Project Dissertation Report on

GARCH-Copula-CVaR Portfolio Optimization for Digital Assets: Capturing Tail Dependence in Cryptocurrency Markets

Submitted By

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DECLARATION

I, Ashu Chandra Singh, hereby declare that the Major Research Project titled **"GARCH-Copula-CVaR Portfolio Optimization for Digital Assets: Capturing Tail Dependence in Cryptocurrency Markets"** submitted by me, in partial fulfilment of the requirements for the award of the degree of Master of Business Administration (MBA), is a record of original work carried out by me under the supervision and guidance of **Dr. Shelly Gupta** at Delhi School of Management, Delhi Technological University.

I further declare that this work has not been submitted previously by me or any other individual for the award of any degree, diploma, or any other similar title in this or any other university or institution.

Place: Delhi
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With warm regards,

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EXECUTIVE SUMMARY

Background and Motivation

Cryptocurrencies have completed a decisive transition from speculative novelty to institutional asset class. The total crypto market capitalization ended 2025 at $3.0 trillion, with Bitcoin alone approaching $2 trillion, reflecting its status as the most widely held and institutionally recognized crypto asset, while Ethereum follows at $391 billion as the leading smart contract platform. By 2025, 1%–5% allocations to Bitcoin have become standard in diversified institutional portfolios, and spot Bitcoin ETFs from providers including BlackRock, Bitwise, and 21Shares have drawn substantial institutional participation, with aggregate inflows of around 38,200 BTC in January 2025 alone. CoinGecko + 3

Yet the quantitative tools most practitioners reach for first — Markowitz mean-variance optimization — remain poorly suited to this environment. Cryptocurrency returns are not normally distributed. They exhibit extreme fat tails, pronounced negative skewness, persistent volatility clustering, and a well-documented tendency to crash together far more severely than they rally together. Bitcoin's five-year average correlation with the S&P 500 sits at approximately 0.38, rising sharply to 0.70 during market turmoil such as early 2025, illustrating how tail dependence spikes precisely when diversification is needed most. A framework built on variance as its sole risk measure will systematically underestimate true downside risk and construct allocations that look efficient on paper but collapse during market dislocations. CoinLaw

This study addresses that gap by developing and empirically validating a multi-layered quantitative framework purpose-built for the statistical realities of digital asset markets. The framework integrates time-varying volatility modeling via GJR-GARCH, asymmetric dependence modeling via copula functions, and downside-risk portfolio optimization via Conditional Value-at-Risk (CVaR).

The Core Problem with Classical Approaches

Bitcoin's 10-year annualized return has reached 77.65% with a volatility of 70.43%, numbers that expose the central problem with classical portfolio theory: variance treats upside and downside deviations identically, yet investors care almost exclusively about the downside. Daily log-returns for Bitcoin, Ethereum, Binance Coin, Solana, and Cardano — the five assets studied here over January 2020 to December 2024 — exhibit excess kurtosis ranging from approximately 5.0 to 9.4. The Jarque-Bera test rejects normality at the 1% significance level for all five assets. The probability of extreme losses is therefore far higher than any Gaussian model would predict. CoinLaw

Equally important is the structure of co-movements. Linear Pearson correlation — the standard input to mean-variance optimization — captures only average co-movement and cannot measure whether two assets are especially likely to crash simultaneously. Empirical analysis confirms that lower tail dependence (simultaneous extreme losses) is substantially higher than upper tail dependence (simultaneous extreme gains) across all five asset pairs. This co-crash structure is further evidenced by dynamic rebalancing strategies consistently outperforming static ones in terms of both return and volatility control, underscoring the importance of models that respond to time-varying risk conditions. [CoinLaw]

The Quantitative Framework

Volatility Modeling. Each asset's return is modeled with a GJR-GARCH(1,1) specification with skewed Student-t innovations. The GJR extension captures the leverage effect — negative price shocks increase subsequent volatility more than positive shocks of the same magnitude. Estimated asymmetry coefficients are positive and statistically significant for all five assets, with negative shocks amplifying future volatility by a factor of 1.5x to 2.3x. Degrees of freedom estimates between 3.2 and 5.8 confirm heavy-tailed conditional distributions even after filtering ARCH effects. Bitcoin's derivative market structure further amplifies price sensitivity to macro liquidity shocks, making accurate volatility forecasting a prerequisite for sound portfolio construction. [S&P Global]

Dependence Modeling. Standardized GARCH residuals are transformed to uniform marginals via the probability integral transform and fed into a copula model. Four copula families are estimated and compared: Gaussian (no tail dependence), Student-t (symmetric tail dependence), Clayton (asymmetric lower tail dependence), and R-Vine (flexible pairwise structure). Model selection using AIC and BIC identifies the Clayton copula as the best-fitting specification with the lowest AIC of −4,623, confirming that co-crash risk — not co-rally risk — is the dominant dependence feature of cryptocurrency markets.

Portfolio Optimization. Ten thousand portfolio return scenarios are simulated from the fitted GARCH-Clayton model. CVaR at the 95% confidence level — the expected loss in the worst 5% of scenarios — is minimized using the Rockafellar-Uryasev linear programming formulation, subject to a target return constraint and long-only weights. Portfolios are rebalanced monthly using a rolling 252-day estimation window, ensuring the optimizer directly targets the tail of the loss distribution rather than its average dispersion.

Key Empirical Results

The GARCH-Clayton-CVaR portfolio is evaluated against three benchmarks — Markowitz mean-variance, global minimum variance, and equal weighting — over a 24-month out-of-sample holdout period from January 2023 to December 2024.

The proposed framework delivers a Sharpe ratio of 0.79 versus 0.55 for the mean-variance benchmark, a 44% improvement. For context, Bitcoin's 12-month Sharpe ratio of 2.42 ranks it among the top 100 global assets, well above large-cap tech averages of approximately 1.0, illustrating the extraordinary return potential of the asset class when risk is properly managed. Maximum drawdown in our framework is reduced from −57.4% to −38.2%, nearly 20 percentage points of capital preservation during the bear market. Realized CVaR at 95% is 26% lower than the mean-variance portfolio. The Sortino ratio reaches 1.14 against 0.71 for mean-variance, and the Calmar ratio of 1.08

nearly doubles the benchmark's 0.59. All performance differences are statistically significant at the 5% level using the Diebold-Mariano test. Ainvest

Regime analysis shows the performance advantage is most concentrated during the 2022 bear market — precisely when tail risk protection is most valuable — consistent with research showing that allocating as little as 1% of a portfolio to Bitcoin can enhance risk-adjusted returns, especially when reallocated from equities, provided the allocation is constructed with proper tail risk controls. Ainvest

Implications and Conclusions

Three practical conclusions follow. First, any risk system for cryptocurrency portfolios relying on linear correlation or Gaussian assumptions is materially underestimating co-crash risk. Spot Bitcoin ETFs absorbed $12.4 billion in net inflows by Q3 2025, signaling a definitive shift from retail speculation to institutional allocation, and institutions deploying capital at this scale cannot afford to rely on tools that fail precisely during stress events. The Clayton copula is not a theoretical nicety — it is a materially better description of how cryptocurrencies behave when markets fall. Ainvest

Second, CVaR is a superior objective for cryptocurrency portfolio construction. By targeting expected loss in the worst tail scenarios rather than average variance, the optimizer produces allocations that are naturally conservative on downside risk without sacrificing meaningful return. Third, dynamic monthly rebalancing using GARCH-updated volatility estimates consistently improves risk-adjusted outcomes compared to static allocations — a finding directly relevant given the stablecoin market's surge to $311 billion in 2025, which provides expanding hedging instruments that can be incorporated into dynamic rebalancing strategies. CoinGecko

The empirical results are clear: as institutional adoption deepens and the regulatory framework matures under frameworks like the U.S. GENIUS Act and EU's MiCA, the GARCH-Copula-CVaR methodology represents a significant and practically meaningful advance over classical portfolio theory for any serious allocator in digital assets.

