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Analysis and Re-solution of the Transportation Problem: A Linear Programming Approach

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DECLARATION

I, ROHIT, 2K20/MSCMAT/25 & MAHENDER HARSOLIA, 2K20/MSCMAT/16 student of M.Sc. Mathematics, hereby declare that the project Dissertation titled "Analysis and Re-solution of the Transportation Problem: A Linear Programming Approach" which is submitted by me to the Department of Applied Mathematics Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Science, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

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CERTIFICATE

I hereby certify that the Project Dissertation titled " Analysis and Re-solution of the Transportation Problem: A Linear Programming Approach " which is submitted by ROHIT, Roll No. 2K20/MSCMAT/25 & MAHENDER HARSOLIA, Roll No. 2K20/MSCMAT/16 [DEPARTMENT OF APPLIED MATHEMATICS], Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Technology/Bachelor of Technology, is a record of the project work carried out by the students under my supervision. To the best of my knowledge this work has not been submitted in part or full for any Degree or Diploma to this University or elsewhere.

Place: Delhi **DR. DHIRENDRA KUMAR**

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ABSTRACT

The constraint structure of the transportation problem is so important that the literature is filled with efforts to provide efficient algorithms for solving it. The intent of this dissertation is to present an efficient approach for finding an initial basic feasible solution of the transportation problem (TP). The proposed method is illustrated with several numerical examples. An analysis to identify the reason of different solutions by different methods is also carried out. Finally a comparative study is made, and we observed that the method presented herein gives a better result.

ACKNOWLEDGEMENT

The satisfaction of successful completion of any task would be incomplete without mentioning the people who made it possible and whose constant guidance and encouragement crown all of my efforts with success. I express my deep sense of gratitude to my parents, my brother and my guide Dr. Dhirendar Kumar, Assistant Professor, Department of Applied Mathematics, Delhi Technological University for her inspiration, constant assistance, valuable suggestions, sympathetic advice, fruitful conversation and unparalleled encouragement made throughout the course of study, without which this piece of work would not have taken its present shape.

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CHAPTER 1

INTRODUCTION

The transportation problem (TP) is a special class of linear programming problems, which deals with the transportation of a single homogeneous product from several sources (production or supply centers) to several sinks (destinations or warehouses). When addressing a TP, the practitioner usually has a given capacity at each supply point and a given requirement at each demand point. The main objective of TP is to determine the amounts transported from each source to each sink to minimize the total transportation cost while satisfying the supply and demand restrictions. The basic steps for obtaining an optimum solution to a TP are :

Step 1: Mathematical formulation of the transportation problem;

Step 2: Determining an initial basic feasible solution

Step 3: To test whether the solution is an optimal one. If not, to improve it further

till the optimality is achieved.

In this study, we have focused on Step 2 above in order to obtain efficient initial basic feasible solutions for the transportation problem. Several heuristics viz. Row Minimum Method (RMM), Column Minimum Method (CMM), Northwest Corner Method (NWCM), Least Cost Method (LCM), Extremum Difference Method (EDM) and Vogel's Approximation Method (VAM) have been developed to find an initial basic feasible solution to a transportation problem. Although some heuristics can find an initial basic feasible solution quickly, often the solution they find is not especially good in terms of minimizing total cost. Other heuristics may not find an initial basic feasible solution as quickly, but the solution they find is often good in terms of minimizing total costs .

VAM is based on the concept of penalty cost. A penalty cost is the difference between the lowest cell cost in the row or column and next to lowest cell cost in the same row or column. It then makes allocation as much as possible to the minimum cost cell in the row or column with the largest penalty cost.

In case of EDM a penalty cost is the difference between the highest cell cost in the row or column and the lowest cell cost in the same row or column. It then makes allocation as much as possible to the minimum cost cell in the row or column with the largest penalty cost.

