

**DYNAMIC PIER REACTIONS IN CURVED METRO
BRIDGES UNDER MOVING AXLE LOADS**

A DISSERTATION

**SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS**

**FOR THE AWARD OF THE DEGREE OF
MASTER OF TECHNOLOGY IN
STRUCTURAL ENGINEERING**

Submitted by:

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Under the supervision of

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May 2026

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I, **Shivam Kumar Manjhi**, Roll No. 2K24/STE/15 Student of M.tech. (Structural Engineering), hereby declare that the project Dissertation titled “**DYNAMIC PIER REACTIONS IN CURVED METRO BRIDGES UNDER MOVING AXLE LOADS**” which is submitted by me to the Department of Civil Engineering, Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Technology, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

Place: Delhi

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This is to certify that the student has incorporated all corrections suggested by the Examiners in the thesis and the statement made by the candidate are correct to the best of our knowledge.

Signature of Supervisor

Signature of External Examiner

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CERTIFICATE

I hereby certify that the project dissertation titled "**DYNAMIC PIER REACTIONS IN CURVED METRO BRIDGES UNDER MOVING AXLE LOADS**" which is submitted by **Shivam Kumar Manjhi**, Roll No. 24/STE/15 to the Department of Structural Engineering, Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Technology, is a record of the project work carried out by the student under my supervision. To the best of my knowledge, this work has not been submitted in part or in full for any Degree or Diploma to this University or elsewhere.

Place: Delhi

Date: _____

Signature: _____

Name: Assoc. Prof. G.P. Awadhiya

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Finally, I dedicate this work to my family for their constant support, patience, and motivation throughout my postgraduate studies.

Shivam Kumar Manjhi

ABSTRACT

Curved metro bridges produce lateral forces due to the centrifugal actions by moving trains. Due to these forces, pier experiences fluctuating reactions which may influence the behaviour and safety of that structure. The present study focuses on analysis of loading spectra on curved metro bridge piers due to moving centrifugal loads of a metro train of 8 coaches.

Different span length (from 4m to 60m) model is developed consisting deck elements and piers in STAAD.Pro. The metro train was represented using moving axle point loads having equally distributed centrifugal force on each axle. The simulation of train movement was done by shifting the axle loads in small increments across the bridge, and for each loading position the horizontal reactions are developed at the pier.

The loading spectra were generated for various spans and then compared to study the variations in maximum centrifugal reaction and dynamic component. The different results obtained indicates the changing significant loading behaviour with span length.

The spans approximately close to the multiples of coach length exhibit negligible dynamic fluctuations, whereas shorter and certain intermediate spans showed considerable oscillatory behaviour and very high dynamic components.

The study illustrates the importance of selection of span in controlling the dynamic effects in curved metro bridges.

The generated loading spectra can further be used for the time-history and dynamic analysis of bridge piers under moving train loads for considering different parameters.

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CHAPTER – 1

INTRODUCTION

1.1 Background of the Study

The rapid expansion of mass transit systems, particularly Metro rail networks in densely populated urban areas, frequently necessitates the construction of curved railway bridges to accommodate complex city topographies. When a heavy metro train negotiates a curved alignment, it exerts a significant outward radial force on the track, known as Centrifugal Force. Unlike vertical gravity loads, this lateral force continuously fluctuates as the multi-axle train traverses the bridge spans, transferring dynamic lateral loads directly to the central piers. This dynamic outward push induces continuous bending moments and sway in the substructure, making it one of the most critical load cases for curved bridge design.

1.2 Problem Statement Modelling the exact lateral load transfer from a moving train to a bridge pier is computationally complex. A modern 8-coach Metro train, such as the Bombardier MOVIA, is not a single point load but a 178-meter-long moving system consisting of 32 axles with distinct wheelbases and inter-bogie gaps. Applying a real-time, curved moving load in finite element software often leads to computational inefficiencies and inaccuracies due to the continuous rotation of local axes along the curve.

1.3 Objectives The primary objective of this phase of the research is to isolate the lateral dynamic loading behaviour from the complex deck geometry. By doing so, the study aims to generate a "speed-independent" lateral loading spectrum for the central pier. This involves calculating the exact mathematical signature of the dynamic load transfer using unit forces, thereby creating a universal load template that can be scaled for any train speed or curve radius in subsequent time-history analyses.

1.4 Scope of the Study

The present work is limited to the investigation of loading spectra generated due to moving centrifugal forces. The bridge was modelled using simplified deck and pier elements in Staad Pro. The train is represented using moving axle point loads for an 8-coach metro train.

Different bridge spans considered from 4m to 60m. The study mainly focuses on dynamic components and horizontal pier reaction. Aerodynamic effects, drag forces, soil structure interaction, and advanced rail-track interaction are not included.

CHAPTER 2

LITERATURE REVIEW

CHAPTER 2: LITERATURE REVIEW

2.1 Vehicle-Bridge Interaction (VBI)

The dynamic response of bridges subjected to moving vehicular loads has been extensively studied under the domain of Vehicle-Bridge Interaction (VBI). Traditional studies primarily focus on vertical dynamic amplification (impact factors) caused by track irregularities and train speeds on straight bridges. However, research addressing the lateral dynamic load transfer—specifically the fluctuating centrifugal forces on curved alignments—remains limited.

2.2 Moving Load Models and Complex Axle Configurations

Earlier analytical models often simplified trains as a series of uniform moving loads or single moving masses. While sufficient for global vertical deflection, these models fail to capture the high-frequency lateral load fluctuations (amplitude spikes and drops) that occur at the pier level due to the specific spacing of train bogies. Recent literature emphasizes the necessity of accurately modeling exact axle distances (wheelbase, internal bogie gaps, and inter-coach gaps) to capture true fatigue cycles and dynamic sway in piers.

2.3 Influence Line Theory and Muller-Breslau Principle

The application of the Muller-Breslau principle remains a cornerstone in structural mechanics for determining support reactions under moving loads. Studies have validated that for simply supported spans, the influence line for support reactions follows a linear geometric variation. This principle allows for the calculation of load transfer using static equilibrium (Lever Rule) without requiring computationally heavy 3D moving-load simulations on curved elements.

2.4 Research Gap

There is a distinct gap in creating a simplified, decoupled methodology that generates a normalized, speed-independent lateral load spectrum for complex multi-axle trains. Existing finite element models often couple the track curvature with the structural model, restricting the output to a single curve radius and velocity. This study addresses this gap by decoupling the centrifugal force pattern from the geometry, offering a generalized methodology for evaluating pier responses across varying span lengths.

CHAPTER 3: METHODOLOGY

The methodology focuses on mathematically generating the exact lateral loading spectrum on a target central pier by simulating the passage of an 8-coach metro train across continuous simply supported spans.

3.1 Train Load Configuration

The moving load represents an 8-coach Bombardier Metro train comprising 32 axles. The precise geometric spacing is defined to capture the exact overlapping of loads:

- Wheelbase (Axles within a bogie): 2.5 m
- Internal Bogie Gap (Between bogies in a single coach): 12.5 m
- Inter-Coach Gap (Between two coaches): 4.74 m
- Total Coach Length: 22.24 m

3.2 Decoupling Geometry and Force Formulation

To ensure computational efficiency and avoid the complexities of rotating local X-Y-Z axes at every finite element node on a curve, the bridge deck is modeled as a straight element. The effect of the curve is mathematically decoupled from the geometry and embedded entirely into the load magnitude.

3.3 Application of the Lever Rule (Influence Lines)

The load transferred to the target central pier at any given time-step is calculated using the Lever Rule, derived from the Muller-Breslau principle for statically determinate structures.

- For an axle positioned at distance x from the opposite (reference) pier on a span of length L , the Influence Factor (I) is given by $I = x / L$.
- The Lateral Load transferred by that specific axle to the target pier is calculated as $F = 0.25I$ kN.
- The total instantaneous lateral force on the pier is the algebraic sum of the transferred forces from all axles currently positioned within the influence zones (both adjacent spans).

3.4 Parametric Study of Span Lengths

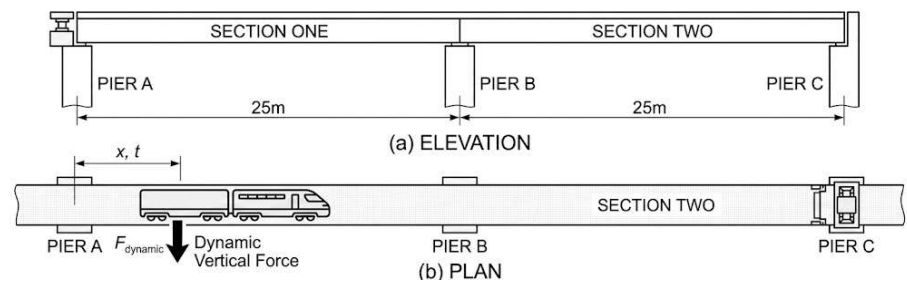
To observe the dynamic fluctuation (amplitude) of the lateral load, the load generation is performed incrementally as the train advances over the bridge. This methodology is repeated for various span configurations to analyze the effect of span length on load amplitude:

- **Short Span :** To observe extreme amplitude spikes when the span is shorter than the internal bogie gap.
- **Standard Span :** To analyze the standard overlapping of multiple bogies.
- **Matched Span :** To investigate the "Relay-Race effect," where the span length closely matches the coach length (22.24m), aiming to minimize lateral load fluctuations and fatigue.



