HIERARCHICAL CLUSTERING OF PICTURE FUZZY RELATION

A DISSERTATION

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE AWARD OF THE DEGREE OF

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IN

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DECLARATION

We, Ranjeeta Kumari, 2K21/MSCMAT/39 & Shivam Sharma, 2K21/MSCMAT/50 students of M.Sc. Mathematics, hereby declare that the project Dissertation titled "Hierarchical Clustering of Picture Fuzzy Relation " which is submitted by us to the Department of Applied Mathematics Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Science, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

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We hereby certify that the Project Dissertation titled " HIERARCHICAL CLUSTERING OF PICTURE FUZZY RELATION " which is submitted by Ranjeeta Kumari, Roll No. 2K21/MSCMAT/39 & Shivam Sharma, Roll No. 2K21/MSCMAT/50 [Department of Applied Mathematics], Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Science, is a record of the project work carried out by the students under my supervision. To the best of my knowledge this work has not been submitted in part or full for any Degree or Diploma to this University or elsewhere.

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ABSTRACT

The paper aims to find the $(\tilde{\alpha},\tilde{\beta}\tilde{\gamma})$ - cuts of picture fuzzy relation and apply it to find its hierarchical clustering. we studied the previous results of Intuitionistic fuzzy relation and its $(\tilde{\alpha},\tilde{\beta},\tilde{\gamma})$ - cuts. We propose a technique of finding $\tilde{\alpha}$ - cuts of picture fuzzy sets using results of intuitionistic fuzzy set. Membership of PFS mainly deals with positive, negative and neutral membership while IFS only deals with positive and negative membership. In this paper, we attempt to study picture fuzzy set more deeply and providing the more results in picture fuzzy relation that contains $\tilde{\alpha}$ - cuts of picture fuzzy set. Further we studied about the various applications of Intuitionistic fuzzy relation and picture fuzzy relation. Intuitionistic fuzzy relation deals with the application on the comparision of distant measures using normalised hamming distance measure. The two applications of picture fuzzy relation is discussed. First application deals with the determination of higher study area selection on the basis of composition of picture fuzzy relations. We found picture fuzzy relation on topic knowledge, study area and the skills. Then, we applied picture fuzzy transformation to obtain new picture fuzzy relation. After defuzzifying the PFR matrix we found the suitable study areas for students to continue higher studies. Second application picture-fuzzy medical diagnosis deals with the medical diagnosis of an illness and patients suffering from a particular disease is identified with the help of composition of picture fuzzy relation.

Keywords : Fuzzy set, Picture fuzzy set, Picture fuzzy relation, Intuitionistic Fuzzy set, Hierarchical Clustering

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CHAPTER-1 INTRODUCTION

A fuzzy set is a group of items with a variety of membership grades. Such a set is defined by a membership (characteristic) function that assigns each item a membership grade between zero and one. Inclusion, union, intersection, complement, relation, convexity, and other concepts are applied to these sets, and different aspects of these concepts are developed in the background of FS. Without requiring the fuzzy sets to be disjoint, a separation theorem for convex fuzzy sets is specifically shown.

In agreement with the dual condition, an element either belong to the set or it does not in standard set theory, which examines membership of elements in binary terms. On the contrary hand,the participation of an element in a set can be evaluated over time thanks to fuzzy set theory. This is expressed by a function of membership having a value within the real value range [0, 1]. Subsequently the characteristic membership functions of fuzzy sets are special cases of functions of classical sets, which can only take values of 0 or 1, fuzzy sets generalise classical sets. Classical bivalent sets are typically referred to as crisp sets in fuzzy set theory. Gradually introducing membership gives us an unlimited expansion to the fundamental ideas in logic and set theory in a way that seems extremely logical. While the logic of fluid-conducting valves is contrasted with boolean logic. A valve can have an endless number of positions between entirely closed and completely open. Language-wise, we identify these postures as "partially closed, almost closed, nearly closed, largely open, etc." As a result, a logical system based on "valve openings" may be able to describe how humans interpret information.

Table 1. Membership values

Linguistic Term	Membership Value
Completely agree	1
Very strongly agree	0.9
Strongly agree	0.7
Agree	0.5
Strongly disagree	0.3
Very Strongly disagree	0.1
Completely disagree	0

A core principle of fuzzy set theory is to offer a rigid mathematical framework so that these ambiguous conceptual occurrences in decision-making can be precisely and systematically examined. It enables a smooth shift from the world of strict, quantitative, and precise phenomena to the world of hazy, qualitative, and imprecise notions. having the help of this theory, it is possible to define imprecision in terms of "fuzziness," a term having a wide range of interpretations, including ambiguity, a collective problem, reasoning with "ball-park" figures, abstraction, and a trait of poorly constructed problems. There is a certain aspect. It is not the same as likelihood, despite the fact that it can be used to define "fuzziness" in very specific situations. Additionally, there is no attempt to maintain continuity for variables and functions that are really discontinuous. Again, in this case, fuzziness may be thought of as offering a topological space in a limited sense. Therefore, one may look at the characteristics of fuzzy sets without attempting to limit the definition of fuzziness to other existing, well-known mathematical structures.

In mathematics, fuzzy sets are whose elements have degree of membership. Mathematically speaking, fuzzy set $F = \{(a, \mu_F (a)) | a \in X\}$ where X is the universal set, $0 \leq \mu_F (a) \leq 1$ depicting the membership value of x. Lofti A. Zedeh introduced fuzzy sets in 1965. The representation of non-membership degree of x in X is $1 - \mu_A(a)$.

Intuitionistic fuzzy set (IFS), which deals with membership degree and non-membership degree, was discovered by Atanasov after fuzzy set. IFS is described as

A ={x, μ_A (x) , ν_A (x) |x ∈ X}, where μ_A, ν_A : X → [0, 1] such that

$0 \leq \mu_A (x) + \nu_A (x) \leq 1$. According to IFS theory, the degree of non-membership is an independent degree that must satisfy the only constraint that it should be less than or equal to 1 - μ_A (x). Various results were introduced for IFS that includes $\tilde{\alpha}$ - cuts or (α, β) - cuts of IFS. The $\tilde{\alpha}$ -cuts of interval valued fuzzy relation for which relation should be an interval valued similarity relation, was developed by Mathematician Guh et al. IVFR is a mapping R : M × N → [0, 1] where μ_R (x, y) = μ_R (x, y) , μ_R (x, y) ∈ M × N. Guh explains that the α cut derived from the resolution form of the interval valued similarity relation is interval valued for $\tilde{\alpha} = (\underline{\tilde{\alpha}}, \tilde{\alpha})$, $0 \leq \underline{\tilde{\alpha}} \leq \tilde{\alpha} \leq 1$ and similarly, for n number of $\tilde{\alpha}$ - cuts

But the same may fails to hold for $\tilde{\alpha}$ - cuts possess the form (0.5,0.9), (0.6,0.7), (1,1) then, although 0.5 < 0.6 < 1 but 0.9/≤ 0.7 ≤ 1. In this paper we take the concept of go Guh et al and improved and extended to PFS. PFS are continuation of the fuzzy sets and IFS. PFS F = {(x, μ_F (x) , η_F (x) , ν_F (x)) | x ∈ X} where μ_F (x) is positive membership, η_F (x) is neutral membership and ν_F (x) is negative membership such that for this paper we find the ($\tilde{\alpha}$, $\tilde{\beta}$, ?) - cuts of picture fuzzy relation. Further we applied it to hierarchical clustering using its resolution matrix.

CHAPTER 2 INTUITIONISTIC FUZZY RELATION

The fuzzy set theory states that a single value among zero and one represents an element's participation in a fuzzy set. The degree of absence from membership of a component in a fuzzy set may not necessarily be equal to 1 minus the membership degree, though, as there may be some hesitation degree. Because of this, Atanassov (1983, 1986) introduced intuitionistic fuzzy sets (IFS), an extension of fuzzy sets that take into account the level of uncertainty defined as the hesitation margins (which is defined as 1 minus the total number of membership and non-membership degrees, respectively). The concept of defining fuzzy sets that are intuitive as extended fuzzy sets is highly intriguing and useful in many application domains.

De et al. (2001) provided a three-step approach to medical diagnosis using intuitionistic fuzzy sets, including symptom determination, formulation of medical knowledge using intuitionistic fuzzy relations and structure of intuitionistic fuzzy relations for diagnosis. Since there is a good chance that there will be a non-null hesitation part present each time an unknown object is being evaluated, intuitionistic fuzzy sets are an effective tool for acting out real-world challenges including market research, the marketing of new products, financial services, the negotiating process, psychological studies, etc. (Szmidt and Kacprzyk, 1997, 2001). Rigid research based on the theory and applications of intuitionistic fuzzy sets was conducted by Atanassov (1999, 2012). IFS is used in numerous situations where distance measurements are used. In this section we will see the IFS which is defined by Bustice and Burillo. we will see the ($\tilde{\alpha}$, $\tilde{\beta}$, ⍰)- cuts for an Intuitionistic fuzzy relation and algorithm used for it and further Hierarchical Clustering of ($\tilde{\alpha}$, $\tilde{\beta}$, ⍰)- cuts of IFR using its resolution matrix.

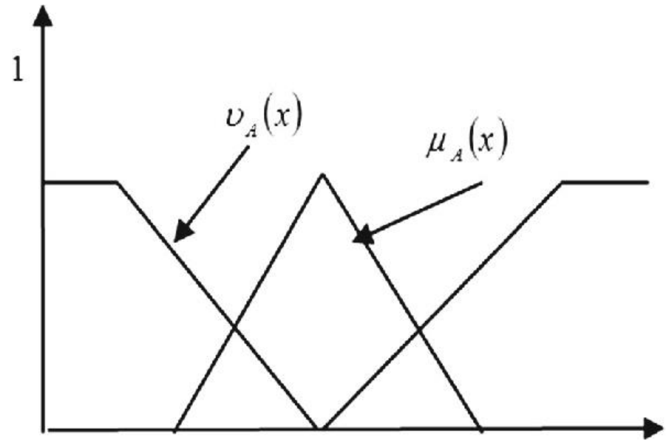


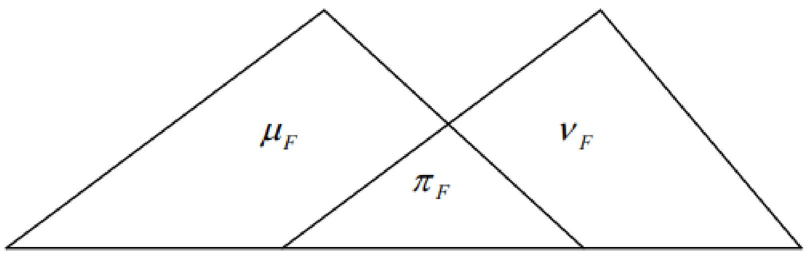
Fig. Intuitionistic fuzzy sets with membership and non-membership function

Definition 1 (Intuitionistic fuzzy relation (IFR))

An IFR R on the two sets M and N is an intuitionistic fuzzy subset of M × N defined by

R ={(a, b), μ_A (a, b) , ν_A (a, b) | a ∈ M, b ∈ N}, where μ_R, ν_R : M × N → [0, 1] such that

$0 \leq \mu_R (a, b) + \nu_R (a, b) \leq 1$, ∀ (a, b) ∈M × N



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Definition 2 (Intuitionistic Fuzzy Proximity Relation)

If an Intuitionistic fuzzy relation R on X has the qualities of

Reflexivity: $\tilde{\mu}_R(a, a) = 1$, $\tilde{\nu}_R(a, a) = 0$, $\forall$ a $\in$ M and

Symmetry: $\tilde{\mu}_R(a, b) = \tilde{\mu}_R(b, a)$, $\tilde{\nu}_R(a, b) = \tilde{\nu}_R(b, a)$, $\forall$ a $\in$ M, b $\in$ N.

