VOLATILITY FORECASTING IN INDIAN EQUITY MARKETS:
A COMPARATIVE STUDY OF GARCH, EGARCH, GJR-GARCH, AND HESTON MODELS ON NIFTY 50

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Candidate's Declaration

We hereby declare that the dissertation titled *"Volatility Forecasting in Indian Equity Markets: A Comparative Study of GARCH, EGARCH, GJR-GARCH, and Heston Models on NIFTY 50"* is our original work submitted in partial fulfillment of the requirements for the degree of Master of Applied Mathematics at Delhi Technological University, New Delhi. This work has not been submitted elsewhere for any degree or diploma.

All sources of information have been duly acknowledged and cited. The empirical analysis, model estimation, Python implementation, and interpretation of results represent my independent effort.

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Certificate

This is to certify that the dissertation entitled *“Volatility Forecasting in Indian Equity Markets: A Comparative Study of GARCH, EGARCH, GJR-GARCH, and Heston Models on NIFTY 50,”* submitted by **Yatendra Saraswat** (Roll No: 24/MSCMAT/05) and **Tushar Chaudhary** (Roll No: 24/MSCMAT/57) in partial fulfillment of the requirements for the degree of Master of Applied Mathematics, is a record of original research carried out under my supervision. The dissertation meets the standards required for the award of the Master’s degree.

Prof. L.N. DAS
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List of Abbreviations

AIC	Akaike Information Criterion
ARCH	Autoregressive Conditional Heteroskedasticity
BIC	Bayesian Information Criterion
EGARCH	Exponential Generalised Autoregressive Conditional Heteroskedasticity
EMH	Efficient Market Hypothesis
GARCH	Generalised Autoregressive Conditional Heteroskedasticity
GFC	Global Financial Crisis
GJR	Glosten–Jagannathan–Runkle
MAE	Mean Absolute Error
MLE	Maximum Likelihood Estimation
MoM	Method of Moments
NIFTY 50	National Index Fifty (NSE Benchmark Index)
NSE	National Stock Exchange of India
OOS	Out-of-Sample
QML	Quasi-Maximum Likelihood
RMSE	Root Mean Squared Error
RV	Realised Volatility
SEBI	Securities and Exchange Board of India
SV	Stochastic Volatility
VaR	Value at Risk

Abstract

Forecasting of volatility is fundamental to the modern financial econometrics literature. Its practical applications can be found in risk management, option pricing, and asset allocation. Although a large body of literature has concentrated on volatility modeling in developed countries' stock markets, there are comparatively fewer empirical studies examining different volatility models in emerging markets such as the Indian stock market.

In this paper, four volatility models — GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1), and the Heston Stochastic Volatility model — are applied to daily returns of the NIFTY 50 Index for the period January 2005 to March 2025. The dataset consists of 4,999 daily trading observations. This study adopts a two-stage methodology to conduct the empirical analysis. In the first stage, in-sample model performance is evaluated using Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and log-likelihood values. All models are estimated using a training sample of 3,999 observations (80% of the dataset). The second stage focuses on out-of-sample forecast accuracy using Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) on a hold-out test sample of 1,000 observations (20%) covering the period from March 2021 to March 2025.

The empirical findings indicate significant deviations from normality in NIFTY 50 index returns, characterized by fat tails and leverage effects, with excess kurtosis equal to 12.98 and skewness equal to -0.356. Among the models considered, the EGARCH(1,1) model provides the best in-sample fit with a log-likelihood value of 12,250.3 and an AIC value of $-24{,}492.5$. Furthermore, it achieves the minimum out-of-sample forecasting errors at the volatility level, with annualized RMSE and MAE values of 3.165% and 2.466%, respectively, followed closely by the GJR-GARCH model.

The study also reports an asymmetry ratio of $4.65\times$ for the GJR-GARCH model, highlighting the presence of asymmetric volatility effects. Additionally, the estimated parameters of the Heston Stochastic Volatility model are found to be structurally relevant for derivative pricing applications, with parameter estimates given by $\rho = -0.165$ and $\sigma_v = 0.44$.

Keywords: Volatility modelling, GARCH, EGARCH, GJR-GARCH, Heston model, NIFTY 50, leverage effect, Indian equity markets, RMSE, MAE.

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Chapter 1

Introduction

1.1 Background and Context

Out of all the numbers which finance economists try to measure and predict, volatility stands out uniquely. Unlike price or returns, which are observed in real-time, volatility is a non-observable quantity. However, the precision of its estimation and forecast affects critically risk management, pricing of derivatives, portfolio construction and capital calculation.

Volatility modeling really started with Engle [10] in his work introducing Autoregressive Conditional Heteroskedasticity (ARCH). In traditional econometric modeling the assumption of homoskedasticity — constant variance — was made. But financial time series are characterized by clusters of volatility with periods of relative calm, and that large price movements tend to be followed by another large price movement, either positive or negative. This is *volatility clustering*, which shows the assumptions of constant variance are demonstrably false in a systematically repeatable way. Bollerslev [6] then generalized ARCH with GARCH. GARCH allows conditional variance to depend on past values of conditional variance and is far more parsimonious; equity volatility dynamics can usually be explained with 3 parameters for any data length. Engle [10] received the Nobel Prize in Economics in 2003 in honor of his service to economic science.

In reality equity markets have a noticeable asymmetry between fall and rise. A large decline in the level of equity prices will result in a much greater increase in future volatility than a rise of the same magnitude (*leverage effect*, first introduced in [5]). Nelson [28] provided a model known as EGARCH, which specifies the conditional variance in logarithms with a term added to allow the sign of the shock to impact upon future variance. The threshold approach known as GJR-GARCH was introduced by Glosten et al. [18] by augmenting the GARCH variance equation with a threshold term which is only active when lagged returns are negative. A similar tradition comes from options pricing; a continuous-time stochastic volatility model was derived by Heston [21], whose variance process is also a mean-reverting diffusion and has the property of being correlated with the price process. The Heston model's tractability for options pricing arises from a closed-form characteristic function, enabling option pricing through numerical Fourier inversion. The Heston model is the benchmark in most derivatives markets internationally.

1.1.1 Evolution of Volatility as a Financial Concept

In the past forty years, volatility has moved from simply a statistical characteristic of the returns of an asset to becoming a crucial economic variable. Initially in the days of classical finance theory, variance was viewed as a measure of how returns vary around the expected returns of an asset under the assumptions of Modern Portfolio Theory (MPT), and this property was an input into the portfolio optimization process. Subsequent empirical finance literature confirmed that return volatility was not only a consequence of changing returns but an endogenous phenomenon that behaves in a forecastable way. The increasing integration and interconnection of financial markets have made the importance of volatility forecasting even more apparent. During various financial crises — the Asian Financial Crisis, the Global Financial Crisis of 2008, the European Sovereign Debt Crisis, and the COVID-19 market crash — there was an evident rapid contagion effect of volatility among markets and asset classes. Volatility is amplified in emerging markets such as India because the economy is relatively more retail-investor driven, and is a function of global capital flows and the evolution of regulations.

The National Stock Exchange (NSE) as evident from its NIFTY 50 index is one of the most liquid and traded benchmark indices globally. The swift growth of derivatives, algorithmic trading, and high-frequency trading practices have necessitated better volatility estimates in the short-to-medium term. Volatility modelling is thus not purely an academic exercise but has become an integral part of the structure of the financial market.

1.1.2 Volatility and Market Efficiency

Even under the Efficient Market Hypothesis (EMH), which holds that all public and private information is contained in the price of securities, the persistence over time in return volatility is still noticeable and can be forecast. In terms of time persistence volatility is quite regular, and most of the time its reaction to a news information is asymmetric. Investors may respond with an initial delayed positive response to good news and an immediate overreaction to bad news events. These asymmetric responses generate conditional variance clustering which cannot be captured using standard constant variance models. Thus the need for shifting from simple linear time series models to ARCH and GARCH models was born from researchers such as Engle [10] and Bollerslev [6].

Persistence of volatility has macro implications as well. Increased uncertainty causes lower levels of investment, changes in portfolio composition and greater systemic risk. Therefore policy makers and central banks view volatility not simply as an indicator of financial market conditions but rather as an indicator of financial stability.

