Dissertation Report

On

Optimal Portfolio Selection

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Certificate

I hereby declare that the work which is being presented in this project work entitled **"Optimal Portfolio Selection"** at Delhi School of Management, Delhi Technological University, New Delhi is an authentic record of my own work carried out during the period of January 2013 to April 2013 under the supervision of **Dr. Archana Singh.**

I am grateful to Prof. Nand Kumar for providing me guidance and support for the fulfilment of the project.

Dr. Archana Singh Saumya Sharma
(Project Mentor)

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Date:

Declaration

I, Saumya, 2K11/MBA/12, student of Delhi School of Management hereby declare that I have pursued a research study on the topic **"Optimal Portfolio Selection"** under the guidance of Dr. Archana Singh, Assistant Professor, Delhi School of Management. I also declare that this work has not been submitted in part or full to this or any other organization/a institute as part of any project work by me.

Acknowledgement

It is indeed a matter of great pleasure to present this project report on the topic **“Optimal Portfolio Selection”** to The Head of the Department, Delhi School of Management. I gratefully acknowledge my profound indebtedness towards my esteemed guide **Dr. Archana Singh, Assistant Professor, Delhi School of Management** for her invaluable guidance, excellent supervision and constant encouragement during the entire the project work. I also take the opportunity to thank **Mr. Nand Kumar, Assistant Professor, Humanities Deptt, Delhi Technological University** for being a constant support in deciding and reviewing the framework of the research study.

Last but not the least; I would like to express my heartfelt gratitude towards my parents for their constant encouragement & support.

Saumya Sharma

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Executive Summary

In this project, the modern portfolio theory (MPT) is written with a primary objective of showing how it aids an investor to classify, estimate, and control both the kind and the amount of expected risk and return in an attempt to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return. A methodology section is included which examined applicability of the theory to real time investment decisions relative to assumptions of the MPT and have plotted the graph . The theories that are used to analyse the problem and the empirical findings provide the essential concepts such as standard deviation, risk and return of the portfolio. Further, diversification, correlation and covariance are used to achieve the optimal risky portfolio. There will be a walk-through of the MPT, with the efficient frontier as the graphical guide to express the optimal risky portfolio.

This paper studies the 25 NIFTY stocks with large market capitalisation and small midcap over a period of 2 years. This study also includes the analysis of long term government bonds with maturity of 10-20 years. We develop a portfolio which allocates financial assets by maximising expected return subject to the constraint that the expected maximum loss should meet the risk limits set by the risk manager. The techniques used take into consideration the return and the risk of each asset in order to build the best portfolio. Three sets of portfolios are considered for investors with different investing styles i.e. which includes only stock, a mix of stocks and bonds in order to diversify the risk and the final one considers only government bonds which is suitable for those investors who would like to take a break from the fixed deposits but would still want a set certain amount of return on investments. The set of all efficient portfolios is called the efficient frontier.

All risk-averse investors who act to maximize expected utility have an optimal portfolio on this frontier. Based on the risk-aversion factor and the investment time horizon of each investor, portfolio optimization is carried for to maximize utility as well as return and minimize risk for all kinds of investors. Thereafter, we would need to plot and analyse these portfolios for three different sets and perform cluster mapping for the same for the same set of stocks and bonds. The findings of the study bring out the importance of the investor's investing pattern and style keeping in mind the utility and the risk aversion factor

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CHAPTER-1

INTRODUCTION

Stock Market efficiency is a matter of great interest for policy makers and investors for designing investment strategy. An efficient stock market fully reflects the available information pertaining to stocks. Accordingly, investors value the stocks taking into account the risk and return prospectus. Such conditions prevent investors to make abnormal return due to market inefficiencies. Fama (1991) described stock market efficiency and subsequent investors' preference for return subject to risk.

Substantial empirical works support risk-return tradeoff in developed stock markets. However, it is an area of great research in emerging stock markets like India.

In his seminal paper, Merton (1973) shows that the conditional expected excess return on the aggregate stock market is a linear function of its conditional variance plus a hedging component that captures the investor's motive to hedge for future investment opportunities. Merton (1980) indicates that the hedging component becomes negligible under certain conditions, and the equilibrium conditional expected excess return on the market can be approximated by a linear function of its conditional variance. This establishes the dynamic relation that investors require a larger risk premium at times when the stock market is riskier.

Despite the importance of the risk–return relationship and the apparent theoretical appeal of Merton's result, the empirical asset pricing literature has not yet reached an agreement on the existence of such a positive risk–return trade-off for stock market indices. Due to the fact that the conditional volatility of stock market returns is not observable, different approaches and specifications used by previous studies in estimating the conditional volatility are largely responsible for the conflicting empirical evidence.

Theory and - perhaps more importantly - financial common sense suggest that there should be a trade-off between a stock's riskiness and its expected returns. On the one hand, standard asset pricing models suggest that systematic risk should be positively rewarded, i.e. stocks with higher betas should earn a higher expected return (Ross's Arbitrage Pricing Theory, 1976). Subsequently, research has underlined the explanatory power of stock-specific or so-called idiosyncratic risk for expected returns (Merton, 1987). Taken together, these results suggest that total volatility, which is the model-free sum of systematic volatility explained by a factor model, and idiosyncratic volatility, should also be positively rewarded (Martellini, 2008).