Definition 3 (Intuitionistic fuzzy (max − min) similarity relation)

An intuitionistic fuzzy proximity relation R on M is referred to as intuitionistic fuzzy (max–min) similarity relation if it holds the below condition:

$$\mu_R(a, b) \geq \{ \min\{\mu_R(a, c), \mu_R(c, b)\}\},$$

$$\nu_R(a, b) \leq \{ \min\{\nu_R(a, c), \nu_R(c, b)\}\}.$$

Definition 4 (Interval valued fuzzy relation (IVFR))

An Interval valued fuzzy relation R on M and N is a mapping R: M ×N → [0, 1], characterised as a fuzzy subset of M × N, with an interval-valued membership degree.

$$\mu_R(a, b) = [\mu_R(a, b), \mu_R(a, b)], \forall (a, b) \in M \times N.$$

Example 1 The (min − max) transitive interval valued proximity relation matrix is shown below.

$[1\ (0.92, 1)\ (0.67, 0.92)\ (0.48, 0.92)\ (0.48, 0.83)\ (0.92, 1)\ 1\ (0.67, 0.92)\ (0.48, 0.92)\ (0.48, 0.83)\ (0.$

R^0 is transitive, symmetric and reflexive. IVFR according to the above matrix. The $\tilde{\alpha}$ cuts are (0.48,0.83), (0.48,0.92), (0.59,0.83), (0.67,0.92) and (0.92,1). The inequality is satisfied by the lower values of $\tilde{\alpha}$ cuts, but the higher values of $\tilde{\alpha}$ cuts do not fulfill because $0.83 < 0.92 \not\leq 0.83 < 0.92 < 1$.

Algorithm to determine $\tilde{\alpha}$ cuts of IVFR.

Algorithm 1

1. To find out all the $\tilde{\alpha}$ cuts of IVFR using methods suggested by Guh et al and let it be represented as a set S.

2. In the set S we will select the element with lower membership value which has maximum value. Then from all the pairs of upper membership value we will select the minimum value. In this way we get first $\tilde{\alpha}$ cuts of IVFR. Reducing the set S by repeating the same procedure.

3. Following above procedure for rest of the element until every element searched.

Using above algorithm, we have extracted the $\tilde{\alpha}$ cuts of set S. The extracted $\tilde{\alpha}$ cuts are given as (1,1), (0.92,1), (0.67,0.92), (0.59,0.83), (0.48,0.83).

Resolution form of the similarity relation can be represented as

R^1 = 1 [1 0 1 0 0 1 0 0 0 1 0 0 0 0 1] + (0.92,1)

[1 1 1 0 0 1 0 0 0 1 0 0 0 0 1]

+ (0.67,0.92)[1 1 1 1 1 1 0 0 0 1 0 0 0 0 1] + (0.59, 0.83)

[1 1 1 1 1 1 0 0 0 1 0 0 0 1 1]

+ (0.48, 0.83) [1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]

Definition 5 ($(\tilde{\alpha}, \tilde{\beta})$-cut of IFS)

The following is the definition of an IFS A's $(\tilde{\alpha}, \tilde{\beta})$ cut:

$A(\tilde{\alpha}, \tilde{\beta}) = \{ a \in X \mid \mu_A(a) \geq \tilde{\alpha}, \nu_A(a) \leq \tilde{\beta} \}$

The $(\tilde{\alpha}, \tilde{\beta})-$ cut for an intuitionistic fuzzy (max-min) similarity relation is determined via an algorithm.

Algorithm 2

1. Select the highest membership value μ_i, say μ_i^* from the pairs of (μ_i, ν_i) in IFR matrix,

2. After that, choose the pair with the highest non-membership value out of all those that could exist. We will get the initial $(\tilde{\alpha}, \tilde{\beta})$ - cuts of Intuitionistic Fuzzy Relation.

3. Strike-off the chosen value from matrix of Intuitionistic fuzzy relation. Then continuing for remaining elements (μ_i, ν_i) of the matrix of IFR.

We can also commence with a non-membership function in place of the membership function. Select the element with the lowest non-membership value for each entry in matrix R, and for this value, select the pair with the lowest membership value. Repeat the procedure for each IFR part until each one has been examined.

Example 2 Considering the following matrix of order 9 :

$B^0 =$

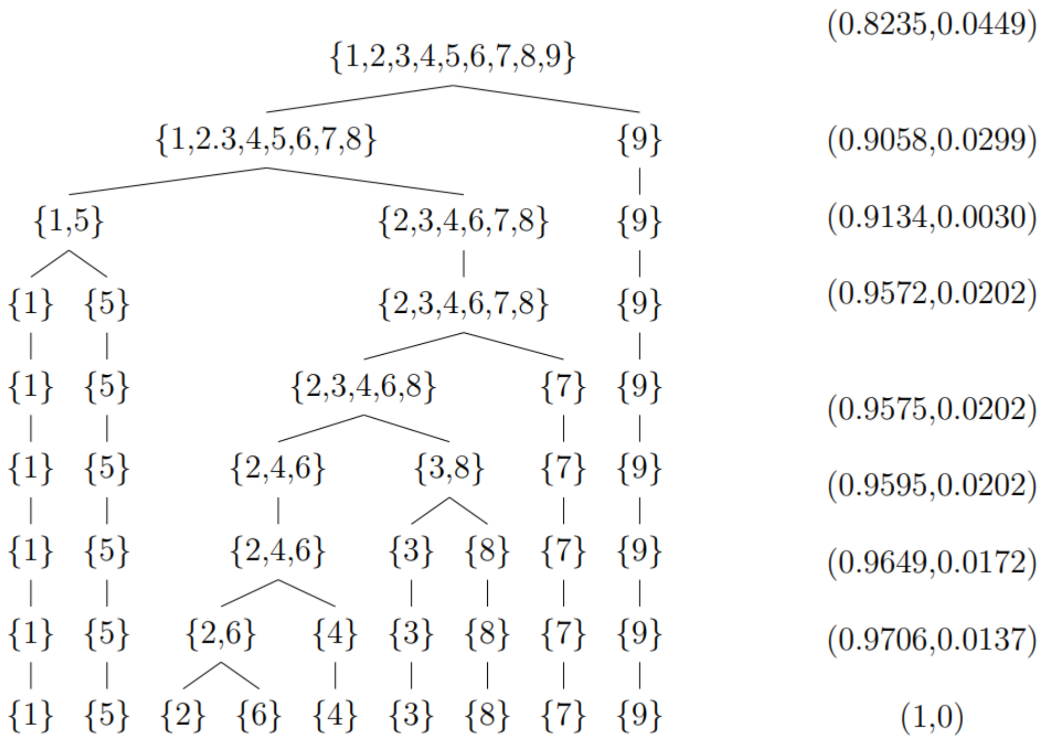
$$\begin{pmatrix}
(1,0) & (0.81472, 0.12874) & (0.90579, 0.02987) & (0.12699, 0.82956) & (0.91338, 0.00298) & (0.63236, 0.1613) & (0.09754, 0.34434) & (0.27849, 0.55232) & (0.54688, 0.36032) \\
(0.81472, 0.12874) & (1,0) & (0.95751, 0.00794) & (0.96489, 0.01719) & (0.15761, 0.37536) & (0.97059, 0.01901) & (0.95718, 0.03037) & (0.48538, 0.38838) & (0.80028, 0.05513) \\
(0.90578, 0.02986) & (0.95751, 0.00794) & (1,0) & (0.14189, 0.58326) & (0.42176, 0.37880) & (0.91574, 0.01370) & (0.79221, 0.02473) & (0.95949, 0.02019) & (0.65574, 0.33040) \\
(0.12699, 0.82956) & (0.96490, 0.01720) & (0.14189, 0.58326) & (1,0) & (0.03571, 0.32823) & (0.84913, 0.08830) & (0.93399, 0.01477) & (0.67874, 0.24136) & (0.75774, 0.06180) \\
(0.91338, 0.00298) & (0.15761, 0.37536) & (0.42176, 0.37880) & (0.03571, 0.32823) & (1,0) & (0.74313, 0.12996) & (0.39223, 0.42488) & (0.65549, 0.30640) & (0.17119, 0.79507) \\
(0.63236, 0.1613) & (0.97059, 0.01901) & (0.91574, 0.01370) & (0.84913, 0.0883) & (0.74313, 0.12996) & (1,0) & (0.70605, 0.16086) & (0.03183, 0.13421) & (0.27692, 0.10795) \\
(0.09754, 0.34434) & (0.95717, 0.03038) & (0.79221, 0.02473) & (0.93393, 0.01477) & (0.39223, 0.42488) & (0.70605, 0.16086) & (1,0) & (0.04617, 0.24562) & (0.09713, 0.75906) \\
(0.27850, 0.55232) & (0.48538, 0.38838) & (0.95949, 0.02019) & (0.67874, 0.24136) & (0.65549, 0.30640) & (0.03183, 0.13421) & (0.04617, 0.24532) & (1,0) & (0.82346, 0.04489) \\
(0.54688, 0.36032) & (0.80028, 0.05513) & (0.65574, 0.33040) & (0.75774, 0.06180) & (0.17119, 0.79507) & (0.27692, 0.10795) & (0.09713, 0.75906) & (0.82346, 0.04489) & (1,0)
\end{pmatrix}$$

B^3 =

$$\begin{pmatrix}
(1,0) & (0.906, 0.030) & (0.906, 0.030) & (0.906, 0.030) & (0.913, 0.003) & (0.906, 0.030) & (0.906, 0.030) & (0.906, 0.030) & (0.823, 0.045) \\
(0.906, 0.030) & (1,0) & (0.958, 0.008) & (0.965, 0.017) & (0.906, 0.030) & (0.971, 0.014) & (0.957, 0.017) & (0.958, 0.020) & (0.824, 0.045) \\
(0.906, 0.030) & (0.958, 0.008) & (1,0) & (0.958, 0.017) & (0.906, 0.030) & (0.958, 0.014) & (0.957, 0.017) & (0.960, 0.020) & (0.824, 0.045) \\
(0.906, 0.030) & (0.965, 0.017) & (0.968, 0.017) & (1,0) & (0.906, 0.030) & (0.965, 0.017) & (0.957, 0.015) & (0.958, 0.020) & (0.824, 0.045) \\
(0.913, 0.003) & (0.906, 0.030) & (0.906, 0.030) & (0.906, 0.030) & (1,0) & (0.906, 0.030) & (0.906, 0.030) & (0.906, 0.030) & (0.823, 0.045) \\
(0.906, 0.030) & (0.970, 0.014) & (0.958, 0.014) & (0.965, 0.017) & (0.906, 0.030) & (1,0) & (0.957, 0.017) & (0.958, 0.020) & (0.824, 0.045) \\
(0.906, 0.030) & (0.957, 0.017) & (0.957, 0.017) & (0.957, 0.015) & (0.906, 0.030) & (0.957, 0.017) & (1,0) & (0.957, 0.020) & (0.824, 0.045) \\
(0.906, 0.030) & (0.958, 0.020) & (0.960, 0.020) & (0.960, 0.020) & (0.906, 0.030) & (0.958, 0.020) & (0.957, 0.020) & (1,0) & (0.824, 0.045) \\
(0.824, 0.045) & (0.824, 0.045) & (0.824, 0.045) & (0.824, 0.045) & (0.824, 0.045) & (0.824, 0.045) & (0.824, 0.045) & (0.824, 0.045) & (1,0)
\end{pmatrix}$$

($\tilde{\alpha}$, $\tilde{\beta}$)-cuts of $\tilde{B}^3$ are (1,0), (0.971,0.014), (0.965,0.0.020), (0.960,0.020), (0.958,0.020), (0.957,0.020), (0.913,0.003), (0.906,0.030), (0.824,0.045)

Hierarchical Clustering of IFR of $\tilde{B}^3$



Hierarchical Clustering

The technique of organising things in a hierarchy is called hierarchical clustering. Depending on how much they resemble one another and how much they differ from objects in other groups, it divides objects into groups.A dendrogram is a type of hierarchical tree that visually depicts clusters.

Two major benifits of hierarchical clustering are as follows:

1. The number of clusters does not need to be predetermined. Instead, the dendrogram can be appropriately split to obtain the required number of clusters.

2. It is straightforward to summarise and arrange information into a hierarchy using dendrograms. It is simple to study and interpret clusters using dendrograms.

Applications of Hierarchical clustering:

Hierarchical clustering has many practical uses. They consist of:

1. **Bioinformatics:** Animals are categorised based on their biological characteristics in bioinformatics in order to reassemble phylogenetic trees.
2. **Business:** creating a pay hierarchy for staff or segmenting consumer bases.
3. **Image processing:** Grouping handwritten characters in text recognition depending on how similar their shapes are in image processing.
4. **Information Retrieval:** Sorting search results into categories based on the search term.