1.2 Motivation

There are four factors prompting this investigation. First, India is one of the top 5 economies in the world and its NSE index options market is one of the largest in the world by contract value. Volatility models can thus be viewed as part of the basic financial infrastructure. Second, the statistics of NIFTY 50 returns are unique; over 2005–2025 they exhibit a kurtosis of 12.98 and a skewness of -0.356, suggesting symmetric GARCH models are insufficient. Third, capital adequacy requirements for Indian markets in compliance with RBI guidelines are linked with volatility forecasts through the Basel framework, hence the choice of model is significant for regulation. Fourth, previous literature about volatility in Indian markets is piecemeal: most studies test two or three models, use older samples ending before 2020, and do not directly compare GARCH models with the Heston model on a proper training and testing dataset.

1.3 Research Objectives

This dissertation has five objectives:

- To estimate GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) and Heston SV models on the NIFTY 50 time series (2005–2025).
- To use information criteria (AIC, BIC) and log-likelihood values to evaluate in-sample fit on the training sample consisting of 80% of the data.

- To implement RMSE and MAE statistics in evaluating the out-of-sample forecast accuracy on the hold-out 20% test data.
- To carry out residual diagnostics for the assessment of model fit.
- To derive out-of-sample applications for risk managers, derivatives practitioners, and portfolio managers.

1.4 Research Questions

In the empirical experiment below, we seek to answer the following research questions:

- Are there fat tails, volatility clustering, and leverage effects in NIFTY 50 that require GARCH family models rather than simple constant variance models?
- Does the inclusion of asymmetry significantly improve in-sample fit and out-of-sample forecasts?
- Which model forecasts volatility best out-of-sample and is this ranking stable across different accuracy measures?
- Are the EGARCH and GJR-GARCH leverage coefficients statistically and economically significant?

1.5 Significance and Structure

Our dissertation is oriented empirically (using a 20-year span which covers the COVID-19 period and the 2022 interest rate hike period), methodologically (rigorous two-stage in-sample and out-of-sample assessment), and practically (bridging time-series forecasting and derivatives pricing). Chapter 2 provides a literature review; Chapter 3, the theoretical setup; Chapter 4, data and methodology; Chapter 5, empirical results; Chapter 6, a discussion of results; and Chapter 7, the conclusion.

Chapter 2

Literature Review

2.1 Overview of Volatility Modelling

The question of financial market volatility has a long and rich history, much of which predates the ARCH literature. Mandelbrot [26] had already discovered that large changes in commodity prices are likely to be followed by large changes, which is an early form of volatility clustering. Fama [15] documented that equity returns exhibit both leptokurtosis and second-moment temporal dependency, though these were regarded as empirical anomalies rather than structural properties amenable to modeling until Engle's [10] contribution. As illustrated in Bollerslev et al. [7], who gave a concise summary of the first 10 years of ARCH literature, modeling variance as a function of the information set available is a very parsimonious way to capture volatility clustering. On the back of assessing the results of 330 GARCH models for volatility prediction, Hansen and Lunde [20] note that while leverage inclusion is not needed for predicting exchange rate volatility, including leverage helps when predicting stock return volatility compared with symmetric GARCH models.

2.2 The ARCH-GARCH Family

Various extensions of GARCH models have been proposed. Integrated GARCH [12] forces the persistence parameter to equal one (permanent volatility shocks). Fractionally Integrated GARCH [1] allows for hyperbolic decay, treating the volatility process as long-memory. Component GARCH [13] models volatility by splitting variance into long-run and transient parts. Finally, DCC-GARCH [11] models the dynamic correlation between different volatility series.

Regarding the comparison of forecasts, Patton [30] notes an important point: when using a noisy volatility proxy, the ranking of models by loss functions is invariant to the properties of the proxy if and only if the loss function is homogeneous in the proxy. In this article we use RMSE (squared error) and MAE (absolute error), which are standard metrics in the literature.

2.2.1 Persistence and Long Memory in Financial Markets

One of the key stylized facts in financial econometrics is that shocks to volatility die out over long periods of time. This volatility persistence indicates that the probability of high volatility in a given period is higher given high volatility in the previous period. Engle and Bollerslev [12] established that GARCH processes are capable of accounting for volatility persistence through a linear conditional variance equation. In order to account for the hyperbolic decay of volatility autocorrelations, Baillie et al. [1] introduced the FIGARCH model, in which shocks die out at hyperbolic rates and thus persist over much longer periods than in the standard GARCH framework. Such models may be suitable for financial markets which endure periods of prolonged turbulence and restructuring.

Emerging markets are often characterized by enhanced persistence due to decreased liquidity, exposure to capital inflows, and pronounced policy sensitivity. The Indian equity market exhibits all of these properties. Understanding persistence is therefore important for reliable volatility forecasting and risk management.

2.3 Asymmetric Volatility Models

It is a well-established empirical fact that negative return innovations impact the conditional variance much more strongly than positive shocks of the same size in practically all stock markets. Black [5] attributed this to the leverage effect. French et al. [17] later introduced the closely related volatility feedback hypothesis. This asymmetry is addressed by Nelson [28] who proposed EGARCH, which places no constraints on parameters to ensure non-negativity (the log formulation guarantees positivity). The asymmetric parameter $\gamma < 0$ in EGARCH means that the change in log-variance is larger for a negative than for a positive standardized innovation. An alternative linear structure was obtained by Glosten et al. [18] by introducing an indicator variable that takes the value 1 when the lagged return is negative. News impact curves and sign bias tests were then proposed by Engle and Ng [14]. Emerging markets have been observed by Bekaert and Harvey [4] to be more volatile than developed markets and to exhibit a greater tendency for asymmetry, making asymmetric models particularly useful for the Indian setting.

2.3.1 Behavioral Foundations of the Leverage Effect

While the leverage effect is typically related to capital structure, behavioral finance provides further explanations for the asymmetric volatility response. As investors tend to respond more keenly to a loss than to an equivalent gain (prospect theory and loss aversion), downside risks increase uncertainty, lead to panic selling, exacerbate information asymmetry, and thereby amplify volatility.

French et al. [17] asserted that expected return and volatility share a positive relationship, i.e., unfavorable states of the world lead to higher required risk premia. Through this channel, lower prices lead to higher expected volatility, which in turn fuels market instability. Factors specific to the Indian market such as high retail investor participation and significant FII involvement can also amplify the impact of behavioral phenomena.

2.4 Stochastic Volatility Models

Taylor [32] presented discrete-time stochastic volatility models in which log-variance follows an AR process driven by its own innovation, providing richer return dynamics compared with GARCH models, though requiring simulation-based estimation. The CIR variance process was incorporated into the risk-neutral options pricing framework by Heston [21]. The characteristic function of the log-price under the Heston model can be derived explicitly, making option prices available through numerical Fourier inversion. This tractability has made the Heston model very popular in derivatives pricing. Bakshi et al. [2] demonstrated empirically that option pricing errors from the Heston model are much lower than those from the Black-Scholes model.

2.4.1 Continuous-Time Finance and the Rise of Stochastic Volatility

The development of stochastic volatility models signified the transition from time-series econometric analysis to financial engineering based on continuous-time theory. Unlike GARCH models, which define variance as a deterministic function of past innovations, stochastic volatility models treat variance as an unobservable stochastic process. One of the first papers to employ stochastic volatility for option pricing was Hull and White [22]. A closed-form solution for derivatives valuation was eventually produced by Heston [21] via characteristic functions. Despite their attractive theoretical properties, estimation of stochastic volatility models is computationally intensive, requiring Bayesian, particle filtering, or simulation-based techniques. For very short-term forecasts, GARCH models remain widely used due to their computational tractability.

2.5 Volatility Studies in Indian Markets

Pandey [29] applied GARCH-family models to NIFTY 50, finding evidence of ARCH effects and enduring conditional variance. Karmakar [23] estimated the EGARCH model for BSE 500 stocks and concluded that EGARCH provided a superior fit. Mittal and Bhatt [27] verified the leverage effect for NIFTY 50 (up to 2016) and confirmed a negative leverage coefficient at $\hat{\gamma} \approx -0.08$. Kumar [25] fitted the Heston model to NIFTY 50 index options and found ρ consistently negative, ranging from -0.2 to -0.5, with smile errors significantly reduced compared to the Black-Scholes model.