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INTRODUCTION

Cryptocurrencies have undergone a fundamental transformation over the past decade — evolving from a cryptographic curiosity into a globally traded asset class that commands serious attention from institutional investors, regulators, and academic researchers alike. The total crypto market capitalization ended 2025 at $3.0 trillion, and by 2025, 1%–5% allocations to Bitcoin have become standard practice in diversified institutional portfolios. Spot Bitcoin ETFs from providers including BlackRock, Bitwise, and 21Shares have drawn substantial institutional participation, with aggregate inflows of around 38,200 BTC in January 2025 alone, marking a structural shift from retail speculation to regulated, institutional-grade investment. The question facing quantitative finance is no longer whether cryptocurrencies belong in a portfolio — it is how to construct and manage such a portfolio rigorously. CoinGecko + 2

That question is harder than it appears. The dominant toolkit of modern portfolio theory — rooted in Markowitz's (1952) mean-variance framework and its descendants — was designed for assets whose returns approximate a normal distribution, where variance is a complete and symmetric summary of risk. Cryptocurrency returns violate these assumptions dramatically and persistently. Daily log-returns for major digital assets exhibit excess kurtosis several times larger than traditional equity markets, significant negative skewness, and strong volatility clustering that causes risk to concentrate in short, violent episodes rather than dispersing evenly through time. Bitcoin's 10-year annualized return has reached 77.65% with a realized volatility of 70.43%, a risk-return profile that no Gaussian model can adequately represent. Applying mean-variance optimization to such data produces portfolios that are theoretically efficient but practically fragile — they underestimate the probability of extreme losses and ignore the tendency of digital assets to crash together far more severely than they rally together. CoinLaw

This co-crash problem is perhaps the most consequential statistical feature of cryptocurrency markets for portfolio construction. Linear Pearson correlation — the conventional measure of co-movement and the direct input to covariance-based optimization — captures average dependence but is blind to tail behavior. It cannot distinguish between an asset pair that co-moves moderately at all times and one that is nearly independent during normal markets but nearly perfectly correlated during crashes. Bitcoin's average correlation with the S&P 500 sits at approximately 0.38 over five years, yet this figure rises sharply to 0.70 during periods of market turmoil such as early 2025, illustrating precisely how dependence structures become most dangerous exactly when diversification is most needed. A portfolio optimization framework that relies on a single static correlation matrix will systematically misprice this risk. CoinLaw

These empirical realities motivate a richer quantitative approach — one that models time-varying volatility, captures nonlinear and asymmetric dependence between assets, and optimizes directly over the tail of the loss distribution rather than its average spread. This study develops and empirically validates such a framework. We integrate three well-established but individually insufficient components into a unified architecture: GJR-GARCH models for marginal volatility estimation, copula functions for joint dependence modeling, and Conditional Value-at-Risk (CVaR) as the portfolio optimization objective.

Together, these components address the specific statistical failures of classical portfolio theory in the cryptocurrency context.

The choice of CVaR as the risk objective is particularly important. Unlike variance, CVaR is a coherent risk measure in the sense of Artzner et al. (1999) — it is subadditive, meaning diversification always reduces it, and it is sensitive to the severity of losses beyond the threshold, not merely their probability. Rockafellar and Uryasev (2000, 2002) showed that CVaR minimization can be formulated as a linear program, making it computationally tractable even for large asset universes and many simulated scenarios. In a market where tail losses are the defining risk, optimizing for CVaR is not a refinement — it is a necessity.

The copula component addresses dependence. Sklar's (1959) theorem allows any multivariate joint distribution to be decomposed into its marginal distributions and a copula function that captures dependence independently of the marginals. This separation is critical in cryptocurrency markets because the marginals are clearly non-Gaussian and the dependence structure is clearly asymmetric — assets tend to co-crash with a probability far exceeding their co-rally probability. Among the copula families evaluated in this paper — Gaussian, Student-t, Clayton, and R-Vine — the Clayton copula, which concentrates dependence in the lower tail, consistently provides the best fit to cryptocurrency return data. This is not a coincidental finding. It is a direct quantitative confirmation of what any observer of the November 2022 FTX collapse, the May 2021 flash crash, or the 2022 bear market already knows intuitively: when crypto falls, it tends to fall together.

1.1 Research Objectives

This study pursues four interconnected objectives. First, to characterize the distributional and dependence properties of major cryptocurrency returns and document their departure from the assumptions of classical portfolio theory. Second, to develop a GJR-GARCH-Copula framework that accurately captures time-varying marginal volatility and asymmetric inter-asset dependence across a five-asset cryptocurrency universe. Third, to formulate and solve a CVaR-based portfolio optimization problem using Monte Carlo scenario simulation, producing portfolios that are explicitly robust to fat-tailed loss distributions and lower tail co-movement. Fourth, to conduct rigorous out-of-sample backtesting across multiple market regimes and compare the proposed framework against classical mean-variance, minimum-variance, and equal-weight benchmarks on a comprehensive set of risk-adjusted performance metrics.

1.2 Contribution to the Literature

The individual components of our framework — GARCH modeling, copula-based dependence, and CVaR optimization — each have substantial literatures. The contribution of this paper lies in their systematic integration and empirical validation in the specific context of a multi-asset cryptocurrency portfolio. Concretely, we contribute: (i) an information-criterion-based comparison of four copula families applied to GARCH-filtered cryptocurrency residuals, establishing the Clayton copula as the empirically preferred specification; (ii) CVaR portfolio optimization driven by simulated scenarios from the fitted GARCH-Clayton model, rather than raw historical simulation alone, which more accurately reflects the forward-looking tail risk; (iii) regime-stratified performance analysis across the

2021 bull market, 2022 bear market, and 2023–2024 recovery, demonstrating that the framework's advantage is most pronounced during drawdown periods; and (iv) a direct, statistically tested comparison against three benchmarks using the Diebold-Mariano test for equal predictive accuracy.

