Image Segmentation

A DISSERTATION

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Submitted by

Diksha Gupta

(2K21/MSCMAT/15)

Sushant Dhingra

(2K21/MSCMAT/52)

Under the supervision of

Prof. Dhirendra Kumar

DEPARTMENT OF APPLIED MATHEMATICS

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

Bawana Road , Delhi - 110042

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DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

Bawana Road , Delhi - 110042

DECLARATION

We , Diksha Gupta (2K21/MSCMAT/15) and Sushant Dhingra (2K21/MSCMAT/52) , students of M.Sc. Mathematics, hereby declare that the project Dissertation titled "Image Segmentation" which is submitted by us to the Department of Applied Mathematics Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Science, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

Place : Delhi — Diksha Gupta

Date : — Sushant Dhingra

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

Bawana Road , Delhi - 110042

CERTIFICATE

We hereby certify that the Project Dissertation titled ”Image Segmentation” which is submitted by Diksha Gupta (2K21/MSCMAT/15) and Sushant Dhingra (2K21/MSCMAT/52) [Department of Applied mathematics] , Delhi Technological University , Delhi in partial fulfillment of the requirement for the award of the degree of Master of Science , is a record of the project work carried out by the students under my supervision . To the best of my Knowledge this work has not been submitted in part or full for any degree or diploma to this university or elsewhere .

Place : Delhi

Date :

Prof. Dhirendra Kumar

SUPERVISOR

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

Bawana Road , Delhi - 110042

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Signature	Signature
Diksha Gupta	Sushant Dhingra

DELHI TECHNOLOGICAL UNIVERSITY
(Formerly Delhi College of Engineering)
Bawana Road , Delhi - 110042

ABSTRACT

Fuzzy C-Means (FCM) is one of the popular techniques used for segmenting scientific images. It is suggested in the literature to use intuitionistic fuzzy c-means (IFCM), which is based on the notion of intuitionistic fuzzy sets (IFSs), to handle the ambiguity and uncertainty related to real data.The hesitation and membership degrees are used to determine the objective function.However, FCM is used to achieve the approximate answer rather than analytically computing the objective function. Even though there are numerous variations of intuitionistic fuzzy set theory, all of them struggle with the issue of noise in images during the segmentation process.In order to address this issue, we have proposed using a picture fuzzy set theoretic approach, which improves the data's ability to be represented and aids in handling the noise structures present in the image. In our proposed work, the picture fuzzy Euclidean distance is swapped out for the Manhattan distance (City Block Distance), as Manhattan distance produces significantly better noise suppression.The method was applied to a fake image that had been "Gaussian" and "salt and pepper" distorted.Partition efficiency, average segmentation accuracy (ASA), and dice score (DS) were the performance metrics used. We can utilize the distance measure and dissimilarity between fuzzy sets to calculate the difference between two fuzzy sets or intuitionistic sets as it can be used for pattern recognition, and image segmentation.Results show that the proposed method gives the better result.

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Chapter 1

INTRODUCTION

The fuzzy theory was introduced by Lotfi Zadeh [2], and the researchers put the fuzzy theory into clustering. The FCM algorithm is introduced by Dunn [3] and later, it is generalized by Bezdek [4] with (fuzzifier) and became very popular. Fuzzy algorithms can assign data objects partially to multiple clusters. The degree of membership in the fuzzy clusters depends on the closeness of the data object to the cluster centers. However, the FCM algorithm has several disadvantages. For example, it performs poorly on data sets that contain clusters with unequal sizes or densities, and it is sensitive to noise and outliers. To overcome these FCM drawbacks, many variants of FCM algorithms have been proposed.

A noise clustering (NC) variation of the FCM was proposed by Dave and Sen [5]. Noisy points are defined as data points whose distances from all cluster centers are greater than a specific threshold. It does not recognize noisy data points that are situated between clusters, and it is not unaffected by the number of clusters present in a given data set. This method clearly detects the outliers and is unaffected by the number of clusters. Although this strategy successfully lessens the impact of outliers, it is less resistant to noise since it frequently distributes these outliers throughout multiple clusters.