In Serdar Korukoğlu and Serkan Balli proposed Improved VAM by introducing total opportunity cost (TOC) matrix. The TOC matrix is obtained by adding the ‘row opportunity cost matrix’ (for each row, the smallest cost of that row is subtracted from each element of the same row) and the ‘column opportunity cost matrix’ (for each column, the smallest cost of that column is subtracted from each element of the same column). It then calculates penalty costs (difference between the lowest cell cost in the row or column and next to lowest cell cost in the same row or column) and makes allocation as much as possible to the cell having lowest unit transportation cost among the row or column containing highest three penalty costs.

In this thesis, we calculate the pointer costs by taking difference of highest cost and next smaller to the highest cost for each row and each column and consider highest three pointer costs. We then make allocation to the cell having lowest unit transportation cost among the row and column containing highest three pointer costs. If more than one cell contain lowest-cost, we allocate to the cell where allocation is maximum. We illustrate the proposed method numerically and ordeal the optimality of the presented method by Modified Distribution method and Stepping Stone method . As a final point, an analysis to identify the grounds of different solutions by different methods is also carried out.

1.1.Background of the study

One of the earliest and most abundant applications of linear programming techniques has been the formulation and solution of the transportation problems as a linear programming problem.

The standard form of the problem was first formulated, along with a constructive solution, by Frank L. Hitchcock in 1941, when he presented a study entitled ‘The Distribution of a Product from several sources to Numerous Localities’. This paper sketched out the partial theory of a technique foreshadowing the simplex method; it did not exploit special properties of a transportation problem except in finding starting solutions.

In 1947, another investigator T.C. Koopmans, as a member of the Combined Shipping Board during World War II, also presented a study called ‘Optimum Utilization of the Transportation System’. This historic paper was based on his wartime experience. Because of this and the work done earlier by Hitchcock, the classical case is often referred to as the Hitchcock Koopmans Transportation Problem.

In 1963, linear programming formulation and the associated systematic method of solution were first developed by G.B. Dantzig .

1.2.Why we choose Transportation Problem

At this moment what is the main problem of our country? It is high cost of commodity. For this purpose we analysis the total production cost of a product and we get two types of cost :

(i) Fixed cost and

(ii) Variable cost.

In fixed cost we have Salaries of Staff, Rent of building, Depreciation of machines and equipment, Interest on capital invested and Insurance.

In Variable cost we have Materials Cost, Transportation Cost, Fuel Cost, Packing Cost, Maintenance Cost, Advertisement Cost and Commission to Salesman.

Total Production Cost	
Fixed Cost	**Variable Cost**
Salaries of Staff	Materials Cost
Rent of building	Transportation Cost
Depreciation of machines and equipment	Commission to Salesman
Interest on capital invested	Packing Cost
Insurance	Maintenance Cost
	Advertisement Cost
	Fuel Cost

Table 1: Total Production Cost Analysis

Among these transportation cost has a great affect to the production cost. If we can reduce the total transportation cost while satisfying the demand and supply restriction, it will be possible to supply product with comparably low cost. As a result the price of commodity will reduce by maintaining the quality of the product.

1.3. Structure of the study

The overall review of this thesis introduces an efficient algorithm for finding an initial basic feasible solution of a transportation problem. This dissertation includes Three chapters. Each Chapter provides the sequential ideas from beginning to end are as follows:

- In Chapter two- Transportation problem which describes, also formulates the transportation problem and its basic integrants.
- Chapter three presents detailed algorithm of existing solution procedure for finding an initial basic feasible solution of a transportation problem and also provides the algorithm to test the optimality of the solution.

Finally this dissertation includes the References of different authors and links from where we collect the important notes and technique to discuss in this dissertation to achieve our maximum goal of minimizing total transportation cost of a transportation problem.

