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Chapter 1

Introduction

1.1 Literature Survey

In the dynamic and competitive landscape of global commerce, organizations must adopt effective supply chain strategies that align with their overarching business objectives and strategic vision. These strategies go beyond traditional cost-efficiency goals and increasingly focus on adaptability, resilience, and sustainability. Supply Chain Management (SCM) encompasses the coordinated integration of all activities and entities involved in the sourcing, production, distribution, and delivery of goods and services [1]. This includes suppliers, manufacturers, distributors, retailers, and end customers, forming a complex network of interactions that facilitate the seamless flow of materials, information, and financial resources.

Traditionally, supply chains were modelled using linear and deterministic frameworks, assuming stable demand patterns and predictable lead times. However, real-world supply chains are far from linear or predictable. Disruptions due to geopolitical instability, pandemics, fluctuating customer preferences, and natural disasters have highlighted the need to re-evaluate traditional approaches. As a result, the supply chain is increasingly understood as a nonlinear dynamical system, wherein small changes at one node can propagate and amplify throughout the network due to interconnected feedback loops, time delays, and nonlinear interactions [2].

A central phenomenon that exemplifies this behaviour is the bullwhip effect, a term that refers to the magnified oscillations in order quantities as one moves upstream in the supply chain. Small fluctuations in consumer demand can result in disproportionately large adjustments in orders placed by retailers, distributors, and manufacturers [3]. This amplification leads to inefficiencies such as excess inventory, increased transportation costs, capacity imbalances, and reduced service levels. The bullwhip effect arises from several root causes, including demand signal processing, order batching, price fluctuations, and lack of communication across supply chain partners.

To address these challenges, researchers are increasingly developing nonlinear mathematical models that more accurately reflect the dynamic nature of supply chains. These models are often expressed as systems of differential equations, capturing time-dependent behaviors and nonlinear dependencies among supply chain elements [4]. By simulating the flow of goods, information, and finances across multiple echelons, these models offer a more realistic view of system behaviour, particularly under volatile market conditions.

Incorporating chaos theory into supply chain modelling provides further insights into the complex, sensitive, and sometimes unpredictable nature of supply chains. Chaos theory, which studies how deterministic systems can exhibit unpredictable behaviour due to sensitivity to initial conditions, is particularly relevant for SCM [5]. This characteristic, known as the "butterfly effect," is analogous to the bullwhip effect in supply chains. A small variation in customer demand or delivery schedule can lead to disproportionate consequences across the network. This suggests that supply chains may possess chaotic attributes, especially when subjected to high variability and delayed feedback.

Understanding supply chains as potentially chaotic systems prompts a shift in managerial focus toward adaptive and robust control strategies. These strategies are designed to cope with the inherent complexity and unpredictability of modern supply chains. They emphasize continuous monitoring, real-time data analytics, and the use of feedback mechanisms to adjust operational decisions dynamically. Synchronization becomes critical in this context—ensuring that supply chain partners operate in coordination despite external shocks and internal delays [4].

A promising method for managing such systems is the application of control theory, particularly through the use of linear feedback controllers. These controllers adjust inputs such as production rates, order quantities, or inventory levels in response to deviations from desired states [6]. When applied to nonlinear supply chain models, linear feedback can reduce demand variability propagation, stabilize inventory dynamics, and improve overall system responsiveness. However, due to the nonlinear and time-varying nature of real-world supply chains, control strategies must often be adaptive—capable of tuning themselves based on system behavior over time[7].

Evaluating the effectiveness and stability of these control mechanisms requires a rigorous mathematical foundation. Lyapunov stability theory offers such a framework. By constructing a Lyapunov function, researchers can determine whether a system will remain stable under perturbations or drift into chaotic behaviour. This theoretical analysis is crucial for validating the robustness of feedback control schemes in supply chain contexts [8]. It helps ensure that even under uncertainty and disturbances, the supply chain can maintain equilibrium or return to it efficiently.

Beyond theoretical modelling, numerical simulations play a crucial role in testing and refining supply chain strategies. Through simulations, supply chain managers and researchers can model various scenarios—such as demand shocks, supplier failures, or lead-time variations—and observe system responses. These simulations help identify optimal control parameters, uncover potential vulnerabilities, and evaluate the performance of synchronization schemes under realistic constraints [9].

In recent years, the integration of digital technologies has significantly enhanced the capacity of supply chains to implement adaptive and data-driven control systems. Technologies such as the Internet of Things (IoT), Artificial Intelligence (AI), and Machine Learning (ML) provide real-time visibility into supply chain operations. IoT-enabled sensors track shipments and monitor inventory, while AI algorithms analyze patterns and predict disruptions. Machine learning techniques continuously refine forecasting and control models based on historical data and system feedback. When combined with nonlinear modelling and chaos-aware controls, these technologies form the backbone of a modern, intelligent supply chain capable of navigating uncertainty and complexity [10].

Another important aspect to consider is the multi-echelon nature of supply chains. Unlike single-echelon systems, where decision-making is localized, multi-echelon supply chains involve coordination among several tiers—each with its unique objectives, constraints, and information sets. Achieving performance optimization across such networks requires balancing conflicting goals, such as minimizing inventory holding costs while maximizing service levels. This necessitates advanced synchronization methods and system-wide optimization strategies that account for interdependencies and feedback delays [11].

Moreover, as sustainability and resilience become key strategic objectives, supply chains must be designed not only for efficiency but also for long-term viability. Resilience refers to a system's ability to withstand shocks and recover quickly, while sustainability encompasses reducing the environmental and social footprint of supply chain operations. Nonlinear models that incorporate resilience metrics and adaptive control policies can help firms proactively design supply chains that meet regulatory standards, stakeholder expectations, and societal needs [12].