However, to overcome the poor performance of FCM caused by noisy data, possibilistic c-means (PCM) was proposed in [6]. PCM interprets the clustering as a possibilistic rather than a fuzzy partition. To overcome these problems, possibilistic-FCM (PFCM) clustering generates both the membership and typicality values. Consequently, it is a combination of PCM and FCM. PFCM performs badly when the input data set contains clusters of different sizes with additional outliers but produces better clustering than FCM and PCM.

In order to enhance performance and increase robustness to noise and outliers, spatial contextual local information of the data must be taken into account in earlier iterations of FCM. There have been introduced improved FCM algorithms for picture segmentation that take into account neighborhood data. [7] proposed bias-corrected FCM (BCFCM) clustering by adding the spatial neighborhood term into the objective function of the FCM algorithm. Although BCFCM is more robust to noise than FCM, it has the disadvantage of higher computational complexity. To overcome this latter drawback, the FCM has been extended to three algorithms: The EnFCM (Enhanced FCM), FCM_S1, and FCM_S2. The first extension EnFCM was proposed by Szilágyi et al. [7]. Chen and Zhang [8] suggested the FCM_S1 and FCM_S2 that compute the neighborhood term based on the mean filtered and median filtered pictures, respectively, with the same objective of making the FCMS fast enough. The computations required to compute the neighborhood term are significantly decreased because the filtered image must be calculated just once, prior to the clustering procedure. In fact, the authors showed that their methods worked in both synthetic and real-world datasets. Fast Generalised Fuzzy -Means (FGFCM) clustering methods were developed by Cai et al. [9] by merging the key concepts of FCM_S1, FCM_S2, and EnFCM and utilizing both the local spatial and the grey information. The authors demonstrated the FGFCM's superiority to all of the aforementioned algorithms, showing that it overcomes many of their shortcomings, including managing the tradeoff between noise immunity and detail preservation and removing the empirically adjusted parameter α, even though it necessitates the adjustment of a new parameter to produce better results. Another uncertainty arises in FCM in defining the membership function. This uncertainty is due to a lack of knowledge. To handle this uncertainty, Atanassov [10] introduced higher order fuzzy set which is called Intuitionistic Fuzzy Set (IFS).IFS takes into account values from both membership and non-membership. On IFS, some early researchers created FCM.

An innovative IFS clustering technique for medical picture segmentation was created by Chaira. [11].In this method to maximize the good points in the class, a new objective function called intuitionistic fuzzy entropy is incorporated into the objective function of conventional FCM. Zhang et al. [12] proposed an intuitionistic fuzzy set clustering method. This method creates an intuitionistic fuzzy similarity matrix using the similarity degree between two intuitionistic fuzzy sets. Xu et al. [13] developed a clustering algorithm for an intuitionistic fuzzy set based on the concept of association matrix and equivalent association matrix.

In order to detect tumors in medical photos, Chaira and Anand [11] created a novel IFS technique. In order to remove undesirable regions from a clustered image, this method employs histogram thresholding. Moreover, the tumor's edge is removed. A fuzzy intuitive possibilistic C means (IFPCM) algorithm was created by Chaudhuri [14]. By broadly defining membership and non-membership with a certain amount of hesitation, IFPCM overcomes the dilemma with regard to the value of membership. Cuong [15] has presented a Picture Fuzzy Set (PFS) which is a generalization of the traditional fuzzy set and IFS. PFS solves real-time problems which require answers like yes, abstain, no, and refusal. Thong and Son., [16] proposed a new Picture Fuzzy Clustering (PFC) proposed by [17]. Experimental results reveal that PFC gives better clustering results. Inspired by the good performance of the PFC, in this paper we proposed a Manhattan distance algorithm to segment MRI brain images . The Euclidean distance metric fails to give good segmentation results on MRI brain images due to noise and intensity inhomogeneity present in the image .

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[2] Reference 2.

