"Optimizing Investment Portfolios in Banking using Integer Programming Techniques"

A DISSERTATION

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IN

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We, **Manan Aggarwal** and **Naman Anand**, Roll No's – **23/MSCMAT/54** and **23/MSCMAT/51**, students of MSc. (**Applied Mathematics**), hereby certify that the work is being presented as the Major Project in the thesis entitled **"Optimizing Investment Portfolios in Banking using Integer Programming Techniques"** in partial fulfilment of the requirement for the award of the Degree of Master of Science in Mathematics and submitted to the Department of Applied Mathematics, Delhi Technological University, Delhi is an authentic record of my work, carried out during the period from January 2025 to May 2025 under the supervision of **Dr. LN Das**

I have not submitted the matter presented in the report for the award of any other degree of this or any other institute/University.

Place: Delhi	**Manan Aggarwal and Naman Anand**
Date:	**(23/MSCMAT/54) (23/MSCMAT/51)**

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CERTIFICATE

I hereby certify that the Project Dissertation titled " **Optimizing Investment Portfolios in Banking using Integer Programming Techniques** " which is submitted by **Manan Aggarwal** and **Naman Anand**, Roll No's - **(23/MSCMAT/54)** and **(23/MSCMAT/51)**, Department of Applied Mathematics, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of Degree of Master of Science in Mathematics, is a record of the project work carried out by the students under my supervision. To the best of my knowledge this work has not been submitted in part or full for any Degree or Diploma to this University or elsewhere.

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"Optimizing Investment Portfolios in Banking using Integer Programming Techniques"

Manan Aggarwal and **Naman Anand**

ABSTRACT

In the modern banking system, optimizing the allocation of capital across various loan aspects is a critical task that directly impacts profitability and risk management. Traditional portfolio optimization methods often rely on linear programming techniques that assume continuous investment decisions. However, real-world banking constraints—such as regulatory limits, discrete investment units, and risk thresholds—demand more realistic and implementable models.

This thesis explores the application of **Integer Programming (IP)** techniques to optimize investment portfolios in the banking domain. The primary objective is to **maximize net profit** from three major loan categories—Home Loans, Personal Loans, and Business Loans—while adhering to operational constraints such as investment caps, borrower creditworthiness, and diversification rules. The dataset used in this study was **manually created to simulate realistic banking scenarios**, including data on expected profits, borrower creditworthiness, and administrative costs. A **Mixed Integer Linear Programming (MILP)** model was formulated to reflect these constraints, with investment decisions modelled in discrete ₹1 lakh units.

The results indicated an optimal allocation of ₹50 lakhs each to Home and Personal Loans, yielding a maximum net profit of ₹8.50 lakhs. Business loans, though offering a competitive return, were excluded from the final allocation due to relatively lower risk-adjusted performance and constraint tightness. Graphical visualizations were used to interpret allocation patterns and profit contributions, while sensitivity analysis highlighted the binding nature of budget and diversification constraints.

The study demonstrates that Integer Programming not only improves the **practical feasibility of financial decisions** but also allows banks to manage risk while achieving profitability. The model's structure provides a robust foundation for extending into multi-period investment strategies, stochastic interest rate environments, or incorporating credit scoring models in future work.

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Chapter 1

INTRODUCTION

Linear programming (LP) is a powerful mathematical model for optimization, widely used in various fields to solve complex decision-making problems. The application of linear programming in finance has gained significant attention due to its ability to optimize resource allocation, minimize costs, and maximize profits under given constraints. This dissertation explores the financial applications of linear programming, focusing on how LP can be used to address real-world challenges in financial decision-making.

1.1 Linear Programming Model

A Linear Programming Problem (LPP) is a mathematical model designed to optimize a linear objective function, subject to a set of linear constraints. Linear programming is used to find the best outcome (such as maximum profit or minimum cost) in a mathematical model whose requirements are represented by linear relationships. These problems are widely applicable in various fields, including economics, engineering, logistics, and finance.

