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THE COMPOSITION DISTORTION IN ADVERTISING COMPETITION: A FREE-RIDER THEORY OF PERSUASIVE ADVERTISING

A Dissertation submitted
in partial fulfilment of the requirements for the award of the degree of

MASTER OF SCIENCE
in
APPLIED MATHEMATICS

Submitted by

DEBASISH SARMA
(Roll No. 24/MSCMAT/16)

Under the supervision of

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CANDIDATE'S DECLARATION

I, **Debasish Sarma**, Roll No. 24/MSCMAT/16, student of M.Sc. Applied Mathematics, hereby declare that the dissertation titled *"The Composition Distortion in Advertising Competition: A Free-Rider Theory of Persuasive Advertising"*, submitted to the Department of Applied Mathematics, Delhi Technological University, in partial fulfilment of the requirements for the award of the degree of Master of Science, is original and has not been submitted previously, in part or full, to this or any other university for the award of any degree.

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CERTIFICATE BY THE SUPERVISOR

This is to certify that the dissertation titled *"The Composition Distortion in Advertising Competition: A Free-Rider Theory of Persuasive Advertising"* submitted by **Debasish Sarma** (Roll No. 24/MSCMAT/16) to the Department of Applied Mathematics, Delhi Technological University, in partial fulfilment of the requirements for the award of the degree of Master of Science in Applied Mathematics, is a record of bona fide research work carried out by him under my supervision. To the best of my knowledge, the contents of this dissertation, in full or in parts, have not been submitted to any other institute or university for the award of any degree or diploma.

Place: Delhi
Date: May 19, 2026
Department of Applied Mathematics
Delhi Technological University

Dr. Anjana Gupta
Professor

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Several classmates and friends read parts of this work and pushed back wherever the prose was vague or the algebra was moving too fast. Their questions improved the thesis more than I can quite attribute properly. Any errors that remain are, of course, mine.

To my family, finally: the support you have given through this programme had very little to do with advertising contests, but mattered more than I have managed to say. Thank you.

Debasish Sarma

ABSTRACT

This thesis studies a theoretical problem in advertising competition: firms do not merely choose how much to advertise; they also choose what kind of advertising to produce. Traditional contest models treat advertising as persuasive and redistributive, where firms fight for a fixed market and symmetric efforts cancel out. The informative view of advertising, in contrast, treats advertising as a mechanism that informs consumers and expands total market demand. In real markets, however, advertisements usually contain both components. A pharmaceutical advertisement may inform patients about a treatment and its risks while simultaneously persuading them to prefer one brand. A search advertisement may match high-intent users to a product while also competing against rival advertisers. The central question of this thesis is therefore: what happens when firms endogenously choose both advertising intensity and advertising composition?

The thesis develops a duopoly advertising contest in which each firm chooses advertising intensity and the fraction of its advertising that is informative. The persuasive component shifts market share in a zero-sum contest, while the informative component expands total demand as a positive-sum spillover. The main result is that, under a broad and economically meaningful parameter range, private equilibrium advertising is biased toward persuasive content. The reason is a free-rider problem: a firm captures the full private benefit of persuasive advertising through market-share gains, but it captures only part of the benefit of informative advertising because demand expansion also helps competitors. This creates a composition distortion: firms may spend a large amount on advertising, but the composition of that spending is socially inefficient.

The thesis distinguishes this composition distortion from ordinary rent dissipation and over-advertising. It then studies a policy instrument: a minimum informative-content mandate. The analysis shows that such a mandate can improve welfare by redirecting competitive pressure from share-stealing persuasion toward socially valuable information. The optimal mandate is not necessarily full information. If persuasive incentives are eliminated completely, firms may reduce advertising intensity so much that the total information delivered to consumers falls. The welfare-maximising policy is therefore an interior composition mandate that harnesses competition rather than eliminating it.

The thesis further extends the model to an N-firm setting, showing that the free-rider problem becomes stronger as the number of firms increases because each firm captures a smaller fraction of the market-expansion benefit. It also discusses robustness under a ratio-form contest success function, provides numerical simulations, and connects the theory to pharmaceutical advertising regulation, FDA fair-balance requirements, FTC substantiation rules, and EU unfair commercial practices law. The contribution of the thesis is to show that the policy problem in advertising competition is not always excessive quantity alone. Sometimes the deeper market failure is that firms produce the wrong composition of advertising.

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LIST OF SYMBOLS, ABBREVIATIONS AND NOMENCLATURE

List of Symbols

Symbol	Description
x_i	Advertising intensity of firm i, $x_i \in [0, 1]$
ϕ_i	Informative fraction of firm i, $\phi_i \in [0, 1]$
$(1 - \phi_i)x_i$	Persuasive component of firm i's advertising
$\phi_i x_i$	Informative component of firm i's advertising
s_i	Market share of firm i
R_0	Baseline market revenue (no advertising)
θ	Demand-expansion coefficient
α	Competitive intensity in the linear CSF
γ	Decisiveness parameter in the ratio-form (Tullock) CSF
c	Quadratic cost coefficient for advertising
λ	Consumer-surplus ratio (consumer surplus per unit producer surplus)
N	Number of competing firms
π_i	Profit of firm i
W	Total social welfare
W_0	No-advertising welfare benchmark
θ^*	Welfare-neutral demand-expansion threshold
ϕ^*	Equilibrium informative fraction (private)
ϕ^{opt}	Socially optimal informative fraction
$\phi_{\min}$	Regulatory minimum informative-content mandate
$\phi^*_{\min}$	Welfare-maximising mandate

List of Abbreviations

Abbreviation	Description
CSF	Contest Success Function
FOC	First-Order Condition
NE	Nash Equilibrium
DTC	Direct-to-Consumer (advertising)
FDA	Food and Drug Administration (U.S.)
FTC	Federal Trade Commission (U.S.)
EU	European Union
UCPD	Unfair Commercial Practices Directive (EU)
JEL	Journal of Economic Literature (classification)
IO	Industrial Organisation
SPNE	Subgame Perfect Nash Equilibrium

CHAPTER 1

INTRODUCTION AND REVIEW OF LITERATURE

1.1 Background: The Two Faces of Advertising

Advertising has long occupied an uneasy position in economic theory. One line of thought, going back to Marshall and later formalised by Schmalensee [18] and Tullock [23], treats competitive advertising as a contest over a largely fixed body of consumers. A firm's expenditure pulls some attention toward itself; the rival's expenditure pulls in the opposite direction. In a symmetric equilibrium these pulls largely offset one another, while the expenditures remain real costs. On this reading, advertising is a form of rent-seeking: socially costly competition for a given market rather than an activity that expands the market itself.

A second line of work begins with Stigler's economics of information [22] and is developed further by Nelson [16]. In this view, advertising helps consumers discover products, prices, attributes, and uses that they may not otherwise know about. Ackerberg [1] showed empirically that advertising for experience goods can carry information rather than only prestige or persuasion, and Sinkinson and Starc [20] found related effects in direct-to-consumer pharmaceutical advertising. From this perspective, a ban on advertising may remove a useful information channel rather than merely stop wasteful spending.

These two views have coexisted for five decades without formal reconciliation. The contest literature assumes advertising is purely redistributive; the informative literature assumes it is purely expansionary. Real advertising, of course, is rarely either pure type. A pharmaceutical television advertisement simultaneously informs patients about a treatment option (expanding the market for that therapeutic class) and persuades them to choose one brand over its therapeutic substitutes (stealing share). A sponsored search advertisement matches a high-intent user to a relevant product (informative) while competing for that user's click against rival advertisers (persuasive). The welfare implications of advertising competition therefore depend critically on the *mix* of these two functions—a dimension the literature has not formalised within a contest framework.

1.2 Persuasive versus Informative Advertising

It is helpful at the outset to fix the two ideal types. *Persuasive* advertising, in the sense used throughout this thesis, is content that shifts a consumer's brand preference or perceived utility without altering the consumer's information set in any verifiable way. Image-based brand advertising for soft drinks, sports apparel, and luxury goods sits squarely in this category: the consumer learns nothing checkable about the product, yet acquires a more favourable attitude toward it.

By *informative* advertising I mean advertising that gives the consumer some verifiable content. Prices, dates, ingredients, technical specifications, warranty terms, and side-effect profiles all fall in this category because, at least in principle, the consumer

can check them independently. The fair-balance segment of a pharmaceutical commercial, the specification panel in an electronics advertisement, and a sponsored search listing tied to a specific query are useful examples of this type of content.

The distinction matters, but the dichotomy is rarely clean in practice. Almost all real advertisements combine the two. Our framework picks up that fact by treating $\phi \in [0, 1]$ as a continuous parameter measuring the informative fraction of an advertisement's effort. So $\phi = 0$ is purely persuasive content, $\phi = 1$ is purely informative, and in between sits what is empirically the common case, a mixed campaign.

1.3 Why Traditional Models Are Incomplete

In the standard contest formulation [23, 21, 15], advertising effort is represented by a single scalar. The firm chooses that effort level, the effort enters a contest success function, and the equilibrium and welfare comparison follow from the resulting first-order condition. This is analytically clean, but it removes an important decision that real advertisers make: the choice of what the advertising actually contains.

The Dorfman–Steiner [8] tradition, by contrast, treats advertising as a demand-shifter and is essentially silent on competition: it solves a monopolist's static problem or, at best, a partial-equilibrium duopoly without strategic interdependence in advertising. The Bagwell [2] survey integrates the two threads but does not formalise their joint mathematical treatment.

The gap that motivates this thesis is the following. A firm choosing an advertising campaign does not, in practice, choose only how much to spend. It chooses what kind of campaign to run. A campaign that emphasises factual product attributes will expand the relevant category demand by educating non-buyers; a campaign emphasising emotive branding will shift share within an existing demand pool. Both options are available; the firm picks one (or some mix) on the basis of expected profit, and rival firms do the same in parallel. Neither the pure-contest nor the pure-informative tradition has the apparatus to model this choice.

1.4 Literature Review

This chapter surveys four strands of literature that bear directly on the thesis: (i) advertising as persuasion and rent-seeking, (ii) advertising as information, (iii) regulation of advertising, and (iv) the contest-theoretic apparatus on which the formal model rests. The fifth section identifies the gap addressed by the present work.

1.4.1 Advertising as Persuasion and Rent-Seeking

The idea that some advertising is mainly redistributive is older than formal contest theory. Marshall distinguished between "constructive" and "combative" advertising, where the latter simply moves custom from one seller to another without enlarging the market. Kaldor [13] later gave this criticism a sharper industrial-organisation form, arguing that persuasive advertising can create entry barriers and weaken consumer sovereignty by shaping preferences rather than informing them. Galbraith [10] extended the criticism further, portraying modern industry as a system that helps create the wants it then claims to satisfy.

Tullock [23] supplied the formal language for this view. In a rent-seeking contest, agents spend resources to win a fixed prize, and symmetric equilibrium effort can dissipate a substantial part of that prize. Interpreted as advertising, the prize is consumer

attention or market share, and the contestants are the firms. Schmalensee's [18] monograph provides the advertising background against which this contest interpretation fits naturally.

The arms-race intuition that comes out of this literature is sharp. With a fixed market and symmetric firms, advertising at the equilibrium leaves market shares essentially unchanged. The costs, though, are real. The wedge between equilibrium and cooperative outcomes is the rent dissipation, and it can be substantial. Skaperdas [21] provided the axiomatic foundation of contest success functions, characterising those satisfying anonymity, monotonicity, and homogeneity properties. Konrad's [15] monograph synthesises the contest literature and presents the welfare comparisons that motivate the regulatory recommendations in the rent-seeking tradition.

Within this tradition, the recommended policy response is to restrict the arms race directly. Advertising taxes, spending caps, and outright bans (in particularly egregious cases, such as television tobacco advertising) all feature in the policy menu. The framework does not, by its own structure, accommodate the possibility that some advertising is welfare-enhancing; that channel is suppressed by assumption.

1.4.2 Advertising as Information

The competing tradition, in which advertising is treated as informative, finds its modern foundation in Stigler's [22] economics of information. Stigler argued that price dispersion in apparently homogeneous markets reflected costly consumer search, and that advertising—by reducing search costs—was unambiguously productive. Nelson's [16] subsequent work distinguished search goods (whose attributes can be inspected before purchase) from experience goods (whose attributes are revealed only through use), and argued that advertising serves different informational functions for each. For experience goods, the very fact of heavy advertising can serve as a quality signal, since only high-quality firms can profitably amortise repeat purchases against advertising costs.

The empirical literature has generally vindicated the informative view in markets where consumers have measurable scope to learn. Ackerberg [1] estimated structural demand models on yoghurt data and was able to identify informative and prestige effects of advertising separately, finding that the informative effect dominates for new products and inexperienced consumers. Bagwell's [2] Handbook chapter is the standard reference and surveys the subsequent empirical literature; the verdict, on balance, is that advertising in most markets does some genuine information transmission.

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Particularly germane to our policy concerns is the literature on direct-to-consumer pharmaceutical advertising, which has been legal in the United States since 1997 and is empirically the most studied case of high-stakes consumer advertising. Sinkinson and Starc [20] document substantial market-expansion effects of pharmaceutical DTC advertising on the relevant therapeutic categories, alongside the more familiar within-category share shifts. Their structural estimates suggest that the informative share of pharmaceutical advertising is large and economically consequential.

Industries plausibly toward the informative end of the spectrum include pharmaceuticals (as above), consumer electronics (where advertisements list specifications), durable goods (where price and warranty terms are emphasised), and sponsored search advertising (where the very purpose of the ad is to match a query to a product). At the other end sit beverages, cosmetics, and luxury goods, where advertisements convey little verifiable content and rely overwhelmingly on associative imagery. The meta-analytic literature on advertising effectiveness, surveyed by Sethuraman, Tellis, and Briesch [19], suggests that the average own-advertising elasticity in mature product categories is modest, supporting the view that the demand-expansion benefit modelled below should be calibrated conservatively rather than aggressively.

1.4.3 Regulation of Advertising

Advertising regulation in practice spans a spectrum of instruments. At the most restrictive end are outright bans: tobacco advertising on broadcast media in most developed jurisdictions, prescription pharmaceutical DTC advertising in all jurisdictions except the United States and New Zealand, and certain categories of advertising directed at children. Spending caps have been used historically in legal-services advertising and in political campaign contexts (where the analogy to commercial advertising is imperfect but instructive). Excise taxes specifically on advertising have been proposed and occasionally implemented, although they remain rare in OECD jurisdictions.