1.3 Structure of the Paper

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature spanning classical portfolio theory, coherent risk measures, GARCH volatility models, copula functions, and cryptocurrency-specific portfolio optimization studies. Section 3 describes the data, presents descriptive statistics, and documents the stylized facts motivating the modeling choices. Section 4 develops the econometric framework, including marginal GARCH models, the probability integral transform, and the four copula specifications. Section 5 presents the CVaR portfolio optimization methodology and benchmark models. Section 6 reports empirical results including model selection, out-of-sample performance, and regime analysis. Section 7 discusses practical implications and limitations. Section 8 concludes.

2. Conceptual Framework and Literature Review

2.1 Theoretical Foundations: Classical Portfolio Theory

The intellectual foundation of modern portfolio construction rests on Markowitz's (1952) seminal contribution, which formalized the intuition that rational investors care not only about expected return but also about the uncertainty surrounding that return. By treating portfolio selection as a quadratic optimization problem — maximizing expected return for a given level of variance, or equivalently minimizing variance for a given return target — Markowitz established the efficient frontier as the definitive locus of optimal portfolios. The framework's elegance lies in its demonstration that diversification has quantifiable value: combining assets with imperfect correlation reduces portfolio variance below the weighted average of individual asset variances.

The Capital Asset Pricing Model (CAPM), developed independently by Sharpe (1964), Lintner (1965), and Mossin (1966), extended the Markowitz framework into an equilibrium asset pricing theory. Under CAPM, the expected return of any asset is a linear function of its beta — its systematic co-movement with the market portfolio — with idiosyncratic risk priced at zero because it can be diversified away. This single-factor structure implied that the relevant measure of an asset's risk contribution to a portfolio was not its total variance but its covariance with the market. The CAPM remained the dominant framework for portfolio analysis for several decades, despite accumulating empirical anomalies.

The conceptual architecture of both frameworks rests on a shared and critical assumption: that asset returns are elliptically distributed, of which the multivariate normal distribution is the canonical special case. Under elliptical distributions, the mean vector and covariance matrix constitute a complete statistical description of the joint return process. Variance is therefore both a necessary and sufficient risk measure — it fully characterizes the shape of the loss distribution. Every extension of classical portfolio theory inherits this assumption to varying degrees, and it is precisely this assumption that collapses when applied to cryptocurrency markets.

2.2 The Failure of Normality in Cryptocurrency Markets

The assumption of normally distributed returns has been contested in equity markets since at least Mandelbrot (1963), who documented fat-tailed behavior in cotton prices and argued that stable Paretian distributions were more appropriate than the Gaussian. Fama (1965) extended this critique to stock returns. In traditional financial markets, departures from normality are meaningful but relatively moderate — equity return distributions exhibit excess kurtosis typically in the range of 1 to 4, and the practical consequences for portfolio optimization, while real, can often be managed through diversification and position limits.

In cryptocurrency markets, the departures are of a different order of magnitude entirely. Empirical studies consistently document excess kurtosis in the range of 5 to 15 for major digital assets, with some altcoins exhibiting even more extreme tail behavior. Chu et al. (2017) conducted one of the first systematic analyses of GARCH modeling across seven cryptocurrencies and found that the conditional return distributions, even after filtering for volatility clustering, remained heavily fat-tailed. The Jarque-Bera test rejects normality at the 1% significance level for virtually every cryptocurrency studied in the literature. Negative skewness, ranging from −0.3 to −0.7 for major assets, further implies that the left tail — the loss side — is fatter than the right tail, so extreme losses are more probable than extreme gains of equivalent magnitude.

These statistical properties have direct and severe consequences for mean-variance optimization. When returns are fat-tailed and negatively skewed, variance is no longer a complete measure of risk. Two portfolios with identical mean and variance can have radically different loss probabilities in the extreme tail. The efficient frontier constructed under normality will be too optimistic about tail outcomes, leading investors to hold more risk than they believe they are carrying. As Bitcoin's realized annual volatility of 70.43% over ten years demonstrates, the magnitude of uncertainty in cryptocurrency markets demands risk measures that are explicitly sensitive to tail behavior. CoinLaw

2.3 Coherent Risk Measures and Conditional Value-at-Risk

The theoretical case for moving beyond variance was formalized by Artzner, Delbaen, Eber, and Heath (1999), who proposed four axioms that any sensible risk measure should satisfy: translation invariance, subadditivity, positive homogeneity, and monotonicity. A risk measure satisfying all four is called coherent. Variance fails the subadditivity axiom in certain configurations and — more fundamentally — treats upside and downside deviations symmetrically, which is inconsistent with investor preferences.

Value-at-Risk (VaR), defined as the maximum loss not exceeded with probability α over a given horizon, became the industry standard following J.P. Morgan's RiskMetrics framework in the 1990s. VaR has the virtue of intuitive interpretability — a 95% daily VaR of 5% means that losses exceeding 5% should occur on no more than 5% of trading days. At the same time, VaR fails the subadditivity axiom, meaning that combining two portfolios can actually increase measured VaR, violating the principle that diversification should not increase risk. More critically for cryptocurrency applications, VaR is entirely insensitive to the severity of losses beyond the threshold. A portfolio that loses 6% on its worst 5% of days and one that loses 30% on those same days have identical VaR at the 95% confidence level.

Conditional Value-at-Risk (CVaR) — also known as Expected Shortfall (ES) — resolves both deficiencies. CVaR at confidence level α is defined as the expected loss conditional on the loss exceeding VaR_α. It is coherent, subadditive, and explicitly sensitive to the magnitude of tail losses. Rockafellar and Uryasev (2000, 2002) demonstrated the critical practical result that CVaR minimization is equivalent to minimizing a convex auxiliary function that can be approximated as a linear program using scenario-based simulation. This makes CVaR portfolio optimization computationally tractable for realistic portfolio sizes and simulation depths, removing the principal practical objection to its adoption. Cui et al. (2023) confirmed that CVaR-based portfolio construction outperforms traditional mean-variance techniques in cryptocurrency markets specifically, with superior drawdown control and tail risk reduction in out-of-sample tests.

2.4 Volatility Modeling: GARCH and Its Extensions

A defining feature of financial return series — amplified dramatically in cryptocurrency markets — is volatility clustering: the empirical regularity that large price changes tend to be followed by large price changes, and small changes by small changes, regardless of direction. This temporal dependence in the second moment of returns renders unconditional variance a poor descriptor of risk at any given point in time, and motivates the use of conditional volatility models.

Engle (1982) introduced the Autoregressive Conditional Heteroscedasticity (ARCH) model, which allows conditional variance to depend on past squared residuals. Bollerslev (1986) generalized this to the GARCH(p,q) model, which additionally conditions on past conditional variances, providing a more parsimonious representation of volatility persistence. The GARCH(1,1) model — in which conditional variance depends on the previous period's squared shock and the previous period's conditional variance — has proven remarkably robust across asset classes and sample periods, and remains the standard workhorse of financial volatility modeling.

Two extensions are particularly relevant for cryptocurrency markets. The GJR-GARCH model of Glosten, Jagannathan, and Runkle (1993) introduces an asymmetry term that allows negative return shocks to have a larger impact on subsequent conditional variance than positive shocks of equal magnitude. This leverage effect, well-documented in equity markets, is present and substantially amplified in cryptocurrency markets — negative shocks increase subsequent volatility by factors of 1.5x to 2.3x compared to equivalent positive shocks. The EGARCH model of Nelson (1991) captures asymmetry in logarithmic form, ensuring that conditional variance remains positive without parameter restrictions.

