EVALUATING THE PERFORMANCE OF IMAGE SEGMENTATION METHODS: RANDOM WAKJER METHOD, MUMFORD SHAH METHOD, MORPHOLOGICAL TRANSFORMATIONS METHOD AND K-MEANS CLUSTERING METHOD

A DISSERTATION
SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE AWARD OF THE DEGREE OF

MASTER OF SCIENCE

IN

APPLIED MATHEMATICS

SUBMITTED BY:

Lakita
(2K22/MSCMAT/57)

UNDER THE SUPERVISION OF
DR. DHIRENDRA KUMAR

DEPARTMENT OF APPLIED MATHEMATICS
DELHI TECHNOLOGICAL UNIVERSITY
(Formerly Delhi College of Engineering)
Bawana Road, Delhi- 110042

May,2024

Delhi Technological University
(Formerly Delhi College of Engineering)
Bawana Road, Delhi-110042

CANDIDATE'S DECLARATION

I,**Lakita (Roll No.2K22/MSCMAT/57)**,student of Master of Science in Applied Mathematics, hereby declare that the dissertation titled, "EVALUATING THE PERFORMANCE OF IMAGE SEGMENTATION METHODS: RANDOM WALKER METHOD ,MUMFORD-SHAH METHOD, MORPHOLOGICAL OPERATIONS METHOD AND K-MEANS CLUSTERING METHOD" submitted to the Department of Applied Mathematics, Delhi Technological University, Delhi, in partial fulfillment of the requirements for the degree of Master of Science,is my original work. Proper citations have been given wherever necessary, and this work has not been submitted previously for any degree, diploma, associateship, or any other similar title or recognition.

Place: Delhi **Lakita**

Date:

DELHI TECHNOLOGICAL UNIVERSITY
(Formerly Delhi College of Engineering)
Bawana Road, Delhi- 110042

CERTIFICATE

I hereby certify that the Project Dissertation titled **"EVALUATING THE PERFORMANCE OF IMAGE SEGMENTATION METHODS: RANDOM WALKER METHOD, MUMFORD-SHAH METHOD,MORPHOLOGICAL OPERATIONS METHOD, AND K-MEANS CLUSTERING METHOD"**, which is submitted by **Lakita** (2K22/MSCMAT/57) [Department of Applied Mathematics], Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of **Master of Science**, is a record of the project work carried out by the student under my supervision. To the best of my knowledge, this work has not been submitted in part or full for any degree or diploma to this university or elsewhere.

Place: Delhi
Date:

Dr. Dhirendra Kumar
SUPERVISOR

Abstract

This paper explores synergistic integration of the semi-automatic segmentation techniques, improved the K-Means clustering algorithms, the morphological transformations, and the Mumford-Shah (MS) functional approximation methods in fields of an image processing and computational geometry. The Semi-automatic segmentation, facilitated by an user interaction, enables precise delineation of the regions of interest within images, offering versatility across the diverse applications. A Random Walker method, is known for its flexibility in segmenting images into the multiple objects and complements traditional binary segmentation approaches. Conversely, MS functional, renowned for modelling images as the piecewise-smooth functions, has seen limited adoption in the geometry processing due to the computational complexities and challenges in a mesh adaptation. To address these issues, advancements have merged algorithms such as the largest minimum distance algorithm with the traditional K-Means clustering and enhancing cluster analysis efficiency. Moreover, an integration of the morphological transformations with a MS functional approximation methods facilitates the noise reduction, an edge detection and the boundary extraction in images. This paper investigates fusion of these methodologies to solve challenges in an image and the geometry processing, offering insights into their applications, the potential advancements in the computational image and geometry processing.

ACKNOWLEDGEMENT

I would like to express my deepest gratitude to Dr. Dhirendra Kumar, my mentor, for his invaluable guidance, support, and encouragement throughout the course of this project. His insightful recommendations and expert advice have been instrumental in the successful completion of this work. I greatly appreciate the time and effort he has devoted to mentoring me over the past year. I would also like to state explicitly that this project was conducted solely by me, without any external assistance. From the initial concept to the final execution, I have worked independently, ensuring the integrity and originality of the research. Thank you once again, Dr. Dhirendra Kumar. Your mentorship has been a cornerstone of this project, and I will always be grateful for your contributions and support.

