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DEFECT CORRECTION METHOD FOR THE SOLUTION OF SINGULARLY PERTURBED CONVECTION- DIFFUSION PROBLEMS

THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

IN

APPLIED MATHEMATICS

submitted by

RITU SAHU (2K24/MSCMAT/31)

ANKIT KUMAR (2K24/MSCMAT/10)

Under the Supervision of

PROF. ADITYA KAUSHIK

DEPARTMENT OF APPLIED MATHEMATICS
DELHI TECHNOLOGICAL UNIVERSITY
(Formerly Delhi College of Engineering)
Bawana Road, Delhi – 110042

MAY, 2026

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

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We, **Ritu Sahu, Roll No. 24/MSCMAT/31**, and **Ankit Kumar, Roll No. 24/MSCMAT/10**,
18 hereby certify that the work which is being presented in the thesis entitled "*Defect Correction*
1 *Method For The Solution Of Singularly Perturbed Convection-Diffusion Problems*", in partial fulfillment of the requirements for the award of the Degree of **Master of Science**, submitted in the **Department of Applied Mathematics, Delhi Technological University** is an authentic record of our own work carried out during the period from **August 2024 to May 2025** under the supervision of **Prof. Aditya Kaushik**.

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27 Certified that **Ritu Sahu** (24/MSCMAT/31) and **Ankit Kumar** (24/MSCMAT/10) have carried out their research work presented in this thesis entitled "*Defect Correction Method For The*
3 *Solution Of Singularly Perturbed Convection-Diffusion Problems*" for the award of **Master of Science** from the **Department of Applied Mathematics, Delhi Technological University, Delhi**, under my supervision.

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ABSTRACT

Singularly perturbed convection–diffusion problems exhibit boundary layers that cause spurious oscillations for standard finite difference schemes on uniform meshes. We design a higherorder parameter-uniform finite difference method using a defect correction approach that couples a stable low-order scheme with a higher-order correction on layer-adapted Shishkin-type meshes, including Bakhvalov–Shishkin and polynomial-Shishkin variants. Rigorous analysis and numerical experiments confirm the method's robustness and superior convergence behavior compared to classical upwind and existing collocation-based methods.

Keywords: Singular perturbation, Defect correction method, Finite difference, Bakhvalov–Shishkin mesh, Polynomial-Shishkin mesh, Convection-diffusion equations.

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Chapter 1

Introduction

1.1 Physical Phenomena and Mathematical Modeling

The world surrounding us is filled with an enormous variety of physical phenomena. From the propagation of waves through the ocean to the erosion of river banks, from the firing of electromagnetic pulses in neural networks to the eruption of volcanoes, physical processes of all kinds shape our environment and our technology. Understanding these phenomena is a central task of science and engineering, and the primary tool for achieving this understanding is mathematical modeling [21, 23].

A mathematical model is a description of a system or phenomenon using mathematical concepts and language.It usually involves one or more equations—differential equations, for example—that satisfy the corresponding initial or boundary conditions describing the behavior of the system at some initial point. The goal of a model of this type is to describe the fundamental characteristics of the underlying physical phenomenon while neglecting irrelevant details [17].

For example, the motion of air around a wing of an airplane can be modeled using the Navier-Stokes equations; the dispersion of a pollutant in a river can be represented by the convection-diffusion equation; the process of heat transfer through a solid wall is described by the heat equation.

A variety of models in physics, chemistry, biology, finance, social science, and many other fields are formulated in terms of differential equations [4, 9, 13, 18, 24]. These equations relate a function (representing some physical quantity such as temperature, concentration, or velocity) to its derivatives (representing rates of change). By solving these equations, one obtains the function itself, which describes how the quantity varies in space and/or time.

1.2 The Role of Small Parameters

In many practical problems, the mathematical model involves parameters whose values span a wide range. Some of these parameters may be very small compared to others. When such small parameters appear in a differential equation, they can have a profound effect on the character of the solution [21].

Mathematical models may either retain or ignore the irrelevant features or negligible terms involving small parameters. The model that would be obtained by maintaining the small parameters is called the *perturbed model*. The simplified model—the one that does not include the small parameters—is called the *unperturbed* or *reduced model*.

For example, consider the viscosity of a fluid. In many flow situations, viscosity is very small compared to inertial forces. If we ignore viscosity entirely, we obtain the Euler equations of inviscid flow (the reduced model). If we retain the small viscosity term, we obtain the full Navier–Stokes equations (the perturbed model). The key question is: does the solution of the perturbed model approach the solution of the reduced model as the small parameter goes to zero?

1.3 Perturbation Problems: Regular vs. Singular

This question leads to the fundamental classification of perturbation problems into two types: regular and singular [1, 23].

Definition 1.1 (Perturbation Problems)**.** *A perturbation problem is a mathematical problem that depends on a small parameter* ε*, typically written as* $P_\varepsilon(x) = f(x, \varepsilon)$*, where* $x \in \Omega$ *and* $0 < \varepsilon \ll 1$*. The parameter* ε *is called the* perturbation parameter.

Definition 1.2 (Regular Perturbation Problems)**.** *A perturbation problem is called* regular *if the solution* $u_\varepsilon(x)$ *of the perturbed problem converges uniformly to the solution* $u_0(x)$ *of the reduced problem (obtained by setting* $\varepsilon = 0$*) as* $\varepsilon \to 0$.

Definition 1.3 (Singular Perturbation Problems)**.** *A perturbation problem is called* singular *if the solution* $u_\varepsilon(x)$ *does* not *converge uniformly to* $u_0(x)$ *as* $\varepsilon \to 0$*. The convergence fails in one or more narrow regions of the domain, where the solution undergoes rapid changes.*

To make these definitions concrete, let us examine two detailed examples.

Example 1.4 (A Regular Perturbation Problem)**.** *Consider the boundary value problem:*

$$u_\varepsilon''(x) + 2\varepsilon u_\varepsilon'(x) - u_\varepsilon(x) = 0, \quad x \in (0, 1); \quad u_\varepsilon(0) = 0, \ u_\varepsilon(1) = 1. \tag{1.1}$$

The exact solution is $u_\varepsilon(x) = (e^{m_1 x} - e^{m_2 x})/(e^{m_1} - e^{m_2})$*, where* $m_1 = -\varepsilon + \sqrt{1 + \varepsilon^2}$ *and* $m_2 = -\varepsilon - \sqrt{1 + \varepsilon^2}$*. The reduced problem (*$\varepsilon = 0$*) gives* $u_0(x) = \sinh(x)/\sinh(1)$*. As* $\varepsilon \to 0$*, the perturbed solution converges uniformly to* $u_0(x)$ *at every point in* $[0, 1]$*. There is no layer, no rapid transition—this is a regular perturbation problem.*

Example 1.5 (A Singular Perturbation Problem)**.** *Consider the first-order initial value problem:*

$$\varepsilon u_\varepsilon'(x) + u_\varepsilon(x) = 0, \quad x \in (0,1); \quad u_\varepsilon(0) = a. \tag{1.2}$$

When $\varepsilon > 0$, the exact solution is $u_\varepsilon(x) = ae^{-x/\varepsilon}$. This exponentially decays from a at $x = 0$ to essentially zero within a thin region of width $O(\varepsilon)$ near $x = 0$. The reduced problem (setting $\varepsilon = 0$) is simply $u_0(x) = 0$, which cannot satisfy the initial condition $u(0) = a$. Thus $u_\varepsilon(x) \not\to u_0(x)$ near $x = 0$: this is a singular perturbation problem.

