QUANTUM TELEPORTATION VIA ENTANGLED FLAVOR STATE OF NEUTRINOS

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By
Mahe-Noor Fatima
2k22/MSCPHY/25
Under the Supervision of

Dr. Mukhtiyar Singh
Department of Applied Mathematics
Delhi Technological University

Dr. Satyabrata Adhikari
Department of Applied Physics
Delhi Technological University

To the
Department of Applied Physics

DELHI TECHNOLOGICAL UNIVERSITY
(Formerly Delhi College of Engineering)
Shahbad Daulatpur, Main Bawana Road, Delhi 110042, India

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DELHI TECHNOLOGICAL UNIVERSITY
(Formerly Delhi College of Engineering)
Shahbad Daulatpur, Main Bawana Road, Delhi 110042, India

I, MAHE-NOOR FATIMA (2K22/MSCPHY/25) student of M.sc physics, hereby certify that the thesis entitled "**Quantum Teleportation via the Entangled Flavor State of Neutrino**" in partial fulfilment of the requirements for the award of the degree of Master in Science, which submitted to the Department of Applied Physics, Delhi Technological University, Delhi is an authentic record of my own work carried out during the period from 2023-2024 under the supervision of Dr Satyabrata Adhikari and Dr Mukhtiyar Singh .

This matter presented in the thesis has not been submitted by me for the award of any other degree of this or any other institute.

Mahe-Noor Fatima
2k22/MSCPHY/25
Department of Applied Physics
Delhi Technology University

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(Formerly Delhi College of Engineering)
Shahbad Daulatpur, Main Bawana Road, Delhi 110042, India

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Dr. Mukhtiyar Singh
Department of Applied Physics
Delhi Technology University

&

Dr. Satyabrata Adhikari
Department of Applied Mathematics
Delhi Technology University

Abstract

Quantum teleportation, a phenomenon well founded in the principles of quantum entanglement has been a subject of extensive theoretical and experimental research. Since Neutrino oscillations exhibits an entangled state thus, we can exploit this property of neutrinos to transfer quantum information encoded within them.

Our main purpose is to find out a state from class of W states which perfectly defines neutrino oscillation and showcase efficient quantum teleportation. Since neutrinos interact very weakly, hence the process of storing and retrieving the information will be difficult which is needed to be tackled. Different inequalities like Leggett Garg inequality and entanglement criteria have been employed to assess the entanglement in the neutrino oscillation. The strength of entanglement is measured using negativity, concurrence and teleportation fidelity is been calculated for different states. Thus, in this study, we propose an approach demonstrating quantum teleportation utilizing the entangled flavor state of neutrinos.

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SYMBOLS /ABBREVIATIONS

EPR – Einstein Podolsky and Rosen Paradox

Qubits – Quantum Bits

QIS – Quantum Information Science

$\otimes$ – Tensor product

NPT – Negative Partial Transpose

PPT – Positive Partial Transpose

Introduction:

One of the best examples of a quantum information processing problem is quantum teleportation, in which an unknown state can be perfectly sent between two locations, i.e. between a sender and a receiver via classical communication, possessing a previously shared entangled pair. This is an excellent usage of entangled states with broader implications for quantum information technology. In Bennett et al. initial protocol, an unknown qubit is transported via an EPR pair i.e. (Einstein –Podolsky-Rosen pair), along with two classical bits of information is sent from Alice to Bob.

Later on, quantum teleportation was further expanded to include scenarios in which the sender and receiver are connected via a channel that is not ideal, but rather affected by noise, instead of using a flawless EPR pair. Quantum teleportation is also possible for infinite dimension Hilbert spaces, and it is known as continuous variable quantum teleportation. Thus, we infer that entangled states have enhanced the capacity for classical information as well as our ability to do a variety of quantum information processing tasks.

There is a significant literature on the study of correlations in quantum systems, and its practical importance is evident from its utilisation in quantum technologies like quantum teleportation and quantum encryption.

There has been a significant movement to broaden these findings to particle physics systems. Neutrino is a special study of interest here because the neutrinos which are characterised by three unique flavor states namely electron, muon, and tau, undergo oscillations as they traverse through space, this has been demonstrated by a number of tests employing both man-made and natural neutrinos. The behaviour described in quantum mechanics is exemplified by neutrino oscillations, which involve a three-flavor oscillation. This indicates that a neutrino initially observed in a specific flavor state has a probability of being detected in a corresponding flavor state as time progresses. The linear superposition of neutrino mass eigenstates that are non-degenerate causes oscillations in the neutrino flavour state. Quantum entanglement and coherence are the two key elements of quantum superposition, making the exploration of quantum entanglement in neutrino systems a valid area of study.

These particles interact very weakly hence the mode entanglement is the innate form of entanglement as is also showcased by its coherent time evolution and also their decoherence in comparison to other particles utilised for quantum information processing is minimal. In a recent study, the concept of a neutrino's state was analysed within the scope of two different configurations: the linear superposition state portraying the neutrino as a two-qubit system, and the three-qubit system representing the three-flavor system. Different quantum correlations such as Bell’s inequality and Bell’s–CHSH inequality violations, teleportation fidelity, geometric discord, Leggett-Garg inequality and other similar inequalities and correlations in neutrino oscillations have been analysed within their respective contexts.

Thus, by using these oscillations and properties of neutrinos, we suggest a new way to entangle the flavor states of the neutrino pairs, thereby building a quantum link that may circumvent traditional obstacles to communication. In this study we have entangled a pair of neutrinos whose flavor states are already entangled and different related quantum correlations such as negativity, concurrence, quantum teleportation fidelity have also been studied regarding the new quantum state so formed.

1 Quantum Computing

Investigating tasks related to information processing that can completed with systems involving quantum mechanics is known as quantum computation and quantum information. The most recent advancement in computing is quantum computing, which does computations using concepts from quantum physics. Information is processed in classical computing using bits that be either in state 0 or state 1. Quantum computing, in contrast, utilizes qubits, also known as quantum bits, which have the ability to simultaneously exist in multiple states due to the concept of entanglement. The domain of quantum computing is currently at an early stage of development, facing significant challenges in establishing and maintaining stable qubits' coherence. However, experts think that once these obstacles are removed, quantum computers might completely transform industries including materials science, cryptography, and optimisation by addressing specific problems exponentially faster than classical computers. Each discipline that has offered vital concepts to the fields of quantum computation and quantum information, including computer science, cryptography, quantum physics and information theory, should be examined in a historical and sequential manner.

