DESIGN OF CONTROLLERS FOR TWIN ROTOR MIMO (MULTIPLE INPUT MULTIPLE OUTPUT) SYSTEM

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Candidate's Declaration

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I hereby certify that the dissertation titled "**Design of Controllers for Twin Rotor MIMO System**" which is submitted by Gurmeet Sehgal, 2K16/C&I/09 in Electrical Engineering, Delhi Technological University, Delhi in partial fulfillment of the requirements for the award of the degree of Master of Technology, is a record of the project work carried out by the student under my supervision. To the best of my knowledge this work has not been submitted in part or full for award of any Degree or Diploma to this University or elsewhere.

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Gurmeet Sehgal

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ABSTRACT

Twin Rotor MIMO System (TRMS) is considered as a replica of Helicopter. The aim of the present work is to study the mathematical model of TRMS and designing the controllers for improving the response of TRMS. It provides a platform for controlling the flight of Helicopter through its pitch and yaw angle control. It is basically providing the turning or rotating action in operation of helicopter. In this project work, a non-linear model of Twin Rotor MIMO system is developed, linearized and represented in state space form. The Linear Quadratic Regulator (LQR) and a fuzzy controller (FC) have been designed for pitch and yaw angle control of a multiple input multiple output Twin Rotor system. Two degree of freedom dynamic model of helicopter is being used for Pitch and Yaw motion control through suitable controller design. In LQR regulator basically the LQR Parameters Q and R matrices have been chosen randomly. LQR method of control is the modern method of system design that uses state space method with full state feedback system to control and to stabilize the system. The performance of LQR controller is better in terms of robustness and good dynamic performance. Fuzzy controller basically consists of four subsystem fuzzifier, fuzzy rule base, inference engine and defuzzifier. Fuzzy controllers are nonlinear controllers in which two different control one for horizontal and other for vertical axis having different Rule base and membership functions are designed. Both the controllers are Mamdani Inference and designed and simulated in MATLAB. Lastly the comparison of performance parameters of TRMS with different controller response have been presented.

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LIST OF SYMBOLS, ABBREVIATIONS AND NOMENCLATURE

M_{V1}	Return torque corresponding to the forces of gravity,
M_{mr}	Mass of the main DC-motor with main rotor
M_m	Mass of main part of the beam,
M_{tr}	Mass of the tail motor with tail rotor
M_t	Mass of the tail part of the beam,
M_{cb}	Mass of the counter-weight,
M_b	Mass of the counter-weight beam,
M_{ms}	Mass of the main shield,
M_{ts}	Mass of the tail shield,
L_m	Length of main part of the beam,
L_t	Length of tail part of the beam,
L_b	Length of the counter-weight beam,
L_{cb}	Distance between the counter-weight and the joint
G	Gravitational acceleration.
I_1	Moment of inertia of vertical rotor
I_2	Moment of inertia of horizontal rotor
b_1	Static characterstics parameter
b_2	Static characteristics parameter
c_1	Static characterstics parameter
c_2	Static characteristics parameter
M_g	Gravity momentum
T_{20}	Motor 2 denominator parameter
T_P	Cross reaction momentum parameter

T_0	Cross reaction momentum parameter
T_{22}	Motor 2 denominator parameter
T_{10}	Motor 1 denominator parameter
T_{11}	Motor 1 denominator parameter
K_{gy}	Gyroscopic momentum parameter
$B_{2\alpha_h}$	Friction momentum function parameter
$B_{1\alpha_h}$	Friction momentum function parameter
$B_{1\alpha_v}$	Friction momentum function parameter
$B_{2\alpha_v}$	Friction momentum function parameter
K_2	Motor gain 2
K_1	Motor gain 1
K_C	Cross reaction momentum parameter gain
M_v	Total moment of forces in the vertical plane
Jv	The sum of moments of inertia relative to the horizontal axis
α_v	The pitch angle of the beam.
M_{v2}	The moment of the propulsive force produced by the main rotor
ω_m	Angular velocity of the main rotor
$F_v(\omega_m)$	Dependence of the propulsive force on the angular velocity of the rotor.
M_{v3}	The moment of centrifugal forces corresponding to the motion of the beam around the vertical axis
Ω_h	The angular velocity of the beam around the vertical axis,
α_h	The azimuth angle of the beam.
M_{v4}	The moment of friction depending on the angular velocity of the beam around the horizontal axis.
Ω_v	The angular velocity around the horizontal axis,
K_v, K_h	Constants
r_{ms}	Radius of main shield
r_{ts}	Radius of tail shield
M_h	The sum of moments of forces acting in the horizontal plane

J h	The sum of moments of inertia relative to the vertical axis
ω_t	The rotational velocity of tail rotor
$F_h(\omega_t)$	Denotes the dependence of the propulsive force on the angular velocity of the tail rotor.
M_{h2}	The moment of friction depending on the angular velocity of the beam around the vertical axis
J_{tr}	Moment of inertia in DC-motor -tail propeller subsystem,
J_{mr}	Moment of inertia in DC-motor -main propeller subsystem
S_v	Angular momentum in vertical plane for the beam,
S_h	Angular momentum in horizontal plane for the beam.
Tmr	Time constant of main motor- propeller system,
Ttr	Time constant of tail motor- propeller system

CHAPTER 1

INTRODUCTION

1.1 GENERAL

The recent development in flight control vehicles such as helicopter and Unmanned Aerial Vehicle (UAV) has emerged as an important area of control system. The control of these vehicles is achieved by modeling the vehicle and then designing suitable linear or non-linear controllers. This Twin Rotor Multiple Input Multiple Output system resembles like a helicopter system having a number of nonlinearities. This is an experimental setup which can be used to study flight dynamics and thus can be used for performing experiments with air vehicles.

TRMS comprises of two types of rotor, main and tail rotor at each end of the beam, which is run by a DC motor and is counter stabilized by an arm with weight at its end pivoted together. The system is movable easily in both horizontal and vertical direction. The four process variables are used to describe the beam vertical and horizontal angles which are monitored by encoders present at pivot and two relative angular velocities which are measured by speed sensor connected to the DC motor.

Still there is some difference in control of helicopter and TRMS. Helicopter control is achieved by changing or varying the rotor angles but in TRMS control it is achieved by rotor speed.

Several research papers are available on modeling and controlling of TRMS. The present work includes design of controller to reduce unmodeled dynamics and disturbances in the output measurement. The controllers are designed which are used in feedback controller model of TRMS for controlling the system. All the states are not reachable in a TRMS system and so a Kalman filter is used to measure the unreachable state variables from the output. This Kalman filter is used along with a LQR controller to get a desired output. This combination of Kalman filter and LQR is termed as LQG control.

The selection of parameters for control of plant is made randomly by hit and trial method which comprises of maintaining a balance between the control effort and the response of the system.
A nonlinear Fuzzy controller is used to control the system.

1.2 TRMS IDENTIFICATION

TRMS system is a replica of the helicopter system and also provides the replication of the flight dynamics. TRMS model gains the popularity because it is difficult to do experiments directly on the air vehicles. The TRMS system has two rotors mounted on a beam together with a balanced weight. The whole unit is connected to the tower which allows safe and fast operation of TRMS. The system rotates in both horizontal as well as vertical direction. The system is evaluated by seven variables which are pitch angle, yaw angle, pitch angular velocity, yaw angular velocity, cross reaction momentum and pitch yaw rotor momentum. Both angular velocities are calculated by encoders present at the pivot for measuring these two speeds sensors are coupled to the dc motors. The electrical part also plays an important role for controlling the system as it helps in transferring of the measured signals to the connected PC via I/O card. The mechanical part and electrical both helpful in controlling the whole unit. Fig 1 shows a picture of TRMS control system.

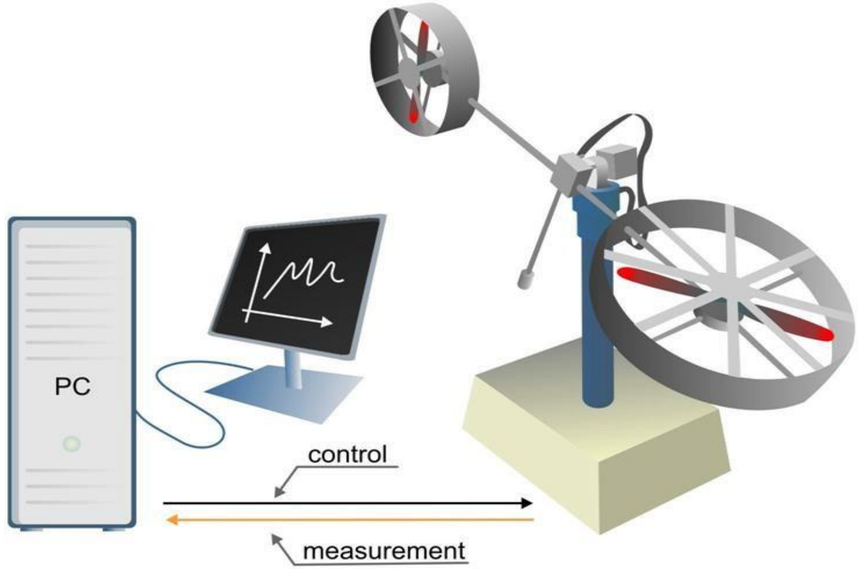

Fig1. TRMS Control System

TRMS MIMO is a nonlinear plant with a cross coupling effect between the main and tail rotor. To simplify and to understand the whole system decoupling need to be done, therefore the system decoupling and designing of two different controllers is required. Four different linear models are considered here one for control input u_1 to α_v , u_2 to α_v , u_1 to α_h and u_2 to α_h.

There are few points to be kept in mind while doing the identification of TRMS, which are listed below:

- Stability problem
- Choice of structure
- Sampling time
- Excitation signal
- Identification method

Usually methods for system identification are least mean square method and instrumental variable method. MainYaw_Ident.mdl. and Main pitch _Ident.mdl. models are used for the identification which are given in the manual also. The input signal is the combination of different sinusoidal signals with different frequencies. The responses are analyzed and recorded for pitch and yaw separately.

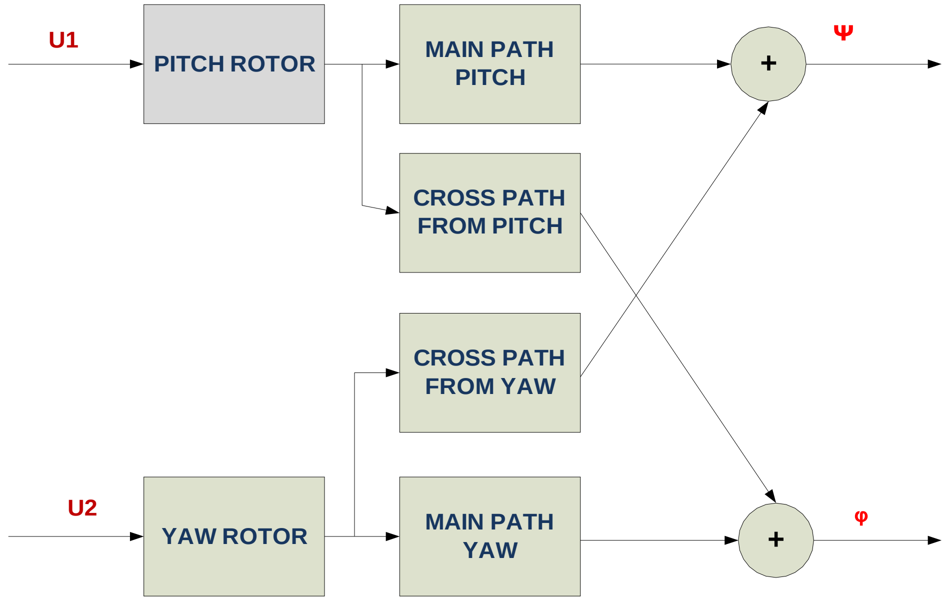

Fig2. TRMS simplified system

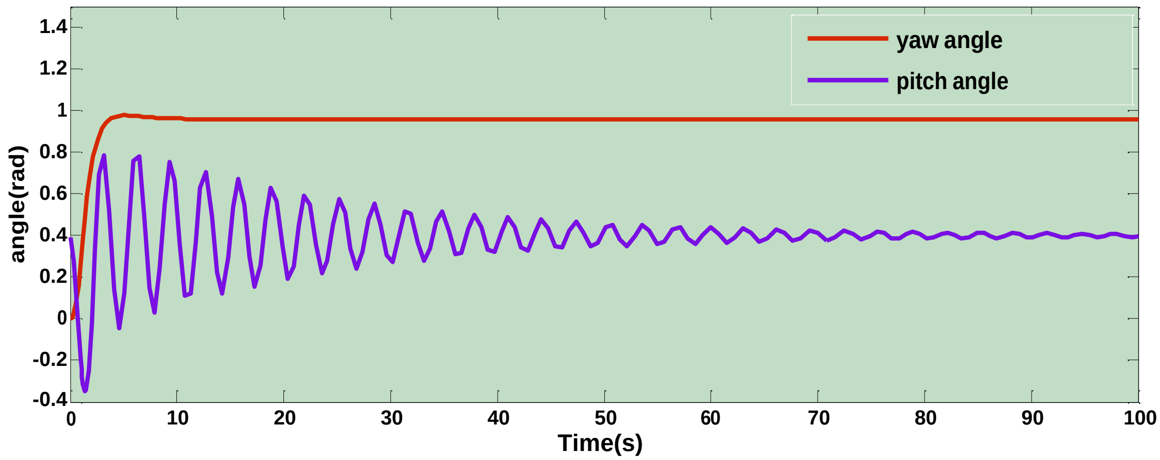

Fig3. Step response of pitch and Yaw angle

Fig.3 shows the step response of Yaw and pitch angle, it presents the model responses to step changes (1 to 0) of control voltages. The response of nonlinear TRMS model varies as the values of control voltages ($u_1$and u_2) varies.

1.2.1 Main Path Pitch Identification

This main path pitch identification model describes the relation between the control voltage u_1 and the pitch angle α_v. For this identification the MATLAB system identification toolbox is used. This identification is carried out by using the model MainPitch_Ident.Modl.

Experiment is carried out with the excitation signal of Sinusoidal nature with the sampling time T_s= 0.1s. This excitation signal excites the model and records its response. We can check the quality of the response by the step response analysis.

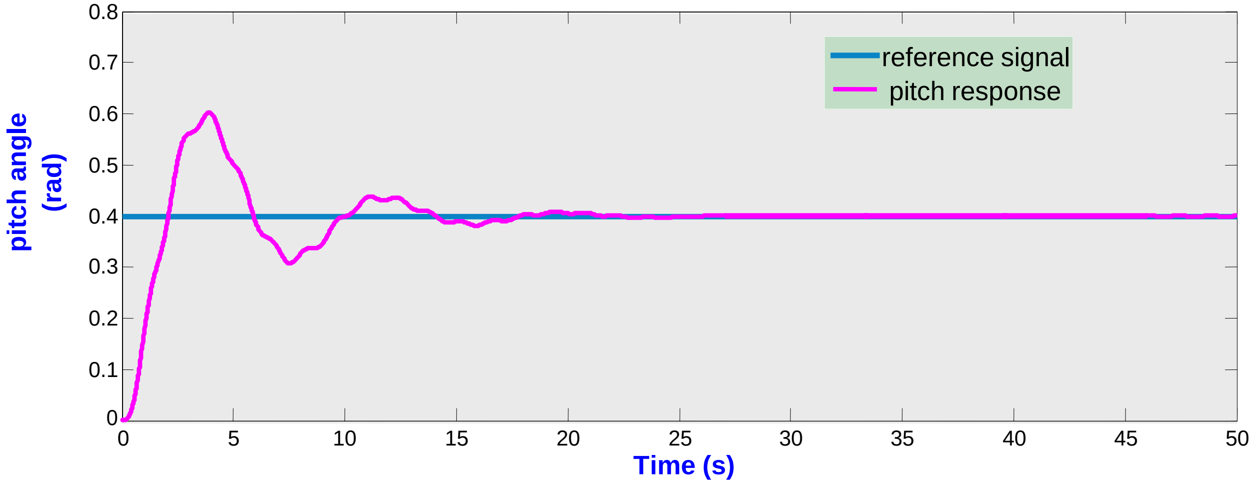

Fig4. Step response of pitch path model

1.2.2 Main Path Yaw Identification

This main path yaw identification model describes the relation between the control voltage u_2 and the pitch angleα_h. For this identification the MATLAB system identification toolbox is used. This identification is carried out by using the model MainYaw_Ident.Modl.

Experiment is carried out with the excitation signal of Sinusoidal nature with the sampling time T_s= 0.1s. This excitation signal excites the model and records its response. We can check the quality of the response by the step response analysis.

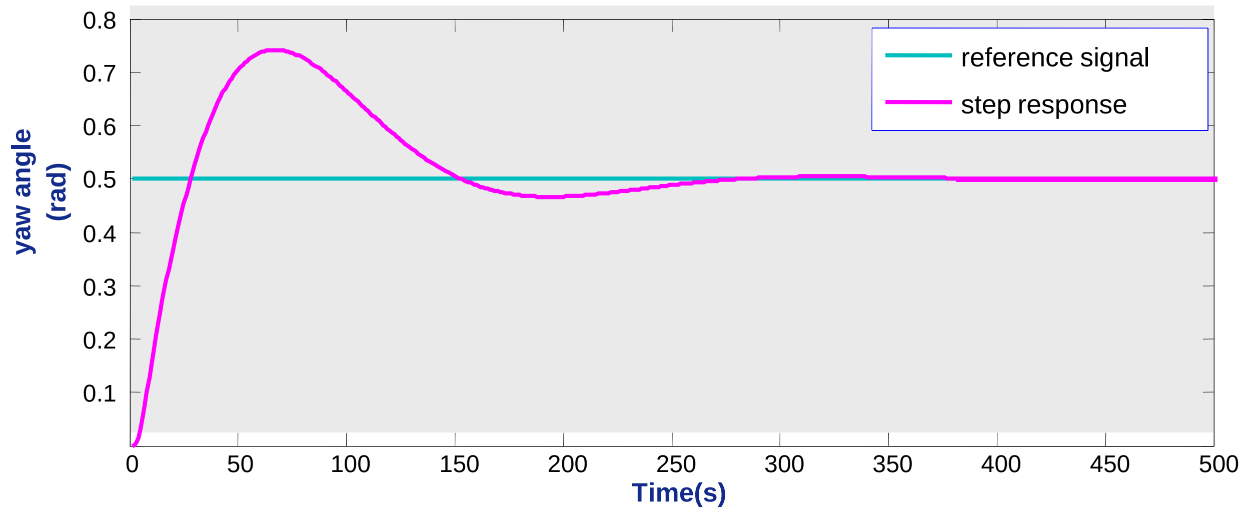

Fig5. Step response of Yaw path model

1.3 Objective of the present work

This thesis mainly focuses on the following aspects of TRMS

1. Identification of parameters of the TRMS system using MATLAB system identification toolbox.
2. Development of mathematical model and linearization of the TRMS system around operating points.
3. Design of LQR model for pitch and yaw control of TRMS system.
4. Design of LQR plus PID model for pitch and yaw control of TRMS system.
5. Design of Fuzzy logic controller for pitch and yaw control of TRMS system.

1.4 Organization of thesis

This thesis is organized in six chapters:

Chapter 1 gives a brief introduction and identification of TRMS system. This chapter also comprises of the mechanical construction of the system.

Chapter 2 presents a literature review of all the references used to carry out the present study.

Chapter 3 presents the mathematical model of the system along with derivation of transfer function and linearization of the system model.

Chapter 4 presents the basic of controller's design and types of controllers. This chapter also defines the responses of PID and LQR controllers.

Chapter 5 presents the design of Fuzzy controller and its responses for main and tail rotors separately.

Chapter 6 gives a conclusion of the complete study on designing of controllers for TRMS system. It also mentions the future scopes of the study.