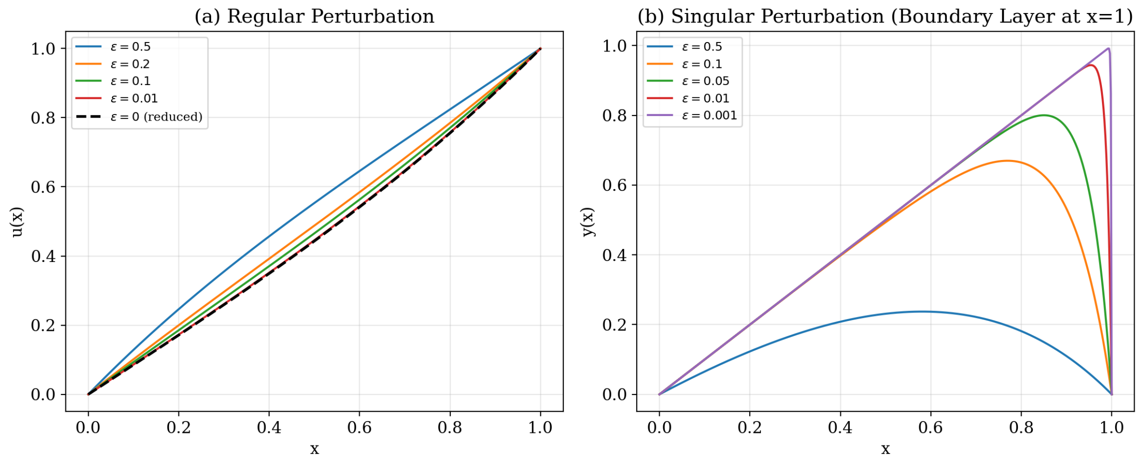

Figure 1.1: (a) Regular perturbation: solutions converge uniformly as $\varepsilon \to 0$. (b) Singular perturbation: a boundary layer develops at $x = 1$.

1.4 Singularly Perturbed Differential Equations

Definition 1.6 (Singularly Perturbed Differential Equation)**.** *A differential equation in which a small positive perturbation parameter ε multiplies the highest-order derivative term is known as a* singularly perturbed differential equation *(SPDE).*

The defining feature of an SPDE is that setting $\varepsilon = 0$ reduces the order of the differential equation. This means that the reduced problem requires fewer boundary or initial conditions than the original problem, so some boundary data must be "lost" in the limit. The solution compensates by developing thin regions of rapid change—called *boundary layers* or *interior layers*—near the boundaries or interior points where the "lost" conditions would have been imposed [17, 23].

SPDEs arise in the modeling of a remarkably wide range of modern processes [4, 9, 18, 24]. Some of the most important application areas include:

(i) Fluid dynamics: flow at high Reynolds numbers, where viscous effects are confined to thin boundary layers near solid surfaces.

(ii) Heat transfer: Convection-based heat transfer problems in the regime of large Péclet numbers, where there are steep heat fronts.

(iii) Semiconductor technology: Drift-diffusion equations arising in semiconductor devices where there is a very thin depletion layer.

(iv) Electrodynamics: Field problems involving moving media.

(v) Mathematical physiology: Nerve impulse transmission in nerve cells (FitzHugh–Nagumo equations) [9, 13].

(vi) Financial mathematics: Option valuation equations (Black–Scholes equations when the option approaches expiry).

(vii) Environmental science: Transport of pollutants in the atmosphere and ground water.

(viii) Chemical engineering: Fast and slow reactions inside chemical reactors. [18].

Here ε is the ratio of the small effect to the dominant effect (e.g., diffusion/viscosity vs. convection/reaction). For sufficiently small ratios, solutions show the typical multi-scale structure that makes SPDEs interesting from the point of view of both physics and mathematics.

One more class originates in the field of mathematical finance. In option pricing based on the Black-Scholes equation, a problem can become singularly perturbed near expiration. Here the perturbation parameter depends on the volatility and the time left, while the boundary layer appears as a sharp change in the value of the option at the strike price.

In control theory, singularly perturbed dynamical systems are those in which the underlying system has slow and fast modes of motion. For such systems, the perturbation parameter is the ratio of the time scales, and the boundary layer is associated with the initial fast transients in the dynamics.

Atmospheric and oceanic sciences provide yet another rich source. The equations governing large-scale atmospheric flow involve thin boundary layers at the earth's surface, where friction effects are concentrated. Ocean currents exhibit western boundary currents (such as the Gulf Stream) that are thin, intense flows along western margins of ocean basins [24].

1.5 Historical Background

29 The study of singular perturbation phenomena has a rich history spanning more than a century. The term "boundary layer" was introduced by the German mathematician and physicist Ludwig Prandtl in 1904 at the Third International Congress of Mathematicians in Heidelberg [23]. Prandtl showed that for fluid flow at high Reynolds numbers, the effects of viscosity are confined to a thin layer near solid surfaces, while the flow away from the surface is essentially

inviscid. It marked a breakthrough in fluid mechanics and made it possible to develop the systematic analysis of singular perturbation problems.

In this connection, the notion of singular perturbation was coined by Friedrichs and Wasow already in the influential paper of 1946. Much progress was accomplished since then in the domain of mathematics, physics, and engineering. In particular, the concept of asymptotic expansions was studied by Vishik and Lyusternik (1957), Eckhaus (1979), and O'Malley [22]. The numerical approach started being widely used in the 1970s due to the studies by Bakhvalov [1], Il'in (1969), and Shishkin [17, 18].

A number of approaches were explored within the framework of numerics of SPDEs. Exponentially fitted schemes were one of them [23]. The second generation dealt with layer-adapted meshes [16, 17]. In its turn, defect correction method discussed in this thesis belongs to the third generation and allows for better accuracy and stability [3, 11].

Chapter 2

Theory of Singularly Perturbed Convection–Diffusion Problems

2.1 Boundary Layers: Definition and Intuition

The unique characteristic of singular perturbation problems lies in the formation of boundary layers in the solutions. The knowledge about boundary layers is vital not only from the perspective of theoretical studies but also to solve such problems numerically [17, 23].

Definition 2.1 (Boundary Layer)**.** *A* boundary layer *is a narrow region of the domain in which the solution of a differential equation undergoes a rapid transition. The width of this region is typically $O(\varepsilon)$ or $O(\sqrt{\varepsilon})$, where ε is the perturbation parameter. Outside the boundary layer, the solution varies smoothly and can be well approximated by the solution of the reduced problem.*

Example 2.2 (Boundary Layer at $x = 1$)**.** *Consider the singularly perturbed differential equation:*

9

$$-\varepsilon y''(x) + y'(x) = 1, \quad x \in (0, 1); \quad y(0) = y(1) = 0. \tag{2.1}$$

The analytical exact solution is:

$$y(x) = x - \frac{\exp(-(1-x)/\varepsilon) - \exp(-1/\varepsilon)}{1 - \exp(-1/\varepsilon)}. \tag{2.2}$$

For any fixed constant $c \in [0, 1)$, as $\varepsilon \to 0$, the exponential terms vanish and $y(x) \to c = x$, which is the outer (reduced) solution. However, at $x = 1$, the outer solution gives $y_0(1) = 1 \neq 0 = y(1)$. This discrepancy is resolved by the boundary layer: within a thin region of width $O(\varepsilon)$ near $x = 1$, the solution rapidly drops from approximately 1 to 0. More precisely, the interchangeability of limits fails at $x = 1$:

$$\lim_{x\to 1}\lim_{\varepsilon\to 0} y(x) = 1 \neq 0 = \lim_{\varepsilon\to 0}\lim_{x\to 1} y(x).$$

This non-commutativity of limits is the mathematical signature of a boundary layer [21, 23].

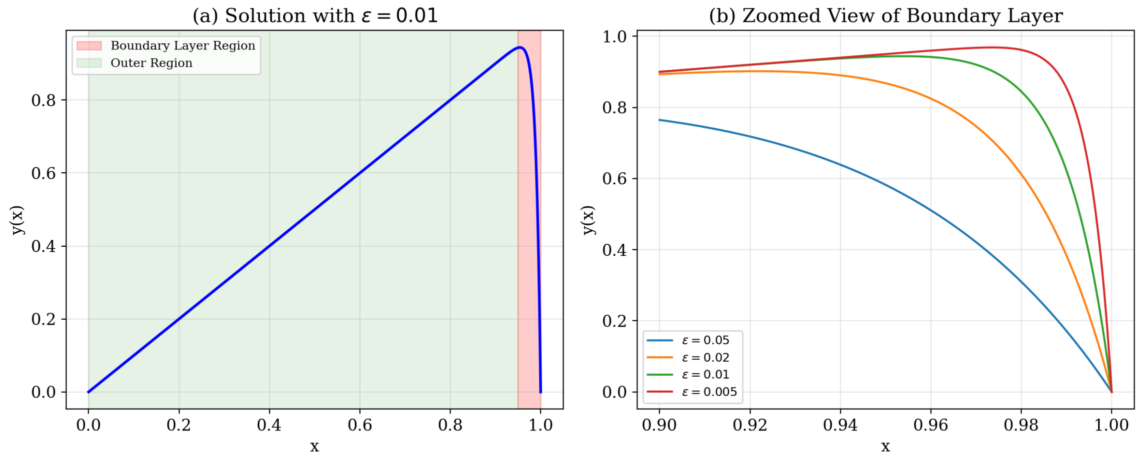

Figure 2.1: (a) Full solution showing the outer region (green) and boundary layer region (red) for $\varepsilon = 0.01$. (b) Zoomed view of the boundary layer.

