INVESTIGATIONS ON MODELING AND CONTROL OF NONLINEAR SYSTEMS

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By Ajit Kumar Sharma

(2K15/Ph.D./EE/05)

Research Scholar

Electrical Engineering Department

Under the Supervision of

Prof. Bharat Bhushan

Professor

Electrical Engineering Department

DEPARTMENT OF ELECTRICAL ENGINEERING

DELHI TECHNOLOGICAL UNIVERSITY

DELHI-110042, INDIA

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Certificate

This is to certify that the thesis entitled **“Investigations on Modeling and Control of Nonlinear Systems”** submitted for the award of the degree of Doctor of Philosophy is original to the best of my knowledge. The work was carried out by Mr. Ajit Kumar Sharma under my guidance and has not been submitted in parts or full to this or any other University for the award of any degree or diploma. All the assistance and help received during the course of study has been duly acknowledged.

Supervisor

Prof. Bharat Bhushan

Department of Electrical Engineering

Delhi Technological University

Delhi-110042, India

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Ajit Kumar Sharma

Abstract

The operation of nonlinear systems to perform complex tasks in dynamic environments has been a crucial area of control. Further, the technological advancements in autonomy, artificial intelligence, and robotics have broad applications across society, bringing both opportunities and risks. Most of these opportunities are directly related to path tracking, speed control, maneuverability, and balancing control which is highly affected by the complexity and unpredictable dynamics of the surroundings. Besides, efficient path tracking and balancing control are particularly important for the robots, in order to achieve autonomy without any collision and disturbances. Consequently, the parameters of the mechanical and electronic components need to be monitored and optimized for performing multiple tasks and maintaining the reliability of the system. In view of these aspects, this research identified the combination of intelligent approaches and machine learning methods to achieve unprecedented path tracking and balancing control, continuous monitoring, and robustness by relying solely on onboard computing. The approaches are developed based on multiple control algorithms and are implemented with two-degree freedom operation of ball balancer, inverted pendulum, TORA system, and two-degree freedom robotic manipulator.

To assess the performance of the various nonlinear systems the different control techniques like sliding mode controller, fuzzy sliding mode controller, and neural sliding mode controller are investigated. A comparative study in terms of setpoint response analysis, convergence analysis, statistical analysis, and trajectory analysis has been done.

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Chapter 1

Introduction

1.1 Overview

Advanced mathematical techniques for control are being developed by researchers working on issues in engineering, computer science, biology and the physical sciences. The application of novel analytical approaches for tackling nonlinear issues has been significantly influenced by technological advancements [1]. The state may not be entirely quantifiable in most situations involving nonlinear control systems, making complicated control engineering problems difficult to address. The employment of a variety of distinct models and ideas, a lack of parameter standardization, a lack of suitable control approaches, external disruptions and the greater level of nonlinearity of the equations that drive processes are all important challenges in the field of control technology. Another difficulty is the lack of understanding of the critical variables since the system's states might significantly affect the nature of the control design stage, allowing for excellent performance. As a result, enhanced forecasting, control and optimization approaches are required to ensure optimal nonlinear system performance. Understanding the system's control needs necessitates knowledge of the system; nevertheless, nonlinearities are frequently so complicated that control design for system performance is challenging [2]. New control techniques have developed over time to maintain optimal system performance that prevents interruptions, pauses, and design flaws. Optimising the nonlinear system's parameters is among the most challenging elements of control theory. Any system's primary requirement depends on the controller's control action. The control action is made precisely to accomplish the goal. Additionally, when system complexity rises, it becomes harder for the researcher to attain optimal control performance. Furthermore, conventional controllers aren't any better at handling plant uncertainty due to their demanding complexity.

To increase the effectiveness of control systems, many types of controllers are used. Based on design and analytical methodology, Linear systems and nonlinear systems are the two broad groups into which the controllers can be divided. The nonlinear controller responds dynamically well and is stable and resilient. Like linear control, the controller's design depends heavily on the feedback concept in nonlinear control. However, compared to linear control, the significance of feed forward is much more obvious. A nonlinear system is frequently impossible to regulate steadily without including feed forward action into the control law. The use of nonlinear feedback makes the control system more robust and less susceptible to load disturbances and changes in the output filter circuit parameters.

1.2 Background and Research Gap

The current section provides a summary of earlier work on mechanical systems and nonlinear control that are pertinent to this study. Our primary concern in this thesis is nonlinear systems control design that results from the regulation of a significant and diverse class of mechanical standard systems. The majority of these mechanical control systems are found to be underactuated since there are fewer actuators (controls) than configuration variables. High order underactuated systems are decomposed into a number of nonlinear systems without compromising their global structure, this is one of the unique contributions of the thesis. The system's analytically tractable Lagrangian is used to obtain the appropriate change of coordinates at the global or the semi-global level. Lower-order nonlinear systems are then employed to control the high-order original system after this transformation. This provided as inspiration for the creation of unique control design methodologies for cascading nonlinear systems as well as other types.

1.2.1 Nonlinear System

To define the characteristics of complex nonlinear systems, the progression of linear systems to that of nonlinear systems in control techniques has been presented. The time-invariant linear control systems are widely established for a while that state or output feedback may be used to controllably, observably, stabilise, and track this system. However, the control design for the system may become fairly challenging if further restrictions or specifications are added to the description of the system. The most common techniques for dealing with mildly nonlinear systems entail a frequency domain analysis or an approach to input-output stability. This idea of feedback connectivity between a linear system and a nonlinearity has been developed in the literature to include feedback between an LTI system and a gain-limited uncertainty (or operator). As a result, integral quadratic constraints [4] and robust stability theory [3] were developed. These techniques work well in the presence of linear, uncertain linear, and slightly nonlinear systems, but they cannot be used with fully nonlinear systems, which are systems where the temporal evolution is nonlinear and that do not include any basic linear parts.

To be more precise, a modification is titled, a saturation-type recurrent neural network (i.e., sigmoidal) nonlinearity, has no fundamental differences from control system of LTI type. or a somewhat nonlinear system in terms of controllability and observability. Additionally, while the system's time-evolution follows a nonlinear rule, its linearity as an output does not make system analysis any easier. The requirements for controllability (just in the discrete-time situation) and observability of dynamic neural networks, which are instances of extremely nonlinear systems, were proposed in [5]. The previous work used a time-domain analysis technique that was somewhat complicated. Given that a frequency domain analysis only addresses systems, then the state development becomes linear in time. For nonlinear systems in a comprehensive local theory on disturbance decoupling, tracking, stability, observability, and controllability may be found in [6]. Differential geometry and Lie theory, which have become widely used in the literature, were the primary techniques used to address these control difficulties. Although these techniques were quite effective for local analyses, but in global studies with affine control based nonlinear systems, local analyses control typically fail. Furthermore, there is no resistance to f, g, and h uncertainty in lying algebraic circumstances. Additionally, in the theory of input-to-state stability [7], contains both theories of absolute stability and theories of robust stability for extremely nonlinear systems. Control Lyapunov functions (CLFs) are the primary instruments in this theory for robustness analysis against disturbances. The issue is that building CLFs for highly nonlinear systems is often not straightforward. Design and analysis of a global/semiglobal control system are necessary in many control applications. Additionally, after applying a specific change of coordinate variables to the nonlinear dynamics, the converted system or one or more of its components may be a nonlinear system that are not affine in terms of control. This inspired us to think about a stabilisation approach with either global or semi-global scope, with the expectation that the research of mechanical systems leads to an understanding of very nonlinear systems that are not homonyms and understated.

1.2.2 Nonlinear Control Problems

The nonlinear control problems which are used in this work for efficiency and efficacy analysis of different existing and proposed algorithms are discussed below:

A. Ball Balancer System

An underactuated, multivariate electromechanical, and nonlinear system may be represented by the use of ball balancer systems. Two servo motors operate simultaneously to regulate the position of the ball in a 2-DOF Ball Balancer. This is a nonlinear system. In many applications and approaches, it is one of the most complicated control benchmarking systems. Users may experiment with a variety of control methods to direct a ball towards a certain spot on a table. It's a horizontal plate with slants in both directions, which allows the ball to roll wherever on the plate. Nonlinear kinematics and control theory are shown dynamically in this system. Control algorithms and technologies are often evaluated using this system because of its inherent nonlinearity, instability and under-actuation [8].

B. Inverted Pendulum

Due to the Inverted Pendulum's broadest range of industrial applications, it serves as a benchmark example of a static unstable nonlinear system. A naturally unstable inverted pendulum is the ideal control benchmark system. The literature has investigated a variety of control techniques to stabilise a pendulum around its shaky equilibrium point. Both theoretically and empirically, the inverted pendulum approximation of input-output linearization has been used to balance a single link inverted pendulum, however this method makes use of sophisticated mathematics that alters the system's response [283].

C. Robotic Manipulator

The movement of an unknown item is directed by a manipulator robot. The following uncertainties need to be resolved in order to control the manipulator:

1) Since the object itself is unknown beforehand, the object's weight is not known;
2) Because they are challenging to accurately describe and measure, the friction and other variables in the dynamics of the manipulator may be unknown.

D. TORA System

In the TORA (Translational Oscillator with Rotational Actuator) system, a pendulum rotation governs an un-damped electromechanical oscillator M. The oscillator is undamped, which causes a lengthy oscillation to be visible in the active path of M in relation to a non-zero starting displacement. A successful oscillation intervention may be made possible by properly controlling the pendulum's motion. The majority of solutions are designed to balance the pendulum's angle and the oscillator's displacement (x).

1.2.3 Research Gap

Nature is nonlinear, therefore nonlinear methods are the easiest way to cope with it. Despite this, linear control has been used effectively for years. The issue with linear systems is that they may not be capable of accommodating recent and innovative technologies. It may be challenging to decide whether to use linear or nonlinear control for a certain application. Linear control has been well researched and industry professionals trust it. For linear systems, there are many good analytic methods available, including the root locus, bode plot, Nyquist stability criterion, Laplace transform, Z-transform, Fourier transform and many more. Nonlinear systems, on the other hand, require more sophisticated numerical methods, such as the Lyapunov stability criteria, the Popov criterion and singular perturbation techniques. For nonlinear systems, implications of controllers are sometimes very intricate and difficult to due to its higher complexity, incomplete system understanding, lack of analysis method and their modelling may be time consuming. Limit cycles, chaos and bifurcation may occur in a system of nonlinear type. Most of the strategies can only guarantee local stability; global stability cannot be assured. Flaws such as time delays and oscillations in the response play vital role. Employment of advanced optimization techniques, different algorithms which precisely estimates the response of output made the nonlinear plants stable in performance and controller design can improve stability and performance.

1.3 Optimization

An important paradigm that is everywhere in addition to a variety of uses is optimization. In practically all application areas such as mathematics, computer science, operation research, industrial and engineering designs, we are continually attempting to upgrade something - regardless of whether to limit the expense and vitality utilization or to expand. The benefit yields execution effectiveness. In all actuality, assets, time and money are consistently restricted; thus, optimization is unmistakably progressively significant [8]. How the optimization algorithm works are shown in Figure 1.1.

Optimization is the study of choosing the best choice among a debilitated hover of choices [1] or it tends to be seen as unitary of the major quantifiable mechanism in a system of dynamics in which judgments must be employed to enhance single or more evaluations in some affirmed set of conditions [9]. Each problem of optimization accompanies some decision variables, certain objective (fitness) functions and few constraints [1]. A literature review of optimization algorithms reveals that, there is no systematic classification is available. In Figure 1.2, the hierarchy of optimization methods is displayed.

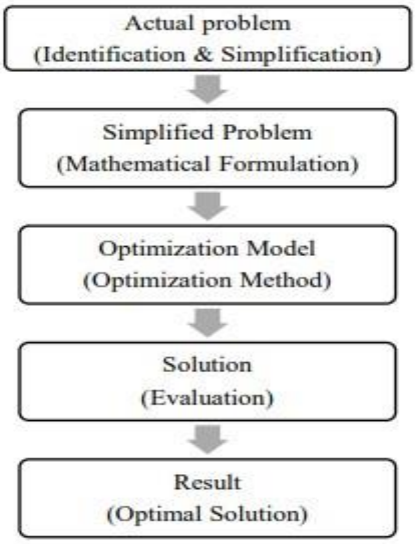


Figure1.1. Flow chart of Optimization

1.3.1 Existing Challenges in Optimization

The effectiveness of an algorithm, the effectiveness and precision of a statistical simulator and assigning the correct methods to the stated problem are the three key challenges in simulation-driven optimization and modelling.

Algorithm's Effectiveness

It's critical to have a good optimizer to get the best results. An optimizer is essentially an optimization technique that has been appropriately built to perform the required search. It may be connected and merged with other modelling elements. No free lunch theorem states that [2], there are several optimization methods in the literature and no one solution is suited for all issues.

Algorithm's Correctness

Choosing the best optimizer or technique for a specific problem is critical from an optimization standpoint. The kind of issue, the structure of the methodology, the desired output quality, the contemporary computing sources, time frame, the method's implementation availability and the selection experience will all influence the algorithm selected for an optimization job [10][11].

1.3.2 Classification of Optimization Techniques

Figure 1.1 displays the categorization of optimization algorithms. To determine the best solution, mathematical optimization relies on gradient information. Although researchers continue to employ similar methods [12]. The primary difficulty with mathematical optimization approaches like Newton's method, steepest descent, etc. is that they have a problem with local optimum. The algorithm did not discover the best answer in this case, assuming that a local solution is also the global solution [13]. It frequently fails to solve issues whose derivations are unknown or expensive to compute. Prior to using metaheuristic algorithms, classical techniques like Hill-Climbing, random search, and simulated annealing were utilised to tackle optimization issues [14]. These techniques begin the search from a single place and use gradient information to guide additional searches to locate global optimums [15]. These algorithms failed to solve real-world applications including the economical optimization problem, engineering design issues, and structural optimization challenge [16] because of several limitations and complexity restrictions. The term "metaheuristic" refers to a directed random search approach that iteratively directs subordinate heuristics to explore and utilise the search space while avoiding becoming snared in local minima [17]. Additionally, it makes clever use of search history to direct future searches toward the best answers [18].

According to [18], Metaheuristic algorithms share the following traits:

- The algorithms are based on natural events or behaviours and they follow specific rules (e.g., biological evolution, physics, social behaviour).
- Probability distributions and random processes are used in the selection phase, which contains random elements.
- They provide several control parameters to modify the search method since they are intended to be general-purpose solvers
- They don't depend on a priori knowledge, which is information about the process that is accessible before the optimization run begins. Nonetheless, such knowledge may be beneficial to them.

1.3.3 Existing issues with Metaheuristic Optimization

The primary challenge for metaheuristics is figuring out how to find the optimal point. Even though many metaheuristics have been suggested, only a handful of metaheuristics have consistently attained the required success rate. Selecting the proper metaheuristic algorithm is a complex thing and recent developments is to liberalize metaheuristic methods to overcome these problems. Figure 1.2 shows the different type of search technique to optimize any problem.

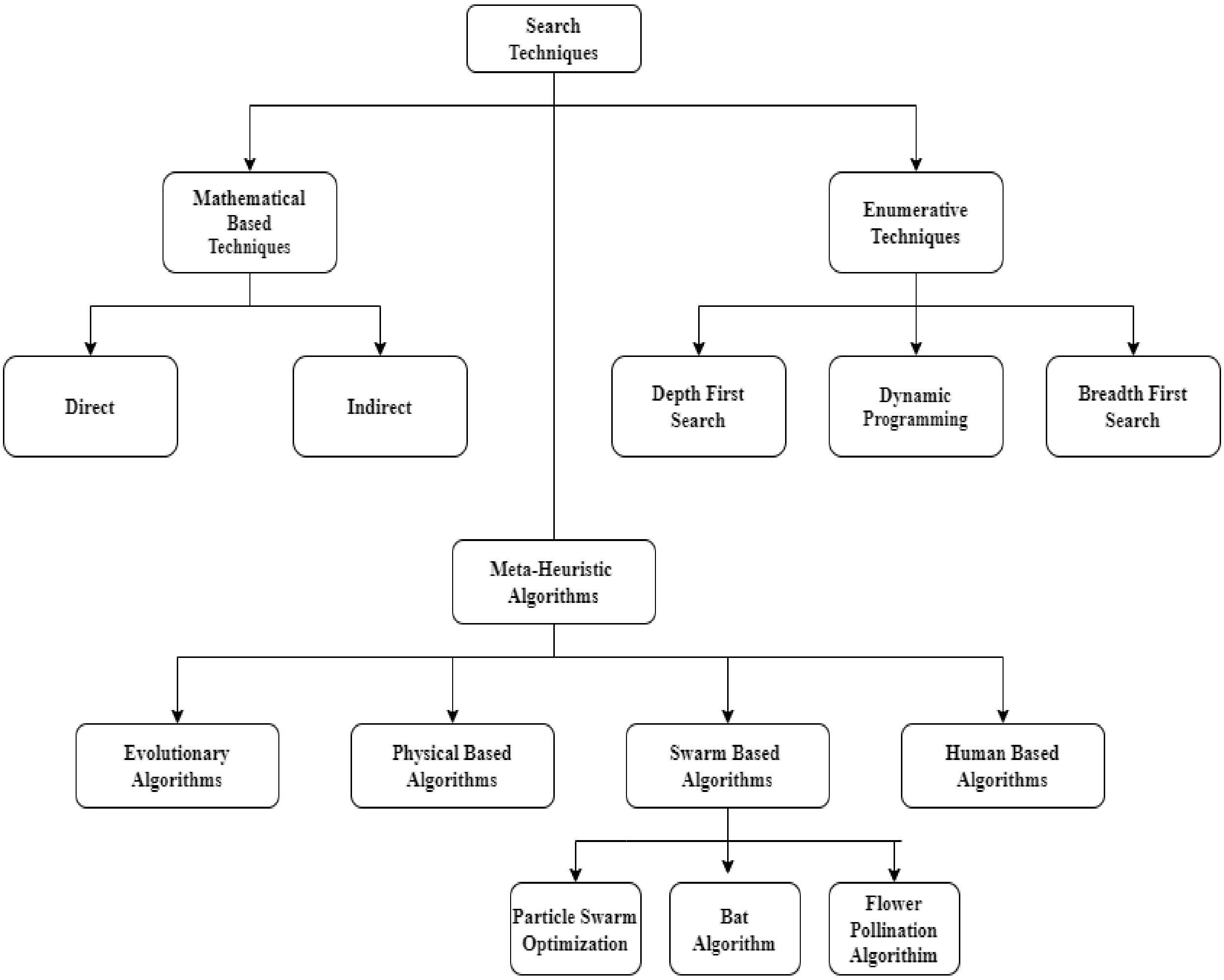


Figure 1.2 Classification of optimization techniques

1.4 Robust Controller

There are five different categories for robust control methods [19]-[22].

1) linear-multivariable methodology;
2) A strategy based on passivity.
3) Controllers with a changeable structure;
4) Dependable saturation strategy;
5) Robust adaptive approach.

Recent studies have addressed parametric uncertainty in [23] and expanded the findings to encompass nonparametric uncertainties in [24]. From all of the aforementioned methods, it is similar to the robust saturation method, in that Lyapunov arguments of some form are employed. Except for the inclusion of an enhanced control to account for the unmatched uncertainty, the robust control issue is converted into an optimum control problem. It is demonstrated in [25] that the answer to the problem of resilient control is the same as the answer to the problem of optimal control with a condition on the size of the unmatched uncertainty. Any system's primary need depends on the controller's control action.

The control action is made precisely to accomplish the goal. Additionally, traditional controllers aren't any better at handling plant uncertainty due to their horrifying complexity. One of the effective methods to design trustworthy controllers for complex, high-order, nonlinear dynamic plants that operate in a range of unpredictable circumstances is a sliding mode control technique. Because sliding mode is less sensitive to changes in plant parameters and disturbances, the demand for precise modelling is relaxed, which is one of its key advantages.

1.4.1 Sliding Mode Control (SMC)

Because of its unique qualities, namely its insensitivity to matched uncertainty, convergence of closed loop systems with zero error and lower order sliding mode equations provide a control of non-linear type. SMC has risen fast as a control in contrast to other robust control schemes. This strategy has a number of significant benefits since it modifies the system's dynamic behaviour by selecting the suitable switching mechanism, making the system's matched uncertainty insensitive to the response of closed loop response. Such an encouraging aspect draws emphasis to the role of the researchers in SMC development. SMC is used as automated controller for many different applications, such as motion control problems, robotics, power electronics, aerospace, and industrial process control. [26]-[29][66].

The primary justification for SMC's use in various applications is that it performs better for nonlinear systems, is suitable for MIMO systems, and also recognises that discrete time systems can benefit from its use with the right design [30]. SMC technique performs better than other recognised techniques like vigorous adaptive control [32], H infinity control [33], and backstepping control [34] whenever bounded uncertainties or disturbances are present along with unmodeled dynamics [31][66][71].

1.4.2 Existing issues with Sliding Mode Control

As it would suggest that the control travels at an infinite frequency, a perfect sliding mode does not actually exist. The feedback control's discontinuity creates a specific dynamic behaviour near the surface known as chattering in the presence of switching flaws, such as switching time delays and tiny time constants of systems. This phenomenon is problematic because, even if it is filtered at the process output, it may stimulate unmodeled high frequency modes that harm the system's performance and potentially cause instability [259]. Additionally, chattering causes significant electrical power circuit heat losses and severe mechanical moving parts wear. Recently, SMC design methods have been used to address important problems including reducing chattering, compensating for the effects of unstructured dynamics, adapting to uncertain systems, and improving the dynamic performance of closed loop systems.

1.5 Motivation

Following are some key insights from the previous research and observations that becomes the motivation for this research:

- All the practical systems are nonlinear in nature and this is the main reason behind the growing interest in nonlinear control. There are many benchmarks example of a static unstable nonlinear system due to its widest scope in industrial application.
- Implementation of less complicated algorithms are preferable, in the same way should be as straightforward as feasible. In practice, a robust algorithm is preferred with a simpler architecture for simplicity of implementation while still being efficient enough for real-world applications.
- By using a standard PID controller, the researchers were able to create a nonlinear system response that was both stable and acceptable. The main benefit of using an SMC controller over a linear controller like PID is that SMC offers stability and robust performance in a variety of uncertain systems, whereas PID fails in the presence of uncertainty.
- While the typical control approaches often adjust the system's control actions in response to changes in its operating conditions, they fall short when it comes to understanding the behaviour of the system. To address the random and parametric uncertainties in the system operation, metaheuristic algorithms have been created. These

algorithms are driven by the desire to handle uncertainties, increase the system's sensitivity to different disturbances, and execute effective control actions.

- To get around the challenges of recognising quick and fleeting changes in system behaviour, fuzzy control and neural network control paired with sliding mode control have been created. These controllers use pre-calculated and fictitious control rules to learn the system's operational condition and provide the relevant control action.

1.6 Methodology of the Research Work

The methodology used to carry out the research work are following:

- System configuration: 11th Gen Intel(R) Core (TM) I3-1125G4 running at 2.00 GHz and 2.00 GHz with 8 GB of RAM.
- MATLAB coding and simulink is used for metaheuristic algorithms like particle swarm optimization, bat algorithm, and flower pollination algorithm.
- MATLAB coding and simulink is used for various sliding mode controllers such as quasi sliding mode control, equivalent sliding mode control, reaching law sliding mode control, and decoupled sliding mode control.
- The fuzzy operation in a single link inverted pendulum is carried out using the fuzzy toolbox.
- In the single link inverted pendulum and two link robotic manipulator, neural operations are carried out using the neural network toolbox.

Simulation is carried out using MATLAB 2015, which is powered by an Intel(R) Core (TM) 2 Duo CPU T6400 running at 2.00 GHz and 1.20 GHz with 1.99 GB of RAM.

1.7 Objectives

The intelligent control of two-degrees-of-freedom mechanical systems (2DoF) is the major topic of this study. The widespread uses of underactuated systems and the conceptually difficult difficulties they provide serve as the driving force behind this. Based on the following identified motivations for the research, the aims of this study are to achieve control and stability of the benchmark systems:

- Mathematical modelling of two degrees of freedom (2DoF) ball balancer. Single link inverted pendulum, two links robotic manipulator, and TORA system.

- For a two-dimensional ball balancer, traditional controlling methods like proportional derivative were designed, controlled, and implemented, along with other meta-heuristic optimization approaches including PSO, BA, and FPA.
- Design, control, and implementation of various sliding mode controls on its reaching law, quasi law, and switching mode for inverted pendulum and compare their performance results.
- Design, control and implementation of different decoupling algorithm-based sliding mode control for single link inverted pendulum and TORA system and compare their results in terms of their pole location.
- Develop, implement, and compare the outcomes of various fuzzy-based sliding mode controls.
- For single-link inverted pendulum and two-link robotic manipulator design, control, and implementation of neural sliding mode control and comparing the outcomes.

1.8 Outline of the Thesis

Following an introductory chapter that describes the basic concept of optimization along with a brief on metaheuristic optimization, robust controller along with various nonlinear control problems, further chapters of this thesis are arranged in the manner described below:

A review of the literature is given in **Chapter 2** on the various optimization strategies and the significant developments like variants, combinations with different controllers, applications in various fields, and hybrid algorithms approach in their respective areas. Literature review on robust control techniques of different sliding mode controller It also covers the various applications and performance evaluation of optimization techniques for nonlinear systems.

Chapter 3 presents position control of a ball-balancing scheme utilising particle swarm optimization, BAT and Flower Pollination Algorithm". In this chapter, the simulation findings of controllers on the control of a ball balancer are present. In this study for the ball balancer system, the ideal parameter choices, mathematical modelling, and controller design are made to solve the existing issues. The main goal is to compare several metaheuristics control strategies used on a ball balancer. The focus is on utilising simulink to develop and implement controllers for ball balancer setup. Adapting a metaheuristics algorithm for optimising controllers proved to be an innovative adaptation, as evidenced by the Ball Balancer findings. Finally, the comparison is performed using various control algorithms.

Chapter 4 focus on different types of sliding mode control techniques, are used in i.e. reaching law sliding mode, quasi-sliding mode, equivalent sliding mode and decoupled sliding mode on different nonlinear systems like inverted pendulum and TORA system to evaluate the effectiveness of these strategies. Maintaining the system's position at the intended position is the goal of control action. Each of these control strategies begins with the construction of a sliding mode surface, following which control functions are created to achieve control goals. Performance metrics are tracked for chattering, convergent time, disturbance rejection, and stability. Based on these performance criteria, a comparative study is then carried out.

Chapter 5 discusses three distinct fuzzy controls using the sliding mode control method for nonlinear systems. A single link inverted pendulum serves as the nonlinear system being addressed here. The system is chatter-free as a set of linguistic based rule is created a result of fuzzy logic control. Fuzzy rules have been used to approximate the nonlinear system’s uncertainties while system parameters are accommodated as per adaptive laws of fuzzy controller. Different kinds of fuzzy controllers based on sliding modes, such as fuzzy sliding mode control methods based on approximation, equivalent control, and switch-gain regulation, have been discussed and shown here. All methodologies have also been compared, as evidenced.

In Chapter 6 discussed about a sliding mode control technique for nonlinear systems that takes various nonlinearities into account. A single link inverted pendulum and two link robotic manipulator make up the nonlinear system under consideration here. Maintaining the system's position at the intended position is the goal of control action. To enhance control performance, the RBF neural network employs an adaptive RBF-based sliding mode control. The RBF neural network compensates the disturbances using a minimal parameter learning technique.

Chapter 7 concludes all the approaches and highlights the current state of potential research directions.

Chapter: 2

Literature Review

2.1 Introduction

For nonlinear systems, several control strategies and optimization techniques have been proposed in the literature. Nearly all real-world systems have dynamic behaviour that evolves over time, which adds to their complexity. Researchers are paying increasing attention to nonlinear systems because of its uncertainties and unpredictable nature. Numerous controllers have been employed to manage nonlinearities and enhance system performance in the literature. The theories' fundamental and most recent advancement for collecting the benchmark's control elements nonlinear systems are summarised in this chapter (inverted pendulum, ball balancer, robotic manipulator, tora system). A thorough rundown of various position tracking and balance control mechanisms are presented.

This chapter is combined into the portion, first portion is to deal with the algorithms and controlling techniques and in the second portion nonlinear systems with their mathematical modelling have been discussed.

2.2 Review of Algorithms and Controllers

2.2.1 Particle Swarm Optimization (PSO)

Kenndy and Eberhart established PSO in 1995 for the purpose of training neural networks and solving non-linear optimization issues. Human cognition of natural behaviour, such as how human learning is influenced by their surroundings, how they interact with others, and how they encode their patterns into their learning methods, are simple findings in PSO. PSO uses this learning phenomenon to find an optimal solution. PSO has become increasingly natural for dealing with non-linear complex optimization problems, especially in a wide range of fields. A swarm in PSO is a population of vector solutions that is probing new search areas while hunting for food, resembling the evolution of a school of fish. To find the global optimum, all particles in the swarm translate information and follow eachother's best experiences as well as their own past best experiences [35]. Each particle must adhere to the basic rule of determining the location of its prior best or neighbour. Particle swarm optimization (PSO), a method based on swarm intelligence, was developed by Kennedy and Eberhart [36]. It has been widely utilized in every discipline since it just requires a small number of parameters and no gradient information from the goal function. Each particle starts off with a velocity and is dispersed at random in the search space so that their fitness may be assessed [37]. Each particle goes to a new place with each iteration, improving the fitness value of the previous position [38]. Three components make form a velocity update in PSO [39]

1. Particle momentum takes into account both the prior and present velocities.
2. Particles are propelled toward their highest possible velocity by cognitive component.
3. Particles are drawn toward their optimal velocity by the social component.

The impact of the previous iteration on this iteration is controlled by inertia. Greater values of "w" enhance global search, whereas smaller values enhance local search functionality. The inertia parameter typically ranges from 0.9 to 0.4 for optimization processes, allowing particles to first investigate neighboring areas at a slower pace [40]. Algorithms are kept out of local minima using random numbers. PSO causes particles to become prematurely caught in local minima, which prevents the optimum solution from being found during the optimization process [41]. There is a periodic algorithm upgrade to address these search issues. The PSO algorithm's performance has been improved by Guochu [42] separating the swarm into better particles, ordinary particles, and the worst particles. To get rid of the influence of premature controlling search, Jau et al. [43] suggested a modified quantum behaving PSO (QPSO), which has been used to the study of the least trimmed squares technique. Gholizadeh and Moghadas [44] evaluated the performance for the best design process using an upgraded QPSO on two numerical cases. On nonlinear benchmark functions, Martins et al. [45] designed a condensed PSO to evaluate performance. This streamlined PSO provides significant performance while also requiring less computing work. Panda et. al. study's [46] evaluated the performance of the PID controller for an automated voltage regulator with that of other algorithms like ABC and DE. For an unstable search process, Vastrakar and Padhy [47] employed a PID controller with a PSO algorithm. The search space is divided into smaller areas in order to locate both local and global minima. For systems with many inputs and multiple outputs, Chang and Chen [48] uses the PSO method to maximise the gain of the PID controller. A new sensor-less system that exhibits a reasonable response for tracking performance was developed by Xiang et al. [49]. PSO algorithms aid the system in tracking under various circumstances. Iterative learning controller and PID controller gains were tuned using enhanced PSO by Huang and Li [50]. On a nonlinear gantry system, Jaafar et al. [51] combined the PSO algorithm with a PID controller to enhance system performance.

2.2.2 Flower Pollination Algorithm (FPA)

A structure known as a flower is designed to house the reproductive organs that involves creating plant's reproductive tissues (ovule and pollen), followed by the creation of seeds containing dormant offspring plants. While bisexual (ideal) flowers include both male and female reproductive parts in the same flower, other flowers feature distinct stamen and carpel blossoms, making them unisexual (not so perfect) flowers. During pollination, the male gamete also known as pollen is transferred to the stigma. [52] so that it can make the female gamete fertile. Flowers may possess scented petals, eye-catching petals, and nectar to attract pollinators like animals, insects, and birds. By having a strong enough attractiveness, certain flowers may persuade pollinators to only visit them, keeping the blossom constant. Additionally, pollinators can secure nectar availability with less exploration by returning to the same species of the flower. Pollen can occasionally be spread only through gravity, water dispersion, or the wind. Cross-pollination and self-pollination are the two basic categories into which pollination may be divided. In the former, pollen is moved from one bloom on one plant to another flower on another plant. In the latter, pollen is moved from one bloom of the same plant to another flower. Additionally, pollination may be divided into two categories: biotic and abiotic, depending on whether the pollinators are living or not. when an insect or mammal visits a flower to eat pollen or sip nectar that contains pollen the grains get attached to its body, this is known as biotic pollination. If the animal makes the same trip to another flower, pollen may be transferred to the stigma and result in flower fertilization. Abiotic pollination is rare since pollen is often spread. When the anthers and stigma are not placed exactly, they can be blown, diffused in water, or crushed by gravity. A brand-new data clustering method called FPAB has been created by researchers to replicate the bee pollination of flowers [52]. FPA or flower pollination algorithm imitates a more comprehensive understanding of the flower reproduction process, was then developed [53]. FPA has recently received a lot of attention due to its successful application to real-world issues. Due to its effectiveness and adaptability, the FPA has been used to manage a number of optimization issues in a variety of real-world circumstances. For certain fields, basic FPA provides the best answers, and it can yet be improved. The complexity of issues, their high dimensionality, the reduction of features in the search space, and the discretization of FPA for combinatorial problems are all addressed through FPA improvements. Continuous optimization difficult issues are solved with basic FPA. Binary versions of FPA are created to address distinct issues. Random numbers are generated via chaotic maps, and these random numbers are employed throughout FPA. The most effective metaheuristic algorithm strikes a compromise between global and local search. While algorithms based on gradients excel at local search, algorithms based on swarms excel at global search. In order to enhance FPA's performance, several algorithms have been hybridised with it. To address issues in the actual world, various approaches of setting parameters and many objectives are also developed. Based on operators [54] – [55], FPA adjustments rely on the lowering characteristics strategy. It solves the optimization problem, designs structures, and offers an elite opposition FPA (EOFPA). The primary goal of the suggested strategy is to handle a sizable search space by achieving equilibrium between global and local search. Three additional initialization steps are enhanced by the forward and backward selection technique. This benefits from balancing local and international searches to discover the optimal answer and select the bare minimum of features in order to improve accuracy. Binarized FPA is known as BFPA [23] was proposed by the researcher for attribute selection. Its effectiveness as an optimization technique was shown on six datasets. a CEED method [56] to address problems with thermal and photovoltaic power generation proposed a method to combine a binary and euclidean economic dispatch FPA that outperforms existing metaheuristic algorithms. By utilising the best members of the population in local optimization methods, it is possible to minimise the fitness function value precisely while consuming little CPU time [56].

If a metaheuristic algorithm can balance local and global search, it is seen to be at its best. Swarm or population-based algorithms are some techniques that are effective for locally exploring the search space. Similar to local search operations, certain algorithms work well for global searches. These algorithms are often gradient-based or trust-region techniques. Compared to other FPP approaches, it has greater performance. Additionally, the Simulated Annealing-Hybridized FPA [58] is used to increase both searches locally and convergent rates, effectively delivering high-quality solutions and rapid global convergence uses a combination of FPA and CSA termed BCFA to address the attribute selection problem by employing a limited set of characteristics quickly and accurately. There are other swarm-based algorithms that, when combined with FPA, produce superior results. One such approach is the FPA Bee Pollinated (BFPA) [60], which solves solar-power parameter problems quickly and robustly by combining artificial bee colony (ABC) with FPA with optimization. The suggested strategy yields the fastest convergence, better solution quality, and convergence to the world's best solution. When FPA and the Firefly algorithm [61] are used to solve multimodal functions, the convergence rate is improved as well as the ability to trap in local minima more quickly. Multi-objective problems are common in real-world issues; therefore, optimization techniques should be changed appropriately. Additionally, multi-objective optimization requires a lot of computation for larger dimensions. The MOFPA, or a random weighted sum technique to solving a multi- objective problem, has also modified FPA [62]. The same two bi-objective test functions are then provided, along with numerous multi-objective functions to provide findings that are more effective. Performance of metaheuristic algorithms is significantly impacted by optimization settings. Some FPA variations are built on fine-tuning its parameters proposes a novel method known as adaptive levy FPA known as ALFPA [63] in order to enhance parameter tweaking of use. Its inclusion of operators for mutation, local lookup, and dynamic reversal distinguishes it from other algorithms as a superior choice. Global mutations give FPA its exploration capability, while improvements to the local search algorithm increase its capacity for exploitation.

2.2.3 BAT Algorithm (BA)

An echolocation property of microbats inspires using a population-based metaheuristic algorithm, the Bat Algorithm (BA) [64]. Researchers from numerous fields start to pay attention to BA after its development in 2010. It has been successfully used to solve several real-world engineering issues. The ability to fly in total darkness allows bats to attack their prey. Bats can do these activities thanks to their ability to use echolocation. They pulse loudly and then watch for the prey's echo [65]. The frequency of their pulses and their predation methods differ among bat species. Their level of noise changes as they look for and approach the prey. Most bats employ brief frequency modulated signals for echolocation, however some bats also use constant- frequency signals. The frequency range of the pulses that bats emit is between 25 and 150 kHz. These pulses may last 8 to 10 milliseconds [66]. While looking for prey, the microbat may produce up to a maximum of 10 to 20 noises per second, or 200 sonic cycles per second. Human ears cannot hear the sound that microbats make. These, however, may be quickly detected using some unique tools [67]. In order to distinguish between background noise and audible signal, bats use a unique acoustic system. Some types of bats are excellent hunters. They often keep an eye on the area while waiting for their prey. There is no prey in this area if the signal is received without any dispersal. If not, prey is present in the targeted area and can be found by vibrating at an echolocation frequency. BA uses frequency tuning while foraging together with adjustments to loudness and pulse emission rates. Since they maximise social interactions and rules with biological inspiration, all of these algorithms may be characterised as swarm intelligence heuristic algorithms [68] - [69].

BA imitates bats' echolocation activity. BA is easy to use, comprehend, and is straightforward. The distinctive quality of echolocation draws researchers from other fields to BA. BA has been extensively employed in a variety of fields, including data mining, pattern recognition, cloud computing, mechanical engineering, and civil engineering. The literature has demonstrated an overview of BA [70] and highlighted applications, problems, and varieties of BA as well as their variations [71]. But in their study, new directions for research were not covered. The uses of BA in the discrete domain [72] together with a comparison of several uses. Researchers have been drawn to BA in recent years to find solutions to practical issues in a variety of fields. Because of this, researchers sought to alter BA's structure in order to improve its functionality. The three main areas have all undergone changes. First, by changing the control settings, BA's performance is improved. Second, parts of the BA are combined with different optimization procedures. Third, the search space and problem type were taken into consideration when modifying BA's structure. The six kinds of BA variants— binary, chaotic, improved, hybrid, Levy fight-based, and multi-objective Bas—are based on the aforementioned characteristics. Four improvements have been suggested in BA [73] to boost performance. These changes were made to the way bats move, their heart rates, their volume levels, and their local search mechanism. The directional BA was the name of the projected BA (DBA). On benchmark tasks, DBA was compared to other algorithms and found to be superior. The Doppler Effect is added to BA to provide a guided BA (GBA) [74]. Bat frequency shift was caused by the Doppler Effect. As a result, bats were able to travel toward the most effective bat recently. A better position around the most recent bat was determined using the search procedure.

