Transportation Problem & Its Application

A Dissertation report

submitted in fulfilment of the requirements

for the award of the degree of

M.Sc Mathematics

Submitted by

Nitish Kumar

(2K19/MSCMAT/17)

Under the Supervision of

Dr. L N Das

DEPT. OF APPLIED MATHEMATICS

DELHI TECHNOLOGICAL UNIVERSITY

Applied Mathematics

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly delhi college of Engineering)

Bawana Road Delhi-110042

CANDITATE'S DECLARATION

I Nitish Kumar ,Roll no. 2K19/MSCMAT/17 students of M.Sc.(Mathematics), hereby declare that the project titled "Transportation Problem & Its Application" which is submitted by us to the Department of Applied Mathematics ,Delhi technological University, Delhi in partial fulfilment of the requirements for the award of the degree of Master of Science , is original and not copied from any source without proper citation. this work has not previously formed the basis for the award of any degree, Diploma Associateship,Fellowship or other similar title or recognition.

Place: Delhi — Name Nitish Kumar

Date-May,2021 — (2K19/MSCMAT/17)

Certificate

I hereby certify that the Project titled ”Transportation Problem & Its Application” which is submitted by Nitish Kumar ,Roll no. 2K19/MSCMAT/17 Delhi Technological University,Delhi in partial fulfillment of the requirement for the award of the degree of Master of Science , is a record of the project work carried out by the students under my supervision. To the best of my knowledge this work has been not submitted in part or full for any Degree or Diploma to this University or elsewhere.

Place: Delhi — Dr. L N Das

Date- May, 2021 — (Department of Applied Mathematics , Delhi)

Abstract: I have briefly reviewed the transportation problem formulation, computational steps to solve the transportation problem model. The method detail steps, such as northwest corner rule, least cost method, Voggel's approximation methods are rewritten to obtain the initial basic feasible solution for the test of optimality is discussed. The modified distribution method is discussed to test the optimality and improvement of the initial basic feasible solution. I have described a transhipment problem model to discuss the nature of problem and solving process of the problem. Finally, I have mentioned the multi modal transportation problem such as the transportation of crude oil, through the train vessels, transportation through ship vessels and either ships or trains which one will be least cost. I have solved the problems by using the software TORA.

ACKNOWLEDGEMENTS

It is an incredible joy for me to communicate my regard and profound feeling of appreciation to my M.Sc supervisor **Dr. L N Das**, Professor, Department of Applied Mathematics , Delhi Technological University , Delhi for his wisdom , vision , ability , direction eager contribution and tireless consolation during the arranging and improvement of this research work. I also thankfully acknowledge his meticuious endeavors in completely going through and improving the original copies without which this work couldn't have been completed.

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Introduction

Transportation Problem

Transportation problem is a special programming problems which have been applied in many real life situation. One of the key feature of the transportation problem is that they need a large number of constraints and variables, so the application of the simplex method will turn out to be highly inefficient. In transportation problem, we transport only single item. The coefficient matrices of transportation problem have a special structure and due to this specific structure, we can develop a specified version of the simplex method for solving the problem that achieves dramatic computational saving in its implementation.

Mathematical Formulations

Sppose that O_1, O_2, O_3 are some origins or supply locations and in general, there could be *m* number of origins. On the other hand, we have D_1, D_2, D_3, D_4 are some destinations or demand locations and in general, there could be *n* number of destinations.

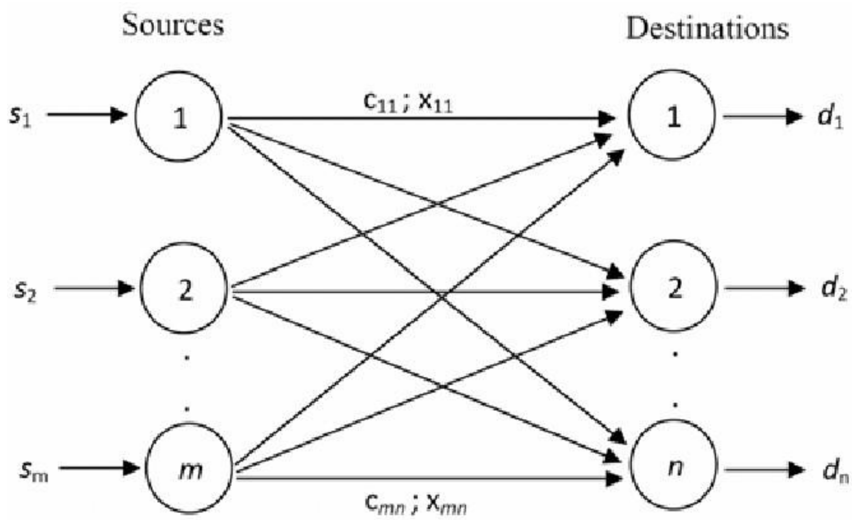


Figure 1: Transportation Problem

A product has to be supplied from the sources to the destinations. Let the cost for supplying the commodity from the *ith* sources to the *jth* destination be C_{ij}. Let the number of units that should be transported, be x_{ij} i.e., the number of units to be transported from *ith* sources to the *jth* destination. We want to determine x_{ij} such that the overall cost is minimized.

Suppose that there are *m* number of sources and *n* number of destinations. Let the availability at the origins be a_i, $i = 1, 2, 3, \ldots, m$ and the demands at the destination be b_j, $j = 1, 2, 3, \ldots, n$ and $a_i \geq 0$, $b_j \geq 0$ $\forall$ i,j. The transportation cost of 1 unit of the goods from the *ith* sources to the *jth* destination is c_{ij} belongs to R.

If $a_1 + a_2 + a_3 + \ldots + a_m = b_1 + b_2 + b_3 + \ldots + b_n$ is satisfied, then it is called a balanced transportation problem. However, if this is not satisfied then it is called a unbalanced problem and in order to convert a unbalanced transportation problem into a balanced transportation problem, we need to add fictitious sources and destinations in such a way that their cots are assigned as zero.

