A GAME-THEORETIC APPROACH TO ANALYZE ACADEMIC INTENT

A DISSERTATION

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE AWARD OF THE DEGREE

OF

MASTER OF SCIENCE

IN

APPLIED MATHEMATICS

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CANDIDATE'S DECLARATION

We, (Himani) 2K20/MSCMAT/13, (Jyoti Lohani) 2K20/MSCMAT/15 students of M.Sc. (APPLIED MATHEMATICS), hereby declare that the Project Dissertation titled "A Game-Theoretic Approach to Analyze Academic Intent" which is submitted by us to the Department of Applied Mathematics, Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Science, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship, or other similar title or recognition.

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CERTIFICATE

I hereby clarify that the Project Dissertation titled "A Game-Theoretic Approach to Analyze Academic Intent" which is submitted by [Himani] 2K20/MSCMAT/13, [Jyoti Lohani] 2K20/MSCMAT/15 [APPLIED MATHEMATICS], Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Science, is a record of the project work carried out by the students under my supervision. To the best of my knowledge this work has not been submitted in part or full for any degree or Diploma to this University or elsewhere.

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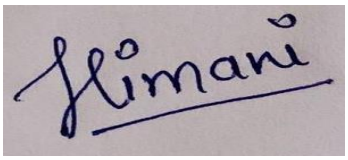
Sincere Thanks,



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ABSTRACT

This research work aims to apply the game theory approach to analyze people's preferences for academic intent. A set of factors influencing people's choice between public and private schools was identified. The approach was to find a person's preference based on those factors. To collect the relevant data, a form was circulated, and then by finding the saddle point using the minimax theorem in a two-person zero-sum game the optimal strategy and the value of the game were determined. The outcome of the approach showed that the value of the game was 4 which concludes that the game was in favor of public schools and people prefer public schools more because of their cost of education while private schools attribute their preferences to the facilities provided by them.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS, ABBREVIATIONS

a_1 is the cost of education in public school

a_2 is Teacher's qualification in public school

a_3 is the quality of education of public school

a_4 is the distance of the public school

a_5 is the facilities of public school

a_6 is the admission process of public school

a_7 is the accountability of public school

a_8 is the use of technology in public school

a_9 is the Academic curriculum of public school

a_{10} is English proficiency of public school

a_{11} is Post school benefits of public school

a_{12} is the extra-curricular activity of the public school

b_1 is the cost of education in private school

b_2 is Teacher's qualification in private school

b_3 is the quality of education of private school

b_4 is the distance of the private school

b_5 is facilities of private school

b_6 is the admission process of private school

b_7 is the accountability of private school

b_8 is the use of technology in private school

b_9 is the Academic curriculum of private school

b_{10} is English proficiency of private school

b_{11} is Post school benefits of private school

b_{12} is the extra-curricular activity of the private school

a_{11} represents row 1 and column 1 strategy

a_{12} represent row 1 and column 2 strategy and further on.

CHAPTER 1 INTRODUCTION

India is one of the world's developing countries and education plays a vital role in improving economic growth, gender equality, peace, health, stability, and reducing poverty [4]. School education is the base of any education system. In India, other than government schools, schools are mainly divided into two categories one is public schools which are funded by the government and another one is private schools which generate their funding from various sources like private grants, school tuition fees, and endowments.

The individual preference for education is based on several factors such as quality of education, teacher's qualification, proximity, cost of education, use of technology, post-school benefits, English proficiency, and facilities such as hygiene, scholarships, etc [1]. The methods for measuring quality school education consist; the structure, process, and outcomes. The context in which education is delivered, together with school buildings, cost of education, staff denotes structure, the coordination between students and education providers in the delivery of education facilities is known as process, and the increase on the literacy status of students is denotes the outcome.

According to, the data released by UDISE (2019-20), there are 84,362 public schools and 3,37,499 private schools in India with 2,74,98,530 and 9,82,09,303 enrolments respectively [7]. As the data shows that there is a big difference between the numbers of students in public and private schools, The only question here is what are the factors that are influencing people to choose between public and private schools.

This research work is mainly based on people's choice between public and private schools based on the elements that make a difference between private and public schools. We have conducted one survey through a google form (Appendix 1) and made people choose one type of school on the bases of the factors such as cost of education,

teacher's qualification, proximity, Admission process, accountability, Use of technology, academic curriculum, English proficiency, post-school benefits, extracurricular activities. There are a lot of controversies when it comes to deciding which type of schools are better and game theory plays a vital role in case of social affairs among competing bodies. The game theory produces optimal decisions based on the strategic setting of independent and competing actors to conceive social situations among competing groups.

Therefore, this research work aims to analyze people's preferences of academic intent between public and private schools by a survey based on a small group of people and then using a game theory approach, and the aims are, To assess people's perceptiveness of quality education services received in their preferred schools between private and public schools, to evaluate the motive for people's perseverance in their preferred schools, to find the factors impacting people's preferences, to find the value of the game and the optimal strategy, and to determine which type of schools between private and public schools give their students the finest education using the value of the game.

CHAPTER 2 LITERATURE REVIEW AND METHODOLOGY

2.1 MATRIX TWO-PERSON GAME

In a two-player game the rules of the game have a numeric representation for both the players. This comprises the strategies, which is a plan depending upon the present state of the game for each stage of the game, and the payoffs, which are consequences of the combination of strategies of both the players.[2][3]

2.1.1 A TWO-PERSON ZERO-SUM GAME WITH PURE STRATEGIES

In a two-person zero-sum game, whatever one player wins the other loses, so if a_{ij} is the amount player 1 wins, then player 2 wins - a_{ij}. Suppose we have player 1 and player 2, player 1 have n possible strategies and player 2 have m possible strategies, So then we have collections of payoffs $\{a_{ij}\}$ where i=1, 2,…, n, j=1, 2,…,m and these values can be presented in matrix. This matrix is called game matrix or payoff matrix.[2][3]

			Player II		
		Strategy 1	Strategy 2	...	Strategy m
	Strategy 1	a_{11}	a_{12}	...	a_{1m}
	Strategy 2	a_{21}	a_{22}	...	a_{2m}
Player I	:	:	:	:	:
	Strategy n	a_{n1}	a_{n2}	...	a_{nm}

In this research work, the row player always wants to maximize his payoff, while to maximize his payoff the column player wants to minimize the payoff of the row player. The rows are called pure strategies for row player and the columns are called pure strategies for column player.