In conclusion, modern supply chains operate in a highly complex, interconnected, and dynamic environment. Traditional linear models are insufficient to describe the behaviours observed in real-world systems, especially in the presence of the bullwhip effect and chaotic dynamics. The use of nonlinear dynamic modelling, coupled with chaos theory, enables a deeper understanding of supply chain behaviour and its sensitivity to internal and external changes. Adaptive control strategies—particularly those leveraging feedback mechanisms and supported by Lyapunov-based stability analysis—offer a robust framework for stabilizing and synchronizing these complex systems.

Furthermore, numerical simulations and real-time data integration provide valuable tools for validating these theoretical models and ensuring their practical applicability. As the supply chain continues to evolve in complexity, integrating intelligent systems, digital technologies, and resilience planning will be critical to maintaining competitiveness, efficiency, and sustainability. The future of supply chain management lies in a multidisciplinary approach—blending systems theory, control engineering, computer science, and business strategy—to navigate uncertainty and drive strategic success.

Beamon in [13] has provided a comprehensive survey of the modelling of SCs. [14] provides the mathematical models of supply chains using non-linear differential equations.

1.2 Chaotic systems

Chaos is a natural phenomenon that appears in numerous systems throughout the universe. It is defined as irregular yet deterministic behavior in dynamic systems that are highly sensitive to their starting conditions. Although the behavior of chaotic systems seems unpredictable, it is actually governed by precise mathematical laws.

Various real-world systems demonstrate chaotic characteristics. These include celestial mechanics (like planetary motion), atmospheric systems (such as weather patterns), financial systems (like the stock market), biological functions (such as brain activity or heartbeat), ecological systems (animal and plant populations), the spread of diseases, cancer development, chemical interactions, power grids, and the Internet. Modeling such systems using the framework of chaos helps us better understand their complex, dynamic nature.

1.2.1 Essential Conditions for Chaos

A system must meet the following three conditions to be classified as chaotic:

1. It must be bounded – the values of the system's variables do not tend toward infinity but stay within a certain range.

2. It must exhibit aperiodic behavior – the system never settles into a regular, repeating cycle.
3. It must show sensitivity to initial conditions – small changes in the starting state result in drastically different outcomes over time.

1.2.2 Properties of Chaotic Systems

1. Though they may appear random, chaotic systems follow specific mathematical equations that determine their evolution over time.
2. Even the slightest difference in initial input can cause large deviations in outcomes, making accurate long-term forecasting nearly impossible. However, short-term predictions are often still feasible.
3. Nonlinearity:
   All chaotic systems are nonlinear, meaning their behavior cannot be represented by a simple sum of their parts. However, not all nonlinear systems are chaotic.
4. In systems defined by autonomous differential equations, chaos arises only when there are at least three independent variables or dimensions.
5. Non-intersecting paths in phase space: The system's path, or trajectory, in phase space never crosses itself. If it did, it would imply repeatability and predictability, which contradicts the nature of chaos.
6. Dependence on Lyapunov exponents: These exponents measure how fast nearby trajectories diverge. Each dimension of the system has a corresponding Lyapunov exponent.
7. At least one positive Lyapunov exponent: A system is considered chaotic if at least one of its Lyapunov exponents is positive. A higher positive value indicates faster divergence of trajectories, which limits how far into the future the system's behavior can be predicted.

1.3 Research Objectives

1. To design a mathematical model of a three-echelon supply-chain with exponential non-linearity.

2. To verify the non-linear dynamic properties of the proposed model.

3. To apply adaptive control synchronization to synchronize the real supply chain with unknown parameters to the desired supply chain.

4. To model the real- time scenario of LPG shortage in India mathematically and control it using adaptive control synchronization.

Chapter 2

Mathematical Modelling of the Supply Chain

2.1 Description of the proposed Model

The proposed model of three echelon SCM , as shown in Fig.1, is given by (2.1) by adding an exponential non-linear term in third equation of the system proposed in [a].

6

$$\dot{x} = my - (n+1)x \quad (1a)$$

$$\dot{y} = rx - y - xz \quad (1b)$$

$$\dot{z} = xy + (k-1)z + e^{x} \quad (1c)$$

In (1), x, y and z represent the actual demand of customer, inventory level of distributor and manufactured quantity respectively. The dynamic changes in x, y and z are represented by $\dot{x}$, $\dot{y}$ and $\dot{z}$ respectively.

The constant m and n in (1a) account for the delivery efficiency of the distributor and rate of customer demand respectively. The constant r in (1b) is the distortion coefficient to show the information distortion. The constant k in (1c) is the safety stock coefficient. The presence of three non-linear terms in (1) shows the extremely unpredictable nature of the supply chains. The bullwhip effect in supply chain has been modelled as an exponential non-linearity because the effect of minor change in demand becomes more pronounced as it moves higher in the supply chain, thus affecting manufacturer the most.

- m = 10
- n = 9
- r = 28
- k = -5/3

The non-linear dynamic system in (1) is simulated in python. The variables x, y and z are plotted against time as shown in Fig.2. The non-repeatability and noise like waveforms in Fig. 2 represent the unpredictive nature of demand, inventory and manufacturer in SC. The two-dimensional state diagrams are also plotted in Fig.3. These diagrams indicate the chaotic nature of SC.