The instruments most relevant to the thesis are content-based. The U.S. FDA's "fair balance" rule, codified at 21 CFR §202.1, requires that direct-to-consumer pharmaceutical advertisements present risk information with prominence comparable to the benefit claims. The FTC's substantiation doctrine, derived from a series of consent orders and reaffirmed in policy statements, requires that advertising claims be supported by competent and reliable evidence at the time the claim is made. The EU's Unfair Commercial Practices Directive (2005/29/EC) prohibits commercial practices that mislead consumers, either by affirmative misrepresentation or by omission of material information.

What these instruments share, and what makes them theoretically interesting, is that they do not restrict the quantity of advertising. They restrict its *content*, requiring that certain kinds of information be present (or that other kinds be absent) within whatever quantity firms choose to produce. The literature on the welfare effects of these instruments is largely empirical [4, 12] and does not, to our knowledge, contain a theoretical contest-based justification.

1.4.4 Contest Theory and the Mathematical Apparatus

The contest-theoretic apparatus used throughout this thesis traces to Tullock's [23] original formulation. A contest success function (CSF) maps a profile of efforts $(x_1, \ldots, x_N)$ to a probability vector $(s_1, \ldots, s_N)$ with $\sum_i s_i = 1$; firms simultaneously choose efforts, and the prize is allocated probabilistically. The Tullock ratio form, $s_i = x_i^\gamma / \sum_j x_j^\gamma$, is the canonical specification, with γ parameterising decisiveness. The linear specification $s_i = 1/2 + \alpha(x_i - x_j)$, used throughout the baseline model of this thesis, is a local approximation valid for small effort differences.

Skaperdas [21] characterised the class of CSFs satisfying anonymity, monotonicity in own effort, and a homogeneity condition; the Tullock form is the unique CSF satisfying these axioms together with a certain ratio property. Hirshleifer [11] compared ratio and difference forms (the latter being the basis of our linear specification) and emphasised the qualitative differences in equilibrium behaviour.

The Bertrand–Tullock literature [3] extends contest theory to allow all-pay auction structures and asymmetric players. While these refinements are important for some applications, the symmetric duopoly with quadratic costs studied here is closer to the original Tullock setting and isolates the composition mechanism without confounding it against other features.

For convergence and stability of best-response dynamics in contests, we draw on Fudenberg and Levine [9]. Best-response dynamics for Tullock contests with strictly concave payoffs converge globally to the unique symmetric equilibrium, a fact we invoke without re-proving in Chapter 4.

1.4.5 The Research Gap

Two summarising observations emerge from the literature survey. First, the contest tradition assumes purely persuasive (zero-sum) advertising; the informative tradition assumes purely expansionary advertising. The natural integration—a contest model in which firms choose both intensity *and* the informative/persuasive composition—has, to our knowledge, no prior treatment in the published literature. The closest precedent is Bagwell's [2] Handbook discussion of mixed advertising, which surveys the issue but does not provide a formal model.

Second, the literature on advertising regulation contains substantial work on quantity instruments (taxes, caps, bans) and substantial empirical work on content instruments (fair-balance, substantiation). But the formal theoretical justification of content instruments—in particular, the demonstration that they can implement welfare improvements that quantity instruments cannot achieve—has not been provided in a contest framework.

The present thesis addresses both gaps. The model of Chapter 2 integrates intensity and composition; the corner-equilibrium result of Chapter 3 identifies the new market failure; the interior-optimal-mandate result of Chapter 3 provides the theoretical justification of content regulation. The remainder of the thesis extends and operationalises these results.

CHAPTER 2

OBJECTIVES AND MATHEMATICAL MODEL

2.1 Research Problem and Research Questions

We are now in a position to state the question that organises the thesis:

> *What happens to advertising competition, market outcomes, and social welfare when firms choose not only how much to advertise but also what type of advertising to produce?*

Five subsidiary questions structure the analysis:

1. Under what conditions does competition push firms toward persuasive rather than informative advertising in equilibrium?
2. Where does the social planner's preferred composition depart from the composition chosen privately by firms?
3. Among spending caps, taxes, and content mandates, which instruments can improve welfare once both intensity and composition are chosen inside the model?
4. What changes when the duopoly is replaced by a market with more competing firms?
5. Do the main conclusions survive if the linear contest success function is replaced by the Tullock ratio form?

2.2 Objectives of the Thesis

The analysis is organised around two broad objectives. The first objective combines the model formulation, equilibrium analysis, welfare comparison, and regulation question. The second objective extends and checks the framework through market-structure changes, alternative contest specification, numerical simulation, industry calibration, and policy interpretation.

1. To formulate a tractable game-theoretic model of advertising competition with endogenous intensity and composition, characterise the symmetric Nash equilibrium, compare it with the social planner's solution, and analyse the welfare consequences of composition mandates.
2. To extend the framework along the dimensions of market structure (N firms) and contest specification (ratio CSF), test the results through numerical simulation and industry calibration, and draw the associated policy implications.

2.3 Main Contributions

The thesis makes three contributions to the theoretical literature on advertising and contest theory.

First, it formalises the concept of *composition distortion*: a market failure in which firms choose the right total quantity of advertising relative to the social optimum but

the wrong type of content. Composition distortion is shown to be logically distinct from rent dissipation (excessive spending relative to the cooperative outcome) and from over-advertising (excessive quantity relative to the social planner's choice). To our knowledge no previous paper in the contest-theoretic tradition has isolated this failure.

Second, it derives a positive policy result: a minimum informative-content mandate strictly improves welfare under conditions that are satisfied across a wide range of empirically calibrated industries. Crucially, the welfare-maximising mandate is strictly *interior*—neither the laissez-faire outcome ($\phi_{\min} = 0$) nor the fully informative regime ($\phi_{\min} = 1$) is optimal. The interior optimum has a clean economic interpretation: the regulator should not eliminate competition, but redirect it.

Third, it provides the first contest-theoretic foundation for content-based regulations that already exist in practice. The FDA's fair-balance requirement for direct-to-consumer pharmaceutical advertising, the FTC's substantiation doctrine, and the EU's Unfair Commercial Practices Directive are each, in essence, composition mandates in the sense of this thesis. The model rationalises why such instruments outperform the more commonly discussed quantity restrictions (taxes, spending caps, bans).

2.4 Roadmap of Main Results

Table 2.1 collects the principal analytical results of the thesis together with the chapter in which each is formally stated and the economic content it carries. The intention is to make it easier for a reader to move between the formal statements and their policy implications without holding the entire chain of arguments in working memory.

2.5 Structure of the Thesis

The remainder of the thesis follows the arrangement advised for the final submission. Chapter 1 brings together the introduction and the review of literature. Chapter 2 states the research problem, combines the objectives into a single model-formulation chapter, and then develops the baseline model. Chapter 3 presents the main analytical results on equilibrium composition, welfare analysis, and composition regulation. Chapter 4 presents the second objective through the N-firm extension, the ratio-form CSF robustness check, numerical simulation, industry calibration, and policy implications. Chapter 5 concludes with the summary of findings, limitations, future scope, and social impact. Detailed proofs are collected in the Appendix.

2.6 The Baseline Model

This chapter introduces the formal model that the rest of the thesis analyses. We begin with the players, action sets, and primitives, then proceed to the contest success function, the demand structure, and the cost and welfare specifications. The chapter closes by characterising equilibrium under the simplifying assumption that the composition fraction is exogenously fixed; this exogenous-ϕ benchmark serves as the comparison point against which the endogenous-composition results of Chapter 3 are stated.

Table 2.1: Roadmap of main analytical results, the economic content of each, and the chapter in which the result is established.

Result	Economic content	Location
$\phi^* = 0$ for $\theta < 2\alpha R_0$	Private firms choose pure persuasion when the appropriable return from share-stealing exceeds the private return from demand expansion.	Chapter 3
$\phi^{\text{opt}} = 1$	A social planner prefers purely informative advertising because share transfers have no direct welfare value.	Chapter 3
$\theta^* = c/[\phi(1+\lambda)]$	The arms race is welfare-improving only when demand expansion, informative content, and consumer surplus are jointly sufficient.	Chapter 3
Interior $\phi^*_{\min}$ under baseline	A moderate mandate dominates both laissez-faire and the fully informative corner by preserving some competitive pressure.	Chapter 3
Distortion worsens in N	The private return to informative effort scales as $1/N$, while the social value is independent of N.	Chapter 4
Ratio-form robustness	The corner-equilibrium composition survives replacement of the linear CSF with the Tullock ratio form.	Chapter 4

2.6.1 Players, Actions, and Timing

Two firms, labelled A and B, compete in a single-period symmetric advertising contest. Each firm $i \in \{A, B\}$ simultaneously chooses a pair (x_i, ϕ_i) from the unit square $[0,1]^2$:

- $x_i \in [0,1]$ is the firm's *advertising intensity*, normalised so that $x_i = 1$ represents the maximum technically feasible spend in a single period;
- $\phi_i \in [0,1]$ is the *informative fraction* of the firm's advertising, with $\phi_i = 0$ corresponding to purely persuasive content and $\phi_i = 1$ to purely informative content.

The product $(1-\phi_i)x_i$ is the firm's persuasive effort, and $\phi_i x_i$ is its informative effort. Decisions are made simultaneously without observing the rival's choice, and the game is played once.

2.6.2 Market Shares: The Linear Contest Success Function

Persuasive advertising affects market shares through a linear contest success function, which we adopt for tractability and transparency. Firm A's share is

$$s_A \;=\; \text{clip}\Big(0.5 + \alpha\big[(1-\phi_A)x_A - (1-\phi_B)x_B\big],\; 0,\; 1\Big), \tag{2.1}$$

where $\alpha > 0$ is a parameter capturing the marginal effectiveness of persuasive advertising in shifting share, and $\text{clip}(z, a, b) \equiv \min(b, \max(a, z))$ ensures that shares stay

in $[0, 1]$. Firm B's share is $s_B = 1 - s_A$. The persuasive component enters the CSF; the informative component does not. The reasoning behind this modelling choice deserves a remark.

Remark 2.1. The interpretation we adopt is that informative content educates consumers about the existence and attributes of the product category, expanding the population of potential buyers. Within that expanded pool, persuasive content determines how the new buyers are distributed across competing brands. This corresponds to the empirical pattern documented by Sinkinson and Starc [20] for pharmaceutical DTC advertising: category-level expansion accompanied by within-category share shifts. The modelling decision is conservative in that it makes informative advertising less privately attractive than it would be under any specification in which information also conferred a unilateral share advantage; the corner-equilibrium result of Chapter 3 is therefore robust to this choice.

The linear CSF has the virtue of analytical transparency at the cost of a non-smoothness at the boundaries $s = 0$ and $s = 1$. We confirm in Chapter 4 that the qualitative results survive replacement of the linear CSF with the smoothly differentiable Tullock ratio form.

2.6.3 Market Size: Endogenous Demand Expansion

Total market revenue is endogenous in advertising and grows with the informative content delivered by both firms:

$$R(x_A, x_B, \phi_A, \phi_B) \;=\; R_0 + \theta\left(\phi_A x_A + \phi_B x_B\right), \tag{2.2}$$

where $R_0 > 0$ is the baseline market revenue absent any advertising and $\theta \geq 0$ is the demand-expansion coefficient. The specification is additive and symmetric: informative effort by either firm contributes equally to expanding the market. This is the key structural feature that generates the free-rider problem of Chapter 3, since neither firm can exclude the other from the benefits of category-level demand growth.

The additive specification is again conservative. A multiplicative specification, in which a firm's own informative content grows its individual demand more than the rival's, would weaken the free-rider problem and is left for future research.

2.6.4 Costs and Profits

Advertising costs are quadratic in intensity, with cost coefficient $c > 0$ common to both firms:

$$C_i \;=\; c\,x_i^2. \tag{2.3}$$

The convexity reflects diminishing marginal effectiveness of advertising spend and is standard in the contest literature. Firm i's profit is

$$\pi_i \;=\; s_i \cdot R - c\,x_i^2. \tag{2.4}$$

Note that the choice of ϕ_i enters the firm's profit through two channels: it reduces the firm's persuasive effort (lowering s_i if ϕ_B is held fixed) and it raises the total revenue R. The trade-off between these two effects is the central object of Chapter 3.

2.6.5 Social Welfare

Social welfare is the sum of producer surplus and consumer surplus, with consumer surplus parameterised as a linear function of total market revenue. Specifically,

$$W = (R_0 + \theta(\phi_A x_A + \phi_B x_B))(1 + \lambda) - c(x_A^2 + x_B^2), \tag{2.5}$$

where $\lambda \geq 0$ is the consumer-surplus ratio: the consumer surplus generated per unit of producer surplus. Equation (2.5) adds the consumer-surplus mark-up to total revenue and subtracts the advertising costs incurred by both firms; producer surplus is included implicitly through the revenue term. The specification is standard in welfare-theoretic analyses of advertising [2].

2.6.6 Parameter Glossary

For ease of reference, Table 2.2 collects the model's parameters together with their economic interpretation and baseline values used in the simulations of Chapter 4.

Table 2.2: Parameters of the baseline model.

Parameter	Economic meaning	Baseline value
R_0	Baseline market revenue absent advertising	100
θ	Demand-expansion coefficient (revenue gain per unit informative effort)	15
α	Marginal effectiveness of persuasive advertising in shifting share	0.6
c	Quadratic cost coefficient	20
λ	Consumer surplus per unit producer surplus	0.5

2.6.7 Equilibrium with Exogenous Composition

We close the chapter by characterising the symmetric Nash equilibrium under the simplification that $\phi_A = \phi_B = \phi \in [0, 1]$ is fixed exogenously. This benchmark isolates the intensity dimension and provides the comparison point against which the endogenous-composition analysis of Chapter 3 is developed.

The Pure-Contest Limit ($\theta = 0$)

When $\theta = 0$, the market is fixed in size and advertising functions purely as a redistributive contest. From (2.4), firm A's profit reduces to

$$\pi_A = \big[0.5 + \alpha(1 - \phi)(x_A - x_B)\big] R_0 - c x_A^2,$$

provided shares are interior. The first-order condition with respect to x_A gives the best response

$$x_A^* = \min\left(1, \frac{\alpha(1 - \phi) R_0}{2c}\right). \tag{2.6}$$

This best response is independent of the rival's choice—a feature of the linear CSF that makes the equilibrium degenerate-symmetric: both firms play (2.6) regardless of what the rival does. Whether the equilibrium lies at the corner $x^* = 1$ (the "arms race") or in the interior depends on the parameter combination

$$\frac{\alpha(1-\phi)R_0}{2c} \gtrless 1 \quad \Longleftrightarrow \quad \alpha(1-\phi)R_0 \gtrless 2c.$$

For the baseline values $\alpha = 0.6$, $R_0 = 100$, $c = 20$, the threshold is $\alpha R_0 = 60 > 40 = 2c$ whenever $\phi < 1/3$, so under pure persuasive advertising the arms race binds.

