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Abstract

The relativistic self-focusing of laser beams in quantum plasma has drawn considerable interest because of its applications in inertial confinement fusion, charged-particle acceleration, and high-energy-density physics. In this dissertation, we study the propagation and self-focusing of non-conventional beam profiles, namely Bessel-Gaussian and Elliptical-Gaussian beams, in magnetized quantum plasmas with spatial density gradients. The paper considers combinations of beam geometry, magnetic fields and plasma inhomogeneity which have never been systematically treated in previous work.

The study uses the quantum hydrodynamic (QHD) model, incorporating the Bohm potential, exchange-correlation effects, relativistic ponderomotive forces, and the influence of an external magnetic field. Applying the paraxial approximation and the Wentzel–Kramers–Brillouin (WKB) method reduces Maxwell's equations to a differential equation governing beam-width evolution along the propagation axis. For Paper-1 on Bessel-Gaussian beams, this results in a second-order nonlinear ordinary differential equation, solved numerically using the fourth-order Runge-Kutta method. Simulations are carried out for high laser frequencies of ($1.78 \times 10^{20} s^{-1}$), electron densities of ($n_0 = 4 \times 10^{19} cm^{-3}$) satisfying ($\chi \geq 1$), an axial magnetic field of ($\omega_c/\omega = 0.3$), and exponential density ramp parameters ($d = 5, 10, 20$).

Key findings from Paper-1 show that: (1) thermal quantum plasma enables much stronger and more sustained self-focusing than classical and cold quantum plasmas; (2) exponential density ramps greatly accelerate self-focusing, maintaining high intensity over multiple Rayleigh lengths; (3) Bessel-Gaussian beams outperform Gaussian beams because of their ring-shaped intensity profile; (4) the transverse wave parameter μ significantly enhances self-focusing; and (5) beyond a threshold, increasing laser intensity reduces self-focusing due to relativistic mass saturation and exchange-correlation effects introducing competing nonlinearities. Paper-2 (Elliptical-Gaussian beams in magnetized quantum plasma with tangential density ramps) is in an advanced stage with mathematical framework and numerical analysis under way.

56

Chapter 1

INTRODUCTION

1.1 Background and Motivation

Laser beams, an electromagnetic radiation, have had a profound impact on modern science and technology. The transformation started in 1960 when Theodore Maiman successfully demonstrated the first operational laser at Hughes Research Laboratories [1]. Since then lasers have become one of the most versatile and powerful tools in physics, engineering, medicine and industry.

A laser (Light Amplification by Stimulated Emission of Radiation) is a source of electromagnetic waves with peculiar combination of properties: high spatial and temporal coherence, excellent monochromaticity, and high directionality. Laser beams are characterized by very small divergences and can be focused to very small spot sizes, leading to very high energy densities and intensities, unlike conventional light sources. These properties enable fine control over light-matter interactions.

These special properties make lasers very useful for many different things. They are important in medicine for surgery, ophthalmology and dermatology [2]. Lasers are widely used in manufacturing for cutting, welding, drilling and surface treatment of materials [3]. They are also important for optical communication systems, high resolution spectroscopy, holography and many fields of scientific research. Lasers have opened new research directions in fundamental physics into nonlinear optical phenomena, plasma dynamics, particle acceleration and high energy density matter. The intense and precise focusing of electromagnetic energy by laser beams has made them essential tools for practical applications, as well as for the study of complex physical processes.

23 Plasma, or the fourth state of matter, is a quasi-neutral medium consisting of free electrons, ions, and often neutral particles, which shows complicated electromagnetic behavior [4]. In contrast to solids , liquids and gases , plasma is highly ionized . This results in a huge variety of collective phenomena due to the long range electromagnetic interaction . These unique properties make the plasma an essential element to understand natural and laboratory systems. This is more than 99 % of all visible (or baryonic) matter in the universe and it is found in environments such as stellar interiors, the solar corona and interstellar space [5]. Plasmas are used extensively in semiconductor manufacturing [6], materials processing [7], fluorescent lighting [8], display technologies [9] and emerging applications in medicine [10] and environmental remediation [11] on Earth.

At very high density and relatively low temperature the quantum-mechanical nature of the plasma particles is important. 8 In such regimes the thermal de Broglie wavelength of electrons becomes comparable to characteristic plasma length scales, such as the Debye length, and classical descriptions become inadequate. The resulting medium, known as quantum plasma, displays phenomena like electron degeneracy pressure, quantum tunneling, and exchange-correlation interactions that can drastically change plasma behavior.

One of the most active fields of modern plasma physics is the interaction of intense laser beams with plasma, usually called Laser-Plasma Interaction (LPI). The field has seen many breakthroughs, including the development of Inertial Confinement Fusion (ICF) [12] or the birth of Laser-Wakefield Acceleration (LWFA) by Toshiki Tajima and John M. Dawson in 1979 [13]. These developments demonstrated that ultra-intense laser pulses can excite large amplitude plasma waves that can accelerate charged particles to very high energies over comparatively short distances. Recent advances in ultrafast laser technology have opened up applications such as laser-driven electron and ion beams for cancer therapy [14], compact X-ray sources for imaging [15] and laboratory studies of matter under extreme conditions [16].

The interaction of a high intensity laser beam propagating in a plasma becomes highly nonlinear. The primary responsible mechanism for this behaviour is the ponderomotive force, a nonlinear force that pushes electrons away from regions of high electromagnetic intensity. This redistribution of charge results in a change of the local electron density and hence of the plasma refractive index. If the refractive index is higher close to the beam axis than at the periphery the plasma behaves as a focusing lens. This results in a contraction of the beam and an increase of the intensity. This phenomenon, called self-focusing, is one of the basic nonlinear effects in laser-plasma interaction and has been studied in detail in classical and relativistic regimes [17–24].

The self-focusing of laser beams in plasma is still a very important research topic, as it reflects the complex interplay of electromagnetic forces, relativistic effects and quantum mechanical corrections. Detailed understanding of self-focusing for various beam profiles, plasma density distributions, and external magnetic field configurations is crucial for optimizing applications in fusion energy, advanced particle acceleration, and high energy density physics. The present work contributes to this area by investigating the relativistic self-focusing of non-conventional laser beam profiles in magnetized quantum plasma with spatially varying density with emphasis on parameter regimes that have received little or no attention in the existing literature.

1.2 Self-Focusing

The self-focusing phenomenon was first predicted by Russian physicist Gurgen Askaryan in 1962 [25]. This theoretical prediction was later confirmed by experiments and numerical studies [26, 27] with the advent of the high power laser systems. Self focusing is one of the most important nonlinear optical effects observed in an intense laser beam propagating through a plasma medium . Unlike the vacuum propagation where the diffraction causes a gradual beam spreading, the self-focusing causes a progressive beam width reduction and a corresponding intensity increase with the propagation in the plasma.

22 The mechanism of self-focusing is the ponderomotive force, a nonlinear force on charged particles in an inhomogeneous electromagnetic field. The force is strongest near the center of the laser beam where the intensity is highest . For a laser beam with a transverse intensity gradient , such as a Gaussian beam , As a result, electrons are driven radially outwards from the axis into regions of lower intensity. This outward motion causes the depletion of electron density near the beam center and accumulation of electrons in the surrounding regions [17–24].

The local electron density directly influences the plasma refractive index. The redistribution of electrons modifies the plasma refractive index. The decrease of density on the axis of the beam leads to a higher refractive index in this region than in the surrounding plasma. Therefore the plasma itself acts as a focusing lens and the electromagnetic wavefront is bent inward toward the axis. As the beam shrinks, the peak intensity increases. This increases the ponderomotive force and hence the density perturbation is deepened. This positive feedback mechanism allows the beam to have a sustained self focusing during propagation.

4 Relativistic effects are dominant for laser intensities of the order of $10^{19} W/cm^2$ and above. In this regime the oscillatory motion of the electrons in the laser field causes an increase of their
3 effective relativistic mass. This reduces the plasma frequency and further alters the refractive index , improving the focusing process . Relativistic self-focusing plays a key role in high power laser systems, such as those used in inertial confinement fusion, laser driven particle acceleration, and high energy density physics [28–34].

The strength of self-focusing depends on several parameters, such as the laser intensity, the wavelength, the plasma electron density, and the initial transverse beam profile. Non-conventional beam shapes like Bessel-Gaussian and elliptical-Gaussian beams can have significantly different focusing characteristics due to their different intensity distributions. External magnetic fields can also affect the dynamics by modifying the electron trajectories and the resulting density perturbations [35].

Self-focusing continues until it is balanced by competing mechanisms such as diffraction, plasma instabilities, or other nonlinear effects that may cause beam defocusing or filamentation. In many cases, the beam undergoes alternating cycles of focusing and defocusing over several
21 Rayleigh lengths. A schematic representation of this process is shown in Figure 1.1.

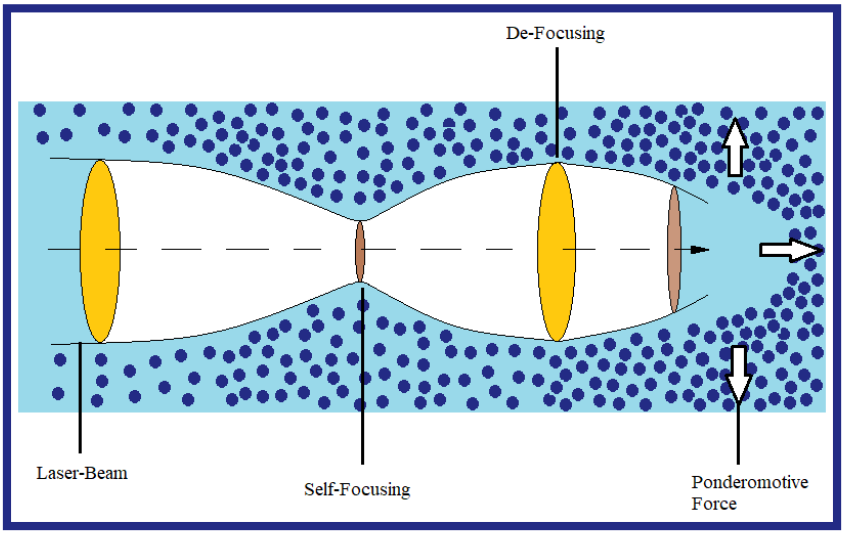

Figure 1.1: Visual representation of the self-focusing effect

9 As a result, the peak regions of the laser beam experience a further increase in intensity. This can be represented by the Figure 1.2.

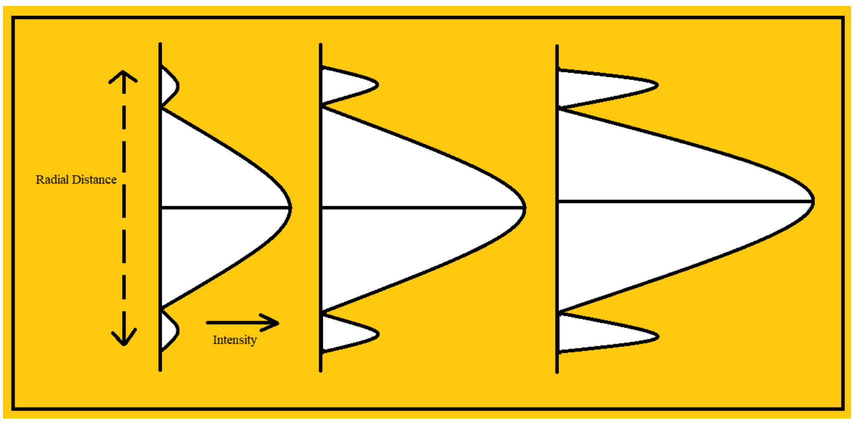

Figure 1.2: Evolution of the laser beam intensity distribution while propagation

7 These figures help to visualize how the laser beam travels through the plasma. We can see a periodic reduction in beam width accompanied by an increase in intensity. This allows the laser

beam to propagate long distances without losing much energy.

