Analysis of Queueing Network Theory in Telecommunication

A DISSERTATION

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF

MASTER OF SCIENCE

IN

APPLIED MATHEMATICS

Submitted by:

Divya (2K21/MSCMAT/16)

Anshu(2K21/MSCMAT/04)

Under the supervision of:

Dr. L.N. Das

Department of Applied Mathematics

Delhi Technological University Shahbad Daulatpur,

Main Bawana Road, Delhi-110042

May 2023

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

Bawana Road, Delhi-110042

Candidate's Declaration

We, Divya (2K21/MSCMAT/16) and Anshu (2K21/MSCMAT/04) of M.Sc. Mathematics, hereby declare that the project dissertation, submitted to the examination coordinator of our Department of Applied Mathematics at Delhi Technological University for the partial fulfillment of the requirement for the Master of Science degree to be awarded. The title of the report, "Analysis of Queuing Network theory in telecommunication," is original and not copied from any source without proper citation. No previous degree, diploma, associate ship, fellowship, or other title or honor has been granted based on this work.

Divya(2K21/MSCMAT/16)
Anshu(2K21/MSCMAT/04)

Place: Delhi

Date: 20/05/2023

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

Bawana Road, Delhi-110042

Certificate

I certify, that the work under the Dissertation Project with the title, "Analysis of Queueing Network theory in telecommunication", submitted by Divya(2K21/MSCMAT/16) and Anshu(2K21/MSCMAT/04), postgraduate students of the M.Sc. Applied Mathematics, Delhi Technology University, Delhi for the partial fulfillment of the requirement for the Master of Science degree, is a record of the students' work under my supervision. This work has not, to the best of my knowledge, been submitted for any degree at this university or elsewhere.

Prof. L.N. Das

SUPERVISOR

Abstract

Queueing network models are cost effective tools to analyze modern computer systems. This analysis shows the basic results using the operational research, which allows the analyst to test whether each assumption is meeting in a provided system. In the early section, we described the nature and basics of queueing network models. The basic performance terminologies like queue length, mean queue lengths, and mean response times, and operational relationships among them are derived. Following this, the concept of Markov chain, birth-death process is introduced. The proportions of time spent in each system state are then linked to job demand parameters and device characteristics using the concepts of state transition balance, one-step behavior, and homogeneity. Eventually, Markov models along with performance probabilities in transition while switching are discussed. Efficient methods for computing optimized path are also described for the transmission.

Key words: Queueing model, network theory, Markov model, servers, birth and death process, Poisson process, synchronizer.

Acknowledgement

We, Divya(2K21/MSCMAT/16) and Anshu(2K21/MSCMAT/04) of M.Sc. Mathematics, would like to express our sincere thanks and appreciations to our guide, Prof. L.N. Das, for his patient guidance, constant encouragement, endless support during our study.

We would like to thank “the Department of Applied Mathematics” of Delhi Technology University for providing a productive scientific research environment.

Preface

The significance of these technologies has been clear since the creation of the earliest telecommunication systems, such as telephone and telegraph networks. People must constantly interact, so the remote interchange of many forms of information is now crucial. Telecommunications are playing and will continue to play a crucial role in society, supporting national growth and the spread of knowledge. Initially, media communications were just considered as connections to send data between two focuses. System of networks; with nodes, where information is processed and appropriately addressed (i.e., switching); and links that connect nodes are currently what distinguish telecommunication systems. The transmission of messages served as the foundation for the initial telegraph networks. Then, the establishment of a physical circuit at call setup served as the foundation for telephone networks, connecting the source and destination throughout a conversation and fostering knowledge dissemination and national advancement. Telecommunications were first thought of as simple lines that could convey data between two locations. Due to telegraphy, message transmission was the foundation of the earliest telecommunications networks. To connect the source and destination (for the duration of the discussion) during call setup, telephone networks have historically relied on the building of a physical circuit. Telecom frameworks now have arrived at an overall dissemination based on the endeavors of global and provincial normalization bodies that have accomplished a critical work, permitting various bits of equipment to interoperate based on obvious guidelines. The digital representation of information has made it possible to efficiently integrate various traffic types, such as voice, video, data traffic, and so on, into the same network rather than having a specialized network for each type of traffic. The Internet, which is a network of networks, is experiencing tremendous success at the moment. The result of this amazing system is that the Web convention results as the paste that can bind together unique organization advancements, from portable to fixed and from earthbound to satellite. The provision of multimedia computing device services with global connectivity (including mobile users) and the guarantee of multiple Quality of Service (QoS) requirements that are differentiated based on the application the user is running (i.e., traffic classes) is the most important aspect of modern telecommunication networks. Additionally, network resources must be utilized effectively because they are pricey and precious. Modern network design necessitates a thorough understanding of the network's characteristics, transit signal transmission media, signal traffic demand statistics, and so on. Analytical techniques can be used to figure out the right number of links, the management strategy for sharing resources among traffic classes, and other things based on these data. The goal is to help establish the foundation for charge-current telecommunication networks' tele signal traffic analysis. The application of these analytical techniques to the study of telecommunication systems is a particular focus of research into queueing systems. In particular, queues can be used at various levels in signal sine receiver systems for communication; They can be used to study the waiting time of a specific request sent to a processor or the waiting time of a message or packet on a particular link or across a network as a whole. Specifically, a suitable queuing procedure enables the modeling of each and every protocol in a telecommunication signal server-receiver system network's nodes. The typical M/M/1 queuing model, which is used in message-switched networks, and the M/M/S/S queue, which is used to characterize the call loss behavior of local offices in telephone networks, will serve as the foundation for our investigation of queueing systems.

Then, more advanced ideas, like embedded Markov chains (M/G/1 theory) and the models used to study the behavior of ATM switches, will be the focus of interest. The provision of QoS is an essential component for both network operators and users who are satisfied with the telecommunications service they are adopting. Future telecommunication services and networks must be modeled and analyzed properly in order to produce an optimized network design that can ensure the right levels of Qi's for various traffic classes. The analytical techniques for telegraphic analysis are essential for telecommunication networks because of this.

The polar curve lines, asymptotic to the polar curves are the signal queue transport line. Transit signal is an exponential curve which is a part of elliptic curve. The exponential curves are one of the asymptotes which is parallel to initial line. There is a correlation with Markov signals with Markov model which we try to discuss in this report.

Contents

List of Figures

Summary of Nota

AWGN
Additive White Gaussian Noise

BER
Bit Error Ratio

BPSK
Binary Phase Shift Keying

CC1
Cochannel Interference

CDF
Cumulative Distribution Function

dB
Decibels

DA
Decision- Aided

DPA
Discrete phase adjustment

DTTL
Data Transition Tracking Loo

FIR
Finite Impulse Response

IID
Independent Identically Distributed

INR
Interference-t0-Noise Power Ratio

ISI
Inversible Interference

LPF
Lowpass Filter

NCO
Number-Controlled Oscillator

NDA
Non-Decision-Aided

OVFLW
Overflow signal from a random walk LF

PAM
Pulse Amplitude Modulation

PDF
Probability Density Function

PMF
Probability Mass Function

PSD
Power Spectral Density

PSK
Phase Shift Keying

QPSK
Quaternary Phase Shift Keying

RMS
Root Mean Square

RW
Random walk

SIR
Signal-to-Interference Power Ratio

SNR
Signal-to-Noise Power Ratio

STR
Symbol Timing Recovery

TDMA
Time-Division Multiple-Access

UNFLW
Underflow signal from a random walk LF

WGN
White Gaussian Noise

Symbols and Functions - Latin

ak Samples used for symbol detection	***bk*** Samples used for zero crossing direction
c Noise covariance matrix	***cij*** Element of the noise covariance matrix
E{.} Expected value	***Eb, E0*** Energy per bit of the desired signal
Ej Energy per bit of the jth cochineals interferer	***Erms*** RMS timing error
dm, d0,m The mth transmitted symbol of the desired signal	***dj, m*** The mth transmitted symbol of the jth
dr The rth sequence of data symbols of the ith interferer	***di,m*** The mth *transmitted complex data symbol of*
d0,m Receiver's estimate of the mth transmitted complex data symbol of the desired signal	***ek*** The kth three-valued filtered timing error estimat***e***
FA(n) Acquisition time CDF	***Fs(n)*** Symbol slip time CDF
fc Carrier frequency	***g(t)*** Baseband pulse shape after the matched filter
gs(t) The shifted baseband pulse shape	***gt(K)*** Gating signal in an STR structure that uses interpolation
I Number of taps in the interpolating filter	***h(t)*** Nyquist pulse shape

