DESIGNING AND ANALYSING A

PETRI NET MODEL OF FOUR FRIENDS SHARING TWO DRINKS

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Place: Delhi
Date: May 20, 2023

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I also want to express my gratitude to all of my classmates for helping me finish this task by providing assistance and exchanging information.

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ABSTRACT

A Petri net is a particular kind of bipartite-directed graph. The notion of Petri net was developed by Carl Adam Petri in 1962. Its application is through modeling. Petri net's theoretical features enable accurate modeling and analysis of system behaviour, and its graphical representation makes it possible to visualize the modeled system. In this paper, a Petri Net model of four friends eating their respective food items but only having two drinks which will be shared among them has been discussed assuming that 1^{st} and 2^{nd} person will be sharing the first drink while 2^{nd}, 3^{rd} and 4^{th} person will be sharing the second drink. Each one of them will be in either eating or drinking state and at a time at most two of them can be in a drinking state. Furthermore, the analysis of this model is done by using the reachability tree and matrix equations. Its Safeness, Conservation, Coverability, Liveness and deadlock have been discussed in detail.

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Chapter 1

Petri nets: Overview

The notion of Petri net was developed by Carl Adam Petri in 1962. Its application is through modeling. In many academic disciplines, a phenomenon is not directly examined but rather indirectly through a model of the phenomenon. Petri net's theoretical features enable accurate modeling and analysis of system behaviour, and its graphical representation makes it possible to visualize the modeled system . In computer science, system engineering, and many other fields, Petri nets are a potent modelling framework.
A Petri net, a mathematical representation of the system, can be used to model a system according to Petri net theory. A Petri net is a particular kind of bipartite directed graphs.

1.1 Petri Net Structure

A specific kind of bipartite directed graph that contains four different types of objects is known as a Petri net. These objects are places, transitions, directed arcs and tokens. Directed arcs connect places to transitions or transitions to places.A Petri net can be expressed in its most basic form by a transition along with an input place and an output place. Many features of the studied systems can be represented by this simple net.
A Petri net is made up of four components: an input function I, an output function O, a collection of places P, and a set of transitions T. The input and output functions relate transitions and places. The input function I maps a transition t_j to a collection of places $I(t_j)$, also referred to as the transition's input places. The output function O maps a transition t_j to a collection of places $O(t_j)$ that are referred to as the transition's output places. To put it another way, a petri net is made up of Places, Transitions, Input Function, and Output Function. Tokens are the most important part of it, they show the movements/changes in the Petri net.
Tokens are created and destroyed in the places (conditions) and can travel in a system under certain parameters that can change the state of the system. A Petri net's places,

transitions, input function, and output function makes its overall structure.

1.2 Places

The places refer to certain set of conditions that are to be satisfied. In simple words, they can be thought of as a box which can hold something in it. These are denoted using circles $\circ$ in the PN structures.

1.3 Transitions

The transitions are the events or activities that occur and lead to the change in the state of the system. These are denoted using a vertical line — or a rectangular bar. The places and transitions are connected via directed edges or arcs.

Definition

The Petri Net structure is made up of four components and is written as a four tupple,

$$PN = (P, T, I, O),$$

where
P = set of all places
T = set of all transitions
I = Matrix that explains the association of input places and the transitions
O = Matrix that explains the association of output places and the transitions
i.e. for a PN consisting of say, M-places and N- transitions

1.4 Example of Petri Net

We have a general Petri Net structure defined as $PN = \{P, T, I, O\}$ let us now consider that for a particular PN structure, where $P = \{p_1, p_2, p\_3, p_4, p_5\}$ and $T = \{t_1, t_2, t_3, t_4\}$ where $I : T \to P^\infty$ and $O : T \to P^\infty$ Let us be given the defined input and output functions as

$$\begin{aligned} I(t_1) &= \{p_1\} & O(t_1) &= \{p_2, p_3, p_5\} \\ I(t_2) &= \{p_2, p_3, p_5\} & O(t_2) &= \{p_5\} \\ I(t_3) &= \{p_3\} & O(t_3) &= \{p_4\} \\ I(t_4) &= \{p_4\} & O(t_4) &= \{p_2, p_3\} \end{aligned}$$

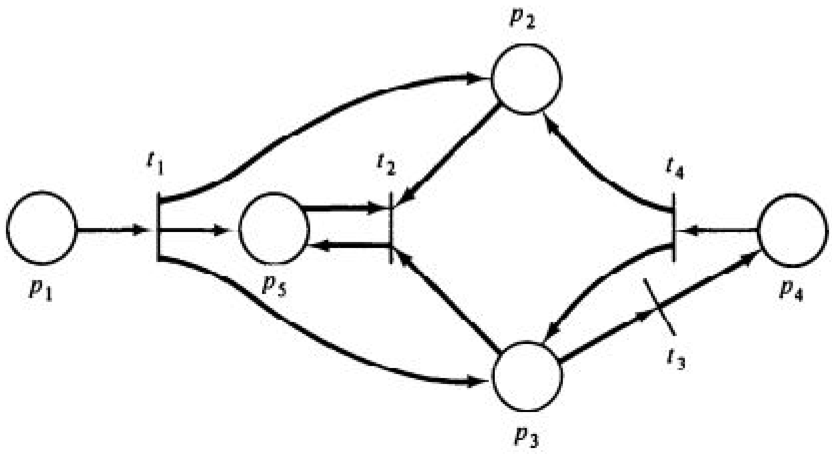

Figure 1.1: Petri Net Model

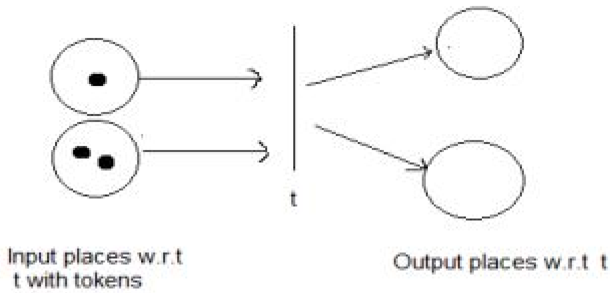

Figure 1.2: Tokens in a Petri Net

The input and the output functions can be extended as $I : T \to P^\infty$ and $O : T \to P^\infty$ such that $(t_j, I(p_i)) = (p_i, O(t_j))$ and $(t_j, O(p_i)) = (p_i, I(t_j))$.The extended input and output functions are:

$$
\begin{aligned}
I(p_1) &= \{\} & O(p_1) &= \{t_1\} \\
I(p_2) &= \{t_1, t_4\} & O(p_2) &= \{t_2\} \\
I(p_3) &= \{t_1, t_4\} & O(p_3) &= \{t_2, t_3\} \\
I(p_4) &= \{t_3\} & O(p_4) &= \{t_4\} \\
I(p_5) &= \{t_1, t_2\} & O(p_5) &= \{t_2\}
\end{aligned}
$$

For the above, Petri Net structure can be drawn as below.

1.5 Tokens

Present in a system are some basic entities called tokens which get created and destroyed in the places (conditions) and can travel in a system under certain parameters that can change the state of the system. With respect to the above simple Petri Net structure, we can deduce that for the transition t, there exist total four places P_1, P_2, P_3, P_4. Further, it is evident that these places and the transition are connected respectively by arcs or the directed edges. The tokens are labelled at the input places P_1 and P_2 for t, where P_1 has a single token and P_2 has 2 tokens.