In contrast to this consensus regarding the existence of an unambiguously positive risk-return relationship from a theoretical perspective, a number of older as well as more recent papers have reported a number of puzzling, or at least, contrasted findings from an empirical perspective. First, the “low beta anomaly” stipulates that the relationship between systematic risk as measured by a stock beta and return is much flatter than predicted by the CAPM (see early papers by Black, 1972, Black, Jensen, and Scholes, 1972) and sometimes even inverted (paper by Haugen and Heins, 1975). More recently, Ang, Hodrick, Xing, and Zhang (2006, 2009) have drawn new attention to these results with a focus on the specific risk component, finding that high idiosyncratic volatility stocks have had "abysmally low returns" in longer U.S. samples and in international markets. This result is now widely known as the “idiosyncratic volatility puzzle”. Yet other papers have documented a rather flat or even negative relationship between total (as opposed to specific) volatility and expected return, an anomaly that some call the “total volatility puzzle” (Haugen and Baker, 2008, Blitz and Van Vliet, 2007, Baker, Bradley, and Wurgler, 2011).

This paper examines the relationship between the risk & return of Bombay stock exchange stocks listed on SENSEX using the application of CAPM. Efficient capital market makes investors’ to realize higher risk premium by shifting from low risky portfolio to higher risky portfolio. The existence of efficient capital market makes investors to earn extra return with respect to bearing extra risk. The present study also attempts to check theory of risk & return in the light of present Indian Stock Market scenario.

The CAPM of Sharpe (1964), Lintner (1965) and Mossin (1966) is a widely used model in modern finance to estimate cost of equity and company performance. CAPM has received considerable attention in financial studies. In its simplest form, the CAPM predicts that the excess return of a stock should be proportional to the market premium. The proportionality factor is known as the ‘systematic risk’ or ‘beta’ of an asset. Early empirical studies on the CAPM such as Black et al. (1972) and Fama and MacBeth (1973) were supportive of the implications of the model. That is, the average return of high beta stocks was higher than the average return of low beta stocks. An early study by Levy (1977) showed that if the analyst used a shorter time horizon then, the beta estimates were biased. Fama (1980, 1981) provided evidence that the power of macroeconomic variables in explaining the stock prices increased with increasing time length. The relationship between the risks of an asset and its expected return serves two vital functions. First, it provides a benchmark rate of return for evaluating possible investments. Second, the model helps us to make an educated guess as to the expected return on assets that have not yet been traded in the market place.

The CAPM equation says that the expected return of any risky asset is a linear function of its tendency to co-vary with the market portfolio. Beta measures the tendency of an asset to co-vary with the market portfolio. It represents the part of the asset's risk that cannot be diversified away, and this is the risk that investors are compensated for bearing. So, if the CAPM is an accurate description of the way assets are priced, this positive linear relation should be observed when average portfolio returns are compared to portfolio betas. Further, when beta is included as an explanatory variable, no other variable should be able to explain cross-sectional differences in average returns. Beta should be all that matters in a CAPM world.

Being risk avert tendency of investors, they have homogenous expectations pertinent to stocks performance and earnings. Accordingly, they value the stocks taking into account risk and return preference. They will prefer risky portfolios with the expectation of extra returns from them.

Studies (Amanulla and Kamaiah, 1998; Altay, 2003; Dhankar and Singh, 2005; Dhankar and Rakesh, 2006; 2007) examine the linear relationship between portfolio beta and portfolio expected return, suggest a proportional relationship between portfolio return and risk for different time intervals. These studies support the linear structure of CAPM equation being a good explanation of security returns. However, Vaihekoski (2000) examines unconditional single and multi factors asset pricing model in Finnish Stock Market wherein a set of portfolios formulated on the basis of market capitalization and industry. The study reports that single factor CAPM provides less explanatory variation in stocks returns, i.e., relationship between risk and return is weak compared to international capital asset pricing model for both size and industry portfolios. Some extensions of the basic CAPM were proposed that relaxed one or more of the assumptions of CAPM (Black, 1972). Instead of simply extending an existing theory, Ross (1976a, 1976b) addresses this concern by developing a completely different model: the Arbitrage Pricing Theory (APT).

Unlike the CAPM, which is a model of financial market equilibrium, the APT starts with the premise that arbitrage opportunities should not be present in efficient financial markets. The APT starts by assuming that there are n factors which cause asset returns to systematically deviate from their expected values. Ross shows that, in order to prevent arbitrage, an asset's expected return must be a linear function of its sensitivity to the n common factors. Further, *Badhani (2007) examines* Intertemporal Capital Asset Pricing Model (ICAPM) which postulates a positive relationship between time-varying conditional risk and conditional return on securities. However, unconditional volatility and returns in two switching regimes are found negatively related. There is strong evidence that volatility increases disproportionately with negative shocks in stock returns. Mohamed (2007), further, examines the CAPM and Fama-French Three Factor Model (FFTFM) in Indian stock market, wherein 200 stocks returns, book values, and market prices are examined. It applies cross sectional regression analysis and evaluation of factor sensitivities to returns in FFTFM. The findings report that FFTFM is more precise in describing returns. It also reports that Indian investors' size factor has no great influence on cross-sectional random stock returns, but the investors are influenced more by the value factor. Studies (Caporale and Gil Alana 2002; Jarrett and Kyper 2005a) found the unit root in stock returns thereby holds seasonal pattern in stock returns. Many empirical research work which investigate the seasonal patterns in stock returns in developed stock exchanges also question efficient market hypothesis and suggest seasonal pattern in these stock markets (Black and Fraser 1995; Clare et al. 1995; Pesaran and Timmermann 1995; Moorkejee and Yu 1999; Caporale and Gil-Alana 2002; Rothlein and Jarrett 2002; Jarrett and Kyper 2005b).

The main objective of the research is to study the relationship between risk and return and selecting the portfolio based on the risk-return trade-off.

Recent research in empirical finance has documented that expected excess returns on stocks and risk shift over time in predictable ways. Furthermore, these shifts tend to persist over long periods of time. A cornerstone in finance theory continues to be the positive relationship between risk and return. Many researchers have done empirical study to find the relationship between risk and return. Still this study remains a controversial one. Bowman (Sloan Management Review 1980, pp. 17–31) studied the relationship from organization theory, and developed a whole research stream known as "Bowman's paradox". This paradox arises from the persistent showing that risk and return are negatively related—i.e., high risk is associated with low return and low risk is associated with high return. This paradox contradicts the view that higher returns are associated with the risk premium.