CHAPTER 2

LITERATURE REVIEW

2.0 GENERAL

Development in flight control vehicles such as helicopter and UAV find great relevance. Helicopters are highly nonlinear with some significant dynamic coupling effect. Due to the complicated aerodynamic nature of the helicopter system there is significant parameter and model uncertainty problem therefore there is major interest in designing of the control system and their implementation. This control is achieved by modeling the vehicle and then controlling using nonlinear control or linear control. In general most controllers designing is based on linearized dynamics of the helicopter system however in recent years there is major interest in helicopter flight control based on non-linearized dynamics. TRMS MIMO system resembles a helicopter system having a number of nonlinearities. This is an experimental setup which can be used to study flight dynamics and thus can be used for performing experiments with air vehicles. Helicopters play the important role in conveyances because these are used to rescue disasters, moving large objects from place to place, to fly over the cities and report on traffic, to surveillance over hard to reach places like mountains and oceans.

2.1 TRMS CONTROL SYSTEM

Feedback Instruments Ltd. Manual [1] refers to the Feedback Instruments Twin Rotor MIMO System. This manual describes useful information and physical behavior of the TRMS system. TRMS system is a nonlinear model and later system identification is done. Control algorithms are developed, tested and analyzed on the real time model. It described the designing of PID controller and Fuzzy PID controller and the result of both the controllers are compared. Fuzzy PID controller is designed for horizontal as well as for vertical part.

P. Wen T.-W. Lu [2] presented identification of nonlinear model of TRMS and decoupling control of a Twin rotor MIMO system using robust deadbeat control technique. So the system can be decoupled into two single-input–single-output (SISO) systems, and the cross couplings are considered as disturbances to each other. Finally, a robust deadbeat control scheme is applied to the two SISO systems and a controller is designed for each of the parts. This design is evaluated in simulations, and the result is tested in real time Twin Rotor MIMO system. Finally, it is compared with the PID controller and it is found that this method is easy, and the results are better than the PID controller in terms of the overshoot, settling time and robustness to the cross coupling.

Rashmi Ranjan Nayak and Asutosh Satapathy [3] have described the nonlinear model identification of TRMS, linearization and designing of state space model for the controller's design. PID, LQR and LQG three controllers are designed in this paper. LQR and LQG both are full state feedback controllers in which the Q and R matrices are calculated which are performance matrices and gain parameter K is calculated and also the Kalman gain is calculated for the LQG controller.

Jatin Kumar Pradhan and Arun Ghosh [4] explained the design and implementation of decoupled compensation for a Twin Rotor Multiple-Input and Multiple-Output system decoupled pitch and yaw separately. The main problem of designing the controllers for this system is coupling effect between its input and output. Similarly all other references mentioned above gives the basics of controllers.

2.2 LQR PLUS PID CONTROLLER DESIGN

Summit Kumar Pander and Vijay Lama [5] have explained control of Twin Rotor MIMO system using PID controller with derivative filter coefficient.

Jisha Shaji and Aswin R [6] have described Pitch control of aircraft using LQR & LQG and pitch control of flight system using dynamic inversion and PID controller. It described two control schemes in detail which are the Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG) Control. LQR is a modern control technique that uses state-space approach to analyze a system. In the Linear Quadratic Regulator (LQR) design method selection of Performance Index (PI) weighting matrices Q and R is the important task, which are also the designing parameters of the regulator. Linear Quadratic Gaussian (LQG) also falls under the category of modern controller. LQG control is the combination of Linear Quadratic regulator (LQR) and Kalman filter

Bhanu Pratap, Abhishek Agrawal, and Shubhi Purwar [7] have illustrated optimal control of Twin Rotor MIMO system using output feedback and have described an optimal state regulator which is known as LQR regulator. The Twin rotor MIMO system (TRMS) is a high order nonlinear system with significant cross couplings effects. Firstly, the nonlinear model is linearised and the controller is designed. The controller gain is changed iteratively, until optimal value is reached.

Sumit Kumar Pandey, Jayati Dey and Subrata Banerjee [8] have illustrated Design and real-time implementation of robust PID controller for Twin Rotor MIMO system (TRMS) based on Kharitonov's Theorem and explained laboratory setup of a helicopter system. TRMS system is linearised and robust PID controller is designed by employing Kharitonov stability theorem.

Andrew Phillips and Dr. Ferat Sahin [9] presented optimal control of a Twin Rotor MIMO system using LQR with Integral action. This paper explained that the Twin rotor MIMO system (TRMS) is a replica of a helicopter like system that is restricted to two degrees of freedom, pitch and yaw. It is a complicated nonlinear and coupled, MIMO system. This paper used a linear quadratic regulator (LQR) with integral action. In this paper LQR controller with integral action (LQI) provided superior performance to existing optimal controllers.

S.M. Ahmad, A.J. Chipperfield and O.Tokhi [10] described the dynamic modelling and optimal control of a Twin Rotor MIMO System.

Zhuoyi Chen, Zhuo Sun and Wenbo Wang [11] explained the design and implementation of Kalman filter as TRMS model is stochastic system due to the presence of process noise and measurement noise, it cannot be modeled by using deterministic model. Thus this noisy plant is a stochastic system, which can be modeled by passing white noise through appropriate linear System While designing the Kalman Filter, the process noise spectral density matrix V and measurement noise spectral density matrix Z are randomly chosen. Similar explanations are given in the other references also.

Howardp Lee and Robertj Dudginski [12] illustrated Kalman filter design for control surface failure detection and isolation.

Ezzara, Mustapha Lafkih and Mohamed Ramzi [13] demonstrated the Design and performance analysis of adaptive linear quadratic Gaussian Controller.

A.Q.Khan and N.Iqbal [14] described Modeling and Design of an optimal regulator for three degree of freedom Helicopter.

S. Juhng-Perng, L. Chi-Ying, and C. Hung-Ming [15] explained robust control of a class of nonlinear systems and its application to a Twin rotor MIMO system,

W.-Y. Wang, T.-T. Lee, and H.-C. Huang[16] explained evolutionary design of PID controller for Twin rotor multi-input multi-output system.

M. Lopez-Martinez, M. G. Ortega, and F. Rubio [17] described an H∞ controller of the Twin rotor laboratory equipment.

M. Lopez-Martinez, C. Vivas, and M. G. Ortega [18] explained a multivariable nonlinear H∞ controller for a laboratory helicopter.

Ahmad, S.M.; Chipperfield, A.J.; Tokhi, MO [19] described Dynamic modeling and optimal control of a Twin rotor MIMO system and described modelling and control of a 2 DOF Twin rotor multi-input multi-output system.

Aldebrez, F.M.; Darus, I.Z.M.; Tokhi, M.O [20] described dynamic modelling of a Twin rotor system in hovering position.

R.A Krohling, H, Jaschek, JP Rey [21] explained designing Pl/PID controllers for a motion control system based on genetic algorithms.

A. P. Ramalakshmi and P. S. Manoharan [22] described non-linear modeling and PID control of Twin Rotor MIMO System.

Nurbaiti Wahid, Mohd Fuaad Rahmat [23] described Pitch control system using LQR and Fuzzy Logic controller.

Labane Chrif , Zemalache Meguenni Kadda [24] described aircraft control system using LQG and LQR Controller with optimal estimation-Kalman filter design.

2.3 Fuzzy logic Controller Design

T. Hosseializade, S.M.J. Hosseini and H. Khaloozadeh [25] presented design and implementation classical, state feedback and Fuzzy controllers on Twin Rotor System. In this paper three controllers PID, state feedback, and fuzzy controllers are designed and implemented to this system. In this paper zero and pole cancellation method is used to design PID controller. In state feedback method state variables for this system are angle position and angular velocity. Using the Mamdani inference engine, singleton fuzzifier and center average defuzzifier, designing is done for the fuzzy. There are not so different from each other from sinusoidal responses point of view although the fuzzy controller has a slower response than others.

Deepak Kumar Saroj and Indrani Kar [26] presented T-S Fuzzy Model Based Controller and Observer Design for A Twin Rotor MIMO System. Twin Rotor MIMO System is the prototype of a helicopter with two degrees of freedom which involves significant cross coupling between the main rotor and the tail rotor. A Takagi-Sugeno (T-S) fuzzy model is used to approximate the nonlinear dynamics of the system. With the derived T-S fuzzy model, a fuzzy controller is designed that guarantees not only stability but also satisfies the specified performance criteria of the closed-loop control system. A set of inequalities is formed based on the Lyapunov sufficient condition to ensure stability of the T-S fuzzy model using a parallel distributor fuzzy compensator. The controller gains are obtained by solving the set of inequalities. A fuzzy observer is also designed to estimate the states of the system. Similar understanding is gained from other references also mentioned above.

B. U. Islam, N. Ahmed, D. L. Bhatti, and S. Khan [27] explained controller design using Fuzzy logic for a Twin Rotor MIMO system.

E.H.Mamdani [28] represented application of Fuzzy logic to approximate reasoning using linguistic Synthesis.

M. Passino and S. Yurkovich [29] presented the designing of Fuzzy controllers for the TRMS system.

2.4 CONCLUSION

An extensive literature review on the TRMS is presented in this chapter and various issues related to the controller design for TRMS system are discussed.

CHAPTER 3

MATHEMATICAL MODELLING OF TRMS

3.1 GENERAL

The TRMS is a multiple input multiple output system. Controlling input parameters of the TRMS are u1 and u2. Both vertical and horizontal rotors are cross coupled with each other which is also a key feature of the system. The position of both the main and tail rotors are calculated using incremental encoders that gives the position signal therefore the setting of initial condition is necessary while running the model in real time.

MIMO which is a nonlinear model with the cross-coupling effect between the vertical and horizontal rotors have been examined through linearization of nonlinear model for control of both rotor dynamics.

3.2 TRMS Mathematical Model

The Electro-Mechanical Model of the TRMS system is shown in Fig.6. It consists of two rotors tail rotor and main rotor placed on a beam. The beam is connected to a counterweight and pivoted on a vertical support or base. The beam can be move freely in both horizontal and vertical direction. Both the rotors are driven by the DC motors separately. Main rotor which is usually known as pitch rotor is used for the vertical movement and the tail rotor which is usually known as yaw rotor is used for the controlling the direction. Usually nonlinear models exist which means that at least one state variable is a nonlinear function. To determine the transfer function and for controller designing for the nonlinear system it has to be linearised. The mathematical model is developed under some simplified assumptions. First, it is assumed that the dynamics of the propeller subsystem can be described by first order differential equations. Further, it is assumed that friction in the system is of the viscous type. It is assumed also that the propeller-air subsystem could be described in accordance with the postulates of flow theory. First, consider the rotation of the beam in the vertical plane that is around the horizontal axis.

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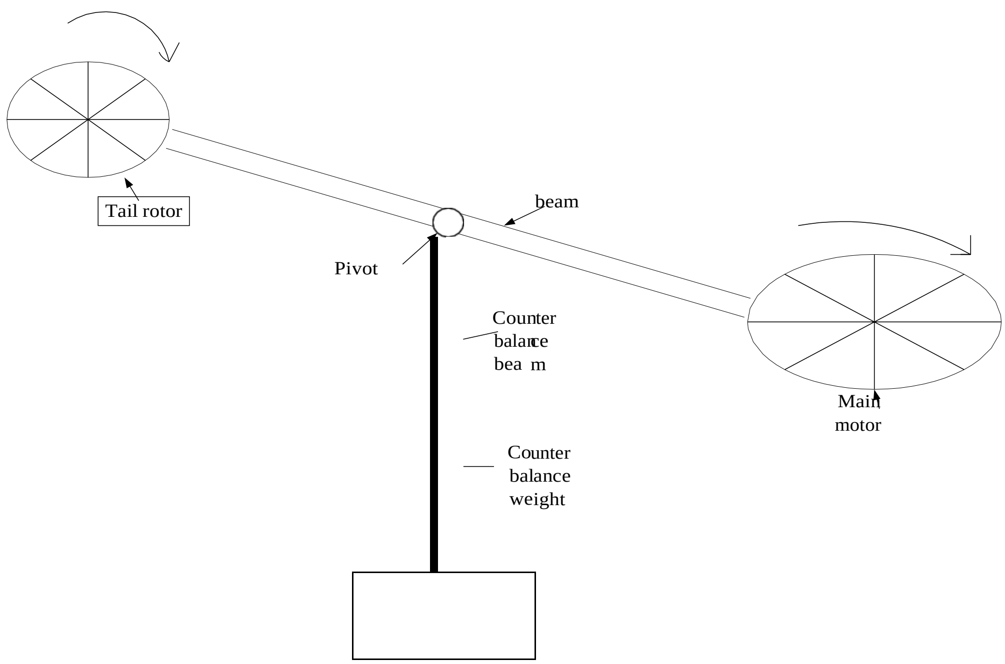


Fig6. TRMS Electro-Mechanical Model

For modeling the nonlinear TRMS system the following notations are used which represents various parameters of the system.

M_{mr} is the main dc motor mass.

M_{mr} is the mass of the main beam.

M_{tr} is the tail motor mass.

M_t is the tail's mass.

M_{Cb} is the counter balance mass.

M_b is the mass of the counter-weight beam.

M_{ms} is the main shield mass.

M_{ts} is the tail shield mass.

L_m is the main part beam's length.

L_t is the tail part beam's length.

L_b is the length related to beam counter-weight.

L_{cb} is the counter-weight and the joint's distance.

g is the acceleration due to gravity.

According to Newton's second law of motion

$$M_v = J_v \frac{d^2\alpha_v}{dt^2} \tag{3.1}$$

$$M_v = \sum_{i=1}^{4} M_{vi} \quad , \qquad J_v = \sum_{i=1}^{8} J_{vi} \tag{3.2}$$

where,

M_v= summation of moment of forces corresponding to the vertical plane,
J_v = summation of moments of inertia corresponding to the horizontal axis,
α_v= pitch angle of the beam.

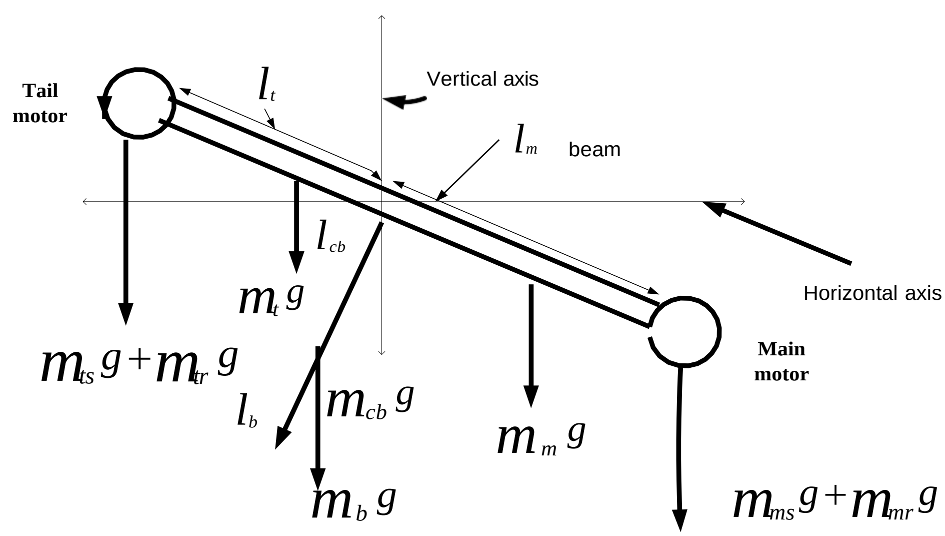


Fig.7 Forces due to gravity related to returning effect

$$M_{v1} = g \left\{\left[\left(\frac{m_t}{2} + m_{tr} + m_{ts}\right) l_t - \left(\frac{m_m}{2} + m_{mr} + m_{ms}\right) l_m\right] \cos \alpha_v - \left(\frac{m_b}{2} l_b + m_{cb} l_{cb}\right) \sin \alpha_v\right\} \tag{3.3}$$

$$M_{v1} = g \{[A - B] \cos \alpha_V - C \sin \alpha_v\} \tag{3.4}$$

Where M_{v1} is the return torque correlated to the forces of gravity.

Where A,B and C are constants and can be expressed as:

$$A = \left(\frac{m_t}{2} + m_{tr} + m_{ts}\right) l_t \text{ , } B = \left(\frac{m_m}{2} + m_{mr} + m_{ms}\right) l_m\text{, } C = \left(\frac{m_b}{2} l_b + m_{cb} l_{cb}\right) \qquad (3.6)$$

Fig.8 shows all forces applied by the gravity which are related to the horizontal axis of the system.

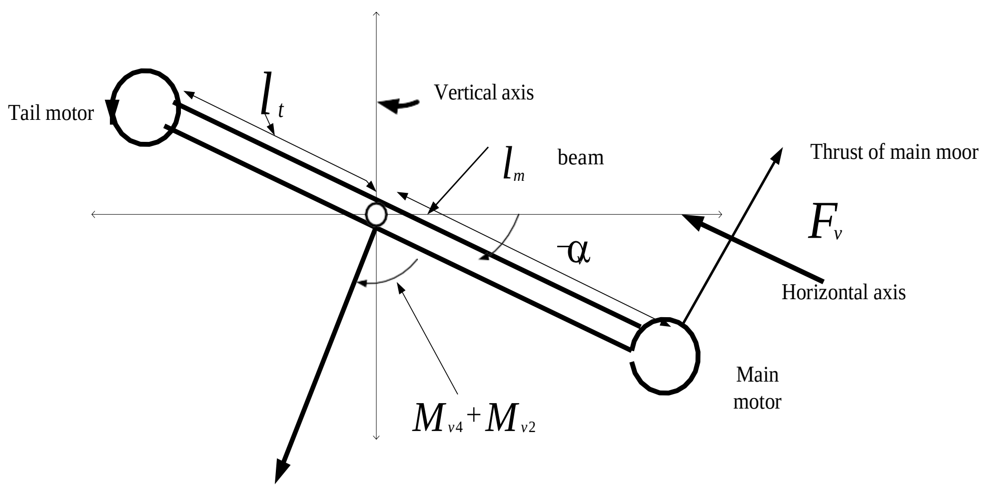


Fig8. Forces due to gravity related to horizontal axis

$$M_{v2} = l_m F_v \omega_m \qquad (3.7)$$

Where,

M_{v2} = propulsive force moment of the main rotor,

ω_m = angular velocity of the main rotor

$F_v\, \omega_m$ = propulsive force dependent on angular velocity

$$M_{v3} = \left\{ \begin{matrix} \left(\frac{m_t}{2} + m_{tr} + m_{ts}\right) l_t \\ + \left(\frac{m_m}{2} + m_{mr} + m_{ms}\right) l_m \left(\frac{m_b}{2} l_b + m_{cb} l_{cb}\right) \end{matrix} \right\} \sin\alpha_v \cos\alpha_v \qquad (3.8)$$

$$M_{v3} = -\Omega_h^2 (A + B + C) \sin\alpha_v \cos\alpha_v \qquad (3.9)$$

Where,

M_{v3}= centrifugal forces momentum of the vertical axis,

$$\Omega_h = \frac{d\alpha_h}{dt} \tag{3.10}$$

Where,

Ω_h= angular velocity of the beam with respect to vertical axis,

α_h = azimuth angle of the beam.

$$M_{v4} = -\Omega_v k_v \tag{3.11}$$

Where,

M_{v4} = moment of friction dependent on angular velocity of the beam around the horizontal axis.

$$\Omega_v = \frac{d\alpha_v}{dt} \tag{3.12}$$

Where,

Ω_v = angular velocity of horizontal axis,

k v = constant

J_{v1}, J_{v2}, J_{v3}, J_{v4}, J_{v5}, J_{v6}, J_{v7} and J_{v8} are the components of the moment of inertia with respect to the horizontal axis (independent of pitch position) are calculated as:

$$J_{v1} = m_{mr} l_m^2 \tag{3.13}$$

$$J_{v2} = m_m \frac{l_m^2}{3} \tag{3.14}$$

$$J_{v3} = m_{cb} l_{cb}^2 \tag{3.16}$$

$$J_{v4} = m_b \frac{l_b^2}{3} \tag{3.17}$$

$$J_{v5} = m_{tr} l_t^2 \tag{3.18}$$

$$J_{v6} = m_t \frac{l_t^2}{3} \tag{3.19}$$

$$J_{v7} = \frac{m_{ms}}{2} r_{ms}^2 + m_{ms} l_m^2 \tag{3.20}$$

$$J_{v8} = m_{ts} r_{ts}^2 + m_{ts} l_t^2 \tag{3.21}$$

Where,

r_{ms}= main shield radius

r_{ts} = tail shield radius

Similarly, we can describe the motion of the beam around the vertical axis having in mind that the driving torques are produced by the rotors and that the moment of inertia depends on the pitch angle of the beam. The horizontal motion of the beam (around the vertical axis) can be described as a rotational motion of a solid mass.

$$M_h = J_h \frac{d^2\alpha_h}{dt^2} \tag{3.22}$$

$$M_h = \sum_{i=1}^{2} M_{hi} \quad , \quad J_h = \sum_{i=1}^{8} J_{hi} \tag{3.23}$$

Where,

M_h= summation of moments of forces related to the horizontal axis,

J_h = summation of moments of inertia respective to the vertical axis.

