A MATHEMATICAL MODEL FOR GIVING UP SMOKING AMONG STUDENTS

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in
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CANDIDATE'S DECLARATION

Certified that we, **Anchal Jaiswal (23/MSCMAT/06)** and **Anjali (23/MSCMAT/72)**, have carried out the research work presented in this thesis, entitled **"A Mathematical Model for Giving Up Smoking among Students"**, in partial fulfillment of the requirements for the award of the degree of **Master of Science**. This thesis, submitted to the Department of Applied Mathematics, Delhi Technological University, is an authentic record of our own work conducted during the period 2024-25 under the supervision of **Dr. Vivek Kumar Aggarwal**.

The matter presented in the thesis has not been submitted by me for the award of any other degree of this or any other institute.

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This is to certify that the student has incorporated all the corrections suggested by the examiners in the thesis and the statement made by the candidate is correct to the best of our knowledge.

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CERTIFICATE BY THE SUPERVISOR

Certified that **Anchal Jaiswal and Anjali** has carried out their research work presented in this thesis entitled **”A MATHEMATICAL MODEL FOR GIVING UP SMOKING AMONG STUDENTS”** for the award of **Master of Science in Applied Mathematics** from the Department of Applied Mathematics, Delhi Technological University, Delhi, under my supervision.

The thesis embodies results of original work, and studies are carried out by the student herself, and the contents of the thesis do not form the basis for the award of any other degree to the candidate or to anybody else from this or any other University/Institution.

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ABSTRACT

Today, students are increasingly affected by smoking. For this reason, this study introduces a mathematical model that describes how 15 years and above students get influenced by tobacco based on a system of ordinary nonlinear differential equations. It is used to analyze the equilibrium points, the basic reproductive number (R_0), their local stability, and their global stability. The system has been solved numerically by the fourth-order Runge - Kutta method using python. The results have been verified with the real data. The observation indicates a concerning increase in smoking behavior, coupled with a decline in quitting rates over a period of time.

Keywords: Mathematical Modeling, Equilibrium Points, Local Stability, Global Stability, Lyapunov function.

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This thesis work accomplished during the 2nd year. It provides an overview of the tasks undertaken, followed by a detailed discussion of the results obtained. In addition, it outlines potential future advancements that could build on our work. We hope that our efforts will culminate in a successful outcome.

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Fig. Figure

w.r.t. with respect to

i.e. that is

viz. namely

S. No. Serial Number

RH Routh Hurwitz

eq. equation

et al. and others

NCR National Capital Region

GYTS Global Youth Tobacco Survey

GATS Global Adult Tobacco Survey

$\mathbf{R}_0$ Reproductive number

Chapter 1

Introduction

1.1 Study Overview

1.1.1 Definition of Smoking

Smoking involves burning tobacco and inhaling its smoke through various methods such as pipes and cigars. The smoke is drawn into the lungs through the mouth and then exhaled. Tobacco consists of a variety of harmful particles and when its smoke is in healed then these particles would enter in to human body and do harm [23].

1.1.2 Prevalence and trends of smoking globally and locally

Tobacco smoking is practiced worldwide by more than one billion people. However, many studies have shown that the prevalence of smoking has been notably reduced in numerous developed countries, but remains high in developing countries. India is the third largest tobacco producer worldwide, following China and Brazil. It is also the second largest consumer of tobacco products worldwide, with a significant portion of the population using smoking and smokeless forms of tobacco.

In India, the Global Youth Tobacco Survey (GYTS) [9] began at the state level in 2003. During its second and third rounds in 2006 and 2009, it evolved into a nationwide survey, engaging over 10,000 students. The results did not show a significant change in tobacco use prevalence among students from 2003 to 2009. In 2009, around 15% of the students used tobacco in some form, a figure that increased to 18% in the fourth round of GYTS in 2019.

The GATS India 2009-10 survey revealed that two out of every five daily tobacco users aged 20-34 began using tobacco daily before the age of 18. Furthermore, the GATS-2 survey conducted in 2016-17 found that 12.2% of daily tobacco users in the age group of 20-34 started smoking before the age of 15, while over one-third (35.8%) of all daily smokers started before turning 18.

1.1.3 Health risks associated with Smoking

The World Health Organization (WHO) [25, 24] highlights that tobacco is the only legally available substance that causes death to many users when consumed as intended by manufacturers. The World Health Organization (WHO) estimates that tobacco and its products are responsible for over 3.5 million deaths globally each year. Projections suggest that by the decade 2020-2030, this number could rise to 10 million annually. In response to this growing issue, WHO introduced the Framework Convention on Tobacco Control (FCTC) in 2005 to safeguard present and future generations from the adverse health, social, environmental, and economic effects of tobacco use and exposure. Tobacco continues to be the leading cause of preventable deaths worldwide.
According to the Global TB Report 2024, India carries the greatest burden of tuberculosis (TB) and Multi-Drug Resistant (MDR) TB

1.1.4 Cultural and Social significance of Smoking

The widespread consumption is driven by cultural, social, and economic factors, despite government efforts to regulate tobacco use and increase awareness of its health risks.
It is evident that many college students engage in smoking as a social activity, often referred to as social smoking [21, 14, 18]. This behavior is characterized in several ways:

a) Non-daily smoking among young adults, typically occurring in bars, restaurants, and nightclubs.

b) Non-daily smoking among adolescents, which happens exclusively in the company of other smokers.

c) Adolescent smoking that commonly takes place in social contexts, around others, rather than in isolation.

Students who smoke but do not label themselves as casual smokers may identify as ”party smokers” or ”weekend smokers,” potentially increasing their likelihood of becoming regular smokers in the future.

1.2 Statement of the Problem

The gymnasium age, between 16 and 20 years old, is a time where many of our attitudes change; this includes the attitude towards smoking [12, 31].

1.2.1 The impact of smoking on Physical and Mental health

Smoking is highly addictive, with nicotine identified as the primary addictive component in tobacco products. Nicotine is also a potential carcinogen, linked to various cancers, including those that affect the lungs, oral cavity, nasal cavity, ureter, oropharynx, hypopharynx, esophagus,larynx, cervix, stomach, liver, pancreas, bladder, kidneys, and myeloid leukemia [23].

Smoking can also lead to osteoporosis, stroke, vision loss, hearing impairment, and back pain. Tobacco smoking is also considered one of the factors for infertility in both men and women [19]. Smoking is now seen as a marker of stress and deprivation, often linked to other drug use.

1.2.2 Challenges individuals face when trying to quit smoking

People initially begin smoking tobacco for enjoyment or entertainment. If this habit persists, it can lead to addiction, which affects both the personal and social aspects of life. Tobacco contains nicotine, a substance known for its addictive properties. Once addicted to smoking, it is difficult to give up [16].The main reasons which prevent giving up smoking include nicotine in the tobacco, psychological conditions and genetics history of the patient, smoking rate, sex, age, education, social and economical states, cigarette price, youth access laws, nicotine replacement therapy, parental smoking, and peer influences [23].

Most smokers develop a strong dependence early on, likely driven more by avoiding withdrawal discomfort than by enjoyment.

The survey was conducted in one of the universities in Delhi/NCR during the academic year 2019 - 2020 and the good part of this study was that a major part of students i.e. 59.5% had intentions / desire to quit smoking due to its ill-effect on their health and remaining wanted to continue with smoking due to their own reasons [22].

1.2.3 Societal and environmental factors influencing dependence on smoking

The report Why children start smoking by Eileen Goddard [15] analyzed many risk factors associated with initiating to smoke and some of the main factors are having family who smoke and students used to think that they might be a smoker in the future. This report also highlights children's attitudes toward smoking and quitting, revealing that while most express a desire not to become smokers in the future, a significant number acknowledge the possibility that they might.