hr(t) Baseband equivalent receiver impulse response	***I*** identity matrix
I1 ,I2 Indices which represent the first and 1st taps of an Interpolating filter	***J*** Number of cochineals interferers
K Ratio of the interpolator output rate to the symbol rate	***Ka*** Number of samples combined in an accumulating LF
Kr,w Number of states in a random walk LF	***L*** Length of the system impulse response at the matched filter
M Ratio of the seraph rate to the symbol rate	***Ms*** Index of the largest ten used in the Fourier series expansion of the Gaussian CDF
mk Base point index	***N*** Number of states in a Markov chain
NP Number of carrier phases used in the approximation of an integral	***Ns*** Number of data sequences producing distinct sets of adjacent signal samples
n(t) Gaussian noise process with unit varia nce	***nw(t)*** Additive white Gaussian noise process
ni Sample of the unit variance Gaussian noise process	***P*** Matrix of transition probabilities
PA Matrix used to compute the acquisition time	***Ps*** Matrix used to compute the symbol slip time

Q(.) Complementary Gaussian distribution	***QA*** Sub matrix of P used to solve for acquisition time
QS Matrix used to solve for symbol slip time	***RK*** Contents of the register of a random walk LF at time tk
r(t) Received signal	***Sk*** The kth accumulator output sample
S(ΘK) Synchronizer S-curve	***Si*** The Ith sample of the noise-free received signal at the matched filter output
S0(t) Desired signal component of the received signal	***Sj(t)*** Component of the received signal due to the jth interferer
T Symbol period	***T0*** Parameter used in the Fourier series expansion for the tram t ion probabilities
TA Acquisition time	***Ti*** Inverse of the average interpolator output rate
Ts Symbol slip time	***topt*** Optimum sampling the of impulse response g(t)
u(n) The nth output sample of a loop filter	***ut(n)*** The nth output sample of the timing error detector
V(k) The kth synchronizer control word	***W(n)*** The nth NCO control word

Greek

α0 Carrier phase of the desired signal	***αj*** Carrier phase of the Jth interferer
β Excess bandwidth h parameter for a NY Quist pulse	***β0*** Carrier phase of the desired signal relative to the phase of the receiver LO
βj Carrier phase of the jth interferer relative to the phase of the Receiver LO	***€0*** Desired signal epoch relative to receiver clock
Өk, Өn Modulo-T timing era at time tk(tn)	***Өk*** second order Markov state at time tk
µk Fractional interval at time tk	***πi*** Steady-state occupancy probability
πA(0) Initial state occupancy vector for the acquisition process	***πS(0)*** Initial state occupancy vector for the symbol slip process
Φn One of the N possible timing error values	***Φk*** one of the 3N possible second order timing error states
Φ(.) Normalized Gaussian CDF	***W*** Parameter used in the Fourier series expansion for the transition probabilities
Φ2(h,k,p) Normalized trivariate Gaussian CDF	***Þ*** Covariance of two adjacent noise samples

CHAPTER 1

QUEUEING THEORY AND NETWORK ANALYSIS

Introduction

A queuing system is not a method of optimization. Instead, it is used to minimizing waiting times and maximizing service.

Stochastic theory of service systems is another name for queueing theory. It also goes by the names of the theory of public causes, tele traffic theory's mathematical approach, etc. Some individuals think that the stochastic theory of service system is the ideal name for it. Since the term indicates both the generality that all research objects may be defined by a single notion of service system and the uniqueness that the questions always have the appearance of randomness from a mathematical perspective. The mathematical analysis of queues or waiting lines is known as queuing theory, which is also frequently spelt as queuing theory. Several interconnected procedures lead to the

The following things characterize the kinds of networks that we are considering and the patterns that can be calculated by queuing theory:

- **Flow balance**: The number of jobs served by the network during a given time period is equal to the number of jobs that entered the network.

- **Homogeneity –** The mean time between service completions at a given node must not be dependent on the lengths of the queues at other nodes. Job routing and servicing at a node are independent of the lengths of the local queues. Because a packet has a fixed length and requires a fixed (and predetermined) amount of service when it enters a queue, a packet moving from one queue to another invariably has a significant impact on the latter. However, uniformity is evident in practice. The response given by gateways, which serve multiple links in the network and must respond to queue traffic, is typically random. This gives the presumption functional legitimacy.

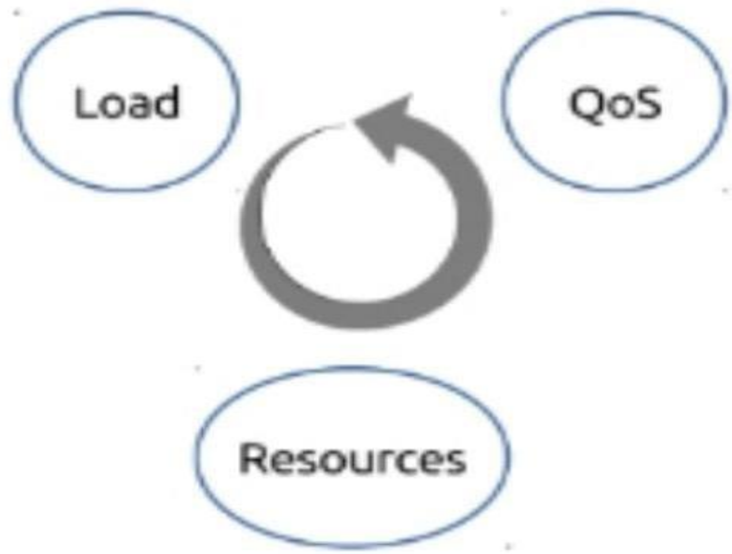
- **Fair share service-** At a node, the service to one packet is either given at random or according to first-come, first-served policy. In a network with access based on a non-preemptive, class-priority scheme and service is first-come, first-served inside a priority class, the same is considered to be true. Nodes with sufficient resources have limitless buffers and quick access to all resources required for service. Neither model effects nor packet retransmission are ever required for bottlenecks at a node.

- **Poisson arrivals:** Jobs arrive at random intervals with an exponential distribution. This is a basic premise for these models.

• **Persistence -** Each node that has a packet to send is tenacious in seeking for service and transmits as soon as it does (an arbitration process is necessary to break ties).

• **Loop less flow**: No job ever needs to be completed at the same node twice. In an open network, a job leaves the network when it reaches its destination.

- **Dependability:** Hardware never breaks down. These effects don't belong in the mode.