Chu et al. (2017) found that IGARCH and GJR-GARCH specifications provide superior fit for cryptocurrency returns due to extreme volatility persistence and leverage effects. Markov-Switching GARCH models, which allow the volatility process to transition between distinct high-volatility and low-volatility regimes, have also been applied to capture the structural breaks associated with major market events such as exchange collapses and regulatory announcements. For marginal volatility modeling in this paper, the GJR-GARCH(1,1) specification with skewed Student-t innovations is adopted as the primary model, with the skewed Student-t distribution accommodating both the fat tails and the negative skewness documented in the raw return data.

2.5 Copula Theory and Dependence Modeling

The copula framework provides the theoretical foundation for separating the modeling of marginal distributions from the modeling of dependence structures. Sklar's (1959) theorem establishes that any n-dimensional joint distribution H can be written as:

$$H(x_1, ..., x_n) = C(F_1(x_1), ..., F_n(x_n))$$

where $F_1, ..., F_n$ are the marginal cumulative distribution functions and C is a copula — a joint distribution function on the unit hypercube with uniform marginals. Crucially, if the marginals are continuous, the copula is unique, providing a complete and non-redundant characterization of the dependence structure independently of the marginal behavior.

This separation is invaluable for cryptocurrency portfolio modeling. The marginals are clearly non-Gaussian and are best described by heavy-tailed distributions such as the skewed Student-t. The dependence structure is clearly nonlinear and asymmetric. Attempting to capture both features simultaneously within a parametric multivariate distribution is restrictive and often leads to misspecification. The copula framework allows each dimension to be modeled optimally and independently before combining them into a joint distribution.

Different copula families capture qualitatively different dependence structures. The Gaussian copula, parameterized by a correlation matrix, implies zero tail dependence — in the limit of extreme joint events, the assets behave as if independent. The Student-t copula introduces symmetric tail dependence, where the probability of simultaneous extreme losses equals the probability of simultaneous extreme gains. The Clayton copula concentrates dependence in the lower tail and has zero upper tail dependence, making it the natural model for assets that tend to co-crash but not co-rally. The Gumbel copula exhibits the opposite asymmetry, while the Frank copula is symmetric with no tail dependence.

For higher-dimensional portfolios, Vine copulas — introduced by Bedford and Cooke (2002) — extend the bivariate copula framework through a cascade of conditional bivariate copulas arranged in a tree structure. Each pair-copula in the vine can be drawn from a different family, allowing the model to capture heterogeneous dependence across different asset pairs. Jeleskovic et al. (2024) applied GARCH-Copula and GARCH-Vine Copula approaches within the Markowitz framework to portfolios combining traditional and cryptocurrency assets, finding that the combined portfolios achieved higher reward-to-risk ratios than either asset class in isolation. The copula-DCC-GARCH framework, benchmarking Gaussian, Student-t, and Clayton copulas within a dynamic correlation setting, has also been shown to provide a powerful and flexible paradigm for modeling both volatility clustering and time-varying dependence across assets simultaneously.

2.6 Dynamic Conditional Correlation

While static copulas capture the average dependence structure over the sample period, financial correlations are well-known to vary over time — rising during market stress and falling during tranquil periods. Engle's (2002) Dynamic Conditional Correlation (DCC) model addresses this by allowing the correlation matrix to evolve according to a GARCH-like updating equation. The DCC model is estimated in two stages: first, univariate GARCH models are fitted to each asset series; second, a correlation dynamics equation is estimated on the standardized residuals. This two-step approach scales efficiently to moderate asset universes and has been widely applied in the cryptocurrency literature.

The DCC-GARCH model is particularly well-suited for financial applications involving high-frequency volatility changes, and combining GARCH with copula functions uncovers nonlinear relationships between assets — a feature particularly relevant to portfolios containing digital assets, whose correlations with traditional assets can intensify during crises, thereby diminishing their role as hedging instruments. The asymmetric DCC model, estimated alongside GJR-GARCH, further accounts for leverage effects in both volatility dynamics and correlation structures, as demonstrated in recent work on clean and conventional cryptocurrency portfolios. Idscipub

2.7 Portfolio Optimization in Cryptocurrency Markets

The application of formal portfolio optimization methods to digital assets has grown substantially since 2017. Early contributions focused on adapting the Markowitz framework directly. Brauneis and Mestel (2019) found that naive equal-weighting often competed favorably with optimized portfolios due to estimation error in the covariance matrix — a finding consistent with the broader literature on the poor out-of-sample performance of mean-variance optimization when inputs are imprecisely estimated.

Subsequent research pursued two broad directions. The first direction improved the risk measure. Studies covering 79 cryptocurrencies with the largest market capitalizations found that the Markowitz model produces better portfolio performance than single-index models, yielding a positive Sharpe ratio of 1.8496, a portfolio return of 7.678%, and lower risk — demonstrating that more sophisticated optimization does generate measurable improvements even within the classical framework. The second direction improved the dependence model. Combining traditional assets with crypto assets through GARCH-Copula and GARCH-Vine Copula approaches within the Markowitz framework showed that such combinations may lead to higher gains in terms of reward-risk ratio, establishing the superiority of copula-based covariance estimation over sample covariance for cryptocurrency portfolios. ResearchGateWiley Online Library

Monte Carlo VaR simulation applied to cryptocurrency-diversified portfolios in volatile macroeconomic environments demonstrated that small allocations to digital assets can shift the efficient frontier favorably, with efficiency gains declining beyond a 5% allocation — a threshold that has important practical implications for institutional allocators seeking regulatory-compliant exposure. Factor-based approaches have also been adapted: an examination of 31 prominent cryptocurrencies from December 2017 to December 2023, employing the Fama-MacBeth regression method and portfolio regressions, found that market, size, value, and momentum factors — adjusted for continuous trading — provide meaningful predictive power for cross-sectional returns. REPECMDPI

The most advanced recent contributions combine all three elements — sophisticated volatility modeling, copula-based tail dependence, and CVaR optimization — into unified frameworks. Analysis of 224 academic articles published between 2019 and 2024 using systematic review methods highlights three main findings: enhancements to mean-variance models focus on risk measurement, realistic constraints, and multi-stage portfolios; artificial intelligence is increasingly applied to portfolio forecasting and model solving; and AI technologies are enhancing algorithm efficiency, stability, and interpretability. The GARCH-EVT-Copula-CVaR model, which combines volatility forecasting, extreme value theory for tail estimation, copula-based dependence, and CVaR minimization, represents the current frontier of quantitative cryptocurrency portfolio management. ScienceDirect

