

**MODELLING, ANALYSIS & CONTROL OF
SINGLE-INDUCTOR DUAL-OUTPUT DC-DC
BUCK CONVERTER SYSTEM**

A DISSERTATION

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CANDIDATE'S DECLARATION

I, MONISHA SINGH, Roll no. 24/C&I/04, student of M. Tech. (Control & Instrumentation), hereby declare that the project Dissertation titled “**Modelling, Analysis & Control of Single-inductor Dual-output DC-DC Buck Converter System**” which is submitted by me to the Department of Electrical Engineering, Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Technology, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

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ABSTRACT

The thesis contains a detailed study on the modelling, analysis and control of SIDO DC-DC buck converter system. The system is described in detail by means of state-space and small-signal modelling, leading to a multi-input multi-output transfer function matrix which represents the coupled system dynamics. These indexes, namely, the Relative Gain Array (RGA), the Hankel Interaction Index Array (HIIA) and the H_2 norm, are used to systematically quantify the interaction between outputs and demonstrate that the coupling is parameter dependent and frequency dependent, and that the coupling is very sensitive to variations in the load. To overcome the limitations of conventional control approaches, two control strategies are studied: a conventional PID controller based on equivalent Single-Input Single-Output (SISO) modelling, and a structured H-infinity based PID controller designed using the full multi-variable system representation.

Many simulation studies have been carried out to evaluate the performance of these controllers such as transient response, frequency domain analysis, stability assessment and load and line regulation. From the results obtained, the classical PID controller is easy to implement but it has some drawbacks when it comes to interaction and disturbance rejection and hence a higher overshoot and cross regulation effects is obtained. The structured H-infinity based PID controller, on the contrary gives better damping, settling time, robustness and cross coupling suppression of the outputs. In conclusion, the paper highlights the importance of interaction-aware modelling and strong control design in multi-output DC-DC converters, and it shows how the structured H-infinity based approach is a suitable and effective solution for achieving high performance applications.

Keywords: SIDO buck converter, MIMO systems, RGA, Hankel norm, H_2 -norm, pairing analysis, Cross-regulation, PID controller, Structured H-infinity Control, Robust Control, Load Regulation, Line Regulation.

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LIST OF SYMBOLS, ABBREVIATIONS AND NOMENCLATURE

V_g	Input supply voltage
V_{o1}	Output Voltage 1
V_{o2}	Output Voltage 2
i_L	Inductor current
L_1	Inductance
C_1	Output Capacitor 1
C_2	Output Capacitor 2
R_1	Load resistance at Output 1
R_2	Load resistance at Output 2
r_{L1}	Inductor series resistance
r_{C1}	ESR of Capacitor C_2
r_{C2}	ESR of Capacitor C_2
d_1	Duty Ratio of switch Q_1
d_2	Duty Ratio of switch Q_2
f_s	Switching Frequency
v_l	Inductor Voltage
x	State Vector
u	Control Input Vector
y	Output Vector
$G(s)$	Transfer Function Matrix
w	Disturbance input
z	Performance Output
K	Controller Gain

γ	H-infinity performance bound
T_{zw}	Closed Loop Transfer Function
T_f	Filter Time constant
H_2	H2 Norm
SIDO	Single-inductor Dual-output
SISO	Single-inductor Single-output
DC-DC	Direct Current to Direct Current
PWM	Pulse Width Modulation
ESR	Equivalent Series Resistance
KVL	Kirchoff's Voltage Law
KCL	Kirchoff's Current Law
MIMO	Multi-input Multi-output
RGA	Relative Gain Array
HIIA	Hankel Interaction Index Array
PID	Proportional-Integral Derivative

CHAPTER 1

INTRODUCTION

As a result of quick progress in modern electronics, there is an urgent need for more effective and compact power conversion technologies. One possible method of solving such issues is the use of multi-output DC–DC converters. This chapter introduces the motivation, background and objectives of the work, with focus on modelling, analysis and control of a Single-Input Dual-Output (SIDO) buck converter.

1.1 Motivation

With the growing demand of small, efficient and low-cost power conversion systems, multi-output DC-DC converters have received great interest for modern electronic applications. In particular, the Single-input Dual-output (SIDO) buck converter provides an attractive solution in the sense that one inductor is shared by two regulated outputs, thus reducing the total component count, size and cost of the overall system. Such converters are especially important in low power embedded systems, portable electronics and distributed supply architectures, where several voltage rails are often required from a single source. But the SIDO topology presents a fundamental control challenge due to the shared-energy transfer mechanism. Since the same inductor is used for both outputs, the regulation of one output has to affect the other output, resulting in large cross-coupling and interaction effects. This adds more complexity to the control problem compared to a standard single-output buck converter. If such interactions are not taken into account in the controller design, the steady-state regulation may be acceptable for nominal conditions, but the performance can be poor for load variations, parameter uncertainty and input disturbances. Therefore, it is required to systematically study the dynamics of the converter and its interaction characteristics for developing an effective control strategy.

Another major motivation of this work comes from the need of going beyond heuristic tuning of controllers and developing robust and practical control strategies. Although classical controllers like PID controllers are still widely used because of their simplicity and ease of implementation, their performance can degrade significantly in multi-output systems with coupled dynamics. On the other hand, robust control techniques such as structured H-infinity PID provide a more systematic framework to improve the closed-loop performance while maintaining a controller structure which is feasible for implementation. This allows the comparison of conventional and robust approaches in a common modelling framework and the assessment of their suitability for SIDO converter applications. Thus, the present work is motivated by two-fold objectives. To establish a clear mathematical model of the SIDO buck converter and to quantify the level of interaction between the output channels. Second, it seeks to explore viable control strategies that can improve regulation performance while maintaining practical

implement-ability. Hence the thesis deals with both the theoretical and practical aspects of SIDO converter analysis and control, emphasizing interaction-aware design and robust voltage regulation.

Thus, this work is motivated by the need for bridging the gap between accurate system modelling, interaction quantification and practical controller design for SIDO DC-DC converters. The goal of this study is to provide a complete picture of the converter behaviour and suggest control strategies which are both effective and implementable by combining these two aspects.

1.2 Literature Review

One of the most interesting areas of investigation in recent years has been the design of multi-output dc/dc converters due to their application in various fields, as they offer an excellent compromise between compactness and efficiency. Many approaches have been explored, but one of the promising options to meet this need is a Single Input Dual Output (SIDO) converter topology. This area of study in the past has been mostly centered around extending the well-established single output buck converters into a multiple output configuration by trying to optimize the design in terms of number of components and increased efficiency [1]. However, a variety of control techniques for SIDO converters have been presented in the literature such as time-multiplexing, duty cycle distribution, etc. But none of these approaches take care of the coupling between the output voltages caused by the inductor. This leads to cross regulation effects where a disturbance or variation of one output affects the other. This complicates the controller design and degrades the performance of the overall system [2]-[4]. To better understand the dynamic behaviour of such systems, researchers have commonly employed modelling techniques based on state-space averaging and small-signal analysis. These techniques facilitate the modelling of SIDO converters as multi-input multi-output (MIMO) systems [5]-[6], enabling a systematic approach for the analysis of the system dynamics. However, many of these works are primarily focused on the stability and transient response of individual outputs, without providing an explicit measure of the level of interaction between channels. To overcome this limitation interaction analysis techniques have been introduced in the study of multi-variable systems. Coupling between inputs and outputs has been assessed using methods like the Relative Gain Array (RGA) to help choose the control structure. In addition, system norm-based measures such as the H2 norm and other interaction indices have been examined to obtain quantitative insight into system behaviour [13]-[14]. The effectiveness of these techniques is well established. However, there are relatively few applications to SIDO DC-DC converter and lack of systematic approach incorporating the controller design methodology [16]-[18].

The controllers used for control in conventional methods of designing control systems using PID control strategy include classical PID. In the process of control in single output, they have been effective and simple to implement. However, the performances deteriorate in case of strong interactions among the various components making up the system. Such

conditions lead to overshooting and longer time taken to settle the output. From this, there arises a need to design controllers that take into account the uncertainty in the behaviour of the system components. Robust controllers and particularly the H-Infinity technique have gained popularity in controlling systems since they are able to guarantee performance even in the worst-case situations. The structured H-Infinity controllers designed using PID technique have the advantages of classical PID control as well as the robustness property associated with modern control. The controllers have an advantage of systematically varying the controller parameters without changing the order. The contemporary research trends have advocated for the integration of modelling, interaction and controller design into one system. This becomes necessary because the isolated application of the three often results in poor performance of the system, particularly for those involving multiple outputs [27]-[29]. A coordinated approach starting with accurate modelling, followed by quantification of interaction and an informed controller design is therefore essential for reliable and high-performance operation. Despite the progress reported in the literature, there is still a lack of comprehensive studies that simultaneously consider modelling accuracy, interaction quantification and robust control design for SIDO buck converters. In particular, comparison between conventional PID control and structured H-infinity based PID control within the same analytical framework is scarce. This gap motivates the present work which aims to provide a systematic and integrated approach for modelling, analysis and control of SIDO DC–DC buck converter systems.

1.3 Research Objective

- The main goal of this work is to develop a systematic and comprehensive framework for modelling, analysis and control of a Single-Input Dual-Output (SIDO) DC–DC buck converter system. The aim of this work is to develop an accurate model of the converter dynamics using state-space and small signal modelling techniques, which can give a clear understanding of the multi-variable behaviour of the converter.
- To study and quantify the interaction between the output channels. The outputs are not independent but are coupled dynamically via a common energy storage element, and it is important to study the extent and nature of this coupling. This is done by using different methods of interaction quantification, for example Relative Gain Array (RGA), HIIA and H_2 norm.
- To create equivalent Single-Input Single-Output (SISO) models from the initial multi-input multi-output formulation of the problem to apply classical control theory while taking into account the interactions in the system. The order reduction of the transformed models is performed where required. As a result, obtained models should be both manageable analytically and suitable for implementation.
- The other goal concerns the design and analysis of control strategies for the purpose of voltage regulation of both converters' outputs. For this reason, the conventional PID controller will be chosen as the basic control strategy because of its wide applicability in power electronic systems. At the same time, the structured H-infinity PID controller

should be designed considering the shortcomings of the first controller in terms of coping with interactions and parametric variations.

Overall, the thesis will help to improve significantly the regulation performance by dealing with interaction effects and provide useful recommendations for further investigations in the field of SIDO buck converters.

1.4 Research Gap

Even though there is an increasing demand for multi-output DC-DC converters, there are various major problems that need to be addressed in the case of Single Input Dual Output (SIDO) buck converter circuits. The majority of researches conducted to date have focused on the expansion of single output topologies into multi-output ones in order to enhance efficiency, compactness, and minimize components' count. Although the research works successfully prove that the SIDO topologies are achievable, they fail to take into account the issues involved in the operation of the shared inductor, which creates an interrelation between the two output loops. State-space averaging and small-signal modelling techniques are widely used to represent converter dynamics, but they are generally used to analyse the stability and transient response of individual outputs. The interaction between outputs, a feature that distinguishes SIDO converters, is often only qualitatively, not quantitatively, discussed. Consequently, the degree of coupling and its effect on system performance are not well understood, which hinders the efficiency of subsequent control design.

In the multi-variable systems' domain there exist a number of well-established tools such as Relative Gain Array (RGA), HIIA and norm-based performance indices such as the H_2 norm that provide a structured way of quantifying interaction and assessing system behaviour. However, their application in power electronic converters, especially SIDO buck converters, is still limited. However, these interaction quantification techniques have rarely been incorporated in the modelling and control framework of existing works. This results in a disconnection between the theoretical analysis and practical implementation of controllers. Another important limitation observed in literature is related to control strategies adopted for SIDO converters. PID controllers are widely used because they are simple and cheap to implement and they are easy to tune. However, these controllers are usually designed through heuristic methods without explicitly considering the coupled nature of the system. Such issues may arise particularly when there is load fluctuation, input perturbations and parametric uncertainty. In such situations, the issue gets worsened by the absence of any interaction aware techniques for tuning, as the controller tuned for one output may end up affecting the other output.

1.5 Thesis Organisation

The structure of the thesis is as follows:

In **Chapter 1** an overview of the importance of efficient, reliable and precise regulation of DC-DC converters is given. It shows the increasing demand of multi-output power conversion systems in the modern power electronics applications. This chapter introduces the Single-Input Dual-Output (SIDO) converter concept and states the main objectives of this study. An extensive review of literature is provided on control strategies, modelling approaches and challenges for voltage regulation and interaction in multi-output converters. The chapter also highlights the research gaps that motivate the present work.

Chapter 2 describes the modelling of SIDO DC-DC buck converter. It starts with a general description of the converter topology, followed by a detailed description of the operating principle and switching modes. Then, a mathematical model of the converter is developed using state space representation and small signal modelling techniques. Finally, the transfer function matrix is obtained which reflects the multi-input multi-output nature of the system and explicitly reveals the coupling between output channels.

Chapter 3 describes the quantification of the interaction of the SIDO converter. It analyses the coupling effect between output channels caused by the common inductor and energy transfer mechanism. Various interaction measures like Relative Gain Array (RGA), Hankel Interaction Index Array (HIIA) and H2 norm are used to measure steady state and dynamic interaction. The effect of parameter variations on the system interaction is investigated and a better understanding of the limitations of independent control approaches is obtained.

Chapter 4 deals with the control strategies of SIDO DC-DC buck converter system. It is started by developing an equivalent SISO model for the purpose of simplifying the controller design. The baseline approach is to design and analyse a traditional PID controller. Then a structured H-infinity based PID controller is designed based on the full multi-variable system representation.

Chapter 5 gives the results and discussion of the proposed work. A detailed comparison between the traditional PID and structured H-infinity based PID controllers is made. The performance is evaluated on the basis of time domain and frequency domain analyses including transient response, stability characteristics and closed loop behaviour. Load and line regulation studies are also conducted to evaluate disturbance rejection and robustness under various operating conditions.

Chapter 6 finally concludes the thesis with a summary of the key findings and contributions of the research. It shows the efficiency of the proposed control strategy in terms of system performance and interaction handling. The chapter also presents possible directions for future work, including experimental validation and extension to more complex multi-output converter system.

CHAPTER 2

MODELLING OF SINGLE-INDUCTOR DUAL-OUTPUT DC-DC BUCK CONVERTER

The modelling of the Single-Input Dual-Output (SIDO) DC–DC buck converter is presented in this chapter. The operating principles and the mathematical representation are combined. The SIDO converter is able to provide several regulated outputs from one input source, but the outputs are coupled by the configuration due to the common energy transfer path [1]-[10]. The chapter begins with a description of the converter topology, its operating principle, and switching modes. We also discuss important design considerations for the selection of passive components to realize stable and efficient operation. Mathematical model of the converter is then developed using state-space and small-signal modelling techniques. The transfer function matrix obtained shows the multi-input multi-output nature of the system and the interaction between the outputs. This model can serve as the foundation for the analysis and design of interactions discussed in subsequent chapters.

2.1 Overview

A Single-Input Dual-Output (SIDO) DC-DC buck converter is an innovative multi-output power conversion structure used to produce several regulated voltages from a single input power source. Unlike conventional multi-output converter structures that require the use of separate inductors and switch networks for generating each output, the SIDO converter topology employs just one common inductor and switching devices that can distribute energy among the outputs. This advantage brings about less component usage, cost reduction, increased power density, and ease of realization of the circuitry, making SIDO converters ideal for implementation into space-constrained electronic systems and low to medium power applications. The SIDO concept basically depends on the controlled allocation of time during which the inductor operates as an energy transfer device between the inputs and outputs. The control mechanism distributes energy from a common source in such a way that the inductor is connected to the source and after that energy is delivered to individual output channels. However, this approach has an inherent coupling between the outputs, as both share the same inductor current to transfer energy. Because of this common energy mechanism, the SIDO converter has characteristics of multi-input multi-output, in which changes of one output or its load conditions affect the behaviour of the other output. This interaction results in cross regulation that presents significant challenges toward achieving precise and stable voltage control. While in single-output converters the control design can be carried out independently for each output, the SIDO topology requires a more complete approach taking into account the coupled dynamics of the system.

