EXPERIMENTAL STUDIES ON FROZEN SOIL

A DISSERTATION

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF

MASTER OF TECHNOLOGY IN GEOTECHNICAL ENGINEERING

Submitted by:

Kshitij Gaur

(2K19/GTE/07)

Under the supervision of

Prof. Anil Kumar Sahu



DEPARTMENT OF CIVIL ENGINEERING DELHI TECHNOLOGICAL UNIVERSITY (Formerly Delhi College of Engineering) Bawana Road, Delhi-110042

AUGUST, 2021

DELHI TECHNOLOGICAL UNIVERSITY (Formerly Delhi College of Engineering) Bawana Road, Delhi-110042

CANDIDATE'S DECLARATION

I, Kshitij Gaur, Roll No. 2K19/GTE/07, student of M.Tech (Geotechnical Engineering), hereby declare that the project dissertation entitled "Experimental Studies on Frozen Soil" which is submitted by me to the Department of Civil Engineering, Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Technology, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition. Responsibility of any plagiarism issue stands solely with me.

Place: Delhi Date : August 2, 2021 (KSHITIJ GAUR)

DEPARTMENT OF CIVIL ENGINEERING DELHI TECHNOLOGICAL UNIVERSITY (Formerly Delhi College of Engineering) Bawana Road, Delhi-110042

CERTIFICATE

I hereby certify that the Project Dissertation entitled "Experimentatal Studies on Frozen Soil" which is submitted by Kshitij Gaur, Roll No. 2K19/GTE/07, to Department of Civil Engineering, Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Technology, is a record of the project work carried out by him under my supervision. To the best of my knowledge this work has not been submitted in part or full for any Degree or Diploma to this University or elsewhere.

Place: Delhi Date : AUGUST 2, 2021. (ANIL KUMAR SAHU) PROFESSOR & SUPERVISOR Department of Civil Engineering

ACKNOWLEDGEMENT

I, Kshitij Gaur (2K19/GTE/07) would like to express my sincere gratitude to Hon'ble Vice-Chancellor, Head of Department, faculties of Geotechnical Engineering, lab staff and friends for supporting me throughout the project. It is indeed a great pleasure and privilege to present this report of Major Project-2 entitled "Experimental Studies on Frozen Soil" under the guidance and supervision of Prof. Anil Kumar Sahu Sir.

I would personally like to thank Prof. Anil Kumar Sahu Sir for his patience, insightful comments, practical advice and motivation which helped me tremendously in the project work. Without his guidance and support, this project would have not been completed. In the times like this where whole world is fighting with the pandemic of COVID-19, I feel really blessed and thankful to my supervisor for his kind nature that helped and eased out the working of this project.

(KSHITIJ GAUR) Roll No. - 2K19/GTE/07

ABSTRACT

Various studies have been carried out for soils at normal room temperature but the studies on frozen soils are meagre. For every construction, soil investigation is the most important and the primary step for a site selection. For constructions at normal room temperature, there are plenty of experimentation and research data on soil is available. But lack of research data exists for colder regions, where the ambient temperature is below zero degree Celsius for most of the time. It is therefore the need to study soil under frost condition to get a better idea about the behaviour of frozen soils. There are few research available on the mechanical behaviour of frozen soil but no study on the very basic parameters like void ratio, bulk density, porosity, and the degree of freezing. Also, how these parameters change as the soil temperature changes from normal room temperature to negative values. The main emphasis is on the study and experimentation on frozen soil and to formulate different relationships amongst individual soil parameters at various temperatures. The methodology used is to model the soil surface (open grounds in colder regions) by taking sand as the soil after sieving. The model samples are taken into beakers with different bulk densities to replicate real site condition in the freezer. Then by calculating factors like density, porosity, void ratio, etc at different negative temperatures (-5, -10, -15, -20 degree Celsius) and forming relationship with same parameters with that on room temperature. The experimental data obtained is used in "Eureqa software" that will utilize the input so provided and will find mathematical relationship that exist in the soil parameters.

CONTENTS

Title	page
-------	------

Can	Candidate's Declaration		
Cert	ificate	ii	
Ackı	nowledgement	iii	
Abst	ract	i v	
Cont	ents	V	
List	of Tables	vi	
List	of Figures	vi	
List	of Symbols, abbreviations	viii	
Chaj	pter 1 INTRODUCTION		
1.1	General	1	
1.2	Objectives of the study	2	
1.3	Organisation of chapters	2	
Chaj	pter 2 LITERATURE REVIEW		
2.1	Introduction	3	
2.2	Literature Review	3	
2.3	Research Gap	5	
Chaj	oter 3 Materials and Methods	5-11	
Chaj	oter 4 Results and Discussion	11-49	
Chaj	oter 5 Conclusion and Recommendations for the future work		
5.1	Conclusions	49	
5.2	Recommendations for the future work	50	
Refe	rences		

List of Tables

Table	Description	Page
No.	Description	No.
3.1	Sieve analysis observations	10
3.2	Observation table for soil at room temperature	10
4.1	Results of soil gradation curve	12
4.2	Weight of dry vs frozen soil at given temperature and water content.	12
4.3	Void ratios for normal (e) and frozen soil (e')	20
4.4	Bulk densities for normal (γ) and frozen soil (γ ')	27
4.5	Degree of freezing.	34
4.6	Porosity at normal (n) and frozen temperature (n')	41
	1	

List of Figures

Figures	Description	Page No.
3.1	Oven dried soil	7
3.2	Soil sieving	7
3.3	Pycnometer test	8
3.4	Frost soil sample weighing	8
3.5	Deep freezer with temperature controls	8
3.6	4-Phase diagram of soil	9
4.1	Gradation curve for the soil.	11
4.2	Data entering in eureqa software for weight function	15
4.3	Preparing data in eureqa software for weight function	16
4.4	Defining search in eureqa software for weight function	17
4.5	Starting search in eureqa software for weight function	18
4.6	Results for weight function in eureqa software	19
4.7	Graph of % change in void ratio (y) vs Temperature	21

4.8	Data entering in eureqa software for void ratio function	22
4.9	Preparing data in eureqa software for void ratio function	23
4.10	Defining search in eureqa software for void ratio function	24
4.11	Starting search in eureqa software for void ratio function	25
4.12	Results for void ratio function in eureqa software	26
4.13	Percentage change in Bulk density Vs Temperature	28
4.14	Data entering in eureqa software for bulk density function	29
4.15	Preparing data in eureqa software for bulk density function	30
4.16	Defining search in eureqa software for bulk density function	31
4.17	Starting search in eureqa software for bulk density function	32
4.18	Results for weight function in eureqa software	33
4.19	Data entering in eureqa software for degree of freezing function	36
4.20	Preparing data in eureqa software for degree of freezing function	37
4.21	Defining search in eureqa software for degree of freezing function	38
4.22	Starting search in eureqa software for degree of freezing function	39
4.23	Results for degree of freezing in eureqa software	40
4.24	Graph of % change in porosity (z) vs Temperature	42
4.25	Data entering in eureqa software for porosity function	43
4.26	Preparing data in eureqa software for porosity function	44
4.27	Defining search in eureqa software for porosity function	45
4.28	Starting search in eureqa software for porosity function	46
4.29	Results for porosity function in eureqa software	47
	·	

List of Symbols, abbreviations

- γ Bulk density of soil
- γ ' Bulk density of frozen soil
- C_c Coefficient of curvature
- C_u Coefficient of uniformity
- F Degree of freezing
- Ht Height
- h Height of sand in beaker
- % Percentage
- x Percentage change in void ratio
- y Percentage change in bulk density
- z Percentage change in porosity
- n Porosity of soil at room temperature
- n' Porosity of frozen soil
- G_s Specific gravity of soil solids
- T Temperature
- γ_s Unit weight of soil solids
- γ_w Unit weight of water
- e Void ratio of soil at room temperature
- e' Void ratio of frozen soil
- V_I Volume of ice
- V_s Volume of soil solids
- Vv Volume of voids
- V_w Volume of water
- w Water content
- Wt Weight
- W Weight of soil at room temperature
- W_s Weight of soil solids
- Ww Weight of water
- W' Weight of frozen soil
- W_I Weight of Ice
- SW Well graded Sand

Chapter 1: Introduction

The following chapters will help to understand frozen soil in a more better way while calculating and comparing soil parameters at different temperatures and establishing relationships among them. The parameters will be calculated at different temperatures based on the data obtained from the laboratory.

1.1 General

The world is expanding in terms of development and construction every day. From construction of high rise bridges to under water structures, the modern civil engineering is leaving no page unturned. The constructions on glacial soils or frozen soils is of concern as the detailed studies are not available similar to that of soils under normal room temperature. The glacial soils behave in a different manner than the usual soil at normal temperature. The properties vary significantly as the temperature changes from normal room temperature to negative values, tested and observed by Baker and Spaans [5]. All the experiments and relations are established for normal soils at room temperature while glacial soils have not been studied in detail as of now. The main objective is to study the behaviour of glacial soils or frozen soil and establish relationships between different characteristics of soil with variation of temperature, studied by Li et al. [15]. Just for example, when the voids of the soil get increased the chances of inter-connectivity also increase and thereby permeability of soil increases, observed by Zhu and Carbee [24]. But in case of glacial soil, the voids get occupied by ice instead of water, which acts as a barrier to flow of water and thereby permeability decreases. This example gives the idea that the governing factors and the formulae applicable to soil under normal room temperature doesn't hold good for frozen soil. In the present study, an attempt has been made to obtain actual data of soil under both room temperature as well as under frozen state. Then using those data as an input to "Eureqa" software[11] which will give the hidden mathematical relationship that exist between the parameter under varying temperature. For example, the porosity of soil sample will be calculated at both room temperature as well as at frozen state. Then by knowing the water content and temperature of frozen soil the relationship can be established. This will be done by taking frozen soil porosity to be a function of water content, porosity at room temperature and frozen soil temperature. The Eureqa software will then give the relationship between both the porosity of same soil

sample with temperature variable. This will help geotechnical engineer to know different property of frozen soil if its data is given for room temperature.

1.2 Objectives of the study

The project deals with the following objectives:

- To classify the soil.
- To establish the relationship between parameters of unfrozen and frozen soil based on variation in temperature.
- To utilize the results for practical purposes.

1.3 Organisation of chapters

The following chapters have been incorporated in this project report:

In chapter 1, the project title have been introduced in detail. The main objectives and outlines of the work have been discussed.

In chapter 2, the study of the research papers and their findings are mentioned. After following various research papers and books, research gaps has been founded and mentioned.

In chapter 3, the materials used in this project and the methodology followed have been described.

In chapter 4, the results obtained in this project have been listed and discussed in detail, followed by solving an example using the formulae obtained.

In chapter 5, the conclusions have been summarized and the recommendations for future work is mentioned.

Chapter 2: Literature review

2.1 Introduction

The purpose of this research is to study the behaviour of glacial soils or frozen soil and establish relationships between different characteristics of soil. By going through several research papers on frozen soil, the lack of knowledge about basic parameters of iced soil can be observed. The parameters like void ratio, bulk density, degree of freezing, porosity of frozen soil and there study is absent creating a void that needs to be filled to get a complete idea about frozen soils. Glacial soils and soils which are frozen for a majority duration of the year can be dealt with the formulae so obtained in this research.