1.3 Plasma Profiles

When analyzing laser-plasma interactions, it is essential to distinguish between the different plasma regimes, since plasma behavior depends strongly on parameters such as density, temperature, and ionization state. The most fundamental classification divides plasmas into classical and quantum plasmas.

Classical Plasma

Classical plasma refers to ionized matter in which quantum mechanical effects are negligible. In this regime, particle dynamics can be described using classical mechanics together with Maxwell's equations. One of the key concepts in classical plasma theory is the Debye length λ_D, which represents the characteristic distance over which electric fields are screened by the redis-
20 tribution of surrounding charges. When the thermal de Broglie wavelength is much smaller than the average interparticle spacing, particle distributions follow Maxwell-Boltzmann statistics and quantum corrections can be ignored.

Classical plasma theory is applicable to a wide range of high-temperature, low-density systems, including laboratory discharges, the solar corona, and many space plasmas. Depending on their thermal properties, classical plasmas are often categorized as thermal (hot) plasmas, where electrons and ions are close to thermodynamic equilibrium, or non-thermal (cold) plasmas, where electrons are much hotter than the heavier species.

Quantum Plasma

Quantum plasma arises when the wave nature and quantum statistics of particles become important for describing plasma behavior. This occurs in extremely dense or low-temperature en-
8 vironments, where the thermal de Broglie wavelength becomes comparable to the interparticle

spacing. In such cases, the condition:

$$n\lambda_{th}^3 \geq 1 \tag{1.3.1}$$

must be satisfied, where $\lambda_{th} = h/\sqrt{2\pi m_e k_B T}$ is the thermal de Broglie wavelength. Under this condition, electrons obey Fermi-Dirac statistics rather than classical Maxwell-Boltzmann statistics. Quantum plasmas are encountered in systems such as white dwarfs, neutron stars, semiconductor nanostructures, and dense laser-compressed matter [5, 36–42].

A defining feature of quantum plasma is electron degeneracy, which becomes significant when the Fermi energy E_F is comparable to or greater than the thermal energy $k_B T$. The resulting degeneracy pressure, arising from the Pauli exclusion principle, can strongly influence plasma dynamics even at very low temperatures. The **degeneracy parameter** characterizing the degree of quantum degeneracy is:

$$\chi = \frac{T_F}{T} = \frac{1}{2}(3\pi^2 n\lambda_{th}^3)^{2/3} \geq 1, \tag{1.3.2}$$

2 where T_F is the Fermi temperature obtained from $k_B T_F = E_F$. Another factor that is significant while treating the electron dynamics in quantum plasma is the **quantum coupling parameter**, represented by,

$$g_Q = \frac{E_{int}}{E_F} = \frac{2e^2 m_0}{(3\pi^2)^{2/3}\hbar^2 \varepsilon_0} \sim \left(\frac{1}{n\lambda_F^3}\right)^{2/3} \leq 1, \tag{1.3.3}$$

where e, $\hbar$, ε_0, and λ_F are the electron charge, reduced Planck's constant, the electric field permittivity in vacuum, and Thomas-Fermi screening radius, respectively. This parameter represents the measure of collisions among the electrons.

The distinction between classical and quantum plasmas is particularly important in laser-plasma interaction studies, because quantum effects such as tunneling, electron degeneracy pressure, and exchange-correlation interactions can substantially modify nonlinear phenomena, including relativistic self-focusing. In quantum plasma physics, there has not been a perfect model for studying collisional quantum plasma. Hence, in my study, I have considered the collisionless case for which the coupling parameters should be,

1. $\chi = T_F/T \geq 1$: (Quantum regime)

2. $g_Q = E_{int}/E_F \leq 1$: (Collisionless QP)

At low temperatures and high electron densities, the classical Debye screening mechanism is replaced by Thomas-Fermi screening, which arises from quantum statistical effects. The corresponding Thomas-Fermi screening length is generally shorter than the Debye length, indicating that degenerate electrons can screen electric fields more effectively. In addition, quantum plasmas exhibit exchange-correlation effects resulting from electron indistinguishability and many-body interactions. These effects can strongly modify wave dispersion collective oscillations and the plasma refractive index and become more important with increasing density and decreasing temperature.

The quantum correction to the plasma models is the Bohm potential also called the quantum pressure term which can be written as:

$$\phi_B = -\frac{\hbar^2}{2m_e\sqrt{n}}\nabla^2\sqrt{n}. \tag{1.3.4}$$

The name is coined due to the quantum kinetic energy of the electron wave function which includes quantum effects like tunneling and wave-packet spreading. It can provide extra focusing or defocusing mechanisms in addition to classical one.

Due to the difficulties in giving theoretical account of fully collisional quantum plasma, the investigations have been performed on collisionless quantum plasmas mostly. There exist three common models for this study: namely Wigner-Poisson model, Schrödinger-Poisson model and Quantum Hydrodynamic (QHD) model. Of all, the QHD model seems more appealing in that, it provides the fluid description of the plasma dynamics, taking into account some essential quantum mechanics effects such as the Fermi pressure and the Bohm potential effect in the plasma.

In the case of a laser beam traveling through quantum plasma, the propagation of the beam will be affected by both classical and quantum nonlinear effects. Generally speaking, the self-focusing effect is strengthened when electron density increases. Hence, the effect of self-focusing in

15 quantum plasma is found to be much stronger than the case for classical plasmas due to the modification of plasma density distribution and hence the refractive index distribution caused by quantum effects.

1.4 Laser Beam Intensity Profiles

It is not only the wavelength and propagation direction of the laser beam that are important parameters, but also the spatial distribution of its intensity. The latter determines how the power of light is distributed over the transverse cross section of the beam, and sometimes even its longitudinal distribution. It is determined by the type of laser source, the geometry and configuration of the resonator, and optics.

The intensity profile is important because it determines the interaction of the beam with matter. Since many nonlinear effects depend on the local intensity, beams of the same power with different intensity distributions may behave very differently. In the case of interaction of the beam with plasma, the transverse profile of the beam determines the value and non-uniformity of the ponderomotive force and thus the induced electron density disturbance and the change in the
25 refractive index. This means that self-focusing of the laser beam is strongly dependent on the initial intensity profile of the beam.

There are several different types of intensity distributions, such as Gaussian, super-Gaussian, Bessel-Gaussian, and elliptical-Gaussian beams. All of them have specific geometric features and energy distribution properties. For instance, in Gaussian beams, there is maximum concentration.

Gaussian Beam Profile

Of all laser beams profiles, the Gaussian beam stands out because it occurs most often and has received considerable attention in theoretical and experimental research related to the study of laser plasma interaction. In fact, many types of lasers operate in the fundamental transverse electromagnetic mode, where a Gaussian distribution of the beam intensity is observed. Thanks to the simplicity of the formula and the good understanding of the beam's propagation, this type of profile can be regarded as a benchmark in laser plasmas studies.

27 The transverse intensity distribution of a Gaussian beam is given by:

$$|\mathbf{E}.\mathbf{E}^*|z \leq 0 = A_{00}^2 \exp\left(\frac{-r^2}{r_0^2}\right), \tag{1.4.1}$$

36 55 where $\mathbf{E}$ is the electric field vector of the laser beam, A_{00} - field amplitude, r is the radial distance coordinate, and r_0 is the beam waist at $z = 0$. From this equation, it becomes clear that the beam reaches the highest intensity at the center point ($r = 0$), and falls off exponentially at greater distances. As a consequence, the greatest part of energy is concentrated around the center of the beam, with intensity approaching zero far away from there. The circular symmetry of the profile makes calculations significantly easier.

Owing to its symmetric and smooth shape, the Gaussian beam acts as an ideal tool for understanding nonlinear effects such as diffraction, self-focusing, and filamentation within the context of a plasma environment. The resulting gradient in the radial intensity gives rise to a ponderomotive force pushing electrons out of the beam axis resulting in changes in the refractive index, and hence the mechanism of self-focusing. While more complicated structures might have improved or even different dynamics, the Gaussian beam still forms the foundation against which other structures are often compared.

Super-Gaussian Beam Profile

The super-Gaussian beams are an extension of the usual Gaussian beam with an even flatter distribution of the intensity towards the center of the beam and steeper edges. This beam type
9 is used when a flatter distribution of energy on the cross-section of the beam is required.

The transverse intensity profile of a super-Gaussian beam is given by

$$|\mathbf{E}.\mathbf{E}^*|z \leq 0 = A_{00}^2 \exp\left[-\left(\frac{r}{r_0}\right)^n\right], \tag{1.4.2}$$

7 where A_{00} is the amplitude of the electric field, r denotes the radial coordinate, r_0 corresponds

to the beam waist radius, and n represents the super-Gaussian parameter. Setting $n = 2$ yields a Gaussian distribution, and a larger value than two will yield flatter distribution at the beam center and steeper edges. As n tends to infinity, the profile evolves into a perfect flat-top beam with almost uniform intensity over the center area.

Due to the flat-top feature of the super-Gaussian profile, super-Gaussian beams have found widespread application in practical applications such as laser machining, material surface processing, and laser ablation of materials, in which uniform energy deposition is required. Super-Gaussian beams produce a different ponderomotive force profile compared to a normal Gaussian beam, which causes significant changes in the density perturbation profile and self-focusing process. Therefore, super-Gaussian beams are typically used for studying the effects of different beam shapes on nonlinear laser-plasma interaction processes.

Cosh-Gaussian and Sinh-Gaussian Beam Profiles

The cosh-Gaussian and sinh-Gaussian beams are special forms of Gaussian beam profiles obtained through the combination of a Gaussian envelope function with hyperbolic cosine or hyperbolic sine functions. In addition to having an amplitude profile like a conventional Gaussian beam, cosh-Gaussian/sinh-Gaussian beams can have other features, such as flat-top, multi-lobe, and/or split-peak profiles. This makes the cosh-Gaussian and sinh-Gaussian beam profiles highly valuable in nonlinear optics and laser-plasma interaction studies.

12 The intensity profile of a cosh-Gaussian beam is represented by:

$$|\mathbf{E}.\mathbf{E}^*|z \leq 0 = A_{00}^2 \exp\left(-\frac{x^2}{r_{0x}^2} - \frac{y^2}{r_{0y}^2}\right) cosh^2\left(\frac{u_x x}{r_{0x}}\right) cosh^2\left(\frac{u_y y}{r_{0y}}\right), \tag{1.4.3}$$

where A_{00} is the amplitude of the field, r_{0x} and r_{0y} are the parameters of the waist along the axes of x and y, respectively, while u_x and u_y are the displacement parameters which dictate the form and separation of the intensity peaks.

In similar fashion, the intensity profile of a sinh-Gaussian beam may be defined by:

$$|\mathbf{E}.\mathbf{E}^*|z \leq 0 = A_{00}^2 \exp\left(-\frac{x^2}{r_{0x}^2} - \frac{y^2}{r_{0y}^2}\right) sinh^2\left(\frac{v_x x}{r_{0x}}\right) sinh^2\left(\frac{v_y y}{r_{0y}}\right), \tag{1.4.4}$$

where v_x and v_y represent the displacement parameters corresponding to the hyperbolic sine term. The cosh-Gaussian beam typically displays a broadened or flattened central peak region, although its intensity profile may resemble a flattened profile in the appropriate case. On the other hand, the sinh-Gaussian beam features an intensity of zero at the center of the beam, with off-axis peaks usually appearing. These unique properties render cosh-Gaussian and sinh-Gaussian beams extremely useful in exploring nonlinear effects under unusual intensity profiles. The altered transverse structure of cosh-Gaussian and sinh-Gaussian beams implies the existence of a modified ponderomotive force distribution and hence modified induced density perturbation profiles. This means that these beams display self-focusing properties that are considerably different from Gaussian beams. They are of interest in the study of nonlinear effects such as harmonic generation and beam shaping.

Q-Gaussian Beam Profile

The Q-Gaussian beams represent a special class of laser beam profiles where the form of the transverse intensity profile is determined by only one parameter, referred to as the quality factor q. Depending on the value of this parameter, the laser beam profile can be changed from a high-intensity profile to a profile with constant or near-constant intensity. This type of beam profile is especially advantageous for researching the impact of the laser intensity distribution on its nonlinear propagation.