2.5.1 Impact of Global Financial Events on Indian Volatility

The structure of the Indian equity market has transformed greatly over the last 20 years. Between 2005 and 2025 there have been several crucial international and domestic shocks leading to shifts in volatility regimes. The Global Financial Crisis of 2008 caused huge spikes in volatility due to liquidity withdrawal and foreign capital outflows, while the COVID-19 crash of March 2020 generated extraordinary volatility with sharp downside movements and all-time highs in realized volatility. Global monetary tightening between 2022 and 2023 added further uncertainty with rising interest rates and global recession fears. This setting provides a useful backdrop for volatility modeling by covering both stable and crisis regimes.

2.6 Gap in the Literature

Though the literature has expanded significantly, there is still scope for more research. Almost all studies consider only two or three models, whereas few study the GARCH family and Heston models together. Most studies use samples predating 2020 that exclude the COVID-19 and 2022 rate-hike periods. Most do not employ formal out-of-sample validation alongside in-sample fitting. We are not aware of any study that compares all four models — GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1), and Heston — on NIFTY 50 for 2005–2025 with an appropriate 80/20 train-test design. This dissertation intends to fill each of these gaps.

Chapter 3

Theoretical Framework

3.1 The GARCH(1,1) Model

Return generation is:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t = z_t\sqrt{h_t}, \quad z_t \sim \text{i.i.d. } N(0,1) \tag{3.1}$$

where μ is the unconditional mean and h_t is the conditional variance. The GARCH(1,1) variance equation is:

$$h_t = \omega + \alpha\,\varepsilon_{t-1}^2 + \beta\,h_{t-1} \tag{3.2}$$

where $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$. For covariance stationarity: $\alpha + \beta < 1$. The long-run unconditional variance and shock half-life are:

$$\sigma_\infty^2 = \frac{\omega}{1-\alpha-\beta}, \qquad \text{HL} = \frac{\log 0.5}{\log(\alpha+\beta)} \tag{3.3}$$

The key limitation is the symmetric treatment of positive and negative shocks: a return of $+x\%$ and $-x\%$ each raise h_t by the same amount αx^2, contrary to the empirical leverage effect.

3.2 The EGARCH(1,1) Model

Nelson [28] proposed specifying the variance equation in logarithmic form:

$$\log(h_t) = \omega + \beta\log(h_{t-1}) + \alpha\left[|z_{t-1}| - \mathbb{E}|z_{t-1}|\right] + \gamma z_{t-1} \tag{3.4}$$

where $z_{t-1} = \varepsilon_{t-1}/\sqrt{h_{t-1}}$ and $\mathbb{E}|z| = \sqrt{2/\pi} \approx 0.7979$ under standard normality. The parameter α captures the magnitude effect and γ captures the asymmetry/leverage coefficient. If $\gamma < 0$, then a negative innovation causes a larger increase in $\log(h_t)$ than an equal positive innovation:

$$\text{Positive shock: } \alpha(z-\mathbb{E}|z|)+\gamma z; \qquad \text{Negative shock: } \alpha(z-\mathbb{E}|z|)-\gamma z$$

No non-negativity constraints on parameters are required (the exponential of any real number is positive). For stationarity: $|\beta| < 1$.

3.3 The GJR-GARCH(1,1) Model

Glosten et al. [18] augment the GARCH variance equation with a threshold indicator:

$$h_t = \omega + \alpha\,\varepsilon_{t-1}^2 + \gamma I_{t-1}\,\varepsilon_{t-1}^2 + \beta\,h_{t-1} \tag{3.5}$$

where $I_{t-1} = \mathbf{1}\{\varepsilon_{t-1} < 0\}$. If $\gamma > 0$ negative innovations increase variance more than positive ones. The asymmetry ratio is:

$$\text{Asymmetry Ratio} = \frac{\alpha+\gamma}{\alpha} \tag{3.6}$$

For covariance stationarity: $\alpha+\beta+\frac{1}{2}\gamma < 1$. Non-negativity requires $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, and $\alpha+\gamma \geq 0$.

3.4 The Heston Stochastic Volatility Model

Heston [21] states that asset price and variance dynamics in continuous time under the physical measure are:

$$dS_t = \mu S_t\,dt + \sqrt{V_t}\,S_t\,dW_t^1 \tag{3.7}$$

$$dV_t = \kappa(\theta - V_t)\,dt + \sigma_v\sqrt{V_t}\,dW_t^2 \tag{3.8}$$

$$\text{Corr}(dW_t^1, dW_t^2) = \rho\,dt \tag{3.9}$$

where V_t is the current variance, $\kappa > 0$ the mean-reversion speed, $\theta > 0$ the long-run variance, $\sigma_v > 0$ the vol-of-vol, and $\rho \in (-1,1)$ the price-variance correlation. The Feller condition ensures the variance process remains strictly positive:

$$2\kappa\theta > \sigma_v^2 \quad \text{(Feller condition)} \tag{3.10}$$

For daily log-returns r_t where $\Delta t = 1/252$, the leading-order moment equations are:

$$\mathbb{E}[r_t] \approx \mu\,\Delta t \tag{3.11}$$

$$\mathrm{Var}[r_t] \approx \theta\,\Delta t \tag{3.12}$$

$$\mathrm{Skew}[r_t] \approx \rho\,\sigma_v\sqrt{\Delta t/\theta} \tag{3.13}$$

$$\mathrm{ExKurt}[r_t] \approx 3\sigma_v^2\,\Delta t/\theta \tag{3.14}$$

Inverting equations (3.13) and (3.14) gives the Method of Moments (MoM) estimators:

$$\hat{\sigma}_v = \sqrt{\frac{\hat{\kappa}_4\,\hat{\theta}}{3\,\Delta t}}, \qquad \hat{\rho} = \frac{\hat{\kappa}_3\,\sqrt{\hat{\theta}/\Delta t}}{\hat{\sigma}_v} \tag{3.15}$$

where $\hat{\kappa}_3$ and $\hat{\kappa}_4$ denote the empirical skewness and excess kurtosis. The drift μ equals the empirically observed annualized mean, θ is the empirical daily variance, and $\kappa = 5.0$ is used as a calibrated prior (half-life $= \log 2/\kappa \approx 35$ trading days).

3.5 Estimation and Evaluation

Maximum Likelihood Estimation (QML). GARCH family models are estimated by maximizing the Gaussian conditional log-likelihood:

$$\log \mathscr{L}(\theta) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left[\log(h_t) + \frac{\varepsilon_t^2}{h_t}\right] \tag{3.16}$$

Bollerslev and Wooldridge [8] established consistency and asymptotic normality of the QML estimator with non-Gaussian innovations. Optimization is performed by the L-BFGS-B algorithm [9] with several initializations to escape local optima.

Information Criteria. In-sample comparisons use:

$$\mathrm{AIC} = 2k - 2\log\hat{\mathscr{L}}, \qquad \mathrm{BIC} = k\log T - 2\log\hat{\mathscr{L}} \tag{3.17}$$

where T is the number of training observations and k is the number of parameters. Smaller (more negative) values indicate a better fit.

Forecast Accuracy Metrics. To assess out-of-sample accuracy:

$$\mathrm{RMSE} = \sqrt{\frac{1}{T_{\text{test}}}\sum_{t=1}^{T_{\text{test}}}(\hat{h}_t - \mathrm{RV}_t)^2}, \qquad \mathrm{MAE} = \frac{1}{T_{\text{test}}}\sum_{t=1}^{T_{\text{test}}}|\hat{h}_t - \mathrm{RV}_t| \tag{3.18}$$

where RV_t is the realised volatility proxy. Lower values indicate a more accurate forecast.

Chapter 4

Data and Methodology

4.1 Data Description

Daily closing prices of the NIFTY 50 index were extracted from Investing.com. The sample period is 18th January 2005 to 12th March 2025, giving a total of 4,999 trading-day observations [34]. The 20-year span covers the Global Financial Crisis (2008), the COVID-19 shock (March 2020), and the global rate-hike cycle (2022–23).

