REVIEW OF FUZZY DECISION TREES AND ITS VARIANTS

A DISSERTATION

This report is submitted in partial fulfillment of the requirements for the degree of

MASTER OF TECHNOLOGY
In
SOFTWARE ENGINEERING

Submitted by
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CANDIDATES' DECLARATION

I, Prathmesh Yakkaldevi, Roll No. 2K22/SWE/12 student of M. Tech (Software Engineering), hereby declare that the project Dissertation titled **"Review of fuzzy decision trees and its variants"** which is submitted by me to the Department of Software Engineering, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Technology is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of and Degree, Diploma Associateship, Fellowship, or other similar title or recognition.

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CERTIFICATE

I hereby certify that the Project Dissertation titled **"Review of fuzzy decision trees and its variants"** which is submitted by Prathmesh Yakkaldevi, 2K22/SWE/12 Department of Software Engineering, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Technology, is a record of the project work carried out by the students under my supervision. To the best of my knowledge, this work has not been submitted in part or full for any Degree or Diploma to this University or elsewhere.

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ABSTRACT

This thesis is about the advancements that have been made in building and applying fuzzy decision trees to solve classification problems characterized by ambiguity and uncertainty. The classification with such kinds of classification methods is essentially exposed with crisp, rigid boundaries and does not capture the vagueness being transmitted from real-world data. In this regard, fuzzy decision trees provide a more flexible approach, in which such vagueness can be captured, hence enabling objects to belong to different classes with varying degrees of membership according to the principles of fuzzy logic.

The thesis also discusses the variants that treat all the variables at every node, such as C-fuzzy decision trees and Neuro-Fuzzy Decision Trees (N-FDTs), which bring in neural learning to improve accuracies further but still be interpretable. The concept of Intuitionistic Fuzzy Decision Trees (IFDTs) is added by incorporating the idea of hesitation parameters so they can handle uncertainty better.

Overall, this work has developed frame for the induction and application of fuzzy decision trees in improving any kind of decision-making process within complex and uncertain environments.

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LIST OF ABBREVIATIONS

1. **DT** - Decision Tree
2. **FDT** - Fuzzy Decision Tree
3. **C-FDT** - C-Fuzzy Decision Tree
4. **N-FDT** - Neuro-Fuzzy Decision Tree
5. **IFDT** - Intuitionistic Fuzzy Decision Tree
6. **GMFMFS** - General Model for Fuzzy Membership Function Selection
7. **DDBFDT** - Data Distribution Based Fuzzy Decision Tree
8. **FRF** - Fuzzy Random Forest
9. **FBDT** - Fuzzy Binary Decision Tree
10. **IDS** - Intrusion Detection Systems
11. **AUC** - Area Under the Curve
12. **LMF** - Linear Membership Function
13. **NLMF** - Nonlinear Membership Function
14. **HNN** - Hybrid Neural Networks
15. **FNN** - Fuzzy Neural Networks
16. **SDT** - Soft Decision Trees
17. **IFRF** - Intuitionistic Fuzzy Random Forest
18. **SAMME** - Stagewise Additive Modeling using a Multiclass Exponential loss function

CHAPTER 1

INTRODUCTION

1.1 OVERVIEW

In the era of data-driven decision-making, how widespread is the recognition of uncertainties and ambiguities of problems in classifications? This thesis brings to the forefront recent advances of fuzzy decision trees: a rather powerful framework in problems of classification that successfully addresses many of the concerns by exploiting the potential of fuzzy logic paradigms. The aim of this paper is to present how FDTs have been developed in an attempt to minimize this classification cardinality and maximize the accuracy of decisions based on works presented by many researchers.

1.2 MOTIVATION

The simplicity and interpretability of traditional decision trees have made them widely acceptable and popular. However, most of the traditional trees fail to address the real-world data very well since they are imprecise and vague. Based on this major reason, they strongly assume that each object belongs uniquely to just one class. This fact consequently supposes hard and often impracticable cutting boundaries. In fact, this hard-edged classification scheme can lead to significant misclassification using conventional models, especially when the data overlap and reveal progressive transitions between classes. Therefore, the major reason for this study is to transform the flexibility of the fuzzy decision tree into a solution that can help it produce something more meaningful, better, and accurate in terms of classification.

1.3 FUZZY SETS AND THEIR NECESSITY

Fuzzy set theory, developed by Lotfi Zadeh in 1965, provides one of the most established and powerful mathematical tools for coping with uncertainty and imprecision. Fuzzy sets allow degrees of membership.

Lotfi Zadeh[13] defined the Fuzzy set as: "A fuzzy set (A) in a universe of discourse (U) is characterized by a membership function (mu_A(x)) which describes to each element (x in U), the grade of membership that varies between 0 and 1". These detailed representations allow for fuzzy sets to describe more accurately the intrinsic uncertainty common in many real-world cases.

1.4 THE NEED FOR FUZZY DECISION TREES

These fuzzy decision trees follow the same principles as a normal decision rule; however, they embody the principles of fuzzy logic. A node in this fuzzy decision tree stands for a fuzzy set or linguistic variable, while the branches represent the fuzzy rules on these linguistic variables or conditions. Therefore, through evaluation of uncertain fuzzy rules at each node, there is a way to determine which branch of a node one should follow, finally resulting in the evidence gathered, in terms of fuzziness, from the tree to make a decision.

The main advantages of Fuzzy Decision Tree are:

• Management of Uncertainty and Vagueness: FDTs manage imprecise information quite well; hence, they are appropriate for applications where clearly defined data boundaries are not available.

• Human-Intuitive Classification: The basis of the fuzzy logic that underpins FDTs is human decision making, which makes the classifications more intuitive.

• Richer Information: In revealing membership degrees and truth level details of rules, FDTs make it possible for the decision-maker to have more informative information.

1.5 RESEARCH OBJECTIVES

The objective of this research will be to describe and discuss methods devised for the induction of fuzzy decision trees. The goals addressed are:

- Analyzing Induction Processes: Analyzing how first of all, the selection of the attributes is based on the classification ambiguity, and then in a recursive build up of the FDT.
- Advanced Models Exploration: Discussion on two advanced models, C-FDT and Neuro-FDT, and how they increase classification levels in both accuracy and robustness.
- Real-Life Applications: The application of FDTs to real-world examples, including credit risk evaluation, intrusion detection, and stock market prediction.

1.6 STRUCTURE OF THE THESIS

This thesis goes as follows:

Chapter 1: Introduction The chapter provides an introduction to the thesis and outlines motivations for current work, fuzzy sets, and their needs, a need for a fuzzy decision tree, research objectives emanating from the need to meet these needs, and the structure of the thesis.

Chapter 2: Literature Review: A appraisal of the existing literature connected with fuzzy decision trees and other significant advances.

chapter 3: Comparison of models: Exploration of fuzzy decision tree models and their potential to improve classification performance.

chapter 4: Conclusion and Future work: Summary of findings, implications, and directions for future research.

1.7 SIGNIFICANCE OF THE STUDY

By discussing the methods developed for inducing fuzzy decision trees, this research aims to contribute to the field by providing a deeper understanding of how FDTs can handle classification ambiguity and improve decision accuracy.