GBA did not use the self-adaptive technique for doppler effect echoes. NBA the new bat algorithm uses a self-adaptive approach to overcome this issue [75]. In NBA, the habitat option was included. NBA delivered superior outcomes to GBA. It has difficulty with parameter adjustment, though. The bat's speed and direction are dynamically altered by an adaptable BA (ABA) [76]. To modernize the location of bats, the inertia weight was added to ABA. RBA is a new algorithm that combines BA and a remembrance mechanism based on a similar idea of ABA [77]. To update the position of the bats, the time-varying inertia weight was also included in RBA. The velocity and location of the bats are changed in a new variation of BA (BBA) [78]. A V-shaped transfer function was used to change the continuous search space into binary search space. For both unimodal and multimodal test functions, BBA was successful in providing the best solutions. A perfect optical bulb design was created by BBA. The knapsack issues are solved using a new BBA (NBBA) [79]. The search was conducted in two stages. BBA's initial task was to identify the most effective solutions. After then, a local search method was used to block results from being in local optimal points. NBBA outperformed BBA in performance. To resolve the multidimensional knapsack issues, a discrete BBA (BinBBA) was used [80]

Binary space is created by transforming the continuous search space. They employed a sigmoidal function. Compared to other binary metaheuristics, BinBBA performed better. A unique BA version built on chaotic map functions has also been addressed in literature [81]. Eleven different chaotic maps were used. Bat noise level was adjusted. Premature convergence was removed using CBA. The chaotic levy fight BA (CLFBA), which is based on the ideas of chaotic dynamics and levy battles [82], is an effective algorithm for identifying the best course of action. Different chaotic maps were employed in a chaos-based BA (CsBA) [83] to replace the control parameters of BA. By altering the frequency, pulse rate, loudness, and location of the bats, four distinct CsBAs were created. There were utilised thirteen different chaotic maps. The experimental findings show that the sinusoidal map-based CsBA outperformed the other three BA variations. The feature selection problem is solved using a global CBA (GCBA) [84]. The population of bats was started using the chaotic approach. The GCBA removed the early convergence. A novel variation of BA called as LFBA was created as a result of the Levy fight (LF) behaviours in BA [85]. Bats' location and speed were altered. Regarding precision and convergence rate, LFBA fared better than the BA. Fractional LFBA (FLFBA) is the name given to research on fractional calculus that was conducted to further improve LFBA performance [86]. Local search was carried out using a random walk based on the Levy distribution. Calculating fractions allowed for the updating of bat velocity. On test functions, FLFBA was assessed, and it outperformed the other algorithms. To speed up convergence, a novel LF variation of BA (DLFBA) based on LF and differential operator is introduced [87]. The variety in BA was preserved by using LF. In comparison to the other Levy battle variation of BA, the suggested method was better. An enhanced variant of BA that made use of the ideas behind OBL and LF random walks [88]. To increase variety in BA, OBL is applied. The OBMLBA algorithm was more effective than the other methods. A double sub-population LFBA (DLBA) is used maintaining variety and intensification in harmony in the BA sub-population [89]. Position updating in internal and exterior subgroups was done using the LF model and the dynamic weight model, respectively. DLBA outperformed the competition in terms of performance.

The performance of BA is enhanced by BA-DE methods (differential evolution) [90]. The mechanics of DE's new local solution creation and solution selection were modified by this hybrid method. For high-dimensional benchmark issues, this method fared better than BA. The computational difficulty of this strategy, however, is not looked into. The optimization issues are solved by combining ABC and BA [91]. To further enhance the search capability, the inertia weight was added to BA. Benchmark test functions were used to assess the hybrid technique, and it was discovered that BA-ABC produced superior outcomes to BA and ABC. In this hybrid technique, BA's (92) and ABC's (4%) contributions were both much larger. However, compared to ABC and BA, BA-ABC has a somewhat larger temporal complexity. Multi objective optimization (MOO) issues are dealt with the BA (MOBAT) extension [92]. MOBAT performed better on Pareto fronts when assessed using benchmark test functions. To address generator planning, a multi-objective shuffled BA (MOShBAT) [93] is developed. ShBAT now includes crowding distance and non-dominated sorting methods. Distributed Pareto Fronts were produced by MoShBAT. Prakash and others multi objective BA (NSBAT-II) was developed using the principles of elitism and non-dominance [94]. When evaluated on a real-world reactor issue, NSBAT-II demonstrated better convergence than NSGA. The association rule mining problem is solved by a multi objective BA (MOB-ARM) [95]. MOB-ARM was successful in retrieving significant and credible regulations. VLSI problems are solved using an expanded form of multi objective BA (MOBA) [96]. and a new Pareto dominance notion in the multi objective improved BA (MOIBA) [97] framework. The crowding distance and non-dominated sorting were applied. MOIBA was put to the test for an electrical issue. The many Pareto Fronts might be produced via MOIBA. The relationship of the dominating solution is established using the new objective BA (MaOBAT) [98]. Based on the dominance connection, non-dominated solutions were chosen. MaOBAT achieved a higher rate of convergence than the alternative methods. The MBA (MOMBA) uses the Pareto dominance strategy to tackle complex problems and improve the capacity for exploration [99]. MOMBA was able to produce a variety of Pareto fronts.

2.2.4 Sliding Mode Control (SMC)

The Soviet Union is where variable structure control (VSC) was initial 1950s appearance. Emelyanov et.al. [100] in their key papers, saw the plant as a linear second-order system described in phase variable form. The capacity of VSC to provide extremely reliable control systems often invariant control systems is its most significant characteristic. When a system is invariant, it totally ignores any external disturbances and parametric uncertainty. In spite of modelling mistakes, positive outcomes have been documented in terms of removing disturbances, resolving nonlinearities, and establishing tolerable control. The primary mode of operation in systems with changing structure is the sliding mode. VSC is still being used in research and development today in several engineering systems. In this field, many VSC articles have been published. The topics under investigation be sure to mention the sliding-mode, its stability, the consequences of changes in the parameters of the system values and external disturbances, and the analysis of systems with unobservable state variables of system. In 1993 and 1999, respectively, two further survey articles [101] – [102] were released. High performance systems that are resistant to noise and parameter uncertainties are produced via variable structure control. Two phases are involved in the design of such systems: (a) selecting a collection of switching surfaces that correspond to a desired motion, and (b) creating a discontinuous control law that assures the convergence to the switching surfaces and the attractiveness of the switching surfaces. When the system's possible trajectories are the switching surfaces and don't leave them for the duration of the motion. Adaptive control is a key strategy for addressing model uncertainty, and sliding mode methods are yet another strategy that may be utilised to address control issues. These methods are currently drawing more attention.

There may be disparities between the mathematical structure and the actual plant used to create the control system. Possible causes for this discrepancy may be a number of circumstances. Instead of such inconsistencies, the engineer's job is to guarantee that the system is performing at the needed levels. To get rid of any errors, a number of reliable control methods have been created in the sliding mode control methodology is one such approach. Any system's primary need depends on the controller's control action. The control action is made precisely to accomplish the goal. Additionally, as system complexity rises, achieving optimal control performance becomes increasingly difficult for researchers. Additionally, because to their horrifying complexity, traditional controllers are not substantially more suited to handling plant uncertainty. SMC method is used as a controller because it has the capacity to function adequately when nonlinear system characteristics are taken into account. Sliding mode control developed from groundbreaking studies done in the former Soviet Union in the 1960s [103]-[104]. Since that time, SMC has offered robust control techniques for all linear and nonlinear systems, including large-scale systems, MIMO systems, and time-variant systems. Due to fascinating characteristics of SMC, it is mostly used in the active control of ambiguous structures. SMC layout and implementation are straightforward and economical. Additionally, order reduction and robustness are ensured by SMC. Against system uncertainties and disruptions [105] The SMC technique is one of the other control methods that is thought to be variable structured [101]. Every time a erratic control architecture flips the system. The stated surface is reached by the system state space and remains there forever. We call the surface of the state space as surface of sliding type. To acquire the progress of the state track solely moving in the direction of switch, several control structures have been developed. As a result, there is no control composition can specify or include a definite path.

The definite path, however, moves in the direction of the controllers' structure boundaries, and the sliding mode [106] is the name given to these system motions that slide around boundaries. The system dynamics should be constrained to move in the direction of the surface of sliding type for the optimum sliding motion. In general, when designing SMC, we adhere to two guidelines. The first rule calls for system responsiveness, which is accomplished by creating a sliding surface. The second group of equations defining the surface of switching of the plant are therefore forced to be satisfied by the state variable of the plant. The second principle calls for the creation of a switched feedback gain that drives the state track of the plant towards the surface of sliding type. Based on the extended Lyapunov stability theory [107], these two principals were developed even while SMC is a powerful tool for control systems, it still has certain limitations. The switching of control must be place with infinitely high frequency in ideal circumstances. Transitioning to the limited sliding type mode's subspace glide pushes the system dynamics trajectory. However, in practice, control switching at high frequencies is not practicable because of the physical constraints of switching devices and the length of time required for control calculation. And the oscillations known as chattering are the effects of this high frequency. Chattering can cause system instability, energy loss, and even plant harm. Another problem mentioned in the literature is the sliding-mode type control method which succeeds in the situation of matched uncertainties but fails in the case of mismatched uncertainties [108] – [109].

2.2.4.1 Quasi-Sliding Mode Control

When direct digital implementation is tried, the sliding-mode type controllers also known as SMCS, that were initially designed for uninterrupted-time methods may not work effectively or may also cause the structure to become unstable. Since immediate execution has its limits, numerous academics have either addressed these restrictions or suggested methods that take the sampling process into consideration. The sampling procedure is said to restrict the occurrence of a real sliding mode in the literature [110]. As a result, definitions of sliding-mode of quasi types have been proposed as QSM, and the prerequisites for their presence are looked and specific attention is paid to the stability problem and the requirements for convergence and sliding [111]. The initial purpose of the sliding mode control systems (SMCS), as it is almost never possible to determine in practice the boundaries of unknown factors [113]-[114], a For the input-output structure, a suggested distinct vigorous adaptive QSM tracing controller is proposed [112]. This controller overcomes the unpractical assumptions without being aware of the parameters' higher and lower limits. On the other hand, a method [115] is provided that, as opposed to driving the system state to a vector with a different form [116], pushes it close to a switching hyperplane in the state space. They described a achieving law-based method for creating the law based on distinct-time sliding-mode type control and outlined the required characteristics of the controlled systems. Later, to ensure higher resilience and increased performance, a revised quasi-sliding mode control of quasi type also known as QSMC technique using achieving law based method was presented [117].

2.2.4.2 Exponential Reaching Law Sliding Mode Control

The flawless tracking performance of sliding-mode type control may potentially be guaranteed regardless of parameter or model uncertainty due to its resilience. Therefore, sliding mode control outperforms other nonlinear approaches in terms of resilience. However, the chattering issue might result in the system oscillating at high frequencies and becoming unstable throughout the real engineering application process. The changing of the reaching rule [118] by producing the discontinuous gain basing their study on this strategy to minimise or even eliminate chattering on control input is an intriguing method in the literature for chattering reduction. The system's approaching process can have better dynamic convergence quality thanks to the approaching legislation technique. Literature has proposed the exponential reaching law [119], power reaching law, and continuous reaching law. Numerous researchers have also studied the reaching law via research, and as a result, numerous new and improved reaching laws have been developed a fast power reaching law [120] that linearly combines the exponential reaching rule with the single power reaching law to reduce the reaching time. A unique exponential reaching law that incorporates the sliding mode variable's exponential term function is suggested in [121]. The exponential term can effectively decrease the chattering issue and react smoothly the changes in the sliding mode variable. In [122], a double-power sliding mode reaching law, which was applied to the robot tracking issue, was created by doubling the order of the reaching rule. Despite the fact that this concept might speed up reaching, it did not provide a qualitative study of the reaching law. On the basis of a double-power sliding mode reaching law and qualitative analysis, a multipower sliding mode reaching law [123] was presented, which may increase the system's dynamic reaction time and successfully suppress the chattering phenomena. An uncertain discrete-time system was given a novel discrete reaching law sliding mode control approach in [124], and the quasi-sliding-mode domain was enhanced by redefining the change rate as the second-order difference of system uncertainties and adopting the continuous-approximate function. A complementary sliding mode control (CSMC) [125] significantly decreased system tracking errors, reduced chattering, and increased system resilience.A control strategy devised for the permanent magnet linear servo motor in [126] that combines complimentary sliding mode control and Elman neural network provides a strong dynamic response and steady-state control precision.

2.2.4.3 Decoupled Sliding Mode Control (DSMC)

The SMC method cannot be applied to nonlinear systems with non-canonical forms, so decoupled sliding mode controllers are used for nonlinear under-actuated systems as a straightforward method to achieve asymptotic stability. Numerous sophisticated features, including strong performance and robustness against parameter variations, are present in this method. Decoupled sliding mode theory has been put out [127], and acquired law design methodology for DSMCs has been provided [128]. Numerous scholars are working to correct this [129] – [130]. A nonlinear model of an inverted pendulum system has been given for DSMC [131], which structures uncertainty in this system by taking into account the possibility of a measurement error in the pendulum angle. For the purpose of simulating the stability of an inverted pendulum system and its design, a Pareto optimum DSMC has been presented [132]. Nonlinear systems built upon a multi-objective GA have been subjected to Pareto design of decoupled sliding-mode controllers [133]. An enhanced particle swarm optimization (PSO) approach has been used to find the ideal DSMC parameters [134]. The Moving Least Squares (MLS) approximation has then been used to adjust the optimum controller to any beginning situation. Finally, a ball and beam system has successfully used the suggested online optimum DSMC. Fourth-order systems have also been used with decoupled sliding-mode controllers based on time-varying sliding surfaces [135]. The input-output mapping of the one-dimensional fuzzy rule bases served as the basis for the linear functions that computed the time-varying sliding surface slope. In addition to having a more straightforward structure than the decoupled control techniques now in use, the suggested approach does so without sacrificing speed [136]. A class of fourth-order nonlinear systems can easily reach asymptotic stability using the decoupling technique. For nonlinear systems, a decoupled sliding-mode with fuzzy-neural network controller has been developed [137]. This technique effectively controls nonlinear systems with a single input and several outputs. Additionally, the system's reaction time will converge more quickly when using this method. The application of GAs optimization to DSMCs is found in [138]. For a class of fourth-order nonlinear systems, a non-singular decoupled terminal sliding-mode control has been used [139]), which offers a significant increase in terms of quicker dynamic reactions. For a class of mechanical systems with underactuated actuators, direct adaptive fuzzy sliding mode decoupling control has been used [140]. Control for the cart-inverted pendulum system has been carried out using a decoupled third-order fuzzy sliding model [141]. On the other hand, in nonlinear systems, the existence of uncertainties such modelling error, outside disturbance, and measurement error can cause the system to become unstable in addition to affecting performance and reaction. The sensor's noise, poor resolution limit, and other issues are among those that might lead to measurement mistake. Therefore, DSMC is modified using a variety of approaches to provide a reliable controller that typically has supervisory performance.

2.2.4.4 Fuzzy sliding mode Control (FSMC)

Zadeh first proposed the fuzzy sets theory in 1965 [19]. Since then, the fuzzy set theory and its applications have advanced quickly as a result of several research initiatives. Mamdani put out the idea for the first productive use of fuzzy set theory in the control in 1974 [142]. In general, fuzzy controllers are very helpful when working with complicated or poorly specified systems, which makes it difficult to employ traditional control methods. The features of the system to be regulated are the foundation for the design of the fuzzy controller. This makes the effective and economical construction of the fuzzy controller a study area in fuzzy controller design. The "self-organizing controller" was subsequently suggested by researchers [143] and others [144], in which the fuzzy controller may be developed through a "learning" process. The creation of the performance index table still necessitates a lot of trial-and-error work, though. Based on the design requirement, it is equally challenging to maximize the performance of the resultant system. The capacity of fuzzy logic to capture the degree of uncertainty in human thought is one of its key characteristics. Consequently, fuzzy logic is an approach to deal with the unknown process when the statistical prototype of one process is absent or exists but is not certain [145]. However, the analysis is difficult due to the enormous number of fuzzy based rules for a system of high order. Nowadays, the fusion of fuzzy logic with SMC has received a lot of attention. Multiple fields have developed composite fuzzy sliding mode controllers [146] – [151]. The membership function must be both broad enough to decrease the effect of sensitivity in noise and sufficiently intense for precision when using fuzzy based logic reasoning systems to get a gain [152]. The performance of a traditional pure fuzzy controller can be enhanced by sliding mode qualities in a composite fuzzy SMC, on the other hand FSMC generate the control output based on sliding conditions of 3 types (approaching, sliding, and stable). Given that the fuzzy inference rules input variable is determined by the sliding surface, rules may be minimized. In terms of eradicating settling time, overshoot, tracking precision, and steady-state error removal, the suggested controller outperformed a pure fuzzy controller [154]. To deal with the chatter issue and attain zero steady-state error, the FSMC, proportional-integral control and state feedback control are used [155]. To allow for the best control attempts when uncertainties are present, membership function parameters cannot be changed. Fuzzy logic is used by the SMC during the reaching phase to reduce chattering without compromising robust performance [156]. The sign function is included in the final control law [157]-[158], There is, however, some chatter in the attempts while control. On the other hand, adding SMC to a fuzzy neural network offers a potential remedy for the chattering phenomenon. Intelligent uncertainty observers were developed in [159] and [160] to assess the constrain of lumped uncertainty, however the network architecture and inference method were overly complicated. By connecting the fuzzy controller's principles to those of a sliding-mode based controller with a bounded layer, a fuzzy sliding mode design method was made possible [161]. This layout can result in a solid closed-loop system while preventing the SMC's chattering issue. A similar paper on fuzzy sliding mode control research may be found in [162].

The desired sliding mode can ensure performance, but the same issues that plagued the fuzzy controller's design still exist, including choosing scaling factors [163] and creating a fuzzy rule base [164]. The computerized layout of fuzzy sliding mode control using a genetic algorithm is accomplished by recasting the issue of establishing the rule-base into one of parameter optimization [165]. It is not ideal for real-time applications since it takes a long time. In [166], two ways of adaptive SMC schemes have been developed that the fuzzy systems have utilised to estimate the new system operates while building the SMC of nonlinear systems. These approaches take into consideration the fuzzy approximation and sliding mode control scheme. In the concepts of fuzzy based control and sliding mode type, the sliding surface of sliding mode control is dynamically optimized by fuzzy control. The controller considerably decreases the control chattering that is inherent to sliding mode controls, as well as uncertainty brought on by mismatched dynamics and disturbances from outside sources. This study compares the outcomes of several sliding mode fuzzy logic controller applications in terms of oscillation and settling time of controller output.

2.2.4.5 Neural Sliding Mode Control

The SMC is a reliable nonlinear control method with a quick transient response and effectiveness in overcoming uncertainty. The chattering and discontinuous control effort, however, might ignite the high-frequency dynamics. The creation of artificially intelligent control for robotic manipulators has drawn a lot of attention during the past 20 years. Neural network control (NNC) and fuzzy control are the two intelligent-control methods that are most often used. A nonlinear function may be learned and approximated by neural networks with any degree of precision. To represent complicated processes and account for unstructured uncertainty, the controller uses this capability. Learning, parallelism, and fault tolerance are characteristics of neural networks [167]. In several situations, NN-based SMC has been employed as the controller [168]–[171]. Wavelet neural networks (WNN) are created by fusing wavelets with neural networks. It blends wavelet decomposition's identification capabilities with artificial NN's online learning capabilities [172]. Its notable features are rapid convergence, excellent accuracy, and a small network size [173]. A approach called adaptive neural tracking control was put forth by certain researchers to handle control systems with dynamic uncertainties [174]. Robust control based on neural networks was used to manage the nonlinear multilayer systems [175]. Any function of nonlinear type over a neat set with random precision may be estimated by an RBF neural network, according to previous research on universal approximation theorems on RBF [176]. To follow a predetermined trajectory, the nonlinear system’s output is controlled. The Lyapunov synthesis method is used to establish an control algorithm adaptive type, which is based on the RBF model. A solid performance may be guaranteed with the chattering action reduced. The manipulator dynamic equation uses an RBF network to estimate the unknown portion, and sliding mode control may also be employed to account for approximation error and disturbance.

2.3 Review of control techniques of nonlinear systems

In this section, a brief view on the implementation of optimization algorithms and controlling techniques is discussed on nonlinear systems with their mathematical modelling. The nonlinear systems which are considered here for the literature review are 2 DOF ball balancer system, inverted pendulum, robot arm manipulator, translational oscillator with rotational actuator (TORA) system.

2.3.1 Review of Control Techniques for 2 DOF Ball Balancer Systems

The modified PD [177] control on ball and beam system uses two configuration of PD controller i.e., serial and parallel. The stability is also analyzed with complete nonlinear model. A coupling free model of ball and beam system is given in [178] with objective to only control the ball position so they suggested separate control for ball and motor positions. The conventional, modern and intelligent controllers has been investigated in terms of step response to observe the performance of system [179]. The tracking of un-modeled dynamics system is control by filter based LQR control [180]. The proposed approach shows the effectiveness and feasibility as compare to existing methods. The decoupled fuzzy sliding mode controllers (DFSMC) with ACO algorithm [181] on ball and beam system. The ACO algorithm used to tune the DFSMC controller parameters and performance is compared with standard ACO algorithm. PSO algorithm with PID and fuzzy controller on the ball and beam [182] with parameter variations have already implemented to control the ball position. Fractional order PID (FOPID) controller on ball and beam model is implemented the both FOPID and conventional PID to control the ball position [183]. The FOPID gave better results as in controlling ball position. 2 DOF feedback controller on the system handles the uncertainty to enhance the tracking [184]. The results obtained were satisfactory compareto existing methods. Fuzzy logic controller (FLC) for ball and beam system is used with modified ACO algorithm (MACO) to tune membership function of FLC [185]. The performance of MACO compared with standard ACO algorithm and results shows better convergence and accuracy in position tracking. The PID controller is use to balance the ball position in ball and beam system and the parameters of controller have been optimized with trial and error method and PSO algorithm [186]. The PSO algorithm not only tuned the controller parameter well but also improves the system response and enhance the system efficiency. The optimal fuzzy controller on ball and beam system with gravitational search algorithm tuned the FLC parameters to improve the control performance [187]. The ball balancer system has intrinsic complexity, making it difficult to stabilise the system such that the ball may be moved to a precise place and retained there while minimising tracking error and time. These problems arise from the system's intrinsic complexity. The goal of this study is to compare various metaheuristics control strategies used on a ball balancer. Results from the Ball Balancer demonstrate the effectiveness of modifying a metaheuristics strategy for controller optimization.

2.3.2 Review of Control Techniques for Inverted Pendulum

The cart pendulum system stabilizes the pendulum position [188] author used an energy control method in which an “energy well” was built within cart to prevent outside motion. When the position of the pendulum came upright, then stabilize controller was activated to hold the same position of the pendulum. From the above proposed method, the Lyapunov stability function was also derived. A FOPID and integer PID controller control the two-wheeled inverted pendulum's pendulum position [189]. The FOPID and integer PID controller parameters were optimized with optimization algorithms like PSO, artificial bee colony (ABC), cuckoo search and GWO. The comparative study showed that ABC based FOPID gave better performance as compared to integer PID controller in stabilizing the pendulum position. Optimization based algorithm with LQR controller is designed to stabilize the position of double inverted pendulum [190]. The linearized system model was used with controller. The controller parameters are required to control the pendulum position. The viscous damping effect introduced into existing inverted pendulum system [191]. The PID controller with optimization algorithms was used to control the position. GSA and Genetic algorithms were used to optimize the controller parameters. The performance of both the algorithms was compared and found that GSA tuned PID control gave more effective and robust performance as compared to other one. The author studied and contributed in the analysis of rotary inverted pendulum system [192] found that most of controllers used on RIP system were based on model and depended integral motion. By using the trial-and-error method, the best linear quadratic controller for a nonlinear inverted pendulum is chosen. [193]. The matrix parameters of the LQR controller were selected by a synthetic bee colony. The ABC algorithms optimized controller parameters and gave better results as compare to conventional LQR controller. Ant colony optimization (ACO) based controller for inverted pendulum is also presented in literature to control the pendulum position [194]. The results found that ACO algorithm was efficient in tuning controller parameters and significantly minimizes the objective function by using the fractional order model (FOM) [195] for inverted pendulum and compared existing identified integer order model (IOM). Sine Cosine algorithm (SCA) was used to tune the coefficients of FOM and IOM model. The FOPID controller of the FOM model outperformed the fuzzy PD controller of the IOM model in terms of precision and pendulum position control. The hybrid PSO based model reference adaptive PID control for rotational inverted pendulum is also implemented to control the pendulum inupward position [196].

2.3.3 *Review of Control Techniques for Robotic Manipulator*

An adaptive fractional order PID sliding mode controller (AFOPIDSMC) with the Bat optimization approach is employed for the Caterpillar robot manipulator to reduce chattering and trajectory tracking inaccuracy [197]. Here FOPID controller is used to improve the trajectory tracking performance while SMC is neutralizing the chattering effect. The Bat algorithm is used to tune proposed controller parameter. The Lyapunov stability is also discussed for the proposed controller. The performance of controllers i.e., PID, FOPID, SMC, AFOPIDSMC, and BA-AFOPIDSMC have been compared and results indicate the effectiveness of system. A multi software platform for robot manipulator systems used 6-DOF robot arm manipulator with optimization algorithm for the analysis [198]. Optimization algorithm gives proper tuning parameter which further enhance in accuracy, efficiency, reliability. Genetic algorithm used to tune parameters of system so that they can model accurately and improves the performance in trajectory tracking. Optimal FOPID controller with adaptive colliding bodies' algorithm reduces the error to converge it and automatically update with next iteration on robotic arm manipulator [199]. The controller parameters are optimized by algorithm which further improves the convergence speed effectively. The 3-DOF robot arm with fractional-order fuzzy PID controller (FOFPID) for trajectory tracking [200], genetic algorithm (GA) has been used to tune controller's parameters. The performance of different controllers like PID, FOPID, Fuzzy PID and FOFPID have been compared. FOFPID gives better performance as compare to other in trajectory tracking. The performance of different controller on nonlinear systems is investigated on three controllers i.e., adaptive nonlinear model predictive control (NMPC), PID based NMPC [201]. These controllers are implemented experimentally on 2 link robot arm system. The comparative study has also been done for all the controllers. ABC based PID controller on robotic arm system is to control the arm position and improve the tracking performance [202]. The ABC algorithm tune the controller parameters and improves the system performance significantly. The optimized controller gives better trajectory response and make system more robust towards external disturbances. The mean square error, tolerance to disturbance, and parameter modifications of the radial basis function network (RBFN) and multilayer feed-forward neural network (MLFFNN) for robot systems are compared [203]. The performance obtained indicates the superiority of RBFN over MLFFNN for both robotic manipulators. 2-DOF robot system with PID controller and adaptive PSO (APSO) algorithm has been used to adjust the gainsof controller. The performance has also been investigated in terms of tracking error and cost function. The tuned controller minimizes the cost function and enhance the trajectory response well as compared to other numerical methods. The PID controller for optimal trajectory tracking control for 2 link robot manipulators with ant colony optimization (ACO) algorithm that optimizes the controller parameter and optimized controller is able to handle uncertainties of the system to very good extent [204]. The optimized controller improves the system trajectory response. A whale optimization algorithm (WOA)-based PID controller for nonlinear systems is used with a two-link robotic arm manipulator with the aim of managing the arm position and minimising the mean square error [205]. The WOA algorithm not only tuned controller parameters well but also controls the arm position subjected to step input [206]. The performance is analyzed in terms of time domain parameters and objective function i.e., MSE. A recently developed algorithms (PSO, WOA and GWO) with PID controller on robot manipulator is used to tune PID controller parameters [207]. They also compared the algorithm based controller performance in terms of settling time, tracking trajectory and ITAE. The results indicates that WOA based control gives better performance. The use of hybrid algorithm DE-TLBO based PID control for robotic arm system is used to optimize the controller parameter [208]. The performance of hybrid algorithm is compared with individual algorithm i.e. DE and TLBO. The results indicates that hybrid control well improves the transients and steady state performance of system.

2.3.4 Review of Control Techniques for TORA system

Cascade controllers and feedback passivating controllers are created to stabilise TORA systems asymptotically [209]. The TORA system is an underactuated system with certain unique properties that can encapsulate the fundamental idea behind dual-spin spacecraft. As a result, it has received a lot of academic interest [210] – [211]. Several researchers have concentrated on the control problems of single TORA systems in recent decades, and they have reported a number of amazing accomplishments. For underactuated TORA systems, a partial feedback linearization-based control approach is offered [212]. To stabilize the oscillation, the three state-feedback controllers for the TORA system [213] are used. On the basis of the function approximation method, a backstepping-like adaptive controller is developed [214]. To stabilize TORA systems, a block-backstepping based control technique is used [215]. To accomplish stabilising control, the sliding mode control (SMC) approach is offered [216]. Incorporating a continuous sliding mode mechanism, a novel nonlinear control method was used to regulate underactuated TORA systems [217]. Several control mechanisms have been used, all based on the cascade and passivity principles of the TORA system [218]. The simultaneous control experiments for a TORA system with a mounted pendulum's total mechanical energy and actuated variables are provided [219]. To avoid velocity feedback, a pseudo-velocity signal-based output feedback controller is suggested [220] - [221]. A multi objective control approach based on linear matrix inequalities is used to address the control problem of TORA systems [222]. The TORA system is demonstrated as a nominal linear system by suggesting an identification-based approach that enables H robust control [223]. In [224], an additive state-decomposition-based tracking control is offered to address the tracking issue for TORA systems. In [225], a worldwide robust output regulation is presented and applied to a family of weakly minimal phase nonlinear systems, including TORA systems. In order to guarantee the robustness with regard to dynamic uncertainties, a stabilizing output feedback controller is built [226]. In addition to these traditional model-based control strategies, numerous intelligent control algorithms are also developed and deployed for underactuated TORA systems [227] – [231]. TORA's decoupled underactuated dynamics were utilised after the development of an adaptive controller to account for various internal uncertainty and external disturbances [232].

Chapter: 3

Position Control of a Ball Balancer System using Particle Swarm Optimization (PSO), Bat Algorithm (BA) and Flower Pollination Algorithm (FPA)

3.1 Introduction

Nonlinear systems with underactuated actuators are approximated with intelligent-control and autonomous decision development methods [233]. They arose in a variety of contexts [234] and were attempted in a variety of ways. The bulk of the research employ the inverted pendulum [235], TRMS system [236], ball and beam system [237], hovercraft [238], furuta-pendulum [239], and, ball and plate system [240] as benchmark examples. In general, linear controllers make closed-loop control for such systems simple to implement [241], but their complex nonlinear dynamics limit the control rules for all generalized applications. This attracted the attention of several nonlinear control approaches [242], however these controllers struggle to handle external load and lagging brought on by extra feedback. For mechanically underactuated systems, the literature has developed feedback-linearization [243] and partial feedback-linearization [244]. However, challenges arising from a lack of resilience have limited their use in a variety of disciplines. Additionally, [245] provides a method for making available the system with a storage function which is based on passivity, at selected equilibrium point. This has the downside of being unable to amplify measurement noises with differential feedback. Additionally, these issues made it impossible to create a control system that could operate steadily. These goals are achieved by using a benchmark issue with an underactuated ball and plate to demonstrate position control and route tracking. Previous studies [246] – [247] presented a PID controller techniques for system control on a point-to-point basis using disturbance rejection controllers [248] and several optimization strategies [249], more evidence is provided to support the intended tracking performance for the systems.

Sliding mode control (SMC) have extensively explored and achieve the self-balancing control [250], and also developed a fractional-order SMC [251] to more effectively reduce the basic SMC chattering issue [252]. The predictive controllers are frequently used with ball balancer systems due to significant advantages when compared to time-varying reference systems [253]. Apart from it a hybrid of intelligent controllers, such as fuzzy logic, are employed to control the ball and plate system's location and trajectory [254], fuzzy cerebellar model articulation controller [255], and particle swarm optimization-based fuzzy-neural controller [256]. In real-world engineering applications, the PID controller is widely used, although their control algorithms for establishing self-balancing control using balancer systems are not present in the literature. The PID controller provides a number of benefits, including a simple design, high dependability, and exceptional stability. On the other hand, traditional PID controllers have a severe problem with parameter tuning. In many engineering and industrial design applications, researchers try to find the solution of a problem while dealing with exceedingly complex limitations. Such limited optimization problems are frequently highly nonlinear, and finding the best solution can be time-consuming. For issues involving nonlinearity and multimodality, traditional optimization does not produce good solutions. To solve such tough problems, the current tendency is to use nature-inspired metaheuristic algorithms, which have been demonstrated to be unexpectedly efficient. Researchers have only used a few natural properties so far, and there is still room for more algorithm improvement. There are a variety of strategies for tweaking PID parameters that may be found in the literature. These strategies make use of a variety of intelligent methods, including fuzzy [257], neural [258], self tuning algorithms [259], genetic [260], and evolutionary algorithms [261].

The development of an ideal multiobjective method for resolving combinatorial optimization problems, including particle swarm optimization (PSO), bat algorithm (BA), and flower pollination algorithm (FPA), has also been brought on by issues with intelligent controller optimizing parameters, memory limitations, fast convergence, poor searches, and excessive computing effort for genetic and other evolutionary algorithms. [262] – [263]. To construct a nonlinear algorithm that can try to address multimodal optimization difficulties, the attraction mechanism was integrated with light intensity fluctuations. An interpretative capability that is present in processing unit communication is known as swarm intelligence (SI) [264]. The theory of swarm explains stochastic manner, plurality, messiness, and unpredictability, but the theory of intelligence suggests that the analytical capacity is successful in some ways [265] – [266]. SI draws inspiration from social animal groups such flocks of birds and schools of fish as well as insects like termites, ants, wasps, and bees [267]-[268]. Individuals in the swarm can be described as simple solutions, yet they have a strong ability to work together to solve complex non-linear problems [269].

In this chapter, the simulation findings of controllers on the control of a ball balancer are present. The ball balancer system struggles with problems including putting the ball on the plate while maintaining balance and stabilized the point control, which enables the ball to be moved to a specific spot and kept there while minimising tracking error and time. These problems arise from the system's intrinsic complexity. This research makes a valuable contribution to the ball balancer system's mathematical modelling, parameter optimization, and controller design. The main goal is to compare the several metaheuristics control strategies used on a ball balancer system. The focus is on utilising simulink to develop and implement controllers for ball balancer setup. Adapting a metaheuristics algorithm for optimising controllers proved to be an innovative adaptation, as evidenced by the Ball Balancer findings. Finally, the comparison is performed using PSO, BA, and FPA.

3.2 2-DOF Ball Balancer System

A ball balancer is a well-defined problem for controlling balancing, monitoring ball position, and controlling visual servos. Creating a control system for a ball balancer utilizing two degrees of freedom is to maintain the ball's position on the balance plate. In order to accomplish this, it is essential to utilise the ball's X-Y position to control the position of the rotating servos connected to the bottom of plate. A ball and plate mechanism is pictured graphically in figure 3.1.

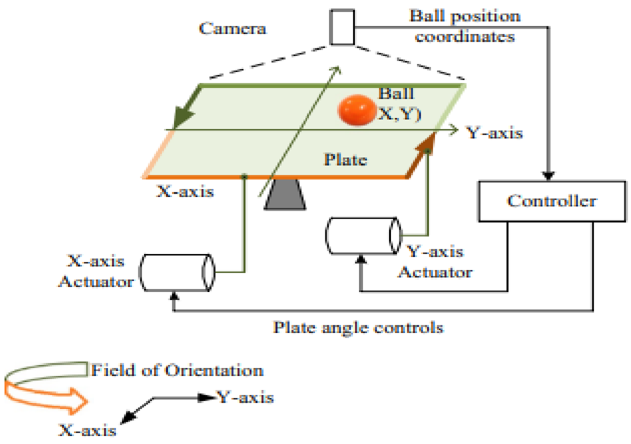


Figure 3.1 Diagrammatic depiction of ball and plate system

The 2 DOF Ball Balancer module comprises a free-moving plate on which a ball can be placed. Two DOF gimbals connect two actuation units to the sides of the plate. The plate can be rotated in both the X- and Y-axis directions. The servo motors are wired together in actuating units, which are controlled by a potentiometer. To balance the ball at a specific planar position, the plate's inclination can be adjusted by adjusting the position of the gear of servo load. The Faulhaber series DC micromotor [270] is utilised to balance the system in both directions using the rotational motion of the plate.

3.2.1 *Mathematical Modeling of Ball Balancer System*

Two spinning servo base units are required to operate two actuators in a two degrees of freedom (DoF) ball balancer. The plate's configuration matches that of the servo devices, and it is anticipated that the dynamics of both devices would not change. As a result, the ball balancer can be modelled by two decoupled ball and beam systems [271].

The transfer function $S_s(s)$ represents the dynamics between the motor load gear $\theta_{gear}(s)$ and the motor (s) input voltage $V_m(s)$. Ratio of the ball's position dynamics $x(s)$ to the load gear angle $\theta_{gear}(s)$ is depicted with $S_b(s)$. Due to similar servo dynamics shared by the ball balancer coordinates, this research proposes a paradigm of control in the X direction. Figure 3.2 shows the ball and plate system's X-axis control.

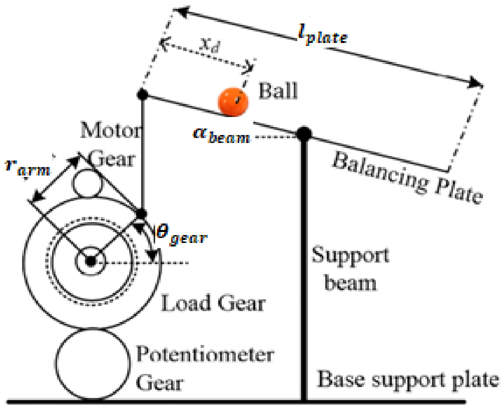


Figure 3.2 2-DOF ball balancer free body diagram

A positive voltage servo motor rotates the gear in the positive direction, causing the beam to rise and the ball to roll in the positive direction. The forces exerted on the ball as it goes down the beam will be, according to Newton's first law of motion:

$$\boldsymbol{m_{ball}\ddot{x}(t) = F_{x,t} - F_{x,r}} \tag{3.1}$$

Where $\boldsymbol{m_{ball}}$ is the ball's mass, $\boldsymbol{x(t)}$ is its displacement, $\boldsymbol{F_{x,r}}$ is its inertia force, and $\boldsymbol{F_{x,t}}$ is its gravitational translational force. When the momentum force and gravitational force are both equal, the ball is considered to be in equilibrium.