To determine the moments of forces and making the beam rotate around the vertical axis considering the situation shown in Figure9, shows the forces due to gravity related to the vertical axis of the system

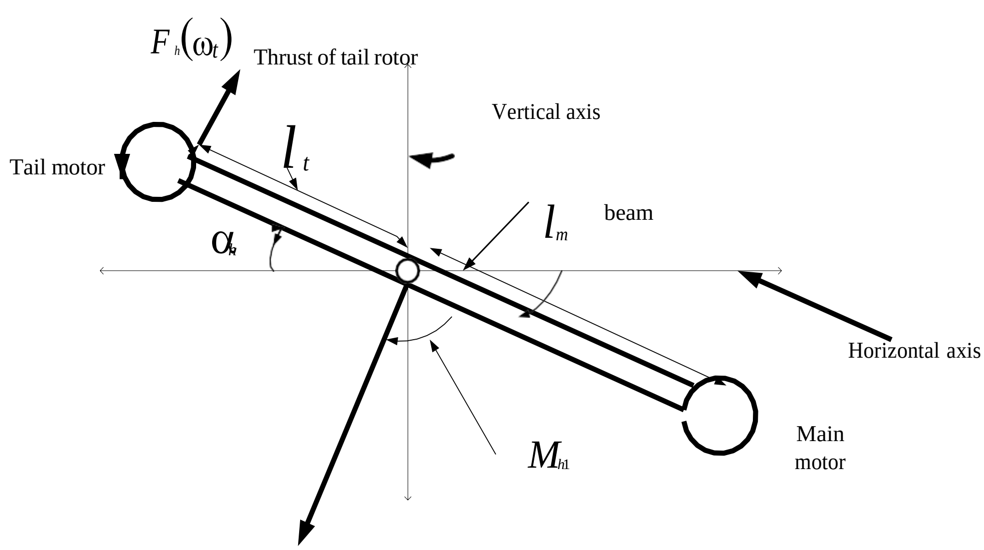


Fig 9. Forces due to gravity related to vertical axis

$$M_{h1} = l_t F_h \omega_t \tag{3.24}$$

Where,

ω_t= rotational velocity of tail part

F_h ω_t = propulsive force and angular velocity are dependent and related to the tail rotor.

$$M_{h2} = -\Omega_h k_h \tag{3.25}$$

M_{h2} = moment of friction and angular velocity are dependent with respect to vertical axis

k_h = constant.

J_{h1} , J_{h2} , J_{h3}, J_{h4}, J_{h5}, J_{h6}, J_{h7} and J_{h8} are the components of the moment of inertia corresponding to the vertical axis (depend on the pitch position) are found as

$$J_{h1} = \frac{m_m}{3}(I_m \cos\alpha_v)^2 \tag{3.26}$$

$$J_{h2} = \frac{m_t}{3}(I_t \cos\alpha_v)^2 \tag{3.27}$$

$$J_{h3} = \frac{m_b}{3}(I_b \sin\alpha_v)^2 \tag{3.28}$$

$$J_{h4} = m_{tr}(I_t \cos\alpha_v)^2 \tag{3.29}$$

$$J_{h5} = m_{mr}(I_m \cos\alpha_v)^2 \tag{3.30}$$

$$J_{h6} = m_{cb}(I_{cb} \sin\alpha_v)^2 \tag{3.31}$$

$$J_{h7} = \frac{m_{ts}}{2} r_{ts}^2 + m_{ts}(I_t \cos\alpha_v)^2 \tag{3.32}$$

$$J_{h8} = m_{ms} r_{ms}^2 + m_{ms}(I_m \cos\alpha_v)^2 \tag{3.33}$$

$$J_h = D\sin^2\alpha_v + E\cos^2\alpha_v + F \tag{3.34}$$

Where,

D, E, F are constants and can be expressed as

$$D = \frac{m_b}{3} I_b^2 + m_{cb} I_{cb}^2 \tag{3.35}$$

$$E = \left(\frac{m_m}{3} + m_{mr} + m_{ms}\right) I_m^2 + \left(\frac{m_m}{3} + m_{tr} + m_{ts}\right) I_t^2 \tag{3.36}$$

$$F = \frac{m_{ts}}{2} r_{ts}^2 + m_{ms} r_{ms}^2 \tag{3.37}$$

Using equations 3.1, 3.10, 3.12 and 3.22 we can describe the motion of the system as follows:

$$\frac{dS_v}{dt} = \frac{l_m F_v \omega_m - \Omega_v k_v + g\{[A-B]\cos\alpha_V - C.\sin\alpha_v\} + (-\Omega_h^2).(A+B+C) + \sin 2\alpha_v}{J_v} \tag{3.38}$$

$$\frac{dS_h}{dt} = \frac{l_t F_h(\omega_t)\cos\alpha_v - \Omega_h k_h}{J_h} \tag{3.39}$$

$$\frac{dS_h}{dt} = \frac{l_t F_h(\omega_t)\cos\alpha_v - \Omega_h k_h}{D\sin^2\alpha_v + E*\cos^2\alpha_v + F} \tag{3.40}$$

$$\frac{d\alpha_h}{dt} = \Omega_h \tag{3.41}$$

$$\Omega_h = S_h + \frac{J_{mr}\omega_{mr}\cos\alpha_v}{J_h} \quad (3.42)$$

$$\Omega_h = S_h + \frac{J_{mr}\omega_{mr}\cos\alpha_v}{D\sin^2\alpha_v + E\cos^2\alpha_v + F} \quad (3.43)$$

Where,

J_{tr} = moment of inertia of motor tail propeller subsystem,

J_{mr}= moment of inertia of motor main propeller subsystem,

S_v= angular momentum of vertical axis,

S_h= angular momentum of horizontal axis.

Angular velocities are the functions of the input voltage of the DC-motor which are nonlinear in nature and thus we have two additional equations:

$$\frac{du_{vv}}{dt} = \frac{1}{T_{mr}}(-u_{vv} + u_v) \ , \quad \omega_m = P_v(u_{vv}) \quad (3.44)$$

$$\frac{du_{hh}}{dt} = \frac{1}{T_{tr}}(-u_{hh} + u_h) \ , \quad \omega_t = P_h(u_{hh}) \quad (3.45)$$

Where,

T_{mr} = main rotor propeller system time constant

T_{tr} = tail rotor propeller system time constant

ω_m = rotational speed of the main DC motor

ω_t = rotational speed of the tail DC motor

The speeds of the motor are controlled by the input command voltage u_v, u_h through the motor armature current u_{vv}, u_{hh} from the supply of the power amplifier.

$$I_1\frac{d^2\alpha_v}{dt^2} = M_1 - M_{FG} - M_{B\alpha_v} - M_G \quad (3.46)$$

$$M_1 = c_1.\tau_1^2 + d_1\tau_1 \quad (3.47)$$

$$M_{FG} = M_g.\sin\alpha_v \quad (3.48)$$

$$M_{B\alpha_v} = B_{1\alpha_v}.\frac{d\alpha_v}{dt} + B_{2\alpha_v}\,\mathrm{sign}.\frac{d\alpha_v}{dt} \quad (3.49)$$

$$M_G = K_{gy}.M_1\frac{d\alpha_h}{dt}.\cos\alpha_v \quad (3.50)$$

The electrical and motor control circuit is proximate by transfer function of a first order therefore the momentum of the rotor in Laplace can be defined as

$$\tau_1 = \frac{k1}{T_{11}s+T_{10}}.u_1 \tag{3.51}$$

Where τ_1= main rotor momentum

u_1 = motor input voltage

Equations related to horizontal axis are:

$$I_2 \frac{d^2 \alpha_h}{dt^2} = M_2 - M_{B\alpha_h} - M_R \tag{3.52}$$

$$M_2 = c_2.\tau_2^2 + d_2.\tau_2 \tag{3.53}$$

$$M_{B\alpha_h} = B_{1\alpha_h}.\frac{d\alpha_h}{dt} + B_{2\alpha_h}.\sin\frac{d\alpha_h}{dt} \tag{3.54}$$

Here, M_R presents the momentum due to cross reaction

$$M_R = \frac{k_c (T_0 s+1)}{T_p s+1}.\tau_1 \tag{3.55}$$

D.C motor with the electrical part

$$\tau_2 = \frac{k2}{T_{21}s+T_{20}}.u_2 \tag{3.56}$$

Where

τ_2 = tail rotor momentum

u_2 = motor input voltage

The parameters of TRMS used in present analysis are described in Table1

TABLE-1 TRMS SYSTEM PHYSICAL PARAMETERS AND VALUES

SYMBOL	**PARAMETERS**	**VALUE**
I_1	Moment of inertia corresponding to vertical rotor	8×10^{-2} kg.m ^{2}
I_2	Moment of inertia corresponding to horizontal rotor	6×10^{-2}kg.m ^{2}

b_1	Static characterstic constant	0.01355
b_2	Static characteristic constant	0.09244
c_1	Static characteristic constant	0.022
c_2	Static characteristic constant	0.099
M_g	Gravity momentum	0.322 Nm.
$B_{1\alpha_v}$	Friction momentum parameter	6×10^{-3} Nms/rad
$B_{2\alpha_v}$	Friction momentum parameter	10^{-3} Nms^2/ rad
$B_{1\alpha_h}$	Friction momentum parameter	10^{-1} Nms /rad
$B_{2\alpha_h}$	Friction momentum parameter	10^{-2} Nms^2/ rad
K_{gy}	Gyroscopic momentum parameter	0.0550 s/rad
K_1	gain of motor 1	1.10
K_2	gain of motor 2	0.80
T_{11}	parameter of motor 1	1.1
T_{10}	Parameter of motor 1	1.0
T_{22}	Parameter of motor 2	1.0
T_{20}	parameter of motor 2	1.0
T_P	Momentum due to Cross reaction	2.0
T_0	Momentum due to Cross reaction	3.55
K_C	Cross reaction momentum parameter gain	-0.20

3.3 LINEARIZATION OF TRMS MODEL

The mathematical model is a nonlinear model so to design any controller for this system firstly it is to linearize it by finding the equilibrium points.

The alternate model is given as-

$$\tau_1 = \frac{K_1}{T_{11}s+T_{10}} u_1 \tag{3.73}$$

$$\tau_2 = \frac{K_2}{T_{21}s+T_{20}} u_2 \tag{3.74}$$

Where τ_1= main rotor momentum

τ_2 = tail rotor momentum

u_1 and u_2 = motor input voltages

K_1 = gain of motor 1

K_2 = gain of motor 2

$$\frac{\partial \tau_1}{\partial t} = -\frac{T_{10}}{T_{11}}\tau_1 + \frac{K_1}{T_{11}}u_1 \quad (3.75)$$

$$\frac{\partial \tau_2}{\partial t} = -\frac{T_{20}}{T_{21}}\tau_2 + \frac{K_2}{T_{21}}u_2 \quad (3.76)$$

$$\frac{\partial x_7}{\partial t} = \frac{c_2x_4^2+d_2x_4}{I_2} \quad (3.77)$$

$$\frac{\partial x_7}{x_4} = \frac{c_2x_4+d_2}{I_2} \quad (3.78)$$

Neglecting c_2x_4

$$= \frac{0.02x4+0.09}{0.02} = 4.5 \quad (3.79)$$

$$\frac{\partial x_7}{\partial t} = \frac{k_c}{T_p}x_3 - \frac{k_cT_0T_{10}}{T_pT_{11}}x_3 - \frac{x_5}{T_p} \quad (3.80)$$

$$x_5 = 0 \quad (3.81)$$

$$\frac{\partial x_5}{x_3} = \frac{k_c}{T_p}dt - \frac{k_cT_0T_{10}}{T_pT_{11}}\partial t = \frac{-0.2*1}{2} + \frac{0.2*3.5*1*1}{2*1.2} = 0.19167 \quad (3.82)$$

$$\frac{\partial x_5}{x_3} = \frac{-dt}{T_p} = \frac{-1}{2} = -0.5 \quad (3.83)$$

$$\frac{\partial x_6}{x_1} = \frac{-M_g}{I_1}\sin x_1 \,\partial t \quad (3.84)$$

Assume $\sin x_1 = 1$

$$\frac{\partial x_6}{x_1} = \frac{-0.32}{6.8x10^{-2}} = -4.705 \quad (3.85)$$

$$\frac{\partial x_6}{x_6} = -B_1 - \frac{B_2\sin x_6}{x_6} = \frac{\sin x_6}{x_6} - 0 = \frac{-6*10^{-3}-1x10^{-3}x1}{I_1} = \frac{7*10^{-3}}{6.8*10^{-2}} = -0.1029 \quad (3.86)$$

$$\frac{\partial x_6}{\partial t} = \frac{c_1 x_3^2 + d_1 x_3 - M_g \sin x_1 - B_1 x_6 - B_2 \sin x_6 - k_{gy}(c_1 x_3^2 + d_1 x_3) - x_7 \cos x_1}{I_1} \tag{3.87}$$

$$\frac{\partial x_7}{\partial t} = \frac{c_2 x_4^2 + d_2 x_4 - B_1 x_6 - B_2 \sin x_7 - x_5}{I_2} \tag{3.88}$$

$$\frac{\partial x_4}{\partial t} = \frac{k_2 u_4 - x_4 T_{20}}{T_{21}} \tag{3.89}$$

$$\frac{\partial x_8}{\partial t} = \frac{k_1 u_1 - x_3 T_{10}}{T_{11}} \tag{3.90}$$

$$\frac{\partial \tau_1}{\partial t} = \frac{1.1 x 1}{1.1} - \frac{1 x \tau_1}{1.1} \tag{3.91}$$

$$\tau_1 = \tau_1 - \frac{\tau_1^2}{2.2} = \tau_1 = 2.2 \tag{3.92}$$

$$\frac{\partial \tau_1}{\partial t} = 1 - \frac{2.2}{1.1} = -1 \tag{3.93}$$

$$\frac{\partial x_8}{\partial t} = -1 \tag{3.94}$$

$$\frac{\partial x_8}{x_8} = -\frac{T_{10}}{T_{11}} = -\frac{1}{1.2} = -0.8333 \tag{3.95}$$

$$\frac{\partial x_4}{x_4} = -\frac{T_{20}}{T_{21}} = -1 \tag{3.96}$$

$$\frac{\partial x_5}{x_3} = -0.2 \tag{3.97}$$

$$\frac{\partial x_6}{x_3} = \frac{c_1 x_3 + d_1 - k_{gy}(c_1 x_3 + d_1)}{I_1} = \frac{0.0924 - 0.05(0.0924)}{6.8 * 10^{-2}} = 1.358 \tag{3.98}$$

$$\frac{\partial x_6}{x_6} = \frac{-B_1 - B_2 \frac{\operatorname{sign} x_6}{x_6}}{I_1} = \frac{-B_1 - B_2}{I_1} = \frac{-6 * 10^{-3} - 1 * 10^{-3}}{6.8 * 10^{-2}} = \frac{7 * 10^{-1}}{6.8} = -0.1029 \tag{3.99}$$

$$\frac{\partial x_3}{u_1} = -\frac{k_1}{T_{11}} \partial t = 1 \tag{3.100}$$

$$\frac{\partial x_4}{u_2} = \frac{k_2}{T_{21}} = 0.8 \tag{3.101}$$

$$\frac{\partial x_5}{u_1} = \frac{K_c T_0 K_1}{T_p T_{11}} \partial t = \frac{-0.2 * 3.5}{2 * 1.1} = -0.35 \tag{3.102}$$

$$\frac{\partial x_3}{u_1} = \frac{k_1}{T_{11}} = \frac{1.1}{1.2} = 0.91666 \tag{3.103}$$

And assuming :

α_v pitch angle

α_h yaw angle

r_1 pitch rotor momentum

r_1 yaw rotor momentum

M_r momentum due to cross reaction effect

$\frac{\partial_{\alpha_v}}{\partial t}$ angular velocity of pitch

$\frac{\partial_{\alpha_h}}{\partial t}$ angular velocity of yaw

$$\alpha_v = x_1 \tag{3.104}$$

$$\alpha_h = x_2 \tag{3.105}$$

$$\tau_1 = x_3 \tag{3.106}$$

$$\tau_2 = x_4 \tag{3.107}$$

$$M_r = x_5 \tag{3.108}$$

$$\frac{\partial_{\alpha_v}}{\partial t} = x_6 \tag{3.109}$$

$$\frac{\partial_{\alpha_h}}{\partial t} = x_7 \tag{3.110}$$

Hence, on linearizing the TRMS system the matrices A, B, C of the system is estimated and given in equations .TRMS system considered is consist of seven states therefore system can be represented as a state space model.

$\dot{X} = Ax + Bu$ and $y = Cx$

Where x is the state vector matrix y is the output matrix u is the input matrix.

A is the state matrix

B is the input matrix

C is the output matrix

D is the feed forward matrix.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -0.833 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0.19167 & 0 & -0.5 & 0 & 0 \\ -4.705 & 0 & 1.358 & 0 & 0 & -0.1029 & 0 \\ 0 & 0 & 0 & 4.5 & -50 & -5 & 0 \end{bmatrix}_{7\times 7}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.917 & 0 \\ 0 & 0.8 \\ -0.320 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}_{7\times 2}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{2\times 7}$$

3.4 MATLAB SIMULATOR OF TRMS ACTUAL NONLINEAR MODEL

Fig. 10 shows the MATLAB Simulink model of the TRMS.