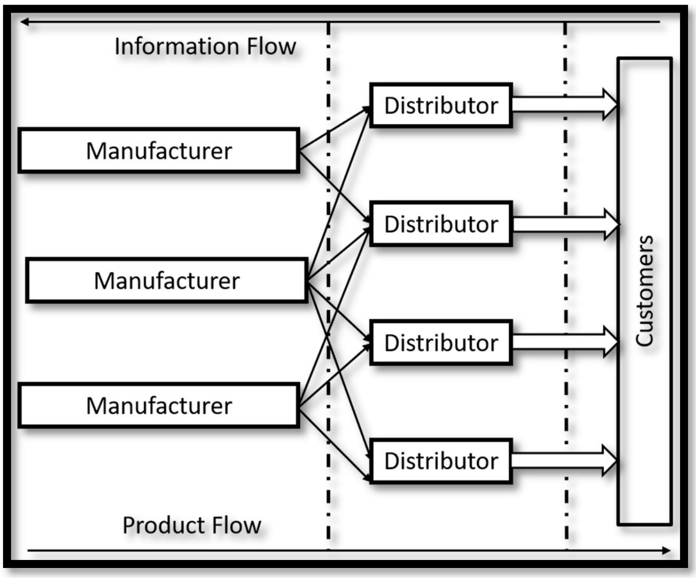


Fig.1. Three- echelon supply chain management model

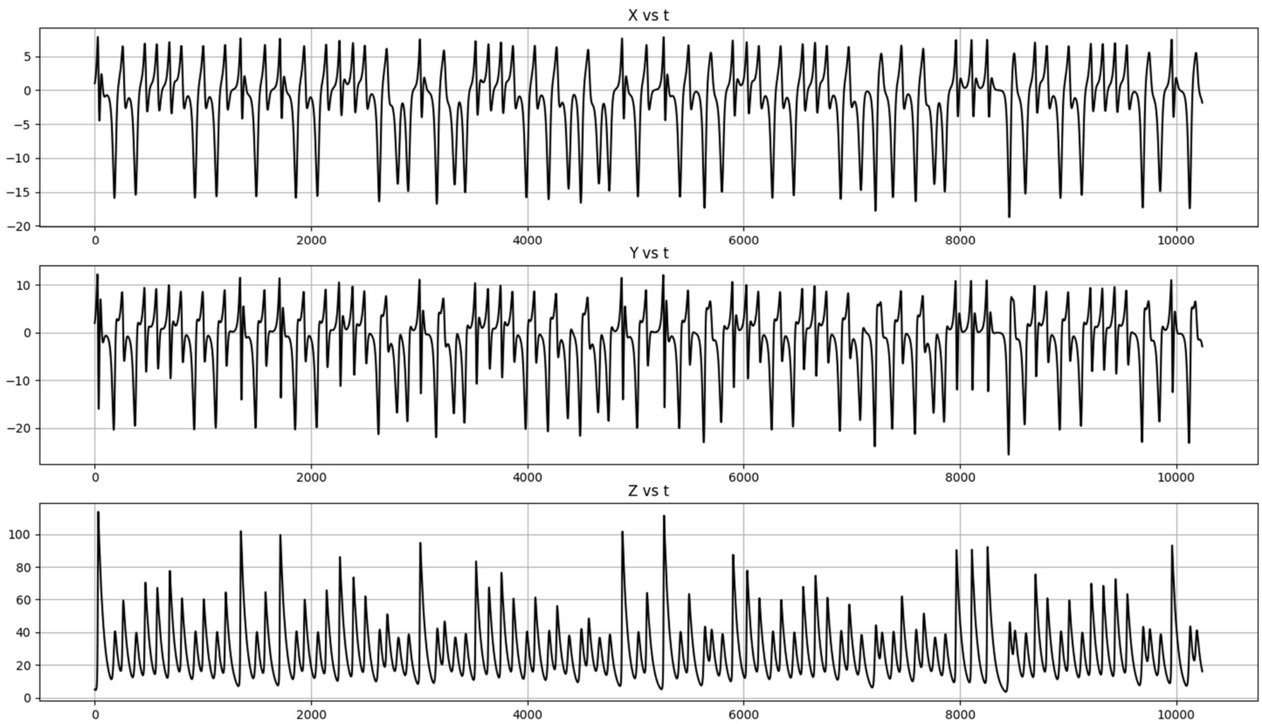


Fig.2. Time-waveforms of the three state variables

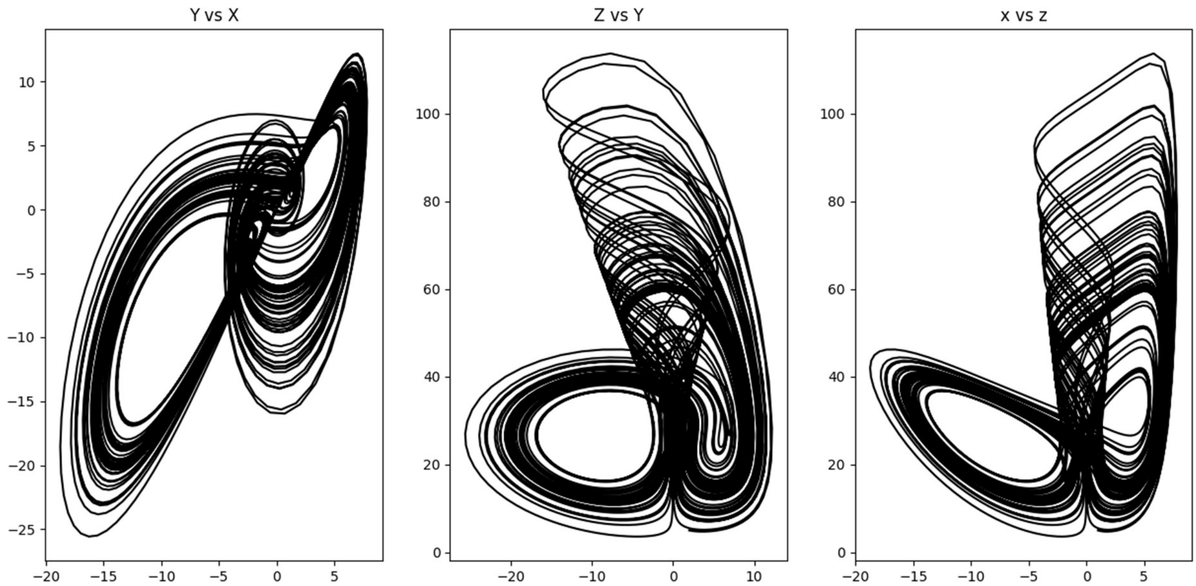


Fig.3. Two-dimensional state diagrams of state variables

Chapter 3

Numerical Analysis

21

3.1 Stability Analysis

3.1.1 Stability analysis

19 In order to check the nature of the proposed model, (2.1) is equated to zero to determine the fixed points, also known as equilibrium points, where the dynamics of the system is absent.

