A multilevel transportation problem with problem with multiple inputs and outputs

A DISSERTATION

SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE AWARD OF

THE DEGREE OF

MASTER OF SCIENCE IN

MATHEMATICS

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May, 2023

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DECLARATION

We ,Jaskirat Kaur (2K21/MSCMAT/23) and Neha Upadhyay (2K21/MSCMAT/31) students of M.Sc. (APPLIED MATHEMATICS),hereby declare that the Project Dissertation titled “Efficiency analysis of extended transportation problem with multiple inputs and outputs using DEA” which is submitted by us to the Department of Applied Mathematics, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Science, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship, or other similar title or recognition.

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CERTIFICATE

We hereby clarify that the Project Dissertation titled "Efficiency analysis of extended transportation problem with multiple inputs and outputs using DEA" which is submitted by [Jaskirat Kaur] 2K21/MSCMAT/23 [APPLIED MATHEMATICS] and [Neha Upadhyay] 2K21/MSCMAT/31 [APPLIED MATHEMATICS] , Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Science, is a record of the project work carried out by the students under my supervision. To the best of my knowledge this work has not been submitted in part or full for any degree or Diploma to this University or elsewhere.

Place: Delhi

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Prof. Anjana Gupta

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ACKNOWLEDGEMENT

The success and final outcome of this dissertation work required a lot of guidance and assistance from many people and we are extremely privileged to have gotten this all along with the completion of our dissertation work. It gives immense pleasure in bringing out this research work entitled “Efficiency analysis of extended transportation problem with multiple inputs and outputs using DEA”. Firstly, we would like to thank our supervisor Prof. Anjana Gupta who gave us her valuable suggestions and ideas when we needed them. She encouraged us to work on this project dissertation and explore the topic and innovate it as per our understanding. We are grateful to our college for giving us such opportunity to work and providing the necessary details for the dissertation. We would also like to thank all of them who have helped us to complete this dissertation. We both are grateful to each other as, without that cooperation and coordination, it would not have been possible to develop the project within the prescribed time.

Sincere Thanks,

JASKIRAT KAUR

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ABSTRACT

Proposed modifications to the traditional multilevel transportation problem include multiple incompatible inputs and outputs for each shipping link, along with a definition of relative efficiency for each shipment connection. To determine the optimal transportation method, two linear programming problems are solved, one for direct transportation and the other for multilevel transportation. A numerical example is provided to illustrate the process.

Keywords: Transportation problem, data envelopment analysis (DEA), input-output, multilevel

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LIST OF SYMBOLS AND ABBREVIATIONS

x_{ij} is DMU Output

y_{ij} is DMU Input

u_r and v_i are Variable Weights

d is destination

ϵ is a non-Archimedean construct

eij* and eij** are relative efficiency indices

e_{ij} is composite efficiency

m is number of warehouses

n is number of destination points

x_1 is shipping cost

x_2 is lot size

y_1 is value of shipment

y_2 is profit

a_i are availabilities

b_j are demands

(i, j) is DMU arc

Chapter 1

1.1 INTRODUCTION

Transportation problem are significant network-structured linear programming problem that appear in a variety of scenarios and have rightfully attracted a lot of attention in the literature. An equivalent transportation problem can be created from any finite-valued, capacitated or incapacitated, minimum-cost network flow problem. As a result of the way the problem is structured, there are numerous shipping routes connecting various supply origins and demand destinations. The objective of transportation models is to determine the optimal quantity of a specific product to be transported from one location to another to meet the demand at each destination center. This needs to be done with the few products and services that are offered at each supply centre and with the least amount of money and/or time spent on transportation. The goal of the transportation algorithm is to reduce the overall cost of moving a homogeneous commodity (product) between supply and demand centres. Moreover, transportation optimization techniques may also be applied to maximize a specific overall value or utility. For instance, financial resources are allocated so as to maximize the beneficial return. The concept of transportation models originated with F L Hitchcock's (1941) presentation of the simplest model, which was later improved upon by T C Koopmans (1949) and G B Dantzig (1951). Over time, numerous advancements and modifications have been made to transportation models and techniques.

The objective is to minimize the expenses of transporting goods from one location to another while simultaneously satisfying the requirements of each receiving area and guaranteeing that each shipping point can operate at its maximum capacity. When solving a transportation problem, it is typical to only consider the cost or revenue associated with each source-destination pair. However, real-world scenarios may require that other variables be factored in for each possible mode of transportation. Additionally, for each potential shipment, decision-makers may have multiple objectives to meet, some of which may be at odds with one another. In these situations, creating a transportation strategy with the highest possible relative efficiency is important to us. Data Envelopment Analysis (DEA) is a mathematical technique used to assess the relative efficiency of Decision-Making Units (DMUs). This method enables the comparison of the effectiveness of a group of DMUs, such as banks, automobile manufacturers, hospitals, universities, municipal corporations, educational institutions, and railways. The DEA methodology was first introduced in academic literature in (Charnes et al., 1978). Numerous DEA applications and research have resulted in numerous significant advancements in DEA-related concepts and approaches. A model called the CCR model, proposed by (Charnes et al., 1978), was proposed to determine the relative efficacy of various DMUs. The conversion of inputs into outputs is carried out by the homogenous DMU. A matrix made up of the inputs, outputs, and complementary components of the sample of DMUs is needed to conduct a DEA analysis. The paper deals with problematic of solving multilevel transportation problem and direct transportation problem with multiple inputs and outputs involving transportation of textile. Our goal is to find a better approach for transportation of textile goods using DEA.

In this paper, the structure is organized as follows: Section 2 provides a review of pertinent literature on the topic. Origin of data envelopment analysis and CCR (Charnes, Cooper and Rhodes) model are introduced in 3rd section. The fourth section of the paper offers an expansion of the transportation problem and its methodology. Next, the fifth section illustrates a numerical example of a multilevel transportation model. Finally, the paper concludes with a summary of findings in the last section.

1.2 ORIGIN OF DATA ENVELOPMENT ANALYSIS

Data Envelopment Analysis (DEA) is a mathematical method used to assess the comparative efficiency of Decision-Making Units (DMUs). The DEA approach can be utilized to evaluate the performance of various DMUs such as banks, hospitals, universities, automobile manufacturers, educational institutions, municipal corporations, and railways. The approach was initially introduced in a publication by (Charnes et al., 1978).

This paper covered similar ground to (Farrell & Fieldhouse, 1962), but the linear programming model developed by CCR was more versatile and showed improved performance in comparison to Farrell's method. The CCR model could be calculated using standard linear programming codes and was computationally efficient. It established a connection between a productivity index and the Farrell technical efficiency measure, which was a significant contribution. The agriculture economists' advancements in the programming approach for piecewise linear frontier production functions went unnoticed until the publication of the CCR paper, which sparked its development. Over time, more economists started adapting this programming approach, especially for empirical applications. Simar and Wilson published a survey of the recent developments in this field in 2000.

The estimation error of DEA estimators is affected by the number of inputs and outputs, as well as the dimensionality of the production set. The sample size must grow exponentially as dimensionality increases to maintain the same level of estimation error. The only practical approach to inference with DEA estimators is to use computationally intensive bootstrap methods, which must be modified with smoothing procedures to produce accurate results.

To conduct a DEA analysis, a matrix consisting of inputs, outputs, and complementary components of the sample DMUs is required. After the DEA model is constructed with specific metrics and orientation, the matrix is applied in the model to be solved. The outcome includes relative efficiency scores and operational benchmarks for each DMU.