Suppose x_{ij} be the number of units that will be transported from ith source to jth destinations and $\forall$ $x_{ij} \geq 0$ should be integral. So, our objective function is to minimize the overall cost i.e.,

$$\text{Minimize} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} cijxij$$

Subject to

$$\sum_{j=1}^{n} xij \leq ai, i = 1 \; to \; m$$

$$\sum_{i=1}^{m} xij \geq bj, j = 1 \; to \; n$$

$$Xij \geq 0, \forall i, j$$

But, $\forall cij \geq 0 \Rightarrow \sum_{i=1}^{m} xij = bj$

And we have a balanced transportation problem,

So $\sum_{i=0}^{m} ai = \sum_{j=0}^{n} bj, \forall j \Rightarrow \sum_{j=1}^{n} xij = ai, \forall i$

Now, LPP will be

$$\text{Minimize} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} cijxij$$

Subject to

$$\sum_{j=1}^{n} xij = ai, i = 1 \; to \; m$$

$$\sum_{i=1}^{m} xij = bj, j = 1 \; to \; n$$

$$xij \geq 0, \forall i, j$$

The objective function can be written as $c^T x$, then constraints will be $Ax = b, x \geq 0$ where A is coefficient matrix,

c = $col(c11, \dots, c1n, \dots, cm1, \dots, cmn) \in R^{m\times n}$,

$x = col(x11, .., x1n, \dots, xm1, \dots, cmn) \in R^{m\times n}$ and

$b = (a1, a2, .., am, b1, b2, \dots, bn)$.

$$x11 + x12 + \cdots + x1n = a1$$

$$x21 + x22 + \cdots + x2n = a2$$

……………………………………………

……………………………………………

$$xm1 + xm2 + \cdots + xmn = am$$

$$x11 + x21 + \cdots + xm1 = b1$$

$$x12 + x22 + \cdots + xm2 = b2$$

…………………………………………..

…………………………………………..

$$x1n + x2n + \cdots + xmn = bm$$

Then, for its dual,introduce dual variable ui, vj for $i = 1,2, \dots, m, j = 1,2, \dots, n$. In primal, all xij appears exactly in 2 constraints. As many dual constraints as a number of primal variables. So, if we have 12 primal variables,
We have 12 dual constraints.
Dual will be

Maximize $\sum_{i=1}^{m} aiui + \sum_{j=1}^{n} bjvj$

Subjet to

$$ui + vj \leq cij \ \ \forall i, j$$

ui, vj are unrestricted

(As many dual variables as a number of primal constraints, and as all the primal constraints have equality, dul variables will be unrestricted.)

BALANCED AND UNBALANCED TRANSPORTATION PROBLEM

A transportation problem is said to be balanced if total supply = total demand. Let us see what this implies. We know that in order to meet the demands of all the destination points, the supplies or the total availability should be greater than or equal to the total requirement , only then sufficient quantities can be sent to meet the requirements.

Therefore to have a feasible solution total supply $\geq$ total demand. If we observe closely we notice that since it is a transportation problem, we must not transfer more than what is required of us since transporting an additional quantity of the goods over and above the demand is only going to increase my transportation cost. Therefore even if total supply $\geq$ total demand it is sufficient to say that in order to obtain a feasible solution and able to meet the demand total supply should be equal to total demand. Thus in this case it would become a balanced transportation problem. In transportation model, if the problem is unbalanced and total supply … total demand or total supply … total demand, thus we would make it into a balanced problem by adding a dummy location so that the supply can get exhausted or demand can be exhausted.

Transportation Algorithm

Step:-1. Identify an initial basic feasible solution.

2. Testing the solution for optimality.

3. Improving solution when it is not optimal.

4. Repeating step 2 and step 3 until the optimal solution is obtained.

We have different method to find IBF solution and to find optimal solution. They are

Identifying initial basic solution
⇒ Northwest Corner Rule

⇒ Vogel's Approximation Method

⇒ Least Cost Method

Optimal solution
⇒ Stepping Stone Method

⇒ MODI Method (Modified Distribution Method) or u-v method

Basic feasible solution to a transportation problem
A Basic feasible solution to a transportation problem satisfies the following conditions:

1. The row column or supply demand constraints are satisfied.

2. The non-negativity constraints are satisfied.

3. The allocations are independent and do not from a loop.

4. There are exactly ($m + n - 1$) allocations.

Northwest Corner Rule

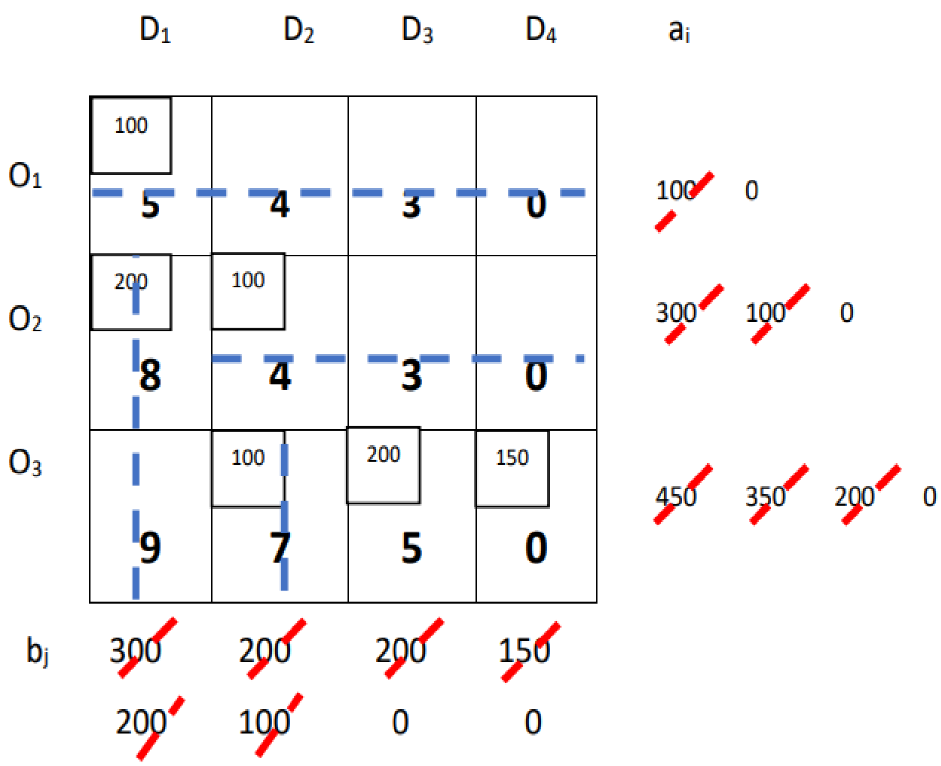
Steps of Northwest corner rule to find initial BFS are:

Step-1: We will start with the northwest cell that is cell (1,1). Then, allocate as much as possible equal to min (a_1 , b_1).

Step-2: If allocation (step-1) is equal to supply available at first source a_1 and move vertically down to cell (2,1) and we will supply step-1 again for our next allocation. OR-If allocation (in step-1) is equal to demand of first destination b_1 and move horizontally to cell (1,2) and we will apply step-1 for next allocation. OR-If $a_1 = b_1$ then allocate $x_{11} = a_1$ or b_1 then move diagonally to cell (2,2).

Step-3: Repeat above steps till all cells are allocated.

Example:-



We have considered an unbalanced transportation problem in which total supply is greater than total demand so we introduce a dummy column say D_4.

So, the initial basic feasible solution is
x_{11} = 100, x_{21} = 200, x_{22} = 100, x_{32} = 100, x_{33} = 200, x_{34} = 150.

Therefore, Total Transportation cost = $(100 \times 5) + (200 \times 8) + (100 \times 4) + (100 \times 7) + (200 \times 5) + (150 \times 0) = 4200$

Hence, the minimum transportation cost obtained through Northwest corner method is 4200.

Least Cost Method

Steps of Least cost method to find initial BFS are:

Step-1: Determine the least cost cell in entire table, and allocate as much as possible to this cell then eliminate that row or column in which either supply or demand is exhausted.

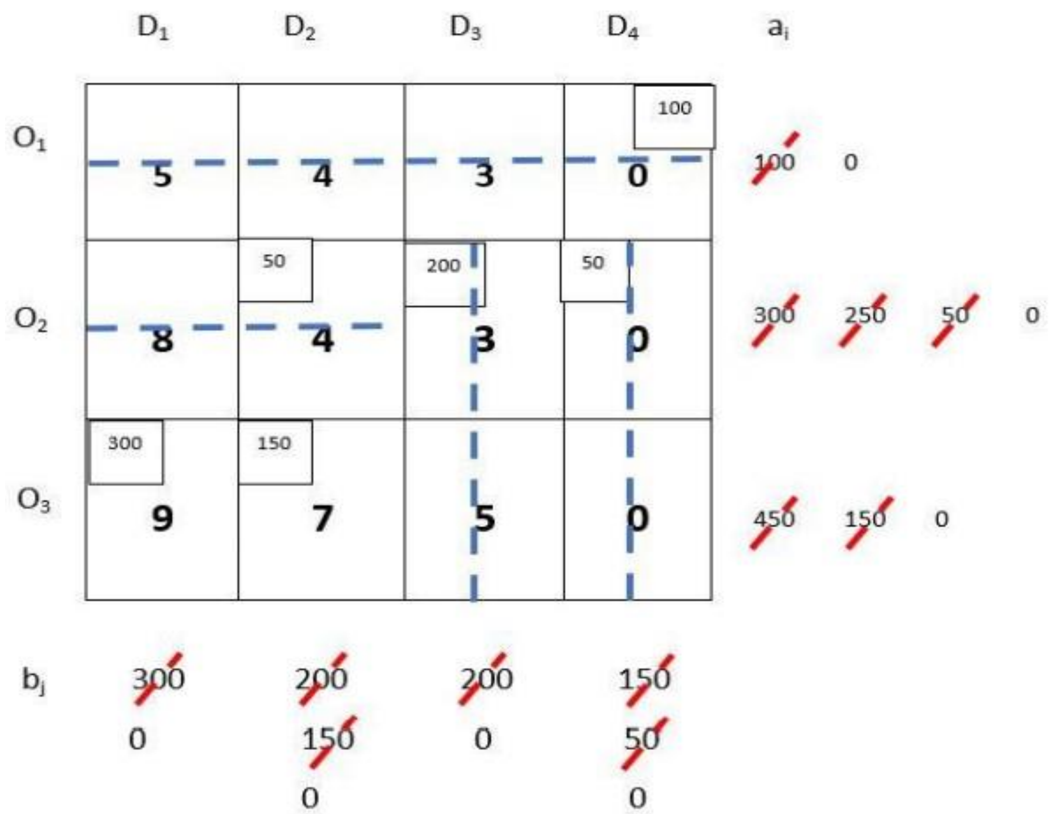
⇒ If we have a situation in which both row and column are satisfied simultaneously then only one may be crossed out.

⇒ In case that smallest cell is not unique, select cell where maximum allocation can be made.

Step-2: Repeat with next lowest unit cost among the remaining rows and column and allocate to this cell as much as possible then eliminate exhausted row and column.

Step-3: Repeat above step until entire supply and demand is satisfied.

Example:-



We have considered an unbalanced transportation problem in which total supply is greater than total demand so we introduce a dummy column say D_4.