2.2 Literature review

Frozen soil is defined as the soil whose water content is converted partially or fully into ice. As the temperatures keeps going down, the water present in the soil converts into crystalline form and slowly it changes to solid ice, thereby, forming fully frozen soil. Glacial or frozen soils behaves in a different way then the soil at normal room temperature as observed by Smith et al. [20] and suggest that in depth study on frozen soil required. Andersland and Ladanyi [3], performed certain tests and observed that the surface area, expandable clay lattice and the activity ratio, was deviated to what assumed for frozen soils. Therefore, following different mechanism. Viklander [22] conducted triaxial drainage shearing test on remolded silt samples with different cycle times of freeze-thaw. It was concluded that the cohesion of remolded silt increases compared with that of non-freeze-thaw group after freeze-thaw cycle. This suggests that frozen soil are more cohesive in nature compared to unfrozen soil. Janoo and Shoop [13], grouped undisturbed fine-grained soil and remoulded them into saturated soil samples of various degrees, and measured the effects of water content and confining pressure on the mechanical properties of fine-grained remolded soil by applying different confining pressures on them. This article reference in Rock And Soil Engineering analysis method for silty clay, and in strict accordance with the Geo-technical Test Method Standard to test the soil sample. Edwards and Cresser [9], performed testing of the soil sample density, water content, particle composition, the basic parameters such as liquid limit and plastic limit is important. Through the analyzing of Grey Theory, this article finally analyses the shearing strength of the silty clay

change rule, provide the reference to the following seasonal frozen area subgrade problems to lay the foundation, observed by Altuhafi and Baudet [1]. Tulaczyk et al. [21], inferred those studies on the behaviour of glacial sediments are limited, and those work are concentrated only on investigating ultimate strength of frozen soil and mechanical behaviour. This shows the importance of studying frozen soil mechanics as a different branch. An elaborative model considering the particle breakage and its plastic shear mechanisms is established by Chang et al. [8]. The model showed the failure planes of frozen soil.

The shear strength and elastic modulus of frozen soil as studied by Khoroshilov et al. [14], observed the shear strength parameters of frozen soil. Also, the influence of freezing and thawing cycles on wind erosion strength of black soil simulations done by Liu et al. [16]. Few other studies also include the three-dimensional compressibility anisotropy of a thawing soil and is determined by the cryogenic constitution of the frozen soil and affects primarily the value of the coefficient of thawing, Bakulin and Zhukov [6]. The mechanical properties of frozen soil, the stress-strains are discussed deeply in Qi et al. [18] the mechanical properties are studied in detail. Shen et al [19], studied the stress paths in frozen soil and how the stress path affects the mechanical properties of frozen soil. The books on the chemical properties, Edwards and Cresser [9], shows that after temperature of soil drops and reaches negative temperature, the chemical composition of frozen soil is also changed and different bonds are formed with greater attraction force between soil particles and water. The mechanical properties of frozen soil studied by Merzlyakov [17] suggests that the properties differ to a great extent and different approach should be use to study the soil in frost state. The creep in frozen soil studied by Fish [12], where he studied the creep by applying different loads for long duration and noted the deformations occurred. Similiar studies done by Eranti and Lee [10] to study the creep in frozen soil. Andersland and Ladanyi [2] suggest that the frozen soil acts in a complete strange way as the results obtained by creep study were challenging and different than those on soil at room temperature.

Further, various research has been done on flow throw partially frozen soil. For example, hydraulic conductivity calculated by Azmatch et al. [4] and found that the hydraulic conductivity of soil decrease sharply as the water content is converted into ice which acts as a barrier to flow of water. Burt and Williams [7] also studied the hydraulic conductivity in partially frozen soil and found the same trend.

Research works are also done on fully frozen soils. For example, strength and deformation characteristics of fully frozen soil by Xu et al. [23] and plotted various stress-strain curves of

frozen soil and estimated its strength. The thermal effects and strain rates are also studied by Zhu et al. [25] in which he studied the thermal effects causing thermal strain on frozen soil. Previously the research on frozen soil was done for the analysis of the mechanical, physical, biological and chemical properties, however the main motive of this study is to obtain the relationship between characteristics of soil like its porosity, void ratio and bulk density, at varying temperature is forming a void in the studies that needs to be fulfilled to get the knowledge about how these characteristics changes as the temperature drops to negative values.

2.3 Research Gap

By going through various research papers and textbooks on frozen soil, the following research gaps came under light:

1. Lack of fundamentals of frozen soil: Frozen soils behave in a different way compared to the unfrozen soil. The basic formulae of normal soil for calculating its properties are not applicable for frozen soils. Therefore, its the need to understand frozen soil in a more detailed way similar to unfrozen soil. The gap is filled by establishing relationships to calculate the properties of frozen soil.

2. Lack of inter-relations: There are no formulae that relates the properties of frozen and unfrozen soil. In this study, this gap is filled by forming relationships of soil properties based on temperature variation.

Chapter 3 Materials & Methods

3.1 Materials and Equipment

1. Soil: locally available soil is used and sieve analysis is performed on it to classify the soil.

2. Beaker: Cylindrical borosil beakers are used to carry soil into it to create the samples.

3. Weighing balance: A digital weighing balance is used to measure weights of samples.

4. Deep freezer: It is used to freeze the soil sample. The temperatures used are -5, -10, and $-15^{\circ}C$

3.2 Methodology

The methodology used in this project is simple and easy to understand. The soil is first oven dried to remove the natural water content of soil and then sieve analysis is done on the soil to find out the classification of soil. The soil available was found to be sand and it is used to find relationships between unfrozen and frozen state which can be further extended to other soil types as well. The soil is then mixed with certain amount of water(i.e., known water content) and then filled in 100ml cylindrical beakers with different bulk densities which can be obtained by temping different number of blows using a glass rod. Sample represents soil condition in open ground at normal room temperature. The sample is then placed in a freezer to replicate the temperature conditions of colder regions. In this study, the working temperatures are 25, -5, -10, -15, -20° Celsius which are achieved in a deep freezer in the laboratory. The soil factors like porosity, void ratio, bulk density, dry density, etc are calculated for frozen soil at above said temperatures. These are then related to the same properties under room temperature. For instance, porosity of frozen soil is a function of porosity at room temperature, water content and the temperature at which it is being calculated for. The degree of freezing is also be calculated using the same approach. After getting data of normal and frozen soil, different relations can be found using eureqa software.

3.3 Experimental Procedure

The test procedures performed in this experimental work is according to Indian Standard codes: IS 2720-2(1973), (IS-2720-PART-3-1980), ISO 11272:2017. The procedure is as follows:

- A) Oven dry the soil at 105° Celsius for 24 hours as per IS 2720-2(1973). Put this soil in plastic bags so that it may not get any moisture from surrounding as shown in Figure 3.1.
- B) Take out samples from this soil bag. Sieve the soil for classification as shown in figure 3.2. For this project the soil gradation curve suggest the soil to be well graded soil as the value of C_c and C_u comes out to be 1.09 ad 8.33 respectively.
- C) Let the dry weight of soil sample= W_s
 Add known amount of water content (2%,4%,6%,8%,...,40%)
 w= (W_w/W_s)(1)
- D) Separately, determine specific gravity(G_s) of soil solids using pycnometer test and relate it with equation (2) to get value of γ_s as shown in Figure 3.3. As per **IS-2720-PART-3-1980.**

$$Gs = (\gamma_s / \gamma_w)$$
(2)

Once, γ_s is calculated volume of soil solids will be further calculated by

$$\gamma_s = (W_s/V_s) \qquad \dots \qquad (3)$$

E) Now, calculate void ratio(e) as per IS : 2720 (Part 2)

$$e = (V_v/V_s)$$
(4)

- F) V_v can be calculated by first measuring the total volume(V_t) of soil sample having different bulk densities($\gamma_{t1}, \gamma_{t2}, \gamma_{t3}...$) in measuring container as per **ISO 11272:2017** and then using: $V_v = (V_t - V_s) \qquad \dots \dots (5)$
- G) Now, place the soil sample container in freezer for various temperatures to record data (-5,-10,-15,-20 degree Celsius) as shown in Figure 3.5.
- H) After 24 hours, take the sample out and weigh it. Also measure heights at five different locations, and the average is taken as shown in Figure 3.4. Do this step as soon as possible to avoid error due to temperature change.

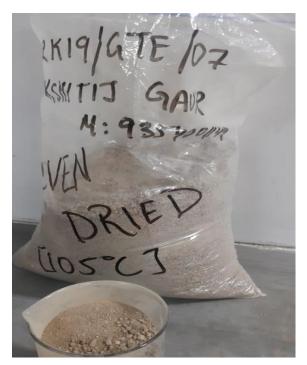


Figure 3.1: Oven dried soil



Figure 3.2: Soil sieving.



Figure 3.3: Pycnometer test



Figure 3.4: Frost soil sample weighing



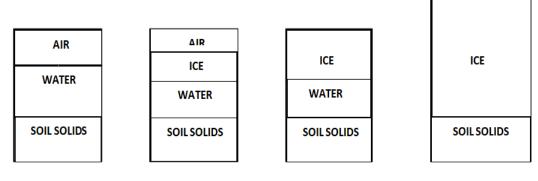
Figure 3.5: Deep freezer with temperature controls.

I) Then the ice content of soil sample will be:

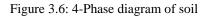
The water content will slowly change to ice content as the temperature is kept on decreasing. Therefore, the ice content is a function of initial water content and the temperature.

$$w' = f(w, T)$$
(6A)

This ice content will reach a maximum value when all the water present in the soil gets converted into ice. The void ratio will also change as volume ice expands filling all the voids at first and then increasing the overall volume of soil sample as shown in Figure 3.6.



Temperature decreases from normal room to negative values



- J) Record the data of frozen weight of soil and the average height of frozen soil to form the observation tables.
- K) Calculate void ratio, degree of freezing, bulk density, porosity of frozen soil from data so obtained for different temperature reading.
- L) Utilize this raw data as input for Eureqa software and formulate different relationships by feeding both input and output parameter reading in the software.

3.4 Observations

The following observations were made during sieve analysis

Sieve . No.	Diameter (mm)	Soil Retained (gm)	Accumulative Retain (gm)	% Mass Retain	% Passing
4	4.75	22.65	22.65	4.5426	95.4574
10	2.00	75.34	97.99	19.6526	80.3474
20	0.85	123.21	221.2	44.3633	55.6367
40	0.43	115.85	337.05	67.5979	32.4021
60	0.25	65.46	402.51	80.7264	19.2736
200	0.075	81.60	484.11	97.0919	2.9081
Pan		14.50	498.61		

Table 3.1 Sieve analysis observations

In this project, the soil parameters at various temperature has been calculated. These parameters have been calculated using the data obtained in the laboratory by applying basic soil mechanics.

The observations and data recorded in lab are as follows:

1. The diameter of cylindrical beaker is 5cm.

2. The weight of dry soil taken for each reading is 100gm.

3. The least count of electronic weighing machine is 0.0001gm.

4. The least count of length measuring scale is 0.1cm.

5. The beakers are of different densities thereby resulting in different empty weights which are recorded accordingly in table 3.2.