Signature
Lakita

Contents

List of Figures

Chapter 1

Introduction

In image processing and computational geometry, the various methodologies have emerged to overcome challenges such as segmentation, noise reduction and feature extraction. Semi-automatic segmentation techniques leverage user interaction to delineate regions of an interest within the images, facilitating tasks ranging from an image editing to the medical imaging and an object recognition. Among these techniques, the Random Walker method stands out for its flexibility in the segmenting images into multiple objects and offering advantages over traditional binary segmentation approaches. On the other hand, Mumford-Shah (MS) functional, originally devised for an image segmentation, has garnered recognition for its ability to model images as the piecewise-smooth functions. Inspite of its success in image processing applications, it has received limited attention in geometry processing. Some challenges such as computational complexity and an adaptation of the MS model to meshes have hindered its widespread adoption in the domain.To address these kinds of challenges, recent advancements have paved a way for the novel approaches. By combining algorithms such as largest minimum distance algorithm with the traditional K-Means clustering, researchers aim to enhance the efficiency and the accuracy of the cluster analysis. In addition to, methodologies like morphological transformations offer versatile tools for tasks such as noise reduction, an edge detection and the boundary extraction in images.
In this paper, we research into the fusion of these methodologies to solve specific challenges in an image processing and the geometry processing domains. We explore an application of the K-Means clustering algorithms in conjunction with the semi-automatic segmentation techniques for further more precise image segmentation. Moreover, we investigate the integration of the morphological transformations with the Random walker Segmentation method and the MS functional approximation methods to enhance the processing of geometric data on meshes. We hope to shed light on developments and future uses in the fields of computational image and geometry processing through the use of these coupled methodologies.

Chapter 2

Related Work

By exploring the frontier of semi-automatic segmentation, researchers have crafted some innovative approaches to empower users in the image analysis. Boykov and Jolly pioneered the paradigm shift with the graph cuts, empowering users to delineate foreground and background regions via intuitive seed points[1], followed by the energy minimization through a Max-flow Min-cut framework. Building upon this foundation,[2] Li et al. merged watershed segmentation with the graph cut optimization, while[3] Nagahashi et al. iteratively refined the segmentation through graph cuts and multiscale smoothing. Shi et al. introduced the groundbreaking methodology based on the normalised cuts, redefining image segmentation as the graph partitioning problem. Further enriching this toolkit, Rother et al. introduced graph cut, ingenious fusion of colour modelling and the local minimization, enhancing the precision of an object delineation.[4] Mortensen and Barrett revolutionized user interaction with the intelligent scissors, enabling swift and accurate object extraction through the intuitive gestures.
In the contrast, the Random Walker method[5], a brainchild of Grady, introduced the dynamic paradigm where user-provided seed points propagate information across an image, allowing for flexible and intuitive segmentation of arbitrary objects. This approach, augmented by subsequent extensions, has emerged as the cornerstone in interactive image segmentation, offering unprecedented versatility and an ease of use.Meanwhile, the[6] Mumford-Shah (MS) functional, originally devised for image segmentation, has transcended the traditional boundaries to find an application in diverse domains. Inspite of its inherent computational challenges, latest advances in approximation techniques and discrete calculus formulations have unlocked its potential for an image restoration, feature extraction, and the mesh processing. From image denoising to mesh segmentation, MS functional has performed an indelible mark on the landscape of image processing and analysis.
Complementing these sophisticated techniques, [7] K-Means clustering algorithm stands as the stalwart in the domain of an unsupervised learning, offering the robust framework for partitioning data into the clusters. However, its reliance on initializations and susceptibility to the local minima highlights the ongoing quest for an improved clustering methodologies.Together, these methodologies form the bedrock of the modern image processing, offering the rich tapestry of tools to overcome the myriad challenges in segmentation and analysis, while continuing to encourage novel approaches and advancements.