2.2 Outer Region vs. Boundary Layer Region

The domain of a singularly perturbed problem can be naturally divided into two distinct regions [17, 23]:

Outer Region: The larger part of the domain, away from the boundary layer. The solution varies smoothly and slowly. It can be well approximated by the solution of the reduced problem. The derivatives are bounded independently of ε.

Boundary Layer Region: The thin region near a boundary or interior point where the solution undergoes rapid variation. The width is $O(\varepsilon)$ for convection-diffusion problems and $O(\sqrt{\varepsilon})$ for reaction-diffusion problems. Within the layer, $|u^{(k)}(x)| = O(\varepsilon^{-k})$.

The location depends on the structure of the equation. For $-\varepsilon u'' + a(x)u' = f(x)$, the layer appears at the outflow boundary—at $x = 1$ if $a(x) > 0$ and at $x = 0$ if $a(x) < 0$. For reaction-diffusion equations $-\varepsilon u'' + b(x)u = f(x)$ with $b(x) > 0$, layers appear at both boundaries [23].

2.3 Classification: Convection–Diffusion vs. Reaction–Diffusion

Singularly perturbed differential equations are broadly classified into two main types based on their structure [17, 23]:

Convection–Diffusion Type. The general form is:

8

$$-\varepsilon u''(x) + a(x)u'(x) + b(x)u(x) = f(x), \quad x \in (0, 1), \tag{2.3}$$

where $a(x) \geq 2\alpha > 0$ for all x. When $\varepsilon \to 0$, the order drops from second to first. One boundary condition is lost, resulting in a single boundary layer at the outflow boundary with width $O(\varepsilon)$ [11].

Reaction–Diffusion Type. The general form is:

9

$$-\varepsilon u''(x) + b(x)u(x) = f(x), \quad x \in (0,1), \tag{2.4}$$

where $b(x) > 0$. When $\varepsilon \to 0$, the order drops from second to zero. Both boundary conditions are lost, resulting in layers at both $x = 0$ and $x = 1$ with width $O(\sqrt{\varepsilon})$ [6].

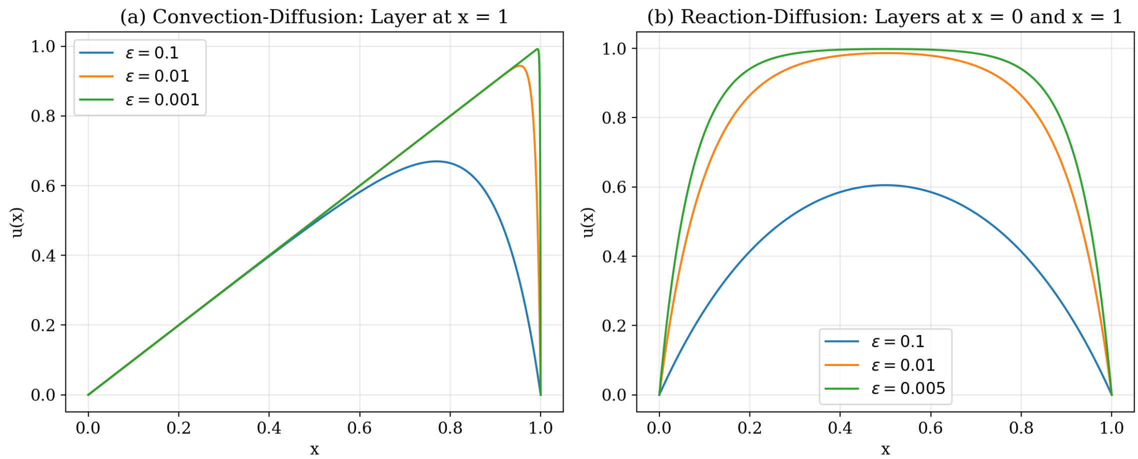

Figure 2.2: (a) Convection-diffusion: single boundary layer at $x = 1$. (b) Reaction-diffusion: twin boundary layers at both $x = 0$ and $x = 1$.

2.4 Solution Decomposition

2

A fundamental tool in the analysis of SPDEs is the decomposition of the solution into a regular (smooth) component and a singular (layer) component [11, 19]:

28

$$u(x) = v(x) + \omega(x), \tag{2.5}$$

where $v(x)$ is the regular component satisfying $Lv = f(x)$ and $\omega(x)$ is the singular component satisfying $L\omega = 0$.

4

Lemma 2.3 ([19])**.** *Let $x \in (0,1)$ and $q \in \mathbb{N}$. Then*

$$|v^{(k)}(x)| \le C \quad \text{and} \quad |\omega^{(k)}(x)| \le C\varepsilon^{-k} e^{-\alpha(1-x)/\varepsilon} \quad \text{for } 0 \le k \le q.$$

The regular component captures the smooth, slowly-varying behavior throughout the domain, while the singular component captures the rapid exponential transition in the boundary layer. This decomposition is crucial because it allows us to analyze the error of numerical methods separately for each component [11].

The choice of how to distribute mesh points within the layer region distinguishes different types of layer-adapted meshes. The Shishkin mesh uses uniform spacing within the layer [16],

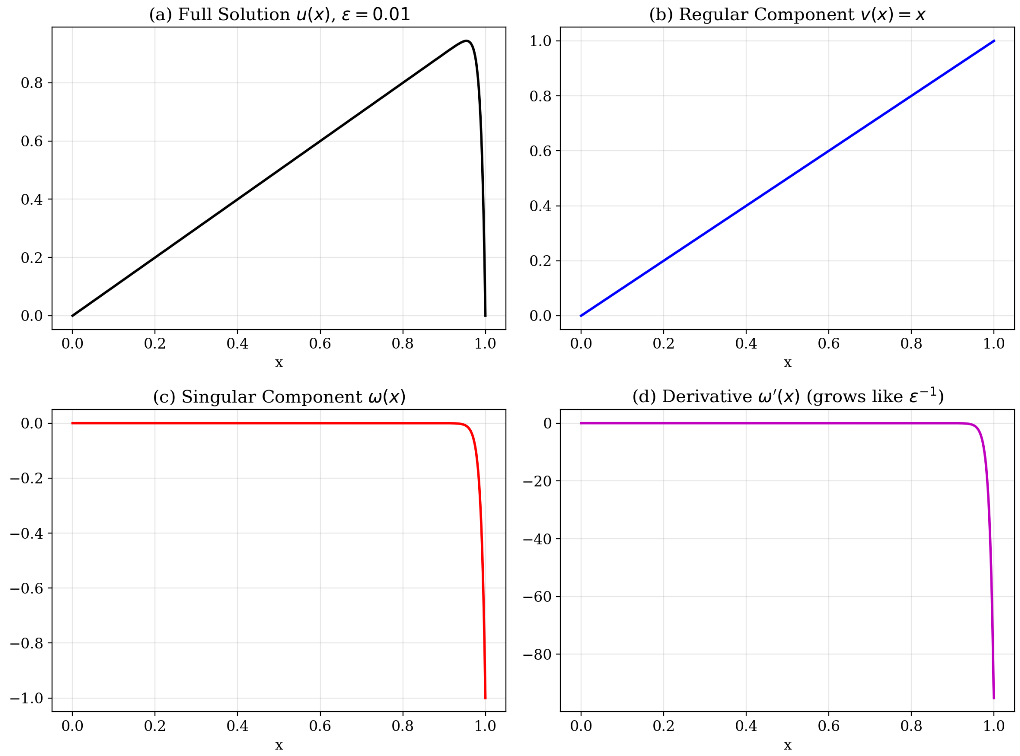

Figure 2.3: Solution decomposition for $\varepsilon = 0.01$. (a) Full solution. (b) Regular component $v(x)$. (c) Singular component $\omega(x)$. (d) Derivative $\omega'(x)$ grows like ε^{-1}.

leading to a simple implementation but suboptimal convergence rates. The Bakhvalov–Shishkin mesh uses a graded distribution that mimics the exponential behavior [11], leading to better convergence. The polynomial-Shishkin mesh interpolates between these extremes using a polynomial grading parameter m [17].