So, the initial basic feasible solution is
$x_{14} = 100$, $x_{22} = 50$, $x_{23} = 200$, $x_{24} = 50$, $x_{31} = 300$, $x_{32} = 150$.

Therefore, Total Transportation cost = $(100 \times 0) + (50 \times 4) + (200 \times 3) + (50 \times 0) + (300 \times 9) + (150 \times 7) = 4550$

Hence, the minimum transportation cost obtained through least cost method is 4550

Vogel's Approximation Method

Steps of Vogel's approximation method to find initial BFS are:

Step-1: Find the difference between smallest and next smallest cost of each row and column called penalties.

Step-2: Select row or column with the largest penalty then allocate as much as possible in cell having least cost in that row or column.

⇒ In case of tie, select maximum allocation row or column.

Step-3: Adjust the supply and demand then strike out the satisfied row or column.
If row and column are simultaneously satisfied then only one row (or one column) is to strike out and the other one column (or row) is located 0.
Any row or column with 0 supply or 0 demand should not be used in future penalties.

Example:-

	D_1	D_2	D_3	D_4	a_i	Row Penalty
O_1	100 5	4	3	0	100	3 1 - -
O_2	8	200 4	100 3	0	300	3 1 1 5
O_3	200 9	7	100 5	150 0	450	5 2 2 4
b_j	300	200	200	150		
Column Penalty	3	0	0	0		
	3	0	0	-		
	1	3	2	-		
	1	-	2	-		

We have considered an unbalanced transportation problem in which total supply is greater than total demand so we introduce a dummy column say *D_4.*

So, the initial basic feasible solution is

$x_{11} = 100$, $x_{22} = 200$, $x_{23} = 100$, $x_{31} = 200$, $x_{33} = 100$, $x_{34} = 150$.

Therefore, Total Transportation cost = $(100 \times 5) + (200 \times 4) + (100 \times 3) + (200 \times 9) + (100 \times 5) + (150 \times 0) = 3900$

Hence, the minimum transportation cost obtained through Vogel's Approximation method is 3900

Optimality Test

Test to check whether the obtained feasible solution is optimal or not. Of course an optimality test is performed on the feasible solution in which:

⇒ No. of allocation should be equal to $(m + n - 1)$ where m is no. of rows and n is no. of columns.

⇒ $(m + n - 1)$ allocations have to be in independent position or should not from a loop.

MODIFIED DISTRIBUTION (MODI) METHOD:

Algorithm for MODI method is

Step-1: Firstly construct the transportation table by filling in the demands of the destinations and supplies of the origin, also fill in the transportation cost in all the cells.

Step-2: Find the initial basic feasible solution of the transportation problem, (generally VOGEL APPROXIMATION METHOD) . Prepare the solution table.

Step-3: Check the Initial Basic Feasible solution is Degeneracy or non-degeneracy (if no. of occupied cells is equal to $m + n - 1$ then IBF solution is non-degeneracy otherwise degeneracy)

Step-4: Assign u_1 , u_2 , u_3 and v_1 , v_2, v_3 , v_4 for rows and columns respectively then for all the basic variables x_{ij} (allocated cells/occupied cells) solve the equation $c_{ij} = x_i + v_j$
For every i and j, starting initially with some $u_i = 0$ and thereafter calculate the other values accordingly.

Step-5: Calculate opportunity cost for each non occupied cells $d_{ij} = c_{ij} - (u_i + v_j)$. This is done to check whether it is advantageous to allocate.

Step-6: If $d_{ij} = c_{ij} - (u_i + v_j)$ is greater than or equal to 0 for all i, j , optimal solution has reached.

Step-7: If $d_{ij} = c_{ij} - (u_i + v_j)$ is less than 0 , then a loop is formed from the most negative value, the allocation are independent. We choose a value α which is the no. of units of goods which would be added or subtracted to minimise the transportation cost . α is assigned the largest possible value such that a basic variable becomes 0.

Step-8: Repeat the above process till all the d_{ij} are greater than equal to 0, and then optimality will be reached.

Checking optimality for VAM for same example:

	D_1	D_2	D_3	D_4	
O_1	[100] 5	4	3	0	$u_1=-4$
O_2	8	[200] 4	[100] 3	0	$u_2=-2$
O_3	[200] 9	7	[100] 5	[150] 0	$u_3=0$
	$v_1=9$	$v_2=6$	$v_3=5$	$v_4=0$	

No. of occupied cells = 6

And $m+n-1=3+4-1=6$

Therefore the IBF solution is non-degeneracy.

Calculate $c_{ij} = u_i + v_j$ we get u_1 , u_2 , u_3 , v_1 , v_2 , v_3 , v_4

Let $u_3 = 0$ (Because, max no. of of occupied cells in third row)

$c_{31} = u_3 + v_1 \Rightarrow 9 = 0 + v_1 \Rightarrow v_1 = 9$

$c_{33} = u_3 + v_3 \Rightarrow 5 = 0 + v_3 \Rightarrow v_3 = 5$

$c_{34} = u_3 + v_4 \Rightarrow 0 = 0 + v_4 \Rightarrow v_4 = 0$

$c_{23} = u_2 + v_3 \Rightarrow 3 = u_2 + 5 \Rightarrow u_2 = -2$

$c_{22} = u_2 + v_2 \Rightarrow 4 = -2 + v_2 \Rightarrow v_2 = 6$

$c_{11} = u_1 + v_1 \Rightarrow 5 = u_1 + 9 \Rightarrow u_1 = -4$

Then Calculate opportunity cost for each non-occupied cells.

$$d_{ij} = c_{ij} - (u_i + v_j)$$

$d_{12} = c_{12} - (u_1 + v_2) = 4 - (-4 + 6) = 2$

$d_{13} = c_{13} - (u_1 + v_3) = 3 - (-4 + 5) = 2$

$d_{14} = c_{14} - (u_1 + v_4) = 0 - (-4 + 0) = 4$

$d_{21} = c_{21} - (u_2 + v_1) = 8 - (-2 + 9) = 1$

$d_{24} = c_{24} - (u_2 + v_4) = 0 - (-2 + 0) = 2$

$d_{32} = c_{32} - (u_3 + v_2) = 7 - (0 + 6) = 1$

Since all of them are positive for non occupied cells,

Hence Solution is Optimal.

i.e., Optimal solution is
$x_{11} = 100$, $x_{22} = 200$, $x_{23} = 100$, $x_{31} = 200$, $x_{33} = 100$, $x_{34} = 150$.