13 The transverse intensity profile of the Q-Gaussian laser beam has the following form:

$$|\mathbf{E}.\mathbf{E}^*|z \leq 0 = A_{00}^2 \exp\left[-\left(\frac{r^2}{r_0^2}\right)^q\right]. \tag{1.4.5}$$

In this formula, A_{00} represents the field amplitude, r represents the radial position, r_0 represents the radius of the beam waist, and q represents the shaping parameter for the profile. Various values of q result in various shapes of the profile. When $q = 0.5$, the profile shape will be the same as that of a standard Gaussian beam. With $q = 1$, the rate of intensity decay along the radial direction becomes higher, leading to the formation of a profile shape akin to a super-Gaussian beam. Higher values of q cause the center of the profile to become flat, while the edge regions become steeper, thus creating a nearly ideal profile shape with a flat top. Inversely, smaller values of q will produce a sharper peak in the profile.

Since the shaping parameter can vary freely, the Q-Gaussian beams make it possible to explore a broad variety of beam shapes through one mathematical expression. This makes them useful when studying laser-plasma interactions because varying the transverse intensity profiles will help in systematically analyzing the effect on the ponderomotive force and plasma density perturbation, which influences the process of self-focusing.

Laguerre-Gaussian Beam Profile

Another significant set of paraxial solutions in cylindrical coordinates are the Laguerre-Gaussian (LG) beams. Different from conventional Gaussian beams, LG beams have a helical phase pattern and orbital angular momentum, which make them highly applicable in topics such as optical traps, particle manipulation, quantum optics, and laser plasmas.

The LG beams can be specified by two integers: the radial index p and the azimuthal index l (also called topological charge). p determines the number of intensity nodes in the radial direction, while l defines the amount of orbital angular momentum carried by the beam. The formula for the transverse intensity pattern of LG beams is expressed by the following formula

$$|\mathbf{E}.\mathbf{E}^{*}|z \leq 0 = A_{00}^{2}\left(\frac{\sqrt{2}r}{r_0}\right)^{2l}\left[L_p^l\left(\frac{2r^2}{r_0^2}\right)\right]^2 \exp\left(\frac{-r^2}{r_0^2}\right), \quad (1.4.6)$$

11 where A_{00} represents the amplitude of the field, r is the radial coordinate, r_0 is the waist radius of the beam, and L_p^l indicates the associated Laguerre polynomial. In the simplest situation ($p = 0$), there is no intensity in the middle of the beam and most of the intensity is concentrated in a circular ring. The ring radius is proportional to the topological charge. In addition, when $p \neq 0$, there will also be several concentric rings. LG beams have a special phase structure due to an azimuthal dependence $\exp(il\phi)$ leading to a phase singularity in the beam center and allowing the LG beam to possess a special orbital angular momentum of $l\hbar$ per photon. It opens up the possibility of transferring orbital angular momentum to a particle and to a plasma medium. Due to their annular intensity distribution and orbital angular momentum, the LG beams generate specific ponderomotive forces during interaction with a plasma. Moreover, these beams can be used in studying vortex formation and rotation of a plasma and also modification of self-focusing properties in comparison with Gaussian beams.

Hermite-Gaussian Beam Profile

51 Another family of solutions to the paraxial wave equation, called the Hermite-Gaussian (HG) beams, can be described by coordinates in the Cartesian system. They have practical importance for analysis of laser modes in rectangular optical resonators or for systems with anisotropic boundary conditions. HG beams possess a nonuniform, structured intensity distribution that differs from the uniformity of Gaussian beams.

HG beams have two parameters defining mode orders along the x and y axes - m and n, respectively. The intensity distribution takes the following form:

$$|\mathbf{E}.\mathbf{E}^*|z \leq 0 = A_{00}^2 \left[H_m\left(\frac{x}{r_{0x}}\right)\right]^2 \left[H_n\left(\frac{y}{r_{0y}}\right)\right]^2 \exp\left(-\frac{x^2}{r_{0x}^2} - \frac{y^2}{r_{0y}^2}\right), \tag{1.4.7}$$

54 Here, A_{00} is the field amplitude, r_{0x} and r_{0y} are the beam waist parameters in the x and y directions, respectively, and H_m and H_n are Hermite polynomials of order m and n, respectively. The fundamental ($m = n = 0$) mode corresponds to the well-known Gaussian mode. As higher-order modes are considered, the beam has more than one intensity lobe that are separated from each other by nodal lines. The total number of lobes in the x and y directions depends on the orders m and n, respectively.

Hermitian-Gaussian beams are extensively used in laser resonators theory, beam manipulation, and mode conversion problems. During interaction of HG beams with plasmas, their inhomogeneous and asymmetric intensity distribution causes spatial variation of ponderomotive force that results in complicated density perturbation profiles and self-focusing phenomena. For these reasons, HG beams can be effectively applied for studying the impact of higher-order transverse modes on nonlinear processes in plasmas.

Elliptical-Gaussian Beam Profile

Elliptical-Gaussian beams are the generalization of Gaussian beams where the widths of the beam in two perpendicular directions are different. Thus, the profile of the beam in transverse plane becomes elliptical rather than circular. The consequence of such asymmetry results in different diffraction and focusing properties of the beam along principal axes, which makes them useful for applications in optics that involve inherently anisotropic geometrical settings. The transverse intensity profile of an elliptic-Gaussian beam is described by the equation:

$$|\mathbf{E}.\mathbf{E}^*|z \leq 0 = A_{00}^2 \exp\left(\frac{-x^2}{r_{0x}^2} - \frac{y^2}{r_{0y}^2}\right). \tag{1.4.8}$$

where A_{00} represents the field amplitude, x and y are the transverse coordinates, while r_{0x} and r_{0y} are the corresponding beam waist radii. In case of $r_{0x} = r_{0y}$, the equation corresponds to the usual Gaussian beam. However, when the radius values are not equal, this results in elongation of the beam along a certain axis, creating an ellipse-shaped intensity distribution. One can consider the electric field as the multiplication of two independent Gaussian functions, each with their own respective waist and diffraction lengths. Elliptic-Gaussian beams are created through astigmatic optical elements, cylindrical lenses, or laser cavities with nonuniform focal lengths in perpendicular directions. Laser beams with asymmetric shapes due to alignment errors or anisotropy are also referred to as elliptic-Gaussian.

In laser-plasma interactions, the different beam width causes anisotropic changes in intensity and, thus, in the ponderomotive force and plasma density distribution. Therefore, elliptical-Gaussian beams provide an interesting approach to studying the effect of asymmetry on nonlinear propagation and may give us extra control over the distribution of energy deposited in the plasma.

Bessel-Gaussian Beam Profile

Bessel-Gaussian beams are created through the superposition of a Bessel function multiplied by a Gaussian envelope. These beams consist of a bright center and concentric rings surrounding it. They have many of the advantages of the ideal Bessel beams while having a finite beam energy. Due to these remarkable properties, Bessel-Gaussian beams have become objects of considerable interest in such diverse applications as optical trapping, material processing, nonlinear optics, and laser-plasma interaction.

The expression for the transverse intensity profile of Bessel-Gaussian beams is:

$$|\mathbf{E}.\mathbf{E}^*|z \leq 0 = A_{00}^2 \left[J_v \left(\frac{\mu r}{\sqrt{2} r_0} \right) \right]^2 \exp \left(-\frac{r^2}{r_0^2} \right), \tag{1.4.9}$$

6 In this expression, A_{00} is the field amplitude, J_v represents the Bessel function of the first kind and vth order, r is the radial coordinate, r_0 is the Gaussian beam waist radius, and μ is the transverse wave number that determines the separation between the concentric rings and their brightness. One of the important characteristics of Bessel-Gaussian beams is their quasi-diffraction-free propagation. In other words, over some finite distance, they retain their transverse intensity profile much better than a regular Gaussian beam would. The low level of diffraction ensures that the beam retains its narrow central core as well as high energy throughout long propagation

distances.

The ring structure also provides the Bessel-Gaussian beams with a notable self-healing feature. Should some part of the beam be blocked or affected, then energy from the outer rings could recreate the central core by further propagating. It thus implies that these beams can handle interference due to their ability to reconstruct themselves.

The rings' structure in the case of interactions between a laser and plasma produces an efficient ponderomotive force. The energy in the outer regions is continually provided to the central region, leading to more powerful and sustainable self-focusing. An essential factor responsible for this process is the wave transverse parameter (μ). Due to the properties outlined above, Bessel-Gaussian beams are best suited for long-range propagation with minimal dispersion and effective self-focusing capabilities within plasma media.

Hermite-Cosh-Gaussian and Other Composite Profiles

The hermite-cosh-gaussian (HChG) beams represent hybrid types of beam profiles, created via the superposition of a hermite polynomials and hyperbolic cosine modulation under the gaussian function envelope. It leads to obtaining the intensity pattern of a beam represented by several lobes with their shapes and separations defined by hermite mode numbers and parameters of cosh modulation, respectively. Thus, the HChG beams give rise to a flexible tool for manipulation with laser intensity distributions.

While the hermite polynomials define the number of lobes and their orthogonality to each other in terms of two transverse directions, the hyperbolic cosine functions are responsible for broadening the beam profile and giving rise to peaks of different intensities and heights. The resultant intensity profile of a beam is more sophisticated compared to a simple HG beam profile.

Such composite beams could be applied for studying the effect of multi-feature beam intensity on non-linear phenomena. The interaction between the several intensity lobes in plasma leads to the ponderomotive force acting on the electron density in plasma and to their complicated refractive index distribution. In this way, it can cause self-focusing of a beam that is different from that in simpler beams.

16 The profile of the laser beam significantly influences the phenomenon of self-focusing in plasma. Because the ponderomotive force is proportional to the local gradient of intensity, different profiles cause different density deformations and refraction index changes. Bessel-Gaussian beams usually demonstrate a stronger effect of self-focusing since their rings keep feeding the core. The anisotropic effect of focusing can be investigated for elliptic Gaussian beams due to their different dimensions across orthogonal axes. Finally, the use of super-Gaussian and Q-Gaussian beams allows systematically studying the influence of the sharpness of the boundary and flatness of the center on self-focusing behavior.

In summary, the choice of the beam shape is an important physical factor, not just a convenient mathematical tool. The control over the transverse distribution of intensity makes it possible to regulate the nonlinear behavior of laser beams in plasma.

1.5 Density Variations

Plasma density is considered one of the essential parameters that affect the process of laser-plasma interaction since plasma density determines the refractive index and the nonlinearity of the reaction to the laser field. The ponderomotive force acts so that the electrons are forced to leave areas with high intensity, meaning that the value of the obtained density perturbation depends on the electron density of the plasma at its initial stage.

Generally speaking, when the electron density increases, the degree of the refractive index change when the electrons move away from the optical axis of the laser beam also grows. It
43 creates a sharper gradient of the refractive index and contributes to self-focusing. The effect becomes even more significant in quantum plasmas with degenerate pressure, Bohm potential, and exchange-correlation, causing the degree of beam compression to increase significantly compared to classical plasmas in identical conditions.

Spatial variations of plasma density mean that the laser propagates through an inhomogeneous medium. The particular nature of the density profiles could cause them to induce the early development of self-focusing, maintain the high intensity over large distances, channel the beam through plasma channels, or even defocus the laser beam. For instance, a rising density profile, such as an exponential one or a tangential ramp profile, could increase the strength of the non-linear response, hence, causing stronger focusing over several Rayleigh lengths. On the other hand, falling density profiles can reduce the gradient of the refractive index and weaken the self-focusing process.

In summary, the careful design of the plasma density profiles can serve as an effective means of managing the beam propagation. It is now possible to control the self-focusing effect based on the specific spatial variation in electron density profiles, thus, optimizing the plasma-laser interaction for various applications including laser-induced particle acceleration and inertial confinement fusion.