Adolescence and early adulthood are considered the most critical stages for smoking initiation in India. As a result, smoking remains a prevalent issue among university students. Smoking becomes ingrained in the daily lives of students, influenced by factors such as stress, peer relationships, and cultural contexts [22].

1.3 Literature survey

In exploring the dynamics of Smoking model, following are the several key research papers provide valuable insights.

- Gul Zaman [31] presented a non-linear model that describes the overall dynamics of the smoking population when the population is not constant, which ensures that the birth rate is different from the death rate.

- In 2000, Carlos Castillo-Garsow, Guarionex Jordan-Salivia, Ariel Rodriguez-Herrera, et al. [11] proposed a general mathematical model for drug abuse in a population of adolescents, with a peer pressure as a key ingredient in the recruitment mechanism of new drug users. They considered to work on the report of the Surgeon General.

- Lady Maribel Benavides [27] presented a mathematical model that represents the population growth dynamics of tobacco consumers by hypothetically assuming the initial population.

- Prince Harvim, Hong Zhang, et al. [18] proposed a compartmental model depicting the spread and cessation of the smoking habit on college campuses which underline the significance of identifying contact processes between habitual-smokers and the other students, in order to impede the spread of cigarette smoking within the university campus via imitation mechanisms and peer pressure.

- Peter R Killeen [20] described a three-state Markov model of smoking cessation and analyzed that quitting smoking involves an initial withdrawal phase with a high relapse risk, followed by a transition to sustained abstinence.

1.4 Objectives of the Study.

In order to understand the dynamics of these diseases caused by smoking, we utilize mathematical modeling, which is essential for analyzing and understanding the changing patterns of smoking behavior within a population over time. Differential equations are widely used to model various natural and social phenomena, such as epidemic spread, economic growth, and ecological interactions. In particular, differential equation-based models have been extensively applied to study smoking initiation, progression, cessation by incorporating factors like peer influence, public awareness campaigns, and governmental policies and relapse process as well [10, 11, 17, 18, 23, 26, 27, 28, 29].

Several continuous variables, such as the rate of smoking initiation, cessation probability, and relapse rate, can be studied together to develop a system of ordinary differential equations and verify the accuracy of these models, it is essential to estimate key parameters accurately, allowing for reliable predictions and the design of effective tobacco control strategies.

Studentship period is a time when most behavioral traits are created and fixed; this is a special time when students can start smoking [17]. This paper utilizes a mathematical model to analyze the transmission of smoking behavior and its impact on the student community, with the aim of assessing the prevalence of tobacco use among students. The model consisting three non-linear ordinary differential equations is used to analyze the equilibrium points, the basic reproductive number (R_0), their local stability, and their global stability. The system is numerically solved using the fourth-order Runge-Kutta method in Python. We considered to work on the real dataset [1, 4, 9] of 15 years and above students for initial values and to predict the correct parameters which help to verify the results.

1.5 Structure of the Thesis

The paper is structured as follows: In Section 1.6, data sources are mentioned that are used to analyze student smoking patterns. Chapter 2 outlines the formulation of the proposed model and determine its equilibrium points. In Chapter 3, the local and global stability of both equilibrium points are investigated. In Chapter 4, the numerical simulations validate our findings. Finally, in Chapter 5, the paper concludes with a summary of key insights.

1.6 Source of Data

- **Global Adult Tobacco Survey GATS 1 (2009-2010)** [1]
- **Statistics of School Education 2009-10** [2]
- **Educational Statistics at a Glance 2012** [3]
- **Global Adult Tobacco Survey GATS 2 (2016-2017)** [4]
- **All India Survey on Higher Education (2017-18)** [5]
- **Educational Statistics at a Glance 2018** [6]
- **Youth in India 2022** [7]
- **All India Survey on Higher Education (2016-17)** [8]
- **Global Youth Tobacco Survey (GYTS-4) India 2019** [9]

Chapter 2

Approach of the Model

2.1 Assumptions of the Model

For this model, the following assumption has been taken into account:

- The model for 15 years and older is analyzed.
- It is assumed that the total population N(t) remains constant over time, which means that every death in the population is offset by a corresponding birth among potential smokers. The mortality rate α is treated as a constant.
- The students who never smoked and are not looking for smoking are not considered in the system.

2.2 Compartmentalization of the model

The students are categorized into three compartments based on their smoking status. This categorization follows the SIR (Susceptible-Infected-Recovered) model, as illustrated in Figure 1. The model is described by a system of three first-order nonlinear ordinary differential equations. The total student population at any time t denoted by N(t), is distributed in three compartments: y_1(t), y_2(t), and y_3(t). This relationship is expressed as:

$$N(t) = y_1(t) + y_2(t) + y_3(t).$$

The notation and corresponding interpretations for each compartmental variable are as follows.

2.2.1 Compartment of Potential Smokers

The variable $y_1(t)$ represents the population of potential smokers at any given time, referring to students who do not currently smoke but have the likelihood of becoming smokers in the future. Students join this group at a natural birth rate α. Some students in this group y_1(t) may leave due to natural death at a rate of α, while others may start smoking and transition to the smoker group y_2(t) at a rate of β/N.

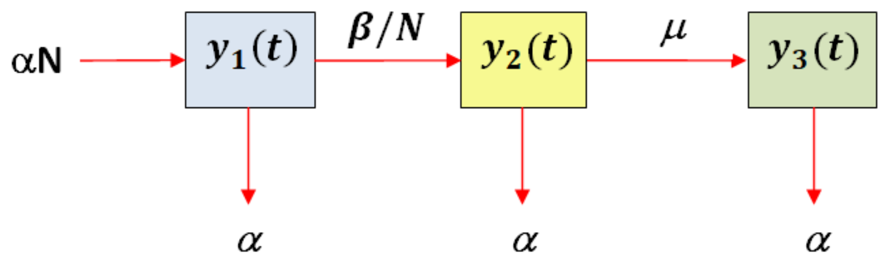
2.2.2 Compartment of Current Smokers

The variable $y_2(t)$ denotes the number of smokers at a given time, representing students who are actively smoking. Potential smokers from the y_1(t) compartment may start smoking and join this group at a rate of β/N. Some smokers in y_2(t) leave the group due to natural death at a rate of α, while others quit smoking permanently and move to the y_3(t) compartment at a rate of μ.

2.2.3 Compartment of Smokers who quit permanently

The variable $y_2(t)$ denotes the number of smokers who permanently quit smoking at any given time, $y_3(t)$, represents those who have transitioned from the $y_2(t)$ compartment by quitting smoking at a rate of μ. Some students in $y_3(t)$ experience natural death at a rate of α.

Figure 2.1: A flow chart illustrating the compartmental classification and directional transitions of students based on their smoking status.



Thus, the mathematical model of the compartments, as depicted in the flowchart in Figure 1, is formulated as a system of three first-order nonlinear ordinary differential equations.

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \alpha N - \beta y_1 \frac{y_2}{N} - \alpha y_1, \tag{2.1a}$$

$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \beta y_1 \frac{y_2}{N} - (\alpha + \mu) y_2, \tag{2.1b}$$

$$\frac{\mathrm{d}y_3}{\mathrm{d}t} = \mu y_2 - \alpha y_3. \tag{2.1c}$$

Table 1 provides a list and description of the variables and parameters employed in the model (2.1).

Table 2.1: Notations and descriptions of the variables and parameter utilized in the model

Variables /Parameters	**Description**
y_1**(t)**	The number of potential smokers at a given time t.
y_2**(t)**	The number of smokers at a given time t.
y_3**(t)**	The number of individuals who successfully quit smoking permanently.
c	The mean number of interactions within a given time period.
p	The likelihood of an individual y_1 starting to smoke after interacting with a smoker.
$\boldsymbol{\beta = pc}$	Per capita influence rate.
$\boldsymbol{\alpha}$	Per capita death Rate.
$\boldsymbol{\mu}$	Per capita recovery Rate.
$N(t)$	Population size at time t.