In this paper, numerous stocks listed on SENSEX have been selected for the purpose of analysis. The risk-return trade-off, based on the selected stocks, was being analysed. Using the stocks, portfolios were constructed. Then risks and returns of various portfolios were calculated. This paper tries to develop the relationship between the risk and return of a portfolio. Also there are various methods to measure the risk of a portfolio. Each of the method comes with its own advantages and disadvantages. The expected return of the stocks is being measured using CAPM. The expected return of the portfolio is given by the weighted average of the expected returns of the stocks in the portfolio. In this paper, an attempt has been made in selecting the portfolio based on the risk-return trade-off.

There is no evidence listing the presence of a positive risk-return tradeoff. There have been various contradicting studies to develop the relationship between risk and return. But in investors' perspective, the analysis of risk is very important as their investment decisions are highly influenced by the degree of risk associated with the investment. For investors, risk is about the odds of losing money. Nowadays, investors are more concerned with a value associated with a particular investment. This value is very well defined by the amount of risk associated with that investment.

The scope of the study is restricted on the measurement of risk based on beta estimation and mean-variance model. Beta gives the systematic risk of the portfolio. It defines the sensitivity of the return of the stocks in the portfolio to the market return. The risk using beta estimation is non-diversifiable. On the other hand, the weighted average risk gives the total risk of the portfolio. It is the non-systematic risk of the portfolio. Investors' can reduce the non-systematic risk through diversification as well.

1.1 Modern Portfolio Theory

Understanding the risky behaviour of asset and their pricing in the market is critical to various investment decisions, is it related to financial assets or real assets. This understanding is mostly developed through the analysis and generalization of the behaviour of individual investors in the market under certain assumptions. The two building blocks of this analysis and generalization are (i) theory about the risk-return characteristics of assets in a portfolio (portfolio theory) and (ii) generalization about the preferences of investors buying and selling risky assets (equilibrium models).

One of the most important and influential economic theories dealing with finance and investment is the Modern Portfolio Theory. MPT says that it is not enough to look at the expected risk and return of one particular stock. By investing in more than one stock, an investor can reap the benefits of diversification - chief among them, a reduction in the riskiness of the portfolio. MPT quantifies the benefits of diversification, also known as not putting all of your eggs in one basket.

For most investors, the risk they take when they buy a stock is that the return will be lower than expected. In other words, it is the deviation from the average return. Each stock has its own standard deviation from the mean, which MPT calls "risk". The risk in a portfolio of diverse individual stocks will be less than the risk inherent in holding any one of the individual stocks (provided the risks of the various stocks are not directly related). Consider a portfolio that holds two risky stocks: one that pays off when it rains and another that pays off when it doesn't rain. A portfolio that contains both assets will always pay off, regardless of whether it rains or shines. Adding one risky asset to another can reduce the overall risk of an all-weather portfolio.

Three Fundamental Assertions of the Portfolio Theory

- Investors seek to maximize utility.
- Investors are risk averse: Utility rises with expected return and falls with an increase in volatility.
- The optimal portfolio has the highest expected return for a given level of risk, or the lowest level of risk for a given expected return.

1.2 Capital Asset Pricing Model

The capital asset pricing model (CAPM) describes the relationship between risk and expected return, and it serves as a model for the pricing of risky securities.

CAPM says that the expected return of a security or a portfolio equals the rate on a risk-free security plus a risk premium. If this expected return does not meet or beat our required return, the investment should not be undertaken.

The Formula

Sharpe found that the return on an individual stock, or a portfolio of stocks, should equal its cost of capital. The standard formula remains the CAPM, which describes the relationship between risk and expected return.

Here is the formula:

$R_p = R_f + \beta (R_m - R_f)$

where,

R_p is the portfolio return or expected return,

R_f is the risk-free return which is 6%,

R_m is the market return which is -12%,

β is the market sensitivity index of individual security

CAPM's starting point is the risk-free rate - typically a 10-year government bond yield. To this is added a premium that equity investors demand to compensate them for the extra risk they accept. This equity market premium consists of the expected return from the market as a whole less the risk-free rate of return. The equity risk premium is multiplied by a coefficient that Sharpe called "beta."

According to CAPM, beta is the only relevant measure of a stock's risk. It measures a stock's relative volatility - that is, it shows how much the price of a particular stock jumps up and down compared with how much the stock market as a whole jumps up and down.

This model presents a very simple theory that delivers a simple result. The theory says that the only reason an investor should earn more, on average, by investing in one stock rather than another is that one stock is riskier.

1.3 Portfolio Selection

The method used in selecting the most desirable portfolio involves the use of *Markowitz Theory*. These curves represent an investor's preferences for risk and return. It can be drawn on a two-dimensional graph, where the horizontal axis usually indicates risk as measured by variance or standard deviation and the vertical axis indicates reward as measured by expected return. Using variance as relevant risk measure comes from Markowitz's paper and is always used in practice, although other possibilities have been considered. This definition gives us the following properties, assuming we have a 'rational investor':

- All portfolios that lie on the same curve are equally desirable to the investor (even though they have different expected returns and variance.)
- An investor will find any portfolio that is lying on a curve that is "further northwest" to be more desirable than any portfolio lying on a curve that is "not as far northwest."

Generally it is assumed that investors are *risk averse*, which means that the investor will choose the portfolio with the smaller variance given the same return. Risk averse investors will not want to take fair gambles (where the expected payoff is zero). These two assumptions of risk aversion cause indifference curves to be positively sloped and convex.

Now that we know about indifference curves and risk aversion, how can we use that to select from an almost infinite number of portfolios available for investment? The key lies in the *efficient set theorem*, which states that an investor will choose a portfolio from the set of portfolios that:

1. Offer maximum expected return for varying levels of risk, and
2. Offer minimum risk for varying levels of expected return.