CHAPTER 2

TRANSPORTATION PROBLEM

2.1 Definition

The transportation problem is a special class of linear programming problems, which deals with the transportation of a single homogeneous product from several sources (production or supply centers) to several sinks (destinations or warehouses). When addressing a Transportation problem, the practitioner usually has a given capacity at each supply point and a given requirement at each demand point. Many decision problems, such as production inventory, job scheduling, production distribution and investment analysis can be formulated as Transportation Problems. Good financial decisions concerning facility location also attempt to minimize total transportation and production costs for the entire system. The transportation problem can be represented as a single objective transportation problem or as a multi-objective transportation problem.

2.2 Main Objectives

The main objective of Transportation Problem is to determine the amounts transported from each source to each sink to minimize the total transportation cost while satisfying the supply and demand restrictions.

2.3 Basic Steps

The basic steps for obtaining an optimum solution to a Transportation Problem are:

Step 1: Mathematical formulation of the transportation problem;

Step 2: Determining an initial basic feasible solution;

Step 3: To test whether the solution is an optimal one. If not, to improve it further till the optimality is achieved.

2.4 Mathematical Formulation

The transportation problem can be stated as an allocation problem in which there are m sources (suppliers) and n destinations (customers). Each of the m sources can allocate to any of the n destinations at a per unit carrying cost c_{ij} (unit transportation cost from source i to destination j). Each sources has a supply of S_i units, $1 \leq i \leq m$ and each destination has a demand of d_j units, $1 \leq j \leq n$. The objective is to determine which routes are to be opened and the size of the shipment on those routes, so that the total transportation cost of meeting demand, given the supply constraints, is minimized.

2.4.1 The decision variables

A transportation problem is a complete specification of how many units of the product should be transported from each source to each destination. Therefore, the decision variables are:

x_{ij} = The amount of the shipment from source i to destination j,

where $i = 1, 2, \ldots\ldots, m$ and $j = 1, 2, \ldots\ldots, n$

2.4.2 The objective function

Consider the shipment from warehouse i to destination j. For any i and any j, the transportation cost per unit is c_{ij} ; and the size of the shipment is x_{ij}. Since we assume the cost function is linear, the total cost of this shipment is given by $c_{ij}\, x_{ij}$. Summing over all i and all j now yields the overall transportation cost for all source-destination combinations.

That is, our objective function is:

$$\text{Minimize:} \qquad z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

2.4.3 The constraints

Considering warehouse i. The total outgoing shipment from this warehouse is the sum $x_{i1}+x_{i2}+\ldots\ldots+x_{in}$. In summation notation, this is written as $\sum_{j=1}^{n} x_{ij}$. Since the total supply from warehouse i is S_i, the total outgoing shipment cannot exceed S_i. That is, we must require

$$\sum_{j=1}^{n} x_{ij} \le S_i \quad ; \qquad i=1,2,\ldots\ldots,m$$

Again, considering destination j. The total incoming shipment at this outlet is the sum $x_{1j}+x_{2j}+\ldots\ldots+x_{mj}$. In summation notation, this is written as $\sum_{i=1}^{m} x_{ij}$. Since the demand at outlet j is d_j, the total incoming shipment should not be less than d_j. That is, we must require

$$\sum_{i=1}^{m} x_{ij} \ge d_j \quad ; \qquad j=1,2,\ldots\ldots,n$$

Of course, as physical shipments, the x_{ij} 's should be non-negative i.e. $x_{ij} \ge 0$.

Then the linear programming model representing the transportation problem is generally given as

$$\text{Minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} .$$

$$\text{subject to} \qquad \sum_{j=1}^{n} x_{ij} \le S_i \quad ; \quad i=1,2,\ldots\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \ge d_j \quad ; \quad j=1,2,\ldots\ldots,n$$

$$x_{ij} \ge 0. \qquad \text{for all } i \text{ and } j .$$

In mathematical terms the above problem can be expressed as finding a set of x_{ij}'s, $i=1,2,\ldots\ldots,m$; $j=1,2,\ldots\ldots,n$ to

$$\text{Minimize } z=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\,x_{ij}.$$

$$\text{subject to } \sum_{j=1}^{n} x_{ij}=S_i \quad ; \quad i=1,2,\ldots\ldots,m$$