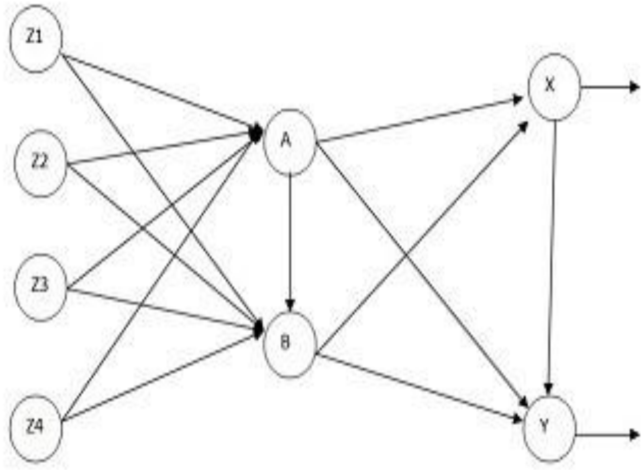
And minimum transportation cost is
$=(100 \times 5) + (200 \times 4) + (100 \times 3) + (200 \times 9) + (100 \times 5) + (150 \times 0) = 3900$

Transhipment Problem

Rubber has been found in one of the thrust area of Tripura. The second largest producer of rubber, after Kerala, is Tripura in India. At present ,Tripura has 58000 hectares area under rubber production and nearly produced 28000 tone of natural rubber but they can only be utilized 6-7% of total production. Tripura export rubber to many states and many countries which includes neighbouring country Bangladesh as one of the major rubber importer from Tripura. We have developed a transhipment model on basis of data collected from local agencies. Tripura has 23 sub-divisions which further divided into 4 zones and two land custom stations has been considered as buffer for transportation . Consider four zones, say z_1 , z_2 , z_3, z_4 , A(Akhaura) and B(Srimantapur) as transit centres for land custum stations and X(Dhaka) , Y(Comilla) as destinations where rubber will be exported. Data that we collected from source is:-

	A	B	X	Y	Supply
Z_1	2750	3500	--	--	1000
Z_2	2675	2500	--	--	700
Z_3	5400	2600	--	--	1200
Z_4	7600	9780	--	--	600
A	0	3400	9000	16750	
B	--	0	15650	1600	
X	--	--	0	7500	
Demand			2000	1500	

Transportation costs in table is in Rubber per 25 tons. We construct a transshipment network between sources and destinations on the basis of data collected from the sources.



Transhipment Network between sources and destination

Model Formulation

We can convert the transshipment model into regular transportation model with 7 sources i.e., (Z_1 , Z_2 , Z_3 , Z_4 , A , B , X) and with 4 destinations i.e., (A , B , X , Y).
The formulated model is

Minimize $Z = \sum\sum c_{ij}x_{ij}$ where i = 1 to 7 ; j = 1 to 4
s.t.

$\sum x_{ij} = a_i$ where i = 1 to 7 ; j = 1 to 4

$\sum x_{ij} = b_j$, j = 1 to 4, i = 1 to 7

$X_{ij} \geq 0 \ \forall$ i,j

Where x_{ij} = no. of units of product (in ton) to be transported from sourses $i \ (i = 1 \ to \ 7)$ to destination $j \ (j = 1 \ to \ 4)$ and c_{ij} = cost of units in Ruppes per 25 tons.

How to compute the amt. of supply and demand at different nodes.

Supply at the pure supply node i.e., Z_1 , Z_2 , Z_3 , Z_4 is equal to original supply

Demand at the pure demand node i.e., X , Y is equal to original demand

Supply at the transshipment node i.e., A , B , X is equal to the sum of original supply and buffer amount

Demand at the transhipment node is the sum of original demand and buffer amount.

In this problem buffer amount is the total supply or demand i.e., 1000 + 700 + 1000 + 800 = 3500 tons.

It is not possible to transport anything from certain sources to certain destinations. But it can be done by assigning very large cost by 10000000.

Solution

We will apply Vogel's Approximation Method to solve the problem.

<table>
<tr><th></th><th>D1</th><th>D2</th><th>D3</th><th>D4</th><th>Supply</th><th>Row Penalty</th></tr>
<tr><td>S1</td><td>2700
(1000)</td><td>3550</td><td>10000000</td><td>10000000</td><td>1000</td><td>850|850|850|850|850|850
850|850|2700| - |</td></tr>
<tr><td>S2</td><td>2670
(200)</td><td>2500
(500)</td><td>10000000</td><td>10000000</td><td>700</td><td>170|170|170|170|170|170|
170|170|2670|2670|</td></tr>
<tr><td>S3</td><td>5450</td><td>2660
(1000)</td><td>10000000</td><td>10000000</td><td>1000</td><td>2790|2790|2790|2790|
2790|2790|- | - | - | - |</td></tr>
<tr><td>S4</td><td>7610
(800)</td><td>9700</td><td>10000000</td><td>10000000</td><td>800</td><td>2090|2090|2090|2090|
2090|2090|2090|- |- |- |</td></tr>
<tr><td>S5</td><td>0
(1500)</td><td>3400</td><td>9000
(2000)</td><td>16750</td><td>3500</td><td>3400|3400|3400|3400|
3400|- |- |- |- |- |</td></tr>
<tr><td>S6</td><td>10000000</td><td>0
(2000)</td><td>15650</td><td>1600
(1500)</td><td>3500</td><td>1600|1600|15650|- |- |- |
- |- |- |- |</td></tr>
<tr><td>S7</td><td>10000000</td><td>10000000</td><td>0
(3500)</td><td>7500</td><td>3500</td><td>7500|- |- |- |- |- |- |- |- |- |</td></tr>
<tr><td>Demand</td><td>3500</td><td>3500</td><td>5500</td><td>1500</td><td>3500</td><td></td></tr>
<tr><td>Column Penalty</td><td>2670
2670
2670
2670
2670
30
30
30
30
2670</td><td>2500
2500
2500
160
160
160
1050
1050</td><td>9000
6650
6650
9991000</td><td>5900
15150</td><td></td><td></td></tr>
</table>