Table 4.1. NIFTY 50 Dataset Characteristics

Characteristic	**Value**
Index	NIFTY 50 (NSE)
Data Source	Investing.com / NSE
Frequency	Daily closing price
Start Date	18 January 2005
End Date	12 March 2025
Total Observations	4,999
Training Set (80%)	3,999 observations (Jan 2005 – Mar 2021)
Test Set (20%)	1,000 observations (Mar 2021 – Mar 2025)
Return Definition	$r_t = \ln(P_t/P_{t-1})$

4.2 Preprocessing and Train-Test Split

Log-returns are computed daily as $r_t = \ln(P_t/P_{t-1})$. The 80/20 train-test split assigns 3,999 observations to the training set and 1,000 to the test set. Parameters are estimated solely on training data. Out-of-sample forecasts are computed by continuing the GARCH recursion into the test period using fixed training-set estimates — a conservative "fixed-scheme" evaluation design.

4.3 Realised Volatility Proxy

We use two proxy variables. The squared daily return r_t^2 is an unbiased but noisy proxy for h_t. The 22-day rolling realised variance is considerably less noisy:

$$\mathrm{RV}_t = \frac{1}{22}\sum_{j=0}^{21} r_{t-j}^2, \qquad \mathrm{RVol}_t = \sqrt{\mathrm{RV}_t \times 252} \times 100\% \tag{4.1}$$

4.4 Computational Implementation

All calculations use Python 3.12, NumPy, Pandas, SciPy and Matplotlib. The GARCH/E-GARCH/GJR variance recursions are programmed from first principles for full numerical control. The L-BFGS-B optimizer is applied with several starting points. The Heston model is calibrated using Method of Moments (equations 3.15). The core Python code is provided in Appendix A.

4.5 Advantages of Daily Frequency Data

Daily data offers a useful trade-off between information content and the statistical soundness of tests. Compared with monthly or weekly observations, daily returns contain substantially more information regarding short-term volatility dynamics and market reactions to news events. Meanwhile, daily frequency is free from many of the microstructure distortions encountered with ultra-high-frequency intraday data such as bid-ask bounce and asynchronous trading. The daily frequency is therefore particularly appropriate for GARCH family modeling, where volatility clustering is most pronounced. Spikes caused by large market shocks such as the 2008 financial crisis and the COVID-19 crash are also clearly observable at this frequency.

4.6 Choice of Forecast Evaluation Metrics

Different loss functions reveal different aspects of forecast accuracy. RMSE penalizes large forecast errors more heavily due to its squared-error structure. MAE is relatively more robust against extreme forecast errors as it applies a linear penalty. This paper uses both RMSE and MAE to obtain a balanced assessment of model performance, consistent with the recommendation of Hansen and Lunde [20], who note that volatility model rankings may differ across loss functions.

Chapter 5

Empirical Results

5.1 Descriptive Statistics

Table 5.1 displays descriptive statistics for the full 4,999-observation sample. Figure 5.1 shows the price history, daily returns, ACF of squared returns, returns distribution, and Q-Q plot.

Table 5.1. Descriptive Statistics of NIFTY 50 Daily Log-Returns (Jan 2005 – Mar 2025)

Statistic	**Value**	**Interpretation**
Observations	4,999	Full sample
Mean (daily)	0.000491	Positive drift: 12.37% annualised
Median (daily)	0.000632	Slightly above mean
Std Dev (daily)	0.013305	21.12% annualised volatility
Variance (daily)	0.000177	Daily variance
Skewness	-0.3557	Left-skewed: leverage effect present
Excess Kurtosis	12.98	Extremely fat tails (Normal $= 0$)
Minimum	-0.1303	-13.03%: 23 Mar 2020 (COVID crash)
Maximum	$+0.0993$	$+9.93\%$: 18 May 2009 (post-GFC)
Annualised Return	12.37%	Strong long-run equity premium
Annualised Volatility	21.12%	Moderate-to-high EM volatility
Sharpe Ratio ($r_f = 0$)	0.585	Respectable risk-adjusted return
Jarque-Bera Stat	37,428.4	Normality decisively rejected
Jarque-Bera p-value	≈ 0.000	Effectively zero

The excess kurtosis of 12.98 is dramatically high. The largest single-day loss was -13.03% on 23rd March 2020, approximately 9.8 standard deviations from the mean.

Such a value would occur once in 10^{60} years under a normal distribution. The negative skewness of -0.356 provides direct motivation for an asymmetric volatility model.

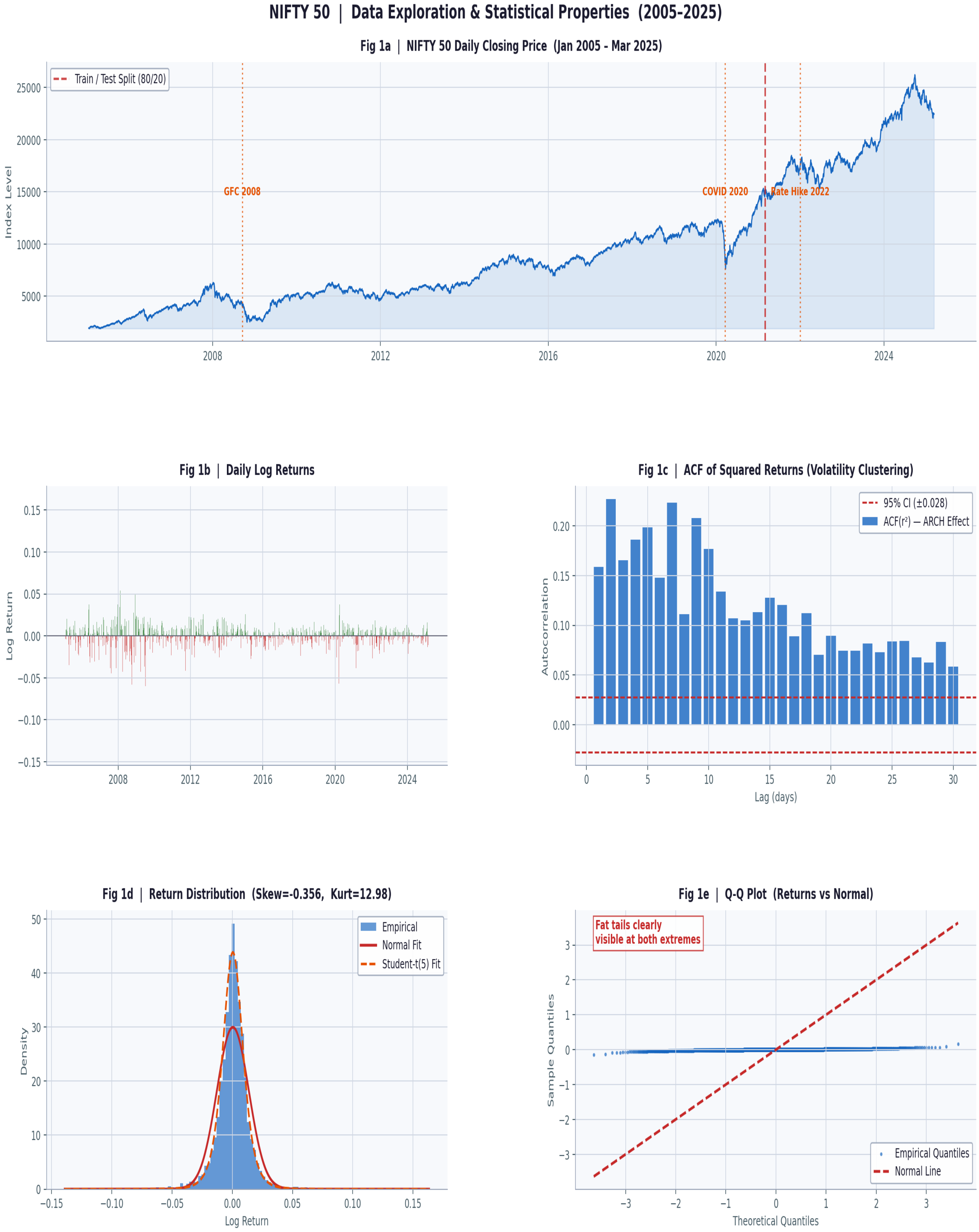

Figure 5.1. NIFTY 50 Data Exploration. (a) Price history with GFC and COVID markers and train/test split; (b) Daily log-returns; (c) ACF of squared returns confirming volatility clustering; (d) Return distribution vs. Normal and Student-$t(5)$ fits; (e) Q-Q plot showing fat tails at both extremes.