1.0 three associated posts is in this system dimensioning

No assumptions have been made about the distribution of service time as they are not necessary; queuing theory is adequately developed to manage general service distributions. Here, the network is loop less and open as well, the end-to-end delay in the packet switching is nothing but the sum of all the individual delays in every sub network and the other network systems, i.e. the network can be decomposed completely which allows us to model them simply in isolation, separately. The only effect of one piece of the network upon another is the induced traffic burden is one thing on another. The amount of traffic must be known in order to predict how a network will behave. The typical behaviour of the network under the specified traffic load can be modeled using the size distribution of packets, the rate of packet transmission, and the service time needed to send a packet. It is possible to determine the transmission delay between two nodes for a file with several packets after the isolated behaviour of the network's individual links and the traffic flow through the network has been determined.

The following variables are used to describe the overall behaviour of network links:

• U: The percentage of time that the network is active. The utilization is what is meant here.

• Q: The average number of packets in the transmission queue.

• R (or the network's reaction time in seconds): This is the duration of a packet's end-to-end delay. It comprises the time spent waiting for the packet as well as the time spent serving it.

The subscript 'a' on a symbol in the following identifies its average value. The values we must extrapolate from the workload are as follows:

• Ja **-** The Average packet size (bytes)

The second moment of a packet size distribution is denoted as S2.

• A **-** The packet arrival rate (packets/sec)

The information about the communication channel required is

- **C**: The capacity of a channel, (bytes/sec)

- **F**: The fixed overhead time for channel usage (seconds) S,
- The service time of a packet say with length of L byte is

$$S=T+F$$

where T is the time of transmission for the L byte packet, is

$$T=L|C$$

Finally, we have the following derived data are of interest to both network users and engineers:

- The capacity of the cable after adjusting for overhead transmission time is the effective transfer rate

$$C.Ta/Sa$$

- The actual traffic or the number of bits transmitted over the network

$$A.Ja$$

- The average waiting time to access the network in seconds

$$Wa$$

- The marginal rate of transfer

$$C.Sa/ (Sa+Wa)$$

The perceived transfer rate of a recently arriving packet is represented by this marginal rate. In other words, it is the data transfer rate that has been reduced by the amount of time spent waiting for the channel to open.

In order to make this report self-contained, the fundamental relationships among the variables described above are presented below. We simply have to apply some specifics of the access protocol and other overheads for particular networks in order to create a model.

Assumedly, the work load consists of a variety of packets with sizes s1, s2,..., sn bytes and relative frequency of occurrences f1, f2,..., fn.

$$\sum_{i=1}^{n} f_i = 1$$

And,

$$J_a = \sum_{i=1}^{n} f_i s_i$$

The time of service on average is denoted by

$$S_a = (J_a \ / C + F)$$

And, the second moment of the service time is

$$S^2 = \sum_{i=1}^{n} f_i \left(s_i/C + F\right)^2$$

$$= \frac{1}{C^2} \sum_{i=1}^{n} (f_i s_i^2) + 2 F J_a / C + F^2$$

The utilization U is

U=A.Sa

The average length of the queue is

$$q_a = U + \frac{A^2 S^2}{2(1 - U)}$$

The average time spent in the system is given by,

R = (qa / A) + any other fixed overheads

And the average waiting time in the system is

$$W_a = \frac{(q_a - U)}{A}$$

All these formulae of a single server queuing system, model the average behavior, where, *qa* is a measure of the average congestion.

1.1. Modeling

Compared to even a few years before, modern computer systems are more complicated, evolve more quickly, and are more crucial to the operation of company. As a result, there is a growing need for methods and strategies that might help us comprehend how complex systems behave. Such comprehension is important to offer perceptive responses to cost and performance queries that surface during the course of a system's life:

• while it is being designed and implemented - A company in the aerospace industry is developing a computer-aided design system that will enable several hundred aircraft designers to access a distributed database at once using graphics workstations. Mechanisms for coordination and communication. Prior to implementation, it is necessary to assess the relative benefits of several approaches.

In order to connect terminals to mainframes via a packet-oriented broadcast communications network, a computer maker is looking at different architectures and protocols. Can terminals be grouped together? Do packets need to include several characters? Should characters coming from many terminals and going to the same mainframe be multiplexed together into one packet?

• During acquisition and sizing

- A productive method of system sizing is required by the maker of a turnkey medical information system in order to prepare bids. This vendor must forecast the reaction times that the system will offer when operating on different hardware configurations based on estimates of the arrival rates of transactions of various categories.

- Twenty offers were received by a university in response to a request for

Suggestions for interactive computing in undergraduate education. Comparing the capacity of these 20 systems is crucial to the procurement since the "cost per port" among those systems meeting certain mandatory specifications is the selection criterion. There is only one month left to make a choice.

• As the workload and configuration change, a stock exchange plans to start trading a new class of options. The overall volume of option trades on the exchange is anticipated to grow by a factor of seven when this happens. When the modification is put into effect, there must be enough staff and computer resources available.

In light of anticipated increases in workload, an energy utility must evaluate the durability of its current setup. Knowing the anticipated system bottleneck and the relative cost effectiveness of several solutions to relieve it is desirable. Since this is a virtual memory system, it is especially important to weigh the trade-offs between memory size, CPU power, and paging device speed.

Unfortunately, these questions are also complex, making it difficult to come up with the right answers despite the fact that they are of enormous significance to the organizations concerned.

One must have a firm understanding of the system, the application, and the study's goals before even starting to think about questions like these. Using this as a foundation, other strategies are possible.

One is the application of trend extrapolation and intuition. There aren't many alternatives to the level of knowledge and wisdom that results in trustworthy intuition, it's true. Unfortunately, people with enough of these attributes are hard to find. The experimental assessment of options is still another. Experimentation is always worthwhile, frequently necessary, and occasionally the best course of action. Additionally, it is costly—often unreasonably so. Another issue is that an experiment is unlikely to provide insight that would enable generalization, just accurate knowledge of system behaviour under one set of assumptions.

These two strategies are, in a way, on opposite ends of a spectrum. Although intuition is quick and adaptable, its accuracy is questionable since it depends on knowledge and understanding that are challenging to get and confirm. Experimentation produces good accuracy but is time-consuming and rigid.

A system is abstracted in a model, which aims to isolate the specific elements necessary for the behavior of the system from the myriad intricacies that make up the system itself. A model can be parameterized to reflect any of the under-consideration alternatives once it has been defined through this abstraction process, and it can then be tested to see how it would behave in that scenario. Because a model is an abstraction that removes pointless detail, it is less time-consuming and more adaptable than experimental to study system behavior. Because it is more methodical, it is more dependable than intuition as a result of the frameworks that each distinct modeling approach provides for the design, parameterization, and evaluation of models. In addition, using a model enhances both intuition and understanding. Therefore, modeling offers a framework for compiling, categorizing, assessing, and comprehending data regarding a computer system.

Queuing Network Model

Queuing network modeling is a specific type of computer system modeling in which the computer system is modeled as an analytically analyzed network of queues. An arrangement of service centers, which stand in for system resources, and customers, which stand in for users or transactions, is referred to as a network of queues. In order to do an analytical evaluation, a set of equations caused by the network of queues and its parameters must be efficiently solved using software.

Centers with a single server

A single service center is shown in the figure below. Customers arrive at the service center, obtain assistance from the server, wait in queue if necessary, and then leave. In actuality, the clients who are arriving at this service center make up a (slightly degenerate) queuing network model.