Chapter 2

Modelling with Petri Net

2.1 Duality

The Petri net (T, P, I, O) that comes from interchanging places and transitions is the dual of a Petri net (P, T, I, O). Just changing out the graph's circles and bars to show where places and transitions change maintains the graph's structural integrity.
The dual of above petri net is:

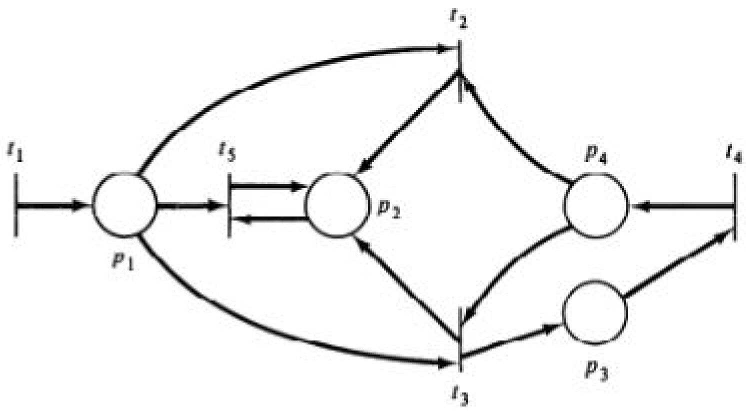

Figure 2.1: Dual of fig.1.1

2.2 Marking

A marking of a Petri Net is the assignment of the tokens to the set of places. It is denoted by $M = M_0, M_1, M_2, ..., M_m$ where M_i gives the number of token that are available at the place p_i.
A marking M is a function defined from P, the set of all places to the non-negative integers i.e., $M : P \to Z^+$. The initial marking is denoted by M_0, $M_0 : P \to Z^+$. A marked Petri Net w.r.t M_0 is a 5-tuple structure where $PN = (P, T, I, O, M_0)$.
It is obvious to realise that the number of token which can be assigned to any place

in a Petri net is not bounded, and thus, there are significantly infinite many number of markings possible for the Petri net. The set of all n vectors, N^n, is the set of all markings for a Petri net with n places. Although infinite, this set is obviously denumerable.

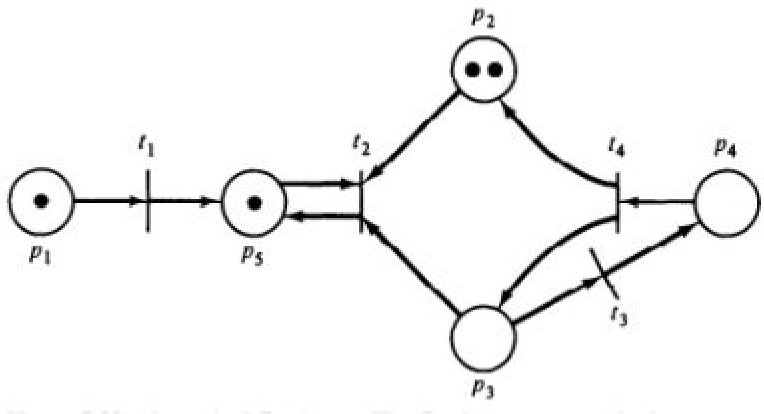

Figure 2.2: Petri Net Model for markings

$M_0 = (1, 2, 0, 0, 1)$
After t_1 fires $M_1 = (0, 2, 0, 0, 2)$

2.3 Transition enabling and firing

Any transition, say t_j in a system, is enabled and can fire with one or multiple input places; if the total number of tokens in all the input places is, for each place's t_j, at least equal to the multiplicity of all the input arcs in those places. We also call this the triggering of t_j. When t_j in a system triggers, a token gets deleted from its input places and eventually gets created in the respective output places,i.e.,a change in marking M will be there when a transition t is enabled to fire, if all $p_i \in P, i = 1 \ldots m$ and

$$M(p_i) \geq (p_i, I(t_j))$$

(a) (a) Before firing of t_j.

(b) (b) After firing of t_j.

Figure 2.3: Transition enabling in a Petri Net

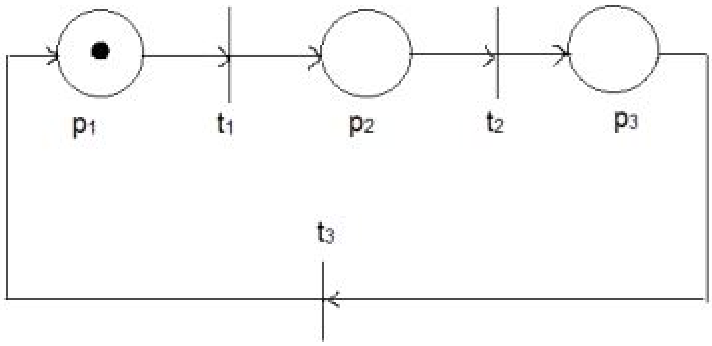

Figure 2.4: Transition enabling in a Petri Net

At time $t = t_0$(initial time), we have initial marking $M(t_0) = (1, 0, 0)$. Correspondingly all other markings.

Chapter 3

Properties of Petri Net

3.1 Safeness

If there are never more than one token in a place then that place is safe. A Petri net is safe if every place within it is safe

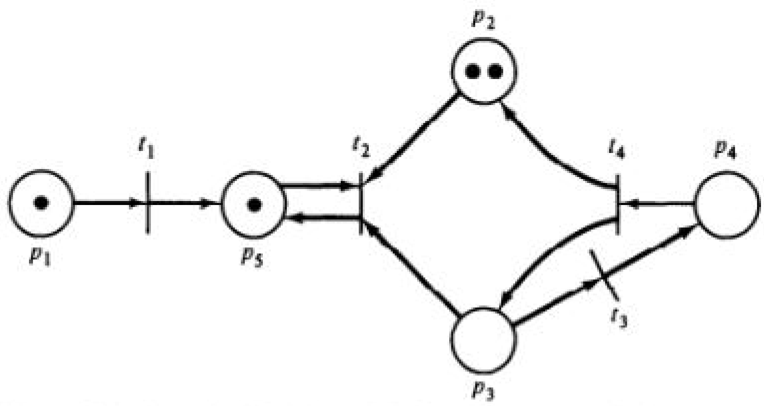

Figure 3.1: Petri Net model which is not safe

Here, p_2 has two tokens so this is not safe.

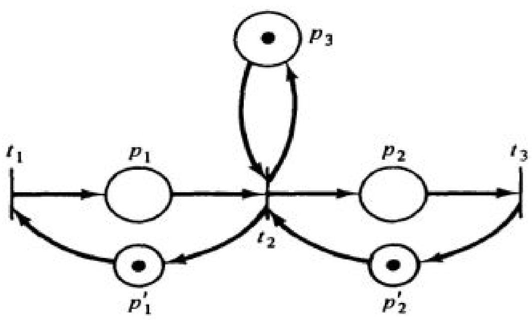

Figure 3.2: Petri Net model which is safe

Here, all places have either no token or one token. so, this is a safe Petri net.

3.2 Boundedness

A Petri net is safe if each place in it is safe, and a place is k-safe or k-bounded, if the place is k-bounded, which means the number of token in that place cannot exceed a positive integer k, then the petri net is k-safe or k-bounded. Here, in fig. 3.1 any place can have at maximum 2 tokens. Hence, fig. 3.1 is $2-$ bounded.

3.3 Conservation

A Petri net with initial marking M_0 is strictly conservative if for all $M^{'}$.

$$\sum_{p_i \in P} M^{'}(p_i) = \sum_{p_i \in P} M_0(p_i)$$

That is, the total number of token in the Petri net at any instant of time must be always equal to number of total token in the initial marking.