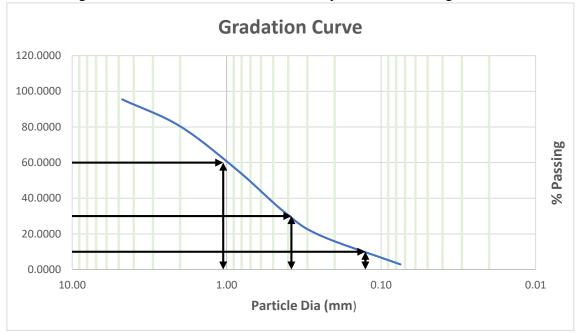
Following are the readings recorded for unfrozen and frozen sand at different temperatures:

	Water	Empty wt. of	Wt. of dry	Ht. of sand [h]	Total
S. No.			-		volume
	content (%)	cylinder (gm)	sand (gm)	(cm)	(cc)
1	6	56.815	100	5.78	113.461
2	6	42.348	100	5.30	104.039
3	10	45.172	100	5.90	115.817
4	10	41.183	100	5.30	104.039
5	14	45.385	100	5.18	101.683
6	14	36.803	100	5.38	105.609
7	18	34.383	100	4.84	95.009
8	18	35.702	100	5.52	108.357
9	22	44.242	100	3.60	70.668
10	22	52.334	100	4.10	80.483
11	26	47.105	100	4.36	85.587
12	26	40.848	100	4.32	84.802
13	30	33.182	100	4.10	80.483
14	30	44.228	100	4.50	88.335

Table 3.2 Observation table for soil at room temperature

Chapter 4 Results and Discussion

4.1 Results



1. The soil gradation curve formed after sieve analysis is shown in figure 4.1.

Fig. 4.1 Gradation curve for the soil.

2. The results obtained from sieve anal	ysis are	given in	n table 4.1.
---	----------	----------	--------------

Table 4.1 Results of soil gradation curve

	Results					
% gravel	4.54	D ₆₀ (mm)	1.053063	$C_u = D_{60}/D_{10}$	8.33	
% sand	92.56	D ₃₀ (mm)	0.381062	$C_c = D_{30}^2 / D_{10}^* D_{60}$	1.09	
% fines	2.90	D ₁₀ (mm)	0.126385			

The soil type is found to be well graded sand (SW) as the coefficient of uniformity is greater than 6 (i.e., $C_u > 6$) and coefficient of curvature is greater than 1 (i.e., $C_c > 1$).

4.1.1 Weight Function

The weight of frozen soil can be found if weight of same soil at room temperature, its water content and the temperature at which weight is to be determined is known. By entering the data obtained in laboratory shown in table 4.2, Eureqa software gives the relationships that may exists between the data set. The weight function would be:

$$W' = f(W_s, w, T)$$
(7)

where w is the water content of soil at room temperature.

W_s is the weight of soil solids at room temperature

T is the temperature of frozen soil.

Table 4.2 Weight of dry	v vs frozen soil at	given temperature an	d water content.

Wt of dry soil (arra)	Water content (0/)	Temperature	Wt. of frozen
Wt. of dry soil (gm)	Water content (%)	(°C)	soil (gm)
Ws	W	Т	W'
100	6	-5	105.818
100	6	-5	105.862
100	10	-5	109.69
100	10	-5	109.767
100	14	-5	113.665
100	14	-5	113.746
100	18	-5	117.645
100	18	-5	117.771
100	22	-5	121.752
100	22	-5	121.738
100	26	-5	125.672
100	26	-5	125.634
100	30	-5	129.545
100	30	-5	129.65
100	6	-10	105.712
100	6	-10	105.783
100	10	-10	109.595
100	10	-10	109.676
100	14	-10	113.554
100	14	-10	113.641
100	18	-10	117.522
100	18	-10	117.657
100	22	-10	121.605

100	22	-10	121.559
100	26	-10	125.506
100	26	-10	125.401
100	30	-10	129.399
100	30	-10	129.454
100	6	-15	105.635
100	6	-15	105.718
100	10	-15	109.432
100	10	-15	109.582
100	14	-15	113.445
100	14	-15	113.593
100	18	-15	117.41
100	18	-15	117.602
100	22	-15	121.45
100	22	-15	121.436
100	26	-15	125.4
100	26	-15	125.188
100	30	-15	129.352
100	30	-15	129.301
	1		

Steps to run search on software:

1. After creating a table of inputs and output for weight function, open eureqa software.

2. An excel sheet will gets open, enter the data in different columns like weight of dry soil at room temperature (W_s) in column 1, water content (w) in column 2, temperature (T) in column 3 and output (W') in column 4 as shown in figure 4.2.

Now, click on "prepare data" tab to smoothen the data points if necessary as shown in figure
 4.3.

4. Now, click on "Define data" tab to define the relationship between input and output data. For example, $W' = f(W_s, w, T)$. Also, select the operators you want to use in your functions like add, subtract, divide, multiply, exponent, etc. as shown in figure 4.4

5. Now click on "start search" tab and click on "Run" to start the search as shown in figure 4.5.

6. Now, click on "Results" tab to see the formulae so obtained in the search with their amount of errors as shown in figure 4.6.

È	🖉 🕼 🖌 🖬	📋 Project D	egree of freezing	Search: 📀 🤇	0 0	How to Enter Data				
	🗄 Enter Data 💀 Prepare Data 🔎 Define Search 📀 Start Search 🔽 Results 🔛 Reports 🛆 Secure Cloud									
	A	В	С	D	E	F	G	Н	I	J
info	Weight of dry soil	Water content	Temperature in negatives	Weight of frozen soil						
name	W	w	Т	W'						
1	100	6	5	105.818						
2	100	6	5	105.862						
3	100	10	5	109.69						
4	100	10	5	109.767						
5	100	14	5	113.665						
6	100	14	5	113.746						
7	100	18	5	117.645						
8	100	18	5	117.771						
9	100	22	5	121.752						
10	100	22	5	121.738						
11	100	26	5	125.672						
12	100	26	5	125.634						
13	100	30	5	129.545						
14	100	30	5	129.65						
15	100	6	10	105.712						
16	100	6	10	105.783						
17	100	10	10	109.595						
18	100	10	10	109.676						
19	100	14	10	113.554						
20	100	14	10	113.641						
21	100	18	10	117.522						
22	100	18	10	117.657						
23	100	22	10	121.605						
24	100	22	10	121.559						
25	100	26	10	125.506						
26	100	26	10	125.401						

Figure 4.2: Data entering in eureqa software for weight function.

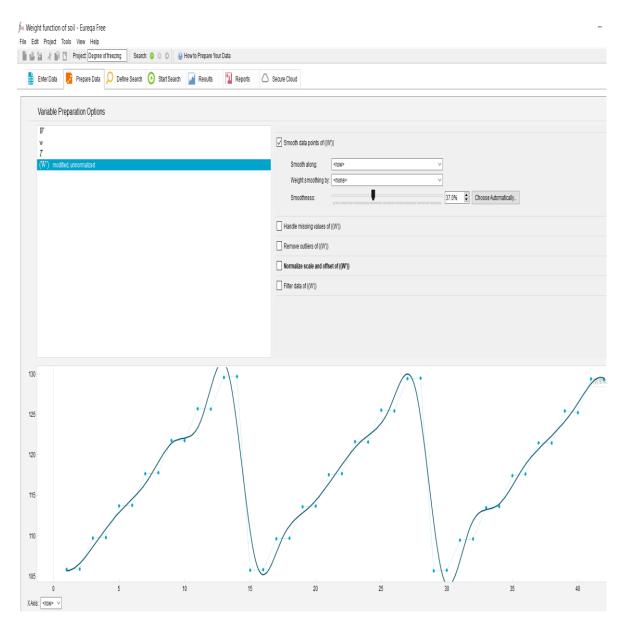


Figure 4.3: Preparing data in eureqa software for weight function.

 ${\it f} \omega$ Weight function of soil - Eureqa Free

Enter Data 🧧 F	repare Data 🔎 Define Search 💽 Start Search	Results 🔛 Reports 🛆 Si	ecure Cloud		
he Target Expressior	Y.				
Search for a formula f () that satisfies the equation See Examples	(W') = f(W, w, T)				
imary Options:					
Formula building-blocks:	Name	Complexity	∧ Currently Selected:		
	Basic		c_, x_, + , - , *, /, sin, cos, tan, exp,	og, ^, sqrt	
	Constant	1			
	Integer Constant	1			
	Input Variable	1			
	Addition Subtraction	1			
	Subtraction	1			
	Multiplication Division	1			
	Division	2			
	Negation	1			
	Trigonometry				
	Sine	3			
	Cosine	3			
	Sine Cosine Tangent	4			
	Exponential				
	Exponential Natural Logarithm	4			
	🗹 Natural Logarithm	4			
	Factorial	4			
	Power	5			
	Square Root	4			
	Squashing				
	Logistic Function	4			
	Step Function	4			
	Sign Function	4			
	Gaussian Function	4			
	Hyperbolic Tangent	4			
	Error Function Complementary Error Function	4	~		
		4	V		
Error metric:	Absolute error (default)				
	Row Weight <none></none>		Y		
	Data Splitting: Treat all data points equally (default)		¥	Set Custom	

Figure 4.4: Defining search in eureqa software for weight function.

v Weight function of soil - Eureqa Free lie Edit Project Tools View Halp Lie La K Lie Project Degree offreezing Search O C Lie Enter Data C Prepare Data O Define Search O S	O O How to Start the Search Iart Search A Reports Secure Cloud	
Run the formula search:		
Progress & performance	Progress over time	
Time Searchtime 1m 17s CPU ores 4 Performance 124.6 Generations/sec 852 M Confidence 852 M Confidence Stability Stability 19.6% Maturity 8.23% Percent converged 19%	0.051 - 0.05 - 0.049 - 0.048 - 0.046 -	
	0.1	1 10 Time (seconds)
Project Log: 11:44:39.4003394M> New solution: Fit: 0.0485313 Size: 23 Solution: (W	1 = 100 + w - 0.00220545an/100.25w) - 0.0240055mHw4T1	
11443 040534M- New solution: FL 0.048213 3 28: 2.3 Solution: (N 114443 041504M- New solution: FL 0.0484705 8: 2:: 3 Solution: (N 114444 040146M- New solution: FL 0.0484756 8: 2:: 3 Solution: (N 114445 040346MI- New solution: FL 0.048456 8: 2:: 3 Solution: (N 114447 040712MI- New solution: FL 0.048456 8: 2:: 3 Solution: (N 114447 040712MI- New solution: FL 0.0484196 8: 2:: 3 Solution: (N 114448 040594MI- New solution: FL 0.0484196 8: 2:: 3 Solution: (N 114448 040594MI- New solution: FL 0.0484196 8: 2:: 3 Solution: (N 114448 040594MI- New solution: FL 0.0484196 8: 2:: 3 Solution: (N 114448 040594MI- New solution: FL 0.0484196 8: 2:: 3 Solution: (N 114448 040594MI- New solution: FL 0.0484196 8: 2:: 3 Solution: (N 114458 9397894MI- New solution: FL 0.0484196 8: 2:: 3 Solution: (N 114450 1413046MI- New solution: FL 0.0484196 8: 2:: 3 Solution: (N 114450 1413046MI- New solution: FL 0.048419 8: 2:: 3 Solution: (N	= 100 + w - 0.003081*tan(1002*w) - 0.03409*sqrt(w*T) = 100 - w - 0.003251*tan(1002*w) - 0.03409*sqrt(w*T) = 100 + w - 0.003397*tan(1002*w) - 0.03409*sqrt(w*T) = 100 + w - 0.003797*tan(1002*w) - 0.03409*sqrt(w*T) = 100 + w - 0.003745*tan(1002*w) - 0.03409*sqrt(w*T) = 100 + 0.0037* - 0.002136*tan(1003*w) - 0.03470*sqrt(w*T) = 100 + 0.003392*tan(1002*w) - 0.03409*sqrt(w*T) = 100 + w - 0.00408*tan(1002*w) - 0.03409*sqrt(w*T) = 100 + w - 0.00408*tan(1002*w) - 0.03409*sqrt(w*T) = 100 + w - 0.00468*tan(1002*w) - 0.03409*sqrt(w*T)	

18

Figure 4.5: Starting search in eureqa software for weight function.