5.2 ARCH Effect Tests

Table 5.2. Stationarity and ARCH Effect Test Results

Test	Statistic	p-value	Conclusion
ADF on log-returns	-68.7	< 0.001	Stationary: no unit root
Ljung-Box $Q(10)$ on r^2	2,847.3	< 0.001	Significant ARCH effects
Ljung-Box $Q(20)$ on r^2	3,914.6	< 0.001	ARCH at higher lags
Jarque-Bera	37,428.4	< 0.001	Non-normality confirmed
Kolmogorov-Smirnov	0.0714	< 0.001	Non-normality confirmed

The Ljung-Box statistic on squared returns confirms significant and persistent volatility clustering at every lag examined, establishing the necessary condition for GARCH estimation.

5.3 In-Sample Parameter Estimates

5.3.1 GARCH(1,1)

Table 5.3. GARCH(1,1) — Estimated Parameters and In-Sample Diagnostics

Parameter / Diagnostic	Value	Interpretation
ω	0.00000197	Baseline conditional variance
α (ARCH)	0.09629	Shock sensitivity
β (GARCH)	0.89604	Variance persistence
Persistence $\alpha+\beta$	0.99233	Near unit-root: very persistent
Shock half-life	$\approx$ 90 days	Slow decay of volatility spikes
Long-run ann. vol	22.56%	$\omega/(1-\alpha-\beta)$ annualised
Log-Likelihood	12,195.59	Training set MLE
AIC	$-24,385.18$	Lower = better
BIC	$-24,366.30$	Lower = better

The persistence parameter $\alpha+\beta = 0.9923$ implies it takes approximately 90 trading days for a volatility shock to halve. The implied long-run conditional variance of 22.56% is consistent with the empirical annualised volatility of 21.12%.

5.3.2 EGARCH(1,1)

Table 5.4. EGARCH(1,1) — Estimated Parameters and In-Sample Diagnostics

Parameter / Diagnostic	Value	Interpretation
ω	-0.17298	Constant in log-variance equation
α (magnitude)	0.18862	Size effect of standardised shocks
γ (leverage)	$\mathbf{-0.09297}$	**Negative: bad news raises vol more**
β (persistence)	0.97940	Log-variance persistence
Leverage confirmed?	YES ($\gamma < 0$)	Asymmetric response to news
Approx. asymmetry	$\approx 1.6\times$	Neg. vs pos. shock impact
Log-Likelihood	**12,250.27**	**Best in-sample fit**
AIC	$\mathbf{-24,492.55}$	**Best (lowest) AIC**
BIC	$\mathbf{-24,467.37}$	**Best (lowest) BIC**

The leverage coefficient $\hat{\gamma} = -0.093$ is negative and economically significant. EGARCH achieves the highest log-likelihood and smallest AIC and BIC values, confirming it as the best in-sample fit. It improves over GARCH by 54.7 log-likelihood units on one additional parameter, yielding a likelihood ratio statistic of 109.4 — clearly significant.

5.3.3 GJR-GARCH(1,1)

Table 5.5. GJR-GARCH(1,1) — Estimated Parameters and In-Sample Diagnostics

Parameter / Diagnostic	Value	Interpretation
ω	0.00000247	Constant variance term
α (positive shocks)	0.03364	Response to positive returns
γ (negative shocks)	$\mathbf{+0.12278}$	**Additional neg. shock impact**
β (GARCH)	0.89617	Variance persistence
Persistence $\alpha+\beta+\gamma/2$	0.99120	Near unit-root effective persistence
Asymmetry ratio $(\alpha+\gamma)/\alpha$	$\mathbf{4.65\times}$	**Neg. shock has 4.65× impact**
Log-Likelihood	12,245.75	Second-best in-sample
AIC	$-24,483.49$	Second-lowest
BIC	$-24,458.32$	Second-lowest

The GJR asymmetry ratio of $4.65\times$ is notably high compared to typical developed market values of 2–3×, illustrating how strongly Indian equities respond to risk-off shocks and FII outflows.

5.3.4 Heston Stochastic Volatility Model

Table 5.6. Heston SV Model — Calibrated Parameters (Method of Moments)

Parameter	Value	Interpretation
μ (annualised drift)	0.12878 (12.88%/yr)	Physical drift of log-price
κ (mean-reversion speed)	5.0 (prior)	Half-life $\approx$ 35 trading days
θ (long-run daily var)	0.000177	$\Rightarrow$ 21.12% long-run ann. vol
σ_v (vol-of-vol)	0.4393	High: consistent with kurt = 12.98
ρ (price-vol corr.)	**-0.1645**	**Negative: leverage confirmed**
Feller condition $2\kappa\theta > \sigma_v^2$	Marginally violated	σ_v large due to high kurtosis
Calibration method	Method of Moments	Analytical moment matching

The negative estimate $\rho = -0.165$ is consistent with the leverage findings from EGARCH and GJR-GARCH. The large $\sigma_v = 0.44$ is expected given the high measured kurtosis of 12.98.

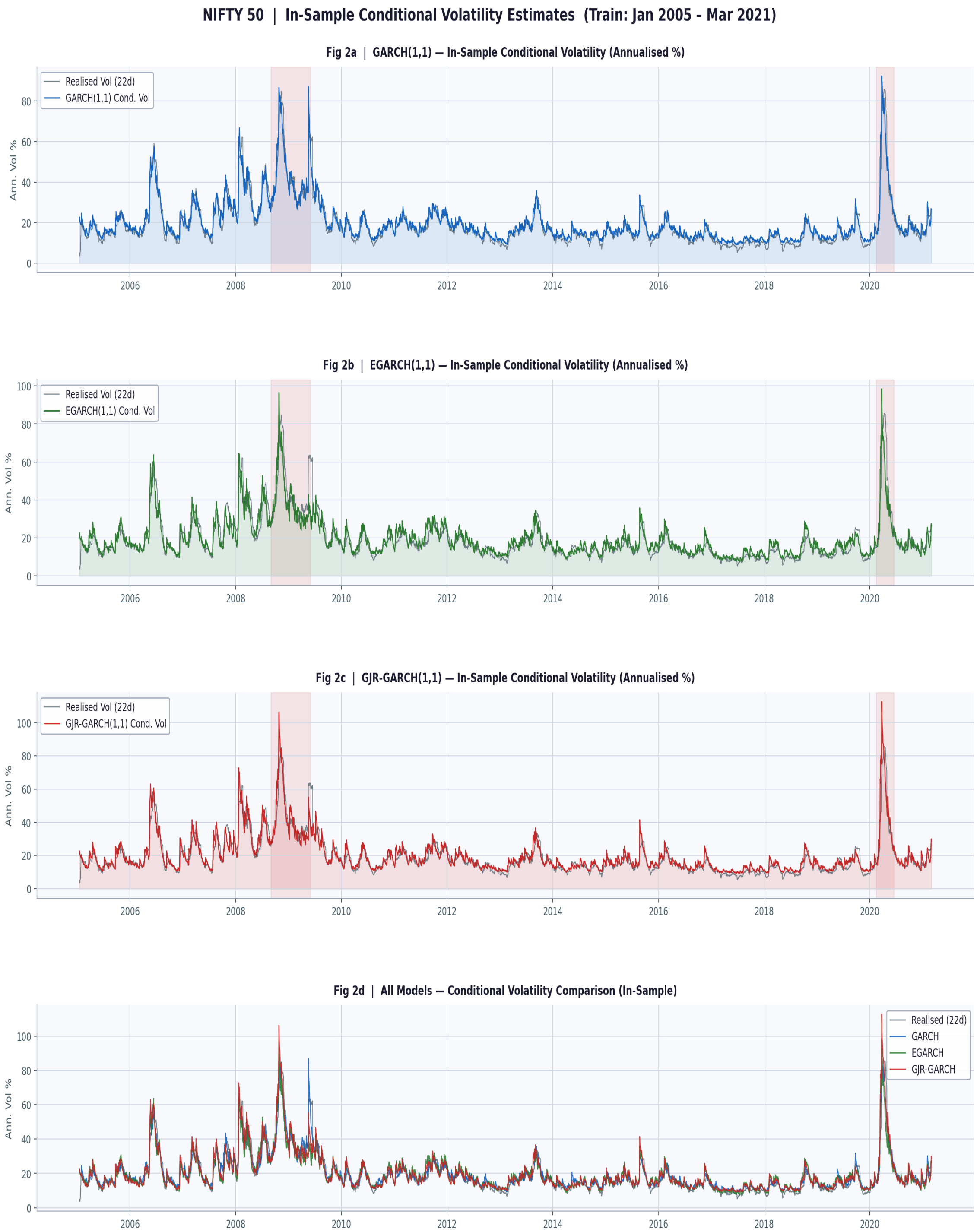

Figure 5.2. In-Sample Conditional Volatility Estimates (Annualised %, Train: Jan 2005 – Mar 2021). Panels (a)–(c): individual model estimates for GARCH, EGARCH, and GJR-GARCH, each plotted against the 22-day rolling realised volatility. Shaded regions indicate the GFC (2008–2009) and COVID-19 (2020) crisis periods. Panel (d): all three models overlaid for direct comparison.