1.1 How Quantum Computing came into being?

The development the origins of quantum computing can be found in several key milestones, with the foundation laid by foundational principles of the quantum domain, including the EPR paradox proposed by Einstein, Podolsky, and Rosen. Here is a brief overview of the journey from the EPR paradox to the development of the quantum computing:

1. **EPR Paradox (1935): -** A study co-authored by Albert Einstein, Boris Podolsky, and Nathan Rosen published a paper that introduced the EPR paradox. They proposed a scenario where two entangled particles, like electrons, would be correlated in situation like this way that the measurement of a single particle state would immediately ascertain the condition of the other,

regardless of how far apart they are. - Einstein is well known for having described this phenomenon as" spooky action at a distance."

2. **Quantum Mechanics and Bell's Theorem (1964):** Physicist John Bell formulated Bell's theorem, which demonstrated that any theory that adheres to certain reasonable assumptions must violate either locality (the concept that events occurring at one place cannot instantly affect events at another place) or realism (the idea that physical systems have pre-existing properties independent of measurement). Experiments conducted following Bell's theorem have subsequently validated the forecasts of quantum mechanics over classical physics.
3. **Quantum Information Theory (1980s):** Quantum information theory's advancement, initiated by physicists like David Deutsch and Richard Feynman, explored the notion of executing tasks with quantum systems computational tasks more efficiently than classical systems. - Richard Feynman suggested the concept in relation to quantum simulation, recommending that a quantum system could replicate the behaviour of another quantum system more effectively than classical computers.

4. **Algorithm of Shor (1994):** A novel quantum method for factoring big numbers tenfold quicker than the most well-known classical algorithms, was created by mathematician Peter Shor. This algorithm showed how quantum computers could perform noticeably better than classical computers for a particular kind of problem.

5. **Grover's Algorithm (1996):** Inventor Lov Grover created an algorithm that achieved a quadratic acceleration in solving unstructured search issues. This algorithm demonstrated that quantum computers could offer speedup advantages for a broader class of problems beyond factorization.
6. **Quantum Gates and Quantum Circuits**: Theoretical work on quantum circuits and gates laid the groundwork for building quantum algorithms. While

they function similarly to classical logic gates, quantum gates with quantum bits (qubits) instead of classical bits.

7. **Experimental Realization (2000s):** Researchers began experimenting with building and manipulating qubits using various physical implementations, such as superconducting circuits, trapped ions, and photonic systems.

8. **Quantum Supremacy (2019)**: Google asserted achieve quantum domination in 2019 By proving that their quantum processor. Sycamore was able to do a task more quickly than even the most sophisticated traditional supercomputers.

9. **Ongoing Developments**: Advances in quantum computing persist, as corporations and academic institutions strive to enhance qubit coherence, error correction, and scalability. Ongoing efforts aim to overcome challenges and make quantum computers more practical for a wider range of applications. The development of quantum computing is a difficult, continuous process that involves contributions from physicists, mathematicians, and computer scientists across several decades. The field continues to evolve, with researchers working on addressing difficulties and exploring the possibilities of quantum information in various applications.

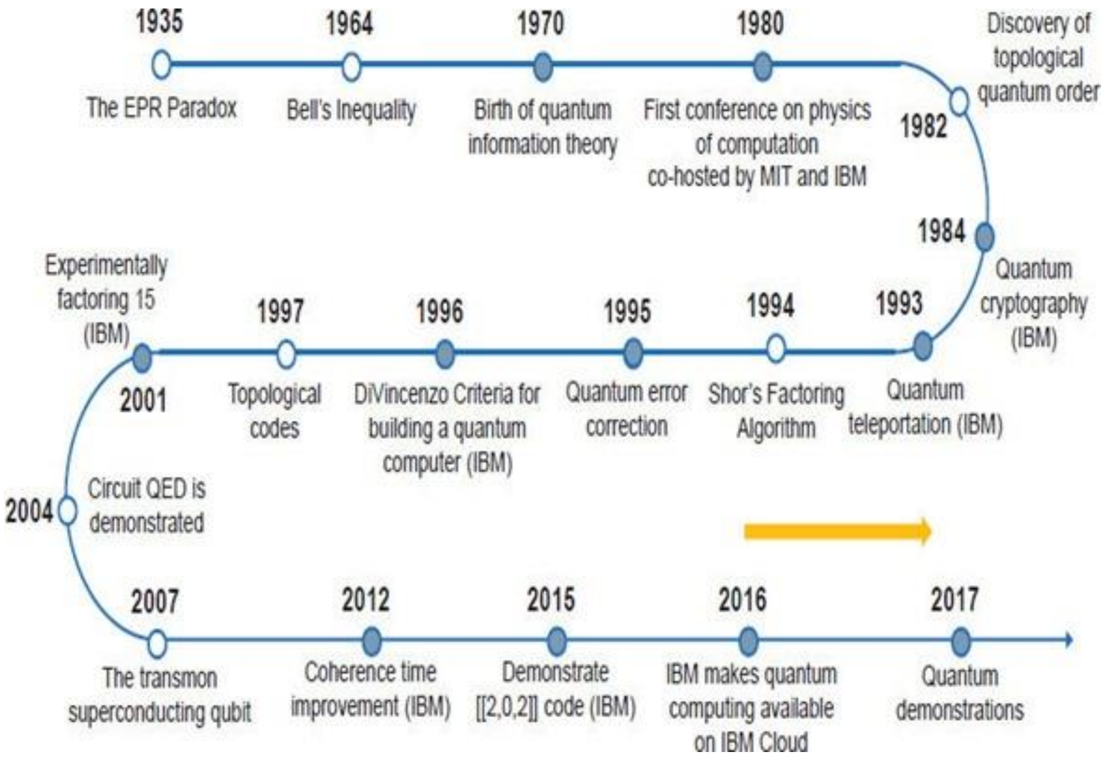

Fig 1: Demonstration on development of quantum computing

2. Entanglement

The concept of quantum entanglement can be defined as the interdependence between one particle in a paired entangled state and specific properties of the other particle, regardless of the distance separating them or the medium in which they exist.. These particles could be electrons or photons, for example, and one characteristic would be how it is" spinning" at the moment. One of the peculiarities of quantum entanglement is that it allows one to immediately learn something about the other particle by measuring one that is millions of light years away from the other. One of the fundamental laws of the universe seems to be broken by this peculiar instantaneous interaction between the two particles. Albert Einstein is credited with describing the phenomenon as" spooky action at a distance." An additional property of quantum states is their capacity to correlate, or have measurements of one state influence observations of another. The interaction of strongly correlated quantum states was investigated in 1935 by Albert Einstein, Boris Podolsky, and Nathan Rosen. They found that when two particles are firmly bonded, they share a single, unified state instead of their individual quantum states.