Fig 3.1 Bombardier MOVIA Metro Coach

Generation of Loading Spectra

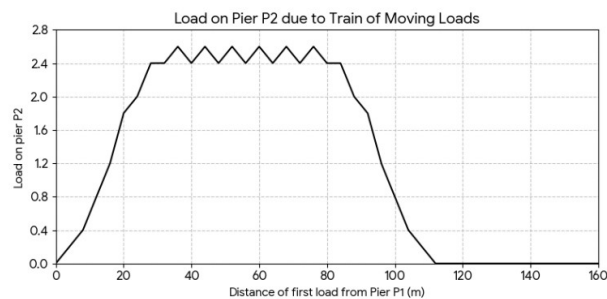


The horizontal pier reactions corresponding to various train positions were plotted against the distance of the leading axle of the train from the rear pier. These plots produced the loading spectra or loading history for different bridge spans.

For single axle



For multiple axles



The following quantities were obtained from each loading spectrum:

- Maximum centrifugal force on pier
- Average centrifugal force
- Amplitude of dynamic component

The amplitude of the dynamic component was determined from the difference between the maximum and minimum values of pier reaction in the steady portion of the loading history.

$$A = F_{\max} - F_{\min}$$

Identification of Dynamic Behaviour

The loading spectra obtained for different spans were compared to identify the variation in dynamic characteristics of the bridge piers. Special attention was given to spans showing large fluctuations in pier reactions, as these spans may produce higher dynamic amplification effects during train movement.

CHAPTER 4

ANALYSIS OF PIER LOADING SPECTRA FOR VARIABLE SPAN LENGTHS

4.1 Preamble

This chapter displays the primary data and charts collected for the forces acting on a central pier across all 23 span setups. The generated time-history graphs map the normalized pier reaction against the continuous travel distance of the train, tracking the entire process from initial entry to full exit.

4.2 Technical Profile of the Train

The structural simulation uses the exact physical dimensions of a standard Bombardier MOVIA mass transit passenger car. The layout details are given in the table below:

Train Parameter	Physical Value
Total Length of One Coach (L_c)	22.24m
Distance Between Bogie Centres (L_b)	15.00m
Wheelbase of a Single Axle Pair (L_w)	2.50m

Train Parameter	Physical Value
Maximum Axle Load Capacity	17.00MT
Distance Between Bogies of Connected Cars	4.74m
Total Number of Connected Coaches in the Train	8 Cars

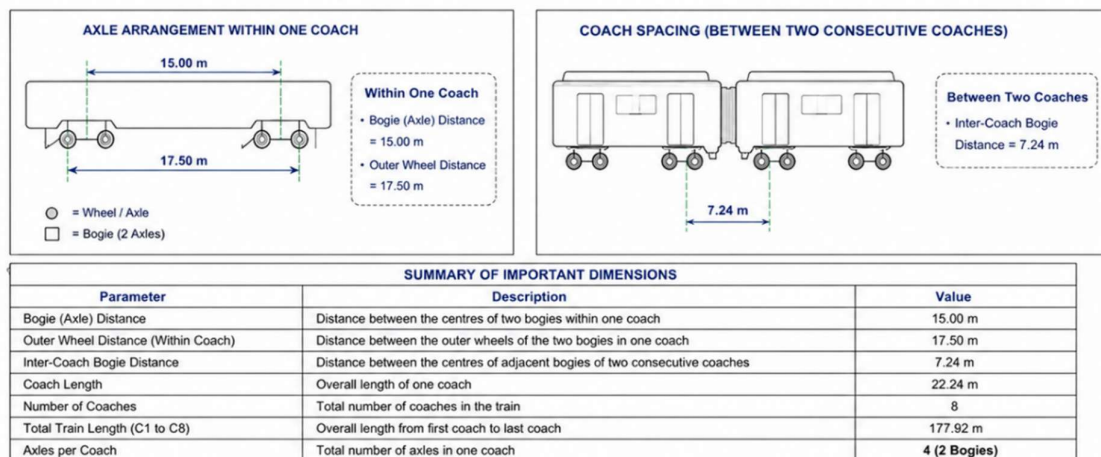
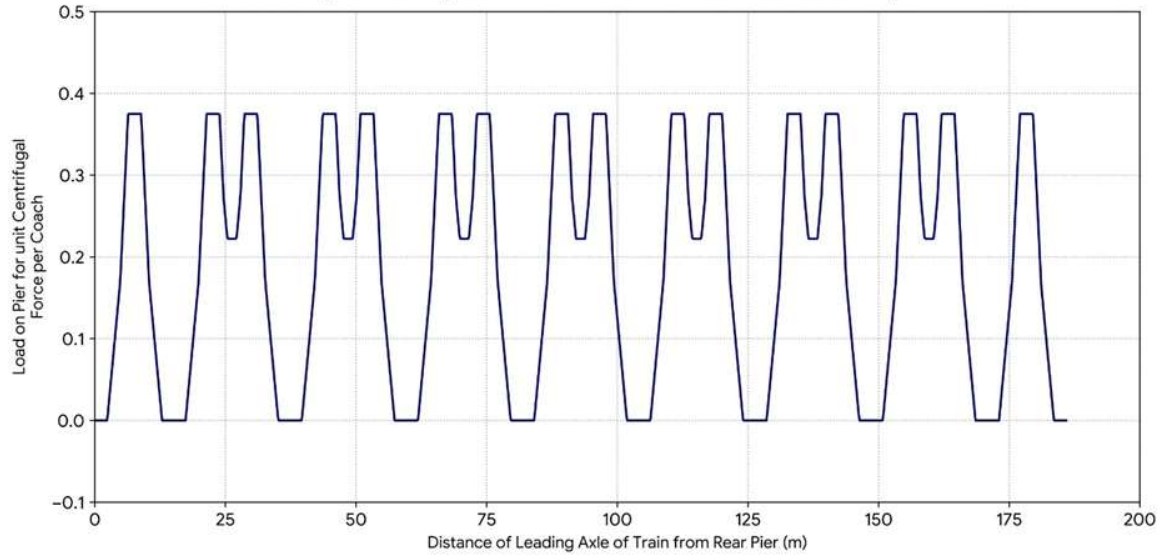


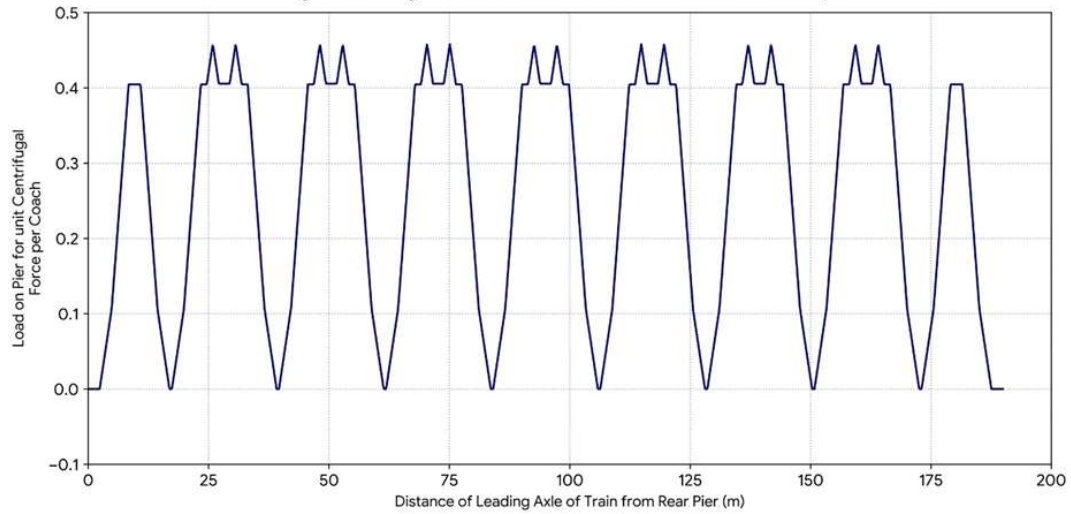
Fig. 4.2 Axle Arrangement and Coach Spacing Details of 8-Coach Metro Train