2.1.2 THE MINIMAX THEOREM

A matrix game with matrix A =(a_{ij}) of order n×m

$$v^- = \max_{i=1,2,\dots,n} \min_{j=1,2,\dots,m} a_{ij}$$ is the lower value

Where v^- is the smallest value player 1 is assured to receive and

$$v^+ = \min_{j=1,2,\dots,m} \max_{i=1,2,\dots,n} a_{ij}$$ is the upper value

Where $v^+$is the largest value player 2 is assured to lose.

A game has a saddle point if and only if $v^- = v^+$ and then the value of the game is $v = v^- = v^+$. [2][3][6]

2.1.3 DOMINANCE PRINCIPLE

The rule of dominance in Game Theory (also known as dominant strategy or dominance method) says that if one strategy of a player dominates over the other strategy for all pay-offs then the latter strategy can be neglected. If a strategy preferable over other in all conditions than the strategy dominates over the other. Generally, the dominance property is used to reduce the size of a large payoff matrix.

Dominant Strategy Rules-

- If every payoff in row i is greater or equal to every corresponding payoff in row j, then the row player would never play row j (since he/she wants the biggest possible payoff). That is Row i is dominated by the row j So we can remove the row j from the matrix.
- If every payoff in column j is less than or equal to every corresponding payoff in column k, then the column player would never play column k (since he/she wants player I to get the smallest possible payoff). That is column k is dominated by the column j So we can remove the column k from the matrix.[2][3]

2.2 METHODOLOGY

This study focuses on people's preferences for schools for quality education. In this research, we assembled the people's choices into public and private schools and assessed them. The public school represents player A who is a row player, while the private school represents player B who is a column player and then we solve the matrix using the minimax theorem. A two-person zero-sum game is adopted in this study. The payoff matrix is a profit matrix for played A and for player B it is a loss matrix; Based on our research we have a 12*12 matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{1,10} & a_{1,11} & a_{1,12} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} & a_{29} & a_{2,10} & a_{2,11} & a_{2,12} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} & a_{39} & a_{3,10} & a_{3,11} & a_{3,12} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} & a_{49} & a_{4,10} & a_{4,11} & a_{4,12} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} & a_{58} & a_{59} & a_{5,10} & a_{5,11} & a_{5,12} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} & a_{68} & a_{69} & a_{6,10} & a_{6,11} & a_{6,12} \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} & a_{79} & a_{7,10} & a_{7,11} & a_{7,12} \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & a_{88} & a_{89} & a_{8,10} & a_{8,11} & a_{8,12} \\ a_{91} & a_{92} & a_{93} & a_{94} & a_{95} & a_{96} & a_{97} & a_{98} & a_{99} & a_{9,10} & a_{9,11} & a_{9,12} \\ a_{10,1} & a_{10,2} & a_{10,3} & a_{10,4} & a_{10,5} & a_{10,6} & a_{10,7} & a_{10,8} & a_{10,9} & a_{10,10} & a_{10,11} & a_{10,12} \\ a_{11,1} & a_{11,2} & a_{11,3} & a_{11,4} & a_{11,5} & a_{11,6} & a_{11,7} & a_{11,8} & a_{11,9} & a_{11,10} & a_{11,11} & a_{11,12} \\ a_{12,1} & a_{12,2} & a_{12,3} & a_{12,4} & a_{12,5} & a_{12,6} & a_{12,7} & a_{12,8} & a_{12,9} & a_{12,10} & a_{12,11} & a_{12,12} \end{bmatrix}$$

CHAPTER 3: PRESENTATION OF DATA AND ANALYSES

3.1 PERCENTAGE DISTRIBUTION OF DEMOGRAPHIC DATA OF RESPONDENTS

The payoff matrix of the people's preferences is represented as,

		Player B											
		b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
	a_1	a_1-b_1	a_1-b_2	a_1-b_3	a_1-b_4	a_1-b_5	a_1-b_6	a_1-b_7	a_1-b_8	a_1-b_9	a_1-b_{10}	a_1-b_{11}	a_1-b_{12}
	a_2	a_2-b_1	a_2-b_2	a_2-b_3	a_2-b_4	a_2-b_5	a_2-b_6	a_2-b_7	a_2-b_8	a_2-b_9	a_2-b_{10}	a_2-b_{11}	a_2-b_{12}
	a_3	a_3-b_1	a_3-b_2	a_3-b_3	a_3-b_4	a_3-b_5	a_3-b_6	a_3-b_7	a_3-b_8	a_3-b_9	a_3-b_{10}	a_3-b_{11}	a_3-b_{12}
	a_4	a_4-b_1	a_4-b_2	a_4-b_3	a_4-b_4	a_4-b_5	a_4-b_6	a_4-b_7	a_4-b_8	a_4-b_9	a_4-b_{10}	a_4-b_{11}	a_4-b_{12}
	a_5	a_5-b_1	a_5-b_2	a_5-b_3	a_5-b_4	a_5-b_5	a_5-b_6	a_5-b_7	a_5-b_8	a_5-b_9	a_5-b_{10}	a_5-b_{11}	a_5-b_{12}
	a_6	a_6-b_1	a_6-b_2	a_6-b_3	a_6-b_4	a_6-b_5	a_6-b_6	a_6-b_7	a_6-b_8	a_6-b_9	a_6-b_{10}	a_6-b_{11}	a_6-b_{12}
A	a_7	a_7-b_1	a_7-b_2	a_7-b_3	a_7-b_4	a_7-b_5	a_7-b_6	a_7-b_7	a_7-b_8	a_7-b_9	a_7-b_{10}	a_7-b_{11}	a_7-b_{12}
	a_8	a_8-b_1	a_8-b_2	a_8-b_3	a_8-b_4	a_8-b_5	a_8-b_6	a_8-b_7	a_8-b_8	a_8-b_9	a_8-b_{10}	a_8-b_{11}	a_8-b_{12}
	a_9	a_9-b_1	a_9-b_2	a_9-b_3	a_9-b_4	a_9-b_5	a_9-b_6	a_9-b_7	a_9-b_8	a_9-b_9	a_9-b_{10}	a_9-b_{11}	a_9-b_{12}
	a_{10}	a_{10}-b_1	a_{10}-b_2	a_{10}-b_3	a_{10}-b_4	a_{10}-b_5	a_{10}-b_6	a_{10}-b_7	a_{10}-b_8	a_{10}-b_9	a_{10}-b_{10}	a_{10}-b_{11}	a_{10}-b_{12}
	a_{11}	a_{11}-b_1	a_{11}-b_2	a_{11}-b_3	a_{11}-b_4	a_{11}-b_5	a_{11}-b_6	a_{11}-b_7	a_{11}-b_8	a_4-b_9	a_{11}-b_{10}	a_{11}-b_{11}	a_{11}-b_{12}
	a_{12}	a_{12}-b_1	a_{12}-b_2	a_{12}-b_3	a_{12}-b_4	a_{12}-b_5	a_{12}-b_6	a_{12}-b_7	a_{12}-b_8	a_4-b_9	a_{12}-b_{10}	a_{12}-b_{11}	a_{12}-b_{12}