Methods for identifying an entangled state: -

- If a composite system is expressed as follows:

  $$\rho_{AB} = \rho_A \otimes \rho_B,$$

  then it is called a separable state, a pure product state. But if

  $$\rho_{AB} \neq \rho_A \otimes \rho_B$$

  then the composite system becomes entangled, it means that ρ_{AB} belongs to H_{AB}, while ρ A belongs to state space A and ρ B belongs to state space B.

- A bipartite pure state exhibits entanglement if the density operator of either subsystem is in a mixed state.

$$\rho_A = tr_B(\rho_{AB}) \text{ mixed}$$

$$\rho_B = tr_A(\rho_{AB}) \text{ mixed}$$

This means that ρ_{AB} = an entangled state.

Now for example we can consider any of the bell states, say

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

The density operator for which can be written as =

$$(\frac{|00\rangle + |11\rangle}{\sqrt{2}})(\frac{|00\rangle + |11\rangle}{\sqrt{2}})$$

$$\frac{|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|}{2}$$

We can find out the reduced density operator of the first qubit by tracing out the second qubit

$$\rho_1 = tr_2(\rho)$$

$$\frac{tr2\,|00\rangle\langle 00| + tr2\,|00\rangle\langle 11| + tr2\,|11\rangle\langle 00| + tr2\,|11\rangle\langle 11|}{2}$$

$$\frac{|0\rangle\langle 0| + |1\rangle\langle 1|}{2}$$

$$= I/2$$

which is a mixed state, thus this implies that the given bell's state is an entangled state.

- **Peres–Horodecki Theorem**:

  A 2-qubit state is entangled if and only if it is NPT. To determine if the density operator ρ of two quantum mechanical systems, A and B, can be separated, this condition is required. PPT stands for positive partial transposition, which is another name for it. The criteria are necessary and sufficient condition for the Hilbert spaces having dimensions as 2×2, 2×3, 3×2 to decide the separability of mixed states. However, for higher dimensional system this is only a necessary condition. For such system NPTness (the negative values of the eigenvalues will imply entanglement) implies entanglement, but there exists entangled state which are PPT. if $(\rho_{AB})^{TB}$ has positive eigenvalues this implies separable state for sure for Hilbert spaces having dimensions less than or equal to 6, but for the negative values the system is in the entangled state.

- **Schmidt Decomposition Criteria**

  $$|\psi_{AB}\rangle = \sum_i \lambda i\ |\psi A\rangle\ |\psi B\rangle$$

  With respect to the Schmidt Criteria, the pure composite state can be expressed as follows: λ_a non-negative real number, sometimes referred to as the Schmidt number or the Schmidt coefficients which indicates the count of non-zero eigenvalues of the states related to systems A and B. $\sum_{i}. \lambda_i^2=1$.

By the Schmidt decomposition $\rho_A = \sum_{i}. \lambda^2 |i\rangle_A\ \langle i|_A$ and $\rho_B = \sum_{i}. \lambda^2 |i\rangle_B\ \langle i|_B$ such that eigenvalues of ρ_A and ρ_B are the same. The eigenvalues of the system's reduced density operator govern many significant aspects of quantum systems; as a result, given a pure state of a composite system, these properties will be identical for both systems. If value of $\lambda = 1$, separable state If value of $\lambda > 1$ entangled state, (for the composite system).

3 Bell’s Inequality

Quantum mechanics introduced Bell’s inequality as it probes for non-classicality phenomena and posits about “Hidden variables” are real.” The inequality has been named after physicist John Bell whose work was featured in an article titled, which was issued in 1964” On the Einstein-Podolsky Rosen paradox”. In summary, the Einstein-Podolsky-Rosen (EPR) paradox represents a theoretical thought experiment developed in 1935 by three individuals known as Albert Einstein, Boris Podolsky, and Nathan. The paradox sought to destabilize some of the tenets in quantum mechanics including the notion of entanglement. Mathematical expression known as bell’s inequality is used as a discriminant which discriminates theoretical prediction of Quantum Mechanics and Classical Physics. This includes a set of correlations among measured parameters taken for entangled pairs, assuming such characteristics as “hidden variables” that could explain behaviour without any involvement of measuring actions themselves. Bell proved that, should there be such unknown variables fulfilling certain common-sense conditionality, the correlation between measurements of any entangled particles will not exceed that fixed quantity. Though quantum mechanics breach of the Bell’s disparity under those specified conditions as a result of which it will be evident that the concept of hidden variables that are classical stands in opposition to quantum mechanical expectations. The validation of Bell's inequality through experimental tests further confirms the support for mechanical theory in all cases, while refuting any notions of classical hidden variable. The experiments establish that the entangled particles are nonlocally connected; hence, whatever happens in one system can be observed instantaneously on the other separated distant system. Bell’s inequality has been fundamental for the quantum mechanics theory development. Without doubt, it had allowed us to explore more about quantum entanglement as well as the boundaries of classical reality when it comes to the particle behaviour at the quantum scale.

Let’s say that we do an experiment in which Charlie prepares two particles in order to comprehend this inequality. After completing the preparation, he sends Bob the second particle and Alice the first. Alice measures her particle as soon as she gets it. The physical attributes that these metrics pertain to are PQ and PR, respectively. Let us now assume that there are two possible outcomes for each measurement: +1 or -1. Consider a scenario where Bob can measure one of the two properties, PS or PT, and discover that the property has an objectively determined value of S or T, each assigning a value of +1 or 1. The experiment is timed such that Bob and Alice can complete their measurements. Because physical influences cannot travel more quickly than light, Alice's measurement cannot change Bob's measurement's outcome, or vice versa.