Uniform Density

Uniform density profile implies that the electron density (n_0) stays constant across the entire space. This kind of density profile is the simplest and most common model used in plasma-laser interaction studies.

In a homogeneous plasma, the refractive index at the beginning remains constant at all locations. With the propagation of a high-power laser beam, the ponderomotive force drives electrons from the high-intensity core of the beam and creates a localized density drop and thus a gradient in the refractive index. This gradient works as a converging lens that resists the diffractive tendency of the beam. There are two main processes that dominate the behavior of the beam: diffusion, which tends to spread the beam, and self-focusing, which tends to converge the beam. If both effects are comparable, there are generally oscillations in the behavior of the beam, which consist of periodic focusing and defocusing. Without any effort in density management, however, the self-focusing process is rather constrained. Although the beam may be focused momentarily, it will diffuse again once the beam reaches its smallest diameter. In consequence, the area of high-intensity laser beam is confined within just a few Rayleigh lengths.

For that reason, the concept of uniform plasma density should be considered to be merely a theoretical benchmark. Although it reflects the physical essence of self-focusing, it lacks the necessary level of control required for optimized laser beam propagation. More advanced plasma density distributions, like exponential or tangential ramps, may dramatically improve and extend the process of focusing due to constant change in the refraction properties of plasma.

Density Ramps

The other type of profile, density ramps, is characterized by an increasing density of electrons along the laser beam propagation. It can be achieved through a linear or nonlinear dependence. These profiles are quite efficient for the enhancement of self-focusing, as they introduce medium with gradually growing refractive properties along the propagation path of the beam. With each step into denser region, the ponderomotive response of the plasma grows up, and therefore the degree of refractive index modification also becomes larger. Density ramp profile can be considered as a kind of optical guide whose cross section is becoming narrower in accordance with its properties.

One more useful feature of density ramps is the decreased oscillations amplitude and increased frequency in relation to uniformly ionized plasma. Consequently, a laser beam propagates through it more stably.

Due to these properties, density ramp profiles are extensively applied in various plasma-related laser technologies such as particle acceleration and inertial confinement fusion.

- Linear Density Ramps are considered one of the simplest plasma inhomogeneities whose electron density varies with distance along the propagation path as follows:

$$n = n_0(1 + \xi/d), \tag{1.5.1}$$

where ξ is the distance along the propagation path and d is the characteristic scale of the density ramp.

Since the density increases in a consistent way, the effect of nonlinearity in the refractive index becomes increasingly significant during the laser propagation. This means that self-focusing occurs easier and the beam width variations are less than in the case of uniform plasma. Due to their relatively simple analytical form and possibility of easy implementation in an experiment, the linear density ramps are often considered as a model of choice for such experiments.

- Exponential Density Ramps are described by the relation

$$n = n_0 \exp(\xi/d), \tag{1.5.2}$$

where ξ represents the propagation distance and d is the characteristic scale length. The plasma density gradually increases in this type of density profile, but as propagation goes on, its increase rate grows rapidly as the laser proceeds deeper into the medium.

Due to the increasing slope of the density profile, the nonlinear refractive response of the plasma continuously builds up. Thus, self-focusing is greatly increased compared to density profiles with linear density increase. This type of density profile creates much less beam width oscillations and provides better beam confinement. It should be noted that, with exponential density increase, lasers can propagate through plasma for several Rayleigh lengths.

- Tangential (or Polynomial) Density Ramps are described by smooth functional dependencies which give gradual change of density but have no abrupt jumps, such as tangent or polynomial profiles. They have relatively steep density changes but mathematically smooth functional forms.

Due to their mathematical smoothness, such density profiles provide no numerical difficulties in laser-plasma modeling. At the same time, they give relatively strong density changes which are useful to increase nonlinearity in laser-plasma experiments. Thus, tangential density profiles can be considered as intermediate between uniform density and sharp jump profiles.

- Density Ripples represent periodic fluctuations of electron density occurring against a homogeneous or smoothly varying plasma background. Usually, they are characterized by

$$n = n_0 + n_{q0} exp(iqz), \tag{1.5.3}$$

where n_{q0} corresponds to the amplitude of the perturbation in density and q is the wave vector of the rippled density component. Density ripples may appear spontaneously as a result of constructive interference of laser pulses or can be created deliberately to obtain periodic distributions of plasma density.

The fundamental phenomenon responsible for the appearance of density ripples is known as the ponderomotive effect produced due to interference of laser pulses. As a consequence, there is a periodic redistribution of electrons which results in creation of density gratings. The characteristic lengths of gratings formed can be orders of magnitude shorter than the laser wavelength. Density ripples can substantially affect laser propagation due to their ability to distort refractive index distribution thus inducing various phenomena such as self-focusing, Bragg scattering, and resonant wave coupling.

- Parabolic Density Profiles are commonly described by

$$n = n_0(1 - \alpha\xi^2), \tag{1.5.4}$$

where n_0 is the electron density at its peak value whereas α specifies the curvature of the profile. Under these conditions, the electron density decreases with increasing distance from the center according to quadratic dependence.

Profiles of this kind can occur, for instance, in nearly thermally equilibrated plasmas or in the presence of externally applied electric or magnetic fields. For the interaction of laser with plasma, profiles of this type may serve as waveguiding media; the interplay of diffraction and self-focusing becomes affected by the profile shape which leads to focusing features different from the ones in the case of linear and exponential density gradients.

1.6 External Fields

In some cases, externally applied electromagnetic fields could significantly affect the physics of self-focusing phenomenon. Of course, field-free situations could be used as an important basis for developing theoretical models but in reality, external fields usually have significant effects on the dynamics of laser-plasma interaction processes.

This effect manifests itself through the influence on the charged particle dynamics, on the dispersion of the plasma, as well as on the refractive index profile created due to the presence of the laser beam. Thus, under certain conditions, externally applied fields can amplify self-focusing while in others they could inhibit this phenomenon.

Role of External Magnetic Fields

External static magnetic fields play a significant role in shaping laser-plasma interaction mechanisms, owing to their impact on the motion of charged particles and the dynamics of the plasma itself. In the presence of a magnetic field, the action of the Lorentz force, which operates perpendicular to both the velocity and the magnetic field, can be observed. The Lorentz force acting on a particle of charge q, having a velocity $\mathbf{v}$ and being in the presence of a magnetic field $\mathbf{B}$, is defined by

$$\mathbf{F}_L = q(\mathbf{v} \times \mathbf{B}). \tag{1.6.1}$$

As the Lorentz force does no work, yet continually deflects the trajectory of the particle, the charged particles move along circular or helical paths, which follow the field lines. The gyration of the particles takes place at the cyclotron frequency

$$\omega_c = q\mathbf{B}/m_e, \tag{1.6.2}$$

where m_e is the electron mass. The presence of this characteristic frequency adds another timescale to the dynamics of the plasma.

An external magnetic field affects the motion of electrons in a plasma perpendicular to the field and thus affects the effect that the electric field from the laser beam has on electron density redistribution. As a result, a modification of the refractive index profile occurs in such interactions, which in turn either strengthens or weakens the self-focusing mechanism. Thus, magnetic fields can be used successfully as a method of controlling laser beams, and there are many examples of research related to magnetized plasmas.

Direction of the Magnetic Field and Its Impact

An important factor when studying the impact of the external magnetic field on plasma interaction with the laser beam is its orientation relative to the direction of the laser beam propagation. 30 Axial magnetic fields oriented parallel to the direction of propagation of the laser have the effect of enhancing electron confinement in the plane perpendicular to the beam. This, in turn, helps to intensify the perturbation created by the ponderomotive force, which causes increased refraction and self-focusing. The increase in the magnetic field leads to a lower critical value of the power necessary for self-focusing, thus resulting in faster focus attainment.

A transverse magnetic field, as the name suggests, is oriented perpendicular to the direction of propagation of the laser. This causes the electrons to be differently confined depending on their orientation. This, in turn, causes the refractive index to be polarization-dependent, hence inducing birefringence. Beam splitting and self-focusing are some of the phenomena that could result from the introduction of a transverse magnetic field in the medium.

35 Therefore, by regulating the strength and direction of the applied magnetic field, it becomes

feasible to manipulate the propagation of lasers within plasma and gain more control over the nonlinear processes involved in particle acceleration, fusion studies, and high-energy density physics applications.

Enhanced Self-Focusing Effects

There exist several physical processes that lead to enhanced self-focusing in magnetized plasmas:

1. **Compression of Density Gradients**: The restriction of electron movement perpendicularly to the magnetic field causes an increase in density gradient due to better confinement of ponderomotively displaced electrons. Consequently, the gradient of refractive index also increases.

2. **Effect on the Plasma Frequency**: The application of an external magnetic field influences plasma characteristics via electron cyclotron motion. Thus, the dispersion relation and refractive index become different in the presence of the magnetic field, resulting in self-focusing enhancement.

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3. **Cyclotron Resonances**: Cyclotron resonance is observed if the frequency of a laser is comparable to that of electron cyclotron resonance or its higher order. This means that interaction between light and a plasma becomes more efficient, which results in stronger nonlinear effects such as self-focusing.

4. **Electron Dynamics in a Magnetic Field**: Electrons subjected to a magnetic field rotate along the Larmor circle while experiencing ponderomotive force of the light wave. This constrained motion contributes to an increase in efficiency of refractive index generation due to electron redistribution.

All of these effects combine to allow external magnetic fields to significantly alter the process of self-focusing, thus enhancing it; magnetized plasma can therefore be employed as an appropriate medium for laser control.

External Electric Fields

External static electric fields are equally important in the interaction between lasers and plasmas since they exert mechanical forces on the charged particles and change the background potential distribution of the plasma. For example, a charged particle of charge q subjected to an electric field

$$\mathbf{F}_E = q\mathbf{E}, \tag{1.6.3}$$

which serves to accelerate the particle in the direction of the electric field. Using this principle, external electric fields may change the electron/ion distribution and create an already existing electric field gradient, which acts on the non-linear fields created by the laser pulse.

Electric fields may be longitudinal or transverse with respect to the laser beam direction. Longitudinal electric fields serve to accelerate or decelerate the electrons in the direction of the beam propagation, affecting the response of plasma to the ponderomotive forces of the laser and, therefore, the efficiency of self-focusing. In case of transverse fields, the density and refractive index profiles become asymmetric in the direction of transverse electric fields, resulting in direction dependent focusing. External electric fields may be used to affect the degree of self-focusing both positively or negatively.

Chapter 2

LITERATURE REVIEW

Laser-plasma interaction has been a popular field of study for many years now, the self-focusing of high intensity laser beams being one of the main nonlinear effects observed therein. This phenomenon is of utmost importance in various applications like inertial confinement fusion, particle acceleration and high energy density physics among others. With time, studies have evolved from those using the relativistic ponderomotive theory in explaining self-focusing dynamics to more complex models where quantum mechanics plays a role. Some of the quantum mechanical effects include the Bohm potential, electron degeneracy pressure and exchange correlation effects. The effects of external magnetic fields as well as various plasma densities and laser intensity distributions have also been looked into.

Nonetheless, many interesting areas of the parameter regime have yet to be fully explored. Specifically, the interplay between advanced beam shapes, for instance Bessel-Gaussian and elliptical-Gaussian beams, and their passage through magneto-quantum plasma characterized by non-uniform density distribution has not been thoroughly investigated. The combination of these elements may give rise to a variety of nonlinear effects, which cannot be explained based on previous findings in the literature. This paper provides a review of the related literature, defines the theoretical background of this thesis, highlights the research gaps, and discusses the key objectives of the study.

2.1 Existing Literature

The interaction of lasers with plasmas has generated a huge volume of literature over the last several decades, and self-focusing stands out as one of the most extensively studied nonlinear processes in this domain. Researchers have investigated the phenomenon of self-focusing in various ways under varied conditions of plasma state, laser intensity, beam shape, and the presence of external fields.

The literature review is presented by organizing previous works on the basis of the regime of physics involved, namely, classical and quantum regimes of plasmas, as well as according to major parameters used for modeling, such as the profiles of plasma density, intensity profiles of lasers, and the magnetic field applied.