From a modelling point of view the SIDO converter is an important case study of multi-variable power electronic systems. The existence of multiple outputs driven by a common inductor calls for advanced modelling techniques able to capture both individual dynamics and interaction effects. This makes the topology particularly relevant for the design and validation of interaction-aware control strategies. In addition, the increasing demand for integrated power management solutions in applications such as embedded systems, portable devices, and distributed power architectures has raised the significance of SIDO converters. These applications usually need multiple voltage levels with the least hardware overhead, making SIDO topologies a practical and efficient solution. However, to realize reliable performance under varying operating conditions, careful consideration of system dynamics, interaction effects and control design is necessary.

In general, the SIDO DC–DC buck converter provides a trade-off between hardware efficiency and control complexity. There are obvious advantages in reducing the number of components, but this leads to new challenges in the form of coupled dynamics that have to be addressed by systematic modelling, interaction analysis and control design. This is the basis for the detailed study described in the following sections of this chapter and the following chapters.

2.2 Operating Principle

The Single-Input Dual-Output (SIDO) DC–DC buck converter operates by distributing energy from a single input source to two output loads using a shared inductor-based switching network. The topology, shown in Fig. 2.1, consists of an input source V_g , a main switch Q_1 , a shared inductor L_1 , an auxiliary switching path involving Q_2 , and two output branches each comprising a capacitor-load pair (C_1, R_1) and (C_2, R_2) . The diodes D_1 and D_2 provide current continuity paths, while parasitic elements such as inductor resistance r_{L1} and capacitor ESRs r_{c1}, r_{c2} contribute to non-ideal behavior and must be considered in practical analysis [7].

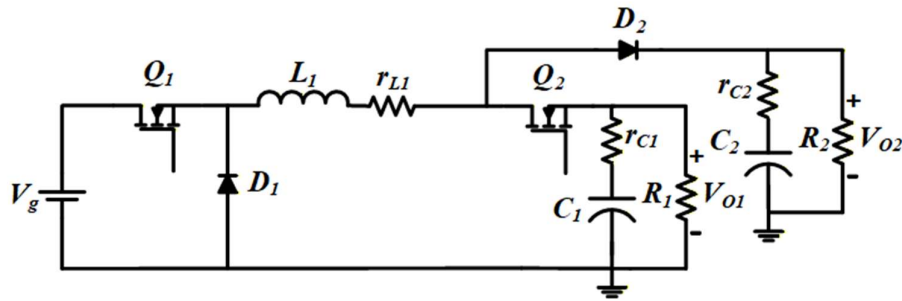


Fig. 2.1: Circuit diagram for SDO DC-DC Buck converter

The In essence, the converter works on the principle of time-multiplexed energy transfer. The inductor stores energy from the input source and then transfers it to one of the output branches alternatively in each switching cycle. In a conventional buck converter, the

energy path is fixed. In the SIDO converter, the energy path is reconfigured dynamically depending on the switching state. This leads to a more complex energy distribution mechanism, as both outputs are inherently coupled to the same inductor current.

During operation the switching elements Q_1 and Q_2 are controlled in a coordinated way, so that the inductor current is first built up and then directed to the desired output. When Q_1 is turned ON, the source at the input is connected directly to the inductor and the inductor current increases linearly (ideally). This phase corresponds to the energy storage in the magnetic field of the inductor. Then Q_1 is turned OFF and the stored energy has to be released. Depending on state of Q_2 and conduction of diodes, inductor current can be fed to output 1 or output 2. The energy distribution among the outputs is achieved by controlling the effective duty ratios associated with each output branch. This means the average output voltages are determined by the input voltage and the percentage of time the inductor is connected to each output. Consequently, the converter has a multi-input multi-output (MIMO) behaviour, i.e. the multiple control inputs affect different outputs simultaneously.

If there is a perturbation in one output branch, e.g. load change, the inductor current trajectory is altered and hence the energy delivered to the other output is influenced. This leads to cross-regulation effects that do not appear in single-output converters [6]. From a control point of view, this coupling requires a more sophisticated approach to attain a steady and accurate voltage regulation. Furthermore, parasitic resistances introduce damping in the system and influence steady-state and transient behaviour. These non-idealities impact the rate of change of the inductor current and the voltage ripple across the output capacitors. Therefore, the operating principle has to be described realistically, taking into account the ideal switching behaviour as well as the practical losses.

To put it briefly, the working principle of SIDO buck converter is based on a coordinated switching strategy, which allows sharing of inductor among multiple outputs in time domain. This is the basic behaviour, which forms the foundation of the subsequent mode-wise analyses and mathematical models presented in the next sections.

2.3 Modes of Operation

The functioning of the SIDO DC-DC buck converter can be mathematically described in terms of the decomposition of the switching period into three operating intervals, corresponding to different modes. These modes specify a specific set-up of the switching elements and define a particular energy transfer mechanism. Not only these modes play a critical role in comprehending the process inside the converter, but they also form the basis for constructing a proper state space model.

The total duration of the switching cycle is denoted by T_s , and the duty cycles corresponding to the three different modes are d_1 , d_2 , and $1-d_1-d_2$. It is clear that in

each switching period, the converter switches between these modes in a certain sequence, and the average behaviour can be computed using proper weightings of these modes.

2.3.1 Mode 1: Switch Q_1 ON

During the first operating mode, switch Q_1 is turned ON, while switch Q_2 is kept OFF. In this case, the input voltage source V_g is directly connected to the inductor L_1 , and thus the inductor current starts increasing. Note that in this mode, the diodes become reverse-biased, and the output branches are isolated from the inductor.

Using Kirchhoff's voltage law (KVL) around the input-inductor circuit loop, the inductor voltage can be written as

$$V_L = V_g - i_L r_{L1} - V_{node} \quad (2.1)$$

Under these assumptions, the initial node voltage can be neglected, further getting

$$V_L \approx V_g \quad (2.2)$$

The corresponding dynamics of current are

$$\frac{di_L}{dt} = \frac{V_g}{L_1} \quad (2.3)$$

which are indicating a linear rise in the inductor current.

The length of time during which this mode continues, $d1T_s$, will be responsible for determining the amount of energy that will be accumulated at each switching period. With an increase in $d1$, there will be an increase in the peak inductor current, and hence an increase in the amount of energy supplied to the loads.

2.3.2 Mode 2: Switch Q_2 ON

In the second mode, switch Q_1 goes OFF while Q_2 is made ON. Thus, a conducting path is created between the inductor and the output terminal 1. The energy that is stored in the inductor is transferred to the capacitor C_1 and resistor R_1 .

Using KVL in the second mode, the voltage across the inductor will be

$$V_L = -V_{o1} - i_L r_{L1} \quad (2.5)$$

Now assuming upon small parasitic resistance, this gets to

$$V_L \approx -V_{o1} \quad (2.6)$$

which then leads to

$$\frac{di_L}{dt} = \frac{-V_{o1}}{L_1} \quad (2.7)$$

This implies that there is a decrease in inductor current with respect to time as energy is delivered to the output 1. The rate of current decrease depends on the value of V_{o1} because the greater the value of V_{o1} , the faster the current decreases.

In this mode, capacitor C_1 has two functions. It filters the ripple in the inductor current and hence ensures the supply of a constant voltage level at the output. The other function performed by the capacitor is that of acting as a source of energy storage.

The energy transfer to output 1 can be expressed as

$$E_{o1} \approx \int_{t_2} i_L V_{o1} dt \quad (2.8)$$

which depends on both the duration of the mode and the instantaneous inductor current. Therefore, the duty ratio d_2 directly influences the average value of V_{o1} .

2.3.3 Mode 3: Switches Q_1 and Q_2 OFF

In the third mode, both switches Q_1 and Q_2 are OFF, and diode D_2 becomes forward-biased, providing a conduction path for the inductor current toward output 2. This mode is structurally similar to the freewheeling interval in a conventional buck converter, but with the key difference that the energy is directed specifically toward the second output branch.

The inductor voltage in this configuration is given by

$$V_L = -V_{o2} - i_L r_{L1} \quad (2.9)$$

which simplifies to

$$V_L \approx -V_{o2} \quad (2.10)$$

leading to

$$\frac{di_L}{dt} = \frac{-V_{o2}}{L_1} \quad (2.11)$$

As in Mode 2, the inductor current decreases as energy is transferred to the output. The capacitor C_2 smooths the current and maintains the output voltage V_{o2} across load R_2 . The energy transferred to output 2 is given by

$$E_{o2} \approx \int_{t_3} i_L V_{o2} dt \quad (2.12)$$

The duration of this mode, d_3T_s , determines the fraction of energy delivered to output 2 within each switching cycle.

2.3.4 Mode Interaction and Duty Allocation

The three modes together satisfy the constraint:

$$d_1 + d_2 \leq 1 \quad (2.13)$$

which implies that the available switching period must be shared among energy storage and delivery to both outputs.

As a result, there is an intrinsic competition between the energies stored in the inductors, whereby the higher the duty cycle of one of the outputs becomes, the less energy will remain for the other one.

The equations of the steady state can be derived from the volt-second balance equation of the inductors:

$$d_1V_g = d_2V_{o1} + (1 - d_1 - d_2)V_{o2} \quad (2.14)$$

The above relationship proves that there is an interaction between the two outputs with regards to their duty ratios.

This effect is known as cross regulation and it is a distinguishing feature of SIDO regulators.

Table 2.1: Design Parameters of SIDO Buck Converter

Parameter	Symbol	Value	Unit	Description
Input Voltage	(V_g)	12	V	Supply Voltage
Output Voltage 1	(V_{o1})	5	V	First Regulated Output
Output voltage 2	(V_{o2})	3.3	V	Second Regulated Output
Inductor	(L_1)	100	μH	Energy storage element
Capacitor 1	(C_1)	220	μF	Output filter Capacitor
Capacitor 2	(r_{C1}, r_{C2})	220	μF	Capacitor resistance
Inductor Resistance	(r_{L1})	0.1	Ω	Inductor resistance

2.4 State-Space Modelling

An exact model of the power converter is indispensable for proper system analysis and control design. Although the working principles of the SIDO buck converter have been explained in the last chapter, the mathematical modelling is necessary to describe the dynamic characteristics of the device. In case of the multi-output power converter, the presence of shared inductor results in the existence of interaction between the outputs that can only be revealed by precise mathematical models. Therefore, to analyse the interaction of the converter, the exact description of the dynamic behaviour must be used. However, unlike ordinary linear systems, the power electronic circuits work according to a switched principle and can change their behaviour during each switching period. Hence, the SIDO DC-DC converter cannot be represented by a simple set of linear differential equations, since the governing equation of the circuit may be completely different in different time periods. Thus, to describe the converter dynamic behaviour mathematically, one should use a mode-dependent representation. Such an approach would allow obtaining the mathematical models of the converter for all possible operating modes that correspond to different switching periods. State-space representation of the SIDO buck converter will be used for modelling the system in this chapter. Using the state space approach, the system equations will be formulated in terms of energy storage elements of the converter, which are the output capacitor voltages and the inductor current. State-space modelling allows taking into account individual dynamics of each output as well as their interaction and will be helpful for the next steps of the analysis and control system design [4]-[6].

The input equation is represented as:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (2.15)$$

The output equation is represented as:

$$y(t) = Ex(t) + Fu(t) \quad (2.16)$$

Firstly, to model SIDO buck converter, mode-wise state equations should be derived for all possible modes. To do that, Kirchhoff voltage and current laws can be used. After writing the differential equations in matrix form, a mode-dependent representation of the converter can be obtained. Next, by applying the state space averaging technique to the mode-dependent models, a continuous-time large-signal model of the SIDO buck converter can be developed. Large-signal model allows considering the overall converter behaviour over the whole switching period and thus provides information about the dynamics of the system on a macro level. Having obtained the large-signal representation of the system, small-signal linearized model can be built to study the behaviour of the converter in case of small disturbances. This linearized model can be used to derive transfer function representation of the system. Transfer function is especially convenient for studying system behaviour in the frequency domain and designing the controllers

based on it. Overall, this chapter develops various mathematical models of the SIDO buck converter, including state-space model, averaged large-signal model, and linearized small-signal model. These models would be very helpful for the development of the following interaction quantification and control strategies.

2.4.1 State, Input, and Output Variables

The process of developing a state space model begins with choosing the state variables needed to describe the behaviour of the system fully. The choice of state variables is normally guided by the fact that they must represent the energy storage elements of the system.

In the SIDO DC-DC buck converter, the main energy storage components are the inductor and the output capacitors. Inductor stores energy in form of magnetic field and capacitors stores energy in form of electric field.

Hence, the inductor current and the capacitor voltages are the natural choice of state variables.

The state vector is defined as:

$$x(t) = \begin{bmatrix} i_L(t) \\ v_{o1}(t) \\ v_{o2}(t) \end{bmatrix} \quad (2.17)$$

This choice ensures that all dynamic energy storage elements are represented. The inductor current $i_L(t)$ captures the flow of energy within the converter, while the capacitor voltages $v_{o1}(t)$ and $v_{o2}(t)$ represent the output states that must be regulated.

The input to the system is the source voltage:

$$u(t) = V_g \quad (2.18)$$

which acts as the driving force for energy transfer. The output vector is defined as:

$$y(t) = \begin{bmatrix} v_{o1}(t) \\ v_{o2}(t) \end{bmatrix} \quad (2.19)$$

Observe that the selection of the state variables makes for a minimal realization of the system (in other words, there is no redundancy in terms of the selected state variables). This produces a neat and useful representation without any unnecessary intricacy.

Furthermore, the selected state variables can actually be measured or approximated using sensors. It is significant that this characteristic will make the representation useful from a practical point of view, particularly in control systems where feedback is required.

2.5 Mode-Based System Representation

The operation of the SIDO converter is naturally dependent on the switching states of the semiconductor devices. As discussed in this chapter, the converter has three different modes of operation within each switching cycle depending upon the configuration of the switches and diodes. Each mode yields a different set of circuit equations which should be derived separately.

For each mode i , the system can be described by a set of first-order differential equations of the form:

$$\dot{x}(t) = A_i x(t) + B_i u(t) \quad (2.20)$$

where A_i and B_i are the state and input matrices for that mode. These matrices are derived by applying the Kirchhoff laws of voltage and current to the circuit configuration associated with the mode.

Therefore, the whole system can be viewed as a switched system switching among different linear subsystems. The switching sequence defines the time intervals during which each subsystem is active, and the system behaviour is obtained by combining these subsystems over one switching period. This representation based on modes is the starting point for further modelling and analysis. The individual mode equations can then be combined using state-space averaging, to obtain a continuous-time model that captures the average behaviour of the converter. The averaged model is used for small-signal analysis and control design.

2.5.1 Mode 1: Switch Q_1 ON

In mode 1, the main switch Q_1 is ON state and the auxiliary switch Q_2 is turned OFF. Under this switching condition the input source V_g is directly connected to the inductor L_1 and the inductor current starts to increase. During this time both output branches are effectively isolated from the input and the load currents are supplied by their own output capacitors.