Figure 2: Schematic representation of experimental setup for bell's inequality

4 Leggett Garg's Inequality

The Leggett-Garg inequalities consist of a series of theoretical inequalities developed by physicists Anthony J. Leggett and Anupam Garg in the 1980s. Leggett-Garg inequality is a concept in the field of quantum mechanics, not in general mathematical or statistical inequalities. These inequalities are designed to test the concept of macro-realism in the quantum mechanical setting. The Leggett-Garg inequality is formulated to assess the validity of classical physics in contrast to quantum mechanics when analyzing the dynamics of systems across time. It involves correlations between measurements performed at different times on a quantum system. In simple terms, the inequality provides a way to distinguish between classical and quantum behaviour by examining the correlations of observables at different times. The concept of macro-realism posits that macroscopic entities possess clearly defined attributes that remain constant regardless of any measurements or observations conducted. In the quantum world, however, the principles of superposition and entanglement challenge this classical notion. The Leggett-Garg inequalities provide a way to test whether a system exhibits behaviour consistent with macro-realism or if it shows quantum correlations that cannot be explained by classical physics. The Leggett-Garg inequalities involve correlations between measurements performed at different times on a single quantum system. Specifically, they consider a system with a binary
observable (a property that can have one of two possible values) measured at two different times. The inequalities express constraints on the correlations between these measurements under the assumption of macro-realism. Here is a simplified explanation of the Leggett-Garg inequalities for a system with measurements at times t1, t2, and t3:

- L1: $C(t1, t2) + C(t2, t3) - C(t1, t3) \leq 1$
- L2: $-C(t1, t2) + C(t2, t3) + C(t1, t3) \leq 1$
- L3: $-C(t1, t2) - C(t2, t3) + C(t1, t3) \leq 1$

In these inequalities:

- C (t1, t2) represents the correlation between measurements at times t1and t2.
- t1, t2, and t3 are the different measurement times.

A system is described as macro-realistic if the correlations between its measurements are satisfied by these inequalities. On the other hand, when the above inequalities are broken, they indicate quantum behaviour that cannot be described within the framework of classical macro realism. They are of a particular kind which deals with macroscopic manifestation of quantum coherence and superposition. The experimental tests of the Leggett-Garg inequalities in different physical systems including superconducting circuits and trapped ions confirm that quantum mechanics does not conform to classical mechanics. These inequalities are violated, illustrating a characteristic feature of quantum systems that differentiate them from general macro-realistic systems. Therefore, the Leggett-Garg inequality is an instrument for studying the edge of a classic and a quantum physics where the probabilities of sequential measuring results have been taken into a consideration.

5 Quantum Teleportation

The reconstruction of a given particular state at a distant place and its subsequent disappearance from its original place, through some correlation is what a quantum teleportation phenomenon sounds like. Let us consider a scenario where we have two spatially distant parties as Alice and Bob and a Referee. Now the referee sends a copy of qubit state say $|\chi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ belonging C^{2} to Alice which is unknown to her. Now Bob has to prepare the same state as that of Alice's at his end and give it back to the referee. However, there is a specific limitation: Alice is unable to transmit the particle to Bob due to the absence of a quantum communication link connecting them. But they can classically communicate to each other, where if, Alice knew the state, she can use the available classical channel to inform Bob about the state and ask him to prepare an identical copy of the state at his end but the method is restricted because of the following reasons namely,

- the state is unknown.
- secondly such a classical communication will require infinite classical bits,
- lastly no-cloning theorem prohibits Bob from creating a copy of the given state at his end.

Protocol: - Consider Alice and Bob sharing some bells' state for example, say

$$|\phi\rangle^{+} = |00\rangle + |11\rangle/\sqrt{2}$$

This implies that Alice will have two particle one from the entangled pair and the other given to her by the referee and Bob will have one particle from the entangled pair. The three-particle state can be presented as

$$|\chi\rangle_{s} \otimes |\phi\rangle^{+}_{AB} = (\alpha\,|0\rangle\,s + \beta\,|1\rangle\,s) \otimes (|0\rangle\,A \otimes |0\rangle\,B + |1\rangle\,A \otimes |1\rangle\,B\,/\sqrt{2})$$

After calculation we'll find that the final state will become as

$$= \tfrac{1}{2} \left(|\phi^{+}_{sA}\rangle \otimes (\alpha\,|0\rangle + \beta\,|1\rangle)_B + |\phi^{-}_{sA}\rangle \otimes (\alpha\,|0\rangle - \beta\,|1\rangle)_B + |\psi^{+}_{sA}\rangle \otimes (\alpha\,|1\rangle + \beta\,|0\rangle)_B + |\psi^{-}_{sA}\rangle \otimes (\alpha\,|1\rangle - \beta\,|0\rangle)_B\right)$$

Now Alice can perform Bell Basis measurement on the new state so formed after the interaction of her unknown state with the entangled pair so that Bob can identify the state so formed and tell it to the referee. The original unknown state can be obtained back by Bob by performing some unitary measurements, the knowledge of which will be communicated by Alice using classical 2-bits. Given below are the action of following Pauli unitary operations on a qubit state: -

$$I(\alpha\,|0\rangle + \beta\,|1\rangle) = \alpha\,|0\rangle + \beta\,|1\rangle$$

$$\sigma_z(\alpha\,|0\rangle - \beta\,|1\rangle) = \alpha\,|0\rangle + \beta\,|1\rangle$$

$$\sigma_x(\alpha\,|1\rangle + \beta\,|0\rangle) = \alpha\,|0\rangle + \beta\,|1\rangle$$

$$\sigma_z\sigma_x(\alpha\,|1\rangle - \beta\,|0\rangle) = \alpha\,|0\rangle + \beta\,|1\rangle$$

The action to be performed by Bob after Alice has performed the Bell's measurement is tabulated below

Projector Clicks	**Alice Communication**	**Bob's Action**	**Bob's Final state**
$|\phi\rangle \langle\phi|^{+}$	00	I	$\alpha\,|0\rangle + \beta\,|1\rangle$
$|\phi\rangle \langle\phi|$ -	01	σ_z	$\alpha\,|0\rangle + \beta\,|1\rangle$
$|\psi\rangle \langle\psi|$ +	10	σ_x	$\alpha\,|0\rangle + \beta\,|1\rangle$
$|\psi\rangle \langle\psi|$ -	11	$\sigma_z\sigma_x$	$\alpha\,|0\rangle + \beta\,|1\rangle$

In each scenario, it is important to note that Bob's state matches the state shared with Alice, and it should be noted that their state remains undisclosed to them at the conclusion of the protocol. Also, the state of system A is not anymore, its initial state as given by the Referee, else it would have violated the no-cloning theorem. So, quantum teleportation exhibits an example of ‘quantum tasks’ which is impossible to accomplish using classical resources but can be done perfectly with the help of quantum entanglement.