2.3 Points of Equilibrium

To determine the equilibrium points of system(2.1),

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = 0, \quad \frac{\mathrm{d}y_2}{\mathrm{d}t} = 0, \quad \text{and} \quad \frac{\mathrm{d}y_3}{\mathrm{d}t} = 0$$

Equation (2.1) can be written as:

$$\alpha N - \beta y_1 \frac{y_2}{N} - \alpha y_1 = 0, \tag{2.2}$$

$$\beta y_1 \frac{y_2}{N} - (\alpha + \mu) y_2 = 0, \tag{2.3}$$

$$\mu y_2 - \alpha y_3 = 0. \tag{2.4}$$

from (2.3)

$$\left(\frac{\beta y_1}{N} - \alpha - \mu\right) y_2 = 0$$

If $y_2 = 0$, then substituting y_2 in equation (2.4)
gives $y_3 = 0$,
and substituting y_2 in equation (2.2)
$\alpha * N = \alpha y_1$
results in $y_1 = N$.
Therefore, the equilibrium point is $E_0 = (N, 0, 0)$.
If there are no current smokers $y_2(t) = 0$, then there will be no students who have quit smoking $y_3(t) = 0$. As a result, potential smokers $y_1(t)$ represent the entire student population N and remain stable, so

$$\mathbf{E_0} = (N, 0, 0) \quad \text{is} \quad \textbf{the smoking-free equilibrium point.}$$

If $y_2 \neq 0$, then from equation (2.3), it follows that:

$$\left(\frac{\beta y_1}{N} - \alpha - \mu\right) * y_2 = 0$$

$$\frac{\beta y_1}{N} - \alpha - \mu = 0$$

Now solving for y_1,

$$y_1 = \frac{N(\alpha + \mu)}{\beta}$$

Now, substituting the value of y_1 from equation (2.2) gives:

$$\alpha N - \beta y_1 \frac{y_2}{N} - \alpha y_1 = 0$$

$$\Longrightarrow \alpha N - \beta \left(\frac{N(\alpha + \mu)}{\beta}\right) \frac{y_2}{N} - \alpha \left(\frac{N(\alpha + \mu)}{\beta}\right) = 0$$

$$\Longrightarrow (\alpha + \mu)\beta y_2 = \alpha N(\beta - \alpha - \mu)$$

Now solving for y_2,

$$y_2 = \frac{\alpha N(\beta - \alpha - \mu)}{\beta(\alpha + \mu)}$$

Now, substituting y_2 into equation (2.4) gives:

$$\mu \left(\frac{\alpha N(\beta - \alpha - \mu)}{\beta(\alpha + \mu)} \right) - \alpha y_3 = 0.$$

$$y_3 = \frac{\mu(\beta - \alpha - \mu)N}{\beta(\alpha + \mu)} \tag{2.5}$$

Therefore,

$$\mathbf{E_1} = \left(\frac{\boldsymbol{N(\alpha + \mu)}}{\boldsymbol{\beta}}, \quad \frac{\boldsymbol{\alpha N(\beta - \alpha - \mu)}}{\boldsymbol{\beta(\alpha + \mu)}}, \quad \frac{\boldsymbol{\mu(\beta - \alpha - \mu)N}}{\boldsymbol{\beta(\alpha + \mu)}} \right)$$

is **endemic equilibrium point**. It is biologically meaningful only when $R_0 > 1$.

Chapter 3

Stability Analysis

Stability analysis is a mathematical approach used to determine the behavior of a system's solutions over time, particularly in response to small perturbations or disturbances. It assesses whether a system will return to equilibrium, remain in a bounded state, or diverge.

3.1 Basic Reproductive Number and Local Stability Analysis

Let,

$$f_1 = \alpha N - \beta y_1 \frac{y_2}{N} - \alpha y_1, \tag{3.1a}$$

$$f_2 = \beta y_1 \frac{y_2}{N} - (\alpha + \mu) y_2, \tag{3.1b}$$

$$f_3 = \mu y_2 - \alpha y_3. \tag{3.1c}$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial y_1} & \frac{\partial f_1}{\partial y_2} & \frac{\partial f_1}{\partial y_3} \\ \frac{\partial f_2}{\partial y_1} & \frac{\partial f_2}{\partial y_2} & \frac{\partial f_2}{\partial y_3} \\ \frac{\partial f_3}{\partial y_1} & \frac{\partial f_3}{\partial y_2} & \frac{\partial f_3}{\partial y_3} \end{bmatrix}$$

The Jacobian matrix (J) associated with the system (3.1) is

$$\mathbf{J} = \begin{bmatrix} -\beta\frac{y_2}{N} - \alpha & -\beta\frac{y_1}{N} & 0 \\ \beta\frac{y_2}{N} & \beta\frac{y_1}{N} - (\alpha + \mu) & 0 \\ 0 & \mu & -\alpha \end{bmatrix}$$

3.1.1 Local stability analysis of the Smoking Free Equilibrium E_0:

Theorem 1. *When $R_0 < 1$, the equilibrium point E_0, representing a smoking-free state in 2.1, is locally asymptotically stable. However, if $R_0 > 1$, it becomes unstable.*

Proof. The Jacobian matrix evaluated at the smoking-free equilibrium E_0 is expressed as:

$$\mathbf{J(N, 0, 0)} = \begin{bmatrix} -\beta\frac{(0)}{N} - \alpha & -\beta\frac{(N)}{N} & 0 \\ \beta\frac{(0)}{N} & \beta\frac{(N)}{N} - (\alpha + \mu) & 0 \\ 0 & \mu & -\alpha \end{bmatrix}$$

$$J(N, 0, 0) = \begin{bmatrix} -\alpha & -\beta & 0 \\ 0 & \beta - (\alpha + \mu) & 0 \\ 0 & \mu & -\alpha \end{bmatrix}$$

For the eigen values,

$$\begin{bmatrix} -\alpha - \lambda & -\beta & 0 \\ 0 & \beta - (\alpha + \mu) - \lambda & 0 \\ 0 & \mu & -\alpha - \lambda \end{bmatrix} = 0$$

$$\implies (-\alpha - \lambda)^2(\beta - (\alpha + \mu) - \lambda) = 0$$

leading to

$$(-\alpha - \lambda)^2 = 0$$

$$\lambda = -\alpha, -\alpha$$

and

$$(\beta - (\alpha + \mu) - \lambda) = 0$$

$$\lambda = (\beta - (\alpha + \mu)$$

For all eigenvalues to be negative, the following condition must be satisfied:

$$\beta - \mu - \alpha < 0 \implies \frac{\beta}{\alpha + \mu} < 1$$

therefore **the Basic Reproductive Number is :**

$$\mathbf{R}_0 = \frac{\beta}{\alpha + \mu}$$

The basic reproductive number, denoted as R_0, is a key epidemiological metric that represents the average number of secondary infections caused by one infected individual in a fully susceptible population.

Mathematically, R_0 helps assess the potential transmission of an infectious disease as follows:

- If $R_0 > 1$, the infection can propagate through the population, potentially causing an outbreak.
- If $R_0 = 1$, the disease persists at a stable level within the population.
- If $R_0 < 1$, the infection will gradually decline and eventually disappear.