Applying Kirchhoff's Voltage Law (KVL) to the input-inductor loop, the voltage across the inductor can be expressed as:

$$V_g = L_1 \frac{di_L}{dt} + r_{L1} i_L \quad (2.21)$$

Rearranging, we obtain:

$$\frac{di_L}{dt} = \frac{1}{L_1} (V_g - r_{L1} i_L) \quad (2.22)$$

This equation represents the change of inductor current during the charging phase. It is evident that the current increases linearly when parasitic resistance is neglected, and the slope is determined by the input voltage and inductance. Next, by applying the Kirchhoff's Current Law (KCL) at the output node 1, since the inductor does not feed current to this node, the load current must be fed from the capacitor:

$$i_{C1} = -\frac{v_{o1}}{R_1} \quad (2.23)$$

Using the capacitor relation:

$$C_1 \frac{dv_{o1}}{dt} = i_{C1} \quad (2.24)$$

we obtain:

$$\frac{dv_{o1}}{dt} = -\frac{v_{o1}}{R_1 C_1} \quad (2.25)$$

Similarly, for output node 2:

$$\frac{dv_{o2}}{dt} = -\frac{v_{o2}}{R_2 C_2} \quad (2.26)$$

Thus, the state equations for Mode 1 can be written as:

$$\dot{x} = A_1 x + B_1 V_g \quad (2.27)$$

$$y(t) = E_1 x(t) + F_1 u(t) \quad (2.28)$$

where the matrix A_1 accounts for the natural decay of the output voltages and the inductor dynamics, and B_1 gives the direct effect of the input voltage on current of inductor. So, the matrices obtained for operating mode 1 is :

$$A_1 = \begin{bmatrix} -\frac{R_L + R_{on}}{L} & -\frac{R_a R_{ca}}{L(R_a + R_{ca})} & 0 \\ \frac{R_a}{C_a(R_a + R_{ca})} & -\frac{1}{C_a(R_a + R_{ca})} & 0 \\ 0 & 0 & -\frac{1}{C_b(R_b + R_{cb})} \end{bmatrix} \quad (2.29)$$

$$B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} \quad (2.30)$$

$$E_1 = \begin{bmatrix} \frac{R_a R_{ca}}{R_a + R_{ca}} & \frac{R_a}{R_a + R_{ca}} & 0 \\ 0 & 0 & \frac{R_b}{R_b + R_{cb}} \end{bmatrix} \quad (2.31)$$

$$F_1 = 0 \quad (2.32)$$

2.5.2 Mode 2: Switch Q_2 ON

In this Mode, the main switch Q_1 is now turned OFF and the another auxiliary switch Q_2 is turned ON. In this type of configuration, the inductor current is redirected further towards output 1, delivering a stored energy to the capacitor C_1 and load R_1 .

Applying KVL around the inductor-output loop:

$$0 = L_1 \frac{di_L}{dt} + r_{L1} i_L + v_{o1} \quad (2.33)$$

Rearranging:

$$\frac{di_L}{dt} = \frac{-v_{o1} - r_{L1} i_L}{L_1} \quad (2.34)$$

This indicates that the inductor current decreases as energy is transferred to the output.

Applying KCL at output node 1:

$$i_L = i_{C1} + \frac{v_{o1}}{R_1} \quad (2.35)$$

Substituting:

$$C_1 \frac{dv_{o1}}{dt} = i_L - \frac{v_{o1}}{R_1} \quad (2.36)$$

$$\frac{dv_{o1}}{dt} = \frac{1}{C_1} \left(i_L - \frac{v_{o1}}{R_1} \right) \quad (2.37)$$

For output node 2, since it is not connected to the inductor in this mode:

$$\frac{dv_{o2}}{dt} = -\frac{v_{o2}}{R_2 C_2} \quad (2.38)$$

Thus, Mode 2 captures the direct coupling between inductor current and output 1 voltage.

So, the obtained matrices for operating mode 2 are:

$$A_2 = \begin{bmatrix} -\frac{R_d + R_L + R_{on}}{L} & -\frac{R_a R_{ca}}{L(R_a + R_{ca})} & -\frac{R_a}{L(R_a + R_{ca})} \\ \frac{R_a}{C_a(R_a + R_{ca})} & -\frac{1}{C_a(R_a + R_{ca})} & 0 \\ 0 & 0 & -\frac{1}{C_b(R_b + R_{cb})} \end{bmatrix} \quad (2.39)$$

$$B_2 = \begin{bmatrix} \frac{V_d}{V_{in}} \\ 0 \\ 0 \end{bmatrix} \quad (2.40)$$

$$E_2 = E_1 \quad (2.41)$$

$$F_2 = 0 \quad (2.42)$$

2.5.3 Mode 3: Switch Q_1 and Q_2 OFF

In Mode 3, both switches Q_1 and Q_2 are OFF, and diode D_2 conducts. The inductor current is now directed toward output 2.

Applying KVL:

$$0 = L_1 \frac{di_L}{dt} + r_{L1} i_L + v_{o2} \quad (2.43)$$

Thus:

$$\frac{di_L}{dt} = \frac{-v_{o2} - r_{L1} i_L}{L_1} \quad (2.44)$$

Applying KCL at output node 2:

$$C_2 \frac{dv_{o2}}{dt} = i_L - \frac{v_{o2}}{R_2} \quad (2.45)$$

$$\frac{dv_{o2}}{dt} = \frac{1}{C_2} \left(i_L - \frac{v_{o2}}{R_2} \right) \quad (2.46)$$

For output 1:

$$\frac{dv_{o1}}{dt} = -\frac{v_{o1}}{R_1 C_1} \quad (2.47)$$

This mode represents energy delivery to output 2 and shows similar structure to Mode 2 but for the second output.

So, the obtained matrices for operating mode 3 are:

$$A_3 = \begin{bmatrix} -\frac{R_d + R_L + R_{bon}}{L} & -\frac{R_b R_{cb}}{L(R_b + R_{cb})} & -\frac{R_b}{L(R_b + R_{cb})} \\ 0 & -\frac{1}{C_a(R_a + R_{ca})} & 0 \\ \frac{R_b}{C_b(R_b + R_{cb})} & 0 & -\frac{1}{C_b(R_b + R_{cb})} \end{bmatrix} \quad (2.48)$$

$$B_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.49)$$

$$E_3 = \begin{bmatrix} 0 & \frac{R_a}{R_a + R_{ca}} & 0 \\ \frac{R_b R_{cb}}{R_b + R_{cb}} & 0 & \frac{R_b}{R_b + R_{cb}} \end{bmatrix} \quad (2.50)$$

$$F_2 = 0 \quad (2.51)$$

2.5.4 Duty Cycle Representation

Let the switching period be denoted by T_s , and let the durations of the three modes be:

$$t_1 = d_1 T_s, t_2 = d_2 T_s, t_3 = d_3 T_s \quad (2.52)$$

where:

$$d_1 + d_2 + d_3 = 1 \quad (2.53)$$

Here:

- d_1 : duty ratio for this Mode 1 (charging)
- d_2 : duty ratio for this Mode 2 (output 1)
- d_3 : duty ratio for this Mode 3 (output 2)

These duty ratios now act as controlling variables, which determines how the inductor energy is distributed among these outputs.

2.5.5 Averaged State Equation

The form generalised for the state equation for each mode is:

$$\dot{x} = A_i x + B_i V_g \quad (2.54)$$

To obtain the averaged model, we compute the weighted sum of these equations over one switching cycle:

$$\dot{x}_{avg} = d_1(A_1 x + B_1 V_g) + d_2(A_2 x + B_2 V_g) + (1 - d_1 - d_2)(A_3 x + B_3 V_g) \quad (2.55)$$

Rearranging:

$$\dot{x}_{avg} = (d_1 A_1 + d_2 A_2 + d_3 A_3)x + (d_1 B_1 + d_2 B_2 + (1 - d_1 - d_2)B_3)V_g \quad (2.56)$$

Thus, the averaged system can be written as:

$$\dot{x} = A_{avg} x + B_{avg} V_g$$

where:

$$\begin{aligned} A_{avg} &= d_1 A_1 + d_2 A_2 + (1 - d_1 - d_2)A_3 \\ B_{avg} &= d_1 B_1 + d_2 B_2 + (1 - d_1 - d_2)B_3 \end{aligned} \quad (2.57)$$

2.5.6 Large-Signal Model Representation

The averaged model derived above represents the large-signal behaviour of the converter. It describes the system dynamics under arbitrary operating conditions and captures the nonlinear dependence on duty ratios.

The complete large-signal state-space model can be written as:

$$\dot{x}(t) = A_{avg}(d_1, d_2)x(t) + B_{avg}(d_1, d_2)V_g \quad (2.58)$$

$$y(t) = Cx(t) \quad (2.59)$$

This model forms the basis for further analysis and is particularly useful for studying transient behaviour and interaction effects.

Physical Insight into Averaged Dynamics

From a physical point of view the averaged model is the net effect of energy transfer in the converter. The model does not consider the individual switching events, but rather describes the distribution of energy over time. The duty ratios control the relative time

that the inductor charges and discharges into each output, and therefore controls the average flow of energy.

System dynamics depend on duty ratios, which highlights the effect of control inputs on system behaviour. The interaction between the outputs can be controlled and the output voltages can be regulated by tuning d_1 , d_2 and d_3 . However, due to the inter-dependency of such duty ratios, careful design is necessary for independent control of outputs.

2.6 Small-Signal Modelling

2.6.1 Introduction

The large-signal averaged model developed in the previous section provides a complete description of the converter dynamics for any operating condition. However, for control system design and stability analysis it is often necessary to study the system response to small perturbations about a nominal operating point. This necessitates the development of a linearised model that approximates the system behaviour for small deviations in state and input. Such a model is often called the small signal model [5].

The principle of linearization is used for Small-Signal modelling in which the nonlinear system equations are approximated by linear equations around a steady-state operating point. This permits the application of classical control techniques such as transfer function analysis, frequency response methods, and stability criteria. Small signal models are needed to design controllers for stable and robust operation of dc–dc converters.

This is even more important for the SIDO DC-DC buck converter, which has multiple outputs with inherent coupling. The outputs interact among themselves adding more dynamics to be captured in the model. The linearization of the averaged state-space equations allows us to study the effect of small perturbations in duty ratios on the output voltages and to quantify the interaction between different channels.

2.6.2 Steady-State Operating Point

The first step in small-signal modelling is the determination of the steady-state operating point. Let the steady-state values of the state variables and inputs be denoted as:

$$\begin{aligned} x(t) &= X + \hat{x}(t) \\ d_1(t) &= D_1 + \hat{d}_1(t), d_2(t) = D_2 + \hat{d}_2(t) \\ V_g(t) &= V_g + \hat{v}_g(t) \end{aligned} \tag{2.60}$$

where:

- X , D_1 , D_2 , and V_g are steady-state values
- $\hat{x}(t)$, $\hat{d}_1(t)$, $\hat{d}_2(t)$, and $\hat{v}_g(t)$ represent small perturbations

The steady-state condition is obtained by setting:

$$\dot{x} = 0 \quad (2.61)$$

which yields:

$$0 = A_{avg}(D_1, D_2)X + B_{avg}(D_1, D_2)V_g \quad (2.62)$$

This equation defines the equilibrium point around which the system is linearized.

2.6.3 Perturbation and Linearization

Substituting the perturbed variables into the averaged model:

$$\dot{x} = A_{avg}(d_1, d_2)x + B_{avg}(d_1, d_2)V_g \quad (2.63)$$

and expanding using:

$$x = X + \hat{x}, d_i = D_i + \hat{d}_i \quad (2.64)$$

we obtain a nonlinear expression. By neglecting higher-order terms (products of small perturbations), the system can be linearized.

After linearization, the small-signal model is expressed as:

$$\hat{\dot{x}} = A\hat{x} + B\hat{u} \quad (2.65)$$

where:

- A is the Jacobian matrix evaluated at the operating point
- B represents the influence of input perturbations

For the SIDO converter, the inputs include:

- duty cycle perturbations $\hat{d}_1, \hat{d}_2$
- input voltage perturbation $\hat{v}_g$

2.6.4 Derivation of Small-Signal Matrices

From your earlier formulation:

$$\begin{aligned} A_{11} &= A \\ B_{11} &= [(A_1 - A_2)X + (B_1 - B_2)V_g, (A_2 - A_3)X + (B_2 - B_3)V_g] \\ E_{11} &= E \\ F_{11} &= [(E_1 - E_2)X + (F_1 - F_2)V_g, (E_2 - E_3)X + (F_2 - F_3)V_g] \end{aligned} \quad (2.66)$$

These matrices represent the **linearized system**, where:

- A_{11} : state matrix
- B_{11} : input matrix (duty perturbations)
- E_{11}, F_{11} : output matrices

Thus, the small-signal model becomes:

$$\begin{aligned}\hat{x} &= A_{11}\hat{x} + B_{11}\hat{d} \\ \hat{y} &= E_{11}\hat{x} + F_{11}\hat{d}\end{aligned}\tag{2.67}$$

where:

$$\hat{d} = \begin{bmatrix} \hat{d}_1 \\ \hat{d}_2 \end{bmatrix}\tag{2.68}$$

Concluding Remarks

In this regard, the following small signal state space model has been developed for the SIDO DC-DC buck converter by linearizing the state equations in the steady-state operating region. This model represents the dynamics of the system and the interactions within it.

2.7 Transfer Function

2.7.1 Introduction

The small signal state space model for the SIDO DC-DC buck converter derived in the previous section is the linearized representation of the system about the steady-state operating condition. Nevertheless, transfer functions are often preferred when analysing systems in the frequency domain.

Transfer functions provide a direct relationship between input perturbations and output responses in the Laplace domain, allowing us to use classical control techniques such as Bode plot analysis, stability margin analysis and controller tuning.

For multi-output systems, such as the SIDO converter, the transfer function representation is particularly relevant, since it allows to quantify explicitly the interaction between different input-output channels. There are multiple control inputs in each output. The transfer function becomes a matrix instead of a scalar function. This matrix form gives insight into system coupling and the basis for the selection of appropriate control strategies.

2.7.2 Small-Signal Model in Laplace Domain

From Section 2.6, the small-signal model is given by:

$$\begin{aligned}\hat{x}(t) &= A_{11}\hat{x}(t) + B_{11}\hat{d}(t) \\ \hat{y}(t) &= E_{11}\hat{x}(t) + F_{11}\hat{d}(t)\end{aligned}$$

Taking the Laplace transform (assuming zero initial conditions):

$$s\hat{X}(s) = A_{11}\hat{X}(s) + B_{11}\hat{D}(s) \quad (2.69)$$

Rearranging:

$$(sI - A_{11})\hat{X}(s) = B_{11}\hat{D}(s) \quad (2.70)$$

Thus:

$$\hat{X}(s) = (sI - A_{11})^{-1}B_{11}\hat{D}(s) \quad (2.71)$$

Substituting into the output equation:

$$\hat{Y}(s) = E_{11}(sI - A_{11})^{-1}B_{11}\hat{D}(s) + F_{11}\hat{D}(s) \quad (2.72)$$

2.7.3 Transfer Function Matrix

The transfer function matrix is therefore defined as:

$$G(s) = E_{11}(sI - A_{11})^{-1}B_{11} + F_{11} \quad (2.73)$$

Thus:

$$\hat{Y}(s) = G(s)\hat{D}(s) \quad (2.74)$$

For the SIDO converter:

$$\hat{Y}(s) = \begin{bmatrix} \hat{v}_{o1}(s) \\ \hat{v}_{o2}(s) \end{bmatrix}, \hat{D}(s) = \begin{bmatrix} \hat{d}_1(s) \\ \hat{d}_2(s) \end{bmatrix} \quad (2.75)$$

Hence:

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \quad (2.76)$$

2.7.4 Individual Transfer Functions

Each element of the transfer function matrix represents a specific input-output relationship:

$$G_{11}(s) = \frac{\hat{v}_{o1}(s)}{\hat{d}_1(s)} \quad (2.77)$$

Effect of duty cycle d_1 on output v_{o1}

$$G_{22}(s) = \frac{\hat{v}_{o2}(s)}{\hat{d}_2(s)} \quad (2.78)$$

Effect of duty cycle d_2 on output v_{o2}

$$G_{12}(s) = \frac{\hat{v}_{o1}(s)}{\hat{d}_2(s)} \quad (2.79)$$

Cross-coupling from d_2 to v_{o1}

$$G_{21}(s) = \frac{\hat{v}_{o2}(s)}{\hat{d}_1(s)} \quad (2.80)$$

Cross-coupling from d_1 to v_{o2}

Evaluated Transfer Functions

$$G_{11}(s) = \frac{7157s^2 + 3.286 \times 10^8 s + 1.464 \times 10^{11}}{s^3 + 2890s^2 + 2.521 \times 10^7 s + 1.085 \times 10^{10}} \quad (2.81)$$

$$G_{12}(s) = \frac{0.0792s^3 + 2851s^2 - 3.446 \times 10^7 s + 4.764 \times 10^{10}}{s^3 + 2890s^2 + 2.521 \times 10^7 s + 1.085 \times 10^{10}} \quad (2.82)$$

$$G_{21}(s) = \frac{4724s^2 + 2.168 \times 10^8 s + 9.663 \times 10^{10}}{s^3 + 2890s^2 + 2.521 \times 10^7 s + 1.085 \times 10^{10}} \quad (2.83)$$

$$G_{22}(s) = \frac{-0.07992s^2 - 4472s^2 - 4.066 \times 10^7 s - 1.139 \times 10^{11}}{s^3 + 2890s^2 + 2.521 \times 10^7 s + 1.085 \times 10^{10}} \quad (2.84)$$

2.7.5 Conclusion

In this section, from small-signal state-space model, the transfer function representation of the SIDO DC–DC buck converter has been derived. As a result, the multi-variable transfer function matrix incorporates both the direct control paths and the interaction between outputs. Such a representation is important for frequency domain analysis and forms the basis for the control strategies developed in subsequent chapters.