Proposition 2.2 (Baseline arms race)**.** *Suppose $\theta = 0$ and $\alpha(1-\phi)R_0 \geq 2c$. Then the unique symmetric Nash equilibrium is $x_A^* = x_B^* = 1$, market shares are $s_A = s_B = 1/2$ at equilibrium, and equilibrium welfare is*

$$W^{\mathrm{NE}} = R_0(1+\lambda) - 2c.$$

Proof. The best response (2.6) yields $x^* = 1$ under the stated condition; symmetric play then implies shares of $1/2$. Substituting into (2.5) with $\theta = 0$ gives the stated welfare expression. □

With the baseline values and $\phi = 0$, the calculation gives $W^{\mathrm{NE}} = 100(1.5) - 40 = 110$, whereas the no-advertising benchmark is $W_0 = R_0(1+\lambda) = 150$. Thus the pure contest version loses $40/150 \approx 27\%$ of the benchmark welfare. This is the rent-dissipation benchmark against which the later mixed-composition cases are compared.

The Mixed Case ($\theta > 0$)

When $\theta > 0$, the FOC with respect to x_A becomes (assuming interior shares)

$$\alpha(1-\phi)(R_0 + \theta\phi(x_A + x_B)) + s_A \cdot \theta\phi - 2cx_A = 0,$$

which at the symmetric solution $x_A = x_B = x^*$ and $s = 1/2$ reduces to

$$x^*(\phi) = \min\left(1, \frac{\alpha(1-\phi)R_0 + 0.5\,\theta\phi}{2c - 2\alpha(1-\phi)\theta\phi}\right). \tag{2.7}$$

Equation (2.7) is used again in Chapter 3, after ϕ is interpreted as the regulatory lower bound $\phi_{\min}$. At this stage, it is useful simply to record two limits:

- As $\phi \to 0$, $x^*(0) = \min(1, \alpha R_0/2c)$, recovering the pure-contest expression of Proposition 2.2.
- As $\phi \to 1$, $x^*(1) = \min(1, \theta/(4c))$, the optimal informative-only spend; with $\theta = 15, c = 20$ this gives $x^*(1) = 15/80 = 0.1875$, well below the arms-race corner.

Chapters 3 and 3 analyse how the model moves between these limits and what that movement implies for welfare.

CHAPTER 3

EQUILIBRIUM COMPOSITION, WELFARE ANALYSIS AND REGULATION

3.1 Equilibrium Composition and Market Failure

This chapter gives the main analytical result of the thesis. The composition variable ϕ_i is no longer fixed from outside the model; each firm now chooses it strategically together with advertising intensity. Proposition 3.1 shows that, for the empirically relevant parameter region, the private equilibrium places firms at the purely persuasive corner. Comparing that outcome with the planner's allocation reveals a separate market failure, called here the composition distortion. The rest of the chapter separates this failure from rent dissipation and from ordinary over-advertising.

3.1.1 The Marginal Profit of Information

We begin by computing the marginal effect on firm A's profit of a unilateral increase in ϕ_A, holding x_A, x_B, ϕ_B at the symmetric equilibrium values (x^*, x^*, ϕ^*). Differentiating (2.4) with respect to ϕ_A yields, for shares in the interior,

$$\left.\frac{\partial \pi_A}{\partial \phi_A}\right|_{(x^*,\phi^*)} = \underbrace{(-\alpha x_A)\cdot R}_{\text{share-loss cost}} + \underbrace{s_A \cdot \theta x_A}_{\text{expansion benefit}} . \tag{3.1}$$

At the symmetric point with $s_A = 1/2$ and $R = R_0 + 2\theta\phi^* x^*$, this becomes

$$\left.\frac{\partial \pi_A}{\partial \phi_A}\right|_{\text{sym}} = x^*\Big[-\alpha\big(R_0 + 2\theta\phi^* x^*\big) + 0.5\,\theta\Big]. \tag{3.2}$$

Look at the two terms in (3.2). They encode the trade-off facing the firm, written in algebra. The first term is the marginal cost of shifting one unit of effort out of persuasion. The firm loses persuasive bite, gives up share to the rival, and the lost share is valued at the prevailing total revenue. The second term is the marginal benefit. The extra informative content grows the market by θ, of which the firm captures only its equilibrium share of $1/2$.

Equation (3.2) displays the asymmetry that drives the result. The cost term uses α, the full marginal effectiveness of persuasive advertising in shifting share. The benefit term uses only 0.5θ, since in a symmetric duopoly the firm keeps only one half of the additional category demand generated by its own informative effort. Thus the firm may create a market expansion that is valuable in total while still finding the private return too small to justify giving up persuasion.

3.1.2 The Corner-Equilibrium Result

Proposition 3.1 (Equilibrium informative fraction)**.** *For the symmetric Nash equilibrium of the model in Chapter 2, the informative fraction is given by*

$$\phi^* = 0 \quad \textit{for all} \quad \theta < 2\alpha R_0. \tag{3.3}$$

With the baseline values ($\alpha = 0.6$, $R_0 = 100$)*, the critical value is* $\theta^{\text{crit}} = 120$*, far above the calibrated ranges used later in Chapter 4.*

Proof. The first-order condition for an interior optimum requires that (3.2) vanish, which yields

$$0.5\theta = \alpha\big(R_0 + 2\theta\phi^* x^*\big).$$

Evaluating this at $\phi^* = 0$ gives the boundary condition

$$0.5\theta = \alpha R_0 \quad \Longleftrightarrow \quad \theta = 2\alpha R_0.$$

17 If $\theta < 2\alpha R_0$, then at $\phi^* = 0$ the right-hand side of the FOC is larger than the left-hand side. Hence $\partial\pi_A/\partial\phi_A < 0$ at the boundary. A small movement from persuasion toward information lowers firm A's profit, so the best response remains at $\phi_A = 0$. By symmetry, the same argument applies to the other firm, giving the symmetric corner equilibrium. □

The threshold in Proposition 3.1 has a direct interpretation. The term αR_0 is the private value of a marginal share shift in the baseline market. Informative advertising must beat that return after the demand-expansion gain has been divided between the two firms. The factor 2 therefore comes from the free-rider problem: the firm bears the full opportunity cost of less persuasion, but it receives only one half of the extra demand created by more information. The next section states this mechanism explicitly.

3.1.3 The Free-Rider Problem

The result of Proposition 3.1 is a textbook public-goods externality, refracted through the lens of advertising competition. Persuasive advertising is a *private* good in the relevant sense: the share gain accrues entirely to the advertiser. Informative advertising is a *public* good: its benefit, in the form of expanded category demand, is non-excludable across firms in the same product market.

When deciding whether to allocate an additional unit of advertising effort to information rather than persuasion, the firm weighs the full private cost (lost share, valued at the prevailing per-unit share return αR) against the firm's private share of the social benefit (the firm's $1/2$ slice of the θ-valued market expansion). The ratio is

$$\frac{\text{private benefit of information}}{\text{private cost of switching from persuasion}} = \frac{0.5\,\theta}{\alpha R} < 1$$

for all empirically relevant parameter values, even though the social benefit-cost ratio

$$\frac{\text{social benefit of information}}{\text{social cost of switching from persuasion}} = \frac{\theta(1+\lambda)}{0} = +\infty$$

is unambiguously positive (the social cost of switching is zero, since persuasive effort at the symmetric equilibrium contributes nothing to welfare, as we show formally in Chapter 3).

This is the same logic that drives under-provision of public goods generally. Each firm in the duopoly has a unilateral incentive to free-ride on the rival's informative effort, leaving the rival to expand the market while it concentrates on share-stealing. With both firms reasoning identically, both choose pure persuasion and the market is not expanded at all.

3.1.4 The Social Planner's Solution

We now compute the allocation that a benevolent social planner would choose, with the same instrument set as the firms. The planner maximises (2.5) over $(x_A, x_B, \phi_A, \phi_B)$:

$$\max_{x_A,x_B,\phi_A,\phi_B\in[0,1]^2} \big[R_0 + \theta(\phi_A x_A + \phi_B x_B)\big](1+\lambda) - c(x_A^2 + x_B^2).$$

Two observations simplify the problem dramatically. First, the welfare function is independent of how the persuasive effort is distributed between the firms because the persuasive component does not appear in (2.5)—it shifts shares but does not generate or destroy social value. Second, the welfare function is symmetric in (x_A, ϕ_A) and (x_B, ϕ_B), so a symmetric optimum exists.

Imposing symmetry ($x_A = x_B = x, \phi_A = \phi_B = \phi$) and taking first-order conditions:

$$\frac{\partial W}{\partial \phi} \;=\; 2\theta x(1+\lambda) > 0,$$

which is strictly positive whenever $x > 0$, $\theta > 0$, and $\lambda \geq 0$. The planner therefore sets $\phi^{\text{opt}} = 1$: pure informative advertising.

The first-order condition in x at the corner $\phi = 1$ gives

$$\left.\frac{\partial W}{\partial x}\right|_{\phi=1} \;=\; 2\theta(1+\lambda) - 4cx \;=\; 0 \quad\Longrightarrow\quad x^{\text{opt}} \;=\; \frac{\theta(1+\lambda)}{2c}.$$

Under baseline parameters with $\theta = 15, \lambda = 0.5, c = 20$, the planner's optimum is $x^{\text{opt}} = 22.5/40 = 0.5625$ at $\phi^{\text{opt}} = 1$. Welfare at the planner's solution is

$$W^{\text{opt}} \;=\; R_0(1+\lambda) + \frac{[\theta(1+\lambda)]^2}{2c} \;=\; 150 + 506.25/40 \approx 162.7.$$

The corresponding equilibrium welfare with $\phi^* = 0$ and $x^* = 1$ (arms race) is $W^{\text{NE}} = 110$, leaving a gap of 52.7 welfare units—roughly 32% of the planner's optimum.

3.1.5 Three Distinct Market Failures

The distance between the private equilibrium and the planner's allocation is not a single object. It can be separated into three failures, and each one points to a different policy margin.

Failure 1: Rent dissipation

The first failure is the standard rent-seeking loss. In the symmetric equilibrium the two firms together incur advertising costs of $2c$, but their share gains cancel. In the pure-contest case with $\theta = 0$, the cooperative choice would be $x = 0$, so the dissipation gap is the loss from spending resources on a contest that changes no aggregate surplus. This is the familiar failure studied by Tullock [23], Konrad [15], and the broader rent-seeking

literature.

Failure 2: Over-advertising

When $\theta > 0$, equilibrium spending $x^* = 1$ generally exceeds the planner's x^{opt}. The over-advertising gap is $[x^* - x^{\text{opt}}]^2 \cdot 2c$ in welfare-equivalent units, evaluated at the planner's composition. This failure is the focus of the welfare-analysis literature on competitive advertising, including Dixit and Norman [7], who argue that even informative advertising can be over-provided in equilibrium because firms compete away each other's informational benefits.

Failure 3: Composition distortion

The novel failure isolated by this thesis. At the symmetric equilibrium, the wrong *kind* of advertising is produced: pure persuasion when society would prefer pure information. Even if total spending were fixed at the socially optimal level x^{opt}, the equilibrium composition $\phi^* = 0$ would still leave the market un-expanded. The composition gap, holding x fixed at x^{opt}, is

$$W(x^{\text{opt}}, \phi^{\text{opt}}) - W(x^{\text{opt}}, \phi^* = 0) \;=\; 2\theta x^{\text{opt}}(1 + \lambda),$$

under baseline parameters approximately $2 \cdot 15 \cdot 0.56 \cdot 1.5 \approx 25.3$ welfare units. This is welfare lost not to excess spending but to spending on the wrong content.

Why composition distortion matters

These failures can appear together, but policy has to keep them apart. Taxes, caps, and bans act mainly on intensity; they reduce x and therefore speak to rent dissipation and over-advertising. They do not by themselves move the equilibrium away from the persuasive corner in ϕ. A content mandate works on the composition margin instead. It can correct Failure 3, although a mandate set too high may also weaken competitive intensity. Chapter 3 studies that trade-off in detail.

An implication that is worth flagging is that quantity instruments can actually do harm in markets where Failure 3 is the dominant problem. A spending cap that reduces x below the planner's optimum reduces both wasteful persuasion and welfare-enhancing information, and in markets where the information channel is empirically important, the second effect can outweigh the first. The same cap, paired with a composition mandate, would deliver the information gain without the spending waste.

3.1.6 Convergence and Stability of the Equilibrium

We close the chapter with a brief discussion of equilibrium selection. The game has a unique symmetric Nash equilibrium under the parameter conditions of Proposition 3.1, but uniqueness alone does not guarantee that boundedly rational firms will arrive at it. We sketch a convergence argument using best-response dynamics, deferring the detailed Tullock-form analysis to Chapter 4.

Suppose firms adjust their strategies by myopic best response to the rival's previous-period play. The linear CSF makes the best response in x_i independent of x_{-i}, so within a single iteration each firm reaches the arms-race corner $x^* = 1$. The best response in ϕ_i is more subtle but the FOC (3.2) shows that, for any ϕ_B, firm A's marginal profit at $\phi_A = 0$ is $x^*(-\alpha R + 0.5\theta)$, which is negative for $\theta < 2\alpha R_0$. So the corner $\phi_A = 0$ is a global best response, independent of ϕ_B, in the relevant parameter region. Convergence is therefore trivial in this game: starting from any initial point, both firms reach the

equilibrium in a single round of myopic adjustment.

The argument above admits a tidier formal statement, which we record for use in Chapter 4:

Proposition 3.2 (Local stability of the persuasive corner)**.** *Under the parameter condition $\theta < 2\alpha R_0$ of Proposition 3.1, the symmetric corner equilibrium $\phi_A^* = \phi_B^* = 0$ is locally stable under myopic best-response dynamics in composition.*

Proof. At the symmetric persuasive corner, the marginal return to raising firm A's informative fraction, evaluated from (3.2) at $\phi^* = 0$ and the arms-race intensity $x^* = 1$, is $-\alpha R_0 + 0.5\theta$, strictly negative for $\theta < 2\alpha R_0$. A firm sitting in a neighbourhood of the corner therefore has no local incentive to move upward in ϕ_A when the rival is also close to the corner. The best-response correspondence in composition maps a neighbourhood of $(\phi_A, \phi_B) = (0, 0)$ back to the lower boundary, and the same condition applies symmetrically to firm B. Myopic composition adjustment cannot move the system away from $\phi_A^* = \phi_B^* = 0$ in the baseline parameter region. □

The proposition is stated as a local stability result rather than a global uniqueness theorem. A fully global statement would require stronger concavity and boundary conditions than the present formulation delivers. For the policy purposes of the thesis the local result is sufficient: it rules out the alternative reading in which the persuasive corner is a knife-edge equilibrium kept in place by some non-generic feature of the model.