The relationship found is:

$$W' = W_s + 0.992 w - 0.000208 W_s(-T)$$
(7A)

This relationship has the maximum error of 0.18 units.

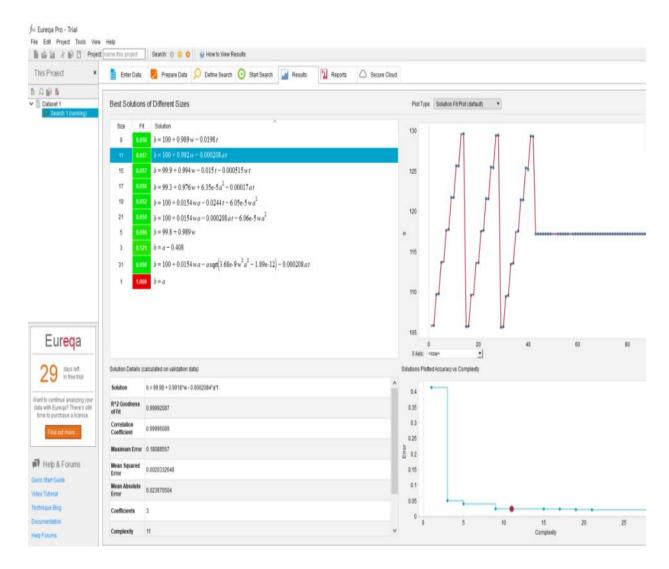


Figure 4.6: Results for weight function in eureqa software.

4.1.2 Void Ratio

Kindly assure that, the void ratio will change with temperature as the volume of water increased as it changes from liquid state to iced state. Thereby increasing the volume of voids while volume of soil solids remains same. Therefore, the void ratio will increase which is confirmed practically too. By entering data obtain in laboratory shown in table 4.3, the following relationship is obtained by the Eureqa Software.

$$e'=f(e, T)$$
(8)

Where T is in degree Celsius.

11.5			E	04
Unfrozen soil	Temperature	Water content	Frozen soil void	% change in
void ratio		(%)	ratio	void ratio
e	Т	W	e'	у
1.909	-5	6	1.949	2.05
1.667	-5	6	1.688	1.24
1.969	-5	10	1.985	0.81
1.667	-5	10	1.688	1.24
1.607	-5	14	1.708	5.91
1.708	-5	14	1.748	2.29
1.436	-5	18	1.532	6.27
1.778	-5	18	1.829	2.79
0.812	-5	22	0.922	11.93
1.063	-5	22	1.073	0.93
1.194	-5	26	1.204	0.83
1.174	-5	26	1.199	2.09
1.063	-5	30	1.073	0.93
1.265	-5	30	1.386	8.73
1.909	-10	6	1.98	3.59
1.667	-10	6	1.698	1.83
1.969	-10	10	1.99	1.06
1.667	-10	10	1.718	2.97
1.607	-10	14	1.713	6.19
1.708	-10	14	1.808	5.53
1.436	-10	18	1.547	7.18
1.778	-10	18	1.829	2.79
0.812	-10	22	1.033	21.39
1.063	-10	22	1.124	5.43
1.194	-10	26	1.104	8.15
1.173	-10	26	1.174	0.09
1.063	-10	30	1.114	4.58
1.265	-10	30	1.476	14.3
1.909	-15	6	1.98	3.59
1.667	-15	6	1.698	1.83
1.969	-15	10	1.99	1.06
1.667	-15	10	1.718	2.97
1.607	-15	14	1.718	6.46
1.708	-15	14	1.813	5.79
1.436	-15	18	1.567	8.36
1.778	-15	18	1.834	3.05
0.812	-15	22	1.033	21.39
1.063	-15	22	1.124	5.43
1.194	-15	22	1.124	7.18
1.174 1.063	-15 -15	26 30	1.179 1.114	0.42

Table 4.3 Void ratios for normal (e) and frozen soil (e')

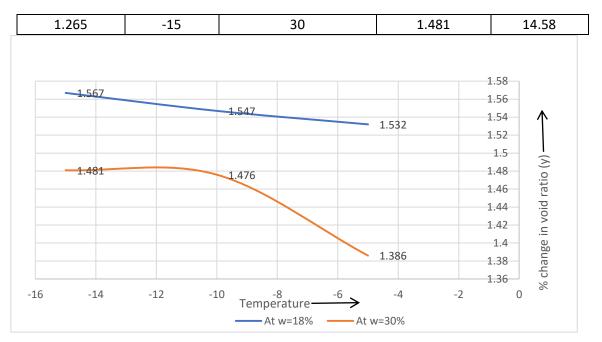


Fig. 4.7 Graph of % change in void ratio (y) vs Temperature

The graph between percentage change in void ratio vs temperature suggests that at higher water content the change in void ratio is more and steep curve is obtained as compared to lower water content as shown in figure 4.7. This is because more water will convert into ice and volume of voids will increase sharply in case of higher water content and volume of ice will be greater than the initial volume of voids.

Steps to run search on software:

1. After creating a table of inputs and output for void ratio function, open eureqa software.

2. An excel sheet will gets open, enter the data in different columns like void ratio of soil at room temperature (e) in column 1, temperature (T) in column 2, water content (w) in column 3 and output (e') in column 4 as shown in figure as 4.8.

3. Now, click on "prepare data" tab to smoothen the data points if necessary as shown in figure 4.9.

4. Now, click on "Define data" tab to define the relationship between input and output data. For example, e'= f(e, w, T). Also, select the operators you want to use in your functions like add, subtract, divide, multiply, exponent, etc. as shown in figure 4.10.

5. Now click on "start search" tab and click on "Run" to start the search as shown in figure 4.11.

6. Now, click on "Results" tab to see the formulae so obtained in the search with their amount of errors as shown in figure 4.12.

Ì (Project vo	oid ratio	Search: 🧿 🔘	0	How to Enter Data											
	Enter Data	Prepare	Data 🔎 Defir	ne Search 🜔 St	tart Search	Results	Reports	Secure Cloud									
	A	В	С	D	E	F	G	Н	I	J	К	L	М	N	0	Р	
0	/oid ratio	Temperature	Water content	Final void ratio													
ne	е	Т	w	e'													_
t	1.909550907	5	6	1.949821508													-
T	1.667927303	5	6	1.688062603													
T	1.969956809	5	10	1.985058284													
t	1.667927303	5	10	1.688062603													
t	1.607521401	5	14	1.708197903													
t	1.708197903	5	14	1.748468504													
t	1.436371348	5	18	1.532014025													
t	1.778671455	5	18	1.829009706													
t	0.812177036	5	22	0.922921188													
)	1.063868291	5	22	1.073935941													
	1.194747743	5	26	1.204815394													
2	1.174612443	5	26	1.199781568													
3	1.063868291	5	30	1.073935941													
	1.265221295	5	30	1.386033097													
5	1.909550907	10	6	1.980024459													
5	1.667927303	10	6	1.698130253													
,	1.969956809	10	10	1.990092109													
3	1.667927303	10	10	1.718265554													
	1.607521401	10	14	1.713231729													
	1.708197903	10	14	1.808874405													
i	1.436371348	10	18	1.5471155													
2	1.778671455	10	18	1.829009706													
3	0.812177036	10	22	1.03366534													
i	1.063868291	10	22	1.124274192													
5	1.194747743	10	26	1.104138892													
5	1.174612443	10	26	1.174612443													
,	1.063868291	10	30	1.114206542													
3	1.265221295	10	30	1.476641949													
	1.909550907	15	6	1.980024459													
)	1.667927303	15	6	1.698130253													
í	1.969956809	15	10	1.990092109													
2	1.667927303	15	10	1.718265554													
3	1.607521401	15	14	1.718265554													
1	1.708197903	15	14	1.813908231													
;	1.436371348	15	18	1.567250801													
;	1.778671455	15	18	1.834043531													
,	0.812177036	15	22	1.03366534													
3	1.063868291	15	22	1.124274192													
	1.194747743	15	26	1.114206542													
)	1.174612443	15	26	1.179646268													

Figure 4.8: Data entering in eureqa software for void ratio function.

fw void ratio - Eurega Free	-
File Edit Project Tools View Help	
	Secure Cloud
Variable Preparation Options	
e T modified	Smooth data points of ((e'))
W (e) modified	Smooth along.
	Weight smoothing by:
	Smoothness
	Hande missing values of ((e'))
	Remove outliers of ((e'))
	Wormalize scale and offset of ((e'))
	Filter data of ((e'))
	20 25 30 35 40
X Avis: 💷 Tower V	

Figure 4.9: Preparing data in eureqa software for void ratio function.

ialk iii⊡ P	offer Internation	- Court	с 🔾 О О 🔒 Но	wie der rarger o	April 11				
Enter Data 🛃	Prepare Data	🔎 Define Search	🕑 Start Search	Results	Reports	Secure Cloud			
he Target Expressio	n:								
Search for a formula <i>f()</i> that satisfies the equation See Examples	(e') = f(e	, T , w)							
rimary Options:									
Formula building-blocks:	Basic Constant Integer Ci Input Vari Subtradit Multiplical Division Negation Trigonential Exponential Power Square R Step Func Step Fu	al iganithm pot unction tion tion tion tion tion tion tion			Complexity 1 1 1 1 1 2 1 3 3 4 4 4 4 4 4 4 4 4 4 4 4 4			Currently Selected: c_ x_ * ,*,(sin, cos	
Error metric:	Absolute error	entary Error Function (default)			-		¥		
	Row Weight	<none></none>						•	
	Data Splitting:	Treat all data points	equally (default)					•	Set Custom

Figure 4.10: Defining search in eureqa software for void ratio function.

🌆 void ratio - Eureqa Free						
File Edit Project Tools View Help						
📗 🍰 🙀 🦌 🛍 🖞 🛛 Project. void ratio	h					
🛔 Enter Data 🗧 Prepare Data 🔎 Define Search 🧿 Start Search 📓 Results	Reports 🛆 Secure Clou	d				
Run the formula search: Run Pause Stop						
Progress & performance	Progress over time					
-	0.05					
Time Search time 24m 29s						
CPU cores 4						
Performance	0.045 -					
Generations 148752						
Generations/sec 79.37						
Formula evaluations 5.1e9	0.04 -					
Evaluations/sec 2.79 M Confidence						
Stability 33.5%	U.035 -					
Maturity 43.7%	E 0.035					
Percent converged 99.2%	li h					
	Mean Abs 0.03 -					
	Ab s					
	e a					
	≥ 0.025 -					
	0.02 -					
	0.015 -					
	0	200	400	600	800	1000 1200
				Time [seconds]		
Project Log:						
1108.48.806527PM New solution: Fit: 0.0148797 Size: 49 Solution: (e') = 0.4637e + 0.004518°T + 0.2149° 1108.56.358562PM New solution: Fit: 0.0469001 Size: 3 Solution: (e') = 0.4637e + 0.004518°T + 0.2149° 1108.56.41322BM New solution: Fit: 0.0469001 Size: 3 Solution: (e') = 0.94637e + 0.004518°T + 0.2149° 1108.56.41322BM New solution: Fit: 0.0148797 Size: 40 Solution: (e') = 0.94637e + 0.004518°T + 0.2149° 1108.56.91323FM New solution: Fit: 0.0148793 Size: 45 Solution: (e') = 0.94637e + 0.004518°T + 0.2149° 1108.09.4033567M New solution: Fit: 0.0148793 Size: 49 Solution: (e') = 0.94637e + 0.049678°T + 0.0214971 1108.09.4033567M New solution: Fit: 0.0148719 Size: 49 Solution: (e') = 0.94637e + 0.049678°T + 0.0214971 1108.09.4033567M New solution: Fit: 0.0148719 Size: 49 Solution: (e') = 0.94637e + 0.049678T + 0.21497 1108.09.4587147 New solution: Fit: 0.0148719 Size: 49 Solution: (e') = 0.94637e + 0.040518T + 0.21497 1109.09.4587147 New solution: Fit: 0.0148719 Size: 49 Solution: (e') = 0.94637e + 0.040518T + 0.21497 1109.21.370747 New solution: Fit: 0.014879 Size: 49 Solution: (e') =	"sin(e) + 0.1086"e*cos(583.3*e) - 0.101 T + 0.07714"e*sin(1.605 - 583.4*e) - 0.101 T + 0.00851"e*cos(583.3*e) - 0.101 T + 0.00851"t*e*T*sin(1.584 - 583.4*e) - "sin(e) + 0.1086"e*cos(583.3*e) - 0.101 sin(e) + 0.1086"e*cos(583.3*e) - 0.101 S83.5*e)e	8 - 0.00569*T*cos(583.3*e) - (162*sin(1.498 - 583.4*e) - 0.0 8 - 0.0569*T*cos(563.3*e) - (0.01862*T*sin(1.505 - 583.4* 8 - 0.00569*T*cos(583.3*e) - (3 - 0.00569*T*cos(583.3*e) - (8 - 0.005692*T*cos(583.3*e) - (8 - 0.005	.1648*sin(1.435 - 583.3)5641*T*sin(1.498 - 583 .1648*sin(1.435 - 583.3 s) .1648*sin(1.435 - 583.3 1648*sin(1.435 - 583.3*	*e) ;4*e) *e) *e)		