$$\dot{x} = my - (n+1)x = 0 \quad (2a)$$

18

$$\dot{y} = rx - y - xz = 0 \quad (2b)$$

$$\dot{z} = xy + (k-1)z + e^{x} = 0 \quad (2c)$$

This gives,

$$my = (n+1)x \quad (3a)$$

$$rx - (n+1)x/m - xz = 0 \quad (3b)$$

From (3b), we can conclude that either x=0 or z = r – (n+1)/m.

Taking x= 0 gives y= 0 and z=1/(1-k).

1 Thus (0,0,1/(1-k)) is one of the fixed points of the proposed chaotic system. In order to analyze the nature of this fixed point, we use Jacobian matrix as follows:

$$J = \begin{matrix} -(n+1) & m & 0 \\ r-z & -1 & -x \\ e^{x}+y & x & k-1 \end{matrix} \quad (4)$$

Putting (x,y,z)= (0,0,1/(1-k)) in (4), we obtain

$$J = \begin{bmatrix} -(n+1) & m & 0 \\ r-1/(1-k) & -1 & 0 \\ 1 & 0 & k-1 \end{bmatrix} \quad (5)$$

Let λ represents the eigen values, writing (5) in the form of (6), we obtain:

$$\begin{vmatrix} -(n+1)-\lambda & m & 0 \\ r-(\frac{1}{1-k}) & -1-\lambda & 0 \\ 1 & 0 & k-1-\lambda \end{vmatrix} = 0 \quad (6)$$

Solving (6), we obtain eigenvalues

$\lambda_1 \approx -2.667$; $\lambda_2 \approx 11.72$; $\lambda_3 \approx -22.725$ as shown in Fig. 4.

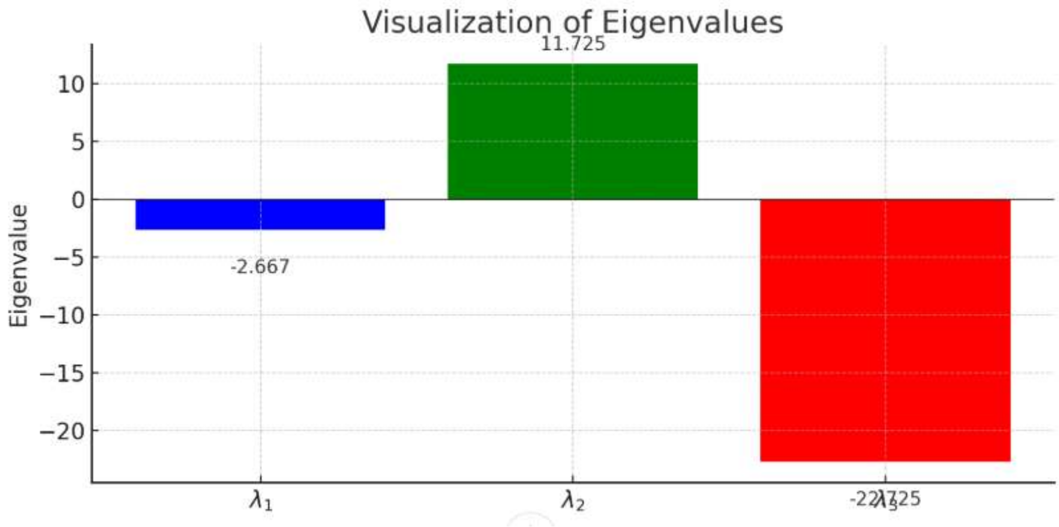
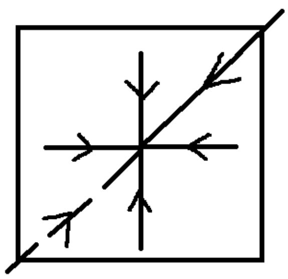
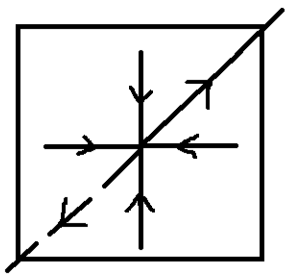
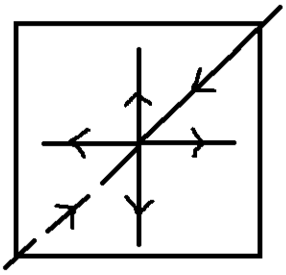
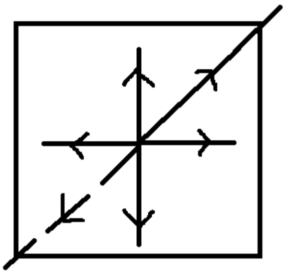
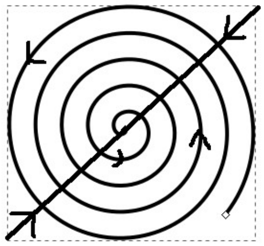
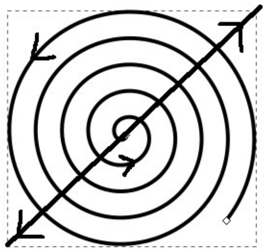
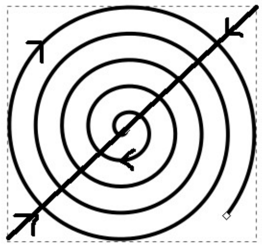


Fig.4. Eigenvalues of the equilibrium point

All the three eigenvalues are real, with two being negative and one positive. From Table 1, we can conclude that this equilibrium point is saddle index-1. This shows one **positive** eigenvalue, indicating potential **instability**, and two **negative** eigenvalues, which suggest **dissipative** behaviour in the corresponding directions.