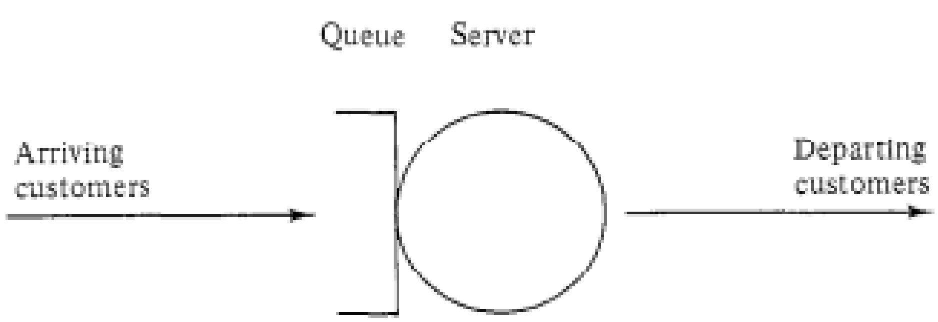


Figure 1.1 — A Single Service Center

The parameters of this model are two. The workload intensity, in this case the rate of customer arrival (e.g., one customer every two seconds, or 0.5 customers/second), must be specified first. The average service demand for a customer is what we need to define next (for example, 1.25 seconds). By resolving a few straightforward equations for particular parameter values, it is possible to evaluate this model and obtain performance metrics like utilization (the percentage of time the server is busy), residence time (the average amount of time a customer spends at the service center, both waiting in line and receiving service), servicing facility). These performance indicators apply to our sample parameter values under the following assumptions:

Utilization is.625

3.33 seconds are spent in residence.

- 1.67 clients are in the queue.

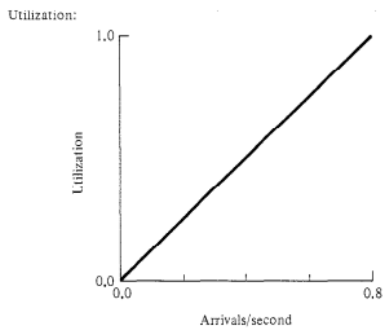
- Throughput: 0.5 clients per second

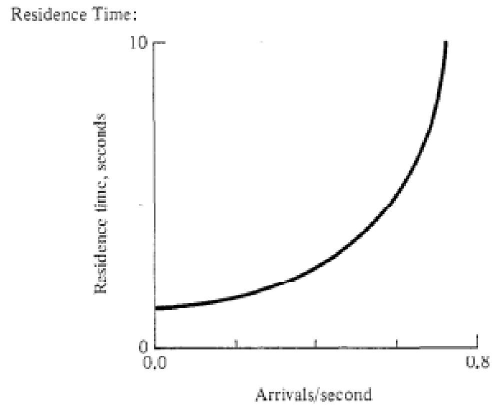
All performance indicators which are shown in the graphs below as the workload intensity varies from 0.0 to 0.8 arrivals/second. That is the interesting range of value for the parameter. It is easy to see that arrival rate can't be less than zero at the lower end.. The optimized rate of service center to handle the customers is one per 1.25 seconds, or 0.8 customers/second; for greater arrival rates, service center will attain its saturation state. This is because the average service requirement for a customer is 1.25 seconds.

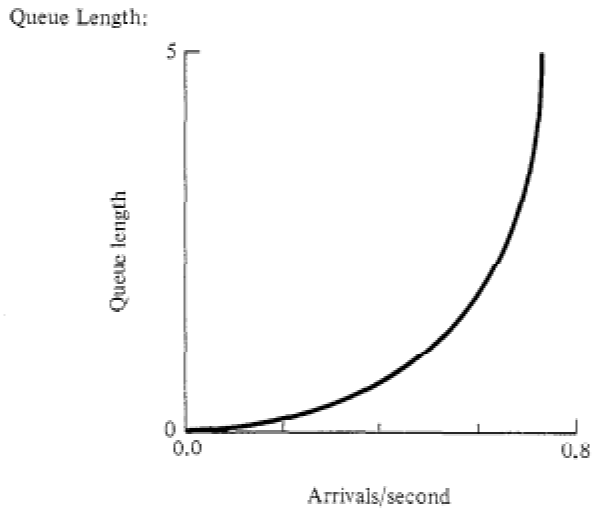
The most important thing to note about Figure 2 is that performance measurements produced by the model's evaluation are qualitatively compatible with intuition and experience. Think about your home time.

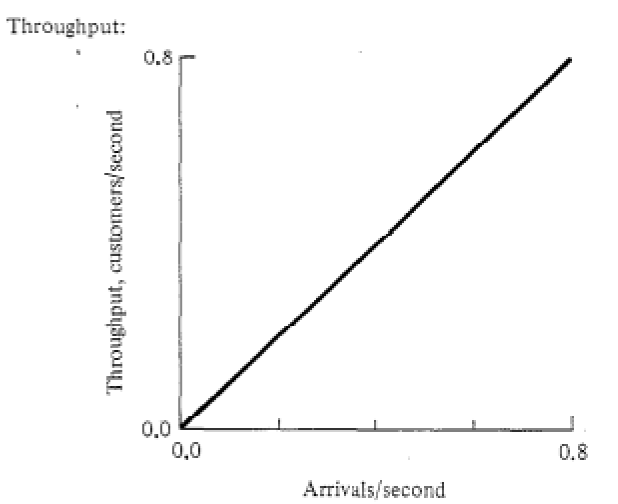
We anticipate that an entering customer will rarely face competition for the service Centre when workload intensity is low, so they will start receiving service right away and have a residence duration that is nearly equivalent to their service requirement.

Congestion and residence time grow in tandem with rising workload intensity. At first, this increase is gradual.







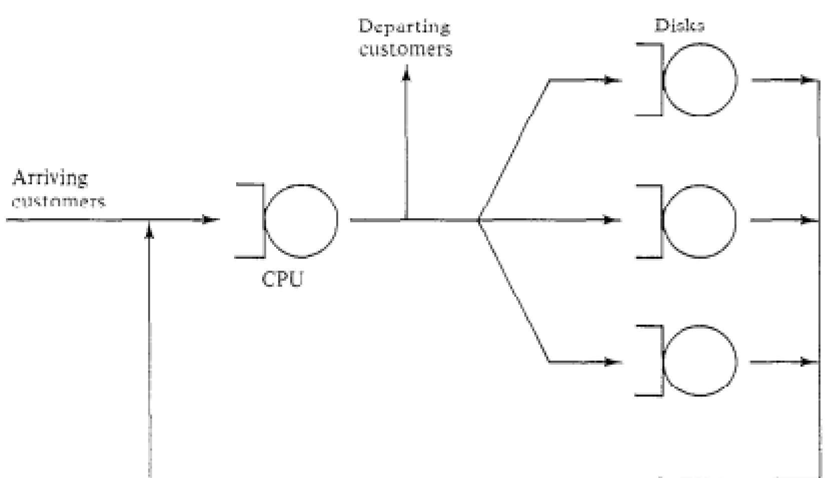


1.2 Performance Measures for the Single Service Center

However, residence time grows more quickly as the load increases, and as the service Centre gets closer to saturation, tiny increases in arrival rate lead to dramatically longer residence times.

• A number of Service Centers

It is difficult to fathom defining a modern computer system by just two parameters as would be necessary to use the figure model. (In actuality, though, this was attempted and accomplished on numerous occasions back in the simpler 1960s.) The model depicted in Figure is more realistic and represents each system resource (in this case, a CPU and three discs) as a single service Centre.