CHAPTER 2

LITERATURE RIVIEW

2.1 INTRODUCTION

Conventional classification techniques are rigid through classic rules that, for each object, stipulate which single class should be chosen mutually exclusively, based on one of the mutually exclusive values to be given to each attribute. This rigidity might result in artificially defined boundaries that are incapable of properly representing the subtleties that define real-world situations, as observed by Quinlan[16]. Only this research has introduced the concept of fuzzy partitioning in relation to fuzzy evidence, which is used to construct the fuzzy decision trees with the goal to minimize the ambiguity of classification. The fuzzy set is defined by Zadehis fuzzy set theory to be those sets for which the membership functions assign values in the range [0, 1]. These membership functions are a far more flexible way of representing data in terms of the degree of class membership of an object in the fuzzy set. Membership functions, in the framework of fuzzy decision trees, thereby allow dealing with cognitive uncertainties, for example vagueness and ambiguity. In 1976, Bellacicco, A.[14] defined the tasks of clustering and classification using fuzzy sets. This gave more development in this direction. One of the early developments of a fuzzy decision tree was conceived in 1980 by Dubois, D. J.[15] in the book Fuzzy Sets and Systems. Since that, many advances have further developed the technique and use of fuzzy decision trees

2.2 LITERATURE RIVIEW

Yuan[1]new way of Induction of Fuzzy Decision Trees is as follows:-.

The induction of fuzzy decision trees follows the next several important steps.

1. Fuzzifying the Data Converting numerical data into words is a process called "fuzzification". In this regard, the crisp numerical data must somehow be converted into categories such as "high", "average", and "low". The membership functions expressing such sets can be determined based on statistical data, expert decisions, or common views. Fuzzy clustering methods, for instance, self-organized learning-based methods, are one

of the ways to determine these membership functions. This conversion renders the data more readable and thus reduces the information overload, increasing interpretability.

2. Building the Tree: The class that has the least classification uncertainty is selected to serve as the root decision node upon which to build the tree being constructed by the induction process. The branches are expanded or closed upon the basis of the thresholds for the truth levels in which they have been pre-set; at the least this is where the tree stops growing. This process is hence iterative. The dimension and accuracy of the tree is so much determined by parameters of such nature as the extent of thresholds for both the truth and significant levels.

3. Rule Extraction and Deployment: Once the building process is complete, the fuzzy decision tree is turned into a series of fuzzy rules. Then these rules are deployed for classification. In fuzzy decision trees, many different fuzzy rules can be simultaneously applied to the objects, which can be categorized into different classes with different grades at the membership; whereas in non-fuzzy decision trees only one rule is applicable to a single item.

Some of the advantages of FDT over crisp traditional DT are

- Uncertainty and Vagueness: Fuzzy decision trees work pretty well in these domains, where imprecise information handling is common, making them more suitable for real-world applications, because the boundaries almost never turn out to be well defined.
- Natural Representation of Knowledge: It takes much closer ways to represent human thought for the representation of classification knowledge.
- More Comprehensive Decision Information: A fuzzy decision tree provides more comprehensive information with the memberships of classifications and truth levels of the rules for the decision-maker.

They propose using a Branch-Bound-Backtrack algorithm for these Algorithmic Enhancements that will handle computational complexity in large fuzzy decision tree sizes. This algorithm enhances efficiency by optimizing the process of applying rules such that each and every rule does not need to apply to each object for classification.

The methods of defuzzification, used to derive crisp numerical values from fuzzy classifications, are discussed further below:

- Max Criterion Method: chose the output value for the output set that maximizes the degree with membership highest.
- Mean of Maximum Method: This method will calculate the mean of all values that have maximum degree membership.
- Center of Area Method: In this method, the center of area is calculated of the fuzzy set in the x-axis where the area under the curve or which would be a plate and it would have a uniform thickness and density.

Such methods, therefore, facilitate the process of analysis or any other decision-making through a transformation of fuzzy outputs into definitive numerical values.

C-fuzzy decision trees, developed by Witold Pedrycz[2]. Unlike existing fuzzy decision trees, at every node, the C-fuzzy decision trees consider all the variables and conclude with a unique partition of the feature space. Experiments have demonstrated that C-Fuzzy Decision Trees have promising classification accuracy and prediction capabilities and hence outperform traditional decision trees, such as C4.5, in compactness, versatility, and ease of interpretation.

Some of the most important features and advantages of C-fuzzy decision trees are:

- All Variables Considered: Most of the classical fuzzy decision trees evaluate each variable individually at the time, while using these trees to examine joint distributions across all the variables under consideration. The results are smaller size trees and flexible feature space partition.
- Clustering Refinement: It also builds up the decision tree structure through clustering, which represents nodes in the decision trees and further breaks down into other granules having minimal variance and maximal homogeneity so that the decision tree can prepare itself by refining as a part of the process.

C-Fuzzy Decision Trees, particularly in terms of design, offer solutions to the following drawbacks: the traditional Decision Trees have been advanced by considering the view of data as a collection of information granules and using fuzzy clustering to capture the continuity of classes.

Therefore, fuzzy clusters are the underlining design blocks of the whole C-Fuzzy Decision Trees and the building blocks of the tree. Prototypes or center points of the training dataset cluster data points into different related data points and then line them up at the highest nodes in the tree structure. This means that the clusters are further refined into granules of less variability and more homogeneity, hence growing in the size of the decision tree.

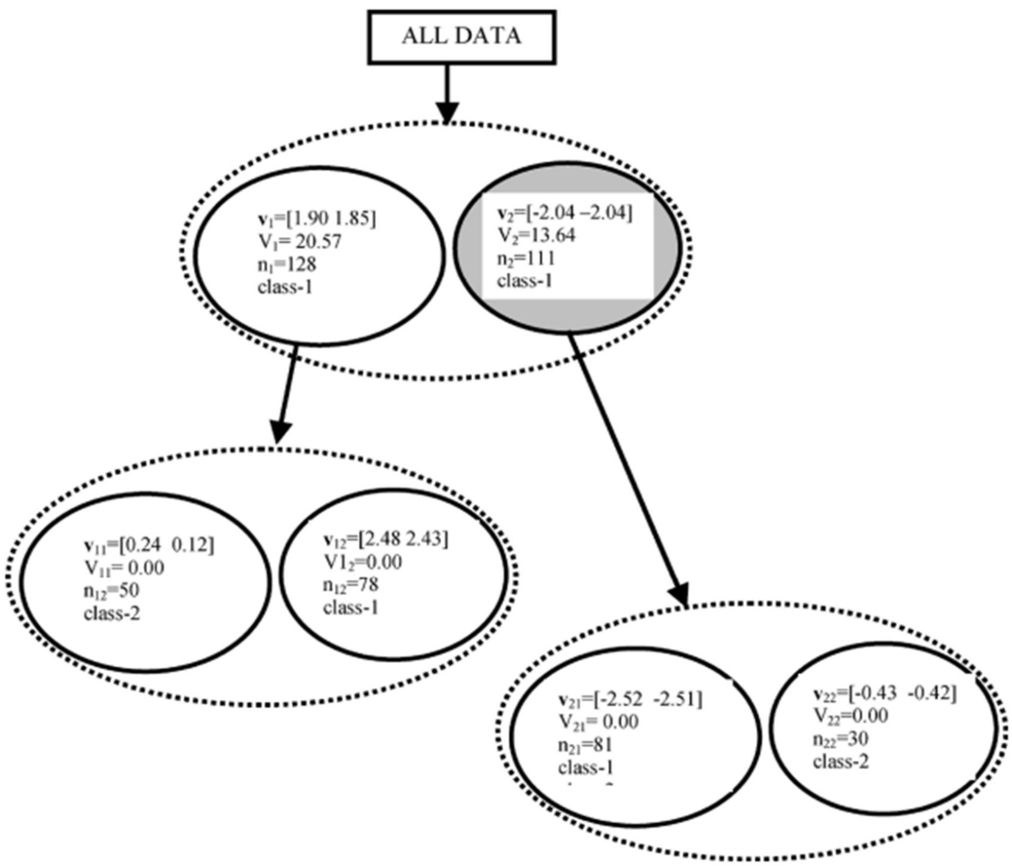


Fig 2.1 C-Fuzzy Decision Tree

We can note that the C-tree is already fully grown: the values of the variability criterion are zeros at all leaves; that is, further growth of the tree is impossible.

