Quantum Entanglement Between Different Flavors of A Neutrino

A DISSERTATION

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE AWARD OF THE DEGREE
OF

MASTER IN SCIENCE
IN
PHYSICS

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June 2024

DECLARATION

I hereby certify that the work which is presented in the Dissertation-I entitled **"Quantum Entanglement Between Different Flavors of A Neutrino"** in fulfilment of the requirement for the award of the Degree of Master of Science in Physics and submitted to Department of Applied Physics, Delhi Technological University, Delhi is an authentic record of my own, carried out during a period from 2023-2024, under the supervision of **Dr Mukhtiyar Singh**, Assistant professor, Department of Applied Physics. Delhi Technological University, Delhi and **Dr Satyabrata Adhikari**, Assistant Professor, Department of Applied Mathematics, Delhi technological University, Delhi.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other Institute/University. The work has been published/accepted/communicated in SCI/SCI expanded/SSCI/Scopus indexed journal OR peer reviewed Scopus indexed conference with the following details:

Title of the Paper: Quantum Entanglement Between Two Flavors of A Neutrino

Author names : Ashish Shrivastava, Mukhtiyar Singh, Satyabrata Adhikari

Name of Conference: National Conference on Apllied Physics and Mathematics

Have you registered for the conference : Yes

Status of paper: Accepted

Date of paper communication: NA

Date of paper acceptance : 29/04/2024

Date of paper publication: NA

Ashish Shrivastava

Roll No.: 2K22/MSCPHY/06

SUPERVISOR CERTIFICATE

To the best of my knowledge, the above work has not been submitted in part or full for any Degree of Diploma to this University or elsewhere. I further certify that the publication and indexing information given by the students is correct.

Place: Delhi

Date: 07/06/2024

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ACKNOWLEDGEMENT

I wish to express my sincere gratitude to **Dr Satyabrata Adhikari**, Assistant Professor (Department of Applied mathematics, DTU) and **Dr Mukhtiyar Singh**, Assistant Professor (Department of Applied Physics, DTU) for their support and continuous guidance throughout the whole year. The mentorship they provided me, helped me to understand and concentrate on the subject and the topic in depth of my study. The path showed by them made me achieve my targets by fulfilling the tasks at regular intervals along with the importance of this project. They were always there to clear the doubts about subjects included in the project and other objections in it.

Place: Delhi

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Date: 07/06/2024

Contents

ABSTRACT

In quantum mechanics, entanglement is an interesting and important concept, serving as the cornerstone for various applications in quantum information science. Non-locality is a crucial quantum correlation to study this quantum mechanical phenomenon. In the field of high energy physics there are numerous phenomenon which exhibits non-local behavior hence in our work we focused in entanglement of different flavor of neutrinos. Neutrino Oscillations is an interesting quantum mechanical phenomenon in which a neutrino changes its flavor.

Using the theoretical framework of neutrino oscillations, entangled state of two flavor and three flavor neutrinos is defined. The density matrix for entangled state is dependent on the oscillation probabilities of the neutrinos hence the quantum correlations are studied in terms of the transition and survival probabilities.

To understand the quantum correlations between two flavors of a neutrino we used the theoretical framework of two flavor neutrino oscillation. The entangled state of a two flavor neutrino is defined by 4x4 density operator. Using the density operator of the entangled state of a two flavor neutrino, we studied the negativity to measure the intra-particle entanglement and quantify the entanglement for different neutrino oscillation experiments between two flavors of a neutrino. The dependence of entanglement on oscillation parameter $\frac{L}{E}$ is expressed analytically in order to analyze the different cases of intra-particle entanglement between the two flavors of a neutrino. We found that the entanglement between two flavors of a neutrino vanishes at extremely high energy. Therefore, we may predict that the two flavor of a neutrino may not be entangled when they pass through a high density matter such as neutron stars, supernovae etc.

For three flavor neutrinos we use the svetlichny inequality and we the genuine tripartite entanglement by showing the genuine non-locality in three qubit state of the neutrinos. Lower bound and upper bound value of the svetlichny operator for the three qubit state violates the svetlichny inequality.

Chapter 1: Introduction to Quantum Information Science

Quantum Information Science or QIS is the study of how an information can be stored, transferred or use by the help of quantum mechanics. Clasicaly the information stored is stored in the form of bits 0 or 1. In QIS the information is stored in the form of quantum bits also known as qubits[1, 2]. These qubits are in the entangled and superposition states which allows them to exist in multiple states quantum mechanically. They can store more information as compared to the classical bits. This enables them to do the fast computation. It is the study of how an information can be stored, transferred or use by the help of quantum mechanics. In QIS we study quantum entanglement which is an interesting phenomenon due to its non-local behavior and can become a useful resource of quantum communication in the near future. More than one qubits can be entangled and can be used to do instant communication[1, 2]. This all is possible due to the quantum mechanics and hence QIS is a direct application of quantum mechanics. It is important to understand the quantum mechanics and how it leads to quantum information science. In this chapter early origin of the quantum mechanics, different interpretations and views of some genius mind are discussed. In the further sections of this chapter, a brief introduction of quantum entanglement and applications of QIS are discussed.

1.1 Origin of Quantum Mechanics

In the advancement of the science, the field of physics has its own race with itself. From Aristote's fallacy to Newton's Gravitational Laws, contribution of Maxwell and revolution by Albert Einstein all are remarkable. With the advancement of the field, scientists have found the Black Body radiation spectrum problem unexplainable with the existing laws of physics. This leads to the emergence of Quantum Mechanics. One more experimental evidence, photo electric effect is also explained using dual behavior of the particle which raises a lot of questions regarding the validity of this new physics. In the year 1925, a conference in Copenhagen Neils Bohr, Heisenberg and many more scientists gave the interpretation of quantum mechanics[3]. The copenhagen interpretation stated that:

- A quantum particle exists in all states and the nature of the particle will be interpretated as the observer wants to observe.
- Observations of a quantum system itself disturbs the measurements results the "Heisenberg Uncertainity Principle".
- For a quantum system it is not possible to separate the behavior of the system from the effect of the measuring instrument with a classical behavior.

- Results obtained in an quantum mechanical observations are always probabilistic.

Although this probabilistic and indeterministic nature of the quantum mechanics raised a question regarding the completeness of this theory and leads to the hidden variable interpretation but this is the most accurate way to describe the universe.

1.2 Postulates of Quantum Mechanics

Postulates of quantum mechanics defined after a large amount of trial and errors by assuming and approximations by the physicists who introduced us to the quantum mechanics. Purpose of these postulates is simple that how to apply and understand the problems which are govern using quantum mechanics. Understanding these postulates is very important to study any quantum mechanical phenomenon. The postulates of quantum mechanics can be stated as[1, 3, 4, 5]

Postulate 1

For any physical system, it belongs to is a complex vector space which is a Hilbert Space known as the state space . This state of the system is completely described by wavefunction $|\psi>$.

Postulate 2

Every physical observable associated with operator $\hat{A}$, and the values that will ever be observed are the eigenvalues a which satisfy $\hat{A}\Psi = a\Psi$.

Postulate 3

Quantum measurements are described by a collection M_n of measurement operators and The average value of the observable corresponding to operator $\hat{A}$ is given by

$$<\hat{A}>=\frac{{}^{+\infty}_{-\infty}\int \psi^* \hat{A}\psi}{{}^{+\infty}_{-\infty}\int \psi^*\psi}$$

Postulate 4

For closed quantum system, it is described by a unitary transformation. That is, the state $|\psi>$ of the system at time t_1 is related to the state $|\psi'>$ of the system at time t_2 by a unitary operator U which depends only on the times t_1 and t_2 as $|\psi'> = U|\psi>$.

Postulate 5

The time evolution of the state of a quantum system is described as,

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$$

1.3 EPR Paradox and Bell's Theorem

A famous thought experiment by Erwin Schrodinger where he stated "The Cat can be dead or alive both at the same time" explains the superposition states of the quantum particle[6]. There were several quantum mechanical phenomenons that could not be explained earlier easily. One of them is "Quantum Entanglement". Quantum Entanglement is a phenomenon where one particle correlates a unique relationship with another particle and can be influenced by each other irrespective of distance between them[7]. These quantum particles can be represented as they store the qubits. It is an instantaneous phenomenon which directs towards the non-local behavior of the universe. Quantum Entanglement shows that if the quantum particles are separable or not and establishes quantum correlation between them.