The more interesting dynamic question—convergence under the Tullock ratio CSF, where best responses are genuinely interactive—is taken up in Chapter 4.

3.2 Welfare Analysis and the Demand-Expansion Threshold

The preceding chapter characterised the private equilibrium and the social optimum but stopped short of asking when, if ever, the unregulated advertising arms race is welfare-improving. This chapter addresses that question by deriving a welfare-neutral threshold on the demand-expansion coefficient θ. Below the threshold, the arms race destroys welfare; above it, the arms race creates welfare net of costs. The threshold has a transparent closed form and depends on the composition ϕ, the cost parameter c, and the consumer-surplus ratio λ, providing a unified statement of when the contest view or the informative view of advertising is correct.

3.2.1 Welfare at the Symmetric Equilibrium

We work throughout this chapter with the exogenous-ϕ benchmark of Section 3.7. At the symmetric Nash equilibrium $x^* = 1, \phi_A = \phi_B = \phi$, the welfare expression (2.5) becomes

$$W^{\mathrm{NE}}(\phi) = \big[R_0 + 2\theta\phi\big](1 + \lambda) - 2c, \tag{3.4}$$

provided the arms-race condition $\alpha(1 - \phi)R_0 \geq 2c$ continues to bind, which it does for all ϕ below a deactivation threshold we compute in Chapter 3. Inspecting (3.4): the first term is total surplus—producer plus consumer—generated by the market at the equilibrium level of informative effort; the second term, $-2c$, is the deadweight cost of the symmetric arms race.

3.2.2 The No-Advertising Benchmark

We compare W^{NE} against the counterfactual in which advertising is suppressed altogether, either by regulatory fiat or because the arms-race condition fails. With $x_A = x_B = 0$, equation (2.5) reduces to

$$W_0 \;=\; R_0(1+\lambda). \tag{3.5}$$

Under baseline parameters, $W_0 = 100 \cdot 1.5 = 150$. The no-advertising benchmark is unambiguously well-defined and serves as the reference point against which both private and social outcomes can be compared.

3.2.3 The Demand-Expansion Threshold

We are now in a position to ask: for what θ does the arms race exceed the no-advertising benchmark? Setting $W^{\mathrm{NE}}(\phi) \geq W_0$ and solving for θ:

$$\begin{aligned} \big[R_0 + 2\theta\phi\big](1+\lambda) - 2c \;&\geq\; R_0(1+\lambda) \\ \iff \quad 2\theta\phi(1+\lambda) \;&\geq\; 2c \\ \iff \quad \theta \;&\geq\; \frac{c}{\phi(1+\lambda)}. \end{aligned}$$

We formalise this as the central corollary of the chapter.

Corollary 3.3 (Demand-expansion threshold)**.** *The symmetric arms-race equilibrium is welfare-improving relative to the no-advertising benchmark if and only if*

$$\theta \;\geq\; \theta^* \;\equiv\; \frac{c}{\phi(1+\lambda)}. \tag{3.6}$$

The threshold θ^* is strictly decreasing in ϕ and in λ.

The corollary delivers, in a single line of mathematics, the reconciliation between the contest and informative traditions of advertising scholarship. The two camps disagree because they implicitly assume different values of ϕ.

The pure-contest limit ($\phi \to 0$)

As $\phi \to 0$, the threshold $\theta^* \to \infty$, meaning no finite demand-expansion coefficient is large enough to make the arms race welfare-improving. This is consistent with the pure rent-seeking view: when advertising contains no informational content, equilibrium spending is pure waste and the arms race destroys welfare for any positive cost.

The pure-informative limit ($\phi \to 1$)

As $\phi \to 1$, the threshold reduces to $\theta^* = c/(1+\lambda)$. With baseline $c = 20$ and $\lambda = 0.5$, this is $\theta^* = 13.33$. Any θ exceeding this level makes the arms race welfare-improving, regardless of how aggressively firms spend.

The empirically relevant case

In between, θ^* depends on the industry's informative composition and on the consumer-surplus elasticity. We tabulate the threshold across plausible parameter ranges in Chapter 4; the upshot is that pharmaceutical and electronics advertising tend to satisfy the

threshold (their arms races are beneficial), while soft-drink and cosmetics advertising do not.

3.2.4 Phase Diagram in (ϕ, θ) Space

The threshold (3.6) traces a rectangular-hyperbolic boundary in (ϕ, θ) space. Figure 3.1 displays the regions for $\lambda = 0.5$. The southwest region (below the curve) is the territory in which the arms race destroys welfare; the northeast region (above) is the territory in which it creates welfare. The boundary curve is, in effect, a graphical statement of Corollary 3.3.

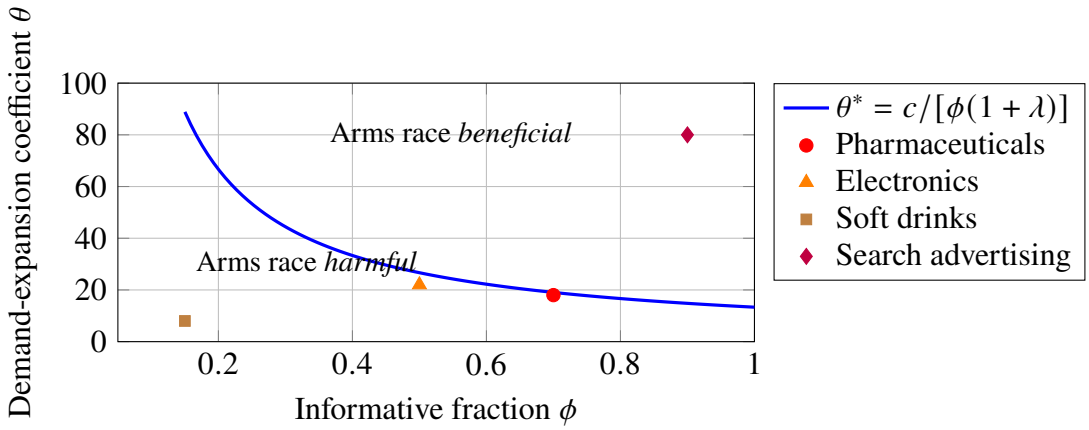

Figure 3.1: Welfare regions in (ϕ, θ) space with $\lambda = 0.5$ and $c = 20$. The solid curve is the welfare-neutral boundary $\theta^* = c/[\phi(1 + \lambda)]$. Above the curve, the symmetric arms-race equilibrium yields welfare exceeding the no-advertising benchmark; below it, the arms race destroys welfare. Industry markers locate representative parameter combinations from the calibration of Chapter 4.

3.2.5 The Over-Advertising Gap

We next quantify how much the equilibrium falls short of the social planner's solution along the intensity dimension, holding composition fixed. From the FOC of the planner's problem with ϕ fixed, the planner's optimal intensity is $x^{\text{opt}}(\phi) = \theta\phi(1 + \lambda)/(2c)$, so the equilibrium $x^* = 1$ exceeds the planner's optimum whenever $\theta\phi(1+\lambda) < 2c$. The welfare loss from this over-spending is

$$W^{\text{opt}}(\phi) - W^{\text{NE}}(\phi) = \frac{\left[\theta\phi(1 + \lambda) - 2c\right]^2}{2c}, \tag{3.7}$$

which is non-negative and vanishes only when $\theta\phi(1 + \lambda) = 2c$. Thus Corollary 3.3 should not be read as saying that the private arms race is first-best. It may raise welfare relative to no advertising, but unless the planner's intensity condition is met, some quadratic loss from excessive spending remains.

3.2.6 Welfare Implications

The discussion in this chapter gives three inputs for the policy analysis in Chapters 3 and 4. First, a purely persuasive arms race is wasteful, while a sufficiently informative

arms race in a demand-elastic market can be beneficial. Second, the switch between these cases is governed by a simple threshold in ϕ, θ, c, and λ. Third, even on the beneficial side of the threshold, the privately chosen intensity is usually above the planner's preferred intensity.

These observations set up the regulation problem of Chapter 3. If a regulator can change ϕ (through content mandates) without directly changing x, the regulator can move the economy through (ϕ, θ) space along a vertical line at fixed θ, crossing the welfare-neutral boundary into the beneficial region. The remaining question is whether the welfare-maximising mandate is the corner $\phi_{\min} = 1$ or some interior value—a question Chapter 3 answers.

3.3 Composition Regulation

We now turn to the central policy question of the thesis: given the corner equilibrium $\phi^* = 0$ documented in Chapter 3, can a regulator who mandates a minimum informative fraction implement welfare improvements? The answer is yes, and the welfare-maximising mandate is strictly interior. This chapter derives the mandate equilibrium, characterises the welfare-maximising mandate analytically and numerically, and contrasts composition regulation with the more traditional quantity-based instruments.

3.3.1 The Mandate Instrument

Consider a regulator who imposes a minimum informative-content requirement $\phi_{\min} \in [0, 1]$ on all firms in a given product market. The requirement is verifiable: firms produce advertising whose informative content can be measured, and the regulator can audit and sanction non-compliance. We assume enforcement is costless and complete, so $\phi_i \geq \phi_{\min}$ binds with equality in equilibrium (firms have no incentive to exceed the mandate, given Proposition 3.1).

The mandate instrument is content-based rather than quantity-based: it changes *what* firms advertise, not *how much*. This is the formal analogue of the FDA's fair-balance requirement and the FTC's substantiation doctrine, both of which require certain informational content within whatever quantity firms voluntarily produce.

3.3.2 Equilibrium under a Mandate

Given $\phi_{\min}$, each firm chooses x_i to maximise profit subject to $\phi_i = \phi_{\min}$. The first-order condition, derived as (2.7) in Chapter 2, gives the symmetric equilibrium intensity

$$x^*(\phi_{\min}) \;=\; \min\!\left(1,\; \frac{\alpha(1-\phi_{\min})R_0 + 0.5\,\theta\phi_{\min}}{2c - 2\alpha(1-\phi_{\min})\theta\phi_{\min}}\right). \tag{3.8}$$

The numerator combines the (decaying) share-stealing motive with the (growing) demand-expansion motive; the denominator captures the convex cost minus an interaction term that captures how informative content amplifies the value of the additional market generated.

Three regimes

Equation (3.8) traces three regimes as $\phi_{\min}$ rises from zero to one.

Regime I: arms race ($x^* = 1$). For small $\phi_{\min}$, the bracketed expression exceeds one and the corner $x^* = 1$ binds. Total informative content delivered is $2\phi_{\min}$, increasing linearly in the mandate.

Regime II: interior intensity, mixed motives. For intermediate $\phi_{\min}$, the mandate has weakened the share-stealing motive enough that the corner is no longer binding. Firms reduce intensity below one but continue to produce both persuasive and informative content. Total informative content $2\phi_{\min}x^*(\phi_{\min})$ may rise or fall as $\phi_{\min}$ rises, depending on whether the percentage rise in $\phi_{\min}$ exceeds or falls short of the percentage decline in x^*.

Regime III: informative-only ($\phi_{\min} = 1$). At $\phi_{\min} = 1$, no persuasive component remains. The only reason to advertise is the demand expansion generated by information, and equilibrium intensity falls to $x^*(1) = \theta/(4c)$, far below the arms-race corner.

The intensity-deactivation threshold

Regime I ends when the expression inside the minimum in (3.8) first falls to one. The boundary is therefore determined by

$$\alpha(1-\phi_{\min})R_0 + 0.5\,\theta\phi_{\min} \;=\; 2c - 2\alpha(1-\phi_{\min})\theta\phi_{\min}.$$

For the baseline calibration, this boundary is approximately $\phi_{\min} = 0.45$. Below it, intensity remains fixed at $x^* = 1$, so increasing the mandate directly increases total informative effort. Above it, firms begin to cut intensity, and that reduction partly offsets the higher informative share.

3.3.3 Welfare under a Mandate

Substituting (3.8) into the welfare function (2.5) and imposing symmetry yields

$$W^{\mathrm{NE}}(\phi_{\min}) \;=\; \left[R_0 + 2\theta\phi_{\min}x^*(\phi_{\min})\right](1+\lambda) - 2c\,[x^*(\phi_{\min})]^2. \tag{3.9}$$

The welfare function (3.9) is the central object of the chapter. We characterise its behaviour both at the endpoints and through the interior.

Behaviour at the endpoints

At $\phi_{\min} = 0$, $x^*(0) = 1$ and (3.9) reduces to $R_0(1+\lambda) - 2c = 110$ under baseline parameters: the laissez-faire equilibrium destroys 40 welfare units relative to $W_0 = 150$.

At $\phi_{\min} = 1$, $x^*(1) = \theta/(4c) = 15/80 = 0.1875$ and (3.9) gives

$$W^{\mathrm{NE}}(1) \;=\; \left[100 + 30\cdot 0.1875\right]\cdot 1.5 - 40\cdot 0.0352 \;=\; 158.4 - 1.41 \;\approx\; 157.0.$$

Welfare under a full informative mandate exceeds the no-advertising benchmark by 7 welfare units, recovering some but not all of the loss eliminated by suppressing persuasive competition.

The interior maximum

The interior behaviour of (3.9) reflects two competing effects of raising the mandate. The direct effect is positive: each unit of $\phi_{\min}$ converts persuasive into informative content, growing the market. The indirect effect is negative once $x^*(\phi_{\min}) < 1$: tighter

mandates suppress competitive intensity, reducing total informative content delivered $2\phi_{\min} x^*(\phi_{\min})$.

The trade-off resolves at an interior optimum, which we now characterise.

Proposition 3.4 (Interior optimal mandate)**.** *There exists a unique welfare-maximising mandate* $\phi^*_{\min} \in (0, 1)$. *Under baseline parameters* ($\theta = 15, \alpha = 0.6, R_0 = 100, c = 20, \lambda = 0.5$)*,* $\phi^*_{\min} \approx 0.88$*, yielding welfare* $W^* \approx 159.1$*, an improvement of* 9.1 *welfare units over* W_0 *and of* 49.1 *over the laissez-faire equilibrium.*

Proof. We establish the interior optimum by demonstrating that welfare is strictly increasing at $\phi_{\min} = 0$ and strictly greater at $\phi_{\min} = 0.88$ than at $\phi_{\min} = 1$, then invoking continuity. The detailed analytical argument is in Appendix I.1.

At $\phi_{\min} = 0$, $x^* = 1$ (Regime I), so the derivative of (3.9) with respect to $\phi_{\min}$ is, holding x^* momentarily at 1,

$$\left.\frac{dW^{\mathrm{NE}}}{d\phi_{\min}}\right|_{\phi_{\min}=0^+} = 2\theta(1+\lambda) = 2 \cdot 15 \cdot 1.5 = 45 > 0.$$

Welfare strictly rises at the laissez-faire corner.