It can be shown that the use of fuzzy clusters in C-fuzzy decision trees employed makes them more compact, smaller, and versatile as compared to traditional decision trees, C4.5.

The results show a C-fuzzy decision tree working so good and stably in training and testing, respectively. Further, the C-fuzzy decision tree is referred to as a high-resolution model which can be created within a locality, and the differences in performance between test and training sets are less with respect to the C4.5 model.

Performance of this model against PIMA Diabetes is shown in Table 2.1 and against Hepatitis as an example in Table 2.2.

Table. 2.1 C-FDT on PIMA Diabetes against C 4.5

DT and Structural Plan	Error	Nodes in Tree
C 4.5	16.01%	43
C-FDT, C = 5, 6 Iteration	10.26%	30
C-FDT, C = 3, 5 Iteration	13.02%	15

Table. 2.2 C-FDT on Hepatitis against C 4.5

DT and Structural Plan	Error	Nodes in Tree
C 4.5	43.86%	45
C-FDT, C = 2, 6 Iteration	36.13%	12
C-FDT, C = 9, 3 Iteration	34.19%	27

The table 2.1 and table 2.2 implies that C-FDT produced better or same result with less complex tree structure.

Hui Zheng[3] introduces a new approach in paper titles: A general model for fuzzy decision tree and fuzzy random forest. The GMFMFS-based FDT first fuzzifies the dataset and then recursively seeks to construct the tree using available features and partitioning points to detect features on the dataset. The method initializes a linear membership function in the application, and further, it extends to a nonlinear membership function to deal effectively with the various data distributions. The FDT construction algorithm consists of tree initialization, best feature and node selection, and recursive generation of subtrees. Correct membership function is the most important step in any success of fuzzy logic techniques.

A general flexible model can adapt to features in data and objectives. This GMFMFS is capable under both FDTs and fuzzy random forests. First, the model builds a linear membership function, after which it contracts into a nonlinear form without increasing computational effort. The important feature allows its application in various applications to improve its effectiveness on risk prediction. In these experiments, the proposed GMFMFS-based FDT will be tested in experiments using real US credit data and UCI Susy datasets. These are followed by experiments on synthetic big data, which may provide the most risk classification and prediction accuracy. In this way, a GMFMFS-based model demonstrates high potential for practical applications and realization of theoretical advantages presented above. Table 2.3 avarage accuracy in C4.5, linear(LMF), and nonlinear function(NLMF) on US credit dataset.

Table 2.3 model accuracy on US Credit dataset

Method	Accuracy
C4.5	0.68
LMF	0.81
NLMF	0.82

NLMF is tested against lot many dataset and it was concluded that it performed well than LMF and C4.5.

The paper concludes that the GMFMFS provides a robust framework for applying fuzzy logic to decision trees and random forests. The proposed models are flexible and adaptable, making them suitable for various risk prediction tasks. The experimental results validate the effectiveness of the GMFMFS-based methods, highlighting their potential for real-world applications. Future research could explore further extensions and optimizations of the proposed models.

Hui Zheng[4] presents a DDBFDT approach on credit risk assessment that improves the classical method of credit classification, providing a new error-related metric for attribute selection, proposing an algorithm considering data distribution in the search of the partition point, and also comes forward with less computational complexity for construction of fuzzy sets with a nonlinear membership function. It reaches high readability when compared to related models of the same kind and shows a considerable ability to resist disturbances over the process of credit risk evaluation. In this paper, the fuzzy decision tree approach is operationalized with the addition of 'impurity' to define uncertainty, the use of non-linear membership functions, and data density in attribute partitioning. The proposed algorithm proves to be quite efficient in intelligent credit scoring processes concerning different datasets of the experiments. Moreover, it will be discussed that the fuzzy logic space has great potential for self-explanation, and a way of profiting from this potential in credit risk evaluation is introduced. The introduction of the DDBFDT supports the conventional credit classification method in the evaluation of credit risk.

The new features to DDBFDT novel appraoch are error-related heuristic to attribute selection, data-distribution aware approach to partition point searching, and less computationally complex non-linear membership function to set building. This upgraded the fuzzy decision tree through dense data distribution, a ranking criterion, and formation of the fuzzy categories into the consideration of the system. "A fuzzy decision tree is built on three steps: selection of attributes, the determination of the point of partition, and the construction of membership functions. The work includes these four main: construction of an algorithm regarding a fuzzy decision tree, finding the best attribute to have the maximum computing of Learning Error; finding a position of points to divide the space of data; and designing fuzzy membership functions to transform attributes.". One of the mechanisms for the methodology is to deal with the construction of non-linear membership functions. Being that data density is reflected as a form of non-linearity with respect to the functions, the approach is better at extracting underlying data patterns; thus, the fuzzy sets obtained are more interpretable. These steps can be further divided into the following: conversion of numeric data into discrete data, discretization of the original data with the aid of SOME selected partition points, and classification of new samples with the integration of the results. The methodology further included the process of designing the membership function, computing for the impurity ratio, and establishing the best attribute with highest suppression of error in decision node generation of fuzzy decision tree. The current paper explains attribute selection process that is error specific; it is done before the searching of partition point, member function design, and synthesized classification to guard against metric imbalance.

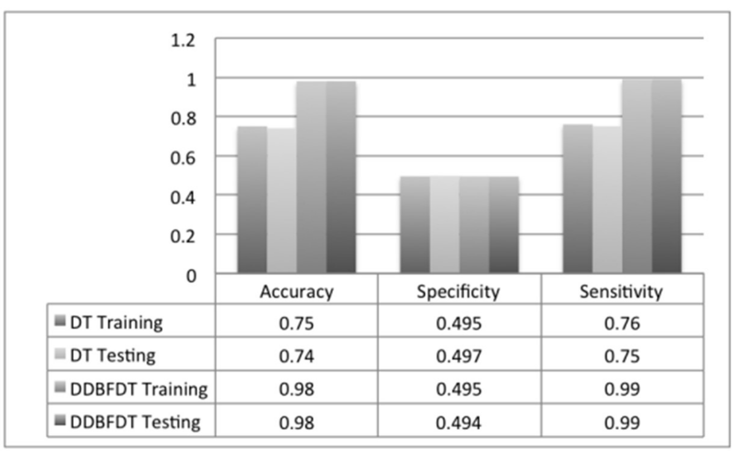
Error represents the overall rate of inaccuracy of the classification results in which inaccuracies are then marked and accumulated into the overall error rate. In building fuzzy decision trees, attributes are selected one at a time based on error; unlike in information gain, that is used for choosing the best attribute. The present error (Errorcurrent) is calculated before any other computations of the error rate and this formula: max |Errorcurrent- Errornodeindex,i| used in the selection of the next internal node giving the best attribute. On the other hand, for the selection of partition points for each incomplete fuzzy decision tree, a similar formula is used. This makes the algorithm for finding partition points for the fuzzy decision tree a very critical feature because it determines to a larger extent the quality of the tree.

It tries to make the distribution of individual datasets throughout the attributes unique, with a minimum possible inaccuracy and relative balance. For this, the algorithm divides each attribute into intervals in such a way that an equal number of data points fall in each interval. First, it assumes a fixed number of intervals and sorts out data points for each attribute. Partition points are there to calculate intervals, and this is conducted for every attribute so that all the attributes are properly partitioned. Need for Membership Function Even though partition point does the important work of effective division of the data, a membership function still needs to be constituted to transform original samples into fuzzy sets.