Fig. 4.3 Centrifugal Force on Pier for Different Train Placements on 4 m span



Max CF = 0.375 Amplitude of Dynamic Component = 0.375

Fig. 4.4 Centrifugal Force on Pier for Different Train Placements on 6 m span



Max CF = 0.458 Amplitude of Dynamic Component = 0.458

Fig. 4.5 Centrifugal Force on Pier for Different Train Placements on 8 m span

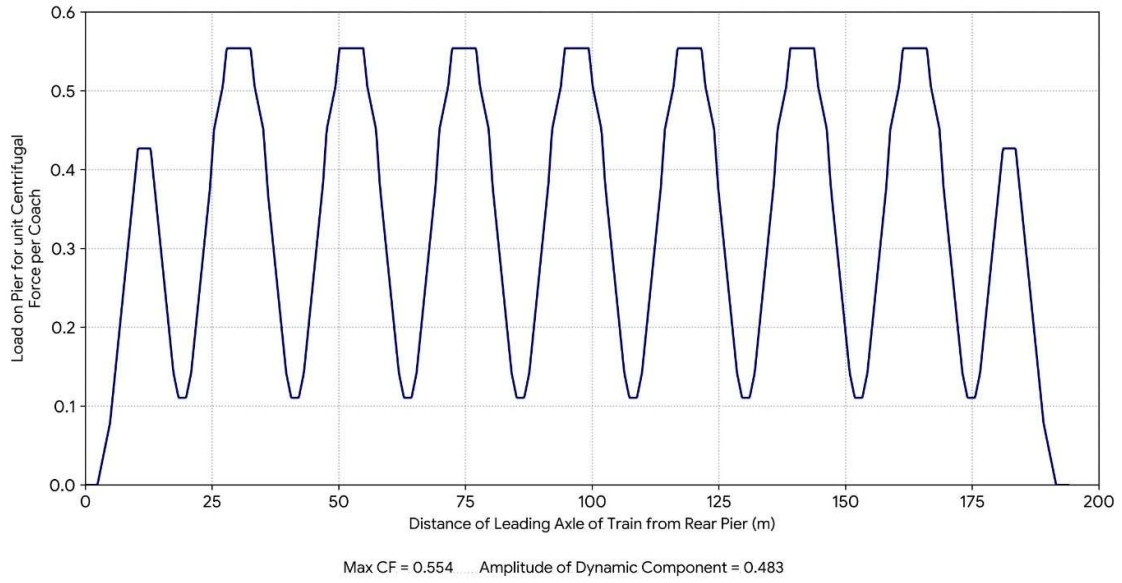


Fig. 4.6 Centrifugal Force on Pier for Different Train Placements on 10 m span