Figure 4.11: Starting search in eureqa software for void ratio function.

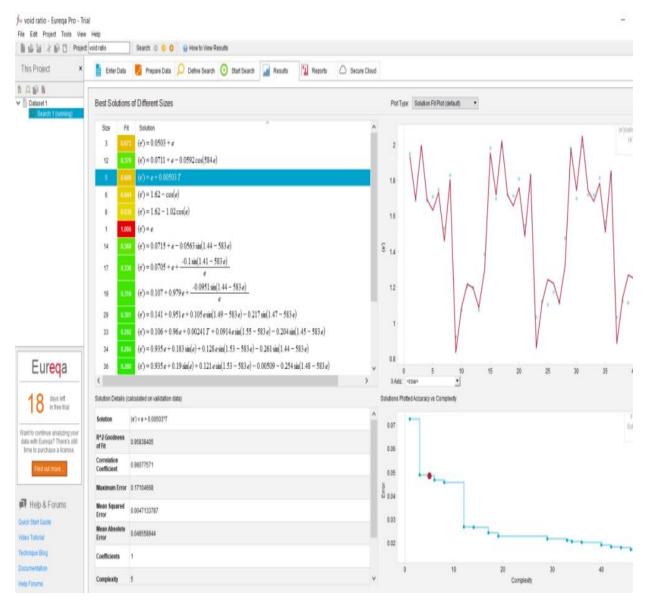


Figure 4.12: Results for void ratio function in eureqa software.

The relationship found is:

$$e' = e + 0.00503(-T)$$
(8A)

4.1.3 Bulk Density of Soil

The weight of soil solids remains constant but the weight of water changes with change of temperature thereby changing the bulk density of soil. Therefore, a relationship may exist between the bulk density at room temperature to the bulk density at frozen temperature.

After entering the data obtained in laboratory shown in table 4.4, the relationship can be obtained in Eureqa software.

The function is expected to be:

$$\gamma_t = f(\gamma_t, w, T)$$
(9)

Bulk Density (KN/m³)	water content (%)	Temperature (°C)	Bulk Density of frozen soil (KN/m3)	% Change in bulk density
γ	w	Т	γ'	х
9.161	6	-5	9.020	1.54
9.991	6	-5	9.903	0.88
9.313	10	-5	9.240	0.78
10.368	10	-5	10.268	0.96
10.994	14	-5	10.554	4.00
10.585	14	-5	10.406	1.69
12.179	18	-5	11.684	4.06
10.679	18	-5	10.468	1.97
16.929	22	-5	15.921	5.95
14.864	22	-5	14.760	0.69
14.436	26	-5	14.333	0.71
14.569	26	-5	14.361	1.43
15.839	30	-5	15.707	0.83
14.431	30	-5	13.664	5.31
9.161	6	-10	8.920	2.63
9.991	6	-10	9.859	1.32
9.313	10	-10	9.217	1.03
10.368	10	-10	10.146	2.14
10.994	14	-10	10.524	4.27
10.585	14	-10	10.173	3.89
12.1789	18	-10	11.602	4.74
10.679	18	-10	10.458	2.07
16.929	22	-10	15.036	11.18
14.864	22	-10	14.389	3.19
14.436	26	-10	14.999	3.89
14.569	26	-10	14.500	0.47
15.839	30	-10	15.390	2.83
14.431	30	-10	13.144	8.92
9.161	6	-15	8.914	2.70
9.991	6	-15	9.853	1.38
9.313	10	-15	9.203	1.18
10.368	10	-15	10.137	2.23
10.994	14	-15	10.495	4.54
10.585	14	-15	10.151	4.10
12.179	18	-15	11.500	5.57

Table 4.4 Bulk densities for normal (γ) and frozen soil (γ ')

10.679	18	-15	10.435	2.28
16.929	22	-15	15.017	11.29
14.864	22	-15	14.375	3.29
14.436	26	-15	14.915	3.32
14.569	26	-15	14.443	0.86
15.839	30	-15	15.385	2.86
14.431	30	-15	13.102	9.21

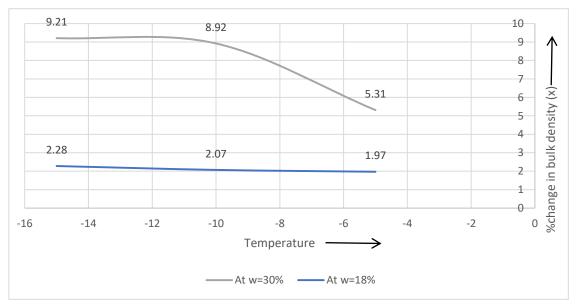


Fig. 4.13 Percentage change in Bulk density Vs Temperature

As the graph in Fig. 4.13 suggests that the change in bulk density will be more steep when working with higher water content. Also, the rate of change in bulk density with temperature decreases as the temperature is decreasing. This is because the density of ice is smaller than that of water.

Steps to run search on software:

1. After creating a table of inputs and output for bulk density function, open eureqa software.

2. An excel sheet will gets open, enter the data in different columns like bulk density of soil at room temperature (γ) in column 1, water content (w) in column 2, temperature (T) in column 3 and output (γ ') in column 4 as shown in figure 4.14.

Now, click on "prepare data" tab to smoothen the data points if necessary as shown in figure
 4.15.

4. Now, click on "Define data" tab to define the relationship between input and output data. For example, $\gamma' = f(\gamma, w, T)$. Also, select the operators you want to use in your functions like add, subtract, divide, multiply, exponent, etc. as shown in figure 4.16. 5. Now click on "start search" tab and click on "Run" to start the search as shown in figure 4.17.

6. Now, click on "Results" tab to see the formulae so obtained in the search with their amount of errors as shown in figure 4.18.

This Project ×		Enter Data	P repare	Data 🔎 Defi	ine Search 🜔	Start Search	📔 Results	🔛 Reports	Secure Cloud						
Q 🗊 🖡	F	A	В	С	D	E	F	G	Н	I	J	К	L	М	N
Dataset 1	info	Bulk Density (g/cc)	water content	Temperature	Bulk Density of frozen soil										
	name	γ	W	Т	Ý										
	1	0.934238428	6	5	0.919902149										
	2	1.018848701	6	5	1.009900386										
	3	0.949774213	10	5	0.942306189										
	4	1.057295822	10	5	1.047153234										
	5	1.121126949	14	5	1.076277301										
	6	1.079449367	14	5	1.06126341										
	7	1.241984987	18	5	1.1914757										
	8	1.088986836	18	5	1.067534078										
	9	1.726382521	22	5	1.623650092										
	10	1.515848067	22	5	1.505250038										
	11	1.472189637	26	5	1.461652442										
	12	1.485821022	26	5	1.464554207										
	13	1.615247941	30	5	1.601781007										
	14	1.471670346	30	5	1.393393819										
	15	0.934238428	6	10	0.909666671										
	16	1.018848701	6	10	1.00538127										
	17	0.949774213	10	10	0.939905079										
	18	1.057295822	10	10	1.034659723										
	19	1.121126949	14	10	1.073231404										
Eurogo	20	1.079449367	14	10	1.037481946										
Eur <mark>eq</mark> a	21	1.241984987	18	10	1.183173291										
· · ·	22	1.088986836	18	10	1.066500726										
n days left	23	1.726382521	22	10	1.533379904										
in free trial	24	1.515848067	22	10	1.467419778										
•	25	1.472189637	26	10	1.529564893										
atta analizua analizia unur	26	1.485821022	26	10	1.478757476										
nt to continue analyzing your ta with Eureqa? There's still	27	1.615247941	30	10	1.569500036										
ime to purchase a license.	28	1.471670346	30	10	1.340386583										
	29	0.934238428	6	15	0.909004075										
Find out more	30	1.018848701	6	15	1.004763498										
	31	0.949774213	10	15	0.938507164										
	32	1.057295822	10	15	1.033772948										

Figure 4.14: Data entering in eureqa software for bulk density function.

∬∾ BULK DENSITY - Eureqa Free Fie Edit Project Tools View Help ■ a a A B T Project BULK DENSITY Search: ● ○ ○ ● ● How to Prepare Your Data	- 0	×
 Enter Data Prepue Data Define Search Stat/Search Results Results Secure Cloud 		
Variable Preparation Options		
(y) I modified I (y) I (y) I (y) I I I I I I I I I I I I I		
	())smooth w 40	
🕂 🔎 Type here to search O 🛱 💽 🛅 🛱 🚖 💟 🏂 👰	ヽ ⑫ ■ <i>億</i> 印) 11:14 23-06-2021	5

Figure 4.15: Preparing data in eureqa software for bulk density function.

	roject. BULK DENSITY Search: 🔾 🔘 🔘 How	to Set Target Options			
Enter Data 🚦	Prepare Data 🔎 Define Search 💽 Start Search 📓	Results 🚹 Reports 🛆 Se	cure Cloud		
Target Expressio	in:				
arch for a formula ƒ() t satisfies the equatio a Examples					
nary Options:					
mula building-blocks:	Name	Complexity	Currently Selected:		Select a minimal set of building bl
	Basic		C_, X_, + , - , *, /, exp, log, ^, sqrt		Double-click to edit complexity val
	Constant Integer Constant	1			Building-blocks Documentation
	Input Variable	1			
	Addition Subtraction Multiplication	1			
	V Subtraction	1			
	✓ Division	2			
	Negation	1			
	Trigonometry Sine	3			
	Cosine	3			
	Tangent	4			
	Exponential				
	 Exponential Natural Logarithm 	4			
	Factorial	4			
	Power	5			
	Square Root	4			
	Squashing Logistic Function	4			
	Step Function	4			
	Sign Function	4			
	Gaussian Function	4			
	Hyperbolic Tangent Error Function	4			
	Complementary Error Function	4	~		
metric:	Absolute error (default)				Error Metrics Documentation
	Row Weight <none></none>		T		
	Data Splitting: Treat all data points equally (default)		•	Set Custom	
e and prior solutions					Enter terms and expressions on separate lines
					Prior Solutions Documentation

Figure 4.16: Defining search in eureqa software for bulk density function.