$$\sum_{i=1}^{m} x_{ij}=d_j \quad ; \quad j=1,2,\ldots\ldots,n$$

$$x_{ij}\geq 0. \qquad \text{for all } i \text{ and } j.$$

2.5 Network Representation

We represent each source and destination by a node, the amount of supply at source i by S_i, the amount of demand at destination j by d_j, the unit transportation cost between source i and destination j by c_{ij} and the amount transported from source i to destination j by x_{ij}. The arcs joining the source and destination represent the route through which the commodity is transported. Then the general transportation problem can be represented by the network as in the following figure.

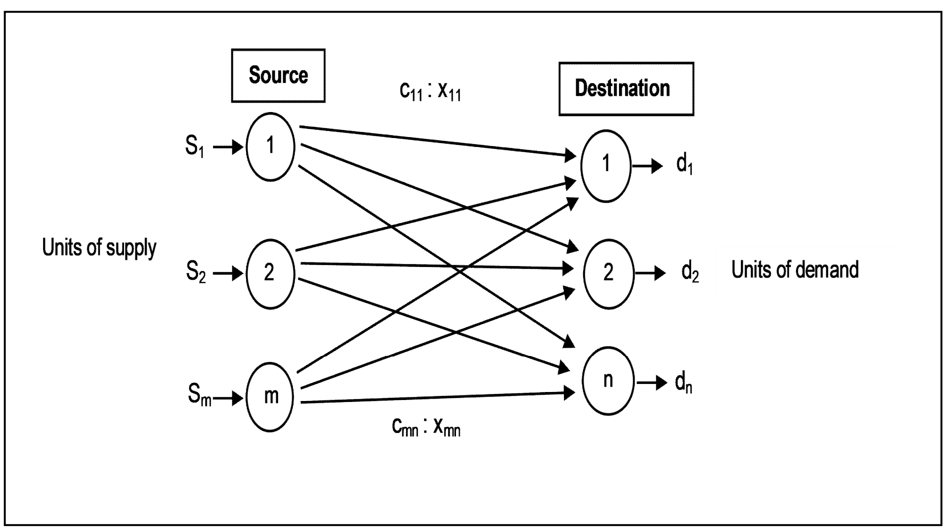


Figure 1: Network Representation of Transportation Problem

2.6 Balanced Transportation Problem

When the total supply amount is equal to the total demand then this transportation problem is called the Balanced Transportation problem. That is in the balance transportation problem

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} d_j$$

where $\sum_{i=1}^{m} S_i$ is the total supply amount and $\sum_{j=1}^{n} d_j$ is the total demand amount.

2.7 Unbalanced Transportation Problem

An unbalanced transportation problem is one in which the total supply of the sources and the total demand at destination centre are not equal. This creates the following two situations:

i. When the total capacity of the origins exceeds the total requirement of destinations, a dummy destination is introduced in the transportation table which absorbs the excess capacity. The cost of shipping from each origin to this dummy destination is assumed to be zero. The insertion of a dummy destination establishes equality between the total sources capacities and total destination requirements.

ii. When total capacity of origins is less than the total requirement of destinations, a dummy origin is introduced in the transportation table to meet the excess demand. The cost of shifting from the dummy origin to each destination is assumed to be zero. The introduction of a dummy origin in this case establishes the equality between the total capacity of sources and the total requirement of destinations.

2.8 Matrix Terminology

The matrix used in the transportation models consists of squares called "cells", which when stacked from 'columns' vertically and rows 'horizontally'. The cell allocated at the intersection of a row and a column is designated by its row and column heading. Thus the cell located at the intersection of row A and column 4 is called cell (A, 4). Unit transportation costs are positioned in each cell. The availabilities of each source are placed in the last column while the requirements are placed in the last row.