6 Neutrino Oscillation

Neutrino flavor oscillation is a quantum mechanical phenomenon in which neutrinos alter their flavor while traveling through space. Neutrinos are extremely elusive and neutral subatomic particles with tiny masses. In 1962, the concept of neutrino oscillation was initially hypothesized by Shoichi Sakata, Masami Nakagawa, and Ziro Mori, who identified three distinct types of neutrinos known as flavors: electron neutrinos (ν_e), muon neutrinos (ν_μ), and tau neutrinos (ν_τ). It was separately rediscovered in 1967 by Bruno Pontecorvo. The key points in neutrino oscillation can be summarized as follows:

Neutrino Flavor States:

- Only in some specific flavor states neutrinos are generated and detected (ν_e,ν_μ,ν_τ).
- Neutrinos interact via weak force interactions, and their flavor is determined at the time of their creation (e.g., in the Sun or in particle collisions).

Neutrino Mass States:

- Neutrinos, according to the theory of neutrino oscillation, have three distinct mass states (ν_e,ν_μ,ν_τ).
- The flavour states and these mass states diverge.

Superposition of Mass States:

- Neutrinos are created in a mass states' superposition.
- Linear combination of the mass states represents the flavor of a neutrino at any given time.

Hamiltonian and Eigenstates:

- The evolution of neutrinos as they propagate through space is described by a Hamiltonian matrix that governs the variation of neutrino flavour states over time.

- The flavor states are not the eigenstates of the Hamiltonian, leading to the phenomenon of oscillation.

Neutrino Oscillation Formula:
The probability of a neutrino created as a specific flavor to be detected as another flavor at a later time is given by the oscillation formula:

$$P\ (\nu_\alpha \rightarrow \nu_\beta) = \sum_{i=1}^{3}\ U_{\alpha i}U*_{\beta i}\, \exp\ (-im_i^2L/2E\ )^2$$

where:

a. $P\ (\nu_\alpha \rightarrow \nu_\beta)$ is the probability of oscillation from flavor α to flavor β,
b. $U_{\alpha i}$ is an element of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, which illustrates how mass and flavour eigenstates mix,
c. m_i as the mass of the ith neutrino mass state,
d. L equals the distance travelled by the neutrino,
e. E defined as neutrino energy.

Experimental Confirmations:

- oscillation has been experimentally confirmed through various experiments, as the Sudbury Neutrino Observatory and Super-Kamiokande experiments. The observation of neutrino flavor transitions that cannot be explained by assuming neutrinos have zero mass provided strong evidence for neutrino oscillation

Our understanding of particle physics is significantly shaped by neutrino oscillation, specifically the requirement that neutrinos possess non-zero masses (in contrast to the initially presumed mass-less nature) the essential introduction of mixing matrix in the neutrino domain. The phenomenon also has implications or astrophysics, such as in understanding solar neutrinos and neutrinos produced in supernovae.

6.1Entanglement in Neutrino Oscillation

A basic principle in quantum mechanics is the entanglement between flavor states and mass eigenstates in neutrino oscillations. However, how the exact event occurs, usually depends on one's theoretical assumptions. Nonetheless, this issue is not unsubstantiated as a number of empirical studies have already verified it. Here are some key aspects and attempts to showcase the entanglement of neutrino flavor states and mass eigenstates:

1. **Theoretical Framework**:
    I. The mixings of the neutrino flavor and mass states are described by quantum field theory as well as Pontecorvo Maki Nakagawa Sakata (PMNS) matrix. The PMNS matrix explains this by connecting the flavour eigenstates (tau neutrino, muon, and electron) to the mass eigenstates

    II. Different inequalities like Leggett Garg inequality and entanglement criteria have been employed for checking the entanglement in the neutrino oscillation. Various other quantum correlations like quantum discord for bipartite state and dissension for multipartite state have been investigated. The entanglement strength is quantified through negativity, concurrence, and the comparison of the neutrino oscillation state to a reference state is determined by the tangle feature. This measure is zero for GHZ states and non-zero for W class of states. Coherence and Decoherence of neutrino entangled state is also being investigated.

1. The discovery of neutrino oscillation proved that the Leggett Garg inequality's classical bounds might be broken, further supporting the notion that quantum coherence can be uniformly applied over macroscopic distances to microscopic objects.

2. The density matrix created demonstrates the presence of negative eigenvalues, indicating that the flavor state of the neutrino in a superimposed state is a bipartite pure entangled state. It appears that there is entanglement between the flavour states based on the negative eigenvalues of the density matrix created for flavour states 2 and 3.
3. The super-positioned neutrino flavour was found to be a bipartite entangled pure state by Von-Neumann entropy, and correlations such as tangle, concurrence, and negativity coincided with the linear entropy.
4. The property of the class of the W states is exhibited by the three-flavour neutrino oscillation.

2. Experimental Observations:

(a) Solar Neutrinos: Electron neutrinos originating from the Sun have been observed transitioning into different varieties as they journey to the earth. The earliest evidence for this was that, in the Homestake experiment, there was a deficiency of electron neutrinos; confirmations of this were other experiments including, Super-Kamiokande and SNO.

(b) Atmospheric Neutrinos: The behaviour of muon neutrinos after interacting with the neutrons generated by cosmic rays was observed using the neutrinos produced in the Earth's atmosphere. This flavor transition has been proven by experiments such as Super-Kamiokande and Ice-Cube.

(c) Reactor Neutrinos: Other experiment examples include Kam-LAND and Daya Bay which concentrated on electron antineutrinos produced by reactors. This indeed verified that there was no more observance of electron antineutrinos while the other tastes became apparent and in agreement with neutrino oscillation.

(d) Accelerator Experiments: Experiments such as MINOS, T2K, and NOvA have used high-energy neutrino beams produced at accelerators to study muon neutrinos and their flavor transitions, providing additional evidence for neutrino oscillations

(e) Precision Measurements: Ongoing experiments, initiatives, like the Deep Underground Neutrino Experiment (DUNE), seek to increase the accuracy of neutrino oscillation parameters. These experiments will enhance our comprehension of the hierarchy of mass and the mixing angles of neutrinos, providing further clarification on the relationship between flavor and mass states.

(f) Global Fits and Constraints: The global fits of neutrino oscillation data involve combining results from various experiments to obtain more precise measurements as well as the mass-squared discrepancies.