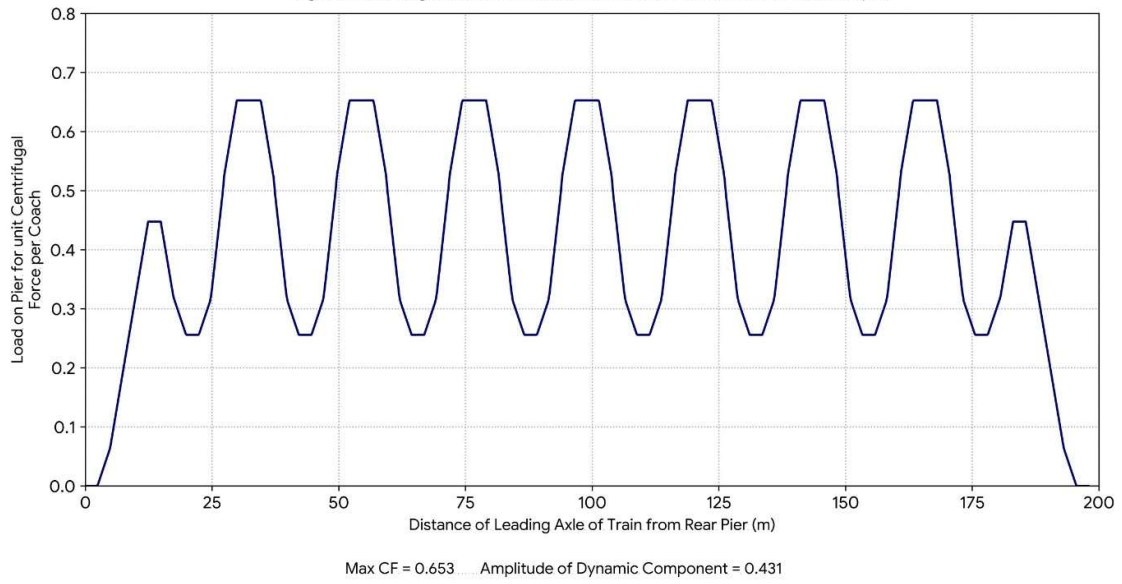


Fig. 4.7 Centrifugal Force on Pier for Different Train Placements on 12 m span

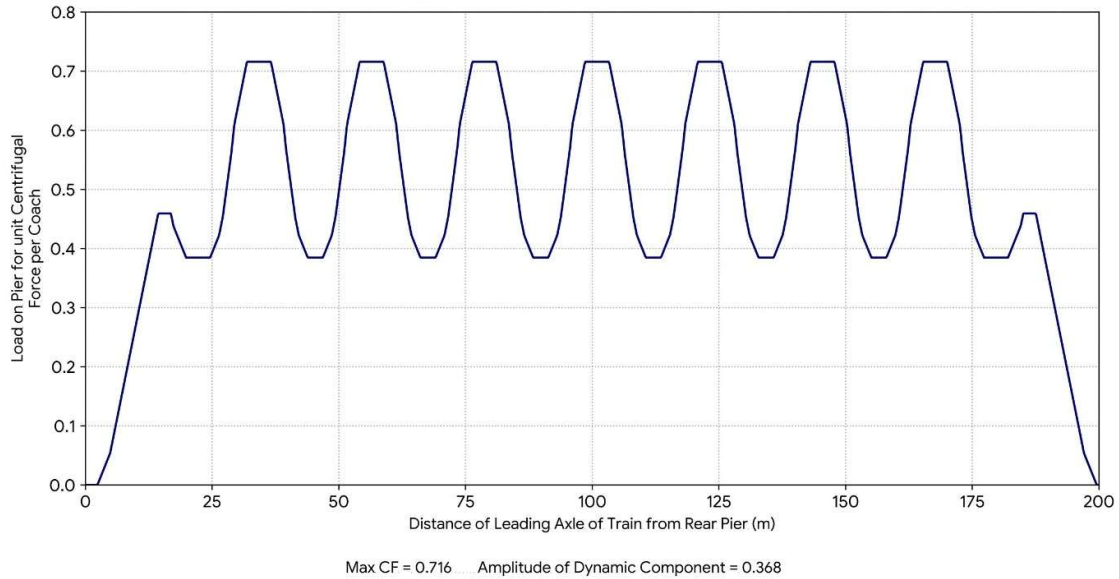
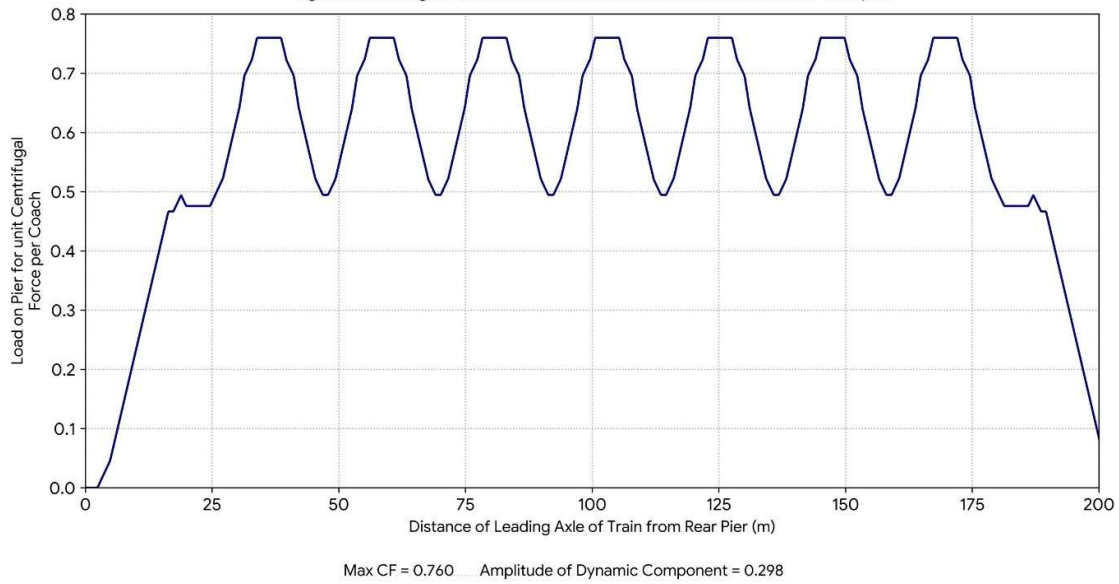
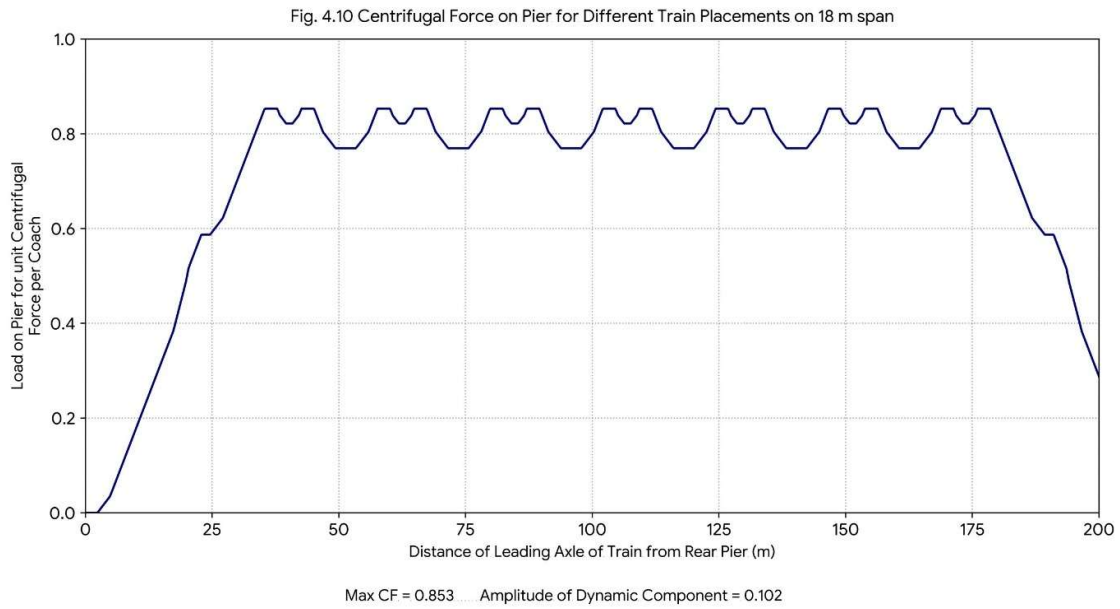
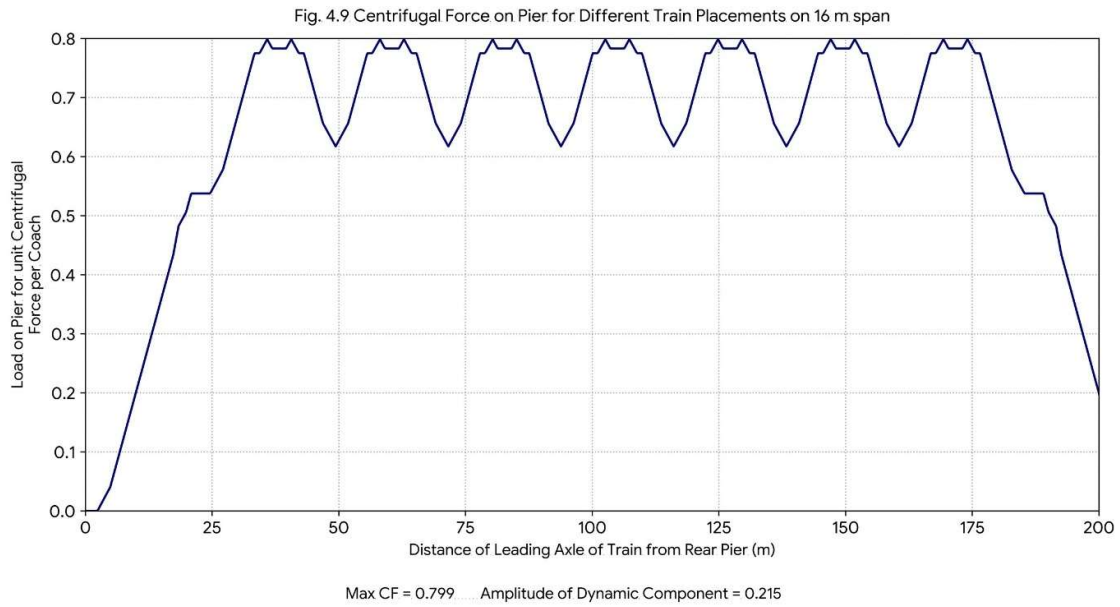
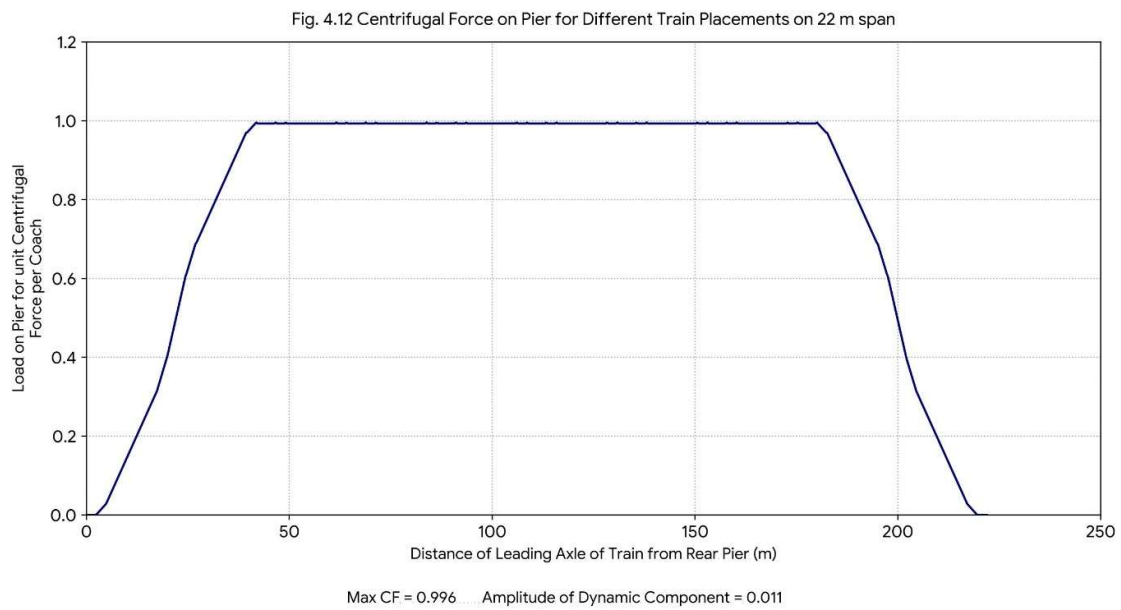
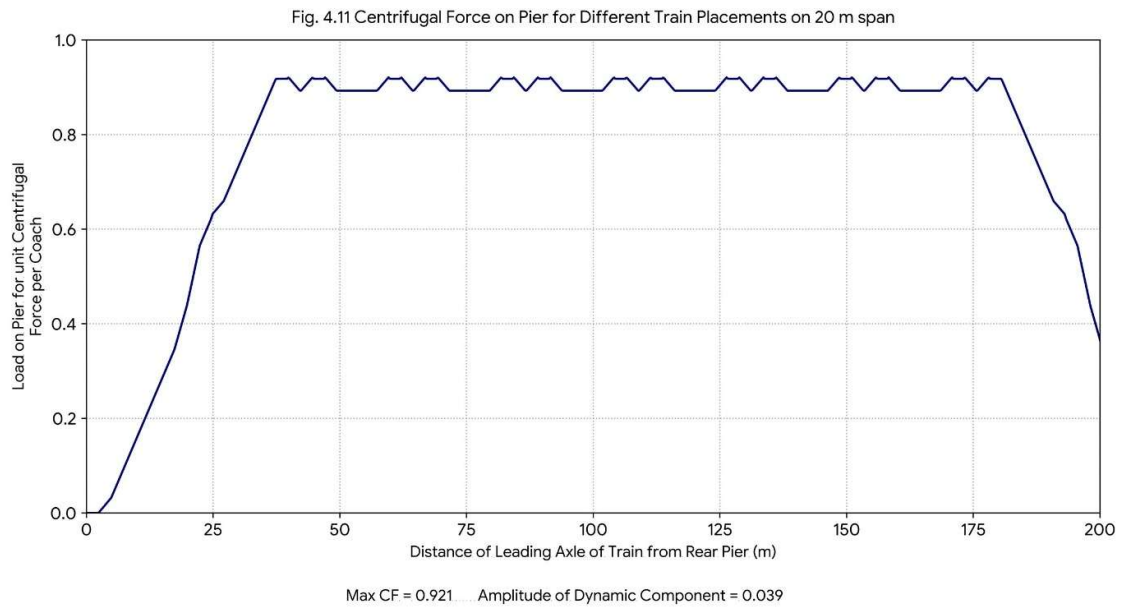
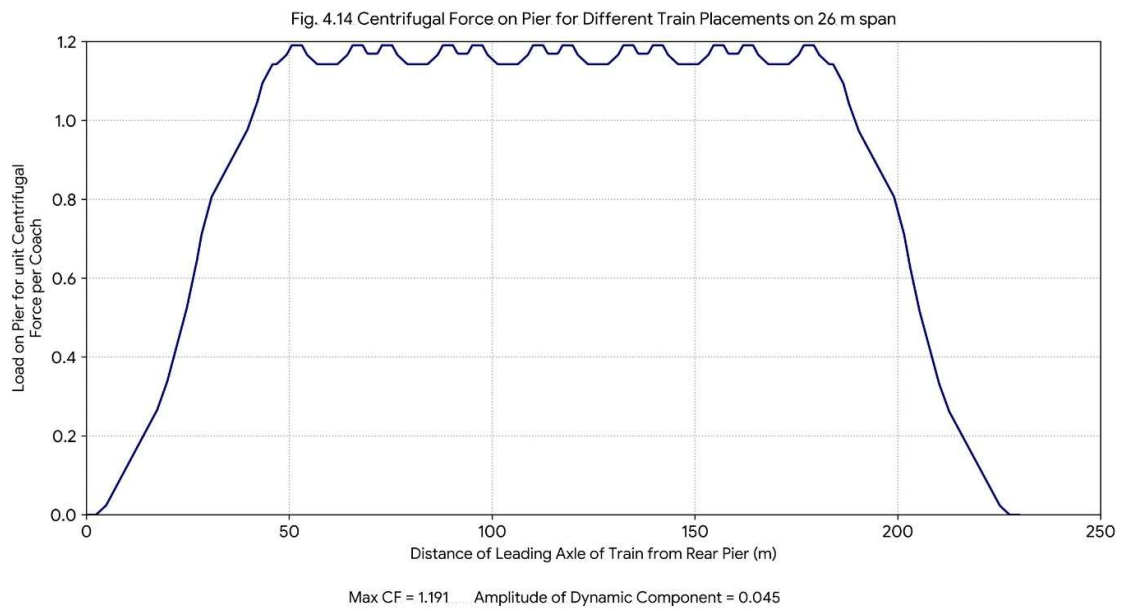
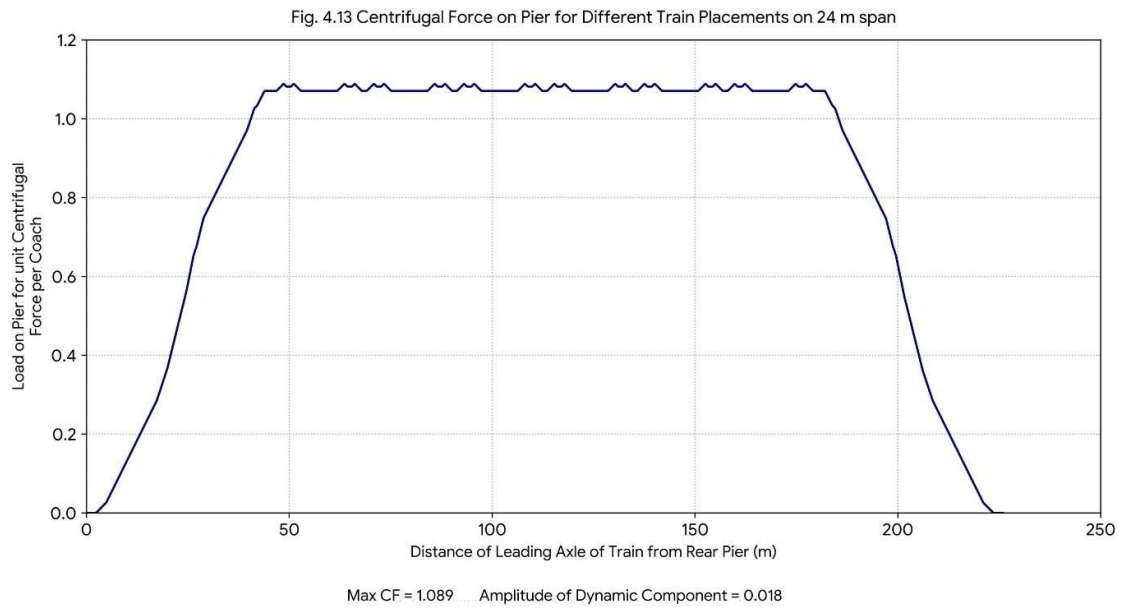