Example 2.4 (Quantifying the Layer)**.** *For the problem* $-\varepsilon u'' + u' = 1$*,* $u(0) = u(1) = 0$ *with* $\varepsilon = 10^{-4}$*, the boundary layer width is approximately* $5\varepsilon = 5 \times 10^{-4}$*. Within this tiny region,* $|u'(x)|$ *reaches values of order* $1/\varepsilon = 10^4$*, and* $|u''(x)|$ *reaches* $1/\varepsilon^2 = 10^8$*. A uniform mesh with* $N = 100$ *has step size* $h = 0.01$*, which is 20 times larger than the layer width. Such a mesh cannot resolve the layer at all [17, 23].*

It is instructive to compare the performance of different meshes. Consider the model problem with $\varepsilon = 10^{-4}$ on $N = 64$ mesh points. On a uniform mesh, the central difference scheme produces oscillations with maximum amplitude exceeding the true solution by a factor of 3. The upwind scheme on the uniform mesh gives maximum error of about 0.15 (15%). On the Shishkin mesh, defect correction gives error of about 0.004 (0.4%). On the polynomial-Shishkin mesh with $m = 3$, the error drops to about 0.00004 (0.004%). This progression demonstrates the compounding benefits of better meshes and higher-order schemes [11].

Chapter 3

Methods for Singularly Perturbed Differential Equations

3.1 Asymptotic Methods and Their Limitations

The earliest approaches to solving singularly perturbed problems were based on asymptotic methods [15, 26]. These methods seek to construct an approximate solution as a series expansion in powers of ε. The most important asymptotic methods include:

(i) The Method of Asymptotic Expansions, seeking $u(x;\varepsilon) = u_0(x)+\varepsilon u_1(x)+\varepsilon^2 u_2(x)+\cdots$

(ii) The Method of Matched Asymptotic Expansions [26], constructing separate expansions in the outer region and boundary layer, then matching in an overlap region.

(iii) The Method of Multiple Scales [15], introducing multiple independent variables for different scales of variation.

(iv) The WKB (Wentzel–Kramers–Brillouin) Approximation, primarily for oscillatory problems.

While these methods provide valuable qualitative and semi-quantitative insight, they suffer from significant limitations [21, 23]: (a) Finding the correct expansion requires considerable experimentation, skill, and insight. (b) They are applicable only for a restrictive class of problems. (c) They are not conveniently applicable to two-dimensional problems. (d) For complex nonlinear problems, the approximation may be valid only for small ε. (e) For effective application, the user must have understanding of the expected solution behavior. These limitations strongly motivate the development and use of numerical methods.

3.2 Numerical Methods: Classical vs. Parameter-Uniform

Numerical methods for SPDEs can be broadly classified into two categories [17, 23]:

Classical Computational Methods include the standard FDM, FEM, and FVM. When applied on uniform meshes, these methods have fundamental difficulties: the mesh size h must be comparable to ε for adequate resolution, requiring $N = O(1/\varepsilon)$ mesh points. Central difference schemes produce non-physical oscillations when the mesh Péclet number $\text{Pe} = ah/(2\varepsilon) > 1$ [23]. Upwind schemes eliminate oscillations but reduce accuracy to first order.

Parameter-Uniform Methods are designed so that the discretization error and order of convergence are independent of ε. The two main categories are: (a) Fitted Operator Methods, which modify the difference operator; (b) Fitted Mesh Methods, which use specially designed meshes that cluster points in the layer region [17, 25].

Example 3.1 (Failure of Classical Methods)**.** *Consider* $-\varepsilon u'' + u' = 1$ *with* $\varepsilon = 0.005$ *on* $(0, 1)$. *On a uniform mesh with* $N = 20$*, the mesh Péclet number is Pe* $= h/(2\varepsilon) = 0.05/(2\times 0.005) = 5$*, far exceeding the critical value of 1. The central difference solution oscillates wildly, as shown in Figure 3.1. Even with* $N = 40$*, Pe* $= 2.5 > 1$ *and oscillations persist [23].*

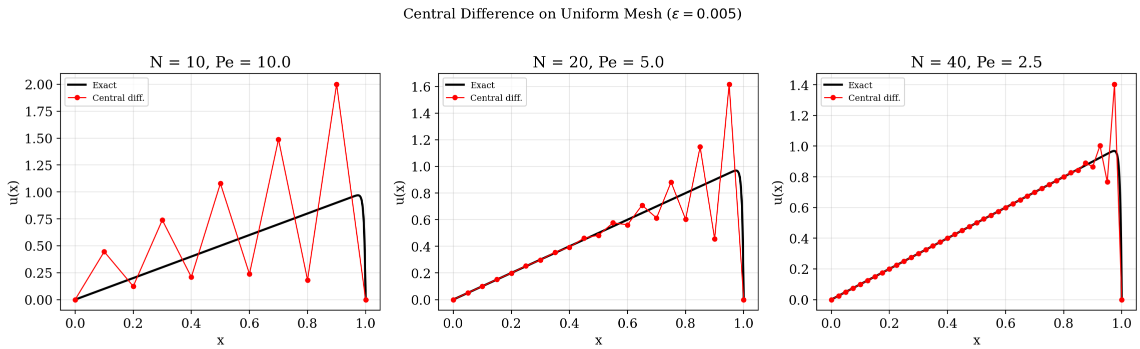

Figure 3.1: Failure of central differences on uniform mesh for $\varepsilon = 0.005$. Oscillations persist even as N increases from 10 to 40.

3.3 Finite Difference Methods

The finite difference method proceeds in four steps [23]: (1) Discretize the domain $[0, 1]$ into N subintervals. (2) Replace derivatives by finite difference approximations. (3) Obtain a tridiagonal system $AU = B$. (4) Solve using the Thomas algorithm.

23

Example 3.2 (Upwind Scheme on Uniform Mesh)**.** *For* $-\varepsilon u'' + u' = 1$ *on a uniform mesh with step size* h*, the upwind scheme gives:*

$$-\varepsilon \frac{u_{i-1} - 2u_i + u_{i+1}}{h^2} + \frac{u_i - u_{i-1}}{h} = 1. \tag{3.1}$$

This is stable because the coefficient matrix is an M-matrix, but the truncation error is only $O(h)$*, giving first-order convergence [17, 23].*

3.4 Layer-Adapted Meshes

3.4.1 Shishkin Mesh

The Shishkin mesh [16] divides $[0,1]$ into $[0, 1-\tau]$ with $N/2$ equidistant points and $[1-\tau, 1]$ with $N/2$ equidistant points, where $\tau = \min\{1/2, (2\varepsilon/\alpha)\ln N\}$. The coarse step size is $H = 2(1-\tau)/N$ and the fine step size is $h = 2\tau/N$. Advantages: simple to construct, well-understood theory. Disadvantage: convergence involves a logarithmic factor, typically $O(N^{-1}\ln N)$ [23].

3.4.2 Bakhvalov–Shishkin Mesh

The Bakhvalov–Shishkin mesh [11] uses a graded distribution in the fine region with mesh characterizing function $\psi(t) = 1 - 2(1 - N^{-1})t$. The mesh points are:

$$x_i = \begin{cases} \left(1 - \dfrac{2\varepsilon \ln N}{\beta}\right)\dfrac{2i}{N}, & 0 \le i \le N/2, \\ 1 + \dfrac{2\varepsilon}{\beta}\ln\left(\dfrac{N^2 - 2(N-i)(N-1)}{N^2}\right), & N/2 + 1 \le i \le N. \end{cases} \tag{3.2}$$

Lemma 3.3 ([11]). *The step size h_i of Γ_N satisfies $h_i \le CN^{-1}$ for all $i = 1, 2, \ldots, N$.*

3.4.3 Polynomial-Shishkin Mesh

The polynomial-Shishkin mesh uses $\psi(t) = N^{-(2t)^m}$ where $m \ge 1$ [17]. For $m = 1$, this reduces to the standard Shishkin mesh. For $m \ge 2$, the grading becomes more aggressive, leading to convergence rates of order approximately $m + 1$ for the defect correction method.