Each DMU is assigned an efficiency score ‘e’ and target values, also known as benchmarks, are computed to transform inefficient DMUs ($e<1$) into efficient ones. DEA helps to identify possible improvements for efficient operational performance and distinguish between efficient and inefficient organizations. The mathematical technique involves calculating the efficiency frontier for the set of DMUs based on the prepared matrix of observed data and DEA model. The production possibility set is defined by the efficiency frontier, and the DMUs located on this border make up the reference set. DEA projects each DMU onto the efficiency frontier and determines the maximum improvements that can be made to the DMU's inputs and/or outputs.

Chapter 2

2.1 LITERATURE REVIEW

Year	Literature Review
(*An Analysis of Production as an Efficient Combination of Activities _ CiNii Research*, n.d.)	According to Koopmans, it is not possible to increase any output and/or decrease any input without a corresponding increase in other outputs and/or decrease in other inputs. Koopmans referred to a feasible input-output vector as efficient. He showed that a vector is only considered efficient when it has a positive normal to the set of production possibilities, using this concept.
(Debreu, 1951)	Koopmans supplied a definition and a description of efficiency, whereas Debreu's "coefficient of resource utilisation" provided a measurement of efficiency. Debreu determined a measure of inefficiency cost by computing a coefficient, which is obtained by subtracting the maximum equiproportionate reduction in all inputs required to maintain present output production from one..
(Shephard, R.W, 1953)	Shephard made use of the distance function to establish a crucial connection between production and costs by demonstrating the unique association between the production process and the cost function. Similarly, Farrell utilized this feature to showcase that his efficiency metrics held a significant relationship with costs.
(Farrell, 1957)	Farrell cites Koopmans and Debreu as influences. Farrell provided the foundation for innovative methods for conducting micro-level productivity and efficiency studies, The insight provides new understanding on two important topics: the definition of productivity and efficiency, and the calculation of benchmark technology and efficiency measurements. The study "The measuring of productive efficiency" by Michael James Farrell was as a major source of inspiration for CCR and the working papers that preceded it.
(Farrell & Fieldhouse, 1962)	In a previous publication, Farrell devised a technique for calculating effective production functions using data on the inputs and outputs of various production units. The method devised by Farrell (1957) for calculating efficient production functions is later used by M. J. Farrell and M. Fieldhouse to the situation of growing returns to scale.

(Aigner & Chu, 1968)	Aigner and Chu's 1968 paper "On Estimating the Industry Production Function" aimed to introduce a new approach to determine the industry production function. They presented a deterministic and parametric framework utilizing econometric techniques to estimate a production function. This approach was a departure from the average function method, which had been the only method used in the past. While acknowledging Farrell's non-parametric approach, the authors opted for the more traditional parametric approach favored by economists. The authors argued that Farrell's method was insufficient because it could not account for various forms of production, including those that followed the Law of Variable Proportions.
(Charnes et al., 1978)	The authors of the paper explored the same concepts related to efficiency measures as Farrell did. Both presented similar measures and a framework for a piecewise linear production technology. However, the paper utilized a more sophisticated linear programming model compared to Farrell's approach, especially in situations of single output production. CCR made a unique contribution by connecting a productivity index, which was based on the combination of inputs and outputs, to Farrell's measure of technical efficiency in cases of constant returns to scale.
(Amirteimoori, 2011)	The transportation problem is a component of supply chain management with the goal of minimizing transportation costs. In this study, the authors used the DEA approach with the CCR model on an extended transportation problem that has multiple inputs (shipping cost, value of shipment) and a single output (profit).
(*1953*, n.d.; Bhardwaj & Gupta, 2021)	The classic multilevel transportation problem was expanded in this paper by concerning multiple input and multiple flexible output for each shipment connection. DEA based proposed solution, with a BCC model based on the relative efficiencies of each potential connection as a performance measure to determine the most efficient transportation strategy.

Chapter 3

3.1 CCR MODEL

The CCR ratio model computes a unit's overall efficiency by combining its pure technical efficiency and scale efficiency into a single value. Because efficiency is always measured relative to the field, it is never absolute. Despite the numerous modified models that have appeared since (Charnes et al., 1978), the CCR model remains the most widely known and used of DEA models.

The (Charnes et al., 1978) versatile performance model is as follows:
The efficiency of any Decision Making Unit (DMU) is calculated as the highest value obtained by dividing the weighted outputs by the weighted inputs, while ensuring that the same ratio for every DMU is less than or equal to 1.

$$\text{Max } h_0 = \frac{\sum_{r=1}^{s} u_r\, y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}}$$

Subject to:

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 ; \qquad j = 1, \dots\dots, n,$$

$$v_r, u_i \geq 0 ; \qquad r = 1, \dots\dots s; \qquad i = 1, \dots\dots, m$$

When evaluating the efficiency of a Decision Making Unit (DMU), the known outputs (x_{ij}) and inputs (y_{rj}) of the DMU are used, and the variable weights (u_r and v_i) are determined through the optimization process. The reference set contains data on all DMUs and the efficiency of each member (j = l, 2.... n) is rated in comparison to the others. The efficiency rating is represented in the functional for optimization and in the constraints, with the selected DMU being distinguished by the subscript '0' in the functional while preserving its original subscript in the constraints. The optimization process maximizes the efficiency of the chosen DMU by assigning it the most favourable weighting allowed by the constraints.The aforementioned fractional program may be converted to a linear programming problem using the Charnes and Cooper (1962) transformation method, which proceeds as follows.

$$\text{Max } h_0 = \sum_{r=1}^{s} u_r y_{r0}$$

Subject to:

$$\sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\sum_{r=1}^{s} u_r y_{ij} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \qquad \text{j= 1,2,……n}$$

3.2 METHODOLOGY

This section proposes an extension to the transportation problem that takes into account multiple inputs and outputs for each possible arc. Consider m warehouses, each of which has a supply of ai units of a specific type commodity. There are also n destination points, with destination j requiring b_j units of the commodity. For each arc(i, j), there are k + s attributes, k inputs $x_{ij}^{(p)}$, p= 1, 2,..., k and s outputs $y_{ij}^{(q)}$, q = 1, 2,..., s
For each warehouse i we consider all possible destinations d, d = 1, 2,..., n, and assume that each arc (i, j) is a DMU. Using the DEA technique and warehouse i as a target, the efficiency of the unit transportation cost from i to j (j = 1, 2,..., n) can be determined. The following linear fractional programme is solved to accomplish this:

$$\text{Max } e_{ij}^{*} = \frac{\sum_{q=1}^{s} u_q\, y_{ij}^{(q)}}{\sum_{p=1}^{k} v_p\, x_{ij}^{(P)}}$$

Subject to:

$$\frac{\sum_{q=1}^{s} u_q\, y_{id}^{(q)}}{\sum_{p=1}^{k} v_p\, x_{id}^{(P)}} \leq 1\,; \qquad d = 1, \ldots\ldots.., n,$$

$$v_p, u_q \geq \epsilon\,; \qquad for\ all\ p, q$$

(E1)

Where $\epsilon \geq 0$ represents a non-Archimedean construct. By changing j in the model, we can determine the comparative effectiveness of transporting goods from warehouse i to each destination j in terms of unit transportation cost as e_{i1}, e_{i2},……, e_{in}. E1 can be reduced to a linear format using the following formula:

$$\text{Max } e_{ij}^{*} = \sum_{q=1}^{s} u_q\, y_{ij}^{(q)}$$

Subject to:

$$\sum_{p=1}^{k} v_p\, x_{ij}^{(P)} = 1,$$

$$\sum_{q=1}^{s} u_q\, y_{id}^{(q)} - \sum_{p=1}^{k} v_p\, x_{id}^{(P)} \leq 0\,, \qquad d = 1,2, \ldots \ldots, n,$$

$$v_p, u_q \geq \epsilon\,; \qquad for\ all\ p, q$$

(E2)