Quantum Mechanics was then discussed by Einstein, Podolsky and Rosen in 1935 and they gave EPR Paradox. EPR stated that quantum entanglement phenomenon violates the special theory of relativity and raised a question over the physical reality of the quantum mechanics. EPR Paradox stated that[8]

- A valid theory must be there for a physical reality.
- Description by a wavefunction is not complete for a physical reality due to its indeterministic nature.
- Operators must commute i.e. no uncertainity for the vaidation of the existence of those physical quantities
- Violates the special theory of relativity

In 1964 after 29 years, Bell in [9] gave the mathematical explaination and obtained an inequality on the basis of hidden variable theory which got violate by quantum mechanical nehavior. He stated that "If a hidden variable theory is local either it will defy quantum mechanics or it will be not be local" and derived Bell CHSH inequality which must be followed as per the hidden variable theory. Further, Bell states were introduced which are the entangled states of qubits. Bell states are also known as EPR pairs and can be shown as

$$|\Psi^+ >= \frac{1}{\sqrt{2}}(|00 > +|11 >)$$

$$|\Psi^- >= \frac{1}{\sqrt{2}}(|00 > -|11 >)$$

$$|\Phi^+ >= \frac{1}{\sqrt{2}}(|01 > +|10 >)$$

$$|\Phi^- >= \frac{1}{\sqrt{2}}(|01 > -|10 >)$$

These bell states are entangled such that one qubit influence the state of another qubit instantly irrespective of the distance between them. All these 4 entangled states violated the Bell's inequality which supported the non local behavior of the quantum mechanics and contradicted the hidden variable theory.

Due to this non local behavior of entanglement QIS has several applications like[10]:

- Quantum Entanglement
- Quantum Cryptography
- Quantum Sensors etc.

In the upcoming chapters quantum entanglement is discussed and try to explain the non-local behavior of the neutrinos by studying the correlations between different flavors of them and the transition probabilities.

Chapter 2: Quantum Entanglement and Quantum Correlations

Scientists have been doing reserch and trying to understand the quantum mechanics since the beginning. Quantum mechanics explained some experimental results which were unexplainable like photoelectric effect, black body radiation etc[11]. So this field of study has always amazed the researchers. Physicists who study compound quantum systems confronted a quantum mechanical phenomenon repeatedly known as enatanglement. As by the name of it, it is leading towards that there is some relation or link between different quantum states of a system. Entanglement between two quantum states shows that if one state is disturbed then other state will be disturbed instantaneously irrespective of the distance between them. This phenomenon shows that the universe is non local since this is a non local behavior of the system[12]. A quantum system can be described using Hilbert spaces as per the postulate 1. This Hilbert space is a complex vector space used in quantum mechanics to describe a quantum system which has completeness with a properly defined inner product of the vectors. Assuming a compound quantum system which has two different quantum systems S_1 and S_2 such that $S_1 \epsilon \mathcal{H}_1$and $S_2 \epsilon \mathcal{H}_2$. If both these quantum systems are entangled then the compound quantum system $S \epsilon \mathcal{H}$ such that $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ i.e. compond system will be described by the tensor product of the individual entangled systems[13]. There is always a possibility that more than one quantum states are entangled then all the states will be tensored in the similar way. These quantum states can be described using qubits on the basis of coordination number.

2.1 Types of Entanglement

Entanglement can be differentiated on the basis of how many quantum states and indivisual are taking part in the phenomenon. It is required to have minimum two quantum states to be entangles. Depending on the no of states entangled number of qubits are used to describe the system and it can be bipartite as well as multipartite[14]. These quantum states may belong to different particles as well as can be the different degree of freedom of a same particle so the entanglement can be interparticle as well as intraparticle[15].

2.1.1 Bi-partite Entanglement

When two quantum states are entangled then the entanglement is called bi-partite entanglement. It is a two qubit system and the states can be as $|00>$,$|01>$, $|10>$and$|11>$. The inear superposition states of these qubit states such that one state of particle influences other instantaneously will be a bi-partite entannglement[14].

The superposition of these states can be the the bell states of section 1.3 are the examples of bi-partite entangled state. This analogy can be used to show the entangled state between two spin half particles which are spatially apart from each other.

2.1.2 Multipartite Entanglement

In the case of multipartite entanglement more than two quantum states are entangled. On the basis of the entanglement there are two classes defined GHZ and W states which have their own properties and definitions[14]. If an entangled state is in GHZ state in a multipartite entanglement if any quantum system is separable then the system will not be entangled any more. In W state enatanglement will always be there in the quantum system unless every quantum system is separable from each other[16].

2.1.3 Interparticle Entanglement

When quantum states of different particles such as spin, polarization etc are entangled then it is said to be an interparticle entanglement. In general, interpartice entanglement occurs when two different particles are spatially separated from each other but their quantum states are entangles. For e.g. EPR pairs is a robust example for interparticle entanglement[15]. Assuming that quantum state of two different particles such that $|1>_1, |0>_1 \epsilon \mathcal{H}_1$ and $|1>_2, |0>_2 \epsilon \mathcal{H}_2$, their linear superposition state can be written as:

$$|\phi^+>= \frac{1}{\sqrt{2}}(|0>_1 \otimes |0>_1 + |1>_2 \otimes |1>_2)$$

In the above state in equation if there is an hermitian operator operated, then the state of both the particles will be influenced which is why the assumed pair is entangled. This phenomenon rises the question of non-local behavior of this phenomenon and this state violates the bell's inequality too.

2.1.4 Intraparticle Entanglement

When the different quantum states of a single particle or degree of freedoms of a single particle are entangled it is said to be an intraparticle entanglement. For e.g. for a photon spin and momentum are entangled. In this case also an entangled system is desribed by tensor product of the individual state, but the spatial separation will not be there. These type of entangled states are known as single particle entangled states (SPE). Although the problem of local behavior is not anymore in intraparticle entanglement, this can also be distinguished very well from the clasical description of a system such as non-contextuality[15]. Assuming the spin-momentum single particle entanglement, where the tensor product will determine the quantum state. Consider the state for degree of freedom momentum, the pure state of momenta can be as $|k_1>, |k_2>$ such as $<k_2|k_1>= 0$ and pure spin state can be described as $|+>, |->$. So their linear superposition entangled state can be written as:

$$|\phi^+>= \frac{1}{\sqrt{2}}(|k_1> \otimes |+> + |k_2> \otimes |->)$$

This equation is very similar to bell's state. The non-contextuality arises when the above state is seen that there is no component defined for the spin state of the particle. In this project, intraparticle entanglement of neutrinos is dealt where non-local behavior is analysed. The intraparticle entanglement study is considered more useful and easy because the entangled pairs can be generate because of less decoherence[15].

2.2 Quantum Correlations

Entanglement phenomenon occurs when there is some unique relationship between the entangles states of the system. These relations are the quantum correlations which are measurable quantities and describes the properties and behavior of the system. These quantum correlations explains the separable properties i.e. whether the states are separable or not, non-local behavior of the entanglement, genuine entanglement etc[17, 18]. Some quantum correlations are mentioned below such as:

- **Non-Local Behavior**: Non-local behavior is a quantum correlation in physics such that when an action at one place can instantaneously effect the state of a system at another location, without any direct interaction through a known medium or force if those two systems were connected in past. It is explained when .quantum entanglement violates the inequalities like bell's Inequality, legett garg Inequality, Svetlichney Inequality etc. [19]
- **Negativity**: It is defined for a two body system or a reduced system to quantify the entanglement. In this work the role of negativity is very important and can be seen further in chapter 4.[20]
- **Teleportation Fidelity**: Not all the quantum entanglement states can take part in teleportation. So the calculation whether the entangled state can be teleport or not is the main key to use it as a mode of instantaneous communication. [21]
- **Concurrence**: Similar to negativity, concurrence is use to quantify the bi-partite entanglement. The concurrence of this inseparable state is given as $C_{e\mu} = max(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0)$ where $\lambda_i$are the square root of of the eigen values of $\rho_{e\mu}\tilde{\rho_{e\mu}}$in decreasing order where $\tilde{\rho_{e\mu}} = (\sigma_y \otimes \sigma_y)\rho^*_{e\mu}(\sigma_y \otimes \sigma_y)$ [22]

Several other quantum correlations like Tangle, Quantum discord, dissonance etc can be used to define the quantum entanglement in a quantum system[1].