We begin by constructing the *feasible set*, which represents all portfolios that could be formed from a group of *N* securities. The efficient set can now be located by applying the efficient set theorem to this feasible set. This demonstrates that all the portfolios in the efficient set are located on the "northwest" boundary of the feasible set, often called the *efficient frontier*. Selecting a portfolio is henceforth easy, by simply plotting the investor's indifference curves on the same figure as the efficient set and then proceeds to choose the portfolio that is on the indifference curve that is "furthest northwest."

The investing pattern of an investor is one of the most important factor as it defines the risk quotient that is associated with different investors. Studies have shown in the past that investors can be classified into three broad categories.

1. Risk Averse
2. Risk Neutral
3. Risk Taker

Risk Averse are the type of investors who have a fixed risk factor and are expecting low return since the risk is low. Risk Neutral are the investors who are neither too conservative about their risk factor nor have the expectation of the highest of return. Risk Taker are the investors who expect a very high return thereby increasing their risk coefficient.

<u>CHAPTER-2</u>

LITERATURE REVIEW

There had been extensive theoretical and empirical studies on asset pricing model, which trying to establish factors that contribute to the expected return of capital asset. These studies contributed towards the development and improvement of the models to explain pricing of capital asset under an equilibrium market.

Edwin J. Elton and Martin J. Gruber (1997), divided the paper into 4 sections. The first section presented a historical review of the basic theory and its current state of development. The second section included issues in estimating the key inputs for portfolio theory. The third one discussed the special issues that arise when portfolio theory is applied to financial institutions. Final section reviewed portfolio evaluation techniques.

Eugene F. Fama (1970), reviewed the theoretical and empirical literature on the efficient markets model. Empirical work concerned with the adjustment of security prices to three relevant information subsets, i.e., weak form test, strong form test & strong form test, were considered.

Holbrook Working (1934), obtained a series by cumulating random numbers for brevity and clarity called a random difference series, since it is the first differences of the series itself which are random. Brief study of the charts showed that in a series composed of purely random changes conspicuous trends will be found.

James Tobin (1958), added money to Markowitz's story and thus obtain the famous "two-fund separation theorem". Effectively, Tobin argued that agents would diversify their savings between a risk-free asset (money) and a *single* portfolio of risky assets (which would be the same for everyone). Different attitudes towards risk, Tobin contended, would merely result in different combinations of money and that unique portfolio of risky assets.

Eugene F. Fama; Lawrence Fisher; Michael C. Jensen; Richard Roll (1969), study attempted to examine evidence on two related questions: (1) Is there normally some "unusual" behavior in the rates of return on a split security in the months surrounding the split? and **(2)** if splits are associated with "unusual" behavior of security returns, to what extent can this be accounted for by relationships between splits and changes in other more fundamental variables?

Paul Milgrom & Nancy Stokey(1982), showed that when risk averse traders begin at a Pareto Optimum(relative to their prior beliefs) and then receive private information(which disturbs the marginal conditions), they can never agree to any non-null trade. This result had implications for the nature of the information transferred among agents in voluntary exchange.

Pete Swisher and Gregory W. Kasten (2005), Post-modern portfolio theory (PMPT) presented a new method of asset allocation that optimizes a portfolio based on returns versus downside risk (downside risk optimization, or DRO) instead of MVO. The core innovation of PMPT was its recognition that standard deviation is a poor proxy for how humans experience risk. PMPT pointed the way to an improved science of investing that incorporates not only DRO but also behavioral finance and any other innovation that leads to better outcomes.

Harry Markowitz (1952), This paper was concerned with the relevant beliefs about future performances and ended with the choice of portfolio. He considered the rule that the investor does (or should) maximize discounted expected, or anticipated, returns. This rule is rejected both as a hypothesis to explain, and as a maximum to guide investment behavior.

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Objective

The objective of this study is to determine the risk return trade off and construct a portfolio with a fair mix of stocks and bonds focussing on the different types of investors and identifying the portfolio for the same. The scope of this study includes:

- Calculating risk coefficient of the portfolio of 25 NIFTY Stocks and 25 G-Sec Bonds
- Creating portfolios with different weightage to stocks and bonds and plotting the efficient frontier curve to study the risk-return pattern followed
- Selection of portfolio depending upon the type of investors and their risk quotient.

CHAPTER-3

RESEARCH METHODOLOGY

3.1 Sample Selection

Consider the feasible set of investment alternatives consisting of all portfolios mixing long-term GSEC bonds and NIFTY stocks. It is assumed that leverage and short sales are not allowed, so each portfolio consists of some percentage x of bonds and 100-x of stocks where $0 < x < 100$.

The data set consists of monthly adjusted closing prices of the market (NIFTY), stock (25 stocks of NIFTY), and risk-free rate of return (from MIBOR) for the period from February 2003 through January 2013 (viz. 10 years).

Long-term GSEC bonds are the Government Securities with years of maturity ranging from 15 years to 25 years. Such bonds are considered to be the risk-free assets. So, using GSEC bonds in portfolio construction can help in producing the portfolios having a mix of risk-free assets and risky assets.

In addition, the choice of the optimal portfolio is based on the investors' risk-aversion factor. In this paper, it is assumed that three types of investors exist in the market. One with low risk-aversion factor, i.e. investors' who can bear more risk for higher expected returns. The second category of investors includes the one with high risk-aversion factor, i.e. investors' who are not willing to take large amounts of risk. The last category is risk neutral investor with a set risk taking ability and constant decent return.