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Nuclear Physics B

inst1]Author One

[inst1]organization=Department One,addressline=Address One, city=City One, postcode=00000, state=State One, country=Country One

inst2]Author Two inst1,inst2]Author Three

[inst2]organization=Department Two,addressline=Address Two, city=City Two, postcode=22222, state=State Two, country=Country Two

Intuitionistic fuzzy sets Fuzzy c-means Manhattan distance Image segmentation Picture fuzzy clustering

8.1 Introduction

8.2 Preliminaries and Related work

8.2.1 Fuzzy set and it's extensions

Fuzzy set

A Fuzzy set is a set in which each member element will have the fractional membership via a membership function

$$\mu_A : X ß[0, 1] \tag{8.1}$$

which gives its degree of belongingness [18]. If A is a fuzzy set defined over a set X, it can be represented as:

$$A = (x, A(x)) : xX \tag{8.2}$$

Intuitionistic fuzzy set

An Intuitionistic fuzzy set B is an extension of a fuzzy set over X which is represented as:

$$B = (x, \mu B(x), \nu B(x)) : xX \; and \; 0 \mu B(x) + \nu B(x) 1 \tag{8.3}$$

where

$$\mu B : X ß[0, 1], \;\; \nu B : X ß[0, 1] \tag{8.4}$$

are membership and non-membership functions of an element x in set B. The IFS B is reduced to FS B when $\mu B(x) + \nu B(x) = 1$ for all x in B.

In an intuitionistic fuzzy set A is represented by a set of ordered triples $(x, \mu A(x), \nu A(x))$, where,

$\mu A(x)$ is the membership degree of x in A, and $\nu A(x)$ is the non-membership degree of x in A. The hesitation degree, denoted as hA(x)

Hesitation Degree:

hA(x) = 1 - $\mu A(x) - \nu A(x)$

The hesitation degree represents the degree of uncertainty or ambiguity associated with the membership and non-membership degrees. It is calculated as the difference between 1 and the sum of the membership and non-membership degrees.

8.2.2 Fuzzy C-Means (FCM)

We define a family of fuzzy set $\{A_i \ : \ i = 1, 2, 3, ...c\}$ as a fuzzy c-partition on a universe of data points X. Now, we can assign membership to the various data points in each fuzzy set (fuzzy class, fuzzy clusters).

Hence a single point can have partial membership in more than one class. $\mu_{ik} = \mu_{Ai}(x_k) \in [0, 1]$ denotes the membership value of the k^{th} data point in the i^{th} class and sum of all membership values for a single data point in all of the classes has to be unity i.e,

$$\sum_{i=1}^{c} \mu_{ik} = 1 \qquad \forall \ k = 1, 2....n$$

and

$$0 \ < \ \sum_{i=1}^{n} \mu_{ik} \ < \ n$$

We can now define a family of fuzzy partition matrices, M_{fc} for the classification involving c classes and n data points.

$$M_{fc} = \left\{ \underset{\sim}{U} \ | \ \mu_{ik} \in [0, 1]; \sum_{i=1}^{c} \mu_{ik} = 1; \ \ 0 < \sum_{k=1}^{n} \mu_{ik} < n \right\}$$

where $i = 1, 2, ...c$ and $k = 1, 2, ...n$

Any $\underset{\sim}{U} \in M_{fc}$ is a fuzzy c-partition.

Since v_i is the i^{th} cluster center which is described m features (m coordinates) and can be arranged in a vector form i.e, $v_i = \{v_{i1}, v_{i2}, ...v_{im}\}$. Each of the cluster coordinates for each class can be calculated in a manner similar to the calculation in the crisp case:-

$$v_{ij} = \frac{\sum_{k=1}^{n} \mu_{ik}^{m'} . x_{ik}}{\sum_{k=1}^{n} \mu_{ik}^{m'}}$$

where j is a variable on the feature space that's j=1,2,...m

To describe a method to determine the fuzzy c-partition matrix $\underset{\sim}{U}$ for grouping a collection of n data sets into c classes, we define an objective J_m for a fuzzy c-partition i.e,

$$J_m \left(\underset{\sim}{U}, V \right) = \sum_{k=1}^{n} \sum_{i=1}^{c} \left(\mu_{ik} \right)^{m'} \left(d_{ik} \right)^2$$

and where $d_{ik} = d\left(x_k - v_i\right) = \left[\sum_{j=1}^{m}\left(x_{kj} - v_{ij}\right)^2\right]^{\frac{1}{2}}$