To assess the efficiency of the unit transportation cost from i to each destination j, the following program can be employed for each destination j as a target.

$$\text{Max } e_{ij}^{**} = \frac{\sum_{q=1}^{s} u_q\, y_{ij}^{(q)}}{\sum_{p=1}^{k} v_p\, x_{ij}^{(P)}}$$

Subject to:

$$\frac{\sum_{q=1}^{s} u_q\, y_{dj}^{(q)}}{\sum_{p=1}^{k} v_p\, x_{dj}^{(P)}} \leq 1\,; \qquad d = 1, \ldots \ldots, m$$

$$v_p, u_q \geq \epsilon\,; \qquad for\ all\ p, q$$

(E3)

We can obtain the relative performance of the unit transportation from each warehouse i to destination j by numerically solving E3 as e_{1j}, e_{2j},……, e_{mj}., by changing i in the model. E3 can be reduced to a linear format using the following formula.

$$\text{Max } e_{ij}^{**} = \sum_{q=1}^{s} u_q \, y_{ij}^{(q)}$$

Subject to:

$$\sum_{p=1}^{k} v_p \, x_{ij}^{(P)} = 1,$$

$$\sum_{q=1}^{s} u_q \, y_{dj}^{(q)} - \sum_{p=1}^{k} v_p \, x_{dj}^{(P)} \leq 0 , \qquad d = 1,2, \ldots \ldots , m.$$

$$v_p, u_q \geq \epsilon ; \qquad for\ all\ p, q$$

(E4)

For each link, two relative efficiency indices eij* and eij**, have now been computed. These two indices are combined to create a new efficiency index.

$$e_{ij} = \frac{e_{ij}^{*} + e_{ij}^{**}}{2}, \quad i = 1,2, \ldots \ldots , m, \quad j = 1,2, \ldots \ldots , n$$

(E5)

Each link (i, j) has been assigned an efficiency measure eij, which is a composite efficiency measure comprised of two types of efficiencies. The eij is the mean of the two indexes eij* and eij**. To formulate a transportation strategy using the maximum efficiency, we solve the following.

$$e = Min \sum_{i=1}^{m} \sum_{j=1}^{n} \left(1 - e_{ij}\right) x_{ij}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1,2, \ldots \ldots , m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1,2, \ldots \ldots , n,$$

$$x_{ii} \geq 0, for\ all\ i, j.$$

(E6)

The aforementioned problem is a traditional transportation problem that can be addressed using the standard simplex algorithm. Solving equation E6 numerically reveals the most efficient transportation plan.

3.3 PSEUDOCODE

The pseudo code for solving CCR model in R using DeaR Library is as follow

```
Begin
Input Data = "book1.xlsx"
Display Data
For CCR Model
Read Data
Set Number of inputs = 2
        Number of outputs = 2
        Row number for DMU's = 1
        Row number for input = 2 and 3
        Row number for output = 4 and 5
Display CCR Model
For Result
Set Model = CCR Model
        Orientation = Input Oriented
        Return to scale = Constant Return to scale
        Number of DMU's for evaluation = 64
Display Result
Display Efficiencies
Display target values
End
```

Chapter 4

4.1 NUMERICAL EXAMPLE

Eight cities ($U_1, U_2, U_3, U_4, U_5, U_6, U_7, U_8$) have assembly lines for textile manufacturers. After being assembled, the bikes are sent to eight warehouses ($V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8$) and 8 clients ($W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8$). The manufacturer takes into account two outputs, namely shipment value and profit, as well as two inputs, namely shipping cost and lot size. Each of the ordered triplets (x_1, x_2, y_1, y_2) represents (shipping cost, lot size, value of shipment, profit). The appropriate input-output, availabilities a_i , and demands b_j are listed in Table 1.

We have solved the problems E2, E4 and E5 and calculated the optimal values of e_{ij}^{*}, e_{ij}^{**} and e_{ij} , Composite efficiency e_{ij} is listed in table 4,5 and 6. After calculating efficiencies, we have solved the transportation problem for the data indicated in Table 7,8 and 9, to determine the transportation plan with maximum efficiency. The entry in each cell is 1-e_{ij} which represent the value of inefficiency associated with particular arc in each table.

We used the simplex method to solve transportation problem of table 7,8 and 9. The optimal solution as $X_{u_1v_2}$=10, $X_{u_2v_3}$=22, $X_{u_3v_8}$= 16, $X_{u_4v_7}$= 12, $X_{u_5v_4}$=24, $X_{u_5v_5}$= 5, $X_{u_5v_8}$ =1, $X_{u_6v_2}$ = 9, $X_{u_6v_8}$=6, $X_{u_7v_1}$ = 17, $X_{u_7v_7}$= 8 , $X_{u_8v_1}$=4 , $X_{u_8v_5}$= 6 and $X_{u_8v_6}$= 8 .Optimal objective value is 12.325 with maximum efficiency 87.674 for table 7 .

The optimal solution as $X_{v_1w_2}$=1 , $X_{v_1w_8}$=20, $X_{v_2w_2}$= 17, $X_{v_3w_3}$= 20, $X_{v_4w_1}$=10, $X_{v_4w_4}$ = 4, $X_{v_5w_4}$= 15, $X_{v_6w_6}$ =10, $X_{v_6w_7}$= 2, $X_{v_7w_1}$=1, $X_{v_7w_2}$=1, $X_{v_7w_7}$ = 11, $X_{v_8w_3}$=2, $X_{v_8w_4}$ = 5 and $X_{v_8w_5}$ = 11. Optimal objective value is 13.182 with maximum efficiency 86.818 for table 8.

The optimal solution as $X_{u_1w_2}$=10, $X_{u_2w_3}$=24, $X_{u_3w_2}$ = 3, $X_{u_3w_5}$ = 1, $X_{u_3w_8}$=12, $X_{u_4w_4}$ = 7, $X_{u_4w_6}$=5, $X_{u_5w_4}$=7, $X_{u_5w_7}$=13, $X_{u_6w_6}$ = 15, $X_{u_7w_1}$=16, $X_{u_7w_2}$ = 7, $X_{u_7w_7}$= 2, $X_{u_8w_3}$=1, $X_{u_8w_5}$= 17. Optimal objective value is 13.397 with maximum efficiency 86.603 for table 9.

The maximum efficiency of transportation from city to warehouse is 87.674 and maximum efficiency for transportation from warehouse to customer is 86.818. So, the maximum efficiency for multilevel transportation is 87.246. Maximum efficiency in direct transportation i.e., from city to customer is 86.603. From the result we can say multilevel transportation is more efficient than direct transportation.