This is required to categorize the data points correctly in the fuzzy decision tree. Non-linear functions require three points to find a function, while linear functions require only two points for determination. For a non-linear membership function, the three points in the defining function are 0, 0.5, and 1. The extra point at 0.5 puts boundaries for a non-linear function, which becomes crucial in depicting the shape and generalities of the function; hence, the three defining points within nonlinear modeling become important. The paper evaluates how the DDBFDT performs and compares it with the traditional ways of classification, on 2 synthesized data sets. The DDBFDT method is also applied to a real-world data set representing UK credit behaviors; it classifies samples as either 'good' or 'bad' credit risk according to certain characteristics.

The test results show the superiority of the DDBFDT model in terms of accuracy, specificity, sensitivity, and overall performance over other traditional classification methods. Through rigorous experiment and comparison with existing models, the proposed DDBFDT reflects excellence in many aspects, such as high readability, robustness, and disturbance resistance in credit risk evaluation processes. The public datasets and synthesized data will be validated according to the DDBFDT model. It will again be able to prove its independent effectiveness in distinguishing between 'good' or 'bad' credit transactions through Credit Score attribute analytics.

Fig 2.1 UK credit dataset shows precision in profit metrics.



	Accuracy	Specificity	Sensitivity
DT Training	0.75	0.495	0.76
DT Testing	0.74	0.497	0.75
DDBFDT Training	0.98	0.495	0.99
DDBFDT Testing	0.98	0.494	0.99

Fig 2.1 Precision in Profit metrics in UK credit dataset

The RAJEN B. BHATT, M. GOPAL [5] present the Neuro-Fuzzy Decision Trees (N-FDTs) as a new model in the literature that fine-tunes the accuracy of FDT by using a backpropagation algorithm put directly on top of this structure; it further extends this work and puts forward a methodology for improving interpretability without any loss of accuracy. Generally, conventional classification schemes aim to convert input patterns into the corresponding class of output relative to available training data, and herein hangs the balance of predictive accuracy versus interpretability. However, FDTs have been criticized for poor accuracy due to local decisions and a priori fuzzification of the feature space, which can otherwise influence tree size and performance. The existing FDT algorithms lack mechanisms for rectification and globally supervised learning that result in initialization bias of parameters. As neural learning could also predict nonlinear boundaries with quite high accuracy, integrating neural learning in FDTs results in N-FDT. The methodology consists of the construction of FDT using the induction algorithms such as fuzzy ID3 in forward cycle and adaptation of tree parameters using a stochastic gradient descent algorithm in feedback cycle. This dual-cycle scheme enhances learning accuracy while maintaining the fragrance of the hierarchical structure and interpretability of trees. N-FDTs are also compared with Hybrid Neural Networks (HNN) and Fuzzy Neural Networks (FNN) to particularly stress the unique contributions made by N-FDTs in improvement of classification accuracy without any loss in comprehensibility.

Neuro-Fuzzy Decision Trees—Architecture

Forward Cycle: FDT are built initially as conventional induction algorithms as fuzzy ID3. This step establishes the initial hierarchical structure for classification, which sets the basic background structure that supports the parameter adaptation in the subsequent phase.

Feedback Cycle Most importantly, the whole process of stochastic gradient descent moves in reverse direction—from leaf back to root—for the refinements in fuzzy decision trees.

When backpropagation is adapted directly into the tree structure, the interpretability and clarity of the model are retained, meanwhile improving learning accuracy.

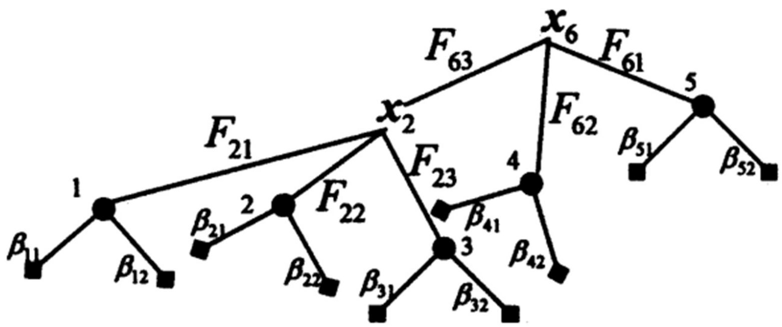


Fig 2.2 Fuzzy decision Tree on Breast Cancer dataset

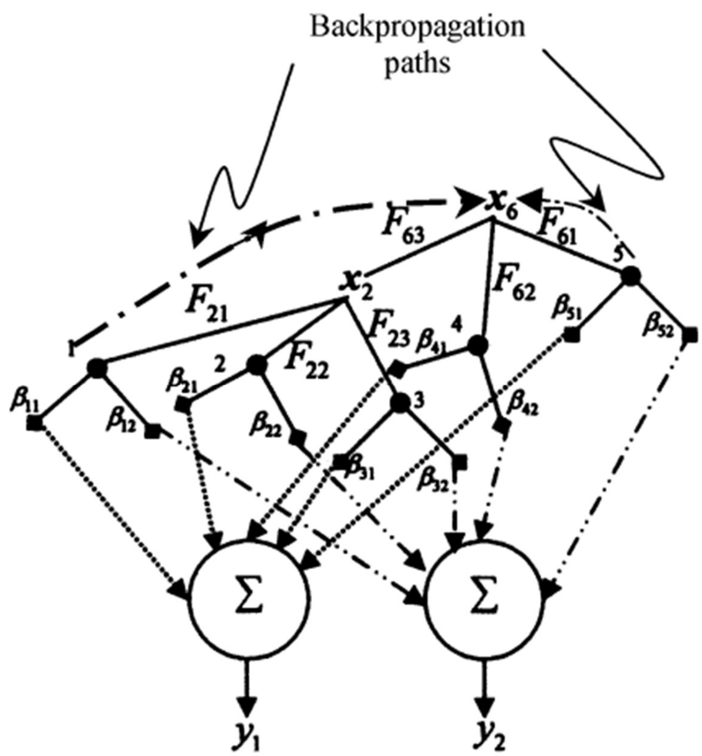


Fig 2.3 FDT Backpropagation

Fig 2.3 shows the working of backpropagation on Fig 2.2 tree structure.

This section provides some experimental results on the real-world datasets of Rice and Satellite images to show the performance of N-FDTs compared to traditional FDTs. The obtained experimental results demonstrated better accuracy with N-FDTs under various scenarios that had different numbers of leaf nodes, along with parameter values. The result underscores the fact that N-FDTs enhance learning accuracies without compromising the interpretability of the model. It also investigates the computational cost of N-FDTs in terms of number of training patterns, number of attributes, number of fuzzy sets per attribute, number of paths, number of classes, number of epochs, fuzziness control parameter (α-cut), leaf selection threshold (β_th), learning rate, and shared membership functions in initial fuzzy partitions. HNN(a) is a hybrid neural network with initial weight set to 1 and HNN(b) is with random initial weight, The final results in terms of accuracy for the Rice Taste dataset were HNN(a) (90%) after 535 epochs, HNN(b) with the same percentage but after 2268 epochs, and the N-FDT model with the best accuracy, at 96.22%, after just 13 epochs.