Chapter 3

Basic idea of the Image Segmentation Methods

3.1 RANDOM WALKER METHOD

This section explains the details of Grady's previously disclosed Random Walker approach [5].G= (V,E) is a representation of an undirected graph, where E and V denote the set of edges and vertices, respectively.The collection of pixels in an image is denoted by the vertex V. Two vertex sets are present.The first group, referred to as "labelled vertices," is always annotated by the user as belonging to several objects, namely the seeds. The other set, referred to as "unlabelled vertices," is located to the left of the image's pixels. The edges are made up of pairs of pixels that are neighbours in a picture; these can be either typical 4- or 8-neighborhood pairs.The weight of an edge "e" could be indicated depending on the difference in the two pixels' colour scale or grayscale intensities.

$$w\left(v_i, v_j\right) = e^{\frac{d(v_i,v_j)^2}{\sigma^2}} or w\left(v_i, v_j\right) = \frac{1}{1 + \frac{d(v_i,v_j)}{\sigma}} \tag{3.1}$$

The value of the parameter σ can be selected appropriately. An edge's weights for similar pixels fall within the range of $(0, 1)$; for dissimilar pixels, the weight will be closer to 0, while for similar pixels, the weight would be closer to 1. The Random Walker method works on the basis of the above-mentioned graph structure. The premise is there are k potential objects in an image, and each and every labelled vertex of V_m belongs to 1 of k objects. Assume that e is the weighted edge with endpoints like v_i and v_j; i.e., $e = (v_i, v_j)$. The weight of the edge $w(e)$ can be understood as a measurement of the random walk's transition probability from one vertex to another, and it falls within the range of $(0, 1)$. Depending on the edge's weight, a random walk from v_i to v_j is more likely to transition if the two have comparable colours or intensities, but is less likely to do so if they don't.

Assuming the specified random walk on a graph, we have the above transition probabilities. For every specific, for every single unlabeled vertex $v \in V_u$, quantity p is the probability that the random

walk, starting at that vertex, will end up at any one of the labelled vertices that correspond to the specific object k. Following that, the probabilities are used to guide the segmentation of the images. More precisely, we determine that every vertex v_i belongs to the segment k if and only if $p_{ik} > p_{ik'}$ for any $k' \neq k$. Because they include a quick shift in hue or intensity, the edges in the image—as opposed to the edges in the graph—correspond to low transition probabilities. As a result, we can state that when doing the segmentation, our algorithm will favour image edges.
By solving the massive, sparse linear system, it becomes possible to compute the probabilities p_{ik} [?]. The combinatorial Dirichlet problem has the same answer as the Random Walker probabilities. The definition of the Dirichlet integral is:

$$D[u] = \frac{1}{2}\int_{\Omega} |\nabla u|^2 \, d\Omega \tag{3.2}$$

for the region Ω and the field u . Finding the harmonic function is the next step, though. The function that fulfils the Laplace equation is known as the harmonic function,i.e.

$$\nabla^2 u = 0 \tag{3.3}$$

The Dirichlet issue is the task of determining the harmonic function, i.e., subject to its boundary values. The Euler-Lagrange equation for the Dirichlet integral is a harmonic function that minimises the integral as long as it meets the boundary constraints. The combinatorial Laplacian matrix is defined by us as

$$L_{ij} = \begin{cases} \delta_i & \text{if } i = j \\ -\omega_{ij} & \text{if } i \text{ and } j \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases} \tag{3.4}$$

where vertices i and j serve as the index for L_{ij}.

By using these definitions, we can determine how to solve for a harmonic function that, while maintaining the fixed seed nodes, determines the probabilities and potentials on the unseeded nodes. The Dirichlet integral can be expressed combinatorially as follows:

$$D[u] = \frac{1}{2}x^T L x = \frac{1}{2}\sum_{c_{ij}\in E} w_{ij}(x_i - x_j)^2 \tag{3.5}$$

The function x that minimizes the integral is the combinatorial harmonic. L is positive semi-definite, hence the only critical points that will be minimum are those in $D[x]$. Next, divide the vertices into two sets: V_m, which are the marked/seed nodes, and V_u, which are the unseeded nodes. This way, $V_m \cup V_u = V$ and $V_m \cap V_u = \emptyset$. Note: All of the seed points, V_m, regardless of their designations, are contained in V_m. Without losing generality, we can suppose that the nodes in L and x are arranged with the seed nodes at the top and the unseeded nodes at the bottom. Thus, the above equation may be broken down into:

$$D[x_u] = \frac{1}{2}\begin{bmatrix} x_m^T & x_u^T \end{bmatrix}\begin{bmatrix} L_m & B \\ B^T & L_u \end{bmatrix}\begin{bmatrix} x_m \\ x_u \end{bmatrix} \tag{3.6}$$

where, respectively ,xm and xu represent the potentials of seeded and unseeded nodes. Regarding determining critical point yields and distinguishing Dxu in relation to xu,

$$L_u x_u = -B^T x_m \tag{3.7}$$

By indicating the (possible) probability that xis assumes for each label,s, at node vi.
On defining the function $Q_{v_j} = s$, $v_j \in V_m$ where $s \in \mathbb{Z}$, $0 < s \leq K$, as the collection of labels for seed points. Upon defining the $V_m \times 1$ vector, at the node $v_j \in V_m$, where $|.|$ indicates cardinality, for each label s:

This is the linear equation system where Vm is unknown. This equation would not be singular if the graph is connected or if there is a seed present in every connected component.

$$m_j^s = \begin{cases} 1 & \text{if } Q(v_j) = s \\ 0 & \text{if } Q(v_j) \neq s \end{cases} \tag{3.8}$$

Therefore, solving for labels s yields a solution to the combinatorial Dirichlet problem.

$$L_u^s x_u^s = -B^T m^s \tag{3.9}$$

3.2 Mumford Shah Method

3.2.1 Mumford Shah Configuration

Mumford and Shah defined the piecewise-smooth approximation function [6] that describes an input image. Reading this functional as follows:

$$MS[u, C] = \alpha \int_{\Omega} (u - g)^2 \, d\Omega + \beta \int_{\Omega \setminus C} |\nabla u|^2 \, d\Omega + \gamma \int_C dS, \tag{3.10}$$

with C being the set of (unknown) curves that characterise the collection of discontinuities,g being an input image on the two-dimensional planar domain Ω,u (unknown) being its approximation. The tightness , smoothness , and length of discontinuities . of a model's approximation to an input image are its intuitive parameters.
Furthermore, C is frequently thought of as the closed curve or the boundary of a space division for segmentation purposes, even if neither the model nor the best solution call for this. Furthermore, this model is not limited to the pictures; in general, g can be a real-valued function over any surface, and Ω need not be the plane. Unfortunately, this functional is non-convex and challenging to optimise outside of the restricted cases.

3.2.2 To Optimise the MS functional.

Convex envelopes or convex approximations to the MS functional are the most common approximations to the MS that have been suggested in order to avoid these problems. By substituting the term accounting for the length of the segment boundaries with the total change of the gradient [8] of the segment binary indicator function, the MS functional could be convexified when this problem is limited to foreground/background extraction. This can be assumed as a piecewise constant function, which eliminates the possibility of isolated discontinuities or open segment boundaries, as opposed to using a piecewise-smooth assumption. Although this might be expanded to include many segments, the discontinuous sets are still inevitably produced.

Although the mesh segmentation [9] process has successfully employed this assumption, it can be appropriate within situations where the interested function is that normal field which varies relatively smoothly across surfaces and may display internal discontinuities.

Similarly, Tsai et al. [10] use various level sets to directly tune MS functional, although they too need closed segment boundaries. As a result, these methods fall short of the desired level of generality. Instead, we use an estimate based on the -convergence results from Ambrosio and Tortorelli, which is likely to converge towards the MS functional.

For example, the Finite Element Method is used by Chambolle, Dal Maso, and Bourdin, together with an adaptive mesh refinement and edge alignment needed for optimisation. These numerical algorithms are quite vulnerable to noise, even with such sophisticated techniques. Fortunately, novel discretization strategies for Ambrosio-Tortorelli (AT) [11] models have been developed on grids; these strategies no longer require adaptive meshes or FEM in order to obtain piecewise smooth solutions.

Applications for this discretization include feature extraction and picture restoration on voxel-based digital geometries. [12] Pokrass et al. used the first discrete differential calculus version of AT to overcome problem of partial matching in order to gain 3D shapes that are rigid. Unlike our pointization, this one evaluates the cross-term on the faces, producing smoother features, while discontinuities and values reside on the vertices. This works effectively for their particular goal but restricts the applications it can be used for.

3.3 Morphological Operations Method

3.3.1 The Opening transformation

It involves performing the erosion followed by the dilation on the input image. It helps in removing the noise from an image by shrinking boundaries of the foreground objects.

3.3.2 The Closing transformation

The closing is opposite of opening, involves a dilation i.e. followed by an erosion. It is quite useful for filling tiny holes or gaps in the foreground objects.

3.3.3 The Morphological gradient

A gradient is obtained by taking the difference between a dilated image and an eroded image. It highlights the boundaries of objects in an image.

3.3.4 The Top hat transformation

This transformation is a difference between an input image and its opening. It highlights small, bright regions in an image.