Based on Table 3.1, it is shown that 61 responders were males indicating 45.9%, and 72 were females indicating 54.1% as shown in Fig 3.1.

The percentage distribution of the age group of responders is indicated in Table 3.2. The table indicates that there were 2 responders below 18 years indicating 1.5%, 92 responders were between 18 - 25 years indicating 69.2%, 34 responders were between 25-45 years indicating 25.6%, 5 responders were above 45 years indicating 3.8%., Thus, Fig 3.2 shows the pie chart showing the description of Table 3.2.

Table 3.1 Percentage distribution of gender of responders.

Gender	Frequency	Percentage (%)
Male	61	45.9
Female	72	54.1
Other	0	0
Total	133	100

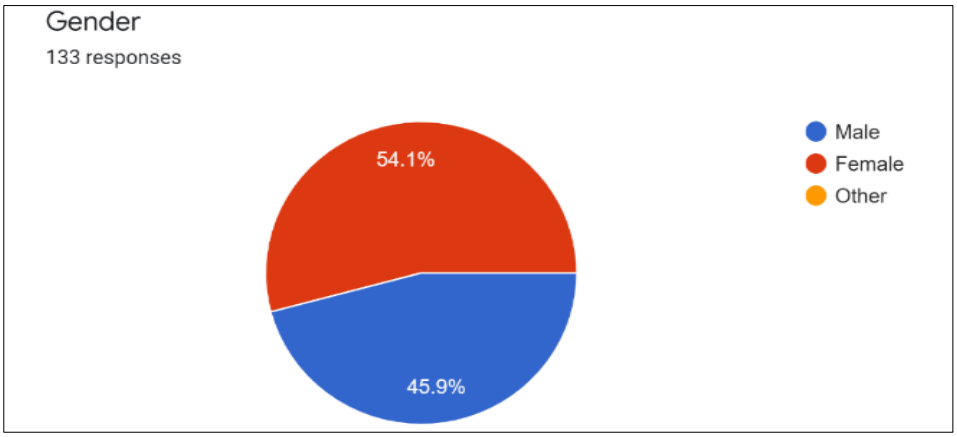

Fig 3.1 Pie chart showing gender of responders

Table 3.2 Age distribution of responders

Age group	Number of respondents	Percentage (%)
Below 18	2	1.5
18-25	92	69.2
25-45	34	25.6
Above 45	5	3.8
Total	133	100

Table 3.3 indicates that 6.8% with 9 responders were government employees, 24.1% representing 32 responders were non-government employees, 60.9% representing 81 responders were students, and 8.3% with 11 responders representing others. Thus, Fig 3.3 represents Table 3.3 using a pie chart.

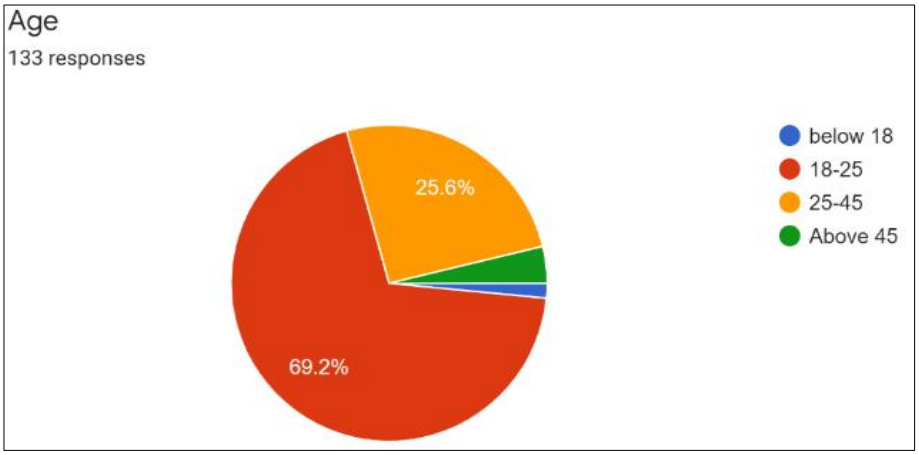

Fig 3.2 Pie chart indicating age group of responders

Table 3.3 Percentage distribution of responders' occupation

Occupation	**Number of respondents**	**Percentage (%)**
Government employee	9	6.8
Non-government employee	32	24.1
Student	81	60.9
Other	11	8.3
Total	133	100