CHAPTER 3

INTERACTION QUANTIFICATION

3.1 Introduction

The dynamic behaviour of the Single-Input Dual-Output (SIDO) DC–DC buck converter is modelled in detail using the modelling framework developed in Chapter 2. In particular, the small-signal transfer function matrix clearly shows that the system has a multi-variable structure, where each output is influenced not only by its own control input but also by the control action on the other output. This coupling arises fundamentally from the use of common inductor as an energy transfer medium between the outputs. Thus, one must recognize the interaction and treat the converter as a multiple-input and multiple-output (MIMO) system. As mentioned before, interaction manifests itself as cross regulation. In particular, this interaction results in voltage regulation errors, additional transient deviation and increases the difficulty of controller design. Therefore, it is important to analyse the strength of the interactions among various output channels, as this is the first step in designing appropriate control technique. Otherwise, control technique design may rely on simplistic assumptions about the dynamics of the system which might not be accurate.

The purpose of this chapter is to quantitatively investigate the interaction characteristics of the SIDO DC-DC Buck converter through well-established analytic methods. Such methods allow to obtain a quantitative numerical assessment of the interaction levels between individual input-output pairs. This will give valuable insight in the system behaviour. Specifically, these analytic tools will be used to analyse the derived transfer function matrix, thereby identifying the strengths and nature of interactions, dominating interaction paths, and assessing the feasibility of decentralized control strategy. For instance, analysis of the interaction between steady-state gains is carried out using Relative Gain Array (RGA). In addition to the steady-state gain-based method, advanced techniques, such as analysis of the Hankel Interaction Index Array (HIIA) and H2 norm, are employed for investigating the effects of dynamic interaction. Thus, the effect of dynamic interaction is captured in a form of an energy-based measure, and provides a comprehensive investigation of the interactions between various output channels both in the steady state and in dynamical regime. Results obtained in this chapter provide crucial groundwork for the following control strategy development in Chapter 5.

3.2 Interaction in SIDO Buck Converter

Since in the SIDO DC-DC buck converter there exists common energy path, there is some natural coupling of the output terminals. Contrary to single output converters where the energy supply element is provided for each terminal, in the SIDO structure, common energy transfer is implemented using the same inductor. This means that the basic

principle of coupling consists in that fact that any energy allocated to one terminal influences the energy supply of the second terminal.

The coupling between the two output terminals arises due to the nature of the time multiplexing scheme used by inductor current waveform. The energy transfer process starts with charging the inductor from the input source and then supplying the energy stored in the inductor into the output terminals with some determined duty ratio, which is different for each terminal. Since the total energy transferred is limited by the amount of energy available during the energy transfer time, any change in allocation causes some variation of the energy supplied to outputs. Since this energy transfer is performed by a common inductor, there should exist some energy sharing mechanism between terminals, which means that a current variation occurs. The increased energy demand at the output terminal requires increasing in the energy delivery time, which causes reducing the time for the other terminal and therefore causes some deviation of its output voltage. This kind of effect is widely known as cross-regulation.

The interaction effect plays a major role in determining the performance of the system. During any transient event, such as changing of loads or input voltage, energy allocation between terminals is carried out with finite times and leads to imbalance in the energy supply between terminals. This is reflected by inductor current, which varies in a short-time manner and affects terminal voltages causing them to be oscillated or having some undershoot or overshoot. The amplitude of these transients is dependent on the value of components, system parameters, load resistance and control technique.

Therefore, the design of control algorithms becomes a problem. In classical decentralized control systems each terminal is controlled independently using single closed-loop control circuits. However, in this case, a poor performance may arise due to ignoring the effect of one control circuit on the other. This will cause the increase in the settling time and overshoot as well as decreased robustness. Therefore, to choose correct control strategy, the knowledge about the system interaction is required.

3.2.1 Cross-Coupling Effects

The interaction within the Single-Input Dual-Output (SIDO) DC–DC buck converter is caused by the existence of a common inductor which acts as the major energy transfer element. This is in contrast with multi-converter topologies where individual energy paths are used to drive multiple outputs. The energy in the SIDO topology is transferred to multiple outputs sequentially through one inductor and this causes cross-coupling since the energy transfer to multiple outputs is controlled by one inductor current.

The existence of a single inductor implies that only one output can be driven using energy supplied through the inductor at any particular time. Another output has to utilize its output capacitor to generate its energy requirements since the inductor cannot supply the energy for both outputs simultaneously. This is achieved by setting the output voltage

control signals accordingly, and this makes the dynamics of the outputs dependant on each other. Cross-coupling occurs when the change in one output affects the energy distribution to another output. For instance, if there is an increase in load demand at output 1 then additional energy will be required to be supplied in its switching intervals. This means that the time allocated to output 1 has to be increased and this can be achieved by increasing the corresponding duty cycle. However, this will cause an equivalent reduction of the time allocated for delivery of energy to output 2 since the total switching period is fixed. As a result, the energy supplied to output 2 will become limited and there will be deviation in the output voltage of the latter due to the load condition. This is referred to as the cross-regulation effect.

From a dynamic analysis perspective, the cross-coupling effect can be illustrated in terms of state equations. In this case, the cross-coupling effect occurs because the common inductor introduces coupling through the inductor current which is common to all switching states. However, when the inductor current is off from the other output then its output voltage is determined by the decaying process which is controlled by its capacitor due to load condition. Therefore, this leads to dynamic interaction that is inherent in the system and cannot be decoupled unless the control approach is changed. Moreover, another source of coupling in the SIDO converter results from the transient responses of the two outputs under changing loads and input voltage conditions. This is because the output capacitors cannot instantly supply extra energy needed during load transient conditions due to their relatively slow dynamics compared to the inductor dynamics. Since the inductor is a common energy storage element then a sudden change in load conditions and input voltage causes a change in the inductor current. As a result, the energy stored in the inductor will influence the response of both outputs and may cause overshooting or undershooting phenomena of their voltages. The amount of interaction depends on several parameters including the output loading levels, relative values of the output voltages and the frequency of switching pulses.

3.3 Interaction Quantification Techniques

The qualitative picture of interaction in the SIDO DC-DC buck converter presented in the previous sections gives useful physical insight into the origin and nature of coupling between outputs. However, in order to study and design control in a rigorous way, this interaction needs to be quantified by well-defined mathematical measures. Such quantification enables an objective assessment of the degree of coupling and provides a basis for comparing different system configurations and control strategies.

In multi-variable systems, the interaction is usually defined as the influence of a change in one input on several outputs. This analysis is based on the transfer function matrix developed in Chapter 3 which takes into account explicitly both direct and cross-coupling dynamics. By employing appropriate analysis tools on this matrix, an assessment of the interaction strength, dominant coupling paths and the potential for decentralized control can be made [16].

Three known methods of interaction quantification are used in this section, including Relative Gain Array (RGA), Hankel Interaction Index Array (HIIA) and H_2 norm as we know. These approaches provide complementary insights into interaction, addressing both steady-state and dynamic aspects of the coupling. RGA is concerned with the steady-state input-output coupling while HIIA and H_2 norm gives information on the energy-based and dynamic interaction characteristics. Taken together, these techniques constitute a complete framework for analysing interaction in the SIDO converter [14]

3.3.1 Relative Gain Array (RGA) Analysis

The Relative Gain Array (RGA) is a widely used technique for analyzing the steady-state interaction in multiple-input multiple-output (MIMO) systems [13]. It measures the degree of coupling between each input-output pair and is widely used for determining proper control loop pairings in decentralized control design.

The RGA matrix is defined as:

$$\Lambda = G(0) \cdot (G(0)^{-1})^T \quad (3.1)$$

where:

- $G(0)$ is the steady-state gain matrix (i.e., $s = 0$)
- $(\cdot)^T$ denotes transpose
- $\circ$ represents element-wise (Hadamard) multiplication

For the SIDO converter:

$$G(0) = \begin{bmatrix} G_{11}(0) & G_{12}(0) \\ G_{21}(0) & G_{22}(0) \end{bmatrix} \quad (3.2)$$

Thus, the RGA matrix becomes:

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \quad (3.3)$$

Each element λ_{ij} represents the relative gain between input j and output i , normalized by the interaction with other loops.

The interpretation of RGA elements is as follows:

- $\lambda_{ij} \approx 1$: strong and desirable pairing
- $\lambda_{ij} \approx 0$: weak interaction
- $\lambda_{ij} < 0$: undesirable pairing (interaction dominant)
- $|\lambda_{ij}| > 1$: strong coupling effects

The non-zero off-diagonal elements of the RGA matrix for the SIDO converter show the presence of the cross-coupling between the outputs. If the diagonal elements are very different from unity it means that independent control of outputs may not be sufficient and interaction compensation may be required.

Thus, RGA analysis is a simple but powerful tool to evaluate the steady-state interaction and a first step in the evaluation of the suitability of the control structure.

3.3.2 Hankel Interaction Index Array (HIIA)

The RGA offers a snapshot of steady-state interaction, but does not include the time-dependent aspects of coupling. To overcome this limitation, the Hankel Interaction Index Array (HIIA) is used to measure the dynamic interaction in multi-variable systems. The HIIA is based on the concept of Hankel singular values, which represent the energy contribution of different modes of the system [17].

The HIIA is defined using the Hankel norm of individual transfer function elements:

$$\Gamma_{ij} = \| G_{ij}(s) \|_H \quad (3.4)$$

where $\|\cdot\|_H$ denotes the Hankel norm of the transfer function. The HIIA matrix is then constructed as:

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \quad (3.5)$$

The Hankel norm reflects the energy of the system response associated with each input-output channel. Larger values of Γ_{ij} indicate stronger dynamic influence of input j on output i .

In the SIDO converter, the diagonal elements Γ_{11} and Γ_{22} represent the dominant control paths, while the off-diagonal elements Γ_{12} and Γ_{21} quantify dynamic cross-coupling. If the off-diagonal elements are comparable in magnitude to the diagonal elements, it indicates significant interaction that cannot be neglected.

The HIIA gives a more complete insight into the interaction, by including the magnitude and dynamic behaviour of transfer functions. Unlike RGA that is evaluated at steady-state, HIIA considers the system dynamics over the entire frequency range, which is particularly useful for the interaction assessment under transient conditions.

3.3.3 H₂ Norm-Based Interaction Measure

Another important interaction quantifying metric is the H₂ norm that provides an energy-based measure of the system response. The H₂ norm of a transfer function is the total energy of the output response to an impulse input. It is defined as

$$\| G_{ij}(s) \|_2 = \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} |G_{ij}(j\omega)|^2 d\omega \right)^{\frac{1}{2}} \quad (3.6)$$

The H_2 norm can be used to construct an interaction matrix similar to HIIA:

$$H = \begin{bmatrix} \| G_{11} \|_2 & \| G_{12} \|_2 \\ \| G_{21} \|_2 & \| G_{22} \|_2 \end{bmatrix} \quad (3.7)$$

This matrix gives a quantitative measure of the transfer of energy from each input to each output. Higher values mean stronger coupling.

The H_2 norm in the SIDO converter highlights the energy distribution between outputs and provides an understanding of how disturbances or control actions propagate through the system. If the off-diagonal elements of the matrix H_2 are significant this means that a significant part of energy is transferred between the channels, this confirms a strong interaction.

The advantage of the H_2 norm is that it can describe the overall system behaviour in a unified way, instead of particular frequencies or steady-state conditions. This makes it good for the performance evaluation under transient conditions and for comparison of different control strategies.

3.4 Conclusion

The interaction analysis using RGA, HIIA and H_2 norm confirms the inherent coupling between the output channels of SIDO DC–DC buck converter. The RGA results indicate that the diagonal elements are relatively dominant, but they are not equal to one and the off-diagonal elements are not zero, which indicates the existence of cross regulation. This means that although approximate input-output pairing is possible, exact decoupling is not.

This is further confirmed by the dynamic interaction measures. The HIIA and H_2 norm analysis show the non-negligible role of the cross-coupling paths in the overall system response, especially in transients. This means that the interaction is not only for the steady state behaviour but for the dynamic performance of the converter as well. The results allow to conclude that, although equivalent SISO models can be used for simplified controller design, the interaction effects are significant enough to affect system performance.

Consequently, classical decentralised control techniques may be not sufficient and this motivates the use of advanced multi-variable control techniques such as structured H_∞ control which explicitly takes into account the coupling between the systems.

CHAPTER 4

CONTROL STRATEGIES FOR SIDO BUCK CONVERTER SYSTEM

4.1 Introduction

From the analysis done in the previous chapters regarding modelling and interaction, it can be seen that the SIDO DC buck converter system is inherently a coupled multi-input, multi-variable system. The state space and transfer function models provide comprehensive insight about the behaviour of the system but, owing to the interaction present amongst output variables, a lot of difficulties arise in implementing an accurate regulation of voltage at both outputs along with a quick response in dynamic situations. The usual control methodologies of independent control of output voltages fail due to the effect of interaction caused by the presence of common inductor and time multiplexing of energy in the system. Hence a proper control methodology needs to be formulated so that the system functions stably and satisfies the design objectives of high efficiency, accuracy, speed, and low ripple. The main purpose of the design will be to regulate the output voltages of the converter with minimum overshoot, rise time, settling time, steady state errors and ripple.

Proportional-Integral-Derivative (PID) controllers have always been popular in power electronics systems because of the simple design, easy implementation and satisfactory performance. The PID controllers are also feasible to design for the SIDO system since it is possible to formulate equivalent SISO models for the SIDO converter. Each output is regulated independently using the SISO model of the corresponding channel in this method. Though it sounds simple, it doesn't include the interaction effect that can adversely affect the system in dynamic operations.

In this chapter, traditional PID control as well as advanced structured PID controller using the concept of H infinity control will be discussed for the control of SIDO converters. While the PID controller is designed using the equivalent SISO models, the H infinity technique takes advantage of the multi-variable nature of the system to incorporate improved performance and robustness in the design procedure. Simulation studies are carried out in order to evaluate the performances of the PID control and H infinity control under various operating conditions. Comparison between PID control and H inf control is done by analyzing the performance of these controls as described in this chapter.

4.2 Equivalent SISO Model Development

The interaction analysis in Chapter 3 has demonstrated that the SIDO DC-DC buck converter exhibits coupled multi-variable behaviour due to the shared inductor and the

time-multiplexed energy transfer mechanism. However, for conventional controller design it is often desirable to reduce the system representation to a form where each output can be controlled independently. This motivates the use of equivalent Single-Input Single-Output (SISO) models approximating the dominant dynamics of each control channel.

The development of the equivalent SISO model is based on the transfer function matrix obtained in Chapter 3. As we mentioned before, the diagonal elements of the matrix are the main control paths, and the off-diagonal elements are the cross-coupling effects. If the system is diagonal dominant to some extent, then the interaction between outputs can be considered as a secondary effect and each output can be approximated by its corresponding diagonal transfer function.