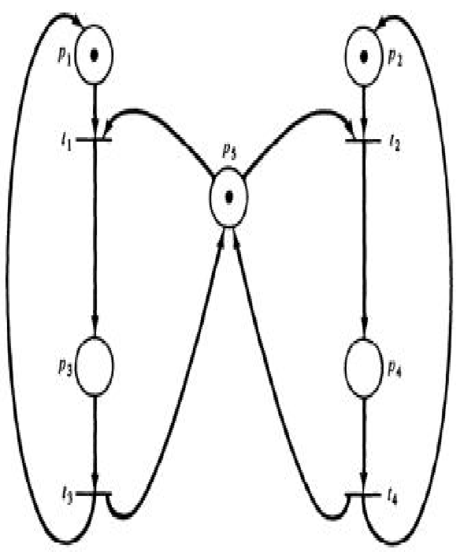

Figure 3.3: Not Strictly conservative Petri net

Here the enabled transitions are t_1 and t_2 The markings are:
$M_0 = (1, 1, 0, 0, 1)$
$M_1 = (0, 1, 1, 0, 0)$ (when t_1 fires)
$M_2 = (1, 1, 0, 0, 1)$ (when t_3 fires)
$M_3 = (1, 0, 0, 1, 0)$ (when t_2 fires)

$M_4 = (1,1,0,0,1)$ (when t_4 fires)
The Petri Net in Fig 3.2 is not strictly conservative since the number of tokens in each of the markings are either increased or decreased by one, on consecutive firing of transitions and hence the token count is not constant.
Remark: A Petri Net is said to be **partially conservative** if the token count is constant for few markings, then changes to another positive integer and remains constant for the next few markings and eventually becomes constant. For a partially conservative Petri Net, the tokens in a place will never become unbounded.

3.4 Coverability

Coverability Problem: Given a Petri Net C with initial marking M_0 and a marking M' is there a reaching marking $M'' \in$R(C,M$_0$) such that $M'' \geq$M$'$. ?

3.5 Liveness

A transition is live in marking M if it can fire. A Petri net is said to be live if in all the marking there is atleast one transition which is live i.e. it is enabled(can fire).

3.6 Deadlock

A deadlock in a Petri net occurs when no transition can fire in any of the marking. That is, there is a marking at which none of the transition is enabled. If a Petri net is live then deadlock cannot occur.

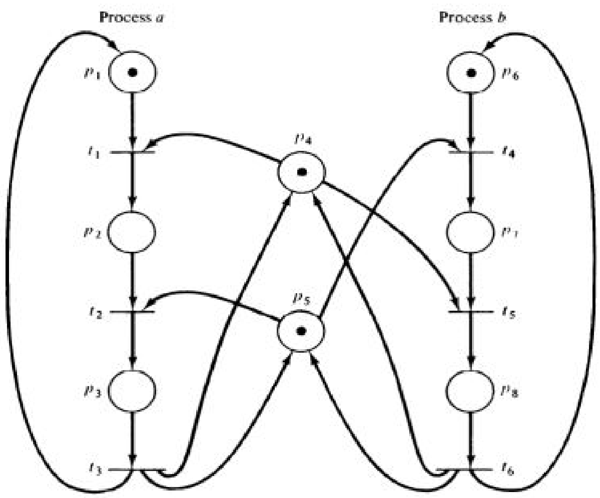

Figure 3.4: Showing Deadlock or Live

In this model illustrated in Fig 3.3 there are two processes , process a and process b. The transition firing sequence $t_1t_2t_3t_4t_5t_6$ and $t_4t_5t_6t_1t_2t_3$ does not produce deadlock.
If we consider the transition firing sequence which starts from t_1t_4 , then process a would have the resources from p_4 and would want resources from p_5 and similarly process b would have resources from p_5 and would be needing resources from p_4.Thus a deadlock condition would be reached and neither of the two processes would be able to proceed further.

Chapter 4

Reachability Tree

4.1 Definition

A reachability tree represents all the reachable markings along with the firing sequence.
Reachability Problem: Given a Petri Net C with marking M and a marking $M^{'}$, is $M^{'}$ $\in$R(C,M).?

4.2 Making reachability tree

The reachability tree/graph is an analytic technique for representing the reachability set of the Petri Net.
If we want to construct the reachability tree for a given Petri Net, then we need to consider the marking of that Petri Net at that time as a node and an arc would represent firing of transition from the initial marking to the subsequent new set of markings.
Consider the Petri Net in the Fig 4.1 given below:

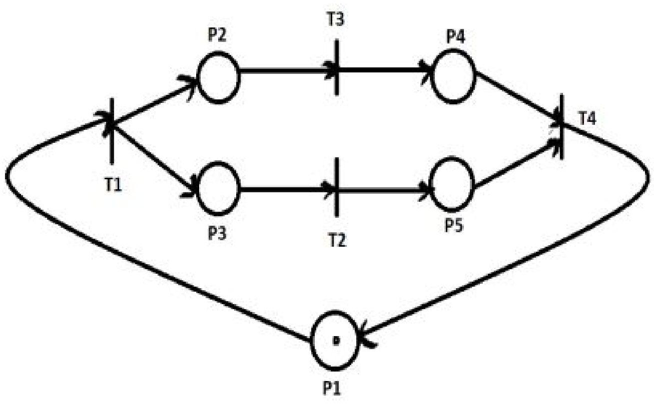

Figure 4.1: Petri Net Model

If we want to draw the reachability graph of the Petri Net, then consider the initial marking $M_0 = (1, 0, 0, 0, 0)$, fire the enabled transition T_1 from here, and the new markings are obtained. Fig 4.2 show how the reachability graph gets constructed following this procedure for the Petri Net model of Fig 4.1.