□

3.1.2 Local stability analysis of the Endemic Equilibrium E_1:

The endemic equilibrium point E_1 s represented in relation to the basic reproductive number R_0 is: $\left(\frac{N}{R_0}, \frac{\alpha N(R_0-1)}{\beta}, \frac{\mu N(R_0-1)}{\beta}\right)$

Theorem 2. *When $R_0 > 1$, the endemic equilibrium point E_1 of System 1 is locally asymptotically stable. However, if $R_0 < 1$, it becomes unstable.*

Proof. The Jacobian matrix, computed at the endemic equilibrium E_1, is expressed as:

$$J\left(\frac{N}{R_0}, \frac{\alpha N(R_0-1)}{\beta}, \frac{\mu N(R_0-1)}{\beta}\right) = \begin{bmatrix} -\frac{\beta}{N}\left(\frac{\alpha N(R_0-1)}{\beta}\right) - \alpha & -\frac{\beta}{N}\left(\frac{N}{R_0}\right) & 0 \\ \frac{\beta}{N}\left(\frac{\alpha N(R_0-1)}{\beta}\right) & \frac{\beta}{N}\left(\frac{N}{R_0}\right) - (\alpha+\mu) & 0 \\ 0 & \mu & -\alpha \end{bmatrix}$$

$$J\left(\frac{N}{R_0}, \frac{\alpha N(R_0-1)}{\beta}, \frac{\mu N(R_0-1)}{\beta}\right) = \begin{bmatrix} -\alpha R_0 & -\beta/R_0 & 0 \\ \alpha(R_0-1) & 0 & 0 \\ 0 & \mu & -\alpha \end{bmatrix}$$

For the eigen values,

$$\begin{bmatrix} -\alpha R_0 - \lambda & -\beta/R_0 & 0 \\ \alpha(R_0-1) & -\lambda & 0 \\ 0 & \mu & -\alpha-\lambda \end{bmatrix} = 0$$

$$\implies (\alpha+\lambda)((R_0)\lambda^2 + (R_0^2\alpha)\lambda + \beta\alpha(R_0-1)) = 0$$

This leads to $\lambda_1 = -\alpha$, and

$$((R_0)\lambda^2 + (R_0^2\alpha)\lambda + \beta\alpha(R_0-1)) = 0$$

Using quadratic equation,

$$\lambda_2 = \left(\frac{-R_0\alpha}{2} + \frac{\sqrt{R_0\alpha(R_0^3\alpha - 4\beta R_0 + 4\beta)}}{2R_0} \right),$$

$$\text{and } \lambda_3 = \left(\frac{-R_0\alpha}{2} - \frac{\sqrt{R_0\alpha(R_0^3\alpha - 4\beta R_0 + 4\beta)}}{2R_0} \right)$$

and all above three eigen values are negative for $R_0 > 1$ as follows.

1. Since $\alpha > 0$, therefore λ_1 is negative.

2. Let $\lambda_2 < 0$

   $$\implies \left(\frac{-R_0\alpha}{2} + \frac{\sqrt{R_0\alpha(R_0^3\alpha - 4\beta R_0 + 4\beta)}}{2R_0} \right) < 0$$

   $$\implies \frac{\sqrt{R_0\alpha(R_0^3\alpha - 4\beta R_0 + 4\beta)}}{2R_0} < \frac{R_0\alpha}{2}$$

   Squaring Both sides,

   $$\implies \frac{R_0\alpha(R_0^3\alpha - 4\beta R_0 + 4\beta)}{R_0^2} < R_0^2\alpha^2$$

   $$\implies (R_0^3\alpha - 4\beta R_0 + 4\beta) < R_0^3\alpha$$

   $$\implies 4\beta < 4\beta R_0$$

   $$\implies R_0 > 1$$

3. Similarly, λ_3 is negative for $R_0 > 1$.

   $$\implies \left(\frac{-R_0\alpha}{2} - \frac{\sqrt{R_0\alpha(R_0^3\alpha - 4\beta R_0 + 4\beta)}}{2R_0} \right) < 0$$

   $$\implies \left(\frac{R_0\alpha}{2} + \frac{\sqrt{R_0\alpha(R_0^3\alpha - 4\beta R_0 + 4\beta)}}{2R_0} \right) > 0$$

   $$\implies \frac{-\sqrt{R_0\alpha(R_0^3\alpha - 4\beta R_0 + 4\beta)}}{2R_0} < \frac{R_0\alpha}{2}$$

   Squaring Both sides,

$$\implies \frac{R_0\alpha(R_0^3\alpha - 4\beta R_0 + 4\beta)}{R_0^2} < R_0^2\alpha^2$$
$$\implies (R_0^3\alpha - 4\beta R_0 + 4\beta) < R_0^3\alpha$$
$$\implies 4\beta < 4\beta R_0$$
$$\implies R_0 > 1$$

□

3.2 Global Stability Analysis

All model parameter values are assumed to be non-negative. To facilitate the analysis of the mathematical model (2.1), the variables in the model are redefined.

$$x_1(t) = \frac{y_1(t)}{N(t)}, \quad x_2(t) = \frac{y_2(t)}{N(t)}, \quad and \quad x_3(t) = \frac{y_3(t)}{N(t)} \tag{3.4}$$

Substituting the time derivatives of system (3.4) using system (2.1) results in a newly simplified system as follows:

$$\frac{dx_1}{dt} = \alpha - \beta x_1 x_2 - \alpha x_1, \tag{3.5a}$$

$$\frac{dx_2}{dt} = \beta x_1 x_2 - (\alpha + \mu)x_2, \tag{3.5b}$$

$$\frac{dx_3}{dt} = \mu x_2 - \alpha x_3. \tag{3.5c}$$

The total number of students remains fixed and does not change.

$$y_1(t) + y_2(t) + y_3(t) = N(t)$$

Using the scaled model of system (2.1), the total student population is calculated.

$$x_1(t) + x_2(t) + x_3(t) = 1$$

Hence, the mathematically and biologically valid domain of system (3.5) is defined as

$$\Omega = \{(x_1, x_2, x_3)\epsilon R_+^3 : x_1 + x_2 + x_3 = 1\}$$

The model retains both mathematical and biological validity provided that the following key lemma is satisfied.

Lemma 1. *The closed set Ω remains positively invariant under the dynamics of the system (3.5).*

Proof. Define $z(t) = (x_1(t), x_2(t), x_3(t))^T$, allowing the system (3.5) to be reformulated as:

$$\frac{\mathrm{d}z(t)}{\mathrm{d}t} = f(z(t)),$$

where $f(z(t)) = [\alpha - \beta x_1 x_2 - \alpha x_1, \beta x_1 x_2 - (\alpha + \mu)x_2, \mu x_2 - \alpha x_3]^T$.
Clearly, the feasible region Ω is a closed set, and the goal is to prove that if the initial condition $f(0) \in \Omega$, the solution $f(t) \in \Omega$ for all $t \geq 0$.
Take $\partial\Omega$ consists four hyperspaces S_1, S_2, S_3 and S_4 such that

$$S_1 = \{(x_1, x_2, 0) : x_1, x_2 \in [0, 1], x_1 + x_2 \leq 1\},$$

$$S_2 = \{(x_1, 0, x_3) : x_1, x_3 \in [0, 1], x_1 + x_3 \leq 1\},$$

$$S_3 = \{(0, x_2, x_3) : x_2, x_3 \in [0, 1], x_2 + x_3 \leq 1\},$$

$$S_4 = \{(x_1, x_2, x_3) : x_1, x_2, x_3 \in [0, 1], x_1 + x_2 + x_3 \leq 1\}.$$

with outer normal vectors $w_1 = (0, 0, -1)$; $w_2 = (0, -1, 0)$; $w_3 = (-1, 0, 0)$; $w_4 = (1, 1, 1)$ respectively.
If the dot product of f(z) and the normal vectors $(w_1; w_2; w_3; w_4)$ of the boundary lines are less than zero then z(t) $\in \Omega$ for all $t \geq 0$. i.e.;

$$f(z(t))|_{z(t)\in S_1} \cdot w_1 = -\mu x_2 + \alpha x_3 \leq 0$$

$$f(z(t))|_{z(t)\in S_2} \cdot w_2 = \beta x_1 x_2 - (\alpha + \mu)x_2 \leq 0$$

$$f(z(t))|_{z(t)\in S_3} \cdot w_3 = \alpha - \beta x_1 x_2 - \alpha x_1 \leq 0$$

$$f(z(t))|_{z(t)\in S_4} \cdot w_4 = 0$$

This implies that all solutions of system (3.5) enter Ω. Therefore, the feasible region Ω is positively invariant, indicating that system (3.5) is both mathematically and biologically well-defined within Ω.

