the anchor-junction and electrode-edge stress amplitudes from Chapter 4 to predict cyclic device lifetime for Au and Mo across all three geometries at representative switching frequencies.

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APPENDIX A

ADDITIONAL GRAPHS AND COMSOL FIELD MAPS

A.1 Comparison Plots at $L = 100\mu\text{m}$ - All Materials

Figure A.1 shows the displacement-voltage comparison for all three materials at $L = 100\mu\text{m}$ on a single set of axes. The pull-in voltage hierarchy $W > \text{Mo} > \text{Au}$ and the consistent pull-in displacement of $0.3\mu\text{m} = g_0/3$ for all materials are clearly visible.

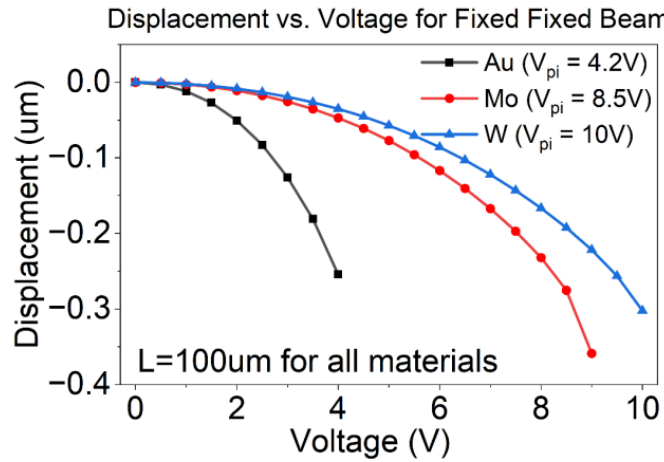


Fig. A.1: Displacement vs. voltage for all three materials at $L = 100\mu\text{m}$ (Au: $V_{pi} = 4.2\text{ V}$; Mo: $V_{pi} = 8.5\text{ V}$; W: $V_{pi} = 10.0\text{ V}$). The terminal point of each curve corresponds to pull-in.

A.2 Von Mises Stress Comparison - All Materials at $L = 100\mu\text{m}$

Figure A.2 shows the von Mises stress comparison for all materials at $L = 100\mu\text{m}$ at pull-in. Figure A.3 overlays all arc-length stress profiles.

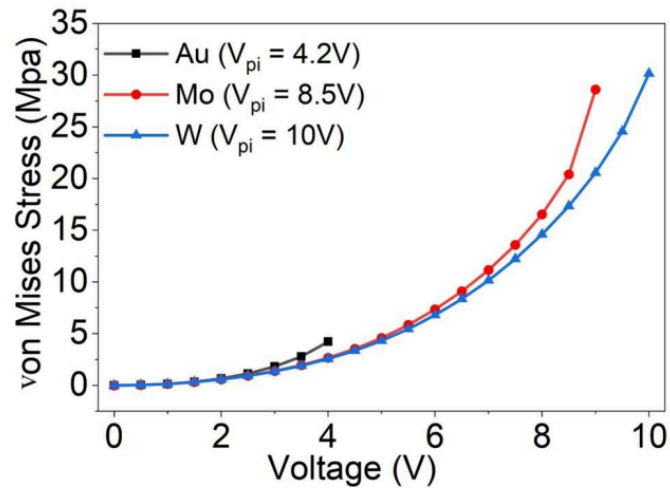


Fig. A.2: Von Mises stress vs. voltage for all materials at $L = 100\mu m$. Peak stress at pull-in: Au = 8.4MPa, Mo = 22.4MPa, W = 28.0MPa - all far below respective yield strengths.

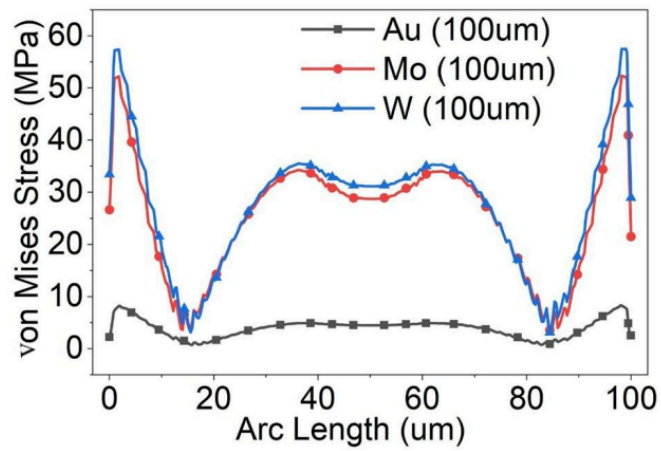


Fig. A.3: Arc-length vs. von Mises stress profiles for Au, Mo, and W fixed-fixed beams at $L = 100\mu m$ at pull-in. Triple-lobe pattern is identical in shape; absolute stress magnitude scales with material modulus.