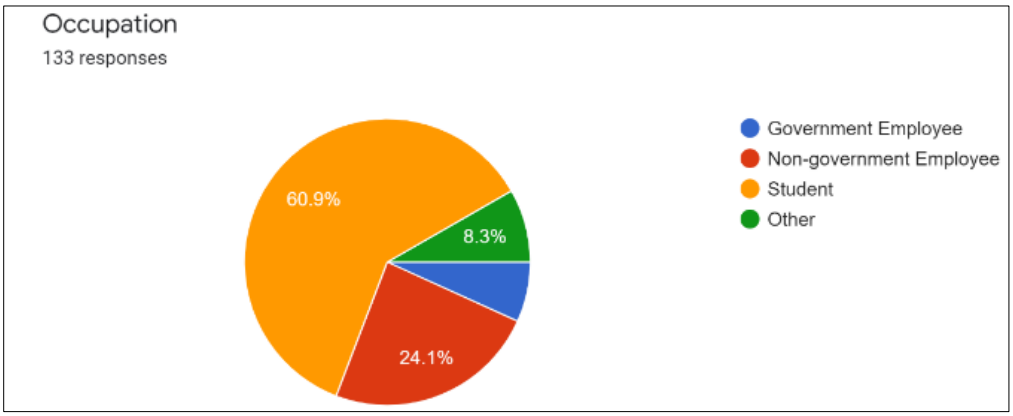

Fig 3.3 Pie chart showing occupation of responders.

Table 3.4 indicates that 50.4% with 67 respondents have zero yearly income, 19.5% representing 26 respondents have yearly income between 0-2 lakh, 11.3% representing 15 respondents have yearly income between 2-6 lakh, 8.3% representing 11 respondents have yearly income between 6-8 lakh and 10.5% with 14 respondents have yearly income more than 8 lakh. Thus, fig 3.4 shows the explanation of Table 3.4 using a pie chart.

Table 3.4 Percentage distribution of yearly income of responders

Yearly Income	**Number of respondents**	**Percentage(%)**
0	67	50.4
0-2 lakh	26	19.5
2 lakh – 6 lakh	15	11.3
6 lakh-8 lakh	11	8.3
>8 lakh	14	10.5
Total	133	100

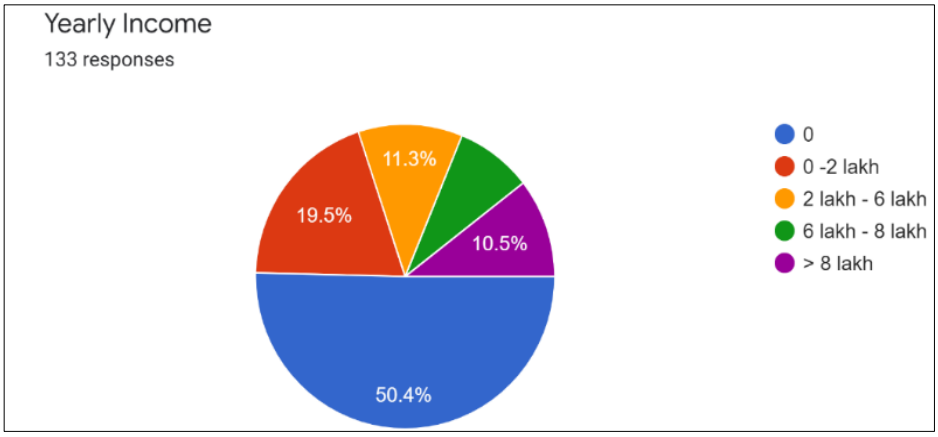

Fig 3.4 Pie chart showing the yearly income of the responders

3.2 DATA REPRESENTATION OF RESEARCH QUESTIONS AND INTERPRETATION

Table 3.5 shows that, 113 people choose public school while 20 people prefer private school on the bases of cost of education, 73 people choose public school while 60 people prefer private school on the bases of teacher's qualification, 41 people choose public school while 92 people prefer private school on the bases of Quality of education, 24 people choose public school while 109 people prefer private school on the bases of Facilities provided by them, 72 people choose public school while 61 people prefer private school on the bases of Admission process of schools, 51 people choose public school while 82 people prefer private school on the bases of Accountability of schools, 26 people choose public school while 107 people prefer private school on the bases of Use of technology, 40 people choose public school while 93 people prefer private school on the bases of Academic curriculum, 26 people choose public school while 107 people prefer private school on the bases of English Proficiency, 53 people choose public school while 80 people prefer private school on the bases of Post-school benefits, 31 people choose public school while 102 people prefer private school on the bases of Extracurricular activity. Fig 3.5 shows the bar graph of table 3.5.

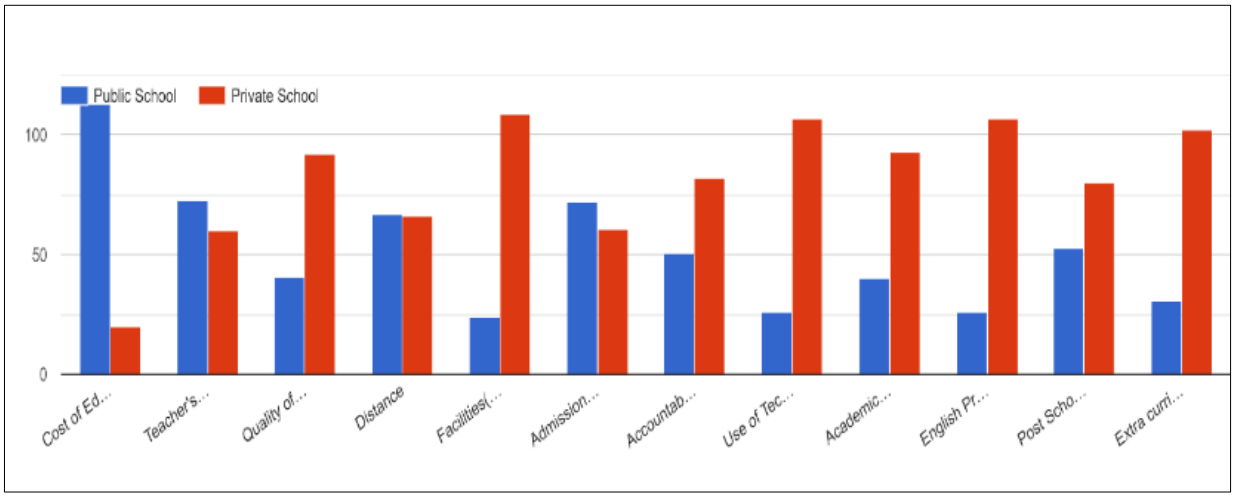

Fig 3.5 Bar graph showing the Frequency distribution of responds on the elements effecting People's choices.