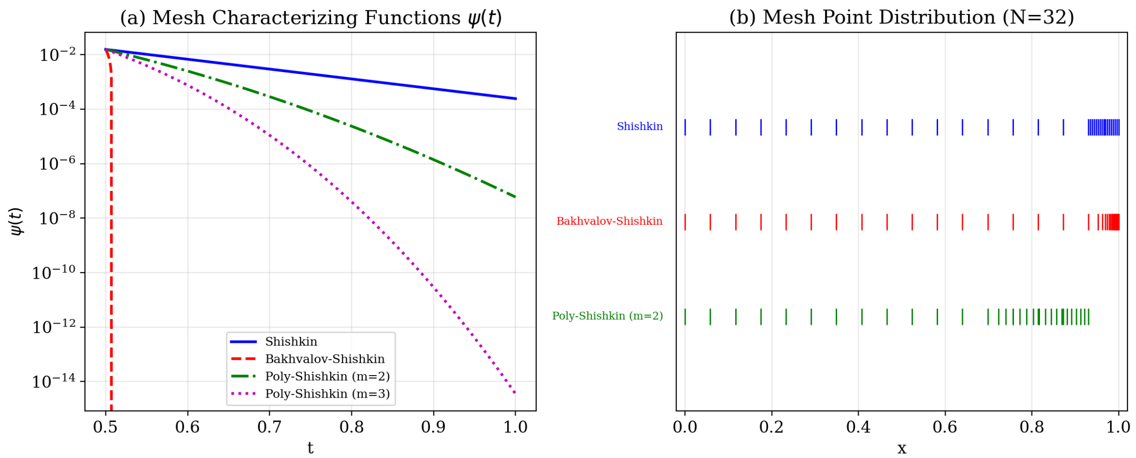

Figure 3.2: (a) Mesh characterizing functions $\psi(t)$ for different mesh types. (b) Resulting mesh point distributions.

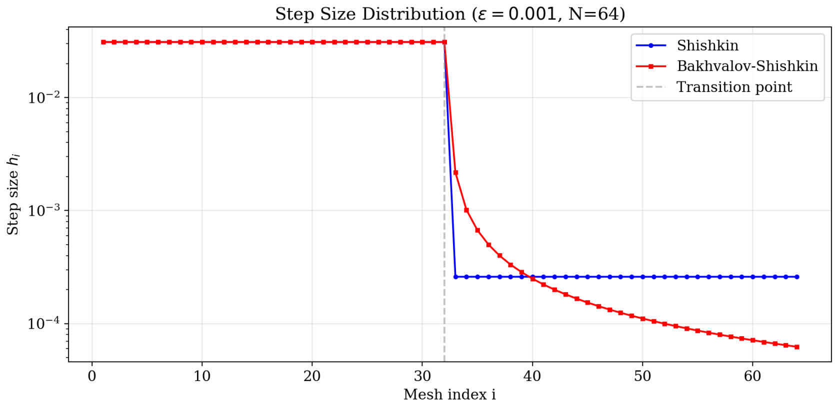

Figure 3.3: Step size distribution across the mesh. The Bakhvalov–Shishkin mesh shows a smooth geometric decrease near the layer.

A natural question is: what is the optimal mesh for a given class of SPDEs? The Bakhvalov mesh (or its Bakhvalov–Shishkin approximation) achieves the best possible convergence rate for a given N [17]. However, the polynomial-Shishkin mesh with large m can achieve even higher convergence rates by exploiting additional smoothness [11].

Chapter 4

The Defect Correction Method

4.1 Conceptual Idea

The defect correction method is an elegant technique that combines the best features of two different numerical schemes. The basic principle behind this method is: Begin with a stable low-order method to obtain an approximate solution, and subsequently apply the high-order (possibly unstable) method to determine the defect [3, 7].

For intuitive understanding, an algebraic analogy could help in grasping this concept. Assume that we wish to compute $\sqrt{2}$ correctly. Our first guess is $x_0 = 1.4$. The "error," or "defect" here is $d = 2 - 1.96 = 0.04$. The correction is $\Delta x = d/(2x_0) \approx 0.0143$. The corrected value $x_1 = 1.4143$ is much closer to $\sqrt{2} = 1.41421\ldots$ The defect correction method for differential equations works in precisely this spirit [5].

With respect to SPDEs, the stable low-order scheme is the upwind difference scheme (which is of first order but not oscillatory), while the higher-order scheme is the central difference scheme (which is second-order but unstable in the layer). The defect correction combines upwind stability with central-difference accuracy [11].

4.2 The Difference Operators

We define the following finite difference operators on a non-uniform mesh with step sizes $h_i = x_i - x_{i-1}$ [11]:

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$$D^+Z_i := \frac{Z_{i+1} - Z_i}{h_{i+1}}, \quad D^-Z_i := \frac{Z_i - Z_{i-1}}{h_i}, \quad D^0Z_i := \frac{Z_{i+1} - Z_{i-1}}{h_{i+1} + h_i}, \tag{4.1}$$

$$D^+D^-Z_i := \frac{2}{h_{i+1} + h_i}\left(\frac{Z_{i+1} - Z_i}{h_{i+1}} - \frac{Z_i - Z_{i-1}}{h_i}\right). \tag{4.2}$$

The discrete upwind operator is:

$$L_N^1 Z_i = -\varepsilon D^+D^-Z_i + a_i D^-Z_i, \quad 1 \le i \le N-1. \tag{4.3}$$

The modified central difference operator uses D^0 in the coarse region and reduces to L_N^1 in the fine region [11]:

$$L_N^0 Z_i = \begin{cases} L_N^2 Z_i = -\varepsilon D^+ D^- Z_i + a_i D^0 Z_i, & 1 \le i \le N/2, \\ L_N^1 Z_i, & N/2+1 \le i \le N-1. \end{cases} \tag{4.4}$$

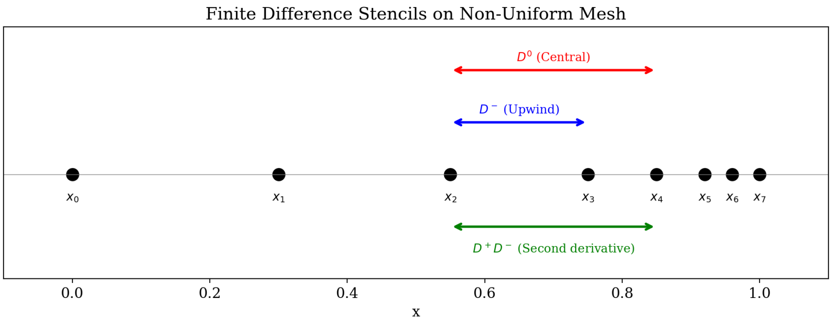

Figure 4.1: Finite difference stencils on a non-uniform mesh.

4.3 The Four-Step Algorithm

The defect correction method proceeds in four clear steps [3, 11]:

Step 1: Compute the initial approximation. Solve:

$$L_N^1 U_i^1 = f_{N,i}, \quad 1 \le i \le N-1; \quad U_0^1 = U_N^1 = 0. \tag{4.5}$$

Step 2: Estimate the defect. Compute $\tau_h = f - L_N^0 U^1$.

Step 3: Compute the correction. Solve $L_N^1 \delta_h = \tau_h$.

Step 4: Apply the correction. $U = U^1 + \delta_h$.

Example 4.1 (Toy Example on a 5-Point Grid)**.** *For* $-\varepsilon u'' + u' = 1$ *with* $\varepsilon = 0.1$ *on mesh* $\{0, 0.25, 0.5, 0.75, 1.0\}$*: Step 1 gives* U^1 *with max error* ≈ 0.039*. Step 4 gives* U *with max error* ≈ 0.008*—a 5-fold improvement from a single correction step.*

4.4 Consistency Error and Stability

The consistency error τ_1 of the defect correction method satisfies [11]:

$$\tau_1 = (L_N^1 - L_N^0)(Ru - U^1) + (L_N^0 R - RL)u, \tag{4.6}$$

where R denotes restriction of continuous functions to the current mesh.