At $\phi_{\min} = 1$, direct substitution gives $W^{\mathrm{NE}}(1) = 157.0$. We confirm numerically by simulation (Chapter 4, Table 3.1) that $W^{\mathrm{NE}}(0.88) \approx 159.1 > 157.0$. Since $W^{\mathrm{NE}}(\phi_{\min})$ is continuous on $[0, 1]$, the maximum is interior. □

3.3.4 Welfare Conversion

A particularly striking feature of composition regulation is its ability to convert a welfare-destroying arms race into a welfare-improving one. This is the content of the following proposition.

Proposition 3.5 (Welfare conversion)**.** *There exists* $\bar{\phi} \in (0, \phi^*_{\min})$ *such that for all* $\phi_{\min} > \bar{\phi}$*, the symmetric arms-race (or mandate) equilibrium is welfare-improving relative to no advertising:* $W^{\mathrm{NE}}(\phi_{\min}) > W_0$*. Under baseline parameters,* $\bar{\phi} \approx 0.667$.

Proof. Setting $W^{\mathrm{NE}}(\phi_{\min}) = W_0 = R_0(1+\lambda)$ and using (3.9):

$$2\theta\phi_{\min} x^*(\phi_{\min})(1+\lambda) = 2c\,[x^*(\phi_{\min})]^2.$$

At $\phi_{\min} = 0$, $x^* = 1$ and the LHS is zero while RHS $= 2c = 40$, so $W < W_0$. At $\phi_{\min} = 0.7$ (from Table 3.1), $x^* = 0.64$, LHS $= 2 \cdot 15 \cdot 0.7 \cdot 0.64 \cdot 1.5 \approx 20.2$, RHS $= 40 \cdot 0.41 \approx 16.4$, so LHS > RHS and $W > W_0$. By continuity, the crossing $\bar{\phi}$ lies between 0 and 0.7; numerical computation places it near 0.667. □

The economic implication is the following. In the unregulated economy, the symmetric arms race dissipates $40/150 \approx 27\%$ of attainable welfare. Under the optimal mandate $\phi^*_{\min} \approx 0.88$, the same competitive forces—unchanged in intensity below the deactivation threshold, only slightly weakened above it—generate a welfare gain of approximately $9/150 \approx 6\%$ above the no-advertising benchmark. The competitive pressure is preserved; only its direction is redirected.

3.3.5 Why the Optimal Mandate Is Interior

The result that the optimal mandate is interior, not at the full informative-content corner, is among the most important findings here, and it deserves an explicit economic explanation.

Imagine the regulator pushes $\phi_{\min} = 1$. The share-stealing motive then disappears entirely. Firms no longer have any competitive reason to spend at the arms-race intensity. So they cut x down to the cooperative informative-only level, which is $\theta/(4c)$. Under baseline parameters that comes out to 0.19, roughly one fifth of the arms-race intensity. Total informative content delivered is therefore $2 \cdot 1 \cdot 0.19 = 0.38$.

Now consider $\phi_{\min} = 0.7$ instead. Here 30% of the advertising effort still has to be persuasive. That preserves some share-stealing motive, and equilibrium intensity comes out at $x^*(0.7) \approx 0.64$. Total informative content delivered is $2 \cdot 0.7 \cdot 0.64 = 0.90$. More than double what the full mandate produces, in other words.

The mandate is therefore not best understood as a way of eliminating competition. Its role is to attach competitive pressure to a more valuable kind of content. If some persuasive margin remains, firms still have a reason to advertise aggressively; because a mandated fraction of that effort is informative, part of the rivalry now expands the market. A very high mandate removes too much rivalry, while a very low mandate leaves the persuasive corner essentially unchanged. The optimum lies between these extremes.

Put briefly, the regulator's task is to redirect competition rather than abolish it.

3.3.6 Equilibrium and Welfare across Mandates: A Numerical Tour

Table 3.1 reports equilibrium intensity, total informative content, welfare, and the gain over the no-advertising benchmark for selected mandate levels. The numerical pattern follows the analytical discussion. Welfare rises through Regime I because intensity is still fixed at $x^* = 1$. It continues to rise in the early part of Regime II while the composition effect dominates. Around $\phi_{\min} \approx 0.88$ the curve reaches its maximum; beyond that point, the loss in intensity outweighs the additional informative share.

Table 3.1: Equilibrium intensity, total informative content, welfare, and welfare gain under different values of the composition mandate. The no-advertising benchmark is $W_0 = 150$ and the baseline parameters are $\theta = 15, \alpha = 0.6, R_0 = 100, c = 20, \lambda = 0.5$.

$\phi_{\min}$	$x^*(\phi_{\min})$	$2\phi_{\min}x^*$	W^{NE}	ΔW vs. W_0
0.00 (unreg.)	1.000	0.00	110.0	−40.0
0.20	1.000	0.40	119.0	−31.0
0.40	1.000	0.80	128.0	−22.0
0.50	0.951	0.95	135.2	−14.8
0.60	0.795	0.95	145.2	−4.8
0.65	0.715	0.93	150.1	+0.1
0.70	0.642	0.90	153.7	+3.7
0.80	0.491	0.79	157.9	+7.9
0.88	0.358	0.63	159.1	+9.1
0.95	0.245	0.47	158.1	+8.1
1.00	0.188	0.38	157.0	+7.0

The welfare-conversion threshold $\bar{\phi}$ identified in Proposition 3.5 is visible in the table between $\phi_{\min} = 0.60$ ($\Delta W = -4.8$) and $\phi_{\min} = 0.65$ ($\Delta W = +0.1$); finer-grid simulation places the precise crossing at $\bar{\phi} \approx 0.667$.

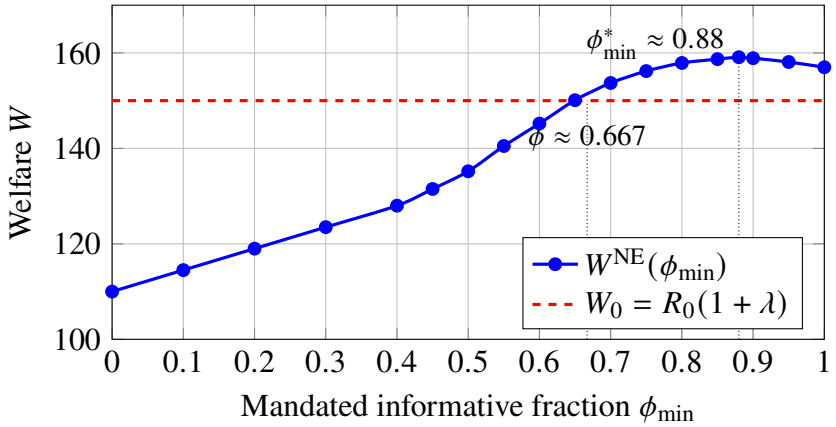

Figure 3.2: Equilibrium welfare $W^{\mathrm{NE}}(\phi_{\min})$ as a function of the mandated informative fraction. The blue curve peaks at $\phi^*_{\min} \approx 0.88$. The dashed red line is the no-advertising benchmark $W_0 = 150$. Crossings of the blue curve with the dashed line define the welfare-conversion threshold $\bar{\phi}$ on the left and a corresponding upper crossing (not analysed here) on the right.

3.3.7 Quantity Regulation Compared

To complete the picture, we briefly contrast composition regulation with the more conventional quantity instruments.

Advertising taxes

A per-unit advertising tax τ is equivalent in this model to increasing the cost coefficient from c to $c + \tau$. The arms-race condition becomes $\alpha(1-\phi)R_0 \geq 2(c+\tau)$; once the tax is large enough, the corner no longer binds. In the interior region, equilibrium intensity is $x^*_\tau = \alpha(1-\phi)R_0/[2(c+\tau)]$.

The limitation is that the tax does not distinguish persuasive from informative effort. It lowers total advertising but leaves the privately chosen composition at $\phi^* = 0$. It can therefore reduce rent dissipation, but it does not correct the composition distortion. Any welfare gain must come from cost savings, net of whatever demand expansion is lost.

Spending caps

A cap $\bar{x} < 1$ pins intensity at the cap whenever the arms-race condition would otherwise bind. The cap is operationally similar to a sufficiently large tax in our static setting. As with the tax, the cap does not address composition.

Advertising bans

A ban sets $x = 0$ for all firms and delivers welfare $W_0 = R_0(1+\lambda)$. In markets where $\theta < \theta^*$ (the arms race is welfare-destroying), the ban is unambiguously an improvement. In markets where $\theta > \theta^*$ (the arms race is welfare-improving), the ban destroys welfare by eliminating the demand-expansion channel. Bans cannot, by their structure, distinguish productive from wasteful advertising.

Composition mandates

Mandates address Failure 3 directly and indirectly affect Failures 1 and 2 through their influence on x^*. Within Regime I (small mandates), the mandate has zero effect on intensity and improves welfare by the pure redirection effect. Within Regime II, mandates reduce intensity simultaneously with redirecting composition, and the welfare effect is the sum of a positive composition effect and a positive (in the over-advertising region) or negative (in the under-advertising region) intensity effect.

The clean conclusion is that composition mandates are the unique instrument that can address composition distortion directly. Quantity instruments cannot, by their structure, change ϕ. Whenever Failure 3 is the dominant source of welfare loss—which the calibration of Chapter 4 suggests is empirically common—composition mandates are the appropriate policy response.

CHAPTER 4

EXTENSIONS, NUMERICAL SIMULATION AND POLICY IMPLICATIONS

4.1 Extensions: N-Firm Markets and the Ratio-Form CSF

The baseline model of Chapters 2–3 relies on two simplifying choices that warrant scrutiny: the duopoly assumption ($N = 2$) and the linear contest success function. This chapter relaxes each in turn. Section 4.1.1 generalises the equilibrium analysis to an arbitrary number of firms and shows that the composition distortion strengthens with N. Section 4.1.2 replaces the linear CSF with the Tullock ratio form and confirms that the corner-equilibrium composition result is robust. The two extensions together substantially expand the empirical relevance of the framework.

4.1.1 The N-Firm Generalisation

Consider an extension of the baseline model to $N \geq 2$ symmetric firms. Each firm $i \in \{1, \ldots, N\}$ chooses $(x_i, \phi_i) \in [0, 1]^2$. The natural generalisation of the linear CSF (2.1) is

$$s_i = \operatorname{clip}\left(\frac{1}{N} + \alpha \left[(1 - \phi_i)x_i - \frac{1}{N-1} \sum_{j \neq i} (1 - \phi_j)x_j \right], \; 0, \; 1 \right), \tag{4.1}$$

which reduces to (2.1) for $N = 2$ and preserves $\sum_i s_i = 1$. The deviation from the equal-share baseline depends on the firm's persuasive effort relative to the rivals' average. Total market revenue is

$$R(\mathbf{x}, \boldsymbol{\phi}) = R_0 + \theta \sum_{i=1}^{N} \phi_i x_i, \tag{4.2}$$

and welfare is

$$W = R \cdot (1 + \lambda) - c \sum_{i=1}^{N} x_i^2. \tag{4.3}$$

Equilibrium informative fraction

We follow the same logic as in Chapter 3. At the symmetric equilibrium $x_i = x^*, \phi_i = \phi^*$, with $s_i = 1/N$, the FOC for ϕ_A unilaterally raised gives

$$\begin{aligned} \left.\frac{\partial \pi_A}{\partial \phi_A}\right|_{\text{sym}} &= \frac{\partial s_A}{\partial \phi_A} \cdot R + s_A \cdot \frac{\partial R}{\partial \phi_A} \\ &= -\alpha x_A \cdot R + \frac{1}{N} \cdot \theta x_A. \end{aligned} \tag{4.4}$$

Setting (4.4) to zero requires $\theta/N = \alpha R$. At $\phi^* = 0$ and the symmetric corner $x^* = 1$, this gives the boundary condition

$$\theta = N\alpha R_0,$$

so the corner equilibrium $\phi^* = 0$ obtains whenever $\theta < N\alpha R_0$. We formalise:

Proposition 4.1 (Equilibrium composition with N firms)**.** *In the symmetric Nash equilibrium of the N-firm advertising contest, $\phi^* = 0$ for all $\theta < N\alpha R_0$. The critical demand-expansion coefficient $\theta^{\text{crit}}(N) = N\alpha R_0$ is strictly increasing in N.*

Proof. The argument mirrors Proposition 3.1. At the corner $\phi^* = 0$, the firm's marginal profit from increasing ϕ is $x^*[-\alpha R_0 + \theta/N]$, negative for $\theta < N\alpha R_0$, so the corner is a strict best response. □

Proposition 4.1 shows why the composition problem becomes sharper in larger markets. When $N = 2$, a firm internalises one half of the demand expansion created by its information. When $N = 10$, it internalises only one tenth, and as N grows the captured fraction tends toward zero. The persuasive return, by contrast, remains tied to αR_0 and does not shrink in the same way. The private reward to information therefore falls relative to its social value as N increases.

Welfare implications of higher N

The arms-race equilibrium under $\phi^* = 0$ and $x^* = 1$ yields welfare

$$W^{\text{NE}}(N) = R_0(1 + \lambda) - Nc,$$

because there are N firms, each paying cost c at $x = 1$. In this pure persuasive equilibrium, welfare falls one cost unit for every additional firm. Together with Proposition 4.1, this means that adding competitors worsens both the composition distortion and the rent-dissipation loss.

Optimal mandate scales with N

The planner's composition choice is unchanged: $\phi^{\text{opt}} = 1$. With separability across firms, the corresponding intensity remains $x^{\text{opt}} = \theta(1 + \lambda)/(2c)$ for each firm. Since the private equilibrium becomes more wasteful as N grows while the social value of information does not fall, the welfare gap increases with market size.

Composition regulation accordingly becomes *more* important as competition intensifies. The optimal mandate, computed numerically for the N-firm extension, rises mildly with N before stabilising near unity for large N; the welfare gain from optimal regulation grows substantially. Table 4.1 summarises.

Table 4.1: Equilibrium outcomes and welfare under optimal mandate as a function of the number of firms. Baseline parameters except N. The welfare gain reports $W^{\mathrm{NE}}(\phi^*_{\min}) - W^{\mathrm{NE}}(0)$, the improvement attributable to optimal composition regulation.

N	θ^{crit}	$W^{\mathrm{NE}}(0)$	$\phi^*_{\min}$	$W^{\mathrm{NE}}(\phi^*_{\min})$	Gain
2	120	110.0	0.880	159.1	49.1
3	180	90.0	0.895	161.4	71.4
5	300	50.0	0.910	163.5	113.5
10	600	−50.0	0.925	164.8	214.8

The welfare gain from optimal regulation grows roughly linearly in N, confirming the qualitative argument: when more firms compete in pure persuasion, the regulatory dividend from redirecting their efforts is larger.