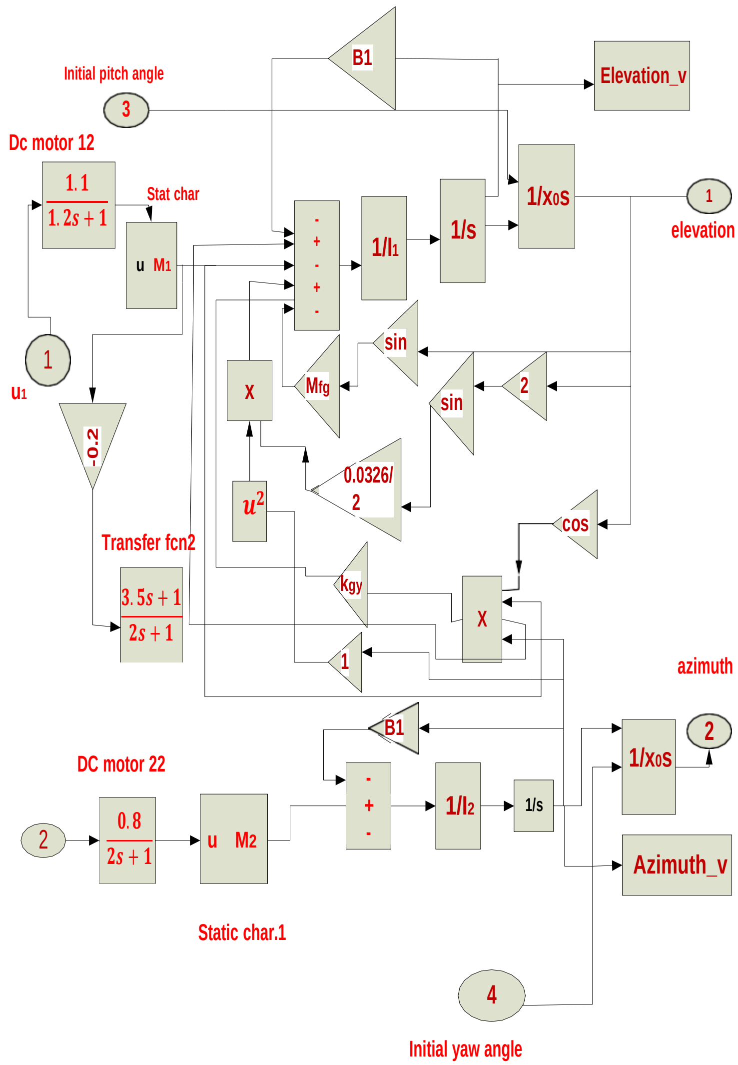


Fig 10. TRMS nonlinear model for finding the transfer function

3.5 TRANSFER FUNCTION OF TRMS MODEL AFTER LINEARIZATION OF THE SYSTEM

Considering our first input u_1

u_1= motor input voltage

$$M_1 = b_1 (\frac{1.1.\times}{1.2s+1}) + a_1 (\frac{1.1\times u_1}{1.2s+1})^2 \quad (3.111)$$

$$A_1 = B_1\ \alpha_v\ d(\alpha_v) \quad (3.112)$$

$$C_1 = (K_{gy}) \times \cos(\alpha_v) \times M_1 \times d(\alpha_h) \quad (3.113)$$

$$D_1 = 0.0163 \times \sin(2\times\alpha_v)(d(\alpha_v))^2 \quad (3.114)$$

$$E_1 = (M_{fg}) \times \sin(\alpha_v) \quad (3.115)$$

$$Y_1 = \int d(\alpha_v) + \alpha_{v_o} \quad (3.116)$$

Considering our second input u_2

u_2=input 2

$$M_2 = b_1(\frac{0.8\ u_2}{s+1}) + a_2(\frac{0.8\ u_2}{s+1})^2 \quad (3.117)$$

$$A_2 = B_1\ \alpha_h\ d(\alpha_h) \quad (3.118)$$

$$C_2 = -0.2M_1 (\frac{3.5s+1}{2s+1}) \quad (3.119)$$

$$Y_2 = \int d(\alpha_h) + \alpha_{h_o} \quad (3.120)$$

Considering our second inputu_2 as zero

When u_2=0

$$d(\alpha_h) = -\frac{1}{I_2}[\int B_1 \times \alpha_h \times d(\alpha_h) + 0.2M_1 \frac{3.5s+1}{2s+1}] \quad (3.121)$$

$$d(\alpha_h) = -\frac{1}{I_2}[\frac{\alpha_h^2}{2} + \int 0.2 \times \frac{3.5s+1}{2s+1} \times (b_1 \frac{1.1\times u_1}{1.2s+1} + a_1 (\frac{1.1\times u_1^2}{1.2s+1})] \quad (3.122)$$

rewriting the above equation again

$$d(\alpha_h) = -\frac{1}{I_2}[\frac{\alpha_h^2}{2} + \int (\frac{(0.2\times 3.5s+1)}{2s+1} \times (b_1 \frac{1.1\times u_1}{1.2s+1} + a_1 (\frac{1.1\times u_1^2}{1.2s+1}))dt] \quad (3.123)$$

$$\alpha_{h_1} = \int d(\alpha_h) + \alpha_{h_o} \quad (3.124)$$

$$d(\alpha_v)=\frac{1}{I_1}[-\int B_1\times\alpha_v\times d(\alpha_v)+\int(b_1\times(\frac{1.u_1}{2s+1})+a_1\times(\frac{1.u_1}{2s+1})^2)dt-\int\{K_{gy}\times\cos(\alpha_v)(b_1\times(\frac{1.1.u_1}{1.2s+1})+a_1(\frac{1.u_1}{2s+1})^2)\times d(\alpha_h)\}+\int\{0.0163\times\sin(2\times\alpha_v)\times(d(\alpha_v))^2\}-(M_{fg})\times\sin(\alpha_v)]$$

(3.125)

$$d(\alpha_v)=\frac{1}{I_1}[-B_1\cdot\frac{\alpha_v^2}{2}+\int(b_1(\frac{1.u_1}{2s+1})+a_1(\frac{1.u_1}{2s+1})^2)dt+\{K_{gy}\cdot b_1(\frac{1.1.u1}{1.2s+1})+a_1(\frac{1.u_1}{1.2s+1})^2\times\sin(\alpha_v)\}+\int\{0.0163\times\sin(2\times\alpha_v)\times(d(\alpha_v))^2\}-(M_{fg}.\sin(\alpha_v)] \quad (3.126)$$

$$\alpha_{v_1}=\int d(\alpha_v)+\alpha_{v_o} \quad (3.127)$$

Considering our first inputu_1 as zero

when $u_1=0$

$$d(\alpha_v)=\frac{1}{I_1}[-\int B_1\times\alpha_v\, d(\alpha_v)-\int K_{gy}\times\cos(\alpha_v)\times 0\times d(\alpha_h)+\int\{0.0163\times\sin(2.\alpha_v)\times(d(\alpha_h))^2\}]-\int(M_{fg})\times\sin(\alpha_v)dt] \quad (3.128)$$

$$(\alpha_v)=\frac{1}{I_1}[-\int B_1.\alpha_v.d(\alpha_v)+\int(0.0163\times\sin(2\times\alpha_v)\times(d(\alpha_h)^2)-\int\{(M_{fg})\times\sin(\alpha_v)\times dt\}]$$

(3.129)

$$\alpha_{v_2}=\int d(\alpha_v)+\alpha_{v_o} \quad (3.130)$$

$$d(\alpha_h)=\frac{1}{I_2}[-\int B_1\times\alpha_h\times d(\alpha_h)+\int M_2\,dt] \quad (3.131)$$

$$d(\alpha_h)=\frac{1}{I_2}[-\int B_1\times\alpha_h\times d(\alpha_h)+\int(b_1(\frac{0.8\,u_2}{s+1})+a_2(\frac{0.8\,u_2}{s+1})^2)dt] \quad (3.132)$$

$$\alpha_{h_2}=\int d(\alpha_h)+\alpha_{h_o} \quad (3.133)$$

From equation (3.310)

$$\alpha_v=\alpha_{v_1}+\alpha_{v_2} \quad (3.134)$$

α_{v_1} is the pitch angle response when u_2 is zero

α_{v_2} is the pitch angle response when u_1 is zero

α_{h_2} is the yaw angle response when u_1 is zero

α_{h_1} is the yaw angle response when u_2 is zero

From equation (3.133)

$$\alpha_h = \alpha_{h_1} + \alpha_{h_2} \tag{3.135}$$

By putting the system parameters values and solving these equations, SISO main pitch rotor transfer function is

Open loop Transfer Function $\frac{Y_1(s)}{U_1(s)} = \frac{-0.0499s^4 - 0.1703s^3 - 0.3625s^2 - 0.6952s + 0.5242}{0.0166s^4 + 0.4234s^3 + 2.6354s^2 + 2.6395s + 12.0313}$

And transfer function of SISO main yaw rotor as,

Open loop Transfer Function $\frac{Y_2(s)}{U_2(s)} = \frac{-0.0225s^4 - 0.4829s^3 - 0.6191s^2 - 0.2154s - 0.1732}{0.0010s^4 + 0.0340s^3 + 0.420v5s^2 + 0.1740s + 0.0887}$

$Y_1(s)$ and $Y_2(s)$ are the outputs i.e. α_v and α_h

$U_1(s)$ and $U_2(s)$ are the input voltages of main and tail motor

3.6 STABILITY ANALYSIS OF THE SYSTEM

SISO Main Pitch Rotor Decoupler

Open loop Transfer function $\frac{\alpha_v(s)}{u_1(s)} = \frac{-0.0499s^4 - 0.1703s^3 - 0.3625s^2 - 0.6952s + 0.5242}{0.0166s^4 + 0.4234s^3 + 2.6354s^2 + 2.6395s + 12.0313}$

Applying Routh's Criteria by taking its characteristics equation,

s^4	0.0166	2.6354	12.0313
s^3	0.4234	2.6395	0
s^2	2.5319	12.0313	0
s^1	0.6275	0	0
s^0	12.0313	0	0

↓

No Sign Change

SISO Main Yaw Rotor Decoupler

$$\frac{\alpha_h(s)}{u_2(s)} = \frac{-0.0225s^4-0.4829s^3-0.6191s^2-0.2154s-0.1732}{0.0010s^4+0.0340s^3+0.420v5s^2+0.1740s+0.0887}$$

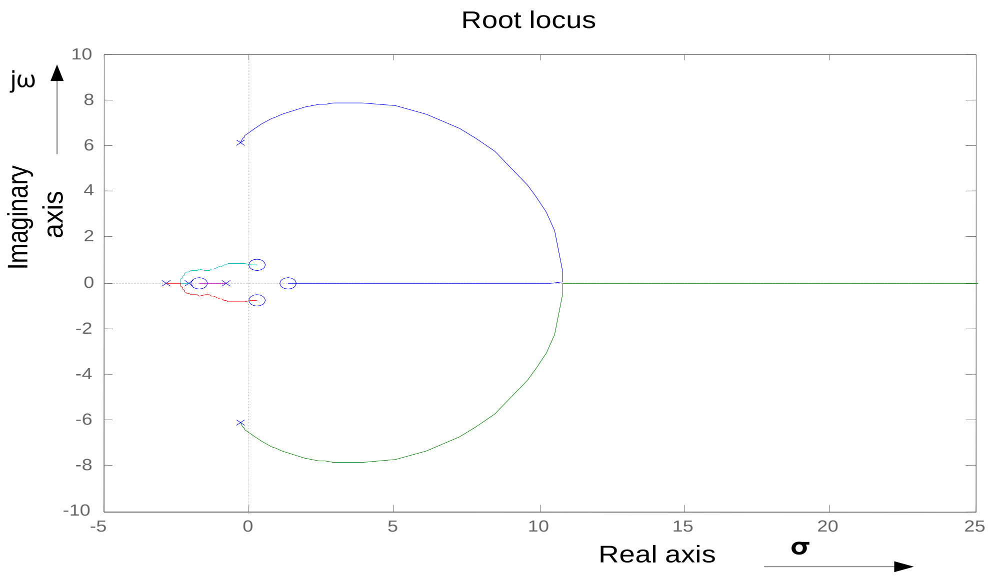
s^4	0.0010	0.4205	0.0887
s^3	0.0340	0.1740	0
s^2	0.4153	0.0887	0
s^1	0.1667	0	0
s^0	0.0887	0	0

↓

No Sign Change

No sign change means system is stable.

Fig. 11 shows the root locus diagram of α_h Yaw angle and it shows that the system is stable according to this location of poles and zeros.



Fig

11.Yaw root locus of the open loop system

Fig. 12 shows the root locus diagram of α_v Pitch angle and the system is stable according to the location of the poles.

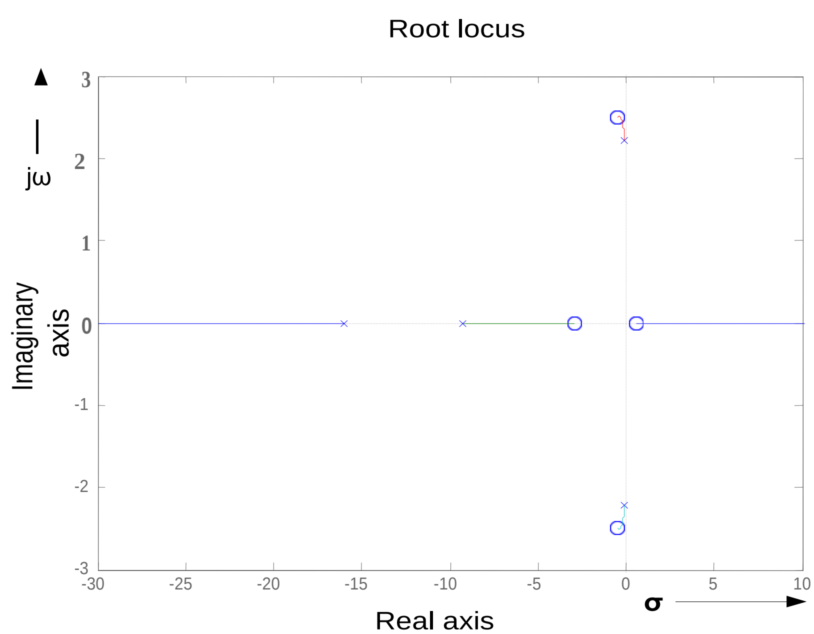


Fig 12.Pitch root locus of the open loop system

3.7 Conclusion

The mathematical model of the TRMS system is studied in detail and linearized model of TRMS is also developed to design controllers. The values of various matrices i.e. A,B,C are also calculated after linearization of the system in this chapter. Stability analysis of pitch angle and yaw angle control is presented using Routh's criteria and root locus method.

CHAPTER 4

DESIGN OF PITCH ANGLE AND YAW ANGLE CONTROLLERS

4.1 GENERAL

Controller is a device, either in the form of analogue circuits or digital circuits which observes and physically changes the parameter and the operating conditions of a given system to acquire the desired output. Controllers are basically used to reduce the difference between the desired and actual value in terms of accuracy and stability.

Controllers are usually used for settings of temperature, pressure, speed, level of liquid, flow etc. controllers are connected in series or in parallel as per the requirement of the plant or system and the conditions. Error signal is the difference between the desired and the actual or measured value which is processed by the controller to drive the system as per command input.

Fig. 13 shows a block diagram of a general closed loop system.

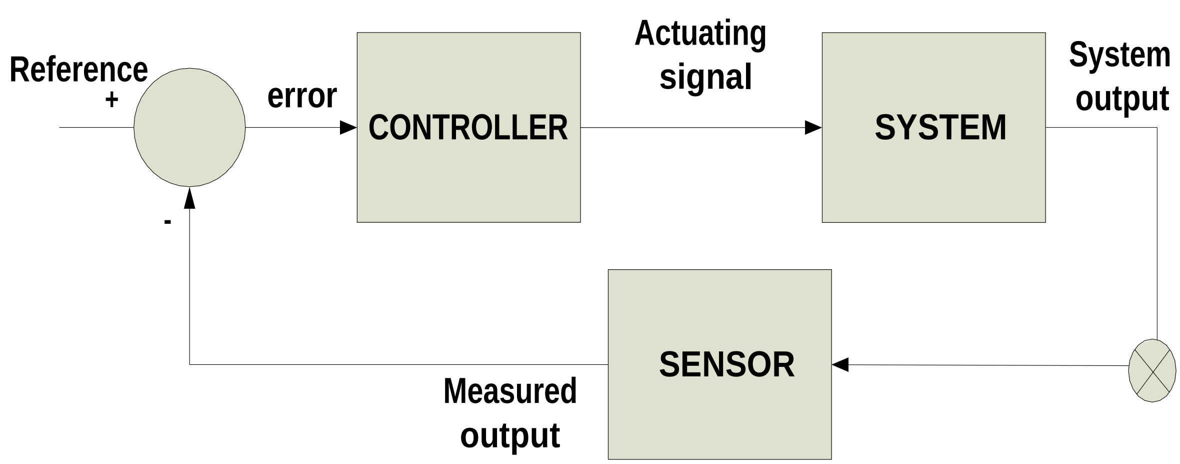


Fig 13. Block Diagram of closed loop control system

Error signal which is the difference between the set point and the measured value is given to the controller as an input to it and regulates it according to the desired condition of plant and accordingly it alters the input value to the plant and gives the desired output. If a plant has multiple input and multiple output it requires multiple controllers. If the system is SISO system with single input and single output than it requires single controller for controlling.

4.1.1 Functions of a controller

Controller improves the steady state accuracy by decreasing the steady state error which in result increases or improves the stability of the system. Controller also helps in reducing the offset produced by the system. Maximum overshoot can be reduced with the help of the controller. Controller helps in reducing the noise produced in the system, it also helps in improving the response that is sluggishness of the system.

4.1.2 Types of Controllers

There are various types of controllers to achieve the desired condition and specification. Basically, all the controllers can be classified in two classifications, feedback and feed forward controller. In feedback controller the input is the signal which is controlled and the controlled variable is given to the controller as a feedback which is compared with the reference signal to generate the error signal which is corrected or filtered by the controller to produce the system control input. Feed-forward control can avoid the slowness of feedback control. Feedforward controllers measure the input variables and take corrective actions before they affect the system or change the process, with retrospective effect.

Controllers are classified as:

a) Proportional controller
b) Proportional plus integral controller
c) Proportional plus derivative controller
d) Proportional plus integral plus derivative controller
e) optimal controller
f) Pole placement controller

4.2 OPEN LOOP ANALYSIS OF TRMS SYSTEM

Fig. 14 shows the MATLAB SIMULINK model of an open loop TRMS system.

The open loop response of a TRMS system in which pitch and yaw control inputs and initial pitch angle taken is π/8 and initial yaw angle is taken as zero.

Open loop response means that the output is not taken as feedback to the input means the response of the system without considering the feedback loop.

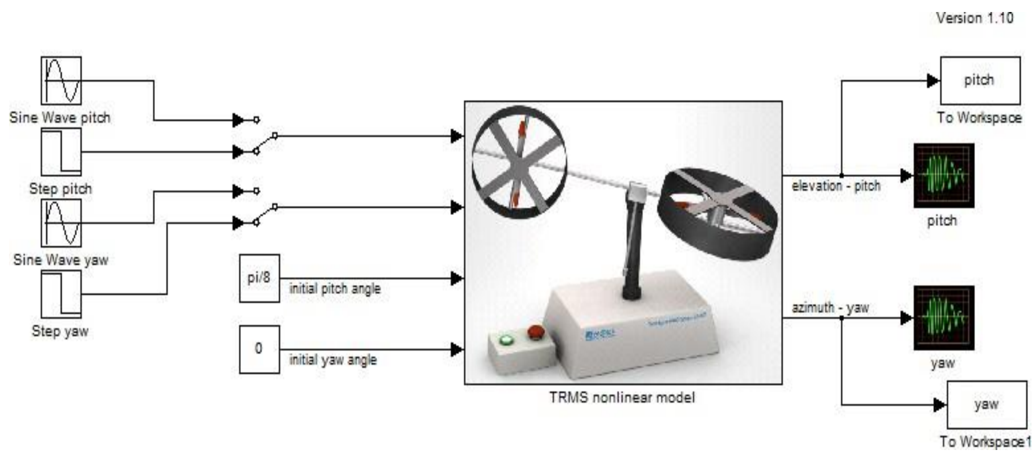
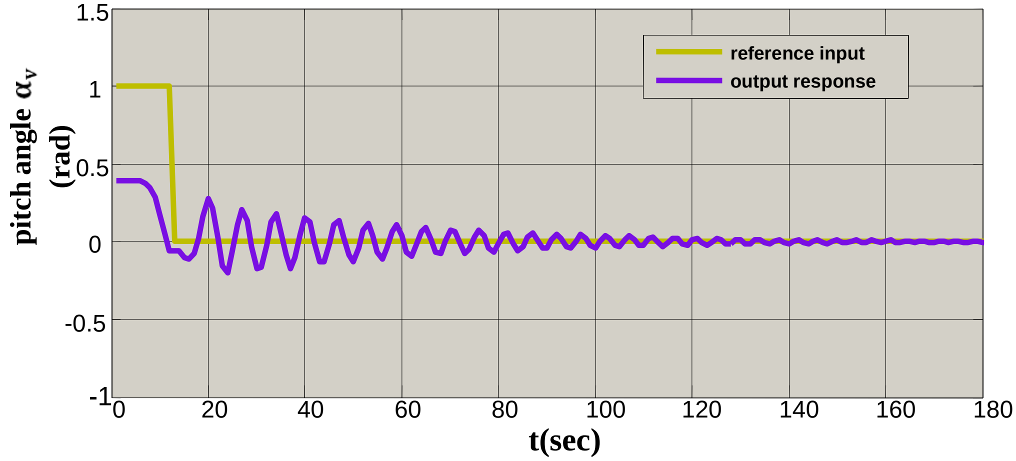


Fig 14.Open loop TRMS SIMULINK Model

Fig. 15 shows the pitch and yaw responses of the open loop model of TRMS system. Initial value or reference input signal is a unit step signal. Pitch response is showing so many oscillations therefore this is not a satisfactory response. Yaw response is approaching towards the reference input signal but this is also not indicating a satisfactory result



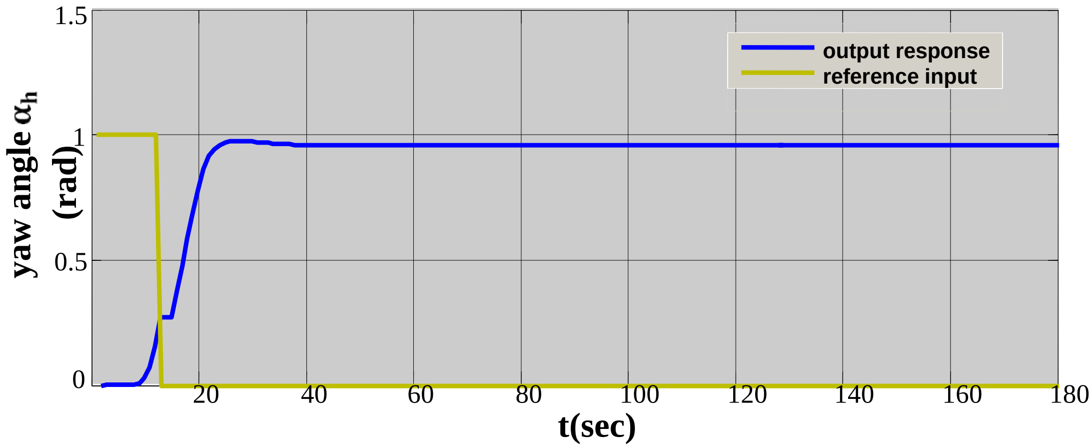


Fig 15. Open loop responses of Pitch and Yaw

4.3 MATLAB MODEL OF LINEARISED TRMS

Fig. 16 shows a SIMULINK MATLAB model of linearised TRMS. Input for both pitch and yaw is step signal. Both signals are given simultaneously to the system as satisfying the name multiple input system. The state space model representing here is similar to the any system satisfying the equations.

$$\dot{X} = Ax + Bu \text{ And } y = Cx$$

Where x is the state vector matrix y is the output matrix u is the input matrix.