Types of hierarchical clustering:

Two main categories of hierarchical clustering exist:

1. **Agglomerative:** The basic assumption is that each object belongs to a certain cluster. The clusters are then gradually integrated using a certain procedure until just a single cluster is left. A cluster including every component will be generated at the conclusion of the cluster combining process.
2. **Divisive:** The Divisive strategy competes with the Agglomerative strategy. At initially, it is assumed that each item is a part of a single cluster. The division process is then applied incrementally until each object forms a distinct cluster.

The technique for splitting or dividing a cluster is carried out in accordance with specific rules that call for the largest distance possible between cluster members. Agglomerative clustering, as opposed to Divisive clustering, is typically the preferable approach. Agglomerative clustering techniques will be the main focus of the example below because they are the most common and straightforward to utilize.

Steps in hierarchical clustering:

In order to create new clusters, HC employs a distance/similarity metric.

The steps of agglomerative clustering are summarised below:

1. Determine the proximity matrix using a specific distance metric.
2. Each data point is allocated to a cluster.
3. Unite the clusters hinge on a cluster similarity metric.
4. Reform the distance matrix.
5. Till there is just one cluster remaining, repeat Steps 3 and 4.

Computing a proximity matrix:

The algorithm's initial step is to produce a distance matrix. Applying a distance function between each pair of objects yields the values of the matrix. For this process, the Euclidean distance function is frequently utilised. For a data collection of elements, the proximity matrix will have the following structure. The values of the distance between p_i and p_j are characterized by $d(p_i,p_j)$.

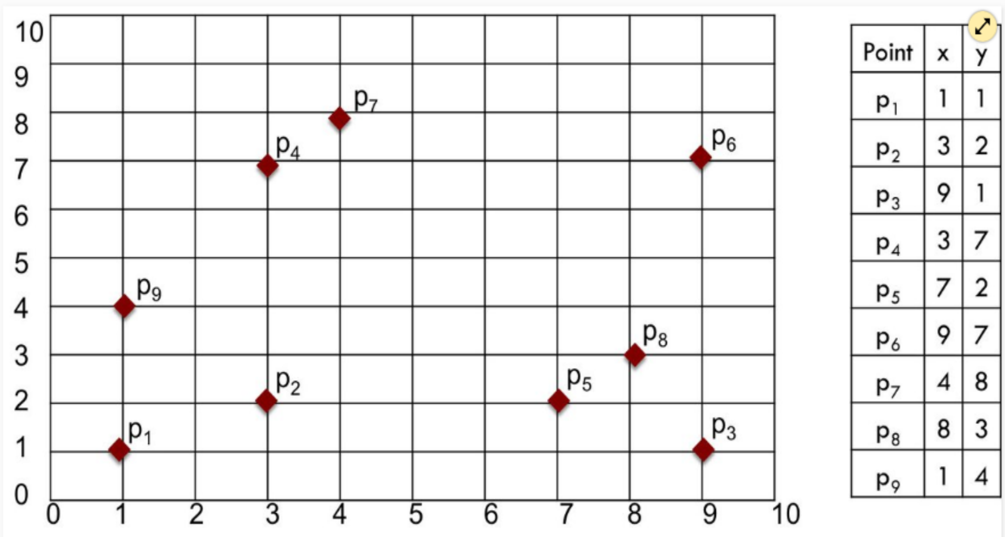
	p_1	p_2	p_3	…	p_n
p_1	$d(p_1,p_1)$	$d(p_1,p_2)$	$d(p_1,p_3)$	…	$d(p_1,p_n)$
p_2	$d(p_2,p_1)$	$d(p_2,p_2)$	$d(p_2,p_3)$	…	$d(p_2,p_n)$

p_3	$d(p_3,p_1)$	$d(p_3,p_2)$	$d(p_3,p_3)$	…	$d(p_3,p_n)$
…	…	…	…	…	…
p_n	$d(p_n,p_1)$	$d(p_n,p_2)$	$d(p_n,p_3)$	…	$d(p_n,p_n)$

Comparability of Clusters:

How to compute the distance between clusters and update the proximity matrix is the primary issue in hierarchical clustering. There are numerous methods for responding to the question. Each strategy has benefits and drawbacks. The decision will be made based on the density of the data points, the shape of the clusters, and whether or not there is noise in the data set.

The techniques and decisions will be demonstrated using a numerical example. Figure 1 shows the little sample data set we'll be using, which consists of just nine two-dimensional points.



Point	x	y
p_1	1	1
p_2	3	2
p_3	9	1
p_4	3	7
p_5	7	2
p_6	9	7
p_7	4	8
p_8	8	3
p_9	1	4

Figure 1: Sample Data

Assume that the sample data set contains two clusters, as depicted in Figure 2. The distance between the clusters can be calculated in a variety of ways. Below is a list of common techniques.

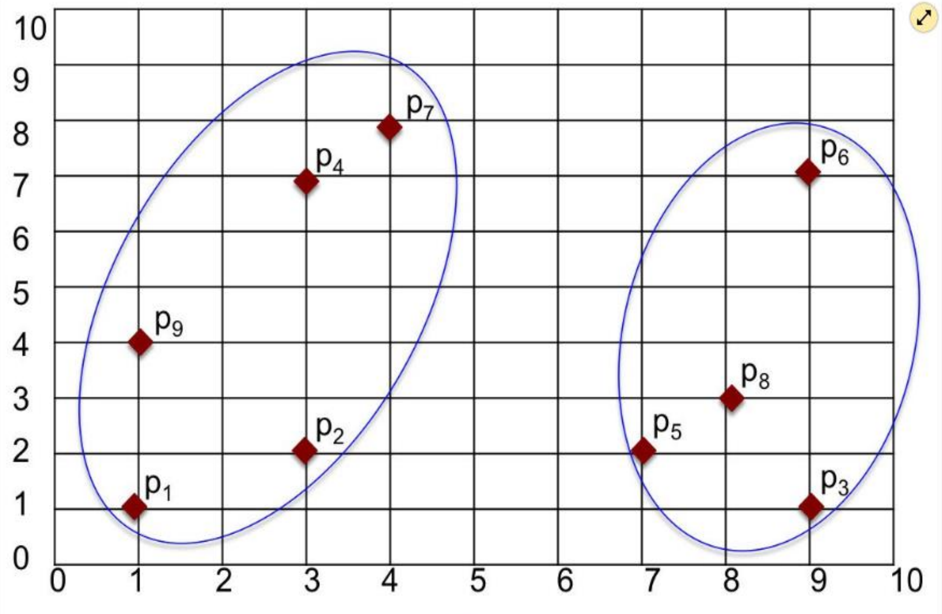


Figure 2: Two clusters

Minimum (Single) Linkage

Finding the shortest distance between points within a cluster is one approach to calculate the distance between them. In other words, we may determine the distance between two places by finding the point in the first cluster that is closest to the other. The nearest points in Figure 2 are p_2 in one cluster and p_5 in another. It is discovered that the distance between those sites, and thus the distance between clusters, is $d(p_2,p_5) = 4$

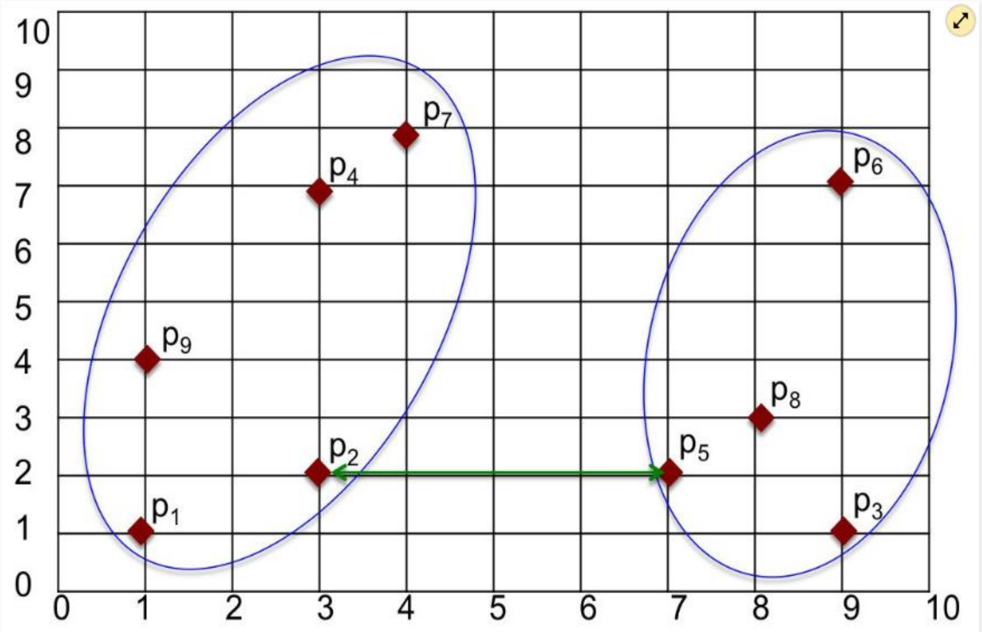


Figure 3: Min Linkage Method

The Min technique has the benefit of accurately handling non-elliptical shapes. The fact that it is sensitive to noise and outliers is a drawback.

Total Maximum Linkage

Finding the greatest distance between points in two clusters is another method of measuring distance. We can determine the distance between the spots in each cluster that are the farthest apart from one another. The greatest separation is shown in Figure 3 between p_1 and p_6. The distance between clusters is determined to be $d(p_1, p_6) = 10$, which is the distance between those two places.

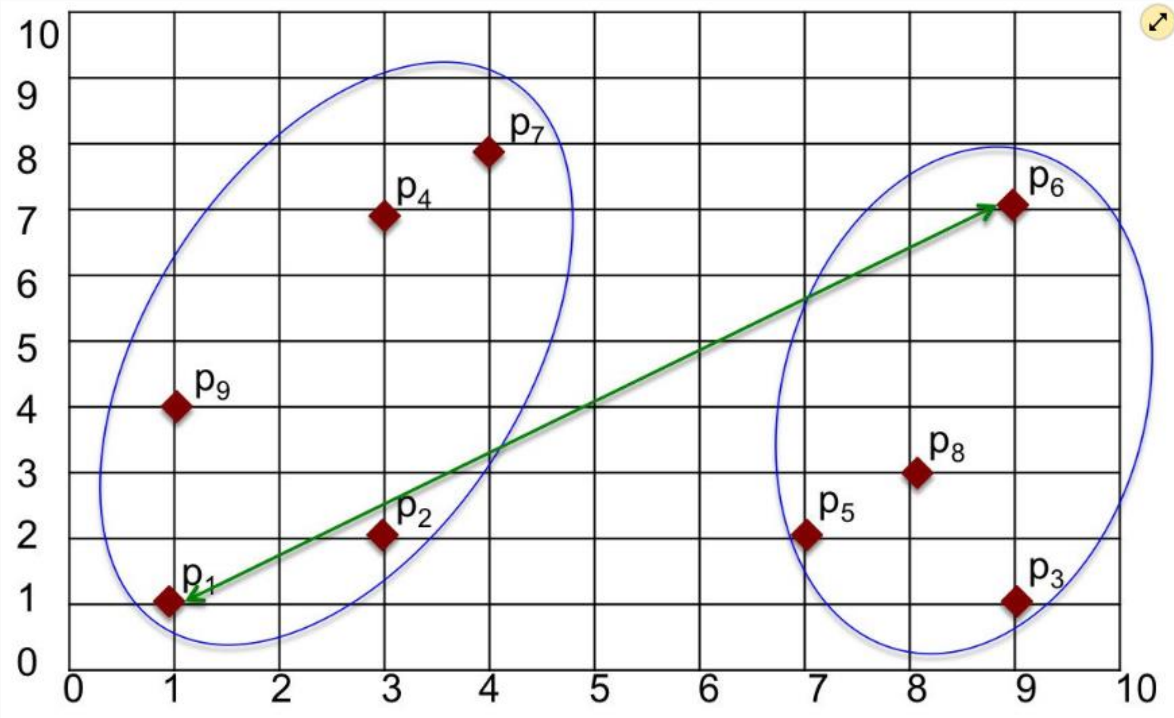


Figure 4: Max Linkage Method

In comparison to the MIN approach, Max is less susceptible to noise and outliers. However, MAX has a tendency to favour globular clusters and has the ability to split substantial clusters.