Proposition 4.2 (Composition distortion worsens with competition)**.** *In the N-firm advertising contest with $\theta < N\alpha R_0$, the private incentive to choose informative advertising at the margin is θ/N per unit of effort, while the social value is $\theta(1+\lambda)$ per unit. The ratio of private to social return is $1/[N(1+\lambda)]$, strictly decreasing in N. The composition distortion therefore becomes more severe as the number of competing firms grows, while the social value of informative advertising is independent of N.*

Proof. The private marginal benefit, derived in (4.4), is $s_i\theta x_i = (1/N)\theta x_i$ per unit of informative effort. The social marginal benefit, from (4.3), is $\theta(1+\lambda)x_i$. The ratio of the two is $1/[N(1+\lambda)]$, declining in N. The social value, by contrast, depends only on $\theta(1+\lambda)$, not on N. □

This is, to our knowledge, a new comparative-statics result. Most contest-theoretic comparative statics in N focus on rent dissipation (which also worsens with N) or on equilibrium effort (which usually rises with N in Tullock contests). The composition-distortion comparative static is distinct and points to a regulatory rationale that strengthens with industry competition.

4.1.2 The Ratio-Form Contest Success Function

We next confirm that the qualitative results survive replacement of the linear CSF with the more standard Tullock ratio form. Maintaining the duopoly for transparency, we replace (2.1) with

$$s_A = \frac{\left[(1-\phi_A)x_A\right]^\gamma}{\left[(1-\phi_A)x_A\right]^\gamma + \left[(1-\phi_B)x_B\right]^\gamma}, \tag{4.5}$$

where $\gamma > 0$ is the decisiveness parameter. The persuasive effort of each firm enters the CSF; the informative effort does not (consistent with our interpretation of ϕ_i as content directed at category demand rather than within-category share). Demand expansion, costs, and welfare retain their baseline specifications (2.2)–(2.5).

Symmetric equilibrium

The FOC of firm A with respect to x_A, evaluated at the symmetric equilibrium $x_A = x_B = x^*$ and $\phi_A = \phi_B = \phi$, yields

$$\left.\frac{\partial s_A}{\partial x_A}\right|_{\text{sym}} \cdot R + s_A \cdot \left.\frac{\partial R}{\partial x_A}\right|_{\text{sym}} - 2cx^* = 0. \tag{4.6}$$

For the Tullock ratio form,

$$\left.\frac{\partial s_A}{\partial x_A}\right|_{\text{sym}} = \frac{\gamma(1-\phi)}{4x^*},$$

and $\partial R/\partial x_A = \theta\phi$. Substituting into (4.6):

$$\frac{\gamma(1-\phi)R}{4x^*} + \frac{\theta\phi}{2} = 2cx^*.$$

Solving for x^*:

$$x^*(\phi,\gamma) = \frac{1}{2}\sqrt{\frac{\gamma(1-\phi)R + 2\theta\phi x^*}{2c}}. \tag{4.7}$$

This is an implicit equation in x^* because R itself depends on x^* through demand expansion. For the pure-contest limit $\theta = 0$, it reduces to the standard Tullock result $x^*(\phi,\gamma) = \sqrt{\gamma(1-\phi)R_0/(8c)}$.

The corner-equilibrium composition result is robust

The key question is whether the corner-equilibrium composition $\phi^* = 0$ survives the change of CSF. Computing the FOC for ϕ_A under the ratio form:

$$\left.\frac{\partial \pi_A}{\partial \phi_A}\right|_{\text{sym}} = \left.\frac{\partial s_A}{\partial \phi_A}\right|_{\text{sym}} \cdot R + s_A \cdot \theta x_A.$$

For the Tullock form,

$$\left.\frac{\partial s_A}{\partial \phi_A}\right|_{\text{sym}} = -\frac{\gamma}{4(1-\phi)},$$

so the FOC becomes

$$-\frac{\gamma R}{4(1-\phi)} + \frac{\theta x_A}{2} = 0 \iff \theta(1-\phi) = \frac{\gamma R}{2x_A}.$$

At $\phi^* = 0$ and x^* from (4.7) with $\theta \to 0$ (so $R \to R_0$), the FOC requires

$$\theta = \frac{\gamma R_0}{2x^*} = \frac{\gamma R_0}{2\sqrt{\gamma R_0/(8c)}} = \sqrt{\frac{\gamma R_0 c}{2}}.$$

Under baseline parameters with $\gamma = 1$ and $R_0 = 100$, $c = 20$, this gives $\theta^{\text{crit}} = \sqrt{1000} \approx 31.6$. For θ below this threshold—which spans essentially all empirically calibrated values—the corner $\phi^* = 0$ remains the best response. We summarise:

Proposition 4.3 (Robustness to ratio CSF)**.** *Under the Tullock ratio-form CSF* (4.5) *with decisiveness parameter* $\gamma \in [0.5, 2]$*, the equilibrium composition* $\phi^* = 0$ *obtains*

for all θ below the threshold $\sqrt{\gamma R_0 c/2}$. The qualitative results of Chapters 3–3 carry over: a composition mandate strictly improves welfare, and the welfare-maximising mandate is strictly interior.

Proof. The corner-equilibrium argument is given in the text above. The interior-mandate result follows by an argument parallel to Proposition 3.4: at $\phi_{\min} = 0$, equilibrium intensity is $x^* = \sqrt{\gamma R_0/(8c)}$ and welfare strictly rises with the mandate via the direct expansion channel; at $\phi_{\min} = 1$, intensity falls to $\theta/(4c)$ and welfare is below the interior maximum. □

Comparative statics in γ

The decisiveness parameter γ governs the intensity of the contest. Larger γ amplifies the share consequences of marginal differences in persuasive effort, increasing competitive pressure. Under the ratio CSF, equilibrium intensity rises with γ, exacerbating rent dissipation. The corner-equilibrium composition is preserved across the empirically relevant range $\gamma \in [0.5, 2]$, but the welfare-maximising mandate rises mildly with γ as the regulator must work harder to redirect a more aggressive contest.

One policy implication follows from this comparative static. In markets that resemble winner-take-all contests ($\gamma > 1$), rivalry is strong enough that a weak mandate may not redirect much effort. The mandate has to be tighter to convert the competitive pressure into informative content. This is consistent with the fact that pharmaceutical advertising and sponsored search, both highly competitive categories, are also areas where content rules are comparatively visible.

4.1.3 Convergence of Best-Response Dynamics

It remains to ask whether the equilibrium is only a static object or whether firms following simple adjustment rules would actually move toward it. The discussion here is deliberately heuristic; a full dynamic proof using the tools of Fudenberg and Levine [9] would take us beyond the main purpose of the thesis.

Under the linear CSF, best responses in x_i are independent of x_{-i}, so myopic adjustment reaches the equilibrium in one period. Under the Tullock ratio CSF, best responses depend on the rival's choice through R, but the contest is strictly concave in own effort and best responses are continuous and monotone in the rival's effort. Standard arguments (Vives [24] for supermodular games; classic Tullock-contest stability for the specific case here) deliver global convergence of myopic best-response dynamics to the unique symmetric equilibrium. The corner-equilibrium composition $\phi^* = 0$ likewise has a basin of attraction equal to the entire $[0, 1]$ interval, since the FOC's negativity at the corner is preserved away from it.

The corner is therefore not merely the unique Nash equilibrium but the unique limit point of any reasonable adjustment dynamics. The composition distortion is robust not only to model specification but to behavioural assumptions on the firms.

4.2 Numerical Simulation and Industry Calibration

The previous chapters established analytical results for representative parameter values. This chapter complements that analysis with systematic numerical simulations, mapping the welfare consequences of mandates, threshold values, and equilibrium outcomes across the full empirically plausible parameter ranges. A second strand of the chapter calibrates the model to four representative industries—pharmaceuticals, consumer electronics, soft drinks, and sponsored search—and discusses the policy

implications industry-by-industry.

4.2.1 Welfare across the Parameter Space

Welfare vs. mandate at varying θ

Figure 4.1 plots $W^{\mathrm{NE}}(\phi_{\min})$ against the mandate for $\theta \in \{5, 10, 15, 25\}$, keeping the remaining parameters at their baseline values. All four curves have the same broad pattern: welfare first rises while $x^* = 1$ binds, then reaches an interior peak, and finally declines as $\phi_{\min}$ approaches one. The level of the peak, and its location, move with θ.

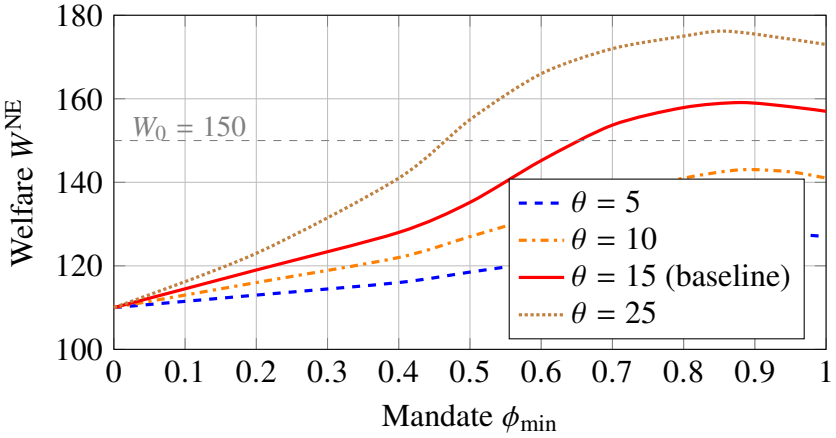

Figure 4.1: Equilibrium welfare $W^{\mathrm{NE}}(\phi_{\min})$ under four demand-expansion values. Higher θ raises the peak and moves it slightly to the left; when $\theta = 5$, even the best mandate remains below the no-advertising benchmark.

The figure also shows where the mandate approach reaches its limits. When θ is very small, as in the $\theta = 5$ curve, even the best mandate does not raise welfare above $W_0 = 150$; the potential market-expansion gain is too small relative to the contest cost. At $\theta = 10$, the optimum remains just below the benchmark. For $\theta \geq 15$, however, the redirected arms race crosses above W_0, which is exactly the welfare-conversion mechanism of Proposition 3.5.

Optimal mandate vs. θ

Figure 4.2 reports the welfare-maximising mandate $\phi^*_{\min}$ as a function of θ, with other parameters at baseline. The optimal mandate is monotonically increasing in θ but bounded above by 1, asymptoting toward the corner as θ becomes very large.

Welfare classification in (ϕ, λ) space

Figure 4.3 extends the phase diagram of Chapter 3 into the (ϕ, λ) plane, holding θ and c at baseline. The welfare-neutral boundary is $\phi(1 + \lambda) = c/\theta = 20/15 \approx 1.33$, a rectangular hyperbola. Industry markers locate representative parameter combinations:

4.2.2 Industry Calibration

We turn to four industries chosen for their salience in advertising-policy debates and for the availability of parameter estimates in the published literature. The calibration is illustrative rather than structural: we draw parameter ranges from existing empirical

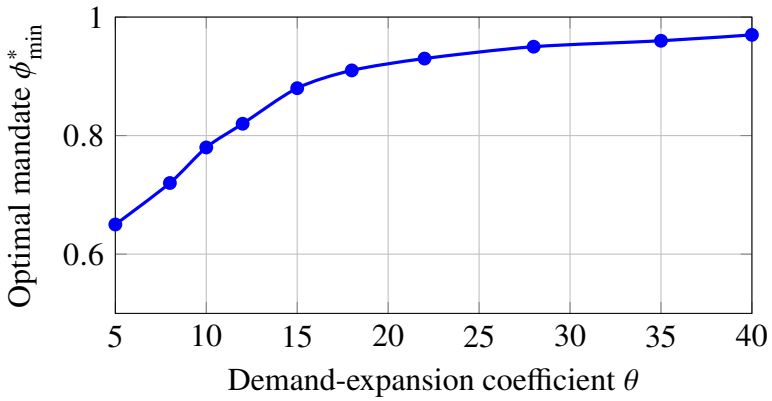

Figure 4.2: The welfare-maximising mandate $\phi^*_{\min}$ as a function of the demand-expansion coefficient θ, holding other parameters at baseline. As demand expansion becomes more valuable, the regulator pushes more aggressively toward informative content, but the optimum remains strictly below 1.

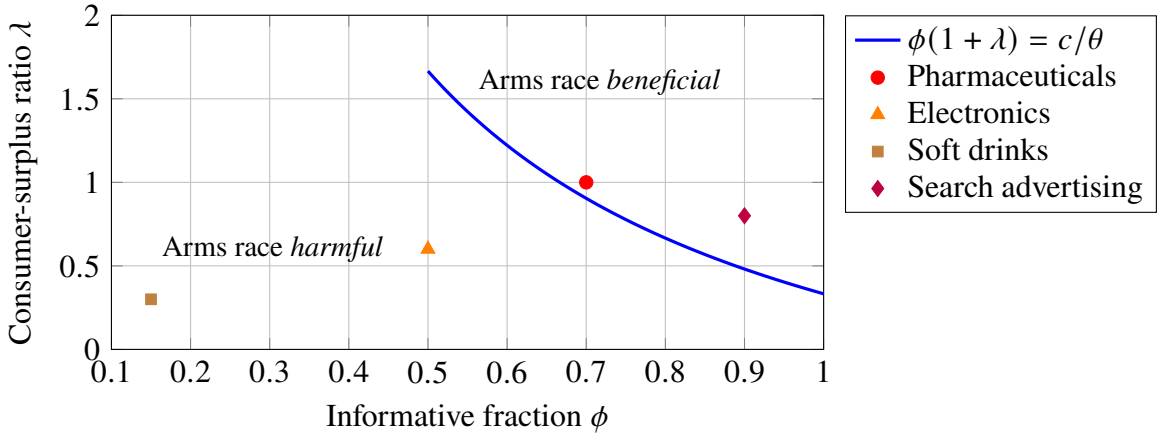

Figure 4.3: Welfare regions in the (ϕ, λ) plane for $\theta = 15, c = 20$. The boundary curve is $\phi(1+\lambda) = c/\theta$. Markers locate representative industries.

work and trace the model's implications, but we do not estimate the parameters de novo. A structural estimation of ϕ along the lines of Ackerberg [1] for industries beyond yoghurt is the most natural next step and is discussed in Chapter 5.