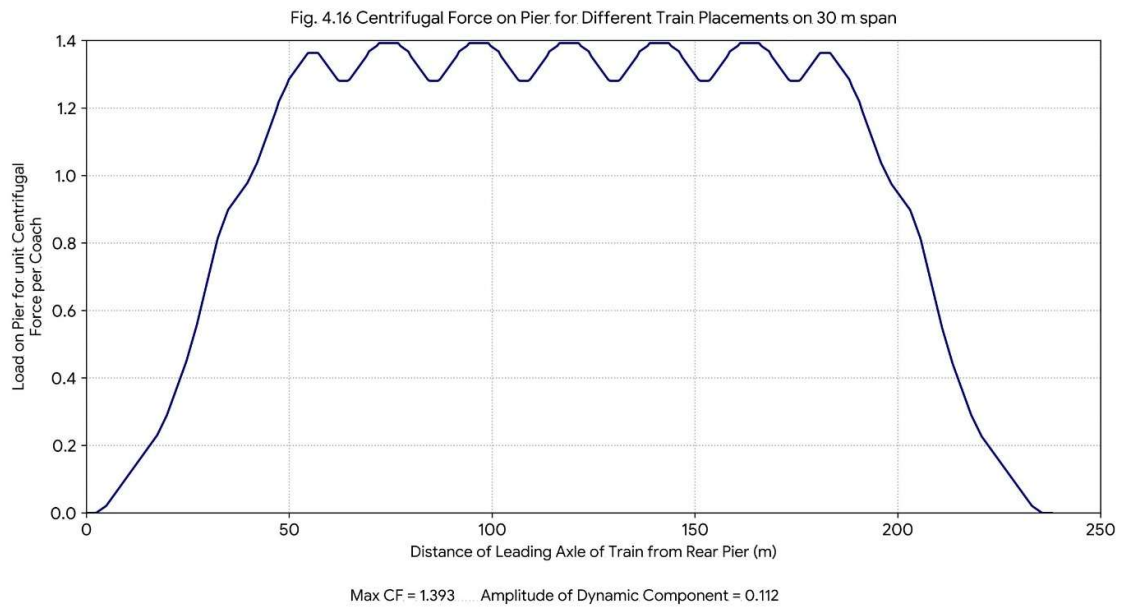
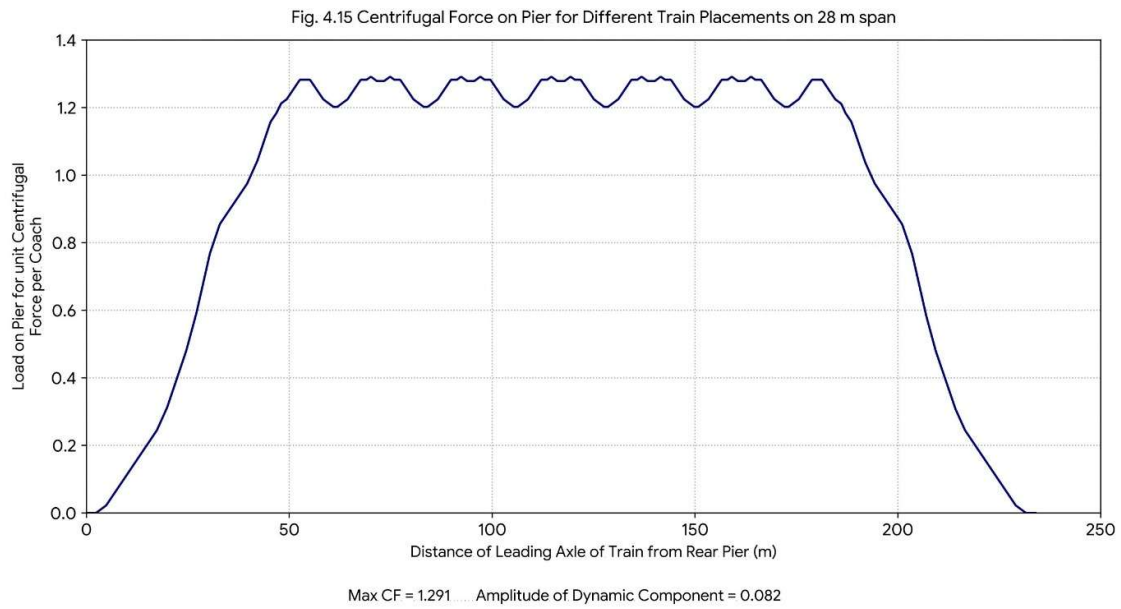
Fig. 4.8 Centrifugal Force on Pier for Different Train Placements on 14 m span