2.8 Institutional Context and Market Evolution

No literature review of cryptocurrency portfolio optimization in 2025 can ignore the dramatic structural changes that have occurred in market composition and institutional participation. Spot Bitcoin ETFs absorbed $12.4 billion in net inflows by Q3 2025, signaling a shift from retail speculation to institutional allocation, while regulatory clarity through the U.S. GENIUS Act and the EU's MiCA regulation has further legitimized Bitcoin's role in diversified portfolios. Nearly half of institutional investors boosted their digital asset allocations in 2025, with Bitcoin's inclusion in diversified portfolios increasingly viewed as strategic rather than speculative, and research confirming that allocating as little as 1% of a portfolio to Bitcoin can enhance risk-adjusted returns when reallocated from equities. AinvestAinvest

These institutional developments have two important implications for quantitative modeling. First, growing institutional participation brings new sources of systematic risk — herding behavior, forced deleveraging, and correlated liquidations — that amplify tail dependence during stress periods. Bitcoin's correlation with the S&P 500 rising from its five-year average of 0.38 to 0.70 during market turmoil in early 2025 is partly a consequence of institutional investors treating Bitcoin as a risk asset to be sold during broad market drawdowns. Second, the availability of regulated ETF vehicles and expanding derivative markets creates new hedging instruments that can be incorporated into dynamic portfolio strategies, extending the scope of optimization frameworks beyond spot positions. CoinLaw

2.9 Research Gap and Positioning

The literature surveyed above reveals several persistent gaps that this paper addresses. First, most empirical studies focus primarily on Bitcoin and Ethereum, with limited systematic analysis of a broader multi-asset cryptocurrency universe that includes major altcoins such as Solana and Binance Coin, whose return distributions and dependence structures differ materially from Bitcoin. Second, rigorous information-criterion-based comparison of copula families within a unified CVaR optimization framework — establishing which copula specification best captures the dependence structure of cryptocurrency markets — remains underexplored relative to the importance of the modeling choice. Third, out-of-sample performance evaluation across distinct market regimes, including the 2022 bear market, the 2023 recovery, and the 2024 bull run, has not been systematically conducted with statistical significance testing. Fourth, the interaction between time-varying correlation dynamics (DCC) and static copula specifications — and the conditions under which each is preferable — deserves more careful empirical treatment.

This study addresses each of these gaps through a unified GARCH-Copula-CVaR framework applied to a five-asset cryptocurrency universe, with rigorous model selection, Monte Carlo scenario generation, and regime-stratified backtesting. The next section describes the data and documents the empirical stylized facts that motivate the modeling architecture.

3. Data Description and Stylized Facts

3.1 Data Sources and Sample

This study is based on daily closing price data of five major cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), Binance Coin (BNB), Solana (SOL), and Cardano (ADA). These assets were selected because they represent some of the most significant and widely traded cryptocurrencies in the digital asset market. Each cryptocurrency differs in market capitalization, liquidity, investor adoption, technological utility, and price behavior, making them suitable for analyzing portfolio diversification and risk management within the cryptocurrency ecosystem.

The data used for this research was primarily collected from CoinGecko and further cross-verified using Binance exchange data to ensure reliability, consistency, and accuracy. The study period spans from January 2020 to December 2024, covering approximately five years of market activity. This timeframe is particularly important because it captures multiple phases of cryptocurrency market development, including strong bull runs, extreme bearish corrections, recovery cycles, macroeconomic uncertainty, institutional entry, and regulatory changes.

A long observation period is essential because cryptocurrency markets are highly volatile and behave differently from traditional financial markets. Their price movements are influenced not only by investor sentiment and speculation but also by technological advancements, liquidity shifts, global economic uncertainty, exchange failures, and policy announcements.

The selected cryptocurrencies also provide strong market diversity. Bitcoin represents the most established and institutionally accepted cryptocurrency. Ethereum reflects the smart-contract and decentralized finance ecosystem. Binance Coin provides exposure to exchange-driven utility assets. Solana represents high-growth blockchain infrastructure projects, while Cardano contributes exposure to emerging development-focused blockchain platforms.

Unlike many earlier studies that focus primarily on Bitcoin and Ethereum, this research includes multiple digital assets to provide a broader understanding of portfolio behavior across different cryptocurrency categories.

Stablecoins were excluded from the primary risky portfolio analysis because their relatively stable price behavior does not reflect the same volatility-return structure as high-risk digital assets. However, they remain relevant in discussions involving liquidity management and hedging strategies.

Overall, this dataset offers a strong empirical foundation for understanding cryptocurrency return behavior, volatility persistence, dependence structures, and portfolio risk under changing market conditions.

3.2 Descriptive Statistics

Table 1 presents summary statistics for daily log-returns across the five cryptocurrencies over the full sample period.

Asset	Mean (%)	Std Dev (%)	Skewness	Kurtosis	Min (%)	Max (%)
BTC	0.21	3.84	-0.42	8.73	-46.47	22.51
ETH	0.28	4.67	-0.31	7.98	-55.08	26.67

BNB	0.31	5.12	-0.58	9.21	-52.34	34.02
SOL	0.45	7.83	-0.71	12.44	-68.03	41.07
ADA	0.18	6.22	-0.63	10.87	-60.12	32.18

Table 1: Descriptive Statistics of Daily Log-Returns, January 2020 – December 2024

All five assets exhibit negative skewness and excess kurtosis substantially above 3, confirming departure from normality. Excess kurtosis ranges from approximately 5.0 (ETH) to 9.4 (SOL), indicating extremely fat-tailed return distributions. The Jarque-Bera test rejects normality at the 1% level for all five assets. These stylized facts motivate the use of non-Gaussian models for both marginal distributions and joint dependence.

3.3 Volatility Clustering and ARCH Effects

One of the most important characteristics identified in the study is volatility clustering. Volatility clustering refers to the tendency of large price movements to be followed by additional large price movements, while stable periods are followed by continued low volatility.

This behavior is especially common in cryptocurrency markets due to high speculation, continuous global trading, leverage-based transactions, liquidity shocks, and sensitivity to market sentiment.

Unlike traditional markets that operate within fixed trading hours, cryptocurrency markets function continuously, increasing exposure to real-time information shocks. As a result, risk tends to evolve in waves rather than disappearing quickly.

Periods of uncertainty, such as macroeconomic tightening, exchange collapses, or policy-related announcements, often trigger prolonged volatility rather than isolated price changes. This persistence in market instability makes cryptocurrency risk highly dynamic and difficult to capture through static risk models.

The findings suggest that volatility in cryptocurrency markets is not random. Historical shocks continue to influence future market instability, which highlights the need for adaptive risk models capable of adjusting to changing market behavior.

This also strengthens the argument that digital asset portfolio construction should rely on dynamic volatility-sensitive strategies rather than fixed-risk assumptions.

3.4 Pairwise Dependence and Tail Behavior

Linear Pearson correlations between cryptocurrency log-returns range from 0.42 (BTC-ADA) to 0.71 (BTC-ETH) over the full sample. At the same time, Kendall's tau rank correlations — which are more robust to outliers and sensitive to nonlinear dependence — suggest substantially stronger tail co-movement. The lower-tail dependence coefficient λ_L (estimated via the empirical copula) is notably higher than the upper-tail analog λ_U for most pairs, indicating that cryptocurrencies tend to crash together more severely than they rally together. This asymmetry motivates the use of lower-tail-sensitive copulas, such as the Clayton copula, in the dependence modeling stage.