Linkage of Centroids:

According to the centroid approach, the separation between clusters is determined by the distance between their centres. Each cluster's centroid is determined, and then a distance function is used to determine how far apart those centroids are from one another.

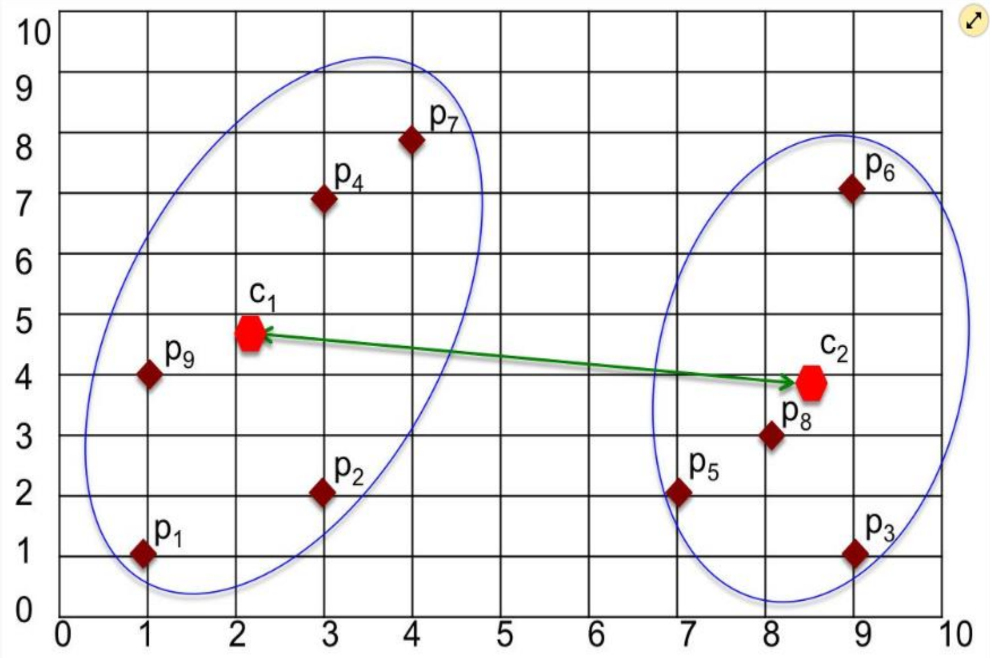


Figure 5: Centroid Linkage Method

Average Linkage

The average pairwise distance between all pairs of points in a cluster is what the Average technique uses to measure the separation between clusters. In Figure 6, only a few of the lines joining point pairs are depicted for simplicity's sake.

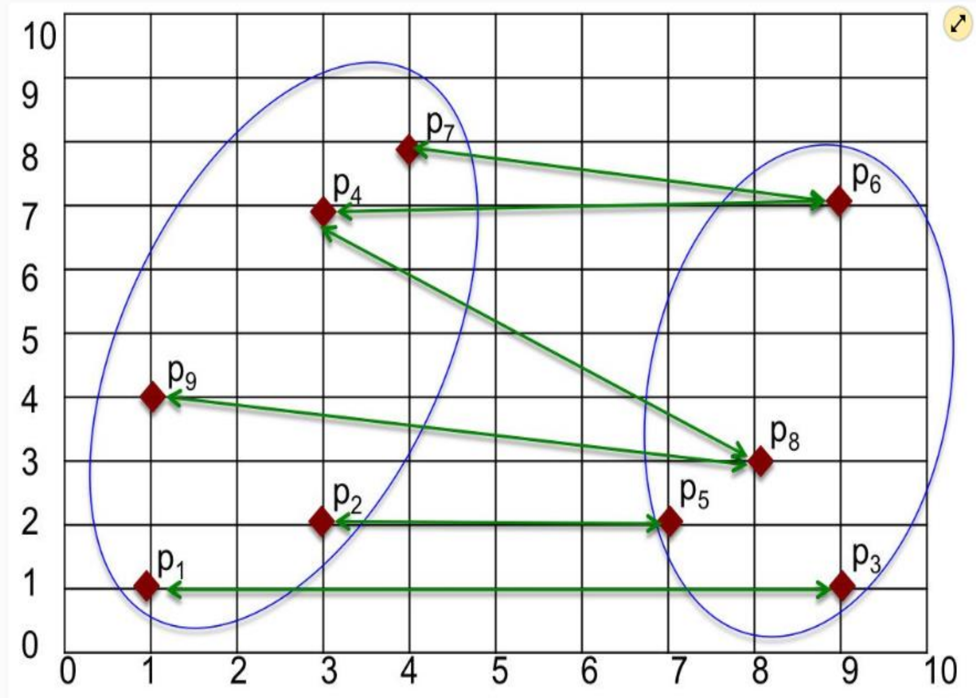


Figure 6: Average Linkage Method

Weak Linkage

Instead of measuring distances directly, the Ward technique decreases the variance between clusters by examining the cluster variance.

With the Ward approach, the amount that the sum of squares (SS) value will upsurge when two clusters are merged is correlated with their distance from one another.

To put it another way, the Ward technique aims to reduce the total squared distances between points and cluster centres. The Ward approach is is less

inclined to noise and outliers than the previously discussed distance-based metrics. Thus, Ward's method is favoured over others in clustering.

Definition 6. Let M = {⟨x, $\mu_M(x)$,$\nu_M(x)$,$\pi_M(x)$⟩| x∈X} and N={⟨x, $\mu_N(x)$, $\nu_N(x)$,$\pi_N(x)$⟩| x∈X} be two IFS in X= $x_1, x_2, \ldots, x_n$, i = 1,2, . . . ,n. established on the geometric interpretation of IFS. Szmidt and Kacprzyk suggested the four distance measures between M and N:

The Hamming distance

$d_H(M, N) = \frac{1}{2}\sum_{i=1}^{n} (|\mu_M(x_i) - \mu_N(x_i)| + |\nu_M(x_i) - \nu_N(x_i)| + |\pi_M(x_i) - \pi_N(x_i)|).$

The Euclidean distance

$d_{n-H\,(A,B)} = \langle \frac{1}{2}\sum_{i=1}^{n} [(u_A(x_i) - u_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2] \rangle^{1/2}$

The Normalised Hamming distance

$d_H(M, N) = \frac{1}{2n}\sum_{i=1}^{n} (|\mu_M(x_i) - \mu_N(x_i)| + |\nu_M(x_i) - \nu_N(x_i)| + |\pi_M(x_i) - \pi_N(x_i)|).$

The Normalized Euclidean distance

$d_{n-H\,(M,N)} = \langle \frac{1}{2n}\sum_{i=1}^{n} [(u_M(x_i) - u_N(x_i))^2 + (\nu_M(x_i) - \nu_N(x_i))^2 + (\pi_M(x_i) - \pi_N(x_i))^2] \rangle^{1/2}$

APPLICATION OF INTUITIONISTIC FUZZY SETS

APPLICATION ON THE COMPARISIOIN OF DISTANCE MEASURES

Depending on the results of the official test, certain students have been searched. These pupils were chosen at random throughout the 2016 academic year. Let H= {H1, H2, H3, H4, H5}be a set of high schools, S={S1, S2,S3,S4,S5,S6,S7,S8,S9,S10} be a set of students.

L= {Turkish, Mathematics, Science, Social, English, Religion} a collection of lessons. For each lesson in L, the base points for high schools were determined for the 2016 academic year.

Table 2

	Turkish	Mathematics	Science	Social	English	Religion
H_1	(0.965, 0.028, 0.007)	(0.985, 0.012, 0.003)	(0.995, 0.004, 0.001)	(0.990, 0.008, 0.002)	(0.975, 0.02, 0.005)	(0.995, 0.004, 0.001)
H_2	(0.91, 0.08, 0.01)	(0.945, 0.044, 0.011)	(0.945, 0.044, 0.011)	(0.92, 0.064, 0.016)	(0.9, 0.08, 0.02)	(0.99, 0.008, 0.02)
H_3	(0.845, 0.124, 0.031)	(0.8, 0.18, 0.02)	(0.825, 0.14, 0.035)	(0.85, 0.12, 0.03)	(0.855, 0.126, 0.019)	(0.95, 0.04, 0.01)
H_4	(0.71, 0.261, 0.029)	(0.57, 0.387, 0.043)	(0.75, 0.2, 0.05)	(0.825, 0.14, 0.035)	(0.65, 0.28, 0.07)	(0.69, 0.248, 0.062)
H_5	(0.64, 0.324, 0.036)	(0.4, 0.48, 0.12)	(0.55, 0.405, 0.045)	(0.61, 0.351, 0.039)	(0.57, 0.345, 0.085)	(0.83, 0.153, 0.017)

Students' official test scores in 2016 academic year have been indicated in Table 2

Table 3

	Turkish	Mathematics	Science	Social	English	Religion
S_1	(0.95, 0.04, 0.01)	(0.98, 0.01, 0.01)	(0.99, 0.005, 0.005)	(0.95, 0.03, 0.02)	(0.98, 0.01, 0.01)	(0.95, 0.04, 0.01)
S_2	(0.9, 0.08, 0.02)	(0.98, 0.01, 0.01)	(0.95, 0.03, 0.02)	(0.99, 0.008, 0.002)	(0.98, 0.015, 0.005)	(0.99, 0.006, 0.004)
S_3	(0.65, 0.28, 0.07)	(0.45, 0.44, 0.11)	(0.6, 0.32, 0.08)	(0.4, 0.48, 0.12)	(0.35, 0.52, 0.13)	(0.65, 0.3, 0.05)
S_4	(0.6, 0.32, 0.08)	(0.45, 0.44, 0.11)	(0.7, 0.24, 0.06)	(0.8, 0.16, 0.04)	(0.8, 0.17, 0.03)	(0.8, 0.16, 0.04)
S_5	(0.7, 0.24, 0.06)	(0.8, 0.16, 0.04)	(0.85, 0.12, 0.03)	(0.75, 0.2, 0.05)	(0.9, 0.08, 0.02)	(0.95, 0.04, 0.01)
S_6	(0.85, 0.12, 0.03)	(0.95, 0.04, 0.01)	(0.95, 0.02, 0.03)	(0.95, 0.05, 0)	(0.85, 0.11, 0.04)	(0.95, 0.02, 0.03)
S_7	(0.9, 0.08, 0.02)	(0.95, 0.04, 0.01)	(0.95, 0.02, 0.03)	(0.95, 0.015, 0.035)	(0.9, 0.09, 0.01)	(0.95, 0.04, 0.01)
S_8	(0.75, 0.2, 0.05)	(0.61, 0.35, 0.04)	(0.72, 0.24, 0.04)	(0.82, 0.15, 0.03)	(0.7, 0.25, 0.05)	(0.7, 0.2, 0.1)
S_9	(0.65, 0.3, 0.05)	(0.68, 0.26, 0.06)	(0.72, 0.24, 0.04)	(0.79, 0.15, 0.06)	(0.75, 0.18, 0.07)	(0.78, 0.15, 0.07)
S_{10}	(0.6, 0.3, 0.1)	(0.55, 0.4, 0.05)	(0.8, 0.15, 0.05)	(0.74, 0.22, 0.04)	(0.9, 0.07, 0.03)	(0.4, 0.51, 0.09)

The shortest distance using the Euclidean distance formula between each high school (Table 2) and each student (Table 3) has been determined in Table 4.