Sources	**Warehouses**					**Supply**
	1	**2**	**3**	**4**	**5**	15
A	2	5	8	1	9	25
B	5	6	8	7	10	20
C	1	2	5	8	2	10
Demand	16	10	18	12	14	

Table 2: Matrix Representation of Transportation Problem

2.9 Some Basic Definitions

i. **Source**

It is a location from which shipment are dispatched.

ii. **Destination**

It is the location to which shipment are transported.

iii. **Unit Transportation Cost**

It is the cost of transporting one unit of the consignment from a source to a destination.

iv. **Feasible solution**

A set of non-negative values x_{ij}, $i = 1, 2, \ldots\ldots, m$; $j = 1, 2, \ldots\ldots, n$ that satisfies the constraints is called a feasible solution to the transportation problem.

v. **Basic feasible solution**

A feasible solution that contains no more than $m + n - 1$ non-negative allocation is called a basic feasible solution to the transportation problem.

vi. **Optimal solution**

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

vii. Non-degenerate basic feasible solution

A basic feasible solution to a $(m \times n)$ transportation problem that contains exactly $m+n-1$ allocations in independent position.

viii. Degenerate basic feasible solution

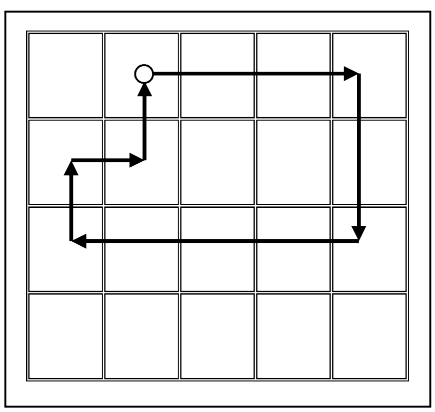
A basic feasible solution that contains less than $m+n-1$ non-negative allocations.

ix. Loop

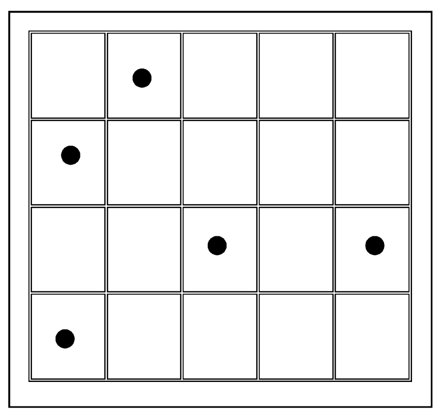
An ordered set of at least four cells in transportation table is said to be a loop provided

a) Any two adjacent cells of the ordered set lie either in the same row or in the same column.

b) No three or more adjacent cells in the ordered set lie in the same row or column.



A loop is formed



Do not form a loop

x. Pointer Cost

Difference of highest cell cost and next smaller to the highest cell cost for each row and column. In case of row, we call it row pointer and for column, it is column pointer.

xi. Independent and Non-independent allocation

If the occupied cells do not make a closed loop, then the allocations are said to be independent. Again a loop may not involve all the allocation. Such allocation with which a loop can be formed are known as non-independent allocation.

xii. Algorithm

A step-by-step problem-solving procedure, especially an established, recursive computational course of action for solving a problem in a finite number of steps is called an Algorithm.

CHAPTER 3

EXISTING SOLUTION PROCEDURE

The transportation problem has been formulated by various investigators and solved to various degrees. The systematic method of solution was first given by Dantzig. There are many methods of determining an initial basic feasible solution. Well-known heuristics are North West Corner Method, Row Minimum Method, Column Minimum Method, Least Cost Method, Vogel's Approximation Method and Extremum Difference Method. These methods differ in the quality of the starting basic solution they produce and better starting solution yields a smaller objective value. Detailed algorithms of these methods are given below:

3.1 Row Minimum Method (RMM)

Row Minimum Method is used when the purpose of completing the warehouse No. 1 and then the next. Row minimum cost is useful in small number of supply and when the cost of transportation on supply. The cost of transportation is less than North West Corner Method. Detailed process is given below:

Step 1: Consider the first row and identify the lowest cost cell of the first row.