Applying to the Iris dataset, HNN(a) and HNN(b), both reached an accuracy of 97%, where HNN(a) took 69 epochs, and HNN(b) took 186 epochs. In contrast, N-FDT exhibited slightly less accuracy—96.22% in only 40 epochs. Applying to the Diabetes dataset, it showed both HNN(a) and HNN(b) gaining an accuracy value of 78%. The total epoch taken to accomplish this for HNN(a) was 535, whereas for HNN(b), it was 5671 epochs. In contrast, the accuracy for N-FDT was much lower, at 72.22%, with only 300 epochs. Confronting these results—i.e. between models—the obtained performance and convergence speed are shown in feature trade-offs between accuracy and training time. The paper concludes that NFDTs improve classification accuracy without losing comprehensibility. NFDTs damp initial parameter impacts by letting the tuning of certainty factors and Gaussian membership functions. They also permit multi-branch trees in order to provide better extraction of rules and online adaptation. Future work will involve the optimization of fuzzy partitions, the enhancement of gradient descent algorithms, and the application of these techniques to real-world problems while classifying with the rule holding the highest fire strength.

The decision tree of fuzzy type can be utilized to conduct different classification tasks, most of all in environments with uncertainty and imprecise data.

The work by N. M. Abu-halaweh [6] builds upon two existing works of FDTs with two key modifications meant to improve the efficiency and effectiveness of these models.

The major changes have to be to eliminate the data items from the nodes depending upon the threshold of a membership value and introduce partial membership values for all available classes in a leaf nodes. These changes have been implemented to obtain more compressed trees while reducing the number of rule bases, the size of the rule base and improve inference times. Key Modifications done by N. M. Abu-halaweh [6]

- Threshold-Based Data Dropping :

  One is to cut off data items with low membership values from a node if the value reflected by the membership of the data is less than a predetermined threshold: ; this helps in tree-pruning, considering only those data with high membership values for the tree structure and, in that way, therefore reducing the complexity of the tree.

- Partial Membership Values Assigned to Leaf Nodes :

  Instead of majority class assignment in the leaf nodes, a minority class assignment scheme using partial membership values of all involved classes is suggested. For instance, three classes of data in a leaf node but only two are there, it will be shown using partial memberships—such as 0.05 for class 1 and 0.95 for class 2. This actually offers a much better representation of class memberships, which carries more information content and hence makes the classification more interpretable and accurate.

A class ratio membership threshold will be set, which the node has to exceed for it to be considered a leaf. This ensures that only leaf nodes are those having a relatively high ratio of class membership.

Another inference method is the method of inferring class by summing up the membership values of each class from the fired rules; hence, select the one having the highest final membership. It is compared with a traditional approach in showing it works good at different methods, but does not show consistent superiority over each other.

The following are the several enhancements realized upon implementation of the above changes:

- Smaller Decision Trees: The produced trees are smaller by pruning data items with low membership values in the result.
- Reduced Rule Bases: It reduces the number of rules because it incorporates the partial memberships and the threshold-based approach.

- Reduced Inference Times: The simplified tree structure together with the number of rules concurrently dropped facilitate faster inference times.

It evaluates the performance of these methods through experiments. Classical FDTs need to build with a modification form, leading to an increment in efficiency without adversarial effects on classification accuracy. Some practical performance gains can be obtained with partial memberships and threshold-based pruning in actual classification applications.

The proposed modifications in the fuzzy decision tree system enhance efficiency and compactness of the model, thereby adopting partial memberships and a threshold-based approach for far better consideration of significant data points during CNF construction. The inference method provides it with flexibility for classification; further research is focused on the refinement of such methods while using fuzzy decision trees in other applications domains.

Table 2.4 Information on Dataset

Dataset	Samples	Attributes	Classes
Breast Cancer Wisconsin (diagnostic)	569	32	2
Waveform Database Generator (V1)	5000	21	3
Statlog (heart)	270	13	2
Ionosphere	351	34	2
Statlog (Landsat Satellite)	6435	36	7
Glass	214	9	6
Protein dataset (RS126)	21834	455	2

Table 2.5 Experiment Results

Data Set	# of rules, previous approach [12]	Accuracy, previous approach [12]	# of rules new approach	Accuracy, new approach	Relative size of rule set in %
Breast	2994	0.977	94	0.981	3.14
Wave	106053	0.832	10952	0.834	10.33
Heart	79	0.819	38	0.823	48.10
Ion	88082	0.940	3284	0.943	3.73
Sat	233306	0.880	16245	0.892	6.96
Glass	291	0.706	222	0.730	76.29
RS232	681	0.759	616	0.780	90.46

Table 2.6 Execution times(seconds) for dataset

Dataset	Execution time for FDT	Execution time for N-FDT
Breast	5.33	<1
Wave	1753.33	106
Ion	425	5.33
Sat	8441	258
RS232	268	256

The proposed modifications in the fuzzy decision tree system enhance efficiency and compactness of the model, thereby adopting partial memberships and a threshold-based approach for far better consideration of significant data points during CNF construction. The inference method provides it with flexibility for classification; further research is focused on the refinement of such methods while using fuzzy decision trees in other

Piero Bonissone[7]) represent an advanced ensemble learning method that integrates the principles of fuzzy logic with the robustness of random forests. This paper explores two distinct strategies for combining individual tree decisions within the FRF ensemble: Strategy 1, which uses Faggre11 and Faggre12, and Strategy 2, which employs Faggre2. These strategies aim to increase the classification accuracy and robustness of the ensemble, particularly in handling imperfect data.

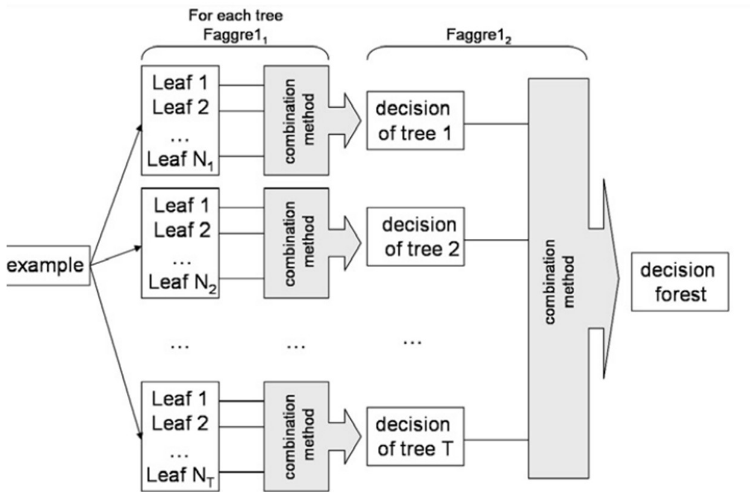


Fig 2.4 Random Forest Strategy 1

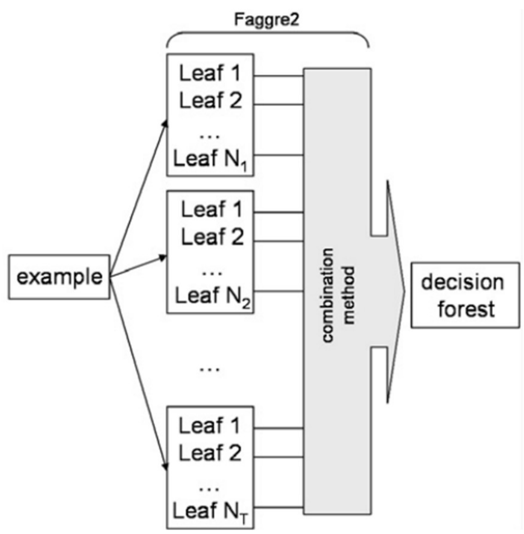


Fig 2.5 Random Forest Strategy 2

Presented below are some of the key strategies and combinations. In Strategy 1, it aggregates the decisions of individual trees according to the schemes Faggre11 and Faggre12. The schemes perform the combination of confidence levels of leaf nodes towards each class with the major purpose of producing the final decision. The process of aggregation combines tree information in such a way as to reflect partial memberships of classes which leads further classification that is full of nuance and more accurate.