Sugeno

A Sugeno class can be generated by using the following generating function :

$$g(\mu) = 1\lambda log(1 + \lambda\mu) \tag{8.5}$$

Using the above definition of negation function, the non-membership values for a given membership values for any element x in IFS B can be defined as follows:

$$\nu B(x) = N(\mu B(x)) = 1\mu B(x)1 + \lambda\mu B(x), \lambda > 0 \tag{8.6}$$

Yager

A Yager class can be generated by using the following generating function:

g(x)= $x^{\alpha}\,\frac{1}{\alpha}$ (8.7)

The negation function or non-membership values using this generating function is calculated as:

$$\nu B(x) = N(\mu B(x)) = (1\mu B(x)^{\alpha})^{\frac{1}{\alpha}}, \alpha > 0 \tag{8.8}$$

8.2.3 Picture Fuzzy Set

Picture Fuzzy Set [15] proposed a Picture Fuzzy Set (PFS), which is generalization of conventional fuzzy set and intuitionistic fuzzy set. A PFS is a non empty set X given by

$$A = hx, uA(x), \nu A(x), \gamma A(x)i|xX \tag{8.9}$$

where µA(x) is the positive membership value of each element, $\nu A(x)$ is the neutral membership degree and $\gamma A(x)$ is the negative membership degree satisfying the constrains,

$$0\mu A(x) + \nu A(x) + \gamma A(x)1 \tag{8.10}$$

The refusal degree of an element is calculated as:

$$\xi A(x) = 1(\mu A(x) + \nu A(x) + \gamma A(x) \qquad (8.11)$$

In case $\xi A(x) = 0$ PFS returns Intuitionistic fuzzy set.

If $\xi A(x) = \nu A(x) = 0$ PFS returns to fuzzy set.

Interval-valued picture fuzzy set

Definition: An interval-valued picture fuzzy set A on a universe X (IvPFS, in short) is an object of the form

$A = (x, M_A(x), L_A(x), N_A(x))|xX$

where

$M_A : X\text{ß}int([0, 1]), M_A(x) = [M_{AL}(x), M_{AU}(x)]int([0, 1])$

$L_A : X\text{ß}int([0, 1]), L_A(x) = [L_{AL}(x), L_{AU}(x)]int([0, 1])$

$N_A : X\text{ß}int([0, 1]), N_A(x) = [N_{AL}(x), N_{AU}(x)]int([0, 1])$

satisfy the following condition:

$$(xX)(supM_A(x) + supL_A(x) + supN_A(x)1) \qquad (8.12)$$

Let IvPFS(X) denote the set of all the interval-valued picture fuzzy set IvPFSs on a universe X.

8.3 Distance between picture fuzzy sets [1]

Distances for two picture fuzzy sets A and B in X = x$_1, x_2, ..., x_n are :$

- The normalized Hamming distance dP(A,B)

$$dP(A, B) = \frac{1}{n}\sum_{i=1}^{n}(|A(xi)B(xi)| + |A(xi)B(xi)| + |A(xi)B(xi)| \qquad (8.13)$$

- The normalized Euclidean distance eP(A,B)mm

$$eP(A, B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}((A(xi))B(xi))2 + (A(xi)B(xi))2 + (A(xi)B(xi))2)} \qquad (8.14)$$

8.4 Code Block Distance

City block distance is a distance metric used in clustering algorithms to assess how distinct or similar data points are to one another. It is also referred to as Manhattan distance or taxicab distance. The city block distance can be used in the context of fuzzy clustering to assess a data point's level of inclusion in several clusters.