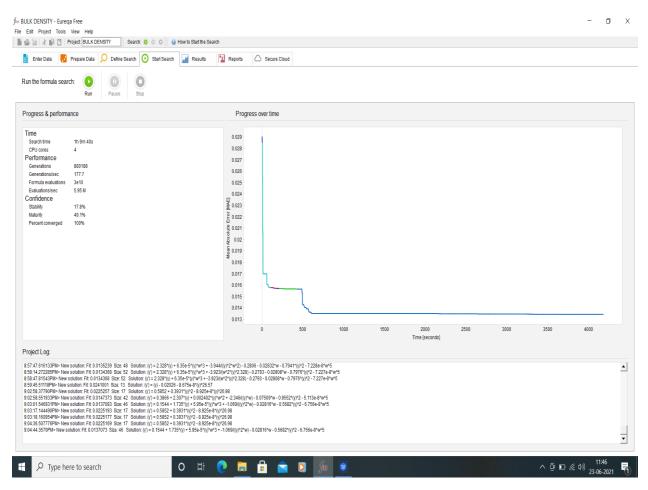


Figure 4.17: Starting search in eureqa software for bulk density function.



Figure 4.18: Results for weight function in eureqa software.

The relationship found is:

$$\gamma_t = \gamma_t - 0.0019(-T)\gamma_t^2$$
(9A)

4.1.4 Degree Of Freezing

Analogous to degree of saturation (S), degree of freezing (F) would tell the degree up to which the soil is frozen. For example, a soil is fully saturated i.e., all the pores are completely filled with water. At -2 degree Celsius, a fraction of water will get crystallized but at -20 degree Celsius, all the water will get frozen. Therefore, degree of freezing will give an idea about the degree up to which the water in pores of soil is frozen. The data obtained from laboratory shown in table 4.5 leads to the following result in eureqa software.

$$S = (w^*G_s)/e$$
(10)

The relationship is given by,

$$F = f(w,e',T)$$
(11)

where F is the degree of freezing computed as,

$$F = V_I / V_v$$
(11A)

Table 4.5 Degree of freezing.

Final void ratio	Temprature	Water content	Specific gravity of soil	Degree of
	(°C)	(%)		Freezing
e'	Т	W	Gs	F
1.95	-5	6	2.803	8.362
1.688	-5	6	2.803	9.732
1.985	-5	10	2.803	13.681
1.688	-5	10	2.803	16.215
1.708	-5	14	2.803	22.420
1.748	-5	14	2.803	22.033
1.532	-5	18	2.803	32.279
1.829	-5	18	2.803	27.230
0.923	-5	22	2.803	66.053
1.074	-5	22	2.803	56.728
1.205	-5	26	2.803	59.717
1.200	-5	26	2.803	59.879
1.074	-5	30	2.803	77.101
1.386	-5	30	2.803	59.953
1.980	-10	6	2.803	8.085
1.698	-10	6	2.803	9.544
1.990	-10	10	2.803	13.512
1.718	-10	10	2.803	15.782
1.713	-10	14	2.803	22.172
1.809	-10	14	2.803	21.135
1.547	-10	18	2.803	31.741

1.829	-10	18	2.803	27.056
1.034	-10	22	2.803	58.578
1.124	-10	22	2.803	53.742
1.104	-10	26	2.803	64.740
1.175	-10	26	2.803	60.606
1.114	-10	30	2.803	73.948
1.477	-10	30	2.803	55.902
1.98	-15	6	2.803	7.9760
1.698	-15	6	2.803	9.437
1.99	-15	10	2.803	13.283
1.718	-15	10	2.803	15.629
1.718	-15	14	2.803	21.929
1.814	-15	14	2.803	21.002
1.567	-15	18	2.803	31.133
1.834	-15	18	2.803	26.897
1.034	-15	22	2.803	58.157
1.124	-15	22	2.803	53.435
1.114	-15	26	2.803	63.889
1.18	-15	26	2.803	59.841
1.114	-15	30	2.803	73.829
1.482	-15	30	2.803	55.422

Steps to run search on software:

1. After creating a table of inputs and output for degree of freezing function, open eureqa software.

2. An excel sheet will gets open, enter the data in different columns like void ratio of frozen soil (e') in column 1, temperature (T) in column 2, water content (w) in column 3, specific gravity (G) of soil at room temperature in column 4 and output i.e., degree of freezing (F) in column 5 as shown in figure 4.19.

Now, click on "prepare data" tab to smoothen the data points if necessary as shown in figure
 4.20.

4. Now, click on "Define data" tab to define the relationship between input and output data. For example, F = f(e', w, T,G). Also, select the operators you want to use in your functions like add, subtract, divide, multiply, exponent, etc. as shown in figure 4.21.

5. Now click on "start search" tab and click on "Run" to start the search as shown in figure 4.22.

6. Now, click on "Results" tab to see the formulae so obtained in the search with their amount of errors as shown in figure 4.23.

i 🥔 🕍 🦂 📫 📋 Proje	ct De	pree of freezing	Search: 📀	0 0 Hov	v to Enter Data									
his Project ×		Enter Data	🧧 Prepare	Data 🔎 Defir	ne Search 🜔	Start Search 🛛	Results	Reports	Secure Cl	oud				
A 🗊 🖡		A	В	С	D	E	F	G	Н		J	K	L	М
Dataset 1	info	Final void ratio	Temperature	Water content	Specific Gracity	Degree of Freezing								
	name	e'	Т	W	G	F								
	1	1.949821508	5	6	2.8025	8.362489556								
	2	1.688062603	5	6	2.8025	9.732266519								
	3	1.985058284	5	10	2.8025	13.68066607								
	4	1.688062603	5	10	2.8025	16.21546351								
	5	1.708197903	5	14	2.8025	22.41961651								
	6	1.748468504	5	14	2.8025	22.03308217								
	7	1.532014025	5	18	2.8025	32.27867133								
	8	1.829009706	5	18	2.8025	27.23030996								
	9	0.922921188	5	22	2.8025	66.05281128								
	10	1.073935941	5	22	2.8025	56.72806052								
	11	1.204815394	5	26	2.8025	59.7167171								
	12	1.199781568	5	26	2.8025	59.87850137								
	13	1.073935941	5	30	2.8025	77.10141448								
	14	1.386033097	5	30	2.8025	59.95257278								
	15	1.980024459	10	6	2.8025	8.084894775								
	16	1.698130253	10	6	2.8025	9.54418659								
	17	1.990092109	10	10	2.8025	13.51227669								
	18	1.718265554	10	10	2.8025	15.78200963								
	19	1.713231729	10	14	2.8025	22.17216494								
Eur <mark>eq</mark> a	20	1.808874405	10	14	2.8025	21.13462388								
Luicya	21	1.5471155	10	18	2.8025	31.74078526								
	22	1.829009706	10	18	2.8025	27.055629								
n days left	23	1.03366534	10	22	2.8025	58.57752859								
in free trial	24	1.124274192	10	22	2.8025	53.74191025								
	25	1.104138892	10	26	2.8025	64.74039966								
nt to continue analyzing your	26	1.174612443	10	26	2.8025	60.60562658								
a with Eureqa? There's still	27	1.114206542	10	30	2.8025	73.94751472								
me to purchase a license.	28	1.476641949	10	30	2.8025	55.90180216								
Find out more	29	1.980024459	15	6	2.8025	7.975907223								
Fille out more	30	1.698130253	15	6	2.8025	9.436911451								
	31	1.990092109	15	10	2.8025	13.28272993								
	32	1.718265554	15	10	2.8025	15.62869122								
Help & Forums	33	1.718265554	15	14	2.8025	21.92942533								
k Start Guide	34	1.813908231	15	14	2.8025	21.00181041								
	35	1.567250801	15	18	2.8025	31.13271521								
o Tutorial	36	1.834043531	15	18	2.8025	26.89732592								
hnique Blog	37	1.03366534	15	22	2.8025	58.15727787								
umentation	38	1.124274192	15	22	2.8025	53.43529793								
umentation	39	1.114206542	15	26	2.8025	63.88880145								

Figure 4.19: Data entering in eureqa software for degree of freezing function.



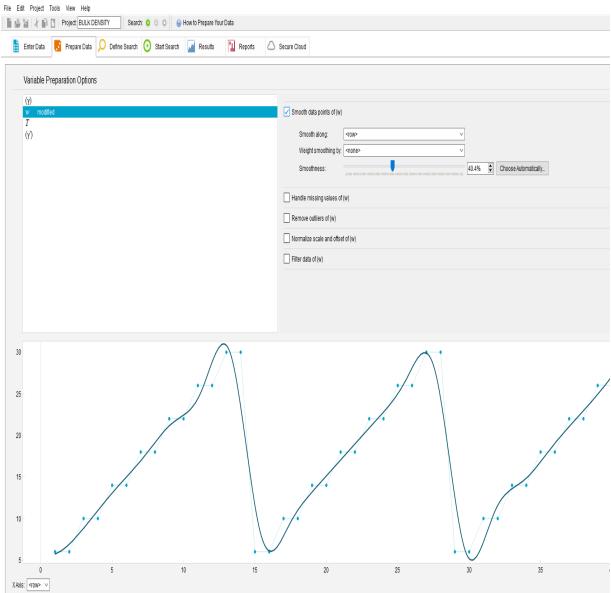


Figure 4.20: Preparing data in eureqa software for degree of freezing function.

🌆 Degree of freezing - Eureqa Free

Enter Data 🚦 F	Prepare Data 🔎 Define Search 🌔	🕑 Start Search 📈 Results	Reports (Secure Cloud			
ne Target Expression	n:						
Search for a formula f()							
hat satisfies the equation	$F = f((\mathbf{e}'), T, w, G)$						
See Examples	·)((•),·,", ", °)						
imary Options:							
ormula building-blocks:	Inditie		Complexity		^	Currently Selected: c_, x_, +, - , *, /, sin, cos, tan, exp, log, ^, sqrt	
	Basic					c_, x_, · , · , · , sin, cos, tan, exp, rog, ; sqit	
	Constant		1				
	Integer Constant		1				
	Input Variable		1				
	✓ Addition ✓ Subtraction		1				
	Multiplication		1				
	Division		2				
	Negation		1				
	Trigonometry		1				
			3				
	Cosine		3				
	✓ Sine ✓ Cosine ✓ Tangent		4				
	Exponential						
	Exponential		4				
	✓ Natural Logarithm		4				
	Factorial		4				
	Power		5				
	Square Root		4				
	Squashing						
	Logistic Function		4				
	Step Function		4				
	Sign Function		4				
	Gaussian Function		4				
	Hyperbolic Tangent		4				
	Error Function		4				
	Complementary Error Function		4		v		
irror metric:	Absolute error (default)						
	Row Weight: <none></none>					•	
	Data Splitting: Treat all data points ec					•	

Figure 4.21: Defining search in eureqa software for degree of freezing function.