Table 4

H	H_1	H_2	H_3	H_4	H_5
S_1	0.0261	0.0458	0.1125	0.2675	0.3623
S_2	0.0257	0.0363	0.107	0.2658	0.365
S_3	0.434	0.4053	0.3375	0.1845	0.1315
S_4	0.2872	0.2418	0.1625	0.0941	0.1183
S_5	0.1465	0.1101	0.056	0.1576	0.2308
S_6	0.0692	0.0333	0.0701	0.2176	0.3184
S_7	0.0539	0.0163	0.08	0.2341	0.333
S_8	0.2519	0.2168	0.1375	0.0415	0.156
S_9	0.2539	0.2051	0.1258	0.0766	0.1513
S_{10}	0.3139	0.2701	0.206	0.1341	0.2266

Table 5

E	H_1	H_2	H_3	H_4	H_5
S_1	0.0568	0.1107	0.2949	0.6491	0.9039
S_2	0.0757	0.0987	0.697	0.6729	0.8889
S_3	0.9633	0.8184	0.61004	0.4247	0.4119
S_4	0.7075	0.6225	0.4371	0.2585	0.3649
S_5	0.4002	0.2949	0.1704	0.4187	0.5935
S_6	0.5424	0.0775	0.2141	0.5296	0.7993
S_7	0.5507	0.0583	0.2142	0.6131	0.8261
S_8	0.6737	0.5254	0.3389	0.1012	0.3927
S_9	0.5839	0.4768	0.2972	0.1915	0.43001
S_{10}	0.8226	0.71707	0.6187	0.5204	0.53906

The shortest distance using the normalised euclidean distance approach has been determined between each student (Table 3) and all of the high schools (Table 2) in Table 5.

Table 6

$n-E$	H_1	H_2	H_3	H_4	H_5
S_1	0.0231	0.045	0.1204	0.2675	0.36903
S_2	0.0308	0.04279	0.27589	0.27471	0.365
S_3	0.43412	0.391	0.3375	0.20901	0.13595
S_4	0.29438	0.37369	0.17615	0.17789	0.14159
S_5	0.22332	0.1204	0.06946	0.17097	0.24231
S_6	0.27333	0.0333	0.0815	0.22134	0.33569
S_7	0.05085	0.03291	0.09209	0.2341	0.33903
S_8	0.27506	0.2168	0.13836	0.0415	0.16034
S_9	0.2383	0.1946	0.1213	0.07818	0.17555
S_{10}	0.33582	0.29274	0.25261	0.21248	0.2266

Shortest distance between each student (i.e. Table 3) and each high school (i.e. Table 2) hasbeen calculated using hamming distance method in Table 6.

Table 7

H	H_1	H_2	H_3	H_4	H_5
S_1	0.0261	0.0458	0.1125	0.2675	0.3623
S_2	0.0257	0.0363	0.107	0.2658	0.365
S_3	0.434	0.4053	0.3375	0.1845	0.1315
S_4	0.2872	0.2418	0.1625	0.0941	0.1183
S_5	0.1465	0.1101	0.056	0.1576	0.2308
S_6	0.0692	0.0333	0.0701	0.2176	0.3184
S_7	0.0539	0.0163	0.08	0.2341	0.333
S_8	0.2519	0.2168	0.1375	0.0415	0.156
S_9	0.2539	0.2051	0.1258	0.0766	0.1513
S_{10}	0.3139	0.2701	0.206	0.1341	0.2266

The shortest route between each high school (Table 2) and each student (Table 3) has been determined using the normalised hamming distance method, and the results are shown in Table 7.

Table 8

$n-H$	H_1	H_2	H_3	H_4	H_5
S_1	0.0043	0.0076	0.0187	0.0445	0.0603
S_2	0.0042	0.006	0.0178	0.0443	0.0608
S_3	0.0723	0.0675	0.0562	0.0307	0.0219
S_4	0.0478	0.0403	0.027	0.0156	0.0197
S_5	0.0244	0.0183	0.0093	0.0262	0.0384
S_6	0.0115	0.0055	0.0116	0.0362	0.053
S_7	0.0089	0.0027	0.0133	0.039	0.0555
S_8	0.0419	0.0361	0.0229	0.0069	0.026
S_9	0.0423	0.0341	0.0209	0.0127	0.0252
S_{10}	0.0523	0.045	0.0343	0.0223	0.0377

Four distinct distance measurements have been used to calculate the distance among every learner and each institution. Table 8 provides a comparison of distance measurements.

Table 8

		H_1	H_2	H_3	H_4	H_5
S_1	d_E	0.0568	0.1107	0.2949	0.6491	0.9039
	d_H	0.0261	0.0458	0.1125	0.2675	0.3623
	d_{n-E}	0.0231	0.045	0.1204	0.2675	0.36903
	d_{n-H}	0.0043	0.0076	0.0187	0.0445	0.0603
S_2	d_E	0.0757	0.0987	0.697	0.6729	0.8889
	d_H	0.0257	0.0363	0.107	0.2658	0.365
	d_{n-E}	0.0308	0.04279	0.27589	0.27471	0.365
	d_{n-H}	0.0042	0.006	0.0178	0.0443	0.0608
S_3	d_E	0.9633	0.8184	0.61004	0.4247	0.4119
	d_H	0.434	0.4053	0.3375	0.1845	0.1315
	d_{n-E}	0.43412	0.391	0.3375	0.20901	0.13595
	d_{n-H}	0.0723	0.0675	0.0562	0.0307	0.0219
S_4	d_E	0.7075	0.6225	0.4371	0.2585	0.3649
	d_H	0.2872	0.2418	0.1625	0.0941	0.1183
	d_{n-E}	0.29438	0.37369	0.17615	0.17789	0.14159
	d_{n-H}	0.0478	0.0403	0.027	0.0156	0.0197
S_5	d_E	0.4002	0.2949	0.1704	0.4187	0.5935
	d_H	0.1465	0.1101	0.056	0.1576	0.2308
	d_{n-E}	0.22332	0.1204	0.06946	0.17097	0.24231
	d_{n-H}	0.0244	0.0183	0.0093	0.0262	0.0384
S_6	d_E	0.5424	0.0775	0.2141	0.5296	0.7993
	d_H	0.0692	0.0333	0.0701	0.2176	0.3184
	d_{n-E}	0.27333	0.0333	0.0815	0.22134	0.33569
	d_{n-H}	0.0115	0.0055	0.0116	0.0362	0.053
S_7	d_E	0.5507	0.0583	0.2142	0.6131	0.8261
	d_H	0.0539	0.0163	0.08	0.2341	0.333
	d_{n-E}	0.05085	0.03291	0.09209	0.2341	0.33903
	d_{n-H}	0.0089	0.0027	0.0133	0.039	0.0555
S_8	d_E	0.6737	0.5254	0.3389	0.1012	0.3927
	d_H	0.2519	0.2168	0.1375	0.0415	0.156
	d_{n-E}	0.27506	0.2168	0.13836	0.0415	0.16034
	d_{n-H}	0.0419	0.0361	0.0229	0.0069	0.026
S_9	d_E	0.5839	0.4768	0.2972	0.1915	0.43001
	d_H	0.2539	0.2051	0.1258	0.0766	0.1513
	d_{n-E}	0.2383	0.1946	0.1213	0.07818	0.17555
	d_{n-H}	0.0423	0.0341	0.0209	0.0127	0.0252
S_{10}	d_E	0.8226	0.71707	0.6187	0.5204	0.53906
	d_H	0.3139	0.2701	0.206	0.1341	0.2266
	d_{n-E}	0.33582	0.29274	0.25261	0.21248	0.2266
	d_{n-H}	0.0523	0.045	0.0343	0.0223	0.0377

Table 8 shows that four distinct distance metrics produce consistent findings. The schools that each kid will enrol in are the same in accordance with the outcomes of each distance measurement. Additionally, there is consistency between these results and the official Ministry of Education data. According to Table 7, Student S1 is expected to enrol in H1 High School, Student S2 is expected to enrol in H1 High School, Student S3 is expected to enrol in H5 High School, Student S4 is expected to enrol in H4 High School, Student S5 is expected to enrol in H3 High School, Student

S6 is expected to enrol in H2 High School, Student S7 is expected to enrol in H2 High School, Student S8 is expected to enrol in H4 High School, Student S9 is expected. The distance measures' validity and reliability are listed in the following order in Table 7 is $d_{n-H}<d_{n-E}<d_E<d_H$. The normalised hamming distance measurement yields the shortest results in this comparison. As a result, the normalised hamming distance measure was used for the applications that were carried out in the following sections of this work.

Hierarchical Clustering

Hierarchical clustering is a method frequently used to organise items. It classifies objects into groups based on how much they resemble one another and how much they differ from objects in other groups. An example of a hierarchical tree that graphically represents clusters is a dendrogram.

Two major benifits of hierarchical clustering are as follows:

1. The number of clusters does not need to be predetermined. Instead, the dendrogram can be appropriately split to obtain the required number of clusters.

2. It is straightforward to summarise and arrange information into a hierarchy using dendrograms. It is simple to study and interpret clusters using dendrograms.

APPLICATION OF INTUITIONISTIC FUZZY SETS IN CAREER DETERMINATION

It is crucial to give pupils the right information so they can make informed professional decisions. This is crucial because the myriad issues related to students' inadequate career guidance have a significant impact on both their profession choice and effectiveness. Therefore, it is important that students receive enough knowledge about choosing a vocation to improve suitable planning, preparation, and proficiency. Academic achievement, interests, personality type, and other elements that can influence a person's job seem to be most important. Since it takes into account the student's membership degree (i.e., their scores for each question to which they correctly responded), non-membership degree (i.e., the scores for the questions to which they failed), and hesitation degree (i.e., the score given to the questions they do not attempt), we use intuitionistic fuzzy sets as a tool.

Let be the set of students S = {s_1, s_2, s_3, s_4 }, C = {medicine, pharmacy, surgery, anatomy} be the set of careers and Su ={Eng, Maths, Bio, Physics, Chemistry} be the group of career-related subjects. We presume that the students listed above take exams (i.e., totaling over 100 points) on the subjects listed above to establish their career choices and preferences. The table below lists the prerequisite courses for various professions.

Table 9 Careers vs Subjects

	Eng	Maths	Bio	Physics	Chemistry
Medicine	(0.7, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.1,0.2 ,0.3)	(0.6, 0.2, 0.1)
Pharmacy	(0.2, 0.3, 0.2)	(0.5, 0.5, 0)	(0, 0.3, 0.5)	(0.9, 0, 0.1)	(0.4, 0.4, 0.1)
Surgery	(0.2, 0.3, 0.3)	(0.3, 0.4, 0.3)	(0.5, 0.5, 0)	(0.5, 02., 0.1)	(0.1, 06., 0.3)
Anatomy	(0.3, 0.3, 0.5)	(0.5, 0.1, 0.1)	(0.3, 0.6, 0.1)	(0.9, 0, 0.1)	(0.2, 0.4, 0.3)

Three numbers, namely membership, non-membership v, and hesitation margin, are used to describe each performance. According to the table below, the students received the following grades as a result of the various exams.

Table 10 Students vs Subjects

	Eng	Maths	Bio	Physics	Chemistry
S1	(0.3, 07., 0)	(0.5, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.2, 0.3, 0.1)	(0.2, 0.4, 0.1)
S_2	(0.2, 0.3, 0.1)	(0.4, 0.4, 0.1)	(0.1, 0.2, 05.)	(0.7, 0.1, 0.1)	(0.5, 0.2, 0.1)
S_3	(0.3, 0.4, 0.1)	(0.1, 0.4, 0.1)	(0.4, 0.1, 0.5)	(0.7, 0.2, 0)	(0.1, 0.3, 0.3)
S_4	(0.3, 0.3, 0.1)	(0.2, 0.6, 0.1)	(0.1, 0.3, 0.5)	(0.5, 0.1, 0.4)	(0.3, 0.3, 0.1)

Using definition of normalized Euclidean distance above to calculate the distance between each student and each career with reference to the subjects, we get the table below.

Table 11 Students vs Careers

	Medicine	Pharmacy	Surgery	Anatomy
S1	0.0021	0.1001	0.0980	0.1009
S_2	0.0321	0.0875	0.0221	0.0212
S_3	0.0912	0.1033	0.0327	0.0431
S_4	0.0310	0.1021	0.0321	0.0324

The shortest distance from the above table gives the appropriate career determination. S1 is learning about anatomy in order to become an anatomist, S2 hopes to become a surgeon, S3 is to become a doctor, and S4 is to become a chemist.

This use of intuitionistic fuzzy sets to determine careers is very crucial because it permits for exact and suitable career selection depend on the performance of test. The decision of a career is a complex one since, if managed incorrectly, it can have a negative impact on competence and efficiency. In the suggested application, we calculated the distance between each student and each career in relation to the subjects using normalised Euclidean distance.