The absolute differences between the associated coordinates of two data points to determine the distance in city blocks between them. For instance, the city block distance in a two-dimensional space between places $A(x_1, y_1)$ and $B(x_2, y_2)$ can be calculated as follows:

$$D = |x_1 - x_2| + |y_1 - y_2| \tag{8.15}$$

In fuzzy clustering, you can use the city block distance to measure the dissimilarity between a data point and the centroid of a cluster.

8.5 Proposed Method

The details of the proposed method is explained in this section.

In this section,a picture fuzzy model for clustering

$$J = \sum_{i=1}^{N} \sum_{J=1}^{C} (u_{ij}(2\xi_{ij}))^m sin||x_i v_j|| + \sum_{i=1}^{N} \sum_{J=1}^{C} \eta_{ij}((log\eta_{ij} + \xi_{ij}) \tag{8.16}$$

When uij is the positive membership degree, ij is the neutral membership degree, and ij is the refusal degree of the element that meets the following requirements, vj is the jth cluster centre.

$$\mu_{ij} + \nu_{ij} + \xi_{ij} 1 \tag{8.17}$$

$$\sum_{j=1}^{C} (\mu_{ij}(2\xi_{ij})) = 1 \tag{8.18}$$

$$\sum_{j=1}^{C} \nu_{ij} + \frac{\xi_{ij}}{c} = 1 \tag{8.19}$$

To determine the optimal solutions of the objective function in Lagrangian method is employed. The optimal solutions of the systems for vj, $\mu_{ij}, \nu_{ij}, \xi_{ij} are :$

$$vj = \frac{\sum_{i=1}^{N}(\mu_{ij}(2\xi_{ij}))^m x_i}{\sum_{i=1}^{N}(\mu_{ij}(2\xi_{ij}))^m} \tag{8.20}$$

$$u_{ij} = \frac{1}{\sum_{i=1}^{N}(\mu_{ij}(2\xi_{ij})\left(\frac{||x_i - v_j||}{||x_i - v_k||}\right)^{\frac{2}{m-1}}} \tag{8.21}$$

$$\nu_{ij} = \frac{e^{-\xi_{ij}}}{\sum_{k=1}^{C} e^{-\xi_{ik}}}\left(1 - \frac{1}{c}\sum_{k=1}^{C}\xi_{ik}\right) \tag{8.22}$$

$$\xi_{ij} = 1(u_{ij} + \nu_{ij})(1(u_{ij} + \nu ij)^{\alpha})^{\frac{1}{\alpha}} \tag{8.23}$$

Proof: taking the derivative of J by $v_j, we have$,
$\frac{\partial J}{\partial v_j} = \sum_{i=1}^{N}(\mu_{ij}(2\xi_{ij}))^m - cos(||x_k - v_j||)^2(2||x_k - v_j||) + \sum_{i=1}^{N}\sum_{J=1}^{C}\eta ij((log\eta_{ij} + \xi_{ij})$
putting above equation =0

$\frac{\partial J}{\partial v_j}$ =0 ,we get

$$vj = -\frac{\sum_{i=1}^{N}(\mu_{ij}(2\xi_{ij}))^m cos((||x_k - v_j||)^2 x_k}{\sum_{i=1}^{N}(\mu_{ij}(2\xi_{ij}))^m cos((||x_k - v_j||)^2} \tag{8.24}$$

The lagrangian function with respect to U is,
L(u) = $\sum_{i=1}^{N}\sum_{J=1}^{C}(u_{ij}(2\xi_{ij}))^m sin||x_i v_j||^2 + \sum_{i=1}^{N}\sum_{J=1}^{C}\eta_{ij}((log\eta_{ij} + \xi_{ij}) - \lambda_k\left(\sum_{J=1}^{C}(u_{ij}(2\xi_{ij})) - 1\right)$
Since $\frac{\partial L(u)}{\partial u_{kj}}$ =0 ,
$\frac{\partial L(u)}{\partial u_{kj}}$ = mu$_{kj}^{m-1}(2\xi_{ij}))^m sin((||x_k - v_j||)^2 - \lambda_k(2\xi_{ij}) = 0$
k=1......N, j= 1.......C,