The ball inertia force $\boldsymbol{F_{x,r}}$ is given as: $\boldsymbol{F_{x,r} = m_{ball}\, g\ \ sin\, sin\, \alpha_{beam}}$ (3.2)

Where, g = gravitational constant, and α = is beam angle. The gravitational translational force $F_{x,t}$ is equal to, $F_{x,t} = \frac{J_{ball}\ddot{x}(t)}{r^2{}_{ball}}$ (3.3)

The nonlinear equation of motion of ball beam is given as.

$$\boldsymbol{m_{ball}\ddot{x}(t) = m_{ball}\, g\ \ sin\, sin\, \alpha_{beam}\ - \frac{J_{ball}\ddot{x}(t)}{r^2{}_{ball}}} \tag{3.4}$$

The acceleration is given as $\ddot{x}(t) = \frac{m_{ball}\, g\, r^2{}_{ball}}{m_{ball}\, r^2{}_{ball} + J_{ball}}$ (3.5)

The beam angle α, is influenced by the ball's placement on the plate, which is further influenced by the servo gear angle. The following is the relationship between gear angle and beam angle:

$$sin\ sin\ \theta_{gear} = \frac{sin\ sin\ \alpha_{beam}\ l_{plate}}{2r_{arm}}$$

Where θ_{gear} = is a gear angle, l_{plate}= plate's length, and r_{arm} = distance between the coupled joint and the servo output gear shaft. The nonlinear equation for ball motion in terms of gear angle is:

$$\ddot{x}(t) = \frac{2\, m_{ball}\, g\, r_{arm}\, r^2{}_{ball}}{l_{plate}(m_{ball}\, r^2{}_{ball} + J_{ball})} sin\ sin\ \theta_{gear} \quad (3.6)$$

The ball's linearized equation of motion is given as at zero angle:

$$\ddot{x}(t) = \frac{2\, m_{ball}\, g\, r_{arm}\, r^2{}_{ball}}{l_{plate}(m_{ball}\, r^2{}_{ball} + J_{ball})} \theta_{gear} \quad (3.7)$$

Also, the following is the transfer function for regulating ball position for input θ_{gear} and output x

$$S_b(s) = \frac{x(s)}{\theta_{gear}(s)} = \frac{K_b}{s^2} \quad (3.8)$$

Where, $K_b = \frac{2\, m_{ball}\, g\, r_{arm}\, r^2{}_{ball}}{l_{plate}(m_{ball}\, r^2{}_{ball} + J_{ball})}$ = Model gain

Similarly, the servo motor's control of plate angle is expressed as a transfer function.

$$S_s(s) = \frac{\theta_{gear}}{V_m(s)} = \frac{K_g}{s(1+s\tau)} \quad (3.9)$$

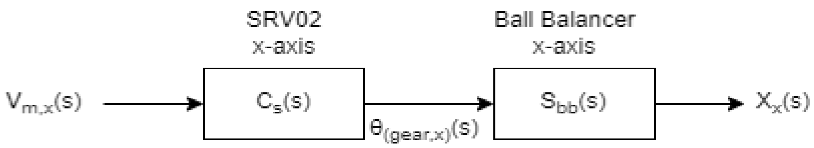
The cascaded path in between the servo motor and ball balancer unit is described as having the following overall transfer action: $S(s) = S_s(s)S_b(s) = \frac{x(s)}{V_m(s)} = \frac{K_b K_g}{s^3(1+s\tau)}$ (3.10)

The following equation of state-space representation is provided:

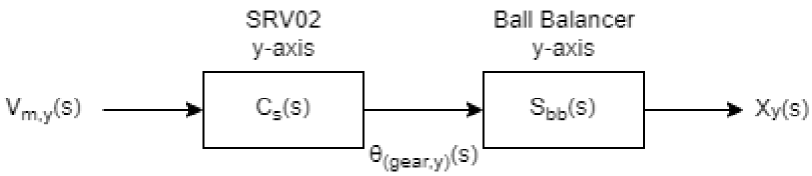
$$\begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \\ \dot{\theta}_{gear}(t) \\ \ddot{\theta}_{gear}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & K_b & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1/\tau \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \theta_{gear}(t) \\ \dot{\theta}_{gear}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_b K_g}{\tau} \end{bmatrix} u(t)$$

3.3 Structure of Ball Balancing Control

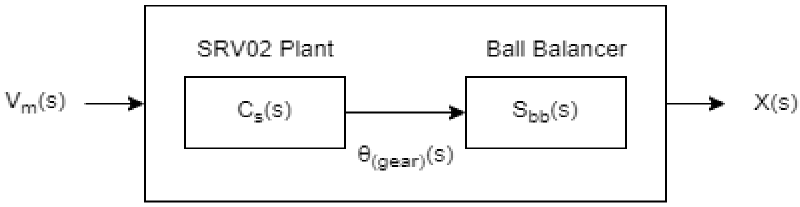
The two-degrees-of-freedom ball balancer's open-loop block diagram is shown in Figure 3.3 as a decoupled approach, in which the x-axis servo has no influence on the y-response axes.



(a) x-axis servo



(b) y-axis servo



(c) one dimensional representation

Figure 3.3 2-DoF ball balancer system open-loop block diagram

Figure 3.4 shows the SRV02's x-axis control model, which is combined with the ball balancer mechanism. The ball balancer block diagram depicts two loops of control. The SRV02 motor model is the first loop, and the 1D ball balancer is the second loop. The inner loop's goal is to control the servo motor's position and estimate the voltage in order to calculate the load's desired angle.

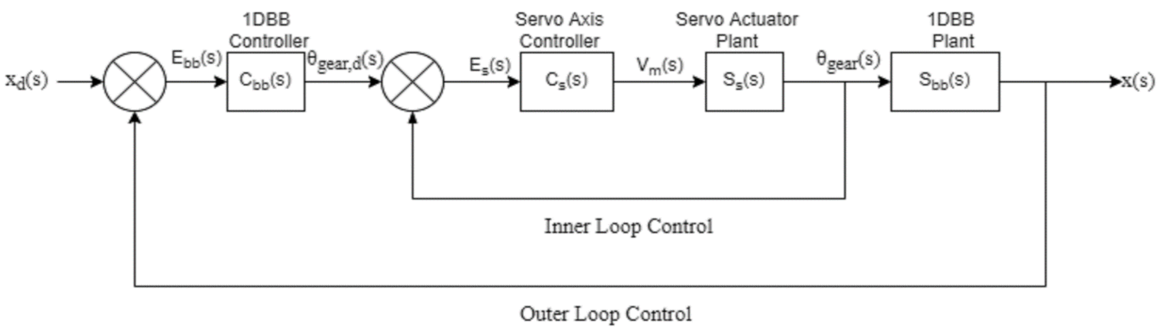


Figure 3.4 Closed loop ball balancer system block diagram

The inner loop for position control system is given as

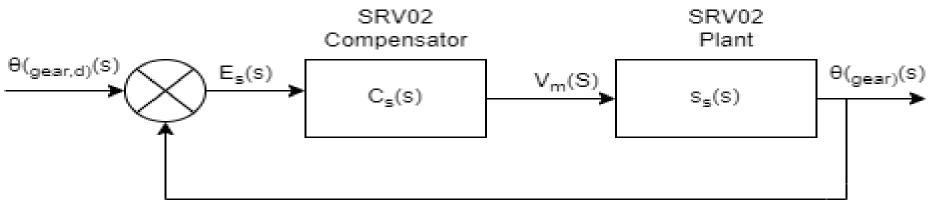


Figure 3.5 SRVO2 closed loop system

The inner loop that manages the SRV02 load shaft's position is finished. It is possible to consider the servo dynamics to be insignificant. It is therefore assumed that the target angle is equal to the actual load angle.

$$\theta_{gear}(t) = \theta_{gear,d}(t) \tag{3.11}$$

The outer loop shown in figure 3.6, will be employed to regulate the ball's position along x-axis of the plate.

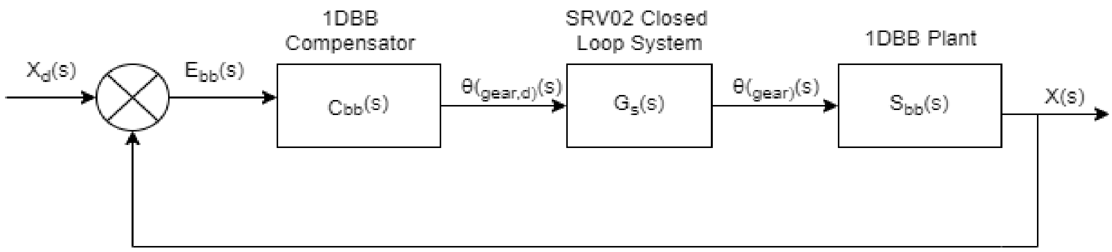


Figure 3.6 x-axis 2DBB closed loop system

The 1DBB controller must be modelled as a PID controller in the time domain in order to determine the first operating gains.

$$\mathbf{\theta_{gear,d}(t) = K_{p,dbb}\big(x_d(t) - x(t)\big) + K_{d,dbb}\left(h_{sd}\left(\frac{d}{dt}x_d(t)\right) - \frac{d}{dt}x(t)\right) + K_{i,dbb}\int\big(x_d(t) - x(t)\big)d(t)} \tag{3.12}$$

Where, $\boldsymbol{K_{p,dbb}}$, $\boldsymbol{K_{d,dbb}}$ and $\boldsymbol{K_{i,dbb}}$ is proportional gain, derivative gain and velocity gain respectively $\boldsymbol{h_{sd}}$ is a velocity weight parameter that is included by a controller to compensate for the derivative error.

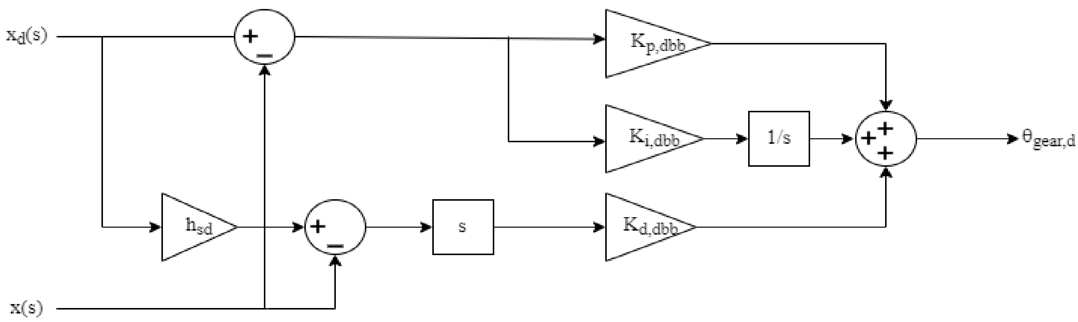


Figure 3.7 PID compensator with derivative set point weight

The closed-loop equation of a ball balancer's outer loop system when servo dynamics are disregarded.

$$\theta_{gear}(s) = \left(K_{p,dbb} + \frac{K_{i,dbb}}{s}\right)\left(x_d(s) - x(s)\right) + K_{d,dbb}s\left(h_{sd}x_d(s) - x(s)\right) \tag{3.13}$$

When the ball rotates along x-axis of the plate, the required gear load are equal ($\theta_{gear,d} = \theta_{gear}$). The 1-D ball balancer system is used to replace the outer loop controller in order to obtain the closed-loop equation:

$$\frac{x(s)}{x_d(s)} = \frac{K_{ball}(K_{p,dbb}\,s + K_{i,dbb} + K_{d,dbb}\,s^2\,h_{sd})}{s^3 + K_{bb}\,K_{p,dbb}\,s + K_{bb}K_{i,dbb} + K_{bb}\,K_{d,dbb}\,s^2} \tag{3.14}$$

Where, K_{ball} is a ball balancer constant.

To calculate the PID constant, the third order prototype equation is given as:

$$(s^2 + 2\zeta\omega_n s + \omega_n{}^2)(s + p_0) \tag{3.15}$$

Where ω_n is the system's natural frequency, ζ is the damping ratio, and p_0 is the pole location.

$$s^3 + (2\zeta\omega_n s + p_0)s^2 + (\omega_n{}^2 + 2\zeta\omega_n p_0)s + \omega_n{}^2 p_0 \tag{3.16}$$

The closed-loop equation's third-order characteristic equation is:

$$s^3 + K_{ball}\,K_{p,dbb}\,s + K_{ball}\,K_{i,dbb} + K_{ball}\,K_{d,dbb}\,s^2 \tag{3.17}$$

Equating equation no. (3.15) and (3.16), the following observations are made:

$$K_{ball}\,K_{p,dbb} = 2\zeta\omega_n s + p_0 \tag{3.18}$$

$$K_{ball}\,K_{i,dbb} = \omega_n{}^2 p_0 \tag{3.19}$$

$$K_{ball}\,K_{d,dbb} = \omega_n{}^2 + 2\zeta\omega_n p_0 \tag{3.30}$$

Moreover, the PID control gains may be computed as follows:

$$K_{p,dbb} = \frac{2\zeta\omega_n + p_0}{K_{ball}} \tag{3.31}$$

$$K_{i,dbb} = \frac{\omega_n{}^2 p_0}{K_{ball}} \tag{3.32}$$

$$K_{d,dbb} = \frac{\omega_n{}^2 + 2\zeta\omega_n p_0}{K_{ball}} \tag{3.33}$$

To meet the specifications of proportional derivative gain, the pole location is adjusted at origin, i.e, $p_0 = 0$.

Hence the control gains of PD controller is given as:

$$K_{p,dbb} = \frac{2\zeta\omega_n}{K_{ball}} \tag{3.34}$$

$$K_{d,dbb} = \frac{\omega_n{}^2}{K_{ball}} \tag{3.35}$$

The optimization the PD controller to balance and regulate the ball balancer system we have used different metaheuristics techniques.

Meta-heuristics are advanced methods for employing heuristics to address a variety of issues. The family of Soft Computing approaches, which allow for partially correct or faulty results, includes meta-heuristics. This comes at the expense of being unreliable in identifying the ideal answer to a specific issue. Such problem-solving approaches are necessary because they produce optimum or nearly optimal solutions in a reasonable amount of time, which justifies the necessity for them given the extreme magnitude of some problems and the failure of precise methods. The swarm intelligence technique family includes some of the algorithms. The primary source of inspiration in this field is the group behaviour of insects, animals, or any other type of species that results in intriguing global behaviours. The local interactions between individuals and their environments produce global decisions since there is no centralised control unit for decision-making.

Some of the techniques are used here to optimize the controller parameters, such as:

- BAT Algorithm (BA)
- Particle Swarm Optimization (PSO) Algorithm
- Flower Pollination Algorithm (FPO)

3.3.1 Particle Swarm Optimization (PSO) Algorithm

The PSO solution is frequently, nonetheless, pretty near to the overall ideal. PSO has been widely used to resolve nonlinear difficult optimization issues in a number of practical applications. A swarm-based intelligent stochastic optimization strategy called particle swarm optimization (PSO) was inspired by the way bees naturally swarm when they're looking for food. As a result, PSO has mostly been utilised to address a variety of optimization difficulties due to the adaptability of numerical experimentation. One of the most renowned swarm intelligence methods that has been widely applied in both science and business is the PSO algorithm. [272] - [273] The PSO is made up of a population of particles and shows a possible solution to the problem $\boldsymbol{K_p}$ and $\boldsymbol{K_d}$ in our situation. Each particle can be represented by an object having a position vector and a vector velocity, with the location relative to the search space and the velocity guiding the particle position during the process execution.

The basic PSO algorithm consists of the equation of velocity and position, respectively:

$$\boldsymbol{v_i(k+1) = w.v_i(k) + c_1 r_1 \big(pbest_i - x_i(k)\big) + c_2 r_2 (gbest - x_i(k))} \tag{3.36}$$

$$\boldsymbol{x_i(k+1) = x_i(k) + \Delta k.v_i(k+1)} \tag{3.37}$$

The population size is given by i=1....n. $\boldsymbol{pbest}$ (personal best) and $\boldsymbol{gbest}$ (global best) are the best positions achieved by a particle in a given position and the entire population in a given neighbourhood, respectively; w is the inertia constant; $\boldsymbol{c_1}$ is a social factor; $\boldsymbol{c_2}$ is the factor cognitive; $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ are random integers produced in the interval using a uniform distribution [0,1]; and t = 1. A social factor of 1.2 and a cognitive factor of 0.12 were employed in the simulation findings. The inertia constant, w, is set to 0.9. Figure 3.8 shows the flowchart of PSO with its initial parameters.

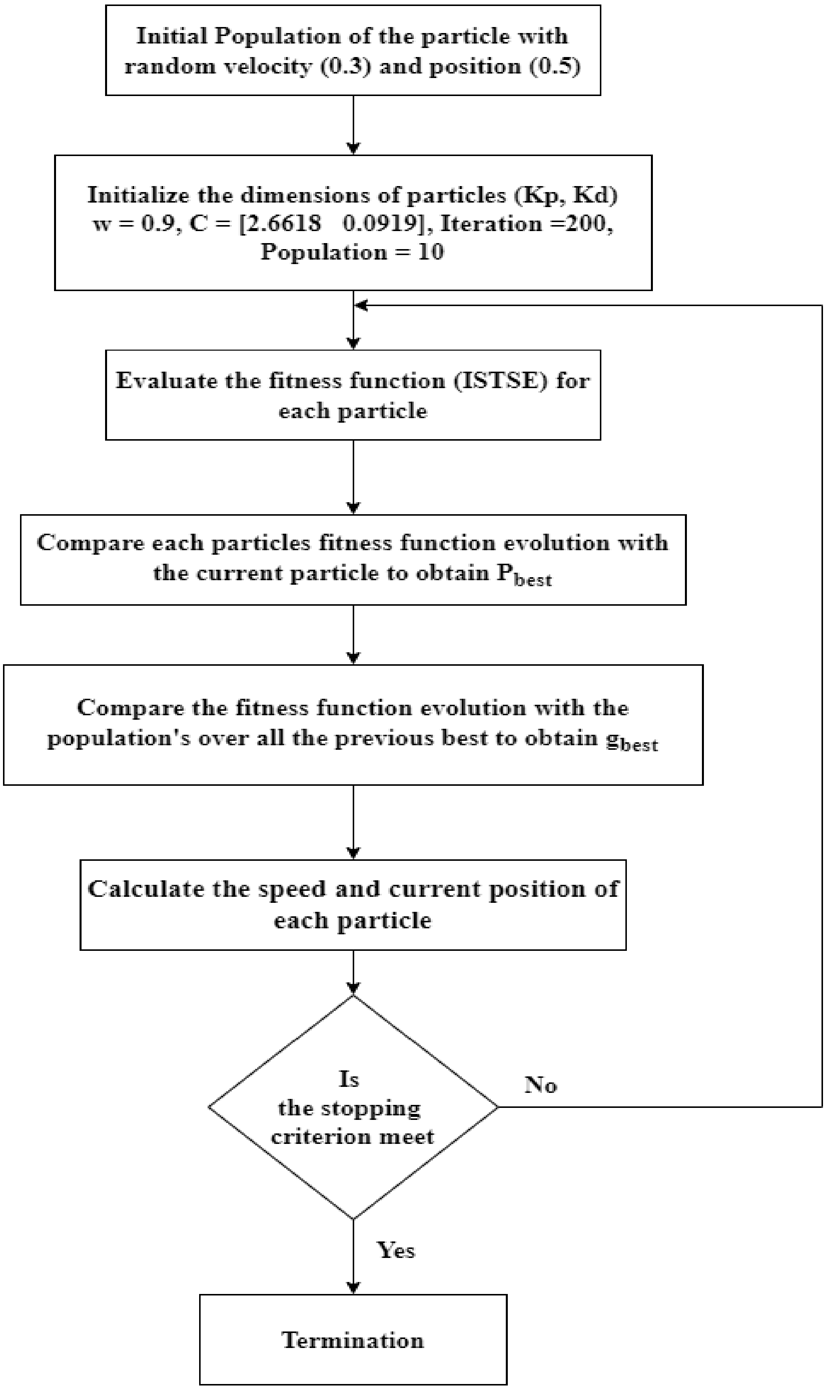


Figure 3.8 Flow chart of PSO algorithm with initial parameters

3.3.2 *Bat Algorithm*

The remarkable echolocation capacity of bats has drawn the interest of researchers from a variety of fields. Bats are attractive creatures. A type of sonar called echolocation is used by bats, primarily microbats, to determine an object's distance by producing a loud, brief pulse of sound that echoes back to their ears. This innovative positioning technique gives bats the ability to distinguish between an obstruction and a target, enabling them to hunt even in complete darkness. [274] - [278]. The fundamental BA algorithm is based on biological principles that mimic bat echolocation or sonar. Additionally, each agent in a swarm is capable of relocating to a prior optimal location discovered by the swarm [279] or discovering the most "nutritious" locations. The Bat method has demonstrated excellent efficacy in solving continuous optimization issues [280].

A set of bats stored in the form of a vector, each representing a candidate solution, is generated in this computational model. The goal is to go to the prey, which is the best approach for minimizing the cost function.

Initially, all n bats $\boldsymbol{x_i}(\boldsymbol{i} = \mathbf{1}, \mathbf{2}, \dots . \boldsymbol{n})$are initialized with the following parameters: pulse rate $\boldsymbol{r_i}$ velocity $\overrightarrow{\boldsymbol{v_i}} = \mathbf{0}$, amplitude $\boldsymbol{A_i}$, frequency $\boldsymbol{f_i}$ and position $\vec{\boldsymbol{x}}_{\boldsymbol{i}}$. For each instant the velocity and position are updated, respectively. The steps of bat algorithim is discussed in flow chart in figure 3.9.

$$\boldsymbol{v_i^j(t)} = \boldsymbol{v_i^j(t-1)} + \left[\boldsymbol{x_{cgbest}^i} - \boldsymbol{x_i^j(t-1)}\right]\boldsymbol{f_i} \tag{3.38}$$

$$\boldsymbol{x_i^j(t)} = \boldsymbol{x_i^j(t-1)} + \boldsymbol{v_i^j(t)} \tag{3.39}$$

The variable $\boldsymbol{\beta} \in [\mathbf{0}, \mathbf{1}]$ is an update using a random number created from a uniform distribution and weight $\boldsymbol{f_i} \in [\boldsymbol{f_{min}}, \boldsymbol{f_{max}}]$. The variable $\boldsymbol{x_{cgbest}^i}$ denotes the current best option available globally for a decision variable d, which is determined by comparing all solutions offered by n bats. In order to explore the domain of candidate solutions to the problem, the algorithm executes a local search in the form of a random walk: $\boldsymbol{x_i^{new}} = \boldsymbol{x_i^{old}} + \boldsymbol{\varepsilon m}$, where $\boldsymbol{m}$ is the mean of the amplitude of all bats at particular time t, and $\boldsymbol{\varepsilon}$ is a random value derived from a uniform distribution. The algorithm comes to stop when $\boldsymbol{r_i}$ hits a predetermined minimum value or after most iterations have been executed, which are known as stopping conditions. An amplitude of 0.5 and an initial pulse rate of 0.5 were used in this job. At maximum and minimum frequencies, 2 and 0 are formed, respectively.

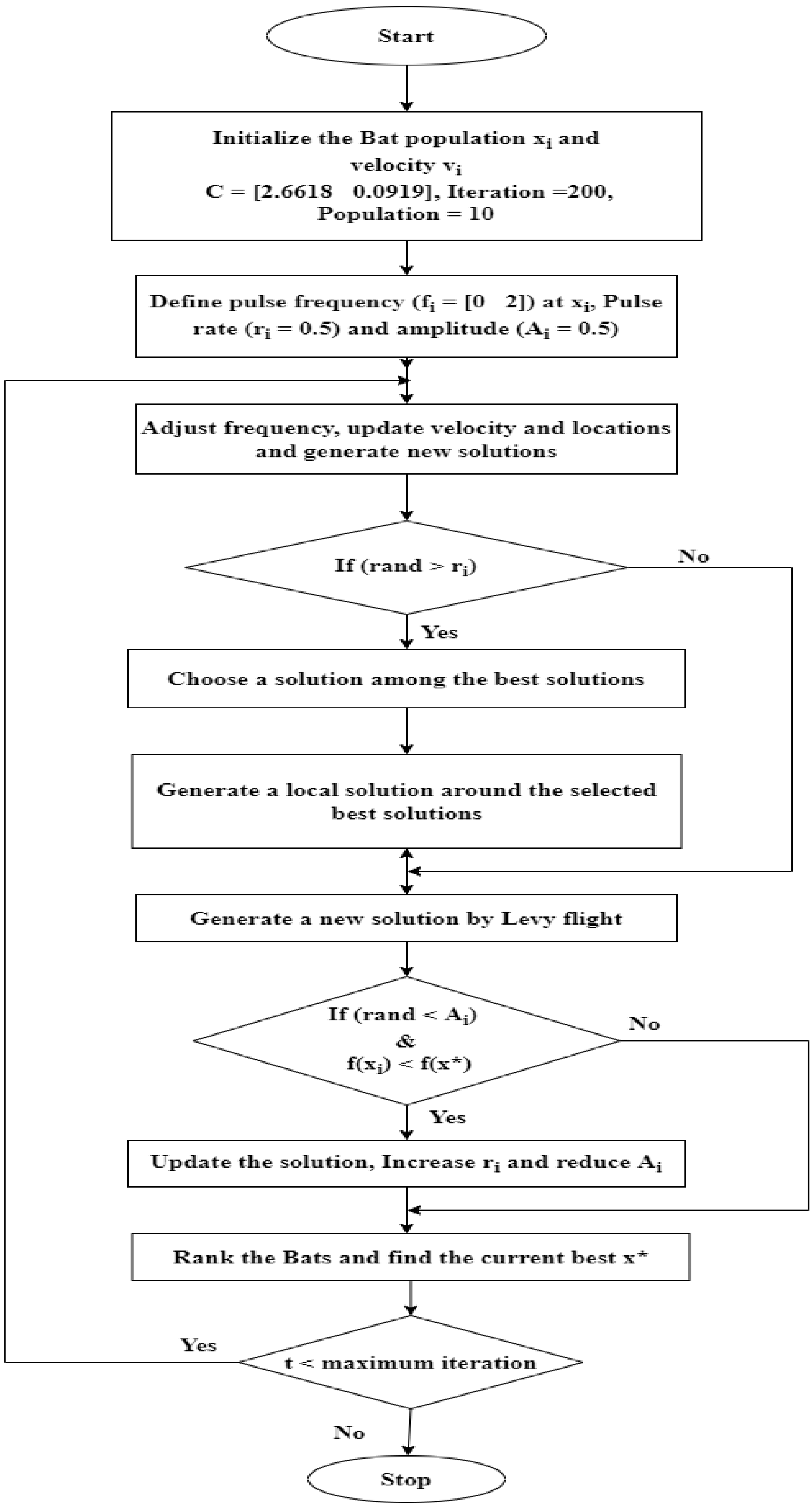


Figure 3.9 Flow chart of bat algorithm with initial parameters

3.3.3 *Flower Pollination Algorithm (FPA)*

The Flower Pollination Algorithm (FPA) is a method for pollinating flowers that was proposed by [281]. Because pollinators may fly great distances, they are classified as global pollinators, and the Lévy probability distribution can be used to describe their behaviors. Two key rules govern the implementation of the method utilizing the Lévy distribution. The direction of travel must be random. A uniform distribution can be used to generate a direction; however, the creation of steps must follow the Lévy distribution.

The following rules [281] were devised by Yang: 1 – Pollen transporters flying from Levy are thought to be part of a worldwide pollination process known as biological pollination and crossover; 2 - local pollination includes self-pollination and abiotic pollination; 3 - Loyalty to a flower can be thought of as having a probability of reproduction proportionate to the similarity of the two plants involved; 4 - pollination on a local and global scale is controlled by a probability $\boldsymbol{p} \in [\mathbf{0}, \mathbf{1}]$. Due to physical proximity and additional factors like wind, local pollination can significantly contribute to total pollination activities.

The best individual represented by$\boldsymbol{g}_*$. The first rule, along with a flower's loyalty, may be expressed mathematically as $\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + \boldsymbol{L}(\boldsymbol{x}_i^t - \boldsymbol{g}_*)$ where $\boldsymbol{x}_i^t$ is the pollen $\boldsymbol{i}$ in the vector of solutions $\boldsymbol{x}_i$ at iteration t, and L is the pollination strength, whose value is determined by the Lévy distribution. The insects can move a long distance with just a few steps away, and this can mimic with a Lévy flight. That is $\boldsymbol{L} > \mathbf{0}$ from a Lévy distribution.

$$\boldsymbol{L} \sim \frac{\lambda r(\lambda) sen\left({}^{\pi\lambda}/_{2}\right)}{\pi} \frac{1}{s+\lambda} (\boldsymbol{s} \gg \boldsymbol{s}_0 > \mathbf{0}) \tag{3.40}$$

Where $\prod(\boldsymbol{\lambda})$ is the Gamma function and $\boldsymbol{s}_0$ a minimum step.

Local pollination (rule 2) and flower loyalty are represented by the equation $\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + \boldsymbol{\epsilon}(\boldsymbol{x}_j^t - \boldsymbol{x}_k^t)$ where $\boldsymbol{x}_j^t, \boldsymbol{x}_k^t$ are pollens from separate plants of the same species in the same iteration. This is similar to a flower's allegiance in a small neighborhood. If $\boldsymbol{x}_j^t, \boldsymbol{x}_k^t$ were from the same species or population, a survey of the neighbourhood random walk (local random walk) would be created by selecting $\prod$ from a uniform distribution. To switch between undertaking global pollination and enhancing local pollination, an exchange probability, or probability of closeness p, is employed according to rule 4. The reason for this characteristic is that the majority of pollination actions are carried out by bees. It might happen on a local or global level. In terms of practicality, flowers that are close by or not too far away from the neighborhood are more vulnerable to pollination than those that are farther away. Figure 3.10 shows the algorithm of flower pollination.

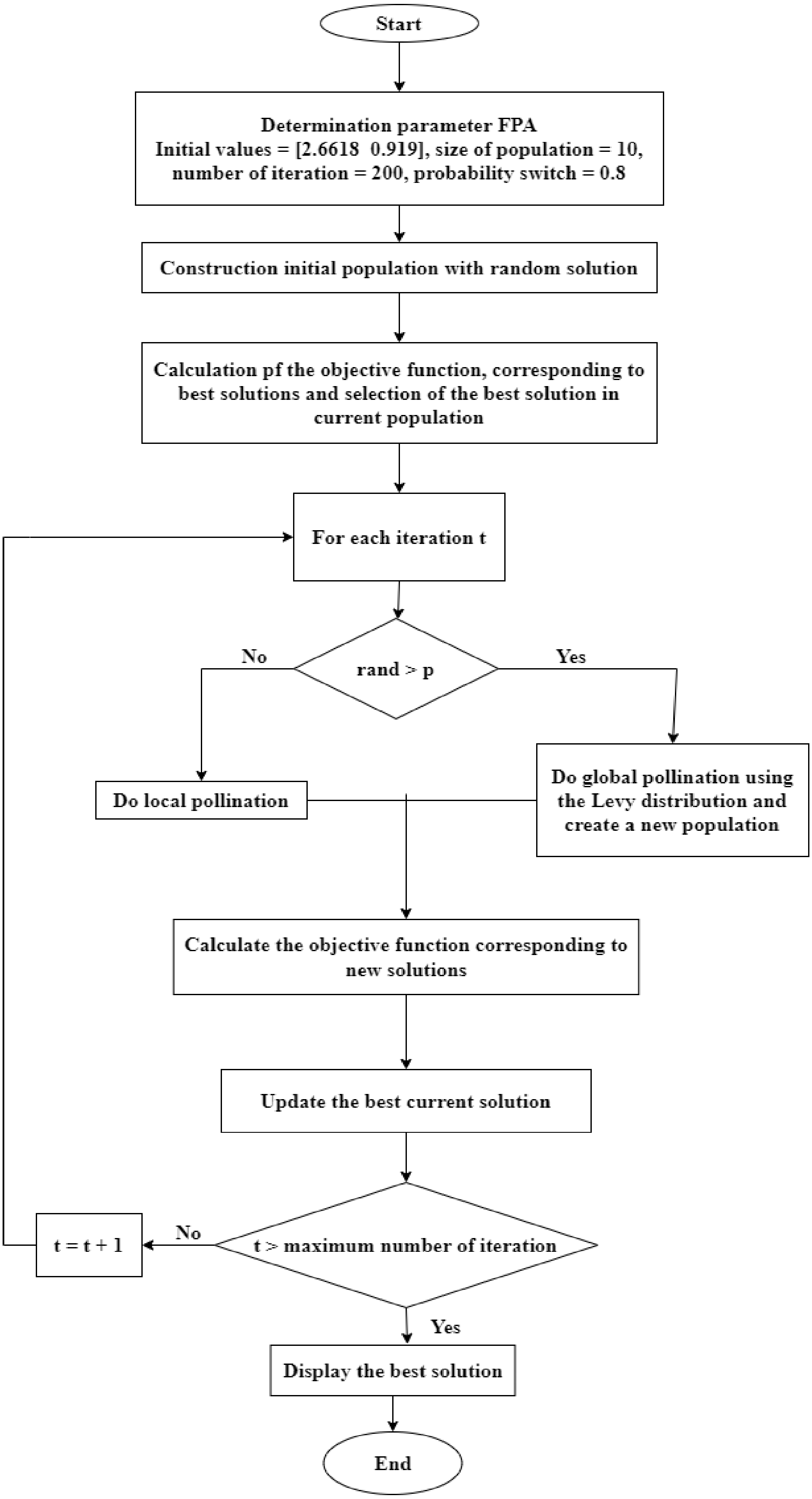


Figure 3.10 Flow chart of flower pollination algorithm with initial parameters

3.4 Simulation Results and Discussion

MATLAB/ Simulink software was used to produce two degrees of freedom ball balancer model numerical simulation described in Section 2. The behaviour of the controller for one servo unit impacts the behaviour of the controller for the second servo unit because the plates of the two servo units are symmetrical. Regardless of the fact that both controllers are developed in a decoupled context, they operate them in a connected environment. The technology is set up to manage the ball's square trajectory on a plate. PD is used to operate the ball balancer by sending a square pulse width with a frequency of 0.08 Hz and an amplitude of 5 volts serves as the reference trajectory. The PD controller's values are originally determined using the method explained in section 3.2. Moreover, the specifications of the PD controller are optimised using three different optimization methodologies (PSO, BA, and FPA), and the difference between the desired and measured ball position is measured as shown in figure 3.11. It has been determined that the plate angle may be adjusted by monitoring the discrepancy between the reference trajectory and the ball location coordinates that were acquired. Therefore, the objective function is chosen to maximise the performance depends on the discrepancy between the measured and desired ball location trajectories. In order to evaluate the effectiveness of PSO, BA, and FPA on the PD controller, the results of the same simulation running over the same trajectory are compared to the behaviour of the usual PD controller. Errors are expressed as integrals square time and absolute and generally said to be an objective function. By calculating the integral of error over a predetermined amount of time, all of these functions represent error as an objective function.

Throughout the optimization process, an objective function called $'\boldsymbol{J}'$ is attached to the best solution. The integral square error, integral absolute error, and integral time absolute error were originally used to define the objective function. But their slow response and large oscillation time, an integral of squared time-multiplied square of the error (ISTSE) is use to improve the error performance in optimization of controller. The objective function $'\boldsymbol{J}'$is given as:

$$\boldsymbol{J = \int_o^T t^2 e^2(t)\, dt} \tag{3.41}$$

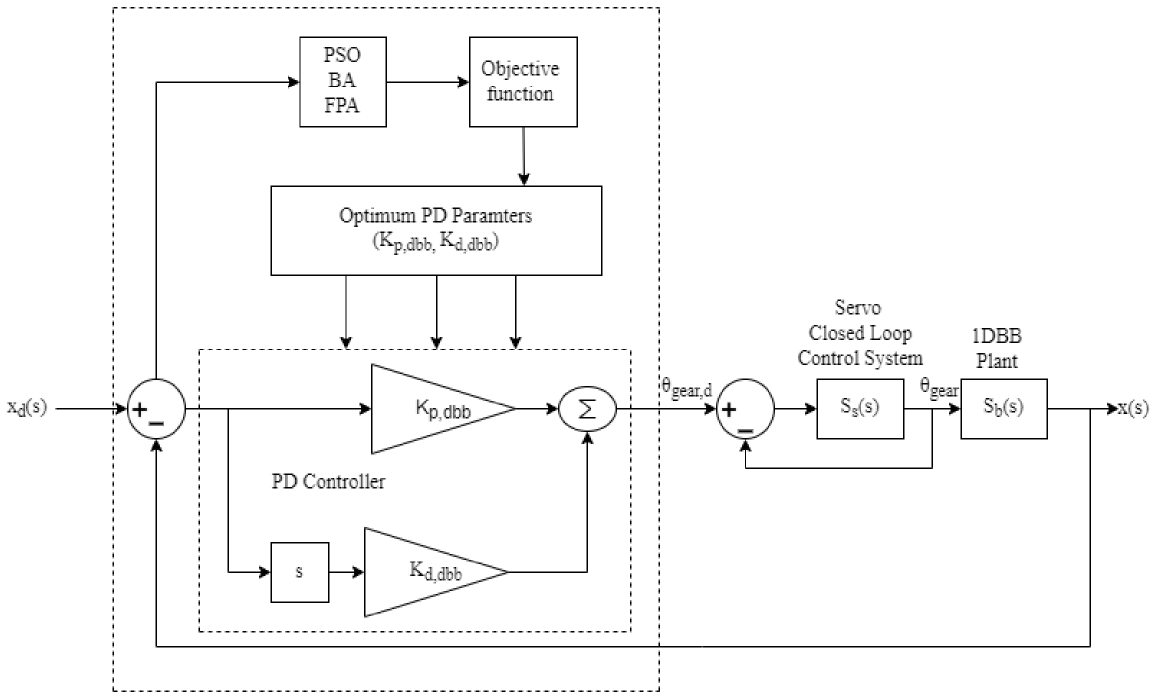
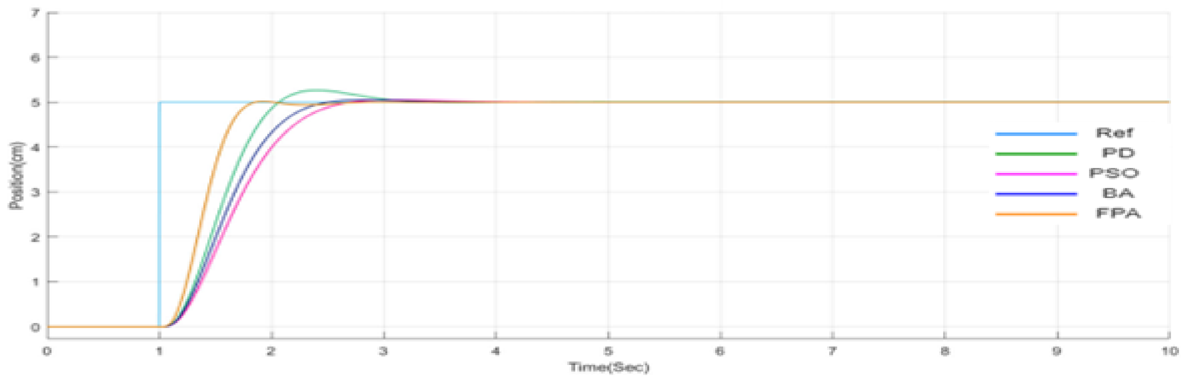
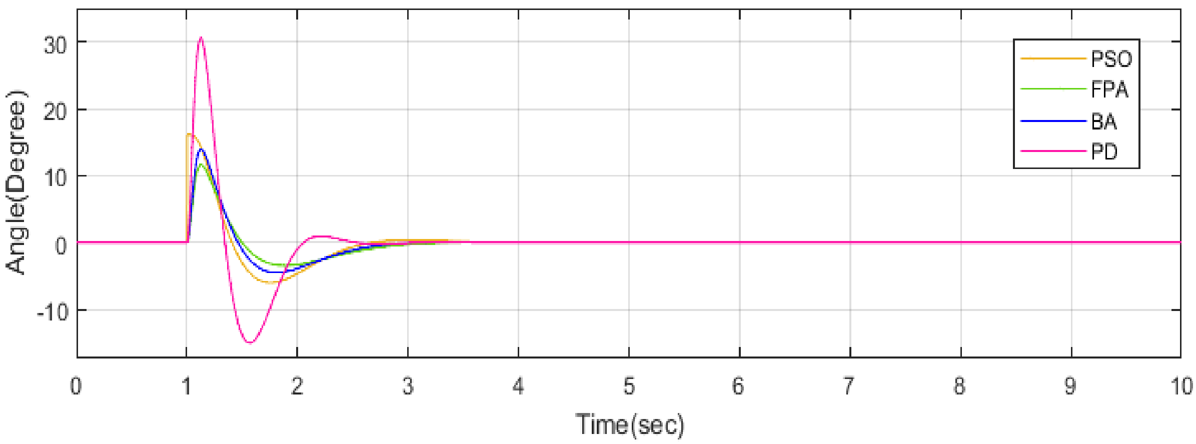


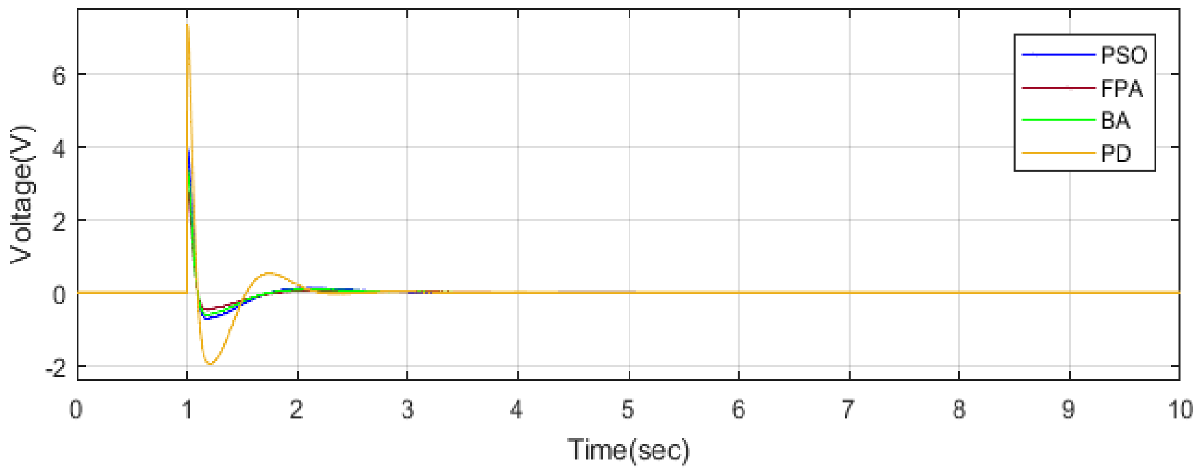
Figure 3.11 Ball position control using PSO, BA, FPA



(a)



(b)



(c)

Figure 3.12 (a) Position of the ball on the x-axis, (b) servo angle response of plate at the x-axis, (c) input voltage applied to the servo motor for the x-axis.