3.2 Data Source

The data has been taken from the website of NSE India. This research is based on the research of secondary data considering 25 NIFTY stocks based on a mix of high market capitalisation and mid cap stocks from various industries. The sample period exhibits a mixed set of economic environment in Indian economy. NIFTY, which covers all industry categories stocks, is value weighted index, and assigns weights to all stocks in proportion to the share of their market capitalization. The market return is tracked for 20 years showing all trends of a market and deriving the optimum market return for efficient portfolio creation.

3.3 Calculation of Variables

3.3.1 Stocks

Calculating Expected Return:

Market Returns are the returns that the investors generate out of the stock market. The return of market is calculated using the relative change in the prices between the two time periods. Let P_t is the price of index in time period t, P_{t-1} is the price of index in preceding time period t-1. The percentage market rate of return, R_m that investors will realize in t time period can be calculated as follows.

Symbolically, it can be written as:

$$R_m = ((P_t - P_{t-1})/P_{t-1})*100 \quad (1)$$

Where

R_m = Market rate of return

P_t = Price of Index in time period t

P_{t-1} = Price of index in preceding time period t

A stock's market price is a function of the market's perception of the value of the future profits a company can create. The method used for calculating the percentage **return on stocks** is same as that of the market rate of return.

Symbolically, it can be written as:

$$R_s = ((P_t - P_{t-1})/P_{t-1})*100 \quad (2)$$

Where,

R_s = Percentage return on a stock

P_t = Closing price of the stock in time period t

P_{t-1} = closing price of the stock in time period t-1.

Beta (β) of a stock or portfolio is a number describing the correlation of its returns with those of the financial market as a whole. Beta is a measure of the volatility, or systematic risk, of a security or a portfolio in comparison to the market. Beta is used in the capital asset pricing model (CAPM), a model that calculates the expected return of an asset based on its beta and expected market returns. Beta is calculated using regression analysis. The calculation of beta through regression is simply the covariance of Stock return and Market Return divided by the variance of the market return.

Symbolically:

$$\beta = [Cov\ (R_s, R_m)]/\sigma_m^2) \qquad (3)$$

Where

β = Beta of Stock or Portfolio

Cov = Covariance

R_s = Return of Stock

R_m = Market Return

σ_m^2 = Variance of market return

It measures the part of the asset's statistical variance that cannot be removed by the diversification provided by the portfolio of many risky assets, because of the correlation of its returns with the returns of the other assets that are in the portfolio.

The capital asset pricing model (CAPM) is used to determine a theoretically appropriate required rate of return of an asset, if that asset is to be added to an already well-diversified portfolio. The model takes into account the asset's sensitivity to non-diversifiable risk (also known as systematic risk or market risk), often represented by the quantity beta (β) in the financial industry, as well as the expected return of the market and the expected return of a theoretical risk-free asset.

The model was introduced by Jack Treynor (1961, 1962), William Sharpe (1964), John Lintner (1965a, b) and Jan Mossin (1966) independently, building on the earlier work of Harry Markowitz on diversification and modern portfolio theory.

An efficient capital market provides investors increasing return for increasing risk. The CAPM is a model for pricing an individual security or a portfolio.

The CAPM equation for the Security Market Line is given as

$$E(R_i) = R_f + \beta_i * [R_m - R_f] \quad (4)$$

Where

$E(R_i)$ = the expected return on security

R_f = the risk-free rate

β_i = the systematic risk

$E(R_m)$ = the expected return on market portfolio

The general idea behind CAPM is that investors need to be compensated in two ways: time value of money and risk. The time value of money is represented by the risk-free (R_f) rate in the formula and compensates the investors for placing money in any investment over a period of time. The other half of the formula represents risk and calculates the amount of compensation the investor needs for taking on additional risk. This is calculated by taking a risk measure (beta) that compares the returns of the asset to the market over a period of time and to the market premium (R_m-R_f).

Calculating Risk:

Risk is the potential that a chosen action or activity (including the choice of inaction) will lead tp a loss (an undesirable outcome). All stocks are subject to two forms of risk – systematic and non-systematic. Risk reflects the chance that the actual return on an investment may be very different than the expected returns.

The standard deviation is often used by investors to measure the risk of a stock or a stock portfolio. The basic idea is that the standard deviation is a measure of volatility the more a stock's returns vary from the stock's average return, the more volatile the stock

Symbolically it can be written as,

$$\sigma = \sqrt{\frac{\sum(\bar{x} - x)^2}{n - 1}}$$

Where,

σ = standard deviation

X = expected return

n = number of observations

3.3.2 Bonds

Calculating Expected Return:

Return on bonds is calculated using Yield to Maturity (YTM). The Yield to Maturity (YTM) is the internal rate of return (IRR, overall interest rate) earned by an investor who buys the bond today at the market price, assuming that the bond will be held till maturity, and that all coupon and principal payments will be made on schedule.

Symbolically,

YTM = Rate (NPER, PMT, PV, FV)

Where,

YTM = Yield to Maturity

NPER = Total number of payments

PVMT = Payment made each period

PV = Present Value

FV = Future Value

Calculation of Risk:

Risk for bonds is calculated in the same manner as the calculation of risk of stocks. Both risk for bonds and risk for stocks use standard deviation as a measure of risk

Symbolically it can be written as,

$$\sigma = \sqrt{\frac{\sum(\bar{x} - x)^2}{n - 1}}$$

Where,

σ = standard deviation

X = expected return

n = number of observations

3.3.3. Portfoliio

The term portfolio refers to any collection of financial assets such as stocks, bonds and cash. In this paper, 21 feasible portfolios are constructed using the mix of stocks and bonds. The most conservative portfolio consists of 0% stocks and 100% bonds whereas the most aggressive portfolio consists of 100% stocks and 0% bonds. In between the conservative and aggressive portfolio, the portfolios are assigned weights like 5% stocks and 95% bonds, 10% stocks and 90% bonds and so on.