CHAPTER 3 PICTURE FUZZY RELATION

Definition 7 (Picture Fuzzy Set)

A picture fuzzy set (PFS) A on a Universal set X is defined by:

$$A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) \mid x \in X\}$$

where: μ_A is a positive membership function, η_A is a neutral membership function, ν_A is a negative membership function of A,in there: $\mu_A(x), \eta_A(x), \nu_A(x) \in [0, 1]$ and

$$0 \leq \mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1, \forall X \in X$$

Definition 8 (Operations on Picture Fuzzy Sets)

$M = \{a, \mu_M (a), \eta_M (a), \nu_M (a)|a \in X\}$ and $N = \{a, \mu_N (a), \eta_N (a), \nu_N (a)|a \in X\}$ are any two Picture fuzzy sets in a set X then

1. $M \subseteq N$ iff $\forall$ $a \in X$, $\mu_M (a) \leq \mu_N (a)$, $\eta_M (a) \geq \eta_N (a)$ and $\nu_M (a) \geq \nu_N (a)$.

2. $M = N$ iff $\forall$ $a \in X$, $\mu_M (a) = \mu_N (a)$, $\eta_M (a) = \eta_N (a)$ and $\nu_M (a) = \nu_N (a)$.

3. $M \cup N = \{(a, \max (\mu_M (a), \mu_N (a))$, $\min (\eta_M (a), \eta_N (a))$, and $\min (\nu_M (a) = \nu_N (a))) \mid a \in X\}$

4. $M \cap N = \{(a, \min, (\mu_M (a), \mu_N (a))$, $\max (\eta_M (a), \eta_N (a))$, $\max (\nu_M (a) = \nu_N (a))) \mid a \in X\}$

Definition 9 (Picture Fuzzy Relation)

Let M be a non-empty set. A PFR R on M is a PFS

$R = \{(a, b) , \mu_M (a, b), \eta_M (a, b), \nu_M (a, b) \mid (a, b) \in M \times M \}$

where $\mu_M (a) : M \times M \rightarrow [0, 1]$, $\eta_M (a) : M \times M \rightarrow [0, 1]$ and

$\nu_M (a) : M \times M \rightarrow [0, 1]$ satisfying the condition

$\mu_M (a, b) + \eta_M (a, b) + \nu_M (a, b) \leq 1 \ \forall (a, b) \in M \times M$.

Definition 10 (Picture Fuzzy Proximity Relation)

A PF relation $R = \{(a, b) , \mu_M (a, b) , \eta_M (a, b) , \nu_M (a, b) \mid (a, b)\} \in M \times M$

is reflexive when $\mu_M (a, a) = 1$, $\eta_M (a, a) = 0$ and $\nu_M (a, a) = 0 \ \forall a \in M$ and symmetric

if $\mu_M (a, b) = \mu_M (b, a)$, $\eta_M (a, b) = \eta_M (b, a)$, $\nu_M (a, b) = \nu_M (b, a)$, $\forall (a, b) \in M \times M$.

Definition 11 (Composition of P F Relation)

If $R_1 = \{((a, b), \mu_1 (a, b), \eta_1 (a, b), \nu_1 (a, b)) \mid (a, b) \in M \times M\}$ and

$R_2 = \{((a, b), \mu_2 (a, b), \eta_2 (a, b), \nu_2 (a, b)) \mid (a, b) \in M \times M\}$ be two PF relations on M

then composition of R_1, R_2 is represented by $R_1 \circ R_2$ is described as

$R_1 \circ R_2 = \{((a, b) , \mu_1 \circ \mu_2 (a, b) , \eta_1 \circ \eta_2 (a, b) , \nu_1 \circ \nu_2 (a, b)) \mid (a, b) \in M \times M \}$

where $\mu_1 \circ \mu_2 (a, b) = \sup\{\min\{\mu_1 (a, c) , \mu_2 (c, b)\}\}, c \in M$

$\eta_1 \circ \eta_2 (a, b) = \inf \{\max\{\eta_1 (a, c) , \eta_2 (c, b)\}\}, c \in M$

$\nu_1 \circ \nu_2 (a, b) = \inf \{\max\{\nu_1 (a, c) , \nu_2 (c, b)\}\}, c \in M$

Distance between two PFS

The distance between two given PFS (μ_M , η_M, ν_M), (μ_N , η_N ,ν_N) can be found by

1. Normalized Hamming distance

   $d_H(M, N)= \frac{1}{n}\sum_{i=1}^{n} \quad (|\mu_M(x_i)-\mu_N(x_i)|+|\nu_M(x_i)-\nu_N(x_i)|+| \eta_M(x_i)- \eta_N(x_i)|).$

2. **The Euclidean distance**

   $d_{n-H\,(M,N)} = \langle\frac{1}{n}\sum_{i=1}^{n} \quad [(u_M(x_i) - u_N(x_i))^2 + (\nu_M(x_i) - \nu_N(x_i))^2 +$

   $(\eta_M(x_i) - \eta_N(x_i))^2]\rangle^{1/2}$

Example 1

Let M = {(0.8, 0.3, 0.2), (0.9, 0.2, 0.2), (0.8,0.2, 0.3)} and N = {(0.7, 0.3, 0.3), (0.9, 0.3, 0.1), (0.9, 0.0,0.2)} are two PFS of dimensions 3. Then its Hamming distance can be calculated as

d_H(M,N)= ⅓ ((|0.8-0.7| + |0.3-0.3| + |0.2-0.3|) + (|0.9-0.9| + |0.2-0.3| + |0.2-0.1|) + (|0.8-0.9| + |0.2-0.0| + |0.3-0.2|))

= ⅓((0.1 + 0.0 + 0.1) + (0.0 + 0.1 + 0.1) + (0.1 + 0.2 + 0.1))

= ⅓(0.2+0.2+0.4)

= ⅓ (0.8)

= 0.27

Operations on PFNs

$M = (\mu_M , \eta_{M,} \nu_M)$ and $N = (\mu_N , \eta_N ,\nu_N)$.

1. $M.N = (\mu_M + \eta_{M,})(\mu_N + \eta_N) - \eta_{M,} \eta_N , \eta_{M,}\eta_N , 1 – (1 - \nu_M)(1 - \nu_N)$
2. $P^{\lambda} = (\mu_N + \eta_N)^{\lambda} - \eta_N^{\lambda}, \eta_N^{\lambda} ,1- (1- \nu_N)^{\lambda}, \lambda > 0$

Example 2

Let M = (0.6,0.1,0.2) and N = (0.7,0.1,0.1) are two PFS and $\lambda = 5$

Solution – M.N = (0.6 + 0.1) *(0.7 + 0.1) – (0.1) *(0.1), (0.1) *(0.1), 1 – (1-0.2) *(1-0.1)

= (0.7) * (0.8) – 0.01, 0.01, (1 – 0.72)

= 0.56 – 0.01, 0.01, 0.28

= 0.55,0.01,0.28

$P^{\lambda} = P^5 = ((0.7 + 0.1)^5 – (0.1)^5, (0.1)^5, 1 – (1 – 0.1)^5)$

= ((0.32768) – 0.00001, 0.00001, 1 – (0.59049))

= (0.32767, 0.00001, 0.40951)

Comparison of PFS

Wang et al with the help of a score function compared two PFS. Let $M = \mu_M , \eta_M , \nu_M , \rho_M$ be a PFN, then a score function S(M) is defined as $S(M) = \mu_M - \nu_M$ & accuracy function is defined as $H(M) = \mu_M + \eta_M + \nu_M$ where $S(M) \in [-1,1]$ and $H(M) \in [0,1]$. Then for the two PFNs M & N

(i) If S(M) > S(N), then M is higher than N, denoted by M > T .

(ii) If S(M) = S (N), then

a. H(M) = H(N) implies that M is equivalent to N, denoted by M = N

b. H M > H(N) implies that M is greater than N, depicted by M > N.

Example 3

Let M = (0.6, 0.1, 0.2) and N = (0.7, 0.1, 0.1) are two PFS.

Now, S(M) = 0.7–0.1 = 0.6,

S(N) = 0.6–0.2 = 0.4

H(M) = 0.7 + 0.2 + 0.1 = 0.9,

H(N) = 0.6 + 0.2 + 0.2 = 1.

Since S(M) > S(N), therefore M > N

Max- min Composition Relation:

Let M be a PFR(X x Y) and N be a PFR(Y x Z), then the max min composition of M and N is the PFR from X to Z defined as

M o N = {((x, z), $\mu_{Ro\ S}$(x, z), $\eta_{Ro\ S}$(x, z), $\nu_{Ro\ S}$(x, z)): x ϵ X, z ϵ Z} where

$\mu_{Ro\ S}$(x, z) = Ú$_{y\epsilon Y}${μ_S(x, z) Ù μ_R(y, z)}

$\eta_{Ro\ S}$(x, z) = Ù$_{y\epsilon Y}${η_S(x, z) Ù η_R(x, z)}

$\nu_{Ro\ S}$(x, z) = Ù$_{y\epsilon Y}${ ν_S(x, z) Úν_R(x, z)}

Whenever 0 ≤ $\mu_{Ro\ S}$(x, z) + $\eta_{Ro\ S}$(x, z) + $\nu_{Ro\ S}$(x, z) ≤ 1 .

Min Max Composition:

Let M PFR(X x Y) and N PFR(Y x Z), then the min max composition of M and N is the PFR from X to Z defined as

$$M * N = \{((x,z), \mu_{R*S}(x,z), \eta_{R*S}(x,z), \nu_{R*S}(x,z)) : x \in X, z \in Z\} \text{ where}$$

$$\mu_{R*S}(x,z) = \acute{U}_{y\in Y}\{\mu_S(x,z)\ \grave{U}\ \mu_R(y,z)\}$$

$$\eta_{R*S}(x,z) = \grave{U}_{y\in Y}\{\eta_S(x,z)\ \grave{U}\ \eta_R(x,z)\}$$

$$\nu_{R*S}(x,z) = \grave{U}_{y\in Y}\{\nu_S(x,z)\ \acute{U}\ \nu_R(x,z)\}$$

Whenever $0 \le \mu_{R*S}(x,z) + \eta_{R*S}(x,z) + v_{R*S}(x,z) \le 1$.

Note:

1.) Let M and N be two elements of Picture Fuzzy Relation (A x A), then $(M \text{ o } P)^c = M^c * N^c$

2.)Let $P_1 \in PFR(AxB)$ and $P_2 \in PFR(BxC)$, then $P_1 \text{ o } P_2 \in PFR(AxC)$

APPLICATION OF PICTURE FUZZY SET:

Application 1

Determination of higher study area selection on the basis of composition of picture fuzzy relations.

It is impossible to choose a future area of higher study based on a numerical figure. In this instance, ambiguity occurs, as when a student informs the student counsellor that his capacity for solving mathematical puzzles is "very high" based on language evaluations. Any numerical figure cannot in this instance accurately describe this student's ability to solve mathematical problems at a "very high" level, but TABLE I may describe this student's capacity to solve mathematical problems.

Table 13

Linguistic Terms	(μ,η,ν)
Extremely High (EH)	(0.9,0.0,0.1)
Very High (H)	(0.8,0.1,0.1)
High (H)	(0.7,0.2,0.1)
Medium (M)	(0.5,0.2,0.3)
Low (L)	(0.4,0.2,0.4)
Very Low (VL)	(0.3,0.1,0.6)
Extremely Low (EL)	(0.1,0.1,0.8)

As a result, we are aware that the membership function of an image fuzzy set can express information about vagueness. The following three tasks are primarily involved with the methodology:

1. Assessment of capabilities.

2. The development of specialised understanding subject to fuzzy image relations.

3. Choosing a higher study area based on the structure of the PFRs.

Procedure:

Let A = $\{a_1, a_2, \ldots a_m\}$ be the set of m students for determining their future higher study areas with a set of n subject knowledge skills C = $\{c_1,c_2,c_3,c_4,c_5\}$ and D = $\{d_1,d_2,\ldots d_k\}$ be the set of k study areas connected to subject knowledge skill. Now we build a PFR R $(A \times C)$ from A to C, where the entries of PFS $a_{ij} = (\mu,\eta,\nu)$ for $1 \leq i \leq m$ and $1 \leq j \leq n$. The student subject knowledge skill relation R$(A \times C)$ is as follows R$(A \times C)$

Algorithm:

Step I: To obtain the student-subject knowledge-skill relation, enter the picture fuzzy sets.