Table 1 Nature of equilibrium points based on eigen values

Category	Nature of eigenvalues	Three-dimensional representation
index- 0	All the eigenvalues are real negative. The fixed point is a stable node. The trajectories move inward linearly in all three directions , i.e. x, y and z.	
index-1 saddle	The dimension of unstable manifold is 1 and all the eigenvalues are real. The trajectories move inward linearly along two directions and outward linearly in third direction.	

index-2 saddle	Two of the three real eigenvalues are positive. The trajectories move outward linearly along two directions and inward linearly along third direction.	
index-3	All the three real eigenvalues are positive. The fixed point is an unstable node. All the trajectories are moving outwards linearly.	
index-0 spiral	One real negative eigenvalue and two complex conjugates with negative real part. This indicates an inward spiral and a stable linear manifold.	
index-1 spiral saddle	One eigenvalue is real positive and two are complex conjugates with negative real part. This indicates an inward spiral and an unstable linear manifold.	
index-2 spiral saddle	One eigenvalue is real negative and two are complex conjugates with positive real part. This indicates an outward moving spiral and a stable linear manifold.	

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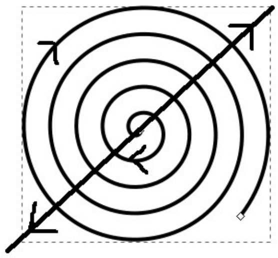
index-3 spiral	One eigenvalue is real positive and two are complex conjugates with positive real part. This indicates an outward moving spiral and an unstable linear manifold.	

The positive eigenvalue indicates that small deviations in some component of the supply chain (e.g., demand, inventory, or production rate) will diverge exponentially, making the system locally unstable. Negative eigenvalues suggest stability along certain directions, where fluctuations are dampened (e.g., rapid correction of stock levels or supply adjustments).

3.2 Lyapunov Exponents

The Lyapunov exponents for the system are approximately:

- $L_1 \approx 1.843$
- $L_2 \approx 0.060$
- $L_3 \approx -16.154$

5 A positive exponent indicates sensitive dependence on initial conditions — a hallmark of chaos. A second exponent near zero corresponds to neutral stability along the trajectory direction. A strongly negative exponent shows contraction in one direction. Thus, this system exhibits chaotic behaviour, as evidenced by the positive Lyapunov exponent.

The largest Lyapunov exponent ($L_1 > 0$) confirms chaotic behaviour, indicating sensitivity to initial conditions—even minor forecasting errors or delivery delays can lead to significantly different outcomes. A second exponent near zero suggests a direction where the system neither grows nor shrinks quickly—this could represent a neutral delay mechanism (e.g., transport lead time). A strongly negative exponent represents convergence—some part of the system (e.g., replenishment) is tightly controlled and resists fluctuation.

The Kaplan–Yorke (Lyapunov) dimension of the system is approximately 2.12, indicating a fractal attractor that lies between a 2D surface and 3D volume — a key hallmark of chaotic dynamics.

Chapter 4

Adaptive Control Synchronization of Chaotic Systems

Synchronization mechanism plays a critical role in supply chains. In most of the synchronization mechanisms, the master-slave method is used. Let the master state variables being (x_1, y_1, z_1) and slave state variables being (x_2, y_2, z_2). The step-by-step process of adaptive control synchronization is as follows:

Equation (4.1) represents the driver or master mathematical model of supply chain.

$$\dot{x}_1 = my_1 - (n+1)x_1 \tag{4.1a}$$

$$\dot{y}_1 = rx_1 - y_1 - x_1z_1 \tag{4.1b}$$

$$\dot{z}_1 = x_1y_1 + (k-1)z_1 + e^{x_1} \tag{4.1c}$$

The corresponding slave or response system is represented as given in (4.2).

$$\dot{x}_2 = my_2 - (n+1)x_2 + u_1 \tag{4.2a}$$

$$\dot{y}_2 = rx_2 - y_2 - x_2z_2 + u_2 \tag{4.2b}$$

$$\dot{z}_2 = x_2y_2 + (k-1)z_2 + e^{x_2} + u_3 \tag{4.2c}$$

Where, u_i, (i=1 to 3) are the non-linear adaptive controllers. The non-linear errors or differences between master and slave system are given in (4.3), and their corresponding dynamics in (4.4).

$$e_x = x_2 - x_1;\ e_y = y_2 - y_1;\ e_z = z_2 - z_1 \tag{4.3}$$

$$\dot{e}_x = \dot{x}_2 - \dot{x}_1;\ \dot{e}_y = \dot{y}_2 - \dot{y}_1;\ \dot{e}_z = \dot{z}_2 - \dot{z}_1 \tag{4.4}$$

With the help of (4.1) and (4.2) and using expressions of (4.3), (4.4) can be rewritten as:

$$\begin{aligned} \dot{e}_x &= \widehat{m}e_y - (\hat{n}+1)e_x + u_1 \\ \dot{e}_y &= \hat{r}e_x - e_y - (x_2z_2 - x_1z_1) + u_2 \\ \dot{e}_z &= x_2y_2 - x_1y_1 + (k-1)e_z + e^{x_2} - e^{x_1} + u_3 \end{aligned} \tag{4.5}$$

The non- linear adaptive control functions u_i, (i =1,2,3) can be obtained from (4.6) as:

$$\begin{aligned} u_1 &= -k_1e_x - \widehat{m}e_y + (\hat{n}+1)e_x \\ u_2 &= -k_2e_y - \hat{r}e_x + e_y + (x_2z_2 - x_1z_1) \\ u_3 &= -k_3e_z - x_2y_2 - x_1y_1 - (\hat{k}-1)e_z - e^{x_2} + e^{x_1} \end{aligned} \tag{4.6}$$

Where, $\hat{m}, \hat{n}, \hat{r}$ *and* $\hat{k}$ are the estimated values of unknown parameter m, n, r and k; and k_1, k_2 and k_3 are positive gain constants, whose values have been chosen as '3'.