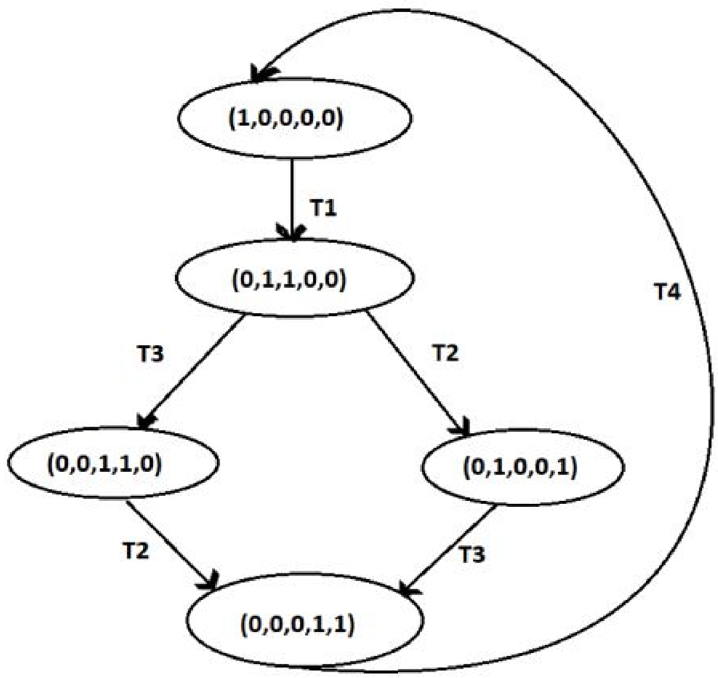

Figure 4.2: Reachability graph of Petri Net model

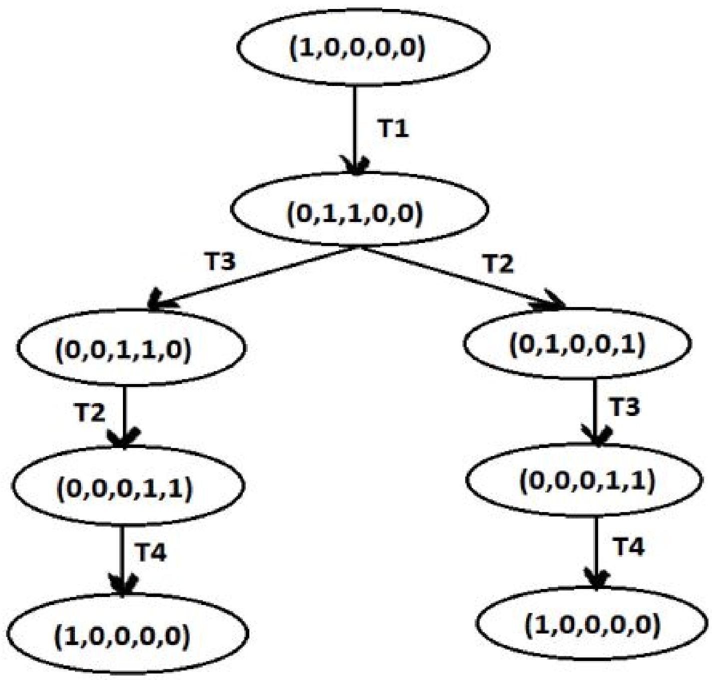

Figure 4.3: Reachability tree with terminaing nodes

Many a times we want to analyse the Petri Net in such a way that the direct relations in the markings are visible and the digraph has no cycles, thus we draw it as a reachability tree, where the nodes might get repeated , thus deleting the existing cycles. The reachability tree for the previous example of Fig 4.1 is shown in Fig 4.3

4.3 Example

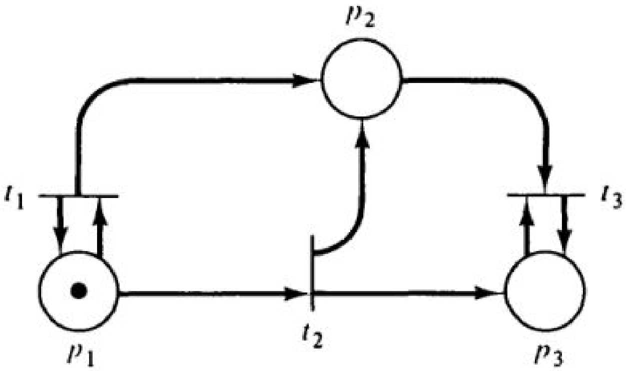

Figure 4.4: Petri Net Model

The reachability tree for the above Petri Net is given by,

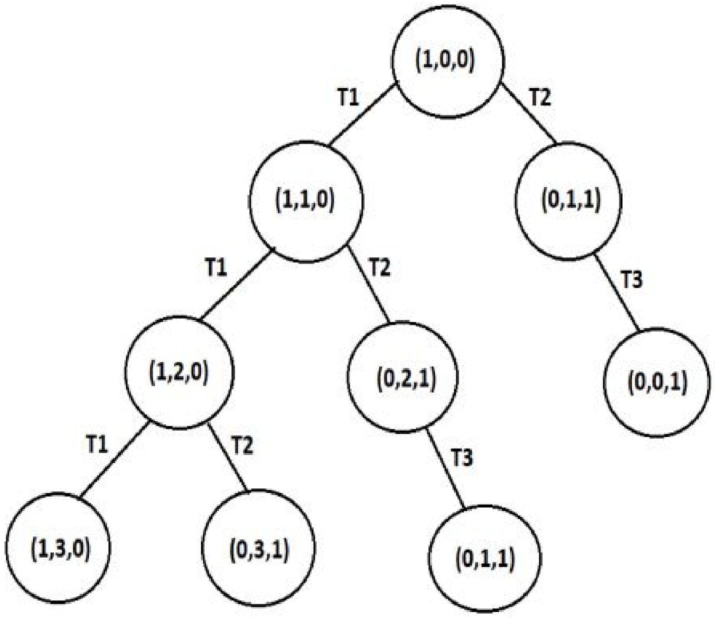

Figure 4.5: Reachability Tree

On repeating this process again and again in order to proceed, at every stage new marking will be produced, and we will get an Infinite Reachability set and thus Infinite Reachability tree.

In order to perform analysis for the Petri Net model we need to limit the size of the tree to a finite one.

4.4 Finite Representation of Infinite Reachability Tree

In order to represent the infinite reachability tree in a finite way, the limitation of new marking is done at each step.The nodes in the infinite tree are categorized in such a way that they limit themselves in a finite manner.

4.4.1 Definition: Terminal Node

They are the nodes that are represented when no transition is enabled..

4.4.2 Definition: Duplicate Nodes

They are represented by duplicate markings. Set of markings that have previously appeared in the tree is called duplicate marking. The markings are not extended further once duplicate nodes are detected since successors of these markings have already been produced in the first occurrence.

4.5 w Representation

Consider the sequence of transition firing , α which starts from M_0 and ends at the marking $M^{'} > M_0$
that is, $M_0 \rightarrow \alpha \rightarrow M^{'}$ The marking $M^{'}$ is same as M_0 except that it has some extra tokens in some of the places.
i.e. $M^{'} = M_0 + (M^{'} - M_0) \rightarrow$ extra tokens (> 0)
If α is fired again ,this time from $M^{'}$, then again $M^{'} - M_0$ tokens will be added to the marking $M^{'}$.
i.e. $M^{'} \rightarrow \alpha \rightarrow M^{''}$
$M^{''} = M^{'} + (M^{'} - M_0) = M_0 + 2(M^{'} - M_0)$ In general if we fire α sequence of transitions n times then we obtain the marking $M_0 + n(M^{'} - M_0)$.Thus for the places which have gained tokens from the sequence α, arbitrarily large number of tokens can be accumulated just by reiterating the sequence of transitions again and again.
This arbitrarily large number of tokens are represented by w. Hence for each marking the number of tokens in a place are either non negative integer or w .Terminal nodes , duplicate nodes along with w representation restrict the infinite tree to a finite one. In the previous example of Fig4.5 , where we were obtaining an infinite reachability tree , after

again applying the finite representation techniques , the new tree that we get is shown in Fig 5.6

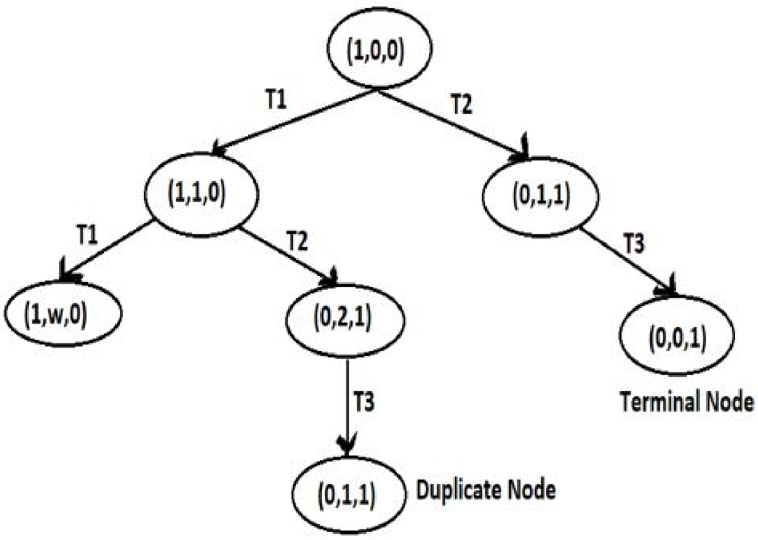

Figure 4.6: Finite reachability tree

Firing t_1 as many times , arbitrary number of tokens can be built in P_2.