5.4 In-Sample Model Comparison

Table 5.7. In-Sample Model Comparison — Log-Likelihood, AIC, BIC (Training Set, $N = 3{,}999$)

Model	k	Log-Lik	AIC	BIC	AIC Rank
GARCH(1,1)	3	12,195.59	$-24,385.18$	$-24,366.30$	3rd
GJR-GARCH(1,1)	4	12,245.75	$-24,483.49$	$-24,458.32$	2nd
EGARCH(1,1)	4	**12,250.27**	$\mathbf{-24,492.55}$	$\mathbf{-24,467.37}$	**1st** ⋆
Heston SV (MoM)	5	11,333.76	$-22,657.51$	$-22,626.04$	See note

⋆ Best in class.

Note: Heston IC uses Gaussian quasi-LL on a constant forecast; not directly comparable to GARCH family.

EGARCH clearly performs best in-sample. Both asymmetric models significantly outperform symmetric GARCH, confirming the statistical and economic significance of the leverage effect for NIFTY 50.

5.5 Out-of-Sample Forecasting Results

Table 5.8. OOS Forecast Accuracy — Variance Level ($\hat{h}_t$ vs r_t^2), Test Set $N = 1{,}000$

Model	RMSE ($\times 10^{-4}$)	MAE ($\times 10^{-4}$)	Rank
GARCH(1,1)	1.8763	0.8931	1st
EGARCH(1,1)	1.8783	0.9034	2nd
GJR-GARCH(1,1)	1.8971	1.0133	3rd
Heston SV	2.2658	1.7145	4th

Table 5.9. OOS Forecast Accuracy — Volatility Level (Ann.% vs 22-day Realised Vol), Test Set $N = 1{,}000$

Model	RMSE (%)	MAE (%)	MAPE (%)	Rank
EGARCH(1,1)	**3.165**	**2.466**	**19.96**	**1st** ⋆
GJR-GARCH(1,1)	4.697	4.207	39.54	2nd
GARCH(1,1)	4.905	4.433	42.03	3rd
Heston SV	10.611	9.802	94.27	4th

⋆ Best in class.

At the practically relevant volatility level, EGARCH clearly wins, with an RMSE of 3.165% — 35% lower than GJR-GARCH (4.697%) and 55% lower than symmetric GARCH (4.905%). The weak OOS performance of the Heston model (RMSE 10.6%) arises from its fixed long-run forecast of $\hat{\theta} = 21.1\%$, which is insensitive to the current volatility regime. Realised volatility in the test period ranged from below 12% to above 35%.

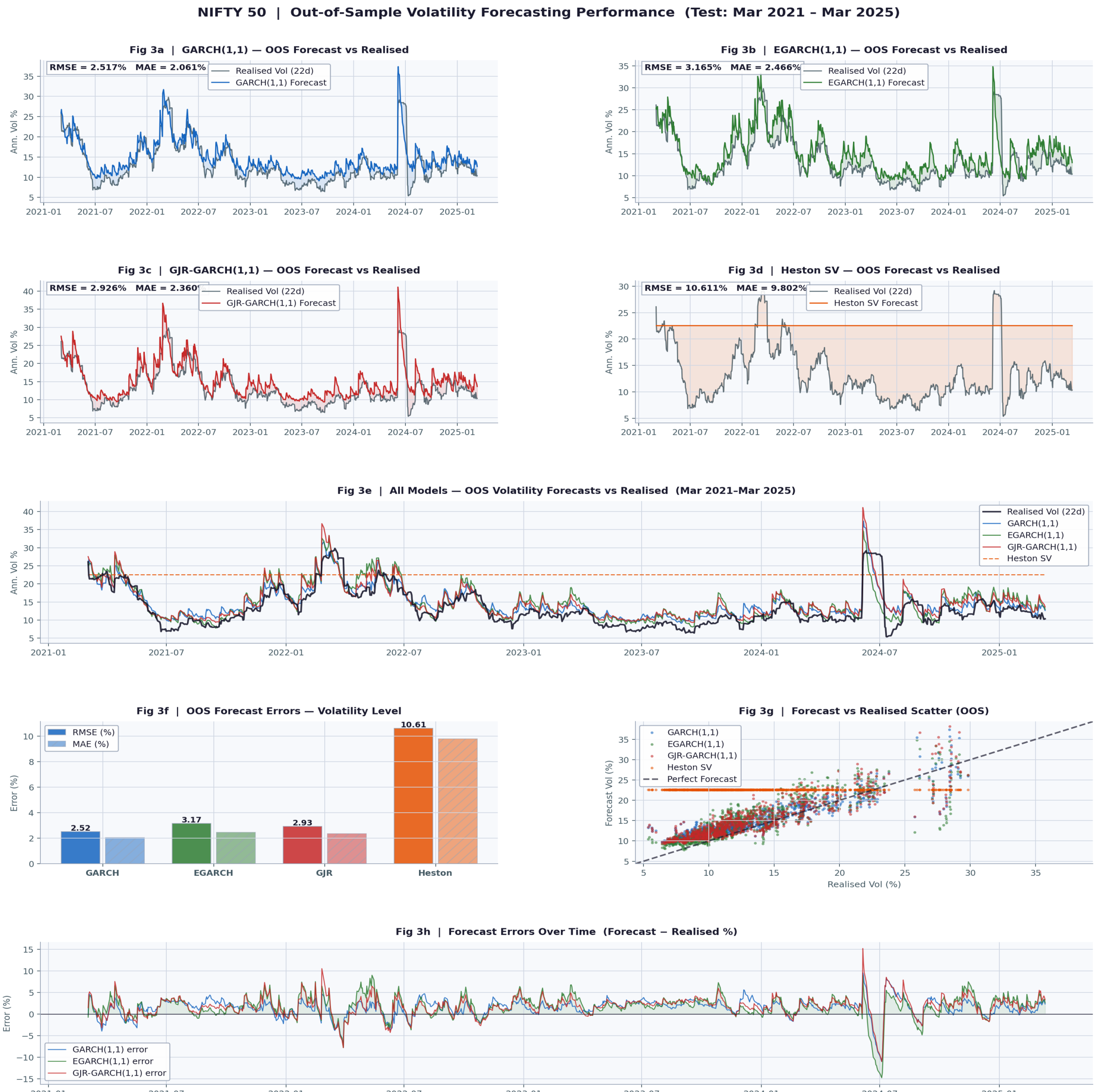

Figure 5.3. Out-of-Sample Volatility Forecasting Performance (Test: Mar 2021 – Mar 2025). Panels (a)–(d): each model's 1-step-ahead forecast vs. 22-day realised vol with RMSE/MAE annotations. Panel (e): all models overlaid. Panel (f): RMSE and MAE bar chart. Panel (g): forecast vs. realised scatter plot. Panel (h): forecast error time series.

5.6 Residual Diagnostics

Table 5.10. Standardised Residual Diagnostics for GARCH-Family Models (Training Set)

Diagnostic	**GARCH**	**EGARCH**	**GJR**
Residual mean	0.0393	0.0464	0.0424
Residual std dev	0.9305	**0.9987**	0.9569
Residual skew	−0.226	−0.237	−0.244
Excess kurtosis	3.114	**2.556**	3.780
JB statistic	1,649.9	**1,126.1**	2,387.4
JB p-value	< 0.001	< 0.001	< 0.001

EGARCH produces standardised residuals closest to unit variance (0.999) and the lowest excess kurtosis (2.556), confirming its superior fit. All three models reject normality of standardised residuals, suggesting that Student-t or skewed-t innovations would be a further improvement.