7 The parameter estimation error, e_m, e_n, e_r *and* e_k are defined by

$$e_m = m - \hat{m}$$
$$e_n = n - \hat{n}$$
$$e_r = r - \hat{r}$$
$$e_k = k - \hat{k}$$

and their dynamics are given as

$$\dot{e_m} = -\dot{\hat{m}}$$
$$\dot{e_n} = -\dot{\hat{n}}$$
$$\dot{e_r} = -\dot{\hat{r}}$$
$$\dot{e_k} = -\dot{\hat{k}}$$

3 Using (4.6), the dynamics of the slave system and error can be simplified as (4.7) and (4.8) respectively

24
$$\dot{x_2} = my_2 - (n+1)x_2 - k_1e_x - \hat{m}e_y + (\hat{n}+1)e_x$$
$$\dot{y_2} = rx_2 - y_2 - k_2e_y - \hat{r}e_x + e_y - x_1z_1 \qquad (4.7)$$
2
$$\dot{z_2} = (k-1)z_2 - k_3e_z + x_1y_1 - (\hat{k}-1)e_z + e^{x_1}$$
$$\dot{e_x} = -k_1e_x;\ \dot{e_y} = -k_2e_y;\ \dot{e_z} = e_ke_z \qquad (4.8)$$

22 Consider a candidate Lyapunov function

$$V = \frac{1}{2}\left(e_x^2 + e_y^2 + e_z^2 + e_m^2 + e_n^2 + e_r^2 + e_k^2\right) \qquad (4.9)$$

Differentiating V provides,

2
$$\dot{V} = e_x\dot{e}_x + e_y\dot{e}_y + e_z\dot{e}_z + e_m\dot{e}_m + e_n\dot{e}_n + e_r\dot{e}_r + e_k\dot{e}_k \qquad (4.10)$$

Putting the values from (4.7) and (4.8) in (4.10) gives:

2
$$\dot{V} = -k_1e_x^2 - k_2e_y^2 + e_ke_z^2 - e_m\dot{\hat{m}} - e_n\dot{\hat{n}} - e_r\dot{\hat{r}} - e_k\dot{\hat{k}} \qquad (4.11)$$

1 According to Lyapunov stability theory [64], the parameter update law derived from (4.11) is

$$\dot{\hat{k}} = e_z^2 \qquad (4.12)$$

By applying the adaptive controller together with the parameter update rule given in equation (4.12), the error dynamics become stable, ensuring that the slave chaotic system—with its own set of parameter values—gradually converges to the behavior of the master system. As the error approaches zero, the two systems achieve synchronization. This synchronization process is illustrated in Fig. 5.

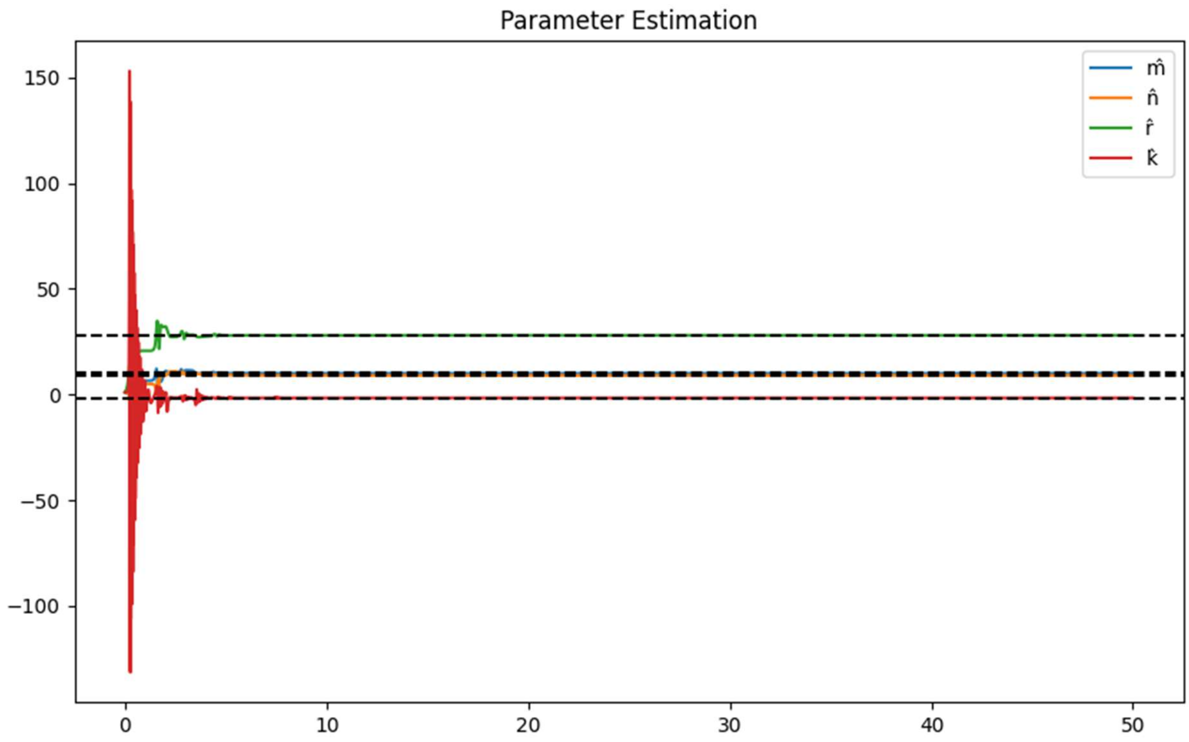


Fig.5. Stabilization of the parameters of the slave system

Chapter 5

Mathematical Modelling of LPG Crisis in India caused by Iran-US War

5.1 Introduction

The proposed model of three echelon SCM , as given in (1) is rewritten in (5) for ready reference.