Table 3.5 Frequency distribution of responds on the Factors effecting People's choices

s/n	Item	Public school	Private school
1	Cost of education	113	20
2	Teacher's qualification	73	60
3	Quality of education	41	92
4	Distance	67	66
5	Facilities (Hygiene, Safety, infrastructure, etc.)	24	109
6	Admission process	72	61
7	Accountability	51	82
8	Use of Technology	26	107
9	Academic Curriculum	40	93
10	English Proficiency	26	107
11	Post-school benefits	53	80
12	Extra-curricular activity	31	102

3.3 DATA ANALYSIS AND INTERPRETATION FOR PEOPLE'S PREFERENCES

		Player B											
		b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
A	a_1	(113,20)	(113,60)	(113,92)	(113,66)	(113,109)	(113,61)	(113,82)	(113,107)	(113,93)	(113,107)	(113,80)	(113,102)
	a_2	(73,20)	(73,60)	(73,92)	(73,66)	(73,109)	(73,61)	(73,82)	(73,107)	(113,93)	(73,107)	(73,80)	(73,102)
	a_3	(41,20)	(41,60)	(41,92)	(41,66)	(41,109)	(41,61)	(41,82)	(41,107)	(41,93)	(41,107)	(41,80)	(113,102)
	a_4	(67,20)	(67,60)	(67,92)	(67,66)	(67,109)	(67,61)	(67,82)	(113,107)	(67,93)	(67,107)	(67,80)	(67,102)
	a_5	(24,20)	(24,60)	(24,92)	(24,66)	(24,109)	(24,61)	(24,82)	(24,107)	(24,93)	(24,107)	(24,80)	(24,102)
	a_6	(72,20)	(72,60)	(72,92)	(72,66)	(72,109)	(72,61)	(72,82)	(113,107)	(72,93)	(72,107)	(72,80)	(72,102)
	a_7	(51,20)	(51,60)	(51,92)	(51,66)	(51,109)	(51,61)	(51,82)	(51,107)	(51,93)	(51,107)	(51,80)	(51,102)
	a_8	(26,20)	(26,60)	(26,92)	(26,66)	(26,109)	(26,61)	(26,82)	(26,107)	(26,93)	(26,107)	(26,80)	(26,102)
	a_9	(40,20)	(40,60)	(40,92)	(40,66)	(40,109)	(40,61)	(40,82)	(40,107)	(40,93)	(40,107)	(40,80)	(40,102)
	a_{10}	(26,20)	(26,60)	(26,92)	(26,66)	(26,109)	(26,61)	(26,82)	(26,107)	(26,93)	(26,107)	(26,80)	(26,102)
	a_{11}	(53,20)	(53,60)	(53,92)	(53,66)	(53,109)	(53,61)	(53,82)	(53,107)	(113,93)	(53,107)	(53,80)	(53,102)
	a_{12}	(31,20)	(31,60)	(31,92)	(31,66)	(31,109)	(31,61)	(31,82)	(31,107)	(31,93)	(31,107)	(31,80)	(31,102)

The payoff matrix based on the above data is-

		Player B											
		b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
	a_1	93	53	21	47	4	52	31	6	20	6	33	11
	a_2	53	13	-19	7	-36	12	-9	-34	-20	-34	-7	-29
	a_3	21	-19	-51	-25	-68	-20	-41	-66	-52	-66	-39	-61
	a_4	47	7	-25	1	-42	6	-15	-40	-26	-40	-13	-35
	a_5	4	-36	-68	-42	-85	-37	-58	-83	-69	-83	-56	-78
	a_6	52	12	-20	6	-37	11	-10	-35	-21	-35	-8	-30
Player A	a_7	31	-9	-41	-15	-58	-10	-31	-56	-42	-56	-29	-51
	a_8	6	-34	-66	-40	-83	-35	-56	-81	-67	-81	-54	-76
	a_9	20	-20	-52	-26	-69	-21	-42	-67	-23	-67	-40	-62
	a_{10}	6	-34	-66	-40	-83	-35	-56	-81	-67	-81	-54	-76
	a_{11}	33	-7	-39	-13	-56	-8	-29	-54	-40	-54	-27	-49
	a_{12}	11	-29	-61	-35	-78	-30	-51	-76	-62	-76	-49	-71

3.3.1 USING MINIMAX THEOREM FOR DATA ANALYSIS AND INTERPRETATION OF THE PAY-OFF MATRIX

The saddle point exists at 4 because the lower value and the upper value of the game is equal which means the value of the game is 4.

		Player B												
		b1	b2	b3	b4	b5	b6	b7	b8	b9	b10	b11	b12	Row min
	a_1	93	53	21	47	4	52	31	6	20	6	33	11	4
	a_2	53	13	-19	7	-36	12	-9	-34	-20	-34	-7	-29	-36
	a_3	21	-19	-51	-25	-68	-20	-41	-66	-52	-66	-39	-68	-68
	a_4	47	7	-25	1	-42	6	-15	-40	-26	-40	-13	-35	-42
	a_5	4	-36	-68	-42	-85	-37	-58	-83	-69	-83	-56	-78	-85
	a_6	52	12	-20	6	-37	11	-10	-35	-21	-35	-8	-30	-37
A	a_7	31	-9	-41	-15	-58	-10	-31	-56	-42	-56	-29	-51	-58
	a_8	6	-34	-66	-40	-83	-35	-56	-81	-67	-81	-54	-76	-83
	a_9	20	-20	-52	-26	-69	-21	-42	-67	-23	-67	-40	-62	-69
	a_{10}	6	-34	-66	-40	-83	-35	-56	-81	-67	-81	-54	-76	-83
	a_{11}	33	-7	-39	-13	-56	-8	-29	-54	-40	-54	-27	-49	-56
	a_{12}	11	-29	-61	-35	-78	-30	-51	-76	-62	-76	-49	-71	-78
	Column max	93	53	21	47	4	52	31	6	20	6	33	11	4

Although using the minimax theorem the game is solvable but it can be the case that we are unable to find the saddle point. In those cases, we use the Dominance rule for solving the game.