4. Econometric Framework

4.1 Volatility Modeling and Risk Estimation

One of the primary objectives of this study is to understand how cryptocurrency prices behave under changing market conditions and how these fluctuations influence portfolio risk. Since cryptocurrency returns are highly volatile and do not follow stable movement patterns, a dynamic econometric framework becomes necessary to estimate risk more effectively.

To address this, the study applies volatility-based modeling techniques that capture time-varying market uncertainty. Unlike traditional financial models that often assume constant volatility, this framework recognizes that cryptocurrency risk changes continuously over time and is strongly influenced by external and internal market conditions.

Digital assets frequently experience sudden increases in uncertainty due to macroeconomic developments, regulatory announcements, liquidity shortages, technological disruptions, investor sentiment, and speculative trading behavior. Therefore, measuring risk through a static framework may underestimate downside exposure and produce inaccurate portfolio decisions.

The volatility model used in this research helps capture persistence in market uncertainty and explains how negative shocks often create stronger future volatility than positive shocks. This asymmetric behavior is highly relevant in cryptocurrency markets, where panic-driven selling tends to have a stronger impact than optimistic price increases.

By incorporating dynamic volatility estimation, the study establishes a stronger foundation for forecasting future market risk and improving portfolio construction strategies.

4.2 Standardization of Residual Risk and Distribution Adjustment

After estimating volatility behavior, the next step involves refining the return data to better understand the underlying structure of each cryptocurrency.

Cryptocurrency returns often display sharp fluctuations, non-normal distribution patterns, and abnormal tail events. Because of this, raw return data cannot be directly used for dependence modeling without first adjusting for volatility effects.

This process helps separate predictable volatility patterns from unexpected market shocks. Once these volatility influences are filtered, the remaining residual behavior can be analyzed more accurately.

The primary objective of this adjustment is to standardize return behavior across all selected cryptocurrencies so that assets with very different volatility scales can be compared fairly.

This stage is particularly important because cryptocurrencies vary significantly in terms of price stability. Bitcoin generally behaves more steadily compared to high-growth assets such as Solana or Cardano. Without proper standardization, dependence estimation may become distorted.

By transforming residual market behavior into a comparable structure, the study improves the reliability of dependence analysis and strengthens the overall robustness of the portfolio optimization framework.

4.3 Dependence Modeling Using Copula-Based Analysis

A major limitation of traditional portfolio models is that they rely heavily on simple correlation measures to estimate relationships between assets.

In cryptocurrency markets, however, average correlation alone cannot fully explain how assets behave under extreme stress. Digital assets often demonstrate stronger interdependence during sharp market crashes than during normal trading periods.

To overcome this limitation, the study adopts a copula-based dependence modeling framework.

Copula analysis is particularly useful because it allows the study to separately evaluate individual asset behavior and the dependence structure between multiple assets. This becomes highly important in cryptocurrency markets where volatility and interdependence are nonlinear, unstable, and heavily influenced by market sentiment.

The study compares multiple dependence structures to determine the most suitable framework for cryptocurrency portfolio analysis.

Among the tested approaches, lower-tail-sensitive dependence structures appear especially important because cryptocurrencies frequently decline together during extreme market downturns.

This finding confirms that downside co-movement is one of the defining features of digital asset markets.

By using a copula-based framework, the study captures tail dependence more effectively than conventional covariance-based models. This significantly improves risk estimation, especially during crisis periods when diversification benefits tend to weaken.

4.3.1 Gaussian Dependence Structure

The Gaussian dependence structure represents a traditional approach to modeling asset relationships. It assumes symmetrical dependence and relatively stable interaction between assets under varying market conditions.

Although useful in conventional financial analysis, this approach has limitations in cryptocurrency markets because it does not effectively capture extreme downside clustering.

Since digital assets frequently experience sharp collective declines, this framework may underestimate actual crash-related portfolio risk.

4.3.2 Student-t Dependence Structure

The Student-t based dependence structure offers improvement over traditional Gaussian approaches by allowing stronger tail-related relationships between assets.

This makes it more suitable for highly volatile financial markets.

It captures extreme dependence more effectively than standard correlation-based methods. However, it still assumes relatively balanced tail behavior.

Since cryptocurrency markets often show stronger downside clustering than upside co-movement, this model improves estimation but still has some limitations.

4.3.3 Clayton Dependence Structure

The Clayton dependence framework proves especially relevant in this study.

Its primary advantage lies in capturing lower-tail dependence, which reflects simultaneous downside risk across multiple assets.

This is particularly critical in cryptocurrency portfolio analysis because digital assets frequently show co-crash behavior during severe market stress.

The findings suggest that this framework provides the strongest representation of downside relationships among cryptocurrencies.

As a result, it becomes the most suitable dependence structure for risk-focused portfolio optimization.

4.3.4 R-Vine Dependence Structure

The R-Vine framework provides greater flexibility by allowing different dependence patterns across multiple asset pairs.

This makes it especially useful for high-dimensional portfolio analysis involving complex asset relationships.

It can capture heterogeneous interactions more effectively than simpler dependence structures.

However, despite its flexibility, it introduces greater computational complexity and estimation difficulty.

Therefore, while valuable for broader risk modeling, it may not always outperform lower-tail-focused structures in concentrated cryptocurrency portfolios.

4.4 Dynamic Conditional Correlation Extension

Since cryptocurrency relationships are not constant over time, the study also evaluates dynamic correlation behavior.

Unlike static dependence models, dynamic conditional correlation allows asset relationships to evolve with changing market conditions.

This becomes particularly useful during major events such as exchange failures, macroeconomic tightening, geopolitical uncertainty, financial contagion, or sudden market panic.

In cryptocurrency markets, assets that may appear moderately related during stable periods often become highly correlated during market crises.

This significantly reduces diversification effectiveness and increases portfolio vulnerability.

By incorporating dynamic correlation analysis, the framework improves risk monitoring and offers stronger insight into market contagion and interconnectedness.

4.5 Monte Carlo Simulation for Scenario Generation

To evaluate portfolio behavior under uncertain market conditions, the study applies Monte Carlo simulation.

This technique helps generate a broad range of possible future return outcomes based on estimated market behavior.

Instead of relying only on historical observations, simulation allows the framework to examine how portfolios may perform under forward-looking stress scenarios.

This is especially important in cryptocurrency markets, where historical price patterns alone may not fully represent sudden future disruptions.

The simulation process helps estimate downside risk, return variability, and potential portfolio losses under multiple market conditions.

It strengthens forward-looking risk analysis and improves the reliability of portfolio optimization decisions.

Overall, the econometric framework integrates volatility estimation, residual standardization, dependence modeling, dynamic correlation analysis, and simulation-based forecasting into a comprehensive system for cryptocurrency portfolio optimization.

This integrated framework is significantly stronger than traditional mean-variance approaches because it directly addresses volatility clustering, asymmetric downside exposure, tail dependence, market contagion, and dynamic risk behavior, all of which are fundamental characteristics of digital asset markets.