Chapter 3: Introduction to Neutrino Physics

3.1 Neutrino Hypothesis

Before the leptons were characterized the discovery of neutrinos is itself a very interesting topic. In 1899 Ernest Rutherford discovered the $\beta - rays$. So many elements showed the $\beta - decay$ in the reaction similar to

$$N_0(A, Z) \rightarrow N(A, Z + 1) + e^-$$

But in this decay, energy spectra of electron were controversial and there was violation in the laws of conservation of angular momentum. The spread in energy of the spectra was observed to be continuous but two body decay would suggest a fixed energy line for electrons. Later, Pauli introduced a light particle with no charge named neutrino to unresolved the mystery and violations in the $\beta - decay$. Neutrinos considered to be massless initially as per the standard model with spin number $\frac{1}{2}$[23, 24, 25].

3.2 Neutrinos in Standard Model

Using Quantum Mechanics and Einstein's Special Theory of Relativity a standard model of particles is described. Leptons and quarks are the basic components of this model. This model anticipated neutrinos of three differnt flavors:electron, muon and tauon. According to the standard model[26]

- The neutrinos are massless leptons
- Chirality and helicity of neutrinos are same
- Neutrinos travel with the speed of light
- Neutrinos are left handed

3.3 Source and Types of Neutrinos

In the reverse $\beta - decay$ process in 1956 neutrinos were first detected in the project "Poltergeist"[27]. Later on neutrinos were detected from differnt sources and named accordingly. Neutrinos can come from atmosphere, sun, reactors etc. These are present everywhere in the universe and on the basis of their origin and their flavor they are categorised[24].

On the basis of orgin different types of neutrinos are:

- Solar Neutrinos
- Atmospheric Neutrinos
- Reacter neutrinos
- Aceelerated Neutrinos and many more

On the basis of flavors they are categorized into 3 types:

- e flavor Neutrino
- μ flavor Neutrino
- τ flavor Neutrino

3.4 Concept of Neutrino Oscillations

Since, the neutrinos were considered to be massless elementary particles in the universe. But due to some experimental and theoretical research it was observed that neutrinos change their flavor and this phenomenon is known as "Neutrino Oscillation"[24]. This phenomenon concluded that neutrinos must have mass and experiment all over the world for the detection of neutrinos predicted the same results. In this chapter we will discuss the some natural ocurring neutrino problems and how their solution led us to the revolutionary concept of neutrino oscillation.

3.4.1 Solar Neutrino Problem

The Sun in the solar system has always been a mystery. One of the most intresting and big mystry of the sun is the source of its energy. It was explained by an British astronomer by the concept of fusion of hydrogen to form helium. Later after the concept of neutrinos came into the field, scientists predicted that the sun should be a producer of the neutrinos and calculation showed that the solar neutrino flux must be 7.5±3 SNU. There are two cycles p-p and CNO cycle explained the Standard Solar Model. According to the Standard Solar Model the fusion p-p reaction is mentioned below:

$p + p \to D + e^+ + \nu_e$ and $p + e^- + p \to D + \nu_e$
$\Longrightarrow D + p \to {}^3He + \gamma$
$\Longrightarrow {}^3He + {}^3He \to \alpha + 2p$ (p-p reaction I)
${}^3He + {}^4He \to {}^7Be + \gamma$

$\Longrightarrow {}^7Be + e^- \rightarrow {}^7Li + \nu_e \Rightarrow {}^7Li + p \rightarrow 2\alpha$ (p-p reaction II)

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

$\Longrightarrow {}^7Be + p \rightarrow {}^8B + \gamma$

$\Longrightarrow {}^8B \rightarrow {}^8Be + e^+ + \nu_e \Rightarrow {}^8Be^* \rightarrow 2\alpha$ (p-p reaction III)

In the p-p cycle of standard solar model reaction I, II, and III occurs 86%, 14% and 0.02% respectively[28].

There reactions shows the conversion of protons into alpha particles and the formation of neutrinos such that they carry 3% of the energy. For the confirmation of the theories experiment were set up by scientists after these calculations to detect the solar neutrinos. The chlorine experiment in the Homestake mine reported that the solar neutrino flux found to be less than 3 SNU. It was difficult to explain such deficiency of solar neutrinos. Experiment SAGE, GALLEX, Kamaikonde used different techniques but only confirmed the neutrino deficit. This problem is known as "Solar Neutrino Problem"[29].

According to the calculation the description was mentioned only for the formation of electron flavor neutrinos hence all these experiments were sensitive for these neutrino only. Using the quantum mechanical approach neutrinos were considered as they have very small mass and different mass eigenstates of different flavor neutrino mixed in each other. Later by the experiments it was confirmed and it was also found that the flavor of the neutrino detected is different as it was predicted. Hence it gives the concept of neutrino oscillation and also the neutrinos have mass[29].

3.4.2 Atmospheric Neutrino Problem

Cosmic rays arrive to the earth from the universe. These are the radiation of high energy particles of the universe. When these rays eneters in atmosphere of the earth, the particles of the rays interact with the nuclei and thus the neutrinos are formed and are known as "Atmospheric Neutrinos"[30]. According to the decay reactions, ratio of muon neutrino flux and electron neutrino flux approximated to be 2 upto few GeV energy. Experiments like Super-Kamaikonde observed the ratio less than expected due to the large zenith angle. This deficit in the ratio is known as "Atmospheric Neutrno Problem"[30, 31]. These results favor the concept of neutrino oscillation and the mixing of neutrinos, violation of CPT symmetry and possiblity of flavor changing.

3.5 Mixing in Neutrinos

Since 1930, the idea of "ghost particles" came, they have always been a mystery. At the start of 21st century SNO also proved the theory of neutrino oscillation and flavor changing possiblity of the neutrinos when they travel. Solar and atmospheric neutrino problem are also solved by having the possiblity of neutrino mixing and neutrino oscillation. To understand this phenomenon we use quantum mechanical approach to understand the mixing and the other calculations like matter effects on neutrinos, probability of flavor changing etc. In this chapter we will discuss about the mixing of neutrinos and use of quantum mechanics in the same.

3.5.1 Neutrino Flavors

As we discussed earlier, there are three flavors of neutrinos initially in the universe. All the neutrios have spin $\frac{1}{2}$ and lepton number 1. Through the solar and atmospheric neutrino studies we have detected electron flavor, muon flavor and tauon flavor neutrino. With each neutrino there is a anti-neutrino associated with this.Since these neutrinos convert into other flavors of their type allow us to consider the point of neutrino having differnt mass eigenstates with significant mixing.

3.5.2 Mixing in Neutrinos

Since it is necessary for having significant amount of mixing with differnt mass eigenstates for the solution of physical phenomenon in the universe. If we define neutrino on the basis of their differnt mass eigenstate as ν_i,where i= 1,2,3....... each is associated with mass m$_i$. On the basis of flavor we define neutrino as $\nu_\alpha$where α represents the flavor of the neutrino[32, 33]. To define mixing we can say that $\nu_\alpha$is the superposition of differnt mass eigenstates ν_i i.e.

$$|\nu_\alpha >= \sum_i U_{\alpha i}|\nu_i >$$

where $U_{\alpha i}$is known as mixing matrix and for i = 1,2,3 and $\alpha = e, \mu, \tau$

Chapter 4: Quantum Entanglement Between Different Flavors of A Neutrino

4.1 Two Flavor Neutrino

4.1.1 Two Flavor Neutrino Oscillation

In two flavor neutrino oscillations, only electron flavor neutrino and muon flavor neutrino are considered and they oscillate from one flavor to another. The two flavor neutrino oscillation has been studied when the solution of solar neutrino deficit problem was given. The possible theoretical solution was provided by explaining that the electron flavor neutrino shows transition and convert itself into muon flavor neutrino[29]. This conclusion makes an important contribution on studying the connection between the flavor of a neutrino. To understand the correlation that may exist between the two flavors of a neutrino, researchers studied the entanglement with the help of existing neutrino oscillation framework[34]. Thus study of entanglement of flavor of a neutrino is important and interesting to understand the quantum correlations between the two flavors of a neutrino and their dependence on oscillation parameter such as baseline length and energy of neutrino.

As the neutrinos travel in the form of mass eigenstates so electron and muon flavor of neutrino can be expressed as superposition of mass eigenstates. In two flavor neutrino oscillation, electron flavor neutrino (ν_e) and muon flavor neutrino (ν_μ) are related to mass eigenstates $\nu_1 and \nu_2$as[35, ?]

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{1}$$

where U denote the unitary mixing matrix for two flavor neutrino oscillations and can be written as $U = \begin{pmatrix} cos\theta_{12} & sin\theta_{12} \\ -sin\theta_{12} & cos\theta_{12} \end{pmatrix}$ and θ_{12} is the mixing angle.