Figure 3.12 contains a detailed description of the ball balancer system's ball position, servo angle, and voltage optimizations for PSO, BA, and FPA. Figure 3.12 (a) presents the comparison of the PSO, BA, and FPA algorithms for the ball's position on the x-axis. The results demonstrate that the x-axis position of the ball is within a defined range. Therefore, the least change between the beginning and end locations illustrates the controller's performance. The FPA has a minimal final position and quickly obtains the goal value, followed by the BA algorithm delivering the minimum position and the PSO algorithm holding the minimum position. In addition, figure 3.12 (b) shows the ball's servo angle reaction on the x-axis, exhibiting the angle of servo motors. The controller accuracy to provide the balancing for a ball balancer system depends on the minimum control angle. In this situation, the FPA has a lower control angle than the BA and PSO controls, but the BA offers a better outcome than the PSO. The FPA algorithm uses a slow-moving plate to balance a ball, enabling the device to respond consistently. Figure 3.12 (c) depicts the servo input voltage fluctuation as a function of the controller action. The servo units for the FPA optimization control are noted for running at a lower voltage and settling down earlier than the other controller. The servo motors' speed is decreased to the smallest value feasible since the enhanced FPA produces the smallest position control and balancing angle. This guarantees that while the ball is moving in the right direction, it stays in its proper place on the plate. As a result, when compared, the FPA performs better than the BA and PSO.

Table 3.1 Performance parameters for various control using PSO, BA and FPA on ball balancer system

Controllers	Peak time $t_p(s)$	Settling time $t_s(s)$	Peak overshoot $M_p(\%)$
Particle Swarm Optimization (PSO)	1.57	2.21	30.2
Bat Algorithm (BA)	1.45	2.178	25.1
Flower Pollination Algorithm (FPA)	1.43	2.16	20.8

In addition, time domain specifications are derived to examine the performance of PSO, BA, and FPA approaches, with the results displayed in table 3.1. The results show that the maximal overshoot of PSO is 30.2 percent, leading to huge oscillations and making it impossible to balance the ball on the plate. In contrast, as seen in the graph, FPA has a good response to peak overshoot of 20.8 percent and exhibits perfect ball-on-plate balance with less oscillation. The integral of squared time-multiplied square of the error (ISTSE) value of PSO, BA, and FPA optimizations is also determined during the ball balancer's operation for square trajectory in terms of ball position. The findings are summarised in table 3.2.

Table 3.2 Integral of squared time-multiplied square of the error (ISTSE) for position during simulation results

Controller	ISTSE
	Ball Position
Particle Swarm Optimization (PSO)	45.337023
Bat Algorithm (BA)	45.446932
Flower Pollination Algorithm (FPA)	44.375275

PSO, BA, and FPA all produced outcomes that were near to one other when compared using the ISTSE performance measure. The FPA, on the other hand, achieved a better outcome and provides great position control of the system's ball balancer's plate with the ball on it.

3.5 Conclusion

In order to accomplish self-balancing and position control of a 2 dof balancer system, this study employs three alternative optimum algorithms to establish the parameters of PD controlller PSO, BA, and FPA. Simulation findings show that the developed strategy improves performance significantly within the context of the standard control structure. On the basis of time response analysis, the outcomes of the established control approaches are validated. On the ball balancer system, the provided controller has adaptability and good control performance. According to the findings, the FPA optimised technique performs better in compare to BA and PSO in terms of ISTSE, settling time, peak time, and peak overshoot.

Chapter: 4

Performance Analysis of Sliding Mode Controllers for Nonlinear systems

4.1 Introduction

Any system's primary need depends on the controller's control action. The control action is made precisely to accomplish the goal. Additionally, when system complexity rises, it becomes harder for the researcher to attain optimal control performance. Additionally, traditional controllers aren't any better at handling plant uncertainty due to their horrifying complexity. The sliding mode control (SMC) technique is used to stablize the system because it may function well when nonlinear system components are taken into account. Pioneering studies conducted in the former Soviet Union in the 1960s led to the development of sliding mode control [102] - [103]. Additionally, SMC is often employed in the dynamic control of uncertain systems due to its intriguing properties. SMC has a straightforward and economical design and execution process. Additionally, the system uncertainties and disruptions are enforced by SMC [100] - [104]. The SMC approach is one of the other control methods that is regarded as variable structured [105]. The state space's surface is referred to as a sliding surface. To acquire the movement of the state trajectory solely move in the same pattern of switching, several control structures have been developed. The absolute trajectory moves in the path of control structure margins. And these system motions that slide around their boundaries are referred as sliding mode [106]. The perfect sliding motion also occurs when the system motions are constrained to roll in the direction of the sliding surface. To design SMC, we typically adhere to two guidelines. Designing a sliding surface allows for the first guideline, which concerns system reaction. As a result, plant dynamics can be forced to change its state variable in order for it to meet the other set of equations that describe its switching plane. The second guideline is to create a switched gain that forces the plant's state trajectory in the direction of the sliding surface. The extended Lyapunov stability theory [282] is the foundation for these two laws. Since it would mean that the control commutes at an infinite frequency, a perfect sliding mode doesn't actually exist. The discontinuous pattern in the feedback control creates a specific dynamic behaviour near the surface known as chattering in the presence of switching flaws, such as switching delays and fractional time constants of systems. This behaviour is problematic because, due to its filtration process of output, it may stimulate unmodeled high frequency modes that harm the system's performance and even cause instability [283]. Some underactuated systems don't meet the requirements for a stabilising smooth feedback rule to exist [106][284]. According to several research, continuous feedback stabilisation may solve the issues that smooth feedback stabilisation faces [285].

When designing the sliding mode control for underactuated systems, the application of a decoupling method will produce results that are adequate in terms of system complexity and improved stability. A well-known resilient control method and common control tactic for nonlinear systems is sliding mode control (SMC) [289]. The following elements explain its appeal in the field of research: In certain circumstances, the systems' behaviours are independent of changes in plant parameters. In addition, the systems' sliding mode regime behaviour is reliable for the intended dynamical features. (iii) The systems' compensator's simplicity in realisation, (iii) Sliding mode regime invariance to shocks, and (iv) Two phases make up the SMC design [290]. In the first step, a sliding surface is selected, and in the second stage, a suitable control rule is developed to direct the system states to the sliding surface's desired states. SMC causes the system's order to decrease, enhancing the system's capacity to reduce the consequences of disruptions and uncertainty [32]. The potential for easy implementation in digital controllers makes the discrete-time sliding mode technique interesting. Discrete SMC design has been investigated by several scholars. [103][292][293]. For discrete SMC, the literature identifies two methods. The primary goal of the first strategy is to transfer continuous-time sliding mode control to discrete-time [294] - [295]. The disturbance observer and equivalent control design [296] – [298] are models for the second strategy. Some writers have reported using higher-order sliding mode controllers for various machines in the literature [32][117][119][299]. Sliding mode controllers with higher orders exhibit additional benefits like low chattering and excellent accuracy. The importance of sliding mode control strategies, including quasi-sliding mode, exponential reaching law sliding mode, equivalent sliding mode, and decoupled sliding mode, is tested in this chapter on a variety of nonlinear systems, including the inverted pendulum and the TORA system. Maintaining the system's position at the intended position is the goal of control action. Each of these control strategies begins with the construction of a sliding mode surface, following which control functions are created to achieve control goals. Performance metrics are tracked for chattering, convergent time, disturbance rejection, and stability. Based on these performance criteria, a comparative study is then carried out.

4.2 Mathematical Modeling of Nonlinear System

4.2.1 Mathematical modeling of TORA System

Figure 4.1 represents a model of the Translational Oscillator Rotational Actuator (TORA) system. A spring holds the cart with mass M to a firm wall and K is the spring stiffness. The cart can only be moved in one direction. The actuator is massed by m and has an inertial moment of I about its centre of mass. L is the distance between the mass's centre and the point it circles around. The movement only takes place in the horizontal plane, hence the gravitational force's impact is ignored. As shown in figure 4.1 the control torque τ applied to m as u. The rotary actuator of mass m is in the angular position indicated by θ, and The disruption force affecting the cart is noted by F.

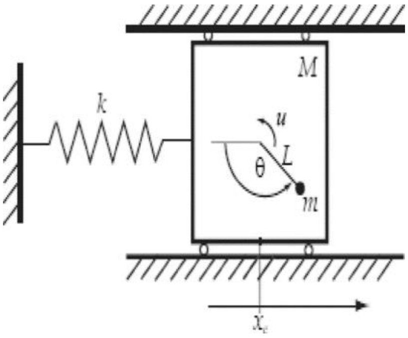


Figure 4.1 Translational operational rotational actuator system

Both the oscillator's and the pendulum's kinetic energy can be expressed as

$$K_{oscillator} = \frac{1}{2}(M+m)\dot{x}^2 \quad (4.1)$$

$$K_{pendulum} = \frac{1}{2}(I+me^2)\dot{\theta}^2 + me\dot{\theta}\, cos\, cos\, \theta \quad (4.2)$$

Where I is the pendulum's moment of inertia, e is the eccentricity of the pendulum, and M and m are the oscillator and pendulum's respective masses. The total kinetic energy is the combination of $K_{oscillator}$ and $K_{pendulum}$ as

$$K = K_{oscillator} + K_{pendulum} = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}(I+me^2)\dot{\theta}^2 + me\dot{\theta}\, cos\, cos\, \theta \quad (4.3)$$

As a result of the system's potential energy being defined

$$V = -mg\, cos\, cos\, \theta + \frac{1}{2}kx^2 \quad (4.4)$$

The Lagrangian equation can be given as:

$$L = K - V = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}(I+me^2)\dot{\theta}^2 + me\dot{\theta}\, cos\, cos\, \theta + mg\, cos\, cos\, \theta - \frac{1}{2}kx^2 \quad (4.5)$$

It is possible to designate the generalised force vector and generalised coordinate vector as

$$q = [q_1\ q_2\ ] = [x\ \theta\ ] \in R^2 \quad (4.6)$$

$$\tau = [F\ N\ ] \in R^2 \quad (4.7)$$

Where F is the disturbance driving the oscillator and N is the control torque given by a motor to drive the pendulum. Let's compute the values below to make it simpler to determine the equation of motion using the Lagrange equation.

$$\left.\begin{aligned}
\frac{\partial L}{\partial \dot{x}} &= (M+m)\dot{x} + ml\dot{\theta}cos\,\theta \\
\frac{\partial L}{\partial x} &= -kx \\
\frac{\partial L}{\partial \dot{\theta}} &= me\dot{x}\,cos\,cos\,\theta\, + (I+me^2)\dot{\theta} \\
\frac{\partial L}{\partial \theta} &= -me\dot{x}\dot{\theta}\,sin\,sin\,\theta
\end{aligned}\right\} \quad (4.8)$$

The transient equation for the TORA system in the X-space may then be obtained using the Lagrange equation.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau \quad (4.9)$$

To have $(M+m)\ddot{x} + me\,cos\,cos\,\theta\,\ddot{\theta} - me\dot{\theta}^2\,sin\,sin\,\theta\, + kx = F$ (4.10)

$$(me^2 + I)\ddot{\theta} + me\,cos\,cos\,\theta\,\ddot{x} = N \quad (4.11)$$

Define the normalized state, $p = \sqrt{\frac{M+m}{I+me^2}}x$, normalized time $\tau = \sqrt{\frac{k}{M+m}}t$, dimensionless control $u = \frac{M+m}{k(I+me^2)}N$ and dimensionless disturbance $w = \frac{1}{k}\sqrt{\frac{M+m}{I+me^2}}F$, then above equation becomes,

$$\ddot{p} + p = \varepsilon\left(\dot{\theta}^2\,sin\,sin\,\theta\, - \ddot{\theta}\,cos\,cos\,\theta\,\right) + w \quad (4.12)$$

$$\ddot{\theta} = -\varepsilon\ddot{p}\,cos\,cos\,\theta\, + u \quad (4.13)$$

Where ε stands for the coupling between the translational and rotational motions with respect to the normalised time

$$\varepsilon = \frac{me}{\sqrt{(I+me^2)(M+m)}}$$

Define the state vector

$$X = [p\;\dot{p}\;\theta\;\dot{\theta}\,]^T = [x_1\;x_2\;x_3\;x_4\,]^T \in R^2$$

And the state space representation for (4.8) becomes

$$\left.\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_2(x) + b_2(x)u \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_4(x) + b_4(x)u
\end{aligned}\right\} \quad (4.14)$$

Where, $f_2(x) = \frac{-x_1+\varepsilon x_4^2\sin x_3+w}{1-\varepsilon^2cos^2x_3}$, $b_2(x_3) = \frac{-\varepsilon\cos x_3}{1-\varepsilon^2cos^2x_3}$, $f_4(x) = \frac{\varepsilon x_1\cos x_3-\varepsilon^2x_4^2\sin x_3-\varepsilon\cos x_3 w}{1-\varepsilon^2cos^2x_3}$

$b_4(x_3) = \frac{1}{1-\varepsilon^2cos^2x_3}$, The oscillator and pendulum dynamics equations in equation (4.14) have control u, as a result, the system consider as underactuated.

4.2.2 Mathematical modeling of Inverted Pendulum

A single link pendulum cart system, which consist a moving cart containing a rod has been shown in figure 4.1. While the data of different parameters related to the inverted pendulum are given in Table 1.

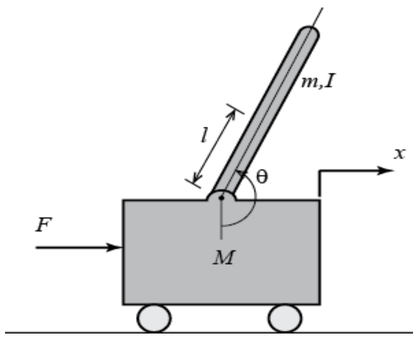


Figure 4.2 Single link inverted pendulum

Table 4.1. Parameter specification of the inverted pendulum

Parameter	Meaning	Unit
θ	Pendulum's angle	rad
x	Cart's displacement	m
m	Pendulum Mass	kg
M	Cart mass	kg
l	Pendulum length	m
g	Acceleration of gravity	m/s^2
b1	Cart coefficient of friction	$Nm^{-1}s^{-1}$
b2	Pendulum's friction coefficient	$Nrad^{-1}s^{-1}$
I	Pendulum's mass moment of inertia	kg/m^2
u	Cart input force	N

Kinetic Energy of cart is

$$E_{KV} = \frac{1}{2}Mx^2 + \frac{1}{2}I\dot{\theta}^2 \qquad (4.15)$$

$$E_{KK} = \frac{1}{2}m\dot{x}^2 + ml\dot{x}\dot{\theta}\ cos\ cos\ \theta\ + \frac{1}{2}ml^2\dot{\theta}^2 \qquad (4.16)$$

Total Kinetic Energy of the system is

$$E_K = L = E_{KV} + E_{KK} = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}(I+ml^2)\dot{\theta}^2 + ml\dot{x}\dot{\theta}\cos\cos\theta \quad (4.17)$$

By using Lagrange derivation for cart position ‘x’

$$\frac{\partial E_K}{\partial \dot{x}} = (M+m)\dot{x} + ml\dot{\theta}\cos\cos\theta \quad (4.18)$$

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{x}}\right) = (M+m)\ddot{x} + ml\ddot{\theta}\cos\cos\theta - ml\dot{\theta}^2\sin\sin\theta$$

Since, $\frac{\partial E_K}{\partial x} = 0$

Hence, $Q_x = F - b_1\dot{x}$ (4.19)

Also, Lagrange derivation for angle θ

$$\frac{\partial E_K}{\partial \dot{\theta}} = \left(I+ml^2\right)\theta + ml\dot{x}\cos\cos\theta \quad (4.20)$$

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{\theta}}\right) = (I+ml^2)\ddot{\theta} + ml\ddot{x}\cos\cos\theta - ml\dot{x}\dot{\theta}\sin\sin\theta \quad (4.21)$$

Since, $\frac{\partial E_K}{\partial \dot{\theta}} = -ml\dot{x}\dot{\theta}\sin\sin\theta$

So, $Q_\theta = -mgl\sin\sin\theta - b_2\dot{\theta}$ (4.22)

According to Lagrange’s equation, pendulum angle ‘θ’ and cart velocity ‘x’ will be,

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{x}}\right) - \frac{\partial E_K}{\partial x} = Q_x \quad (4.23)$$

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{\theta}}\right) - \frac{\partial E_K}{\partial \theta} = Q_\theta \quad (4.24)$$

After simplifying above equations, the resultant equations of motions.

$$\ddot{x} = -\frac{ml}{(M+m)}\ddot{\theta}\cos\cos\theta - \frac{ml}{(M+m)}\dot{\theta}^2\sin\sin\theta - \frac{b_1}{(M+m)}\dot{x} + \frac{F}{(M+m)} \quad (4.25)$$

$$\ddot{\theta} = \frac{ml}{(I+ml^2)}\ddot{x}\cos\cos\theta - \frac{b_2}{\left(I+ml^2\right)}\dot{\theta} - \frac{mgl}{(I+ml^2)}\sin\sin\theta \quad (4.26)$$

Equation (4.25) and (4.26) represents dynamic equations for pendulum to stabilize at upright equilibrium position of cart pendulum system. For controller design the above equation must be represented in state space form.

From equation (4.25) and (4.26), the state equations are,

$$\begin{cases} \dot{x_1} = x_2 \\ x_2 = \dfrac{g\sin x_1 - mlx_2{\dot{}}^2\cos x_1\sin x_1/(M+m)}{l\left(\frac{4}{3} - \frac{mcos^2 x_1}{M+m}\right)} \\ \qquad + \dfrac{\cos x_1/(M+m)}{l\left(\frac{4}{3} - mcos^2 x_1/(M+m)\right)} \end{cases} \quad (4.27)$$

Let, the inverted pendulum is having unwanted disturbance and parameter uncertainties

Hence, the above equation will be-

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{g\sin x_1 - mlx_2{}^2\cos x_1 \sin x_1/(M+m)}{l\left(\frac{4}{3}-\frac{m\cos^2 x_1}{M+m}\right)} + \frac{\cos x_1/(M+m)}{l\left(\frac{4}{3}-m\cos^2 x_1/(M+m)\right)} + d(t) \end{aligned}\right\} \quad (4.28)$$

Where d(t) is unknown disturbance.

4.3 Sliding Mode Controller (SMC)

Since model uncertainties can have a negative impact on nonlinear control systems, sliding mode control addresses this issue. Adaptive control is a key strategy for addressing model uncertainty, and additional strategies that may be utilised to address control issues include sliding mode methods. These methods are currently drawing more attention. The actual plant and the mathematical model that was used to develop the controller may not be exactly the same. A variety of factors could be accountable for this discrepancy. SMC has been used for a variety of systems, including stochastic systems, large-scale and infinite-dimension systems, discrete models, and nonlinear systems. To external disturbances and parametric errors, SMC is fully insensitive. VSC achieves two goals using a switching control rule. The sliding or switching surface, a predetermined surface in the state space, is initially projected using the nonlinear plant's state trajectory to control the state trajectory at the switching surface. And maintains the state trajectory of the plant on the same surface. To emphasise the significance of the sliding mode, the control is sometimes referred to as the sliding mode control. The system is designed to first drive the system state and then force it to stay close to the switching function when employing a sliding mode control technique

Now since,

$$s = cx_1 + \dot{x_1}, c > 0 \quad (4.29)$$

The first order linear time invariant system described as:

$$\dot{x_1} = -cx_1 \quad (4.30)$$

High frequency switching, commonly referred to as the chattering effect, will really occur, implying that the states repeatedly move across the surface even though there shouldn't be any sliding mode. A nonlinear control method that assures that trajectories always move in the direction of a switching condition, change the dynamics of a nonlinear system. As a result, the ultimate trajectory won't be totally contained by a single control structure. There is no continuous function of time for the state-feedback control law. Instead, dependent on the location in the state space, it transitions from one continuous structure to another. To ensure that trajectories always go in the direction of a switching condition, control structures are created. The final trajectory will thus not be entirely contained by a single control structure. The final trajectory will instead veer close to the confines of the control structures. Sliding mode is the name given to the system's motion as it slides along these limits, and sliding (hyper) surface is the name given to the geometric locus formed by the boundaries. Sliding mode control employs almost limitless gain to compel the paths a dynamic system would take to slide along a small sliding mode subspace. This reduced-order sliding mode yields control trajectories that have certain favourable characteristics (for instance, the system will inevitably slide along it until it reaches the required equilibrium.).

The control doesn't have to be precise or sensitive to parameter mistakes because it can simply be a switch between two states. The sliding mode can be attained in a limited amount of time since the control rule is not a real-valued function (i.e., better than asymptotic behaviour). The SMC design involves two steps, to construct a sliding surface on which plant of system is attached then direct the plant's state trajectory toward the sliding surface is the second phase. The generalised Lyapunov stability theory serves as the foundation for these creations. In the state space, a sliding surface is a preset surface that a controller aids a system in reaching and then remaining on. The system's movement while being held to the surface is referred to as its sliding motion. The advantage of achieving this motion is that the system's order will be reduced, and the sliding motion is unaffected by changes in a plant's parameter. This technique is appealing for constructing robust control for various uncertain systems due to the latter attribute.

The design process consists mostly of these two steps:

1. Constructing a sliding surface that satisfies the system's requirements for lower order sliding motion in the state space.
2. Control rule synthesis to steer closed-loop motion's paths toward the sliding surface.

The generated closed loop dynamical behaviour is controlled by a variable structure control rule that has two different forms of motion. As the states are pushed toward the sliding surface, the first phase—also known as the reaching phase—occurs. The external disturbances have an impact on this sort of motion. Only when the states reach the surface and lose sensitivity to uncertainty is this abolished. There is a sliding surface if the state velocity vectors in its immediate vicinity are pointed in its direction. The graphic provides the clearest explanation of the sliding mode (4.3)

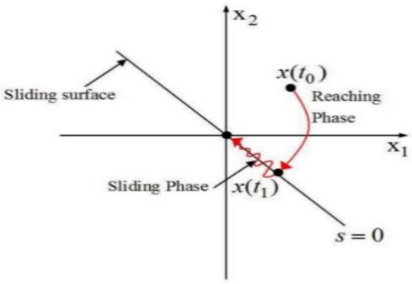


Figure 4.3 Sliding mode phases

The reachability criterion that ensures a single input system's motion state trajectory, x(t).

$$\dot{x} = f(x, u, t) \tag{4.31}$$

Now $\sigma = 0$ on either side of sliding surface,

$$\dot{\sigma} < 0; \dot{\sigma} > 0 \tag{4.32}$$

The result of combining the two equations above is

$$\sigma\dot{\sigma} < 0 \tag{4.33}$$

$$\lim_{\sigma \to 0} \sigma \frac{d\sigma}{dt} < 0 \tag{4.34}$$

The path always follows the hypersurface s.

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dx}\right)^T \dot{x} = 0 \tag{4.35}$$

*T*herefore, if the global reachability requirement is met and therefore

$$\frac{1}{2}\frac{d}{dt}\sigma^2 = \sigma\dot{\sigma} < 0 \tag{4.36}$$

Which gives, V (σ) =$\frac{1}{2}\sigma^2$ (4.37)

A Lyapunov function for $\sigma(t)$.

A high gain control input is applied once the aforementioned processes have been finished. Additionally, the input chattering caused by this high gain control results in a high frequency in plant dynamics, which has the potential to cause unforeseen instabilities [18] – [19]. Few of the several sliding mode control techniques that may be used to preserve the system state on a sliding surface and the chattering issue are discussed.

4.3.1 Reaching Law Control SMC

The reaching phase and sliding phase of the system are defined by the controller in the reaching law approach [16]. The system is kept stable by the reaching phase, and it is brought into equilibrium by the sliding phase.

Let the plant output is given as,

$\ddot{\theta}(t) = -f(\theta, t) + bu(t) + d(t)$ (4.38)

Where $f(\theta, t)$ and $bu(t)$ are plant constant and d(t) is external disturbance

The sliding function may be described as

$s(t) = ce(t) + \dot{e}(t)$ (4.39)

The hurwitz condition must be met by the constant "c.", which states that "c > 0."

Following is a definition of the tracking error and its derivative*:*

$e(t) = x(t) - \theta(t)$ (4.40)

$\dot{e}(t) = \dot{x}(t) - \dot{\theta}(t)$ (4.41)

Where x(t) is ideal positional signal.

Therefore, we have

$\dot{s}(t) = c\dot{e}(t) + \ddot{e}(t)$

$= c\left(\dot{x}(t) - \dot{\theta}(t)\right) + \left(\ddot{x}(t) - \ddot{\theta}(t)\right)$

$= c\left(\dot{x}(t) - \dot{\theta}(t)\right) + (\ddot{x}(t) + f(\theta, t) - bu(t) - d(t))$ (4.42)

The exponential reaching law,

$\dot{s} = -\epsilon\ sgn(s) - ks$ (4.43)

Where, $\epsilon > 0, k > 0$

From equation (4.44) & (4.46),

$\left(\dot{x}(t) - \dot{\theta}(t)\right) + (\ddot{x}(t) + f(\theta, t) - bu(t) - d(t)) = -\epsilon\ sgn\ s - k$ (4.44)

The sliding mode controller has been designed as:

$u(t) = \frac{1}{b}\left(\epsilon\ sgn(s) + ks + c\left(\dot{x}(t) - \dot{\theta}(t)\right) + \ddot{x}(t) + f(\theta, t) - d(t)\right)$ (4.45)

Since the disturbance'd' in the equations above is an unknown variable, it must be substituted with a conservative known quantity, such as "dc," to solve the issue.

The controller's model equation based on sliding mode can therefore be expressed as;

$u(t) = \frac{1}{b}\left(\epsilon\ sgn(s) + ks + c\left(\dot{x}(t) - \dot{\theta}(t)\right) + \ddot{x}(t) + f(\theta, t) - d_c\right)$ (4.46)

By simplifying it,

$\dot{s}(t) = -\epsilon\ sgn(s) - ks + d_c - d$ (4.47)

Where reaching condition is d_c ensures that d is bounded, therefore:

$d_L \leq d(t) \geq d_U$ (4.48)

Where the bounds d_L and d_U are known.

Hence,

i. When $s(t) > 0, \dot{s}(t) = -\epsilon\, sgn(s) - ks + d_c - d$ and we want $\dot{s}(t) < 0$ so $d_c = d_L$

ii. When $s(t) < 0, \dot{s}(t) = -\epsilon\, sgn(s) - ks + d_c - d$ and we want $\dot{s}(t) > 0$ so $d_c = d_U$

Therefore, if we define $d_1 = \frac{d_U - d_L}{2}$,

$$d_2 = \frac{d_U + d_L}{2} \tag{4.49}$$

$$d_c = d_2 - d_1 sgn(s) \tag{4.50}$$

4.3.2 Quasi-Sliding Mode SMC

The SMC is a reliable control method but a drawback is chattering. Due to chattering, an unwanted oscillation phenomenon, the system's components are being harmed. It has been suggested to use continuous control to create a boundary layer around a switching surface in order to prevent chattering. The system requires its state to remain within range to nearby as part of the proposed strategy to decrease chattering.

The sliding function defined as

$$s(t) = ce(t) + \dot{e}(t) \tag{4.51}$$

Where ‘c’ is a constant, c > 0.

The tracking error and its derivative are described as follows:

$$e(t) = x(t) - \theta(t) \tag{4.52}$$

$$\dot{e}(t) = \dot{x}(t) - \dot{\theta}(t) \tag{4.53}$$

The ideal positional signal is x(t) in this case.

Therefore, $\dot{s}(t) = c\dot{e}(t) + \ddot{e}(t)$

$$= c\dot{e} + \left(\ddot{x}(t) - \ddot{\theta}(t)\right)$$

$$= c\dot{e} + (\ddot{x}(t) + f(\theta, t) - bu(t) - d(t)) \tag{4.54}$$

Hence, the proposed controller will be,

$$u(t) = \frac{1}{b}(c\dot{e} + \ddot{x}(t) + f(\theta, t) + \eta\, sgn(s)) \tag{4.55}$$

Lyapunov function is defined as,

$$L = \frac{1}{2}s^2 \tag{4.56}$$

Therefore, $\dot{L} = s\dot{s} = s(\ddot{x}(t) - f(\theta, t) - bu(t) + c\dot{e})$

$$= s\big(\ddot{x}(t) - f(\theta, t) - (-f(\theta, t) + c\dot{e} + \ddot{x}(t) + \eta\, sgn(s))\big)$$

$= s\big(-d(t) - \eta \; sgn(s)\big)$

$-sd(t) - \eta \; |s|$ (4.57)

A saturated function, or sat(s) function, is used to control chattering.

$$sat(s) = \begin{cases} 1 & s > \Delta \\ ks & |s| \leq \Delta, k = \frac{1}{\Delta} \\ -1 & s < -\Delta \end{cases} \quad (4.58)$$

Where Δ is define as boundary layer, a switch control is employed for the outer boundary layer, and linear feedback is used for the inner boundary layer.

The sliding variable and state variables have a smooth control rule, thus they do not converge to zero but rather, under the impact of the disturbance, converge to areas around the origin. The aforementioned smooth control rule is sometimes referred to as quasi-sliding mode control.

4.3.3 Equivalent Control SMC

In equivalent control the SMC controller devided into the switching control u_{sw} and the equivalent control u_{eq}. By switching between equivalent and different controls, the system state is controlled, and the system is kept on a sliding surface. If we disregard the plant's uncertainty and outside disruption, the plant may be described as:

$$x^{(n)} = f(x,t) + bu(t) \quad (4.59)$$

The tracking error vector is defined as

$$e = x_d - x = [e \; \dot{e} \; e^{(n-1)}]^T \quad (4.60)$$

And switch function

$$s(x,t) = ce = c_1 e + c_2 \dot{e} + \cdots e^{(n-1)} \quad (4.61)$$

Where $c = [c_1 \;\; c_2 \cdots c_{n-1} \;\; 1]$

Let $\dot{s} = 0$

Then, $\dot{s}(x,t) = c_1 \dot{e} + c_2 \ddot{e} + \cdots e^n$

$= c_1 \dot{e} + c_2 \ddot{e} + \cdots c_{n-1} e^{n-1} + x_d^{(n)} - x^{(n)}$

$$\sum_{i=1}^{n-1} c_i e^{(i)} + x_d^n - f(x,t) - bu(t) = 0 \quad (4.62)$$

Hence, the control input is designed as

$$u_{eq} = \frac{1}{b} [\sum_{i=1}^{n-1} c_i e^{(i)} + x_d^n - f(x,t)] \quad (4.63)$$

To fulfil the sliding mode's reaching criterion

$s(x,t).\dot{s}(x,t) \leq -\eta |s|, \;\; \eta > 0$

The switching control law

$$u_{sw} = \frac{1}{b} Ksgn(s) \tag{4.64}$$

Where $K = D + \eta$

Hence sliding mode controller

$$u = u_{eq} + u_{sw} \tag{4.65}$$

Where, u_{eq} is equivalent control input and u_{sw} is switching control input.

4.3.4 Decoupled Sliding mode control

The sliding mode control system is divided into two subsystems in decoupled sliding mode control in order to achieve asymptotic stability for higher class nonlinear systems. A subsystem is said to have a primary purpose and a sub-control purpose if it is separated into two second-order systems. Through the decoupled subsystem's state variables, two sliding surfaces are created. We define the primary and sub-target criteria for these sliding surfaces and add an intermediate variable obtained from the sub-sliding surface condition.

The decoupling sliding mode control have following advantages:

a) It is possible to control complicated systems effectively without being familiar with the particular mathematical models.
b) The decoupled sliding surface may roughly dominate the dynamic behaviour of the controlled system.
c) The decoupled sliding mode control increases resilience to system uncertainties while reducing the chattering phenomenon of the traditional sliding-mode controller.

4.3.4.1 Decoupled Sliding Mode Control for a Tora System

A standardised system is created to evaluate the effectiveness of potential controllers on a TORA. The system is further enhanced and provides the following generic dynamics [23] for a TORA system:

$$\left.\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \frac{-z_1 + \varepsilon\theta_2^2 sin\theta_1}{1 - \varepsilon^2 cos^2\theta_1} - \frac{\varepsilon cos\theta_1}{1 - \varepsilon^2 cos^2\theta_1} v \\ \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= \frac{\varepsilon cos\theta_1 (z_1 - \varepsilon\theta_2^2 sin\theta_1)}{1 - \varepsilon^2 cos^2\theta_1} + \frac{1}{1 - \varepsilon^2 cos^2\theta_1} v \end{aligned}\right\} \tag{4.66}$$

Where v denotes control input, z_1 is platform's normalized displacement from the equilibrium position $z_2 = \dot{z}_1$. The objectives of control are:

$z_1,\ \dot{z}_{1,}\ \theta_1,\ \dot{\theta}_1 \rightarrow 0$, for, t $\rightarrow\infty$

Apply the decoupling algorithm to the above equation

$f_1 = \frac{-z_1+\varepsilon\theta_2^2 sin\theta_1}{1-\varepsilon^2 cos^2\theta_1},\ \ g_1 = \frac{-\ \varepsilon cos\theta_1}{1-\varepsilon^2 cos^2\theta_1},\ f_2 = \frac{\varepsilon cos\theta_1(z_1-\varepsilon\theta_2^2 sin\theta_1)}{1-\varepsilon^2 cos^2\theta_1},\ g_2 = \frac{1}{1-\varepsilon^2 cos^2\theta_1}$, We have, $\frac{g_1}{g_2} = -\varepsilon cos\theta_1$

$$\left.\begin{aligned} &\text{Let, } x_1 = z_1 + \varepsilon\ sin\ sin\ \theta_1 \\ &x_2 = z_2 + \varepsilon\theta_2\ cos\ cos\ \theta_1 \\ &x_3 = \theta_1 \\ &x_4 = \theta_2 \end{aligned}\right\} \quad (4.67)$$

From equation (4.66), the control goals $z_1,\ \dot{z}_{1,}\ \theta_1,\ \dot{\theta}_1 \rightarrow 0$ are equivalent to $x_i \rightarrow 0$, i=1, 2, 3, 4.

Since, $\dot{x_2} = \dot{z_2} + \varepsilon\dot{\theta}_2 cos\theta_1 - \varepsilon\theta_2^2 sin\theta_1$

$$= \frac{-z_1+\varepsilon\theta_2^2 sin\theta_1}{1-\varepsilon^2 cos^2\theta_1} - \frac{\varepsilon cos\theta_1}{1-\varepsilon^2 cos^2\theta_1} v + \varepsilon\left(\frac{\varepsilon cos\theta_1(z_1-\varepsilon\theta_2^2 sin\theta_1)}{1-\varepsilon^2 cos^2\theta_1} + \frac{1}{1-\varepsilon^2 cos^2\theta_1} v\right) cos\theta_1 - \varepsilon\theta_2^2 sin\theta_1$$

$$= \frac{-z_1+\varepsilon\theta_2^2 sin\theta_1}{1-\varepsilon^2 cos^2\theta_1} + \frac{\varepsilon^2\ cos\theta_1\ (z_1-\varepsilon\theta_2^2 sin\theta_1)}{1-\varepsilon^2 cos^2\theta_1} - \varepsilon\theta_2^2 sin\theta_1$$

$$= \frac{-z_1(1-\varepsilon^2\ cos\theta_1)+\varepsilon\theta_2^2 sin\theta_1(1-\varepsilon^2\ cos\theta_1)}{1-\varepsilon^2 cos^2\theta_1} - \varepsilon\theta_2^2 sin\theta_1 \quad (4.68)$$

Let, $u = \frac{\varepsilon cos\theta_1(z_1-\varepsilon\theta_2^2 sin\theta_1)}{1-\varepsilon^2 cos^2\theta_1} - \frac{1}{1-\varepsilon^2 cos^2\theta_1} v$

i.e., $v = \varepsilon cos\theta_1(z_1 - \varepsilon\theta_2^2 sin\theta_1) - (1-\varepsilon^2 cos^2\theta_1)u$ (4.69)

From equation (4.67) z_1 can be written a

$$z_1 = -x_1 + \varepsilon sin\ x_3 \quad (4.70)$$

From equation (4.69) and (4.70), the control input is written as,

v = ε cos$x_3(x_1 - (1 + x_4^2)\varepsilon sin x_3) - (1-\varepsilon^2 cos^2 x_3)u$ (4.71)

The above equation must be satisfied with three assumptions as follows for any kind of underactuated system:

Assumption 1: $f_1(0,0) \rightarrow 0$;

Assumption 2: $\frac{df_1}{dx_3}$ *is invertible*;

Assumption 3: If $f_1(0, x_3) \rightarrow 0$, then $x_3 \rightarrow 0$

$$\left.\begin{aligned} &\dot{x}_1 = x_2 \\ &\dot{x}_2 = f_1(x_1, x_3) = -x_1 + \varepsilon\ sin\ sin\ x_3 \\ &\dot{x}_3 = x_4 \\ &x_4 = u \end{aligned}\right\} \quad (4.72)$$

If equation (4.72) is not satisfying assumption 2, Then the function $f_1(x_1, x_3)$ may be written as:

$$f_1(x_1, x_3) = -x_1 + \varepsilon sin\, x_3 + 11\varepsilon x_3 \quad (4.73)$$

The equation (4.74) may be written as with the help of equation (4.75),

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(x_1, x_3) = -x_1 + \varepsilon sin\, x_3 + 11\varepsilon x_3 \\ \dot{x}_3 &= x_4 \\ x_4 &= u \end{aligned}\right\} \quad (4.74)$$

The primary goal in this case is to reach the target and create a sliding surface such that the tracking error vector's state response results in a satisfying response. The tracking mistake is described as

$$\left.\begin{aligned} \dot{e}_1 &= x_2 \\ e_2 &= \dot{e}_1 = x_2 \\ e_3 &= f_1(x_1, x_3) \\ e_4 &= \dot{e}_3 = \dot{f}_1(x_1, x_3) = \frac{df_1}{dx_1}x_2 + \frac{df_1}{dx_3}x_4 \end{aligned}\right\} \quad (4.75)$$

The sliding mode function is defined as:

$$s = c_1 e_1 + c_2 e_2 + c_3 e_3 + e_4 \quad (4.76)$$

where, c_1, c_2, c_3 are positive constant, from $\frac{d}{dt}\left(\frac{df_1}{dx_1}\right) = 0$,

$$\dot{s} = c_1\dot{e}_1 + c_2\dot{e}_2 + c_3\dot{e}_3 + \dot{e}_4$$

$$= c_1 x_2 + c_2(f_1 - 11\varepsilon x_3) + c_3\frac{d}{dt}\left(\frac{df_1}{dx_1}x_2 + \frac{df_1}{dx_3}x_4\right) + \dot{e}_4 \quad (4.77)$$

where, $\frac{d}{dt}\left(\frac{df_1}{dx_1}x_2 + \frac{df_1}{dx_3}x_4\right) = \frac{df_1}{dx_1}(f_1 - 11\varepsilon x_3) + \frac{d}{dt}\left(\frac{df_1}{dx_3}x_4\right) + \frac{df_1}{dx_3}u$ (4.78)

From equation (4.77) & equation (4.78),

$$\dot{s} = c_1 x_2 + c_2(f_1 - 11\varepsilon x_3) + c_3 + \frac{df_1}{dx_1}(f_1 - 11\varepsilon x_3) + \frac{d}{dt}\left(\frac{df_1}{dx_3}x_4\right) + \frac{df_1}{dx_3}u + \dot{e}_4 \quad (4.79)$$

Let, $M = c_1 x_2 + c_2(f_1 - 11\varepsilon x_3) + c_3 + \frac{df_1}{dx_1}(f_1 - 11\varepsilon x_3) + \frac{d}{dt}\left(\frac{df_1}{dx_3}x_4\right) + \dot{e}_4$ (4.80)

Then, the sliding mode controller is designed as,

$$U = \left[\frac{df_1}{df_3}\right]^{-1}(-M - nsgns - ks) \quad (4.81)$$

Where n and k are positive constant. To get stable and distortion less performance, the systems, and the sliding mode controller must obey the similar parameter functions in the algorithm.