□

The **Lyapunov function** is defined as follows:

$$L(x_1, x_2, x_3) = v_1(x_1 - x_1^*)^2 + v_2(x_2 - x_2^*)^2 + v_3(x_3 - x_3^*)^2 \tag{3.6}$$

where v_1, v_2 and v_3 are positive constants.

The time derivative of $L(x_1, x_2, x_3)$ corresponding to the solution of the system (3.5) is given by:

$$\frac{\mathrm{d}L}{\mathrm{d}t} =2v_1(x_1 - x_1^*)\left(\frac{\mathrm{d}x_1}{\mathrm{d}t} - \frac{\mathrm{d}x_1^*}{\mathrm{d}t}\right)t + 2v_2(x_2 - x_2^*)\left(\frac{\mathrm{d}x_2}{\mathrm{d}t} - \frac{\mathrm{d}x_2^*}{\mathrm{d}t}\right) + 2v_3(x_3 - x_3^*)\left(\frac{\mathrm{d}x_3}{\mathrm{d}t} - \frac{\mathrm{d}x_3^*}{\mathrm{d}t}\right)$$

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} - \frac{\mathrm{d}x_1^*}{\mathrm{d}t} = -\beta x_1 x_2 - \alpha x_1 + \beta x_1^* x_2^* + \alpha x_1^*$$

$$\frac{\mathrm{d}x_2}{\mathrm{d}t} - \frac{\mathrm{d}x_2^*}{\mathrm{d}t} = \beta x_1 x_2 - \beta x_1^* x_2^* - (\alpha + \mu)(x_2 - x_2^*)$$

$$\frac{\mathrm{d}x_3}{\mathrm{d}t} - \frac{\mathrm{d}x_3^*}{\mathrm{d}t} = \mu(x_2 - x_2^*) - \alpha(x_3 - x_3^*)$$

$$\begin{aligned}\frac{\mathrm{d}L}{\mathrm{d}t} =& 2v_1(x_1 - x_1^*)[-\alpha(x_1 - x_1^*) - \beta x_1 x_2 + \beta x_1^* x_2^*] + \\ & 2v_2(x_2 - x_2^*)[\beta x_1 x_2 - \beta x_1^* x_2^* - (\alpha + \mu)(x_2 - x_2^*)] + \\ & 2v_3(x_3 - x_3^*)[\mu(x_2 - x_2^*) - \alpha(x_3 - x_3^*)]\end{aligned}$$

Adding and subtracting the term $\beta x_1^* x_2$ in the first and second square brackets gives:

$$\begin{aligned}\frac{\mathrm{d}L}{\mathrm{d}t} =&2v_1(x_1-x_1^*)[-\alpha(x_1-x_1^*)-\beta x_1x_2+\beta x_1^*x_2^*+\beta x_1^*x_2-\beta x_1^*x_2]+\\&2v_2(x_2-x_2^*)[\beta x_1x_2-\beta x_1^*x_2^*-(\alpha+\mu)(x_2-x_2^*)+\beta x_1^*x_2-\beta x_1^*x_2]+\\&2v_3(x_3-x_3^*)[\mu(x_2-x_2^*)-\alpha(x_3-x_3^*)]\end{aligned}$$

Thus, it follows that:

$$\begin{aligned}\frac{\mathrm{d}L}{\mathrm{d}t} =&2v_1(x_1-x_1^*)[(-\alpha-\beta x_2)(x_1-x_1^*)-\beta x_1^*(x_2-x_2^*)]+\\&2v_2(x_2-x_2^*)[\beta x_2(x_1-x_1^*)+(\beta x_1^*-(\alpha+\mu))(x_2-x_2^*)]+\\&2v_3(x_3-x_3^*)[\mu(x_2-x_2^*)-\alpha(x_3-x_3^*)]\end{aligned}$$

from where,

$$\begin{aligned}\frac{\mathrm{d}L}{\mathrm{d}t} =&-2v_1(\alpha+\beta x_2)(x_1-x_1^*)^2-2v_1\beta x_1^*(x_1-x_1^*)(x_2-x_2^*)\\&2v_2\beta x_2(x_1-x_1^*)(x_2-x_2^*)+2v_2(\beta x_1^*-(\alpha+\mu))(x_2-x_2^*)^2\\&+2v_3\mu(x_2-x_2^*)(x_3-x_3^*)-2v_3\alpha(x_3-x_3^*)^2\end{aligned}$$

Assuming that,

$$\frac{\mathrm{d}L}{\mathrm{d}t} = N(VA + A^TV^T)N^T \tag{3.7}$$

where $N = (x_1 - x_1^*, x_2 - x_2^*, x_3 - x_3^*)$, $V = diag(v_1, v_2, v_3)$ and

$$A = \begin{bmatrix} -(\alpha+\beta x_2) & -\beta x_1^* & 0 \\ \beta x_2 & \beta x_1^* - (\alpha+\mu) & 0 \\ 0 & \mu & -\alpha \end{bmatrix} \tag{3.8}$$

Lemma 2. *[13]The matrix A is negative definite if*

1. $A_{11} < 0$,
2. $A_{11}A_{22} - A_{21}A_{12} > 0$ *and*
3. $Det(A) < 0$.

Proof. Clearly above all three conditions followed by both equilibrium points having values:

$$A_{11} = -(\alpha + \beta x_2)$$

$$A_{11}A_{22} - A_{21}A_{12} = (-\alpha - \beta x_2)(\beta x_1^* - (\alpha + \mu)) - (-\beta x_1^*)(\beta x_2)$$

$$= -\alpha\beta x_1^* + \alpha^2 + \alpha\mu - \beta 62 x_2 x_1^* + \beta x_2 \alpha + \beta x_2 \mu + \beta^2 x_1^* x_2$$

$$= -\alpha\beta x_1^* + \alpha^2 + \alpha\mu + \beta x_2 \alpha + \beta x_2 \mu$$

$$DetA = (-\alpha - \beta x_2)((\beta x_1^* - (\alpha + \mu))(-\alpha) - 0) - (-\beta x_1^*)(-\beta x_2 \alpha - 0) + 0$$

$$= \alpha^2 \beta x_1^* - \alpha^3 - \alpha^2 \mu + \alpha\beta^2 x_2 x_1^* - \beta x_2 \alpha^2 - \alpha\beta x_2 \mu - \beta^2 x_1^* x_2 \alpha$$

$$\det(A) = -\alpha^3 - \alpha^2\mu - \alpha^2\beta x_2 + \alpha^2\beta x_1^* - \alpha\beta x_2 \mu$$

1. For the smoke-free equilibrium, given values: $\alpha = 0.058$, $\beta = 0.05$, $\mu = 0.005$, $x_2 = 0.66$, and $x_1^* = 1$