3.3.5 The Black hat transformation

This transformation is a difference between closing of an input image and an input image itself. This highlights dark regions or features in an image.

3.3.6 The Boundary extraction

It involves taking the difference between an input image and its eroded version. This highlights the boundaries of objects in an image.

3.4 K-Means Clustering

The K-Means algorithm was first proposed by J.B. MacQueen [13] is clustering process that depends on splitting. an algorithm that's generally utilised for pattern recognition or mining of data. Foundations of technique are error criterion or square error with the goal of minimising the performance index of cluster.

The process seeks out "K" divisions that require the set of criteria for optimising output. In order to obtain the initial classification, we must first select some dots to represent initial focal points of cluster (typically,first K sample income dots are chosen to present the initial cluster focal point). Next, we must gather the remaining sample dots and arrange them in accordance with least distance criterion. If the initial classification we felt unreasonable,must modify them by recalculating every cluster focal point. This process must be repeated until the desired classification is obtained.

The certainty, efficiency, and briefness of the K-Means algorithm are its advantages. As a result, this method heavily relies on the starting dots and the variations in beginning sample selection, which consistently produce distinct results.

Chapter 4

Algorithms

4.1 Algorithm for Random Walker Method

For applications of real world involve segmenting an interested object that has different shades of colour from "distant" portions of the colour spectrum. Because of this, Random Walker is subject to an unusual constraint: it must determine the weight between two pixels entirely by calculating the Euclidean distance between each pixel's colour vector. Since $\|A_1 - A_2\|$ is "the large value," for example, if the object is primarily composed of two colour regions $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$, thereafter any pixel pair of an interested object that are adjacent, have the colour values A_1, A_2 respectively would get connected by weight's edge $w = e^{-\beta\|A_1 - A_2\|^2}$. Since this value is almost zero, Random Walker would not be able to "transition" between these pixels. This presents a special challenge when there is object segmentation. Our goal in this situation is to deal with the difficulties brought on by such barriers.

4.1.1 Graph based weight

To tackle or get around this issue, weight based on probability distribution has been employed. Consequently, the mean-shift based segmentation process might come before the Random Walker based segmentation. An algorithm's complexity is increased by mean-shift based methods, so we looked for any segmentation techniques that may eliminate the need for over-segmentation in image preprocessing. When it comes to semi-supervised segmentation, the user has access to "seeds," or previous knowledge that provides crucial details about the object's profile. Typically, when a user "scratches" an object of interest, they are able to identify the colour spectrum regions that make up the object.

Ideally, this method should be granted great probability when crossing 2 pixels with colour values from set $A = \{A_i\}_{i=1}^{n}$, if the object is composed of clustered parts of the colour spectrum, such as $A_1, A_2, \ldots, A_n$. Predefining that the distance between A_i and A_j must be considered as 0, as well as that the colour vectors near A_i and A_j have a tiny distance between them, are two ways to accomplish this.

Visualising colour space as the completely linked graph is one method to accomplish the aforementioned goal. Since discrete colour values are typically used in digital image processing, each and every colour vector can be seen as a node in a fully connected graph. $d_{ij} = |B_i - B_j|$ is the weight of an edge that connects the colour values B_i.

First, we create this kind of graph G and set the weights. Now, the user just needs to adjust the weight of an edge connecting any A_i, A_j after assigning seed information, which is given in the form of set A. Using this redesigned graph G, now, we use the Floyd-Warshall pairs shortest path algorithm, which yields least distance between any two colour valence. We now create a new graph G', where minimum path's weight between two nodes, B_jB_i and B_j in G equals the distance between them, represented as d_{ij}'s. Take note of a formulation that meets both criteria, d''_{ij}.

4.1.2 To use Random Walk

Take the task to be segmented k objects, that an user has provided seed data in sets' form $A_i^{(l=1 \text{ to } k)}$, where l represents each sign, to be performed for each l between 1 and k. We take the subsequent actions:

1. Create the network H whose vertices are an image's pixels. If comparable pixels are 4-connected, then two vertices are connected by an edge.

2. Using the set A_i^4, construct G'_i and construct the graph G_i on the colour space. A d_{ij}^{χ} is the distance between colours A_i and A_j in G'_i.

3. The weight of an edge linking two linked vertices of H, v_i and v_j with colour values A_i, A_j is determined by $w_{ij}^{\chi} = e^{-\beta d_{ij}^{\chi}}$.