6

$$\dot{x} = my - (n+1)x \quad (5.1a)$$

$$\dot{y} = rx - y - xz \quad (5.1b)$$

$$\dot{z} = xy + (k-1)z + e^{x} \quad (5.1c)$$

x represents the household or commercial demand of LPG, y represents the LPG stock at distributors and z represents imported supply. Because of Iran -US war, the Strait of Hormuz was blocked through which the shipments of imported LPG from Gulf region pass, resulting in the supply shock in the system. In (5.1a), 'my' represents that when cylinders are available, people continue using LPG normally. The negative sign in (5.1a) accounts for reduction in usage when shortage occurs.

Equation (5.1b) is an inventory equation, where the term 'rx' accounts for inventory being proportional to demand of LPG because of information distortion of shortage in availability. '-xz' represents the mismatch between demand and supply, leading to inventory collapse.

Finally, equation (5.1c) shows the changes in imports based on demand and stock situation, and 'e^{x}' shows the explosive response to rising demand, in other words, the bullwhip effect. This has accounted for emergency imports from multiple countries.

Thus, because of war uncertainty, shipping disruptions and policy changes has lead to panic buying of LPG cylinders by households and commercial units, which in turn lead to non-linear rise in imports.

5.2 Stabilization of Demand and Supply

11 Adaptive control synchronization can play a crucial role in stabilizing the LPG supply situation in India during disruptions caused by US–Iran conflict by dynamically aligning demand, inventory, and production in real time.

In the given nonlinear supply chain model, instability arises due to sudden demand spikes, distorted information , represented by parameter ‘r’, and non-linearities ‘xz’, ‘xy’ and ‘e^x’, which leads to shortages, overreactions, and chaotic fluctuations.

Adaptive control introduces corrective inputs into each equation that continuously adjust based on the deviation between actual and desired states. This corresponds to policy actions such as rationing LPG usage, regulating prices, monitoring distributor inventories, preventing hoarding, and increasing or diversifying supply through imports and domestic production. The “adaptive” aspect ensures that these interventions are not fixed but evolve according to real-time conditions. Synchronization, in this context, means ensuring that demand signals, inventory levels, and production decisions are consistently aligned across all stakeholders, including government agencies, oil companies, and distributors. By doing so, adaptive control synchronization reduces the bullwhip effect, minimizes volatility, and guides the system back toward a stable equilibrium, thereby preventing extreme shortages and inefficiencies in the LPG supply chain during crisis situations.

Using adaptive control synchronization scheme, as shown in Fig. 6 below, the slave system or the real time system with unknown parameters and initial values can synchronize with the original ideal or desired system within maximum 8 days, as the fluctuations in parameters settle down.

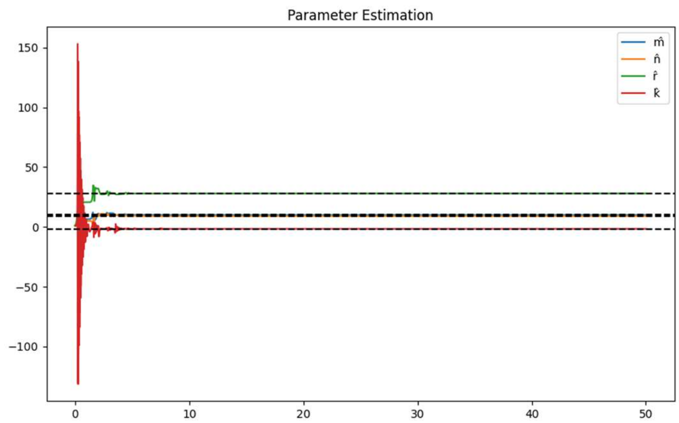


Fig 6. Stabilization of parameters

Chapter 6

Conclusion and Future Scope

Supply chains are complex, dynamic systems that often span multiple levels, including manufacturers, distributors, and retailers. The coordination and flow of goods, information, and finances across these tiers involve intricate interactions and dependencies that are highly sensitive to fluctuations in demand and supply. In particular, a three-echelon supply chain—comprising these three distinct levels—poses significant analytical challenges due to the compounded variability at each stage. In recent years, researchers have sought to understand and manage this complexity using advanced mathematical models. One such approach involves the use of differential equations with embedded nonlinearities to simulate the system's behavior under real-world constraints.

A prominent feature of this modelling framework is the incorporation of both quadratic and exponential nonlinearities in the differential equations. These nonlinearities serve specific roles in capturing the true nature of interactions within the supply chain. Quadratic terms often represent cost or inventory dynamics that do not scale linearly with production or storage levels. For instance, holding costs, lead times, or penalty costs might increase at a rate that accelerates with inventory surplus or shortage, justifying a quadratic component in the model. Exponential nonlinearities, on the other hand, are particularly suited to represent phenomena that exhibit rapid growth or decay—one of the most critical being the bullwhip effect.

2 The bullwhip effect is a well-documented challenge in supply chain management, characterized by increasing fluctuations in inventory and order quantities as one moves upstream from retailers to manufacturers. Small variations in consumer demand can become amplified due to order batching, price fluctuations, demand forecasting errors, and lack of coordination among supply chain partners. This cascading amplification leads to inefficiencies such as overstocking, stockouts, increased operational costs, and reduced service levels. Accurately modelling this internal nonlinear behaviour is crucial for any meaningful analysis and improvement of supply chain performance.