A is the 7x7 state matrix

B is the 2x7 input matrix

C is the 2x7 output matrix

D is the feed forward matrix.

Scope 1 represents the pitch response and scope 2 represents yaw response.

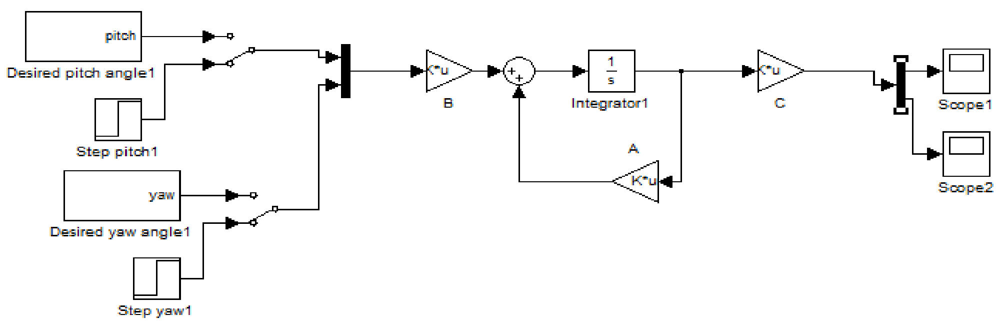


Fig 16. State space linearised model of TRMS

Fig. 17 shows the response of pitch angle of TRMS system and Fig 18 shows the response of the yaw. Pitch response showing so many oscillations which is not desirable. The response must be follow the reference input signal without any oscillations and settle to steady state value.

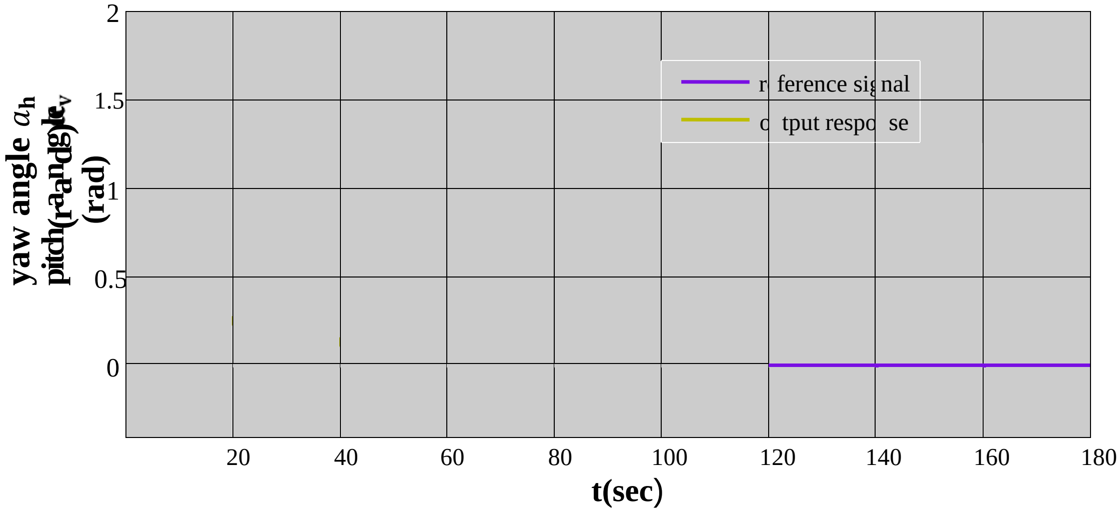


Fig 17.Response of pitch

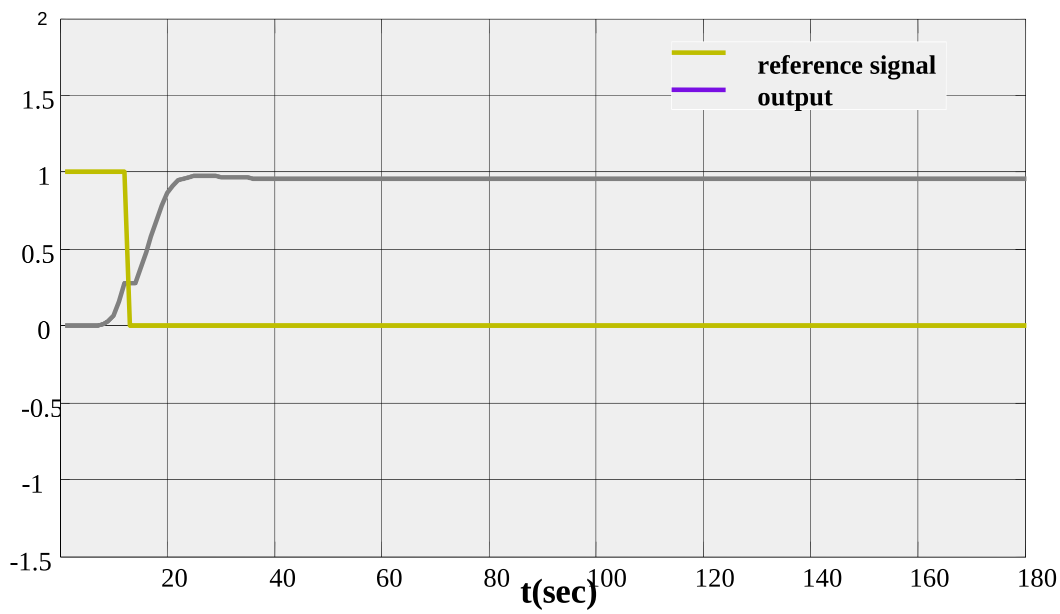


Fig18.Response of yaw

4.4 PID CONTROLLER FOR TRMS SYSTEM

4.4.1 PID controller with non linear model of TRMS

Fig. 19 shows the nonlinear SIMULINK model of PID pitch controller. Two separate PID controllers for Pitch and Yaw are used for controlling the angles. Initial pitch and yaw angle are taken as zero. PID parametres K_p , K_i and K_d values by trial and error method are 0.375,0.99 and 3 respectively. Where K_p is the proportional gain K_i is the integral gain and K_d is the derivative gain. In the beginning the value of K_p, proportional gain set as 0. As proportional gain is known to cause overshoot problem so it is tuned slowly and as small as possible. Then K_i and K_d , integral and derivative gain added respectively to deal with the steady state error and overshoot problem. Reference input value is 0.4rad.

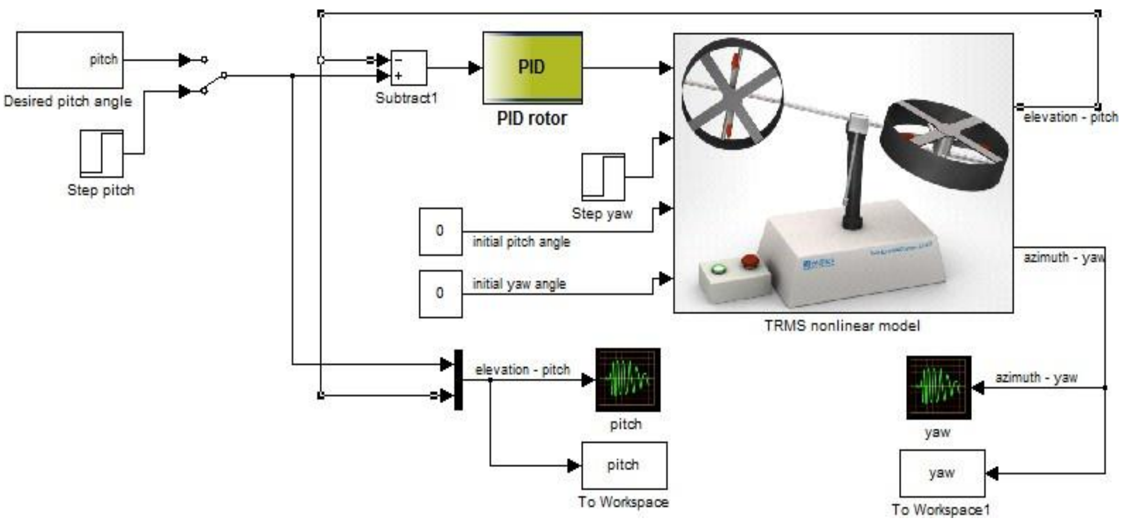


Fig 19. SIMULINK model of PID pitch controller

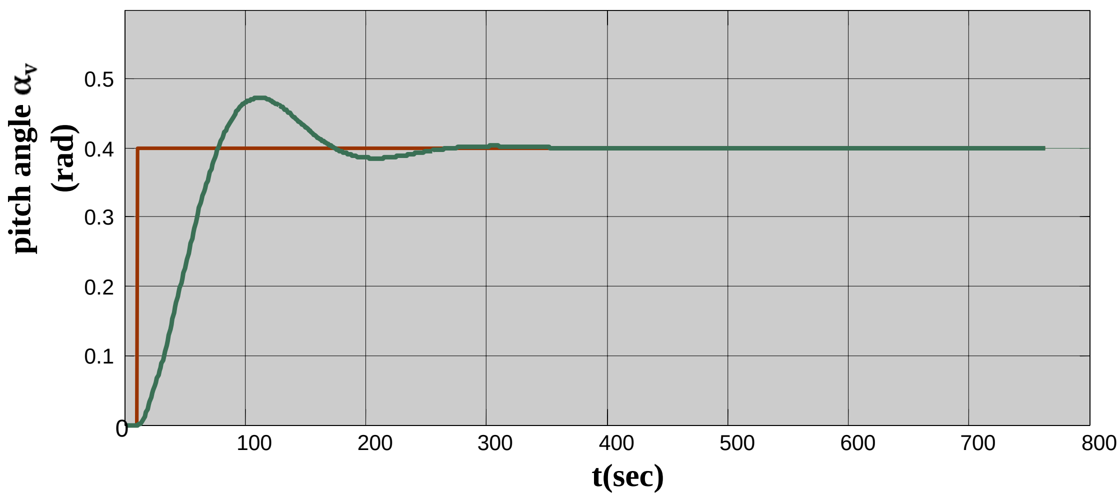


Fig 20. Response of pitch with PID controller

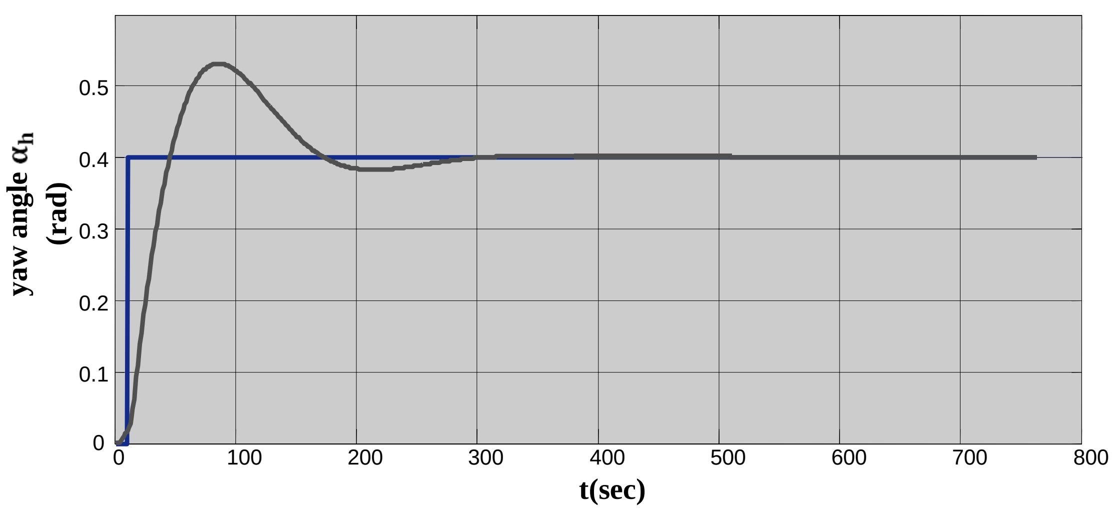


Fig. 21 Response of yaw with PID controller

Fig.20 and Fig.21 representing here the responses of the Pitch and yaw with the PID controller. As compared to the nonlinear response and linearised open loop response without any controller these responses of pitch and yaw are better than the other two in terms of no. of oscillations and settling time.

Fig.22 shows the response of state space linearised model with PID controller with the parameter values K_p, K_i and K_d values are 0.375,0.99 and 5 respectively. Where K_p is the proportional gain K_i is the integral gain and K_d is the derivative gain.

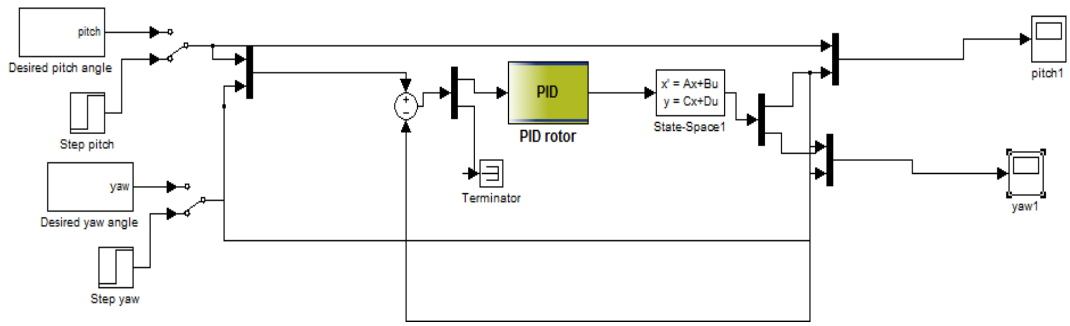


Fig 22.State space model of TRMS with PID controller

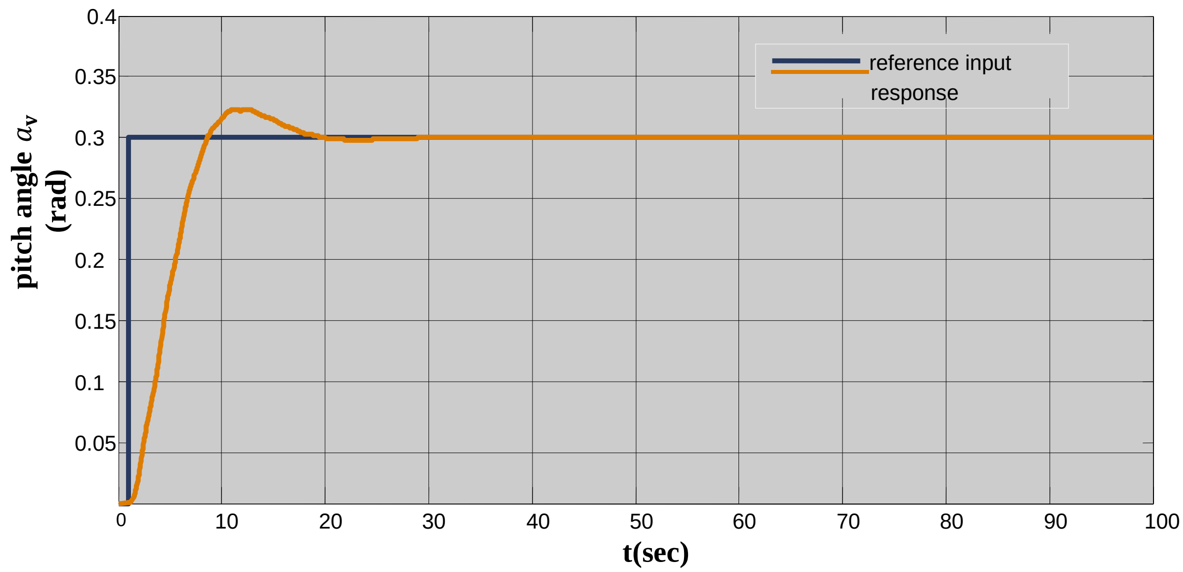


Fig 23. State space linearised model PID pitch response

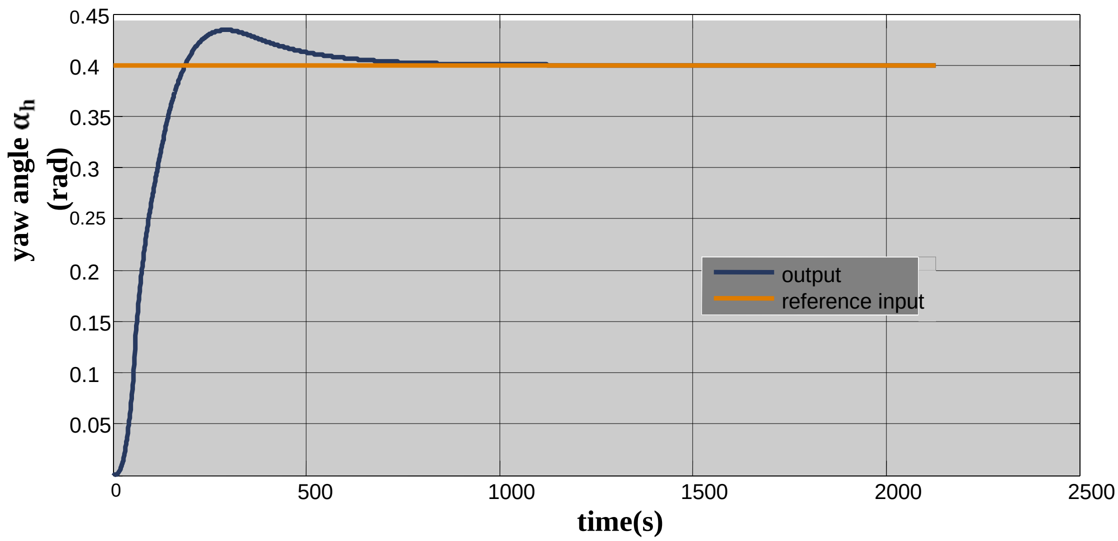


Fig 24. State space linearised model PID yaw response

Fig 23 and Fig 24 shows the responses of Pitch and Yaw with the state space linearised model.