For stable operations, the sliding mode controller must follow the Lyapunov function.

To get this Lyapunov function, $\dot{s} = -nsgn(s) - ks$ (4.82)

The Lyapunov function is define as, V=$\frac{1}{2}s^2$

Then, $\dot{V} = s\dot{s} = -n|s| - ks^2$, must be negative.

From equation (4.76) and equation (4.81) we can obtain that system stay at its switching manifold $s = 0$ in finite time, on the switching manifold the error equation is given as

$e_4 = c_1e_1 + c_2e_2 + c_3e_3$

Let the Hurwitz vector A is given as $A = [0\ 1\ 0\ 0\ 0\ 1\ -c_1\ -c_2\ -c_3]$ makes that the values of c_i to stable. Then the error state equation is given as:

$$\left.\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 - 11\varepsilon x_3 \\ \dot{e}_3 &= -c_1e_1 - c_2e_2 - c_3e_3 \end{aligned}\right\} \quad (4.83)$$

Let the error signal is defined as $E = [e_1\ e_2\ e_3]^T, d_2 = -11\varepsilon x_3$

From equation (4.75) the error e_3 is given as $e_3 = f_1(x_1, x_3) = -x_1 + \varepsilon\, sin\, sin\, x_3 + 11\varepsilon x_3$

Then d_2 is defined as $d_2 = -11\varepsilon x_3 = -e_3 - x_1 + \varepsilon\, sin\, sin\, x_3$ (4.84)

If we choose that $sin\, sin\, x_3 < x_3$, then we have $|d_2| = 11\varepsilon|x_3|$

$= |-e_3 - x_1 + \varepsilon\, sin\, sin\, x_3| \leq |e_3| + |e_1| + \varepsilon|x_3|$ (4.85)

Then $10\varepsilon|x_3| \leq |e_3| + |e_1|$, and $|d_2| = 11\varepsilon|x_3|$

or, $1.1 * 10\varepsilon|x_3| \leq 1.1(|e_3| + |e_1|) \leq 2.2\|E\|_2$ (4.86)

Let $D = [0\ e_2\ 0]^T$, then $\|D\|_2 \leq \gamma\|E\|_2$, where $\gamma = 2.2$

Error E is define as:

$\dot{E} = AE + D$ (4.87)

To guarantee that $E = [e_1\ e_2\ e_3]^T \rightarrow 0$, vector A must be Hurwitz, hence the real part of the characteristic's equation of A must be negative.

From $|\boldsymbol{A} - \lambda\boldsymbol{I}| = \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ -c_1 & -c_2 & -c_3 - \lambda \end{vmatrix} = \lambda^2(-c_3 - \lambda) - c_1 - c_2\lambda$

$= -\lambda^3 - c_3\lambda^2 - c_2\lambda - c_1 = 0$, i.e $\lambda^3 + c_3\lambda^2 + c_2\lambda + c_1 = 0$ (4.88)

From function $(\lambda + a)^3 = 0$, then we have $\lambda^3 + 3a\lambda^2 + 3a^2\lambda + a^3 = 0$ (4.89)

By comparing equation (4.88) and equation (4.89) we get,

$c_1 = a^3, c_2 = 3a^2, c_3 = 3a$, and let $a > 0$, then vector A will be Hurwitz.

For the system should be controllable defining a term $Q = Q^T > 0$, then there exists a Lyapunov equation $A^TP + PA = -Q, P = P^T > 0$.

To satisfy the above condition let the Lyapunov function is designed as $V_1 = E^T PE$,

or, $\dot{V}_1 = \dot{E}^T PE + E^T P\dot{E} = (AE + D)^T PE + E^T P(AE + D)$

$= E^T(A^T P + PA)E + D^T PE$+$ET$$= -E^T QE + D^T PE + E^T PD \leq -\lambda_{min}(Q)\|E\|_2^2 +$
$2\lambda_{max}(P)\gamma\|E\|_2^2$

$$= (-\lambda_{min}(Q) + 2\lambda_{max}(P)\gamma)\|E\|_2^2 \quad (4.90)$$

Where $D^T PE + E^T PD \leq 2\lambda_{max}(P)\gamma)\|E\|_2^2, \lambda_{min}(Q)$ is the minimum eigen value of positive definite matrix Q and $\lambda_{max}(P)$ is the maximum eigen value of positive definite matrix P.

To get the desired response from nonlinear system, the system states must be approaches to zero i.e, $X = [x_1\ x_2\ x_3\ x_4\ ] \rightarrow 0$, and it will be achieved by making error signals to zero $E = [e_1\ e_2\ e_3\ ]^T \rightarrow 0$, and from $s \rightarrow 0$ we have $e_4 \rightarrow 0$. The condions of error signal only will be satisfied if vector A is design in such manner that have real positive values and it must be Hurwitz.

4.3.4.2 Decoupled Sliding Mode Control for an Inverted Pendulum System

The decoupled model of above system is given as:

Let, $t_1 = \theta, t_2 = x$ then $w_1 = \dot{\theta}, w_2 = \dot{x}$

Define $f_1(t,w) = \frac{m(m+M)gl}{(M+m)I+Mml^2} t_1,\ g_1(w_2) = \frac{ml}{(M+m)I+Mml^2}$, $f_2(t,w) = -\frac{m^2gl^2}{(M+m)I+Mml^2} t_1$,

and $g_2(w_2) = \frac{I+ml^2}{(M+m)I+Mml^2}$ Then, $\frac{g_1(w_2)}{g_2(w_2)} = \frac{\frac{ml}{(M+m)I+Mml^2}}{\frac{I+ml^2}{(M+m)I+Mml^2}} = -\frac{ml}{I+ml^2}$

By using decoupling algorithm,

$$\left.\begin{aligned} \gamma_1 &= t_1 - \int_0^{t_2} \frac{g_1(s)}{g_2(s)} ds = t_1 + \frac{ml}{I+ml^2} t_2 \\ \gamma_2 &= t_1 - \frac{g_1(t_1)}{g_1(t_2)} w_2 = w_1 + \frac{ml}{I+ml^2} t_2 \delta_1 = t_2 \\ \delta_1 &= t_2 \\ \delta_2 &= w_2 \end{aligned}\right\} \quad (4.91)$$

The system under stable point when both pendulum angle and cart position both comes at rest position, means$\theta = 0, x = 0, \dot{\theta} = 0, \dot{x} = 0.$ Then $\gamma_1 = 0, \gamma_2 = 0, \delta_1 = 0, and\ \ \delta_2 = 0$

By using decoupling algorithm, the state equation of inverted pendulum is written as,

$$\left.\begin{aligned} \dot{\gamma}_1 &= \gamma_2 \\ \dot{y}_2 &= \left(\frac{m(m+M)gl}{(M+m)I+Mml^2} - \frac{ml}{I+ml^2} \frac{m^2gl^2}{(M+m)I+Mml^2}\right) w_1 \end{aligned}\right\} \quad (4.92)$$

$\dot{\delta}_1 = \delta_2$

$\dot{\delta}_2 = -\frac{m^2gl^2}{(M+m)I+Mml^2}w_1 + \frac{I+ml^2}{(M+m)I+Mml^2}\big(u + d(t)\big)$

Let, $T_1 = \left(\frac{m(m+M)gl}{(M+m)I+Mml^2} - \frac{ml}{I+ml^2}\frac{m^2gl^2}{(M+m)I+Mml^2}\right)$, $T_2 = -\frac{m^2gl^2}{(M+m)I+Mml^2}$, and $T_3 = \frac{I+ml^2}{(M+m)I+Mml^2}$

Then the equation (4.92) may be written as,

$$\left.\begin{aligned}\dot{\gamma}_1 &= \gamma_2\\ \dot{\gamma}_2 &= T_1 w_1\\ \dot{\delta}_1 &= \delta_2\\ \dot{\delta}_2 &= -T_2 w_1 + T_3\big(u + d(t)\big)\end{aligned}\right\} \quad (4.93)$$

Considering $w_1 = \gamma_1 - \frac{ml}{I+ml^2}w_2 = \gamma_1 - \frac{ml}{I+ml^2}\delta_1 = \gamma_1 - T_4\delta_1$, and $T_4 = \frac{ml}{I+ml^2}$, then equation (4.92) may be written as,

$$\left.\begin{aligned}\dot{\gamma}_1 &= \gamma_2\\ \dot{\gamma}_2 &= T_1\gamma_1 + T_1T_4\delta_1\\ \dot{\delta}_1 &= \delta_2\\ \dot{\delta}_2 &= -T_2\gamma_1 + T_2T_4\delta_1 + T_3\big(u + d(t)\big)\end{aligned}\right\} \quad (4.94)$$

Equation (4.94) may be define in terms of $\mu_1 = \delta_2, \mu_2 = [\gamma_1\ \gamma_2\ \delta_1]^T$, then the surface of sliding mode controller is design as,

$$\sigma = \mu_1 - C\mu_2 \quad (4.95)$$

where $C = [c_1\ c_2\ c_3]$, then the surface equation may be written as,

$$\dot{\sigma} = \dot{\mu}_1 - C\dot{\mu}_2 = T_2\gamma_1 + T_2T_4\delta_1 + T_3\big(u + d(t)\big) - C\dot{\mu}_2 \quad (4.96)$$

Hence the control input is designed as

$$u = \frac{1}{T_3}\big(-T_2\gamma_1 - T_2T_4\delta_1 + C\dot{\mu}_2 - hsgn(\sigma)\big) \quad (4.97)$$

where $h \geq |d(t)|$.

The stability of sliding mode controller is defined by Lyapunov function. The Lyapunov function is designed as, $V = \frac{1}{2}\sigma^2$,

Then, the sliding function is written as,

$$\sigma\dot{\sigma} = \sigma\big(-hsgn(\sigma) + d(t)\big) \leq 0 \quad (4.98)$$

From equation (4.98) it is clear that, system can reach and thereafter stay on the manifold $\sigma = 0$ in finite time t_S. On the sliding manifold the sliding function is given as,

When $t \geq t_S$, we have

$$\dot{\mu_2} = \begin{bmatrix} \gamma_2 \\ T_1\gamma_2 + T_1T_4\delta_1 \\ \delta_2 \end{bmatrix} = A_1\mu_1 + A_2\mu_2 = (A_1C + A_2)\mu_2 \tag{4.99}$$

where $A_1 = [0 \quad 0 \quad 1]^T, A_2 = \begin{bmatrix} 0 & 1 & 0 \\ T_1 & 0 & T_1T_4 \\ 0 & 0 & 0 \end{bmatrix}$.

For stable operation μ_2 must be tends to zero, for that $A_1C + A_2$ should be Hurwitz. Thus we can say that $\mu_1 = C\mu_2 \to 0$.

From $\mu_1 = \delta_2, \mu_2 = [\gamma_1 \quad \gamma_2 \quad \delta_1]^T$, since we know that if $t \to \infty$, then we have $\gamma_1 \to 0, \gamma_2 \to 0, \delta_1 \to 0$, and $\delta_2 \to 0$, hence $\theta \to 0, \dot{\theta} \to 0, x \to 0$, and $\dot{x} \to 0$.

The equation $A_1C + A_2$ is define as,

$$A_1C + A_2 = [0 \quad 0 \quad 1]^T[c_1 \quad c_2 \quad c_3] + \begin{bmatrix} 0 & 1 & 0 \\ T_1 & 0 & T_1T_4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c_1 & c_2 & c_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ T_1 & 0 & T_1T_4 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ T_1 & 0 & T_1T_4 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

The poles of $A_1C + A_2$ can be designed by the following:

$$|sI - (A_1C + A_2)| = \begin{vmatrix} s & -1 & 0 \\ -T_1 & s & -T_1T_4 \\ -c_1 & -c_2 & s - c_3 \end{vmatrix} = s^3 - c_3s^2 - T_1T_4c_1 - c_2T_1T_4s -$$

$$T_1(s - c_3) = s^3 - c_3s^2 - (c_2T_1T_4 + T_1)s - T_1T_4c_1 + T_1c_3 = 0 \tag{4.100}$$

By comparing above equation with $(s + k)^3 = \begin{cases} -c_3 = 3k \\ -c_2T_1T_4 - T_1 = 3k^2 \\ -T_1T_4c_1 + T_1c_3 = k^3 \end{cases}$

To make $A_1C + A_2$ Hurwitz, the sliding mode parameters are designed as follows:

$$\begin{cases} c_3 = -3k \\ c_2 = \dfrac{-3k^2 + T_1}{T_1T_4} \\ c_1 = -\dfrac{k^3 - T_1c_3}{T_1T_4} \end{cases}$$

4.4 Simulation Results and Discussion

To assess the outcomes for various controller situations, simulation is carried out in the MATLAB/Simulink environment. The system's algorithm needs to be implemented, according on the simulation findings.

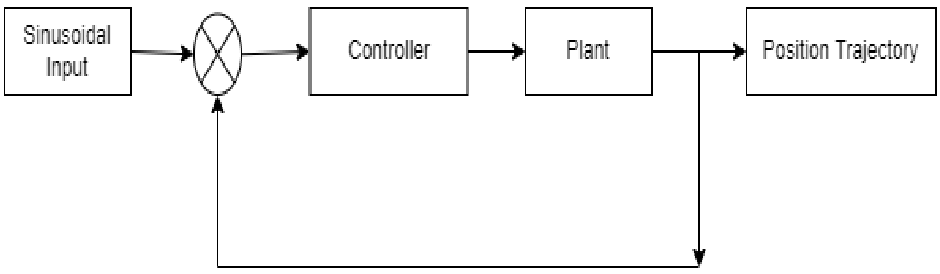
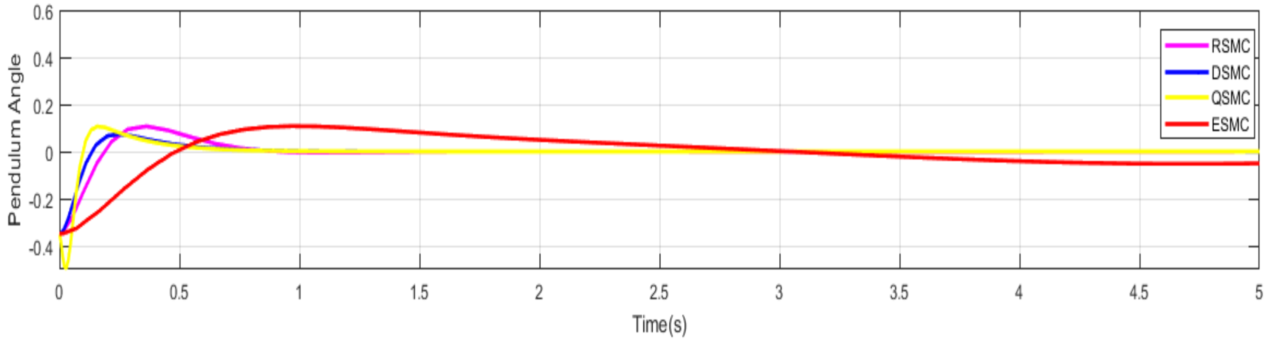


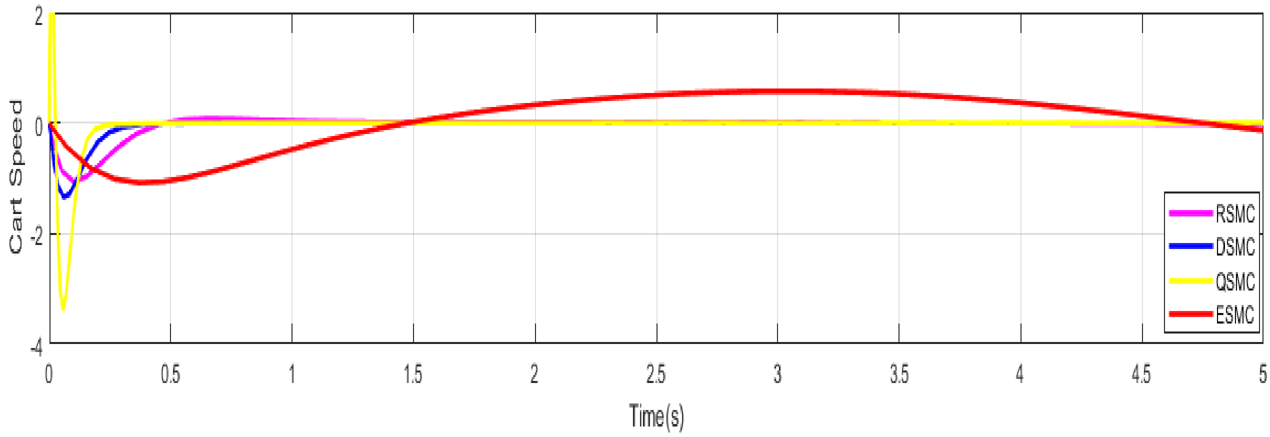
Figure 4.4 Block diagram of simulink model

Simulation Results of Inverted Pendulum for Reaching Sliding Mode Control (RSMC), Decoupled Sliding Mode Control (DSMC), Quasi Sliding Mode Control (QSMC) and Equivalent Sliding Mode Control (ESMC) Algorithm

The following parameters were chosen for simulation: M=1 gm; m=.010gm ; L=0.50m, g=9.8m/s^2. The other constants are given as c=15, $\epsilon = 0.5$, k=10. Figure 4.5, gives the pendulum angle and cart speed for different algorithm of sliding mode control and the control input for plant is shown in figure 4.6.



(a)



(b)

Figure 4.5 (a) Pendulum angle (b) Cart speed of inverted pendulum

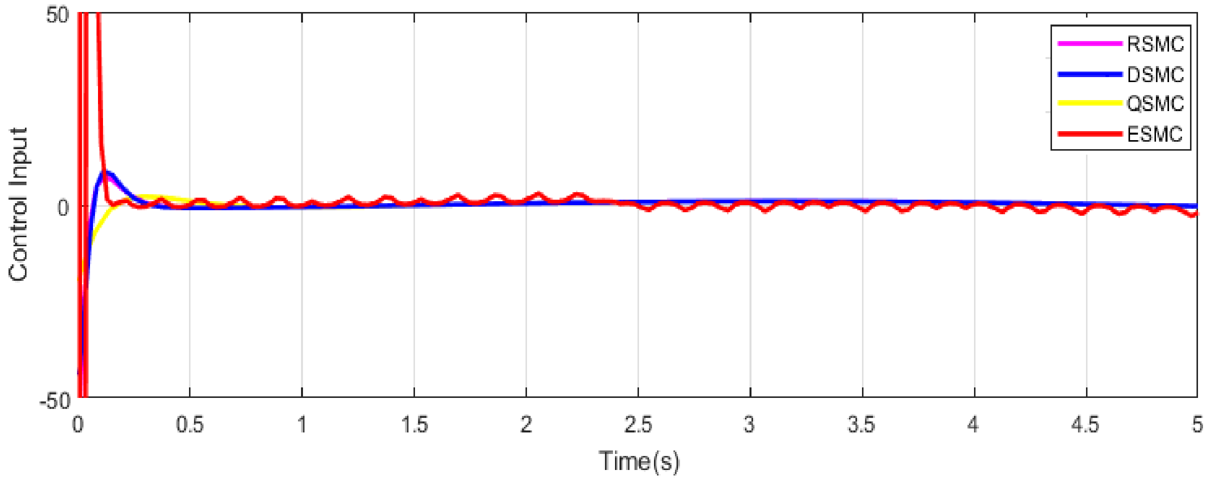


Figure 4.6 Control input of inverted pendulum

Table 4.2. Comparative analysis of equivalent, reaching, quasi, and decoupled sliding mode control

Parameters	**Rise Time (s)**	**Peak Time (s)**	**Max Overshoot (%)**	**Settling Time (s)**
Equivalent SMC	0.5	0.7	17.9	0.25
Reaching SMC	0.2	0.35	18.1	0.6
Quasi SMC	0.1	0.9	18.6	0.4
Decoupled SMC	0.15	0.2	17.9	0.25

The aforementioned findings suggest that all four control options function satisfactorily in terms of stability. Due to time domain measurement, sliding mode controller with decoupled law stands out, with quasi mode mode producing the superior results. As has already been mentioned, the plant's discrete time implementation and unmodeled dynamics make the chattering a bad characteristic. Also, the mechanical components' control precision and strength are being compromised by this issue. And in all of the approaches mentioned above, the decoupled algorithm dramatically lessens chattering than the quasi, equivalent, and sliding mode controller.

Simulation Results of TORA System for Decoupled Sliding Mode Controller (DSMC) Algorithm

For TORA system: The initial states are [1 0 π 0], to satisfy$\lambda_{ref}(-A) > \boldsymbol{\gamma}$**,** a=5, η=0.50, and switch function Δ=0.10, when the poles are moves far away from imaginary axis its resonant frequency comes to zero, so its damping coefficient and transient response time are reduced to zero as shown in the figure 4.8 and figure 4.9, the control input of TORA system is shown in figure 4.7.

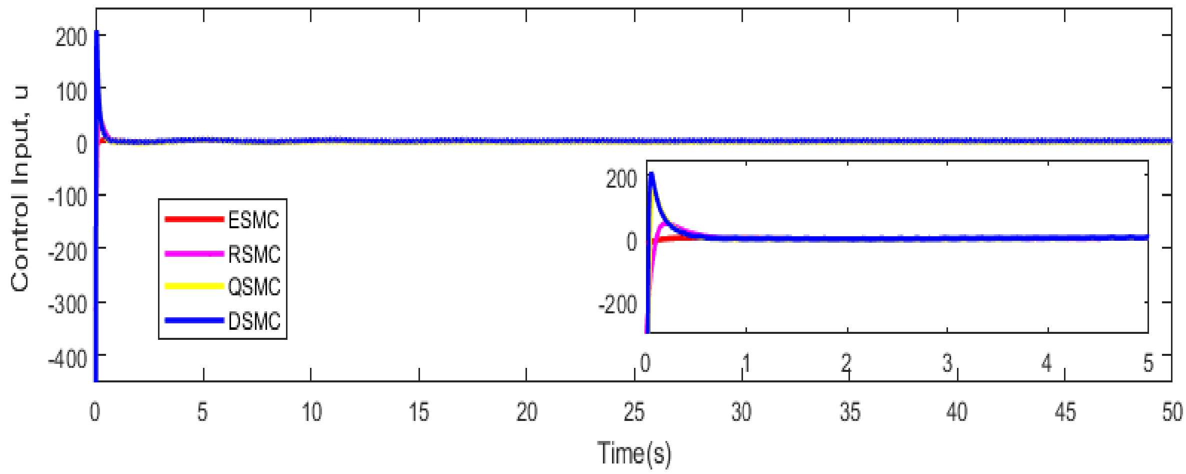
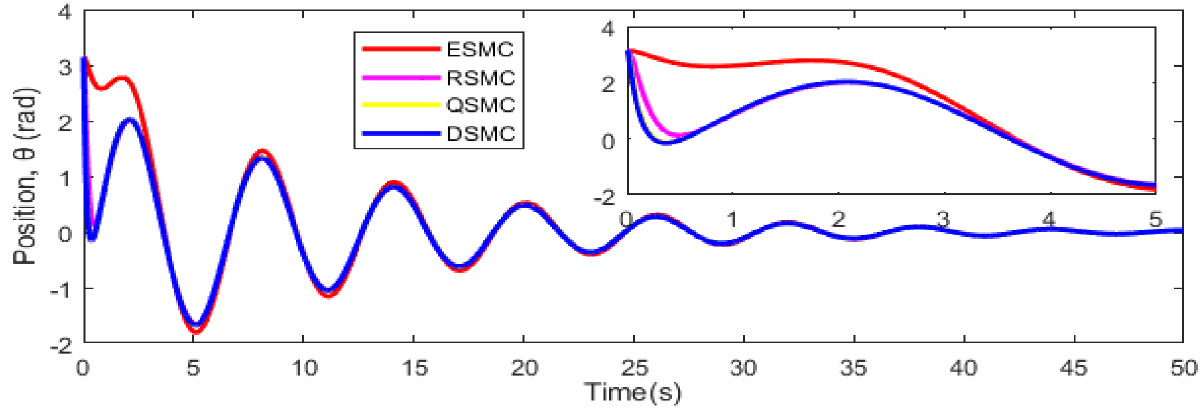
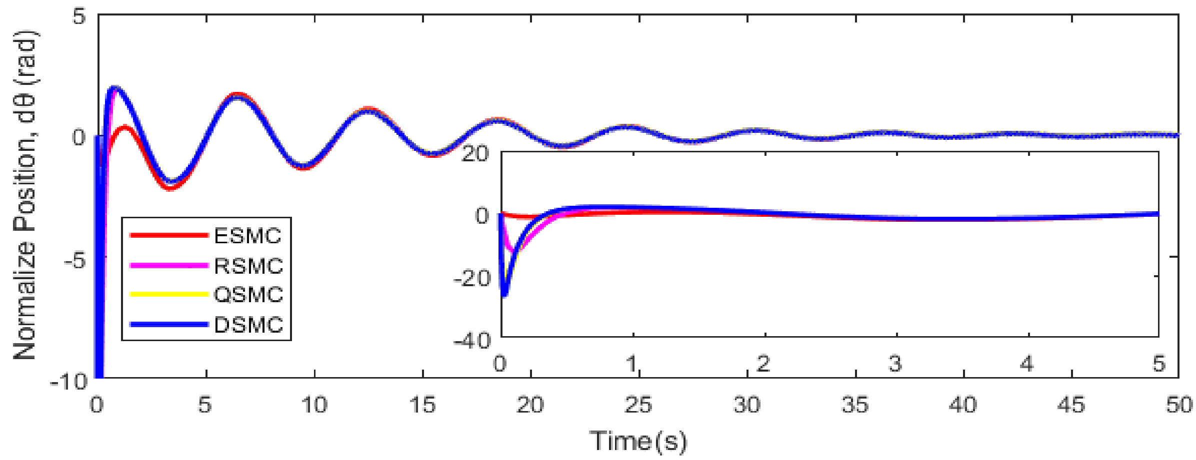


Figure 4.7 Control input of TORA system for decoupled sliding mode control

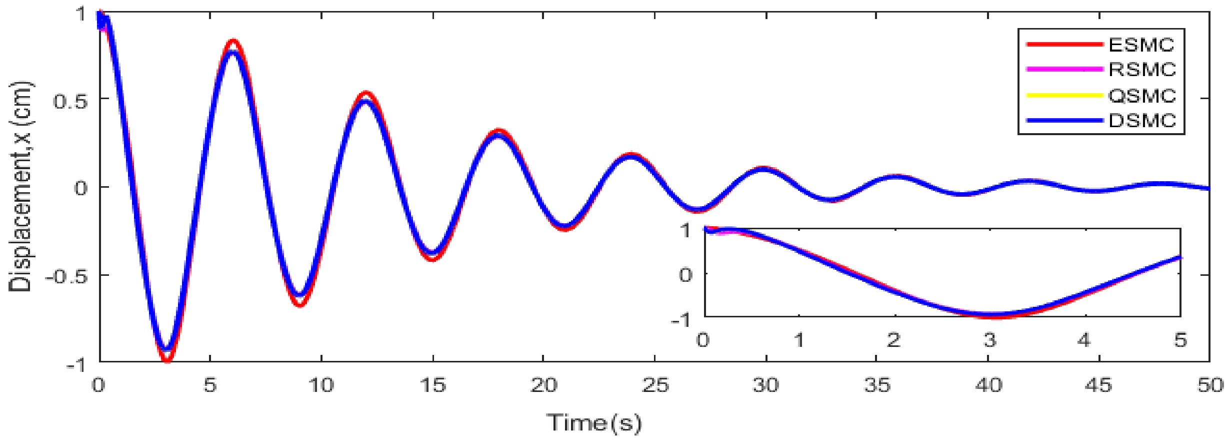


(a)

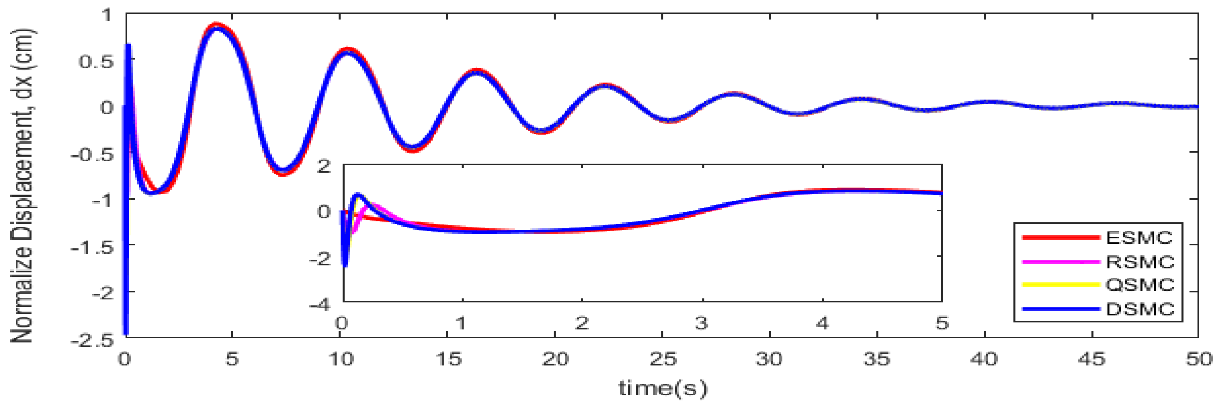


(b)

Figure 4.8 (a) Actuator angle, and (b) derivative of actuator angle of TORA system



(a)



(b)

Figure 4.9 (a) Actuator displacement, and (b) derivative of actuator displacement of TORA system

It is clear from above results that the system experiences oscillations and overshoot, which further increased chattering in case of equivalent sliding mode control and reching sliding mode control and improved in quasi sliding mode control and decoupled sliding mode mode control.

4.4 Conclusion

In this study, reaching sliding mode control (RSMC), decoupled sliding mode control (DSMC), quasi sliding mode control (QSMC) and equivalent sliding mode control (ESMC) algorithm is used to improve the performance of inverted pendulum systems and TORA system and reduces the chattering. The above study is divided into two parts, in first part implementation of switching sliding mode controller, quasi sliding mode controller, equivalent sliding mode controller, and decoupled sliding mode controller is applied on inverted pendulum to check the system performance. And as per the result decoupled sliding mode controller gives best result among all four.

While in second part a decoupled algorithm was investigated to stabilize TORA systems. The performance of the control scheme was evaluated based on transient performance and stability. Simulation results showed that proposed sliding mode controller was able to give a robust performance in a wide range of operations and disturbances. Further the chattering phenomenon is also discussed and found that decoupled algorithm reduces the chattering sufficiently compare to other methods i.e., quasi sliding mode controller, equivalent sliding mode controller, switching sliding mode controller.

Chapter: 5

Comparison of Approximation, Equivalent, and Switch-Gain Regulation Fuzzy Sliding Mode based Controller for Single Link Inverted Pendulum

5.1 Introduction

All the practical systems are nonlinear in nature and this is the main reason behind the growing interest in nonlinear control. The Inverted pendulum is a benchmark example of a static unstable nonlinear system due to its widest scope in industrial application. The best control benchmark system is an inverted pendulum which is inherently unstable. To stabilize a pendulum around its unstable equilibrium point, different control approaches have been reviewd in the literature. To balance a single link inverted pendulum, approximation of input-output linearization have been used theoretically and experimentally both, but this scheme uses complex mathematics which affects the response of the system [300]. Many controls theorem such as Lasalle's invariant, feedback linearization on the basis of input-output, and second theorem of Lyapunov's have been already used but at zero input case. [301]. Use of conventional controllers like PID based control tuned by trial and error method [302], optimization tools like a linear quadratic regulator and soft computing technique like neural and fuzzy control [303]-[304] has been given in the literature. But the common problem with all the said technique is that the system gives optimal result for certain specific operations. To handle the uncertainty in terms of external disturbance sliding mode control shows promising result which further enhance the system robustness [305] – [307]. In this method, a sliding surface is developed on the basis of state space, which works as a switch to control and observe plant behaviours with the input-output condition along with the constraints of modeling uncertainties and disturbance.

Despite the fact, sliding mode control is the best techniques for controlling, but it suffers from a few drawbacks. In ideal conditions switching of control should occur at high frequency. Due to these switching forces, the dynamic system changes there trajectories and slide along the given restricted subspace of sliding mode. But practically, it is impossible to control the switching at high speed due to the physical limitations of switching devices and time delay computational control. This high speed switching produces a oscillation at very high frequency known as chattering in sliding mode control. Chattering may result in energy loss, system instability and sometimes it may lead to plant damage. Various techniques have been reviewed in the literature which focuses on chattering. A boundary layer is created and overlap in nearby region of the sliding surface [308], boundary layer solution is proposed [101], higher-order sliding mode technique [309]-[312] estimation techniques of disturbance [313]-[314] control technique based on adaptive sliding mode [315], and adaptive fuzzy based sliding technique [316]-[317] are proposed in the literature to minimize the chattering. Each proposed technique has its advantages and disadvantages, but the selection criterion of the controller is depending on the ability to reduce the chattering which is related to the order of the mathematical model, plant uncertainty, and the type of application. Hence in all mentioned techniques fuzzy sliding mode control is simpler access to accord with chattering and plant uncertainties. The relation in the sliding mode controller and fuzzy is a physical phenomenon [318], which forces a sliding surface to follow the fuzzy rules. Hybrid control approach of fuzzy and SMC gives more stability and robustness against external disturbances due to unmatched plant dynamics. Fuzzy controller requires expert knowledge for designing of the controller which is a model-free, insensitive peripheral disturbances and variations in parameters [319]. The concept of fuzzy sliding mode control has been adapted by combining both fuzzy logic and SMC. The suitable controller not only attenuates uncertainties caused by unmatched dynamics and disturbances due to external sources but also significantly reduces the control chattering which is inherently found in the sliding mode control's structure. This paper focus on application of sliding mode on fuzzy controller and compare their results in terms of oscillation and settling time of controller output. By comparing the output results of the three controllers it has been observed that the results corresponds to equivalent control based fuzzy sliding mode controller gives better response than approximation theory based fuzzy sliding mode controller and switching gain based fuzzy sliding mode control in order to reduce the chattering.

In this chapter, the approximation, equivalent, and switch-gain regulation fuzzy sliding mode based controller for inverted pendulum systems is address keeping external disturbances and uncertainty into considerations. The system is chatter-free because a set of linguistic rules are created in fuzzy logic control. Fuzzy rules have been used to approximation the uncertainties of the nonlinear system while system parameters are accommodated according to adaptive laws fuzzy controller.

5.2 Implementation of fuzzy sliding mode controller (FSMC)

In many areas, fuzzy control (FC) has replaced traditional approaches. The capacity of fuzzy logic to capture the degree of uncertainty in human thought is one of its key characteristics. Thus, FC is a different approach to dealing with the unknown process when the mathematical model of the process is absent or exists but has uncertainties. The study is complicated, however, due to the enormous number of fuzzy rules for high-order systems. The FSMC has so received a lot of attention. In this chapter, a fuzzy inference engine is employed to arrive at the phase and a fuzzy sliding mode control approach is suggested in order to solve the chattering problem. The primary benefit of this approach is the assurance of the system's resilient behaviour. The second benefit of the suggested plan is that it outperforms the identical SMC approach without FLC in terms of the system's ability to eliminate chattering.

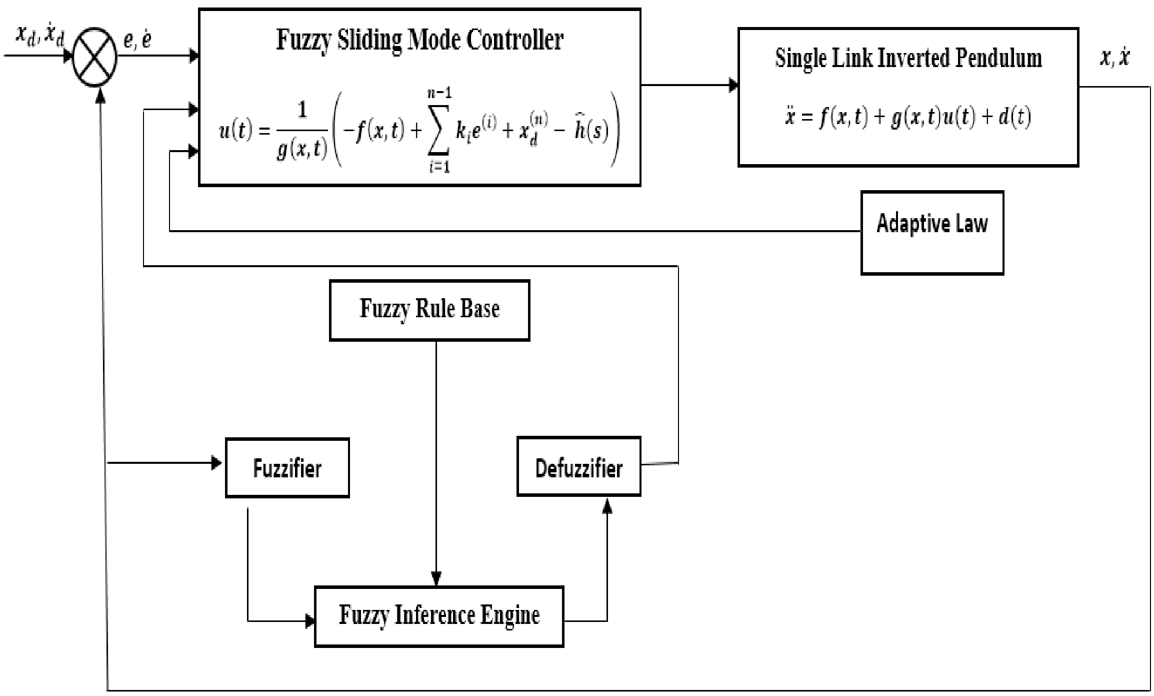


Figure 5.1. Control structure of fuzzy logic based sliding mode control

Sliding control mode is robust to parameter variation and designs an accurate function of the system. The real continuous system is easily approximated by fuzzy logic [320]-[323]. Therefore, logic rules are designed and realized accordingly on a fuzzy sliding mode controller, based on experience. Fuzzy system based on the universal theory of approximation is used to design the model and to realized its external disturbances.