Accordingly, the equivalent SISO models for the SIDO converter are defined as:

$$G_{11}(s) = \frac{\hat{v}_{o1}(s)}{\hat{d}_1(s)}, G_{22}(s) = \frac{\hat{v}_{o2}(s)}{\hat{d}_2(s)} \quad (4.1)$$

The transfer functions represent the dominant input-output relationship for output 1 and output 2 respectively, and are the basis for independent controller design. This approximation does not explicitly account for the cross-coupling terms $G_{12}(s)$ and $G_{21}(s)$, but implicitly considers them as disturbances acting on each channel.

The interaction analysis results further support the validity of this SISO approximation. The RGA suggested a good input-output pairing with fairly dominant diagonal elements. The HIIA and H 2 norm analysis showed that although there is interaction, the main dynamics are dominated by the diagonal paths. Hence, the use of equivalent SISO models is justified for the design of conventional PID controllers. But we need to be aware of the limitations of this approach. The performance of controllers designed based on SISO models can be degraded under conditions where the coupling becomes significant, for example, under large load variations or fast transients, since the interaction effects are not explicitly considered. This restriction suggests the need for more advanced control strategies that take into account the complete multi-variable nature of the system.

In this work the equivalent SISO models found above are used for the design of standard PID controllers applied separately to each output channel. These controllers are then compared with structured H-infinity based controllers that explicitly take into account system interaction and robustness requirements.

4.3 Conventional PID Controller

The Proportional-Integral-Derivative (PID) controller is one of the most simple and popular control strategies in industrial and power electronic applications. Its popularity is mainly due to its simple structure, intuitive tuning method and the ability to achieve

satisfactory performance over a wide range of systems. In the DC-DC converter application, PID controllers are used for voltage regulation. The duty cycle of the switching device is controlled so that the output voltage is kept at a desired reference value.

The PID controller works on the principle of error minimization i.e. the control action is based on the difference between the reference signal and the measured output. The control law in time domain can be expressed as:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \quad (4.2)$$

where $e(t) = v_{ref}(t) - v_o(t)$ represents the control error. The proportional term is an immediate corrective action proportional to the magnitude of the error and thus improves the response speed. The integral term integrates the error over time and reduces steady state offset, which is important for voltage regulation applications. The derivative term gives an anticipation of the future error trend by looking at the rate of change of error. This improves the damping characteristic and reduces the overshoot.

In the Laplace domain, the PID controller transfer function is given by:

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s \quad (4.3)$$

This clearly shows the frequency dependent behaviour of this controller. At low frequencies the integral term dominates and provides good steady state tracking. At high frequencies the derivative term dominates and improves transient response. In practical implementations the ideal derivative term is often modified by a first-order low pass filter to mitigate noise amplification. This results in the PID with derivative filtering (PIDF) structure:

$$G_c(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{1 + T_f s} \quad (4.4)$$

where T_f is the time constant of the derivative filter. This modification ensures that high frequency measurement noise does not adversely affect the control signal which results in better robustness. The PID controller design for the SIDO DC-DC buck converter is based on the equivalent SISO models obtained in Section 4.2. In particular, two independent control loops are realized:

$$G_{11}(s) = \frac{\hat{v}_{o1}(s)}{\hat{d}_1(s)}, G_{22}(s) = \frac{\hat{v}_{o2}(s)}{\hat{d}_2(s)} \quad (4.5)$$

Each loop regulates one output voltage by changing its duty cycle. The controller gains K_p , K_i and K_d are tuned to obtain a desired trade-off between transient performance and steady-state accuracy. Usually tuning is done on the basis of frequency domain specifications such as phase margin, crossover frequency or time domain specifications such as settling time and overshoot.

However, the implementation of the PID control on the SIDO converter is not straightforward because of the presence of interaction between outputs. The SISO approximation assumes that each output is driven only by its corresponding input, and the cross-coupling terms are considered as disturbances [24]-[25]. In practice, the shared inductor creates a dynamic coupling so that the variations in one output influence the other. This results in performance restrictions, especially under transient conditions.

For example, if one output suddenly experiences a higher load, it may require an increase in its duty cycle, and thus, less time for energy delivery to the other output. The PID controller is ignorant of this coupling, and tries to compensate independently, which can lead to oscillations or slow convergence. This tells a fundamental limitation of this decentralized control for multi-variable systems.

Even with these challenges, the PID controller provides a useful baseline for performance comparison. The simplicity, ease of tuning and acceptance makes it a good reference for more advanced control strategies. In this work the PID controller is applied separately to each output channel and its performance is evaluated with respect to transient response, steady-state error, ripple characteristics and cross-regulation [26]-[28].

4.4 Structured H-infinity based PID Controller

The drawbacks of conventional PID control for interaction and robustness in multi-variable systems provide the impetus for the use of advanced control design methodologies. The inherent coupling of the output in the SIDO DC–DC buck converter demands a control strategy that can simultaneously handle multi-variable dynamics, interaction effects and robustness requirements. Structured H-infinity control provides a systematic framework to achieve these objectives.

Generally, the state feedback for H-infinity control is given by following set of equations:

$$\dot{x} = Ax + B_1w + B_2u \quad (4.6)$$

$$z = C_1x + D_{11} + D_{12}u \quad (4.7)$$

$$y = C_2x + D_{21} + D_{22}u \quad (4.8)$$

Where B_1 : Disturbance Matrix, B_2 : Control Input Matrix, C_1 : Performance output Matrix, C_2 : Measured output Matrix. D_{11} , D_{12} , D_{21} and D_{22} : feedthrough matrices.

The H-infinity control paradigm is based on the minimization of the worst-case amplification of disturbances and uncertainties of the system. Classical control techniques often assume nominal models, whereas H-infinity control explicitly accounts for model uncertainties and attempts to optimize the system performance under worst-case scenarios. The control objective is expressed in terms of the H-infinity norm of the closed-loop transfer function:

$$\| T_{zw}(s) \|_{\infty} < \gamma \quad (4.9)$$

where $T_{zw}(s)$ represents the transfer function from disturbance input w to controlled output z , and γ is the desired performance bound.

The Riccati Equations for obtaining controller gains are given below:

First Equation:

$$A^T X + XA - XB_2 R^{-1} X + C_1^T C_1 = 0 \quad (4.11)$$

Second equation:

$$AY + YA^T - YC_2^T S^{-1} C_2 Y + B_1 B_1^T = 0 \quad (4.12)$$

Where,

$$R = D_{12} D_{12}^T \quad (4.13)$$

$$S = D_{21} D_{21}^T \quad (4.14)$$

The H-infinity norm is defined as:

$$\| T(s) \|_{\infty} = \sup_{\omega} \bar{\sigma}(T(j\omega)) \quad (4.15)$$

where $\bar{\sigma}$ denotes the maximum singular value of the transfer function matrix. This formulation ensures that the system has an acceptable performance for all frequencies, thus providing robustness against uncertainties and disturbances.

In structured H-infinity control, the controller is constrained to a particular structure, e.g. a PID form, and the parameters of the controller are optimized to satisfy the H-infinity performance criterion. This approach takes advantage of the merits of classical and modern control techniques, keeping the controller simple and implementable form while optimizing systematically.

For the SIDO converter, the controller is parameterized as:

$$K(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{1 + T_f s} \quad (4.16)$$

The parameters K_p , K_i , K_d and T_f are obtained by an optimization procedure that minimizes the H-infinity norm of the closed loop system. The plant model used here is the multi-variable transfer function matrix developed in Chapter 3, which includes the interaction between outputs.

The controller gain is given by:

State Feedback:

$$K = R^{-1} B_2^T X \quad (4.17)$$

Observer Gain:

$$L = Y C_2^T S^{-1} \quad (4.18)$$

So, the final Controller Form for H-infinity Control is given by:

$$\dot{x} = (A - B_2 K - L C_2) \hat{x} + L y \quad (4.19)$$

$$u = -K \hat{x} \quad (4.20)$$

The approach of design is based on the construction of a generalized plant which includes the model of the system and suitable weighting functions. These weighting functions are used to specify the desired performance specifications like tracking accuracy, disturbance attenuation and noise rejection. For instance, low frequency weights are used to provide accurate steady-state tracking and high frequency weights are used to limit the control effort and reduce the sensitivity to noise. One of the main advantages of the structured H-infinity approach is its ability to take into account the interaction between outputs. The PID control is based on independent SISO models, while the H-infinity design considers the full multi-variable system. In this way, the controller can coordinate the control efforts for all outputs, minimizing the cross-coupling effect and enhancing the performance of the whole system.

Moreover, the optimization process automatically optimizes for various performance metrics without the need for any adjustments. It ensures that the controller design parameters are optimized according to the selected performance criteria. This is particularly useful in complex systems like the SIDO converter, where manually optimizing the PID parameters can be both inefficient and ineffective. The use of derivative filters in the optimization procedure is necessary due to practical reasons for realizable designs and robust designs at high frequencies. In addition, the structured approach helps in designing a lower order controller, which would be easy to implement on a digital platform.

Based on this understanding, it is expected that the designed structured H-infinity based PID controller will improve the transient response, minimize the overshoot, reduce the settling time, and enhance the disturbance rejection characteristics compared to traditional PID control methods [23]. Most importantly, it can efficiently counteract the output interactions and improve the cross-regulation performance.

4.5 Conclusion

Control strategies for the considered SIDO DC-DC converter are developed in this chapter, aiming at overcoming difficulties associated with the multi-variable nature of the dynamics and interaction effects. The modelling and analysis of the previous chapters gave a foundation for developing this chapter where a unified framework for controller design and evaluation will be presented. The first step in developing control strategies has been the reduction of the system into its equivalent SISO models by considering only the dominant diagonal elements of the transfer function matrix. Such reduction makes possible to use classical techniques for designing the controller, decoupling the system into separate controllable subsystems. Each output channel is designed with conventional PID controllers which is based on the equivalent SISO model. Proportional-Integral-Derivative controller (PID), combining proportional, integral and derivative actions, provides a balance between transient performance and steady-state accuracy.

The controllers were tuned in order to obtain desirable characteristics such as reduction in overshoot, fast settling time and minimum of steady state error. However, this control strategy is decentralized in nature which means that the effects of interaction cannot be managed directly and lead to certain degradation of system performance especially when it comes to dynamic regimes. In order to overcome above mentioned drawbacks, a structured PID-H-infinity control is used. Structured PID-H-infinity design approach is advantageous in combining advantages of simplicity and practicability of PID design with robustness of optimal H-infinity control. PID control problem in the presence of worst-case disturbance is formulated as minimization of H-infinity norm of the closed loop system. The structured nature of the controller allows its systematic optimization and practical implementation. Structured H-infinity PID control design allows incorporating the complete multivariable dynamics of the system. Unlike the conventional PID controller, which bases on the consideration of separate SISO models, structured H-infinity design considers all elements of the transfer function matrix, i.e. interaction.

The control strategies considered in this chapter will be further investigated through simulations described in the next chapter. Performance of the developed strategies will be evaluated through several important criteria such as overshoot, settling time, ripple, cross-regulation and load regulation.

CHAPTER 5

RESULTS & DISCUSSION

5.1 Introduction

In this chapter, a detailed simulation-based analysis of Single-Input Dual-Output (SIDO) DC–DC buck converter is presented. The goal is to validate the modelling, interaction quantification and control strategies developed in the previous chapters. The results are summarized in two major parts: interaction analysis with respect to parametric variations and control performance assessment.

The first part concerns the quantification of the interaction between output channels by varying the main system parameters such as the load resistances, the output capacitances and the inductance. The steady state and dynamic coupling effects are evaluated by calculating and analysing the metrics such as relative gain array (RGA), Hankel interaction index array (HIIA) and H_2 norm. Furthermore, the dynamic RGA sigma plots in the frequency domain are used to incorporate the interaction behaviour over a wide frequency range. The second part of the chapter (presented in subsequent sections) evaluates the performance of the conventional PID and structured H-infinity based PID controllers. Their transient response, robustness, and cross-regulation characteristics are compared by using both time-domain and frequency-domain analyses.

5.2 Interaction Analysis under Parametric Variations

5.2.1 Fixed System Parameters

Table 6.1 summarizes the baseline parameters used for all simulations. These values characterize the nominal operating condition of the SIDO buck converter, and are fixed in the evaluation of the effect of single parameter variations.

Table 5.1: Fixed Circuit Parameters

Parameter	Value
V_g (V)	12
V_{o1} (V)	5
V_{o2} (V)	3.3
r_{C1} (Ω)	0.1
r_{C2} (Ω)	0.1
r_{L1} (Ω)	0.1

5.2.2 Effect of Load Variation on Output 1 (R_1)

The effect of load variation on output 1 is investigated by varying R_1 over a large range of values and fixing other parameters. The corresponding interaction metrics are summarized in Table 6.2.

Table 5.2: Pairing Analysis with Variations in R_1 (Ω)

R_1 (Ω)	RGA	HIIA	H ₂ norm
1	$\begin{bmatrix} 0.9647 & 0.0353 \\ 0.0353 & 0.9647 \end{bmatrix}$	$\begin{bmatrix} 0.8594 & 0.1406 \\ 0.1263 & 0.8737 \end{bmatrix}$	$\begin{bmatrix} 0.8015 & 0.1985 \\ 0.1286 & 0.8714 \end{bmatrix}$
5	$\begin{bmatrix} 0.8800 & 0.1200 \\ 0.1200 & 0.8800 \end{bmatrix}$	$\begin{bmatrix} 0.8794 & 0.1206 \\ 0.4453 & 0.5547 \end{bmatrix}$	$\begin{bmatrix} 0.8031 & 0.1969 \\ 0.3573 & 0.6427 \end{bmatrix}$
10	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8495 & 0.1505 \\ 0.7002 & 0.2998 \end{bmatrix}$	$\begin{bmatrix} 0.7547 & 0.2453 \\ 0.4592 & 0.5408 \end{bmatrix}$
15	$\begin{bmatrix} 0.7023 & 0.2977 \\ 0.2977 & 0.7023 \end{bmatrix}$	$\begin{bmatrix} 0.8052 & 0.1948 \\ 0.7871 & 0.2129 \end{bmatrix}$	$\begin{bmatrix} 0.7083 & 0.2917 \\ 0.5075 & 0.4925 \end{bmatrix}$
20	$\begin{bmatrix} 0.6344 & 0.3656 \\ 0.3656 & 0.6344 \end{bmatrix}$	$\begin{bmatrix} 0.7511 & 0.2489 \\ 0.8228 & 0.1772 \end{bmatrix}$	$\begin{bmatrix} 0.6664 & 0.3336 \\ 0.5357 & 0.4643 \end{bmatrix}$

The RGA results indicate a clear reduction in diagonal dominance as R_1 increases. For smaller values of R_1 the diagonal elements are close to unity, indicating strong input-output pairing and little interaction. However, as R_1 increases the diagonal terms decrease and the off-diagonal terms increase indicating an increase in the cross-coupling between outputs. This trend is further confirmed with the HIIA and H₂ norm value.

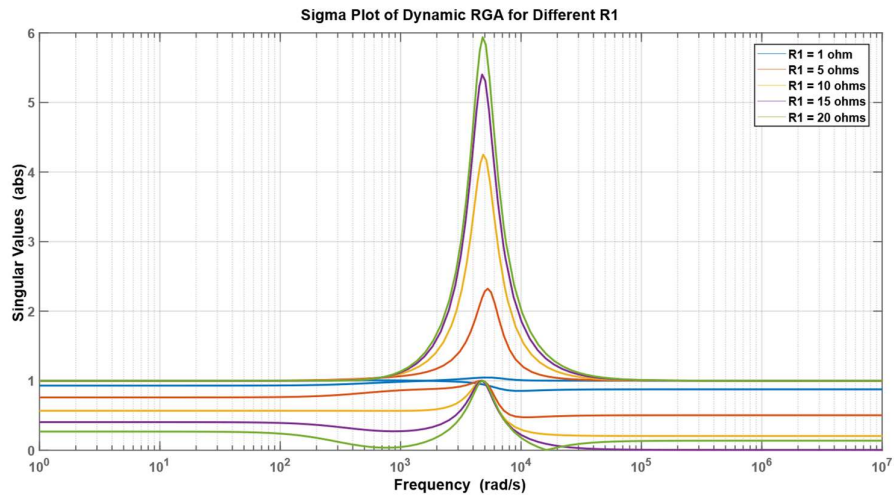


Fig. 6.1 Simulation results when R_1 Changes

The dynamic RGA sigma plot shows that the interaction is very frequency dependent. Mid-frequencies range has a prominent peak. This is related to the dominant dynamics of the converter. The peak magnitude increases significantly with the increase of R_1 , indicating that the interaction effects are more severe under higher load resistance conditions.