4.5 LINEAR QUADRATIC REGULATOR (LQR)

LQR controller is a modern controller which uses state space method to analyze multiple input multiple output system.LQR controller uses full state feedback to provide robust and good performance. A control system which minimizes the cost function associated with generating control inputs is called an optimal control system. Optimal controllers are controllers which operates a control system at a minimal cost. TRMS system is a dynamic system which is modeled by set of differential equations and quadratic cost function which has to be minimized. Basically an optimal controller is that controller which optimizes or minimizes the cost function related to its controller input. The performance objective function should be time integral of control and transient energy in terms of time, for optimally controlling the system. Maximum overshoot can be defined by maximum value of transient energy and time taken by transient response to decay to zero represent the settling time. Thus, acceptable value of settling time and maximum overshoot can be defined by including the transient energy in objective function. In similar way, control energy should also be comprised in objective function to minimize the control energy of system.

Kalman filter provides the procedure for designing the obserever having multiple inputs and outputs, combination of LQR and Kalman filter known as LQG regulator.

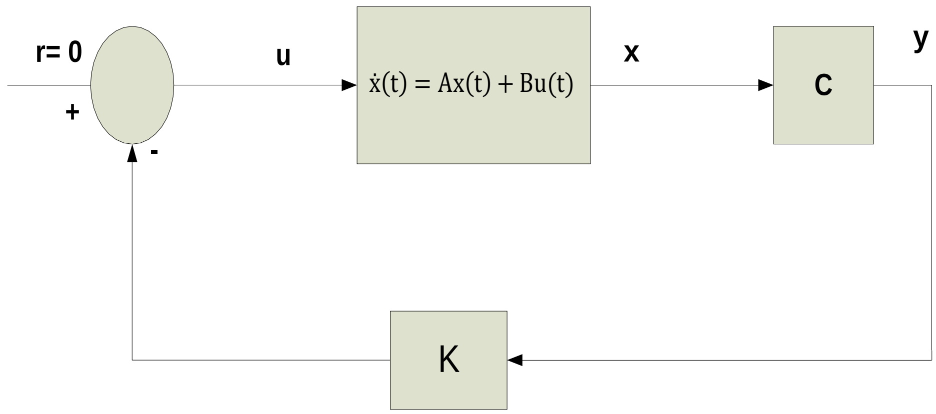


Fig 25. Block diagram of Linear Quadratic Regulator

Linear plant defined by the state equation –

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{4.1}$$

Control input vector is given below

$$u(t) = -K(t)x(t) \tag{4.2}$$

given control input is linear, because the plant is also linear. The control energy can be expressed by $u^T(t) \times R(t) \times u(t)$

Where R (t) is control cost matrix which is square and symmetric in nature.

Transient energy is given as $x^T(t) \times Q(t) \times x(t)$

Where Q (t) is state weighing matrix which is square and symmetric.

Thus cost objective function can be defined as –

$$J(t, t_f) = \int_t^{t_F} (x^T(\tau) \times Q(\tau) \times x(\tau) + u^T(\tau) \times R(\tau) \times u(\tau) \times d\tau \tag{4.3}$$

Where,

t and t_f are the initial and final time value. The main purpose of this optimal controller is to find the gain K matrix so that the objective function should be minimized.

4.5.1 Optimal Control Gain K

For designing the LQR controller it is necessary to determine the value of the optimal control gain K of the LQR controller. This optimal gain can be calculated as follows:

The closed loop state equation of the plant is given as –

$$\dot{x}(t) = (A - BK(t)) \times x(t) \tag{4.4}$$

$$\dot{x}(t) = A_c \times x(t) \tag{4.5}$$

$A_c = (A - BK(t))$ Is closed loop state matrix.

The solution can be defined as

$$x(t) = \theta_c(t, t_0) \times x(t_0) \tag{4.6}$$

where,

$\theta_c(t, t_0)$ state transition matrix

Now putting the value of x (t) in cost objective function we get

$$J(t, t_f) = \int_t^{t_F} (x^T(t)\, \theta_c^T(\tau, t)[Q(\tau) + K^T(\tau)R(\tau)K(\tau)]\theta_c^T(\tau, t)x(t)d\tau \tag{4.7}$$

$$J(t, t_f) = x^T(t)M(t, t_f)x(t) \tag{4.8}$$

where

$$M(t, t_f) = \int_t^{t_F} \theta_c^T(\tau, t)(Q(\tau) + K^T(\tau)R(\tau)K(\tau))\theta_c^T(\tau, t)d\tau \tag{4.9}$$

This problem also known as linear quadratic problem because the cost objective function is a quadratic function of initial state.

$$J(t, t_f) = \int_t^{t_F} x^T(t)\,(Q(\tau) + K^T(\tau)R(\tau)K(\tau))x(t)d\tau \tag{4.10}$$

Partial differentiation of equation 4.10 w.r.t time

$$\frac{\partial J(t, t_F)}{\partial t} = -x(t)(Q(\tau) + K^T(\tau)R(\tau)K(\tau))x^T(t) \tag{4.11}$$

$$\frac{\partial J(t,t_F)}{\partial t} = (\dot{x}(t)^T)M(t,t_f)x(t) + x^T(t)\frac{\partial M(t,t_F)}{\partial t}x(t) + x^T(t)M(t,t_f)\dot{x}(t)$$

(4.12)

On combining equation 4.9 and 4.12 we get –

$$\frac{\partial J(t,t_F)}{\partial t} = x^T(t)\{A_c^T M(t,t_f) + \frac{\partial M(t,t_F)}{\partial t} + M(t,t_f)\}x(t) - \frac{\partial M(t,t_F)}{\partial t} = \{A_c^T M(t,t_f) + M(t,t_f)A_c^T + Q(t) + K^T(t)R(t)K(t)\} \quad (4.13)$$

This equation is called as Riccati equation by solving this K matrix is determined

$$K(t) = R^{-1}(t)B^T(t)M \quad (4.14)$$

The cost objective function for infinite final time, $J_\infty(t)$ is represented the same

$$J_\infty(t) = \int_t^\infty (x^t(\tau)Q(\tau)x(\tau) + u^T(\tau)R(\tau)u(\tau))d\tau \quad (4.15)$$

Here, $M(t,\infty)$is constant. So,

$$\frac{dM}{dt} = 0 \quad (4.16)$$

The final Riccati equation is –

$$0 = A^T M + MA - MBR^{-1}(t)B^T M + Q(t)$$

(4.17)

The equation 4.17 is algebraic that's why it is known as Algebraic Riccati equation. Condition for finding the solution of Riccati equation is either asymptotically stable with output

$$y(t) = C(t) * x(t) \quad (4.18)$$

$Q(t) = C^T(t) \times C(t)$ and R (t) is positive symmetric and semi-definite also, which are chosen randomly. At the time of designing the LQR regulator Q and R are taken such that it reduces the steady state error.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 \end{bmatrix}_{7\times7} \qquad R = \begin{bmatrix} 0.0395 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$$

$q_5 = 0.001$ due to the cross coupling effect which requires to be minimized, so its value is taken to be minimum.

The optimal controller gain evaluated as given as

```
A =[0 0 0 0 0 1 0;0 0 0 0 0 0 1;0 0 -0.833 0 0 0 0;0 0 0 -1 0 0 0;0 0 0.19167
0 -0.5 0 0;-4.705 0 1.358 0 0 -0.1029 0;0 0 0 4.5 -50 -5 0];
B =[0 0;0 0;0.917 0;0 0.8;-0.320 0;0 0;0 0];
C =[1 0 0 0 0 0 0; 0 1 0 0 0 0 0];
D =[0 0; 0 0];
Q = [1 0 0 0 0 0 0;0 0.1 0 0 0 0 0;0 0 1 0 0 0 0;0 0 0 1 0 0 0;0 0 0 0 0.001
0 0;0 0 0 0 0 0.4 0;0 0 0 0 0 0 0.4]
R = [0.0395 0;0 1];
[K s e] = lqr(A,B,Q,R)
sys_cl = ss(A-B*K, B, C, D);
step(0.2*sys_cl)
ylabel('yaw angle (rad),pitch angle');
title('Closed-Loop Step Response: LQR');
```

$$K = \begin{bmatrix} 22.7532 & 1.5743 & -6.4137 & 3.9206 & -51.7449 & 0.2944 & 3.6661 \\ 2.3186 & 0.0458 & -0.8768 & 0.5702 & -2.8996 & -0.5182 & 0.1207 \end{bmatrix}_{2\times 7}$$

4.5.2 SIMULINK MODEL FOR LQR CONTROLLER

Fig. 26 shows the SIMULINK model of the LQR controller for the TRMS system. Scope 1 is the pitch angle response and scope 2 is the yaw angle response.

$[A]_{7\times 7}$ matrix is the state matrix

$[B]_{7\times 2}$ is the input matrix

$[C]_{2\times 7}$ is the output matrix

$[D]_{0\times 0}$ is the feed forward matrix.

$[K]_{2\times 7}$ is the optimal LQR controller gain.

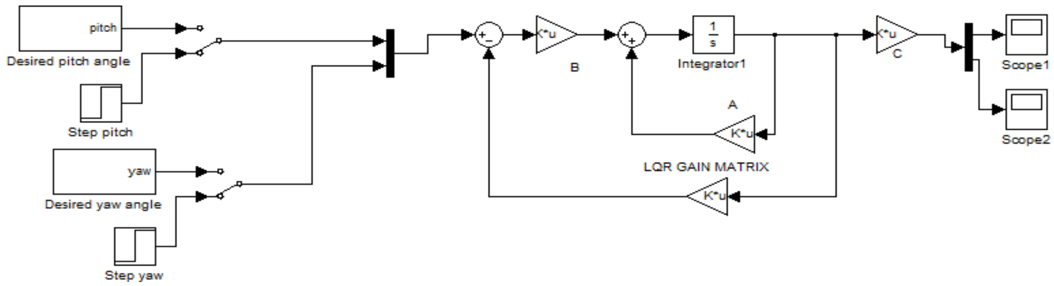


Fig 26.SIMULINK Model for LQR

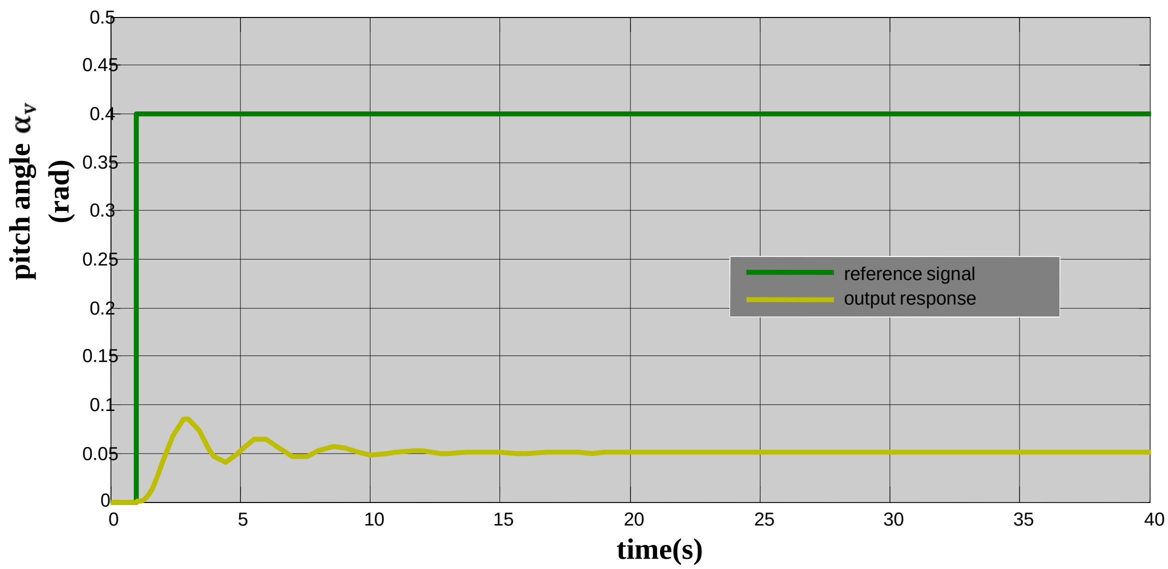


Fig 27.Pitch LQR response

Fig. 27 shows the pitch angle response with the LQR controller and Fig. 28 shows the Yaw angle response with LQR controller. The refrence input signal for these model is 0.4. The responses of the scopes are settles to 0.4.

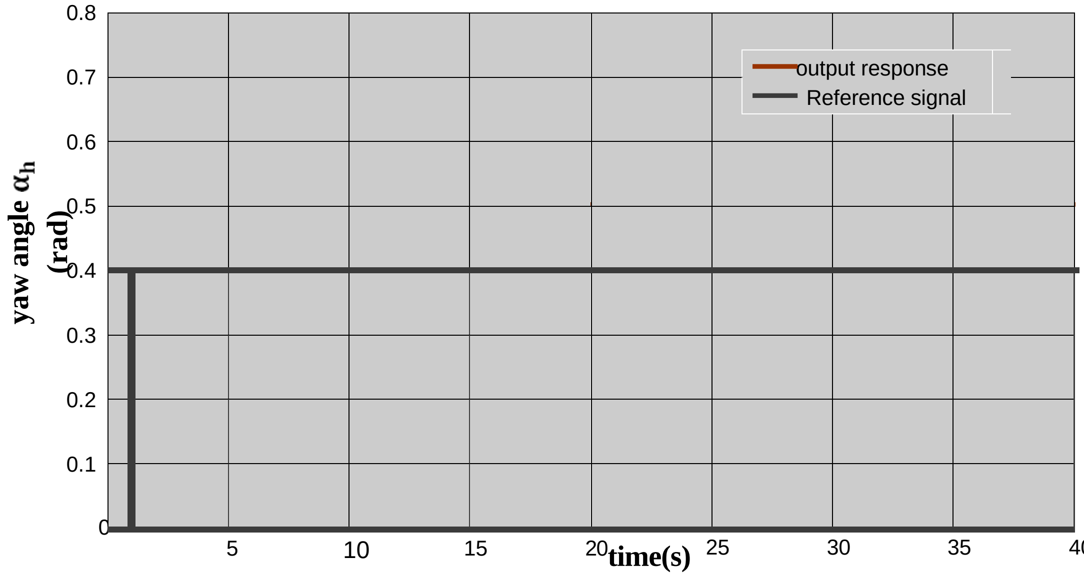


Fig 28.Yaw LQR response

4.6 LQR PLUS PID CONTROLLER

Fig 29 shows the LQR plus PID model for pitch controlling. Fig 30 and Fig 31 shows the responses of both the pitch and yaw. PID controller parameter values K_p, K_i and K_d values are tuned to 7,5 and 13 respectively. Where K_p is the proportional gain K_i is the integral gain and K_d is the derivative gain. K is the optimal controller gain of the LQR controller which is a 7x2 matrix. The refrence input signal value is 0.3. Fig 30 and 31 represents the responses of pitch and yaw which are better than the responses with only PID controller.

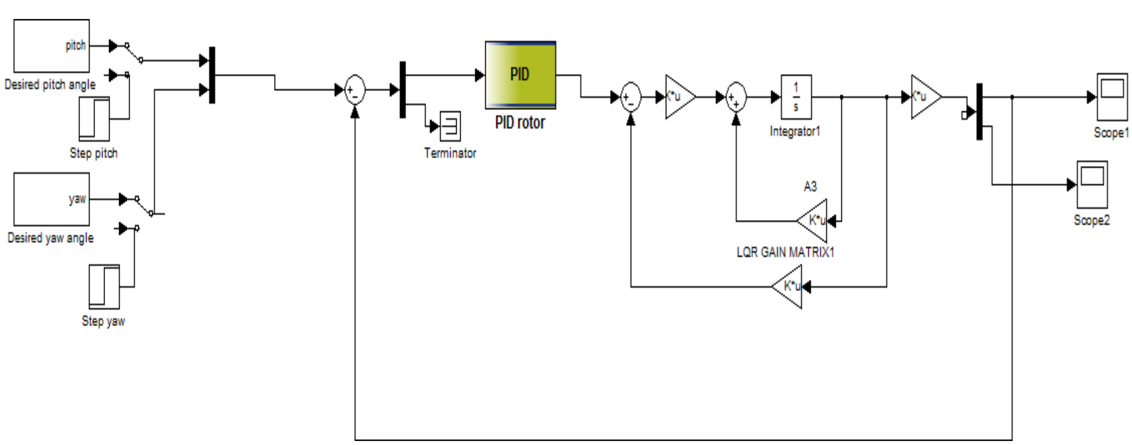


Fig 29. LQR plus PID state space linearised model

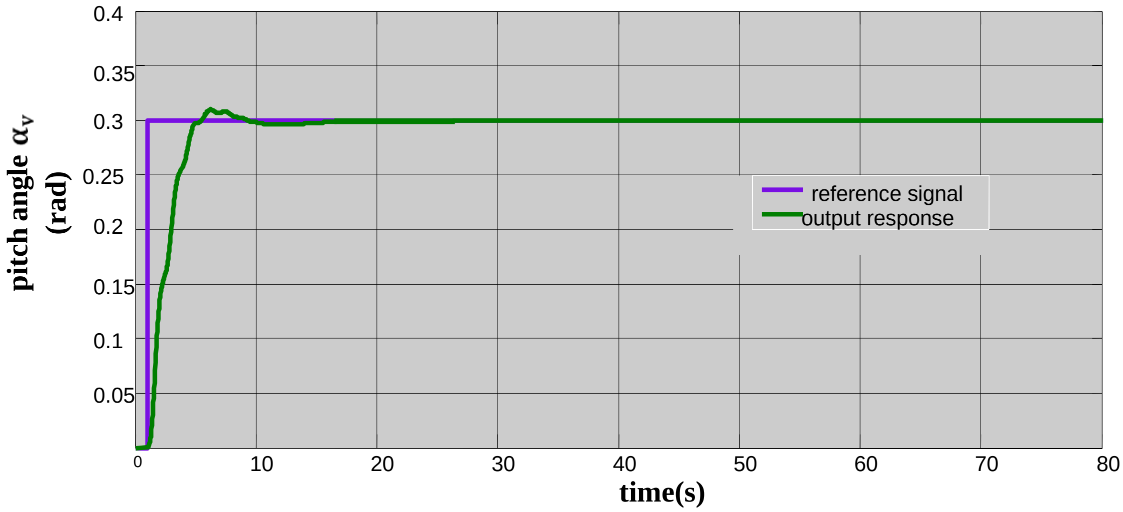


Fig 30. Response of PID plus LQR Pitch

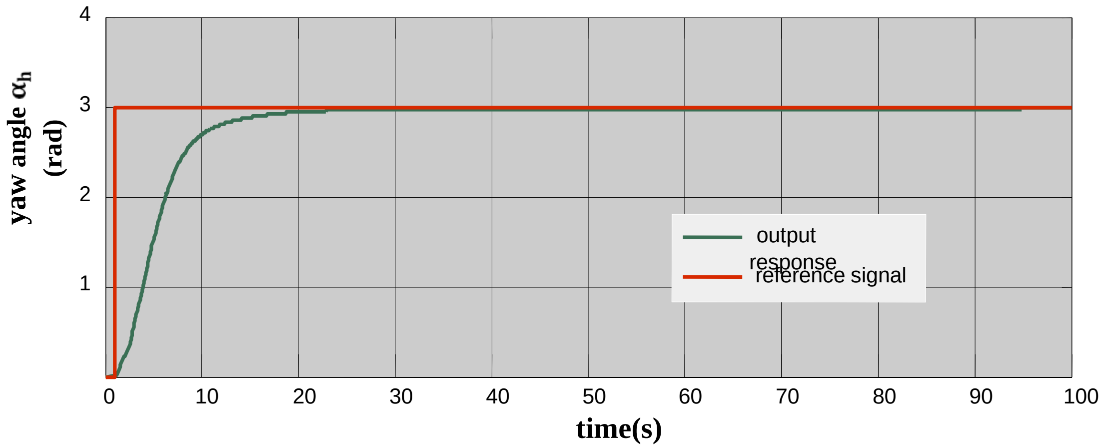


Fig.31 Response of PID plus LQR Yaw

4.7 Conclusion

Open loop response of nonlinear TRMS model, PID controller and PID plus LQR controller are designed for the both main and tail rotor separately. PID plus LQR controller response is better than the other two controller responses.