3.3.2 USING DOMINANCE RULE FOR DATA INTERPRETATION AND ANALYSIS OF THE PAY-OFF MATRIX

We use the dominance principle for reducing the size of the payoff matrix-

Step 1- Row-12 ≤ Row-11, so remove Row-12,

($a_{12} \leq a_{11}$:11≤33, -29≤-7, -61≤-39, -35≤-13, -78≤-56, -30≤-8, -51≤-29, -76≤-54, -62≤-40, -76≤-54, -49≤-27, -71≤-49)

		Player B											
		b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
Player A	a_1	93	53	21	47	4	52	31	6	20	6	33	11
	a_2	53	13	-19	7	-36	12	-9	-34	-20	-34	-7	-29
	a_3	21	-19	-51	-25	-68	-20	-41	-66	-52	-66	-39	-61
	a_4	47	7	-25	1	-42	6	-15	-40	-26	-40	-13	-35
	a_5	4	-36	-68	-42	-85	-37	-58	-83	-69	-83	-56	-78
	a_6	52	12	-20	6	-37	11	-10	-35	-21	-35	-8	-30
	a_7	31	-9	-41	-15	-58	-10	-31	-56	-42	-56	-29	-51
	a_8	6	-34	-66	-40	-83	-35	-56	-81	-67	-81	-54	-76
	a_9	20	-20	-52	-26	-69	-21	-42	-67	-23	-67	-40	-62
	a_{10}	6	-34	-66	-40	-83	-35	-56	-81	-67	-81	-54	-76
	a_{11}	33	-7	-39	-13	-56	-8	-29	-54	-40	-54	-27	-49

Step 2- Row-11 ≤ Row-6, so remove Row-11,

($a_{11} \leq a_6$: 33≤52, -7≤12, -39≤-20, -13≤6, -56≤-37, -8≤11, -29≤-10, -54≤-35, -40≤-21, -54≤-35, -27≤-8, -49≤-30)

		Player B											
		b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
Player A	a_1	93	53	21	47	4	52	31	6	20	6	33	11
	a_2	53	13	-19	7	-36	12	-9	-34	-20	-34	-7	-29
	a_3	21	-19	-51	-25	-68	-20	-41	-66	-52	-66	-39	-61
	a_4	47	7	-25	1	-42	6	-15	-40	-26	-40	-13	-35
	a_5	4	-36	-68	-42	-85	-37	-58	-83	-69	-83	-56	-78
	a_6	52	12	-20	6	-37	11	-10	-35	-21	-35	-8	-30
	a_7	31	-9	-41	-15	-58	-10	-31	-56	-42	-56	-29	-51
	a_8	6	-34	-66	-40	-83	-35	-56	-81	-67	-81	-54	-76
	a_9	20	-20	-52	-26	-69	-21	-42	-67	-23	-67	-40	-62
	a_{10}	6	-34	-66	-40	-83	-35	-56	-81	-67	-81	-54	-76

Step 3- Row-10 ≤ Row-9, so remove Row-10,

($a_{10} \leq a_9$: 6≤20, -34≤-20, -66≤-52, -40≤-26, -83≤-69, -35≤-21, -56≤-42, -81≤-67, -67≤-23, -81≤-67, -54≤-40, -76≤-62)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
a_1	93	53	21	47	4	52	31	6	20	6	33	11
a_2	53	13	-19	7	-36	12	-9	-34	-20	-34	-7	-29
a_3	21	-19	-51	-25	-68	-20	-41	-66	-52	-66	-39	-61
a_4	47	7	-25	1	-42	6	-15	-40	-26	-40	-13	-35
a_5	4	-36	-68	-42	-85	-37	-58	-83	-69	-83	-56	-78
a_6	52	12	-20	6	-37	11	-10	-35	-21	-35	-8	-30
a_7	31	-9	-41	-15	-58	-10	-31	-56	-42	-56	-29	-51
a_8	6	-34	-66	-40	-83	-35	-56	-81	-67	-81	-54	-76
a_9	20	-20	-52	-26	-69	-21	-42	-67	-23	-67	-40	-62

Step 4- Row-9 ≤ Row-6, so remove Row-9,

($a_9 \leq a_6$: 20≤52, -20≤12, -52≤-20, -26≤6, -69≤-37, -21≤11, -42≤-10, -67≤-35, -23≤-21, -67≤-35, -40≤-8, -62≤-30)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
a_1	93	53	21	47	4	52	31	6	20	6	33	11
a_2	53	13	-19	7	-36	12	-9	-34	-20	-34	-7	-29
a_3	21	-19	-51	-25	-68	-20	-41	-66	-52	-66	-39	-61
a_4	47	7	-25	1	-42	6	-15	-40	-26	-40	-13	-35
a_5	4	-36	-68	-42	-85	-37	-58	-83	-69	-83	-56	-78
a_6	52	12	-20	6	-37	11	-10	-35	-21	-35	-8	-30
a_7	31	-9	-41	-15	-58	-10	-31	-56	-42	-56	-29	-51
a_8	6	-34	-66	-40	-83	-35	-56	-81	-67	-81	-54	-76