$$u_{kj} = \frac{1}{((2\xi_{ij})}\left(\frac{\lambda_k}{msin||x_i - v_k||^2}\right)^{\frac{1}{m-1}} \tag{8.25}$$

k=1......N, j= 1.......C,

the solution of U are as follows,

$$\sum_{j=1}^{C}\left(\frac{\lambda_k}{msin||x_i - v_k||^2}\right)^{\frac{1}{m-1}} = 1 \tag{8.26}$$

k=1......N, j= 1.......C,

$$\lambda_k = \left(\frac{1}{\sum_{j=1}^{C} msin||x_i - v_k||^2)^{\frac{1}{m-1}}} \right)^{m-1} \tag{8.27}$$

k=1......N, j= 1.......C,

plugging

$$u_{ij} = \frac{1}{\sum_{i=1}^{N} ((2\xi_{ij}) \left(\frac{sin||x_k - v_j||^2}{sin||x_k - v_i||^2} \right)^{\frac{1}{m-1}}} \tag{8.28}$$

k=1......N, j= 1.......C,

Similarly, The lagrangian function with respect to η is,

$$\mathrm{L}(\eta) = \sum_{i=1}^{N} \sum_{J=1}^{C} (u_{ij}(2\xi_{ij}))^m sin||x_i v_j|| + \sum_{i=1}^{N} \sum_{J=1}^{C} \eta_{ij}((log\eta_{ij} + \xi_{ij}) - \lambda_k \left(\sum_{J=1}^{C} (\eta_{ij} + \frac{\xi_{ij}}{C}) - 1 \right) \tag{8.29}$$

$\frac{\partial L(\eta)}{\partial \eta_{kj}} = \log\eta_{kj} + 1 - \lambda_k + \xi_{kj} = 0,$

k=1......N, j= 1.......C,

$\eta_{kj} = exp\{\lambda_k - 1 - \xi_{kj}\}$

k=1......N, j= 1.......C,

8.5.1 Picture Fuzzy set representation of Image

The fuzzy complement generator developed by Yager is used to create the fuzzy image. Take a look at the image X = x1, x2,.....xNi, which consists of N pixels with intensities ranging from 0 to L - 1. The image's PFS representation can be specified as follows:

I = (xij, $\mu I(x_{ij})$, $\nu I(x_{ij})$, $\gamma I(x_{ij})$, $\xi I(x_{ij})$)

where I represents the refusal degree of the pixel, I represents the neutral membership value, I represents the negative membership value, and uI represents the positive membership value. Each pixel in an image has a corresponding intensity value. We compute the normalised intensity level for each pixel to translate the intensity data into membership values. i.e:

$$\mu I(x_{ij}) = \frac{x_{ij}}{L - 1} \tag{8.30}$$

In this paper, we calculated the negative membership value using Yager's fuzzy complement generator. The following describes Yager's fuzzy complement generator:

$$\gamma I(x_{ij}) = (1(\mu I(x_{ij}) + \nu I(x_{ij}))^{\alpha})^{\frac{1}{\alpha}} \tag{8.31}$$

Thus after applying Yager's fuzzy complement generator the PFS image becomes:

$$I_{\alpha}^{PFS} = (x_{ij}, \mu I(x_{ij}), \nu I(x_{ij}), (1(\mu I(x_{ij}) + \nu I(x_{ij})))^{\frac{1}{\alpha}, \xi I(x_{ij}))} (8.32)$$

The refusal degree of the pixel is calculated as:

$$\xi I(x_{ij}) = 1(\mu I(x_{ij}) + \nu I(x_{ij}))(1(\mu I(x_{ij}) + \nu I(x_{ij})^{\alpha}))^{\frac{1}{\alpha}} \tag{8.33}$$

where is exponent, the value varies between 0 and 1.