Self-Focusing in Classical Plasma

Self-focusing in classical plasmas can be attributed to a synergic effect of two effects: relativistic and ponderomotive nonlinearities. At large laser intensities, electrons acquire high enough velocities at which they start moving relativistically, meaning that their effective masses increase and the plasma frequency drops. As a result, one observes the dependency of the refractive index of a medium on the intensity of light. The second factor that contributes to self-focusing is the ponderomotive force that excludes electrons from the central part of the beam where the intensity is large. The combination of both effects results in more pronounced self-focusing than any of them taken separately; the threshold intensity depends on several parameters, such as laser intensity, plasma density, and its wavelength [43, 44].

In the development of the laser beam within the above systems, parabolic wave equation combined with Wentzel-Kramers-Brillouin (WKB) approximation and high order paraxial approximation are employed to study the behavior of the beam-width parameter and the self-focusing phenomenon. It has been shown that the relativistic self-focusing can amplify the peak laser intensity by 5 to 15 times that of the intensity in vacuum state and that amplification is most pronounced when the laser intensity is near critical self-focusing point.

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Self-Focusing of Gaussian Beams in Classical Plasma

The Gaussian beam of lasers is one of the well-studied beam profiles with respect to the problem of self-focusing and hence it has been taken as the base for the theoretical analysis of self-focusing phenomena due to nonlinear interaction of lasers with plasmas. The Gaussian beam has a radially symmetric intensity profile which makes it easier to analyze the mechanism of relativistic and ponderomotive self-focusing.

For the case of classical plasma, self-focusing phenomena in a uniform-density medium usually display a periodic nature as the result of the interaction between diffraction and non-linear refraction properties, which causes focusing and defocusing phases to alternate during Gaussian beam propagation [45]. The period and amplitude of such oscillations can be estimated as functions of laser power, plasma density, and waist of the initial beam. More modern research works employed the advanced analytical tools, including the source-dependent expansion technique, for a more accurate study of Gaussian beams in magnetized plasma.

Higher-Order Beam Profiles and Composite Beams

In addition to Gaussian-type beams, other structured types of laser beams can demonstrate self-focusing properties similar to those of conventional Gaussian beams, or even more intense. HG and HChG modes were found to allow sustained high-power propagation at distances much greater than the Rayleigh length, provided that the choice of the appropriate mode parameters. The peculiarities of their intensity distribution give rise to multi-lobe patterns of the ponderomotive force, and hence affect the non-linear beam propagation.

It has been shown that the cosh-Gaussian beams propagating in plasma under magnetic field conditions are prone to undergo self-focusing process more effectively and demonstrate higher efficiency of second harmonic generation than Gaussian beams. Q-Gaussian beams were analyzed too, and it was concluded that sharper intensity distribution (which corresponds to smaller value of the profile parameter) results in self-focusing effect enhancement. The Laguerre-Gaussian

(LG) beams, carrying orbital angular momentum and demonstrating ring-like intensity distribution, reveal specific properties of self-focusing and absorption, which differ noticeably from properties inherent in Gaussian beams [46–49].

Self-Focusing in Quantum Plasma

It was clear that quantum mechanical phenomena begin to play an essential role in the case of dense and cold plasmas, resulting in the generalization of the self-focusing theory into the quantum regime. When such plasma conditions occur, classical fluid description is not applicable anymore and the QHD description should be employed. QHD model involves several quantum features, such as the Bohm potential and the Fermi pressure, responsible for quantum diffraction and quantum degeneracy of electrons, respectively. As one may see, introduction of such quantum corrections results in increased efficiency of self-focusing compared with classical model [50, 51].

Another possible quantum correction stems from many body interaction of electrons, leading to exchange-correlation effects. Such quantum phenomena result in modification of dispersion relation and dependence of plasma wave refractive index from intensity of electromagnetic field, thus affecting the process of self-focusing.

It is also revealed from recent research that combining quantum effects with customized density profiles leads to improved self-focusing capability. Research on relativistic self-focusing in a thermal quantum plasma having ramped density profiles revealed earlier starting point, enhanced beam compression, increased oscillation frequency, and decreased oscillation amplitude for ramped density profiles when compared with constant and/or classical plasma.

Effects of External Magnetic Field

It has been observed from numerous researches that application of external magnetic fields to
12 the system results in enhanced self-focusing of intense laser beams in plasma. Enhancement is due to multiple factors such as electron confinement, modification of the plasma frequency,

and change in magnetized electron behavior. Ponderomotive force is restricted to perpendicular direction by magnetic field which results in steepness of density gradient and higher variation in refractive index. Consequently, stronger self-focusing behavior and higher beam compression are obtained as a result of increasing magnetic field strength. Gaussian beams in magnetized plasma experience increased self-focusing strength with enhanced magnetic field strength along with lower critical power for self-focusing and lower self-focusing length [48, 49].

Orientation of the magnetic field is another factor that has significant importance. Axial magnetic fields, where the alignment is along the laser propagation direction, are most likely to offer higher enhancement due to better confinement in the transverse plane without disturbing beam symmetry. The transverse magnetic fields are more likely to cause anisotropies, thereby affecting the focusing properties. If relativistic and ponderomotive nonlinearities operate together
38 under the influence of magnetic field in the plasma, then the resulting self-focusing could be far more intense, making extreme beam compression possible at high laser intensities.

Density Profiles

It has been shown that density profiles in plasmas offer an effective way of managing self-focusing. In cases where the density of the plasma increases along the propagation direction, the nonlinearity of the medium becomes higher and helps achieve stronger focusing. These kinds of density profiles can limit the oscillations in beam width significantly and make the frequency of such oscillations higher. This allows laser self-focusing to occur over greater distances in comparison to the case of uniform plasma.

Out of all profiles investigated, those with exponential density ramps lead to the strongest effect due to the fact that the plasma density, and therefore the nonlinearity, grows faster compared to linear ramps. There are other structural density profiles, such as density ripples and preformed plasma channels, which serve as efficient methods for manipulating the propagation of the laser beam. Density ripples create periodic changes in the refractive index that increase the efficiency of nonlinear phenomena like harmonic generation and Raman scattering, whereas plasma channels with specially designed density profiles are capable of steering intense laser beams over long distances without diffraction.

2.2 Research Gaps

52 Although considerable effort has been dedicated to studying the interaction of intense laser beams with plasma and their self-focusing, a number of significant issues still need to be addressed. Whereas both classical and quantum approaches have been thoroughly investigated, most of the studies consider either a basic beam profile, homogeneous plasma, or individual nonlinear processes. By contrast, a combination of more sophisticated beam profiles, quantum contributions, the effect of an external magnetic field, and inhomogeneous plasma is much less frequently discussed.

Among these effects, one can identify the interaction of Bessel-Gaussian and elliptical-Gaussian beams with magnetized quantum plasma having an inhomogeneous density profile. Such beams are characterized by a special intensity distribution which affects the distribution of the ponderomotive force and, therefore, influences the process of self-focusing. Meanwhile, the presence of quantum factors (such as the Bohm potential and exchange-correlations), in addition to the magnetic field and density ramp, introduces new mechanisms of beam evolution. This scarcity of research dealing with all these problems at once clearly demonstrates a gap in the current literature, thus forming the main rationale for this work.

Gap 1: Bessel-Gaussian Beams in Magnetized Quantum Plasma with density variations

Even though the Bessel-Gaussian (B-G) beam has already been analyzed in the classical plasma regime, its self-focusing phenomenon within the framework of the quantum plasma regime is almost untouched by researchers. Specifically, there has not been much analysis done on how the quantum mechanics properties such as Bohm force and exchange-correlation potential effect the self-focusing process. It is a crucial issue because, due to the unique ring-shaped structure of the B-G beam's intensity, one can expect a different ponderomotive force distribution.

Most research done recently regarding the Bessel-Gaussian beam in cold magnetized quantum collisional plasma is related to the problem of second-harmonic generation. The study involving relativistic self-focusing of the Bessel-Gaussian beam in a collisionless quantum plasma considering the complete quantum hydrodynamic (QHD) approach including Bohm potential and exchange-correlation potential has not yet been carried out.

Gap 2: Elliptical-Gaussian Beams in Magnetized Quantum Plasma with density variations

The nature of Elliptical-Gaussian (E-G) beams, which are beams having unequal widths in two mutually orthogonal planes in transverse plane, has been explored extensively using classical plasma as a tool to analyze the phenomenon of asymmetric self-focusing and anisotropic evolution of the beams. The reason being its noncircular shape leading to different rates of diffraction and nonlinear focusing along different axes. However, their performance within quantum plasma environment has not been well-explored yet.

Just a little literature is available on related aspects. One such piece of work is the study of generation of second harmonic efficiency of elliptically polarized Bessel-Gaussian beams in magnetized collisional quantum plasma. But still no effort has been made for a detailed study on the relativistic self-focusing of elliptical-Gaussian beams in collisionless quantum plasma including electron degeneracy, Bohm and exchange-correlation potentials.

In summary, two main voids can be identified from the existing literature regarding the studies carried out in this area: (i) the lack of investigations of Bessel-Gaussian beams in the magnetized quantum plasma environment with spatially varying density, and (ii) the lack of relevant studies on elliptical-Gaussian beams in such an environment. In fact, these two areas have been left relatively untouched due to the potential contributions that can be made in terms of developing novel beams which possess higher focusing properties in such environments.

The current dissertation fills these voids by conducting an extensive theoretical and numerical analysis of these beams in a magnetized quantum plasma environment with spatially varying density profiles.

2.3 Objectives of the Study

The major goals of this dissertation are:

1 1. **To study the effect of relativistic self-focusing of Bessel-Gaussian laser beams in the presence of a magnetic field in quantum plasma**. In this regard, the present work aims at exploring the nature of interaction of ring-shaped Bessel-Gaussian laser beams with the quantum and magnetic properties of plasma.

1 2. **To study the effect of relativistic self-focusing of elliptical-Gaussian laser beams in magnetized quantum plasma**. In particular, we intend to explore the nature of interaction of asymmetrically shaped elliptical Gaussian laser beams with magnetized quantum plasma.

3. To derive analytical and numerical models taking into account the role of **relativistic effects, quantum hydrodynamics, magnetic field, and plasma inhomogeneity** in the case of both beam profiles.

Thus, the above-listed goals correspond to the main gaps in the literature, which we would like to fill during this work. The findings of this project will allow us to obtain important insights into nonlinear laser-plasma interaction in magnetized quantum plasmas.

2.4 Limitations of the Work

1. **Collisionless plasma assumption**: The analysis is restricted to collisionless quantum plasma ($g_Q \leq 1$), and therefore does not account for strongly collisional effects or collision-induced damping.

2. **One-dimensional density variation**: Plasma density is assumed to vary only along the laser propagation direction. More complex two- or three-dimensional density structures are beyond the scope of the present work.

3. **Uniform external magnetic field**: The applied magnetic field is taken to be static and spatially uniform, excluding the effects of time-dependent or non-uniform magnetic field configurations.

4. **Paraxial approximation**: The theoretical model assumes paraxial beam propagation, which is valid when the beam divergence is small, and the wavelength is much smaller than the characteristic beam dimensions.

5. **Finite computational domain**: Numerical simulations are performed over propagation distances sufficient to capture the self-focusing dynamics, but may not include phenomena that arise over extremely long distances.

6. **Neglect of ionization and thermal effects**: Processes such as ionization, collisional heating, and hydrodynamic expansion are not included, under the assumption that their influence remains negligible over the timescales considered.

Chapter 3

RESEARCH METHODOLOGY

3.1 Theoretical Analysis

3.1.1 Approach and Hypotheses

The theoretical analysis is based on the paraxial approximation and the Wentzel–Kramers–Brillouin (WKB) method to describe the propagation of intense laser beams in magnetized quantum plasma. Under the paraxial approximation, Maxwell's electromagnetic equations are reduced to a parabolic wave equation for the slowly varying envelope of the electric field. This approximation is valid when the laser wavelength is much smaller than the characteristic transverse dimensions of the beam and the beam divergence remains small.