The above block diagram, figure 5.1 explains the control operation for the position of an inverted pendulum through a fuzzy sliding mode controller. The above diagram shows two main blocks namely, ‘single link inverted pendulum’ and ‘fuzzy sliding mode controller’. The fuzzy sliding mode controller operates on the IF-THEN rules of the fuzzy system. Moreover, the detailed steps of fuzzy operation has been explained through the block diagram mentioned as fuzzification, fuzzy inference engine, fuzzy rule, and defuzzification. Besides the fuzzy controller, an ‘adaptive law’ has also been incorporated with the fuzzy sliding controller. In the adaptive law, the parameters corresponding to the fuzzy system have been controlled and set to a new value. The adaptive law of the controller makes the system trajectory to track the reference trajectory.

5.2.1 Classical sliding mode control

This section describes a nonlinear system that contains uncertainty and outside disturbances. The nonlinear system's state equation is the following second-order differential equation:

$$\ddot{x} = f(x,t) + g(x,t)u(t) + d(t) \tag{5.1}$$

Where $f(x,t)$ and $g(x,t)$ are nonlinear functions representing the uncertain term of the unmodeled dynamics or structural variation of system and $d(t)$is the disturbance of the system. In general, the uncertain term $f(x,t)$, $g(x,t)$ and the disturbance term $d(t)$ are assumed to be bounded.

The control problem is to get the system to track the desired position of inverted pendulum $x_d(t), = [x_{d1}(t)\ x_{d2}(t)\ x_{d3}(t) \dots \dots \dots \dots]^T \epsilon R^n$ (5.2)

Let the tracking error be

$$e(t) = x(t) - x_d(t) = [x_1(t) - x_{d1}(t)\ x_2(t) - x_{d2}(t) \dots \dots \dots x_n(t) - x_{dn}(t)]^T = [e_1(t) e_2(t) \dots \dots e_n(t)]^T \tag{5.3}$$

The main aim is here to achieve the target $x_d(t)$, and design a sliding surface, such that the resulting state response of the tracking error vector satisfies

$$\|e(t)\| = \|x(t) - x_d(t)\| \to 0 \tag{5.4}$$

Due to its order reduction feature and low susceptibility to disturbances and changes in plant parameters, ISMC is an effective instrument for controlling complex high-order dynamic plants that are working under uncertainty. In SMC, the controlled system's states are initially directed to dwell on a sliding surface in state space before being constrained there by a shifting law (based on the system states). The variable $s(x,t)$, specified is equated to zero to define a time-varying surface $s(t)$ in the state space.

The sliding function is designed as, $s = \dot{e} + ke$

Where $k > 0$, then

$$\dot{s} = \ddot{e} + c\dot{e} = \ddot{x}_d - \ddot{x} + c\dot{e}x_1(t) - x_{d1}(t) = \ddot{x}_d - f(x,t) - g(x,t)u(t) - d(t) + c\dot{e} \quad (5.5)$$

The control law is given as,

$$u = \frac{1}{g(x,t)}[-f(x,t) + \ddot{x}_d + c\dot{e} + \delta sgn(s)] \quad (5.6)$$

In control law equation (5.6), the switch term $\delta sgn(s)$ is a robust term, which is used to overcome $d(t)$

The sliding mode surface is designed as,

$$\dot{s} = \ddot{e} + c\dot{e} = \ddot{x}_d - \ddot{x} + c\dot{e}$$

$$\ddot{x}_d - f(x,t) - g(x,t)u(t) - d(t) + c\dot{e} = \delta sgn(s) - d(t) \quad (5.7)$$

5.2.2 Approximation theory based FSMC

The fuzzy approximation theory shows that a fuzzy system can be represented as any continuous system. In this theorem, the unknown plant nonlinearities are approximated by fuzzy set IF-THEN rules of the fuzzy system. According to online adaptive law the parameters corresponding to fuzzy system have been controlled and set to a new value.

The adaptive law of the controller makes the system trajectory track the reference trajectory, and at the same time also reduces its chattering.

If function $f(x,t)$ is unknown of a nonlinear system, then a fuzzy estimation $\hat{f}(x)$ is used to get feedback control. The universal approximation theorem is given as:

1. For $x_i (i = 1,2 \dots . n)$ define the fuzzy sets $A_i^{l_i}, l_i = 1,2, \dots . , q_i$.
2. Follow $\prod_{i=1}^{n} q_i$ fuzzy rules to construct fuzzy system $\hat{f}(x|\theta_f)$.

If x_1 is $A_1^{l1} \dots .. \ x_n$ is A_1^{ln} then $\hat{f}$ is $E^{l_1 \dots \dots l_n}$, where $l_i = 1_i, 2_i, \dots . . q_i$ and $i = 1,2, \dots . . , n$

The fuzzy output can be described as

$$\hat{f}(x|\theta_f) = \frac{\sum_{l_1=1}^{p_1} \cdots \sum_{l_n=1}^{p_1} y_f^{-l_1,\dots,l_n} (\prod_{i=1}^{n} \mu_{A_i^{l_i}}(x_i))}{\sum_{l_1=1}^{p_1} \cdots \sum_{l_n=1}^{p_n} (\prod_{i=1}^{n} \mu_{A_i^{l_i}}(x_i))} \quad (5.8)$$

Where $\mu_{A_i^{l_i}}(x_i)$ is the membership function of x_i.

In the above equation, a column vector €(x) is introduced due to free parameter $y_f^{-I_1,..I_n}$. Hence the fuzzy estimation is written as:

$$\hat{f}(x|\theta_f) = \hat{\theta}_f^T €(x) \quad (5.9)$$

Where $€(x)$ is the $\prod_{i=1}^{n} p_i$- a dimensional column vector.

$$€_{l_1,\dots..l_n}(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^{l_i}}(x_i)}{\sum_{l_1=1}^{p_1} \cdots \sum_{l_n=1}^{p_n} (\prod_{i=1}^{n} \mu_{A_i^{l_i}}(x_i))} \quad (5.10)$$

Assuming, the optimum solution for fuzzy approximation,

$$\theta_f^* = arg\ arg\ (sup|\hat{f}(x|\theta_f)\ \ - f(x)| \tag{5.11}$$

Function $f(x,t)$ represents the unmatched uncertainty component of the nonlinear system which appears in the input and α_f is the set of θ_f.

The unknown function $f(x,t)$ is written as, $f = \theta_f^{*T} €(x) + \varepsilon$ (5.12)

In the above equation the term x is the input signal of fuzzy system, $€(x)$ is set of fuzzy vectors and ε is the fuzzy error, $\varepsilon \leq \varepsilon_N$.

The unknown function $f(x,t)$ is approximated by fuzzy system. The input of fuzzy system is selected as $x = [e\ \dot{e}]^T$, and the output of the fuzzy system is given as, $\hat{f}\left(\theta_f\right) = \hat{\theta}_f^T €(x)$

The control equation no (5.6) may be written as to design the sliding mode controller,

$$u = \frac{1}{g(x,t)}\left[-\hat{f}(x,t) + \ddot{x}_d + c\dot{e} + \delta sgn(s)\right] \tag{5.13}$$

The equation of fuzzy sliding surface,

$$\dot{s} = -f - d(t) - \delta sgn(s) \tag{5.14}$$

Where, $f = f - \hat{f} = f = \theta_f^{*T} €(x) + \varepsilon$ (5.15)

To drive the plant trajectory on a sliding surface Lyapunov function is designed as,

$$L = \frac{1}{2}s^2 + \frac{1}{2}\gamma\tilde{\theta}_f^T\tilde{\theta}_f \tag{5.16}$$

From equation (5.14), (5.15) and (5.16)

$$\dot{L} = s\dot{s} + \gamma\tilde{\theta}_f^T\dot{\tilde{\theta}}_f$$

$$= \tilde{\theta}_f^T\left(s€(x) + \gamma\dot{\tilde{\theta}}_f\right) - s(\varepsilon + d(t)\delta sgn(s) \tag{5.17}$$

The adaptive rule of fuzzy controller $\dot{\tilde{\theta}}_f = \frac{1}{\gamma}s€(x)$ (5.18)

$$\dot{L} = -s\big(\varepsilon + d(t) + \delta sgn(s)\big) = -s\big(\varepsilon + d(t)\big) - \delta|s| \tag{5.19}$$

Since the approximation error ε is so small hence $\delta \geq \varepsilon_N + D$, the Lyapunov function is approximated as $L \dot{\leq} 0$.

5.2.3 Switching gain based FSMC

In this section, FSMC controller has been implemented for the purpose of switching gain based operation. The switching control technique in sliding mode control derive the system states on sliding surface in the presence of unwanted external disturbances. And the fuzzy controller maintains system states on surface to improve the control performance and reduces the chattering in the sliding mode.

In this method there is a discontinuous switch pattern of sliding surface which has been substituted with fuzzy logics for further control.

The switching function of sliding mode be,

$$s = -(k_1 e + k_2 \dot{e} + \cdots + k_{n-1} e^{(n-1)} + e^{(n-1)} \quad (5.20)$$

The error of switching function e,

$$e = x_d - x = \left[e\ \dot{e} \ldots\ e^{(n-1)}\right]^T, k_1, k_2 \ldots\ k_{n-1} \quad (5.21)$$

The control equation no (5.6) may be written as to design the sliding mode controller,

$$u(t) = \frac{1}{g(x,t)}\left(-f(x,t) + \sum_{i=1}^{n-1} k_i e^{(i)} + x_d^{(n)} - u_{sw}\right) \quad (5.22)$$

The switched input u_{sw} of the controller is written as:

$$u_{sw} = \delta sgn(s), \delta > D \quad (5.23)$$

The sliding surface of switching gain fuzzy controller is designed as:

$$\dot{s} = \sum_{i=1}^{n-1} k_i e^{(i)} + x^{(n)} - x_d^{(n)}$$

$$= \sum_{i=1}^{n-1} k_i e^{(i)} + f(\theta, \dot{\theta}) + g(\theta, \dot{\theta}) u(t) + d(t) - x_d^{(n)} \quad (5.24)$$

$$\dot{s} = d(t) - \delta sgn(s) \quad (5.25)$$

i.e., $s\dot{s} = d(t)s - \delta|s| \leq 0$ (5.26)

Where'd' disturbance and switching term $'\delta'$ both are large, which increases chattering. Then $\hat{h}$ is used to approximate $\delta sgn(s)$ which reduces the switching function, which will further reduce chattering.

Using product deduction and centre average fuzzy, the equation (5.22) may be written as:

$$u(t) = \frac{1}{g(x,t)}\left(-f(x,t) + \sum_{i=1}^{n-1} k_i e^{(i)} + x_d^{(n)} - \hat{h}(s)\right) \quad (5.27)$$

Where $\hat{h}(\theta_h)$ output is for the universal approximation based fuzzy controller of the equation (5.9), $\varphi(s)$ is the fuzzy vector, which will vary as per the rules adopted.

The ideal $\hat{h}(\theta_h)$ is given as $\hat{h}(\theta_h) = \delta sgn(s)$

The adaptive rule to optimized the fuzzy controller is given as $\dot{\theta}_h = \gamma s \varphi(s)$, where $\gamma > 0$.

The optimization parameter of adaptive law is

$$\theta_h^* = \arg \min_{\theta_h \epsilon \Omega_h} [\sup|\hat{h}(s|\theta_h) - \delta sgn(s)|] \quad (5.28)$$

Sliding surface $\dot{s}$ of the controller is designed as per switched gain regulation algorithims:

$$\dot{s} = -\sum_{i=1}^{n-1} k_i e^{(i)} + x^{(n)} - x_d^{(n)}$$

$$= \sum_{i=1}^{n-1} k_i e^{(i)} + f(\theta, \dot{\theta}) + g(\theta, \dot{\theta})u(t) + d(t) - x_d^{(n)}$$

$$= -\hat{h}(s|\theta_h) + d(t) + \hat{h}(s\,|\,\theta_h^*) - \hat{h}(s|\theta_h^*)$$

$$= \tilde{\theta}_h^T \varphi(s) + d(t) - \hat{h}(\theta_h^*) \quad (5.29)$$

Lyapunov function for the system can be defined as,

$$V = \frac{1}{2}(s^2 + \frac{1}{\gamma}\tilde{\theta}_h^T \tilde{\theta}_h)$$

$$\dot{V} = s\tilde{\theta}_h^T \varphi(s) + \frac{1}{\gamma}\tilde{\theta}_h^T \dot{\tilde{\theta}}_h + s(d(t) - \hat{h}(\theta_h^*)$$

$$\dot{V} = \frac{1}{\gamma}\tilde{\theta}_h^T \left(\gamma s \varphi(s) - \dot{\theta}_h\right) + sd(t) - \delta|s| \quad (5.30)$$

From equations (5.29) and (5.30), $\dot{V} = sd(t) - \delta|s| < 0$ (5.31)

5.2.4 Equivalent control based FSMC

A switch control equation and an equivalent control equation make up the control law of an equivalent sliding mode controller. Switch control forces the system state to stay on the sliding surface while analogous control drives the system states on the sliding surface. In order to lessen the chattering phenomena of sliding mode controllers, fuzzy rules of fuzzy controller are established based on comparable control and switch control. When disturbance magnitudes are small, switch control with a little gain is used, and when disturbance magnitudes are big, a switch control with a large gain is used.

The tracking error of the switching surface is given as:

$$e = x_d - x \quad (5.32)$$

The switching function of sliding mode is given as:

$$s = ce + \dot{e} \quad (5.33)$$

By applying the reverse fuzzification method, the fuzzy control is designed as:

$$u = u_{eq} + \mu * u_s \quad (5.34)$$

The chattering phenomenon of sliding mode controller is reduced by varying the membership function μ .

5.3 Simulation Results and Discussion

The performance of fuzzy sliding mode control on the inverted pendulum is evaluated via numerical simulation. The parameters for single link inverted pendulum are given as, M=1.0kg, m=0.1kg, l=0.5 m, g=9.8m/s^2. Simulation is carried out while considering the movement of the cart and pendulum both are in one plane. The objective of the controller is to stabilize the pendulum position in upward.

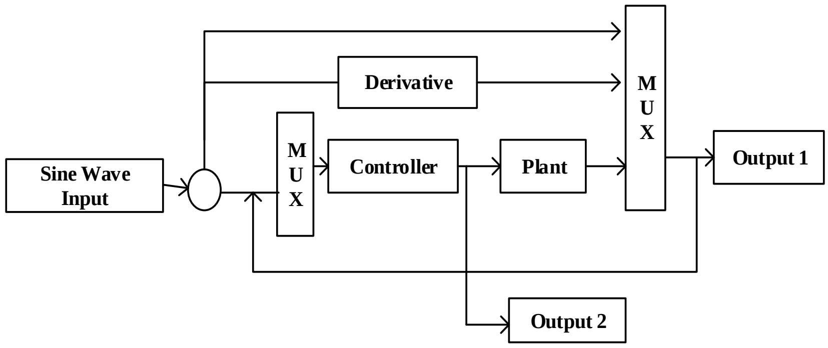


Figure 5.2 Block diagram of Simulink model

5.3.1 *Simulation Results of Approximation theory based FSMC*

In the case of approximation theory based FSMC the initial states of the system are $\theta(0) = -60^0, \dot{\theta}(0) = 0, x(0) = 5.0, \dot{x}(0) = 0$ and the desired states $\theta(0) = 0, \dot{\theta}(0) = 0, x(0) = 0, \dot{x}(0) = 0$, $\theta_f = 0.5$, $\delta = 0.1$, $k_1 = 20$, $k_2 = 10$ and the adaptive parameter is $\gamma = 0.005$.

To fuzzify the state vectors x_1 and x_2, five membership function is selected and figure 5.3 shows the membership function.

$$\mu_{HN} = \exp\left[-\left(\frac{x_i + \pi/3}{\pi/12}\right)^2\right]$$

$$\mu_N = \exp\left[-\left(\frac{x_i + \pi/6}{\pi/12}\right)^2\right]$$

$$\mu_Z = \exp\left[-\left(\frac{x_i}{\pi/12}\right)^2\right]$$

$$\mu_P = \exp\left[-\left(\frac{x_i - \pi/3}{\pi/12}\right)^2\right]$$

$$\mu_{HP} = \exp\left[-\left(\frac{x_i - \pi/6}{\pi/12}\right)^2\right]$$

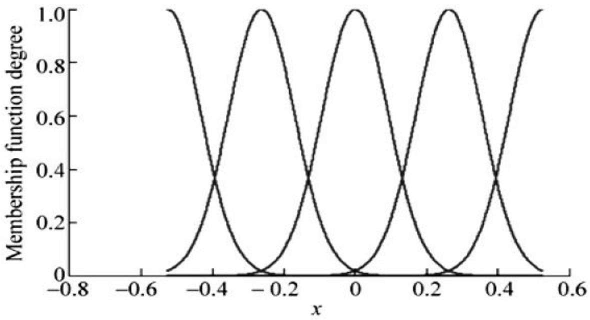


Figure 5.3 Membership function of input error for approximation theory based FSMC

5.3.2 Simulation Results of Switched gain based FSMC

The switching function sliding surface is given as, $s = -k_1 e - \dot{e}$, where $k_1 = 50$ and the adaptive parameter is $\gamma = 0.005$, the initial states of the system are $\theta(0) = -60^0, \dot{\theta}(0) = 0, x(0) = 5.0, \dot{x}(0) = 0$ and the desired states $\theta(0) = 0, \dot{\theta}(0) = 0, x(0) = 0, \dot{x}(0) = 0$.

Membership functions developed and considered for the switching gain-based method is defined as

$$\mu_{NB} = \frac{1}{(1 + e^{(5(x+2))})}$$

$$\mu_{NM} = e^{(-(x+1.5)^2)}$$

$$\mu_{ZO} = e^{\ (-(x^2))}$$

$$\mu_{PM} = e^{(-(x-1.5)^2)}$$

$$\mu_{PB} = \frac{1}{(1 + e^{(5(x-2))})}$$

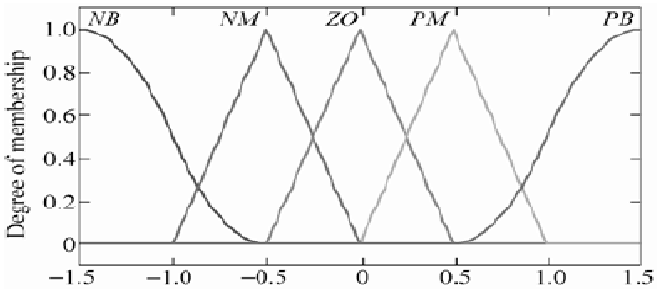


Figure 5.4 Membership function of input error for switched gain based FSMC

Fuzzy logic based controller approximated discontinuous switching gain by the, the membership function adapted as shown in figure 5.4.

5.3.3 Simulation Results of Equivalent control based FSMC

Let the disturbance d(t) is given in form of gaussian function is given as, $d(t) = 5e^{\left(-\frac{(t-c_i)^2}{2b_i^2}\right)}$, where $b_i = 0.050, c_i = 5.0$ and $\gamma = 0.5$ the initial states of the system are $\theta(0) = -60^0, \dot{\theta}(0) = 0, x(0) = 5.0, \dot{x}(0) = 0$ and the desired states $\theta(0) = 0, \dot{\theta}(0) = 0, x(0) = 0, \dot{x}(0) = 0$.

Generally, sliding mode-based control operates using switch rule and equivalent rule. The logics of equivalent rule has been replaced by a fuzzy equivalent rule, which are mentioned below as follows;

IF x is HN THEN μ is HP

IF x is N THEN μ is P

IF x is Z THEN μ is Z

IF x is P THEN μ is P

IF x is HP THEN μ is P

Where the fuzzy sets *HP, P, HN, N* and, *Z* denote highly positive, "positive", "highly negative" "negative", and "zero", respectively.

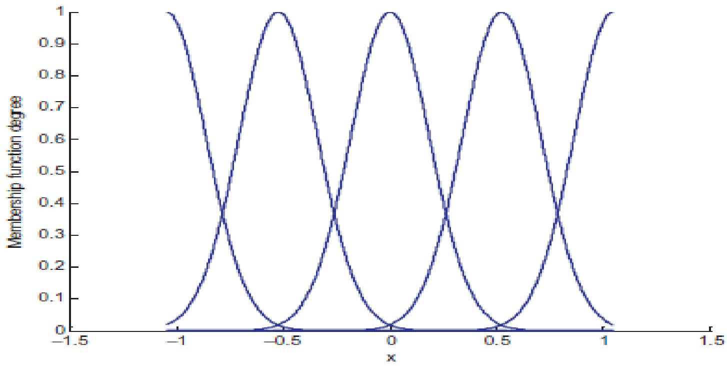


Figure 5.5 Membership function of input error for equivalent control based FSMC

Membership function μ equivalent control method is shown in figure 5.5. Control input is divided in two steps, first step is equivalent control u_{equ} which takes the system states toward the sliding surfaces, and second step ensure the system trajectory well on sliding surface by switch control u_s. Further, fuzzy control method is also devoloped as $u = u_{equ} + \mu . u_s$, by the implementation of reverse fuzzification.

Where u_{equ}= equivalent control law and u_s= switch control law.

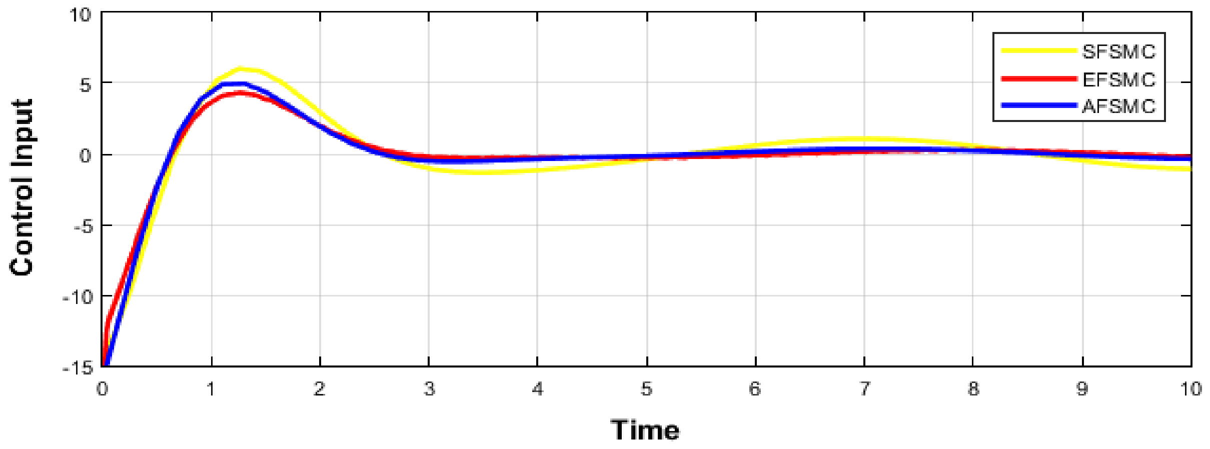
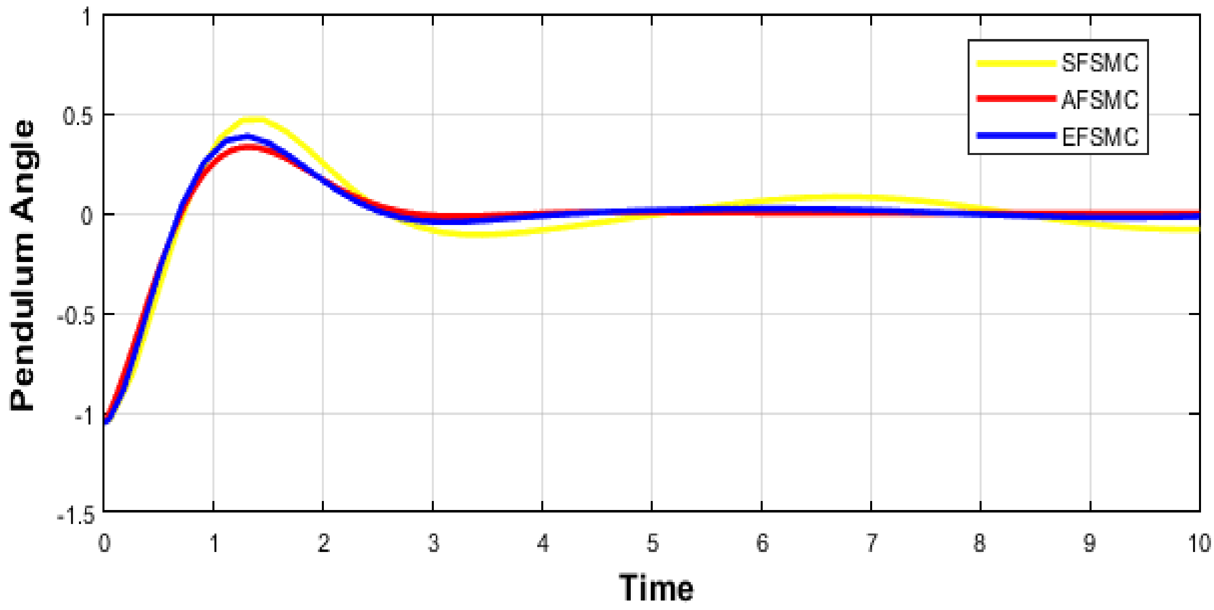
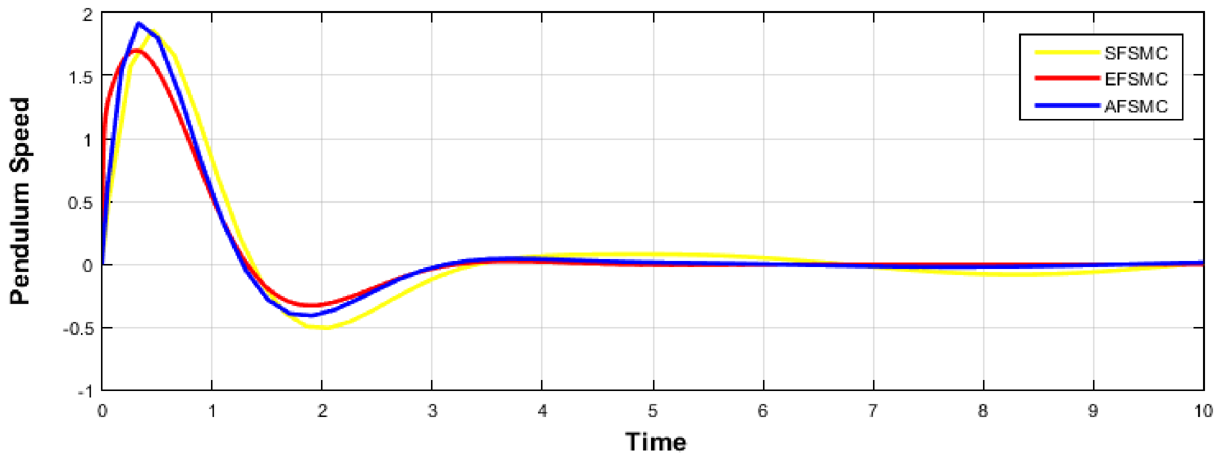


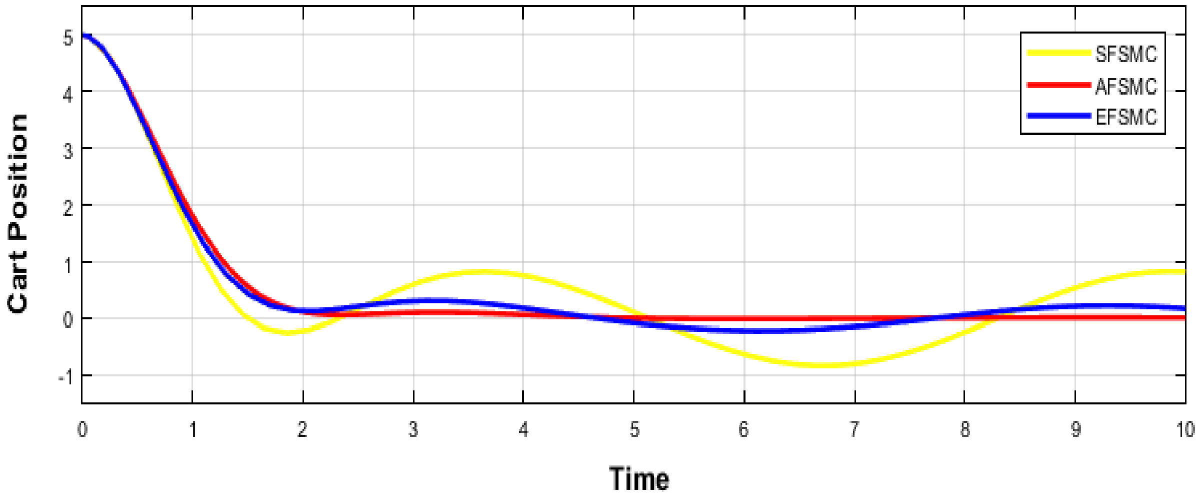
Figure 5.6 Control input of inverted pendulum



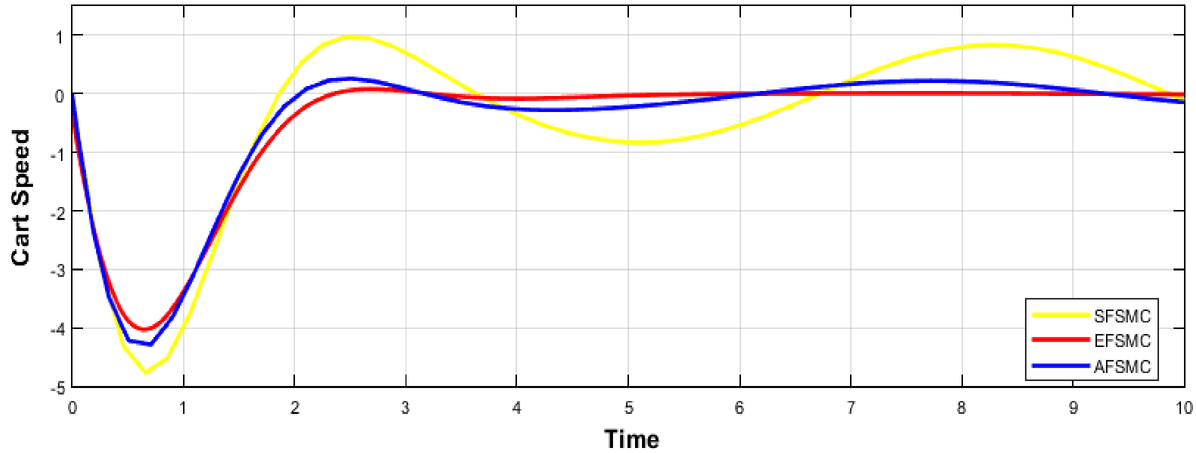
(a)



(b)



(c)



(d)

Figure 5.7 (a) Pendulum angle (b) Pendulum speed of inverted pendulum (c) Cart Position (d) Cart Speed

Figure 5.6 shows control input of inverted pendulum for switching, approximation and equivalent fuzzy sliding mode controller and figure 5.7 shows the response of inverted pendulum such as pendulum angle, pendulum speed, cart position and cart speed for switching, approximation and equivalent fuzzy sliding mode controller. The equivalent control based fuzzy sliding method reduces oscillation of the system. The reduced oscillation sufficiently minimizes the chattering phenomenon. Hence equivalent control based method is the most suitable method to eliminate chattering over approximation based sliding control mode and switching gain based sliding control mode.

In equivalent control based sliding mode is directly replaced by the fuzzy controller, whenever the trajectory of system is trying to leave the switching surface, large control force is provided by fuzzy rules to drive the trajectory again on the switching surface. Whereas approximation based fuzzy sliding mode and switching gain based sliding mode controller has been approximated with fuzzy values. Also, the equivalent control based sliding mode controller obtained a stable position through tracking of an inverted pendulum. This has been achieved with zero steady error within a specified bounded area.

5.4 Conclusion

The current work depicts three types of fuzzy sliding mode controller named as approximation based method equivalent control based method, and switching gain based method have been implemented to stabilize the tracking issue of inverted pendulum system. The equivalent control based sliding mode controller shows effective results in terms of system uncertenities and disturbances. A control strategy of fuzzy sliding mode involves a chattering free and stabilized nature for an inverted pendulum.

Chapter: 6

Sliding Mode Control of Nonlinear Systems using RBF Neural Network

6.1 Introduction

Every physical system has a nonlinear character by definition. The failure of the superposition principle, which underlies linear control analysis, in the nonlinear situation necessitates the study of nonlinear systems. Improvements in linear control systems, nonlinearity analysis, and the necessity to deal with model designs are the main drivers of the increased interest in nonlinear system analysis. Nonlinear systems are challenging to examine since there is no single method that can be used to study all nonlinear systems. Since it is challenging to locate the direct solutions to nonlinear differential equations, laborious attempts have been undertaken to build suitable instruments for doing so. Sliding mode approaches have been cited as a superior option for robust control of non-linear systems due to its basic structure and straightforward implementation. Numerous strategies have been employed in the literature to control non-linear systems and determine their stability. Additionally, order reduction and adaptation against system uncertainties and disruptions are enforced via control of sliding mode type [324]–[325]. There are several proposals for non-linear control approaches based on sliding mode control in the literature [326]-[331]. It has been established that sliding mode approaches are a superior option for handling uncertainty and disturbances. Due to the discontinuous signum function of the control rule, the main problem with traditional sliding mode control is inherent chattering. Many methodologies described in the literature can be used to improvise the functionality of classical sliding mode control [332]. The first and second phases of classical sliding mode control are called the reaching mode phase and the sliding phase, respectively. Yet only the sliding mode phase demonstrates immunity to changes in the system's parameters. The SMC is able to handle the chattering issue in classical sliding because it is based on the innovative reaching law. A global SMC that avoids the reaching mode phase, preserves the resilience of the system, and is present during the whole reaction period was also presented by a few researchers. Some control systems have been subjected to this methodology [333] – [334].

Sliding mode control combines with other intelligence controllers, including fuzzy, neural, etc., to eradicate the chattering phenomena [335] – [339]. Every nonlinear function over a RBF neural network can approximate an efficient set with arbitrary precision, according to earlier research on the universal approximation theorem [340]. There have been many different control strategies for neural sliding mode control developed, such as using a neural network as an observer for equivalent control estimation. In addition, a synthetic NN is employed in tandem with a modified switching function of the sliding mode controller for online diagnosis of model flaws. This improves controller performance and lessens chattering. To approximate the system's unknown nonlinearities, the RBF neural network can be utilised. Its weight value parameters are updated in real time in accordance with adaptive rules with the aim of directing the nonlinear system's output to create a specific trajectory. The chattering activity is reduced by using the Lyapunov function to create an adaptive control mechanism based on the RBF model.

In this chapter, a resilient sliding mode control method for nonlinear systems taking into account various nonlinearities has been explored. A single link inverted pendulum and a robotic manipulator are the nonlinear systems that are taken into account here. The aim of control action is to keep the system's position where it is meant to be. To enhance control performance, an adaptive sliding mode control based on RBF compensation is used by the RBF neural network.

6.2 Mathematical Modeling of Nonlinear System

6.2.1 Mathematical Modeling of Robotic Manipulator

The nonlinearities in a nonlinear system may be cancelled, converting the system dynamics into a linear form. This is demonstrated in the control scheme for the two-link robot in figure 6.1.

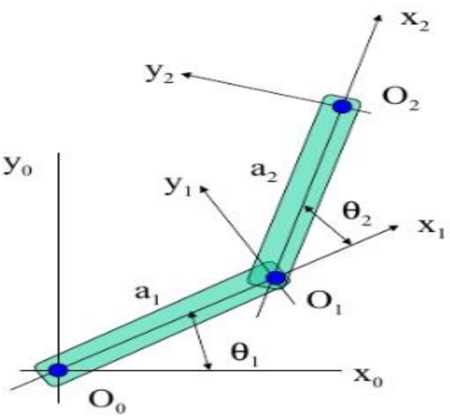


Figure 6.1 Two link robotic maniputaor

Two-link planar robot arm manipulator's arm dynamics are determined by assuming that the link masses m_1 and m_2 are primarily found at the terminals of the length linkages.a_1 and a_2

respectively. The initial link's angle θ_1, θ_2 are with regard to the initial link's oriented torques τ_1 and τ_2 respectively.