Calculating Expected Return:

As per Modern Portfolio Theory, Portfolio Return is the proportion-weighted combination of the constituent assets' returns. Portfolio Return is the monetary return experienced by a holder of a portfolio. Portfolio returns are calculated using the weighted average method where the portfolios are assigned weights based on the composition of stocks and bonds.

Symbolically it can be calculated as,

$$R_P = W_S R_S + W_B R_B$$

Where,

R_P = Portfolio Return

W_S = Weight assigned to stocks

R_S = Return on Stocks

W_B = Weights assigned to bonds

R_B = Return on bonds

Calculating Risk:

The risk indicates that σ of a portfolio return depends not only on the standard deviation of the returns of the individual assets themselves but also on how common factors affect them via the covariance of returns. If a common factor, like interest rates, affects all assets similarly, covariance rises and so will the portfolio risk.

Portfolio risk depends upon:

- The proportion of funds invested in each stock.
- The standard deviation of each stock.
- The covariance between the two stocks.

Symbolically,

$$\sigma_P = W_S\sigma_S^2 + W_B\sigma_B^2 + 2W_SW_BCOV_{SB}$$

where,

σ_P = Portfolio risk

W_S = Weight given to stocks

σ_S^2 = Variance of stocks

W_B = Weight assigned to bonds

σ_B^2 = Variance of bonds

COV_{SB} = Covariance between bond and sock

3.3.4 Efficient Frontier

A portfolio is referred to as "efficient" if it has the best possible expected level of return for its level of risk. Here, every possible combination of risky assets, without including any holdings of the risk free asset, can be plotted in risk-expected return space, and the collection of all such possible portfolios defines a region in this space. The efficient frontier is the positively sloped portion of the opportunity set that offers the highest expected return for a given level of risk.

CHAPTER-4

DATA ANALYSIS AND INTERPRETATION

4.1 Efficient Portfolio Construction

The feasible set of stocks and bonds plots as a curve between the risk and the expected return. The most conservative portfolio, 100% bonds, has both lower expected return and lower risk than does the most aggressive portfolio, 100% stocks.

The portfolio with smallest standard deviation (equivalently, the smallest variance), is 100% bonds and 0% stocks. The portfolios which are more conservative than this minimum efficient portfolio are inefficient, since the minimum variance portfolio has both a greater expected return and a smaller standard deviation. This phenomenon and the general shape of the curve are typical when a more conservative asset is mixed with a more risky asset. All of the portfolios which are more aggressive than the minimum variance portfolio are efficient.

In the table, the different weights assigned to the stock and bonds to construct 21 set of portfolios are listed. As the Portfolio Theory states, risk and return are directly proportional and increase or decrease in tandem. In the set considered in this paper, the most conservative portfolio (0% stocks, 100% bonds) has a portfolio return of 8.2116% and the 1-year portfolio risk of 0.5011% as shown in Table 4.1. On the other hand, the most aggressive portfolio (100% stocks, 0% bonds) has a portfolio return of 19.662% return and 1-year portfolio risk of 2.013%

The portfolio risk is being calculated using the mean-variance model. When two or more securities are combined in the portfolio there is need to consider interactive risk or covariance. In this case, portfolio risk depends upon the proportion of funds invested in each stock, the standard deviation of each stock, and the covariance between the two stocks. Symbolically, portfolio risk can be written as:

$$\sigma^2 = \sum\sum w_i w_j Co\text{var}inace_{ij}$$

$$\sigma^2 = \sum\sum w_i w_j r_{ij} \sigma_i \sigma_j$$

Also, the portfolio return is the weighted average of the expected rate of return of the individual stocks in the portfolio. Symbolically, portfolio return can be obtained as:

$$E\ (R_p) = \sum wi^* E(R_i)$$

Table 4.1 Portfolio Risk and Return

Portfolio No.	Percentage of Stocks	Percentage of Bonds	Portfolio Return	Portfolio Risk
1	0%	100%	8.2116%	0.5011%
2	5%	95%	8.7841%	0.5036%
3	10%	90%	9.3566%	0.5252%
4	15%	85%	9.9291%	0.5638%
5	20%	80%	10.5016%	0.6161%
6	25%	75%	11.0742%	0.6790%
7	30%	70%	11.6467%	0.7498%
8	35%	65%	12.2192%	0.8265%
9	40%	60%	12.7917%	0.9077%
10	45%	55%	13.3642%	0.9921%
11	50%	50%	13.9368%	1.0791%
12	55%	45%	14.5093%	1.1681%
13	60%	40%	15.0818%	1.2586%
14	65%	35%	15.6543%	1.3504%
15	70%	30%	16.2268%	1.4431%
16	75%	25%	16.7994%	1.5367%
17	80%	20%	17.3719%	1.6310%
18	85%	15%	17.9444%	1.7258%
19	90%	10%	18.5169%	1.8212%
20	95%	5%	19.0894%	1.9169%
21	100%	0%	19.6620%	2.0130%

4.2 Efficient Frontier

The set of all efficient portfolios is called the efficient frontier. In general, it consists of the portfolios which lie on the northwest boundary of the feasible set. All risk averse investors who act to maximize expected utility have an optimal portfolio on this frontier.

Figure 5.1 below shows the curve efficient frontier. The slope of the Efficient Frontier at any point depicts how much extra expected return is obtained by taking some more risk. This is called the Return/Risk Trade-off.

Return/Risk Trade-off = Change in R_P / Change in σ_P

The Trade-off between Return and Risk of this relationship is the Slope of the curve. This is called Sharpe Ratio (S). It is a measure of Risk Adjusted Return.