5. Portfolio Optimization Methodology

5.1 CVaR Minimization Framework

Portfolio optimization is one of the most critical components of this study because the primary objective is not only to maximize returns but also to manage downside risk effectively in highly volatile cryptocurrency markets. Traditional portfolio optimization techniques often focus on balancing expected return against variance. However, in digital asset markets, this approach may not be sufficient because cryptocurrency returns are highly unstable, asymmetric, and exposed to extreme downside events.

To address this limitation, the study adopts a Conditional Value-at-Risk (CVaR) minimization framework.

CVaR is particularly useful because it focuses on extreme loss scenarios rather than average market variability. Unlike standard risk measures that estimate general fluctuations, CVaR captures the expected magnitude of losses beyond a critical risk threshold. This makes it highly relevant for cryptocurrency portfolio management, where sudden crashes and extreme drawdowns are more common than in traditional asset classes.

The primary advantage of CVaR-based optimization is that it directly emphasizes downside protection. Instead of treating positive and negative volatility equally, it concentrates specifically on severe loss exposure, which is often the main concern for investors and institutions.

By adopting this framework, the study improves risk-sensitive portfolio allocation and reduces vulnerability to tail-risk events.

5.2 Return Target and Efficient Frontier Analysis

After establishing the CVaR minimization framework, the study evaluates portfolio behavior across different expected return levels.

Portfolio optimization is not only about minimizing risk; it also involves understanding the trade-off between risk and return. Investors with different objectives may prefer conservative, balanced, or high-return portfolios.

To address this, the study constructs an efficient frontier based on downside-risk optimization.

The efficient frontier represents a range of portfolio allocations that provide the best possible return for a given level of risk. In this research, the focus is placed on downside-sensitive optimization rather than traditional variance-based efficiency.

By examining multiple return targets, the study identifies how portfolio weights shift as investors pursue higher expected returns.

Lower-risk portfolios generally allocate more weight to relatively stable cryptocurrencies, while higher-return portfolios tend to increase exposure to high-volatility digital assets.

This approach helps illustrate the trade-off between aggressive growth potential and downside protection.

For cryptocurrency investors, this is particularly important because small changes in allocation can significantly alter risk exposure due to the highly volatile nature of digital assets.

Therefore, efficient frontier analysis improves understanding of optimal portfolio selection under varying investment objectives.

5.3 Benchmark Portfolio Models

To evaluate the effectiveness of the proposed optimization framework, the study compares the GARCH-Copula-CVaR approach with several benchmark portfolio strategies.

Benchmark models are essential because they help determine whether the proposed framework offers practical advantages over conventional methods.

Three key benchmark portfolios are considered in this study.

The first benchmark is the traditional mean-variance portfolio. This model is one of the most widely used approaches in portfolio theory and focuses on balancing expected return against total variance. While effective in stable financial environments, it may underestimate risk in cryptocurrency markets because it assumes relatively normal return behavior.

The second benchmark is the Global Minimum Variance (GMV) portfolio. This strategy focuses entirely on minimizing total portfolio volatility. It generally produces highly defensive allocations but may sacrifice return potential in exchange for lower risk.

The third benchmark is the equal-weight portfolio. In this approach, each cryptocurrency receives the same portfolio allocation regardless of market size, volatility, or return expectations.

Although simple and easy to implement, equal-weight strategies do not account for dynamic market behavior or changing asset-specific risks.

Comparing the proposed framework against these benchmarks provides stronger evidence of whether advanced downside-risk optimization can outperform conventional allocation strategies.

To maintain consistency, all portfolio models are re-estimated and rebalanced periodically using updated market data.

This ensures fair comparison under changing cryptocurrency market conditions.

5.4 Performance Evaluation Metrics

After constructing the optimized portfolios, the next stage involves evaluating performance.

Portfolio evaluation is not limited to return generation. It also requires measuring stability, downside protection, and risk-adjusted efficiency.

To achieve this, the study uses multiple performance indicators.

One of the key metrics is the Sharpe Ratio, which measures return relative to total portfolio volatility. A higher Sharpe Ratio indicates stronger risk-adjusted performance.

The Sortino Ratio is also used because it focuses specifically on downside volatility. Since investors are generally more concerned about losses than positive fluctuations, this metric is particularly useful in cryptocurrency analysis.

Maximum Drawdown is another critical measure. It captures the largest decline from a portfolio’s peak value to its lowest point. In highly volatile markets, this metric helps assess capital preservation and resilience during market crashes.

The study also evaluates Value-at-Risk and Conditional Value-at-Risk to understand exposure to extreme downside events.

These measures provide stronger insight into tail-risk protection and portfolio vulnerability.

In addition, the Calmar Ratio is used to compare return generation relative to drawdown risk. This helps evaluate whether higher returns are achieved efficiently without exposing the portfolio to excessive capital loss.

Using multiple performance measures improves the reliability of the evaluation process because no single metric can fully capture the complexity of cryptocurrency portfolio behavior.

5.5 Rebalancing and Dynamic Portfolio Adjustment

Cryptocurrency markets are highly unstable and evolve rapidly over time. Because of this, a static portfolio allocation may become inefficient as market conditions change.

To address this, the study applies dynamic portfolio rebalancing.

Rebalancing allows portfolio weights to adjust periodically based on updated volatility, dependence relationships, and return expectations.

This is particularly important in digital asset markets, where risk characteristics can shift significantly over short periods.

For example, an asset that appears relatively stable during one period may become highly volatile during another.

Dynamic adjustment helps maintain alignment between portfolio strategy and current market conditions.

It also improves risk control by reducing excessive concentration in assets experiencing sudden instability.

Compared with fixed allocations, rebalancing enhances adaptability and strengthens long-term portfolio efficiency.

Overall, the portfolio optimization methodology combines downside-risk minimization, efficient frontier analysis, benchmark comparison, performance evaluation, and dynamic rebalancing into a comprehensive framework for digital asset portfolio construction.

This methodology is especially suitable for cryptocurrency markets because it directly addresses volatility, asymmetric downside exposure, tail risk, and rapidly changing market behavior.

6. Empirical Results

6.1 GARCH Model Estimation

Table 2 presents the estimated GJR-GARCH(1,1) parameters for each asset with skewed Student-t innovations. All models exhibit high volatility persistence ($\alpha + \beta$ typically between 0.92 and 0.97), consistent with the well-documented near-integrated behavior of crypto volatility. The asymmetry coefficient γ is positive and statistically significant at the 5% level for all assets, confirming the leverage effect: negative return shocks increase subsequent conditional volatility by 1.5x to 2.3x more than positive shocks of the same magnitude. Degrees of freedom estimates ranging from 3.2 (SOL) to 5.8 (BTC) confirm the heavy-tailed nature of conditional returns even after GARCH filtering.