CHAPTER 5

FUZZY LOGIC CONTROLLER

5.1 General

Fuzzy controller is an adaptive intelligent control scheme designed for complex, nonlinear and time variant system for providing better and robust response under the presence of large disturbances compared to other conventional PID and LQR controllers. Fuzzy controllers are easy to design and easy to implement than the traditional methods of controlling. Fuzzy controller basically consists of four subsystems fuzzifier, inference engine, fuzzy rule base and de-fuzzifier .The fuzzifier converts the input value or crisp value into fuzzy values. The fuzzy rule base consists of basic data and if and then rules. The main and important part of fuzzy controller which is also called as brain of the fuzzy controller which helps to take human decision based on the fuzzy logic. Finally the conversion of the fuzzy values into the real values is achieved. For TRMS system two fuzzy controllers are required one for horizontal and the other for vertical to analyze motion control which works at the same time to give the desired response.

The main requirement to design the fuzzy controller is the full knowledge of the system and system should be controllable and observable. Firstly the system is examined with the step input along with the PID converter. The important function while designing fuzzy logic controller is to find out the fuzzy system inputs. In Twin rotor MIMO system the two input error and rate of change of error and range of these two inputs should be defined. After setting the inputs have to choose the membership function and by considering this system we select the triangular membership function .both Inputs and outputs are classified as zero, positive large and negative levels.

One out of two controller controls travel while the other controls the pitch. Both these controllers are Mamdani type Inference system and designed in MATLAB. Each controller has two different inputs and produces one output.

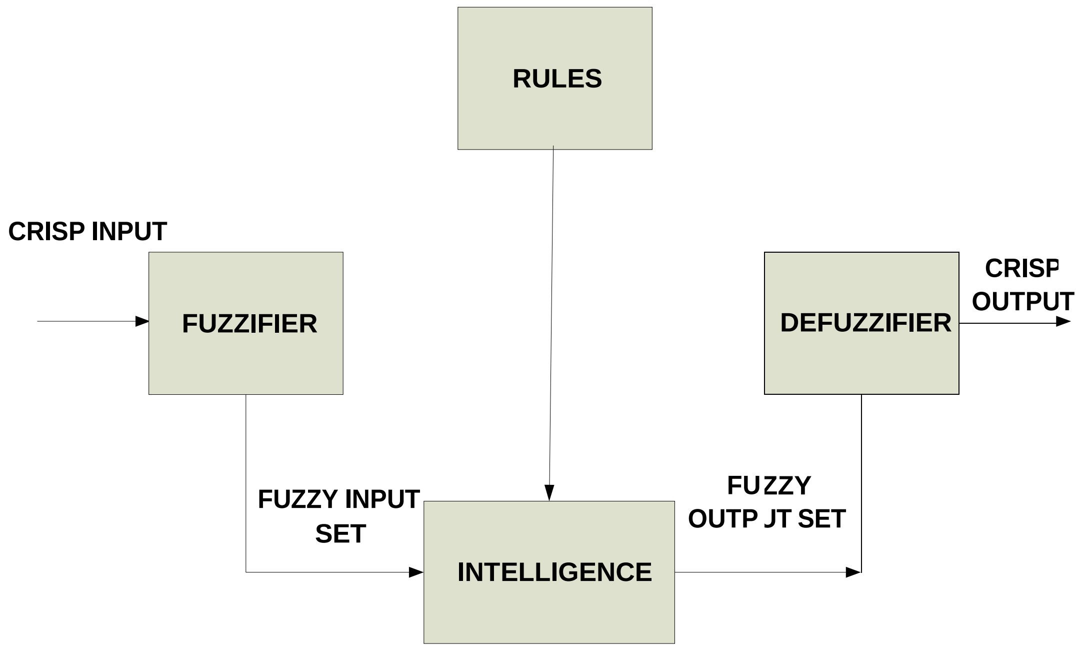


Fig32. Basic fuzzy model

5.1.1 Rule Base And Membership Functions

The inputs to the both fuzzy controllers are error in beam angle and error in change in beam angle. The most important part of fuzzy controller design is the establishment of rule base. The fuzzy rule base is established by studying the whole system with respect to both horizontal and vertical axis. Triangular membership functions are used for this system in fuzzification process. Almost same rule bases have been used for vertical and horizontal fuzzy controllers. The input range is from (−0.5 to 0.5) for the horizontal part and from (−0.6 to 0.6)for the other three output of the four membership functions is Positive very large(PVL), Positive large(PL), Positive(P), Positive Small(PS), Zero(Z), Negative Small(NS), Negative(N), Negative large(NL), and Negative very large(NVL) range from (−2.5 to 2.5).

Rate of error/error	Negative Large	Negative Medium	Negative Small	Zero	Positive Small	Positive Medium	Positive Large
Negative Large	NVL	NVL	NL	NM	NS	NS	Z
Negative Medium	NVL	NL	NM	NM	NS	Z	PS
Negative Small	NL	NM	NS	NS	Z	PS	PM
Zero	NM	NS	NS	Z	PS	PS	PM
Positive Small	NM	NS	Z	PS	PS	PM	PL
Positive Medium	NS	Z	PS	PM	PM	PL	PVL
Positive Large	Z	PS	PS	PM	PL	PVL	PVL

Table 3. Rule Base for vertical and horizontal fuzzy controller

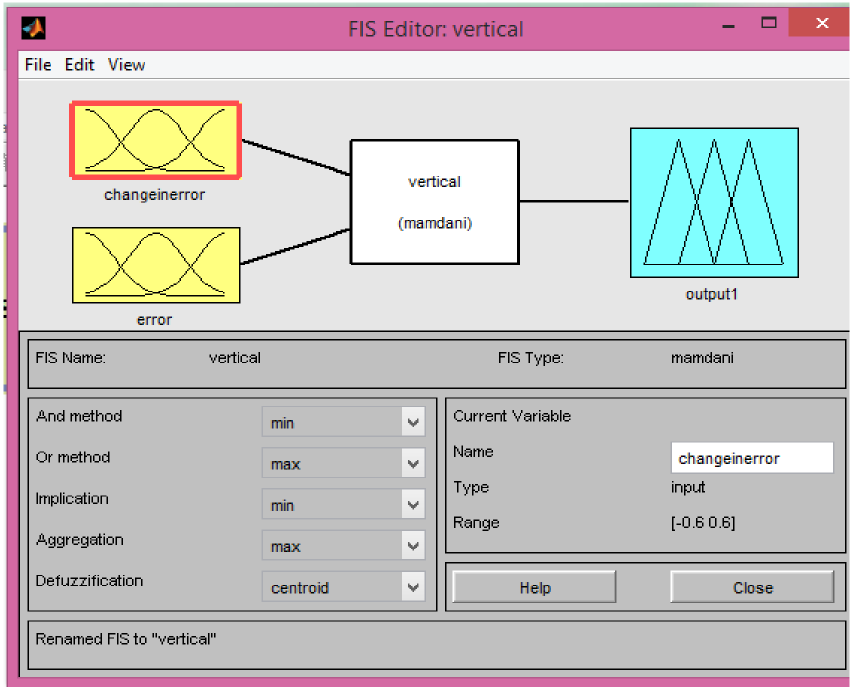


Fig 33. FIS model for vertical part of TRMS

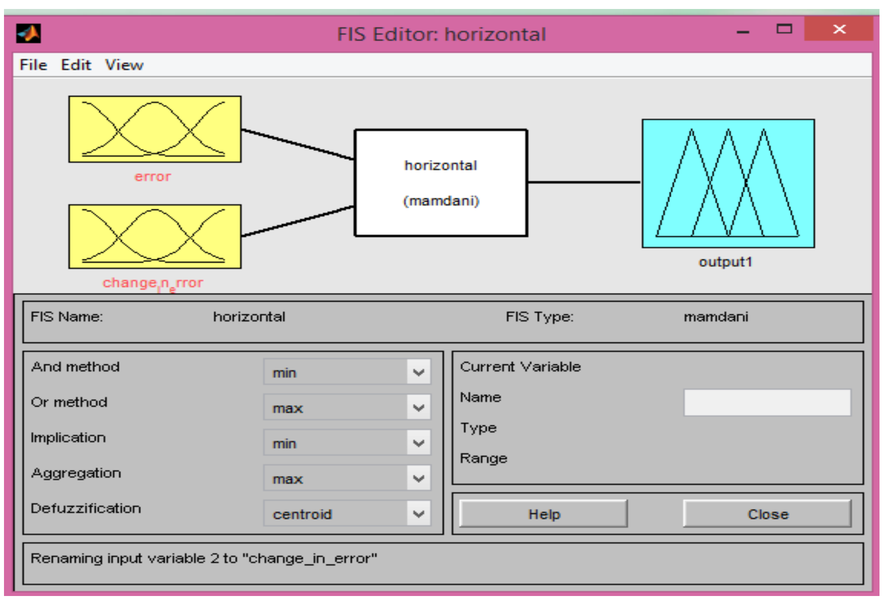


Fig34.FIS model for horizontal part of TRMS

Fig 33 and 34 shows the FIS editor file for horizontal and vertical part. Two inputs i.e. error and change in error are considered while the output is the control signal. Mamdani type inference is used in this fuzzification system. Inputs for horizontal and vertical fuzzy controller are same.

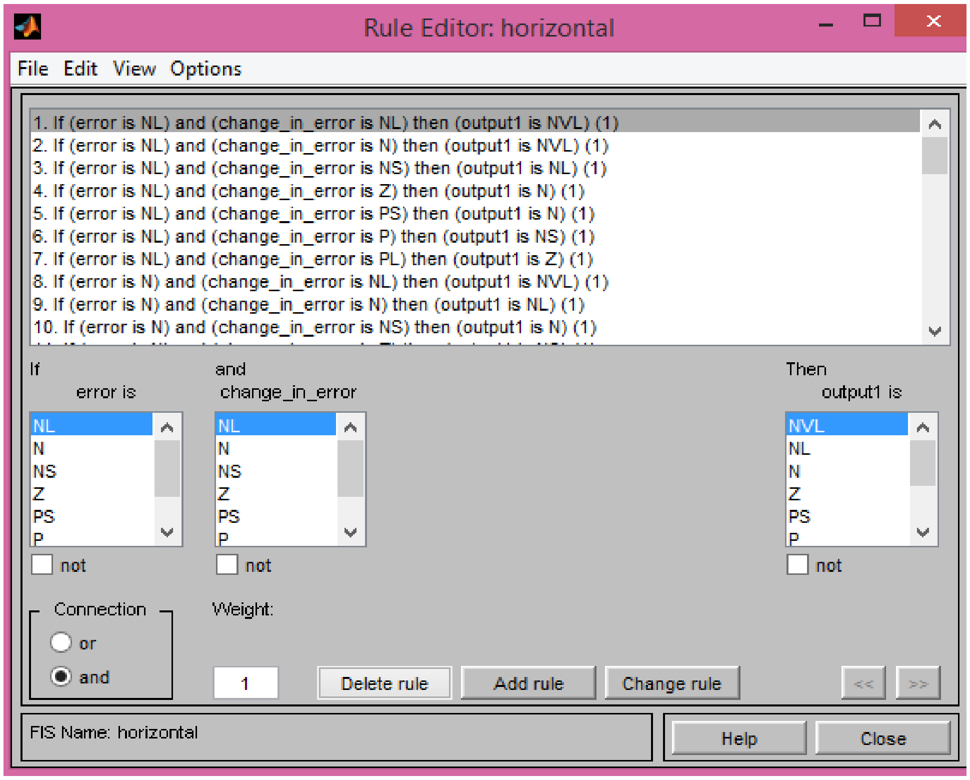


Fig 35.Rule base for horizontal part

This rule editor representing above error and change in error are two inputs and the rule base as shown in the Fig35.

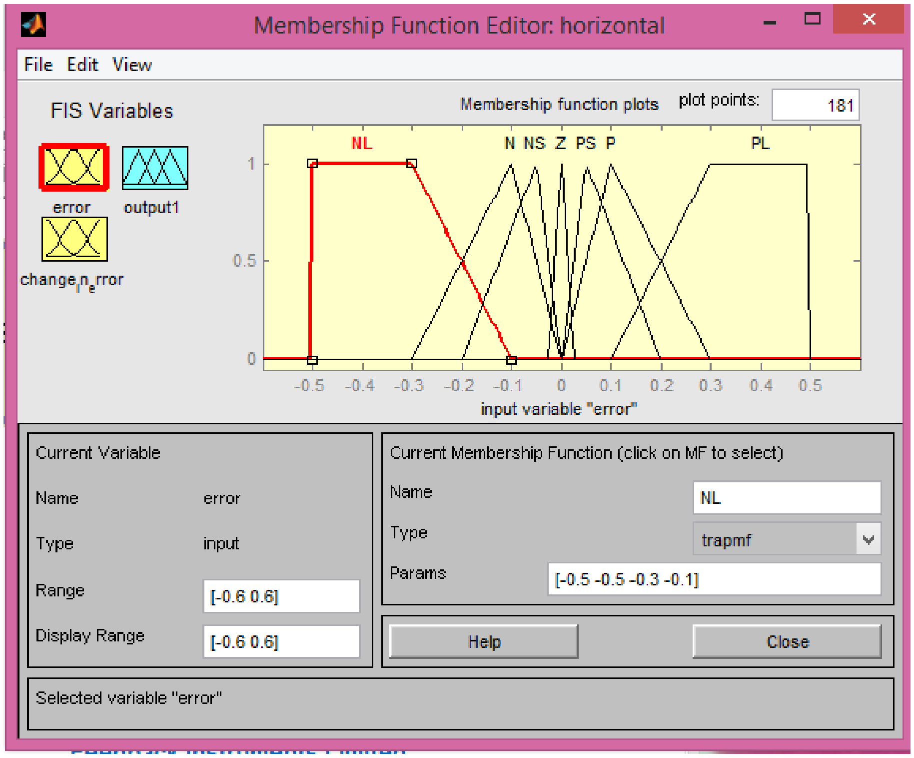


Fig 36.Membership function of error for horizontal part

FIS variables of Fuzzy controller for horizontal part of the TRMS system are shown in Fig36 i.e. error and rate of error. Triangular and trapezoid Membership functions are used in the designing the controller which are stated as NL, N, NS, Z, PS, P and PL. The range of the input variable is -0.5 to 0.5.

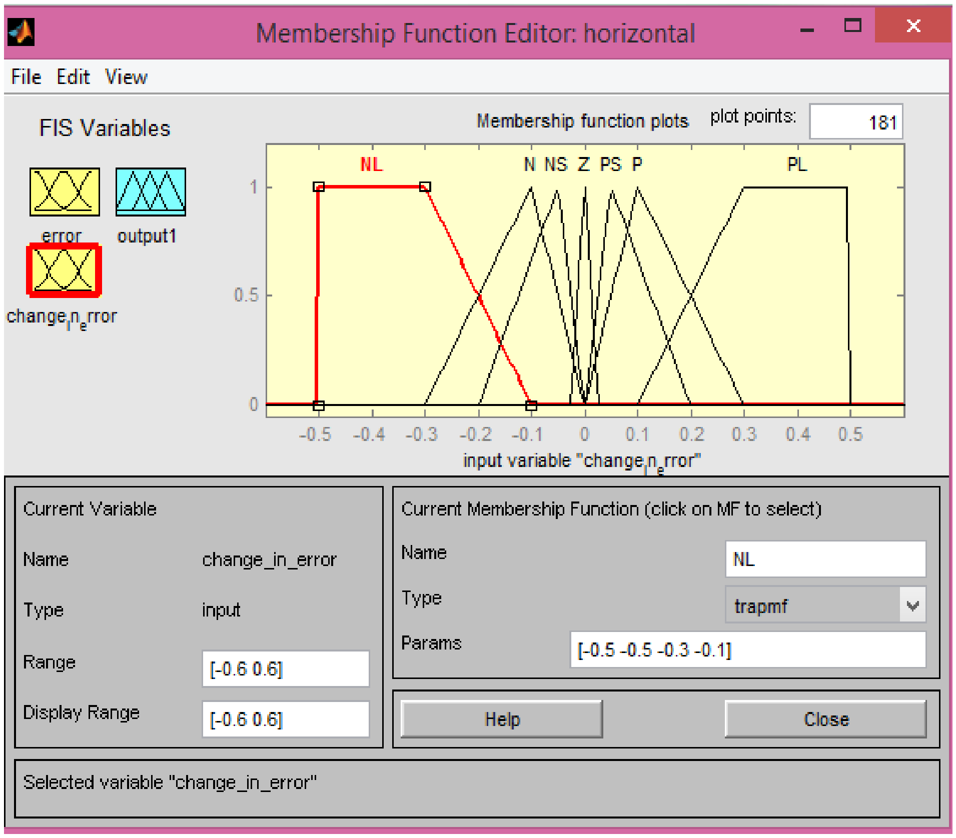


Fig 37. Membership function of change in error for horizontal part

Similarly membership function for the input variable change in error is shown in Fig37. Triangular and trapezoid Membership functions are used in the designing the controller which are stated as NL, N, NS, Z, PS, P and PL. The range of the input variable is -0.5 to 0.5.

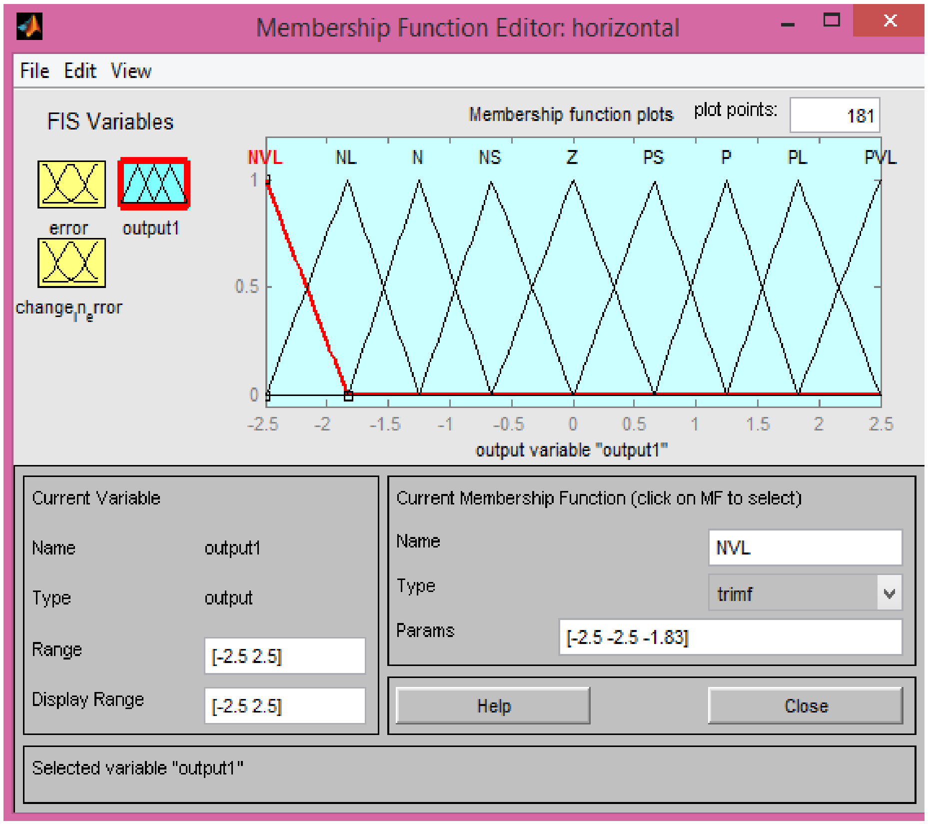


Fig 38 Membership function of output for horizontal part

Membership function for the output variable for horizontal means for Yaw controller is shown in the Fig38. Nine Triangular Membership function is used in the designing the controller which are stated as NVL, NL, N, NS, Z, PS, P, PVL and PL. The range of the output variable is -2.5 to 2.5.