For two flavor neutrino oscillation, the oscillating probability from ν_e to ν_μ is given by:

$$Prob(\nu_e \to \nu_\mu) = P_{e\mu} = -4U^*_{e2}U_{\mu 2}U_{e1}U^*_{\mu 1}sin^2(\frac{\Delta_{21}}{4E}L) \tag{2}$$

where Δ_{21}, L and E denote mass square difference, baseline length and energy of neutrino respectively.

From different neutrino oscillation experimental results, it is found that the mixing angle $\theta_{12} = 33.4^0$ and mass square difference $\Delta_{21} = 7.4 \times 10^{-5} eV^2$[37, 38, 39]. The oscillation probability given in (2) can be re-expressed in terms of the mixing angle as

$$P_{e\mu} = 4cos^2\theta_{12}sin^2\theta_{12}sin^2(\frac{\Delta_{21}}{4E}L) \tag{3}$$

The relation between oscillation probability and survival probability in two flavor neutrino oscillation is given by

$$P_{ee} + P_{e\mu} = 1 \tag{4}$$

Using the values of mixing angle $\theta_{12} = 33.4^0$ and mass square difference $\Delta_{21} = 7.4 \times 10^{-5} eV^2$, equation (3) and equation (4) can be re-written as

$$P_{e\mu} = 0.84481 sin^2(9.4357 \times 10^{-5}\frac{L_{(km)}}{E_{(GeV)}}) = 0.84481 sin^2(9.4357 \times 10^{-5}X) \quad (5)$$

$$P_{ee} = 1 - 0.84481 sin^2(9.4357 \times 10^{-5}X) \quad (6)$$

where $X = \frac{L}{E}$.
Since the transition and survival probability given by (3) and (4) depends upon the mixing angle, mass square difference, baseline length and energy of neutrino so we can discuss the dependency of oscillation probability with $\frac{\Delta_{21}L}{E}$ in the Figure 1.

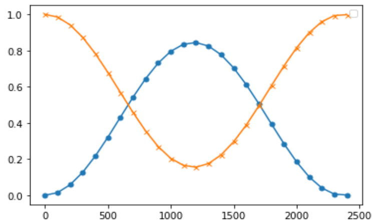

Figure 1: The variation of oscillation probability with the $\frac{\Delta_{21}L}{E}$ is shown. Y-axis represents oscillation probability where P_{ee}(Orange Line) is survival probability, $P_{e\mu}$(Blue Line) is transition probability and x-axis represents $\frac{\Delta_{21}L}{E}$.

The intesection points on the graph gives the value of $\frac{\Delta_{21}L}{E}$ where the survival probability is equal to the transition probability i.e. the chances of neutrino being in electron flavor or muon flavor are equal. We will find in the later section that these two flavor of a neutrino are maximally entangled.

4.1.2 Quantum Entanglement between two flavors of a neutrino

The neutrinos are linear superposition of their mass eigenstates and a neutrino can be in any flavor depending upon the oscillation parameters. In this section, we will study the correlation between the electron and the muon flavor of a neutrino. To start with, let us represent the electron and muon flavor at t=0 as the two qubit states, which can be constructed on the basis of occupation number assigned to each flavor. Therefore, the electron and muon flavor in terms of a qubit can be defined as[40, 41]

$$|\nu_e >= |1 > \otimes |0 > \equiv |10 > \quad (7)$$

$$|\nu_\mu >= |0 > \otimes |1 > \equiv |01 > \quad (8)$$

Let us consider the initial state of neutrino in electron flavor. Then it will either oscillate into muon flavor or it will retain its original flavor as time passes. Thus, the time evolution of the composite two qubit system can be written as

$$|\nu_e(t) >= A_{ee}|10 > + A_{e\mu}|01 > \quad (9)$$

where $|A_{ee}|^2 = P_{ee} and |A_{e\mu}|^2 = P_{e\mu}$are the survival and transition probabilities. Hence, the state in (9) can be described by the density operator

$$\rho_{e\mu} = |\nu_e(t) >< \nu_e(t)| = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & P_{ee} & \sqrt{P_{ee}P_{e\mu}} & 0 \\ 0 & \sqrt{P_{ee}P_{e\mu}} & P_{e\mu} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

To measure the entanglement in the system described by the density operator $\rho_{e\mu}$, we will use the negativity(Ref. Werner), which can be defined as[42]

$$N_{e\mu} = max\{0, -2\lambda_{min}\} \tag{11}$$

where $\lambda_{min}$is the minimum eigenvalue of $\rho_{e\mu}^{T_\mu}$. The matrix $\rho_{e\mu}^{T_\mu}$ denote the partial transpose of the density matrix (10), which is given by

$$\rho_{e\mu}^{T_\mu} = \begin{bmatrix} 0 & 0 & 0 & \sqrt{P_{ee}P_{e\mu}} \\ 0 & P_{ee} & 0 & 0 \\ 0 & 0 & P_{e\mu} & 0 \\ \sqrt{P_{ee}P_{e\mu}} & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

Eigenvalues of $\rho_{e\mu}^{T_\mu}$ are $P_{ee}, P_{e\mu}, \sqrt{P_{ee}P_{e\mu}}, -\sqrt{P_{ee}P_{e\mu}}$. One negative eigenvalue of $\rho_{e\mu}^{T_\mu}$ suggests that the state $\rho_{e\mu}$ is an entangled state. Therefore, the degree of entanglement is given by

$$N_{e\mu} = 2\sqrt{P_{ee}P_{e\mu}} \tag{13}$$

It can be easily seen that when $P_{ee} = P_{e\mu} = \frac{1}{2}$, then $N_{e\mu} = 1$. This reflects the fact that the state described by the density operator $\rho_{e\mu}$ is maximally entangled.

4.1.3 Quantification of entanglement between two flavors of a neutrino in different neutrino oscillation experiments

Here, we will quantify the entanglement that may exist between two flavors of neutrino in different neutrino oscillation experiments. Quantum entanglement is a non-classical phenomenon and thus it is a very challenging task to quantify the entanglement between different degrees of freedom of a neutrino.
To achieve this task, we will use negativity for the quantification purpose. Let us recall (13) and squaring it both sides, we get

$$N_{e\mu}^2 = 4(P_{e\mu} - P_{e\mu}^2) \tag{14}$$

Using (5), the equation (14) reduces to

$$N_{e\mu}^2 = 3.3792sin^2(\phi) - 2.8548sin^4(\phi)$$

$$\Rightarrow 2.8548sin^4(\phi) - 3.3792sin^2(\phi) + N_{e\mu}^2 = 0 \tag{15}$$

where $\phi = 9.4357 \times 10^{-5}X$
Equation (15) represents the quadratic equation in $sin^2(\phi)$ and by solving it, we get

$$sin^2(\phi) = 0.5918(1 - \sqrt{(1 - N_{e\mu}^2})$$

$$\Rightarrow \phi = sin^{-1}(\sqrt{0.5918(1 - \sqrt{(1 - N_{e\mu}^2})}) \tag{16}$$

Using $\phi = 9.4357 \times 10^{-5}X$ in (16), we get

$$\frac{L}{E} = \frac{1}{9.4357 \times 10^{-5}} sin^{-1}(\sqrt{0.5918(1 - \sqrt{(1 - N_{e\mu}^2})}) \tag{17}$$

Baseline Length (L) is the distance between the source and the detector in an experiment. We will now consider different neutrino oscillation experiments K2K, T2K, ICARUS and NoνA [43] to quantify the entanglement (generated between the different flavors of a neutrino, provided the baseline length (L) and energy of neutrino (E) is given.