The WKB method is then used to separate the rapidly oscillating phase from the slowly varying amplitude of the electromagnetic field. This results in a second-order differential equation that defines the behavior of the beam-width parameter r in the propagation direction z. This equation accounts for relativistic changes in the mass, the ponderomotive force, quantum effects due to the Bohm potential and the exchange-correlation potential, and the modification of the refractive index of the plasma under the influence of the applied magnetic field.

The above-mentioned nonlinear ordinary differential equation forms the core of this thesis. Its analytical and numerical solutions provide the basis for evaluating self-focusing behavior, intensity amplification, and beam stability for both Bessel-Gaussian and Elliptical-Gaussian laser beams propagating in magnetized quantum plasma with non-uniform density profiles.

3.1.2 Nonlinear Model of Laser-Plasma Interaction

A laser beam is fundamentally an electromagnetic wave, and its propagation through plasma is governed by Maxwell's equations. For a monochromatic laser field, the electric field satisfies the wave equation [52]

$$\nabla^2 \mathbf{E} + \frac{\omega^2}{c^2}\epsilon(r,z)\mathbf{E} = 0, \tag{3.1.1}$$

where $\nabla = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)$, ω is the laser angular frequency, and $\epsilon(r,z)$ is the dielectric function of the plasma. The electric field is expressed as

$$\mathbf{E}(r,z,t) = A(r,z)\exp[i(kz - \omega t)]\hat{\mathbf{x}}, \tag{3.1.2}$$

where $A(r,z)$ is the slowly varying complex amplitude and $k = (\omega/c)\sqrt{\epsilon_0}$ is the wave number associated with the linear dielectric constant ϵ_0.

Substituting Eq. (3.1.2) into Eq. (3.1.1) and applying the paraxial approximation reduces the full wave equation to the nonlinear parabolic equation

$$2ik\frac{\partial A(r,z)}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A(r,z)}{\partial r}\right) + \frac{\omega^2}{c^2}\Phi(\mathbf{E}.\mathbf{E}^{)}A(r,z), \tag{3.1.3}$$

where $\Phi(\mathbf{E}.\mathbf{E}^*)$ represents the nonlinear part of the dielectric function, which comes from the general expression of the dielectric constant for collisionless plasma, given as

$$\epsilon = \epsilon_0 + \Phi(\mathbf{E}.\mathbf{E}^*). \tag{3.1.4}$$

This equation is formally analogous to the Schrödinger equation and forms the basis for analyzing nonlinear beam propagation in plasma. To obtain the beam-width evolution, the Wentzel–Kramers–Brillouin (WKB) approximation is employed by writing the field amplitude as [53]

$$A(r,z) = A_0(r,z)\exp[-ikS(r,z)], \tag{3.1.5}$$

where $A_0(r,z)$ and $S(r,z)$ denote the real amplitude and eikonal functions, respectively. The eikonal is chosen as

$$S(r,z) = S_0(r,z) + \frac{r^2}{2f}\frac{df}{dz}, \tag{3.1.6}$$

with f representing the beam-width parameter and S_0 representing the axial phase. Substitution of this form into the parabolic wave equation leads to a second-order nonlinear ordinary differential equation governing the evolution of $f(z)$, which serves as the central equation for analyzing self-focusing

$$2\left(\frac{\partial S}{\partial z}\right) + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) + \frac{\Phi(\mathbf{E}.\mathbf{E}^*)}{\epsilon_0}. \tag{3.1.7}$$

Now, substituting the eikonal function from Eq. (3.1.6) into Eq. (3.1.7), we get,

$$\frac{r^2}{f^2}\left(\frac{d^2 f}{dz^2}\right) = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\left(\frac{\partial A_0}{\partial r}\right)\right) + \frac{\Phi(\mathbf{E}.\mathbf{E}^*)}{\epsilon_0}. \tag{3.1.8}$$

For thermal quantum plasma, the dielectric function is modeled as [50, 51, 54–57]

$$\epsilon = 1 - \frac{\omega_p^2}{\gamma\omega^2}\left[1 - \frac{\delta}{\gamma} - \beta\right]^{-1}, \tag{3.1.9}$$

where $\beta = k^2 v_F^2/\omega^2$ represents the contribution of Fermi pressure, $\delta = \overline{h}^2 k^4/4m_e^2\omega^2$ accounts for the Bohm potential, and γ is the relativistic factor. For an exponential density ramp, the plasma density is taken as $n = n_0 \exp(\xi/d)$. In the cold quantum limit, $v_F \rightarrow 0$, and therefore $\beta \rightarrow 0$. Now, if an external magnetic field is applied axially to the thermal quantum plasma, the relativistic factor gets modified to [49]

$$\gamma = \left[1 + \frac{\alpha\mathbf{E}.\mathbf{E}^*}{\left(1 - \frac{\omega_c}{\gamma_0\omega}\right)^2}\right]^{1/2}, \tag{3.1.10}$$

where $\gamma_0 = \sqrt{1+a^2}$ with $a^2 = e^2A_0^2/(m_e^2c^2\omega^2)$ being the normalized vector potential, and $\omega_c = eB_s/(m_e c)$ being the electron cyclotron frequency. Hence, the final relativistic factor comes out to be

$$\gamma = \left[1 + a^2 + 2a^2\frac{\omega_c}{\omega}\left(\frac{1}{\sqrt{1+a^2}}\right) + 3a^2\frac{\omega_c^2}{\omega^2}\left(\frac{1}{1+a^2}\right)\right]^{1/2}, \tag{3.1.11}$$

Now, in the dielectric function in Eq. (3.1.9), the linear term of can be separated as

$$\epsilon_0 = 1 - \frac{\omega_p^2}{\omega^2}, \tag{3.1.12}$$

and the nonlinear term as

$$\Phi(\mathbf{E}.\mathbf{E}^*) = \frac{\omega_p^2}{\omega^2}\left[1 - \frac{1}{\gamma}\left(1 - \frac{\delta}{\gamma} - \beta\right)^{-1}\right]. \tag{3.1.13}$$

Now, according to [35], the nonlinear dielectric function can also be obtained by partially differentiating the total dielectric function with respect to r^2 at the value of $r = 0$, i.e.,

$$\Phi(\mathbf{E}.\mathbf{E}^*) = \left[\frac{\partial\epsilon}{\partial r^2}\right]_{r=0} \tag{3.1.14}$$

This formulation provides the theoretical foundation for the present thesis. By incorporating relativistic effects, quantum corrections, density inhomogeneity, and external magnetic fields into the nonlinear dielectric response, the resulting beam-width equation enables a comprehensive analytical and numerical investigation of the self-focusing behavior of both Bessel-Gaussian and Elliptical-Gaussian laser beams in magnetized quantum plasma.

Paper-1

For a Bessel-Gaussian beam, the transverse intensity distribution given in Eq. (1.4.9) must be modified to account for changes in beam width during propagation. Introducing the beam-width parameter $f(\xi)$, the field amplitude can be written as

$$A_0^2 = \frac{A_{00}^2}{f^2}\left[J_v\left(\frac{\mu r}{\sqrt{2}r_0 f}\right)\right]^2 \exp\left(\frac{-r^2}{r_0^2 f^2}\right), \tag{3.1.15}$$

26 where f represents the normalized beam width, μ is the transverse wave parameter, and J_v is the Bessel function of the first kind of order v. For the lowest-order case, the Bessel function may be expanded near the beam axis, yielding

$$A_0^2 = \frac{A_{00}^2}{f^2}\left[1 - \frac{\mu^2 r^2}{8 r_0^2 f^2}\right]^2 \exp\left(\frac{-r^2}{r_0^2 f^2}\right). \tag{3.1.16}$$

This expression provides a convenient form for evaluating the nonlinear dielectric response in the vicinity of the beam axis. Substituting the magnetically modified relativistic factor from Eq. (3.1.11) into the dielectric function of Eq. (3.1.9), and applying the derivative relation in Eq. (3.1.14) yields the nonlinear contribution

$$\begin{aligned}\Phi(\mathbf{E}.\mathbf{E}^*) \;=\; & \frac{\partial}{\partial r^2}\left[1 - \frac{\omega_p^2}{\omega^2}\left(1 + a^2 + 2a^2\frac{\omega_c}{\omega}\frac{1}{\sqrt{1+a^2}} + 3a^2\frac{\omega_c^2}{\omega^2}\frac{1}{(1+a^2)}\right)^{-1/2}\right. \\ & \left.\times\left(1 - \delta\left(1 + a^2 + 2a^2\frac{\omega_c}{\omega}\frac{1}{\sqrt{1+a^2}} + 3a^2\frac{\omega_c^2}{\omega^2}\frac{1}{(1+a^2)}\right)^{-1/2} - \beta\right)^{-1}\right],\end{aligned} \tag{3.1.17}$$

which, after simplifying, comes out to be

$$\begin{aligned}\Phi(\mathbf{E}.\mathbf{E}^*) \;=\; & -\frac{\omega_p^2 P_0}{2\omega^2 r_0^2 f^4}\left(1 + \frac{\mu^2}{4}\right)\left[\gamma_{r=0}\left(1 - \frac{\delta}{\gamma_{r=0}} - \beta\right)^{-1}\right. \\ & \times\left[1 + \frac{\delta}{\gamma_{r=0}}\left(1 - \frac{\delta}{\gamma_{r=0}} - \beta\right)\right] \\ & \left.\times\left(1 + \frac{2\omega_c}{\omega}\left(\frac{1}{\sqrt{1+a^2}}\right) - \frac{3\omega_c^2}{\omega^2}\left(\frac{1}{\sqrt{1+a^2}}\right)^2\right)\right],\end{aligned} \tag{3.1.18}$$

where

$$\gamma_{r=0} = \left[1 + \frac{P_0}{f^2} + 2\frac{P_0\omega_c}{\omega f^2}\left(\frac{1}{\sqrt{1+P_0/f^2}}\right) + 3\frac{P_0\omega_c^2}{\omega^2 f^2}\left(\frac{1}{\sqrt{1+P_0/f^2}}\right)\right]^{-1/2}, \tag{3.1.19}$$

and $P_0 = \alpha\mathbf{E}.\mathbf{E}^*$ is the dimensionless intensity parameter, and we have also ignored the higher-
48 order terms in the binomial expansion. After simplification and neglecting higher-order terms in the binomial expansion, the nonlinear dielectric term is obtained in the compact form of Eq. (3.1.4), which incorporates the effects of quantum corrections, relativistic mass variation, the external magnetic field, and the transverse beam parameter μ.

Finally, substituting Eqs. (1.4.9), (3.1.12), and (3.1.18) into Eq. (3.1.8) and equating the coefficients of r^2 leads to the beam-width evolution equation

$$\begin{aligned}\frac{d^2 f}{d\xi^2} &= \frac{(\mu^2+2)}{2f^3} - \frac{\rho_0^2 P_0}{2f^3}\left(1+\frac{\mu^2}{4}\right)\gamma_{r=0}^{-3}\left[1-\frac{\delta}{\gamma_{r=0}}-\beta\right]^{-1} \\ &\times\left[1+\frac{\delta}{\gamma_{r=0}\left(1-\frac{\delta}{\gamma_{r=0}}-\beta\right)}\right] \\ &\times\left[1+\frac{2\omega_c}{\omega}\left(\frac{1}{\sqrt{1+P_0/f^2}}\right)-\frac{3\omega_c^2}{\omega^2}\left(\frac{1}{\sqrt{1+P_0/f^2}}\right)^2\right],\end{aligned} \tag{3.1.20}$$

where $\rho_0 = r_0\omega_p/c$, $\xi = z/R_d$ is the normalized propagation distance, and $R_d = kr_0^2$ is the Rayleigh length.