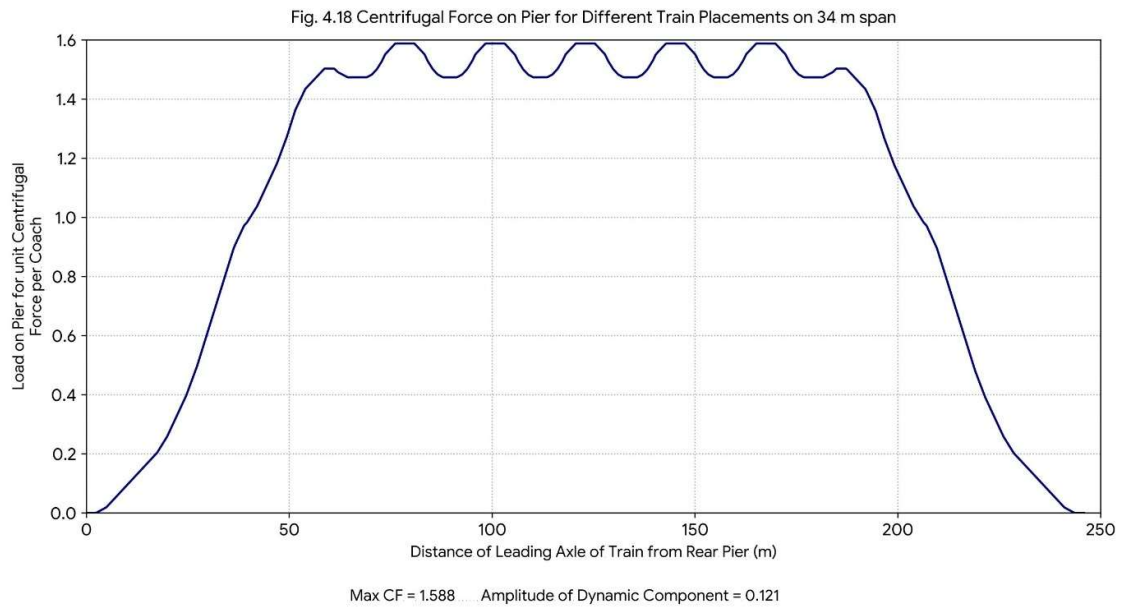
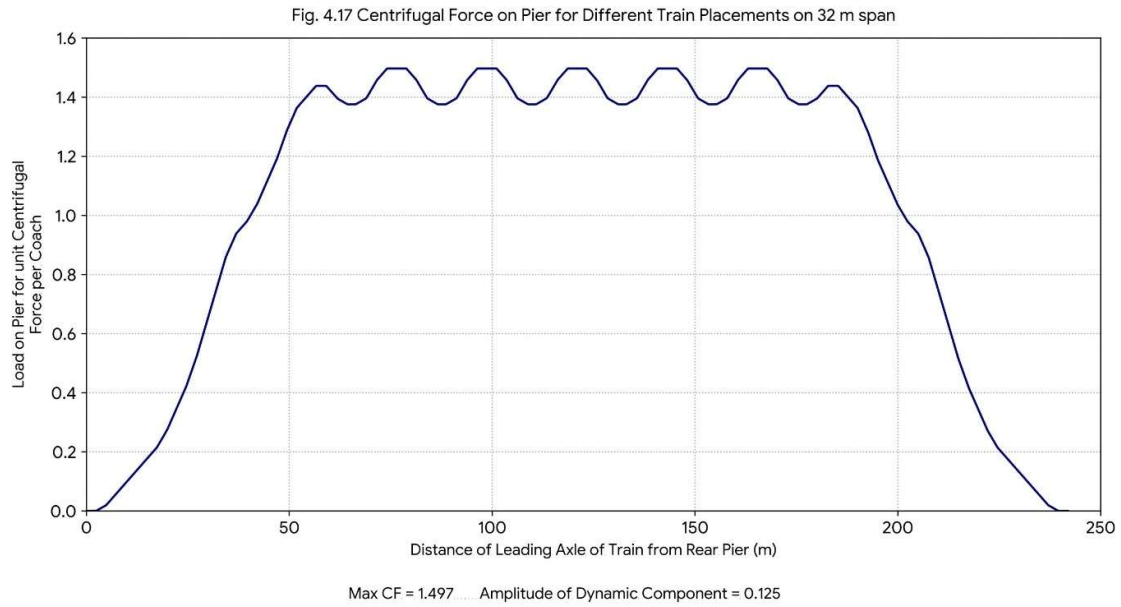












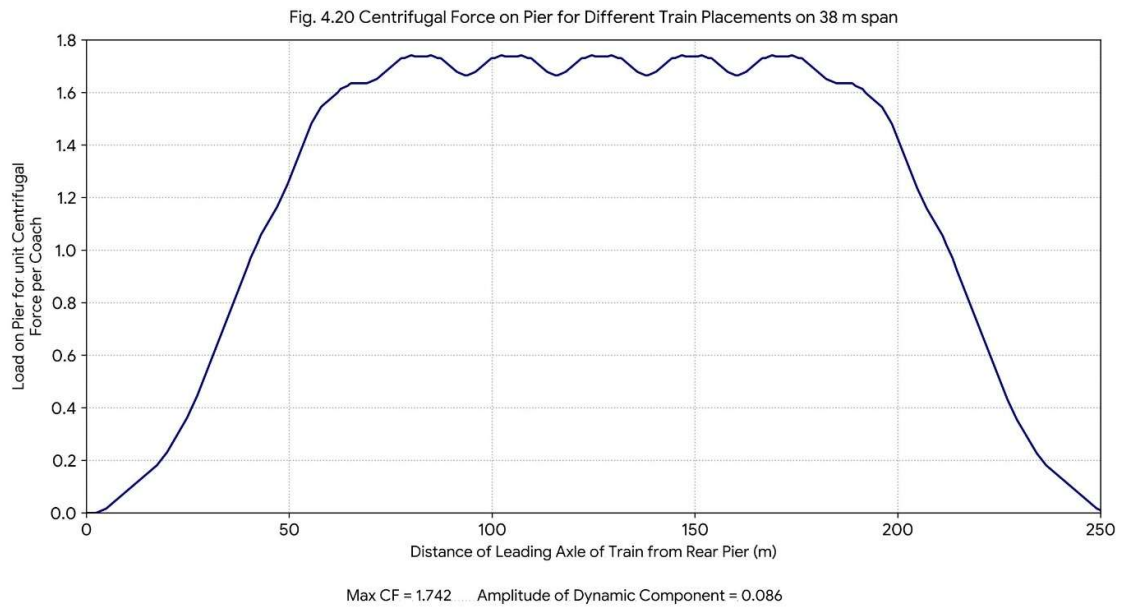
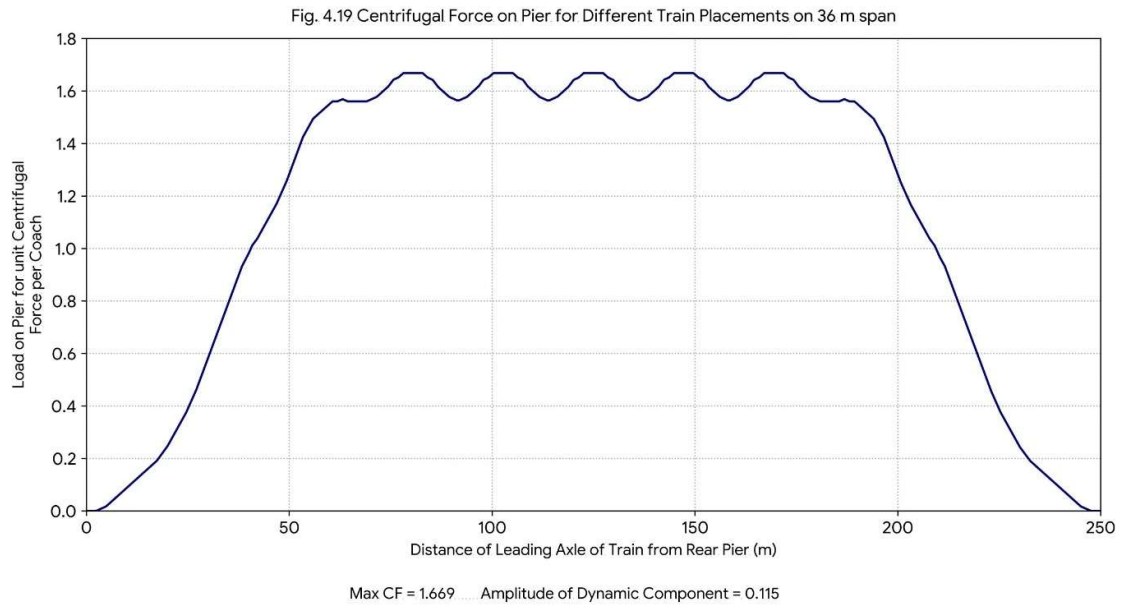