4. Build the Laplacian L_l whose ijth element is defined as ...

$$L_{ij} = \begin{cases} \delta_{i,\chi} & \text{if } i = j \\ -\omega_{ij,\chi} & \text{if } i \text{ and } j \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases} \tag{4.1}$$

5. We solve the system $LuP = B^T m$, just like in the case of an ordinary Random Walker.

6. We classify v_j as belonging to segment l if $P_i > P_i^{\chi}$ for every $l' \neq l$. Take note that P is known as the $(P_1, P_2, ...)^T$ where P_i is vertex probability v_i belongs to vertex set with label l to vertex v_i.

4.2 Algorithm for Mumford Shah Method

The Mumford-Shah algorithm is the technique used for an image segmentation and an edge detection, aiming to create the piecewise smooth representation of an image. Here is the simplified outline:

- Initialization:-Begin with the initial guess for an image segmentation. Set parameters to balance the smoothness of an image and how closely it fits an original image.

- Energy Functional:-Define the function that measures the quality of segmentation. This function considers the difference between the original and the segmented images, the smoothness and segmented image and the total length of an edge.

- Gradient Descent:-Use an iterative process to minimise an energy functional .And update the segmented image and the edges using the gradient descent approach, that iteratively adjusts them to minimise the energy functional.

- Edge Detection:-Identify edges by finding the regions where an image changes rapidly. Update set of edges to reflect these detected changes.

- Regularisation:-Regularise an edge set to avoid having many edges.Try to Smooth the edges using techniques like the curve evolution or the level set methods to ensure they are not fragmented.

- Iteration:-Repeat process of updating an image and the edges until the changes become minimal, indicating that the segmentation has stabilised.

- Final Output:Produce final segmented image and an edge set, achieving the piecewise smooth representation of an original image.
  Throughout the process, an algorithm balances fit to an original image and smoothness of segmentation by adjusting parameters. This balance helps in achieving the meaningful segmentation where the edges are very clear and segments are very smooth.

4.3 Algorithm for Morphological Transformations Methods

4.3.1 Dilation

- Input: Take the binary image and the structuring element

- Process:For each and every pixel in an image, superimpose structuring element centred on pixel.If any pixel in structuring element overlaps with the foreground pixel in an image, set the centre pixel to the foreground.
- Output: Generate dilated image, where the objects are expanded.

4.3.2 Erosion

- Input: Take the binary image and the structuring element.
- Process: For each and every pixel in an image, superimpose structuring elements centred on the pixel. If all pixels in the structuring element overlap with the foreground pixels in an image, set the centre pixel to the foreground; otherwise, set it to the background.
- Output:Generate an eroded image, where the objects are shrunk.

4.3.3 Opening

- Input: Take a binary image and a structuring element.
- Process: Perform erosion on an image to remove the small objects and the noise.Perform dilation on an eroded image to restore size of remaining objects.
- Output: Generate an opened image, where the small objects and the noise are removed but main structures remain intact.

4.3.4 Closing

- Input:-Take the binary image and the structuring element.
- Process: Perform dilation on an image to fill the small holes and the gaps. Perform erosion on an dilated image to restore size of the original objects.
- Output: Generate closed image, where the small holes and the gaps are filled, and main structures are preserved.

4.4 Algorithm for K-means Clustering Method

Let us consider N pattern x1,x2,,xN ,samples that need to be classified.They must be grouped into K cluster.

1. Select value from $\{x_1, x_2, \ldots, x_n\}$ to serve as first focal point's cluster z_1. For example, $z_1 = x_1$.

2. Select a different place to serve like the second cluster's focus point that is equally spaced from z_1. Next, measure distant between every point and z_1.

$$x_i - z_i, \quad i = 1, 2, \ldots, n \tag{4.2}$$

If:

$$\|x_j - z_1\| = \max \|x_i - z_1\|, \quad i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n \tag{4.3}$$

Next, designate x_j as the second cluster's focal point, with $z_2 = x_j$.