1.1.1 Key Components of a Linear Programming Problem

1. **Decision Variables**:
    - These are the variables that represent the decisions to be made. The objective is to determine the values of these variables that will optimize the objective function.
    - For example, in a production problem, decision variables could be the quantities of different products to manufacture.
2. **Objective Function**:
    - This is the function that needs to be maximized or minimized. It is a linear equation in terms of the decision variables.
    - For example, in a business scenario, the objective could be to maximize profit or minimize costs.

- **Maximize $Z = c_1x_1 + c_2x_2 + ... + c_nx_n$**, where $c_1, c_2, ..., c_n$ are coefficients and $x_1, x_2, ..., x_n$ are the decision variables.

3. **Constraints**:
   - These are the restrictions or limitations on the decision variables. Constraints are also linear equations or inequalities involving the decision variables.
   - Constraints could represent limitations like available resources (e.g., labour hours, raw materials) or other conditions (e.g., market demand, budget limits).
   - $\mathbf{a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1}$ (and similar for other constraints).
4. **Non-negativity Restriction**:
   - Typically, the decision variables must be non-negative because negative values may not make sense in certain practical situations (e.g., negative quantities of products or services).
   - $\mathbf{x_1 \geq 0, x_2 \geq 0, ..., x_n \geq 0}$

1.1.2 Standard Form of a Linear Programming Problem

An LPP is usually presented in a standard form:

- **Objective Function**: Maximize or Minimize $\mathbf{Z = c_1x_1 + c_2x_2 + ... + c_nx_n}$
- **Subject to Constraints**:
  - $\mathbf{a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1}$
  - $\mathbf{a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leq b_2}$
  - $\mathbf{x_1, x_2, ..., x_n \geq 0}$ (non-negativity constraint)

1.1.3 Example of a Linear Programming Problem

Consider a company that produces two products: **Product A** and **Product B**. The company wants to maximize its profit, given that:

- Product A yields a profit of \$3 per unit, and Product B yields a profit of \$4 per unit.

- The company has 100 units of raw material and 80 hours of labor available. Each unit of Product A requires 2 units of raw material and 1 hour of labor, while each unit of Product B requires 3 units of raw material and 2 hours of labor.

The linear programming model can be formulated as follows:

Objective Function:
Maximize profit:
$\mathbf{Z = 3x_1 + 4x_2}$,
where:

- x_1 = number of units of Product A produced,
- x_2 = number of units of Product B produced.

Constraints:

1. $\mathbf{2x_1 + 3x_2 \leq 100}$ (Raw material constraint),
2. $\mathbf{x_1 + 2x_2 \leq 80}$ (Labor constraint),
3. $\mathbf{x_1, x_2 \geq 0}$ (non-negativity constraint).

The goal is to find the values of $\mathbf{x_1}$ and $\mathbf{x_2}$ that maximize the objective function, subject to the constraints.

1.1.4 Methods for Solving LPP

Linear programming problems can be solved using various methods, including:

Graphical Method:

- Used for problems with two variables. The feasible region is plotted, and the objective function is evaluated at the corner points of the feasible region.
- This method is limited to problems with only two decision variables.

Steps:

- **Formulate** the LPP (objective function and constraints).
- **Convert** constraints into equations and plot them on a 2D graph.
- **Identify** the **feasible region** (intersection of all constraints, including non-negativity).
- **Plot the objective function** (e.g., draw lines for different values of $Z = c_1x + c_2y$).

- **Find the optimal point** by moving the objective function line parallelly until it touches the last point in the feasible region (for maximization or minimization).
- **Evaluate Z** at corner points (vertices) of the feasible region to find the maximum or minimum.

Simplex Method:

- An algorithm used for solving LPPs with more than two variables. It iterates through the vertices of the feasible region to find the optimal solution.