5.2.3 Effect of Load Variation on Output 2 (R_2)

Also, the load resistance R_2 has a significant influence on the interaction behaviour of the converter. The metrics calculated are shown in Table 6.3.

Table 5.3: Pairing Analysis with Variations in R_2 (Ω)

R_2 (Ω)	RGA	HIIA	H ₂ norm
1	$\begin{bmatrix} 0.2443 & 0.7557 \\ 0.7557 & 0.2443 \end{bmatrix}$	$\begin{bmatrix} 0.3363 & 0.6637 \\ 0.7966 & 0.2034 \end{bmatrix}$	$\begin{bmatrix} 0.3399 & 0.6601 \\ 0.6146 & 0.3854 \end{bmatrix}$
5	$\begin{bmatrix} 0.6312 & 0.3688 \\ 0.3688 & 0.6312 \end{bmatrix}$	$\begin{bmatrix} 0.7405 & 0.2595 \\ 0.7998 & 0.2002 \end{bmatrix}$	$\begin{bmatrix} 0.6644 & 0.3356 \\ 0.5366 & 0.4634 \end{bmatrix}$
10	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8495 & 0.1505 \\ 0.7002 & 0.2998 \end{bmatrix}$	$\begin{bmatrix} 0.7547 & 0.2453 \\ 0.4592 & 0.5408 \end{bmatrix}$
15	$\begin{bmatrix} 0.8493 & 0.1507 \\ 0.1507 & 0.8493 \end{bmatrix}$	$\begin{bmatrix} 0.8752 & 0.1248 \\ 0.5770 & 0.4230 \end{bmatrix}$	$\begin{bmatrix} 0.7907 & 0.2093 \\ 0.4011 & 0.5989 \end{bmatrix}$
20	$\begin{bmatrix} 0.8851 & 0.1149 \\ 0.1149 & 0.8851 \end{bmatrix}$	$\begin{bmatrix} 0.8842 & 0.1158 \\ 0.4670 & 0.5330 \end{bmatrix}$	$\begin{bmatrix} 0.8100 & 0.1900 \\ 0.3560 & 0.6440 \end{bmatrix}$

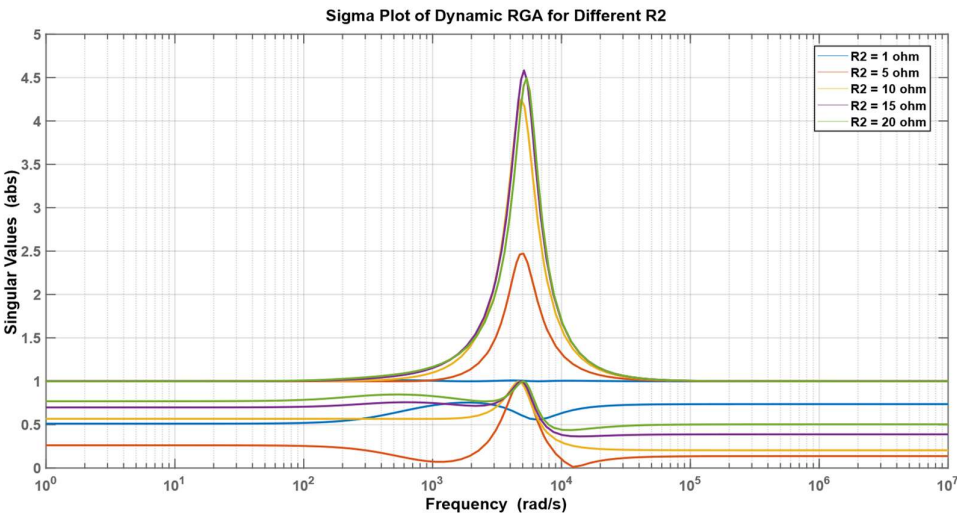


Fig. 6.2 Simulation results when R_2 Changes

The dynamic RGA plots show that the peak magnitude increases with increasing R_2 , while the frequency of the peak almost remains the same. This means that the frequency

of system interaction is governed by system dynamics whereas the interaction strength is controlled by the load conditions.

5.2.4 Effect of Capacitance Variation (C_1)

The effect of output capacitance C_1 on interaction is analyzed by varying its value over a wide range. The results are summarized in Table 6.4.

Table 5.4: Pairing Analysis with Variation in C_1 (μF)

C_1 (μF)	RGA	HIIA	H_2 norm
10	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8166 & 0.1834 \\ 0.4483 & 0.5517 \end{bmatrix}$	$\begin{bmatrix} 0.7548 & 0.2452 \\ 0.4589 & 0.5411 \end{bmatrix}$
47	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8706 & 0.1294 \\ 0.4524 & 0.5476 \end{bmatrix}$	$\begin{bmatrix} 0.7548 & 0.2452 \\ 0.4589 & 0.5411 \end{bmatrix}$
100	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8762 & 0.1238 \\ 0.5823 & 0.4177 \end{bmatrix}$	$\begin{bmatrix} 0.7548 & 0.2452 \\ 0.4590 & 0.5410 \end{bmatrix}$
220	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8495 & 0.1505 \\ 0.7002 & 0.2998 \end{bmatrix}$	$\begin{bmatrix} 0.7547 & 0.2453 \\ 0.4592 & 0.5408 \end{bmatrix}$
470	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.7986 & 0.2014 \\ 0.7710 & 0.2290 \end{bmatrix}$	$\begin{bmatrix} 0.7547 & 0.2453 \\ 0.4599 & 0.5401 \end{bmatrix}$

Unlike load variation, the RGA values are almost constant for different values of C_1 which shows that the steady state interaction is not much affected by the output capacitance. The HIIA values, however, show a significant variation

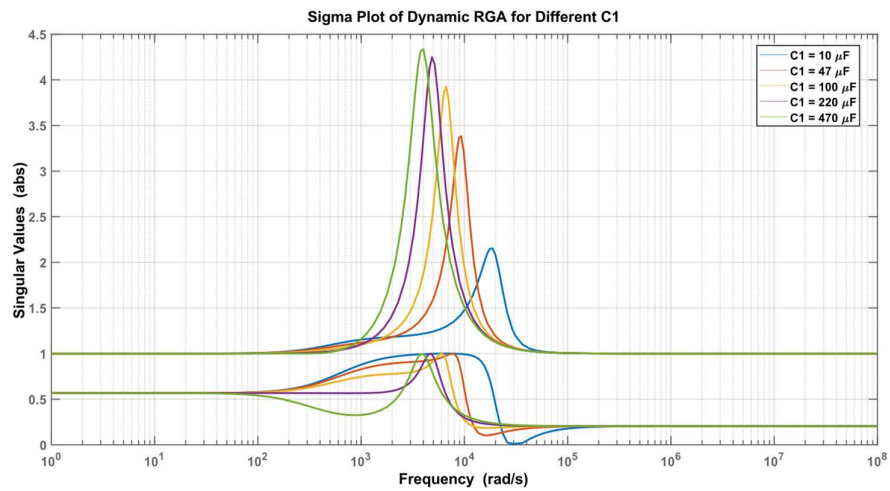


Fig. 6.3 Simulation results when C_1 Changes

The dynamic RGA plot clearly demonstrates the effect of capacitance on the frequency response of interaction. The increase of the C_1 results in a smoothing of the interaction peaks and a slight modification of their features, indicating a better dynamic decoupling. Thus, capacitance is a key parameter for transient interaction

5.2.5 Effect of Capacitance Variation (C_2)

The influence of capacitance C_2 on interaction characteristics is presented in Table 6.5.

Table 5.5: Pairing Analysis with Variation in C_2 (μF)

C_2 (μF)	RGA	HIIA	H_2 norm
10	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.7382 & 0.2618 \\ 0.8333 & 0.1667 \end{bmatrix}$	$\begin{bmatrix} 0.7545 & 0.2455 \\ 0.4597 & 0.5403 \end{bmatrix}$
47	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.7740 & 0.2260 \\ 0.8340 & 0.1660 \end{bmatrix}$	$\begin{bmatrix} 0.7545 & 0.2455 \\ 0.4596 & 0.5404 \end{bmatrix}$
100	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8248 & 0.1752 \\ 0.7967 & 0.2033 \end{bmatrix}$	$\begin{bmatrix} 0.7546 & 0.2454 \\ 0.4594 & 0.5406 \end{bmatrix}$
220	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8495 & 0.1505 \\ 0.7002 & 0.2998 \end{bmatrix}$	$\begin{bmatrix} 0.7547 & 0.2453 \\ 0.4592 & 0.5408 \end{bmatrix}$
470	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8589 & 0.1411 \\ 0.5493 & 0.4507 \end{bmatrix}$	$\begin{bmatrix} 0.7550 & 0.2450 \\ 0.4591 & 0.5409 \end{bmatrix}$

Similar to C_1 , the RGA values remain unchanged, confirming that steady-state interaction is unaffected by capacitance variation. However, the HIIA results indicate variations in dynamic interaction, particularly in off-diagonal terms.

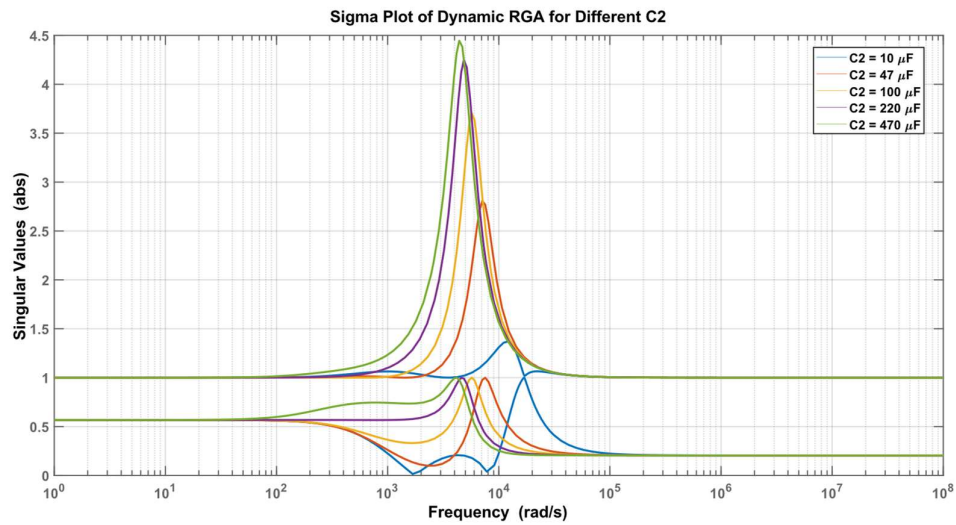


Fig. 6.4 Simulation results when C_2 Changes

The dynamic RGA plot shows that increasing C_2 reduces the sharpness of interaction peaks and improves damping characteristics. This indicates that higher capacitance enhances the ability of the system to absorb energy fluctuations, thereby reducing dynamic coupling between outputs.

5.2.6 Effect of Inductor Variation (L_1)

This inductor plays a vital role in energy transfer within the specified SIDO converter, and its variation is significantly affecting interaction dynamics. The results are summarized here in Table 6.6.

Table 5.6: Pairing Analysis with Variation in L_1 (μH)

L_1 (μH)	RGA	HIIA	H ₂ norm
10	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.7463 & 0.2537 \\ 0.5518 & 0.4482 \end{bmatrix}$	$\begin{bmatrix} 0.7547 & 0.2453 \\ 0.4592 & 0.5408 \end{bmatrix}$
50	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8243 & 0.1757 \\ 0.6615 & 0.3385 \end{bmatrix}$	$\begin{bmatrix} 0.8243 & 0.1757 \\ 0.6615 & 0.3385 \end{bmatrix}$
100	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8495 & 0.1505 \\ 0.7002 & 0.2998 \end{bmatrix}$	$\begin{bmatrix} 0.7547 & 0.2453 \\ 0.4592 & 0.5408 \end{bmatrix}$
200	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8660 & 0.1340 \\ 0.7247 & 0.2753 \end{bmatrix}$	$\begin{bmatrix} 0.7548 & 0.2452 \\ 0.4592 & 0.5408 \end{bmatrix}$
500	$\begin{bmatrix} 0.7837 & 0.2163 \\ 0.2163 & 0.7837 \end{bmatrix}$	$\begin{bmatrix} 0.8758 & 0.1242 \\ 0.7281 & 0.2719 \end{bmatrix}$	$\begin{bmatrix} 0.7548 & 0.2452 \\ 0.4593 & 0.5407 \end{bmatrix}$

The RGA values remained unchanged all across different inductance values further indicating that this steady-state interaction is not influenced by L_1 . However, now the HIIA results are showing a clear variation.

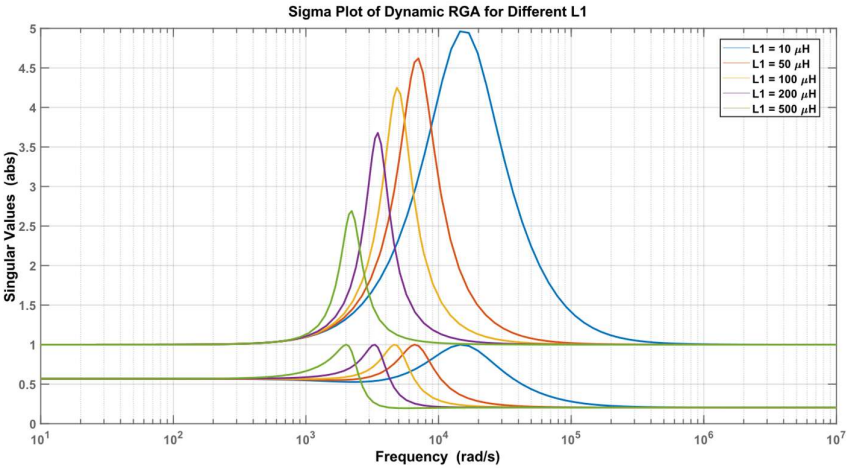


Fig. 6.5 Simulation results when L_1 Changes

The dynamic RGA plots reveal that increasing inductance shifts the interaction peak toward lower frequencies and reduces its magnitude. This indicates that higher inductance slows down system dynamics and improves decoupling. Conversely, lower inductance leads to sharper peaks and stronger interaction, making the system more sensitive to coupling effects.

Conclusion

The dynamic RGA analysis shows a strong frequency dependence of the interaction with significant peaks in the mid-frequency range associated with the converter dynamics. The size and distribution of these peaks are sensitive to the system parameters, reflecting the interaction's response to the load and the component values.

Finally, the results show that some operating conditions may be approximately diagonally dominant, but the interaction effects cannot be ignored especially in dynamic situations. These results motivate the need for control strategies that explicitly take into account coupling between the outputs and hence motivate the use of advanced multi-variable control approaches in the following sections.

5.3 Open Loop Frequency Domain Analysis

5.3.1 Open Loop Transfer Function

The multi-input multi-output (MIMO) transfer function matrix $G(s)$ of the open-loop dynamics of the SIDO DC–DC buck converter, derived in Chapter 3, is used to study the system. This matrix describes the relation between duty cycle inputs and output voltages and includes direct and cross-coupling effects between channels. The plant's general structure is as follows:

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

$$G_{12}(s) = \frac{v_{o1}(s)}{d_2(s)}, G_{21}(s) = \frac{v_{o2}(s)}{d_1(s)}$$

where the diagonal elements are the main control paths, and the off-diagonal elements are the interactions between outputs. The non-zero off-diagonal elements indicate that the system has coupled dynamics that should be carefully studied in the frequency domain.