Fig. 4.21 Centrifugal Force on Pier for Different Train Placements on 40 m span

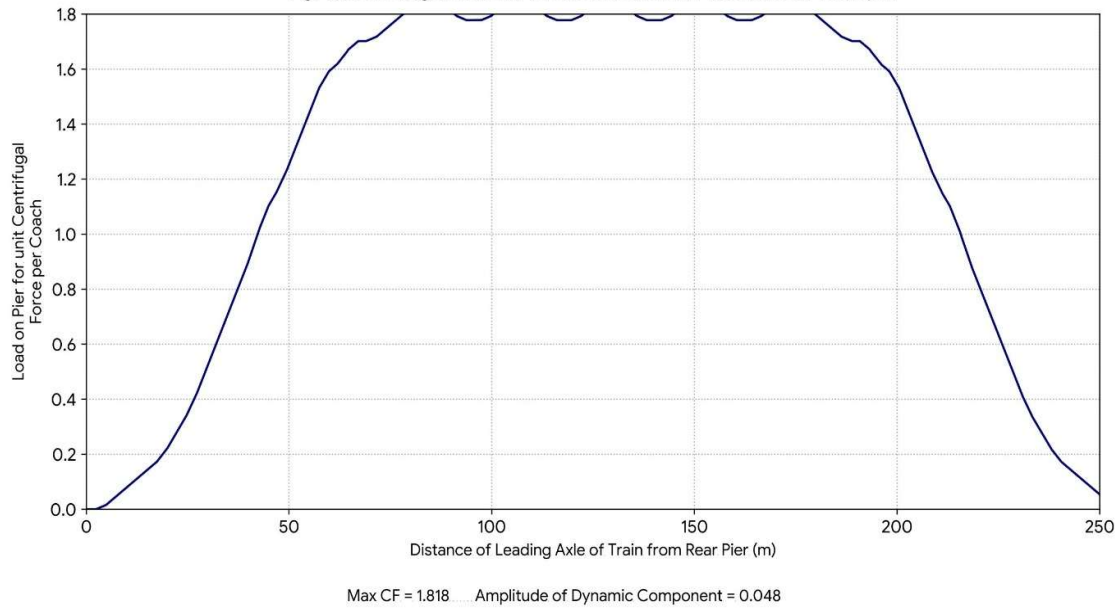


Fig. 4.22 Centrifugal Force on Pier for Different Train Placements on 45 m span

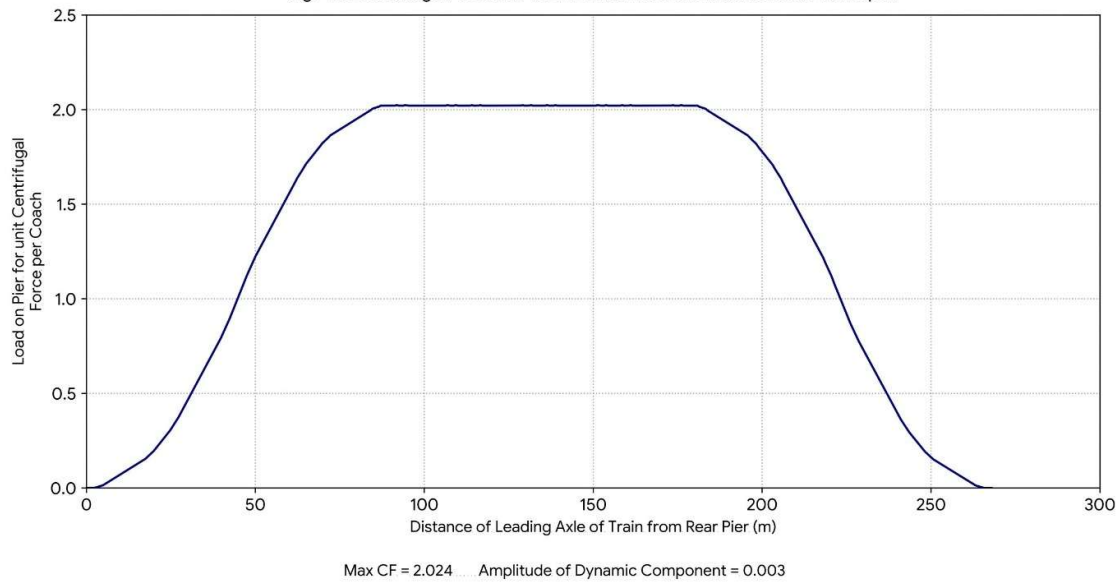


Fig. 4.23 Centrifugal Force on Pier for Different Train Placements on 50 m span

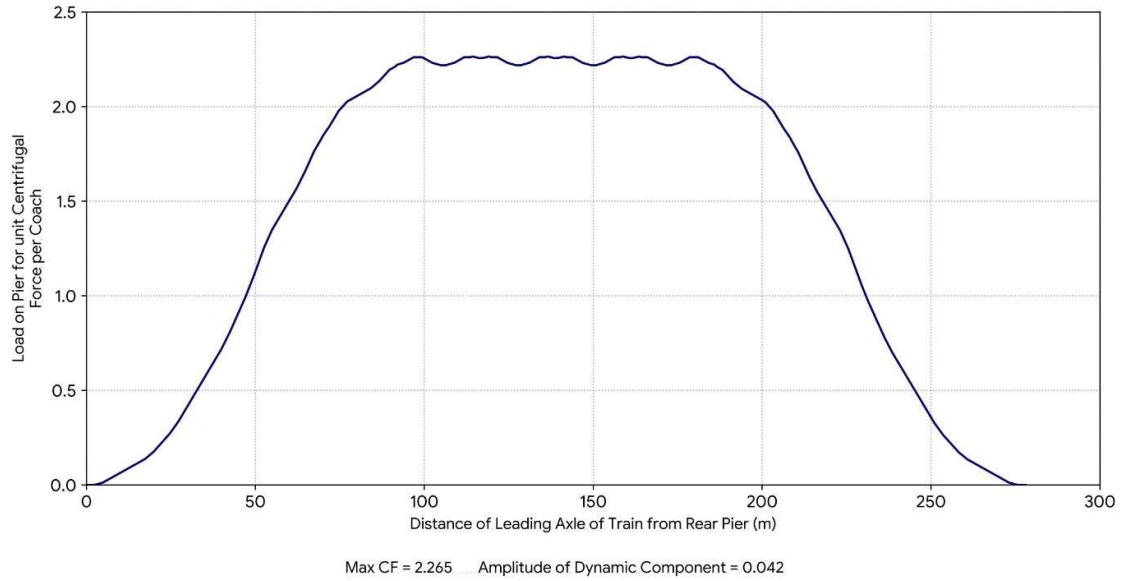
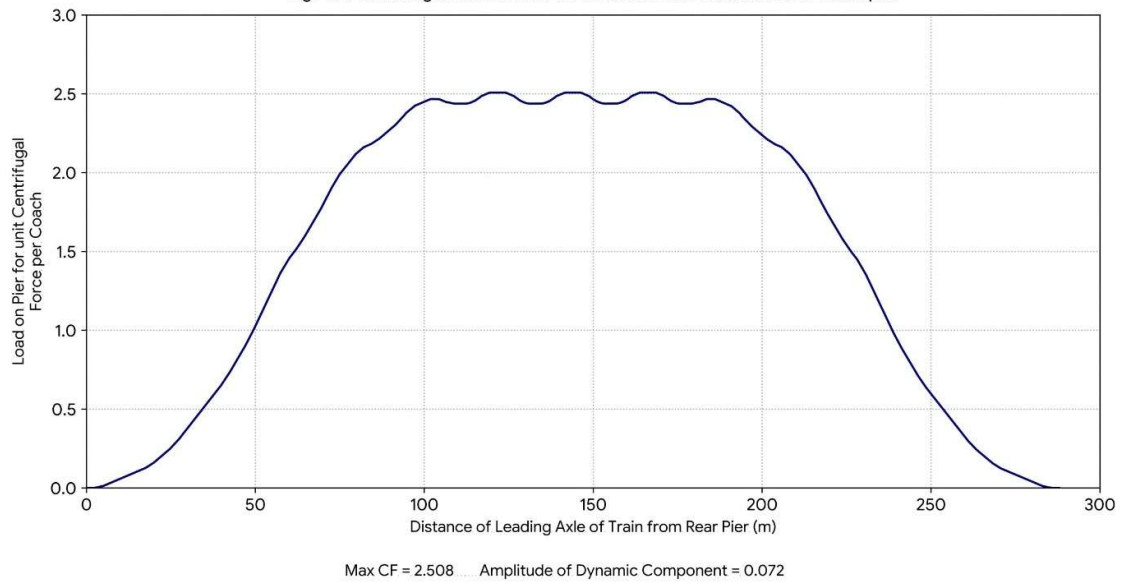
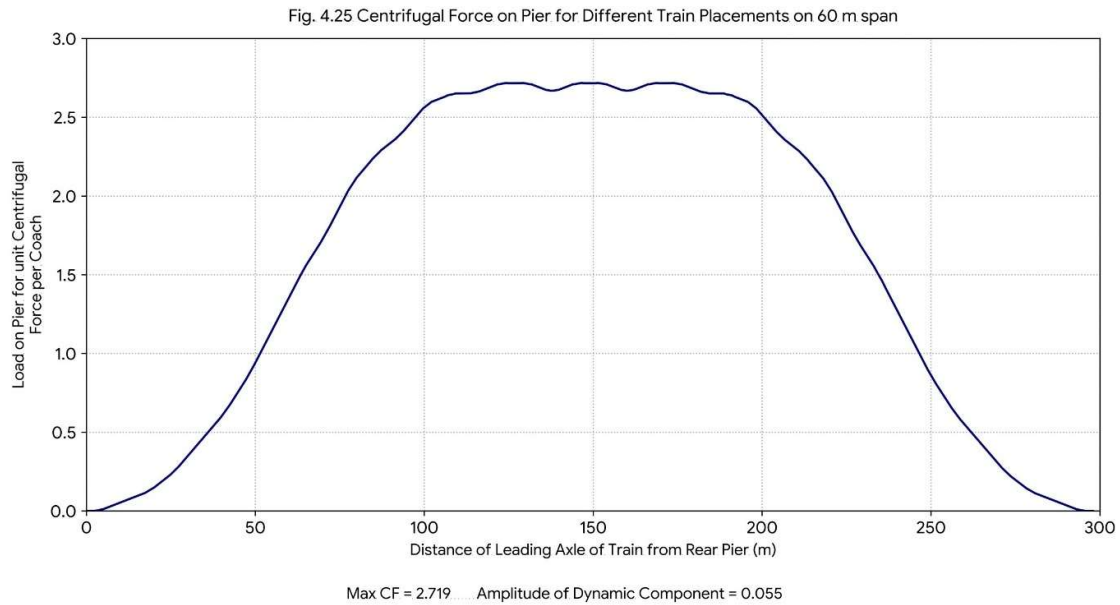