4.6 Analyzing properties of Petri Nets through Reachability Tree

- A Petri Net is considered to be bounded if the symbol w never appears in the reachability tree.. Hence, the Petri Net represents the finite state system if it is bounded.Thus to determine the bound for a particular place , we need to draw the reachability tree and examine the tree for the largest value of the tokens in the markings corresponding to that place.
  Thus the reachability tree helps in determining the boundedness or safeness property for individual places in the Petri Net , or the entire Net .
- Given that the reachability tree is finite, it is simple to test for **conservation** by computing the weighted sum of tokens at each marking. If this weighted sum is constant and the same for all subsequent markings, the Petri Net is strictly conservative.
- If the symbol w appears for any place in a marking and the corresponding element of the weighing vector for that place is 0 , then there can be a scope for the Petri Net to be conservative , but if the weight is positive of the element of the weighing vector for any place then the Net will not be conservative under any circumstances.

Since now the symbol w tells us that the number of tokens for some place can be arbitrarily increased, thus clearly the Petri Net won't be conservative.

- **Coverability Problem** can also be solved through reachability tree as all the reachable markings will be shown in the reachability tree.
- A Petri Net has **deadlock** iff the reachability graph of the Petri Net has atleast one node(represented by a marking) without an outgoing arc.
- A Petri Net is **live** if a path can be found to connect each node in its related reachability graph.

Chapter 5

Matrix Equations

Another technique for analysis of Petri Nets is Linear Algebra methods known as Matrix Equations. A Petri Net $PN = (P, T, I, O)$ can be defined as the tupple $PN = (P, T, D^-, D^+)$ where D^- is the matrix for input function and D^+ is the matrix for output function of the Petri Net.
The matrix D^- and D^+ have transitions as the rows $(t_1, t_2, \ldots t_m)$ and the places of the Petri Net as columns of the matrix $(p_1, p_2, p_3, \ldots, p_n)$. Thus both input and output matrix are matrices of order $m * n$
Then $D = D^+ - D^-$ is known as the incidence matrix.

5.1 Example

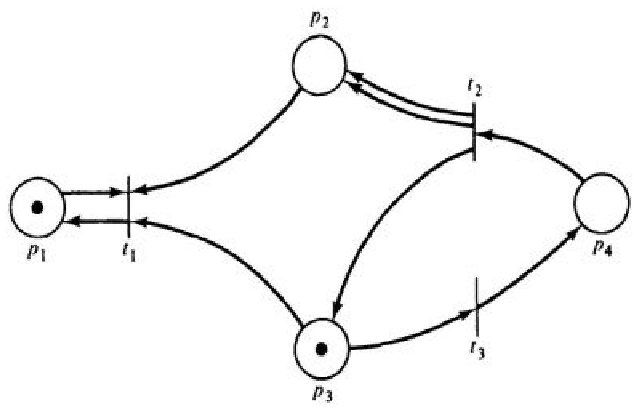

Figure 5.1: Petri Net Model

Here the input and output matrices will be of order $3 * 4$ as the petri net has 3 transitions and 4 places. The matrices are as follows:
$D^- = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $D^+ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The incidence matrix is $D = D^{+} - D^{-}$

$$D^{+} = \begin{bmatrix} 0 & -1 & -1 & 0 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

5.2 Conservative behaviour

A Petri Net is conservative iff there exists a positive vector w such that $D.w = 0$
and
A Petri net is partially conservative iff there exists a non-negative vector w such that $D.w = 0$

5.3 To find reachable marking using given firing sequence

There exists a firing sequence (σ) of transitions if any marking $M^{'}$ is reachable from any marking M_i. If the net takes from M_i to $M^{'}$ as a result of the firing sequence of transitions, then
$M^{'} = M + f(\sigma).D$,
where,
$f(\sigma)$ is the non- negative firing vector of the firing sequence.
Here in fig. 5.1, t_3 is enabled so let the firing sequence $\sigma = t_3$ then the firing vector $f(\sigma) = (0, 0, 1)$ from the initial marking $M_0 = (1, 0, 1, 0)$
$M^{'} = M_0 + f(\sigma).D$
$M^{'} = (1, 0, 1, 0) + (0, 0, 1).D$
$M^{'} = (1, 0, 1, 0) + (0, 0, -1, 1)$
$M^{'} = (1, 0, 0, 1)$
That means $(1, 0, 0, 1)$ is the reachable marking from initial marking when t_3 is fired.

5.4 To check whether the given marking is reachable or not?

Let us check that the marking $(1, 8, 0, 1)$ is reachable from $M_0 = (1, 0, 1, 0)$ or not.
Using the formula, $M^{'} = M_0 + f(\sigma).D$, we need to determine the firing sequence to reach this $M^{'}$
$M^{'} = M_0 + f(\sigma).D$
$(1, 8, 0, 1) = (1, 0, 1, 0) + x.D$
$(0, 8, -1, 1) = (x_1, x_2, x_3).D$

$(0, 8, -1, 1) = (0, -x_1 + 2x_2, -x_1 + x_2 - x_3, -x_2 + x_3)$
That gives $2x_2 - x_1 = 8$, $-x_1 + x_2 - x_3 = -1$ and $-x_2 + x_3 = 1$
That gives $x_1 = 0$, $x_2 = 4$, $x_3 = 5$.
$x = (0, 4, 5)$
This corresonds to the sequence $\sigma = t_3t_2t_3t_2t_3t_2t_3t_2t_3$
That means this marking $M^{'} = (1, 8, 0, 1)$ can be reached using the firing sequence $\sigma = t_3t_2t_3t_2t_3t_2t_3t_2t_3$

Chapter 6

Labeled Place-Transitive Matrix

A definition of the Labeled Place-Transitive matrix is $L_{BP} = [D^{-}]^{T}.D_{t}.D^{+}$ of order $|P| * |P|$ where,
D^{-} is the input matrix
D^{+} is the output matrix
D_t is the diagonal-transition matrix for $T = t_1, t_2, ..., t_n$ of order $n * n$

6.1 Example

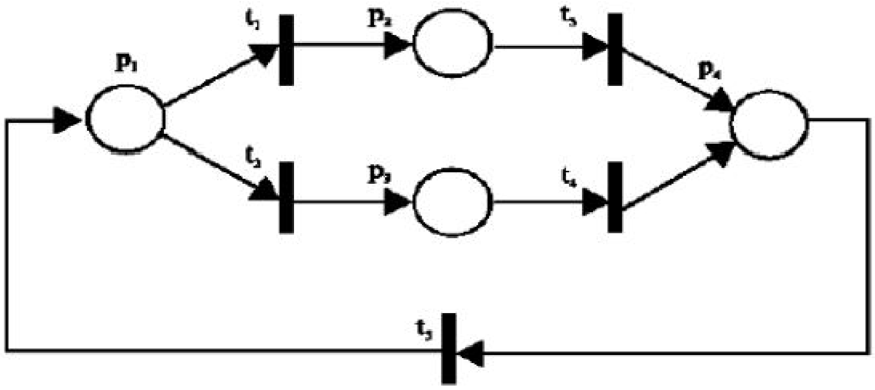

Figure 6.1: Petri Net Model

Here,

$$D^{t} = \begin{bmatrix} t_1 & 0 & 0 & 0 & 0 \\ 0 & t_2 & 0 & 0 & 0 \\ 0 & 0 & t_3 & 0 & 0 \\ 0 & 0 & 0 & t_4 & 0 \\ 0 & 0 & 0 & 0 & t_5 \end{bmatrix}$$

$$D^- = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } D^+ = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