In Strategy 2, Faggre uses all of the trees' information directly to guide its final decision.

Faggre tallies the membership of all the trees in the ensemble and identifies which class has the most rain falling on it. This strategy allows an ensemble to gain access to combined knowledge of the forest, for better classification accuracy. The ensembles of FRFs can be combined in two ways: trainable and non-trainable methods.

Non-Trainable Methods There is no further training present in such methods, for the reason that these predefined rules combine tree decisions. The training techniques are explicit and data-dependent, such as weighted majority vote-by-leaf, in which a weight is assigned for the leaf nodes according to their relevance. Implicitly Data-Dependent Techniques Majority Vote adjusted by Membership Values is another. In this case, the weights are derived from the fuzzy set membership functions. In adaptive techniques, the parameters of the classifier may be learned either concurrently or after training each individual classifier within the ensemble. Extra training is undertaken for the determination of decision weights, along with the reconsidering of the importance of each individual tree according to errors found within the Out-of-Bag dataset.

It does experimental comparisons in two main groups

1. Comparison to other ensembles

The assessment for the performance of the FRF ensemble comes when compared against other ensembles using the base fuzzy decision tree classifier and Breiman's Random Forest. The main objective of this comparison was to ensure the accuracy and robustness of the new FRF ensemble in comparison with these methods.

2. Baseline comparison with existing classifiers

The FRF ensemble is also benchmarked with different possible combinations of classifiers and ensembles that have been done previously. This is important to get a baseline idea of the effectiveness of the FRF ensemble in handling imperfect data. All the experiments confirm the amplitude of effectiveness of the FRF ensemble in cases of imperfect data as opposed to other classifiers and ensembles in respect to accuracy and robustness. The error rates concerning trees, explicit calculations of errors in out-of-the-sample samples, and errors at diverse tree sizes support the performance of the FRF ensemble. This underlies that the ensemble is excellent in dealing with imperfect data, and its application value is quite high. In this regard, it is clear from this paper that the FRF ensemble can be nominated as a potentially strong and effective method for handling classification tasks, especially in cases where the data may not be perfect. The two strategies of combining tree decisions offer flexibility and adaptability in various scenarios through the use of different methods for that work. The experimental results hence justified the effectiveness in using the proposed FRF ensemble classifier and its probable usefulness in practice. Future studies may further develop such methods or apply them to other domains, as follows.

Marco Barsacchi [8] presented a model called FDT-Boost: A new boosting technique based on a fuzzy binary decision tree (FBDT) as the base classifier for multiclass classifiers. The design of the proposed model was based on the principle adopted in the SAMME-AdaBoost scheme, which illustrated the capability of such a model to handle complex classification tasks.

Further on, the paper demonstrates an in-depth experimental evaluation recording the performance of the FDT-Boost using a benchmark dataset consisting of eighteen classification datasets. Such an assessment does not only measure the performance of FDT-Boost but compares it with other fuzzy classifiers like FURIA, FBDT, and fuzzy multi-way decision trees, hence making one have a full view of its efficiency and effectiveness.

The study also looks in favor of preserving the compactness of FBDT, whereby, through this, the restriction of depth will put a limitation in regard to compromising classification accuracy. It also brings to focus that efficient and accurate classification models in expert systems will lend more credence to the value of the approach in FDT-Boost.

The comparison with conventional classifiers is highly emphasized. The application of FDT-Boost in systems with memory constraints is assessed. This indicates practical significance of FDT-Boost across many areas that demand efficient and robust classification models.

The article elaborates on the novel ensemble-based fuzzy classifier—FDT-Boost. In FDTBoost, the weak learner is a depth-limited binary fuzzy decision tree, while in the continuous-type attributes, a fuzzy discretizer is used. The approach presented here builds on the framework of SAMME-AdaBoost and is an extensive study of this topic. As the experimental results proved, the SAMME-AdaBoost model could have good accuracy and the binary fuzzy decision tree ensemble can be effectively trained if the individual trees are constrained to be very small in depth. These ensembles often perform in practice much better than the majority of other state-of-the-art fuzzy classifiers. Indeed, even if hyperparameters have been optimized on particular datasets, FDT-Boost always performs substantially better than both the baseline FURIA and a fuzzy multi-way decision tree via evaluations conducted over eighteen datasets. Moreover, results show that FDT-Boost also builds models which are of less complexity and more compact than their non-fuzzy analog. 30% average reduction of node and leaf count with roughly equivalent accuracy. As a result, FDT-Boost is an appropriate algorithm when the memory capacity is restricted. Boosting algorithm does not resist noise well in general. The results in the noise injection tests give settings in which FDT-Boost confirmed to work even better than SAMME-AdaBoost in very few datasets and for difficult situations of noise. For 30% label noise, the gains were modestly placed, but that further assured the confirmation of the effectiveness as a whole.

The Bujnowski [9] developed a new classifier- IFDT, which is the abbreviation of Intuitionistic Fuzzy Decision Tree, in which Atanassov's intuitionistic fuzzy sets are used to make decisions allowing imprecision and hesitation in data.

Intuitionistic fuzzy sets add the extra parameter of hesitation to the classical definition of fuzzy sets and hence can be used to model uncertainty in a more comprehensive manner. This effort is to improve accuracy in classification and efficiency, particularly in those datasets where the decision boundaries are quite vague and very hard to handle with traditional classifiers.

Krassimir T. Atanassov[17] defines intuitionistic fuzzy.

An Intuitionistic Fuzzy Set (A) in a universe X is characterized by two functions:

Membership Function $\mu A(x)$: That is (x) to a certain extent is (A).

Non-Membership Function $\nu A(x)$: Indicates how much (x) does not belong to (A).

The degree of hesitation $\pi A(x)$ s: $\pi A(x) = 1 - \mu A(x) - \nu A(x)$.

This framework allows for a vaguer uncertainty approach, so it is worthwhile for the recognition in decision making, pattern recognition, and image processing.

Bujnowski [9] proposed the use of Atanassov's intuitionistic fuzzy sets in IFDT based classifiers for the treatment of imprecision in pro and con type statements and hesitation;

the aim is to increase classification accuracy and efficiency with mathematics that govern datasets showing uncertainty and vagueness in decision boundaries by incorporating the following new approach.

The IFDT was developed using the following methodology:.

fuzzifying the dataset: It will require representing the dataset as an intuitionistic fuzzy set in order to capture the membership, non-membership degrees, and degree of hesitation together.

Building tree: Intuitionistic fuzzy sets are used at each step to recursively build the decision tree, thus selecting attributes and partitioning points based on the information provided by intuitionistic fuzziness.

Classify New Instance: Classification of the new instance is done based on the constructed IFDT with partial membership of every class, not to a single major class.

These benchmarking experiments showed that IFDT classifier performance was better than all the other algorithms, with the exception of Soft Decision Trees. For this reason, logistic regression was carried out with WEKA software to inspect the classifier behavior in different scenarios. In addition, the proposed one has outperformed the fuzzy decision trees and classical random forest classifiers; for this reason, it was more accurate and robust dealing with uncertain and imprecise data.

The experimental results suggest that IFDT classifiers should be very useful with datasets that contain both uncertainty and vagueness, thus outperforming traditional classifiers not only in relation to accuracy but also robustness. Intuitionistic fuzzy sets can capture more detailed information associated with the representations of class membership, based on which better classification performance can be achieved.

Finally, the paper concludes that Intuitionistic Fuzzy Decision Tree (IFDT) is a strong and effective approach for the treatment of classification tasks in both environments where uncertainty is added and environments in which data are unsure and imprecise.