   (a) $A_{11} = -(\alpha + \beta x_2) = -(0.058 + 0.05 \times 0.66) = -0.091 < 0$

   (b) $A_{11}A_{22} - A_{21}A_{12} = -\alpha\beta x_1^* + \alpha^2 + \alpha\mu + \beta x_2 \alpha + \beta x_2 \mu$

   $$= -0.058 \times 0.05 + (0.058)^2 + 0.058 \times 0.005 + 0.05 \times 0.66 \times 0.058 + 0.05 \times 0.66 \times 0.005$$

   $$= 0.002833 > 0$$

   (c) $\det(A) = -\alpha^3 - \alpha^2\mu - \alpha^2\beta x_2 + \alpha^2\beta x_1^* - \alpha\beta x_2 \mu$

   $$= -0.058^2 \times 0.005 - (0.058)^3 - 0.058^2 \times 0.05 \times 0.66 + 0.05 \times 0.058^2 - 0.05 \times 0.66 \times 0.005 \times 0.058$$

   $$= -0.000164314 < 0$$

2. For the endemic equilibrium, given values: $\alpha = 0.058, \beta = 2.3, \mu = 0.005, x_2 = 0.66$, and $x_1^* = 0.027$

   (a) $A_{11} = -(\alpha + \beta x_2) = -(0.058 + 2.3 \times 0.66) = -1. < 0$

   (b) $A_{11}A_{22} - A_{21}A_{12} = -\alpha\beta x_1^* + \alpha^2 + \alpha\mu + \beta x_2\alpha + \beta x_2\mu$

   $$= -0.058{\times}2.3{\times}0.027{+}(0.058)^2{+}0.058{\times}0.005{+}2.3{\times}0.66{\times}0.058{+}2.3{\times}0.66{\times}0.005$$

   $$= 0.0956862 > 0$$

   (c) $\det(A) = -\alpha^2\mu - \alpha^3 - \alpha^2\beta x_2 + \alpha^2\beta x_1^* - \alpha\beta x_2\mu$

   $$= -0.058^2{\times}0.005{-}(0.058)^3{-}0.058^2{\times}2.3{\times}0.66{+}2.3{\times}0.027{\times}0.058^2{-}2.3{\times}0.66{\times}0.005{\times}0.058$$

   $$= -0.0055497996 < 0$$

Thus, the following conclusions are derived concerning the global stability of the smoking-free equilibrium and the endemic equilibrium.

□

Theorem 3. *If $R_0 < 1$, the smoking-free equilibrium E_0 of system (3.5) is globally stable in Ω.*

Proof. Lemma 2 ensures the global stability of the smoking-free equilibrium in system (3.5). □

Theorem 4. *If $R_0 > 1$, the endemic equilibrium E_1 of system (3.5) is globally stable in Ω.*

Proof. Lemma 2 guaranteed that the endemic equilibrium of the system (3.5) is globally stable. □

Chapter 4

Numerical Simulation

The study of dynamical systems often involves visualizing their evolution using phase portraits. A phase portrait is a visual depiction of the paths followed by a dynamical system within the phase plane. It illustrates how the system evolves over time from different initial conditions, showing the stability and behavior of the equilibrium points.[30]

The following information are considered from the report of the Global Adult Tobacco Survey India 2009-10[1] and the Global Adult Tobacco Survey 2 India 2016-17[4] and it is found that,

Year	$y_1(t)$ (%)	$y_2(t)$ (%)	$y_3(t)$ (%)
2009-10	0.95	66.63	32.42
2016-17	3.31	73.05	23.63

Table 4.1: Values of $y_1(t)$, $y_2(t)$, and $y_3(t)$ for different years

The data of 2009-10 are taken as an initial value and parameters $\alpha = 0.058$, $\beta = 2.3$ and $\mu = 0.005$ validate the proposed mathematical model that is for $R_0 > 1$. Python software is used to program the following plots for the numerical simulation of system (1):

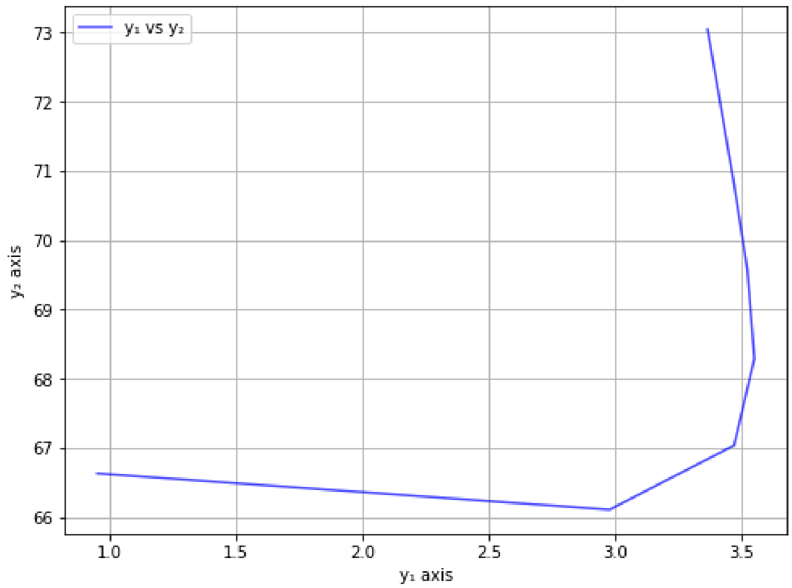


Figure 4.1: **Phase portrait of Potential smokers y_1 vs Current smokers y_2**

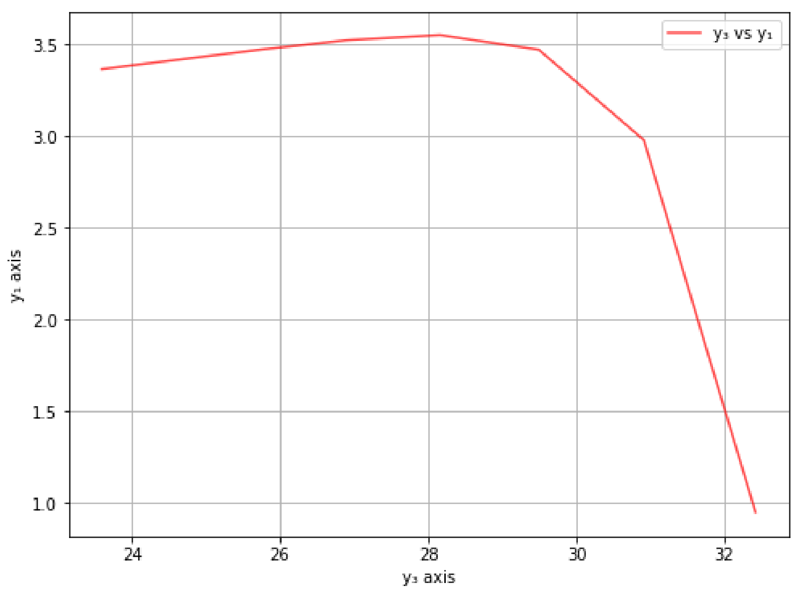


Figure 4.2: **Phase portrait of Smokers who quit permanently y_3 vs Potential smokers y_1**

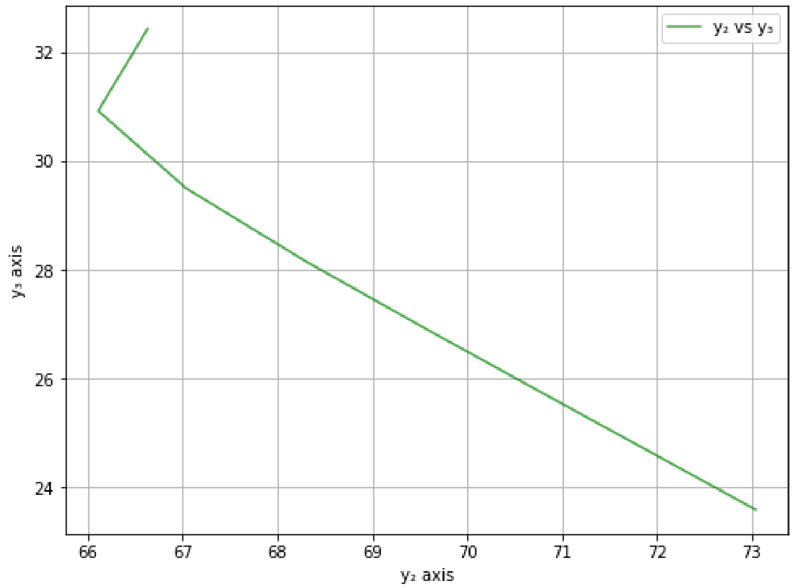


Figure 4.3: **Phase portrait of Current smokers y_2 vs Smokers who quit permanently y_3**