5.3.2 Bode Plot of the Open Loop System

The open loop system frequency response is analysed from the Bode plot of the transfer function matrix $G(s)$. This gives us a feeling for the dynamics of the system such as gain characteristics, phase lag, resonance behaviour and bandwidth limitations.

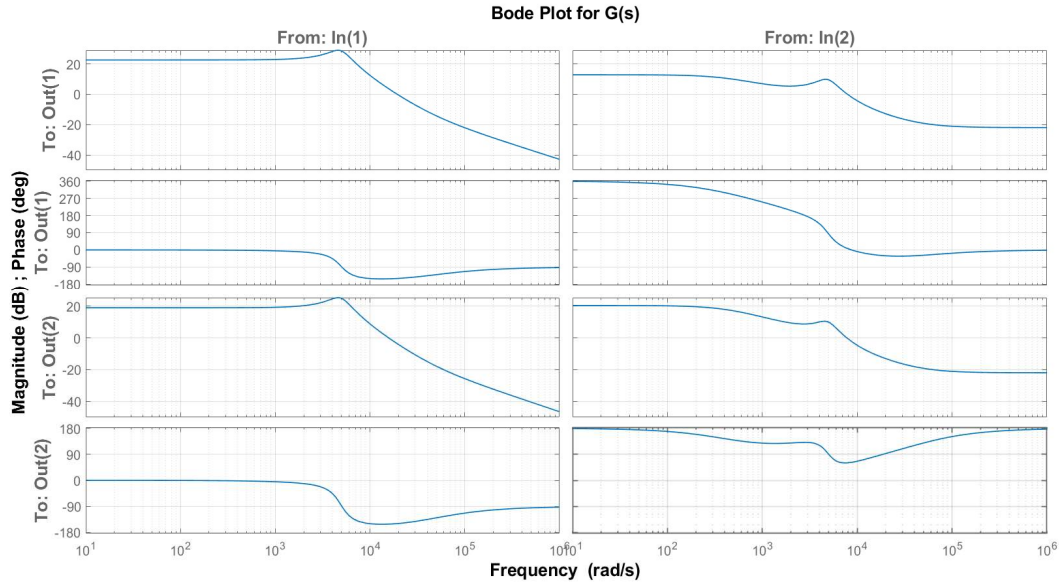


Fig 6.6: Bode plot of the open-loop transfer function matrix $G(s)$

The magnitude plots show that both output channels have a high gain at low frequencies, which is desirable for good steady-state regulation. The gain decreases with the increase of frequency, which means that the high frequency components are attenuated. This behaviour is a reflection of the inherent low-pass characteristics of the converter.

Both outputs show a clear resonance peak in the mid-frequency range. This peak is associated with the natural dynamics of the energy storage elements, especially the inductor and output capacitors. The observation of this resonance suggests that the system is susceptible to oscillatory behaviour if not properly controlled.

From the phase plots, there is a considerable amount of phase lag as the frequency increases, with phase lag values tending towards those equal to or greater than -180° in some channels. This means that the system has a small phase margin and can therefore be unstable when used in an open-loop configuration. The high phase lag is common to higher order systems, mainly due to the presence of the energy storage devices.

5.3.3 Interaction Analysis in Frequency Domain

Another piece of information that is obtained from open loop frequency analysis is the interaction between output channels. As shown in the figures, the magnitude and phase characteristic for each element of the transfer function is not the same. This implies that the degree of interaction varies at various frequencies. There is minimal interaction

between the channels at low frequencies, making the behaviour of the system relatively stable. However, at mid-frequencies (where resonance occurs), the interactions have a bigger role to play. The variations of the gain and phase characteristics between the channels indicate different behaviour to the input variations for each output. This indicates that the system cannot be considered as two independent SISO systems without introducing inaccuracies.

Implications for Control Design

The open loop analysis clearly demonstrates that the SIDO converter exhibits complex dynamic behaviour, including resonance, phase lag and coupling between channels. These characteristics limit the performance that can be achieved and suggest the need for feedback control. In particular, the existence of resonance peaks and high phase lag reveals that the system needs to be compensated in order to improve the stability margins and the damping. Furthermore, the coupling between channels in the results suggests that decentralized control methods might fail under certain operating conditions.

Therefore, the results of the open-loop frequency analysis are a good motivation for the control strategies developed in Chapter 4. The need of controllers to take care of both dynamic behaviour and interaction effects becomes evident, thus motivate the comparative study between conventional PID and structured H-infinity based PID control.

5.4 Controller Performance and Comparative Analysis of Conventional PID vs H- infinity PID Controller

Table 5.7: Controller parameters for Conventional PID Controller

Controller	K_i	K_p	K_d	T_f
K11	0.452	316e+03	2.96e-05	3.03e-06
K22	-1.6	-2.85e+04	1.42e-05	1.5e-05

Table 5.8: Controller parameters for H-infinity PID Controller

Controller	K_i	K_p	K_d	T_f
K11	0.33	881	3.45e-05	2.68e-05
K22	-2.52	-3.91e+03	0.000119	1.18e-05

5.4.1 Transient Response Analysis

The transient performance of the SIDO converter is investigated with the closed-loop MIMO step responses for both control strategies. These responses show the dynamic

behaviour of each output with respect to change in the control inputs and hence give an idea about the system stability, damping characteristics and interaction effects.

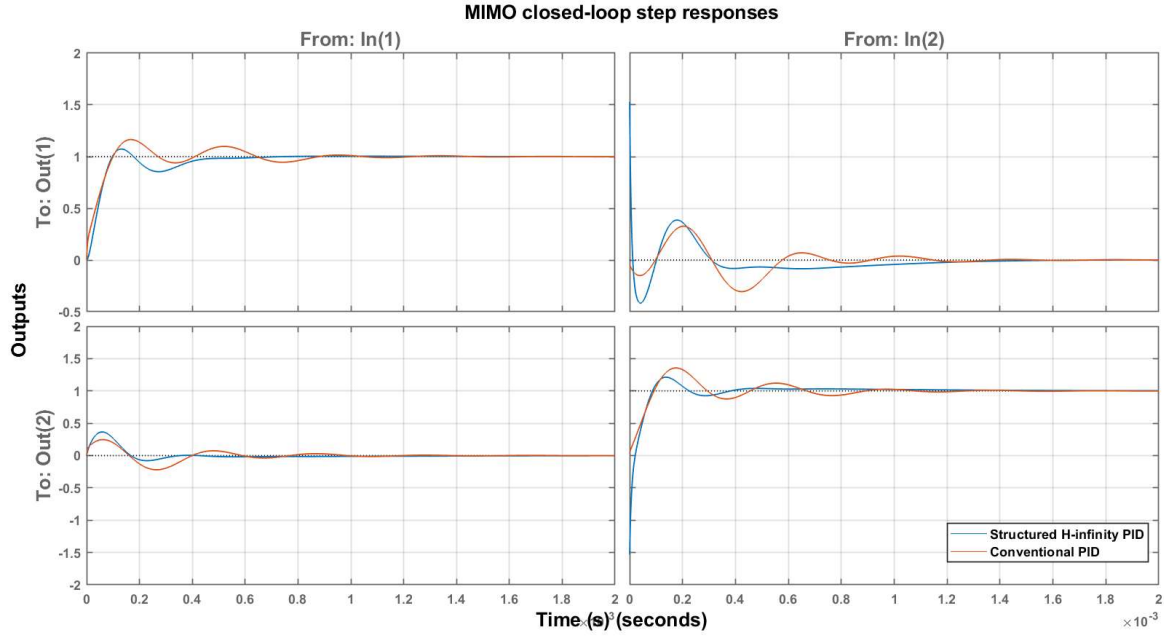


Fig 6.7: Closed Loop MIMO Step Response

The diagonal responses, corresponding to the main control paths, ($d_1 \rightarrow v_{o1}$ and $d_2 \rightarrow v_{o2}$) show that both controllers can perform stable voltage regulation. But their transient properties are definitely different. The structured H-infinity based controller has less overshoot and faster settling time compared to the conventional PID controller. The response is smoother and more damped which means improved stability margins.

On the other hand, the conventional PID controller shows relatively more oscillations and slower convergence to the steady state value. This behaviour is a signature of lack of damping and robustness particularly in the presence of higher order dynamics.

The off-diagonal responses ($d_1 \rightarrow v_{o2}$ and $d_2 \rightarrow v_{o1}$) directly measure the cross-coupling between outputs. In these channels, the conventional PID controller has larger deviations, which indicates that there are more interaction effects. On the contrary, the structured H-infinity controller suppresses such deviation to a large extent, which demonstrates the effectiveness of the controller in cross-regulation suppression.

From the transient response analysis, it is obvious in general that the structured H-infinity based PID controller provides better dynamic performance with improved damping, fast response and less interaction between the output channels.

5.4.2 Closed Loop Frequency Domain Analysis

The closed loop Bode plots are utilized to analyse the frequency domain behaviour of the controlled SIDO converter. These plots provide insight into the system bandwidth, robustness, disturbance attenuation and interaction behaviour for both the control strategies.

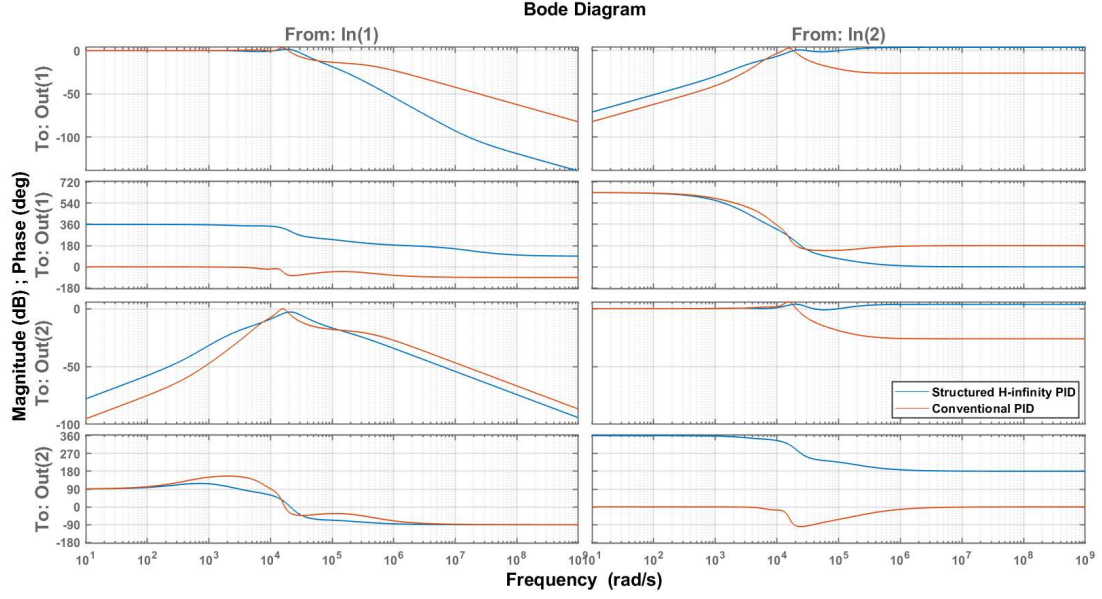


Fig 6.8: Closed Loop Bode Plot Comparison

The magnitude plots of the main control paths show that both controllers have sufficient low frequency gain, which guarantees good steady-state tracking performance. However, the high-frequency behaviour shows distinct differences. The structured H-infinity based controller has a smoother roll-off and much higher attenuation at high frequencies than the conventional PID controller. On the other hand, the H-infinity based controller has a more uniform and well-shaped frequency response which demonstrates its ability to control system dynamics in a better way. The differences between the two controllers can also be seen in the phase characteristics. The structured H-infinity controller exhibits better phase behaviour which implies better stability margins and lesser chances of oscillatory response. The conventional PID controller, on the other hand, has relatively larger phase variations which are consistent with the oscillatory transient behaviour observed in the time domain response.

An important observation can be made from the off-diagonal frequency responses which represent the cross-coupling between the outputs. These channels have lower gain over a wide frequency band, which indicates a good suppression of the effects of the interaction, for the structured H-infinity controller. On the other hand, the traditional PID controller has larger gains in the cross-coupling paths, especially in the mid frequency region, which demonstrates its poor ability to deal with multi-variable dynamics.

5.4.3 Stability Analysis

The stability characteristics of the closed-loop SIDO converter are investigated by means of pole-zero maps of the system transfer functions. The position of poles in the complex plane directly explains the stability, damping behaviour and transient response of a system.

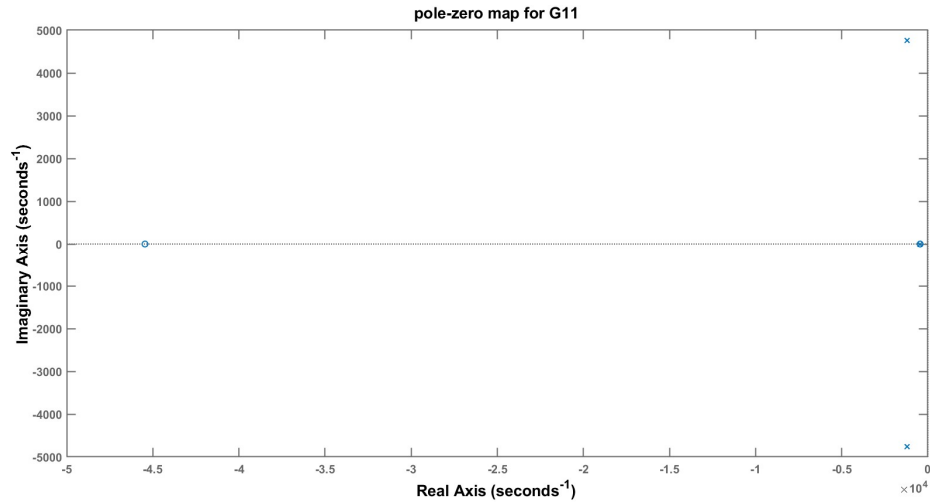


Fig 6.9: Pole-Zero map for G11

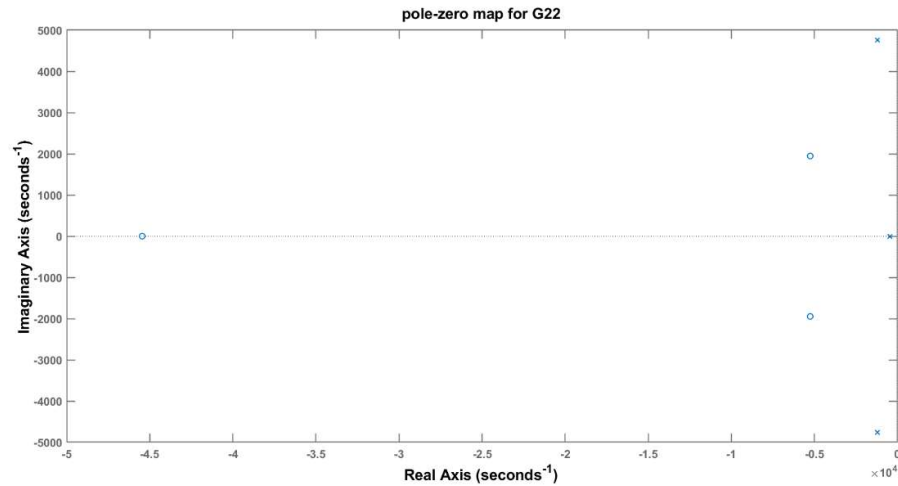


Fig 6.10: Pole-Zero Map for G22

From The pole-zero maps show that all poles are located in the left-half of the complex plane, which confirms that the closed-loop system is stable for both control strategies. However, the relative location of poles provides further insight into the dynamic performance of the system. The dominant poles are close to the imaginary axis which means that the system has an oscillatory behaviour with finite damping. This is in agreement with the transient responses discussed earlier, where oscillations are observed

before the system settles to its steady state value. It should be noted that the structured H-infinity based controller results in pole locations that are effectively pushed deeper into the left-half plane than the conventional PID controller. This shift implies more damping and faster decay of transient oscillations. Thus, the system shows less overshoot and faster settling as seen in the step response analysis.