The Labeled Place-Transitive Matrix is

$$L_{BP} = \begin{bmatrix} 0 & t_1 & t_2 & 0 \\ 0 & 0 & 0 & t_3 \\ 0 & 0 & 0 & t_4 \\ t_5 & 0 & 0 & 0 \end{bmatrix}$$

The row-transition vector T_R is the sum of corresponding columns of L_{BP} and the column-transition vector T_C is the sum of corresponding rows of L_{BP}
The components/entries of T_R and T_C are finite linear combinations of transitions with positive integer coefficients.The number of incoming/outgoing arcs from the transitions are represented by the coefficients of transitions that are present in the transition vectors $T_C(T_R)$.
For fig. 6.1. the corresponding row and column transition vectors are:
$T_R = \begin{bmatrix} t_5 & t_1 & t_2 & t_3 + t_4 \end{bmatrix}$
and
$T_C = \begin{bmatrix} t_1 + t_2 & t_3 & t_4 & t_5 \end{bmatrix}^T$

6.2 Structural properties

i. **Cyclic/Acyclic Nature**:
A Petri Net is acyclic if atleast one entry in T_R or T_C is zero. The T_R and T_C corresponding to our proposed model has no zero component.
<u>The cycles are computed as</u>:
The transition vectors (row and column) are inputs.

(a) Select any i^{th} entry(place) of T_C

(b) Select any transition in the i^{th} component of T_C

(c) locate this transition in the components of T_R

(d) Choose an entry (place) in T_R that corresponds to the transition we chosen in step (b)

(e) If this selected entry is same as previously selected entry then end the cycle otherwise go back to step(b).

ii. **Conflict**:
If every component of T_C has exactly one transition, or if any component has more than one transition, then the same transitions must be present in the corresponding component of T_R, then Petri Net is conflict-free.

iii. **Self-loop(free)** :
If and only if the corresponding components of both T_R and T_C have no transitions that are the same or identical at a certain place pk, the Petri Net is said to be self loop free.

iv. **Structural Concurrency**:
If at least one component of the column transition vector T_C has a coefficient of 2 or higher, the Petri net is said to be structurally concurrent.

For Fig. 6.1,

- Since the transition vectors don't have any zero component thus, the PN in the given example is cyclic.
- The Petri Net graph is in conflict since the first component of column transition vector has two transitions.
- The Petri Net graph is self loop free since the corresponding distinct components of row transition vector and column transition vector are different.
- Some of the possible cycles are:
$p_1 \to t_1 \to p_2 \to t_3 \to p_4 \to t_5 \to p_1$
$p_1 \to t_2 \to p_3 \to t_4 \to p_4 \to t_5 \to p_1$

Chapter 7

Petri net model of four friends sharing two drinks

In day to day life, we eat food items along with some shakes/drinks. It happens many times that the same drink is being shared between some of us. In this section, a Petri net model of four friends has been discussed where the four friends are eating their respective food items but only 2 drinks are shared among them in such a way that the first drink will be shared between 2 of them while the second drink will be shared among 3 of them such that only one person will be having both the drinks.
WLOG one can assume that 1^{st} and 2^{nd} person will be sharing first drink while 2^{nd}, 3^{rd} and 4^{th} person will be sharing second drink. Each one of them will be in either eating or drinking state and at a time at most 2 of them can be in drinking state as when the drink is kept on table then only one can have it.
The Petri net corresponding to this situation is:

Depiction of Places(conditions) and Transitions(activities) is as follows:

p_{1e}	First person in the eating state.
p_{1d}	First person in the drinking state.
p_{2e}	Second person in the eating state.
p_{2d1}	Second person having 1^{st} drink.
p_{2d2}	Second person having 2^{nd} drink.
p_{3e}	Third person in the eating state.
p_{3d}	Third person in the drinking state.
p_{4e}	Fourth person in the eating state.
p_{4d}	Fourth person in the drinking state.
p_{d1}	First drink which will be shared by 1^{st} and 2^{nd} person.
p_{d2}	Second drink which will be shared by 2^{nd}, 3^{rd} and 4^{th} person.

Table 7.1: List of places of the proposed model

t_1	First person takes the drink.
t_2	First person puts the drink back on the table.
t_3	Second person takes the 1^{st} drink.
t_4	Second person puts the 1^{st} drink back on the table .
t_5	Second person takes the 2^{nd} drink.
t_6	Second person puts the 2^{nd} drink back on the table .
t_7	Third person takes the drink.
t_8	Third person puts the drink back on the table.
t_9	Fourth person takes the drink.
t_{10}	Fourth person puts the drink back on the table.

Table 7.2: List of transitions of the proposed model

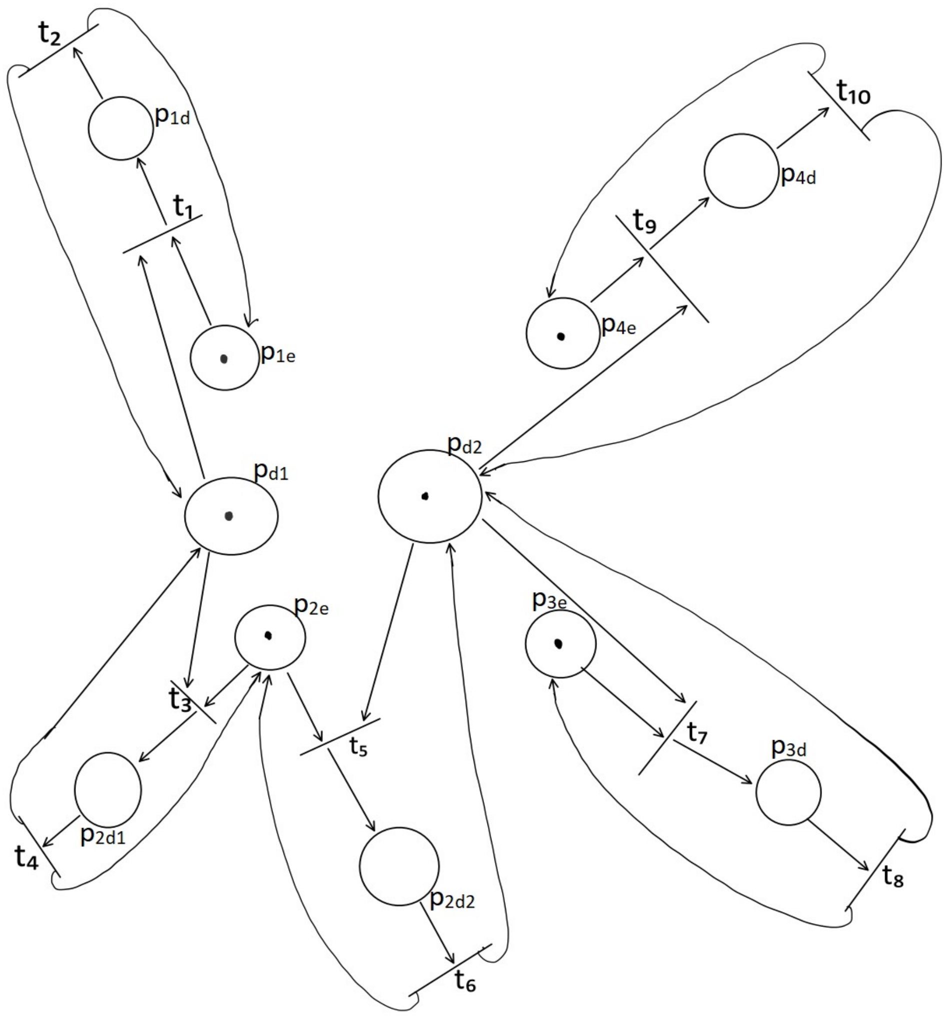

Figure 7.1: Petri Net model of 4 friends sharing 2 drinks

Whenever there is a token in p_{d1} or p_{d2} that means the 1^{st} or the 2^{nd} drink is kept on the table, respectively. Tokens in p_{1e}, p_{2e}, p_{3e} and p_{4e} denote that the respective person is eating the food and the token in p_{1d}, p_{2d1} denote that the respective person is having 1^{st} drink while token in p_{2d2}, p_{3d} and p_{4d} denote that the respective person is having 2^{nd} drink.
The analysis of this Petri net structure can be done by both reachability tree and matrix equations.