IBFS are:-

$x_{11} = 1000$, $x_{21} = 200$, $x_{22} = 500$, $x_{32} = 1000$, $x_{41} = 1500$, $x_{51} = 1500$
$x_{53} = 2000$, $x_{62} = 2000$, $x_{64} = 1500$, $x_{73} = 3500$

The minium total transportation cost is $2700 * 1000 + 2670 * 200 + 2500 * 500 + 2660 * 1000 + 7610 * 800 + 0 *$

$$1500 + 9000 * 2000 + 0 * 2000 + 1600 * 1500 + 0 * 3500 = 33632000$$

Optimality Test MODI Test

	D1	D2	D3	D4	Supply	u_i
S1	2700 (1000)	3550 (1020)	10000000 (9988300)	10000000 (9995870)	1000	2700
S2	2670 (200)	2500 (500)	10000000 (9988330)	10000000 (9995900)	700	2670
S3	5450	2660 (1000)	10000000 (9988170)	10000000 (9995740)	1000	2830
S4	7610 (800)	9700 (2260)	10000000 (9983390)	10000000 (9990960)	800	7610
S5	0 (1500)	3400 (3570)	9000 (2000)	16750 (15320)	3500	0
S6	10000000 (9999830)	0 (2000)	15650 (6480)	1600 (1500)	3500	170
S7	10000000 (10009000)	10000000 (19170)	0 (3500)	7500 (15070)	3500	-9000
Demand	3500	3500	5500	1500	3500	
v_j	0	-170	9000	1430		

Since all $d_{ij} \geq 0$, so final optimal solution is arrived.

	D1	D2	D3	D4	Supply
S1	2700 (1000)	3550	10000000	10000000	1000
S2	2670 (200)	2500 (500)	10000000	10000000	700
S3	5450	2660 (1000)	10000000	10000000	1000
S4	7610 (800)	9700	10000000	10000000	800
S5	0 (1500)	3400	9000 (2000)	16750	3500
S6	10000000	0 (2000)	15650	1600 (1500)	3500
S7	10000000	10000000	0 (3500)	7500	3500
Demand	3500	3500	5500	1500	3500

The minium total transportation cost is $2700 * 1000 + 2670 * 200 + 2500 * 500 + 2660 * 1000 + 7610 * 800 + 0 * 1500 + 9000 * 2000 + 0 * 2000 + 1600 * 1500 + 0 * 3500 = 33632000$

Transportation : Case Study

There is a petroleum company which is responsible for shipping of crude oil from its oil refineries to distribution centres.The availability of oil barrels in refineries and the demand of oil barrels in distribution centres are given in the table below along with the unit transportation costs .In the past the company use to ship oil barrels through train as a means of transport , but now due to increasing prices of the rails the company is set to investigate an alternate shipping plan through ships (when feasible).Shipping cost for both means of transport are: The new alternative of transporting through

	1	2	3	4	5	6	7	8	Supply (in barrels)
1	30	32	44	35	30	19	30	29	1200
2	31	37	19	29	17	21	42	19	550
3	25	12	16	30	42	34	17	33	1300
4	19	28	34	26	39	31	35	45	400
5	36	22	26	28	50	27	20	28	1150
6	30	33	40	17	28	13	39	34	400
demand	500	700	1000	600	450	800	450	500	

Figure 1: Unit cost through train in hundreds.

	1	2	3	4	5	6	7	8	Supply (in barrels)
1	18	19	15	-	18	16	14	-	1200
2	-	12	21	19	20	14	-	21	550
3	12	17	17	19	-	25	23	19	1300
4	21	19	-	15	16	21	15	15	400
5	-	15	16	23	25	17	-	16	1150
6	15	12	-	17	21	23	14	-	400
demand	500	700	1000	600	450	800	450	500	

Figure 2: Unit cost through ship in hundreds.

	1	2	3	4	5	6	7	8
1	100	120	130	-	170	90	89	-
2	-	190	195	190	110	120	-	110
3	210	327	120	140	-	105	170	130
4	130	128	-	125	150	170	90	70
5	-	150	180	128	170	119	-	140
6	106	171	-	170	150	132	170	-

Figure 3: Investment for ships in hundreds

Ships require some investment. The capital investment in thousands of rupees for ships for transporting oil barrels annually along each rout is given as follows: Considering the good life expectancy of the ships and the time value of money, the equivalent uniform annual cost of these investment is one-tenth the amount given in the table. Being part of the management the task is to make a shipping plan to determine which rout should give the minimal transportation cost. The problem can be divided into 3 scenarios.

Scenario 1: Continue shipping crude oil barrels entirely through rails.

Scenario 2: Move to new shipping route exclusively by water (except where it is not feasible).

Scenario 3: Ship by either train or ships, depending on which is less expensive for the particular rout.

SCENARIO-1

Objective – to minimize the transportation cost by shipping of oil through trains.

Decision variables- x_{ij} where i = 1 to 6 and j = 1 to 8 and x_{ij} is the amount shipped from ith refinery to jth distribution centre.