Figure 4.1

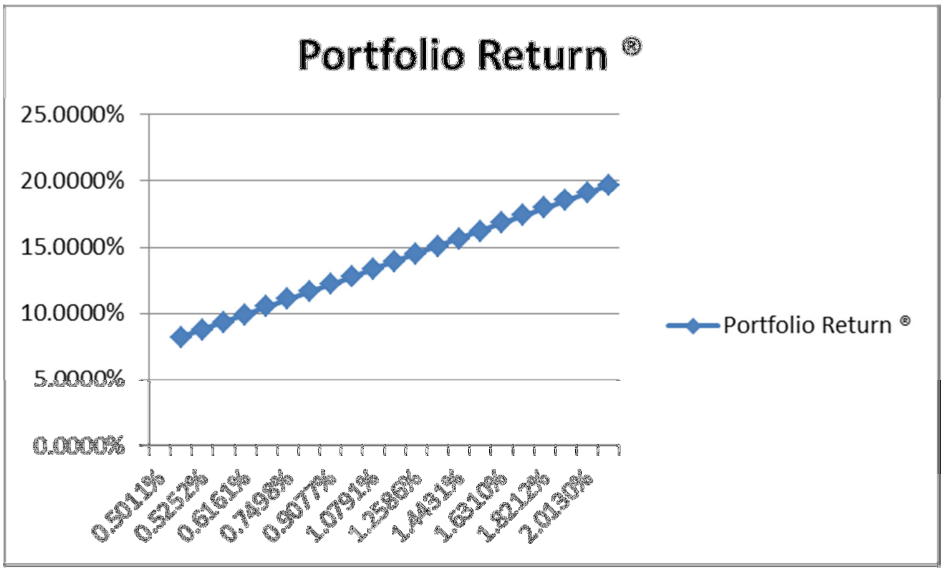

Figure 4.1 clearly shows that there is a direct relationship between the portfolio risk and the portfolio return. This means that with the increasing value of the portfolio return, the portfolio risk is will also increase.

CHAPTER-5

FINDINGS AND DISCUSSION

5.1 Findings

Table I shows the positive relationship between the portfolio return and the portfolio risk. Thus, this result is in accordance with the Portfolio Theory that the investor attempts to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return, by carefully choosing the proportions of various assets.

The Slope of the Efficient Frontier at any point depicts how much extra expected return is obtained by taking some more risk. The curve efficient frontier also shows that for taking any extra amount of risk the investor must be rewarded with a higher return.

Based on the risk-aversion factor, A More risk-averse investors prefer the investment which has lower risk, while less risk-averse investors prefer the investment with a higher expected return for very small values of the coefficient of risk aversion A (near zero), the investor is primarily concerned with maximizing expected return, and has little concern for risk. Conversely, for very large values of A, the investor is primarily concerned with minimizing risk.

5.2 Discussion/Recommendation

From the Analysis and Portfolio creation and plotting the efficient frontier, we recommend following portfolio to the different kinds of investors.

The risk averse investor should invest in Portfolio 1 and 2 with minimum weightage given to stocks and more focus towards bonds. This gives 8% to 9% return with the lowest possible risk.

The risk neutral investor should put in close to equal weightage on both stocks and bonds. This will give them 13% to 15% returns with 1% to 1.5% risk.

The investors who have low risk aversion i.e. the risk takers should prefer investing more on stocks then on bonds as it will give them 18% to 19% returns

CHAPTER-6

CONCLUSION AND IMPLICATIONS

6.1 Conclusion

This paper attempts to select an optimal portfolio for an investor based on the risk-aversion factor and the investment time horizon of that particular investor. The study reports important implications for investors and policy makers. This paper tries to establish a relationship between the risk and return of a portfolio. Also, based on the risk-return trade-off an attempt is being made in selecting the optimal portfolio.

The set of all efficient portfolios is called the efficient frontier. All risk-averse investors who act to maximize expected utility have an optimal portfolio on this frontier. Given a utility function for an individual investor, the portfolio optimization problem is to find the indifference curve which is tangent to the efficient frontier. The optimal portfolio for the investor is the one located at the tangency point.

In this paper it is clearly shown that no Rational Investor will invest in any portfolio unless its utility exceeds the risk free rate. Investor will not opt for risky portfolios unless their returns exceed the risk free rate by an amount that is sufficient to overcome the risk scaled by a factor related to his risk-aversion factor. In addition, the choice of the optimal portfolio is based on the investors' risk-aversion factor and the investment time horizon.

With the investment period of the investor increasing from 1-year to 5-years, the risk taking capacity of the investor also increases. The choice of the optimal portfolio changed from 80% bond/20% stock portfolio to 55% bond/45% stock portfolio.

More risk-averse investors prefer the investment which has lower risk, while less risk-averse investors prefer the investment with a higher expected return. The more risk-averse an investor is, the lower will be the optimal portfolio on the return/risk spectrum defined by the efficient frontier for very small values. The investor is primarily concerned with maximizing expected return, and has little concern for risk.

6.2 Implication

The purpose of this study was to cater to the different investors and how they can maximize their returns. The study was based on Markowitz theory which explains the relationship of risk and return and the trade-off between them.

However, there are many theories that explain optimal portfolio selection; this is considered to be the most implemented theories. This analysis will be of help to the Fund Managers of various mutual funds, Research Analysts and investors.