Experiment	L(km)	E(GeV)	X=$\frac{L}{E}$	$N_{e\mu}$
K2K	250	1.3	192.31	0.03316
T2K	295	0.6	491.67	0.08514
ICARUS	730	17	42.94	0.00744
NoνA	810	2	405	0.07014

Table 1: Values of $N_{e\mu}$for different accelerator neutrino oscillation experiment

Analytically equation (17) shows the relation between $\frac{L}{E}$ and $N_{e\mu}^2$. For the special cases where either $N_{e\mu} = 0$ or 1, we get the value of oscillation parameter $\frac{L}{E}$ as:
Case 1: For $N_{e\mu} = 0$, equation (17) reduces to

$$\frac{L}{E} = \frac{n\pi}{9.4357 \times 10^{-5}}, n = 0, \pm 1, \pm 2, ... \tag{18}$$

(a) For $n = 0$, we have $\frac{L}{E} = 0$. This implies that if the negativity of the entangled state $\rho_{e\mu}$ is vanishing then we can conclude that for a fixed baseline length L, the energy of neutrino is very high.
(b) For large n, the energy of a neutrino is very small for a given baseline length L and therefore, in this case, the entanglement of the state $\rho_{e\mu}$ will vanish.
Case 2: For $N_{e\mu} = 1$, equation (17) reduces to

$$\frac{L}{E} = \frac{n\pi + sin^{-1}(\sqrt{0.5918})}{9.4357 \times 10^{-5}}, n = 0, \pm 1, \pm 2, ... \tag{19}$$

To achieve the maximally entangled state between electron and muon flavor of the neutrino, the ratio of oscillation parameter baseline length L and energy of neutrino E i.e. $\frac{L}{E}$, must hold the equation (19).

4.2 Three Flavor Neutrino

4.2.1 Three Flavor Neutrino Oscillation

There are three flavor of neutrinos, electron flavor (ν_e), muon flavor (ν_μ) and tauon flavor (ν_τ). Analogous to two flavor neutrino oscillation, three flavor neutrino also shows transition between electron flavor-muon flavor or electron flavor-tauon flavor or muon flavor-tauon flavor. This phenomenon is known as three flavor neutrino oscillation[44]. To understand the correlation that may exist between the three flavors of a neutrino, we study the entanglement between different flavor of a neutrino in the subsequent section.
In three flavor neutrino oscillation, $\nu_e, \ \nu_\mu$ and ν_τ are related to mass eigenstates $\nu_1, \ \nu_2 \ and \ \nu_3$as[44, 45]

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{20}$$

where U denote the unitary mixing matrix for three flavor neutrino oscillation and can be written as $U = U^{(3)}U^{(2)}U^{(1)}$

(i)$U^{(1)}$is produced by euler rotation performed at the angle θ_{12}

$U^{(1)} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(ii)$U^{(2)}$is produced by euler rotation performed at the angle θ_{13}introduces a CP phase δ

$U^{(2)} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix}$

(iii)$U^{(3)}$is produced by euler rotation performed at the angle θ_{23}

$U^{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$

where, $c_{ij} = cos\theta_{ij}$and $s_{ij} = sin\theta_{ij}$and $i, j\epsilon\{1, 2, 3\}$. For simplicity, we consider $\delta = 0$.

Therefore, the unitary matrix U is given by

$$U = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{23}s_{13} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23} & -c_{12}s_{23} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \tag{21}$$

For three flavor neutrino oscillation, oscillation probability can be written as

$$Prob(\nu_\alpha \to \nu_\beta) = P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{i>j} U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j} sin^2(\frac{\Delta_{ij}}{4E}L) \tag{22}$$

where α,β $\epsilon\{e, \mu, \tau\}$ and i, j $\epsilon\{1, 2, 3\}$

From different neutrino oscillation experimental results, it is found that in three flavor neutrino oscillation, there are three differnt mixing angles $\theta_{12} = 33.4, \theta_{13} = 8.6^0 and\ \theta_{23} = 49.6^0$ and mass square differences $\Delta_{21} = 7.4 \times 10^{-5}eV^2, \Delta_{31} = 2.457 \times 10^{-3}eV^2, \Delta_{32} = \Delta_{31} - \Delta_{21} \approx \Delta_{31}$. [46, 47]

Let us consider that initially there is an electron flavor neutrino (ν_e) that may transit into another flavor. The probability on transiting from one flavor to another flavor is given by (22)

$$P_{e\mu} = -4\{(U_{e2}U_{\mu 2}U_{e1}U_{\mu 1})sin^2(9.4357\times 10^{-5}\frac{L_{(km)}}{E_{(GeV)}})+(U_{e3}U_{\mu 3}U_{e1}U_{\mu 1})sin^2(3.1329\times 10^{-3}\frac{L_{(km)}}{E_{(GeV)}})+(U_{e3}U_{\mu 3}U_{e2}U_{\mu 2})sin^2(3.1329\times 10^{-3}\frac{L_{(km)}}{E_{(GeV)}})\} \tag{23}$$

$$P_{e\tau} = -4\{(U_{e2}U_{\tau 2}U_{e1}U_{\tau 1})sin^2(9.4357\times 10^{-5}\frac{L_{(km)}}{E_{(GeV)}})+(U_{e3}U_{\tau 3}U_{e1}U_{\tau 1})sin^2(3.1329\times 10^{-3}\frac{L_{(km)}}{E_{(GeV)}})+(U_{e3}U_{\tau 3}U_{e2}U_{\tau 2})sin^2(3.1329\times 10^{-3}\frac{L_{(km)}}{E_{(GeV)}})\} \tag{24}$$

$$P_{ee} = 1 - (P_{e\mu} + P_{e\tau}) \tag{25}$$

The transition and survival probabilities given by (23), (24) and (25) depends upon the mixing angle, mass square difference, baseline length and energy of neutrino so we can discuss the dependency of oscillation probability with $\frac{L}{E}$ in the Figure 2.

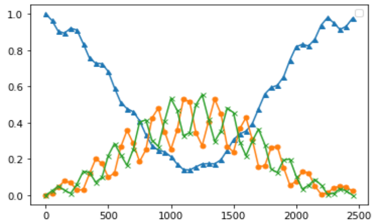

Figure 2: The variation of oscillation probabilities with the parameter $\frac{L}{E}$ is shown. Y-axis represents oscillation probability where P_{ee}(Blue Line) is survival probability, $P_{e\mu}$(Orange Line) and $P_{e\tau}$(Green Line) are transition probabilities.

The intersection of the $P_{ee}, P_{e\mu}\ and\ P_{e\tau}$ is the probability of a neutrino being in any flavor is equal. Although muon flavor and tauon flavor are equally probable at various energy if baseline length is constant unlike electron flavor.

4.2.2 Quantum Entanglement of three flavor of a neutrino

In this section, we will study the correlation among three different flavor of a neutrino. To start with, let us represent the electron and muon flavor at t=0 as the three qubit states, which can be constructed on the basis of occupation number assigned to each flavor. Therefore, the electron, muon and tauon flavor of a neutrino in terms of a qubit can be defined as[41, 48]

$$|\nu_e >= |1 > \otimes|0 > \otimes|0 >= |100 > \tag{26}$$

$$|\nu_\mu >= |0 > \otimes|1 > \otimes|0 >= |010 > \tag{27}$$

$$|\nu_\tau >= |0 > \otimes|0 > \otimes|1 >= |001 > \tag{28}$$

Considering initial state of neutrino in electron flavor. As time passes, it will either oscillate to different flavor or remain in its original flavor. Thus, the time evolution of the composite system for state (26), (27) and (28) can be written as

$$|\nu_e(t) >= A_{ee}|100 > +A_{e\mu}|010 > +A_{e\tau}|001 > \tag{29}$$

where $|A_{ee}|^2 = P_{ee}, |A_{e\mu}|^2 = P_{e\mu}$ and $|A_{e\tau}|^2 = P_{e\tau}$ are the survival and transition probabilities.
Hence, the state in (29) can be described by the density operator as

$$\rho_{e\mu\tau} = |\nu_e(t) >< \nu_e(t)| = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{ee} & 0 & \sqrt{P_{ee}P_{e\mu}} & \sqrt{P_{ee}P_{e\tau}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{P_{ee}P_{e\mu}} & 0 & P_{e\mu} & \sqrt{P_{e\mu}P_{e\tau}} & 0 \\ 0 & 0 & 0 & \sqrt{P_{ee}P_{e\tau}} & 0 & \sqrt{P_{e\mu}P_{e\tau}} & P_{e\tau} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{30}$$

4.2.3 Reduced Density Matrices and Concurrence

This density matrix is dependent on the neutrino oscillation probabilities, hence the quatum corrrelations are dependent on the mixing parameters of neutrino oscillations. Entangled state $\rho_{e\mu\tau}$ is a tri-partite system and it is very complex to deal with 8x8 matrix. Therefore, to study the entanglement between different flavors of a neutrino, the reduced state can be written as

$$Tr_\tau(\rho_{e\mu\tau}) = \rho_{e\mu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & P_{ee} & \sqrt{P_{ee}P_{e\mu}} & 0 \\ 0 & \sqrt{P_{ee}P_{e\mu}} & P_{e\mu} & 0 \\ 0 & 0 & 0 & P_{e\tau} \end{bmatrix} \tag{31}$$

$$Tr_\mu(\rho_{e\mu\tau}) = \rho_{e\tau} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & P_{ee} & \sqrt{P_{ee}P_{e\tau}} & 0 \\ 0 & \sqrt{P_{ee}P_{e\tau}} & P_{e\tau} & 0 \\ 0 & 0 & 0 & P_{e\mu} \end{bmatrix} \tag{32}$$

$$Tr_e(\rho_{e\mu\tau}) = \rho_{\mu\tau} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & P_{e\mu} & \sqrt{P_{e\mu}P_{e\tau}} & 0 \\ 0 & \sqrt{P_{e\mu}P_{e\tau}} & P_{e\tau} & 0 \\ 0 & 0 & 0 & P_{ee} \end{bmatrix} \tag{33}$$