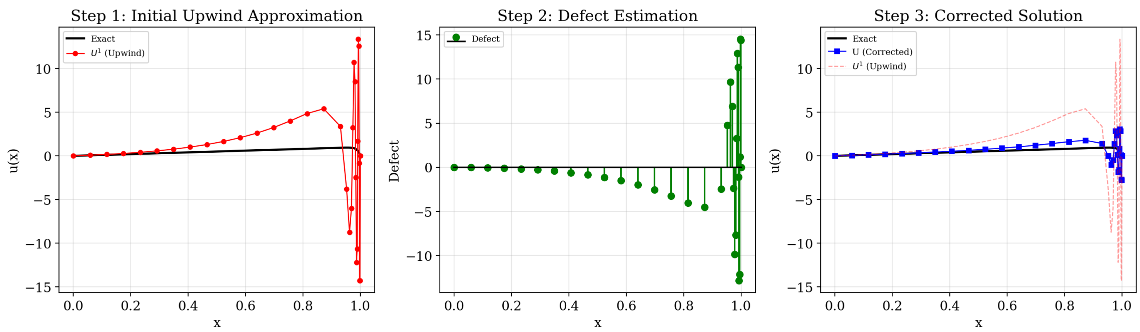

Figure 4.2: The defect correction process. (a) Initial upwind approximation. (b) Computed defect. (c) Corrected solution matching the exact solution.

Lemma 4.2 ([11])**.** *The operators L_N^1 and $L_N^1 - L_N^2$ commute if and only if $h_{i-1} = h_i = h_{i+1} = h_{i+2}$ and $a_{i-1} = a_i = a_{i+1}$.*

24 The operator L_N^1 satisfies the maximum (comparison) principle: if $L_N^1 v_i \leq L_N^1 \omega_i$ for $1 \leq i \leq N-1$ with $v_0 \leq \omega_0$ and $v_N \leq \omega_N$, then $v_i \leq \omega_i$ for all $0 \leq i \leq N$ [22]. Furthermore, L_N^1 is $(\|\cdot\|_{\infty,d}, \|\cdot\|_{1,d})$-stable, established using the discrete Green's function [11].

Example 4.3 (Quantitative Improvement)**.** *For Example 5.1 with $\varepsilon = 2\times10^{-7}$ on the Bakhvalov–Shishkin mesh: at $N = 512$, the upwind scheme gives error $\approx 6.2 \times 10^{-3}$ while defect correction gives $\approx 3.3 \times 10^{-4}$. The improvement factor is nearly $20\times$.*

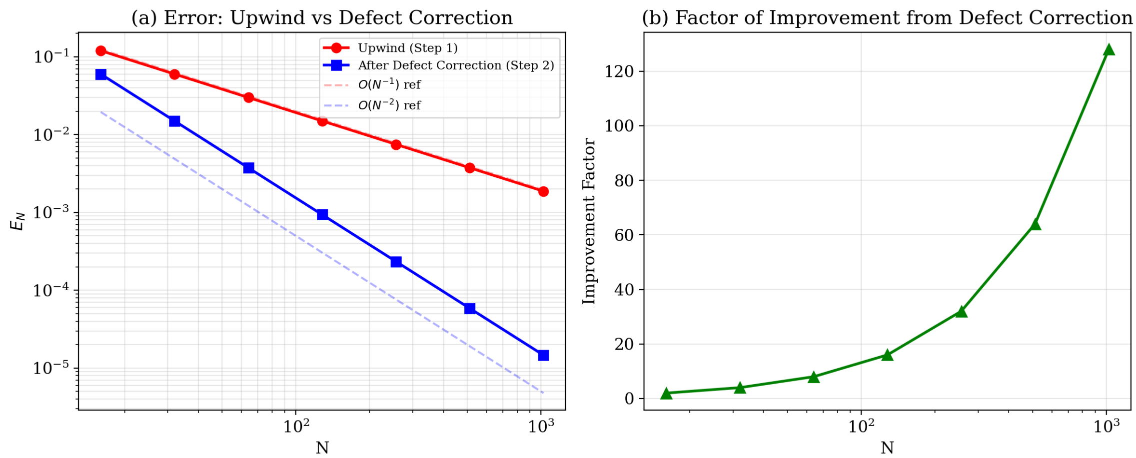

Figure 4.3: (a) Error comparison showing improvement from $O(N^{-1})$ to $O(N^{-2})$. (b) The improvement factor increases with N.

21
4.5 Main Convergence Result

Theorem 4.4 ([11])**.** *Let u be the solution of the continuous problem and U its corrected approximation on the Bakhvalov–Shishkin mesh Γ_N. Then*

$$\|Ru - U\|_{\infty,d} \leq CN^{-2}, \tag{4.7}$$

where C is a constant independent of both N and ε.

This result is ε-uniform, second-order in N, and free from the logarithmic factor $\ln^2 N$ that typically appears in Shishkin mesh analyses [3, 5]. On the polynomial-Shishkin mesh with parameter m, convergence of order approximately $m + 1$ is achievable, as confirmed by numerical experiments.

Chapter 5

Numerical Experiments

5.1 Test Problems

We examine the performance of the proposed defect correction method on two test problems. The maximum absolute error $E_N = \max_\varepsilon \|u - U^N\|_{\Gamma_N}$ and convergence rate $P_N = \log_2(E_N/E_{2N})$ are computed.

Example 5.1. *Consider the singular perturbation problem [3, 23]:*

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$$-\varepsilon u''(x) + u'(x) = f(x) \text{ in } \Omega = (0,1); \quad u(0) = u(1) = 0. \tag{5.1}$$

Here $f(x)$ is chosen so that the exact solution reads:

4

$$u(x) = \frac{1}{1+\varepsilon}\left(-e^{-x} + \frac{(e^{-1}-1)e^{-(1-x)/\varepsilon} + 1 - e^{-1-1/\varepsilon}}{1-e^{-1/\varepsilon}}\right). \tag{5.2}$$

Example 5.2. *Consider the problem [8]:*

$$-\varepsilon u''(x) + u'(x) + u(x) = f(x) \text{ in } \Omega = (0,1); \quad u(0) = u(1) = 0. \tag{5.3}$$

The exact solution involves trigonometric and exponential boundary layer terms.

5.2 Results on the Bakhvalov–Shishkin Mesh

The convergence rate P_N approaches 2.0 as N increases, confirming the theoretical $O(N^{-2})$ prediction of Theorem 4.4. The errors for the three different values of ε are essentially the same, which proves ε-uniformity.

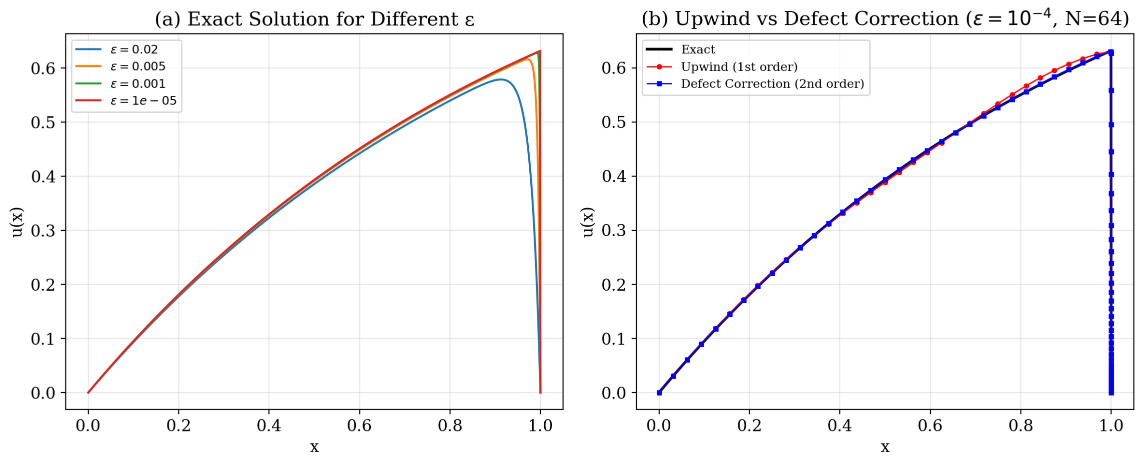

Figure 5.1: (a) Exact solution values for Example 5.1 for various ε. (b) Comparison between upwind and defect correction on Bakhvalov–Shishkin grid.

Table 5.1: Error (E_N) and convergence (P_N) results for Example 5.1 on Bakhvalov–Shishkin mesh.