Table 4.1: city to warehouse

	V1	V2	V3	V4	V5	V6	V7	V8	ai
U1	20,50,160,100	19,100,245,110	25,150,190,180	27,125,152,220	30,55,157,180	44,200,129,230	36,195,147,120	24,80,138,167	10
U2	25,135,256,168	42,60,198,147	22,50,167,256	55,90,256,163	33,155,233,230	28,170,158,250	39,145,140,220	48,75,159,116	24
U3	23,100,258,254	27,120,156,251	40,80,246,267	30,90,129,252	29,110,152,147	23,135,157,195	42,160,245,170	26,200,194,180	16
U4	35,50,190,264	25,195,255,249	36,190,175,250	34,60,232,247	31,90,198,187	21,200,147,249	45,50,265,252	25,140,190,147	12
U5	30,130,160,164	24,60,198,159	34,75,167,267	23,50,264,252	25,100,264,187	36,160,243,259	32,140,164,254	41,175,265,180	30
U6	28,190,130,252	24,135,255,258	27,200,139,269	25,145,254,294	34,50,154,250	23,90,256,247	34,75,154,241	31,100,225,265	15
U7	22,120,160,260	36,145,232,140	42,160,156,252	28,190,167,267	45,55,126,194	31,80,134,123	27,50,236,195	46,200,169,169	25
U8	23,140,159,251	29,145,121,152	49,130,125,252	40,75,158,258	25,80,168,256	27,65,160,264	21,140,131,169	40,200,151,195	18
bj	21	19	22	24	11	10	20	23	

Table 4.2: warehouse to customer

	W1	W2	W3	W4	W5	W6	W7	W8	ai
V1	30,50,180,115	29,100,250,120	31,150,200,190	35,125,190,200	45,55,210,210	42,200,155,230	33,195,172,155	29,80,200,140	21
V2	22,135,288,150	45,60,222,155	30,55,187,250	41,85,239,160	55,150,255,255	50,160,199,290	37,150,190,200	50,90,175,105	17
V3	55,120,286,245	54,100,193,225	30,120,267,287	35,80,155,231	22,100,185,165	31,90,157,200	44,200,290,110	30,150,220,190	20
V4	29,55,197,245	35,170,275,206	46,185,180,205	52,50,230,259	36,55,200,195	29,155,130,255	41,90,274,230	40,150,210,175	14
V5	42,120,186,197	40,90,205,159	35,80,178,248	30,100,299,259	41,150,270,190	31,120,240,214	33,50,190,275	40,150,260,165	15
V6	25,100,158,196	48,130,268,231	51,210,130,248	30,150,250,254	38,90,179,210	25,100,250,276	30,80,150,276	45,150,279,247	12
V7	25,155,187,268	33,130,268,130	48,120,184,278	41,170,194,280	45,50,136,168	35,100,145,160	55,55,249,115	50,210,179,200	13
V8	40,155,150,255	42,140,116,146	47,120,148,287	35,90,150,246	44,75,160,278	50,80,157,268	48,150,177,120	52,190,189,200	18
bj	11	19	22	24	11	10	13	20	

Table 4.3: city to customer

	W1	W2	W3	W4	W5	W6	W7	W8	ai
U1	25,55,155,105	24,110,225,115	30,120,155,185	29,130,197,210	42,60,200,220	37,169,150,225	30,183,165,175	27,130,195,128	10
U2	30,148,280,143	42,83,210,138	25,60,155,230	40,120,230,180	49,190,235,210	45,193,210,273	40,175,225,184	42,110,180,110	24
U3	46,140,275,200	50,110,190,215	36,130,260,255	39,124,150,215	20,80,175,155	28,81,149,185	50,174,255,105	26,148,200,145	16
U4	25,88,178,250	38,193,257,200	40,205,155,202	50,75,200,227	44,86,189,163	25,174,155,238	34,137,267,217	38,200,205,196	12
U5	39,160,180,183	37,135,200,159	34,100,178,200	25,130,258,220	44,125,254,170	29,145,228,204	30,80,224,258	44,125,247,165	20
U6	28,152,135,183	35,110,220,190	50,190,125,238	25,135,237,218	35,125,180,200	22,80,237,237	24,69,173,218	39,184,247,238	15
U7	20,173,190,217	35,146,263,155	50,138,165,258	32,159,158,267	36,90,125,139	29,125,119,149	50,84,238,149	45,200,148,210	25
U8	35,150,128,238	37,155,162,150	40,110,183,290	32,95,119,246	39,80,127,270	46,100,135,260	44,125,163,100	41,200,200,169	18
bj	16	20	25	14	18	20	15	12	

Table 4.4: Composite Efficiency for city to warehouse

	V1	V2	V3	V4	V5	V6	V7	V8
U1	0.688008	1	0.661089	0.693419	0.636892	0.44426	0.345061	0.606049
U2	0.8343	0.623711	1	0.538021	0.627942	0.755439	0.481371	0.400894
U3	0.995987	0.79047	0.654878	0.719306	0.460302	0.720027	0.479423	0.646286
U4	1	0.901093	0.589384	0.804502	0.55523	1	1	0.636102
U5	0.485577	0.709508	0.6942	1	0.870783	0.629915	0.675475	0.529417
U6	0.760718	0.959527	0.841688	1	0.949182	0.976082	0.6266	0.748047
U7	1	0.522736	0.511763	0.806306	0.668044	0.373591	0.893324	0.329378
U8	0.923362	0.446524	0.44114	0.664422	0.876351	0.839722	0.682657	0.415076

Table 4.5: Composite Efficiency for warehouse to customer

	W1	W2	W3	W4	W5	W6	W7	W8
V1	0.965854	0.853975	0.604935	0.565519	0.917596	0.496032	0.478408	0.792959
V2	1	0.918736	0.97065	0.811512	0.539102	0.570384	0.506301	0.534425
V3	0.700507	0.53503	0.884926	0.719528	0.774422	0.655298	0.582355	0.661967
V4	1	0.687102	0.40367	1	0.947867	0.796477	0.896726	0.506611
V5	0.515598	0.676675	0.77248	1	0.637552	0.749198	1	0.623138
V6	0.710145	0.650271	0.440466	0.803642	0.620615	1	0.959569	0.626863
V7	0.971014	0.763116	0.615735	0.618593	0.633618	0.490333	1	0.362319
V8	0.584656	0.337042	0.64522	0.741105	0.739866	0.636145	0.387892	0.374968

Table 4.6: Composite Efficiency for city to customer

	W1	W2	W3	W4	W5	W6	W7	W8
U1	0.903506	0.870253	0.572136	0.671383	1	0.563737	0.53972	0.670417
U2	0.866385	0.799639	1	0.636048	0.445191	0.562625	0.522152	0.535511
U3	0.652751	0.574479	0.674723	0.540921	0.812236	0.67235	0.492896	0.714054
U4	0.941203	0.627804	0.46778	0.821427	0.693498	0.879531	0.728965	0.500777
U5	0.435283	0.501768	0.617125	0.957975	0.670697	0.729812	0.96863	0.652213
U6	0.605139	0.666627	0.441753	0.88	0.534539	1	0.926471	0.587904
U7	1	0.697529	0.534275	0.773052	0.467939	0.476479	0.873967	0.432692
U8	0.630638	0.406432	0.751668	0.774678	0.880435	0.678261	0.430408	0.452815

Table 4.7: 1- Efficiency for city to warehouse

	V1	V2	V3	V4	V5	V6	V7	V8
U1	0.311992	0	0.338911	0.306581	0.363108	0.55574	0.654939	0.393951
U2	0.1657	0.376289	0	0.461979	0.372058	0.244561	0.518629	0.599106
U3	0.004013	0.20953	0.345122	0.280694	0.539698	0.279973	0.520577	0.353714
U4	0	0.098907	0.410616	0.195498	0.44477	0	0	0.363898
U5	0.514423	0.290492	0.3058	0	0.129217	0.370085	0.324525	0.470583
U6	0.239282	0.040473	0.158312	0	0.050818	0.023918	0.3734	0.251953
U7	0	0.477264	0.488237	0.193694	0.331956	0.626409	0.106676	0.670622
U8	0.076638	0.553476	0.55886	0.335578	0.123649	0.160278	0.317343	0.584924