3. Determine the distance, one by one, between every example in $\{x_1, x_2, \ldots, x_N\}$ and $\{z_1, z_2\}$:

$$d_{i1} = \|x_i - z_1\|, \quad i = 1, 2, \ldots, N \tag{4.4}$$

$$d_{i2} = \|x_i - z_2\|, \quad i = 1, 2, \ldots, n \tag{4.5}$$

Select shortest distance between the results:

$$\min(d'_{i1}, d'_{i2}), \quad i = 1, 2, \ldots, n \tag{4.6}$$

Collect minimums from each model, $\{z_1, z_2\}$ sample. Select greatest from the list of least to serve as focal point z_3 of the cluster. If:

$$\min(d'_{i1}, d'_{i2}) = \max\{\min(d_{i1}, d_{i2})\}, \quad i = 1, 2, \ldots, N \tag{4.7}$$

Then:

$$z_3 = x_j \tag{4.8}$$

4. Assume that the cluster focal points $\{z_i, i = 1, 2, \ldots, r\}$ have a value of r $(r < k)$. Finding $r+1$th focal point of the cluster is now necessary:

$$\min\{d_{j1}, d_{j2}, \ldots, d_{jr}\} = \max\{\min\{d_{i1}, d_{i2}, \ldots, d_{ir}\}\}, \quad i = 1, 2, \ldots, N,\ j = 1, 2, \ldots, N \tag{4.9}$$

5. Continue until $r + 1 = K$ is reached.

6. As of right now, $z_{1,1}, z_{2,1}, \ldots, z_{k,1}$ are our chosen initial cluster focal points. The serial numbers utilised in an iterative process to find the cluster locations are the numbers in parentheses.

7. Assign $\{x_1, x_2, \ldots, x_N\}$ to one of K clusters in accordance with the distance minimization rule:

$$\|x - z_j(t)\| = \min \|x - z_i(t)\|, \quad i = 1, 2, \ldots, K,\ j = 1, 2, \ldots, k \tag{4.10}$$

Thereafter:

$$x \in S_j(t) \tag{4.11}$$

The serial number of an iterative operation is represented by the symbol t in the formula; S_j reprents for jth cluster, and z_j represents focal point of the clusters.

8. Determine the updated values ny vector for every focal point of cluster individually:

$$z_j(t+1), \quad j = 1, 2, \ldots, k \tag{4.12}$$

Determine sample vectors by mean for every cluster:

$$z_j(t+1) = \frac{1}{N} \sum_{x \in S_j(t)} x, \quad j = 1, 2, \ldots, K \tag{4.13}$$

The number of samples of the jth cluster is represented by the symbol in the formula above. Determine the sample mean vectors for each of the K clusters. The cluster criterion function J_j could be minimized by using mean vectors to create new clusters.

$$J_j = \sum_{x \in S_j(t)} \|x - z_j(t+1)\|^2, \quad j = 1, 2, \ldots, K \tag{4.14}$$

9. Classify all the samples of pattern one by one again and executive an iterative process if:

$$z_j(t+1) \neq z_j(t), \quad j = 1, 2, \ldots, K \tag{4.15}$$

then go back to step 7. An algorithm has completed its convergence if:

$$z_j(t+1) = z_j(t), \quad j = 1, 2, \ldots, K \tag{4.16}$$

Chapter 5

Result

5.1 for the Random Walker Method

Figure 5.1: segmented image using random walker method

Here, this segmentation result image shows the segmented image where the different regions are highlighted that are based on the labels created by Random Walker algorithm. Regions are differentiated based on markers set in grayscale image.There will be two segments, one for areas with intensity less than 0.4 and one for areas with intensity greater than 0.4.

5.2 for the Mumford Shah Method

Figure 5.2: segmented image using mumford shah method

Here,Segmentation starts by initializing the level set function with the circular region, applies Gaussian smoothing to grayscale image, and iteratively evolves level set to segment an image. The output shows images: the original grayscale image, the edges, and the segmented image, highlighting the distinct regions based on the intensity variations.

5.3 for the Morphological Operations Methods

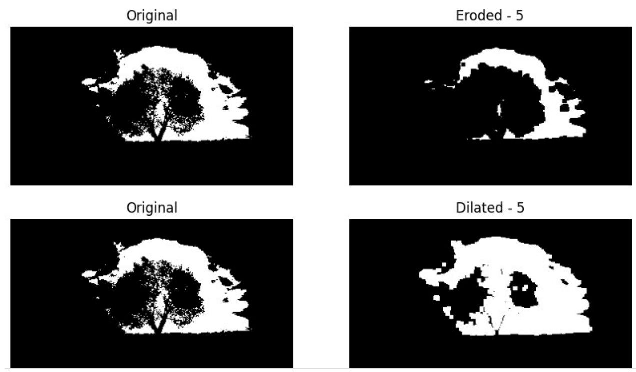

Figure 5.3: segmented image using morphological operations method

Figure 5.4: segmented image using morphological operations method

Here,the segmented images consists of the four processed images, each demonstrating the effect of one of the morphological operations, helping to understand how these operations modify.