🌆 Degree of freezing - Eureqa Free

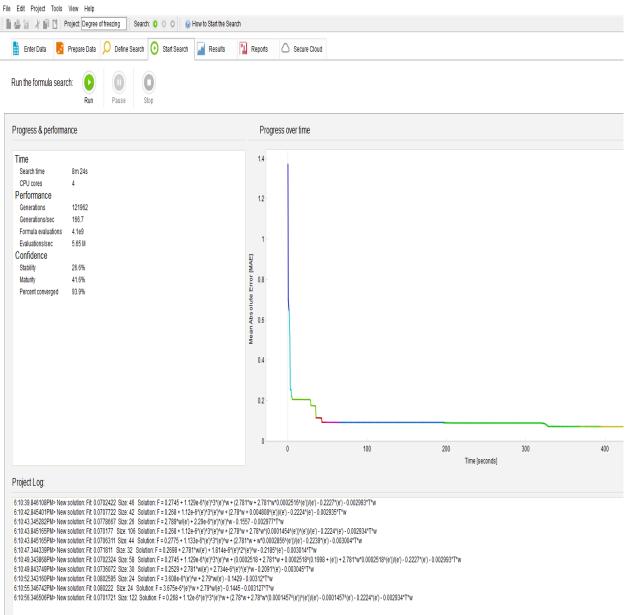


Figure 4.22: Starting search in eureqa software for degree of freezing function.

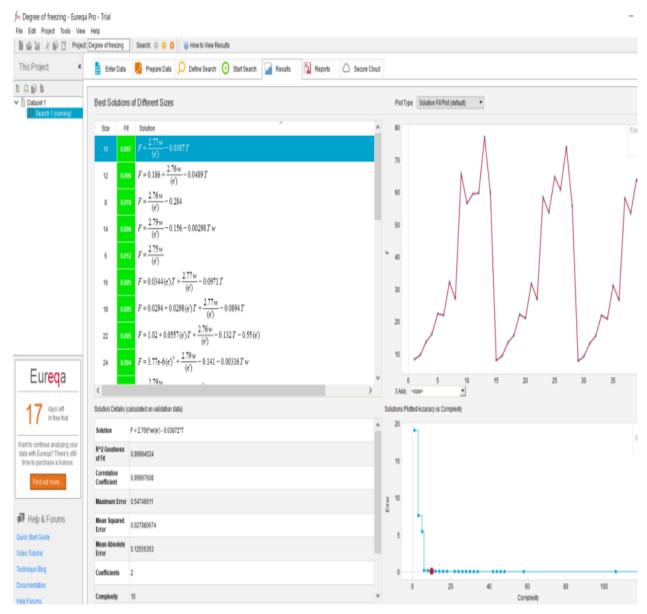


Figure 4.23: Results for degree of freezing in eureqa software.

The relationship found is:

$$F = (2.77 \text{ w/e}') - 0.0387(T)$$
(11B)

4.1.5 Porosity

Porosity of soil is a function of temperature. When temperature becomes negative, the water present in the pores converts into ice which expands in volume. The increased volume of ice at first try to occupy all the void space. Once occupied, on further increase in water content, more water turns into ice which expands more than the void space. This thereby increases the volume of voids. When volume of voids increases, the porosity will also increase. By entering

data obtained from laboratory shown in table 4.6 in the Eureqa software, relationship can be obtained. The function of porosity with temperature can be given by;

$$n' = f(n, w, T)$$
(12)

Unfrozen soil	Water	Temperature	Frozen soil	% change in
porosty	content (%)	(°C)	porosity	porosity
n	w	Т	n'	Z
0.656	6	-5	0.661	0.66
0.625	6	-5	0.628	0.63
0.663	10	-5	0.665	0.67
0.625	10	-5	0.628	0.63
0.616	14	-5	0.631	0.63
0.631	14	-5	0.636	0.64
0.589	18	-5	0.605	0.61
0.640	18	-5	0.647	0.65
0.448	22	-5	0.480	0.48
0.515	22	-5	0.518	0.52
0.544	26	-5	0.546	0.55
0.540	26	-5	0.545	0.55
0.515	30	-5	0.518	0.52
0.558	30	-5	0.581	0.58
0.656	6	-10	0.664	0.66
0.625	6	-10	0.629	0.63
0.663	10	-10	0.666	0.67
0.625	10	-10	0.632	0.63
0.616	14	-10	0.631	0.63
0.631	14	-10	0.644	0.64
0.589	18	-10	0.607	0.61
0.640	18	-10	0.647	0.65
0.448	22	-10	0.508	0.51
0.515	22	-10	0.529	0.53
0.544	26	-10	0.525	0.53
0.540	26	-10	0.540	0.54
0.515	30	-10	0.527	0.53
0.558	30	-10	0.596	0.60
0.656	6	-15	0.664	0.66
0.625	6	-15	0.629	0.63
0.663	10	-15	0.666	0.67
0.625	10	-15	0.632	0.63
0.616	14	-15	0.632	0.63
0.630	14	-15	0.645	0.65
0.589	18	-15	0.610	0.61
0.640	18	-15	0.647	0.65
0.448	22	-15	0.508	0.51

Table 4.6 Porosity at normal (n) and frozen temperature (n')

0.515	22	-15	0.529	0.53
0.544	26	-15	0.527	0.53
0.54	26	-15	0.541	0.54
0.515	30	-15	0.527	0.53
0.559	30	-15	0.597	0.60

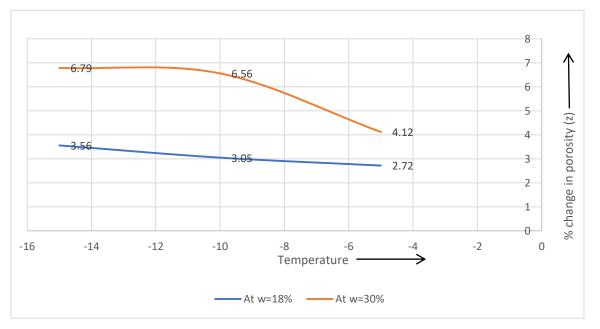


Fig. 4.24 Graph of % change in Porosity (z) vs Temperature.

In figure 4.24, a graph between percentage of change in porosity vs temperature is plotted. The trend of the curve suggest that at more water content, the change is more visible marked with a steep slope. This is because more water will convert into ice and volume of voids will increase sharply in case of higher water content.

Steps to run search on software:

1. After creating a table of inputs and output for porosity function, open eureqa software.

2. An excel sheet will gets open, enter the data in different columns like porosity of soil (n) at normal room temperature in column 1, water content (w) in column 2, temperature (T) in column 3, porosity of frozen soil (n') in column 4 as shown in figure 4.25.

Now, click on "prepare data" tab to smoothen the data points if necessary as shown in figure
 4.26.

4. Now, click on "Define data" tab to define the relationship between input and output data. For example, n' = f(n, w, T). Also, select the operators you want to use in your functions like add, subtract, divide, multiply, exponent, etc. as shown in figure 4.27.

5. Now click on "start search" tab and click on "Run" to start the search as shown in figure 4.28.

6. Now, click on "Results" tab to see the formulae so obtained in the search with their amount of errors as shown in figure 4.29.

i (📋 🛛 Project Po	rosity	Search: 🗿 🔘	0 8 0	How to Enter Data										
	Enter Data	🧾 Prepare I	Data 🔎 Defi	ine Search 🜔 S	tart Search	Results	🚹 Reports	Secure Cloud	đ							
	A	В	С	D	Е	F	G	Н	I.	J	K	L	М	N	0	Р
fo	Porosity	Water content	Temperature	New Porosity												
me	п	w	Т	n'												
1	0.656304347	6	5	0.660996438												-
2	0.625177193	6	5	0.627984855												
3	0.663294767	10	5	0.664998166												
4	0.625177193	10	5	0.627984855												
5	0.616494039	14	5	0.630750767												
6	0.630750767	14	5	0.636161012												
7	0.589553538	18	5	0.60505748												
B	0.640115783	18	5	0.646519417												
9	0.448177534	22	5	0.479957886												
0	0.515472957	22	5	0.51782503												
1	0.544366772	26	5	0.546447289												
2	0.540147945	26	5	0.54540941												
3	0.515472957	30	5	0.51782503												
4	0.558542028	30	5	0.58089433												
5	0.656304347	6	10	0.664432284												
6	0.625177193	6	10	0.629372971												
7	0.663294767	10	10	0.665562142												
8	0.625177193	10	10	0.632118356												
9	0.616494039	14	10	0.63143583												
0	0.630750767	14	10	0.643985506												
1	0.589553538	18	10	0.607399036												
2	0.640115783	18	10	0.646519417												
3	0.448177534	22	10	0.508277011												
4	0.515472957	22	10	0.529250977												
5	0.544366772	26	10	0.524746202												
6	0.540147945	26	10	0.540147945												
7	0.515472957	30	10	0.527009315												
8	0.558542028	30	10	0.596227464												
9	0.656304347	6	15	0.664432284												
0	0.625177193	6	15	0.629372971												
1	0.663294767	10	15	0.665562142												
2	0.625177193	10	15	0.632118356												
3	0.616494039	14	15	0.632118356												
4	0.630750767	14	15	0.644622384												
5	0.589553538	18	15	0.61047826												
6	0.640115783	18	15	0.647147269												
7	0.448177534	22	15	0.508277011												
8	0.515472957	22	15	0.529250977												
9	0.544366772	26	15	0.527009315												

Figure 4.25: Data entering in eureqa software for porosity function.

🌆 Porosity - Eureqa Free

File Edit Project Tools View Help	
🖺 🍰 🌡 🕼 📋 Project Porosity 💿 💿 💿 🚱 How to Prepare Your Data	
🖺 Enter Data 🚦 Prepare Data 🔎 Define Search 🧿 Start Search 📓 Results 📲 Reports 🛆	Secure Cloud
Variable Preparation Options	
n modiled, unnormalized	Smooth data points of ((r1))
Т	
(n) modified	Smooth along: <pre>dvv> </pre>
	Weight smoothing by: enone>
	Smoothness: Choose Automatically_
	Handle missing values of ((n'))
	Remove outliers of ((n'))
	Normalize scale and offset of ((n'))
	Filter data of ((n'))
	$\lambda \qquad \lambda \sim \lambda$
0.64	
0.62	
0.6	
0.58 -	
0.56	
054	
0.52	
0.5	
0.48	
0 5 10 15	20 25 30 35 40
XAxis: <row> ∨</row>	

Figure 4.26: Preparing data in eureqa software for porosity function.

∱≈ Porosity -	Eureqa Free
---------------	-------------

Enter Data 🛃 P	repare Data	🔎 Define Search	Start Search	Results	🚹 Reports	Secure Cloud					
e Target Expression	n;										
earch for a formula <i>f()</i> at satisfies the equation ee Examples	(n') = f(n	.w,T)									
mary Options:											
ormula building-blocks:	Basic	Name			Complexity		^	Currently Selected: c_, n_, X_, + , - , *, /, sin, cos, exp,	log, ^, sqit		
	 ✓ Integer Co ✓ Input Varia ✓ Addition 	nstant ble			1						
	Subtractio Multiplicat Division				1 2						
	Negation Trigonometry Sine Cosine				3						
	Tangent Exponential	1			4						
	V Natural Lo Factorial Power Square Ro				4 4 5 4						
	Squashing Logistic Fi Step Fund	nction			4						
	Sign Fund Gaussian Hyperbolic Error Fund	Function Tangent			4 4 4						
	Complem	entary Error Function			4		v				
rror metric:	Absolute error							•			
		Treat all data points	equally (default)					•		Set Custom	

Figure 4.27: Defining search in eureqa software for porosity function.