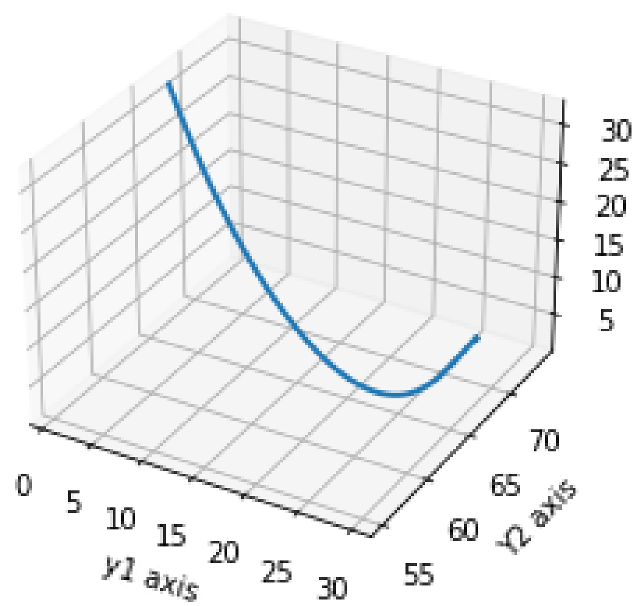


Figure 4.4: **Phase portrait of non linear ODE system (1)**

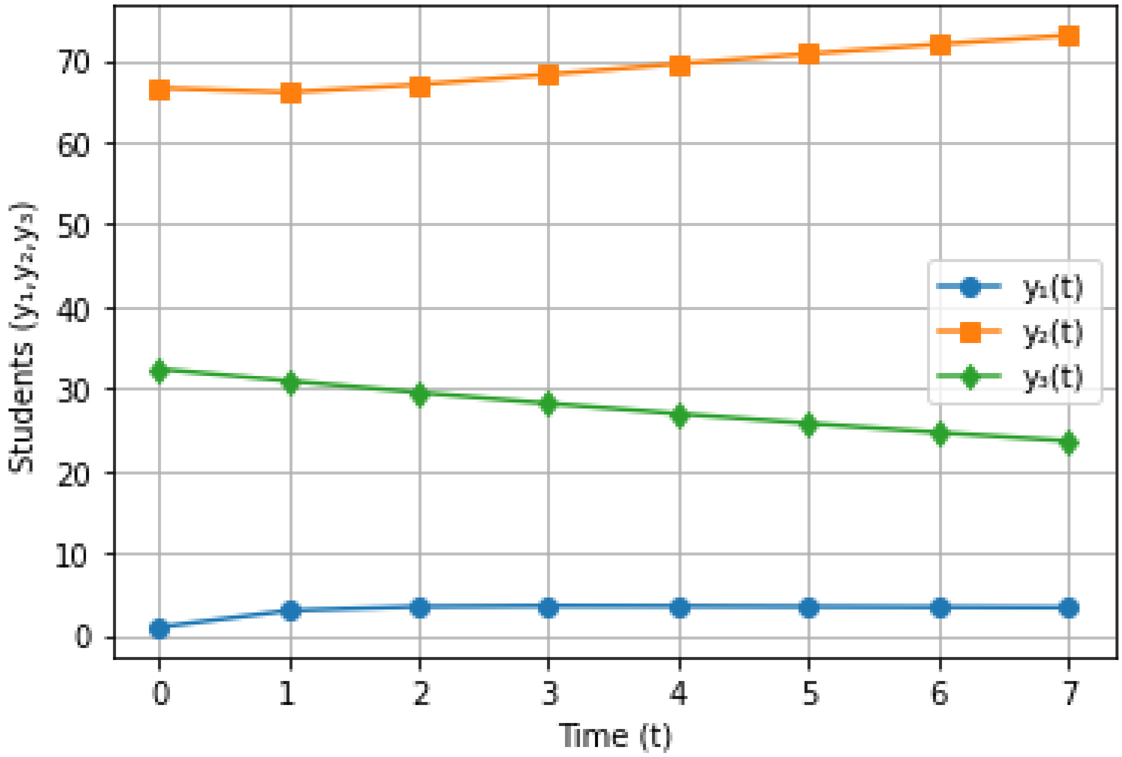


Figure 4.5: **Numerical simulation of system (1) for seven years from 2009-10 to 2016-17.**

In Figure 4.5, t=0 shows the data of year 2009-10 and after seven years that is t=7 shows the data of year 2016-17.

Moreover, this indicates that the population of current smokers rises while the number of individuals who quit smoking permanently declines over time.

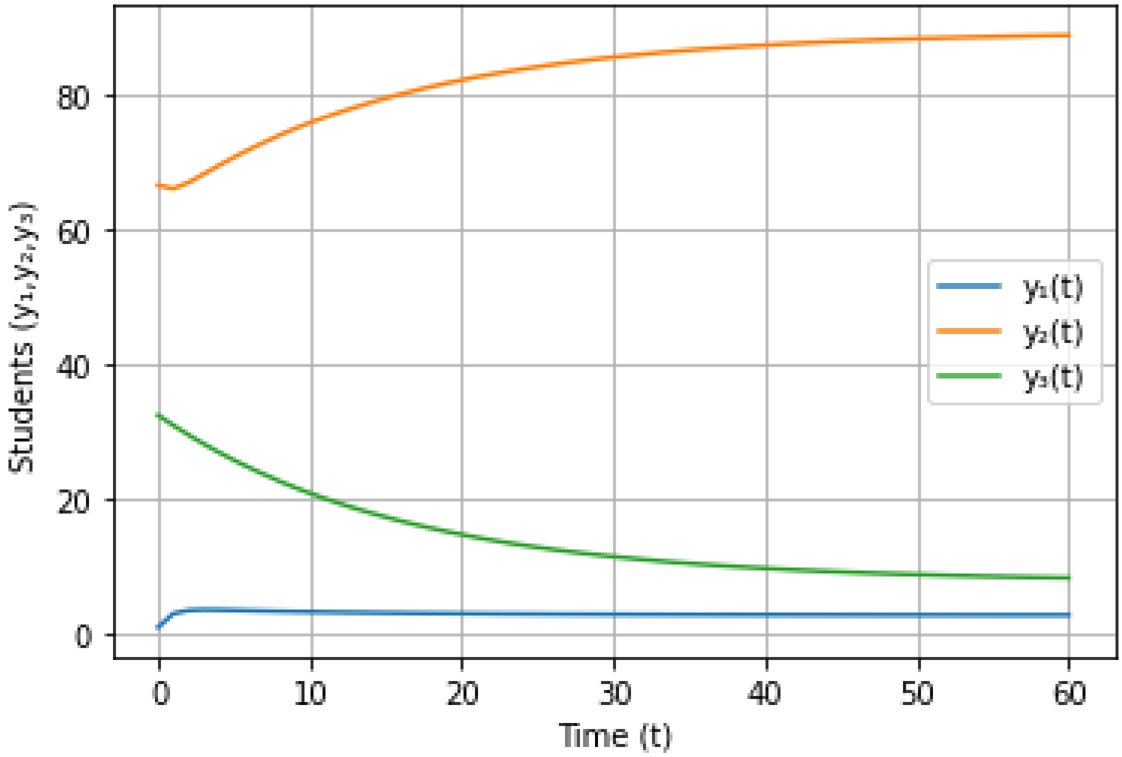


Figure 4.6: **Dynamics of system (1) for the endemic equilibrium point.**

Based on Figure 4.6, as time progresses, the potential smoker population converges to $2.74 \approx 3$, the current smoker population to $89.54 \approx 89$, and the population of smokers who quit permanently to $7.72 \approx 8$. This implies that the endemic equilibrium $E_1(3, 89, 8)$ is asymptotically stable when $R_0 = 36.5 > 1$.