A.3 Arc-Length vs. Displacement Profiles for All Materials at $L = 100\mu\text{m}$

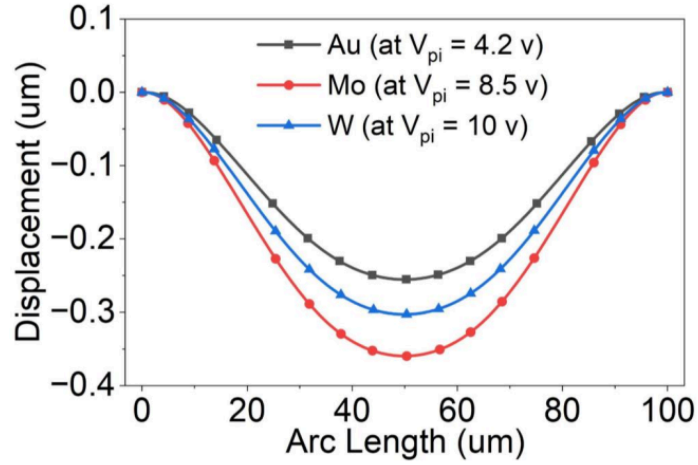


Fig. A.4: Arc-length vs. displacement profiles at pull-in for Au ($V_{pi} = 4.2\text{ V}$), Mo ($V_{pi} = 8.5\text{ V}$), and W ($V_{pi} = 10.0\text{ V}$) fixed-fixed beams at $L = 100\mu\text{m}$. All profiles terminate at mid-span deflection $= 0.3\mu\text{m} = g\alpha/3$.

A.4 COMSOL Model Geometry - U-Shape Cantilever (Reference)

Figure A.5 shows the COMSOL geometry of the U-shape cantilever configuration investigated in the companion publication, provided here for reference to illustrate the geometric contrast between the two actuator types.

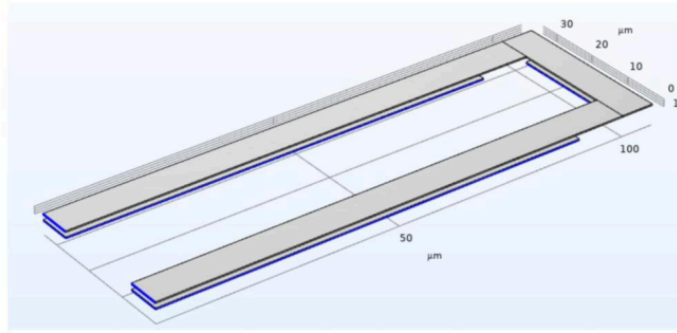


Fig. A.5: COMSOL geometry of the U-shape cantilever actuator (230 μm arm length). Fixed constraints are applied at the closed bridge end. The open arm tips deflect freely toward the substrate electrode. Compared with the fixed-fixed beam (Fig. 3.1), the U-shape geometry provides 14–50% higher pull-in voltage through the geometric stiffening contribution of the bridge.

APPENDIX B

COMSOL SIMULATION SETTINGS

B.1 Physics and Solver Settings

Table B.1: COMSOL Multiphysics simulation settings

Setting	Value / Description
COMSOL version	6.0 (or later)
Physics 1	Solid Mechanics (solid)
Physics 2	Electrostatics (es)
Multiphysics coupling	Electromechanical Forces
Moving mesh formulation	Arbitrary Lagrangian-Eulerian (ALE)
Study type	Stationary (quasi-static)
Parametric sweep	Voltage V: 0 to $V_{pi} + 0.5$ V, step 0.1 V
Nonlinear solver	Newton-Raphson (damped)
Convergence criterion	Relative residual $< 10^{-6}$
Damping factor (initial)	1.0 (reduced automatically on convergence issues)
Continuation method	Previous solution as initial guess at each step
Pull-in detection	Last converged step before divergence
Output	Displacement, von Mises stress, electric field at each step

B.2 Material Property Inputs

Table B.2: COMSOL material property entries for all three structural materials

Property	Au	Mo	W	Unit
Young's Modulus	79e9	329e9	411e9	Pa
Poisson's Ratio	0.44	0.31	0.28	—
Density	19300	10200	19300	kg/m ³
Relative permittivity (beam)	1.0 (conductor)	1.0 (conductor)	1.0 (conductor)	—
Air permittivity	8.854e-12	8.854e-12	8.854e-12	F/m
Initial gap g_0	0.5e-6	0.5e-6	0.5e-6	m

Journal Publication

Part of the work presented in this thesis has been submitted for publication as:

P. Gupta, V. Gautam, B. Prasad, and P. Sharma, "Pull-in Voltage Analysis of Fixed-Fixed Beam and U-Shape Cantilever MEMS Actuators: A Comparative Finite Element Study," *Journal of Nano- and Electronic Physics*, submitted May 2026.

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