$R(A \times C)$

Step II: To find the relationship between topic knowledge, skill, and study areas, enter the picture fuzzy sets. $R_1(C \times D)$.

Step III: To obtain the student study area relation, perform the transformation procedure.

$R_2(A \times D) = R(A \times C) \circ R_1(C \times D)$

Step IV: Defuzzify all the elements of the matrix $R_2(A \times D)$ to get $R_3(A \times D)$.

Example 4: Let's say there are four students $A = \{a_1, a_2, a_3, a_4\}$in a college who want to choose their higher study areas based on their subject knowledge skills in the following areas: Functional Analysis, Operational Research, Fuzzy Logics, Statistics, and Discrete Mathematics. Possible study areas based on the mentioned subject knowledge skills include Real Analysis, MATLAB, Coding, and Computer vision. Set $C = \{c_1,c_2,c_3,c_4,c_5\}$ is a set of subject knowledge skills and c_1,c_2,c_3,c_4,c_5 symbolise Functional Analysis, Operational Research, Fuzzy Logics, Statistics, and Discrete Mathematics respectively. The student counsellor can create the following table using four students' evaluations and their topic knowledge abilities after observing their subject knowledge abilities.

TABLE 14

$R(A \times C)$	c_1: Functional Analysis	c_2: Operational Research	c_3: Fuzzy Logics	c_4: Statistics	c_5: Discrete Mathematics
a_1	VL	H	EL	M	M
a_2	H	VH	L	M	EH
a_3	EH	M	M	H	VL
a_4	L	H	M	EH	M

PFR $R(A \times C)$, By rule of conversion between linguistic terms and numerical values from Table 1. The following are the details of the relationship known as the student-subject knowledge skill relation:

$R(A \times C)$ =

[(0.34,0.11,0.66) (0.76,0.25,0.11) (0.11,0.11,0.87) (0.54,0.25,0.34) (0.54,0.25,0.34) (0.76,0.25,0.11) (0.87,0.11

The career advice officer might create the following table with the four students' linguistic evaluations after once more identifying their subject knowledge skills.

TABLE 15

R_1 ($C\times D$)	d_1: Real Analysis	d_2: MATLAB	d_3: Coding	d_4: Computer vision.
c_1	EL	H	VH	VL
c_2	H	M	H	VH
c_3	M	H	EH	H
c_4	H	M	M	L
c_5	EH	EH	H	M

We now create PFR $R_1(C \times D)$. This relation is known as subject knowledge skill - study areas relation.

$R_1(C \times D)$ =

(0.11,0.11,0.87)	(0.76,0.25,0.11)	(0.87,0.11,0.11)	(0.34,0.11,0.66)
(0.76,0.25,0.11)	(0.54,0.25,0.34)	(0.76,0.25,0.11)	(0.87,0.11,0.11)
(0.54,0.25,0.11)	(0.76,0.25,0.11)	(0.92,0.00,0.11)	(0.76,0.25,0.11)
(0.76,0.25,0.11)	(0.54,0.25,0.34)	(0.54,0.25,0.34)	(0.46,0.25,0.46)
(0.92,0.00,0.11)	(0.92,0.00,0.11)	(0.76,0.25,0.11)	(0.54,0.25,0.34)

Using composition of PFS, we get the following PFR

$$R_2(A\times D) = R(A \times C) \text{ o } R_1(C \times D)$$

Student - study areas matrix R_2 is given below:

$R_2(A \times D)$ =

$$\begin{bmatrix} (0.76,0.00,0.11) & (0.54,0.00,0.34) & (0.76,0.00,0.11) & (0.76,0.00,0.11) \\ (0.92,0.00,0.11) & (0.92,0.00,0.11) & (0.76,0.00,0.11) & (0.87,0.00,0.11) \\ (0.92,0.00,0.11) & (0.76,0.00,0.11) & (0.87,0.00,0.11) & (0.54,0.00,0.34) \\ (0.76,0.00,0.11) & (0.54,0.00,0.34) & (0.76,0.00,0.11) & (0.76,0.00,0.11) \end{bmatrix}$$

Defuzzifying the above matrix , we get $R_3(A \times D)$

$R_3(A \times D)$ =

$$\begin{bmatrix} 0.66 & 0.25 & 0.66 & 0.76 \\ 0.87 & 0.87 & 0.66 & 0.76 \\ 0.87 & 0.66 & 0.76 & 0.25 \\ 0.66 & 0.25 & 0.66 & 0.66 \end{bmatrix}$$

a_1 can study either Real Analysis or Coding, a_2 can study either Real Analysis or MATLAB, a_3 can only study Real Analysis, and a_4 can study any subject except MATLAB, according to the relationship shown above. The institutional policies and the student counsellor's evaluation technique in Table 3 could have an impact on the outcomes.

Example 5

Let P = {p_1,p_2,p_3}, Q = {q_1,q_2,q_3,q_4}, R = {r_1,r_2,r_3}, A ∈ PFR $(P \times Q)$ & B ∈ PFR$(Q \times R)$ provided in the table.

TABLE 16: A is PFR between P and Q

A	q_1	q_2	q_3	q_4
p_1	(0.76,0.25,0.11)	(0.11,0.05,0.66)	(0.02,0.66,0.25)	(0.07,0.34,0.46)
p_2	(0.54,0.46,0.01)	(0.87,0.03,0.05)	(0.25,0.25,0.54)	(0.76,0.15,0.08)
p_3	(0.35,0.54,0.15)	(0.92,0.05,0.01)	(0.45,0.54,0.01)	(0.11,0.11,0.46)

TABLE 17: B is a PFR between Q and R

B	r_1	r_2	r_3
q_1	(0.75,0.11,0.15)	(0.54,0.25,0.01)	(0.45,0.46,0.01)
q_2	(0.25,0.46,0.34)	(0.36,0.66,0.05)	(0.25,0.25,0.66)
q_3	(0.06,0.24,0.46)	(0.55,0.09,0.34)	(0.76,0.11,0.11)
q_4	(0.34,0.04,0.66)	(0.46,0.34,0.25)	(0.46,0.25,0.11)

Let $T_\chi : [0,1]^2 \to [0,1]$ is a t-norm defined by

$$T(x,y) = \begin{cases} 0 & \text{if } x+y \le 1 \\ x+y-1 & \text{if } x+y > 1 \end{cases}, \text{ for all } (x,y) \in [0,1]^2 .$$

Then composed relation is given as BC_3A, $\beta_1 = T_X$, $\beta_2 = ϶$

TABLE 18: BC_3A relation with $\beta_1 = T_X$, $\beta_2 = ϶$

BC_3A	r_1	r_2	r_3
p_1	(0.45,0.04,0.15)	(0.25,0.05,0.11)	(0.15,0.05,0.11)
p_2	(0.25,0.03,0.15)	(0.15,0.03,0.01)	(0.11,0.03,0.01)
p_3	(0.11,0.04,0.15)	(0.25,0.05,0.05)	(0.15,0.05,0.11)

Application 2

Determination of diagnosis on the basis of composition of picture fuzzy relations.

We demonstrate a PFR application in Sanchez's approach [8], [9] for medical diagnostics in this part. In this method, S stands for a group of symptoms, D for a group of diagnoses, and P for a group of patients.

We characterized picture medical knowledge as a PFR R among a set of symptoms S and a set of diagnoses D that shows the strength of any positive association, strength of any neutral association, and strength of any negative association among the symptoms and the diagnosis.

Let's talk about picture-fuzzy medical diagnosis now. In keeping with the conventional approach, the process also entails the three tasks listed below:

1. Identifying the symptoms.

2. The development of medical knowledge based on hazy image relations.

3. Making a diagnosis based on the fuzzy relational makeup of the image.

Let $R \in PFR\ (P \times S)$ and $Q \in PFS\ (D \times S)$, clearly, the composition T of R and Q ($T = Ro\ Q$) describes the state of patients in terms of the diagnosis. For sample, the state of patients can be defined as a max – min composed relation T from P to D:

$$\mu_T (p,d) = \{\mu_Q(p,s) \ni \mu_R(s,d)\};$$

$$\eta_T (p,d) = \{\eta_Q(p,s) \ni \eta_R(s,d)\};$$

$$\nu_T(p,d) = \{ _Q(p,s) \ni \nu_R(s,d)\}; \quad \forall\ p_i \in p,d \in D.$$

Consider the following four patients: p_1 , p_2, p_3 and p_4. Their signs include temperature, headache, stomach pain, coughing, and chest pain. Once the patients are determined to be $P = p_1$, p_2, p_3 and p_4, the symptoms are determined to be S = Temperature, Joint Pain, Stomach pain, cold, and chest pain. The hypothetical PFR is shown in Table.

Let D stand for the following diagnoses: viral fever, Dengue, Arthritis, Gas, and heart issue. Table 3 provides the picture fuzzy relation R in PFS $S \times D$. As a result, Table 5's constructed connection $T = R\square\ Q$ is as stated.

The terms $\mu_T(p,d)$, $\eta_T(p,d)$, $\{_T(p,d)$ denotes the patient p and diagnosis d as a triple.

$p,d \in P\times D$, we have $S_T(p,d)$:

$S_T(p,d) = \mu_T(p,d) \; - {}_T(p,d) \, . \, \pi_T(p,d)$

And $\pi_T(p,d) = 1 - [\mu_T(p,d) + \eta_T(p,d) + \{_T(p,d)]$

If the condition $\mu_T(p,d) + \eta_T(p,d) + \{_T(p,d) = 1$,

Then it is obvious that $S_T(p,d) = \mu_T(p,d)$

If $S_T(p,d) \geq 0.5$ then, it is claimed that patient p had sickness d. Therefore, it is clear from Table 7 that, if the doctor concurs, then p_1, p_3 and p_4 have malaria, p_1 and p_3 have typhoid and p_2 have stomach issues.

TABLE-19 Q is PFR among sets of patients P and symptoms S

Q	Temperature	Joint Pain	Stomach pain	Cold	Chest Pain
p_1	(0.81,0.03,0.11)	(0.72,0.05,0.21)	(0.11,0.21,0.66)	(0.72,0.15,0.11)	(0.21,0.34,0.54)
p_2	(0.01,0.21,0.72)	(0.54,0.05,0.34)	(0.65,0.11,0.11)	(0.05,0.21,0.72)	(0.07,0.21,0.66)
p_3	(0.75,0.15,0.05)	(0.81,0.11,0.08)	(0.15,0.35,0.54)	(0.34,0.05,0.66)	(0.11,0.45,0.54)
p_4	(0.66,0.25,0.11)	(0.45,0.15,0.45)	(0.21,0.45,0.34)	(0.66,0.21,0.15)	(0.35,0.21,0.21)

TABLE-20 S_R is PFR among sets of symptoms S and Diagnosis D

S_R	Viral Fever	Dengue	Arthritis	Gas	Heart issue
Temperature	(0.45,0.45,0.05)	(0.81,0.11,0.11)	(0.34,0.34,0.34)	(0.15,0.05,0.66)	(0.05,0.15,0.72)
Joint Pain	(0.45,0.25,0.34)	(0.11,0.21,0.66)	(0.75,0.05,0.03)	(0.34,0.05,0.05)	(0.01,0.11,0.81)

Stomach pain	(0.11,0.25,0.66)	(0.01,0.03,0.91)	(0.11,0.21,0.72)	(0.81,0.11,0.01)	(0.11,0.15,0.75)
Cold	(0.45,0.21,0.11)	(0.65,0.54,0.05)	(0.21,0.15,0.66)	(0.25,0.25,0.54)	(0.15,0.21,0.72)
Chest Pain	(0.05,0.25,0.66)	(0.03,0.07,0.81)	(0.01,0.01,0.85)	(0.11,0.11,0.72)	(0.91,0.02,0.05)

TABLE 21: T is PFR among sets of Patients P and Diagnosis D

T	Viral Fever	Dengue	Arthritis	Gas	Heart issue
p_1	(0.45,0.03,0.1)	(0.8,0.03,0.1)	(0.7,0.01,0.2)	(0.3,0.03,0.2)	(0.2,0.02,0.5)
p_2	(0.4,0.05,0.3)	(0.1,0.03,0.6)	(0.5,0.01,0.3)	(0.65,0.05,0.1)	(0.1,0.02,0.5)
p_3	(0.4,0.05,0.05)	(0.75,0.03,0.1)	(0.75,0.01,0.08)	(0.3,0.05,0.08)	(0.15,0.02,0.5)
p_4	(0.45,0.15,0.1)	(0.6,0.03,0.1)	(0.4,0.01,0.3)	(0.3,0.05,0.3)	(0.35,0.02,0.2)

TABLE 22 : S_T

S_T	Viral Fever	Dengue	Arthritis	Gas	Heart issue
p_1	0.4081	0.7934	0.6821	0.2066	0.066
p_2	0.3254	-0.0621	0.4434	0.6344	-0.0911
p_3	0.3754	0.7381	0.73721	0.26445	-0.0154
p_4	0.4211	0.5734	0.3134	0.1954	0.2645

The idea of an image fuzzy set has only recently been introduced, thus additional theoretical and practical research is needed. One of the first important concepts to be addressed is the picture fuzzy relation. In this study, the validity of max - min composition and max - prod are first looked at. The definition of a more generic composition based on two arbitrary t-norms follows. Finally, a real-world example is provided where picture fuzzy relations are used as a knowledge representation method.