This is in contrast to the poles of the traditional PID controller which are comparatively closer to the imaginary axis showing less damping and slower convergence. This accounts for the larger overshoots and longer settling times in the transient response. The presence of complex conjugate pole pairs also shows the second order dominant behaviour of the system dynamics, which is typical of power electronic converters with energy storage elements. The position of these poles further strengthens the idea that the structured H-infinity controller offers better damping while not significantly changing the fundamental system dynamics.

The pole-zero analysis is consistent with the time- and frequency-domain results, and shows that the structured H-infinity based PID controller has better stability characteristics, better damping and improved transient response than the conventional PID controller.

5.4.4 Load Regulation Analysis

The load regulation performance of the SIDO DC–DC buck converter is evaluated by introducing step changes in the load resistances of both output channels at different time instants. Specifically, the load resistance R_1 is reduced to half of its nominal value at $t = 0.1$ s, followed by a similar reduction in R_2 at $t = 0.2$ s. These changes represent sudden increases in load demand and are used to assess the disturbance rejection capability and dynamic response of the control strategies.

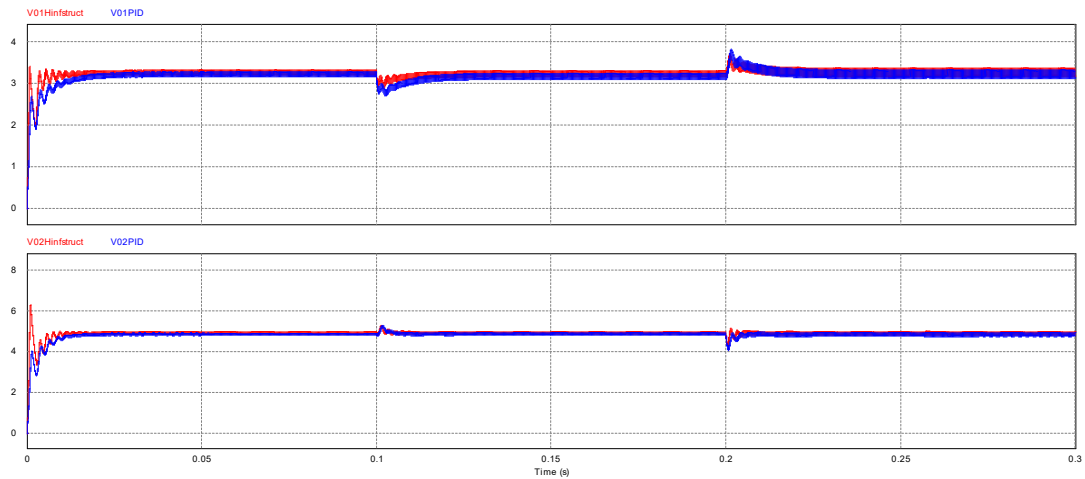


Fig 6.11: Load regulation comparison for conventional PID vs structured H-infinity PID controller for SIDO buck converter

When R_1 is reduced to a half, a clear disturbance in the output voltage v_{o1} is seen at the time $t = 0.1$ s. The standard PID controller suffers from large voltage dip and oscillatory behaviour before slowly converging to steady state value. The deviation extent and settling time show that the damping is not high and the recovery is slow under sudden load variation.

On the other hand, the structured H-infinity based PID controller exhibits a much more regulated response. With small oscillations, the voltage dip is much smaller and the system recovers faster. This shows a better ability to reject disturbances and better damping characteristics. At this point an important observation is the effect on the second output, v_{o2} .

The SIDO converter has a common inductor and inherent coupling that passes the disturbance in R_1 to the second output channel. Obviously, this cross-regulation effect is more pronounced in the case of the conventional PID controller where v_{o2} shows visible fluctuations. The structured H-infinity controller effectively suppresses the interaction and hence the disturbance in the unaffected output is much smaller.

A similar trend is observed at $t = 0.2$ s, when the load resistance R_2 is halved. The conventional PID controller again exhibits a greater transient deviation and oscillatory response in v_{o2} and a measurable disturbance in v_{o1} due to coupling. This is an indicator of sensitivity to load variations, since the recovery is relatively slow.

But the H-infinity based controller, which is structured, provides a more stable response. With better damping, the deviation in v_{o2} is less and the system soon returns to steady-state. Also, the disturbance induced in v_{o1} is greatly reduced, indicating effective suppression of cross-coupling effects.

The overall load regulation analysis has shown the superiority of the structured H-infinity based PID controller to deal with the load variations in a dynamic manner. It provides lower voltage deviation, faster recovery and better decoupling between output channels. These features make it very attractive for multi-output converter applications which need independent regulation over various load conditions.

5.4.5 Line Regulation Analysis

The line regulation performance of the SIDO DC–DC buck converter is tested by applying a step change to the input voltage. In this work, the input voltage V_{in} is increased from 12 V to 18 V at around $t = 0.1$ s. This disturbance is used to evaluate the capability

of the control strategies to maintain stable output voltages under supply variations.

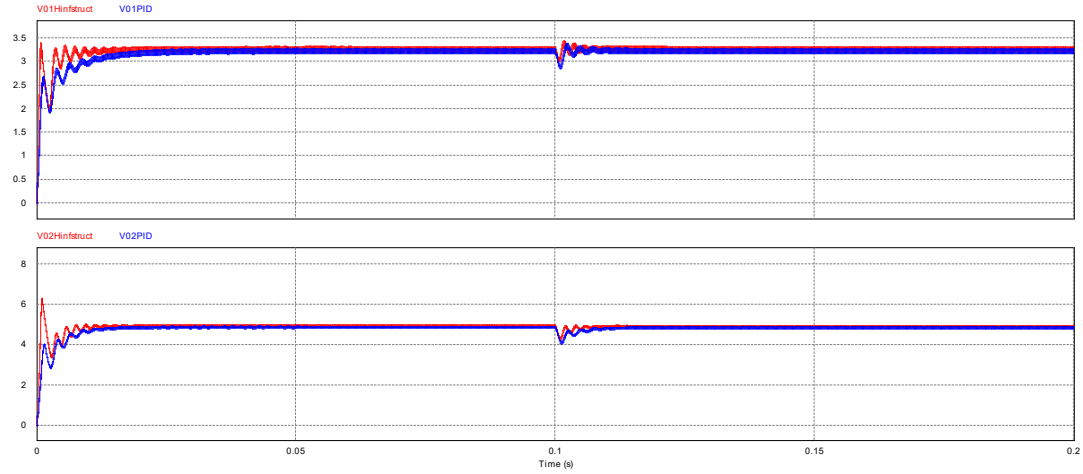


Fig. 6.12. Line regulation comparison for conventional PID vs structured H-infinity PID controller for SIDO buck converter

At the moment of the input voltage change, both output voltages v_{o1} and v_{o2} are transiently disturbed due to the abrupt increase of available input energy. For the classical PID controller, both outputs show large variations with overshoot and oscillatory behaviour before reaching the desired steady-state values. These fluctuations are large enough to suggest a higher sensitivity to the inputs and a limited robustness to line disturbances. On the other hand, the structured H-infinity based PID controller has a more controlled response. The transient deviation in both outputs is reduced greatly and the system settles faster with small oscillations. This leads to better disturbance rejection capability and regulation performance under different input conditions. Also, it is important to note that the effect of the input voltage change is distributed between the two output channels due to the shared energy path in the converter. The conventional PID controller shows more prominent fluctuations in both the outputs indicating its poor ability to handle this coupled disturbance effectively. The structured H-infinity controller, however, is more tightly regulated so that the disturbance does not propagate to other outputs. In summary, the result of line regulation analysis shows that the H-infinity structured PID controller gives better robustness against the variations in the input voltage. It has reduced overshoot, faster settling and better voltage stability and is therefore more suitable for applications where supply conditions may vary.

5.5 Conclusion

In this chapter, the interaction behaviour and control performance of the SIDO DC–DC buck converter was analysed. The interaction study proved the existence of coupling between output channels. Load variations had a significant effect on system interaction and dynamic behaviour. Conventional PID and structured H-infinity based PID controllers were compared using time-domain and frequency-domain metrics. The results showed that the H-infinity based controller gives better transient performance

CHAPTER 6

CONCLUSION AND FUTURE SCOPE

6.1 Conclusion

This thesis presented an in-depth study on the modelling, analysis and control of a Single-Input Dual-Output (SIDO) DC–DC buck converter system with emphasis on tackling the issues of multi-output operation. The SIDO topology, unlike conventional single output converters, inherently introduces coupling between output channels due to the common energy transfer path, making the independent regulation a non-trivial problem. The work started with the development of an accurate mathematical representation of the converter using state-space and small signal modelling techniques. The derived transfer function matrix was able to sufficiently describe the dynamic interactions between the inputs and outputs, highlighting the multi-variable characteristic of the system. This modelling framework was the basis for further analytical and control-oriented studies. A key contribution of this work is the systematic quantification of the interaction between output channels. The study demonstrates the parameter-dependence and frequency-dependence of interaction using well-established metrics such as Relative Gain Array (RGA), Hankel Interaction Index Array (HIIA) and H2 norm. The results clearly indicated that load variations have significant impact on both steady-state and dynamic coupling, while passive components mainly affect the transient interaction characteristics. These results proved that the hypothesis of independent output behaviour is not valid for SIDO converters under practical conditions.

To meet these challenges, two control strategies were studied and compared. The baseline approach was a traditional PID controller designed on the basis of equivalent SISO models. Although it provided acceptable steady-state regulation, it performed poorly in interaction and disturbance handling, which led to higher overshoot, oscillations and cross-regulation effects.

Contrarily, the structured H-infinity based PID controller was designed based on the full multi-variable system model considering explicitly the robustness and interaction. The simulation results have shown that this method can effectively optimize the performance of the system. The H-infinity based controller exhibited better damping, lower overshoot, faster settling time and enhanced robustness to parameter variations and disturbances. It also effectively suppressed the cross-coupling between the outputs, thus achieving better decoupling and independent regulation.

Both the controllers' performance has been validated by extensive analysis in terms of transient response, frequency domain behaviour, stability assessment and load and line regulation studies. The structured H-infinity based controller outperformed the

conventional PID approach in all evaluation metrics and proved to be suitable for the multi-output converter applications.

In conclusion, this work lays the ground that interaction-aware modelling and robustness-oriented control design are needed for high-performance regulation of SIDO DC-DC converters. The proposed structured H-infinity based PID framework offers a practical and effective solution that combines the simplicity of PID control with the benefits of modern robust control techniques.

6.2 Future Scope

The current paper presents a detailed analysis and a feasible control solution for the SIDO DC-DC buck converter. Nevertheless, there is room for further research and development to improve applicability and performance of such systems in practical scenarios. One of the most important directions for future work is the experimental validation of the proposed control strategies. The present study is based on simulation results, which, although extensive, do not fully take into account practical non-idealities such as switching losses, parasitic elements, sensor inaccuracies and noise. The implementation of the proposed controllers on a hardware prototype would provide a valuable insight of the real-time performance and help to identify implementation challenges thus strengthening the practical relevance of the work.

Another interesting direction is the extension of the control framework for more complex converter topologies. The methodology developed in this thesis can be applied to Single-Input Multiple-Output (SIMO) converters with more than two outputs, where interaction becomes even more relevant. An important step towards practical deployment in advanced power electronic applications would be to investigate scalability of the structured H-infinity based approach in such systems. Furthermore, optimization-based tuning techniques for controller parameter selection are also another possible research area. The validation through experiments of the suggested approach will depend on the development of a special-purpose testing facility that allows for the experimental implementation of a particular class of benchmark problems.

Also, worth mentioning is the implementation of the suggested digital control algorithms. Aspects such as discretization, sampling, computational limitations and quantization errors may seriously influence system performance and need to be taken into account with suitable compensators designed for the problem at hand.

In conclusion, although the thesis provides some solid foundations regarding the modelling and control of SIDO converters, further studies can be conducted in order to improve its performance.

LIST OF PUBLICATIONS

Publisher	Name of Paper	Scopus Indexed	Status
NE-IECCE 2026 (NIT Meghalaya)	Pairing Analysis and Control Structure Selection with Parametric Variations in SIDO DC-DC Buck Converter System	YES	Accepted
Springer PICS 2026 (NIT Hamirpur)	Decentralized Robust PID Controller Design for Single-Inductor Dual-Output DC-DC Buck Converter System	YES	

APPENDICES

APPENDIX 1: CONFERENCES



2026 IEEE North-East India International Energy Conversion Conference and Exhibition

(NE-IECCE 2026)








12th to 14th June 2026

Organised by Department of Electrical Engineering
National Institute of Technology Meghalaya, Saitsohpen, Sohra (Cherrapunjee)
East Khasi Hills District, Meghalaya – 793108, India

Figure A.1: NE-IECCE 2026



PICS-2026

2nd International Conference on Power and Intelligent Control System

July 02-03, 2026

(Hybrid Mode)

Department of Electrical Engineering
National Institute of Technology, Hamirpur

▶ The Power and Intelligent Control Systems (PICS-2026) is the second international conference dedicated to power engineering, control systems, and artificial intelligence. It brings together researchers, academicians, and practitioners to share ideas, discuss emerging technologies, and inspire transformative advancements. Join us in shaping the future of these dynamic fields!

It is planned to publish the peer reviewed and selected papers of conference as proceedings with Springer in their prestigious "Lecture Notes in Electrical Engineering" series. For detailed instructions for authors and editors of conference proceedings, kindly visit the following link: <https://www.springer.com/us/authors-editors/conference-proceedings>. Select papers from the conference will be published by Springer as a proceedings book volume. Springer will conduct quality checks on the accepted papers and only papers that pass these checks will be published. Springer Nature does not charge any money for publication of Non-Open Access content. Abstracts/extended abstracts and short papers (less than 4 pages) are not considered for publication. All communication regarding the proceedings must be directed to the conference organizers or volume editors. Paper authors should not contact Springer Nature directly.



Figure A.2: Springer PICS 2026

APPENDIX 2: PAPER ACCEPTANCE PROOF

5/30/26, 9:15 PM

Delhi Technological University Mail - NE-IECCE 2026: Notification of Acceptance on Paper ID: 805



24/C&I/04 MONISHA SINGH <monishasingh_24ci04@dtu.ac.in>

NE-IECCE 2026: Notification of Acceptance on Paper ID: 805

2 messages

Microsoft CMT <noreply@msr-cmt.org>
To: Monisha Singh <monishasingh_24ci04@dtu.ac.in>

Thu, Apr 2, 2026 at 9:13 PM

Dear Monisha Singh,

Greetings from 2026 IEEE North-East India International Energy Conversion Conference and Exhibition (NE-IECCE 2026)

Congratulations!

In reference to Paper ID 805: "Pairing Analysis and Control Structure Selection with Parametric Variations in SIDO DC-DC Buck Converter System" submitted at the 2026 IEEE North-East India International Energy Conversion Conference and Exhibition to be organized by the Department of Electrical Engineering at the National Institute of Technology Meghalaya, Saitsohpen, Sohra (Cherrapunjee), Meghalaya - 793108, India, it is our pleasure to inform you that the paper has been Accepted for presentation (Oral/Poster). Please revise your paper according to the comments of the reviewers (can be found at CMT).

You are instructed to follow the NE-IECCE 2026 website: <https://www.ne-iecce2026.org> for instructions on preparing the final Camera Ready Paper and submitting the Copyright Form.

Mode of presentation: In-person.

IMPORTANT!!!!!!

1. The registration, camera-ready paper submission, and IEEE Electronic Copyright Form (eCF) submission for all accepted articles are open until April 15, 2026.
2. Please submit your final camera-ready paper and IEEE eCF through both your CMT account and the designated Google Form during registration.

We welcome you to 2026 IEEE North-East India International Energy Conversion Conference and Exhibition at NIT Meghalaya Sohra (Cherrapunjee), India.

Regards,
TPC, Chair(s)
NE-IECCE 2026

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Figure A.3: Acceptance Proof for NE-IECCE 2026

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