Step 5- Row-8 ≤ Row-7, so remove Row-8,

($a_8 \leq a_7$: 6≤31, -34≤-9, -66≤-41, -40≤-15, -83≤-58, -35≤-10, -56≤-31, -81≤-56, -67≤ -42, -81≤-56, -54≤-29, -76≤-51)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
a_1	93	53	21	47	4	52	31	6	20	6	33	11
a_2	53	13	-19	7	-36	12	-9	-34	-20	-34	-7	-29
a_3	21	-19	-51	-25	-68	-20	-41	-66	-52	-66	-39	-61
a_4	47	7	-25	1	-42	6	-15	-40	-26	-40	-13	-35
a_5	4	-36	-68	-42	-85	-37	-58	-83	-69	-83	-56	-78
a_6	52	12	-20	6	-37	11	-10	-35	-21	-35	-8	-30
a_7	31	-9	-41	-15	-58	-10	-31	-56	-42	-56	-29	-51

Step 6- Row-7 ≤ Row-6, so remove Row-7,

($a_7 \leq a_6$: 31≤52, -9≤12, -41≤-20, -15≤6, -58≤-37, -10≤11, -31≤-10, -56≤-35, -42≤-21, -56≤-35, -29≤-8, -51≤-30)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
a_1	93	53	21	47	4	52	31	6	20	6	33	11
a_2	53	13	-19	7	-36	12	-9	-34	-20	-34	-7	-29
a_3	21	-19	-51	-25	-68	-20	-41	-66	-52	-66	-39	-61
a_4	47	7	-25	1	-42	6	-15	-40	-26	-40	-13	-35
a_5	4	-36	-68	-42	-85	-37	-58	-83	-69	-83	-56	-78
a_6	52	12	-20	6	-37	11	-10	-35	-21	-35	-8	-30

Step 7- Row-6 ≤ Row-2, so remove Row-6,

($a_6 \leq a_2$: 52≤53, 12≤13, -20≤-19, 6≤7, -37≤-36, 11≤12, -10≤-9, -35≤-34, -21≤-20, -35≤-34, -8≤-7, -30≤-29)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
a_1	93	53	21	47	4	52	31	6	20	6	33	11
a_2	53	13	-19	7	-36	12	-9	-34	-20	-34	-7	-29
a_3	21	-19	-51	-25	-68	-20	-41	-66	-52	-66	-39	-61
a_4	47	7	-25	1	-42	6	-15	-40	-26	-40	-13	-35
a_5	4	-36	-68	-42	-85	-37	-58	-83	-69	-83	-56	-78

Step 8- Row-5 ≤ Row-4, so remove Row-5,

($a_5 \leq a_4$: 4≤47, -36≤7, -68≤-25, -42≤1, -85≤-42, -37≤6, -58≤-15, -83≤-40, -69≤-26, -83≤-40, -56≤-13, -78≤-35)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
a_1	93	53	21	47	4	52	31	6	20	6	33	11
a_2	53	13	-19	7	-36	12	-9	-34	-20	-34	-7	-29
a_3	21	-19	-51	-25	-68	-20	-41	-66	-52	-66	-39	-61
a_4	47	7	-25	1	-42	6	-15	-40	-26	-40	-13	-35

Step 9- Row-4 ≤ Row-2, so remove Row-4,

($a_4 \leq a_2$: 47≤53, 7≤13, -25≤-19, 1≤7, -42≤-36, 6≤12, -15≤-9, -40≤-34, -26≤-20, -40≤-34, -13≤-7, -35≤-29)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
a_1	93	53	21	47	4	52	31	6	20	6	33	11
a_2	53	13	-19	7	-36	12	-9	-34	-20	-34	-7	-29
a_3	21	-19	-51	-25	-68	-20	-41	-66	-52	-66	-39	-61

Step 10- Row-3 ≤ Row-2, so remove Row-3,

($a_3 \leq a_2$: 21≤53, -19≤13, -51≤-19, -25≤7, -68≤-36, -20≤12, -41≤-9, -66≤-34, -52≤-20, -66≤-34, -39≤-7, -61≤-29)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
a_1	93	53	21	47	4	52	31	6	20	6	33	11
a_2	53	13	-19	7	-36	12	-9	-34	-20	-34	-7	-29

Step 11- Row-2 ≤ Row-1, so remove Row-2,

($a_2 \leq a_1$: 53≤93, 13≤53, -19≤21, 7≤47, -36≤4, 12≤52, -9≤31, -34≤6, -20≤20, -34≤6, -7≤33, -29≤11)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
a_1	93	53	21	47	4	52	31	6	20	6	33	11

Step 12- Column-12 ≥ Column-10, so remove Column-12. ($b_{12} \geq b_{10}$: 11≥6)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}
a_1	93	53	21	47	4	52	31	6	20	6	33

Step 13- Column-11 ≥ Column-10, so remove Column-11. ($b_{11} \geq b_{10}$: 33≥6)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}
a_1	93	53	21	47	4	52	31	6	20	6

Step 14- Column-10 ≥ Column-8, so remove Column-10. ($b_{10} \geq b_8$: 6≥6)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
a_1	93	53	21	47	4	52	31	6	20

Step 15- Column-9 ≥ Column-8, so remove Column-9. ($b_9 \geq b_8$: 20≥6)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
a_1	93	53	21	47	4	52	31	6

Step 16- Column-8 ≥ Column-5, so remove Column-8. ($b_8 \geq b_5$: 6≥4)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6	b_7
a_1	93	53	21	47	4	52	31

Step 17- Column-7 ≥ Column-5, so remove Column-7. ($b_7 \geq b_5$: 31≥4)

Player A \ Player B	b_1	b_2	b_3	b_4	b_5	b_6
a_1	93	53	21	47	4	52

Step 18- Column-6 ≥ Column-5, so remove Column-6. ($b_6 \geq b_5$: 52≥4)

		b_1	b_2	b_3	b_4	b_5
		Player B				
Player A	a_1	93	53	21	47	4

Step 19- Column-4 ≥ Column-5, so remove Column-4. ($b_4 \geq b_5$:47≥4)

		b_1	b_2	b_3	b_5
		Player B			
Player A	a_1	93	53	21	4

Step 20- Column-3 ≥ Column-4, so remove Column-3. ($b_3 \geq b_5$: 21≥4)

		b_1	b_2	b_5
		Player B		
Player A	a_1	93	53	4

Step 21- Column-2 ≥ Column-3, so remove Column-2. ($b_2 \geq b_5$: 53≥4)

		b_1	b_5
		Player B	
Player A	a_1	93	4

Step 22- Column-1 ≥ Column-2, so remove Column-1. ($b_1 \geq b_5$: 93≥4)

		b_5
		Player B
Player A	a_1	4

Hence the value of the game by the dominance rule is 4.