By embedding exponential nonlinearities into the mathematical model, researchers are able to more accurately simulate the dynamics of the bullwhip effect. Exponential growth terms can mimic the rapid amplification of order variance, while exponential decay terms may represent stabilization efforts, such as inventory smoothing or adaptive control mechanisms. The resulting system of nonlinear differential equations provides a highly detailed and realistic portrayal of supply chain behaviour, capturing both the macro-level dynamics and micro-level variations that emerge from individual decision-making processes.

This modelling approach is particularly effective in three-echelon supply chains, where complexity is multiplied by the interactions across three levels of the distribution process. Unlike two-tier models, which might focus on a single supplier-retailer interaction, three-tier systems reflect a more realistic scenario found in most global supply networks. The behaviour at the manufacturing level can be drastically influenced not only by retailer demand but also by distributor decisions, lead times, transportation constraints, and market uncertainties.

Through such mathematical modelling, strategic decision-makers gain critical insights into the state and evolution of the entire supply chain system. Visualization tools derived from model simulations allow supply chain managers to identify bottlenecks, forecast system responses to changes in demand or supply conditions, and evaluate the effectiveness of various control strategies. For instance, by simulating different demand scenarios, decision-makers can assess how quickly the system returns to equilibrium, which segments are most sensitive to disruptions, and what kind of inventory policies may help mitigate risks. This level of visibility and foresight is indispensable in today's volatile, uncertain, complex, and ambiguous (VUCA) business environment.

Moreover, the model enables the exploration of "what-if" scenarios. Strategic interventions such as adjusting reorder points, lead times, batch sizes, or implementing information-sharing mechanisms can be tested in a virtual environment before actual deployment. This proactive capability not only improves decision quality but also reduces the costs and risks associated with trial-and-error strategies in real operations.

Adaptive control synchronization is a method to synchronize two chaotic systems, master and slave, when one or more parameters are unknown or time-varying. In this work, the adaptive control synchronization for the proposed model of supply chain has been used, and the maximum time it takes for parameter to stabilize is 4.5 seconds.

A comparison has been made with the existing designs as shown in Table 2.

Table 2 Comparison of the stabilization time in different synchronization techniques

Reference	Synchronization Technique	Stabilization time
[15]	RBF Control	5 sec

[16]	Active Control	7 sec
[17]	Backstepping Control	5 sec
[18]	Non-linear control	6 sec
Proposed System	Adaptive Control	4.5 sec

5.1 Advantages and Strategic Importance

One of the major benefits of this modelling framework is its **predictive capability**. While chaotic or nonlinear systems often defy long-term prediction due to sensitivity to initial conditions, they can still yield valuable short-term forecasts that are more accurate than those derived from linear models. This is crucial for inventory control, procurement planning, and logistics coordination, especially in industries with perishable goods, fast-changing demand, or highly competitive markets.

Additionally, the model facilitates **resilience planning**. By examining the supply chain's response to perturbations—such as sudden demand spikes, supplier failures, or transportation delays—firms can develop robust strategies to maintain continuity and service levels. In this context, modeling chaos and nonlinear dynamics is not merely an academic exercise but a practical tool for navigating real-world uncertainties.

10 This approach also aligns well with the increasing adoption of **digital twins** in supply chain management. A digital twin is a virtual replica of a physical system that allows continuous monitoring and simulation. The differential equation-based model with nonlinearities can serve
11 as the computational engine of such a digital twin, offering real-time insights and facilitating automated decision-making through integration with AI and machine learning tools.

5.2 Future Scope

While current models represent a significant advancement, there remains ample scope for future research and development. Some of the key areas for exploration include:

1. Extension to Hyperchaotic Modelling

As supply chains grow in complexity—due to globalization, e-commerce, and multi-channel distribution—the underlying dynamics may no longer be adequately captured by simple chaotic models. The future lies in hyperchaotic systems, which are characterized by having more than one positive Lyapunov exponent. These systems can capture more intricate and less predictable behaviours, making them highly suitable for modelling modern, interconnected supply networks.

2. Incorporation of Stochastic Elements

Real-world supply chains are influenced not only by deterministic factors but also by random events—such as natural disasters, political upheavals, and market crashes. Introducing stochastic differential equations into the model can bridge the gap between deterministic chaos and random variability, providing a more holistic understanding of risk and uncertainty.

3. Integration with Machine Learning

Combining mathematical modelling with machine learning algorithms offers exciting possibilities. For instance, AI can be used to estimate model parameters, identify nonlinear patterns, and adaptively update the model as new data becomes available. This hybrid approach can lead to more flexible, data-driven supply chain systems.

4. Optimization Under Uncertainty

While the current model allows visualization and simulation, future models could be enhanced to support real-time optimization. This would involve identifying optimal strategies for inventory, pricing, and logistics in dynamically changing environments. Incorporating multi-objective optimization techniques that balance cost, service level, and resilience would be particularly valuable.

5. Customization for Industry-Specific Applications

Different industries face different supply chain challenges. Future work could focus on tailoring the modelling framework to specific sectors—such as pharmaceuticals, agriculture, automotive, or technology—each of which has unique dynamics, regulations, and risk profiles.

6. Policy Modelling and Governance

Supply chains are increasingly influenced by regulatory policies, trade agreements, and sustainability requirements. The model can be expanded to include policy variables, allowing governments and large enterprises to simulate the impact of environmental regulations, carbon taxes, or trade tariffs on supply chain performance.

7. Human Behaviour and Decision Bias

Supply chain decisions are made by people, and human behavior often deviates from rational models due to cognitive biases, emotions, and limited information. Future models could incorporate elements of behavioral economics or agent-based modelling to simulate more realistic decision-making processes within the supply chain.