In summary, experimental observations in various scientific settings - including solar, atmospheric, reactor, and accelerator experiments - have consistently illustrated the intricate relationship between different neutrino flavors and their corresponding mass eigenstates. These experiments have significantly enhanced our comprehension of neutrino physics and the broader implications of quantum mechanics in the subatomic domain.

7. Entanglement in flavor states of neutrinos:

The three flavor neutrino states are the linear superposition of the mass eigenstates and are given as

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j}\, |\nu_j\rangle$$

Where $|\nu_\alpha\rangle$ (α = e, μ, τ) are the flavor eigenstates, $|\nu_j\rangle$ (j = 1, 2, 3) are the mass eigenstates and $U_{\alpha j}$ are the elements of a leptonic mixing matrix called the PMNS (Pontecorvo-Maki-Nakagawa-Sakita) matrix, characterised by three mixing angles (Θ_{12}, Θ_{13}, Θ_{23}) and CP violating phase (charge conjugation and parity).

In a plane wave picture the time evolution of the neutrino flavor state is

$$|\nu_\alpha(t)\rangle = \sum_j e^{-itE_j}\, U_{\alpha j}\, |\nu_j\rangle$$

The neutrino flavor state in an evolved state of coherent superposition of flavor basis can be written as

$$|\nu_\alpha(t)\rangle = U_{\alpha e}\, |\nu_e\rangle + U_{\alpha \mu}\, |\nu_\mu\rangle + U_{\alpha \tau}\, |\nu_\tau\rangle$$

Time evolution of neutrino flavor state in three mode system is written as

$$|\psi\rangle = a_1\, |100\rangle + a_2\, |010\rangle + a_3\, |001\rangle$$

Where the neutrino modes in occupation basis are identified as

$$|\nu_e\rangle = |1\rangle_e \otimes |0\rangle_\mu \otimes |0\rangle_\tau = |100\rangle_e$$

$$|\nu_\mu\rangle = |0\rangle_e \otimes |1\rangle_\mu \otimes |0\rangle_\tau = |010\rangle_\mu$$

$$|\nu_\tau\rangle = |0\rangle_e \otimes |0\rangle_\mu \otimes |1\rangle_\tau = |001\rangle_\tau$$

7.1 Entangling the two neutrinos

In this section we will analyse two such neutrinos to investigate their entanglement. The degree of this entanglement will be quantified through negativity calculations, while the suitability of the resulting state for quantum teleportation will be assessed by measuring fidelity and singlet fraction.

The flavor state of the two neutrinos in mode system is

$$|\psi_1\rangle = a_1\,|100\rangle + b_1\,|010\rangle + c_1\,|001\rangle$$

$$|\psi_2\rangle = a_2\,|100\rangle + b_2\,|010\rangle + c_2\,|001\rangle$$

Entangling the neutrinos via obtaining their tensor product first (creating a separable state).

$$|\psi_1\rangle \otimes |\psi_2\rangle = (\,a_1|100\rangle + a_2|010\rangle + a_3|001\rangle) \otimes (b_1|100\rangle + b_2|010\rangle + b_3|001\rangle)$$

$$= a_1a_2|100100\rangle + a_1b_2|100010\rangle + a_1c_2|100001\rangle + b_1a_2|010100\rangle + b_1b_2|010010\rangle$$
$$+ b_1c_2|010001\rangle + c_1a_2|001100\rangle + c_1b_2|001010\rangle + c_1c_2|001001\rangle$$

The action of CNOT gate is defined as

$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |11\rangle$$
$$|11\rangle \rightarrow |10\rangle$$

Taking up different cases where the CNOT gate is applied to different flavors of the two neutrinos.

Case1: applying to tau flavor of first neutrino and electron flavor of second neutrino (3rd and 4th qubits)

After the CNOT operation the $|\psi_1\rangle \otimes |\psi_2\rangle$ looked like

$$a_1a_2|100100\rangle + a_1b_2|100010\rangle + a_1c_2|100001\rangle + b_1a_2|010100\rangle + b_1b_2|010010\rangle + b_1c_2|010001\rangle + c_1a_2|001000\rangle + c_1b_2|001110\rangle + c_1c_2|001101\rangle$$

The density matrix of the above state containing 81 elements was constructed and then the matrix was reduced to 2 qubit state from 6 qubit state and different 2 flavor qubit states was studied. Considering first the tau-electron flavor state the reduced matrix took the form as

$$\begin{pmatrix} A & 0 & 0 & B \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ C & 0 & 0 & D \end{pmatrix}$$

Which resembles to that of an entangled state, thus we conclude the tau-electron flavor are entangled. Thus, we say that the neutrinos are showing intra and inter entanglement here.

Similarly checking for other different flavors, we found out that the $\tau - e, \tau - \tau$, e-e, $\mu - \mu, \mu$-e, $\mu - \tau$ and other different flavors also represented an entangled state and the negative eigenvalues were of the form $\sqrt{BD}$ where A, B, C and D are the constants representing the coefficients of the $|00\rangle\langle 00|$, $|00\rangle\langle 11|$, $|11\rangle\langle 00|$ and $|11\rangle\langle 11|$.

The strength of the entanglement can be calculated using negativity where negativity is defined as

$$N = 2 \max \{-\lambda_i, 0\}$$
$$= 2\sqrt{BD}$$

The negativity values for

$$\tau - \tau = 0.0825$$
$$\tau - e = 0.26706$$
$$e - e = 0.6382$$
$$\mu - \mu = 0.9387$$

Since we know two distant partners sharing an entangled state ρ can employ quantum teleportation usefully if

$$\lambda_{max}(\rho) > \frac{1}{d}$$

where 'd' is the Hilbert space dimension and an upper bound to the maximum achievable teleportation fidelity from a given bipartite state ρ in d ⊗ d dimensional Hilbert space is given by

$$f_{tel}(\rho) \leq \frac{\lambda max(\rho) d+1}{d+1}$$

hence calculating the usefulness and upper bound on the flavor states so formed we found out that

For $\tau - e$

Since the dimension of the Hilbert space is $2 \otimes 2$ so $d=2$

And from calculation

$$\lambda_{max} = 0.9213$$

$$\Rightarrow \lambda_{max}(\rho) > \frac{1}{d}$$

and also

$$f_{tel}(\rho) \leq 0.9475$$

similarly for e−e

$$\lambda_{max} = 0.31191$$

$$\Rightarrow \lambda_{max}(\rho) \ngtr \frac{1}{d}$$

$$f_{tel}(\rho) \leq 0.54$$

similarly, for $\mu - \mu$

$$\lambda_{max} = 0.9387$$

$$\Rightarrow \lambda_{max}(\rho) > \frac{1}{d}$$

$$f_{tel}(\rho) \leq 0.9591$$

Similar kind of calculations were performed for different cases and their negativities, usefulness of their entangled state for quantum teleportation and upper bound on maximum achievable teleportation fidelity were calculated.