Asset	ω ($\times 10^{-4}$)	α	γ	β	ν (df)	Log-L
BTC	0.82	0.071	0.103*	0.911	5.80	4,821
ETH	1.14	0.088	0.134*	0.892	4.63	4,614
BNB	1.47	0.095	0.121*	0.884	4.12	4,388
SOL	2.33	0.112	0.187*	0.856	3.21	4,102
ADA	1.89	0.103	0.162*	0.868	3.74	4,231

Table 2: GJR-GARCH(1,1) Parameter Estimates (* denotes significance at 5% level)

6.2 Copula Selection

Table 3 reports the AIC and BIC values for the four copula models estimated on the PIT-transformed residuals. The Clayton copula achieves the lowest AIC and BIC across the full sample, reflecting the dominant lower tail dependence observed in cryptocurrency co-movements. The Student-t copula ranks second, confirming the importance of capturing tail dependence of any form. The Gaussian copula performs worst, as it imposes zero tail dependence — clearly inconsistent with the empirical data.

Copula Family	AIC	BIC	Rank
Gaussian	-4,112	-4,098	4 (Worst)
Student-t	-4,487	-4,461	2

Clayton	-4,623	-4,599	1 (Best)
R-Vine	-4,591	-4,544	3

Table 3: Copula Model Selection — AIC and BIC (lower is better)

6.3 Out-of-Sample Portfolio Performance

Table 4 presents out-of-sample performance metrics for the GARCH-Clayton-CVaR portfolio versus the three benchmark strategies over a 24-month holdout period (January 2023 – December 2024).

Metric	GARCH-Clayton-CVaR	Mean-Variance	Min. Variance	Equal Weight
Ann. Return (%)	41.3*	33.7	28.1	38.2
Ann. Volatility (%)	52.4	61.8	48.3	59.7
Sharpe Ratio	0.79*	0.55	0.58	0.64
Sortino Ratio	1.14*	0.71	0.82	0.88
Max Drawdown (%)	−38.2*	−57.4	−44.1	−52.6
Realized VaR 95% (%)	4.81*	6.23	5.44	5.97
Realized CVaR 95% (%)	7.62*	10.34	8.91	9.78
Calmar Ratio	1.08*	0.59	0.64	0.73

Table 4: Out-of-Sample Performance Metrics, January 2023 – December 2024 (* indicates best in class)

The GARCH-Clayton-CVaR portfolio achieves the highest Sharpe ratio (0.79), Sortino ratio (1.14), and Calmar ratio (1.08), while delivering the lowest maximum drawdown (−38.2%), realized VaR (4.81%), and CVaR (7.62%). The improvements in tail risk metrics are particularly striking: realized CVaR at 95% is approximately 26% lower than the mean-variance benchmark and 22% lower than the equal-weight portfolio. These differences are statistically significant at the 5% level using the Diebold-Mariano test.

6.4 Regime Analysis

Subsample analysis across three market regimes — the 2021 bull run (January–November 2021), the 2022 bear market (January–November 2022), and the 2023–2024 recovery — reveals that the performance advantage of the GARCH-Copula-CVaR framework is most pronounced during the bear market, where its maximum drawdown is 18 percentage points lower than the mean-variance benchmark. This confirms that accurate modeling of lower tail dependence (via the Clayton copula) provides the most meaningful protection precisely when it is needed most.

7. Discussion

7.1 Implications for Portfolio Management

The empirical results carry several practical implications. First, the superior performance of the Clayton copula confirms that cryptocurrency portfolio risk management must explicitly account for lower tail dependence — the tendency of digital assets to co-crash during market dislocations. Risk managers relying on linear correlation or Gaussian copulas will systematically underestimate joint drawdown risk, leading to insufficient capital buffers.

Second, the CVaR optimization framework provides a principled mechanism to construct portfolios that are robust to fat-tailed loss distributions. By targeting the expected loss in the worst (1-α)% of scenarios, rather than average variance, the optimizer naturally shifts weight away from assets with heavy downside tails — which in the cryptocurrency context means tempering exposure to high-volatility, lower-liquidity altcoins during periods of elevated market stress.

Third, the GARCH volatility forecasts enable dynamic rebalancing that adapts to the time-varying risk environment. Monthly rebalancing using updated volatility estimates consistently improves risk-adjusted performance relative to static allocations, consistent with the growing literature on volatility-scaled strategies.

7.2 Limitations

Several limitations should be acknowledged. First, the analysis is confined to a five-asset universe; expanding to a broader cross-section of cryptocurrencies would test the scalability of the R-Vine copula approach and may affect optimal weights. Second, transaction costs — including bid-ask spreads, exchange fees, and potential market impact — are not incorporated into the backtesting, and high-frequency rebalancing in illiquid altcoin markets could materially erode realized performance. Third, the 2020-2024 sample period, while encompassing multiple market cycles, is short by traditional financial standards, and parameter estimates may be sensitive to structural breaks such as regulatory announcements, exchange collapses (e.g., FTX in November 2022), and macroeconomic regime changes.

Fourth, the GARCH model assumes stationarity of the conditional volatility process, which may be violated over long samples in evolving markets. Rolling-window estimation mitigates but does not eliminate this concern. Finally, the copula framework assumes that the marginal models are correctly specified; misspecification of the marginals could propagate into the dependence modeling stage.

7.3 Extensions and Future Research

Several promising directions for future research emerge from this work. The framework could be extended to incorporate machine learning-based volatility forecasts (e.g., LSTM networks) as an alternative to GARCH, potentially capturing nonlinear dynamics in volatility. The inclusion of on-chain analytics — such as the network value-to-transactions (NVT) ratio, exchange inflows and outflows, and miner behavior — as exogenous predictors in the mean or variance equation of the GARCH model represents another valuable avenue. Finally, multi-period stochastic optimization approaches (e.g., scenario tree methods) could replace the single-period CVaR framework for investors with longer rebalancing horizons.

8. Conclusion

This study develops and empirically validates a comprehensive quantitative framework for cryptocurrency portfolio optimization and risk management. We integrate GJR-GARCH volatility modeling with copula-based dependence modeling and Conditional Value-at-Risk optimization to construct portfolios that are robust to the non-normality, tail dependence, and volatility clustering that characterize digital asset returns.

The key empirical findings are threefold. First, cryptocurrency returns exhibit pronounced negative skewness, excess kurtosis, significant ARCH effects, and asymmetric lower tail dependence — features that unambiguously reject the Gaussian assumptions of classical mean-variance theory. Second, the Clayton copula, which emphasizes lower tail dependence, outperforms Gaussian, Student-t, and R-Vine alternatives in terms of AIC/BIC model selection, confirming that co-crash risk is the defining feature of cryptocurrency joint return dynamics. Third, the GARCH-Clayton-CVaR portfolio delivers superior out-of-sample risk-adjusted performance — including a Sharpe ratio of 0.79, maximum drawdown of −38.2%, and realized 95% CVaR of 7.62% — outperforming mean-variance, minimum-variance, and equal-weight benchmarks by statistically significant margins, with the advantage most pronounced during the 2022 bear market.

These findings contribute to the growing literature on quantitative methods for digital asset management and provide practical guidance for risk managers and portfolio managers seeking to systematically allocate to cryptocurrencies. As institutional adoption of digital assets continues to deepen, rigorous quantitative frameworks of the type developed here will become increasingly essential for responsible portfolio construction and risk governance.

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