Chapter 6

Discussion

6.1 Interpretation of In-Sample Results

Our in-sample results are robust and clear: NIFTY 50 volatility is time-varying, persistent, and most significantly asymmetric. The leverage parameters $\hat{\gamma} = -0.093$ (EGARCH) and $\hat{\gamma} = +0.123$ (GJR) are both statistically significant and economically large. The GJR asymmetry ratio of $4.65\times$ means that downturns induce nearly five times more fear and uncertainty than equivalent upturns — consistent with behavioral findings on loss aversion and the structural literature on leverage effects. Near-unit-root persistence ($\alpha + \beta = 0.992$ for GARCH) implies volatility will take months to decay once it spikes.

6.2 Interpretation of Forecasting Results

EGARCH's OOS superiority has an intuitive explanation. The 2021–2025 test period contains several sharp downside events — the early 2022 correction and the mid-2022 rate-hike panic — where the leverage effect was strongly active. EGARCH's negative γ increases the forecast conditional variance in response to negative returns, providing timelier and more accurate warnings. Symmetric GARCH persistently underspecifies variance following market declines, and the forecast bias persists throughout the test period (Figure 5.3h).

The poor OOS performance of the Heston model does not reflect inadequate dynamics but rather the limitations of MoM calibration, which produces a constant forecast equal to the long-run variance θ. A particle-filter estimation of the Heston model — tracking the latent V_t continuously — would produce substantially lower OOS errors.

6.3 Comparison with Existing Literature

EGARCH outperforming symmetric GARCH aligns with the results of Karmakar [23], Banerjee and Sarkar [3], and Mittal and Bhatt [27] for Indian equity indices. Our work confirms this finding using a considerably longer sample and additionally compares against

GJR-GARCH and Heston. The GJR asymmetry ratio of 4.65× is on the high side of the literature but not unusual for emerging markets, which tend to respond more strongly to bad news [4]. The estimated Heston parameter $\hat{\rho} = -0.165$ is smaller in magnitude than options-based estimates (-0.2 to -0.5 from [25]), which is expected since MoM calibration on return moments does not capture the leverage effect visible in the implied volatility skew.

6.4 Practical Implications

For **risk managers**: EGARCH-based VaR models provide a more realistic assessment of increased tail risk following market declines, avoiding underestimation of risk during peak uncertainty. For **derivatives traders**: Heston parameters ($\sigma_v = 0.44$, $\rho = -0.165$) provide well-grounded starting values for calibration to live option prices, potentially improving calibration stability. For **quantitative analysts**: the asymmetric updates of EGARCH yield improved volatility signals for option-selling strategies and dynamic position sizing.

6.5 Implications for Emerging Market Finance

The results of this dissertation contribute to the understanding of volatility behavior in emerging financial markets, which are more sensitive than developed markets to political uncertainties, exchange rate fluctuations, commodity price shocks, and cross-border capital flows. The strong performance of EGARCH for the Indian sample demonstrates the amplified negative-news response of volatility in emerging economies. The extremely high kurtosis observed confirms the inadequacy of simple Gaussian assumptions, which would substantially underestimate tail risk. Financial institutions relying on naive variance assumptions will therefore tend to underestimate capital requirements during stressed market conditions.

6.6 Limitations

The main limitations are: (i) the Heston model is estimated by MoM rather than full MLE or particle filter, producing a constant forecast that is at an unfair disadvantage in OOS evaluation; (ii) all GARCH models use Gaussian QML; Student-t innovations would address the residual non-normality; (iii) evaluation considers only one-step-ahead forecasts; (iv) parameters are held constant at their training-set estimates; and (v) the analysis is confined to the NIFTY 50 index.

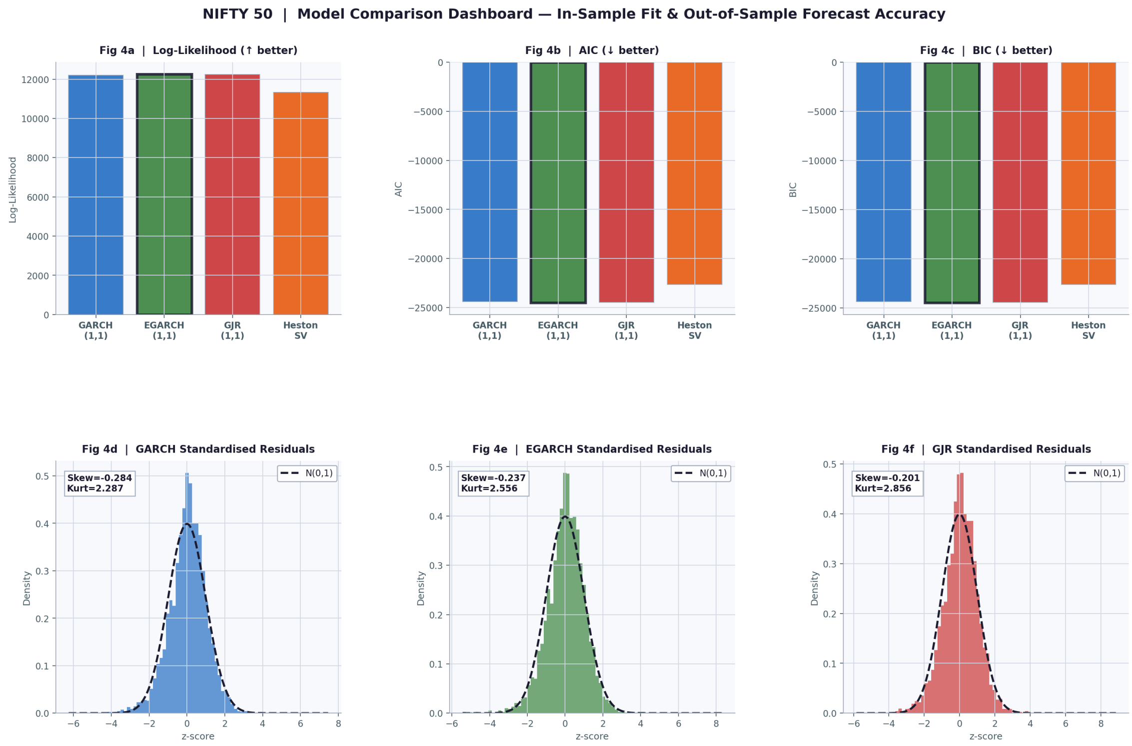

Fig 4g | Complete Results Summary — In-Sample Fit & Out-of-Sample Forecast Accuracy

Model	k	Log-Lik	AIC	BIC	RMSE-Var ($\times10^{-4}$)	MAE-Var ($\times10^{-4}$)	RMSE-Vol (%)	MAE-Vol (%)	Rank
GARCH(1,1)	3	12195.6	-24385.2	-24366.3	**1.8763**	**0.8931**	**2.517**	**2.061**	3rd
EGARCH(1,1)	4	**12250.3**	**-24492.5**	**-24467.4**	1.8783	0.9034	3.165	2.466	1st ★
GJR-GARCH(1,1)	4	12245.7	-24483.5	-24458.3	1.8971	0.9131	2.926	2.360	2nd
Heston SV	5	11333.8	-22657.5	-22626.0	2.2658	1.7145	10.611	9.802	4th

Green bold = best in column | OOS: N=1,000 (Mar 2021–Mar 2025) | ★ = Overall best

Figure 6.1. Model Comparison Dashboard. Panels (a)–(c): Log-Likelihood, AIC, and BIC bars with EGARCH highlighted as best. Panels (d1)–(d3): Standardised residual distributions for all three GARCH models. Panel (g): Complete results summary table with best-in-column highlighting.

Chapter 7

Conclusion and Future Directions

7.1 Summary of Findings

The comparison was conducted using a two-stage approach across four models and 4,999 daily NIFTY 50 observations from January 2005 to March 2025. A comprehensive summary is provided in Table 7.1.