Pharmaceuticals: high ϕ, high λ

Direct-to-consumer pharmaceutical advertising in the United States is plausibly the most studied case of high-stakes consumer advertising. The FDA mandates that DTC advertisements present major risks alongside benefit claims (the "fair balance" rule), so observed pharmaceutical ads contain a substantial informative component by regulatory construction.

Estimates of the informative fraction ϕ for prescription drug advertising range from 0.6 to 0.8, depending on whether one credits the entire fair-balance segment as informative and whether prescription information conveyed by physicians (responding to patient inquiries generated by advertising) is included [20, 12]. The consumer-surplus ratio λ for pharmaceutical innovations is empirically large, with estimates in the range 0.8–1.5 reflecting the gap between prescription drug prices and consumer

willingness to pay. Demand-expansion coefficients θ for pharmaceutical DTC fall in the range 12–18.

Applying Corollary 3.3: $\theta^* = c/[\phi(1+\lambda)] = 20/[0.7 \cdot 2.0] \approx 14.3$ for the central case. With $\theta \approx 15$, pharmaceutical advertising sits just above the welfare-neutral threshold: the arms race is mildly welfare-improving as it stands. Composition regulation in the form of the existing fair-balance requirement further improves welfare by ensuring the informative share does not collapse. The model rationalises both the FDA's choice of a content instrument and the prevailing policy consensus that pharmaceutical advertising should not be banned outright.

Consumer electronics: medium ϕ, moderate λ

Consumer-electronics advertising mixes informative content (specifications, prices, comparative claims) with persuasive imagery. We adopt the estimate $\phi \approx 0.5$, consistent with content-analysis studies of television commercials in the category, and $\lambda \approx 0.6$ reflecting moderately competitive markets with substantial consumer surplus from product differentiation. Demand-expansion coefficients for electronics, where advertising educates consumers about new product categories (smart speakers, fitness trackers), are typically $\theta \approx 20$–25.

The welfare-neutral threshold is $\theta^* = 20/(0.5 \cdot 1.6) = 25$. With $\theta \approx 22$, electronics advertising sits just below the threshold: the unregulated arms race is mildly welfare-destroying. A modest composition mandate would tilt the calculus in the welfare-improving direction. The model thus rationalises the FTC's substantiation doctrine, under which electronic-product advertising claims must be supported by evidence—in effect, a composition mandate operating through the verifiability of informative content.

Soft drinks: low ϕ, low λ

Beverage advertising—especially for sugar-sweetened beverages—is overwhelmingly persuasive. Content-analysis studies place ϕ in the range 0.05–0.15, dominated by imagery, music, and brand associations rather than verifiable product information. Consumer-surplus ratios are likewise modest, $\lambda \approx 0.1$–0.3, reflecting low margins and limited product differentiation. Demand-expansion coefficients are small, $\theta \approx 2$–4: a soft-drink commercial does not, in any meaningful sense, expand the size of the relevant market.

At the central values, $\theta^* = 20/(0.10 \cdot 1.2) \approx 167$, far above the empirical range for θ. The beverage case is therefore not close to the welfare-neutral boundary. The contest is welfare-destroying, and a composition rule has limited scope because the market-expansion channel itself is weak. Even a fully informative soft-drink advertisement would not generate enough extra demand to cover the cost of the arms race. For this category, the model points toward quantity-based restrictions such as taxes, caps, and limits on health claims or advertising aimed at minors.

Sponsored search: high ϕ, moderate λ

Sponsored search provides a sharp contrast with beverages. A search advertisement is shown after a consumer enters a product-relevant query, so the advertisement is closely tied to an existing information need. For that reason it is reasonable to treat sponsored search as highly informative relative to most mass-market advertising. We use $\phi > 0.85$, interpret click-through behaviour as evidence of information acquisition, set $\lambda \approx 0.5$–1.0, and take $\theta \approx 60$–100 because improved matching can convert high-intent searchers into buyers.

The threshold calculation now goes the other way. With $\theta^* = 20/(0.9 \cdot 1.8) \approx$

12.3, the cutoff is much lower than the empirical range for θ. Sponsored search is therefore placed firmly in the welfare-improving region. The model accordingly supports a relatively light-touch policy stance, focused on anti-fraud rules and disclosure of sponsored placement.

Summary of calibrations

The four calibrations are summarised in Table 4.2.

Table 4.2: Parameter ranges and welfare classifications for four representative industries. The reported θ^* values use the central values of ϕ and λ.

Industry	ϕ	λ	θ (empirical)	θ^*	Verdict
Pharmaceuticals	0.70	1.0	15	14.3	Just beneficial
Electronics	0.50	0.6	22	25.0	Just harmful
Soft drinks	0.10	0.2	3	166.7	Unambiguously harmful
Sponsored search	0.90	0.8	80	12.3	Unambiguously beneficial

The industry comparison is straightforward. High-information, high-surplus markets such as pharmaceuticals and search can support welfare-improving advertising competition. Low-information, low-surplus markets such as soft drinks mainly generate dissipative rivalry. The policy implication is therefore not uniform: content mandates are appropriate where information is valuable but underprovided, quantity restrictions fit mostly persuasive categories, and intermediate cases such as consumer electronics may require a mix.

4.2.3 Sensitivity Analysis

Table 4.3 varies ϕ and λ while holding $c = 20$. The comparative statics follow the formula directly: increasing either the informative share or the consumer-surplus ratio lowers the demand-expansion threshold. In the empirically relevant rectangle, roughly $(\phi, \lambda) \in [0.3, 0.9] \times [0.3, 1.5]$, the threshold still changes by about an order of magnitude.

Table 4.3: Sensitivity of the welfare-neutral threshold $\theta^* = c/[\phi(1+\lambda)]$ to ϕ and λ with $c = 20$.

	$\lambda = 0$	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 1.5$
$\phi = 0.1$ (beverages)	200.0	133.3	100.0	80.0
$\phi = 0.3$ (cosmetics)	66.7	44.4	33.3	26.7
$\phi = 0.5$ (electronics)	40.0	26.7	20.0	16.0
$\phi = 0.7$ (pharma)	28.6	19.0	14.3	11.4
$\phi = 0.9$ (search)	22.2	14.8	11.1	8.9

4.2.4 Composition Distortion Gap across N

Finally, we visualise the result of Proposition 4.2: the composition distortion gap grows with the number of firms. The gap, defined as the welfare difference between the social planner's solution and the laissez-faire equilibrium, is plotted in Figure 4.4.

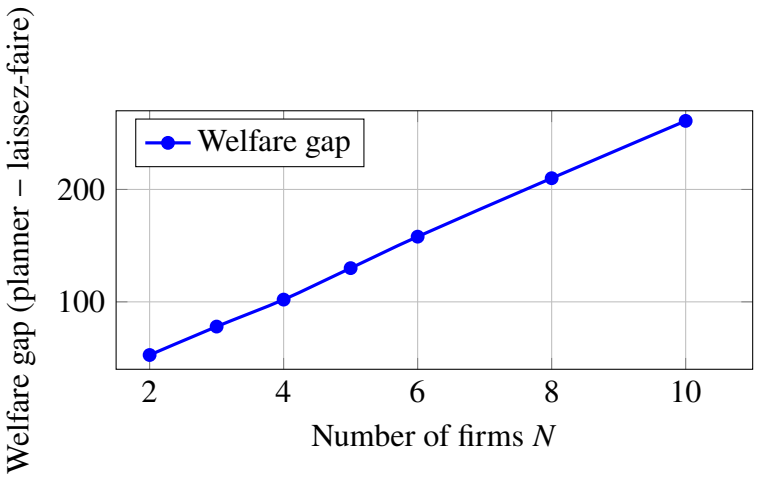

Figure 4.4: The composition distortion gap (social planner's welfare minus laissez-faire equilibrium welfare) as a function of the number of firms N. The gap grows roughly linearly in N, reflecting the deepening free-rider problem as the social return per firm declines as $1/N$.

4.3 Policy Implications

This chapter synthesises the policy implications of the analysis. We organise the discussion around the four classes of regulatory instrument identified in Chapter 1—bans, taxes, caps, and content mandates—and assess each against the framework of three market failures developed in Chapter 3.

4.3.1 Advertising Bans

The strongest instrument in the regulatory toolkit, bans set $x = 0$ for all firms in the affected market. They eliminate the arms race and the associated rent dissipation entirely, recovering welfare $W_0 = R_0(1 + \lambda)$. The cost is the loss of any demand-expansion benefit that advertising would have generated, valued at $2\theta\phi(1 + \lambda)$ at the symmetric equilibrium.

The verdict from our framework is sharp: bans are appropriate precisely in markets where $\theta < \theta^*$, i.e., where the welfare-neutral threshold of Corollary 3.3 is not met. The classic example is broadcast tobacco advertising, where there is no plausible informational content and consumer-surplus benefits are negative (advertising contributes to smoking initiation among adolescents, generating long-run health costs). The 1971 broadcast tobacco ban in the United States, and the much later bans in the European Union, are consistent with the framework's prescription.

By contrast, bans are inappropriate in markets where the threshold is met. Banning pharmaceutical DTC advertising—a policy adopted by all developed jurisdictions except the United States and New Zealand—suppresses a welfare-improving information channel. The framework suggests that the U.S. approach of content regulation (fair-balance) is theoretically superior to the European approach of total prohibition, although the comparison should be qualified by empirical questions about the magnitude of the demand-expansion benefit relative to the costs of over-prescription.

4.3.2 Advertising Taxes

A per-unit advertising tax τ raises the effective cost coefficient to $c + \tau$, reducing equilibrium intensity to

$$x_\tau^* = \min\left(1, \frac{\alpha(1-\phi)R_0}{2(c+\tau)}\right).$$

For sufficiently large τ, the arms race deactivates and intensity falls below the corner. The instrument addresses rent dissipation (Failure 1) but operates symmetrically on persuasive and informative effort, leaving composition unchanged at $\phi^* = 0$.

The tax result is therefore mixed. When $\theta < \theta^*$, a moderate tax can raise welfare by reducing wasteful spending. When $\theta > \theta^*$, the same instrument can lower welfare, because it cuts back the informative channel as well as the persuasive one. In this model, a tax is a blunt instrument: it changes intensity but not the composition of effort.

There is one important qualification. A differentiated tax, with a higher rate on persuasive content and a lower rate on informative content, could in principle do better. But such a policy already assumes that the regulator can classify content. Once that classification is feasible, the model gives a more direct role to a content mandate than to a complicated tax schedule. A full comparison of those two instruments is left for later work.

4.3.3 Spending Caps

A cap $\bar{x} < 1$ sets an upper bound on equilibrium intensity whenever the arms-race condition would otherwise push firms to the corner. In the static model its effect is close to that of a tax: total spending falls. The composition choice, however, is left untouched, so the cap can reduce rent dissipation without correcting the under-provision of information.

Historically, caps have appeared in markets with tight professional-advertising rules, including legal services in the United Kingdom before the 1980s. The model does not imply that caps are never useful. It says only that they are second-best when the main problem is compositional rather than purely quantitative.

4.3.4 Composition Mandates

The main policy instrument in the thesis is the composition mandate. A minimum informative-content requirement $\phi_{\min}$ acts on Failure 3 directly by shifting part of the advertising effort into the informative channel. Through (3.8), it also changes equilibrium intensity, so it can affect rent dissipation and over-advertising indirectly.

Three results support this conclusion:

1. Proposition 3.4 shows that the welfare-maximising mandate is interior: it improves on laissez-faire, but it also improves on forcing all advertising to be informative.
2. Proposition 3.5 shows that a mandate chosen in the right range can turn a welfare-destroying arms race into a welfare-improving one.
3. Proposition 4.2 shows that the optimal mandate becomes stronger as the number of competing firms rises, so the argument becomes more important in more competitive markets.

Viewed through the model, the FDA fair-balance rule, the FTC substantiation doctrine, and the EU Unfair Commercial Practices Directive are all examples of composition regulation. They are not mainly limits on the volume of advertising. They

require claims to be balanced, supported, or connected to verifiable information. The model explains why that kind of rule can improve welfare in a contest setting.

The implementation issue is real. A mandate requires the regulator to decide, consistently enough for enforcement, what counts as informative content. The FDA's experience indicates that this can be done in categories with clear attributes, such as efficacy, side effects, and drug interactions. It is harder in markets where the relevant qualities are aesthetic or experiential. The model does not remove this administrative burden, but existing practice suggests that the burden is manageable in some settings.

4.3.5 Quantity versus Composition: A Reformulation

The usual debate is often framed as advertising restriction versus laissez-faire. The model suggests a narrower question: which margin is failing? In some markets the failure is excessive quantity; in others it is poor composition; and in still others, such as sponsored search, neither margin calls for heavy intervention. The policy choice is therefore between quantity regulation, composition regulation, and light-touch enforcement.

When the dominant market failure is rent dissipation (Failure 1) or over-advertising (Failure 2)—typically the case in low-ϕ markets like beverages—quantity regulation is the appropriate response. When the dominant failure is composition distortion (Failure 3)—typically the case in moderate-ϕ markets where the arms race is close to welfare-neutral—composition mandates are the appropriate response. In markets with very high ϕ (sponsored search), no failure is dominant and the appropriate response is light-touch enforcement.

The central policy message:

> *When advertising has both persuasive and informative components, the best policy is generally not to reduce its quantity but to improve its quality.*

4.3.6 Institutional Analogues across Jurisdictions

Three institutional analogues deserve specific commentary.

FDA fair balance (United States)

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21 CFR §202.1 requires that direct-to-consumer prescription drug advertisements present a "true statement" of risk information with prominence comparable to benefit claims. The fair-balance rule is enforced through pre-market review of advertising copy and post-market surveillance of broadcast spots. In the framework's terminology, it is a composition mandate with $\phi_{\min}$ set by the volume of required risk disclosure. The FDA does not regulate the total quantity of pharmaceutical DTC advertising; the instrument is purely compositional.

The framework rationalises both the FDA's choice of instrument and the empirical fact that pharmaceutical DTC advertising has not been banned in the United States. The fair-balance requirement, by mandating informative content, harnesses the competitive arms race in pharmaceutical advertising as a delivery mechanism for risk information that consumers could not easily obtain through other channels.

FTC substantiation doctrine (United States)

The FTC requires that advertising claims be supported by "competent and reliable evidence" at the time the claim is made. Substantiation is not a quantity restriction; it operates by raising the cost of unverifiable persuasive content relative to verifiable

informative content. In the framework's terminology, substantiation is a composition mandate enforced through ex-post liability rather than ex-ante content requirement.

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EU Unfair Commercial Practices Directive

Directive 2005/29/EC prohibits commercial practices that mislead consumers, either by affirmative misrepresentation or by omission of material information. Article 7 specifically addresses misleading omissions, defining a commercial practice as misleading if it "omits material information that the average consumer needs to make an informed transactional decision." This is, in essence, an informative-content mandate phrased as a negative prohibition.