Conclusion

From the above calculations we concluded that

- All the flavor states of the 2 neutrinos are entangled with each other, thus these 2 neutrinos represent a beautiful example of inter and intra entanglement.
- Not all entangled states are useful quantum teleportation as some of states are strongly correlated as compared to others and some are firmly correlated.
- Like from the calculation of case 1 we can see $\mu - \mu$ is very strongly entangled and upper bound on maximum achievable teleportation fidelity is also very high.

Summary and Future scopes

Various different inequalities were studied for checking the entanglement of flavor states of neutrinos and based upon that 2 neutrinos were entangled and there inter entanglement was studied which revealed their usefulness for quantum teleportation. Only some of the all-entangled states were useful for quantum teleportation and some states were not and how could teleportation be achieved using those states is a part of further studies and discussion. The search for correlation between different cases of entangled flavor states of the two neutrinos and their reason for not showing same kind of usefulness is a topic of open discussion.

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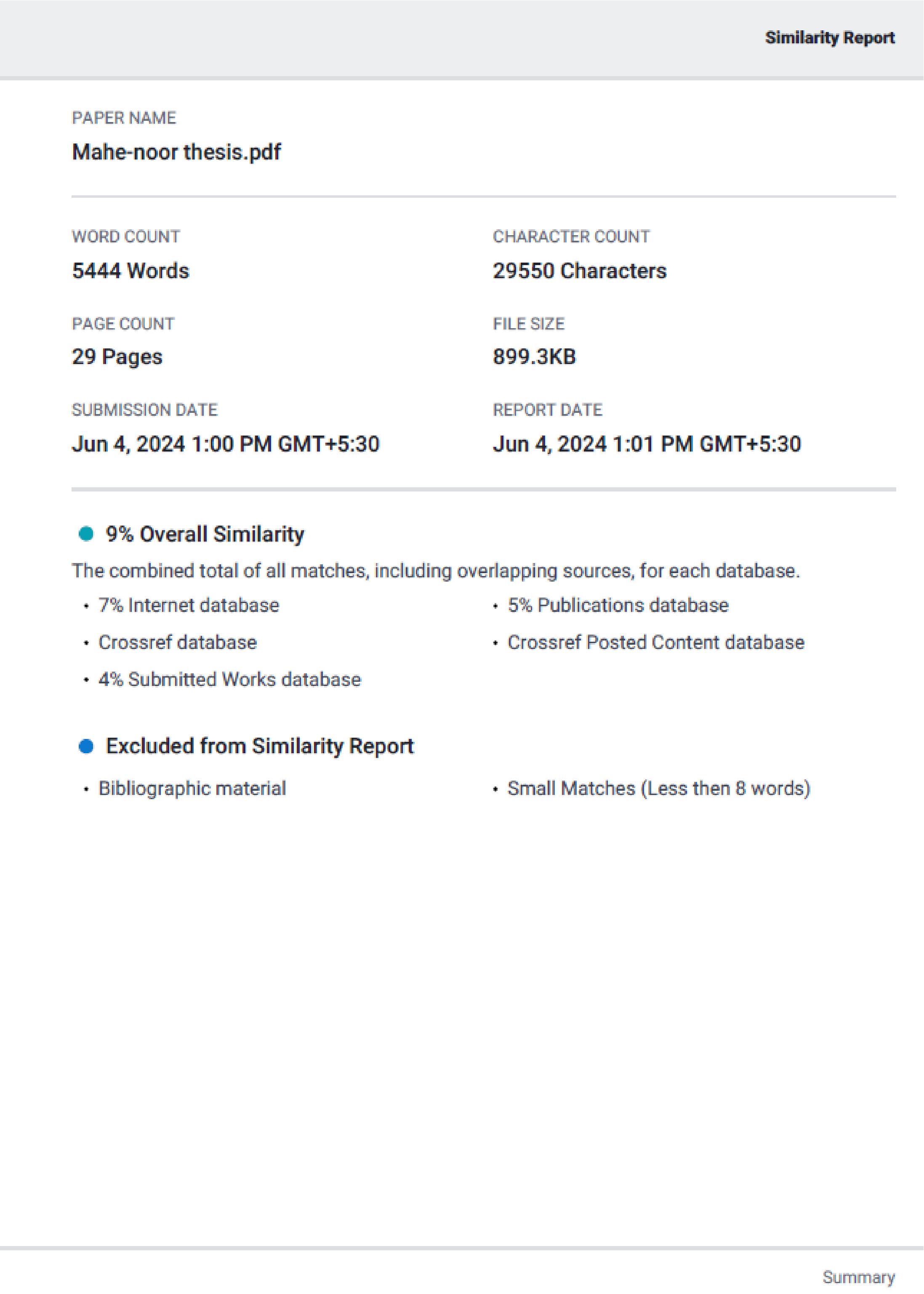