Table 4.8: 1- Efficiency for warehouse to customer

	W1	W2	W3	W4	W5	W6	W7	W8
V1	0.034146	0.146025	0.395065	0.434481	0.082404	0.503968	0.521592	0.207041
V2	0	0.081264	0.02935	0.188488	0.460898	0.429616	0.493699	0.465575
V3	0.299493	0.46497	0.115074	0.280472	0.225578	0.344702	0.417645	0.338033
V4	0	0.312898	0.59633	0	0.052133	0.203523	0.103274	0.493389
V5	0.484402	0.323325	0.22752	0	0.362448	0.250802	0	0.376862
V6	0.289855	0.349729	0.559534	0.196358	0.379385	0	0.040431	0.373137
V7	0.028986	0.236884	0.384265	0.381407	0.366382	0.509667	0	0.637681
V8	0.415344	0.662958	0.35478	0.258895	0.260134	0.363855	0.612108	0.625032

Table 4.9: 1- Efficiency for city to customer

	W1	W2	W3	W4	W5	W6	W7	W8
U1	0.096494	0.129747	0.427864	0.328617	0	0.436263	0.46028	0.329583
U2	0.133615	0.200361	0	0.363952	0.554809	0.437375	0.477848	0.464489
U3	0.347249	0.425521	0.325277	0.459079	0.187764	0.32765	0.507104	0.285946
U4	0.058797	0.372196	0.53222	0.178573	0.306502	0.120469	0.271035	0.499223
U5	0.564717	0.498232	0.382875	0.042025	0.329303	0.270188	0.03137	0.347787
U6	0.394861	0.333373	0.558247	0.12	0.465461	0	0.073529	0.412096
U7	0	0.302471	0.465725	0.226948	0.532061	0.523521	0.126033	0.567308
U8	0.369362	0.593568	0.248332	0.225322	0.119565	0.321739	0.569592	0.547185

4.2 TARGET VALUES:

The values of the inputs and outputs which would result in an inefficient unit becoming efficient.

Table 4.10: Target value City to warehouse

	V1	V2	V3	V4	V5	V6	V7	V8
U1	13.8,34.4,160,143.3	19,100,245,110	16.5,36.6,190,180	18.7,86.7,152,220	19.1,35,157,180	19.5,88.9,143.9,230	12.4,36.6,147,120	14.5,48.5,138,167
U2	20.9,81.5,256,168	26.2,37.4,198,188.6	22,50,167,256	29.6,48.4,256,244	20.7,58.4,233,230	21.2,128.4,158,250	18.8,69.8,140,220	19.2,30.1,159,151.5
U3	22.9,63.4,258,254	21.3,94.9,157.4,251	26.2,52.4,246,267	21.6,64.7,162.2,252	13.3,32.4,152,147	16.6,97.2,157,195	20.1,74,245,170	16.8,39,194,180
U4	35,50,190,264	22.5,58.8,255,249	21.2,112,175,250	27.4,48.3,232,247	17.2,38.4,198,187	21,200,147,249	45,50,265,252	15.9,50.9,190,147
U5	14.6,51.5,160,164	17,42.6,198,177.3	23.6,52.1,175.1,267	23,50,264,252	21.8,78.1,264,187	22.7,96.9,243,259	21.6,94.6,164,254	21.7,81.8,265,180
U6	21.3,144.5,153,252	23,75.7,255,258	22.7,168.3,162.3,269	25,145,254,294	32.3,47.5,178.7,250	22.4,53.4,256,247	21.3,47,158.1,241	23.2,74.8,225,265
U7	22,120,160,260	18.8,75.8,232,147.8	21.5,81.9,159.9,252	22.6,153.2,167,267	25.7,36.7,139.6,194	11.6,27.5,134,123	24.1,44.7,236,225.1	15.2,46.4,169,169
U8	21.2,129.3,159,251	12.9,64.7,121,152	21.6,57.3,163.3,252	26.6,49.8,174.7,258	21.9,70.1,168,256	22.7,54.6,171.8,264	14.3,95.6,131,169	16.6,83,151,195

Table 4.11: Target Value Warehouse to customer

	W1	W2	W3	W4	W5	W6	W7	W8
V1	29,48.3,180,197.6	24.8,85.4,250,212	18.8,83.6,200,190	19.8,70.7,190,200	41.3,50.5,210,210	20.8,83.3,208.3,230	15.8,72.9,172,155	23,63.4,200,196.8
V2	22,135,288,150	41.3,55.1,222,184.2	29.1,53.4,187,250	33.3,69,239,281.8	29.7,80.9,255,255	28.5,91.3,246.2,290	18.7,75.9,190,200	26.7,48.1,175,207.5
V3	38.5,84.1,286,326.9	28.9,53.5,193,234.9	26.5,106.2,267,287	25.2,57.6,178.1,231	17,77.4,185,165	20.3,59,165.1,200	25.6,116.5,290,201.1	19.9,94.2,220,190
V4	29,55,197,245	24,116.8,275,206	18.6,74.3,185.7,205	52,50,230,259	34.1,52.1,200,199.1	23.1,92.4,231,255	36.8,80.7,274,312	20.3,76,210,175
V5	21.7,61.9,186,197	27.1,60.9,205,229.9	27,61.8,191.2,248	30,100,299,259	26.1,95.6,270,220.2	23.2,89.9,240,214	33,50,190,275	24.9,93.5,260,208.5
V6	17.8,71,177.5,196	31.2,84.5,268,267	22.5,89.9,224.6,248	24.1,102.6,250,254	23.6,55.9,179,210	25,100,250,276	28.8,76.8,222.3,276	28.2,94,279,247
V7	24.3,97.1,242.8,268	25.2,99.2,268,207.5	29.6,73.9,219.9,278	25.4,101.4,253.6,280	28.3,31.7,136,168	17.2,49,145,160	55,55,249,115	18.1,72.5,181.2,200
V8	23.4,90.6,228.9,255	14.2,47.2,125.4,146	30.3,77.4,228.3,287	25.9,66.7,196.1,246	32.6,55.5,198,278	31.8,50.9,187.7,268	18.6,58.2,177,160.2	19.5,71.2,189,200

Table 4.12: Target Value City to customer

	W1	W2	W3	W4	W5	W6	W7	W8
U1	22.6,49.7,155,162	20.9,75.9,225,225	17.2,68.7,183.3,185	19.5,87.3,205.6,210	42,60,200,220	20.9,95.3,219.8,225	16.2,96,165,175	18.1,65.8,195,195
U2	26,94.5,280,280	33.6,66.4,210,222	25,60,155,230	25.4,76.3,230,233.5	21.8,79.3,235,235	25.3,108.6,268.6,273	20.9,75.9,225,225	22.5,58.9,180,184.9
U3	30,91.4,275,278.8	28.7,63.2,190,215	24.3,87.7,260,260.1	21.1,67.1,191.6,215	16.2,59.1,175,175	18.8,54.5,151,185	24.6,85.8,255,255.8	18.6,67.5,200,200
U4	23.5,82.8,243.4,250	23.9,86.8,257,257	18.7,95.9,194.5,202	41.1,61.6,200,227	30.5,59.6,189,200.1	22,153,218.3,238	24.8,90.1,267,267	19,69.2,205,205
U5	17,69.6,180.9,183	18.6,67.5,200,200	21,61.7,178,200	23.9,87.1,258,258	29.5,83.8,254,259.1	21.2,77,228,228	29.1,77.5,224,258	28.7,81.5,247,251.9
U6	16.9,92,174.8,183	23.3,73.3,220,222.5	22.1,83.9,237,238	22,80,237,237	18.7,66.8,197.1,200	22,80,237,237	22.2,63.9,176.9,218	22.9,83.4,247,247
U7	20,173,190,217	24.4,88.8,263,263	26.7,73.7,201.2,258	24.7,122.9,258.1,267	16.8,42.1,125,139	13.8,59.6,146.5,149	43.7,73.4,238,256.4	19.5,86.5,205.8,210
U8	22.1,94.6,234.1,238	15,54.7,162,162	30.1,82.7,225.3,290	24.8,73.6,205.8,246	29.3,70.4,182,270	28.3,67.8,175.2,260	18.9,53.8,163,166.2	18.6,67.5,200,200