5.2 FUZZY PLUS PID PITCH CONTROLLER DESIGN

SIMULINK model of Fuzzy plus PID controller for pitch or main rotor control is represented in Fig 39. PID controller parametersK_p, K_i K_d selected as 0.1,0.2 and 0.5 respectively. Initial pitch and yaw angle both are zero in this controller designing process.

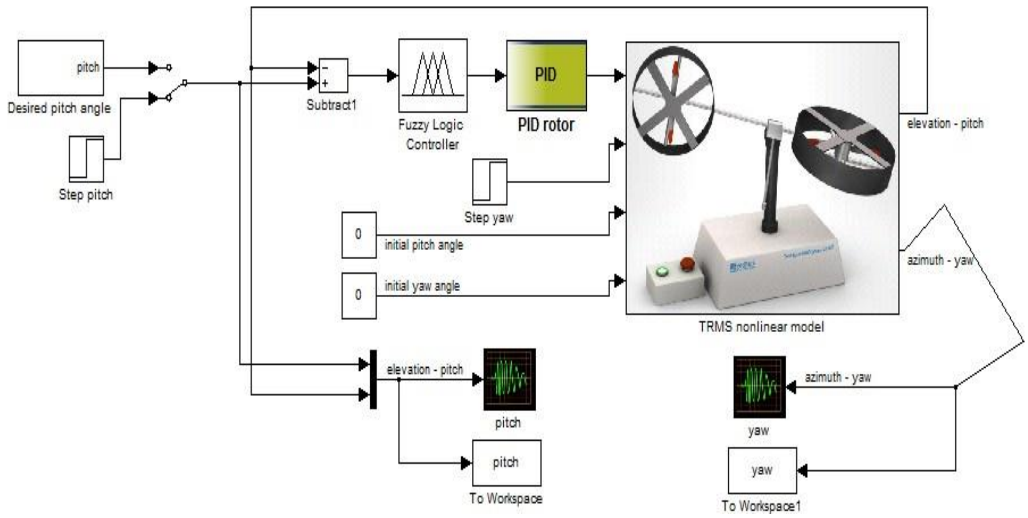


Fig 39 Fuzzy plus PID pitch controller design

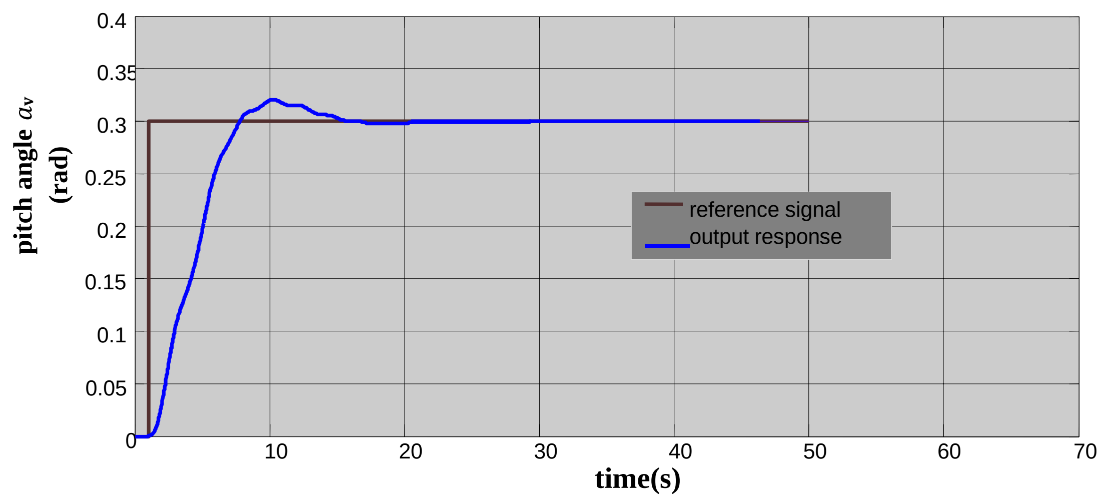


Fig 40 Pitch Response of Fuzzy plus PID

5.3 FUZZY PLUS PID YAW CONTROLLER

SIMULINK model of Fuzzy plus PID controller for yaw or tail rotor is shown in Fig.41. PID controller parameters K_p, K_i and K_d selected as 0.8, 0.01 and 0.5 respectively. To improve the overall performance of this system Hybrid controller is selected. Initial pitch and yaw angle are selected as zero in thisf process controller designing.

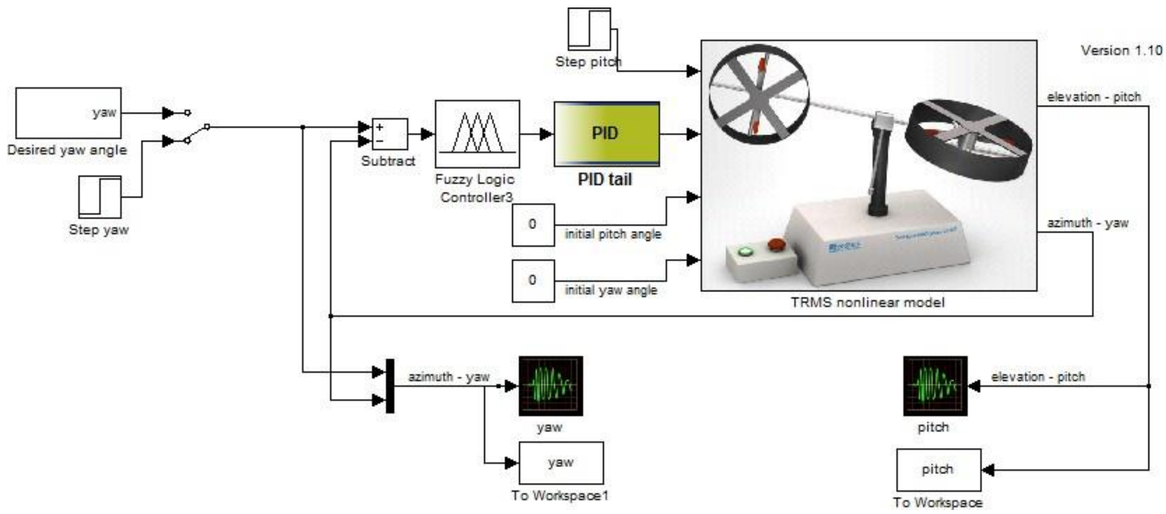


Fig 41 SIMULINK Model of FUZZY Plus PID

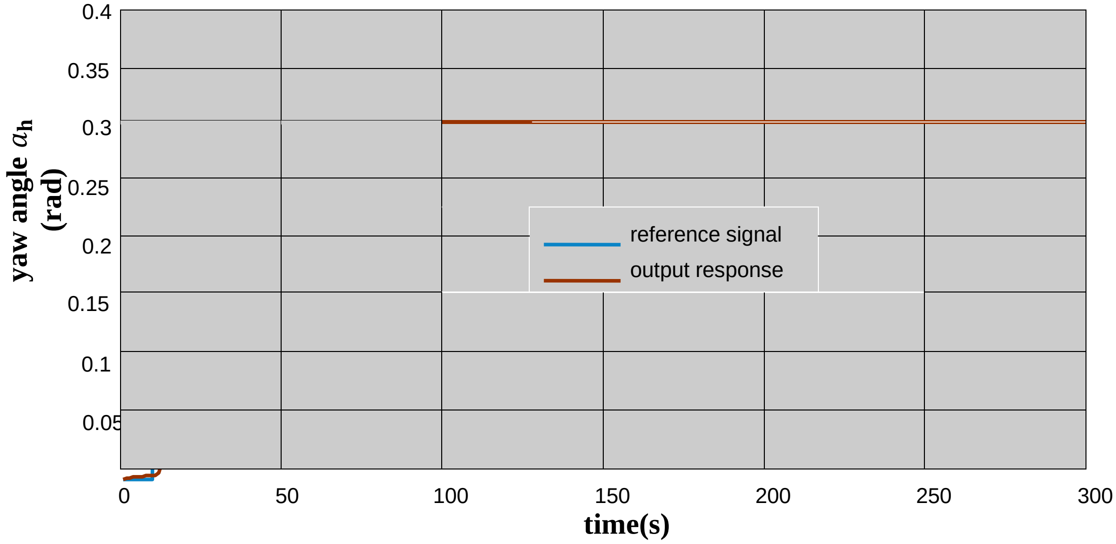


Fig 42 YAW Response of Fuzzy plus PID

Fig 42 represents the response of the Fuzzy plus PID controller for the tail rotor. The reference input step signal of value 0.3 is selected. Yaw output signal is feedback to the input as a feedback signal.

5.4 FUZZY PLUS PID PITCH AND YAW CONTROLLER

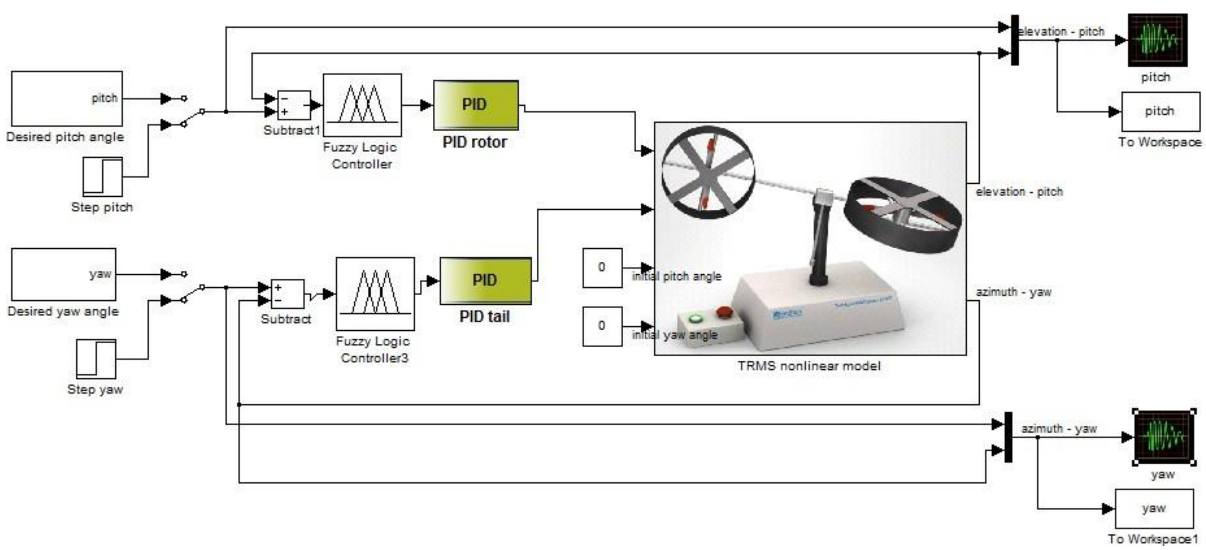


Fig 43.SIMULINK Model of FUZZY Plus PID Pitch and Yaw

Fuzzy plus PID controller SIMULINK model is designed for the main and tail rotor and implemented as shown in Fig43. Two different fuzzy controllers are used with different PID controllers with different parameters. Both outputs are used as feedback signal.

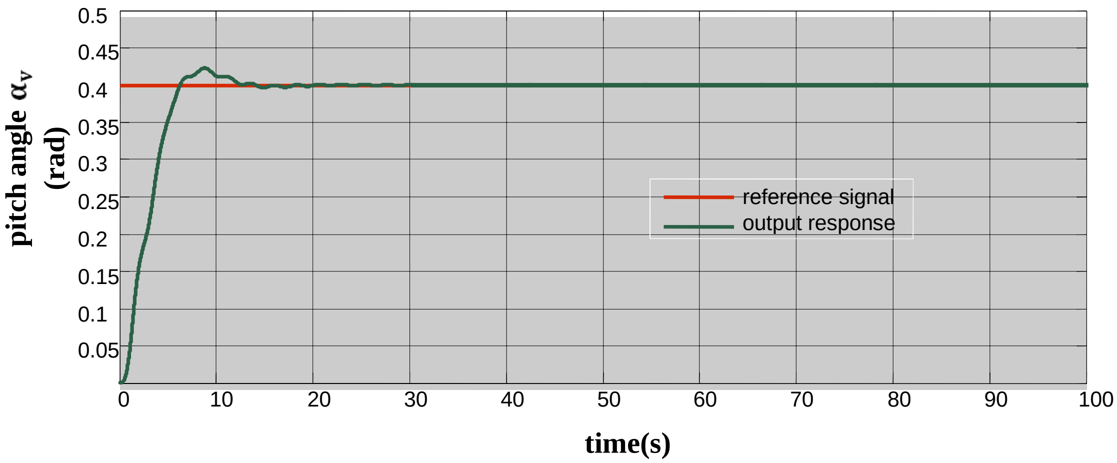


Fig 44. Response of FUZZY Plus PID Pitch

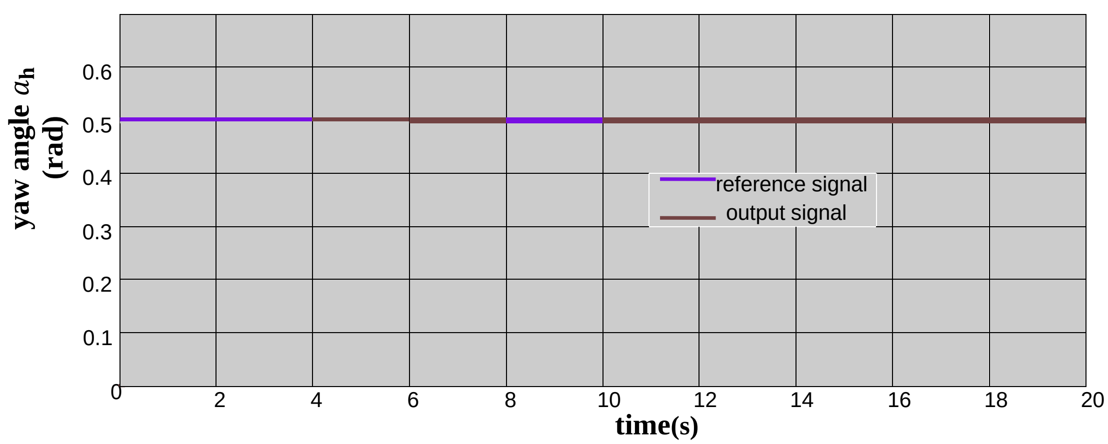


Fig 45. Response of FUZZY Plus PID for Yaw

5.5 COMPARISON OF PERFORMANCE OF TWIN ROTOR MIMO SYSTEM WITH DIFFERENT CONTROLLERS.

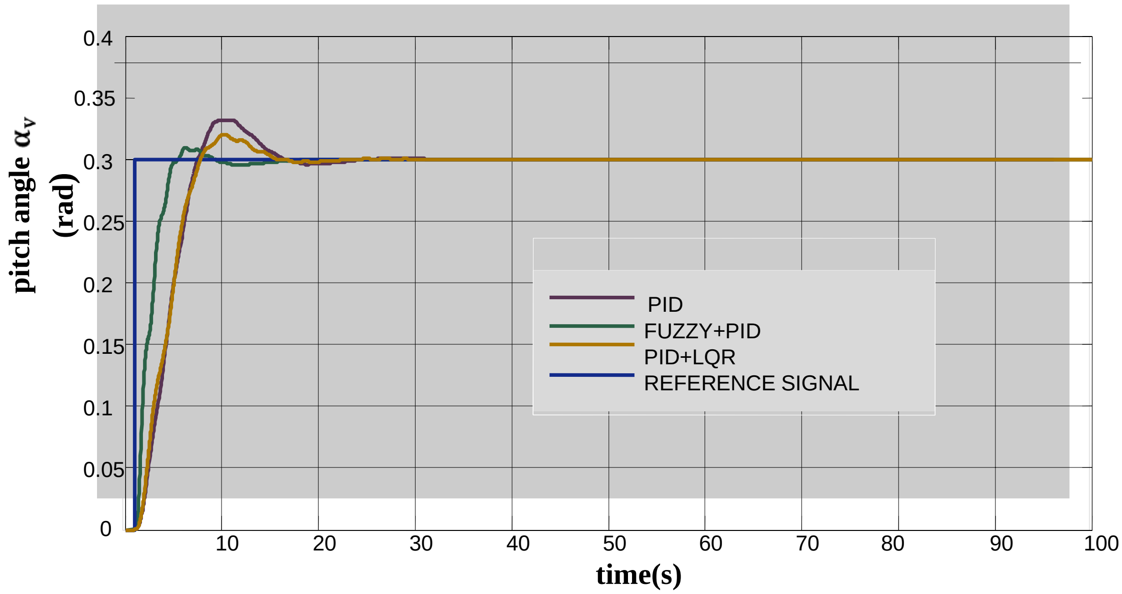


Fig 46 Comparison of responses

Type of controller	Performance Parameter				
	t_d(sec)	**t_r(sec)**	**t_p(sec)**	**t_s(sec)**	**M_P(sec)**
PID	4.55	7.18	12.105	24.5	0.030
FUZZY	3.0415	7.794	12.995	21.5	0.0116
LQR PLUS PID	1.83	5.54	6.4302	14.9	0.0117

Table 4 Comparison of responses

Response of three different controllers and performance parameters are shown in the Fig 46 and Table 4 respectively. Peak overshoot of PID controller is higher compared to other two and the response is also settling after taking some time unlikely PID Plus LQR response which is much better compared to PID controller as its response overshoot is lesser and settles to the reference input in lesser time and no. of oscillations are also less.

5.6 CONCLUSION

In this chapter a brief theory of fuzzy logic control is discussed and its implementation on TRMS system for pitch and yaw control is demonstrated through simulation study in MATLAB.

CHAPTER 6

CONCLUSIONS AND FUTURE SCOPE OF WORK

6.1 MAIN CONCLUSIONS

The TRMS Twin rotor multiple-input and multiple-output (MIMO) system that resembles the characteristics of a helicopter, with two-input and two-output. It is highly non-linear and unstable system with strong coupling effect between its input and output with two degree of freedom. Therefore in this work the efforts have been made to identify, linearise and to design the robust controller. This complex TRMS system has been linearised about an operating point and two decoupled systems have been developed .Two single input and single output (SISO) controllers have been employed for these decoupled units to achieve desired closed-loop performances and robustness. Different types of linear and nonlinear controller were designed to analyze the TRMS system. Performances of the system with conventional and intelligent controllers have been evaluated. All controllers have been designed, simulated, implemented experimentally and compared against existing optimal and robust controllers in the literature.

The simulation result shows that the PID controller has good tracking performance, but a lack of dynamic damping. PID tuning turned out to be inadequate for this complex system, resulting in a poor performance and its settling time was also very high.

Fuzzy controllers are also good in terms of stability but have slower response. The coupling effect is also brought to minimal. Steady-state error is as low at highest values. System input tracking is excellent. The response is fast and no effective overshoot when increasing the angle over the zero value, while the response is over-damped otherwise.

LQR plus PID controller was also designed which gives further improvement in performance such as fast dynamic response, lesser steady state error, and overshoot and rising time. Linear Quadratic Regulator (LQR) and Kalman Filter (KF) provide practical solutions to the full-state feedback and state estimation problems, respectively. Kalman filter loop gives good robustness and tracking properties but LQR controller does not perform well except away from the equilibrium point for which it has been linearized. Therefore from the simulation results and analysis of performance parameter values it is concluded that LQR plus PID control performs better as compared to PID or Fuzzy controller.

6.2 FUTURE SCOPE OF WORK

In the present work different type of controllers such as PID, LQR plus PID and fuzzy controllers were designed and implemented for the TRMS system. LQR plus PID controller is a linear controller while the Fuzzy controller is a nonlinear comparison of responses with the linear and nonlinear controllers for TRMS pitch and yaw control is summarized. The effectiveness and performance of the controllers have been shown in the simulation result. For advanced work, effort can be devoted in developing intelligent controllers with more robust control techniques that can control pitch and yaw angle of the system and also to implement the proposed control algorithm to real plant for validation of the theoretical result. In the next step as a future work another nonlinear controller H-infinity and BFO approaches will be implemented. Implementing the sliding mode control via back stepping control can also be considered as a recommended future work, since parametric uncertainty and external disturbance can be mitigated within a single model, which can stabilize the TRMS in a more robust way.

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