Equation (3.1.20) is the governing equation for the evolution of the normalized beam width of a lowest-order Bessel-Gaussian beam propagating through an axially magnetized thermal quantum plasma with an exponential density ramp. The first term on the right-hand side represents diffraction, which tends to broaden the beam, while the second term describes the nonlinear self-focusing contribution arising from relativistic, quantum, and magnetic effects. The balance between these terms determines whether the beam diverges, remains stable, or undergoes self-focusing during propagation.

Paper-2

The second stage of the present thesis work is underway with the theoretical model formulation nearing completion and the numerical analysis in progress. The main subject of this study is the propagation and relativistic self-focusing of elliptical-Gaussian laser beams in magnetized thermal quantum plasma with a preformed tangential density ramp.

Specifically, the elliptical-Gaussian beams have attracted much attention owing to their different beam widths along two mutually perpendicular directions which may induce anisotropic self-focusing properties and thereby provide for more intense beam compression. The proposed 33 theoretical model is based on the presence of an external static magnetic field along the beam 34 propagation axis. The introduction of the magnetic field is due to the additional self-focusing provided by the enhanced electron confinement and increased nonlinearity of the refractive index of the plasma. Another important ingredient of the model is the presence of a preformed tangential density ramp in the medium which leads to further self-focusing by means of gradual increase of the electron density along the propagation axis.

With the increase of the electron density, the nonlinear interaction becomes stronger providing for the increase of beam convergence and intensity amplification.

3.2 Numerical Analysis

42 For the numerical analysis of the first study, Eq. (3.1.20) is solved using the fourth-order Runge-2 Kutta method in MATLAB, subject to the initial conditions $f = 1$ and $df/dz = 0$ at $z = 0$. These conditions correspond to a beam entering the plasma with its initial waist and no initial convergence or divergence.

An Nd:YAG laser with wavelength $\lambda = 1064nm$ is used as the incident source. The simulations are carried out for transverse wave parameter values $\mu = 0.5, 1, 1.5, 2, 2.5$, exponential density ramp parameters $d = 5, 10, 20$, laser intensities $P_0 = (0.1, 0.2, 0.3, 0.4) \times 1.2 \times 10^{18}, Wcm^{-2}$, an external magnetic field characterized by $\omega_c/\omega = 0.3$, and a Fermi temperature of $T_F = 10^9 K$. The resulting beam-width profiles are plotted in MATLAB to examine the influence of each parameter on the self-focusing dynamics and to identify the conditions that produce the strongest beam compression.

3.2.1 MATLAB Codes

The MATLAB code that has been used to plot the respective graphs is illustrated in the following figures.

```
function dy = rigid(t,y)
c=3*10^10;
TF=10^9;
% TF=1
w0=1.78*10^20;
wwc=0
wc=wwc*w0;
% wwc=wc/w0;
lambada=2*pi*c/w0;
p0=.3;
r0=20*10^-4;
d=10;
h=6.626*10^-27;
% h=0;
kB=1.38*10^-16;
m=9.1*10^-28;
e=4.8*10^-10;
n0=4*10^19;
ne=n0*exp(t/d);
delta=(4*h^2*pi^4/(m*w0*lambada^2))^2;
wp=(4*pi*e^2*ne/m)^.5;
e0=1-(wp/w0)^2;
k=e0^.5*w0/c;
Rd=k*r0^2;
wp0=(4*pi*e^2*n0/m)^.5;
rrr=wp0*r0/c;
www=w0/wp0;
mu=1
vF=(2*kB*TF/m)^.5;
beta=(k*vF/w0)^2
```

Figure 3.1: Preamble code

```
delta=(2*pi^2*h/(m*w0*lambada^2))^2
ro0=(r0*wp/c);

dy = zeros(2,1)

dy(1)=y(2);
gama0=(1+p0/y(1)^2+2*p0*wwc*(1+p0/y(1)^2)^-0.5/y(1)^2+3*p0*wwc^2*(1+p0/y(1)^2)^-1/y(1)^2);

se1=-ro0^2*p0*(1+mu^2/4)*gama0^-3/(2*y(1)^3);
se2=(1-delta/gama0-beta)^-1;
se3=1+delta*(1-delta/gama0-beta)^-1/gama0;
se4=1+2*wwc*(1+p0/y(1)^2)^-0.5-3*wwc^2*(1+p0/y(1)^2)^-1;
se=se1*se2*se3*se4;
dy(2)=(mu^2+2)/(2*y(1)^3)+se;
```

Figure 3.2: Preamble code (cont.)

```
options = odeset('RelTol',1e-17,'AbsTol',[1e-17 1e-17]);
[T,Y] = ode45(@rigid,[0 1],[1 0],options);
plot(T,Y(:,1))

hold on
```

Figure 3.3: Run code

Chapter 4

RESULTS and DISCUSSION

Using the parameter values from the previous section (3), MATLAB was used to plot graphs of Eq. (3.1.20) using numerical analysis.

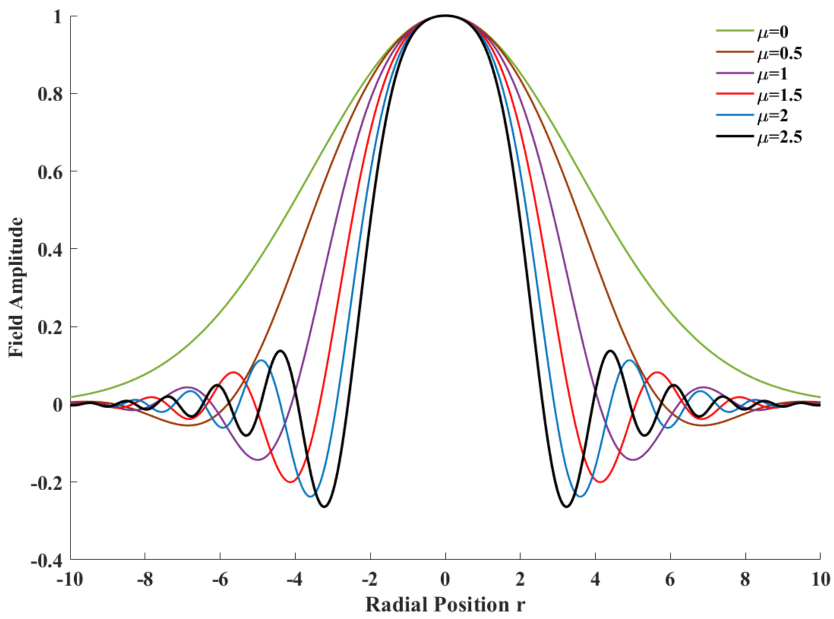


Figure 4.1: Intensity variation of Bessel-Gaussian laser beam with different values of μ

From Fig. (4.1), it is evident that the transverse wave parameter μ has a significant influence on the radial intensity distribution of the Bessel-Gaussian beam. With an increase in μ, the main peak sharpens and narrows, whereas a series of concentric side lobes (vortices) is formed around the beam. This suggests that the increase in μ results in higher beam energy localization at the beam center, which in turn contributes to an increased self-focusing effect of the laser radiation propagating through the plasma medium.

If $\mu = 0$, then the Bessel function takes on the value of unity, thus making the intensity distribution coincident with the distribution for the ordinary Gaussian beam. It is worth mentioning that in the current case, the paraxial approximation is still appropriate due to the fact that we are primarily concerned with the central peak, where all the beam energy is located.

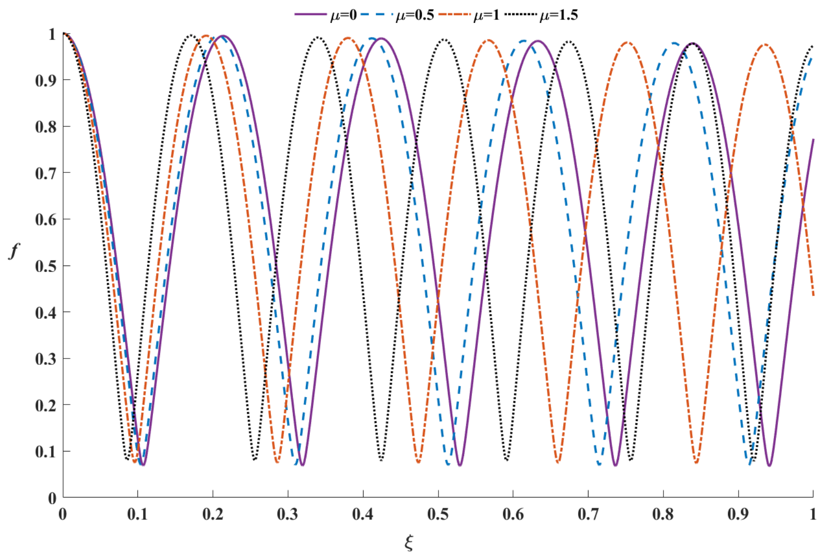


Figure 4.2: Effect of different values of transverse constant of the wave parameter on the beam width evolution

5 Fig. (4.2) illustrates the dependence of the normalized beam width on propagation distance with respect to various values of the wave transverse parameter μ, when the remaining parameters are kept constant ($d = 10$, $\omega_c/\omega = 0.3$, and $P_0 = 0.3$). As observed from the plot, an increase in the parameter μ results in faster onset of self-focusing, accompanied by a reduction in the minimum beam width.

These findings could be understood from the facts that a higher value of the parameter μ leads to sharper peaks and rings in the profile of the Bessel-Gaussian beam, which causes greater ponderomotive force around the beam center, resulting in a higher refractive index gradient and, thus, an enhancement in the nonlinearity induced focusing. The findings prove that self-focusing of the Bessel-Gaussian beams is much faster compared with the Gaussian beams under similar plasma and magnetic field environments.

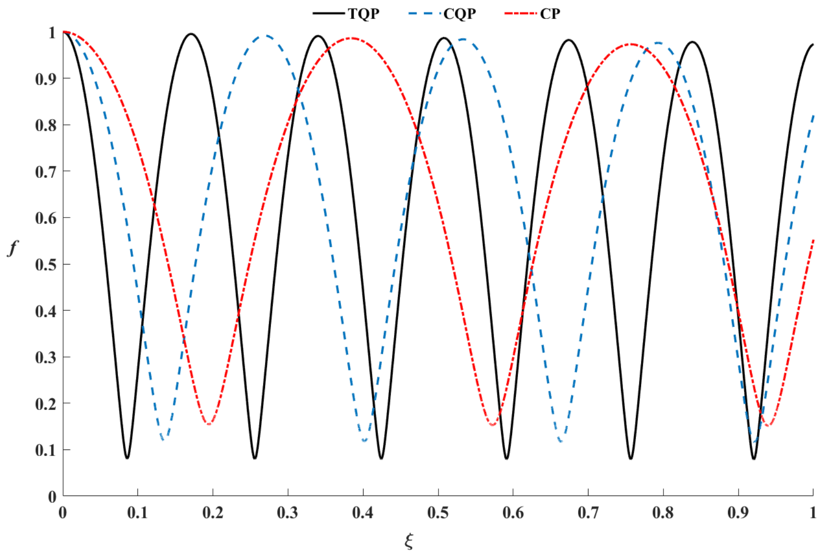
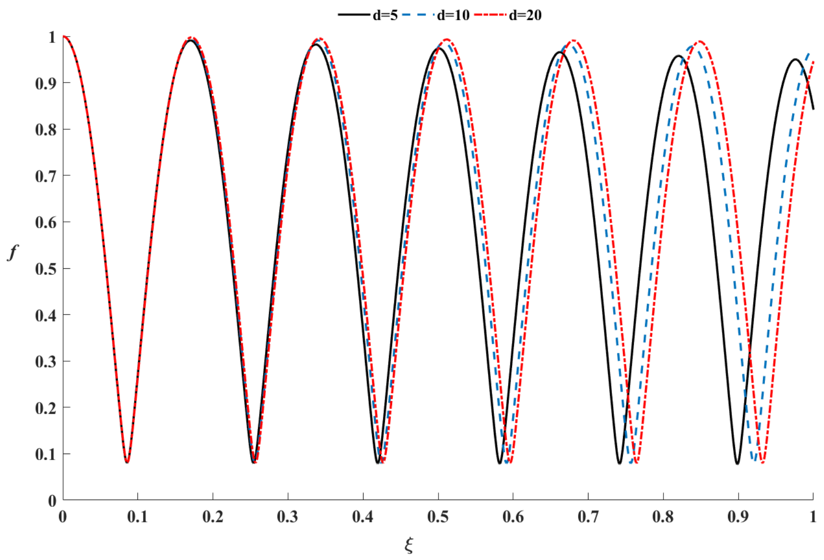


Figure 4.3: Evolution of the beam width parameter f as a function of the propagation distance parameter ξ for classical, thermal quantum, and cold quantum plasma

Fig. (4.3) illustrates the evolution of the normalized beam width in the classical, cold quantum, and thermal quantum plasma environments. Here, the values of the parameters are considered to be $d = 10$, $\mu = 1.5$, $P_0 = 0.3$, and $\omega_c/\omega = 0.3$. It can clearly be seen from the results shown in Fig. (4.3) that in terms of self-focusing capability, the thermal quantum plasma offers the maximum self-focusing with the minimum value of the beam width and the largest propagation distance.