Step 2: Allocate as many units as possible equal to the minimum between available supply and demand so that either the capacity of the first supply is exhausted or the demand of the jth distribution center is satisfied or both. Then three cases arises:

Case 1: If the capacity of the first supply is satisfied, cross off the first row and move below to the second row.

Case 2: If the demand of the jth distribution center is satisfied, cross off the jth column and reconsider the first row with the remaining demand.

Case 3: If the capacity of the first supply as well as the demand of the jth distribution center is completely satisfied, make a zero allocation in the second lowest cost cell of the first row. Cross off the row as well as the jth column and move below to the second row.

Step 3: Continue the process until all rows and column are satisfied.

3.2 Column Minimum Method (CMM)

Column Minimum Method is used when the purpose of completing the demand No. 1 and then the next. Column minimum cost is useful in small number of demand and when the cost of transportation on demand. The cost of transportation is less than North West Corner Method. The standard instructions are paraphrased below:

Step 1: Consider the first column and identify the lowest cost cell of the first column.

Step 2: Allocate as many units as possible equal to the minimum between available supply and demand so that either the demand of the first distribution center is satisfied or the capacity of the ith supply is exhausted or both. Then three cases arises:

Case 1: If the demand of the first distribution center is satisfied, cross off the first column and move right to the second column.

Case 2: If the capacity of ith supply is satisfied, cross off the ith row and reconsider the first column with the remaining demand.

Case 3: If the demand of the first distribution center as well as the capacity of the ith supply is completely satisfied, make a zero allocation in the second lowest cost cell of the first column. Cross off the column as well as the ith row and move right to the second column.

Step 3: Continue the process until all columns are satisfied.

3.3 North West Corner Method (NWCM)

A particularly simple method of determining an initial basic feasible solution of a transportation problem is the so-called North West Corner Method, introduced by Charnes and Cooper, where the basic variables are selected from the North West Corner

(i.e. top left corner). The standard North West corner rule instructions are paraphrased below.

Step 1: Select the north west (upper left hand) corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand.

Step 2: Adjust the supply and demand numbers in the respective rows and columns.

Step 3: If the demand for the first cell is satisfied, then move horizontally to the next cell in the second row.

Step 4: If the supply for the first row is exhausted, then move down to the first cell in the second row.

Step 5: If for any cell, supply equals demand, then the next allocation can be made in either in the next row or column.

Step 6: Continue the process until all supply and demand values are exhausted.

North west corner method provides quick solution because computations take short time but yields a bad solution because it is very far from optimal solution.

3.4 Least Cost Method (LCM)

The heuristics we describe for finding an initial basic feasible solution to a transportation problem is called the least cost method. It is also known as Matrix minima Method or Minimum Cost method or Best Cell Method. In this method, the basic variables are chosen according to the unit transportation cost. This heuristics strikes a compromise between finding a feasible solution quickly and finding a feasible solution that is optimal or very close to the optimal solution. Detailed processes are given below:

Step 1: Identify the box having minimum unit transportation cost in the cost matrix of the transportation table. If a tie occurs, choose any one of them randomly.

Step 2: Allocate as many units as possible equal to the minimum between available supply and demand.

Step 3: Adjust the supply and demand numbers in the respective rows and columns.

Step 4: No further consideration is required for the row or column which is satisfied. If both the row and column are satisfied at a time, delete only one of the two, and the remaining row or column is assigned zero supply (or demand).

Step 5: Continue the process for the resulting reduced transportation table until all the rim conditions are satisfied.

3.5 Vogel's Approximation Method (VAM)

The Vogel Approximation (Unit penalty) method is an iterative procedure for computing an initial basic feasible solution of a transportation problem. This method is preferred over the three methods i.e. North West Corner Rule, Row minimum Method and Column Minimum Method, because the initial basic feasible solution obtained by this method is either optimal or very close to the optimal solution. The standard instructions are paraphrased below:

Step 1: If either (total supply > total demand) or (total supply < total demand), balance the transportation problem.