Steps:

- Convert LPP to standard form (introduce slack/surplus/artificial variables).
- Set up the initial simplex tableau.
- Identify entering and leaving variables:
- Entering variable: most negative coefficient in objective row (for maximization).
- Leaving variable: ratio of RHS to pivot column (minimum positive ratio).
- Pivot operation to update the tableau.
- Repeat until no negative coefficient in the objective row (for maximization) → optimal solution reached

Interior Point Methods:

- Used for large-scale linear programming problems. They work by traversing the interior of the feasible region rather than the vertices.

Idea: Instead of moving along the edges of the feasible region (as in simplex), it moves through the interior of the feasible region towards the optimal point.

How it works:

- Starts from an interior feasible point.
- Uses iterative improvement based on mathematical optimization (usually Newton-type methods).
- Approaches the optimal point directly through the interior (not just corner points).

1.1.5 Applications of Linear Programming

Linear Programming (LP) is a powerful optimization technique widely used across various industries and academic disciplines. Its ability to model real-life situations using linear relationships makes it an essential tool in decision-making, resource allocation, and planning.

1. Operations Research & Industrial Engineering

- Production Planning: Optimizing product mix to maximize profit or minimize cost given constraints like labor, materials, and machine time.
- Inventory Management: Minimizing holding and shortage costs while satisfying demand and space limitations.
- Transportation and Logistics: Determining the most efficient route or schedule to minimize transportation costs (e.g., the transportation problem and assignment problem).

2. Finance and Banking

- Portfolio Optimization: Maximizing return or minimizing risk subject to investment constraints.
- Asset Allocation: Distributing capital among different financial instruments while satisfying constraints like risk exposure or investment limits.
- Loan Optimization: Minimizing total interest payments under constraints on available funds and loan structures.

3. Agriculture

- Crop Planning: Selecting optimal crop combinations that maximize yield or profit, given land, water, fertilizer, and labor constraints.
- Livestock Feeding: Formulating low-cost feed blends that meet nutritional requirements for animals.

4. Marketing

- Media Planning: Allocating advertising budget across various media (TV, radio, online, print) to maximize reach or impressions under cost constraints.

- Product Mix Optimization: Determining the best combination of products to offer that maximizes sales or profit under marketing budget and demand constraints.

5. Manufacturing

- Blending Problems: Finding the least-cost combination of raw materials that meet quality standards (e.g., oil refinery or alloy production).
- Workforce Scheduling: Assigning shifts and tasks to workers to meet production targets while minimizing labor costs.

6. Transportation and Network Design

- Optimal Routing: Planning shortest or least-cost paths in a transportation network (e.g., airline scheduling, freight delivery).
- Supply Chain Optimization: Balancing supply and demand across locations while minimizing operational costs.

7. Telecommunications

- Bandwidth Allocation: Distributing bandwidth among various users or services to optimize utilization and minimize cost.
- Network Flow Problems: Designing optimal flow of data through a communication network.

8. Energy Sector

- Power Generation Planning: Minimizing generation costs while meeting demand and regulatory constraints.
- Resource Allocation: Optimizing fuel mix or renewable resource usage under environmental and economic constraints.

9. Health Care and Pharmaceuticals

- Hospital Resource Allocation: Scheduling doctors, nurses, and operating rooms to maximize efficiency.
- Drug Production: Planning production batches under quality, quantity, and cost constraints.

Chapter 2

LITERATURE REVIEW

2.1 Foundations of Portfolio Optimization

Portfolio optimization is a fundamental concept in financial decision-making and investment strategy. It involves selecting the best mix of assets to achieve specific financial goals, typically balancing expected return and risk. The mathematical formulation of this problem forms a core application of optimization techniques, including linear and quadratic programming.

1. Historical Background

The foundations of modern portfolio optimization were laid by Harry Markowitz in 1952 through his Modern Portfolio Theory (MPT). Markowitz proposed that investors can construct an "efficient frontier" of optimal portfolios that offer the maximum expected return for a given level of risk, or equivalently, the minimum risk for a given level of return.