The state described by the density operator $\rho_{e\mu\tau}$ is W state. To define the state expressed in (27) we must calculate concurrence to find the value of tangle where, $tangle = C^2_{e(\mu\tau)} - C^2_{e\mu(\tau)} - C^2_{e\tau(\mu)}$[49] such that, $C_{e(\mu\tau)}, C_{e\mu(\tau)} and C_{e\tau(\mu)}$are the concurrence for their respective reduced state using the formula given in section 2.2 of chapter 2.
Concurrence of the reduced state are calculated as

$$C_{e(\mu\tau)} = 2\sqrt{P_{ee}(P_{e\mu} + P_{e\tau})} \tag{34}$$

$$C_{e\mu(\tau)} = 2\sqrt{P_{ee}P_{e\mu}} \tag{35}$$

$$C_{\mu\tau(e)} = 2\sqrt{P_{e\mu}P_{e\tau}} \tag{36}$$

$$C_{e\tau(\mu)} = 2\sqrt{P_{ee}P_{e\tau}} \tag{37}$$

Using above equations

$$tangle = 4[P_{ee}(P_{e\mu} + P_{e\tau}) - P_{ee}P_{e\mu} - P_{ee}P_{e\tau}] = 0 \tag{38}$$

The calculated value of tangle is 0. Thus, the entangled state is in W state.

4.2.4 Negativity

Partial Transpose of above reduced density matrix as:

$$\rho^{T_\mu}_{e\mu(\tau)} = \begin{bmatrix} 0 & 0 & 0 & \sqrt{P_{ee}P_{e\mu}} \\ 0 & P_{ee} & 0 & 0 \\ 0 & 0 & P_{e\mu} & 0 \\ \sqrt{P_{ee}P_{e\mu}} & 0 & 0 & P_{e\tau} \end{bmatrix} \tag{39}$$

$Tr(\sqrt{(\rho^{T_\mu}_{e\mu(\tau)})^\dagger \rho^{T_\mu}_{e\mu(\tau)}}) = P_{ee} + P_{e\mu} + \sqrt{P_{ee}P_{e\mu} + P^2_{e\tau}}$ Hence, the negativity will be be given as:

$$N(\rho_{e\mu(\tau)}) = -P_{e\tau} + \sqrt{P_{ee}P_{e\mu} + P_{e\tau}^2} \tag{40}$$

Similarly the Negativity of other reduced density matrices as:

$$N(\rho_{e\mu(\tau)}) = -P_{e\mu} + \sqrt{P_{ee}P_{e\tau} + P_{e\mu}^2} \tag{41}$$

$$N(\rho_{\mu\tau(e)}) = -P_{ee} + \sqrt{P_{e\mu}P_{e\tau} + P_{ee}^2} \tag{42}$$

4.2.5 Detection of Genuine Entanglement

Violation of Bell's Inequality confirms the non-locality of a two qubit state. Thus, using the correlation matrix T where $T_{xy} = Tr(\rho(\sigma_x \otimes \sigma_y))$. Bell-CHSH inequality states that $M(\rho) \leq 1$ and $M(\rho) = max(\lambda_i + \lambda_j)$ for λ_i *and* λ_j be the maximum eigenvalues of $T^{\dagger}T$[41, 40].
For reduced state $\rho_{e\mu}$

$$M(\rho_{e\mu}) = 4P_{ee}P_{e\mu} + 1 + P_{e\tau}^2 - 4P_{e\tau} \tag{43}$$

For reduced state $\rho_{e\tau}$

$$M(\rho_{e\tau}) = 4P_{ee}P_{e\tau} + 1 + P_{e\mu}^2 - 4P_{e\mu} \tag{44}$$

For reduced state $\rho_{\mu\tau}$

$$M(\rho_{\mu\tau}) = 4P_{e\tau}P_{e\mu} + 1 + P_{ee}^2 - 4P_{ee} \tag{45}$$

From (43), (44) and (45) it is clear that $M(\rho)$ is dependent on oscillation probability. Thus, we need to find the region where the reduced entangled states violate the Bell-CHSH inequality i.e. $M(\rho) > 1$.

For reduced state $\rho_{e\mu}$

$$4P_{ee}P_{e\mu} + 1 + P_{e\tau}^2 - 4P_{e\tau} > 1$$

$$\Rightarrow P_{e\tau}^2 - 4P_{e\tau} + C_{e\mu}^2 > 0 \tag{46}$$

Solving equation (46) for $P_{e\tau}$, we get

$$2 - \sqrt{4 - C_{e\mu}^2} > P_{e\tau} \geq 0 \tag{47}$$

For reduced state $\rho_{e\tau}$

$$4P_{ee}P_{e\tau} + 1 + P_{e\mu}^2 - 4P_{e\mu} > 1$$

$$\Rightarrow P_{e\mu}^2 - 4P_{e\mu} + C_{e\tau}^2 > 0 \tag{48}$$

Solving equation (48) for $P_{e\mu}$, we get

$$2 - \sqrt{4 - C_{e\tau}^2} > P_{e\mu} \geq 0 \tag{49}$$

For reduced state $\rho_{\mu\tau}$

$$4P_{e\mu}P_{e\tau} + 1 + P_{ee}^2 - 4P_{ee} > 1$$

$$\Rightarrow P_{ee}^2 - 4P_{ee} + C_{\mu\tau}^2 > 0 \tag{50}$$

Solving equation (50) for P_{ee}, we get

$$2 - \sqrt{4 - C_{\mu\tau}^2} > P_{ee} \geq 0 \tag{51}$$

Equation (47), (49) and (51) gives the required range of the oscillation probabilities where, reduced state of the entangled state represent by density operator $\rho_{e\mu\tau}$ violates the Bell-CHSH inequality.
It is difficult to check the genuine entanglement in a tri-partite entanglement. Bell-CHSH does not show the genuine entanglement for a tri-partite state. Although Mermin inequality can be used to check the non-localilty but it also

detects the biseparabe states[50]. Thus, to check the genuine entanglement it is required to use some other method. Violation of Svetlichhny inequality confirms the genuine entanglement in a tri-partite entanglementand Svetlichny operator is given as[51, 52]

$$Sv = |E(ABC)+E(ABC^{'})+E(A'BC)-E(A'BC')+E(AB'C)-E(AB'C')-E(A'B'C)-E(A'B'C')| \leq 4, \tag{52}$$

Where, where E(ABC) represents the expectation value of the product ABC in (52).

According to Svetlichny inequality, expectation value of the svetlichny operator is defined as[53]

$$| < S_v > | \leq 4 \tag{53}$$

In three flavor framework of neutrinos, entanglement is among the different flavors of the neutrinos. Optimization and calculation of the Svetlichny operator is complicated problem. Hence, to check the genuine entanglement the expectation value of the Svetlichny operator is calculated to find upper bound and lower bound of inequality. There are two possibilities whether $\rho_{e\mu(\tau)}$is detected by W_{CHSH}witness operator or not i.e. $\frac{Tr(W_{CHSH}\rho_{\alpha\beta(\gamma)})}{8} < 0$ or not. Here, $\rho_{\alpha\beta(\gamma)}$is the reduced state and $W_{CHSH} = 2I-(A_0 \otimes B_0 + A_0 \otimes B_1 + A_1 \otimes B_0 - A_0 \otimes B_0)$ such that A_0, B_0, A_1, B_1 are hermitian operators. All three reduced states in equation (x,y,z) $\frac{Tr(W_{CHSH}\rho_{\alpha\beta(\gamma)})}{8} > 0$. Hence the state is not detected by the witness operator. For this condition, as per Result 3 given by Anuma Garg, Satyabrata Adhikari [53] the lower bound and upper bound of the expectation value of Svetlichny operator will be given as:

$$S_v^{(l)} = \frac{8(1-p)}{p\lambda_{max}(I_2 \otimes \rho_{\alpha\beta(\gamma)})} \times L \tag{54}$$

$$S_v^{(u)} = \frac{2(1-q)}{p\lambda_k(I_2 \otimes \rho_{\alpha\beta(\gamma)})} \times U \tag{55}$$

where,

$S(\rho_{\alpha\beta(\gamma)}) = r(P - \frac{3}{4}) + (1-r)(\frac{Tr(W_{CHSH}\rho_{\alpha\beta(\gamma)}(\rho_{\alpha\beta(\gamma)}^{T_\beta})}{4N(\rho_{\alpha\beta(\gamma)})})$ is the strength of non-locality, $N(\rho_{\alpha\beta(\gamma)}) = \frac{(Tr(\sqrt{(\rho_{\alpha\beta(\gamma)}^{T_\beta})^\dagger \rho_{\alpha\beta(\gamma)}^{T_\beta}})-1)}{2}$ is the negativity and $r \,\epsilon\, [0,1)$ such that $S(\rho_{\alpha\beta(\gamma)}) > 0$, $P = \frac{3}{4} - \frac{Tr(W_{CHSH}\rho_{\alpha\beta(\gamma)})}{8}$