Chapter 5

5.1 COMPARISON

There is a wealth of literature on this subject that utilizes DEA-based approaches. (Chen & Lu, 2007) expanded the assignment problem by taking multiple inputs and outputs into account. (Amirteimoori, 2011) also added to the transportation problem using a DEA-based approach. (Pathan, 2019) further enhanced the transportation problem by using the BCC model for each potential shipping link and considering multiple inputs and outputs. To the best of our knowledge, there hasn't been any study that uses the strategy proposed in this work. The present research contributes to the development of the multilevel transportation problem with multiple inputs and outputs by utilizing the CCR model. The relative efficiency of each potential shipment link is determined, and the most effective shipment strategy is deemed the best solution to the transportation problem.

5.2 CONCLUSION

This paper expanded the conventional multilevel transportation problem by introducing multiple inputs and flexible outputs for each shipment connection. To determine the most efficient transportation strategy, a DEA-based proposed solution with a CCR model based on the relative efficiencies of each possible connection was used as a performance measure. Decision makers use different methods to achieve goals with each conflict potential shipment connection, and these goals may conflict with one another in the case of multilevel transportation. This paper reveals that a multilevel transportation approach is more effective than a direct transportation approach. When dealing with transportation issues that involve multiple inputs and versatile outputs, we suggest employing multilevel transportation rather than direct transportation as a problem-solving approach.

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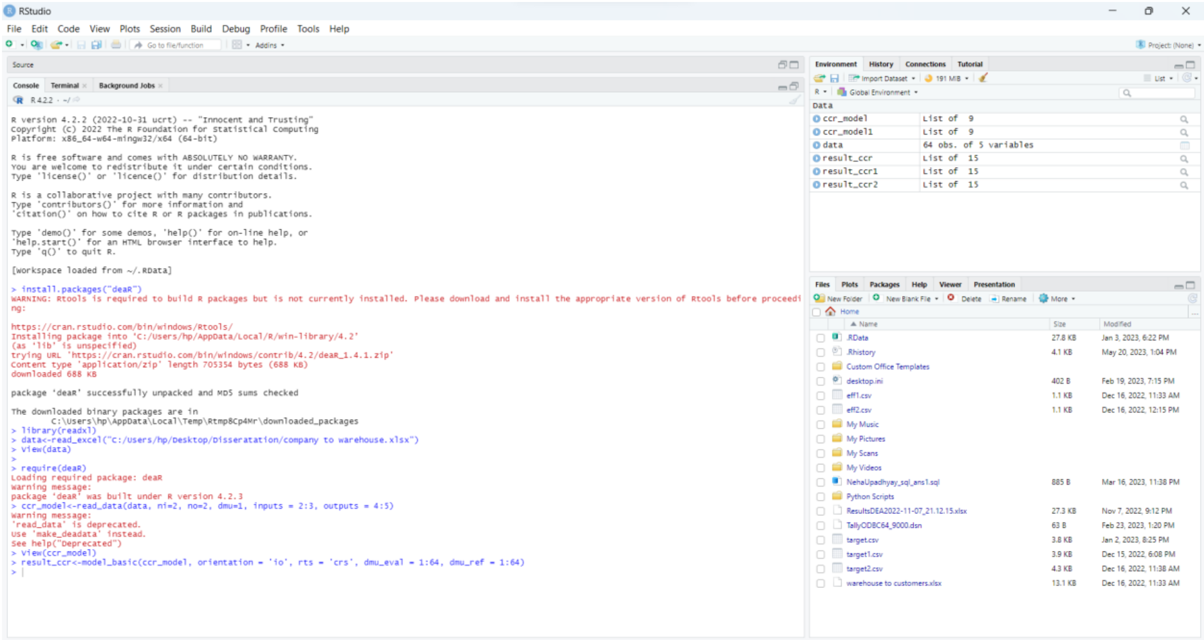
Appendix A

APPENDIX

A.1 R CODE




```
> install.packages("deaR")
WARNING: Rtools is required to build R packages but is not currently installed. Please download and install the appr
ng:

https://cran.rstudio.com/bin/windows/Rtools/
Installing package into 'C:/Users/hp/AppData/Local/R/win-library/4.2'
(as 'lib' is unspecified)
trying URL 'https://cran.rstudio.com/bin/windows/contrib/4.2/deaR_1.4.1.zip'
Content type 'application/zip' length 705354 bytes (688 KB)
downloaded 688 KB

package 'deaR' successfully unpacked and MD5 sums checked

The downloaded binary packages are in
        C:\Users\hp\AppData\Local\Temp\Rtmp8Cp4Mr\downloaded_packages
> library(readxl)
> data<-read_excel("C:/Users/hp/Desktop/Disseratation/company to warehouse.xlsx")
> View(data)
>
> require(deaR)
Loading required package: deaR
Warning message:
package 'deaR' was built under R version 4.2.3
> ccr_model<-read_data(data, ni=2, no=2, dmu=1, inputs = 2:3, outputs = 4:5)
Warning message:
'read_data' is deprecated.
Use 'make_deadata' instead.
See help("Deprecated")
> View(ccr_mode efficiencies(x, ...)
> result_ccr<-model_basic(ccr_model, orientation = 'io', rts = 'crs', dmu_eval = 1:64, dmu_ref = 1:64)
> efficiencies(result_ccr)
```

dmu.R × | ccr_model × | data ×

Filter

	DMU	I1	I2	O1	O2
1	(1,1)	20	50	160	100
2	(1,2)	19	100	245	110
3	(1,3)	25	150	190	180
4	(1,4)	27	125	152	220
5	(1,5)	30	55	157	180
6	(1,6)	44	200	129	230
7	(1,7)	36	195	147	120
8	(1,8)	24	80	138	167
9	(2,1)	25	135	256	168
10	(2,2)	42	60	198	147
11	(2,3)	22	50	167	256
12	(2,4)	55	90	256	163
13	(2,5)	33	155	233	230
14	(2,6)	28	170	158	250
15	(2,7)	39	145	140	220
16	(2,8)	48	75	159	116
17	(3,1)	23	100	258	254
18	(3,2)	27	120	156	251
19	(3,3)	40	80	246	267
20	(3,4)	30	90	129	252
21	(3,5)	29	110	152	147
22	(3,6)	23	135	157	195
23	(3,7)	42	160	245	170
24	(3,8)	26	200	194	180
25	(4,1)	35	50	190	264
26	(4,2)	25	195	255	249
27	(4,3)	36	190	175	250
28	(4,4)	34	60	232	247
29	(4,5)	31	90	198	187
30	(4,6)	21	200	147	249
31	(4,7)	45	50	265	252