Now, applying the Lagrangian method to calculate the optimal solution of the above model , we get:

8.5.2 Picture Fuzzy Clustering

This section presents the Picture Fuzzy Clustering (PFC) technique for segmenting MRI brain images, which clusters the picture by looking for local minima of the following objective function:

$$J = \sum_{i=1}^{N}\sum_{J=1}^{C}(u_{ij}(2\xi_{ij}))^{m}||x_i v_j|| + \sum_{i=1}^{N}\sum_{J=1}^{C}\nu ij((log\nu_{ij} + \xi_{ij}) \tag{8.34}$$

When uij is the positive membership degree, ij is the neutral membership degree, and ij is the refusal degree of the element that meets the following requirements, vj is the jth cluster centre.

$$\mu_{ij} + \nu_{ij} + \xi_{ij}1 \tag{8.35}$$

$$\sum_{j=1}^{C}(\mu_{ij}(2\xi_{ij})) = 1 \tag{8.36}$$

$$\sum_{j=1}^{C}\nu_{ij} + \frac{\xi_{ij}}{c} = 1 \tag{8.37}$$

To determine the optimal solutions of the objective function shown in equation 19 Lagrangian method is employed. The optimal solutions of the systems for vj, $\mu_{ij}, \nu_{ij}, \xi_{ij} are:$

$$vj = \frac{\sum_{i=1}^{N}(\mu_{ij}(2\xi_{ij}))^m x_i}{\sum_{i=1}^{N}(\mu_{ij}(2\xi_{ij}))^m} \tag{8.38}$$

$$u_{ij} = \frac{1}{\sum_{i=1}^{N}(\mu_{ij}(2\xi_{ij})\left(\frac{||x_i-v_j||}{||x_i-v_k||}\right)^{\frac{2}{m-1}}} \tag{8.39}$$

$$\nu_{ij} = \frac{e^{-\xi_{ij}}}{\sum_{k=1}^{C} e^{-\xi_{ik}}}\left(1 - \frac{1}{c}\sum_{k=1}^{C}\xi_{ik}\right) \tag{8.40}$$

$$\xi_{ij} = 1(u_{ij} + \nu_{ij})(1(u_{ij} + \nu ij)^{\alpha})^{\frac{1}{\alpha}} \tag{8.41}$$

where i = 1.,,,,N, k = 1.,,,,,n, j = 1,.....c.

The membership value in fuzzy set-based clustering methods depends on the distance metric. If the pixel intensity is nearer to the cluster centre value, the pixel has a higher membership value. The membership value is therefore extremely noise-sensitive. Due of the noise and intensity inhomogeneity present in the MRI brain pictures, the euclidean distance measure does not produce satisfactory segmentation results. In this paper, the distance between the cluster centre and the pixel was calculated using the image euclidean distance function to account for noise and intensity inhomogeneity. The formula for picture euclidean distance is:

$$d(x_i, v_j) = ((u(x_i)u(v_j)) + (\nu(x_i)\nu(vj)) + (\gamma(xi)\gamma(v_j)))^{\frac{1}{2}} \tag{8.42}$$

Now, we are modifying Manhattan distance method in picture fuzzy set which is way better than euclidean distance method which is defined by

$$d(x_j, v_i) = sin|x_j - v_i| \tag{8.43}$$

8.6 Proposed Method

The details of the proposed method is explained in this section.

8.6.1 Picture Fuzzy set representation of Image

The fuzzy complement generator developed by Yager is used to create the fuzzy image. Take a look at the image X = x1, x2,.....xNi, which consists of N pixels with intensities ranging from 0 to L - 1. The image's PFS representation can be specified as follows:

I = (xij, $\mu I(x_{ij})$, $\nu I(x_{ij})$, $\gamma I(x_{ij})$, $\xi I(x_{ij})$)

where I represents the refusal degree of the pixel, I represents the neutral membership value, I represents the negative membership value, and uI represents the positive membership value. Each pixel in an image has a corresponding intensity value. We compute the normalised intensity level for each pixel to translate the intensity data into membership values. i.e:

$$\mu I(x_{ij}) = \frac{x_{ij}}{L-1} \quad (8.44)$$

In this paper, we calculated the negative membership value using Yager's fuzzy complement generator. The following describes Yager's fuzzy complement generator:

$$\gamma I(x_{ij}) = (1(\mu I(x_{ij}) + \nu I(x_{ij}))^{\alpha})^{\frac{1}{\alpha}} \quad (8.45)$$