CHAPTER 4 RESULT, SUMMARY, AND CONCLUSION

4.1 RESULT

On the basis of above two methods, the saddle point is 4 since the upper value of the game is equal to the lower value, and also by the dominance rule, the value of the game is 4. This concludes that player A, the public school prefers the a_1 strategy, that is indicating the cost of education, and player B, the private school prefers the b_5 strategy, which represents facilities (Hygiene, Safety, infrastructure, etc.) provided by private schools. The value of the game is 4. Which concludes that people prefer public schools more than private schools as the value of the game is positive. To be specific, the game is in favor of public schools.

4.2 SUMMARY

This research work was based on people's preference for quality education between public and private schools, in which the survey was done with the help of a google form. A cross-sectional explanatory study together with a purposive sampling method to collect the appropriate data for this work. Data were gathered using a google form which was distributed among a small group of people and 133 responses were received. Questionnaires were executed to obtain people's preferences on quality education received and the reasons behind their favoured school between private and public schools. The received data were analyzed using a two-person-zero sum game. On the basis of this study, the minimax theorem is used to find the saddle point of the payoff matrix and also the dominance rule to reduce the size of the payoff matrix and find the value of the game. The result in both the method showed that the value of the game is= 4 and is in favor of public schools as we get positive value of the game. However, based

on the study we can conclude that public schools were favoured because of their less cost of education, and private schools were favoured because of the facilities (Hygiene, Safety, infrastructure, etc.) they provide.

4.3 CONCLUSION

However, there are factors such as the quality of education, teacher's qualification, academic curriculum, admission process, use of technology, and others, the most preferred factor by people is the cost of education for public schools and facilities (Hygiene, Safety, infrastructure, etc.) for private school. It was shown that public schools were favoured by more people because of the cost of education and private schools were favoured by people because of the facilities (Hygiene, Safety, infrastructure, etc.) provided by them. People preferred to prefer public schools more than private schools as the game has positive value.

APPENDIX 1 GOOGLE FORM

Public or private school prefrence survey

Let me know what you choose either a private school or a public school with respect to the factors mentioned.

* Required

1. Name *

2. Age *

Mark only one oval.

- ◯ below 18
- ◯ 18-25
- ◯ 25-45
- ◯ Above 45

3. Gender *

Mark only one oval.

- ◯ Male
- ◯ Female
- ◯ Other

4. Occupation *

Mark only one oval.

- ◯ Government Employee
- ◯ Non-government Employee
- ◯ Student
- ◯ Other

5. Yearly Income *

Mark only one oval.

- ◯ 0
- ◯ 0 -2 lakh
- ◯ 2 lakh - 6 lakh
- ◯ 6 lakh - 8 lakh
- ◯ > 8 lakh

6. What would you prefer considering following factors . *

Mark only one oval per row.

	Public School	Private School
Cost of Education	◯	◯
Teacher's Qualification	◯	◯
Quality of Education	◯	◯
Distance	◯	◯
Facilities(Hygiene,Safety,infrastructure etc.)	◯	◯
Admission Process	◯	◯
Accountability	◯	◯
Use of Technology	◯	◯
Academic curriculum	◯	◯
English Proficiency	◯	◯
Post School benefit	◯	◯
Extra curricular activity	◯	◯

APPENDIX 2 LIST OF PUBLICATION

PUBLICATION DETAILS:

JOURNAL NAME: International Journal of Special Education

ISSN: 0827-3383

MANAGER: Institute for Scientific and Engineering Research (ISER)

STATUS: ACCEPTED

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EVENT NAME: International Conference on Engineering & Technology, Computer, Basic & Applied Sciences (ICETCBAS-22)

EVENT PLACE: Kolkata, India

DATE: 23 April 2022

STATUS: CERTIFIED for presentation

MODE OF PRESENTATION: ONLINE

REFERENCES

[1] B. Grossberg, “Public vs. private schools: 5 major differences,” *ThoughtCo*, 10-Nov-2019. [Online]. Available: https://www.thoughtco.com/major-differences-between-public-and-private-2773898. [Accessed: 02-May-2022].

[2] E.N. Barron, Game Theory: An Introduction.2nd ed., Chicago: Wiley,2013.

[3] G. Bonanno, Game Theory.2nd ed., California: CreateSpace Independent,2018.

[4] *The importance of education in developing countries*. [Online]. Available: https://www.kudroli.org/blogs/the-importance-of-education-in-developing-countries#:~:text=The%20role%20of%20education%20in%20poverty%20reduction%20is%20huge.,as%20there%20are%20huge%20benefits. [Accessed: 02-May-2022].

[5] Pradeep, Siddhartha (2019) : Game theory, Strategies and the convoluted triangle - India, Pakistan, Kashmir, ZBW – Leibniz Information Centre for Economics, Kiel, Hamburg

[6] Scarvalone, Mira. “Game theory and the minimax theorem”,(2008).

[7] “Unified District Information System for Education (UDISE+)2019-20 Report,” *https://udiseplus.gov.in/#/home* , 2020. [Online]. Available: https://udiseplus.gov.in/assets/img/dcf2021/UDISE+2019_20_Booklet_English.pdf

[Accessed: 02-May-2022].

[8] V. U. Udeme and U. C. Orumie, “Patients’ preferences of healthcare facilities for quality healthcare services in Akwa Ibom State: A game theory approach,” *American Journal of Operations Research*, vol. 11, no. 03, pp. 181–198, 2021.