Consider the figure-1, Let the entries in the table which are the transportation cost be denoted by a_{ij} and the supply entries be denoted by s_i and demand entries by d_j . The objective is to

Objective function : Minimise $\sum_{i=1}^{j=6} \sum_{i=1}^{j=8} a_{ij}x_{ij}$

Constraints

$\sum x_{ij} \leq s_i$

$\sum x_{ij} \geq d_i$

Non-negativity $x_{ij} \geq 0$

Solving the above linear programming problem as a transportation model in TORA software would give final iteration table as

	Name		v1=30.00	v2=23.00	v3=27.00	v4=29.00	v5=27.00	v6=19.00	v7=21.00	v8=29.00	
			30.00	32.00	44.00	35.00	30.00	19.00	30.00	29.00	
S1		u1=0.00	100					800		300	1200
			0.00	-9.00	-17.00	-6.00	-3.00	0.00	-9.00	0.00	
			31.00	37.00	19.00	29.00	17.00	21.00	42.00	19.00	
S2		u2=-10.00					450			100	550
			-11.00	-24.00	-2.00	-10.00	0.00	-12.00	-31.00	0.00	
			25.00	12.00	16.00	30.00	42.00	34.00	17.00	33.00	
S3		u3=-11.00		700	600						1300
			-6.00	0.00	0.00	-12.00	-26.00	-26.00	-7.00	-15.00	
			19.00	28.00	34.00	26.00	39.00	31.00	35.00	45.00	
S4		u4=-11.00	400								400
			0.00	-16.00	-18.00	-8.00	-23.00	-23.00	-25.00	-27.00	
			36.00	22.00	26.00	28.00	50.00	27.00	20.00	28.00	
S5		u5=-1.00			400	200			450	100	1150
			-7.00	0.00	0.00	0.00	-24.00	-9.00	0.00	0.00	
			30.00	33.00	40.00	17.00	28.00	13.00	39.00	34.00	
S6		u6=-12.00				400					400
			-12.00	-22.00	-25.00	0.00	-13.00	-6.00	-30.00	-17.00	
	Demand		500	700	1000	600	450	800	450	500	

Figure 4: Final iteration optimality table

Thus the final computations are given as follows:

Title: Scenario-1
Final Iteration No.: 4
Objective Value (minimum cost) =96650.00

From	To	Amt Shipped	Obj Coeff	Obj Contrib
S1:	D1:	100	30.00	3000.00
S1:	D6:	800	19.00	15200.00
S1:	D8:	300	29.00	8700.00
S2:	D5:	450	17.00	7650.00
S2:	D8:	100	19.00	1900.00
S3:	D2:	700	12.00	8400.00
S3:	D3:	600	16.00	9600.00
S4:	D1:	400	19.00	7600.00
S5:	D3:	400	26.00	10400.00
S5:	D4:	200	28.00	5600.00
S5:	D7:	450	20.00	9000.00
S5:	D8:	100	28.00	2800.00
S6:	D4:	400	17.00	6800.00

Conclusion The total minimised cost of transportation would be Rs 9665000 when opted transportation by rail.

SCENARIO-2 Objective - to minimize the transportation cost by shipping of oil through ships wherever feasible. Decision variables- xij where i = 1 to 6 and j = 1 to 8 and xij is the amount shipped from ith refinery to jth distribution centre.Consider the figure -2 ,Let the entries in the table which are the transportation cost be denoted by bij and the supply entires be denoted by si and demand entries by dj .Let yij be the investment required for ships on the route.The objective is to minimise the total transportation cost along with investment costs attached to it

Objective function Minimise $\sum_{i=1}^{j=6} \sum_{i=1}^{j=8} (\boldsymbol{b}_{ij} + 0.1y_{ij})$

Constraints $\sum x_{ij} \leq s_i$

$\sum x_{ij} \geq d_i$

Non-negativity $x_{ij} \geq 0$

The new transportation table for this scenario can be constructed by adding the investments and the transportation costs . For the routes where shipping via ships is not feasible it is assumed that the transportation will be made by rails only and those values are represented in red in the table.

	1	2	3	4	5	6	7	8	Supply (in barrels)
1	28	31	28	35	35	25	22.9	29	1200
2	31	31	40.5	38	31	26	42	32	550
3	33	49.7	29	33	42	35.5	40	32	1300
4	34	31.8	34	27.5	31	38	24	24	400
5	36	30	34	35.8	42	28.9	20	30	1150
6	25.6	29.1	40	34	36	36.2	31	34	400
demand	500	700	1000	600	450	800	450	500	

Solving the above linear programming problem as a transportation model in TORA software would give final iteration

	Name										
			v1=28.00	v2=30.00	v3=28.00	v4=32.00	v5=30.00	v6=25.00	v7=20.00	v8=29.00	
			28.00	31.00	28.00	35.00	35.00	25.00	22.90	29.00	
S1		u1=0.00	100		300			700		100	1200
			0.00	-1.00	0.00	-3.00	-5.00	0.00	-2.90	0.00	
			31.00	31.00	40.50	38.00	31.00	26.00	42.00	32.00	
S2		u2=1.00		0			450	100			550
			-2.00	0.00	-11.50	-5.00	0.00	0.00	-21.00	-2.00	
			33.00	49.70	29.00	33.00	42.00	35.50	40.00	32.00	
S3		u3=1.00			700	600					1300
			-4.00	-18.70	0.00	0.00	-11.00	-9.50	-19.00	-2.00	
			34.00	31.80	34.00	27.50	31.00	38.00	24.00	24.00	
S4		u4=-5.00								400	400
			-11.00	-6.80	-11.00	-0.50	-6.00	-18.00	-9.00	0.00	
			36.00	30.00	34.00	35.80	42.00	28.90	20.00	30.00	
S5		u5=0.00		700					450		1150
			-8.00	0.00	-6.00	-3.80	-12.00	-3.90	0.00	-1.00	
			25.60	29.10	40.00	34.00	36.00	36.20	31.00	34.00	
S6		u6=-2.40	400								400
			0.00	-1.50	-14.40	-4.40	-8.40	-13.60	-13.40	-7.40	
	Demand		500	700	1000	600	450	800	450	500	