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ANNEXURES

Annexure I

Stocks	SD	R_f	COV	beta	R_m	E(r)
ACC	0.0156	8.00%	17.7031	0.79	14.1900000%	19.08%
Asian Paints	0.0142	8.00%	35.2108	0.66	14.1900000%	18.28%
BHARTIARTL	0.0202	8.00%	-5.7462	0.82	14.1900000%	19.27%
BHEL	0.0424	8.00%	-90.4072	1.47	14.1900000%	23.29%
BPCL	0.0290	8.00%	-10.1528	0.63	14.1900000%	18.09%
Cairn	0.0187	8.00%	2.2304	0.65	14.1900000%	18.21%
Cipla	0.0147	8.00%	2.5492	0.54	14.1900000%	17.53%
COALIND	0.0172	8.00%	-1.7407	0.54	14.1900000%	17.53%
DLF	0.0273	8.00%	-5.2218	1.68	14.1900000%	24.59%
DR Reddy	0.0129	8.00%	8.1585	0.28	14.1900000%	15.92%
HCL	0.0129	8.00%	3.7962	0.28	14.1900000%	15.92%
HDFC Bank	0.0385	8.00%	-64.1436	1	14.1900000%	20.38%
HUL	0.0164	8.00%	9.5137	0.47	14.1900000%	17.10%
ICICI	0.0203	8.00%	-14.4683	1.65	14.1900000%	24.40%
INFY	0.0206	8.00%	9.7619	0.77	14.1900000%	18.96%
ITC	0.0137	8.00%	5.0225	0.65	14.1900000%	18.21%
Jindal Steel	0.0240	8.00%	-14.5216	1.58	14.1900000%	23.97%
M&M	0.0174	8.00%	-12.1803	0.89	14.1900000%	19.70%
Maruti	0.0191	8.00%	5.7782	0.91	14.1900000%	19.82%
NTPC	0.0148	8.00%	0.1315	0.65	14.1900000%	18.21%
ONGC	0.0170	8.00%	1.8175	0.93	14.1900000%	19.95%
REL	0.0176	8.00%	-8.6724	1.01	14.1900000%	20.44%
SBI	0.0204	8.00%	9.6868	1.49	14.1900000%	23.41%
TCS	0.0171	8.00%	9.6589	0.36	14.1900000%	16.42%
Tata Steel	0.0212	8.00%	-10.0351	1.4	14.1900000%	22.86%
Average SD of Stocks	**0.0201**				**Average Portfolio Return**	**19.66%**

Annexure II

S.N	Security Name	Coupon	Maturity Date	Market Price	FV	Coupon Interest	Maturity Period	PV	YTM	SD
1	8.40% GSEC 29-03-2026 (OIL BOND)	8.40%	29-Mar-26	97.92	100	8.40	9	-97.92	8.7433%	0.002021
2	8.00% GoI Sec OIL SPECIAL Bonds23-03-2026	8.00%	23-Mar-26	101.22	100	8.00	8	-101.22	7.7894%	0.004724
3	10.18% GoI Sec11-09-2026	10.18%	11-Sep-26	132.84	100	10.18	11	-132.84	6.0133%	0.017283
4	6.01% GoI Sec 25-03-2028	6.01%	25-Mar-28	77.13	100	6.01	7	-77.13	10.8380%	0.016832
5	7.40% GoI Sec 09-09-2035	7.44%	09-Sep-35	87.92	100	7.44	8	-87.92	9.6778%	0.008629
6	7.40% GoI Sec 09-09-2035	7.40%	09-Sep-35	94.45	100	7.40	8	-94.45	8.3797%	0.00055
7	6.01% GoI Sec 25-03-2028	6.01%	25-Mar-28	77.13	100	6.01	7	-77.13	10.8380%	0.016832
8	6.01% GoI Sec 25-03-2028	6.01%	25-Mar-28	86.04	100	6.01	7	-86.04	8.7616%	0.00215
9	7.40% GoI Sec 09-09-2035	7.40%	09-Sep-35	87.92	100	7.40	8	-87.92	9.6343%	0.008321
10	6.01% GoI Sec 25-03-2028	6.01%	25-Mar-28	86.04	100	6.01	7	-86.04	8.7616%	0.00215
11	8.00% GoI Sec OIL SPECIAL Bonds23-03-2026	8.00%	23-Mar-26	101.23	100	8.00	8	-101.23	7.7877%	0.004736
12	6.01% GoI Sec 25-03-2028	6.01%	25-Mar-28	86.04	100	6.01	7	-86.04	8.7616%	0.00215
13	10.18% GoI Sec11-09-2026	10.18%	11-Sep-26	121.35	100	10.18	11	-121.35	7.2914%	0.008246
14	8.40% GSEC 29-03-2026 (OIL BOND)	8.40%	29-Mar-26	97.96	100	8.40	9	-97.96	8.7366%	0.001974
15	8.24% GoI Sec 15-02-2027	8.24%	15-Feb-27	105.57	100	8.24	30	-105.57	7.7566%	0.004957
16	7.50% GoI SEC 10-08-2034.	7.50%	10-Aug-34	98.62	100	7.50	44	-98.62	7.6093%	0.005997
17	7.95% GoI Sec 28-08-2032	7.95%	28-Aug-32	100.88	100	7.95	40	-100.88	7.8772%	0.004104
18	7.95% NATIONAL FERTILIZERS GOI SPECIAL BONDS 2026	7.95%	18-Feb-26	95.62	100	7.95	28	-95.62	8.3594%	0.000694
19	8.32% GOI SEC 02-08-2032	8.32%	02-Aug-32	97.74	100	8.32	40	-97.74	8.5202%	0.000443
20	8.32% GOI SEC 02-08-2032	8.32%	02-Aug-32	97.74	100	8.32	40	-97.74	8.5202%	0.000443
21	6.13% GoI Sec 04-06-2028	6.13%	04-Jun-28	86.57	100	6.13	32	-86.57	7.2159%	0.00878
22	8.26% GoI Sec 02-08-2027	8.26%	02-Aug-27	97.95	100	8.26	30	-97.95	8.4499%	5.4E-05
23	8.33% GOI SEC 07-06-2036	8.33%	07-Jun-36	101.99	100	8.33	48	-101.99	8.1637%	0.002078
24	8.28% GoI Sec 15-02-2032	8.28%	15-Feb-32	96.9	100	8.28	40	-96.9	8.5556%	0.000693
25	8.33% GOI SEC 07-06-2036	8.33%	07-Jun-36	99.23	100	8.33	48	-99.23	8.3960%	0.000435
								Average	8.4575%	0.005011