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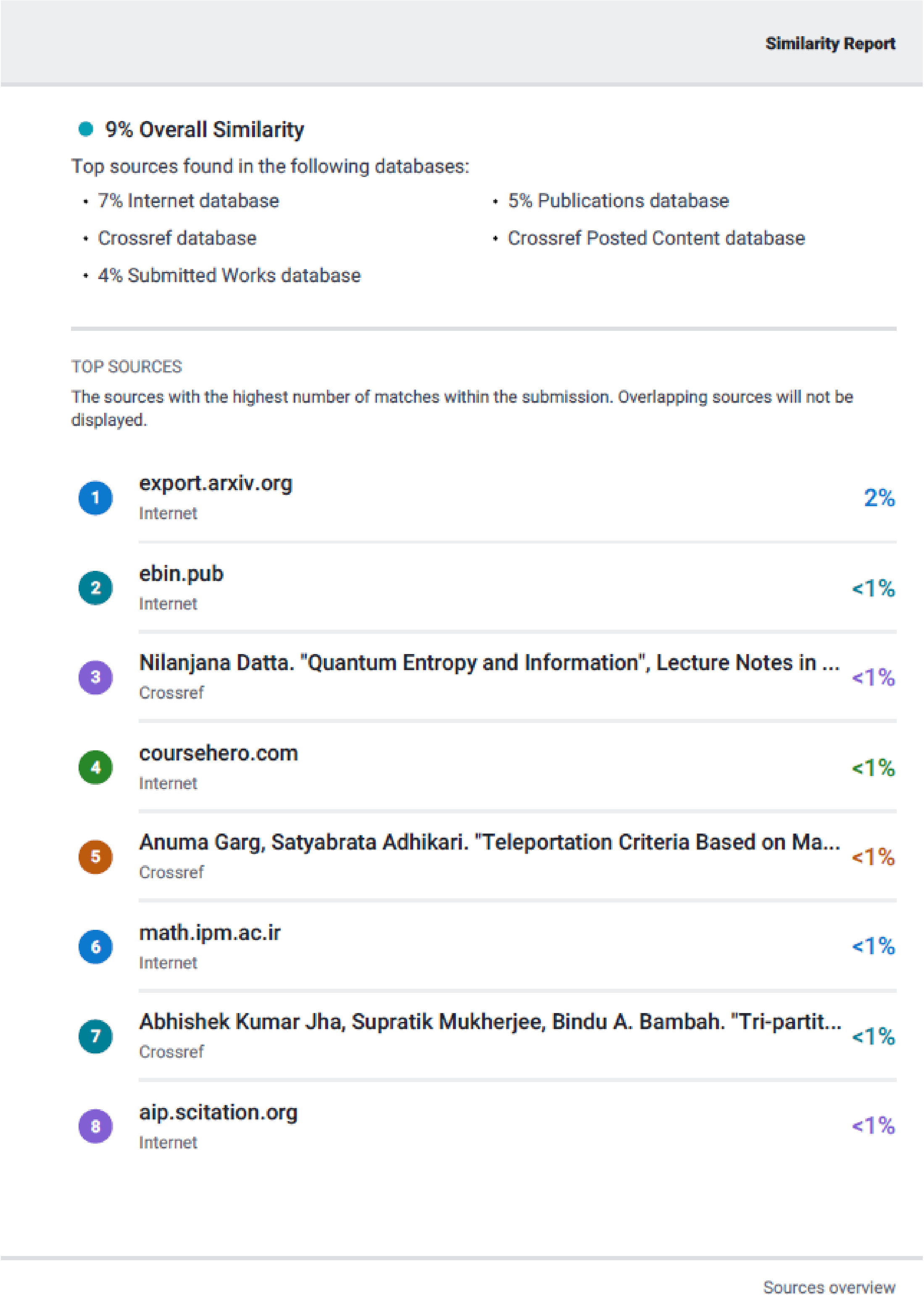

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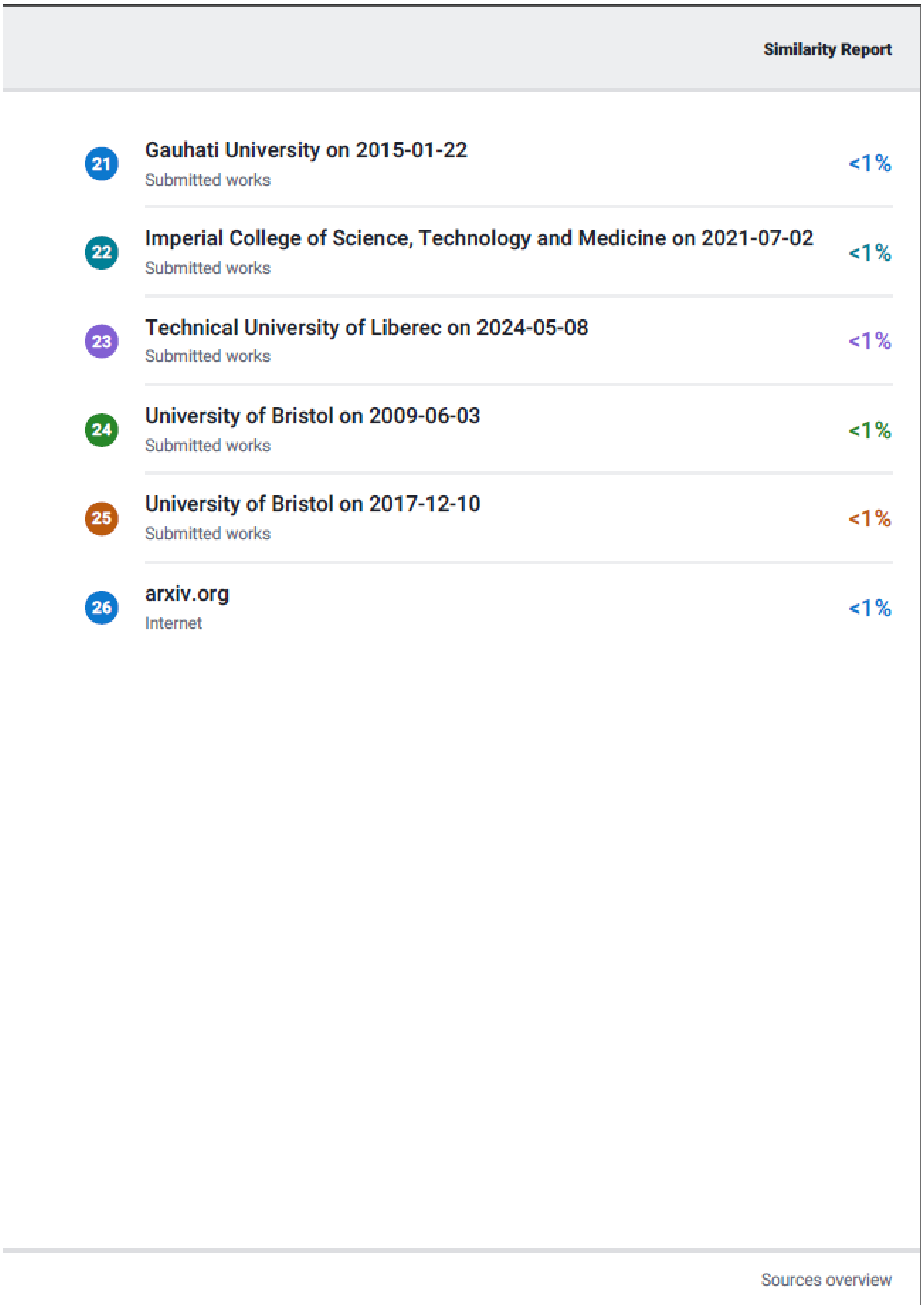

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