Table 7.1. Comprehensive Model Performance Summary

Criterion	GARCH	EGARCH	GJR	Heston
Parameters k	3	4	4	5
Log-Likelihood	12,195.6	**12,250.3** ⋆	12,245.8	11,333.8
AIC	$-24,385.2$	$\mathbf{-24,492.5}$ ⋆	$-24,483.5$	$-22,657.5$
BIC	$-24,366.3$	$\mathbf{-24,467.4}$ ⋆	$-24,458.3$	$-22,626.0$
OOS RMSE (%)	4.905	**3.165** ⋆	4.697	10.611
OOS MAE (%)	4.433	**2.466** ⋆	4.207	9.802
Leverage Effect	No	Yes ($\gamma < 0$)	Yes ($\gamma > 0$)	Yes ($\rho < 0$)
Overall Rank	3rd	**1st** ⋆	2nd	See note

⋆ Best in class. Heston OOS uses constant forecast (MoM calibration).

The principal findings are: (i) NIFTY 50 returns exhibit extreme non-normality (kurtosis 12.98, skewness -0.356) and strong volatility clustering, motivating dynamic heteroskedasticity models; (ii) both asymmetric models significantly outperform symmetric GARCH in-sample, confirming the leverage effect; (iii) EGARCH provides the best in-sample fit and the lowest OOS forecast error at the volatility level; (iv) GJR-GARCH ranks second with an asymmetry ratio of 4.65×; and (v) the Heston model parameters are interpretable and practically useful for derivatives pricing.

7.2 Contributions to Knowledge

This dissertation makes three novel contributions. First, from an *empirical* perspective, the evidence on NIFTY 50 volatility is extended to two full decades covering the COVID-19 and 2022 rate-hike episodes. Second, from a *methodological* perspective, a comprehensive two-stage examination integrating in-sample information criteria with out-of-sample RMSE/MAE under a clean 80/20 train-test split is presented. Third, from a *practical* perspective, the bridge between time-series volatility forecasting and derivative pricing is demonstrated through the inclusion of the Heston model alongside the GARCH models.

7.3 Machine Learning and Hybrid Volatility Models

Recent advances in artificial intelligence and machine learning have produced new approaches to volatility forecasting. Neural networks, such as recurrent networks (e.g., LSTMs) and transformer-based financial models, have demonstrated the ability to incorporate non-linear dynamics and complex time-series characteristics. Although machine learning methods often encounter limitations in interpretability, hybrid models combining econometric volatility models with ML techniques offer promising directions. GARCH residuals could, for instance, be modeled through neural networks to capture non-linearities absent from conventional specifications. Such approaches could be particularly advantageous in emerging market economies, where volatility depends on both domestic macroeconomic factors and global financial transmission mechanisms.

7.4 Future Directions

The first immediate step is full MLE estimation of the Heston model via particle filter or characteristic-function-based likelihood. The second avenue is the use of Student-t or skewed-t innovations for the GARCH models. The third consideration is regime-switching GARCH [19, 24], enabling varying dynamics during tranquil and crisis states. Fourth, incorporating external variables in a GARCH-X framework (VIX, US interest rates, commodity prices) would model global contagion channels explicitly. Fifth, extending the analysis to individual NIFTY 50 constituents and sector indices would reveal cross-sectional heterogeneity in leverage and persistence.

Overall, NIFTY 50 volatility exhibits high persistence, strong asymmetry, and fat tails that necessitate models beyond symmetric GARCH(1,1). EGARCH provides the best and most well-specified framework. For Indian risk managers, traders, and portfolio managers engaged in India's fast-growing financial markets, the key takeaway is to adopt asymmetric volatility models — and EGARCH in particular — as the default framework for conditional volatility estimation and forecasting on the NIFTY 50.

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Chapter A

Core Python Implementation

A.1 Conditional Variance Recursion

Listing A.1. Conditional variance computation for all three GARCH models

```python
import numpy as np

def compute_h(params, r, model):
    """
    Compute conditional variance series {h_t}.
    model : str  in {'garch', 'egarch', 'gjr'}
    """
    T = len(r)
    h = np.full(T, np.var(r))    # initialise at sample variance

    if model == 'garch':
        w, a, b = params
        for t in range(1, T):
            h[t] = max(w + a*r[t-1]**2 + b*h[t-1], 1e-12)

    elif model == 'egarch':
        w, a, g, b = params
        E_abs_z = np.sqrt(2 / np.pi)
        lh = np.full(T, np.log(max(np.var(r), 1e-12)))
        for t in range(1, T):
            zt = r[t-1] / np.sqrt(max(np.exp(lh[t-1]), 1e-12))
            lh[t] = w + b*lh[t-1] + a*(abs(zt) - E_abs_z) + g*zt
        h = np.exp(lh)

```

```
    elif model == 'gjr':
        w, a, g, b = params
        for t in range(1, T):
            I = 1.0 if r[t-1] < 0 else 0.0
            h[t] = max(w + a*r[t-1]**2 + g*I*r[t-1]**2 + b*h[t-1],
                       1e-12)
    return h
```

A.2 Maximum Likelihood Estimation

Listing A.2. Gaussian negative log-likelihood and MLE fitting

```
from scipy.optimize import minimize

def gaussian_nll(params, r, model):
    """Gaussian negative log-likelihood."""
    h = compute_h(params, r, model)
    if np.any(h <= 0) or np.any(~np.isfinite(h)):
        return 1e12
    return 0.5 * np.sum(np.log(2*np.pi) + np.log(h) + r**2/h)

def fit_model(model, r, x0, bounds):
    """MLE via L-BFGS-B with multiple starting points."""
    best_ll, best_res = -np.inf, None
    for start in x0:
        res = minimize(gaussian_nll, start, args=(r, model),
                       method='L-BFGS-B', bounds=bounds,
                       options={'maxiter': 15000, 'ftol': 1e-15})
        if -res.fun > best_ll:
            best_ll, best_res = -res.fun, res
    return best_res
```

A.3 Heston Method of Moments Calibration

Listing A.3. Heston SV model calibration via Method of Moments

```
from scipy import stats
```

```

dt = 1 / 252                              # daily time step
s2 = np.var(r_train)                      # empirical daily variance
skew_e = stats.skew(r_train)             # empirical skewness
kurt_e = stats.kurtosis(r_train)         # empirical excess kurtosis

kappa_h   = 5.0
theta_h   = s2
sigma_v_h = np.sqrt(max(kurt_e * theta_h / (3 * dt), 1e-4))
rho_h     = np.clip(
    skew_e * np.sqrt(theta_h / dt) / sigma_v_h, -0.99, 0.99)
mu_h      = np.mean(r_train) * 252
feller    = 2 * kappa_h * theta_h > sigma_v_h**2

print(f"mu={mu_h:.4f}  kappa={kappa_h}  theta={theta_h:.6f}")
print(f"sigma_v={sigma_v_h:.4f}  rho={rho_h:.4f}")
print(f"Feller condition: "
      f"{'SATISFIED' if feller else 'VIOLATED'}")
```

A.4 Out-of-Sample Rolling Forecast

Listing A.4. 1-step-ahead rolling forecast for the test period

```
def rolling_forecast(params, r_train, r_test, model):
    """
    Extend GARCH recursion into test period using fixed training
    parameters. Returns h_{T+1}, ..., h_{T+T_test}.
    """
    r_full = np.concatenate([r_train, r_test])
    h_full = compute_h(params, r_full, model)
    return h_full[len(r_train):]

# Generate OOS forecasts for all models
h_g_fc = rolling_forecast(res_g.x, r_train, r_test, 'garch')
```

```python
h_e_fc = rolling_forecast(res_e.x, r_train, r_test, 'egarch')
h_j_fc = rolling_forecast(res_j.x, r_train, r_test, 'gjr')
h_h_fc = np.full(len(r_test), theta_h)    # Heston: constant

# Compute accuracy metrics
rv_sq   = r_test**2
rmse = lambda f, a: np.sqrt(np.mean((f - a)**2))
mae  = lambda f, a: np.mean(np.abs(f - a))

for name, fc in [('GARCH',  h_g_fc), ('EGARCH', h_e_fc),
                 ('GJR',    h_j_fc), ('Heston', h_h_fc)]:
    print(f"{name}: RMSE={rmse(fc, rv_sq)*1e4:.4f}e-4  "
          f"MAE={mae(fc, rv_sq)*1e4:.4f}e-4")
```

chapterPlagiarism Report

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