Adding some level of hesitation into the IFDT allows the modelling of more implicative uncertainty in every node of the constructed tree, which will increase classification efficiency and accuracy. Experimental results support how effective the suggested IFDT is, and what potential it has for applying in real applications. This will be a furrowed path for future research in further refining the methodology and working out its application in another sphere.

Yingtao Ren[10] introduces the Intuitionistic Fuzzy Random Forest (IFRF), a novel ensemble method that combines IFDT with the robustness of random forests. This approach utilizes intuitionistic fuzzy information gain and accounts for hesitation in the transmission of information, aiming to enhance classification accuracy and robustness. By integrating the randomness of random forests with the adaptability of fuzzy logic and the strength of multiple classifier systems, the IFRF offers a powerful tool for handling complex and uncertain data.

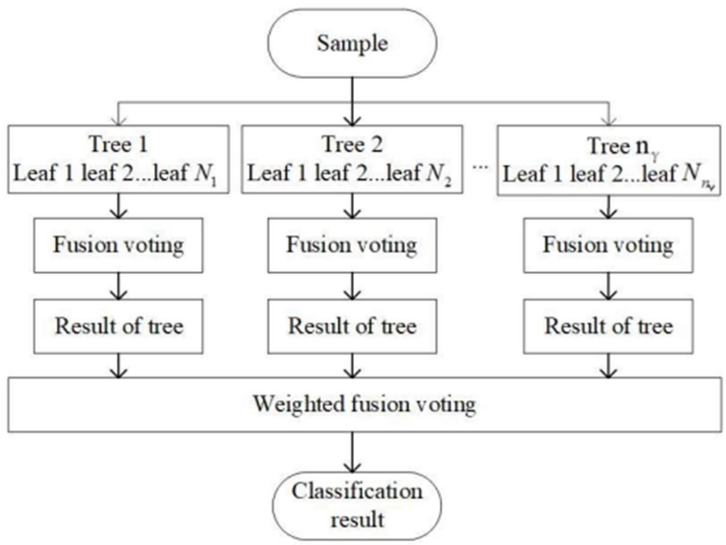


Fig 2.6 IFRF Tree-based Voting (Scheme 1)

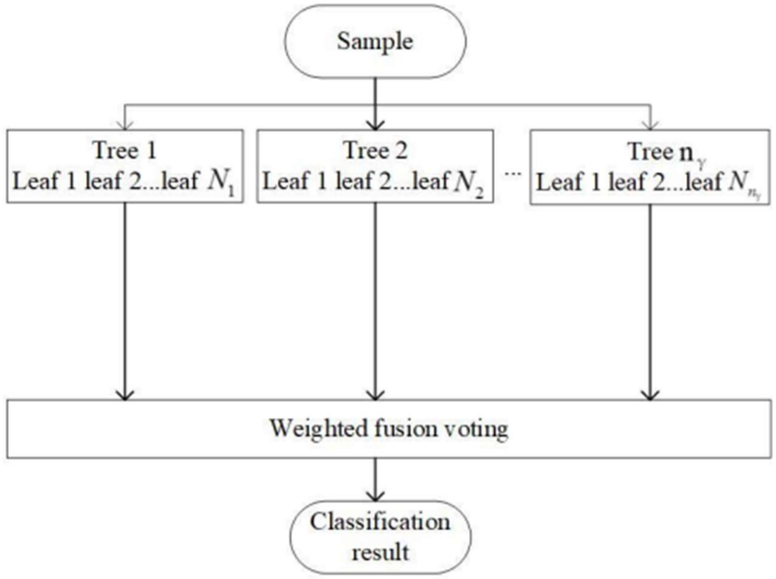


Fig 2.7 IFRF leaf-based Voting (Scheme 2)

IFRF Voting Classifications

Principle of Majority Vote:

It is finally decided, after aggregation, which class a case belongs to in the IFRF, according to the majority voting principle. That is, the class that gets more votes from individual decision trees in the committee will be the ultimate predicted class for the data point.

1. Scheme 1 Tree Voting:

Now, at each leaf node within trees, fusion voting of the weights assigned to different classes is carried out, and for every class, a classification decision is taken. And so the decisions in these trees are then passed through weighted fusion voting to arrive at the final class partitioning result of the IFRF forest ensemble.

2 Leaf-Based Voting (Scheme 2)

Here, in weighted fusion voting, the information provided by all the leaf nodes in the decision trees of the ensemble is directly combined. The final decision does not partially pass through that voting aggregation at the tree level before the end of the whole process.

Scheme 1 Combined Voting Add up the vote weights of a class from leaf nodes of all trees The projected class for that tree would be the one considering having the highest accumulated weight. In order to compute the winning class, total vote weights of each tree will be added in ascending orders to get the accumulated weight of the class with most accumulated weight.

Scheme 2 Voting by Weighted Fusion Normalize, or class-weight, the vote from the leaf-nodes by their parent tree. Aggregate these values; they are of class-weight. The final result is the class with the most weight all together.

The performance of the proposed IFRF had compared with some state-of-the-art fuzzy classifiers and ensemble methods, applying the same test configuration. It has been proved that IFRF gained the highest accuracy and outperformed the best one of them on two thirds of test datasets as their performances are different. A non-parametric statistical test has shown differences in the performances of IFRF by improving the ranking first by accuracy (p-value is below 0.001). Such similar findings have been further proved fair by the pairwise Friedman test with Holm adjustment.

Key Features of that new approach are as follows:.

1. Hesitation Parameter

This increases the handle on uncertainty by adding one more parameter of hesitancy in the intuitionistic fuzzy sets.

2. Imperfect Data

The system is robust in real-world applications since it is able to handle datasets with missing/fuzzy values.

3. Ensemble Method

Improved by random feature selection and with the characteristic of multi-split support, it adds to further accuracy and robustness, thereby combining the power of both random forests and fuzzy logic.

4. Membership

The study used trapezoidal intuitionistic fuzzy membership functions; however, other more advanced functions have been put forward to extract higher performance.

5. Fuzzy Clustering

They chose to use k-means clustering because of its simplicity, but other techniques, such as fuzzy C-means, could be applied in subsequent studies.

They show that IFRF is superior to other methods in classification accuracy and robustness, proving the fitness of an intuitionistic fuzzy set using a decision tree ensemble method. Although the proposed one shows less sufficiency in some issues, such as further research on the voting scheme and the membership function, it shows a great capability in handling complex and unclear data. Future work can be directed at fine-tuning these methods for applications across domains.

Eyke Hullermeier[11] compared the performance of ranking by fuzzy decision trees with that by conventional trees. Though FDTs have been primarily used as classifiers for the identification of objects under one of the predefined set of classes, the potential of FDTs to be also used for ranking has not been exploited. Results showed that the ability of this FDT performs well in Ranking-the task of sorting instances from most likely positive to most likely negative. The strength of FDTs is mainly the ability to obtain fine-grained membership degrees as scores, which eventually resolve the "tie-breaking" problem encountered by conventional decision trees. It helps motivate first by pointing out the ranking deficiencies of CDTs, with most deficits being observed when there is a tie among instances having the same leaf node. Later, it develops fuzzy decision trees in which fuzzy logic is used to model uncertainty around feature splits, leading to soft splits and a more nuanced ranking of instances. Theoretic analysis relates the AUC to the number of distinct scores assigned by the decision tree showing that FDTs can achieve superior ranking performance. Experiments on benchmark datasets confirm that our FDTs provide improved performance over methods and, at the same time, also remain interpretable and comprehensible in the structure of a decision tree. The most important conclusion is that the fine-grained scores produced by FDTs make them a good ranker, hence providing a very strong case in favor of their use in any ranking scenarios where the ability to handle uncertainty and fine-grained discrimination are required. I

Kavita Sachdeva [12] revisited various real-life applications of FDTs in the fields of data mining and stock markets, information retrieval, biometrics, human-computer interaction, intrusion detection, cognitive processes, and support of parallel processing.

intrusion Detection Systems

Effective intrusion detection methods are essential for bettering the quality of decision tree-based IDSs and system security. It is pertinent to note here that this particular feature has been cited to reduce the complexities and uncertainties associated with the operation of various intrusion detection systems.