1.3 A Network of Queues

These model's parameters are similar to those of the preceding one. The workload intensity, which is the frequency of customer arrivals, must be specified. The service demand must be specified as well, but this time we give a different service demand for each service Centre. Considering that the model's customers match to

Service demand at each service Centre corresponds to the overall service required per transaction at the relevant resource in the system, and the workload intensity at each service Centre relates to the pace at which users submit transactions to the system. We may imagine clients entering, moving about the service centers, and finally leaving, as shown by the lines in the diagram. Only the total service demand at each Centre matters; the pattern of circulation between the centers is irrelevant.) For instance, we could define that each transaction requires an average of 3 seconds of CPU service, 1, 2, and 4 seconds of memory service, and that transactions occur at a rate of one every five seconds. of service at the three discs, in that order. Similar to the single service Centre example, this model can be evaluated for certain parameter values by resolving a few straightforward equations. Under specific assumptions that will be given later, parameter values for our example performance measurements consist of:

CPU utilization: 0.60; average system response time as perceived by users: 32.1 seconds; average number of concurrently active transactions: 6.4; system throughput: 0.2 transactions/second (We consistently refer to residence time as the amount of time a customer spends in a service Centre, and response time as the intuitive concept of perceived system response time. Instead of distributional data (such as the 90th percentile of response times), average values (such as average response time) are the most common performance metrics derived from queuing network models. Thus, even if the word "average" is eliminated, the meaning should still be clear.)

1.1.1. Markov Model

A (first order) Markov model depicts a series of stochastic events where each event's probability of transition is solely determined by the previous event's final state. There is therefore no "memory" other than what happened before. A Markov process is a series of subsequent events that can either be discrete or continuous, with transitions occurring only at specific times. When a component's failure rate is constant and the likelihood that it will perform dependably at time t, Pi(t), declines with time as Pi(t) =λ it, the continuous Markov process works well. The process is referred to as homogeneous if all i are the same, but typically for a system this will not be the case, in which case the process is referred to as semi-Markov. The following equation expresses the probability of a homogeneous process's P system states as a function of time t:

$$P_j(t+\Delta t) = \sum_{k\neq j} P_k(t)\lambda_{kj}(\Delta t) + P_j(t)\left(1 - \sum_{k\neq j} \lambda_{jk}\Delta t\right)$$

Where Ki is the system's rate of transition from state k to state j and j to state k. To determine all possible states of the system while using a semi-Markov model to solve a system with n components, n simultaneous differential equations must be solved. This can only be accomplished for a large system using Markov Chain Monte Carlo (MCMC) simulation. Because Markov modeling can simulate redundancy and repair times, it is frequently used in reliability analysis and is a useful tool for predicting a system's predicted dependability and availability over time. Repair rates can be added in the same way as failure rates in order to restore component functionality.

Birth and death process

The birth-and-death processes represent a significant subclass of the class of all continuous-time Markov chains. Every time a transition takes place from one state to another, these processes are characterized by the fact that this transition can only take place to a neighboring state. If the state space is S = 0, 1, 2,..., i,..., then a transition from state i can only take place to a neighboring state (i- 1) or (i + 1).

A continuous-time Markov chain with state space, X(t), t, and T S = {0,1, 2,…} and with rates

$$q_{i,i+1} = \lambda_i(\text{say}), \quad i = 0, 1, \ldots,$$
$$q_{i,i-1} = \mu_i(\text{say}), \quad i = 1, 2, \ldots,$$
$$q_{i,j} = 0, \quad j \neq i \pm 1, \quad j \neq i, \quad i = 0, 1, \ldots, \quad \text{and}$$
$$q_i = (\lambda_i + \mu_i), \quad i = 0, 1, \ldots, \quad \mu_0 = 0,$$

If i = 0 for i = 1, 2,..., then the process is known as a pure birth.

- a birth-and-death process if some of the i's and some of the i's are positive. - a pure death process if i = 0, i = 0, 1, etc.

The Chapman-Kolmogorov forward equations for the birth-and-death process can be obtained using the aforementioned equations.

For *i*, *j* = 1,2, …,

$$p'_{ij}(t) = -(\lambda_j + \mu_j)p_{ij}(t) + \lambda_{j-1}p_{i,j-1}(t) + \mu_{j+1}p_{i,j+1}(t)$$

and,

$$p'_{i0}(t) = -\lambda_0 p_{i0}(t)\mu_1 p_{i,1}(t).$$

The boundary conditions are

$$p_{i,j}(0+) = \delta_{ij}, \quad i, j = 0, 1, \ldots.$$

Denote

$$P_j(t) = Pr\{X(t) = j\}, \quad j = 0, 1, \ldots, t > 0$$

and assume that at time *t* = 0, the system starts at state *i*, so that

$$P_j(0) = Pr\{X(0) = j\} = \delta_{ij},$$

then

$$P_j(t) = p_{ij}(t),$$

and the forward equations can be written as

$$P'_j(t) = -(\lambda_j + \mu_j)P_j(t) + \lambda_{j-1}P_{j-1}(t) + \mu_{j+1}P_{j+1}(t), j = 1, 2, \ldots$$

$$P'_0(t) = -\lambda_0 P_0(t) + \mu_1 P_1(t).$$

-eqn.(1&2)

Suppose that all the λi's and μi,'s are nonzero. Then the Markov chain is irreducible. It can be shown that such a chain is non-null persistent and that the limits

$$\lim_{t\to\infty} p_{ij}(t) = p_j$$

exist and are independent of the initial state *i*. Then Eqs. (1) and (2) become

$$0 = -(\lambda_j + \mu_j)p_j + \lambda_{j-1}p_{j-1} + \mu_{j+1}p_{j+1}, \quad j \geq 1$$

$$0 = -\lambda_0 p_0 + \mu_1 p_1.$$

Define

$$\pi_j = \frac{\lambda_0\lambda_1 \ldots \lambda_{j-1}}{\mu_1\mu_2 \ldots \mu_j}, \quad j \geq 1, \quad \text{and}$$

$$\pi_0 = 1;$$

then the solution of the above can be obtained by induction. We have from (1.4.8)

$$p_1 = \left(\frac{\lambda_0}{\mu_1}\right)p_0 = \pi_1 p_0$$

and assuming $pk = \pi k p0, k = 1,2,\ldots,j$, we get from above equation

$$p_{j+1}\mu_{j+1} = \lambda_i \pi_j p_0, \quad \text{or}$$

$$p_{j+1} = \pi_{j+1}p_0.$$

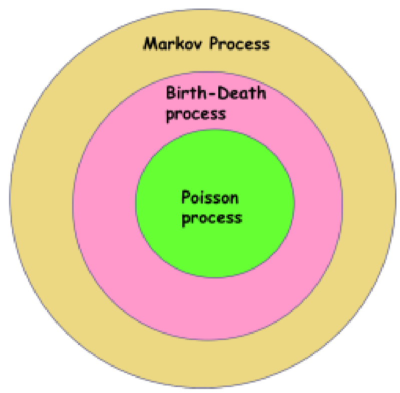
Thus, it Σk=0 to ∞ $\pi k < \infty$, then

$$p_j = \frac{\pi_j}{\sum \pi_k}, \quad j \geq 0.$$

In addition, the existence of k is a necessary requirement for the birth and death process to have all non-null persistent states (and, hence, for the process to be ergotis).

Given that many queuing systems may be modeled as birth-and-death processes, queuing theory is particularly interested in this process.

The Poisson Process is a particular type of birth-death process where the population size is initially zero and the birth probability per time unit is constant.