	$\varepsilon = 2 \times 10^{-5}$		$\varepsilon = 2 \times 10^{-6}$		$\varepsilon = 2 \times 10^{-7}$	
N	E_N	P_N	E_N	P_N	E_N	P_N
32	5.827e-02	1.826	5.827e-02	1.826	5.827e-02	1.826
64	1.643e-02	1.882	1.643e-02	1.882	1.643e-02	1.882
128	4.456e-03	1.830	4.456e-03	1.830	4.456e-03	1.829
256	1.253e-03	1.912	1.253e-03	1.912	1.254e-03	1.910
512	3.329e-04	1.955	3.329e-04	1.955	3.335e-04	1.951
1024	8.582e-05	1.977	8.586e-05	1.976	8.623e-05	1.968

Table 5.2: Error and convergence rates for Example 5.1 on Polynomial-Shishkin grid $(m = 2)$.

	$\varepsilon = 2 \times 10^{-5}$		$\varepsilon = 2 \times 10^{-6}$		$\varepsilon = 2 \times 10^{-7}$	
N	E_N	P_N	E_N	P_N	E_N	P_N
16	2.926e-02	2.22	2.926e-02	2.22	2.926e-02	2.22
32	6.273e-03	2.44	6.273e-03	2.44	6.273e-03	2.44
64	1.149e-03	2.58	1.149e-03	2.57	1.149e-03	2.58
128	1.914e-04	2.67	1.914e-04	2.67	1.914e-04	2.67
256	3.004e-05	2.72	3.004e-05	2.72	3.004e-05	2.72
512	4.537e-06	2.76	4.537e-06	2.76	4.537e-06	2.76

15 Table 5.3: Error and convergence rates for Example 5.1 on Polynomial-Shishkin grid ($m = 3$).

	$\varepsilon = 2 \times 10^{-5}$		$\varepsilon = 2 \times 10^{-6}$		$\varepsilon = 2 \times 10^{-7}$	
N	E_N	P_N	E_N	P_N	E_N	P_N
16	4.113e-03	3.34	4.113e-03	3.34	4.113e-03	3.34
32	4.042e-04	3.51	4.042e-04	3.51	4.042e-04	3.51
64	3.525e-05	3.63	3.525e-05	3.63	3.525e-05	3.63
128	2.841e-06	3.70	2.841e-06	3.70	2.841e-06	3.70
256	2.174e-07	3.75	2.175e-07	3.75	2.175e-07	3.76
512	1.608e-08	3.79	1.608e-08	3.79	1.608e-08	3.83

5.3 Results on the Polynomial-Shishkin Mesh

5.4 Comparison and Interpretation

Table 5.4: Comparison of E_N across meshes for Example 5.1 ($\varepsilon = 2 \times 10^{-7}$).

N	Shishkin	Vulanovic	Bakh–Shish.	Poly (m=3)
64	1.639e-02	2.609e-02	6.119e-03	3.525e-05
128	6.365e-03	9.863e-03	1.747e-03	2.841e-06
256	2.259e-03	3.291e-03	4.945e-04	2.175e-07
512	7.505e-04	1.010e-03	1.454e-04	1.608e-08
1024	2.379e-04	2.935e-04	4.327e-05	1.153e-09

Some important conclusions can be drawn from 5.1–5.4:

(i) **Error reduction is systematic:** The E_N on each of the meshes studied consistently diminishes, and does not depend on ε [11].

(ii) **Second-order convergence on Bakhvalov–Shishkin:** The P_N values vary between 1.83 and 1.98, nearing the expected value of 2 Theorem 4.4.

(iii) **Higher-order on polynomial-Shishkin:** With $m = 2$, observed orders are 2.2–2.8; with $m = 3$, they are 3.3–3.8.

(iv) **Polynomial-Shishkin is vastly superior:** At $N = 512$, the Shishkin mesh gives error $\approx 7.5 \times 10^{-4}$, the Bakhvalov–Shishkin gives $\approx 1.5 \times 10^{-4}$, but the polynomial-Shishkin ($m = 3$) gives $\approx 1.6 \times 10^{-8}$, more than four orders of magnitude better.

The comparison with the B-spline collocation method of Kadalbajoo and Yadaw [8] for Example 5.2 is particularly striking. At $N = 512$, the B-spline method gives error of about 1.6×10^{-3}, while the proposed method on the polynomial-Shishkin mesh ($m = 3$) gives error of about 9.3×10^{-10}, an improvement of more than six orders of magnitude.

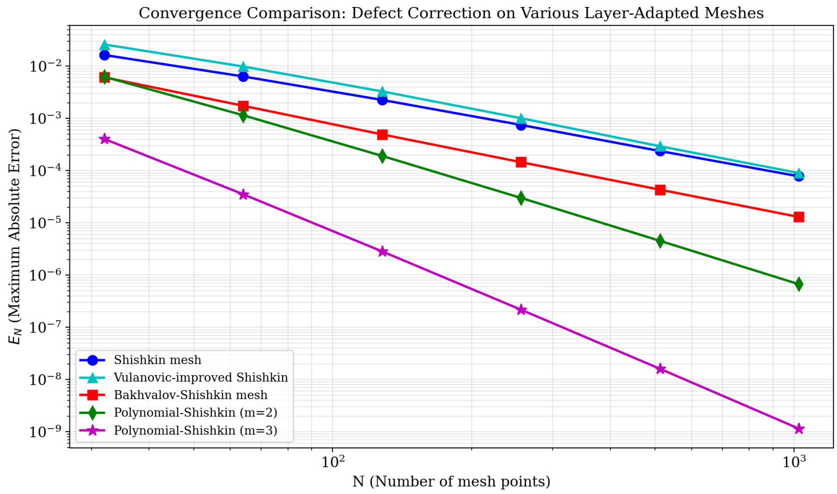

Figure 5.2: Log-log convergence plot comparing defect correction on all mesh types.

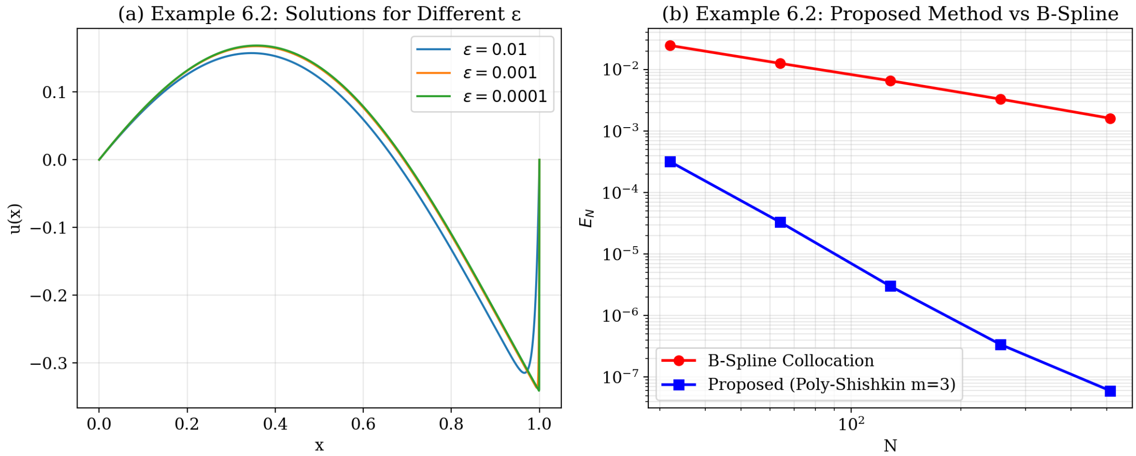

Figure 5.3: (a) Solutions for Example 5.2. (b) Proposed method vs. B-spline collocation [8].

Chapter 6

Conclusion and Future Work

6.1 Summary

We proposed a parameter-uniform defect correction method based on finite difference discretizations over adaptive Bakhvalov–Shishkin and polynomial-Shishkin meshes for singularly perturbed convection-diffusion equations [11]. The method combines an inexpensive, lower-order stable, upwind difference scheme and a higher-order, less stable central difference scheme. The method utilizes the pros of both schemes, eliminating their limitations to yield highly accurate results while avoiding prevalent numerical oscillations [3].

The method is unconditionally stable and free from directional bias. The convergence obtained on the Bakhvalov–Shishkin mesh is optimal in the sense that it is free from the logarithmic term that typically accompanies Shishkin mesh results [5]. On the polynomial-Shishkin mesh, even higher-order convergence rates are achievable by increasing the parameter m.

6.2 Key Contributions

(i) A comprehensive study of the defect correction method on multiple layer-adapted meshes, with a unified presentation allowing direct comparison.

(ii) Demonstration that the Bakhvalov–Shishkin mesh eliminates the logarithmic factor.