As per theorem-3a to violate the Svetlichny inequality values of p and q must satisfy:

$$\frac{\sqrt{2}L}{\sqrt{2}L - \lambda_{max}(I_2 \otimes \rho_{\alpha\beta(\gamma)})} < p < \frac{2L}{2L - \lambda_{max}(I_2 \otimes \rho_{\alpha\beta(\gamma)})} \tag{56}$$

and

$$\frac{U}{U - \lambda_k(I_2 \otimes \rho_{\alpha\beta(\gamma)})} < q < \frac{U}{U - \lambda_k(I_2 \otimes \rho_{\alpha\beta(\gamma)})} \tag{57}$$

where,

$$L=\lambda_{min}(\overline{\rho_{\alpha\beta\gamma}(I_2\otimes\rho_{\alpha\beta(\gamma)})})-\frac{\{(S(\rho_{\alpha\beta(\gamma)})-r(P-\frac{3}{4})(N(\rho_{\alpha\beta(\gamma)})\lambda_{max}(\rho^{T_\beta}_{\alpha\beta(\gamma)})\lambda_{max}(\rho_{\alpha\beta\gamma}))\}}{(1-r)\lambda_{min}((\rho^{T_\beta}_{\alpha\beta(\gamma)})^2)} \tag{58}$$

and

$$U=4\lambda_{max}(\overline{\rho_{\alpha\beta\gamma}(I_2\otimes\rho_{\alpha\beta(\gamma)})})-\{\frac{\lambda_{min}(\rho_{\alpha\beta\gamma})\times(\frac{8N(\rho_{\alpha\beta(\gamma)})(S(\rho_{\alpha\beta(\gamma)})-r(P-\frac{3}{4})}{1-r}-Tr((\rho^{T_\beta}_{\alpha\beta(\gamma)})^2))}{\lambda_{max}(W_{CHSH})\lambda_{max}(\rho_{\alpha\beta(\gamma)})}\} \tag{59}$$

Using above equations, we have kept the values of r,p and q constant and the lower bound and upper bound bound is plotted against the parameter $\frac{\Delta_{21}L}{E}$ for the observed mixing angles in 3-flavor neutrino oscillations.

For $\nu_e-\nu_\mu$:

For the reduced state $\rho_{e\mu(\tau)}$in equation (31) calculations are performed according to equation (54) and (55). Before evaluating the value of equation (54) and (55) simplify the equation (58) and (59) as:

$$L_{e\mu(\tau)}=\lambda_{min}(\overline{\rho_{e\mu\tau}(I_2\otimes\rho_{e\mu(\tau)})})-H_{e\mu(\tau)} \tag{60}$$

where $H_{e\mu(\tau)}=\frac{(2(P^2_{ee}+P^2_{e\mu}+P^2_{e\tau})+\sqrt{2}(P^2_{ee}+P^2_{e\mu}-P^2_{e\tau})-\sqrt{2}\sqrt{P_{ee}P_{e\mu}})\lambda_{max}(\rho^{T_\mu}_{e\mu(\tau)})\lambda_{max}(\rho_{e\mu\tau})}{4\lambda_{min}((\rho^{T_\mu}_{e\mu(\tau)})^2)}$

and

$$U_{e\mu(\tau)}=4\lambda_{max}(\overline{\rho_{e\mu\tau}(I_2\otimes\rho_{e\mu(\tau)})})-F_{e\mu(\tau)} \tag{61}$$

where $F_{e\mu(\tau)}=\frac{\lambda_{min}(\rho_{e\mu\tau})\times\{3(P^2_{ee}+P^2_{e\mu}+P^2_{e\tau})+2\sqrt{2}(P^2_{ee}+P^2_{e\mu}-P^2_{e\tau})-2\sqrt{2}\sqrt{P_{ee}P_{e\mu}}-2P_{ee}P_{e\mu}\}}{4.82843\lambda_{max}(\rho_{e\mu(\tau)})}$

The values of equation (60) and (61) are used to calculate lower bound and upper bound of the svetlichny operator as:

$$S^{(l)}_v=\frac{8(1-p)}{p\lambda_{max}(I_2\otimes\rho_{e\mu(\tau)})}\times L_{e\mu(\tau)} \tag{62}$$

$$S^{(u)}_v=\frac{2(1-q)}{p\lambda_k(I_2\otimes\rho_{e\mu(\tau)})}\times U_{e\mu(\tau)} \tag{63}$$

From equation (62) and equation (63) the lower bound and upper bound of expectation value of Svetlichny operator are dependent on oscillation probabilities i.e. oscillation parameters. Hence the plot of both lower and upper bound is shown in Figure 3 as the value of $\frac{\Delta_{21}L}{E}$ varies.

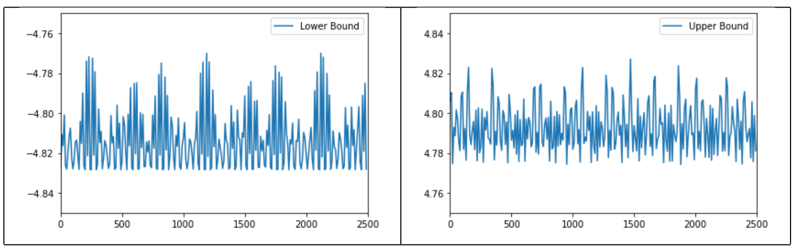

Figure 3: Lower bound and upper bound of the expectation value of svetlichny operator v/s $\frac{\Delta_{21}L}{E}$ for $\rho_{e\mu(\tau)}$reduced sentangled state at mixing angles $\theta_{23}=49.6^0$, $\theta_{13}=8.6^0$, $\theta_{12}=33.4^0$

For $\nu_e - \nu_\tau$:

For the reduced state $\rho_{e\tau(\mu)}$in equation (32) calculations are performed according to equation (54) and (55). Before evaluating the value of equation (54) and (55) simplify the equation (58) and (59) as:

$$L_{e\tau(\mu)} = \lambda_{min}(\overline{\rho_{e\mu\tau}(I_2 \otimes \rho_{e\tau(\mu)})}) - H_{e\tau(\mu)} \tag{64}$$

where $H_{e\tau(\mu)} = \frac{(2(P_{ee}^2+P_{e\mu}^2+P_{e\tau}^2)+\sqrt{2}(P_{ee}^2-P_{e\mu}^2+P_{e\tau}^2)-\sqrt{2}\sqrt{P_{ee}P_{e\tau}})\lambda_{max}(\rho^{T_\tau}_{e\tau(\mu)})\lambda_{max}(\rho_{e\mu\tau})}{4\lambda_{min}((\rho^{T_\tau}_{e\tau(\mu)})^2)}$

and

$$U_{e\tau(\mu)} = 4\lambda_{max}(\overline{\rho_{e\mu\tau}(I_2 \otimes \rho_{e\tau(\mu)})}) - F_{e\tau(\mu)} \tag{65}$$

where $F_{e\tau(\mu)} = \frac{\lambda_{min}(\rho_{e\mu\tau})\times\{3(P_{ee}^2+P_{e\mu}^2+P_{e\tau}^2)+\sqrt{2}(P_{ee}^2-P_{e\mu}^2+P_{e\tau}^2)-\sqrt{2}\sqrt{P_{ee}P_{e\tau}}-2P_{ee}P_{e\tau}\}}{4.82843\lambda_{max}(\rho_{e\tau(\mu)})}$

The values of equation (64) and (65) are used to calculate lower bound and upper bound of the svetlichny operator as:

$$S_v^{(l)} = \frac{8(1-p)}{p\lambda_{max}(I_2 \otimes \rho_{e\tau(\mu)})} \times L_{e\tau(\mu)} \tag{66}$$

$$S_v^{(u)} = \frac{2(1-q)}{p\lambda_k(I_2 \otimes \rho_{e\tau(\mu)})} \times U_{e\tau(\mu)} \tag{67}$$

From equation (66) and equation (67) the lower bound and upper bound of expectation value of Svetlichny operator are dependent on oscillation probabilities i.e. oscillation parameters. Hence the plot of both lower and upper bound is shown in Figure 4 as the value of $\frac{\Delta_{21}L}{E}$ varies.