7.1 Reachability Tree

A reachability tree [2, 4, 5] represents all the reachable markings along with the firing sequence. The reachability tree of Petri net in Fig.7.1 is depicted in Fig.7.2

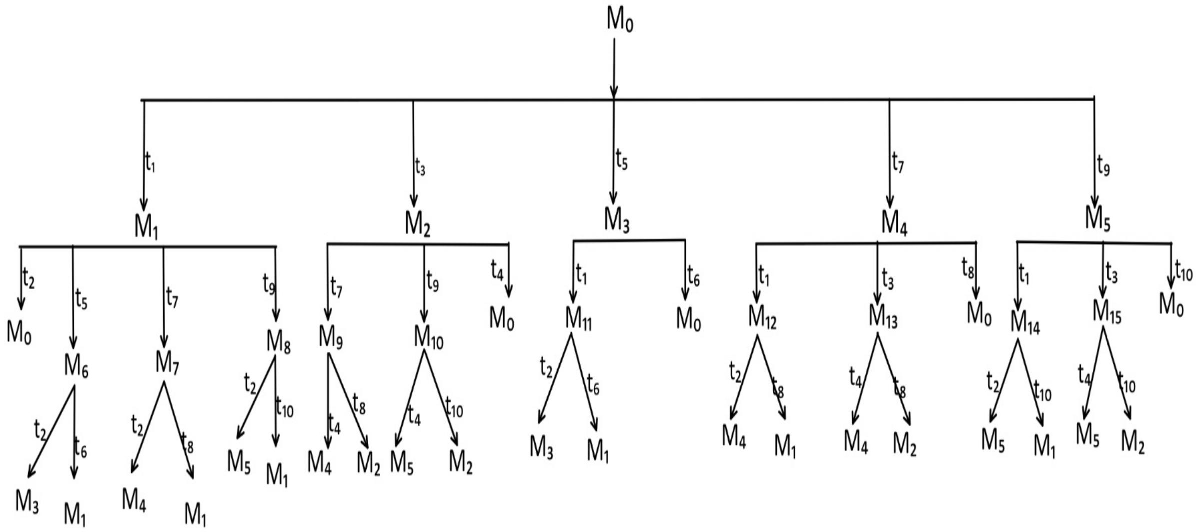

Figure 7.2: Reachability tree for the Petri net structure, in Fig.7.1

The markings in Fig.7.2 are as follows:

M_0	$(1,0,1,0,0,1,0,1,0,1,1)$
M_1	$(0,1,1,0,0,1,0,1,0,0,1)$
M_2	$(1,0,0,1,0,1,0,1,0,0,1)$
M_3	$(1,0,0,0,1,1,0,1,0,1,0)$
M_4	$(1,0,1,0,0,0,1,1,0,1,0)$
M_5	$(1,0,1,0,0,1,0,0,1,1,0)$
M_6	$(0,1,0,0,1,1,0,1,0,0,0)$
M_7	$(0,1,1,0,0,0,1,1,0,0,0)$
M_8	$(0,1,1,0,0,1,0,0,1,0,0)$
M_9	$(1,0,0,1,0,0,1,1,0,0,0)$
M_{10}	$(1,0,0,1,0,1,0,0,1,0,0)$
M_{11}	$(0,1,0,0,1,1,0,1,0,0,0)$
M_{12}	$(0,1,1,0,0,0,1,1,0,0,0)$
M_{13}	$(1,0,0,1,0,0,1,1,0,0,0)$
M_{14}	$(0,1,1,0,0,1,0,0,1,0,0)$
M_{15}	$(1,0,0,1,0,1,0,0,1,0,0)$

Table 7.3: List of markings for the proposed model

7.1.1 Analysis of the model using reachability tree

i. **Safeness and Boundedness** [3, 4, 5] : If there are never more than one token in a place then that place is safe. A Petri net is safe if each place in it is safe, and a

place is k-safe or k-bounded, if the place is k-bounded, which means the number of token in that place cannot exceed a positive integer k, then the petri net is k-safe or k-bounded.
Here, one can clearly see in reachability tree or in all possible markings that all the places have either 0 or 1 token. Hence, this Petri net is safe or $1-$bounded.
For example, in M_0 places p_{1e}, p_{2e}, p_{3e}, p_{4e}, p_{d1} and p_{d2} have one token each while all other remaining places have no token.

ii. **Conservation**[3, 4, 5]: A Petri net with initial marking M_0 is strictly conservative if for all $M^{'}$.

$$\sum_{p_i \in P} M^{'}(p_i) = \sum_{p_i \in P} M_0(p_i)$$

That is, the total number of token in the Petri net at any instant of time must be always equal to number of total token in the initial marking. This Petri net is not strictly conservative as $\sum_{p_i \in P} M_0(p_i) = 6$ while after t_1 fires $\sum_{p_i \in P} M_1(p_i) = 5$ and we know $5 \neq 6$.
Hence, this Petri net is not strictly conservative.

iii. **Coverability** [3, 5]: All the coverable markings are shown in the reachability tree. The sequence of transitions leading from the initial marking to the covering marking is given by the path from the root to the covering marking. A marking like $(1, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0)$ is not in the reachability tree means that **this marking will not be covered or cannot be reached**.

iv. **Liveness**[5]: **A transition is live in marking M if it can fire**. A Petri net is said to be live if in all the marking there is atleast one transition which is live i.e. it can fire. In this Petri net, one can easily see in the reachability tree that in all the possible marking, there is always atleast one transition which can fire, so this Petri net structure is live.

For example,
in marking M_0, transitions t_1, t_3, t_5, t_7 and t_9 can fire,
in marking M_1, transitions t_2, t_5, t_7 and t_9 can fire,
in marking M_3, transitions t_1 and t_6 can fire,
in marking M_{12}, transitions t_2 and t_8 can fire.

v. **Deadlock** [5]: A deadlock in a Petri net occurs when no transition can fire in any of the marking. In all the markings of this Petri net, there is always atleast one transition which can fire. Hence, no deadlock comes in this Petri net structure.
If a Petri net is live then deadlock cannot occur.