1.4. poison process

A Poisson Process is a mathematical model for a sequence of discrete events where the exact timing of events is random but the average duration between events is known. An event's arrival is unrelated to the event that came before it (the interval between events is memory less). For instance, let's say we run a website that, according to our content delivery network (CDN), averages one outage every 60 days, but the likelihood of the next one is unaffected by the previous one. The average interval between failures is all we are aware of. This Poisson process appears to be:

1.5 Failures over time

1. The crucial distinction is that occurrences are stochastically spaced yet we know the typical interval between them. Due to the randomness of the process, we can experience back-to-back failures or years between them.

2. The following characteristics—many events that are modeled as Poisson processes don't actually match these requirements—define a Poisson Process:

3. 1. Things don't depend on one another. The likelihood that one event will happen has no bearing on the likelihood that another event will happen.

4. 2. There is consistently the same number of events per unit of time. Two events cannot occur at the same time.

The final feature, that events are not simultaneous, allows us to think of each Poisson process sub-interval as a Bernoulli trial, in which there is either a success or a failure. When it comes to our website, the overall period may be 600 days, yet every sub-interval of one day either results in a website outage or not.

Customers calling a help line, website visits, radioactive decay in atoms, photons arriving at a space observatory, and changes in stock prices are typical examples of the Poisson process. Poisson processes typically involve time, however this is not a requirement. We may have a Poisson process for the number of trees on an acre (events per area) in the stock scenario, in addition to knowing the average daily movements (events per time) for the stock.

(An example of a Poisson Process that is usually cited is the arrival of buses, trains, or, more recently, Umbers. The arrivals are not independent of one another, hence this is not a real Poisson process. Even with bus networks that don't run on time, a bus' tardiness influences when the following bus will arrive.

Markov Modeling to synchronize digital signals

To analyses DPLLs and symbol synchronizers with discontinuous phase adjustment (DPA), first order Markov chains are frequently employed. When a binary quantized DTTL and a DPLL were examined in the literature in 1972, Markov models of DPA tracking circuits were first introduced. Analyzing an analogous analogue loop is an alternative to performing a Markov analysis of DPA circuits; this method was best described in. The linear technique has the drawback that some performance statistics are significantly simpler to derive from a Markov model and that it is only valid for moderate and large SNRs. Since the method's inception, several Markov assessments of DPA tracking loops have been published. DPA symbol synchronizers' Markov analyses were published. Although it has been established that a first order model is an approximation of the actual performance, the literature on some of the synchronizers lacks an explanation of why the models are approximations. In this part, we discuss the shortcomings of a form order model of the binary quantized DTTL. We also describe how the generated transition probability matrix may be understood as an accurate computation of the transition probabilities at the beginning of the synchronization procedure. We compute a noise-adjusted transition probability matrix and discuss the key performance metrics that can be derived from the first order model.

CHAPTER 2

FIRST ORDER MARKOV MODEL

2.1 First Order Model Descriptions

The random process satisfies the condition given below if it is a first order Markov chain, where k varies in natural numbers.

$$\Pr\{\theta_{k+1} | \theta_k, \theta_{k-1}, \ldots, \theta_0\} = \Pr\{\theta_{k+1} | \theta_k\},$$

Relation Pr, for all k is greater than zero. If it is also homogeneous, and θ_k can assume at most N values, than $P_r \{\theta_{k+1} \backslash \theta_k\}$ is independent of k, and can be written as on N x N matrix of single step transition probabilities, P. Each element of P, ij, represents the probability that the next state θ_j, given that the current state is θ_i. In practice, a timing error sequence does not satisfy the above equation, because the next timing error value θ_{k+1} is a function of not only θ_k, but also of θ_{k-1} and other previous timing errors. In general, $\Pr\{\theta_{K+1} \backslash \theta_k)$ is also a function of k. Therefore, a first order the timing error process has a rough Markov model. No choice of P provides exact results; however, some P options may yield correct performance outcomes for the relevant statistics. we use a first order Markov chain model to determine how well the synchronizer is working. Due to the aforementioned factors, the model is approximate. For each of the N possible sampling phases φ_i and conditional on a single symbol sequence, d, the author calculates the joint PDF of three successive samples of the received signal to arrive at the answer, P. Here, the symbol d stands for a series of data symbols (d-1, d0, d1, etc.). The unconditional transition probabilities are then determined by taking an average over d potential sequences.

We now discuss the interpretation of this calculation procedure for calculating the Pr as a precise computation of $P_r\{\theta_1 \backslash \theta_0\}$ the process's initial set of state transition probabilities. We also clarify why $P_r\{\theta_{k_{+1}} \backslash \theta_k\}$a synchronizer in Fig. as a function of k is. The input data sequence d is present, for a certain period of time before the synchronization procedure starts. As a result, ISI from past and future symbols can be seen in the synchronizer's initial sample data. When the synchronizer is turned on, its first sampling phase, Bo, randomly chooses one of the N possible values &. To determine the first timing error estimate, y0, the synchronizer collects three initial samples with phase Bo. Since O0, solely depends on a random value in a synchronizer register and not on any decisions involving the received signal, it is independent of the data sequence d in this initial stage. θ_1 , $P_r\{\theta_1 \backslash \theta_0\}$, involves determining the n conditioning transition probabilities under the premise that each data sequence has an equal likelihood and the transition probabilities conditioned on a specific data sequence. Two further signal samples are obtained after the synchronizer's initial phase adjustment, this time with sampling phase _1. The second timing error estimate, y1, is calculated using these two data along with the final sample from the three original samples. Contrary to the random variable &, O1 is dependent on the data sequence, d; hence, not every sequence has the same likelihood of producing a specific timing mistake BI. Pr4[01) calculations that rely on the independence of O1 and the data sequence are consequently approximations. This holds true for d additional distributions as well.$P_r(\theta_1$), k > 2. To $P_r\{\theta_1 \backslash \theta_0\}$ in summary, a first order Markov chain can be used to approximate the performance of the synchronizer in Fig. but does not perfectly represent the timing error process "_k" of the synchronizer. Additionally, it is possible to interpret the method employed to compute the transition probabilities of the estimated fist order chain mode1 as an exact method of calculation. Bo is first calculated precisely with this method; however the filtered noise samples are the link to was then deemed independent to reduce computational complexity, meaning that $P_r\{\theta_1 \backslash \theta_0\}$is no longer exact. In the next section, we derive an expression for

$P_r\{\theta_1 \backslash \theta_0\}$ It takes into account noise correlation for the significant particular case where only nearby noise samples are present.

2.2 Transition Probabilities of the First Order

first order of transition probabilities Pr{$\theta_1|\ \theta_0$) depend on error esthete y, a property of the first three samples of x(t), which is a function. These three examples are designated as

$$x_i = x[(i-1)/2 + \tau_0], \qquad i = 1, 2, 3.$$

Since the timing phase is updated every period and x(t) is sampled twice every symbol interval, only two subsequent samples should be dependent on one another. Since three samples are needed to provide the first error estimate y_0, the situation of _0 is an exception. Additionally, one can infer that $\tau_0 - \epsilon_o \in \left(-\frac{1}{2}, \frac{1}{2}\right]$without loss of generality. Therefore, mod($\tau_0 - \epsilon_o$)=($\tau_0 - \epsilon_o$) and. Each ($\tau_0\ = \theta_0 + \epsilon_0$) For each i, xi = si+ ni because the sample is the total of the signal and noise components. The signal elements come from

$$s_i = \sqrt{2E_b/N_0} \sum_m d_m\, g(\theta_0 + \epsilon_0 + (i-1)/2 - m), \qquad i = 1, 2, 3,$$

where g(t) is considered to be zero outside the range [O, L], resulting in the sum in the previous equation having L nonzero components. Keep take mind that is contains ISI from nearby symbols. _o = 0.0 for the Integrate-and-Dump filter and the NY Quist pulse with 100% extra bandwidth. _o = 0.223 for the Butterworth filter of the third order. For the square pulse, L = 2 is taken as the response length, while for the NY Quist pulse, L = 4, while L = 5 for the Butterworth pulse. The components of the noise covariance matrix, C, are assumed to be Gaussian with unit variance for the noise samples and

$$C_{ij} = E\{n_i n_j\} = \begin{cases} 1, & i = j \\ \rho, & |i-j| = 1 \\ 0, & \text{elsewhere.} \end{cases}$$