Let's use the fundamentals of Lagrange's equation of motion to calculate the dynamics of the two-link arm:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = \tau;\ \theta = [\tau_1\ \tau_2\ ]^T \quad (6.1)$$

The Lagrangian L is defined as follows using the kinetic energy K and potential energy P:

$$L = K\left(\theta, \dot{\theta}\right) - P(\theta) \quad (6.2)$$

Locations and velocities for connection 1 are:

$$\left.\begin{aligned} x_1 &= a_1\ cos\,cos\,\theta_1 \\ y_1 &= a_1\ sin\,sin\,\theta_1 \\ \dot{x}_1 &= -a_1\,\dot{\theta}_1\,sin\,sin\,\theta_1 \\ \dot{y}_1 &= a_1\,\dot{\theta}_1\,cos\,cos\,\theta_1 \\ v_1^2 &= \dot{x}_1^2 + \dot{y}_1^2 = a_1^2\,\dot{\theta}_1^2 \end{aligned}\right\} \quad (6.3)$$

For link 1, the kinetic and potential energy are

$$K_1 = \frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1 a_1^2\dot{\theta}_1^2 \quad (6.4)$$

$$P_1 = m_1 g y_1 = m_1 g a_1\,sin\,sin\,\theta_1 \quad (6.5)$$

For link 2, the positions and velocities are given as:

$$x_2 = a_1\,cos\,cos\,\theta_1 + a_2\,cos\,cos\,(\theta_1 + \theta_2) \quad (6.6)$$

$$y_2 = a_1\,sin\,sin\,\theta_1 + a_2\,sin\,sin\,(\theta_1 + \theta_2) \quad (6.7)$$

$$\dot{x}_2 = -a_1\dot{\theta}_1\,sin\,sin\,\theta_1 - a_2(\dot{\theta}_1 + \dot{\theta}_2)\,sin\,sin\,(\theta_1 + \theta_2) \quad (6.8)$$

$$\dot{y}_2 = a_1\dot{\theta}_1\,cos\,cos\,\theta_1 + a_2(\dot{\theta}_1 + \dot{\theta}_2)\,cos\,cos\,(\theta_1 + \theta_2) \quad (6.9)$$

$$v_2^2 = \dot{x}_2^2 + \dot{y}_2^2 = a_1^2\dot{\theta}_1^2 + a_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2a_1a_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)\,cos\,cos\,\theta_2 \quad (6.10)$$

Therefore, kinetic energy for link 2 is

$$K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_2a_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2a_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + m_2a_1a_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)\,cos\,cos\,\theta_2 \quad (6.11)$$

The potential energy for link 2 is

$$P_1 = m_2g(a_1\,sin\,sin\,\theta_1 + a_2\,sin\,sin\,(\theta_1 + \theta_2) \quad (6.12)$$

The complete arm's Lagrangian equation is

$$L = K - P = K_1 + K_2 - P_1 - P_2 \quad (6.13)$$

$$= \frac{1}{2}(m_1 + m_2)a_1^2\dot{\theta}_1^2 + m_2a_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + m_2a_1\,a_2\left(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2\right)\cos\theta_2 - (m_1 + m_2)ga_1\sin\theta_1 - m_2ga_2\sin(\theta_1 + \theta_2) \quad (6.14)$$

The vector equation, which is made up of two scalar equations, is equation (6.14). To put out these two equations, the individual terms are

$$\frac{\partial L}{\partial \dot{\theta_1}} = (m_1+m_2)a_1^2\dot{\theta}_1^2 + m_2 a_2^2(\dot{\theta}_1+\dot{\theta}_2)m_2 a_1 a_2(2\dot{\theta_1}+\dot{\theta}_2)\ cos\ cos\ \theta_2 \quad (6.15)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta_1}} = (m_1+m_2)a_1^2\ddot{\theta}_1 + m_2 a_1 a_2\left(2\ddot{\theta}_1+\ddot{\theta}_2\right)\ cos\ cos\ \theta_2\ - m_2 a_1 a_2(2\theta_1\dot{\dot{\theta}}_2+\dot{\theta}_2^2)$$

$$sin\ sin\ \theta_2 \quad (6.16)$$

$$\frac{\partial L}{\partial \theta_1} = -(m_1+m_2)g a_1\ cos\ cos\ \theta_1\ - m_2 g a_2\ cos\ cos\ (\theta_1\ + \theta_2) \quad (6.17)$$

$$\frac{\partial L}{\partial \dot{\theta_2}} = m_2 a_2^2\left(\dot{\theta}_1+\dot{\theta}_2\right) + m_2 a_1 a_2\dot{\theta}_1\ cos\ cos\ \theta_2 \quad (6.18)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta_2}} = m_2 a_2^2\ \left(\dot{\theta}_1+\dot{\theta}_2\right) + m_2 a_1 a_2\ddot{\theta}_1\ cos\ cos\ \theta_2\ - m_2 a_1 a_2\dot{\theta}_1\ sin\ sin\ \theta_2 \quad (6.19)$$

$$\frac{\partial L}{\partial \theta_2} = -m_2 a_1 a_2\left(\dot{\theta}_1^2+\dot{\theta}_1\dot{\theta}_2\right)\ sin\ sin\ \theta_2\ - m_2 g a_2\ cos\ cos\ (\theta_1\ + \theta_2) \quad (6.20)$$

The two linked nonlinear differential equations provide the arm dynamics according to Lagrange's equation.

$$\tau_1 = [(m_1+m_2)a_1^2 + m_2 a_2^2 + 2m_2 a_1 a_2\cos\theta_2]\ddot{\theta}_1 + [m_2 a_2^2 + m_2 a_1 a_2\cos\theta_2]\ddot{\theta}_2 -$$

$$m_2 a_1 a_2\left(2\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_2^2\right)\sin\theta_2 + (m_1+m_2)g a_1\cos\theta_1 + m_2 g a_2\cos(\theta_1\ +\theta_2) \quad (6.21)$$

$$\tau_2 = [m_2 a_2^2 + m_2 a_1 a_2\cos\theta_2]\ddot{\theta}_1 + m_2 a_2^2\ddot{\theta}_2 + m_2 a_1 a_2\dot{\theta}_1^2\sin\theta_2 + m_2 g a_2\cos(\theta_1\ +\theta_2)$$

$$(6.22)$$

The manipulator dynamics in vector form are:

$$\begin{bmatrix}(m_1+m_2)a_1^2 + m_2 a_2^2 + 2m_2 a_1 a_2\cos\theta_2 & m_2 a_2^2 + m_2 a_1 a_2\cos\theta_2 \\ m_2 a_2^2 + m_2 a_1 a_2\cos\theta_2 & m_2 a_2^2\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} +$$

$$\begin{bmatrix}-m_2 a_1 a_2\left(2\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_2^2\right)\sin\theta_2 \\ m_2 a_1 a_2\dot{\theta}_1^2\sin\theta_2\end{bmatrix} + \begin{bmatrix}(m_1+m_2)g a_1\cos\theta_1 \\ m_2 g a_2\cos(\theta_1\ +\theta_2)\end{bmatrix}\ = \begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} \quad (6.23)$$

The dynamics of two link manipulator is given as:

$$M(\theta)\dot{\theta} + V\left(\theta,\dot{\theta}\right) + G(\theta) = \tau \quad (6.24)$$

Where the symmetric inertia matrix

$$M(\theta) = \begin{bmatrix}\alpha+\beta+2\mu\cos\theta_2 & \beta+\mu\cos\theta_2 \\ \beta+\mu\cos\theta_2 & \beta\end{bmatrix} \quad (6.25)$$

And nonlinear terms

$$N\left(\theta,\dot{\theta}\right) = V\left(\theta,\dot{\theta}\right) + G(\theta) \quad (6.26)$$

$$V\left(\theta,\dot{\theta}\right) = \begin{bmatrix}\mu(2\theta_1\dot{\dot{\theta}}_2+\dot{\theta}_2^2) \\ \mu\dot{\theta}_1^2\sin\theta_2\end{bmatrix} \quad (6.27)$$

$$G(\theta) = \begin{bmatrix}\alpha e_1\cos\theta_1 + \mu e_1\cos(\theta_1+\theta_2) \\ \mu e_1\cos(\theta_1+\theta_2)\end{bmatrix} \quad (6.28)$$

$$\alpha = (m_1+m_2)a_1^2; \beta = m_2 a_2^2; \mu = m_2 a_1 a_2; e_1 = \frac{g}{a_1} \quad (6.29)$$

Finding an appropriate state space transformation will allow the general state to be converted to Brunovsky form.

Defining the state vector as

$$x = [x_1\ x_2] = [x_{11}\ \ x_{12}\ x_{21}\ \ x_{22}] = [\theta\ \dot{\theta}] = [\theta_1\ \theta_2\ \dot{\theta}_1\ \ \dot{\theta}_2] \tag{6.30}$$

we get the following state equations:

$$\dot{x}_1 = \dot{\theta} = x_2$$

$$\dot{x}_2 = \ddot{\theta} = -M^{-1}(\theta)\left[V\left(\theta, \dot{\theta}\right) + G(\theta)\right] + M^{-1}(\theta)\tau = -M^{-1}(x_1)[V(x_1, x_2) + G(x_1)] + M^{-1}(x_1)\tau = f(x) + g(x)\tau \tag{6.31}$$

Where, f(x)= $-M^{-1}(x_1)[V(x_1, x_2) + G(x_1)];\ g(x) = M^{-1}(x_1)$

The control law is given as,

$$\tau = g^{-1}[-f(x) + u] \tag{6.32}$$

The linearized state equation is given as,

$$\begin{bmatrix} \dot{x_{11}} \\ \dot{x_{12}} \\ \dot{x_{21}} \\ \dot{x_{22}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{6.33}$$

Equation (6.33) may be also expressed as,

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u \tag{6.34}$$

6.2.2 Mathematical Modelling of Inverted Pendulum

In Chapter 4, the inverted pendulum mathematical model was already covered.

6.3 RBF Neural Network

Neural network techniques have advanced significantly since the 1940s, when the idea of networks composed of fundamental neuronal models was first put forth, and have found successful applications in a variety of fields, including modelling, signal processing, learning, pattern recognition, and system control. The employment of neural networks in the detection and control of nonlinear systems is greatly encouraged by their key advantages, which include learning capability, nonlinear function approximation, fault tolerance, and well implemented analogue VLSI for real-time applications [342]. Numerous factors, including the following important ones, have sparked intense research interest in the neural networks as alternatives to conventional control approaches.

- Any function may be taught to neural networks. As a result, many classic adaptive and optimal control approaches do not utilise the intricate and challenging mathematical analyses that neural networks are capable of.

- The incorporation of activation function in multilayered neural networks hidden neurons provides nonlinear mapping capabilities for tackling extremely nonlinear control issues for which conventional control techniques have not yet produced a workable solution.
- Traditional adaptive and optimum control strategies require extensive a priori knowledge about the plant to be managed, such as mathematical modelling, before they can be put into practise. Such extensive knowledge is not necessary for neural controllers because of neural networks capability for self-learning. Therefore, it appears that neural controllers may be used in a larger range of uncertain situations.
- The remarkable parallel computing of neural networks, when used in conjunction with neural chips or parallel hardware, provides extremely quick multiprocessing.

The creation of reliable adaptive neural network control methods has engaged numerous research teams. Many research studies [343] – [347] have shown that neural networks are capable of approximation, and several books [348] – [354] propose numerous adaptive neural network controllers based on approximation skills. Radial basis function (RBF) networks are a prominent type of artificial neural network for problems involving function approximation. Radial basis function networks have a universal approximation, unlike other neural networks creating a faster rate of studying. A feed-forward neural network, also referred to as an RBF network, has three layers: output, hidden, and input.

A single hidden layer of linearly independent functions makes up the RBF feed-forward connection topology $f_m: R^N \to R$ on which an m-dimensional function space is built [355]. Several nonlinear mappings i.e., $f_m \in span\{\emptyset_1, \emptyset_2, \cdots \emptyset_m\}$ RBF network architecture as shown in figure 6.2. The linear regression equation below yields its transfer function:

$$f_m(x) = \sum_{k=1}^{m} w_k \emptyset_k(x) + w_0 \tag{6.35}$$

Where $\emptyset_k(x) = \emptyset\left(\frac{|x-c_k|}{s_k}\right)$ are the basis functions being translated to increase the size of a radially-symmetric prototype function $\emptyset: R^N \to R$ and $w_k, k = 0,1,2, \dots, m$ are the adjustable weight coefficients of linear regression.

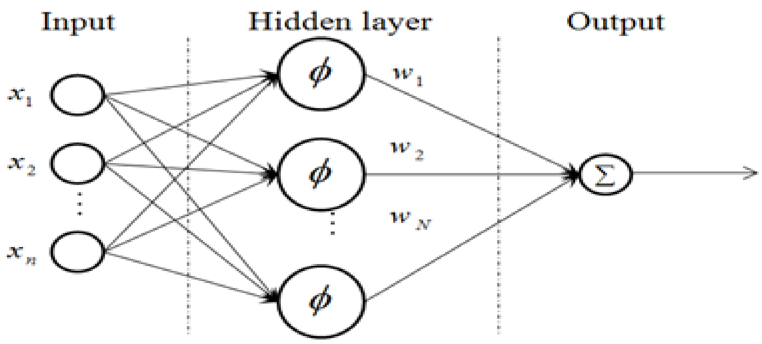


Figure 6.2 RBF neural network architecture

Once the computed error has achieved the desired values (i.e. 0.01) or the required number of training iterations (i.e. 500), the RBF model's training is finished. It is decided to use an RBF network with 10 nodes or more in the hidden layer. In computing units, the transfer function is a Gaussian function. Depending on the situation, it is frequently seen that the RBF network took less time to complete training.

An appropriate radial basis function relies on the system and area of interest [356], and a variety of functions, including the Gaussian function, are provided.
$\emptyset(r) = exp\ (-r^2)$, inverse multiquadric $\emptyset(r) = \frac{1}{\sqrt{r^2+1}}$, cubic $\emptyset(r) = r^3$, liner $\emptyset(r) = r$, multiquadric $\emptyset(r) = \sqrt{r^2+1}$, etc. The Gaussian and the remaining radial functions are referred to as non-local, whereas the Gaussian and inverse multiquadric are referred to as localised functions. Localized radial functions are closely related to receptive fields, which are responsive to inputs situated within a restricted range of the input space and are seen in biological neuronal systems. However, compared to localised RBFs, non-local radial functions are more effective in approximating smooth input-output mappings [357]. The idea behind RBF neural networks was to reframe the stringent interpolation condition as the function approximation issue, where the number of radial basis functions is purposefully smaller than the number of training points that are available.
Multivariate interpolation in high-dimensional areas leads to the development of the radial basis function neural network approximation approach [359] and potential functions [358] shown that for a given set of $\{(x_i, y_i) \in R^d \times R\}_{i=1}^{P}$ interpolation knots y_i, i.e.,x_i scalar values given at $f_m(x) = \sum_{i=1}^{m=P} w_i \emptyset(\|x - x_i\|)$ coordinates, the approximation function can be used $f_m(x_i) = y_i, i = 1,2, \dots\dots, m = P$ satisfies the interpolation condition $f(x)$. According to the universal theory of approximation for any continuous input-output mapping function m. It is possible to divide the multi-output RBF network into a number of

independent networks that share the same set of RBFs. RBF networks may therefore be thought of as universal approximators with many inputs and outputs. Three applications of RBF network approximation include identification of nonlinear dynamic systems, control, and time-series prediction.

Models are used to determine the relationships between system inputs u(t) and system responses y(t),

$$y(t+1) = S(y(t), y(t-1), \ldots . y(t-k); u(t+1), u(t), \ldots . u(t-n)) \tag{6.36}$$

Where S(.) is the multivariate function defined as dimensional vector inputs which will be approximated by the network.

6.4 RBF Neural Sliding Mode Control

Due to its reliable control methods, SMC offers the finest answer for any plant uncertainty and external disruption. The most notable characteristic of SMC is that, while in sliding mode, it is fully indifferent to plant uncertainty and outside perturbations [360]. It offers a Switched control law for high speed that allows the plant's state trajectory of non-linear type to be moved onto a chosen sliding or switching surface of state space and maintained there for the duration of the period. The system state is intended to control the state and then confine it to be in close proximity to the switching function. Its two main benefits are the ability to change the system's dynamic behaviour by choosing a switching criterion and the response of closed-loop becoming insensitive to a certain class of uncertainty. SMC is appealing from a design standpoint since it allows for direct performance specification. A sliding mode controller stabilises a system's trajectory.

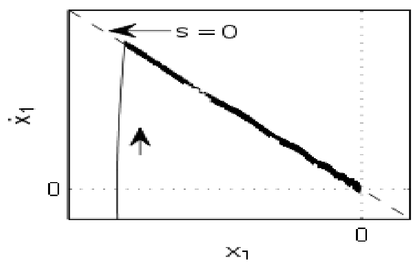


Figure 6.3 Ideal concept of sliding mode

Figure 6.3 illustrates how throughout the line, the mechanism "slides" s=0 once it reaches state trajectory of the plant on the chosen surface. The surface at s=0 was picked because, when limited to it, it exhibits good reduced-order dynamics. The SMC design has two stages. The first stage in getting the plant that is restricted to it to behave creating a surface of sliding type as intended. It implies that a supplementary equations' set defining the 'switching surface' must be satisfied by the dynamics of the state variables of the plant. The second stage involves making the switching feedback gains necessary to orient the plant's state trajectory towards the surface of sliding type. These inventions are based on the generalised Lyapunov stability theory. One issue in SMC design is ensuring that the system state converges to the sliding surface. Another challenge is assuring control target achievement on the sliding surface (i.e., providing robustness). As shown in figure 6.3, the artificial intelligence integration (AI) into sliding mode control (SMC) represents a milestone for the more than 50-year-old sliding mode theory. For increased responsiveness, various sliding surface configurations have been suggested for sliding mode control. The various integration strategies are looked at. Technical descriptions of composite SMC and computational intelligence controllers are available.[361]. Also, emphasis has been made on the particular advantages and disadvantages of integrating AI methods into SMC. SMC and AI were both incorporated into sliding mode controllers for a number of reasons, including enhancing the performance of the controller by merging the two, enhancing the primary AI controllers by utilising SMC's advantages, and reducing chattering. In a few studies, SMC, AI, and adaptive control methodologies have all been combined. Adaptive control systems' independence from precise prior knowledge of dynamic characteristics is one of its key advantages. Their capacity to efficiently suppress mistakes induced by uncertainties of parameters, and their capability to partially neutralise the effects of SMC high-frequency switching.

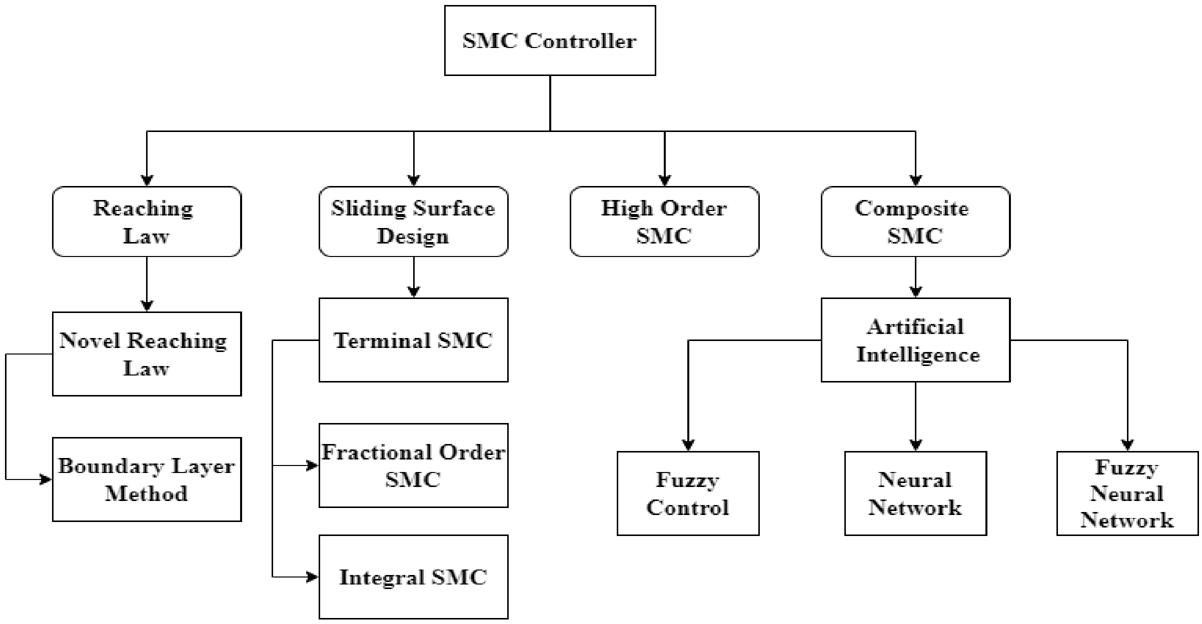


Figure 6.4 Performance improvement techniques of SMC

For the approximate solution of non-linear functions, neural networks are employed. Any function of practical importance can have its input-output behaviour approximated by a neural network with sufficient neurons in at least one hidden layer and acceptable starting weight values. Given that they execute and learn the mapping from input data to output data, neural networks may be thought of as instruments for pattern recognition. The primary drawback of using neural networks in control applications is the traditional back propagation technique, which only allows for a delayed and ineffective learning process.

Learning, parallelism, and fault tolerance are characteristics of neural networks [169]. In several situations, NN-based SMC has been employed as the controller [170]–[174]. Its notable traits include rapid convergence, great accuracy, and a small network size [176]. To address the problem of uncertainty bound in SMC design, position control of nonlinear, uncertain system dynamics was anticipated using a NN bound observer. For a variety of operating situations, this composite NN-SMC controller enabled reliable position control.

6.4.1 RBF Based Neural Sliding Mode Control for Inverted Pendulum

Let the state space representation of an inverted pendulum is given as –

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{g sinsin\, x_1 - mlx_2^2 coscos\, x_1\, sinsin\, x_1 /(m_c+m)}{l(4/3 - mcos^2 x_1/(m_c+m))} + \frac{coscos\, x_1 /(m_c+m)}{l(4/3 - mcos^2 x_1/(m_c+m))} u + d(t) \end{aligned}\right\} \quad (6.37)$$

Where $f(x_1, x_2) = \frac{g sinsin\, x_1 - mlx_2^2 coscos\, x_1\, sinsin\, x_1 /(m_c+m)}{l(4/3 - mcos^2 x_1/(m_c+m))}, g(x_1, x_2) = \frac{coscos\, x_1 /(m_c+m)}{l(4/3 - mcos^2 x_1/(m_c+m))}$ is an unknown function of any nonlinear plant, x_1, x_2 are the angle and speed of inverted pendulum respectively, $d(t)$ is the disturbance $|d(t)| \le D$.

The control problem's objective is to obtain the nonlinear system to track the target position at an angle defined by x_{1d}, the tracking inaccuracy is therefore stated as:

$$\left.\begin{aligned} e &= x_1 - x_{1d} \\ \dot{e} &= x_2 - \dot{x}_{1d} \end{aligned}\right\} \quad (6.38)$$

The sliding mode function is defined as,

$$s = ce + \dot{e}, c > 0 \quad (6.39)$$

The above sliding function may be represented in term of state equation,

$$\dot{s} = c\dot{e} + \ddot{e} = c\dot{e} + x_2 - \ddot{x}_{1d} = c\dot{e} + f(x_1, x_2) + g(x_1, x_2)u + d(t) - \ddot{x}_{1d} \quad (6.40)$$

From equation (6.39), we can say that, if $s \to 0$, then $e \to 0$ and $\dot{e} \to 0$.

The algorithm of neural RBF network is given as:

$h_{ij} = exp\ exp\left(-\frac{\|x_i - c_{ij}\|^2}{b_{ij}^2}\right), j = 1, \ldots, m$ (6.41)

$f(x_1, x_2) = w_i^T h_i + \epsilon_i$ (6.42)

Where $x_i = [e_i\ \dot{e}_i\ q_{di}\ \dot{q}_{di}\ \ddot{q}_{di}\ ]$ are input of RBF, $h_i = [h_{i1} h_{i2} \ldots . h_{im}]^T$ is a gaussian output, ϵ_i is the neural network's approximation error, $\epsilon \leq \epsilon_N$ and w_i is ideal neural network weight value, j is the hidden layer nodes in the network.

The controller's sliding mode design was created in such a way that,

$u = \frac{1}{g(x,t)}\left[-\frac{1}{2}s\widehat{\emptyset}h^T h + \ddot{x}_{1d} - c\dot{e} - \eta sgn(s) + d - \mu s\right]$ (6.43)

The Lyapunov function is defined as,

$L = \frac{1}{2}s^2 + \frac{1}{2\gamma}\tilde{\phi}^2$ (6.44)

Where $\gamma > 0$.

From equation 6.9 and 6.10, the Lyapunov function may be written as,

$$\dot{L} = s\dot{s} + \frac{1}{\gamma}\tilde{\phi}\dot{\hat{\phi}} = s\left(W^T h + \epsilon - \frac{1}{2}s\widehat{\emptyset}h^T h - \eta sgn(s) + d - \mu s\right) + \frac{1}{\gamma}\tilde{\phi}\dot{\hat{\phi}}$$

$$\leq \frac{1}{2}s^2\emptyset h^T h + \frac{1}{2} - \frac{1}{2}s\widehat{\emptyset}h^T h + (\epsilon + d)s - \eta + \frac{1}{\gamma}\tilde{\phi}\dot{\hat{\phi}} - \mu s^2$$

$$= -\frac{1}{2}s^2\widetilde{\emptyset}h^T h + \frac{1}{2} + (\epsilon + d)s - \eta|s| + \frac{1}{\gamma}\tilde{\phi}\dot{\hat{\phi}} - \mu s^2$$

$$= \widetilde{\emptyset}(-\frac{1}{2}s^2 h^T h \frac{1}{\gamma}\dot{\hat{\phi}}) + \frac{1}{2} + (\epsilon + d)s - \eta|s| - \mu s^2$$

$$\leq \widetilde{\emptyset}(-\frac{1}{2}s^2 h^T h \frac{1}{\gamma}\dot{\hat{\phi}}) + \frac{1}{2} - \mu s^2 \quad (6.45)$$

The adaptive law of RBF function is derived as:

$\dot{\widehat{\emptyset}} = \frac{\gamma}{2}s^2 h^T h - k\gamma\widehat{\emptyset}$, where $k > 0$.

Then,

$\dot{L} \leq -k\widetilde{\emptyset}\widehat{\emptyset} + \frac{1}{2} - \mu s^2 \leq -\frac{k}{2}\left(\widetilde{\emptyset}^2 - \emptyset^2\right) + \frac{1}{2} - \mu s^2 = -\frac{k}{2}\widetilde{\emptyset}^2 - \mu s^2 + (\frac{k}{2}\emptyset^2 + \frac{1}{2})$ (6.46)

Let us define, $k = \frac{2\mu}{\gamma}$, then equation (6.46) may be written as:

$\dot{L} \leq -\frac{\mu}{\gamma}\widetilde{\emptyset}^2 - \mu s^2 + + \left(\frac{k}{2}\emptyset^2 + \frac{1}{2}\right) = -2\mu\left(\frac{1}{2\gamma}\widetilde{\emptyset}^2 + \frac{1}{2}s^2\right) + \left(\frac{k}{2}\emptyset^2 + \frac{1}{2}\right) = -2\mu L + Q$ (6.47)

Where $Q = \left(\frac{k}{2}\emptyset^2 + \frac{1}{2}\right)$

As per the Lemma: Let $f, V: [0, \infty] \in R$, then $\dot{V} \leq -\alpha V + f, \forall t \geq t_0 \geq \tau$

Implies that $V(t) \leq e^{\alpha(t-t_0)}V(t_0) + \int_{t_0}^{t} e^{-\alpha(t-\tau)}f(\tau)d\tau$ For any finite constant α. The solution of equation 6.46is given as:

$L \leq \frac{Q}{2\mu} + \left(L(0) - \frac{Q}{2\mu}\right)e^{-2\mu t}$, (6.48)

Then, $\lim_{t\to\infty} L = \frac{Q}{2\mu} = \frac{\frac{K}{2}\emptyset^2+\frac{1}{2}}{2\mu} = \frac{k\emptyset^2+1}{4\mu} = \frac{\frac{2\mu}{\gamma}\emptyset^2+1}{4\mu} = \frac{\emptyset^2}{2\gamma} + \frac{1}{4\mu}$ (6.49)

It is obvious from equation (6.49) that γ and μ have an impact on the convergence precision.

6.3.2 RBF Based Neural Sliding Mode Control for Manipulators

Let the dynamic equation of two-link manipulator is given as:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \tau_d = \tau \tag{6.50}$$

$D(q)$ (Inertia Matrix) = $\begin{bmatrix} p_1 + p_2 + 2p_3\cos q_2 & p_2 + p_3\cos q_2 \\ p_2 + p_3\cos q_2 & p_2 \end{bmatrix}$

$C(q,\dot{q})$ (Centripetal Matrix) = $\begin{bmatrix} -p_3\dot{q}_2\sin q_2 & -p_3(\dot{q}_1 + \dot{q}_2)\sin q_2 \\ p_3\dot{q}_1\sin q_2 & 0 \end{bmatrix}$

$G(q)$ (Gravity Vector) = $\begin{bmatrix} p_4 g\cos q_1 + p_5 g\cos(q_1 + q_2) \\ p_5 g\cos(q_1 + q_2) \end{bmatrix}$

$F(\dot{q})$ (Friction force) = $0.2sgn(\dot{q})$, τ_d (Unknown Disturbance) = $[0.2\sin(t) \quad 0.2\sin(t)]^T$ and τ is the control input.

The control problem is to get the system to track the desired position of nonlinear system, the tracking error is defined as:

$$e(t) = q_d(t) - q(t) \tag{6.51}$$

To get the desired position the control goals are $e(t) \to 0$, and $\dot{e}(t) \to 0$ as $t \to \infty$.

The sliding mode function is defined as: $s = \dot{e} + ce$

Where $c = c^T > 0$, then $\dot{q} = -r + \dot{q}_d + ce$

$$D\dot{s} = D(\ddot{q}_d - \ddot{q} + c\dot{e}) = D(\ddot{q}_d + c\dot{e}) - D\ddot{q}$$

$$= D(\ddot{q}_d + c\dot{e}) + C\dot{q} + G + F + \tau_d - \tau$$

$$= D(\ddot{q}_d + c\dot{e}) - C_r + C(\dot{q}_d + ce) + G + F + \tau_d - \tau$$

$$= -C_r - \tau + f + \tau_d \tag{6.52}$$

Where $f = D(\ddot{q}_d + c\dot{e}) + C(\dot{q}_d + ce) + G + F$

Since f is unknown function, because of this, the RBF neural network is utilised with minimal parameter learning to approximate function.

Let the $\widehat{w}_i$ as an estimation of w_i, then the weight function is approximated as

$$\widetilde{w}_i = w_i - \widehat{w}_i,\ \|w_i\|_F \le w_{imax} \tag{6.53}$$

Let the minimum parameter function is defined as $\emptyset = max_{1\le i\le n}\{\|w_i\|^2\}$, where $\emptyset$ is a positive constant, and $\widehat{\emptyset}$ is an estimation of $\emptyset, \widetilde{\emptyset} = \widehat{\emptyset} - \emptyset$.

Define $W = \begin{bmatrix} w_1 \\ \vdots \\ w_2 \end{bmatrix}, H = \begin{bmatrix} h_1 \\ \vdots \\ h_2 \end{bmatrix}, \widetilde{W} = W - \widehat{W}$, according to GL operator, we define:

$WoH = \begin{bmatrix} w_1^T h_1 \\ \vdots \\ w_n^T h_n \end{bmatrix}, sos = \begin{bmatrix} s_1^T s_1 \\ \vdots \\ s_n^T s_n \end{bmatrix}$ and $HoH = \begin{bmatrix} h_1^T h_1 \\ \vdots \\ h_n^T h_n \end{bmatrix}$, then f can be expressed as

$$f = WoH + \epsilon \tag{6.54}$$

The control law is designed as: $\tau = \frac{1}{2}\widehat{\emptyset} ro(HoH) + K_v s - v$ (6.55)

Where v is the robust term to overcome approximation error ϵ.

The robust term v is designed as $v = -(\epsilon_N + b_d)sgn(s)$ (6.56)

Where b_d is constant and $\|\tau_d\| \le b_d$

From equation (6.52) and (6.56), we get:

$$D\dot{s} = -(K_v + C)s - \frac{1}{2}\widehat{\emptyset} so(HoH) + (f + \tau_d) + v \tag{6.57}$$

Let the Lyapunov function is defined as

$L = \frac{1}{2}s^T Ds + \frac{1}{2\gamma}\widetilde{\emptyset}^2$, where $\gamma > 0$.

Then, $\dot{L} = s^T D\dot{s} + \frac{1}{2}s^T \dot{D} + \frac{1}{\gamma}\widetilde{\emptyset}\dot{\widehat{\emptyset}}$

$$= s^T \left[-(K_v + C)s - \frac{1}{2}\widehat{\emptyset} so(HoH) + (f + \tau_d) + v\right] + \frac{1}{2}s^T \dot{D} + \frac{1}{\gamma}\widetilde{\emptyset}\dot{\widehat{\emptyset}}$$

$$= s^T \left[-K_v s - \frac{1}{2}\widehat{\emptyset} so(HoH) + WoH + (v + \epsilon + \tau_d)\right] + \frac{1}{2}s^T\left(\dot{D} - 2C\right)s + \frac{1}{\gamma}\widetilde{\emptyset}\dot{\widehat{\emptyset}}$$

$$= s^T \left[-\frac{1}{2}\widehat{\emptyset} so(HoH) + WoH +\right] - s^T K_v s + s^T(v + \epsilon + \tau_d) + \frac{1}{\gamma}\widetilde{\emptyset}\dot{\widehat{\emptyset}} \tag{6.56}$$

Since $s^T\left(\dot{D} - 2C\right)s = 0$

$$s^T(v + \epsilon + \tau_d) = s^T(\epsilon + \tau_d - (\epsilon_N + b_d)sgn(s)) \le 0 \tag{6.57}$$

$$s^T WoH = [s_1 \quad \cdots \quad s_n] \begin{bmatrix} w_1^T h_1 \\ \vdots \\ w_n^T h_n \end{bmatrix} = s_1 w_1^T h_1 + \cdots + s_n w_n^T h_n = \sum_{i=1}^n s_i w_i^T h_i \tag{6.58}$$

$s_i^2 \emptyset h_i^T h_i + 1 \ge s_i^2 \|w_i\|^2 \|h_i\|^2 + 1 \ge 2 s_i w_i^T h_i$

$s_i w_i^T h_i \le \frac{1}{2} s_i^2 \emptyset h_i^T h_i + \frac{1}{2} = \frac{1}{2} s_i^2 \emptyset h_i^T h_i + \frac{1}{2}$

$$s^T[WoH] \le \frac{1}{2}\emptyset \sum_{i=1}^n s_i^2 h_i^T h_i + \frac{n}{2}$$

$$s^T\left[-\frac{1}{2}\widehat{\emptyset} so(HoH)\right] = -\frac{1}{2}\widehat{\emptyset}[s_1 \quad \cdots \quad s_n]\left(\begin{bmatrix} s_1 \\ \vdots \\ s_n \end{bmatrix} o \begin{bmatrix} h_1^T h_1 \\ \vdots \\ h_n^T h_n \end{bmatrix}\right) \tag{6.59}$$

$$= -\frac{1}{2}\widehat{\emptyset}[s_1 \quad \cdots \quad s_n] \begin{bmatrix} s_1 h_1^T h_1 \\ \vdots \\ s_n h_n^T h_n \end{bmatrix}$$

$$= -\frac{1}{2}\widehat{\emptyset}(s_1^2\|h_1\|^2 + \cdots + s_n^2\|h_n\|^2) = -\frac{1}{2}\widehat{\emptyset}\sum_{i=1}^{n} s_i^2\|h_i\|^2 \qquad (6.60)$$

Where n denotes number of joints, for second link manipulators, $n = 2$.

Then we have $\dot{L} \leq -\frac{1}{2}\widehat{\emptyset}\sum_{i=1}^{n} s2_i^2\|h_i\|^2 + \frac{1}{2}\emptyset\sum_{i=1}^{n} s_i^2 h_i^T h_i + \frac{n}{2} + \frac{1}{\gamma}\widetilde{\emptyset}\dot{\widehat{\emptyset}} - s^T K_v s$

$$-\frac{1}{2}\widehat{\emptyset}\sum_{i=1}^{n} s_i^2\|h_i\|^2 + \frac{n}{2} - s^T K_v s \qquad (6.61)$$

As per design of adaptive law,

$$\dot{\widehat{\emptyset}} = \frac{\gamma}{2}\sum_{i=1}^{n} s_i^2\|h_i\|^2 \qquad (6.62)$$

Then, $\dot{L} \leq \frac{n}{2} - s^T K_v s$

To guarantee $\dot{L} \leq 0$, one must ensure $\frac{n}{2} \leq s^T K_v s$, then

$$\|s\| \leq \sqrt{\frac{n}{2K_v}} \qquad (6.63)$$

6.5 Simulation Results and Discussion

Control in neural sliding mode performance such as inverted pendulum and manipulator is evaluated via numerical simulation. Simulation is carried out while considering to stabilize the position of nonlinear system.

Simulation Results of Results of Single Link Inverted Pendulum

The parameters for single link inverted pendulum are given as, M = 1.0 kg; m = 0.1 kg; l = 0.5 m, g = 9.8 m/s2. Simulation is carried out while considering the movement of the cart and pendulum both are in one plane. The objective of the controller is to remain the pendulum in an upward position for stable operation.

Simulation Results of Results of Classical Sliding Mode Control of Inverted Pendulum

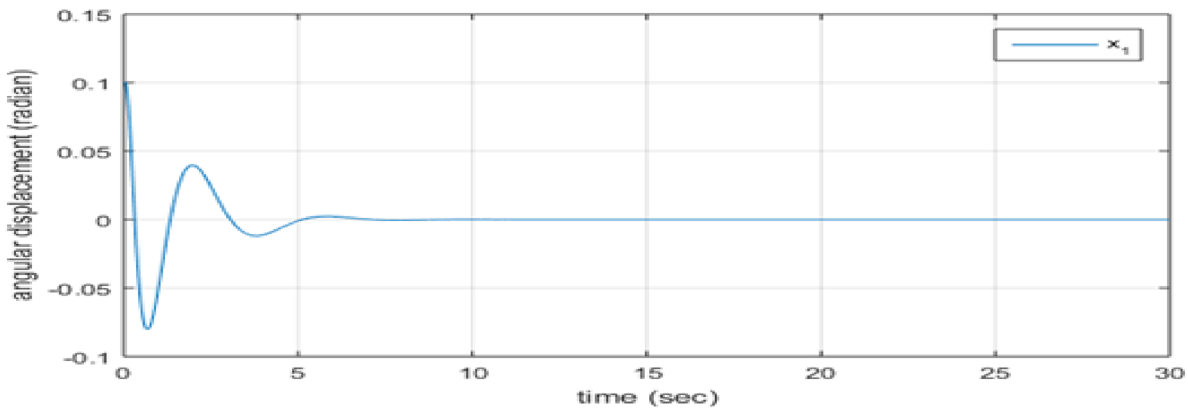


Figure 6.5 Angular displacement of the pendulum system with SMC

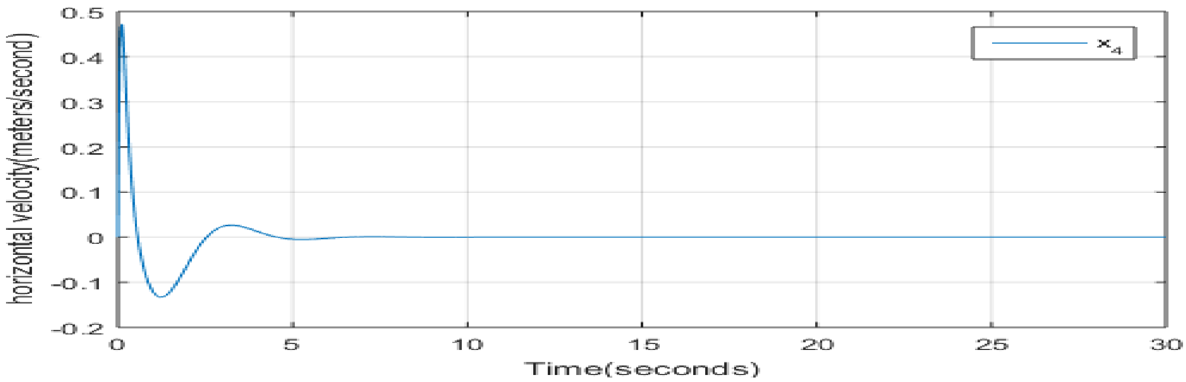


Figure 6.6 Angular velocity plot of the cart with SMC

By observing the pendulum angle and cart velocity, the simulation study of the conventional SMC controller on a single inverted pendulum system is performed.. Figure 6.6 represents the angular displacement of the pendulum system and figure 6.7 gives the cart velocity.