This revolutionary idea introduced the concepts of portfolio diversification and quantitative risk assessment using statistical tools, laying the groundwork for modern financial engineering and asset management.

2. Key Concepts in Portfolio Optimization

a. Portfolio Return

The expected return of a portfolio is the weighted average of the expected returns of the individual assets:

$$R_p = \sum_{i=1}^{n} w_i R_i$$

Where:

- R_p: Expected portfolio return
- W_i: Proportion of investment in asset i
- R_i: Expected return of asset i

b. Portfolio Risk (Variance / Standard Deviation)

Risk is typically measured using the variance or standard deviation of portfolio returns:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

Where:

- σ_{ij}: Covariance between asset i and j

c. Diversification

Combining assets that are not perfectly correlated can reduce overall portfolio risk, even if individual assets are risky.

3. Mathematical Formulation

Objective:

Minimize portfolio risk (variance) or maximize expected return subject to certain constraints.

Common Optimization Models:

a. Mean-Variance Optimization (Markowitz Model):

$$\min_{w} \quad w^T \Sigma w$$

subject to:

$$w^T \mu \geq R_t \quad \text{(target return)}$$
$$\sum w_i = 1$$
$$w_i \geq 0 \quad \text{(no short-selling)}$$

Where:

- w: Weight vector of asset allocations
- Σ: Covariance matrix of asset returns
- μ: Expected return vector

- R_t: Minimum acceptable return

b. Linear Programming Approach (used in simplified portfolio models): When returns and risk are simplified to piecewise-linear functions or constraints:

$$\max \sum_{i=1}^{n} w_i R_i$$
$$\text{subject to: } \sum w_i = 1, \quad \sum w_i \text{Risk}_i \leq R_{max}, \quad w_i \geq 0$$

This form is especially useful for **integer or binary programming** (e.g., when choosing a limited number of assets).

4. Constraints in Real-World Portfolio Optimization

- Budget constraints (total allocation = 100%)
- Minimum/maximum investment limits per asset
- Sector or asset class exposure limits
- Transaction costs and tax implications
- Liquidity and risk tolerance constraints

5. Extensions and Modern Approaches

- Quadratic Programming (QP): Used in the classical Markowitz model due to the quadratic nature of risk (variance).
- Robust Optimization: Addresses uncertainty in expected returns and covariances.
- Integer and Mixed-Integer Programming: Used when portfolio choices are discrete (e.g., buy or not buy).
- Multi-Objective Optimization: Incorporates goals like minimizing risk, maximizing return, and ethical investing.
- Machine Learning Models: Used to forecast returns and volatility before optimization.

2.2 Integer Programming in Portfolio Optimization

Integer Programming (IP) is useful when decisions are binary (yes/no), categorical, or whole-number constrained. It is particularly important in banking where:

- Some assets may require a minimum buy-in
- Portfolio must include specific asset types
- Banks must satisfy regulatory constraints like Basel III norms

In banking, portfolio optimization must consider:

- Capital allocation regulations
- Credit exposure limits
- Stress testing and Value-at-Risk (VaR) constraints

Why Integer Programming in Banking Portfolios?

In a banking context, when allocating capital across various loan products (e.g., home, personal, business loans), the decision must account for:

- Regulatory caps or floors (e.g., not more than 50% in any one category)
- Risk thresholds based on creditworthiness or default history
- Indivisible units (e.g., investing in chunks of ₹1 lakh or ₹5 lakh)
- Operational feasibility, like not splitting loans into fractional units

Benefits of Integer Programming in Portfolio Decisions

- Feasibility: Ensures allocations are in actionable, whole-number units
- Customizability: Incorporates hard constraints like regulatory or policy limits
- Precision: Models real-world banking rules like credit rating thresholds, branch-level allocations
- Risk Management: Easily integrates risk-return trade-offs using additional constraints

Challenges in Integer Programming

- Computational Complexity: Integer programs are NP-hard; large-scale problems may require specialized solvers or heuristics
- Solution Time: Compared to LP, IP solutions can be slower, especially with many binary / integer constraints
- Scalability: Additional dimensions like time, stochastic elements, or multi-objective trade-offs add further complexity