Definition 12 (($\tilde{\alpha}$, $\tilde{\beta}$, ?) - cuts of a Picture Fuzzy Relation)

For any PFS M = {a, μ_M (a), η_M (a), ν_M (a) | a ∈ X} of a set X, we define a

($\tilde{\alpha}$, $\tilde{\beta}$, ?) − cut of M as a crisp set {a ∈ X | $\mu \geq \alpha$, $\eta \leq \beta$, $\nu \leq \gamma$} of X and it is denoted

by $C_{\alpha,\beta,\gamma}$ (M).

Definition 13 (Picture fuzzy Transitive Relation)

A Picture Fuzzy Relation R on M is transitive if R ∘R ⊆ R

Definition 14 (Picture Fuzzy Equivalence Relation)

A PF Relation R on M is referred to as Picture Fuzzy Equivalence Relation if R is reflexive, symmetric and transitive.

Definition 15 (Picture Fuzzy Equivalence Class)

Let R = {((m, n) , μ_A (m, n) , η_A (m, n) , ν_A (m, n)) | (m, n)} ∈ X × X be a picture fuzzy equivalence on a set X . Let, a be an element of X. Then the picture fuzzy set characterised by aR = {(m, aμ_R (m), aη_R (m) , aν_R (m)) |m ∈ X} where (aμ_R) (m) = η_R (a, m) and (aν_R) (m) = ∀m ∈ X is referred to be a PF equivalent class of a with respect to R.

Definition 16

Let R = {((m, n) , μ_A (m, n) , η_A (m, n) , ν_A (m, n)) | (m, n) ∈ X × X } be a relation on set X. Then A is a PF equivalence on X iff $C_{\alpha,\beta,\gamma}$ (R) is a equivalence relation on X with $0 \leq \alpha, \beta, \gamma \leq 1$ and $\alpha + \beta + \gamma \leq 1$, $C_{\alpha,\beta,\gamma}$ (aR) = a the equivalence class of a with the equivalence relation $C_{\alpha,\beta,\gamma}$ (R) in X .

Note Let R ={((m, n) , μ_A (m, n) , η_A (m, n) , ν_A (m, n)) | (m, n)} ∈ X × X be a picture fuzzy equivalence relation on a set X. Then [a] = [b] iff where [a],[b] are equivalence classes of a and b with respect to the equivalence relation $C_{\alpha,\beta,\gamma}$ in X for (a, b) ∈ $C_{\alpha,\beta,\gamma}$ (R) $0 \leq \alpha, \beta, \gamma \leq 1$ and $\alpha + \beta + \gamma \leq 1$,

Note:

1. Intersection of any two-picture fuzzy equivalence relation is also an equivalence relation on the set.
2. Union of two picture fuzzy equivalence relation is not necessarily an equivalence relation.

Algorithm 3

1. From all pair of (μ_i, η_i, γ_i) in the matrix representation of picture fuzzy relation, first choose the maximum value of the membership μ_i, say μ_i^*

2.Then among all possible tuples (μ^*_i , η_i) choose the one with maximum neutral membership value say η^*_i .

3.Now repeat the same for non-membership function. This gives the first $(\tilde{\alpha}, \tilde{\beta}, \tilde{\tilde{?}})$ -cuts of picture fuzzy relation.

4. Cross-out this chosen α- cut value from matrix of picture fuzzy relation. Then repeat the process for rest of the elements (μ_i, η_i, γ_i) of the matrix of picture fuzzy relation.

<u>Example 1:</u>

Let be the fuzzy picture proximity relation matrix

$$A^{(0)}= [< 1,0,0 >< 0,0,0 >< 0.4,0.3,0.2 >< 0,0,0 >< 1,0,0 >< 0.2,0,0.1 >< 0.4,0.3,0.2 >< 0.2,0,0.1 >< 1,0,0 >]$$

First we will check similarity

Applying the max-min composition

$A^{(1)}= A^{(0)}OA^{(0)}$

$A^{(1)}=[< 1,0,0 >< 0,0,0 >< 0.4,0.3,0.2 >< 0,0,0 >< 1,0,0 >< 0.2,0,0.1 >< 0.4,0.3,0.2 >< 0.2,0,0.1 >< 1,0,0 >]$

$A^{(0)}= A^{(1)}$

Here $A^{(0)}$ is similarity matrix

Using above algorithm $< 0,0,0 >, < 0.2,0,0.1 >, < 0.4,0.3,0.2 >$ *and* $< 1,0,0 >$ are

(〈 ® ©)-cut of above matrix

Resolution form of above matrix:

$< 0,0,0 > [1 \ \ 1\ 1\ \ 1\ 1\ 1\ ] + < 0.2,0,0.1 > [1\ \ 0\ 1\ \ 1\ 1\ 1\ ] + < 0.4,0.3,0.2 >$

$[1\ \ 0\ 1\ \ 1\ 0\ 1\ ] + < 1,0,0 > [1\ \ 0\ 1\ \ 0\ 0\ 1\ ]$

Hierarchical Clustering of PFR in Example 1

<u>Example 2:</u>

Let be the fuzzy picture proximity relation matrix

$A^{(0)}= [< 1,0,0 > < 0.4,0.1,0.2 > < 0.47,0.11,0.21 > < 0.5,0.12,0.22 > < 0.51,0.2,0.25 > < 0.4,0.1,0.2 > <$

$1,0,0 > < 0,0,0 > < 0,0,0 > < 0.47,0.11,0.21 > < 0.47,0.11,0.21 > < 0,0,0 > < 1,0,0 > < 0.4,0.1,0.2 > <$

$0.54,0.23,0.22 > < 0.5,0.12,0.22 > < 0,0,0 > < 0.4,0.1,0.2 > < 1,0,0 > < 0.54,0.23,0.22 > <$

$0.51,0.2,0.25 > < 0.47,0.11,0.21 > < 0.54,0.23,0.22 > < 0.54,0.23,0.22 > < 1,0,0 >]$

First we will check similarity

Applying the max-min composition

$A^{(1)}= A^{(0)} O A^{(0)}$

$A^{(1)}=[< 1,0,0 > < 0.4,0.1,0.2 > < 0.47,0.11,0.21 > < 0.5,0.12,0.22 > < 0.51,0.2,0.25 > < 0.4,0.1,0.2 > <$

$1,0,0 > < 0,0,0 > < 0,0,0 > < 0.47,0.11,0.21 > < 0.47,0.11,0.21 > < 0,0,0 > < 1,0,0 > < 0.4,0.1,0.2 > <$

$0.54,0.23,0.22 > < 0.5,0.12,0.22 > < 0,0,0 > < 0.4,0.1,0.2 > < 1,0,0 > < 0.54,0.23,0.22 > <$

$0.51,0.2,0.25 > < 0.47,0.11,0.21 > < 0.54,0.23,0.22 > < 0.54,0.23,0.22 > < 1,0,0 >]$

$A^{(0)}= A^{(1)}$

Here $A^{(0)}$ is similarity matrix

$Using\ above\ algorithm\ < 0,0,0 >, < 0.4,0.1,0.2 >, < 0.47,0.11,0.21 >,$

$< 0.5,0.12,0.22 >< 0.51,0.2,0.25 >, and \; < 1,0,0 >$ are (〈 ® ©)-cut of above matrix

Resolution form of above matrix:

$< 0,0,0 >$ [1 1 1 1 1 1 1 1 1 1 1 1 1 1 1] + $< 0.4,0.1,0.2 >$

[1 1 1 1 0 1 1 0 1 1 1 1 1 1 1] +

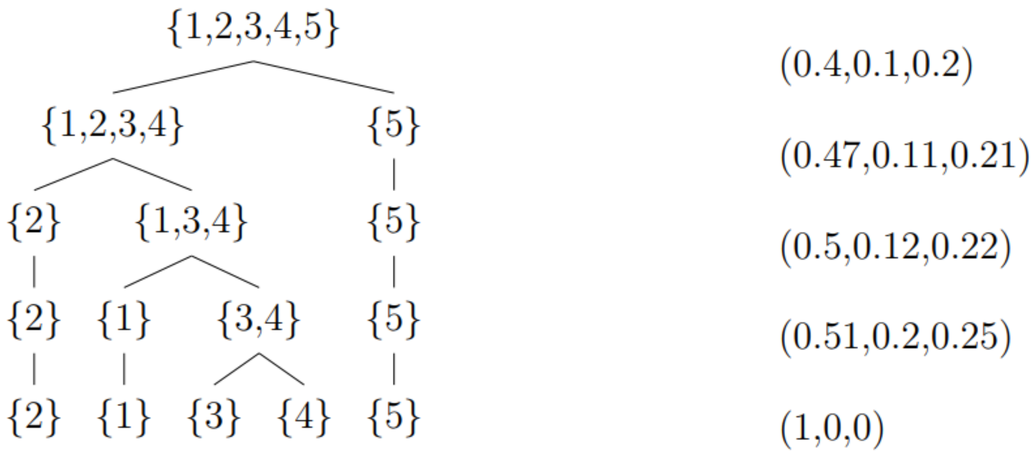
$< 0.47,0.11,0.21 >$ [1 0 1 1 0 1 1 0 0 1 1 1 1 1 1] + $< 0.5,0.12,0.22 >$

[1 0 1 0 0 1 1 0 0 1 1 0 1 1 1]+ $< 0.51,0.2,0.25 >$

[1 0 1 0 0 1 0 0 0 1 1 0 1 1 1] +

$< 1,0,0 >$ [1 0 1 0 0 1 0 0 0 1 0 0 0 0 1]

Hierarchical Clustering of PFR in Example 2



CHAPTER 4 CONCLUSION

In this paper, we started with some basic concepts of fuzzy sets. Later we get the idea of intuitionistic fuzzy set and intuitionistic fuzzy relation. we see the α cuts of Intuitionistic fuzzy relation which is also known as ($\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$) - cuts of IFR. We studied similarity metric for picture fuzzy sets that takes into account not just their four membership functions but also the connections between them. in particular, the connection between the rejection membership as well as positive, neutral, and negative membership. The suggested similarity metric complies with the axiom's definition of the term. The proposed similarity metric is superior to other approaches in differentiating picture fuzzy sets, according to a numerical comparison. It can overcome the shortcomings of some current similarity measures. Additionally, the findings of multi-attribute decision studies demonstrate the validity and superiority of the proposed similarity measure.

Hence, we introduced the ($\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$) - cuts of PFR. We developed the algorithm for α - cuts of PFR. With the help of algorithm, one can find the α - cuts of picture fuzzy set using relation and converting it to the picture fuzzy similarity relation. Finally, we applied the ($\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$α, β, γ) cuts of PFR in the Hierarchical clustering. We cluster the ($\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$) cuts of PFR using its resolution form. Further Hierarchical Clustering of PFR can be used to find the interval valued PFR. There we can deal with ($\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$)- cuts of PFR in interval form.

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