7.2 Matrix Equations

To represent the input and output functions, two matrices D^- and D^+ [2, 4] can be defined as an alternative method of defining a Petri net, respectively.
The input(D^-) and output(D^+) matrices of Fig. 7.1 will be of order $(10 * 11)$, the two matrices are as follows:

$$D^- = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

and

$$D^+ = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The incidence matrix $D = D^{+} - D^{-}$:

$$D = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

- **Conservative behaviour** :
  ”A Petri Net is conservative iff there exists a positive vector w such that $D.w = 0$”
  and
  ”A Petri net is partially conservative iff there exists a non-negative vector w such that $D.w = 0$”
  Here, the system of equations w.r.t. the incidence matrix D by taking
  $w = (w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8, w_9, w_{10}, w_{11})^T$ are:
  $-w_1 + w_2 - w_{10} = 0$
  $w_3 - w_4 + w_{10} = 0$
  $w_3 - w_5 + w_{11} = 0$
  $w_6 - w_7 + w_{11} = 0$
  $w_8 - w_9 + w_{11} = 0$

Solution of these equations gives the weighting vector:

$$w = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

which is a non-negative vector. Hence, the Petri net is partially conservative.

- **To find reachable marking using given firing sequence:**
  If any marking $M^{'}$ is reachable from any marking M_i then there exist a firing sequence (σ) of transitions. If firing sequence of transitions takes the net from M_i to $M^{'}$ then
  $M^{'} = M + f(\sigma).D$,
  where,
  $f(\sigma)$ is the non- negative firing vector of the firing sequence.
  For example: If there are 4 transitions in the net t_1, t_2, t_3, t_4 and the firing sequence is $\sigma = t_1 t_3$ then the firing vector is $f(\sigma) = (1, 0, 1, 0)$.
  In the Petri net structure, Fig. 7.1; from the initial marking M_0 using firing sequence, $t_7 t_3$ the reachable marking will be:
  $M^{'} = M_0 + f(\sigma).D$
  $M^{'} = (1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0) = M_{13}$

- **To check whether the given marking is reachable or not**?
  Let us check that the marking $M^{'} = (1, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0)$ is reachable from initial marking M_0 or not.
  Using the formula, $M^{'} = M_0 + f(\sigma).D$, we need to determine the firing sequence to reach this $M^{'}$
  $M^{'} = M_0 + f(\sigma).D$
  $(1, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0) = (1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 1) + x.D$

  Here, no such $x = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10})$ can be calculate. That means this marking $M^{'} = (1, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0)$ cannot be reached.

7.2.1 Analysis using Labeled Place-Transitive Matrix

For studying properties like cyclic/acyclic, conflict, self-loop, we compute Labeled Place-Transitive Matrix
The Labeled Place-Transitive matrix is defined as: $L_{BP} = [D^{-}]^{T}.D_{t}.D^{+}$ of order $|P|*|P|$ where,
D^{-} is the input matrix
D^{+} is the output matrix
D_t is the diagonal-transition matrix for $T = t_1, t_2, ..., t_n$ here, $n = 10$.

The Labeled Place-Transitive Matrix[2] is:

$$L_{BP} = \begin{bmatrix} 0 & t_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_2 & 0 \\ 0 & 0 & 0 & t_3 & t_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & t_4 & 0 & 0 & 0 & 0 & 0 & 0 & t_4 & 0 \\ 0 & 0 & t_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & t_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & t_8 & 0 & 0 & 0 & 0 & t_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{10} & 0 & 0 & t_{10} \\ 0 & t_1 & 0 & t_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & t_5 & 0 & t_7 & 0 & t_9 & 0 & 0 \end{bmatrix}$$

Corresponding row and column transition vectors are:

$$T_R = [t_2 \quad 2t_1 \quad t_4 + t_6 \quad 2t_3 \quad 2t_5 \quad t_8 \quad 2t_7 \quad t_{10} \quad 2t_9 \quad t_2 + t_4 \quad t_6 + t_8 + t_{10}]$$

and

$T_C = [t_1 \quad 2t_2 \quad t_3 + t_5 \quad 2t_4 \quad 2t_6 \quad t_7 \quad 2t_8 \quad t_9 \quad 2t_{10} \quad t_1 + t_3 \quad t_5 + t_7 + t_9]^T$

where, the row-transition vector T_R is the sum of corresponding columns of L_{BP} and the column-transition vector T_C is the sum of corresponding rows of L_{BP}
The components/entries of T_R and T_C are finite linear combinations of transitions with positive integer coefficients.The number of incoming/outgoing arcs from the transitions are represented by the coefficients of transitions that are present in the transition vectors $T_C(T_R)$.

i. **Cyclic/Acyclic Nature** :
A Petri Net is acyclic if atleast one entry in T_R or T_C is zero. The T_R and T_C corresponding to our proposed model has no zero component. Hence, this Petri net is cyclic in nature.

The cycles are computed as:

The transition vectors (row and column) are inputs.

(a) Select any i^{th} entry(place) of T_C

(b) Select any transition in the i^{th} component of T_C

(c) locate this transition in the components of T_R

(d) Choose an entry (place) in T_R that corresponds to the transition we chosen in step (b)

(e) If this selected entry is same as previously selected entry then end the cycle otherwise go back to step(b).

Some of the possible cycles are:
$p_{1e} \rightarrow t_1 \rightarrow p_{1d} \rightarrow t_2 \rightarrow p_{1e}$
$p_{1d} \rightarrow t_2 \rightarrow p_{1e} \rightarrow t_1 \rightarrow p_{1d}$
$p_{2e} \rightarrow t_3 \rightarrow p_{2d1} \rightarrow t_4 \rightarrow p_{2e}$
$p_{d2} \rightarrow t_7 \rightarrow p_{3d} \rightarrow t_8 \rightarrow p_{d2}$

ii. **Conflict** :
If every component of T_C has exactly one transition, or if any component has more than one transition, then the same transitions must be present in the corresponding component of T_R, then Petri Net is conflict-free.
Here, 3^{rd} component of T_C has two different transitions that are t_3 and t_5 and the corresponding component of T_R has different transitions implying that there is conflict between t_3 and t_5. Hence, this Petri net is in conflict.

iii. **Self-loop(free)** :
A Petri Net is said to be self loop free if and only if the corresponding components of both T_R and T_C have no transitions that are same or identical at a certain place p_k. In our proposed model, there are no identical corresponding components of T_R and T_C so this Petri net is self-loop free.

iv. **Structural Concurrency** :
A Petri Net is said to be structurally concurrent iff at least one component of the column transition vector T_C has coefficient 2 or greater than 2. Our Petri net structure is concurrent as in T_C, components corresponding to places $p_{1d}, p_{2d1}, p_{2d2}, p_{3d}, p_{4d}$ has coefficient 2.

Chapter 8

Conclusion

In chapter 7, we have designed the model of four friends sharing two drinks while they have their respective eating items. The model has been analysed and observed the properties like safeness and boundedness, conservation, coverability, liveness using reachability tree and cyclic/acyclic nature, conflict, self-loop(free), structural concurrency using matrix equations.
The Figures in Chapter$1-6$ has been taken from [3, 4]

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The Paper titled **”Designing and Analysing a Petri Net Model of four friends sharing two drinks”** was accepted and presented in Scopus Indexed conference titled ”International Conference on Applied Science, Mathematics and Statistics(ICASMS-23), held on 18^{th} February, 2023.
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Paper Title: A Petri Net Model of friends sharing drinks

Authors Name: Anshu Choudhary, Sangita Kansal

Dear Authors,

With heartiest congratulations I am pleased to inform you that based on the recommendations of the reviewers and the Technical Program Committees, your paper identified above has been accepted for publication and oral presentation by **International Conference on Applied Science, Mathematics and Statistics (ICASMS-23)**

(ICASMS-23) conference received over 80 submissions from countries and regions, reviewed by international experts and your paper cleared all the criteria, got accepted for the conference. Your paper will be published in the conference proceeding after registration.

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Herewith, the conference committee sincerely invites you to come to present your paper at (ICASMS-23) to be held on 18th February`2023, Mumbai, India.

Regards,

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George Mathew
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