5.4 for the K-Means Clustering Method

Figure 5.5: segmented image using k-means clustering method

Here,the segmented image displays regions of an image grouped into 2 dominant colors, highlighting the major color-based segments in an image.

Chapter 6

Conclusion

In the conclusion, the presented algorithm exemplifies a systematic approach to computational image processing and geometry, showcasing robust experimental validation, the efficient implementation, and an innovative techniques for an image enhancement and the clustering. Through the qualitative evaluation on an image, the algorithm's segmentation performance surpassed that of the ordinary Random Walker method, demonstrating notable accuracy improvements across various scenes and the objects.By leveraging morphological transformations, follow as opening, closing, gradient, top hat, black hat, and boundary extraction, an algorithm achieves the superior noise reduction, an object highlighting, and the boundary delineation, contributing to an overall image quality and the segmentation accuracy.

Moreover, the evaluation of the K-Means clustering algorithm enhances the algorithm's versatility and the effectiveness in the cluster analysis tasks. By mitigating random cluster focal points and optimising clustering criteria, the algorithm outperforms the standard K-Means method, offering stable and accurate clustering outcomes.In the essence, combined results highlight the algorithm's comprehensive approach, positioning it as the valuable tool for diverse applications in the computational image processing and geometry, characterised by its performance, efficiency and an innovation in handling complex image data and geometric structures.

Chapter 7

References

[1] Y. Boykov and M.-P. Jolly. Interactive organ segmentation using graph cuts. In Proceedings of the International Conference on Medical Image.Computing and Computer Assisted Intervention (MICCAI), pages 276–286, 2000.
[2] Y. Li, J. Sun, C.K. Tang, and H.Y. Shum. Lazy snapping. In Proceedings of the International Conference on Computer Graphics and Interactive Techniques (SIGGRAPH), volume 23, pages 303–308, 2004
[3] T. Nagahashi, H. Fujiyoshi, T. Kanade: Image Segmentation Using Iterated Graph Cut Based on Multi-scale Smoothing. Computer Vision ICCV-2007. Lecture Notes in Computer Science Volume 4844, 2007, pp 806-816.
[4] E.N. Mortensen and W.A. Barrett. Intelligent scissors for image composition. In Proceedings of the International Conference on Computer Graphics and Interactive Techniques (SIGGRAPH), pages 191–198, 1995.
[5] Leo Grady, Random walks for Image segmentation. IEEE TRANSACTION ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 28, NO. 11, NOV. 2006
[6] MUMFORD D., SHAH J.: Optimal approximation by piecewise smooth functions and associated variational problems. Commun. Pure Applied Mathematics (1989), 577–685.
[7] BianZhaoQi and ZhangXuegong Pattern Recognition Beijing Tsinghua University Press 2000
[8] CHAN T. F., ESEDOGLU S., NIKOLOVA M.: Algorithms for finding global minimizers of image segmentation and denoising models.SIAM Journal of Applied Mathematics 66, 5 (2006), 1632–1648.
[9] ZHANG J., ZHENG J., WU C., CAI J.: Variational mesh decomposition. ACM Trans. Graph. 31, 3 (2012), 21:1–21:14
[10] TSAI A., YEZZI A., WILLSKY A. S.: Curve evolution im-plementation of the Mumford-Shah functional for image segmentation,denoising, interpolation, and magnification. IEEE Trans. on Image Pro-cessing 10, 8 (2001), 1169–1186.
[11] AMBROSIO L., TORTORELLI V. M.: Approximation of functional depending on jumps by elliptic functional via -convergence.Comm. Pure and App. Math. 43, 8 (1990).
[12] POKRASS J., BRONSTEIN A. M., BRONSTEIN M. M.: A correspondence-less approach to matching of deformable shapes. In Int. Conf. Scale Space and Variational Meth. in Comp. Vis.

(2011).
[13] JAIN A K, DUBES R C. Algorithms for clustering data[M].New Jersey:Prentice-Hall,1988.