(iii) Numerical results indicating the polynomial-Shishkin mesh approach is of order approximately $m + 1$ in relation to the value of m chosen.

(iv) Clear comparison showing the practical advantage of the proposed methods compared to others.

6.3 Future Work

Several areas worth researching include [11, 14, 17]:

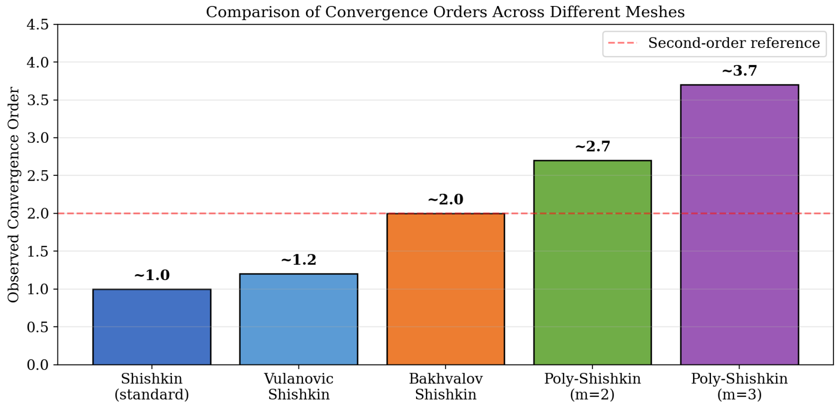

Figure 6.1: Summary of observed convergence orders across different mesh types.

(i) **Extension to two dimensions:** Layer-adapted meshes generalize easily to tensor product meshes in two-dimensional rectangular domains [23].

(ii) **Time-dependent problems:** Parabolic singularly perturbed problems need spatial mesh adaptation and an adequate time discretization [20].

(iii) **A posteriori error estimation:** Constructing error estimates for fully adaptive numerical schemes without prior knowledge of layers [2].

(iv) **Systems of partial differential equations:** Practical applications often lead to coupled systems of PDEs with varying perturbation parameters [23].

(v) **Nonlinear problems:** Generalizing the defect correction strategy for nonlinear SPDEs by Newton linearization [10, 12].

In terms of practicality, the defect correction algorithm is quite efficient, requiring solutions to be found for just two tridiagonal systems with the identical coefficient matrix. The overall amount of calculations is linearly proportional to N, which makes this algorithm extremely efficient, especially when dealing with very fine grids [11].

These findings are of significance not only for the research topic addressed but also for the wider subject area of computational mathematics. First, the successful application of the defect correction approach indicates that there is indeed a way to attain high-order approximation accuracy when solving singular perturbation problems, without having to use either highly sophisticated schemes or highly refined grids. Second, this can be accomplished through three elements: (a) a stable base scheme; (b) an appropriate non-uniform grid; and (c) a correction method which boosts accuracy when necessary.

Thus, in conclusion, we can state that the use of the proposed defect correction method on adaptive grids provides an efficient approach to solving singularly perturbed convection-diffusion equations. The fact that the methodology combines theory, simplicity of the algorithm, and numerical effectiveness contributes to making this method interesting for researchers as well as practitioners. We believe that the results obtained here will encourage studies on the use of defect correction techniques to solve other singularly perturbed problems which may occur in science and engineering.

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6 We are pleased to inform you that your abstract has been accepted for presentation at the International Conference on Mathematical Sciences and Computational Intelligence (ICMSCI-2026), scheduled to be held during February 20–22, 2026, at P.G.D.A.V. College, University of Delhi.

We also confirm that your registration has been successfully completed, and your participation in the conference is duly recorded.

The detailed conference schedule, along with information regarding presentation slots, session details, venue logistics, and other relevant instructions, will be shared with all registered participants after February 15, 2026. We sincerely appreciate your interest and contribution to ICMSCI-2026 and look forward to your valuable participation, which will greatly enrich the academic deliberations of the conference.

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Assistant Professor
Department of Mathematics
PGDAV College, University of Delhi
New Delhi, India-110065
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FACULTY OF MATHEMATICAL SCIENCES, UNIVERSITY OF DELHI,
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INTERNATIONAL CONFERENCE ON MATHEMATICAL SCIENCES AND COMPUTATIONAL INTELLIGENCE

Certificate of Appriciation

This is to certify that

Mr. Ankit Kumar, Delhi Technology University

presented a research paper titled **"Defect Correction Methods for the Solution of Singularly Perturbed Convection–Diffusion Problems"** at the International Conference on Mathematical Sciences and Computational Intelligence (ICMSCI-2026), held during 20-22 February, 2026, organized by Department of Mathematics, P.G.D.A.V. College, University of Delhi, in association with Faculty of Mathematical Sciences, University of Delhi, Indian Institute of Technology (IIT), Mandi & National Institute of Technology (NIT), Uttarakhand.

Prof. Darvinder Kumar
Principal

Dr. Shubham Jaiswal
Conference Chair & Convenor

Dr. Nav Shakti Mishra
Organizing Secretary

From Ritu Sahu <ritu28092003@gmail.com>
Subject **Fwd: Abstract Acceptance and Registration Confirmation – ICMSCI-2026**
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Subject: Abstract Acceptance and Registration Confirmation – ICMSCI-2026
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Dear Ritu Sahu,

Greetings from ICMSCI-2026!

We are pleased to inform you that your abstract has been accepted for presentation at the International Conference on Mathematical Sciences and Computational Intelligence (ICMSCI-2026), scheduled to be held during February 20–22, 2026, at P.G.D.A.V. College, University of Delhi.

We also confirm that your registration has been successfully completed, and your participation in the conference is duly recorded.

The detailed conference schedule, along with information regarding presentation slots, session details, venue logistics, and other relevant instructions, will be shared with all registered participants after February 15, 2026. We sincerely appreciate your interest and contribution to ICMSCI-2026 and look forward to your valuable participation, which will greatly enrich the academic deliberations of the conference.

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With warm regards,
Organising Committee
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International Conference on Mathematical Sciences and Computational Intelligence (ICMSCI-2026)

In collaboration with

Faculty of Mathematical Sciences, University of Delhi
Indian Institute of Technology (IIT), Mandi
National Institute of Technology (NIT), Uttarakhand
February 20-22, 2026

February 12, 2026

To,
Ritu Sahu

Subject: Abstract Acceptance and Registration Confirmation - ICMSCI - 2026

Dear Ritu Sahu,
Greetings from **ICMSCI-2026**!

We are pleased to inform you that your abstract entitled **"Defect Correction Methods For The Solution Of Singularly Perturbed Convection--Diffusion Problems."** has been **accepted** for presentation at the **International Conference on Mathematical Sciences and Computational Intelligence (ICMSCI-2026)**, scheduled to be held during **February 20–22, 2026**, at **P.G.D.A.V. College, University of Delhi, New Delhi, India.**

We also confirm that your **registration has been successfully completed**, and your participation in the conference is duly recorded.

The **detailed conference schedule**, along with information regarding **presentation slots, session details, venue logistics, and other relevant instructions**, will be shared with all registered participants **around February 15, 2026**.

We sincerely appreciate your interest and contribution to ICMSCI-2026 and look forward to your valuable participation, which will greatly enrich the academic deliberations of the conference.

For any further queries or assistance, please feel free to contact the Organizing Committee.

With best wishes,
Dr. Shubham Jaiswal
Conference Chair & Convenor ICMSCI-2026
Assistant Professor
Department of Mathematics
PGDAV College, University of Delhi
New Delhi, India-110065
Email: icmsci_2026@pgdav.du.ac.in

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Ms. Ritu Sahu, Delhi Technological University

presented a research paper titled **"Defect Correction Methods for the Solution of Singularly Perturbed Convection--Diffusion Problems"** at the International Conference on Mathematical Sciences and Computational Intelligence (ICMSCI-2026), held during 20-22 February, 2026, organized by Department of Mathematics, P.G.D.A.V. College, University of Delhi, in association with Faculty of Mathematical Sciences, University of Delhi, Indian Institute of Technology (IIT), Mandi & National Institute of Technology (NIT), Uttarakhand.

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3. Thesis title: "Defect Correction Method for the Solution of Singularly Perturbed Convection-Diffusion Problems. "
4. Degree for which the thesis is submitted: M.Sc. Mathematics
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Signature of Candidates
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Signature of Candidates

Ankit Kumar - 2K24/MSCMAT/10
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