Biometric Authentication:

This paper addresses the importance of user authentication in the context of biometrics and the process entailed in a biometric authentication system. Therein lies a significant point: the choice of optimal thresholds for security is paramount, and FDTs are highly effective in coping with the consequent uncertainties.

Stock Market Forecast :

The paper opens a new FDT model for databases of stock exchange based on WFPR in the prediction of trends of the stock market. This application obviously opens the possibility of being used in the domain of financial forecasting. Robert K. Lai[18] also suggest a FDT model for stock prediction with time series.

Human-Computer Interaction

The study identifies user modelling in HCI; it focuses on making the systems more usable and providing adaptive user experiences through cognitive process analysis and adaptive hypermedia design. Effective analysis of the user and modeling user behavior is achieved by employing FDTs.

Medical Diagnosis

Emma L[19] showcase that FDT can handle the uncertainty in medical Diagnosis very well than the other classifiers. Chin-Yuan Fan[20] suggested a FDT model to identify the diseases from medical data, it outperform the other classifiers.

CHAPTER 3

CONCLUSION AND DISCUSSION

Chapter 2 provided an in-depth analysis of various models and approaches. The key insights are as follows:

Ensemble Techniques on Base Classifiers of Fuzzy Sets techniques were found to be exceptionally effective.

The choice of membership functions is a critical factor in developing an effective model.

There is potential for further development by hybridizing C-FDT (Crisp Fuzzy Decision Trees) with IFDT (Intuitionistic Fuzzy Decision Trees).

There is a noted absence of comparative studies on the performance of random forests comprising Fuzzy Decision Trees (FDTs).

Random Forest Ensemble of Intuitionistic Fuzzy Decision Trees By Y. Ren[10] (2024) approach was identified as the most advanced and noise-tolerant classifier to date.

Table 3.1 provides a summary of the model discussed in Chapter 2.

Table 3.1 Summary of the model discussed in Chapter 2

Paper	Overview	Literature Survey	Methods Used	Results
Induction of fuzzy decision trees by Yufei Yuan[1]	FDT handle cognitive uncertainties in classification problems effectively.	Discusses cognitive uncertainties in classification problems.	FDT induction method based on reduction of classification ambiguity and incorporating	Improvement in FDT, with incorporating of cognitive uncertainties

	They represent knowledge naturally, tolerating imprecise, conflict, and missing information.		cognitive uncertainties.	
C–Fuzzy Decision Trees by W. Pedrycz[2]	Introduces C-FDT based on information granules. Explores new geometry of feature space partition compared to standard trees.	Compares C-FDT with standard decision trees like C4.5. Conducts experiments on synthetic and machine learning datasets for comparison.	Cluster-based decision tree architecture using fuzzy clusters as building blocks. Expansion of nodes based on heterogeneity criterion and structural dependencies. Utilization of synthetic and machine learning datasets for experiments.	C-FDT compared with standard decision trees using experiments. Classification error was zero for training and testing sets. Results showed the impact of different node expansion criteria.

A general model for fuzzy decision tree and fuzzy random forest by Zheng[3]	Proposes GMFMFS for FDT and random forest methods. Demonstrates effectiveness with US credit, Susy dataset, and big data.	Fuzzy logic in decision trees, fuzzy random forests, and membership functions.	GMFMFS in FDT and fuzzy random forest. Extension to nonlinear membership function in GMFMFS.	GMFMFS-based fuzzy decision tree outperforms C4.5 method in accuracy. Linear and nonlinear methods achieve 90% average accuracy on testing. Proposed methods show better testing results compared to C4.5 method.
A fuzzy decision tree approach based on data distribution construction by Hui Zheng[4]	Proposes DDBFDT approach for credit risk evaluation with enhanced performance. Demonstrates better performance and readability	Fuzzy decision tree methods, improvements, and applications in classification problems.	Utilizes new error-related metric and data-distribution aware algorithm for classification.	Experimental performance evaluation compared with classic methods and real-world application dataset.

	through experiments.			
NEURO-FUZZY DECISION TREES by Bhatt RB[5]	N-FDTs combine FDT with neural parameter adaptation for accuracy. Backpropagation on FDT enhances learning without losing interpretability.	N-FDTs improve learning accuracy without compromising interpretability. Previous work used FDTs to initialize structure of Gaussian RBF networks.	N-FDTs with backpropagation for learning accuracy. Hybrid Neural Networks (HNN) and Fuzzy Neural Networks (FNN) comparison.	N-FDTs improve learning accuracy without compromising interpretability
Rule set reduction in fuzzy decision trees by N. M. Abu-halaweh[6]	ID3 Fuzzy algorithm modification reduces rules, runtime, and improves accuracy. New threshold and syntax for fuzzy rules in decision trees.	Examines categorization ambiguity metrics and information gain in relation to split attribute selection.	Fuzzy ID3 algorithm modification for better accuracy and efficiency. Introduction of new threshold values for membership	Huge reduction in generated rules, better accuracy than previous work. New approach executes faster, significant reduction in rule base size.

			and syntax rules. Utilization of classification ambiguity and information gain for split attribute selection.	
A fuzzy random forest by Piero Bonissone[7]	Paper introduces a random forest of FDT.	Compares FRF ensemble with other classifiers and ensembles.	Trainable and non-trainable combiners for classifier combination.	FRF ensemble accuracy compared with other ensembles and classifiers. Transformation method applied to FRF ensemble matrix for error rate.
An analysis of boosted ensembles of binary fuzzy decision trees by Marco Barsacchi[8]	FDT-Boost uses fuzzy binary decision trees for accurate classification models.	Focus on boosting with fuzzy weak learners for classification accuracy.	FDT-Boost with FBDT as base classifiers.	FDT-Boost outperformed FURIA and other fuzzy classifiers statistically.

	Outperforms FURIA and other fuzzy classifiers in experimental evaluation. Offers similar performance to SAMME-AdaBoost but with less complexity.		Comparison with FURIA, FBDT, and fuzzy multi-way decision tree.	FDT-Boost showed similar performance to SAMME-AdaBoost with less complexity.
An Approach to Intuitionistic Fuzzy Decision Trees by Bujnowski[9]	Intuitionistic fuzzy decision tree classifier performance compared with popular algorithms.	Compares classifier performance with popular algorithms on benchmark data.	Constructing intuitionistic fuzzy decision trees for classification analysis. Comparing results with popular classification algorithms like logistic regression.	Intuitionistic fuzzy decision tree outperformed other classifiers in accuracy and stability.
A New Random Forest Ensemble of Intuitionistic	IFRF combines random forest with	Defines intuitionistic fuzzy set and entropy,	approach involves the utilization of IFS techniques	IFRF outperforms other fuzzy and ensemble

Fuzzy Decision Trees by Y. Ren[10]	intuitionistic fuzzy theory for classification accuracy. Intuitionistic fuzzy information gain is used by IFDT in IFRF to choose features.	reviews classic fuzzy decision trees.	in the processes of discretization, decision tree construction, and random forest modeling	algorithms in classification accuracy.

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