🌆 Porosity - Eureqa Free

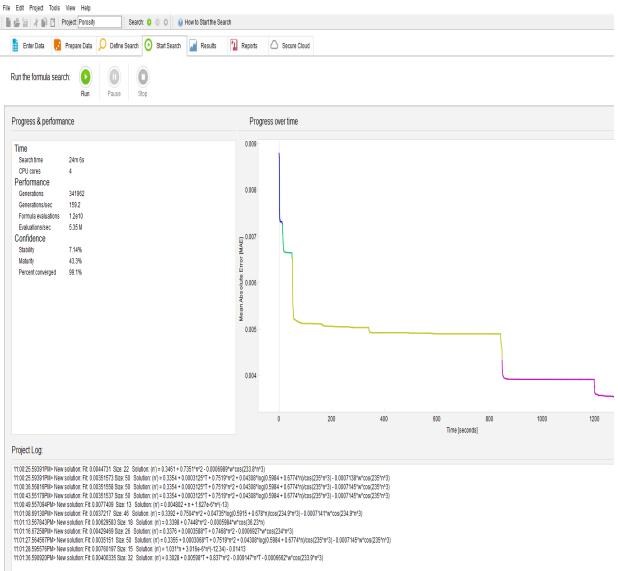


Figure 4.28: Starting search in eureqa software for porosity function.



Figure 4.29: Results for porosity function in eureqa software.

Where,

- n'= Porosity of frozen soil
- T= Temperature in degree Celsius
- n= Porosity at room temperature

The above function has the maximum error of 0.021.

After performing the soil parameters analysis on Eureqa software above, the following relationships are obtained.

a) W' = W + 0.992 w - 0.000208 W(-T)

Using this formula, one can obtain the water content of frozen soil if weight of frozen soil, weight of soil at room temperature and Temperature of frozen soil is known. This formula thereby helps in calculating the water content of soil sample theoretically.

b) e' = e + 0.00503(-T)

Using this formula, one can obtain the void ratio of frozen soil if the void ratio of same soil sample at room temperature is known and the temperature of frozen soil is also known.

c) $\gamma_t' = \gamma_t - 0.0019(-T)\gamma_t^2$

Using this formula, one can calculate the bulk density of frozen soil sample, if the bulk density of same soil sample under room temperature is known along with the temperature at which frozen bulk density is required.

d) F = (2.77 w/e') - 0.0387(T)

Where, F is the degree of freezing which will give the fraction of water which is converted into ice.Using this formula, one can calculate the degree of freezing can be calculated which will tell the amount of freezing occurred in the frozen sample. This term is analogous to degree of saturation in normal room temperature soil which tells the amount of saturation of soil sample.

e) $n' = 0.3376 + 0.0003*T + 0.74*n^2 - 0.00069*w*cos(234*n^3)$

Using this formula, one can obtain the porosity of frozen soil if the porosity of same soil sample at room temperature is known and the temperature of frozen soil is also known.

Where,

W'= Weight of Frozen soil.

W = Weight of soil at room temperature.

T = Temperature at which soil is being investigated.

W = Water content of soil.

- γ_t ' = Bulk unit weight of frozen soil.
- γ_t = Bulk unit weight of soil at room temperature.
- F = Degree of freezing.
- n = Porosity of soil at room temperature.
- n' = Porosity of frozen soil.

4.2 Discussion

There are five relationships that have been established between unfrozen and frozen soils. Formulae so obtained are valid up to -20 degree Celsius and are tested on sandy soil. Below this temperature, the soil parameter becomes constant. Therefore at -20 degree Celsius, the soil becomes fully frozen, and the water present in the voids of the soil is fully transformed into ice by going through crystal phase. The leads of this study can help other authors for detail study of frozen soils and to take the results of this research to dig further into the newly studied branch of Frozen Soil mechanics.

The Novelty of this work lies in the methodology of the work performed, as this it includes a new idea of relating the properties of frozen soil with that of unfrozen soil. From these relationships one can found properties of frozen soil if the data of unfrozen soil is available and vice-versa. While all the research that have been done on frozen soil prior to this is about mechanical, physical, biological and chemical properties but the basic idea about relationship between characteristics of soil like its porosity, void ratio, bulk density, etc at different temperature is forming a gap in the studies that needs to be fulfilled to get the knowledge about how these characteristics changes as the temperature drops to negative values.

In various regions of Northern India, the temperature goes as high to 20 degrees Celsius but for a small period of time, while throughout the major period of time it remains negative. For such sites and for other colder regions, if data is obtained for soil at higher temperature (say, 20 degrees Celsius), then when the same soil becomes frozen then the relationships developed in this research would be helpful to determine the characteristics at frozen temperature.

For example, if the value of void ratio is 0.6 at 25° Celsius, its water content is 25% and it is required to calculate void ratio and degree of freezing of same soil at same density at -20° Celsius. To calculate this, one needs not to measure void ratio of frozen soil if he have a handy formula with him.

$$e' = e + 0.00503(-T)$$
(from 8A)

And,

$$F = (2.77 \text{ w/e}') - 0.0387(T)$$
(from 11B)

Therefore, the value of void ratio at -20 degree Celsius of the given soil sample will be 0.7006 and the degree of freezing will be 99.617% which means at -20 degree Celsius, almost all the water content is converted into ice and the soil tends to be fully frozen.

Likewise, formulae for porosity, weight of soil, bulk density developed in this research will help the site engineers to have reliable formulae to measure these parameters of frozen soil if unfrozen soil data is available. The formulae will also help in determining several other functions that uses these parameters.

Chapter 5 Conclusions and Recommendations

5.1 Conclusions

- The formulae obtained will help cover the gaps in the literature review by providing basic information about frozen soil and its parameters.
- This study can be used for practical purpose as the data obtained by the author in the laboratory is true and self-verified. The constructions done in colder regions can use the formulae so obtained to carry out soil investigation and understand the frozen soils in a better way. The results so obtained will help to know any parameter of soil at different freezing temperature if there values at normal room temperature is known or vice-versa.
- The study can play a vital role in determining the parameters of frozen soil and can also prove helpful in further studies on frozen soils.
- The study inculcate all the references and thorough reviewing of the literature reviews related to the field of frozen soil.

5.2 Recommendations for the future work

As this study is for the locally available soil i.e., sand, the work can be further extended to different soil types like clay, silt and gravels. By following same experimental procedure and similar simulation technique, the results for different soil types can be found.

Also, this work opens up a door to different new possibilities. By using these results as to calculate different parameters and then utilizing those parameters to estimate the settlement, bearing capacity and other such properties of soil. For example, void ratio is used in the calculation of soil settlement, therefore, settlements of frozen soil may depend on its void ratio too.

References

- Altuhafi, F.; Baudet, B.. A hypothesis on the relative roles of crushing and abrasion in the mechanical genesis of a glacial sediment. Engineering Geology. 2011. 10.1016/j.enggeo.2011.03.002.
- 2. Andersland, O. B., and Ladanyi, B.; Frozen Ground Engineering. New York: Wiley; 2004.
- 3. Andersland, O.B., Ladanyi, B. An introduction to frozen ground engineering. Chapman and Hall, New York. **1984**.
- Azmatch, T.F., Sego, D.C., Arenson, L.U., and Biggar, K.W. Using soil freezing characteristic curve to estimate the hydraulic conductivity function of partially frozen soils. Cold Reg. Sci. Technol. 2012. 83–84: 103–109.
- Baker, J.M., and Spaans, E.J.A. Mechanics of meltwater above and within frozen soil. In: CRREL Spec. Rep. 79. US Army Corps of Eng., Cold Regions Res. Eng. Lab., Hanover, NH.1997. p. 31–36.
- 6. Bakulin, F.G. and Zhukov; Deformations of Frozen Dispersed Soils during Thawing; 1955.
- Burt, T.P., and Williams, P.J. Hydraulic conductivity in frozen soils. Earth Surf. Processes. 1976. 9: 411–416.
- Chang, Dan; Lai, Yuanming; Gao, Jianqiang. An investigation on the constitutive response of frozen saline coarse sandy soil based on particle breakage and plastic shear mechanisms.Cold Regions Science and Technology. 2018. 10.1016/j.coldregions.
- 9. Edwards, A. C., Cresser, M. S.; Freezing and Its effect on Chemical and Biological Properties of Soil; **1992**.
- 10. Eranti, E., and Lee, G. C.. Cold Region Structural Engineering. USA: McGraw Hill. 2000.
- 11. Eureqa Software Created by: Michael Schmidt and Hod Lipson. 2009.
- 12.Fish, A.M.: Strength of frozen soil under a combined stress state. In: Proc., 6th Int. Symp. on Ground Freezing, Beijing, China, **1991**. vol. 1, pp. 135–145.
- 13.Janoo, V.; Shoop, S. Study of shear strength of silty clay under freeze-thaw cycle. Earth Environment Science. **2019.** 330 042009.
- 14. Khoroshilov, K., Katarov, V.; Gavrilov, Timmo; Kolesnikov, Gennady. Strength and Elastic Modulus of Frozen Sandy Soil as a Material for a Logging Road. Resources and Technology. 2019. 10.15393/j2.art.2019.4742.
- 15.Li, D.W., Wang, R.H., Zhao, Y.H., Hu, P. Research on parabolic yield-surface creep constitutive model of artificial frozen soil. Rock Soil Mech. **2007**. 28(9), 1943–1948.

- 16.Liu, Tiejun; Xu, Xiangtian; Yang, Jie. Experimental study on the effect of freezing-thawing cycles on wind erosion of black soil in Northeast China. Cold Regions Science and Technology. 2017. 10.1016/j.coldregions.
- Merzlyakov, V. Physical and Mechanical Conditions for Primary Frost Crack Formation.
 Soil Mechanics and Foundation Engineering. 2016. 10.1007/s11204-016-9389-1.
- 18.Qi, Jilin; State-of-art of research on mechanical properties of frozen soils; Springer Series in Geomechanics and Geoengineering, pp. 129-133, **2010.**
- Shen, Mingde; Zhou, Zhiwei; Zhang, Shujuan. Effect of stress path on mechanical behaviors of frozen subgrade soil. Road Materials and Pavement Design. 2021. 10.1080/14680629.2020.1869583.
- 20.Smith; Chen; Haofeng. Lower and Upper Bound Shakedown Analysis of Structures With Temperature-Dependent Yield Stress. Journal of Pressure Vessel Technology.2010. 132. 10.1115/1.4000369.
- 21. Tulaczyk, Slawek; Kamb, W.; Engelhardt, Hermann. Basal mechanics of Ice Stream B, West Antarctica 1. Till mechanics. Journal of Geophysical Research. 2000. 10.1029/1999JB900329
- 22. Viklander, P. Influence of cycles of freezing and thawing on the permeability in soils. Lulea University of Technology. **1995**. SE-971 87.
- 23.Xu, Xiangtian; Lai, Yuanming; Dong, Yuanhong; Qi, Jilin. Laboratory investigation on strength and deformation characteristics of ice-saturated frozen sandy soil. Cold Regions Science and Technology. 2011. 10.1016/j.coldregions.2011.07.005.
- 24.Zhu, Y.L., Carbee, D.L. Creep behavior of frozen silt under constant uni-axial stress In: Proc., 4th Int. Conf. on Permafrost, Fairbanks, Alaska, pp. **1984**. 1507–1512.
- 25.Zhu, Zhi-Wu; Ma, Yue; Zhang, Hai-Dong; Song, Weidong; Gan, Yuan-Chao. Evaluation of thermal effects and strain-rate sensitivity in frozen soil. Thermal Science. 2014. 10.2298/TSCI1405631Z.

Appendix

Publication: Geotechnical and Geological Engineering journal in Springer Publication. The status of the paper is 'communicated'.

Kshitij gaur; Anil kumar sahu. Experimental studies on frozen soil. Geotechnical and geological engineering.