Showing 1 to 31 of 64 entries, 5 total columns dmu.R × ccr_model × data ×

Show Attributes

Name	Type	Value
ccr_model	list [9] (S3: deadata)	List of length 9
input	double [2 x 64]	20 50 19 100 25 150 27 125 30 55 44 200 36 195 24 80 25 135 42 60 ...
output	double [2 x 64]	160 100 245 110 190 180 152 220 157 180 129 230 147 120 138 167 256 168 198 147 ...
dmunames	character [64]	'(1,1)' '(1,2)' '(1,3)' '(1,4)' '(1,5)' '(1,6)' ...
nc_inputs	NULL	Pairlist of length 0
nc_outputs	NULL	Pairlist of length 0
nd_inputs	NULL	Pairlist of length 0
nd_outputs	NULL	Pairlist of length 0
ud_inputs	NULL	Pairlist of length 0
ud_outputs	NULL	Pairlist of length 0

```
> library(readxl)
> data<-read_excel("C:/Users/hp/Desktop/Disseratation/company to warehouse.xlsx")
> View(data)
>
> require(deaR)
Loading required package: deaR
Warning message:
package 'deaR' was built under R version 4.2.3
> ccr_model<-read_data(data, ni=2, no=2, dmu=1, inputs = 2:3, outputs = 4:5)
Warning message:
'read_data' is deprecated.
Use 'make_deadata' instead.
See help("Deprecated")
> View(ccr_model)
> result_ccr<-model_basic(ccr_model, orientation = 'io', rts = 'crs', dmu_eval = 1:64, dmu_ref = 1:64)
> efficiencies(result_ccr)
    (1,1)     (1,2)     (1,3)     (1,4)     (1,5)     (1,6)     (1,7)     (1,8)     (2,1)     (2,2)     (2,3)     (2,4)     (2,5)     (2,6)     (2,7) 
0.6880076 1.0000000 0.6610887 0.6934191 0.6368920 0.4442605 0.3450612 0.6060487 0.8342997 0.6237113 1.0000000 0.5380206 0.6279417 0.7554387 0.4813714 
    (2,8)     (3,1)     (3,2)     (3,3)     (3,4)     (3,5)     (3,6)     (3,7)     (3,8)     (4,1)     (4,2)     (4,3)     (4,4)     (4,5)     (4,6) 
0.4008939 0.9959870 0.7904703 0.6548777 0.7193060 0.4603023 0.7200266 0.4794233 0.6462856 1.0000000 0.9010935 0.5893839 0.8045020 0.5552303 1.0000000 
    (4,7)     (4,8)     (5,1)     (5,2)     (5,3)     (5,4)     (5,5)     (5,6)     (5,7)     (5,8)     (6,1)     (6,2)     (6,3)     (6,4)     (6,5) 
1.0000000 0.6361015 0.4855771 0.7095079 0.6942000 1.0000000 0.8707829 0.6299154 0.6754753 0.5294175 0.7607181 0.9595275 0.8416883 1.0000000 0.9491822 
    (6,6)     (6,7)     (6,8)     (7,1)     (7,2)     (7,3)     (7,4)     (7,5)     (7,6)     (7,7)     (7,8)     (8,1)     (8,2)     (8,3)     (8,4) 
0.9760818 0.6266000 0.7480474 1.0000000 0.5227362 0.5117628 0.8063064 0.6680441 0.3735908 0.8933242 0.3293779 0.9233617 0.4465242 0.4411404 0.6644216 
    (8,5)     (8,6)     (8,7)     (8,8) 
0.8763509 0.8397224 0.6826574 0.4150763 
> 
```