We now derive the transition probabilities. Note that

$$\Pr\{\theta_1|\theta_0\} = \sum_{r=1}^{N_S} \Pr\{\theta_1, \mathbf{d}_r|\theta_0\},$$

where Ns is the number of data sequences that produce distinct sets of the three samples, and d is a sequence of data symbols. x = [x1, x2, x3] Each xi is a function of L symbols, with symbol periods of one between x1 and x3. Ns is 2L+1 as a result, and we have

$$\Pr\{\theta_1|\theta_0\} = \frac{\sum_{r=1}^{N_S} \Pr\{\theta_1|\theta_0, \mathbf{d}_r\} \Pr\{\theta_0, \mathbf{d}_r\}}{\Pr\{\theta_0\}},$$

and since the initial timing phase θ_0 and the data sequence dr, are independent, becomes

$$\Pr\{\theta_1|\theta_0\} = \sum_{r=1}^{N_S} \Pr\{\theta_1|\theta_0, \mathbf{d}_r\} \Pr\{\mathbf{d}_r\}.$$

the a priori probability of each sequence Pr{dr) is equal to l/Ns . Therefore, we have

$$\Pr\{\theta_1|\theta_0\} = \frac{1}{N_S} \sum_{r=1}^{N_S} \Pr\{\theta_1|\theta_0, \mathbf{d}_r\}.$$

The transition probabilities for operating without an LF are found in this section. The likelihood of delaying the sample phase is the probability that yk = -1 in the absence of an LF. The retard probability can also be stated as Pr if the first sampling phase is i.$\{\emptyset_i, \emptyset_I + |\theta_0 = \emptyset_N\}$ or more concisely, , Pi ,i+1 .], the subscript (i + 1) should be interpreted as a rnodulo-N quantity, so that if i = N, the probability of retarding the phase is Pr{$\emptyset_i = \emptyset_I| = \emptyset_N$ or This also applies to the probability of advancing the sampling phase, pi ,i-i

$$p_{N,1} = \Pr\{\theta_1 = \phi_1|\theta_0 = \phi_N\}.$$

This also applies to the probability of advancing the sampling phase, Pi ,i-1 , we have

$$p_{i,i+1} = \frac{1}{N_S}\sum_{r=1}^{N_S}[\Pr\{x_1 < 0, x_2 < 0, x_3 > 0 \,|\, \theta_0 = \phi_i, \mathbf{d}_r\} + \Pr\{x_1 > 0, x_2 > 0, x_3 < 0 \,|\, \theta_0 = \phi_i, \mathbf{d}_r\}] .$$

Some manipulation is given by

$$p_{i,i+1} = \frac{1}{N_S}\sum_{r=1}^{N_S}\int_{-\infty}^{-s_1}\int_{-\infty}^{-s_2}\int_{-s_3}^{\infty}\phi_3(n_1,n_2,n_3)dn_3dn_2dn_1 + \int_{-s_1}^{\infty}\int_{-s_2}^{\infty}\int_{-\infty}^{-s_3}\phi_3(n_1,n_2,n_3)dn_3dn_2dn_1$$

$$= \frac{1}{N_S}\sum_{r=1}^{N_S}[\Phi(-s_3)\,\Phi(s_1) + \Phi_2(-s_1,-s_2,\rho) - \Phi_2(-s_2,-s_3,\rho)] ,$$

where $\emptyset_3(n_1, n_2, n_3)$ is the trivariate normalized Gaussian PDF, and its covariance matrix is (. For the sake of conciseness, the dependence of s1, s2, and s3 on SNR and the data sequence is suggested rather than explicitly demonstrated. From this point forward, the dependence of the is terms on the data sequence will be suppressed. Given by is the Gaussian univariate CDF.

$$\Phi(x) = (\sqrt{2\pi})^{-1}\int_{-\infty}^{x}\exp[-t^2/2]\,dt,$$

and the normalized Gaussian bivariate CDF is a2(h, k, p) = ~h, &(z, y, p) dz dy, where

$$\phi_2(x,y,\rho) = (2\pi\sqrt{1-\rho^2})^{-1}\exp\left[-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right].$$

The probability that the phase remains unchanged is given by

$$p_{i,i} = \frac{2}{N_S}\sum_{r=1}^{N_S}\Phi(s_1)\cdot\Phi(s_3).$$

The probability of advancing the sampling phase is

$$p_{i,i-1} = 1 - p_{i,i} - p_{i,i+1}.$$

The matrix P has nonzero entries on the main diagonal, the first upper and lower diagonals, and elements P1,N and PN,1. It is N x N in size. Fig. displays a state diagram for the Markov mode

2.3 Calculating the Performance Statistics

The root-mean-square (RMS) timing error, the mean time to acquire synchronization, and the mean symbol slip time are three crucial synchronizer performance statistics that may be solved for using the state transition probability matrix P. The steady state timing error distribution, Pr, is first solved for in order to determine the RMS timing error.

$\{P_r\,\{\theta_k = \emptyset_i\}$). This is a vector containing N elements, denoted

$\boldsymbol{\pi} = [\pi_1, \pi_2, \ldots, \pi_N]$.

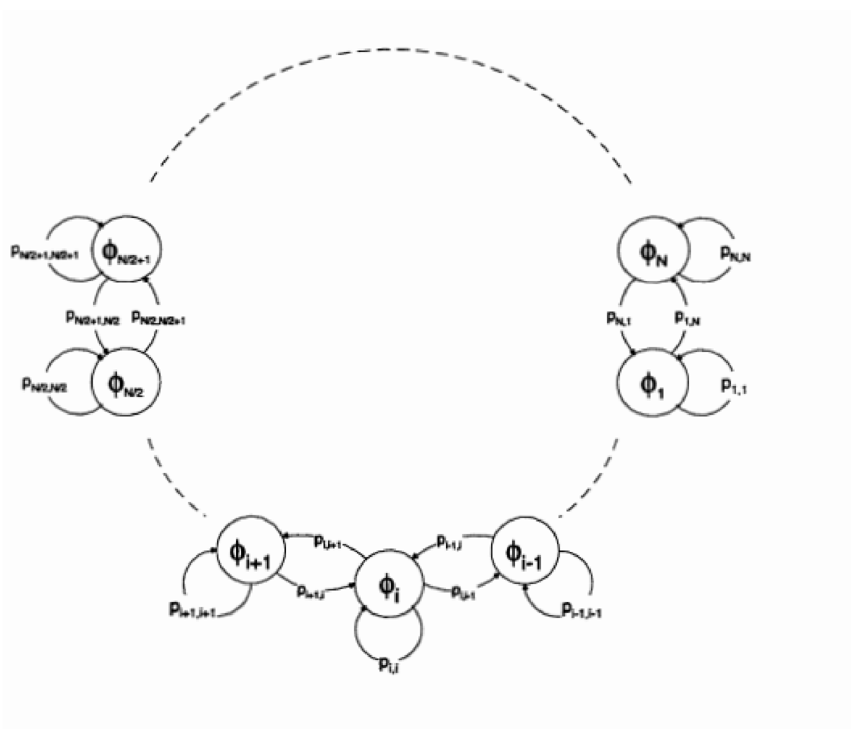


Figure 2.1 Block diagram of the first order Markov chain model.