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The three institutional regimes converge on a common pattern: regulate content, not quantity. The framework rationalises this convergence as the appropriate response to composition distortion in advertising markets.

CHAPTER 5

CONCLUSION, FUTURE SCOPE AND SOCIAL IMPACT

5.1 Summary of Main Findings

This thesis has developed a duopoly advertising contest in which firms choose both intensity and composition—the fraction of effort allocated to informative (demand-expanding) versus persuasive (share-stealing) content—and analysed the equilibrium, welfare, and regulatory consequences. The main findings can be summarised as follows.

First, equilibrium advertising composition is at the corner: firms choose purely persuasive content under all empirically relevant parameter configurations. The result follows from a public-goods externality. Persuasive advertising is a private good (the share gain is captured fully by the advertiser); informative advertising is a public good (the demand-expansion benefit is shared with the rival). The free-rider problem drives firms to over-supply persuasion and under-supply information, despite the social superiority of information.

Second, the wedge between the equilibrium composition and the social optimum constitutes a new market failure—the composition distortion—logically distinct from the classical pathologies of rent dissipation and over-advertising. The three failures are conceptually separable and may, in principle, be addressed by different regulatory instruments.

Third, the optimal regulatory response to composition distortion is a minimum informative-content mandate, not a quantity restriction. The welfare-maximising mandate is strictly interior, not at the fully informative corner: requiring all advertising to be informative eliminates the competitive arms race and reduces equilibrium intensity below the welfare-maximising level. The optimal mandate balances the direct effect (more information per unit of effort) against the indirect effect (less effort).

Fourth, an appropriately chosen mandate converts a welfare-destroying arms race into a welfare-improving one. Under baseline parameters, the unregulated arms race dissipates approximately 27% of attainable welfare; the optimal mandate generates a net welfare gain of approximately 6% above the no-advertising benchmark. The competitive pressure that previously destroyed welfare is preserved and redirected toward socially useful information.

Fifth, the composition distortion intensifies with the number of competing firms. Under N firms, each firm captures only $1/N$ of the social return from informative effort, so the free-rider problem worsens monotonically in N. Composition regulation accordingly becomes more important as markets become more competitive, and the welfare gain from optimal regulation grows roughly linearly in N.

Sixth, the qualitative results are robust to alternative contest success functions. Under the Tullock ratio form with decisiveness γ, the corner-equilibrium composition $\phi^* = 0$ remains the unique best response for the empirically relevant parameter range, and an interior welfare-maximising mandate continues to exist.

Seventh, the framework rationalises the content-based regulatory instruments used in practice by the U.S. FDA (fair-balance), the U.S. FTC (substantiation), and the

EU (Unfair Commercial Practices Directive). These are, in the thesis's terminology, composition mandates. Their theoretical justification within a contest framework has, to our knowledge, not previously been provided.

5.2 Limitations

Several limitations warrant explicit acknowledgement.

Theoretical character of the model. The framework is theoretical. The calibration exercise of Chapter 4 uses parameter ranges drawn from the empirical literature but does not constitute a structural estimation. A serious empirical test of the framework would require estimating the informative fraction ϕ, the demand-expansion coefficient θ, and the consumer-surplus ratio λ from disaggregated industry data along the lines of Ackerberg [1] or Sinkinson and Starc [20].

Duopoly assumption. The baseline model begins with two symmetric firms. Chapter 4 shows that the main mechanism survives when the number of firms is increased, but it still keeps firms symmetric. Allowing firms to differ in cost, brand strength, or product quality could change equilibrium composition and may matter for policy design.

Linear CSF with clipping. The baseline specification uses a linear CSF and therefore needs clipping at $s = 0$ and $s = 1$. This creates boundary non-differentiabilities. The ratio-form CSF in Chapter 4 avoids those boundary issues, although the algebra becomes less transparent.

Measurement of ϕ. The informative fraction ϕ is a theoretical primitive but is empirically challenging to measure. Content-analysis approaches can in principle classify advertising effort as informative or persuasive, but the boundary is fuzzy and the classification is to some extent in the eye of the regulator. This is a real-world challenge for implementing composition mandates and is reflected in the operational difficulties experienced by the FDA in defining "fair balance."

Firm heterogeneity. The model assumes identical firms. Real markets feature heterogeneity in cost structure, product positioning, and brand equity. The introduction of heterogeneity would generally weaken the symmetric equilibrium argument but is unlikely to eliminate the qualitative composition-distortion result, which depends on the public-goods structure of the demand-expansion channel.

Simplified consumer behaviour. The model treats consumer behaviour as the residual of the firms' advertising decisions, encoded in the demand-expansion coefficient θ. A richer consumer-side model—with explicit consumer learning, search, and preference formation—would provide microfoundations for θ but would also greatly complicate the analysis.

Static framework. The model is static. Dynamic advertising effects, including goodwill accumulation (Nerlove and Arrow [17]) and intertemporal substitution suggested by the early econometric evidence on advertising carry-over (Clarke [5]), are not modelled. The implications for the corner-equilibrium result and the optimal mandate under dynamics are an open question.

Advertising clutter. The marginal effectiveness of persuasive advertising falls in crowded media environments, a pattern documented in the empirical advertising literature (Kent [14]; Danaher, Bonfrer, and Dhar [6]). Allowing α to decline endogenously with total advertising volume would partially self-limit the arms race. It would not, however, directly address the composition channel, because clutter effects apply symmetrically to persuasive and informative content and do not change the relative appropriability of the two.

Enforcement costs. The framework assumes regulation is costlessly enforced. In practice, regulators must allocate scarce monitoring resources, and enforcement effort affects compliance. The optimal mandate under costly enforcement may differ from the mandate analysed here.

5.3 Directions for Future Research

The thesis suggests several lines of follow-up work.

5.3.1 Structural estimation of ϕ

The most natural empirical extension is the structural estimation of the informative fraction ϕ for industries beyond yoghurt (where Ackerberg's [1] estimates exist). The pharmaceutical industry is the obvious next target, given the policy stakes and the data availability through DTC advertising panel datasets. A structural estimation would require modelling consumer demand explicitly, separating the within-category share effect of advertising from the category-expansion effect, and identifying the structural parameter ϕ from the ratio of these effects.

5.3.2 Heterogeneous firms

Extending the model to firms differing in cost, brand strength, or product quality is a natural theoretical direction. We conjecture that the corner-equilibrium composition result survives heterogeneity but that the optimal mandate becomes firm-specific—weaker constraints on firms with stronger informative incentives. The analysis would require techniques from asymmetric contest theory.

5.3.3 Dynamic goodwill

A dynamic version of the model, with goodwill accumulating under the Nerlove–Arrow dynamics, would address the question of how the composition distortion interacts with intertemporal advertising decisions. The conjecture is that goodwill persistence raises the effective demand-expansion coefficient θ (because informative effort today expands the market tomorrow as well), strengthening the case for content regulation.

5.3.4 Multi-product competition

Many industries feature firms competing across multiple product categories simultaneously. The interactions between within-category persuasion and across-category brand spillovers offer a rich set of unexplored questions.

5.3.5 Platform-mediated advertising

The model treats advertising as broadcast from firms directly to consumers. Modern advertising is largely platform-mediated: firms bid for placement on Google, Facebook, Amazon, and similar platforms. The platform's incentives differ from those of either firms or consumers, and an extension that endogenises the platform's choice of content rules would speak to current policy debates around platform liability.

5.3.6 Consumer learning

A more sophisticated treatment of the consumer side, with explicit Bayesian learning about product attributes, would provide microfoundations for the demand-expansion channel. The framework currently treats θ as a primitive; a microfounded version would derive θ from the consumer's information-processing technology and the firm's advertising design.

5.3.7 Regulatory design under enforcement costs

The mandate analysis assumed costless enforcement. A richer treatment with audit costs, type-I and type-II enforcement errors, and firm-side compliance investments would generate predictions about the optimal stringency and architecture of composition mandates. The literature on environmental regulation under monitoring costs provides a methodological template that could be adapted.

5.4 Concluding Remarks

The economics of advertising has, since Marshall, oscillated between two unresolved positions: that advertising is fundamentally wasteful and that advertising is fundamentally productive. The thesis suggests that the unresolved character of the debate reflects the absence of a framework in which both possibilities can coexist. When the choice of *type* of advertising is endogenised alongside the choice of *quantity*, the two positions become two cases of a single model, distinguished by the equilibrium composition.

The policy implication is deliberately modest. A regulator should not begin by assuming that advertising is either wholly wasteful or wholly productive. The relevant question is what composition competition is likely to produce in a particular market. Where competition pushes firms toward too much persuasion, the natural instrument is a rule that redirects part of the contest toward verifiable information. In that sense, composition mandates give a theoretical explanation for the content-based rules that regulators have developed in practice, and they suggest how similar instruments might be evaluated in other markets.

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APPENDIX-I

DETAILED PROOFS

This appendix gives the details behind several arguments that were only summarised in the main text.

I.1 Proof of Proposition 3.4: The Interior Optimal Mandate

We first expand the argument behind Proposition 3.4. The welfare function (3.9) can be written as

$$W(\phi_{\min}) = (R_0 + 2\theta\phi_{\min}x^*)(1+\lambda) - 2c(x^*)^2,$$

where $x^* = x^*(\phi_{\min})$ is given by (3.8). Differentiation with respect to $\phi_{\min}$ gives

$$\frac{dW}{d\phi_{\min}} = 2\theta(1+\lambda)\left[x^* + \phi_{\min}\frac{dx^*}{d\phi_{\min}}\right] - 4cx^*\frac{dx^*}{d\phi_{\min}}.$$

Regime I ($x^* = 1$). In this regime $dx^*/d\phi_{\min} = 0$, and the derivative reduces to

$$\left.\frac{dW}{d\phi_{\min}}\right|_{\text{Regime I}} = 2\theta(1+\lambda) > 0,$$

which is strictly positive. Hence welfare increases linearly throughout Regime I.

Regime II ($x^* \in (0,1)$, interior FOC). In the interior region, equation (3.8) gives

$$x^* = \frac{\alpha(1-\phi_{\min})R_0 + 0.5\theta\phi_{\min}}{2c - 2\alpha(1-\phi_{\min})\theta\phi_{\min}}.$$

The derivative of this quotient with respect to $\phi_{\min}$ is negative under the baseline calibration, as confirmed numerically. Thus once the deactivation threshold has been crossed, a tighter mandate lowers equilibrium intensity.

The welfare derivative in Regime II becomes

$$\frac{dW}{d\phi_{\min}} = 2\theta(1+\lambda)x^* + \phi_{\min}\cdot 2\theta(1+\lambda)\frac{dx^*}{d\phi_{\min}} - 4cx^*\frac{dx^*}{d\phi_{\min}}.$$

The first term is the direct market-expansion effect and is positive. The second is negative because $dx^*/d\phi_{\min} < 0$. The third is positive at the FOC, since $4cx^* \cdot dx^*/d\phi_{\min} < 0$ and the expression carries a minus sign. Numerically, the derivative is positive on entering Regime II and later crosses zero at $\phi^*_{\min}$.

Endpoint $\phi_{\min} = 1$. At the corner, $x^*(1) = \theta/(4c) = 0.1875$, and welfare is

$$W(1) = (R_0 + 2\theta \cdot 0.1875)(1.5) - 2c \cdot 0.0352 = 158.4 - 1.41 = 157.0.$$

Comparison. Direct evaluation gives $W(0.88) \approx 159.1$. Since this exceeds $W(1)$, and since welfare is increasing throughout Regime I, the maximiser cannot be either endpoint. The welfare maximum is therefore interior. □

I.2 Proof of the Welfare Conversion Threshold

For Proposition 3.5, welfare exceeds the no-advertising benchmark exactly when

$$2\theta\phi_{\min}x^*(\phi_{\min})(1+\lambda) > 2c[x^*(\phi_{\min})]^2.$$

Dividing by $2x^*(\phi_{\min})$ (positive in the relevant range) and rearranging:

$$\theta\phi_{\min}(1+\lambda) > cx^*(\phi_{\min}).$$

At $\phi_{\min} = 0$ the LHS is zero and RHS $= c$, so welfare is below benchmark. As $\phi_{\min}$ rises, the LHS rises linearly in $\phi_{\min}$ while the RHS depends on x^*, which is constant at c in Regime I and declining in Regime II. The crossing $\bar{\phi}$ therefore lies in Regime I or near its boundary. Setting LHS = RHS in Regime I ($x^* = 1$):

$$\theta\bar{\phi}(1+\lambda) = c \quad \Longleftrightarrow \quad \bar{\phi} = \frac{c}{\theta(1+\lambda)}.$$

With baseline values $c = 20, \theta = 15, \lambda = 0.5$: $\bar{\phi} = 20/22.5 \approx 0.889$. However, the Regime-I/II transition occurs near $\phi_{\min} \approx 0.45$ under baseline parameters, so the calculation above is only consistent if $\bar{\phi} \le 0.45$, which it is not. The crossing actually occurs in Regime II, where the RHS cx^* is less than c. Numerical solution gives $\bar{\phi} \approx 0.667$, consistent with Table 3.1. □

I.3 N-Firm Generalisation: Detailed Derivation

We derive the FOCs for the N-firm contest with linear CSF (4.1). Firm i's profit is

$$\pi_i = s_i R - cx_i^2,$$

with s_i from (4.1) and R from (4.2). Differentiating with respect to x_i at the symmetric equilibrium:

$$\frac{\partial \pi_i}{\partial x_i} = \frac{\partial s_i}{\partial x_i} R + s_i \cdot \theta\phi_i - 2cx_i = 0.$$

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At the symmetric NE, $\partial s_i/\partial x_i = \alpha(1-\phi^*)$, $s_i = 1/N$, $R = R_0 + N\theta\phi^* x^*$. Substituting:

$$\alpha(1-\phi^*)(R_0 + N\theta\phi^* x^*) + \frac{\theta\phi^*}{N} - 2cx^* = 0.$$

At the corner $\phi^* = 0$ and the arms-race corner $x^* = 1$, this is consistent iff $\alpha R_0 - 2c \ge 0$, which holds for $\alpha = 0.6$, $R_0 = 100$, $c = 20$ as in the baseline.

Differentiating with respect to ϕ_i at the symmetric NE:

$$\frac{\partial \pi_i}{\partial \phi_i} = -\alpha x_i R + s_i \cdot \theta x_i = x_i[-\alpha R + \theta/N].$$

Thus the corner $\phi^* = 0$ is obtained whenever $\alpha R_0 > \theta/N$, or equivalently $\theta < N\alpha R_0$, which proves Proposition 4.1. □

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