Chapter 3

PROBLEM STATEMENT AND OBJECTIVES

3.1 Problem Statement

In the modern banking environment, portfolio management is a critical task involving complex decision-making under various financial, regulatory, and operational constraints. Traditional portfolio optimization models, such as those based on Modern Portfolio Theory, often rely on simplifying assumptions like fractional asset allocation, the absence of transaction costs, and continuous variables. These assumptions are not realistic in the context of banking, where decisions often involve discrete choices, minimum investment thresholds, regulatory compliance, and strict risk exposure limits.

Therefore, there is a need for a more practical and robust model that can incorporate such real-world constraints. Integer programming offers a powerful framework to handle discrete decision variables and complex constraints, making it suitable for portfolio optimization in banking scenarios. This research seeks to develop and apply an integer programming model for optimizing investment portfolios within a banking context, aiming to enhance decision-making effectiveness while adhering to real-world financial and regulatory boundaries.

3.2 Objective

- To understand and analyze the limitations of traditional portfolio optimization models in the context of banking investment decisions.
- To formulate a mathematical model using Integer Programming that captures realistic banking constraints, including:
    i. Minimum and maximum investment bounds
    ii. Sector-wise allocation limits
    iii. Regulatory compliance constraints
- Binary or integer decision variables (e.g., select/do not select an asset)
    i. To apply the model to a real or simulated investment dataset using appropriate computational tools (such as Python).
- To solve and evaluate the performance of the integer programming model in terms of:

i. Return optimization
ii. Risk minimization
iii. Asset diversification
iv. Regulatory compliance

- To compare the results of the integer programming model with traditional (continuous) optimization models and analyze the trade-offs in terms of accuracy, feasibility, and computation.
- To provide insights into how integer programming can be integrated into banking portfolio management strategies for improved investment decisions.

3.3 Scenario Overview

A bank has a fixed investment budget and must decide how to allocate its funds across different types of loans or investment opportunities. Each option yields:

- Expected profit (e.g., interest earned)
- Administration costs
- Risk of default or payment issues (represented by a "paying ability" score)

The bank wants to maximize its total expected net profit (interest income minus administrative cost) while:

- Staying within a total investment budget
- Limiting exposure to low-credit borrowers
- Diversifying across loan types

3.4 Decision Variables and Parameters

Let:

- x1: Amount invested in home loans (in ₹ lakhs)
- x2: Amount invested in personal loans
- x3: Amount invested in business loans

Loan Type	Expected Interest Income (%)	Admin Cost (%)	Paying Ability Score (0–1)
Home Loans	8%	1.5%	0.95
Personal Loans	14%	3.5%	0.65
Business Loans	12%	2.5%	0.80

- Total available investment budget: ₹100 lakhs
- Minimum paying ability score (average): ≥ 0.8
- Diversification: no more than 50% of total budget in a single loan type

3.5 Objective Function and Constraints

Maximize net expected profit (interest – admin cost):

Maximize $Z = (0.08-0.015)\ x_1 + (0.14-0.035)\ x_2 + (0.12-0.025)\ x_3$

$\Rightarrow Z = 0.065x_1 + 0.105x_2 + 0.095x_3$

1. Budget Constraint

$x_1 + x_2 + x_3 \leq 100$

2. Paying Ability Constraint (Weighted Average ≥ 0.8)

$$\frac{0.95x1+0.65x2+0.80x3}{x1 + x2 + x3} \geq 0.8$$

Multiply both sides by $x_1+x_2+x_3$ (assuming non-zero investment):

$0.95x_1 + 0.65x_2 + 0.80x_3 \geq 0.8(x_1 + x_2 + x_3)$

Simplify:

$(0.95-0.8)\ x_1 + (0.65-0.8)\ x_2 + (0.80-0.8)\ x_3 \geq 0$

$\Rightarrow 0.15x_1 - 0.15x_2 + 0x_3 \geq 0$

3. Diversification Constraint (no more than 50% in one loan type)

$x_1 \leq 50, x_2 \leq 50, x_3 \leq 50$

4. Non-negativity Constraints

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

3.6 Complete LP Model

We aim to **maximize** total net profit, calculated as:

$Z=0.065x_1 + 0.105x_2 + 0.095x_3$

Subject to:

$x_1 + x_2 + x_3 \leq 100$ (Budget)

$x_1 \geq x_2$ (Paying ability)

$x_1 \leq 50, x_2 \leq 50, x_3 \leq 50$ (Diversification)

$x_1, x_2, x_3 \geq 0$

Where:

- Home Loan: 8.0% interest − 1.5% admin cost = 6.5% net
- Personal Loan: 12.0% − 1.5% = 10.5% net
- Business Loan: 11.0% − 1.5% = 9.5% net

Chapter 4

METHODOLOGY

The methodology of this research is rooted in Operations Research and Quantitative Finance, focusing on the application of Integer Programming (IP) techniques to solve a real-world financial allocation problem in banking. The study follows a structured workflow from problem definition to model implementation and solution analysis.

4.1 Research Design

This study employs a quantitative analytical framework using mathematical programming techniques to simulate and optimize banking investment decisions. The decision environment is modelled through a constrained optimization problem, where the goal is to allocate limited capital resources across different loan categories to maximize net returns.

4.2 Loan Categories and Financial Parameters

Three types of loans are considered for investment:

- Home Loans (Low risk, moderate return)
- Personal Loans (High return, higher risk)
- Business Loans (Moderate return, moderate risk)

Each loan category is associated with:

- Interest Rate (IR) – representing income
- Administrative Expense (AE) – representing cost
- Net Profit per Unit = IR − AE
- Paying Ability (PA) – a measure of creditworthiness on a scale of 0 to 1

These were modelled based on industry averages from RBI and financial reports.

4.3 Mathematical Formulation

Let the decision variables be:

- X_1: Amount invested in Home Loans (₹ Lakhs)
- X_2: Amount invested in Personal Loans (₹ Lakhs)
- X_3: Amount invested in Business Loans (₹ Lakhs)

Objective Function:

Maximize total net profit:

Maximize $Z=0.065x_1 + 0.105x_2 + 0.095x_3$

Where the coefficients represent net profit percentages (interest rate − admin cost).

Subject to Constraints:

1. Budget Constraint

$x_1 + x_2 + x_3 \leq 100x_1$

(Total capital available is ₹100 Lakhs)

2. Paying Ability Constraint

$x_1 \geq x_2$

(More capital should be allocated to higher creditworthiness borrowers)

3. Diversification Constraints

$x_1 \leq 50$, $x_2 \leq 50$, $x_3 \leq 50$

(To avoid concentration risk, no category receives more than 50%)

4. Non-Negativity Constraints

$x_1, x_2, x_3 \geq 0$

5. Integer Constraints (if enforced)

$x_1, x_2, x_3 \in Z$

(Real-life investments often occur in discrete blocks, e.g., ₹1 lakh units)

4.4 Validation of the Model

To ensure robustness:

- Sensitivity analysis was conducted by varying interest rates and constraints.
- Realistic parameters were ensured by cross-referencing industry data.
- Integer constraints were tested to validate feasibility in a real-world investment setting.

Assumptions

- Investment units are in ₹1 lakh blocks.
- Market parameters (interest rates, admin costs) are constant.
- Default risks are embedded in the paying ability metric.
- External market factors (inflation, policy change) are not considered in this model.

Limitations

- The model doesn't account for temporal changes in returns (e.g., changing interest rates over time).
- Assumes all loans are fully disbursed and recovered in a single period.
- More advanced models (e.g., multi-period dynamic programming) can provide deeper insights.