Thus after applying Yager's fuzzy complement generator the PFS image becomes:

$$I_{\alpha}^{PFS} = (x_{ij}, \mu I(x_{ij}), \nu I(x_{ij}), (1(\mu I(x_{ij}) + \nu I(x_{ij}))^{)^{\frac{1}{\alpha}}, \xi I(x_{ij}))} (8.46)$$

The refusal degree of the pixel is calculated as:

$$\xi I(x_{ij}) = 1(\mu I(x_{ij}) + \nu I(x_{ij}))(1(\mu I(x_{ij}) + \nu I(x_{ij})^{\alpha}))^{\frac{1}{\alpha}} \quad (8.47)$$

where is exponent, the value varies between 0 and 1.

8.6.2 Picture Fuzzy Clustering

This section presents the Picture Fuzzy Clustering (PFC) technique for segmenting MRI brain images, which clusters the picture by looking for local minima of the following objective function:

$$J = \sum_{i=1}^{N} \sum_{J=1}^{C} (u_{ij}(2\xi_{ij}))^{m} ||x_i v_j|| + \sum_{i=1}^{N} \sum_{J=1}^{C} \nu ij((log\nu_{ij} + \xi_{ij}) \quad (8.48)$$

When uij is the positive membership degree, ij is the neutral membership degree, and ij is the refusal degree of the element that meets the following requirements, vj is the jth cluster centre.

$$\mu_{ij} + \nu_{ij} + \xi_{ij}1 \tag{8.49}$$

$$\sum_{j=1}^{C}(\mu_{ij}(2\xi_{ij})) = 1 \tag{8.50}$$

$$\sum_{j=1}^{C}\nu_{ij} + \frac{\xi_{ij}}{c} = 1 \tag{8.51}$$

To determine the optimal solutions of the objective function shown in equation 19 Lagrangian method is employed. The optimal solutions of the systems for vj, $\mu_{ij}, \nu_{ij}, \xi_{ij} are:$

$$vj = \frac{\sum_{i=1}^{N}(\mu_{ij}(2\xi_{ij}))^m x_i}{\sum_{i=1}^{N}(\mu_{ij}(2\xi_{ij}))^m} \tag{8.52}$$

$$u_{ij} = \frac{1}{\sum_{i=1}^{N}(\mu_{ij}(2\xi_{ij})\left(\frac{||x_i - v_j||}{||x_i - v_k||}\right)^{\frac{2}{m-1}}} \tag{8.53}$$

$$\nu_{ij} = \frac{e^{-\xi_{ij}}}{\sum_{k=1}^{C} e^{-\xi_{ik}}}\left(1 - \frac{1}{c}\sum_{k=1}^{C}\xi_{ik}\right) \tag{8.54}$$

$$\xi_{ij} = 1(u_{ij} + \nu_{ij})(1(u_{ij} + \nu ij)^{\alpha})^{\frac{1}{\alpha}} \tag{8.55}$$

where i = 1.,,,,N, k = 1.,,,,,n, j = 1,.....c.

The membership value in fuzzy set-based clustering methods depends on the distance metric. If the pixel intensity is nearer to the cluster centre value, the pixel has a higher membership value. The membership value is therefore extremely noise-sensitive. Due of the noise and intensity inhomogeneity present in the MRI brain pictures, the euclidean distance measure does not produce satisfactory segmentation results. In this paper, the distance between the cluster centre and the pixel was calculated using the image euclidean distance function to account for noise and intensity inhomogeneity. The formula for picture euclidean distance is:

$$d(x_i, v_j) = ((u(x_i)u(v_j)) + (\nu(x_i)\nu(vj)) + (\gamma(xi)\gamma(v_j)))^{\frac{1}{2}} \tag{8.56}$$

Now, we are modifying Manhattan distance method in picture fuzzy set which is way better than euclidean distance method which is defined by

$$d(x_j, v_i) = sin|x_j - v_i| \tag{8.57}$$

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