Thus the final computations are given as follows:

Title: scenario 2
Final Iteration No.: 3
Objective Value (minimum cost) =138090.00

From	To	Amt Shipped	Obj Coeff	Obj Contrib
S1:	D1:	100	28.00	2800.00
S1:	D3:	300	28.00	8400.00
S1:	D6:	700	25.00	17500.00
S1:	D8:	100	29.00	2900.00
S2:	D2:	0	31.00	0.00
S2:	D5:	450	31.00	13950.00
S2:	D6:	100	26.00	2600.00
S3:	D3:	700	29.00	20300.00
S3:	D4:	600	33.00	19800.00
S4:	D8:	400	24.00	9600.00
S5:	D2:	700	30.00	21000.00
S5:	D7:	450	20.00	9000.00
S6:	D1:	400	25.60	10240.00

Conclusion The total minimised cost of transportation would be Rs 13809000 when opted transportation by ships wherever feasible..

SCENARIO-3

Objective - to minimize the transportation cost by selecting the less expensive mode of transportation for each route. Decision variables- xij where i = 1 to 6 and j = 1 to 8 and xij is the amount shipped from ith refinery to jth distribution centre.

Objective function Minimise $\sum_{i=1}^{j=6}\sum_{i=1}^{j=8} \boldsymbol{minimum}(\boldsymbol{a}_{ij}x_{ij}, (b_{ij} + 0.1y_{ij})\, x_{ij})$

Constraints $\sum x_{ij} \leq s_i$

$\sum x_{ij} \geq d_i$

Non-negativity $x_{ij} \geq 0$

The new transportation table for this scenario can be constructed by considering the minimum of the costs incurred in the above two scenarios for each route. For each of the routes the minimum cost is given in the following table.

	1	2	3	4	5	6	7	8	Supply (in barrels)
1	20	31	28	35	30	19	22.9	29	1200
2	31	31	19	29	17	21	42	19	550
3	25	12	16	30	42	34	17	32	1300
4	19	28	34	26	31	31	24	24	400
5	36	22	26	28	42	27	20	28	1150
6	25.6	29.1	40	17	28	13	31	34	400
demand	500	700	1000	600	450	800	450	500	

Solving the above linear programming problem as a transportation model in TORA software would give final iteration Thus the final computations are given as follows:

	Name										
			v1=20.00	v2=19.00	v3=23.00	v4=25.00	v5=23.00	v6=19.00	v7=17.00	v8=25.00	
			20.00	31.00	28.00	35.00	30.00	19.00	22.90	29.00	
S1		u1=0.00	400					800			1200
			0.00	-12.00	-5.00	-10.00	-7.00	0.00	-5.90	-4.00	
			31.00	31.00	19.00	29.00	17.00	21.00	42.00	19.00	
S2		u2=-6.00					450			100	550
			-17.00	-18.00	-2.00	-10.00	0.00	-8.00	-31.00	0.00	
			25.00	12.00	16.00	30.00	42.00	34.00	17.00	32.00	
S3		u3=-7.00		700	600						1300
			-12.00	0.00	0.00	-12.00	-26.00	-22.00	-7.00	-14.00	
			19.00	28.00	34.00	26.00	31.00	31.00	24.00	24.00	
S4		u4=-1.00	100							300	400
			0.00	-10.00	-12.00	-2.00	-9.00	-13.00	-8.00	0.00	
			36.00	22.00	26.00	28.00	42.00	27.00	20.00	28.00	
S5		u5=3.00			400	200			450	100	1150
			-13.00	0.00	0.00	0.00	-16.00	-5.00	0.00	0.00	
			25.60	29.10	40.00	17.00	28.00	13.00	31.00	34.00	
S6		u6=-8.00				400					400
			-13.60	-18.10	-25.00	0.00	-13.00	-2.00	-22.00	-17.00	
	Demand		500	700	1000	600	450	800	450	500	

Title: scenario-3
Final Iteration No.: 5
Objective Value (minimum cost) =94450.00

From	To	Amt Shipped	Obj Coeff	Obj Contrib
S1:	D1:	400	20.00	8000.00
S1:	D6:	800	19.00	15200.00
S2:	D5:	450	17.00	7650.00
S2:	D8:	100	19.00	1900.00
S3:	D2:	700	12.00	8400.00
S3:	D3:	600	16.00	9600.00
S4:	D1:	100	19.00	1900.00
S4:	D8:	300	24.00	7200.00
S5:	D3:	400	26.00	10400.00
S5:	D4:	200	28.00	5600.00
S5:	D7:	450	20.00	9000.00
S5:	D8:	100	28.00	2800.00
S6:	D4:	400	17.00	6800.00

Conclusion The total minimised cost of transportation would be Rs 9445000 when opted for transportation by train.

Analysis Comparing the costs in all the three scenarios we arrive at the conclusion that changing the transportation mode from trains to ships entirely is not advisable as it would cost even larger amounts.But it can be seen that for each route opting for a lesser transportation cost among train or ships would give an optimal minimal cost of transportation so the shipping should be done in this way.

Chapter conclusion:

I have briefly mentioned the transportation problem formulation, steps to solve the transportation problem model. The detail steps, such as northwest corner rule, least cost method, Voggel's approximation methods are rewritten to obtain the initial basic feasible solution for the test of optimality is discussed. The modified distribution method is discussed to test the optimality and improvement of the initial basic feasible solution. I have described a transhipment problem model to discuss the nature of problem and solving process of the problem. Finally I have mentioned the multi modal transportation problem such as the transportation of crude oil, through the train vessels, transportation through ship vessels and either ships or trains which one will be least cost. I solved the problems by using the software TORA.

References

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