Step 2: Determine the penalty cot for each row by taking difference of lowest cell in the row and next to lowest cell cost in same row and put in front of the row on the right. In a similar fashion, calculate the penalty cost for each column and write them in the bottom of the cost matrix below corresponding columns.

Step 3: Choose the highest penalty costs and observe the row or column to which this corresponds. If a tie occurs, choose any one of them randomly.

Step 4: Make allocation min (S_i,d_j) to the cell having lowest unit transportation cost in the selected row or column.

Step 5: No further consideration is required for the row or column which is satisfied. If both the row and column are satisfied at a time, delete only one of the two, and the remaining row or column is assigned zero supply (or demand).

Step 6: Calculate fresh penalty costs for the remaining sub-matrix as in Step 2 and allocate following the procedure of Steps 3, 4 and 5. Continue the process until all rows and columns are satisfied.

Step 7: Compute total transportation cost for the feasible allocations using the original balanced transportation cost matrix.

3.6 Extremum Difference Method (EDM)

Step 1: Determine the penalty cost for each row by taking difference of highest cell cost in the row and lowest cell cost in the same row and put in front of the row on the right. These numbers are called row penalties. In a similar fashion, calculate the column penalties for each column and write them in the bottom of the cost matrix below corresponding columns.

Step 2: Choose the highest penalty costs and observe the row or column to which this corresponds. If a tie occurs, choose any one of them randomly.

Step 3: Make allocation min (S_i,d_j) to the cell having lowest unit transportation cost in the selected row or column.

Step 4: Ignore the row and column, for further consideration, which is satisfied. If both the row and column are satisfied at a time, delete only one of the two, and the remaining row or column is assigned zero supply (or demand).

Step 5: Calculate fresh penalty costs for the remaining sub-matrix as in Step 2 and allocate following the procedure of Steps 2, 3 and 4.

Step 6: Continue the process until all rows and columns are satisfied.

Step 7: Compute total transportation cost for the feasible allocations using the original balanced transportation cost matrix

This technique is applicable to find an initial basic feasible solution of a transportation problem which is also very much nearer to optimal solution and in most of the case it yields directly the optimal solution. The following steps summarize the approach.

3.7 Optimality Test

Make an optimality test to find whether the obtained feasible solution is optimal or not. An optimality test can, of course, be performed only on that feasible solution in which

a. Number of allocation is m+n-1, where m is the number of rows and n is the number of columns.
b. These m+n-1 allocation should be in independent positions.

Now test procedure for optimality involves examination of each vacant cell to find whether or not making an allocation in it reduces the total transportation cost. The two methods used for this purpose are the *Stepping-Stone Method* and *The Modified distribution Method (MODI) or u-v method*.

3.7.1 Stepping Stone Method

It is a method for finding the optimum solution of a transportation problem. The detailed process is given here:

Step 1: Determine an initial basic feasible solution using any one of the following method:

i. Row Minimum Method;
ii. Column Minimum Method;
iii. North West Corner Method;
iv. Least Cost Method;
v. Vogel's Approximation method;
vi. Extremum Difference Method;
vii. Proposed Method.

Step 2: Make sure that the number of occupied cells is exactly equal to m+n-1, where m is the number of rows and n is the number of columns.

Step 3: Select an unoccupied cell. Beginning at this cell, trace a closed path, starting from the selected unoccupied cell until finally returning to that same unoccupied cell.

Step 4: Assign plus (+) and minus (-) signs alternatively on each corner cell of the closed path just traced, beginning with the plus sign at unoccupied cell to be evaluated.

Step 5: Add the unit transportation costs associated with each of the cell traced in the closed path. This will give net change in terms of cost.

Step 6: Repeat steps 3 to 5 until all unoccupied cells are evaluated.

Step 7: Check the sign of each of the net change in the unit transportation costs. If all the net changes computed are greater than or equal to zero, stop; an optimal solution has been reached. Otherwise, go to Step 8.