Simulation Results of Results of RBFNN Sliding Mode Control of Sliding Mode Control Inverted Pendulum

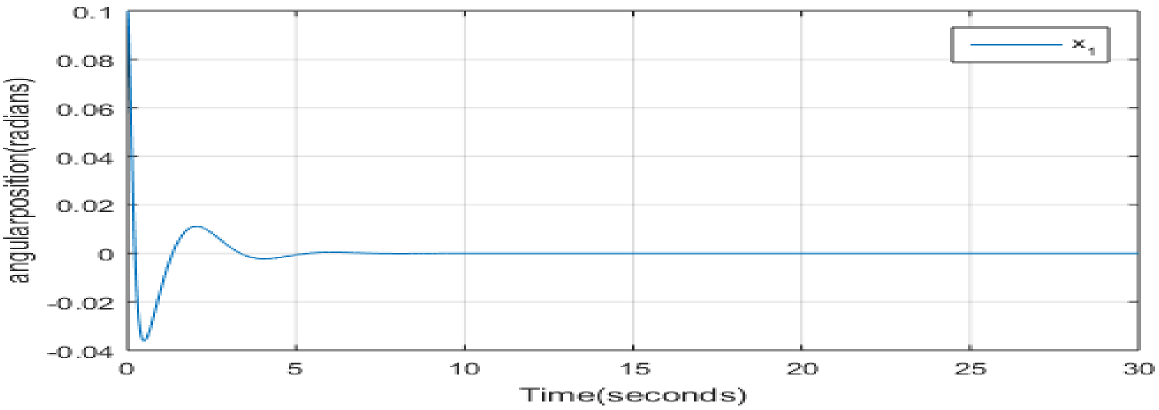


Figure 6.7 Angular displacement of the pendulum system with RBFNN-SMC

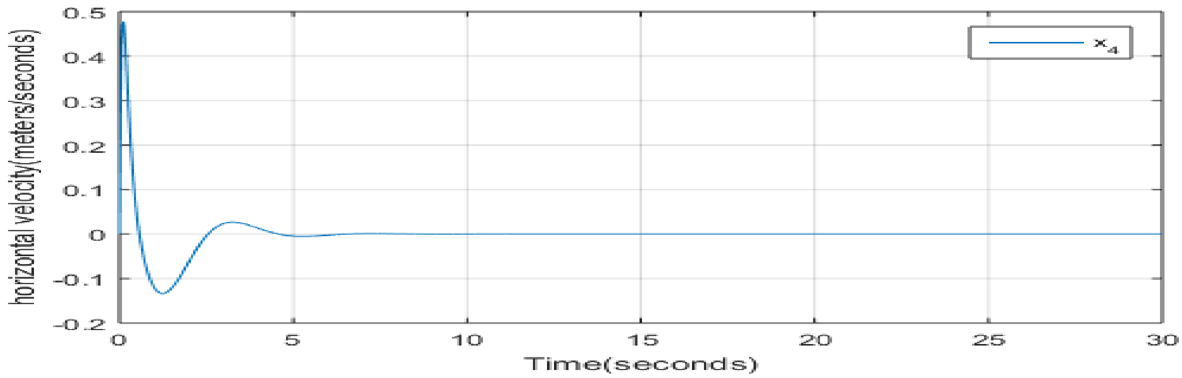


Figure 6.8 Angular velocity of the cart with RBFNN-SMC

Table 6.1 Comparision of single link inverted pendulum for classical SMC and RBFNN SMC

Parameters	Classical SMC		RBFNN SMC	
	Position	**Velocity**	**Position**	**Velocity**
Delay Time (sec)	1.28	0.7	1.37	0.49
Rise Time (sec)	1.9	0.92	1.73	0.65
Peak Time (sec)	2.35	1.07	2.09	0.88
Settling Time (sec)	7.2	5.1	5.46	4.35
Max. Overshoot (%)	3.9	6.4	1.1	5.20

The position of pendulum and its velocity is controlled and stabilized by function of radial type on the basis of neural network-based sliding-mode controller as shown in figure 6.7 and 6.8 respectively. In phrases of rise time, settling time, delay time, peak time, and peak overshoot, table 6.1 compares classical SMC and RBFNN SMC. It is evident from the table above that RBFNN SMC performs better than classical SMC while controlling the position and velocity of a single link inverted pendulum.

Simulation Results of Results of Two Link Robotic Manipulator

Simulation of the two-connect robot arm's controller of sliding mode type (l_1=1m, l_2=1m, m_1=1kg, m_2=1kg, g= 9.8kgm/s^2, $\theta_{d1} = sin\, sin\,(\pi t)\,, \theta_{d2} = cos(\pi t)$) was carried using MATLAB.

Physical Specifications of Two Link Robot Manipulator:

Link 1’s mass; m_1= 1kg, Mass of link 2 m_2= 1kg

Link 1’s length l_1= 1m, Length of link 2 l_2= 1m

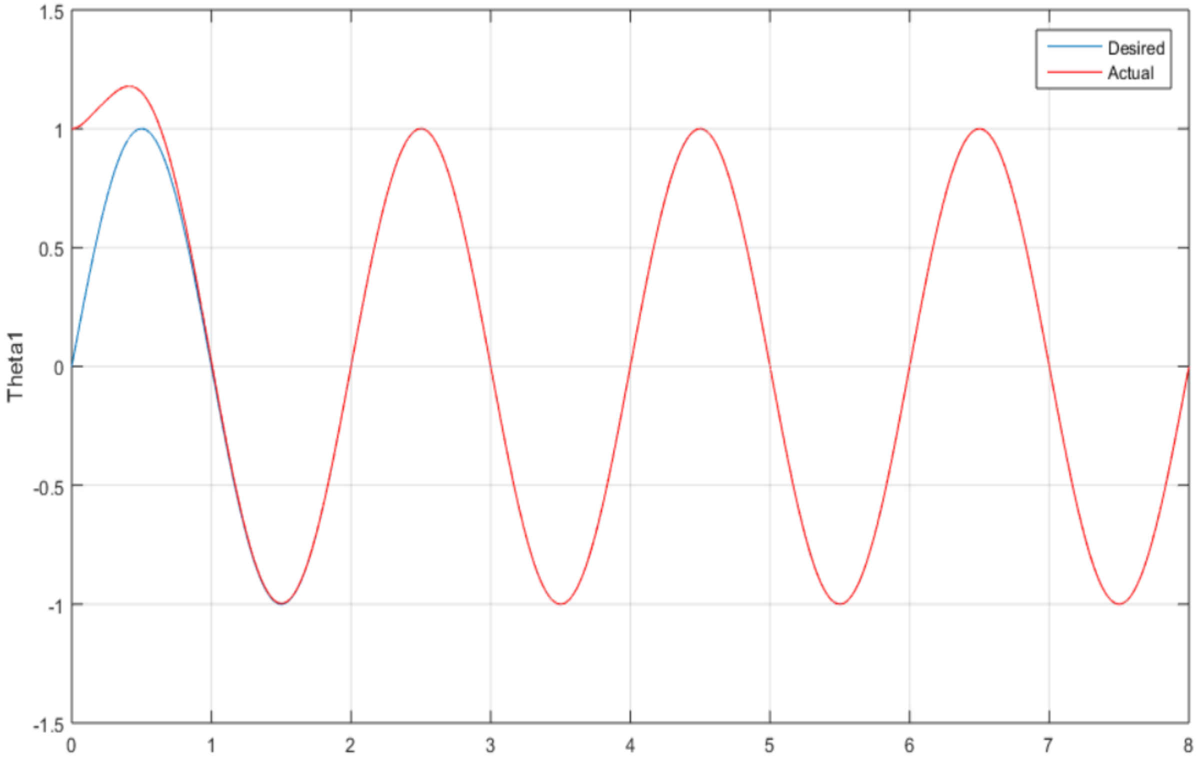


Figure 6.9 Desired and actual trajectory of link 1(θ_1) using SMC

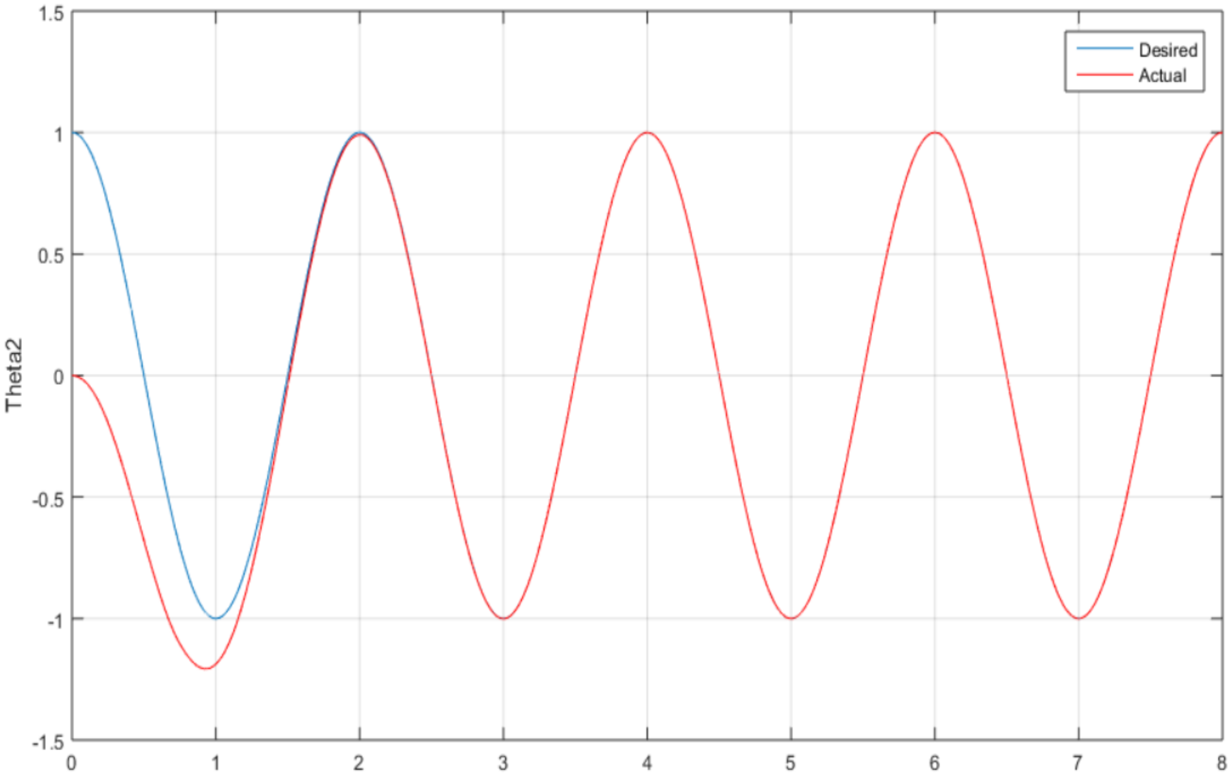


Figure 6.10 Desired and actual trajectory of link 2 (θ_2) using SMC

Simulation Results of Results of RBF NN Sliding Mode Control of Two Link Robotic Manipulator

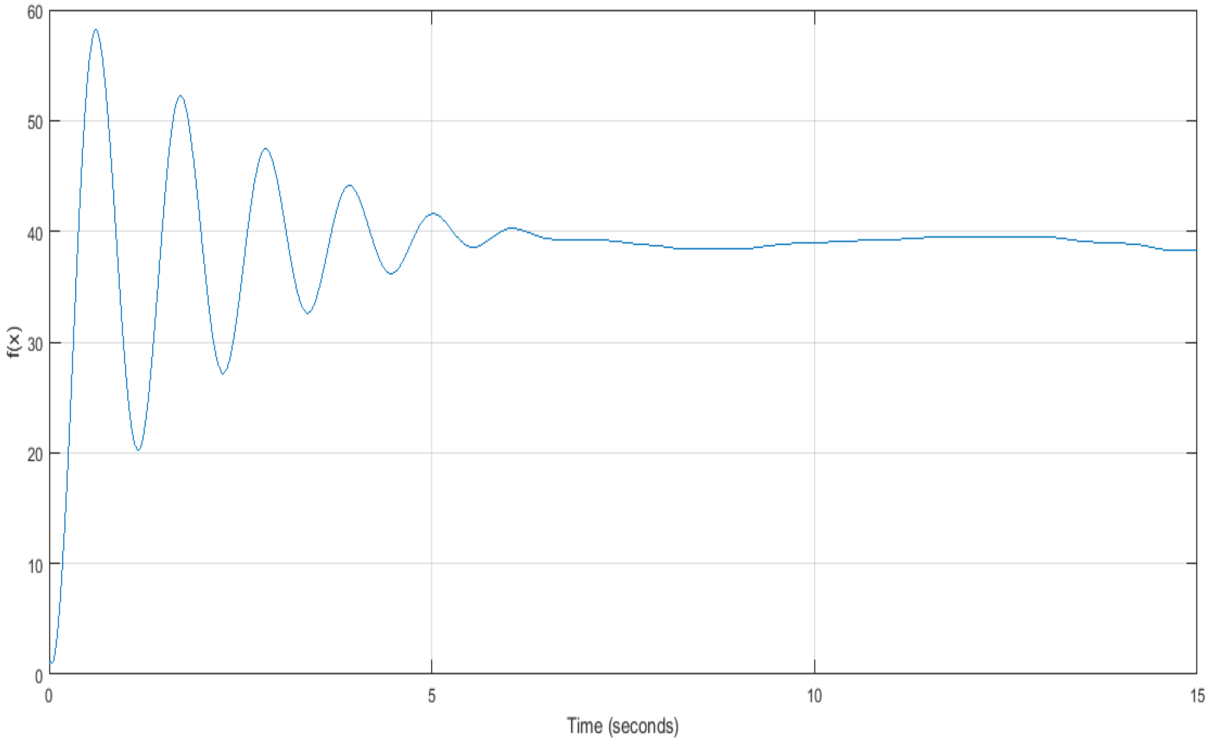


Figure 6.11 Desired and actual trajectory of link 1(θ_1) using RBFNN-SMC

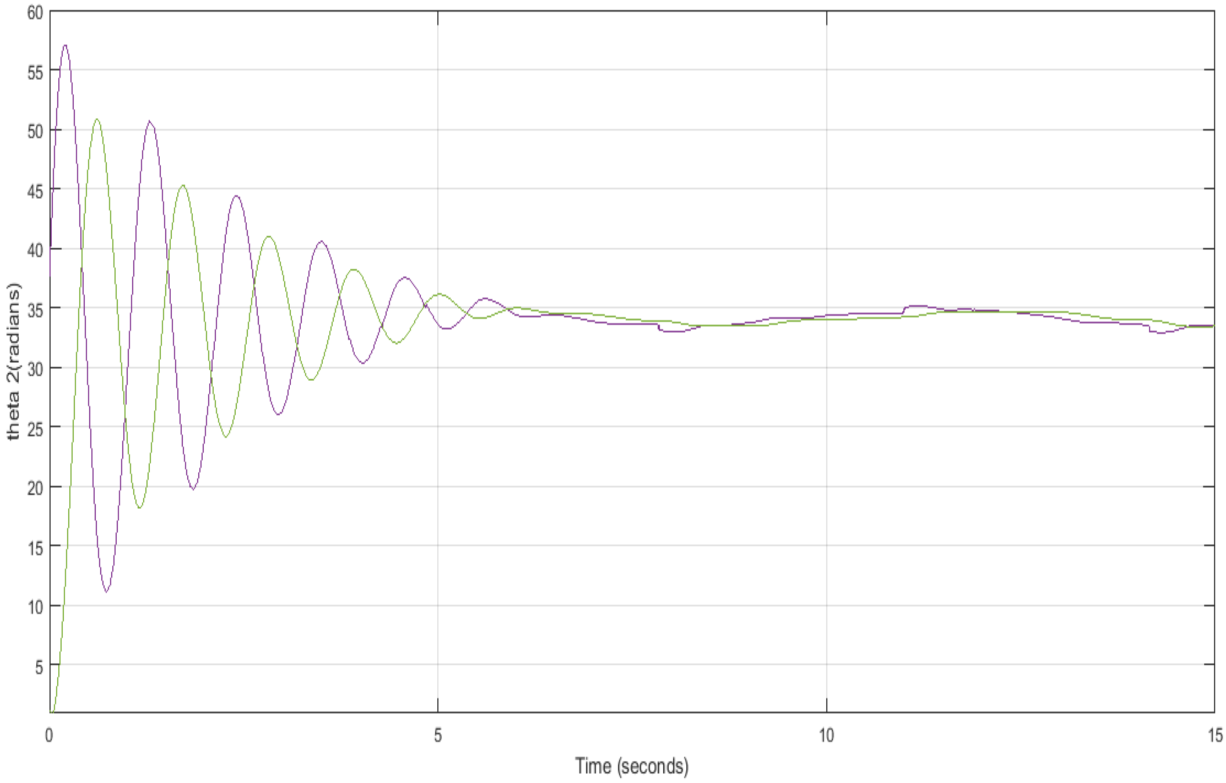


Figure 6.12 Desired and actual trajectory of link 2 (θ_2) using RBFNN-SMC

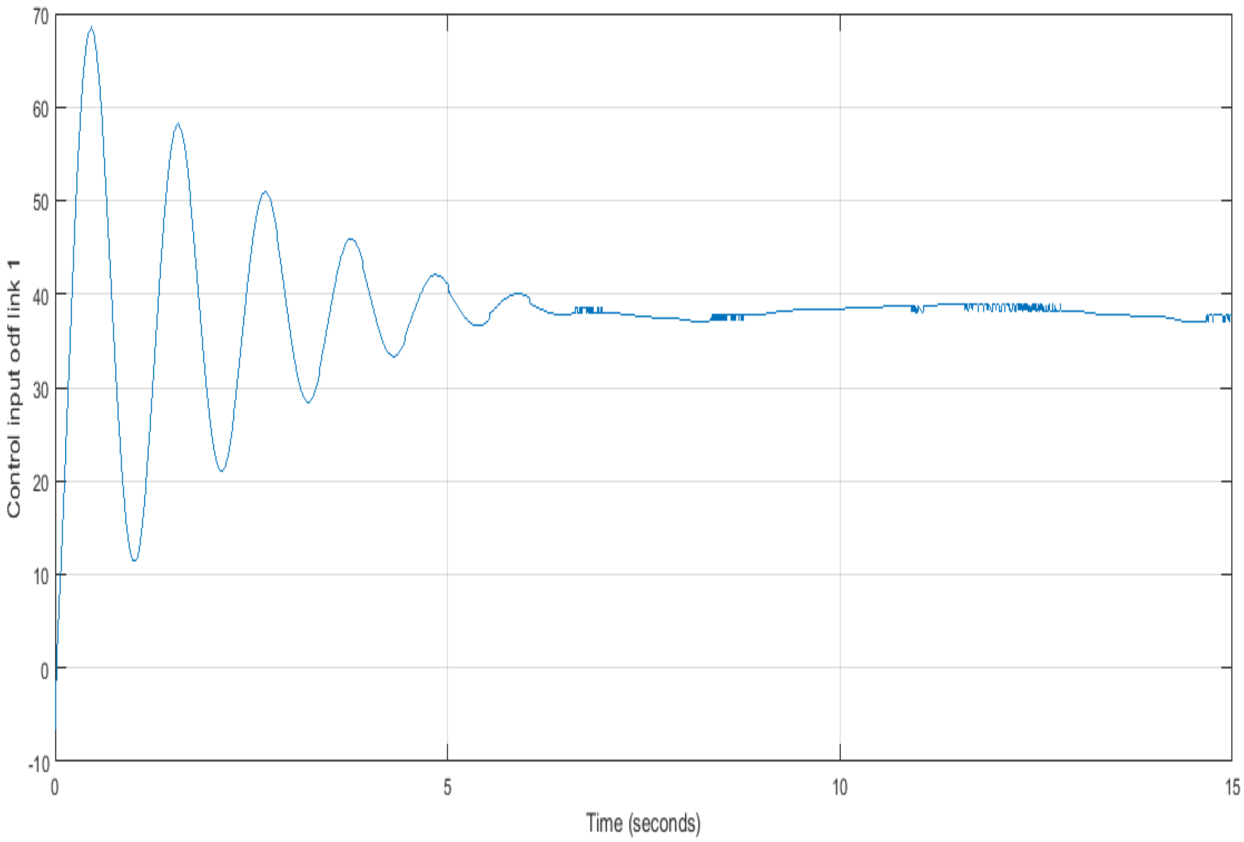


Figure 6.13 Control Input (torque) of θ_1 using RBFNN-SMC

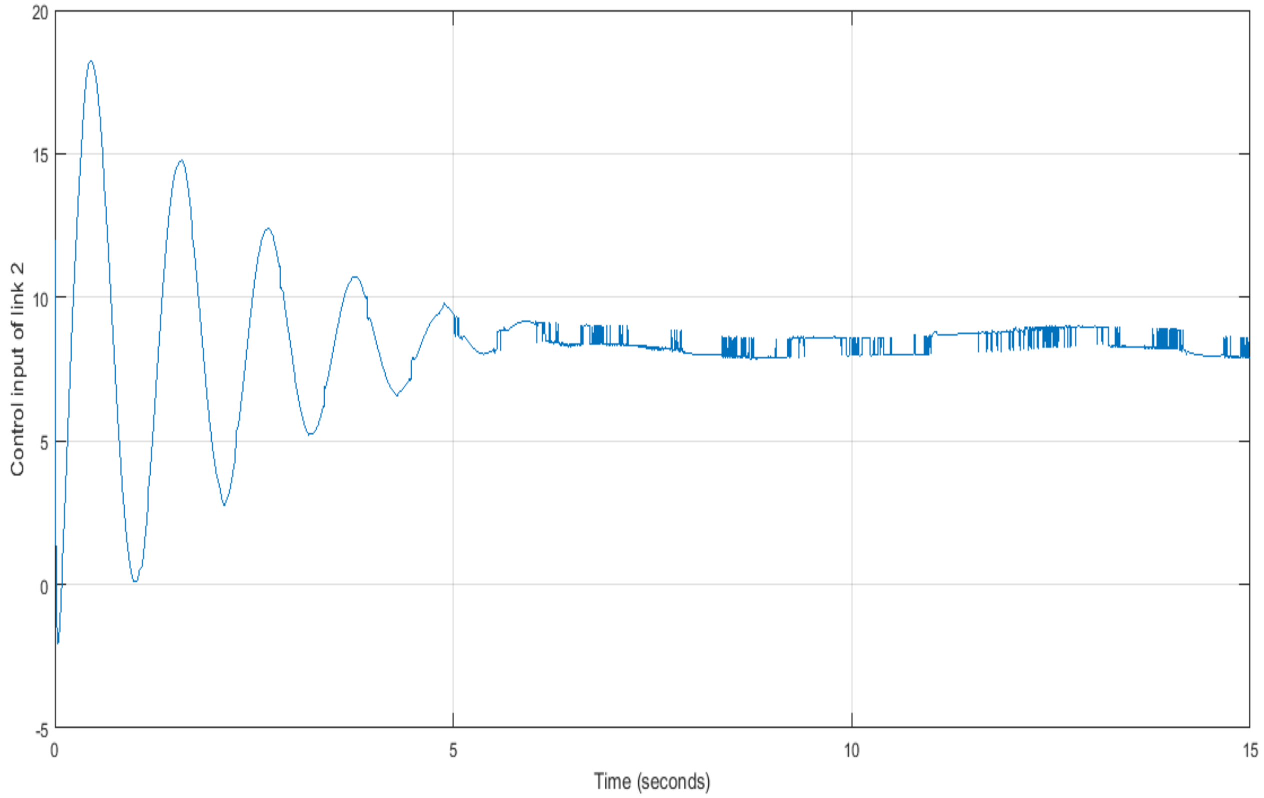


Figure 6.14 Control Input (torque) of θ_2 using RBFNN-SMC

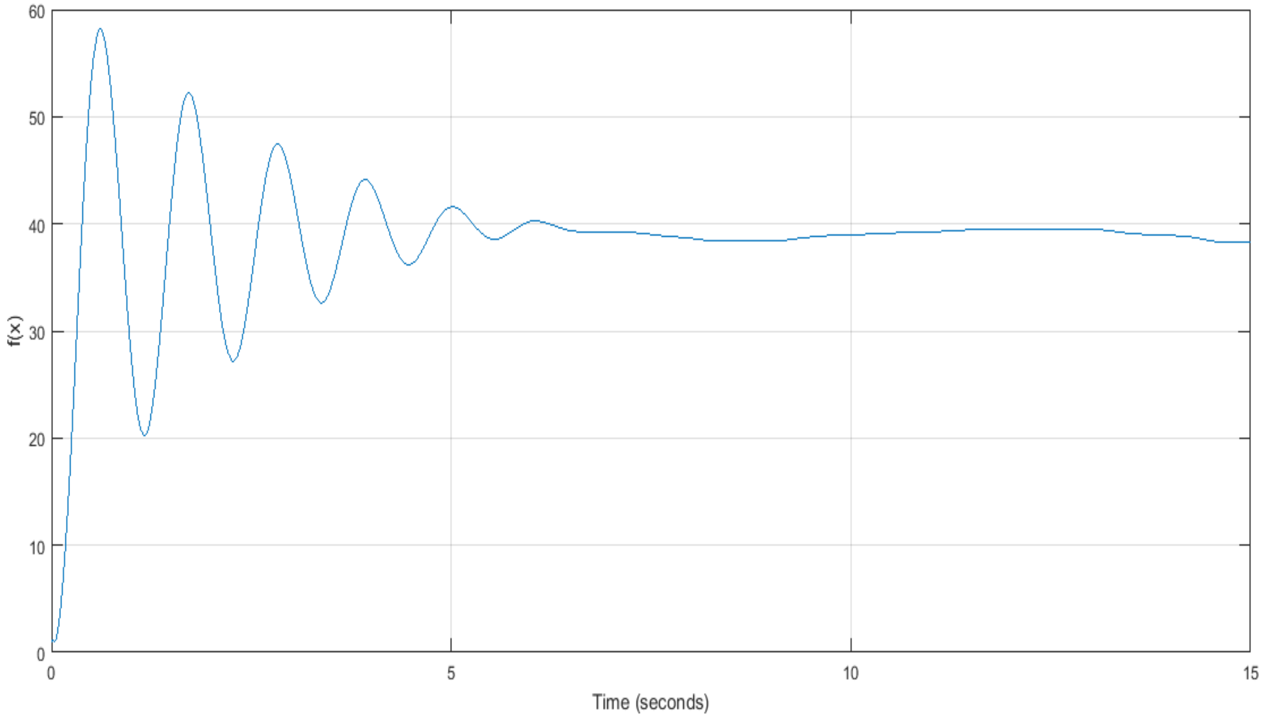


Figure 6.15 ||f(x)|| using RBFNN

Table 6.2 Comparison of two link robotic manipulator for position of link 1 and link 2 for classical SMC and RBFNN SMC

Parameters	**Classical SMC**		**RBFNN SMC**	
	Link 1	**Link 2**	**Link 1**	**Link 2**
Delay Time (sec)	0.02134	0.1969	0.01874	0.1472
Rise Time (sec)	0.1524	0.7317	0.1224	0.6283
Peak Time (sec)	0.4113	0.9213	0.3216	0.8163
Max. Overshoot (%)	17.9	0.9978	14.18	12.44

The simulation results of two link robotic manipulator is carried out by applying classical SMC and RBFNN SMC. Classical sliding mode control technique has been applied first and desired actual trajectory of link 1 and link 2 are shown in figure 6.9 and figure 6.10, respectively. Then RBFNN SMC is applied which gives better result for both link trajectory control and also for control torque of both links as shown in figure 6.11, figure 6.12, figure 6.13 and figure 6.14 respectively. Figure 6.15 gives the result of approximation of RBF approximation function. The performance of both methods are shown in table 6.2 in terms of system delay time, rise time, peak time and maximum overshoot, depict that RBFNN gives better results.

6.6 Conclusion

The objective of the study is to improve the tracking performance of the nonlinear mechanical systems to reduce the nonlinear phenomenon's occurring which drastically effects the performance and stability of these nonlinear system. The chattering effect produced due to the presence of nonlinearities has been reduced to a greater extend by applying the control techniques on these nonlinear systems.

First of all, mathematical model of two DOF robot manipulator model has been designed for the first system in the form of differential equation then a feedback linearization technique has been applied to obtain the plant model for the manipulator. After getting the exact model for the two link robot manipulator, the two control techniques are implemented to study the effect of tracking performance of the robot manipulator system. The sliding mode control technique has been applied first and parameters are evaluated and then RBFNN-SMC scheme is carried out on the model to improve the result of the nonlinear model. Similar system equations have been obtained for the second system, an inverted pendulum system, providing a physical model on which controller techniques may be used. Same two techniques are carried out on this system as well. After studying the effect on both the system, it can easily be deduced that after applying the RBFNN-SMC, tracking performances of the nonlinear system can be improved and it gives the best robust control for the systems which is insensitive to the parameter variations and hence achieve stability effectively.

Chapter 7
Conclusion & Future Scope of Work

7.1 Introduction

Based on observation and simulation findings, a concluding remark on the control approaches are presented in thesis. Modeling the benchmark systems was initially used to identify the issues with the mechanical systems. Also, the difficulties are resolved using a variety of nonlinear control theory methodologies. By addressing the drawbacks caused by uncertainty and failure modes, controller development is primarily focused on attaining velocity, and position control for the nonlinear systems. In subsequent parts, a thorough summary of the work covered in Chapters 1 through 6 is provided.

7.2 Principal Elements of the Work

The thesis can be summed up as follows:

The chapter 1 gives an introduction to nonlinear system, as well as the fundamentals of nonlinear control theory and their governing strategies, and optimization, are covered. The need for advancements in nonlinear control theory is acknowledged, and the necessary goals are established. In addition, the thesis' significant contributions are emphasised, followed by other chapters outline.

The chapter 2 gives the background information of nonlinear systems such as, TORA system, 2DoF ball balancer, single link inverted pendulum, and two link robotic manipulators. A thorough analysis of the literature is also provided, bearing on sliding mode controller and meta-heuristic optimization methods. The complex nonlinear dynamics constrain the broad applications of control principles and limit the capacity to offer a believable solution. To address the issues with underactuated systems, a number of nonlinear control approaches were developed as a result of this. In the past few years, many nonlinear controllers, including Lagrangians, the lambda technique, and back-stepping controllers, have been developed. Throughout the early stages of their development, these controllers encountered some issues, including the lambda method's limits in handling external load. By doing partial feedback linearization for mechanical systems as well as feedback linearization, these problems in the standard nonlinear approaches were resolved. By eliminating the nonlinearities through a feedback control, these techniques converted the nonlinear system into linear. However, when adapting the partial feedback linearization strategy and its approach, the issue with lack of robustness is viewed as a significant disadvantage. The sliding mode approach is viewed as a viable option for achieving robustness while regulating The findings showed how the control techniques worked and highlighted their shortcomings as a result of how external disruptions, parametric uncertainties, and defects affected the operation of the plant. In Chapters 3 to 6, several intelligent, and learning algorithms are created in response to these disadvantages.

Chapter 3 discuss three (PSO, BA, and FPA) optimal strategies used to set the parameters of convention controller such as PD to stabilize the position control of a two-degree-of-freedom ball balancer system. Simulation shows the developed strategy and improve performance significantly within the context of the standard control structure. On the basis of time analysis, the outcomes of the established control approaches are validated. On the ball balancer system, the provided controller has adaptability and good control performance. According to the findings, the FPA optimised technique performs BA and PSO in terms of ISTSE, settling time, peak time, and peak overshoot.

In Chapter 4 quasi-sliding mode, exponential reaching law sliding mode, equivalent sliding mode, and decoupled sliding mode controller improve the performance of two nonlinear systems i.e., single link inverted pendulum and TORA ststem, and also to reduces the chattering. The above study is divided into two parts, in first part comparison of three different sliding mode controller like switching SMC, quasi SMC and equivalent SMC is applied on inverted pendulum to check the system performance in terms of stability, chattering, disturbance rejection and convergence. As per the result obtained quasi sliding mode controller gives best result among all three.

A decoupling technique was looked into in the second section to stabilise underactuated systems. Two test cases were used: the inverted pendulum and the TORA system. On the basis of transient performance, stability, overshoot, and settling response, the control scheme's performance was assessed. The results of the simulations demonstrated that the suggested sliding mode controller could provide a reliable performance in a variety of operations and disturbances. With increment type control input to the system, the system's stability, accuracy, and transient performance metrics like overshoots and response time increased.

In Chapter 5 three types of fuzzy sliding mode controller named as approximation based method, equivalent control based method, and switching gain based method have been implemented to stabilize the tracking issue of inverted pendulum system. The equivalent control based sliding mode controller with fuzzy shows effective results in terms of system uncertainties and disturbances. A control strategy of fuzzy sliding mode involves a chattering free and stabilized nature for an inverted pendulum.

In Chapter 6, a mathematical model of a two-DOF robot manipulator has been developed for the first system in the form of a differential equation. The plant model for the manipulator has then been obtained by using a feedback linearization technique. Using a single-link inverted pendulum and a two-link robotic manipulator, a radial basis function neural sliding control is used, and the performance characteristics are described its performance indices. The results of both classical SMC and RBFNSMC have been compared, and it has been determined that RBF neural sliding mode control produces better system stabilisation and chattering phenomenon removal results.

7.3 Suggestions for further work

With various research recommendations for future investigations, this study may be expanded and improved.

- For the evaluation of highly computational systems, novel metaheuristic algorithms, such as bio-simulated and nature-inspired algorithms, can be used with intelligent controllers. There is still a lot of possibility for exploration because this field has not yet been properly examined.
- The sliding mode controller allows some work to be done in terms of developing Observers to lessen the chattering features. Many complex system control issues can be resolved with the aid of higher-order sliding mode control and higher-order differentiators.

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Publication Details

The work conducted in this research has resulted in the following major publications:

Journal

1. Ajit Kumar and Bharat Bhushan, "Position Control of a Ball Balancer System using Particle Swarm Optimization, BAT and Flower Pollination Algorithm." International Journal of Parallel, Emergent and Distributed Systems, vol. 38(3), pp. 213-228, March 2023, DOI: 10.1080/17445760.2023.2190972. (**ESCI Indexed**)

2. Ajit Kumar Sharma and Bharat Bhushan, “Comparison of Various Fuzzy Sliding Mode Based Controller on Single Link Inverted Pendulum”, Journal of Intelligent & Fuzzy System, vol. 42 (2), pp. 679-688, 2022. (DOI: 10.3233/JIFS-189740) (**SCI Indexed**)

3. Ajit Kumar Sharma and Bharat Bhushan, “Sliding Mode Control of Inverted Pendulum with Decoupling Algorithm”, International Journal of Computer Applications, vol. 181(27), pp. 1-5, November 2018. (DOI:10.5120/ijca2018918044)

4. Ajit Kumar Sharma, Bharat Bhushan, and Ankita Yadav, “Sliding Mode Control with RBF Neural Network for Two Link Robot Manipulator”, International Journal of Computer Applications, vol. 178(52), pp. 31-36, September 2019. (DOI: 10.5120/ijca2019919408)

5. Ajit Kumar Sharma and Bharat Bhushan, “Sliding Mode Cruise Control Based on High Gain Observer”, Journal of Emerging Technologies and Innovative Research (JETIR), ISSN-2349-5162, volume 6, issue 6, pp. 300-304, June 2019.

Book Chapters

1. Ajit Kumar Sharma and Bharat Bhushan, “Performance Analysis of Different Sliding Mode Controller on Single Link Inverted Pendulum”, Lecture Notes in Electrical Engineering, LNEE, volume 723, pp.247-261, 2021. (DOI: 10.1007/978-981-33-4080-0_24) **(Scopus Indexed)**

2. Ajit Kumar Sharma and Bharat Bhushan, “Performance Evaluation of Sliding Mode Control for Underactuated Systems Based on Decoupling Algorithm”, Machine Learning, Advances in Computing, Renewable Energy and Communication, volume 768, pp. 75-87,2021. (DOI: 10.1007/978-981-16-2354-7_8) **(Scopus Indexed)**

Curriculum Vitae

AJIT KUMAR SHARMA
Experience: **16+ yrs.**
sharmaajit01@gmail.com
Mobile: +91- 9899912688, +91-8076129897

Aspiring for a challenging career, where I would direct my knowledge and skills into a meaningful contribution towards meeting organizational and personal goals.

EMPLOYMENT HISTORY

- **Sr. Associate Patent**, Ennoble IP. Noida, Uttar Pradesh
  Sep. 2022 – Present

- **Assistant Professor**, ADGITM, Delhi (GGSIP University). Delhi, Delhi
  Aug. 2008 - Sep. 2022

- **Execution Engineer**, BSES Yamuna Power Ltd. Delhi, Delhi
  Aug. 2006 - Aug. 2008

ACADEMIA

- **Ph. D (pursuing)** from **Delhi Technological University, Delhi (*Formerly Delhi College of Engineering, Delhi*)**

- **M. Tech (**2011-2014**) (Control & Instrumentation**) with **I**st Div. from **Delhi Technological University, Delhi (*Formerly Delhi College of Engineering, Delhi*)**.

- **B. Tech**. (2002-2006) **(Electrical Engineering)** with **I**st Div. from **Uttar Pradesh Technical University, Lucknow (U.P)**

PROJECTS

- Ph.D. Topic "**Investigations on Modeling and Control of Nonlinear Systems".**
- M. Tech Thesis - **"Position Control of Two Link Robot System by using PD & Fuzzy Logic Controller"**
- B. Tech Project title **"Mobile Telephonic Controller of Electrical and Electronic Equipment".**

RESEARCH PAPERS

Research Papers
International Journal (SCI/SCOPUS INDEX)

1. **Ajit Kumar Sharma** and Bharat Bhushan "Comparison of different types of fuzzy controller based sliding mode control on single link Inverted Pendulum" Journal of Intelligent & Fuzzy Systems: Applications in Engineering and Technology. Volume 42, Issue 2, pp 679–688, 2022, DOI: https://doi.org/10.3233/JIFS-189740

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4. **A. Sharma,** T. Som**,** D. Thakur**,** **"**Effect of Solar Tilt Angles on Photovoltaic Module Performance: A Behavioural Optimization Approach" Artificial Intelligence Evolution, Universal Wiser Publisher, vol 1, issue 2, pp: 90-101, 2020, DOI: https://doi.org/10.37256/aie.122020505.

5. **A. Sharma**, T. Som, M. Dwivedi. C. Dubey, "Parametric Studies on Artificial Intelligence Techniques for Battery SOC Management and Optimization of Renewable Power", Procedia Computer Science, Elsevier, vol 167, pp. 353-362, 2019, DOI: 10.1016/j.procs.2020.03.235.

6. **Ajit Kumar Sharma**, Ankita Yadav, and Bharat Bhushan, "Sliding Mode Control with RBF Neural Network for Two Link Robot Manipulator", International Journal of Computer Applications, vol. 178(52), pp. 31-36, September 2019, DOI: 10.5120/ijca2019919408**.**

7. **Ajit Kumar Sharma**, Bharat Bhushan, "Sliding Mode Cruise Control Based on High Gain Observer", Journal of Emerging Technologies and Innovative Research (JETIR), ISSN-2349-5162, volume 6, issue 6, pp. 300-304, June 2019**.**

8. **Ajit Kumar Sharma** and Bharat Bhushan. "Sliding Mode Control of Inverted Pendulum with Decoupling Algorithm". International Journal of Computer Applications, vol. 181(27), pp. 1-5, November 2018, DOI: 10.5120/ijca2018918044.

International Conference:

1. **A. K. Sharma,** B. Bhushan, and D. Singh, "Fuzzy & ANFIS based temperature control of water bath system," *2016 IEEE 1st International Conference on Power Electronics, Intelligent Control and Energy Systems (ICPEICES)*, Delhi, 2016, DOI: 10.1109/ICPEICES.2016.7853729.

2. **Sharma, A. K**, Bhushan, B. (2014, December). Position control of two-link robot system by PD & fuzzy controller. In Power India International Conference (PIICON), 2014 6th IEEE, pp. 1-6, IEEE, DOI: 10.1109/POWERI.2014.7117753**.**

3. **Ajit Kumar Sharma**, Swati Paliwal, Piyush Sharma, "Dynamic Stability Enhancement of Power System Using Intelligent Power System Stabilizer." In Proceedings of Fourth International Conference on Soft Computing for Problem Solving, pp. 571-583. Springer, New Delhi, 2015, DOI: 10.1007/978-81-322-2217-0_46.

National Conference/Seminar:

1. **Ajit Kumar Sharma**, Vandana Arora, Juhi Priyani and Amit Patel, "Design, Control and Simulation of PMSG- Based Wind Energy Conversion System, proceedings of the National Conference on "Innovative and Emerging Trends in Engineering and Technology (IETET-2019)", pp 158-162, ISBN 978-93-5351-529-4, New Delhi, 2019.

(**AJIT KUMAR SHARMA)**