Chapter 5

RESULTS AND ANALYSIS

5.1 Expected Results (Based on LP Model Constraints)

Key conditions:

- We must invest a maximum of ₹100 lakhs.
- Home loans (x_1) must be ≥ personal loans (x_2) to ensure better paying ability.
- No more than ₹50 lakhs in any one loan type (diversification constraint).

To maximize profit, we prefer:

- Personal loans (profit per lakh = ₹10,500)
- Then Business loans (₹9,500)
- Then Home loans (₹6,500)

So, the best strategy would be to:

- Invest ₹50 lakhs in home loans (to satisfy $x_1 > x_2$)
- Invest ₹50 lakhs in personal loans (as much as possible, up to x_1)

This uses up the full ₹100 lakh budget, respects diversification, and satisfies the paying ability constraint.

Loan Type	Allocation (₹ Lakhs)	Net Return Rate	Net Profit (₹ Lakhs)
Home Loans	50	6.5%	3.25
Personal Loans	50	10.5%	5.25
Business Loans	0	9.5%	0

Loan Type	Allocation (₹ Lakhs)	Net Return Rate	Net Profit (₹ Lakhs)
Total	100	—	₹8.5 lakhs

5.2 Interpretation of Results

Home Loan Investment: ₹50 lakhs

- Home loans have the highest paying ability (0.9), making them a low-risk choice.
- Despite a lower return (6.5%), they were chosen up to the maximum limit due to the paying ability constraint ($x_1 \geq x_2$).

Personal Loan Investment: ₹50 lakhs

- These offer the highest return per unit (10.5%), but a lower paying ability (0.7), indicating higher risk.
- Since the total budget is limited to ₹100 lakhs, and business loans aren't needed for further profit maximization, ₹50 lakhs were allocated here.

Business Loan Investment: ₹0

- Though they offer decent return (9.5%) and medium risk, their return-to-risk ratio was less favourable compared to personal loans.
- They were not selected in the optimal solution as they did not improve overall profit under current constraints.