A.2 EXCEL CODE

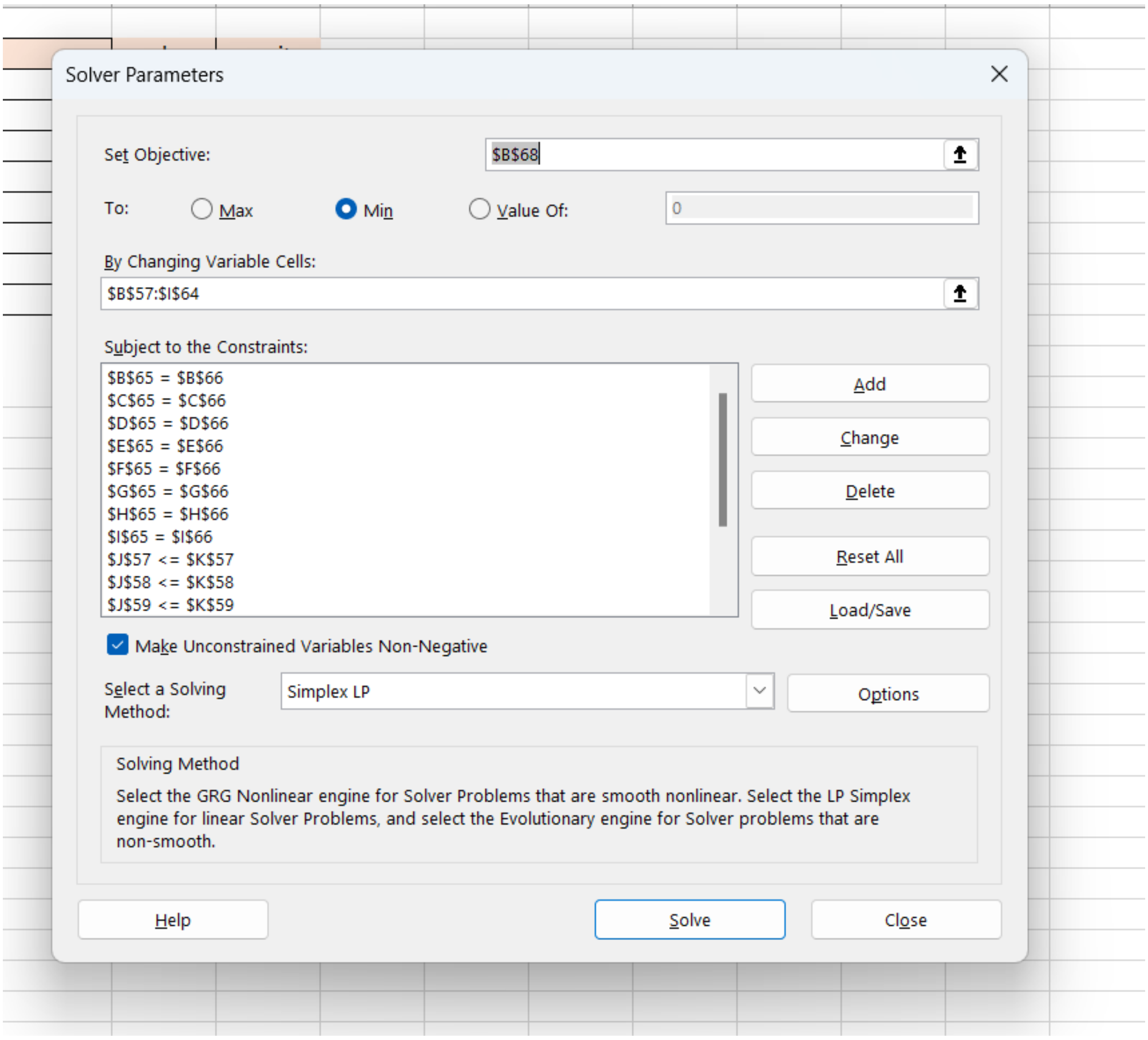
PROBLEM STATEMENT

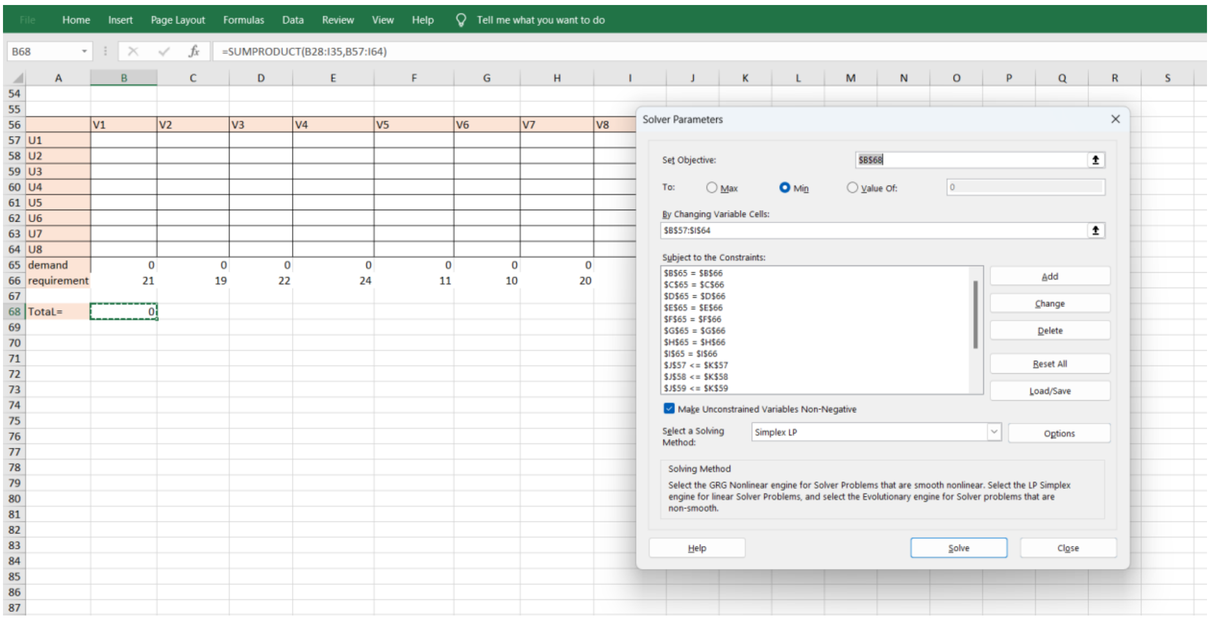
	A	B	C	D	E	F	G	H	SUPPLY
I	20,50,160,10	19,100,245,11	25,150,190,1	27,125,152,220	30,55,157,180	44,200,129,2	36,195,147,12	24,80,138,167	10
J	25,135,256,1	42,60,198,147	22,50,167,2	55,90,256,163	33,155,233,230	28,170,158,2	39,145,140,22	48,75,159,116	24
K	23,100,258,2	27,120,156,25	40,80,246,2	30,90,129,252	29,110,152,147	23,135,157,1	42,160,245,17	26,200,194,18	16
L	35,50,190,26	25,195,255,24	36,190,175,2	34,60,232,247	31,90,198,187	21,200,147,2	45,50,265,252	25,140,190,14	12
M	30,130,160,1	24,60,198,159	34,75,167,2	23,50,264,252	25,100,264,187	36,160,243,2	32,140,164,25	41,175,265,18	30
N	28,190,130,2	24,135,255,25	27,200,139,2	25,145,254,294	34,50,154,250	23,90,256,24	34,75,154,241	31,100,225,26	15
O	22,120,160,2	36,145,232,14	42,160,156,2	28,190,167,267	45,55,126,194	31,80,134,12	27,50,236,195	46,200,169,16	25
P	23,140,159,2	29,145,121,15	49,130,125,2	40,75,158,258	25,80,168,256	27,65,160,26	21,140,131,16	40,200,151,19	18
DEMAND	21	19	22	24	11	10	20	23	

	V1	V2	V3	V4	V5	V6	V7	V8
U1	0.688008	1	0.661089	0.693419	0.636892	0.44426	0.345061	0.606049
U2	0.8343	0.623711	1	0.538021	0.627942	0.755439	0.481371	0.400894
U3	0.995987	0.79047	0.654878	0.719306	0.460302	0.720027	0.479423	0.646286
U4	1	0.901093	0.589384	0.804502	0.55523	1	1	0.636102
U5	0.485577	0.709508	0.6942	1	0.870783	0.629915	0.675475	0.529417
U6	0.760718	0.959527	0.841688	1	0.949182	0.976082	0.6266	0.748047
U7	1	0.522736	0.511763	0.806306	0.668044	0.373591	0.893324	0.329378
U8	0.923362	0.446524	0.44114	0.664422	0.876351	0.839722	0.682657	0.415076

INEFFICIENCIES OF DMU's

	V1	V2	V3	V4	V5	V6	V7	V8
U1	0.311992	0	0.338911	0.306581	0.363108	0.55574	0.654939	0.393951
U2	0.1657	0.376289	0	0.461979	0.372058	0.244561	0.518629	0.599106
U3	0.004013	0.20953	0.345122	0.280694	0.539698	0.279973	0.520577	0.353714
U4	0	0.098907	0.410616	0.195498	0.44477	0	0	0.363898
U5	0.514423	0.290492	0.3058	0	0.129217	0.370085	0.324525	0.470583
U6	0.239282	0.040473	0.158312	0	0.050818	0.023918	0.3734	0.251953
U7	0	0.477264	0.488237	0.193694	0.331956	0.626409	0.106676	0.670622
U8	0.076638	0.553476	0.55886	0.335578	0.123649	0.160278	0.317343	0.584924


Solver Parameters
Set Objective:
$B$68
To:
Max
Min
Value Of:
0
By Changing Variable Cells:
$B$57:$I$64
Subject to the Constraints:
$B$65 = $B$66
$C$65 = $C$66
$D$65 = $D$66
$E$65 = $E$66
$F$65 = $F$66
$G$65 = $G$66
$H$65 = $H$66
$I$65 = $I$66
$J$57 <= $K$57
$J$58 <= $K$58
$J$59 <= $K$59
Add
Change
Delete
Reset All
Load/Save
Make Unconstrained Variables Non-Negative
Select a Solving Method:
Simplex LP
Options
Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
Help
Solve
Close


=SUMPRODUCT(B28:I35,B57:I64)
V1
V2
V3
V4
V5
V6
V7
V8
U1
U2
U3
U4
U5
U6
U7
U8
demand
requirement
21
19
22
24
11
10
20
Total=
Solver Parameters

SOLVING TRANSPORTATION PROBLEM TO FIND OBJECTIVE VALUE

	A	B	C	D	E	F	G	H		SUPPLY
I	0	10	0	0	0	0	0	0	10	10
J	0	0	22	0	0	2	0	0	24	24
K	0	0	0	0	0	0	0	16	16	16
L	0	0	0	0	0	0	12	0	12	12
M	0	0	0	24	5	0	0	1	30	30
N	0	9	0	0	0	0	0	6	15	15
O	17	0	0	0	0	0	8	0	25	25
P	4	0	0	0	6	8	0	0	18	18
	21	19	22	24	11	10	20	23		
DEMAND	21	19	22	24	11	10	20	23		

OBJECTIVE VALUE= 12.325267

MAXIMUM EFFICIENCY= 87.674733

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International Conference on Mathematics and its Applications in Technology

March 24 – 25, 2023

Organized by

Department of Mathematics
&
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Srinivasa Ramanujan Centre
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Submission of Full paper	**: February 15, 2023**
Acceptance Notification	**: February 28, 2023**
Conference Registration	**: March 10, 2023**

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