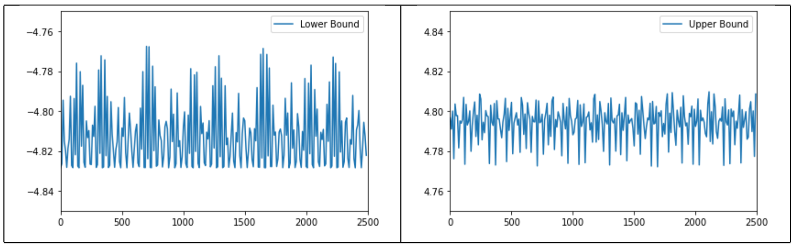

Figure 4: Lower bound and upper bound of the expectation value of svetlichny operator v/s $\frac{\Delta_{21}L}{E}$ for $\rho_{e\tau(\mu)}$reduced sentangled state at mixing angles $\theta_{23} = 49.6^0$, $\theta_{13} = 8.6^0$, $\theta_{12} = 33.4^0$

For $\nu_\mu - \nu_\tau$:

For the reduced state $\rho_{\mu\tau(e)}$in equation (33) calculations are performed according to equation (54) and (55). Here, simplifying the equation (58) and (59) as:

$$L_{\mu\tau(e)} = \lambda_{min}(\overline{\rho_{e\mu\tau}(I_2 \otimes \rho_{\mu\tau(e)})}) - H_{\mu\tau(e)} \tag{68}$$

where $H_{\mu\tau(e)} = \frac{(2(P_{ee}^2+P_{e\mu}^2+P_{e\tau}^2)+\sqrt{2}(-P_{ee}^2+P_{e\mu}^2+P_{e\tau}^2)-\sqrt{2}\sqrt{P_{e\mu}P_{e\tau}})\lambda_{max}(\rho^{T_\tau}_{\mu\tau(e)})\lambda_{max}(\rho_{e\mu\tau})}{4\lambda_{min}((\rho^{T_\tau}_{\mu\tau(e)})^2)}$

and

$$U_{\mu\tau(e)} = 4\lambda_{max}(\overline{\rho_{e\mu\tau}(I_2 \otimes \rho_{\mu\tau(e)})}) - F_{\mu\tau(e)} \tag{69}$$

where $F_{\mu\tau(e)} = \frac{\lambda_{min}(\rho_{e\mu\tau})\times\{3(P_{ee}^2+P_{e\mu}^2+P_{e\tau}^2)+2\sqrt{2}(-P_{ee}^2+P_{e\mu}^2+P_{e\tau}^2)-2\sqrt{2}\sqrt{P_{e\mu}P_{e\tau}}-2P_{e\mu}P_{e\tau}\}}{4.82843\lambda_{max}(\rho_{\mu\tau(e)})}$

The values of equation (68) and (69) are used to calculate lower bound and upper bound of the svetlichny operator as:

$$S_v^{(l)} = \frac{8(1-p)}{p\lambda_{max}(I_2 \otimes \rho_{\mu\tau(e)})} \times L_{\mu\tau(e)} \tag{70}$$

$$S_v^{(u)} = \frac{2(1-q)}{p\lambda_k(I_2 \otimes \rho_{\mu\tau(e)})} \times U_{\mu\tau(e)} \tag{71}$$

From equation (70) and equation (71) the lower bound and upper bound of expectation value of Svetlichny operator are dependent on oscillation probabilities i.e. oscillation parameters. Hence the plot of both lower and upper bound is shown in Figure 5 as the value of $\frac{\Delta_{21}L}{E}$ varies.

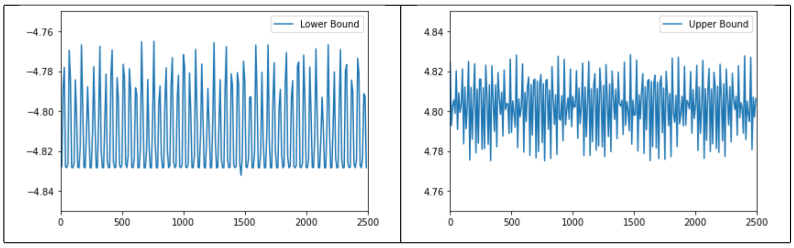

Figure 5: Lower bound and upper bound of the expectation value of svetlichny operator v/s $\frac{\Delta_{21}L}{E}$ for $\rho_{\mu\tau(e)}$reduced sentangled state at mixing angles $\theta_{23} = 49.6^0$, $\theta_{13} = 8.6^0$, $\theta_{12} = 33.4^0$

For all the Figure(3) to Figure(5) by varying the value of $\frac{\Delta_{21}L}{E}$, the lower bound of the expectation value of Svetlichny operator is less than 4 while the upper bound of the expectation value of Svetlichny operator is greater than 4. Svetlichny inequality states that

$$|<S_v>| \leq 4 \tag{72}$$

Hence, the Svetlichny inequality is violated by the reduced entangled state of the 3-flavor neutrino oscillations.

Conclusion

In this study, we have seen that the flavor of a neutrino is entangled and studied the neutrino oscillation and its parameters. To study the entanglement of flavor of neutrinos and studied negativity by comparing it with structured negativity for two flavor neutrino. The entanglement is intra particle and studied as two qubit system for two flavor neutrinos. By considering initial flavor of the neutrino to be electron flavor the relation of the oscillation parameter $\frac{L}{E}$ is understood as very crucial to observe the entanglement.
Physical system such as binary system of neutron starts, supernovae, white dwarf etc are having high density. When neutrinos travel through high density matter, their potential will increase hence, the energy of the neutrino will be very high therefore, the entanglement will start to vanish.
For three flavor neutrino, the density matrix of tripartite entangled state is dependent on oscillation probability i.e. oscillation parameter hence the relation of the oscillation parameter $\frac{L}{E}$ is understood as very crucial to observe the entanglement. In three flavor neutrino we defined the necessary crteria for the entanglement for the reduced bipartite state. For the reduced state, Bell CHSH inequality is violated for a particular region. Lower bound and upper bound of expectation value of svetlichney operator for three flavor neutrino also violated the svetlichney inequality concluded that three flavor neutrino is genuinely entangled.

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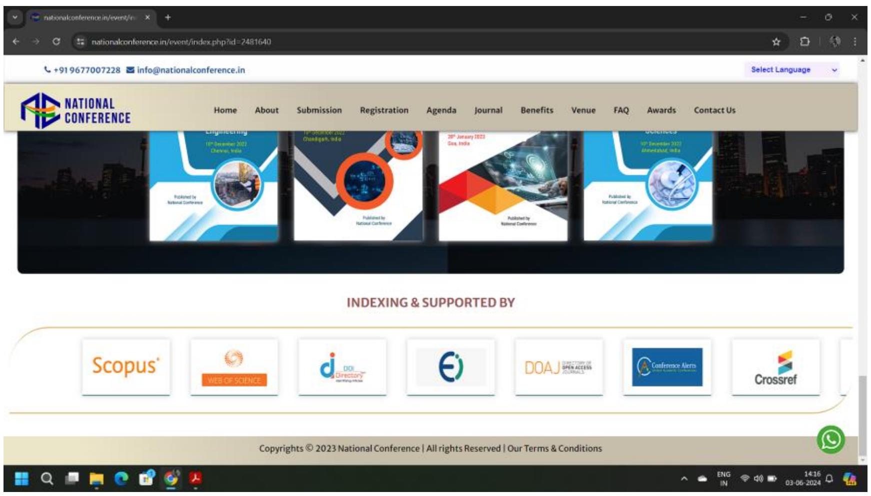

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