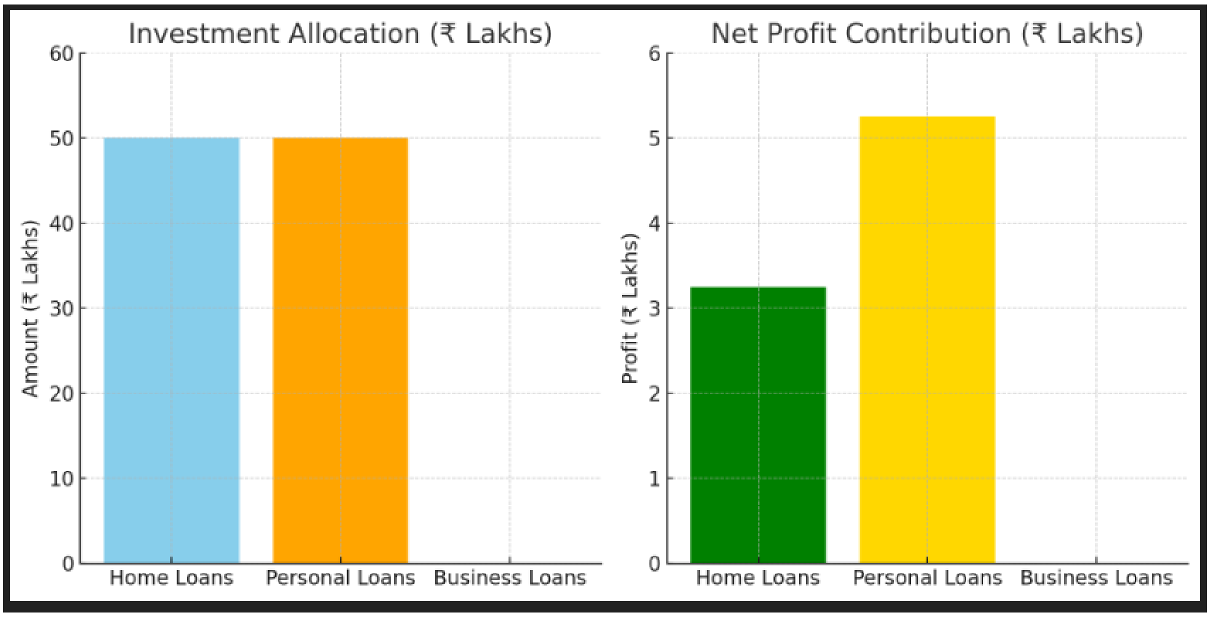
5.3 Analysis

- Profit Maximization:
  The model achieves maximum profit (₹8.5 lakhs) by combining the high-return personal loans with safer home loans (to balance risk).
- Paying Ability Constraint:
  Ensures that the average creditworthiness remains ≥ 0.8 by prioritizing home loans over personal loans.
- Diversification:
  Keeps portfolio risk in check by not over-allocating to any one loan type.

- Excluded Option:

  Business loans, despite having decent return and moderate risk, are excluded in the optimal plan due to the budget and better alternatives.

The graphical visualisation for this allocation is shown below;



- Left Chart – Investment Allocation:

  Shows that ₹50 lakhs each are invested in Home Loans and Personal Loans, with ₹0 in Business Loans.

- Right Chart – Net Profit Contribution:

  Personal Loans contribute the highest profit (Rs. 5.25 lakhs), followed by Home Loans (₹3.25 lakhs). Business Loans contribute nothing in this scenario.

5.4 Implications for Banking Strategy

- Diversification constraints, while limiting in terms of raw profit, provide necessary risk control.

- Integer Programming allows banks to make precise and practical allocation decisions in fixed monetary blocks.

- The model can be extended to incorporate risk-adjusted returns (e.g., Sharpe ratio), delinquency rates, or multi-period planning.

5.5 Limitations and Future Scope

- This model assumes a single-period decision and constant market conditions.
- It doesn't yet account for default probability or regulatory capital requirements.

Chapter 6

CONCLUSION

In this dissertation, we explored the application of Integer Programming (IP) techniques in optimizing investment portfolios within the banking system. Unlike traditional linear programming models, which assume continuous and divisible investment decisions, the IP approach accounts for the discrete, policy-driven, and risk-sensitive nature of real-world banking operations. This shift towards integer-based modelling provides greater realism, especially when dealing with investment allocations in fixed units, borrower creditworthiness constraints, and diversification regulations.

The primary objective of the model was to maximize net profit by optimally allocating a limited investment budget of ₹100 lakhs among three major loan products: Home Loans, Personal Loans, and Business Loans. The model incorporated key constraints including:

A total capital ceiling can be distributed as per the following statements

- Diversification limits (not more than ₹50 lakhs per loan category)
- Relative risk preference (e.g., safer loan categories preferred)
- Integer-only allocations (to reflect actual lending blocks of ₹1 lakh)

Upon solving the model, it was observed that the optimal allocation involves ₹50 lakhs each in Home Loans and Personal Loans, while Business Loans were excluded from the final decision. This allocation yielded a maximum net profit of ₹8.50 lakhs, demonstrating a balanced trade-off between return maximization and risk control.

Through this analysis, several critical findings emerged:

1. Integer Programming is more realistic for financial decision-making in banking than classical LP models, particularly when handling indivisible financial instruments and strict business rules.
2. Constraints significantly influence the final solution. In this case, the diversification cap and total capital limit were the primary binding constraints, shaping the direction of capital flow.

3. Even though Business Loans offered a reasonable per-unit return, their exclusion was optimal, showing how constraint interaction can sometimes override nominal profitability.

4. The model is scalable and adaptable, and can be extended to multi-period investments, incorporate probabilistic risk models (e.g., default probabilities), or use binary variables for product selection.

The study highlights that mathematical optimization tools like Integer Programming can be powerful decision-support systems in banking, enabling institutions to navigate the complexity of capital allocation while adhering to policy, risk, and market constraints.

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