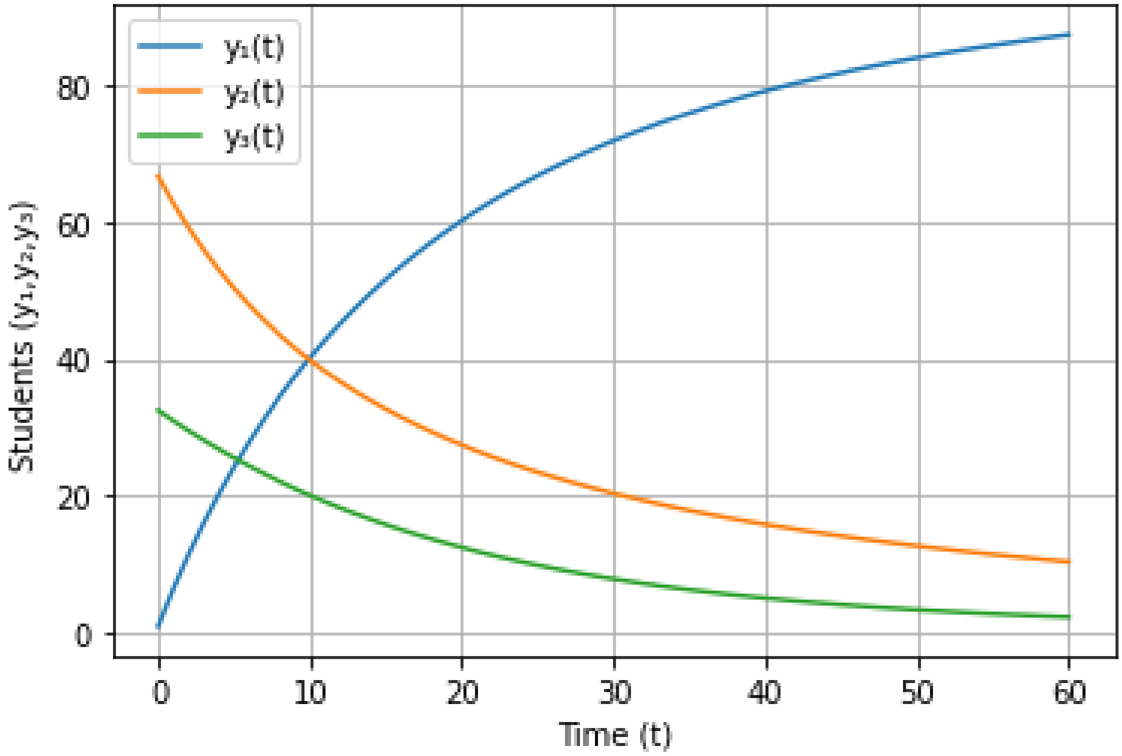


Figure 4.7: **Dynamics of system (1) for the smoking-free equilibrium point.**

Based on Figure 4.7, as time progresses, the potential smoker population converges to $N = 100$, while both the current smoker population and the population of smokers who quit permanently approach zero. This suggests that the smoking-free equilibrium $E_0(100, 0, 0)$ is asymptotically stable when $R_0 = 0.79 < 1$.

Chapter 5

Discussion

5.1 Conclusion

- The proposed model of Giving up Smoking has two equilibria, namely the Smoking-free equilibrium and the Endemic equilibrium that is Smoking - present equilibrium.
- When the Basic reproductive number is below one, that is a single current smoker produces less than one new tobacco smoker then the population of smokers decreases over time.
- When the Basic reproductive number is more than one that is a single current smoker produces more than one new tobacco smoker then the smoking population persists.
- The condition of students of India is very dreadful as these lies under endemic equilibrium.
- Since any nation wants their youth smoking free and for this R_0 should be less than one which is practically very difficult to achieve.

Hence, this study introduces a nonlinear mathematical model to analyze the spread of smoking among students and concludes that reducing the contact rate below the natural death rate (α) can lead to a decline in the number of smokers.

5.2 Future Scope

This study further suggests to classify students into more compartments such as active smokers, occasional smokers and recovered smokers who may relapse to obtain more significant results.Control programs can be implemented to analyze their impact on the progression of smoking spread.

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Date: November 16, 2024

Sub: Invitation for Participation and Oral Presentation

Dear **Ms. ANCHAL JAISWAL,**

Greetings from South Asian University, New Delhi!

South Asian University is an international university set up jointly by the eight member countries of the SAARC. It is a unique academic institution that aims at providing a vibrant educational platform for the collective South Asian community. We have students and teachers from across the region working together to fulfil this objective.

The Department of Mathematics, South Asian University, New Delhi, India is organising an Annual Conference of Indian Society for Mathematical Modeling and Computer Simulation (ISMMACS) and International Conference on Differential Equations: Theory, Computation and Applications during November 29 - December 1, 2024. The conference is expected to attract leading applied and computational mathematicians across the globe, working in the direction of mathematical modelling of real-life problems. It is also intended to expose young researchers to international experts working in these areas.

On behalf of the Organizing Committee, it gives us great pleasure to invite you to participate in this conference. We have already received and approved the abstract of article titled "**A MATHEMATICAL MODEL FOR GIVING UP SMOKING**" that you submitted in the conference, and will schedule your oral presentation in one of the Special Session. Due to the paucity of funds, we are unable to provide any kind of travel support to you. **Moreover, please note that we do not have any guest house/hostel rooms available, but a few dormitories are available at INR 800 per person per day, which you can avail of by paying at the link https://forms.gle/tvpdnhUSLwiopbkb9**. We very much hope that you will accept our invitation and look forward to seeing you in New Delhi. Please contact us if you need further assistance or information.

Before I conclude, I would like to extend a very warm welcome to you on behalf of all my colleagues and hope your stay here will be academically rewarding.

With best regards,

FACULTY OF MATHEMATICS & COMPUTER SCIENCE
SOUTH ASIAN UNIVERSITY
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Dear **ANJALI**,

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FACULTY OF MATHEMATICS & COMPUTER SCIENCE
SOUTH ASIAN UNIVERSITY
NEW DELHI, INDIA

Saroj Kumar Sahani
On Behalf of Organizing Team
ICDETCA-2024

Conference Certificate

Annual Conference of Indian Society for Mathematical Modelling and Computer Simulation (ISMMACS) and International Conference on Differential Equations: Theory, Computation and Applications

29 November 2024 – 1 December 2024

Department of Mathematics
South Asian University, New Delhi

CERTIFICATE OF APPRECIATION

This is to certify that **Ms. Anchal Jaiswal** of Department of Applied Mathematics, Delhi Technological University, Delhi, has presented a research article titled "**A Mathematical Model for Giving up Smoking.**" in the Annual Conference of ISMMACS and International Conference on Differential Equations: Theory, Computation and Applications organised by the Department of Mathematics, South Asian University, New Delhi, during 29 November - 1 December, 2024.

Prof. Deepa Sinha	**Prof. Kapil K Sharma**	**Dr. Navnit Jha**	**Dr. Saroj K Sahani**
Chair	Convener	Convener	Convener

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Prof. Deepa Sinha	**Prof. Kapil K Sharma**	**Dr. Navnit Jha**	**Dr. Saroj K Sahani**
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