As compared to the classical and the cold quantum plasmas, self-focusing effect in the case of thermal quantum plasma is the highest, whereas, it is relatively weaker in the case of the cold quantum plasma as compared to the classical one. On the other hand, the classical plasma offers the minimum self-focusing capability due to larger oscillations of the beam width. This study has proven that quantum mechanical effects, like Fermi pressure, Bohm potential, and exchange correlation effects, enhance the nonlinearity in the plasma.



2 Figure 4.4: Variation of the beam width parameter f as a function of the propagation distance parameter ξ for different density ramp constants

The effects of the exponential density ramping constant on the self-focusing process of the Bessel-Gaussian beam are demonstrated in Fig. (4.4), where other constants are considered fixed values such as $\omega_c/\omega = 0.3$, $\mu = 1.5$, and $P_0 = 0.3$. As seen from the figure, the decrease in the value of the ramp constant (d) leads to a shorter time interval for self-focusing to take place and also a smaller minimal beam radius.

This effect is due to the fact that, as the laser propagates through the medium, it sees an increasing electron density, making the nonlinear refraction phenomena become stronger and hence focusing the laser beam more effectively. This effect plays a crucial role especially when using Bessel-Gaussian beams due to their long propagation distance. By accelerating the focusing process, the exponential density ramp enables the beam to maintain high intensity over a substantially greater distance.

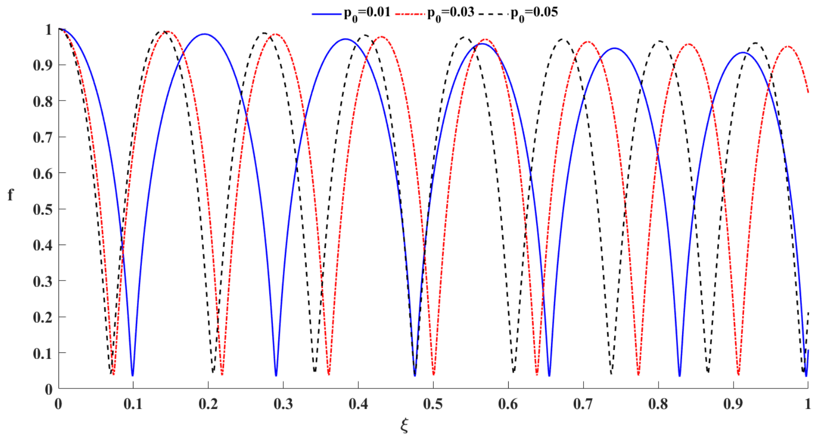


Figure 4.5: Evolution of beam width through propagation for different values of laser beam intensity parameters

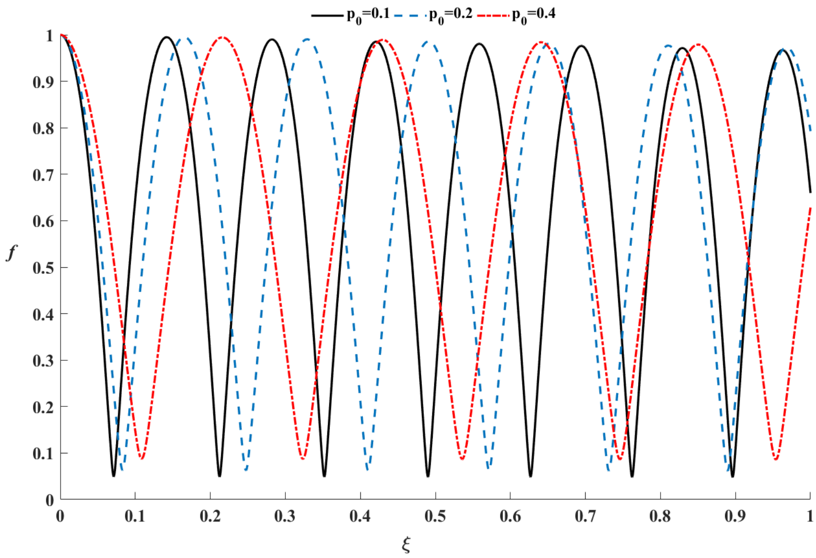


Figure 4.6: Evolution of beam width through propagation for different values of laser beam intensity parameters

Here, Fig. (4.5) and (4.6) illustrate the influence of the initial laser intensity parameter P_0 on
10
the self-focusing behavior of the Bessel-Gaussian beam. Figure (4.5) corresponds to the low-to-moderate intensity regime, with $P_0 = 0.01, 0.03$, and 0.05. In this range, increasing the laser intensity enhances the nonlinear refractive response of the plasma, causing the beam to self-focus more rapidly and attain a smaller minimum beam width.

In contrast, Fig. (4.6) examines the ultra-relativistic intensity regime with $P_0 = 0.1, 0.2$, and 0.4. In this case, a further increase in laser intensity leads to weaker and slower self-focusing. This behavior is attributed to relativistic saturation, wherein the relativistic factor γ becomes sufficiently large that the nonlinear dielectric response approaches a limiting value. Consequently, additional increases in intensity produce only marginal changes in the refractive index. At these extreme intensities, the highly energetic oscillatory motion of plasma electrons also introduces stronger radial dynamics, which partially counteract the focusing effect and reduce the overall beam compression.

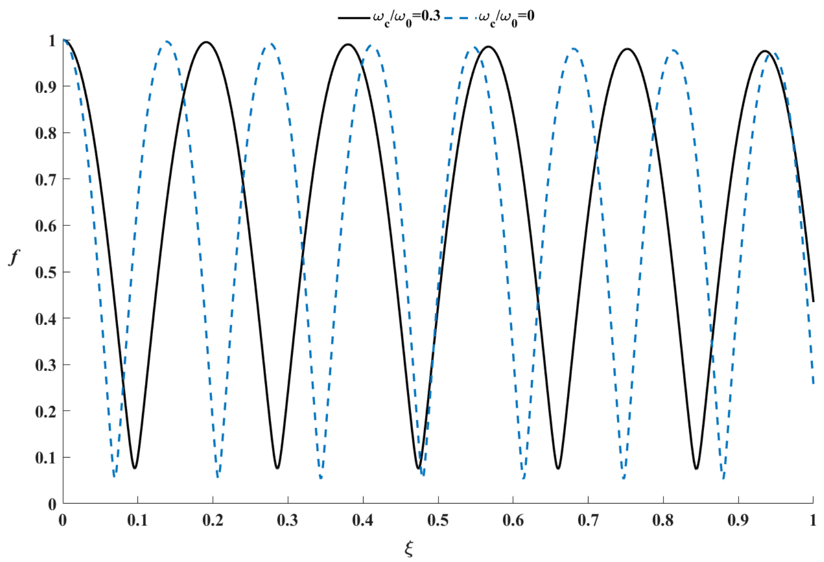


Figure 4.7: Comparison between self-focusing with magnetic field and density variation, and self-focusing without magnetic field and density variation

Fig. (4.7) shows one of the important conclusions obtained from this investigation. It can be seen that due to the existence of an external magnetic field there is a considerable decrease in both self-focusing time and self-focusing degree. In comparison with a situation without magnetic fields, in their presence, the minimal radius of the beam occurs at a larger propagation distance and has a smaller degree of contraction.

The reason for such a phenomenon lies in the fact that the action of an external magnetic field causes electrons in plasma to move according to the Lorentz force law. The new motion of these electrons impacts their relativistic oscillations under influence of the laser pulse and consequently modifies the dependence between the refraction index and the intensity of the beam.

A detailed analysis of all numerical results has now been completed, and the observed trends are found to be in strong agreement with the predictions of the theoretical model. The findings of this study have been compiled into a research manuscript and submitted to a peer-reviewed journal of international repute in the field of optics for publication.

Chapter 5

CONCLUSION

5.1 Conclusion and Current Work Status

So far, I have completed one research paper, titled **”Relativistic Self-Focusing of Bessel-Gaussian Laser Beam in Magnetized Thermal Quantum Plasma under Exponential Density Gradient”**, and I am currently working on a second paper, **”Self-Focusing of Elliptical-Gaussian Laser Beam in Quantum Plasma”**. These projects were initiated during my 2^{nd} and 3^{rd} semesters, respectively. Together, they have involved an extensive literature review of more than 50 research articles, analytical derivations, numerical simulations in MATLAB, and the preparation of a scientific manuscript.

- **Self-Focusing of Bessel-Gaussian Laser Beam in Quantum Plasma:** My first study investigates the relativistic self-focusing of a lowest-order Bessel-Gaussian laser beam propagating through magnetized thermal quantum plasma in the presence of an exponential density ramp. In the theoretical model, the effects of Bohm potential, exchange correlation, relativistic ponderomotive nonlinearity, and axial magnetic field have been included. Post obtaining the beam width equation, numerical simulation analysis was done using fourth order Runge-Kutta scheme in MATLAB for the variation in some key parameters like laser intensity, magnetic field intensity, density ramp scale length, and transverse wave parameter μ. It has been found that the effect of thermal quantum plasma on self-focusing is far more superior than that of cold quantum and classical plasma. With the help of exponential density ramp, the beam is confined to a certain region which enables it to remain in its high intensity form through many Rayleigh lengths whereas the Bessel Gaussian profile helps it focus strongly owing to its intense ring shaped form and larger beam width. This research work has already been completed and a paper has been submitted to ***Journal of Optics***, ***Springer Nature*** with manuscript ID **OPTI-D-26-00232**.

- **Self-Focusing of Elliptical-Gaussian Laser Beam in Quantum Plasma:** The second investigation is dedicated to relativistic self-focusing of elliptical Gaussian laser beams in a magnetized thermal quantum plasma with a tangential density gradient. In this research, we try to understand the impact of asymmetry of the beam, quantum effects, magnetization, and plasma non-uniformity on beam self-compression in an anisotropic manner. Most of the theoretical part is already done and, specifically, the beam-width equations in two different transverse directions are derived.

  Simulation studies will be performed soon to analyze the effect of the ellipticity of the beam, density gradient characteristics, laser beam intensity, and magnetic field strength.

5.2 Scope of Future Work

The current work can be further expanded by accounting for extra physical effects not considered in this work, including collisions, ionization, collisional heating, and hydrodynamic expansion. Future research will examine the effect of time-varying or inhomogeneous magnetic field distributions, as well as other plasma density configurations, such as two- or three-dimensional density profiles and pre-formed channels of plasma density. These modifications will lead to a more accurate model of the laser-plasma interaction in experimental setups.

Moreover, one should explore other types of structured beams, such as Laguerre-Gaussian, Hermite-Gaussian, or Airy beams, based on the methodology proposed in this thesis. The effect of nonparaxiality or vectoriality can also be accounted for in the case of ultra-short pulses. Particle-in-cell (PIC) modeling and experimental tests are needed to validate the results obtained and improve beam propagation optimization in such areas as particle acceleration and inertial confinement fusion.