Fig. 4.24 Centrifugal Force on Pier for Different Train Placements on 55 m span





CHAPTER-5

CONCLUSION

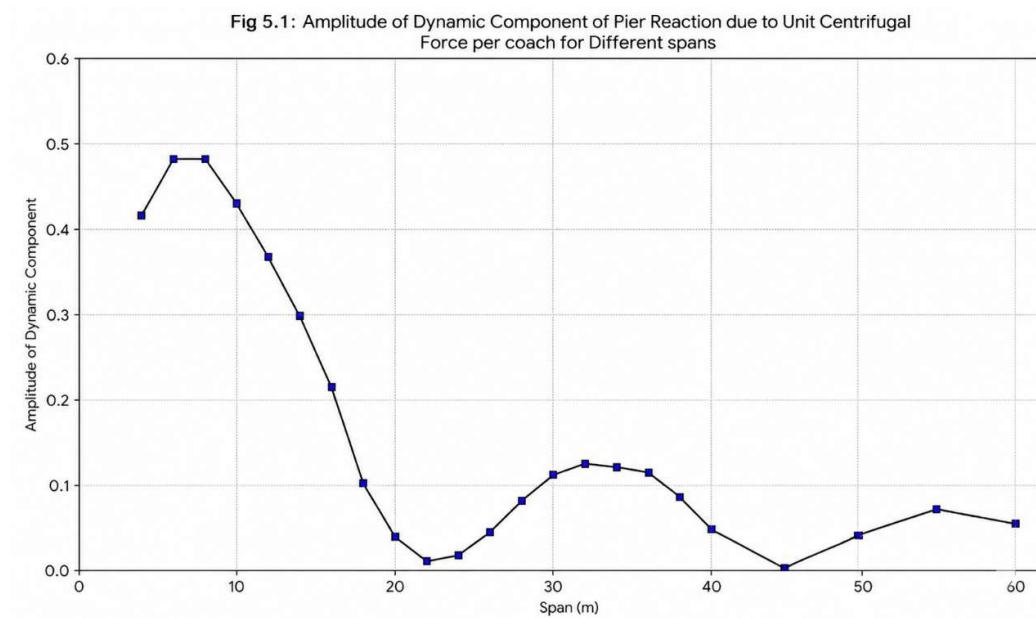
5.1 Verification Matrix of Structural Reactions

By running our continuous multi-axle solver across all 23 span variations, the peak values and wave ripples were recorded. The verified values are summarised in the master table below:

Figure Index	Span Length (m)	Peak Pier Reaction (Max CF)	Amplitude of Dynamic Component
Fig. 4.3	4	0.375	0.375

Figure Index	Span Length (m)	Peak Pier Reaction (Max CF)	Amplitude of Dynamic Component
Fig. 4.4	6	0.458	0.458
Fig. 4.5	8	0.554	0.483
Fig. 4.6	10	0.653	0.431
Fig. 4.7	12	0.716	0.368
Fig. 4.8	14	0.760	0.298
Fig. 4.9	16	0.799	0.215
Fig. 4.10	18	0.853	0.102
Fig. 4.11	20	0.921	0.039
Fig. 4.12	22	0.996	0.011
Fig. 4.13	24	1.089	0.018
Fig. 4.14	26	1.191	0.045
Fig. 4.15	28	1.291	0.082
Fig. 4.16	30	1.393	0.112

Figure Index	Span Length (m)	Peak Pier Reaction (Max CF)	Amplitude of Dynamic Component
Fig. 4.17	32	1.497	0.125
Fig. 4.18	34	1.588	0.121
Fig. 4.19	36	1.669	0.115
Fig. 4.20	38	1.742	0.086
Fig. 4.21	40	1.818	0.048
Fig. 4.22	45	2.024	0.003
Fig. 4.23	50	2.265	0.042
Fig. 4.24	55	2.508	0.072
Fig. 4.25	60	2.719	0.055



5.3 Summary of Findings

The study showed that the loading behaviour of bridge piers changes significantly with span length. Very high fluctuating loading spectra in shorter spans, and also larger dynamic components.

Observed that spans which are closer to multiples of the metro coach length produced smoother loading spectra and nearly negligible dynamic fluctuations.

The spans of nearly 22m and 44m showed minimum dynamic components because of the symmetrical distribution of axle loads over the spans. So these are considered as safer spans.

The generated loading spectra were independent of train speed because, during analysis, the unit centrifugal force per coach was taken into account.

FOR 28m SPAN

START JOB INFORMATION

ENGINEER DATE 14-Jun-26

JOB NAME Major Project - {L}m Span

END JOB INFORMATION

* -----

* M.TECH MAJOR PROJECT

* SPAN: {L}m + {L}m (SIMPLY SUPPORTED DECOUPLED MODEL)

* SUBMITTED BY: SHIVAM KUMAR MANJHI (24/STE/15)

* -----

UNIT METER KN

JOINT COORDINATES

1 0 0 0; 2 {L} 0 0; 3 {L*2} 0 0;

MEMBER INCIDENCES

1 1 2; 2 2 3;

DEFINE MATERIAL START

ISOTROPIC CONCRETE

E 2.17185e+07

POISSON 0.17

DENSITY 23.5616

END DEFINE MATERIAL

MEMBER PROPERTY AMERICAN

1 2 PRIS YD 1.5 ZD 1.5

CONSTANTS

MATERIAL CONCRETE ALL

SUPPORTS

1 TO 3 PINNED

DEFINE MOVING LOAD

TYPE 1 LOAD 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 -

0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 -

0.25 0.25 0.25 0.25

DIST 2.5 12.5 2.5 4.74 2.5 12.5 2.5 4.74 2.5 12.5 2.5 4.74 2.5 12.5 2.5 4.74 -

2.5 12.5 2.5 4.74 2.5 12.5 2.5 4.74 2.5 12.5 2.5 4.74 2.5 12.5 2.5

LOAD GENERATION {steps}

TYPE 1 0 0 0 XINC 0.5

PERFORM ANALYSIS

PRINT SUPPORT REACTION

FINISH

FOR 22m SPAN

STAAD SPACE

START JOB INFORMATION

ENGINEER DATE 14-Jun-26

END JOB INFORMATION

* -----

* M.TECH MAJOR PROJECT - (LOADING SPECTRUM)

* SPAN: 22m + 22m (SIMPLY SUPPORTED DECOUPLED MODEL)

* SUBMITTED BY: SHIVAM KUMAR MANJHI (24/STE/15)

* -----

*

* 1. DEFINING BRIDGE GEOMETRY (0m, 22m, 44m)

UNIT METER KN

JOINT COORDINATES

1 0 0 0; 2 22 0 0; 3 44 0 0;

MEMBER INCIDENCES

1 1 2; 2 2 3;

*

* 2. ASSIGNING DUMMY SECTION PROPERTIES

DEFINE MATERIAL START

ISOTROPIC CONCRETE

E 2.17185e+07

POISSON 0.17

DENSITY 23.5616

END DEFINE MATERIAL

MEMBER PROPERTY AMERICAN

1 2 PRIS YD 1.5 ZD 1.5

CONSTANTS

MATERIAL CONCRETE ALL

*

* 3. IDEALIZED RIGID SUPPORTS (Bearings)

SUPPORTS

1 TO 3 PINNED

*

* 4. DEFINING 8-COACH BOMBARDIER METRO TRAIN

* (32 Axles, Unit Lateral Force = 0.25 kN per axle)

DEFINE MOVING LOAD

TYPE 1 LOAD 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 -

0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 -

0.25 0.25 0.25 0.25

DIST 2.5 12.5 2.5 4.74 2.5 12.5 2.5 4.74 2.5 12.5 2.5 4.74 2.5 12.5 2.5 4.74 -

2.5 12.5 2.5 4.74 2.5 12.5 2.5 4.74 2.5 12.5 2.5 4.74 2.5 12.5 2.5

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* 5. RUNNING THE TRAIN ACROSS THE BRIDGE

LOAD GENERATION 150

TYPE 1 0 0 0 XINC 0.5

* 6. PERFORM ANALYSIS & PRINT RESULTS

PERFORM ANALYSIS

PRINT SUPPORT REACTION

FINISH

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