We solve the equation z = xP with the restriction zc, ri = 1 in order to determine the steady state distribution. In Appendix A, we cover two methods for doing this work. The method employed to determine the steady state distribution Pr,k) converges rather slowly for the low SNR values examined in this chapter. We employ the stochastic complementation method, which is efficient and precise for SNR values below 0 dB. Appendix B contains the stochastic complementation approach's specifics. Given is the RMS timing error. $E_{rms} = \left[\sum_{i=1}^{N} \phi_i^2 \pi_i\right]^{1/2}$

Where is defined in and has the value. A first passage time problem can be solved using the mean time to acquire and mean symbol slip times, respectively. The initial passage time for a first order chain with transition matrix P is given by $T = \pi(0)[\mathbf{I} - \mathbf{Q}]^{-1}\mathbf{1},$

Where 1 is the identity matrix, 1 is a column vector of ones, and Q is a matrix that only contains the rows and columns of P that correspond to non-absorbing states. The initial non-absorbing state distribution is 0 in number. First, think about the mean symbol slide the, TVs, which is the average number of steps needed to go from state to state straight from state &v or state to state directly from state &. $\frac{\emptyset_n}{2}\ and \frac{\emptyset_n}{2+1}$. The process begins with a timing error that is one of the two minimum values. The initial distribution is given by

$$\Pr\{\theta_0 = \phi_i\} = \begin{cases} 1/2, & i = N/2, N/2+1 \\ 0, & \text{otherwise.} \end{cases}$$

We add a new stage, known as the slip state, to the previously stated chain in order to mode1 the symbol slip process. This indicates the slip state. The transition from state #N to state dl is thus replaced by a transition from state q5N to Similarly, the transition from state di to state #N is replaced by a transition from state to. The vector of non-absorbing states is designated as _s=0, and it has a length of N. Given by, the mean slip time is

$$T_S = \pi_S(0)[\mathbf{I} - \mathbf{Q}_S]^{-1}\mathbf{1},$$

Qs is identical to P except that Pn,1 and Pn,2 zero. we consider now the mean time to acquire synchronization, TA, which is defined as the mean number of steps required to reach either state π_{A_0}N/2 or state 4NIZ+1 for the first time. The acquisition process is equal likely to begin in any of the N states, so the initial distribution is given by

$$\Pr\{\theta_0 = \phi_i\} = 1/N, \quad 1 \le i \le N.$$

No additional states are needed to represent the acquisition process as a Markov chain, unlike the symbol slip analysis above. The initial state occupancy vector,$\emptyset_N$ and $\emptyset_{n+2}$contains N - 2 elements, and each element is equal to 1/N. The mean time to acquires is given by $\overline{T}_A = \pi_A(0)[\mathbf{I} - \mathbf{Q}_A]^{-1}\mathbf{1},$, where Q_A is the sub matrix of P that lacks the two central rows and columns.

Summary and Conclusions

A binary quantized DTTL's performance can be misrepresented by a first-order Markov chain at low and moderate SNRs. When noise samples separated by one symbol period or more are uncorrelated, a second order chain may be an accurate model of the DTTL phase error process in the same region of received SNR. In this chapter, we looked at how well a first-order Markov model performed and found that a second-order Markov model or another model will accurately calculate mean acquisition time and mean symbol slip time.

References

1. https://en.wikipedia.org/wiki/Telecommunications_network
2. Galant, D. (1989). Queuing theory models for computer networks. *ResearchGate*. https://www.researchgate.net/publication/24287912_Queuing_theory_models_for_computer_networks
3. Benard Tonui, Reuben C. Lang'ant (November2014) · On Markovian Queing Model https://www.researchgate.net/publication/268338497_On_Markovian_Queuing_Models
4. **David Galant (March 1989)** Queuing theory models for computer network https://www.researchgate.net/publication/24287912_Queuing_theory_models_for_computer_networks
5. Hans J. Pasman, William J. Rogers, in Methods in Chemical Process Safety, 2020
6. Teletraffic: Theory and Applications - Haruo Akimaru, Konosuke Kawashima - Google Books
7. Packet Switching in Computer Networks - Studytonight
8. Saad Mneimnech , computer Networks A gentle introduction to Queuing Theory note9.pdf (cuny.edu)
9. **David Galant (March 1989)** Queuing theory models for computer network https://www.researchgate.net/publication/24287912_Queuing_theory_models_for_computer_networks
10. Sci-Hub | Queueing Models for Performance Evaluation of Computer Networks—Transient State Analysis. Analytic Methods in Interdisciplinary Applications, 51–80 | 10.1007/978-3-319-12148-2_4

11. J. MEDHI, in Stochastic Models in Queueing Theory (Second Edition), 2003
12. Kyle Siegrist,The poisson Process 14: The Poisson Process - Statistics LibreTexts
13. Boris Bellalta and Simon Oechsner ,Analysis of Packet Queueing in Telecommunication Networks

https://www.upf.edu/documents/221712924/231130239/NetworkEngBook-2020/46de69a2-8cb7-4618-a9c4-4d52d572c149

14. David W.Paranchych (june 1997) Markov Modelling of Digital Symbol Synchronizers in Noise and Interference https://central.bac-lac.gc.ca/.item?id=nq20576&op=pdf&app=Library&oclc_number=1159595247

PAPER NAME

report_.pdf

WORD COUNT

7249 Words

CHARACTER COUNT

37666 Characters

PAGE COUNT

47 Pages

FILE SIZE

1.5MB

SUBMISSION DATE

May 25, 2023 10:40 AM GMT+5:30

REPORT DATE

May 25, 2023 10:41 AM GMT+5:30

19% Overall Similarity

The combined total of all matches, including overlapping sources, for each database.

- 15% Internet database
- 7% Publications database
- Crossref database
- Crossref Posted Content database
- 2% Submitted Works database

Excluded from Similarity Report

- Bibliographic material
- Quoted material
- Cited material
- Small Matches (Less then 14 words)

19% Overall Similarity

Top sources found in the following databases:

- 15% Internet database
- 7% Publications database
- Crossref database
- Crossref Posted Content database
- 2% Submitted Works database

TOP SOURCES

The sources with the highest number of matches within the submission. Overlapping sources will not be displayed.

#	Source	Database	Similarity
1	central.bac-lac.gc.ca	Internet	5%
2	cs.washington.edu	Internet	3%
3	vdoc.pub	Internet	2%
4	researchgate.net	Internet	2%
5	dspace.dtu.ac.in:8080	Internet	2%
6	J. MEDHI. "Stochastic Processes", Elsevier BV, 2003	Crossref	2%
7	D.W. Paranchych, N.C. Beaulieu. "Use of second order Markov chains t...	Crossref	<1%
8	mafiadoc.com	Internet	<1%

9	S.A. Afolalu, O.M. Ikumapayi, A. Abdulkareem, M.E Emetere, O. Adejum... Crossref	<1%
10	Deakin University on 2022-08-28 Submitted works	<1%
11	Indian Institute of Technology on 2022-07-24 Submitted works	<1%
12	coursehero.com Internet	<1%
13	Study Group Worldwide on 2020-05-26 Submitted works	<1%
14	ipcsit.com Internet	<1%