Step 8: Select the unoccupied cell having the most negative net cost change and determine the maximum number of units that can be assigned to this cell. The smallest value with a negative position on the closed path indicates the number of unit that can be shipped to the entering cell. Add this number to the unoccupied cell and to all other cells on the path marked with a plus sign. Subtract this number from cells on the closed path marked with a minus sign.

Step 9: Go to Step 3.

3.7.2 The Modified Distribution Method (MODI) or u-v method

The Modified Distribution Method also known as MODI method or u-v method provides a minimum cost solution to the transportation problem. In the Stepping Stone Method, we have to draw as many closed paths as equal to the unoccupied cells for their evaluation. To the contrary, in MODI method, only closed path for the unoccupied cell with most negative opportunity cost is drawn. The method, in outline, is:

Step 1: Determine an initial basic feasible solution using any one of the following method:

i. Row Minimum Method;
ii. Column Minimum Method;
iii. North West Corner Method;
iv. Least Cost Method;
v. Vogel's Approximation Method;
vi. Extremum Difference Method;
vii. Proposed Method.
viii.

Step 2: Make sure that the number of occupied cells is exactly equal to m+n-1, where m is the number of rows and n is the number of columns.

Step 3: Compute the row indices, u_i and column indices v_j.

Step 4: Determine the opportunity cost for non basic variable using $\Delta_{ij} = c_{ij} - (u_i + v_j)$.

Step 5: If $\Delta_{ij} \geq 0$ for all unoccupied cells, Stop; the solution is optimal. Otherwise, Go to Step 6.

Step 6: Identify the unoccupied cell with the negative opportunity cost and select it as the incoming cell. If more than one unoccupied cell contains negative opportunity cost, select the cell with most negative opportunity cost.

Step 7: Draw a closed loop or stepping stone path associated with the incoming cell.

Step 8: Assign an alternate plus and minus signs on the corner points of the closed path with a plus sign at the cell selected as the incoming cell.

Step 9: The smallest value among the cell with a minus sign on the closed path indicates the number of units that can be shipped to the incoming cell. Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs, and subtract it from those cells marked with minus signs. If there is a tie, select any one of the tied cell. The tied cells that are not selected will be artificially occupied with a zero allocation.

Step 10: Go to Step 3.

TRANSPORTATION MODEL

The general and usual form of the TP is available by the following Model:

Destination / Source	D_1	D_2	D_3	$\ldots D_J \ldots\ldots$	D_n	SUPPLY a_i

S_1	x_{11} c_{11}	x_{12} c_{12}	x_{13} c_{13}	.	.	a_1
S_2	x_{21} c_{21}	c_{22}	c_{23}	.	.	a_2
....S_I.....				$\ldots x_{ij}\ldots$		a_i
S_m	x_{m1} c_{m1}	x_{m2} c_{m2}	x_{m3} c_{m3}		x_{mn} c_{mn}	a_m
DEMAND b_j	b_1	b_2	b_3	b_j	b_n	

Figure 2: Transportation Problem Model

Suppose there are n destinations and m sources. Let a_i be the number of supply units presented at sources i and let b_j be the number of demand units required at destination , c_{ij} represents the cost of transporting one unit of commodity from source i to destination j. If $(x_{ij} \geq 0)$ is the number of units transported from source i to destination , then the equal linear programming representation will be determine non-negative value of x_{ij} satisfyingboth the availability constraints

$$\sum_{j=1}^{n} x_{ij=a_i} \ (i = 1,2,3 \ldots\ldots m) \ldots\ldots\text{(a)}$$

as well as the requirement constraints

$$\sum_{i=1}^{m} x_{ij=b_j} \ (j = 1,2,3 \ldots\ldots n) \ldots\ldots\text{(b)}$$

and minimizing the total cost of transportation

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(c)}$$

It is also assumed that total availabilities $\sum a_i$ satisfy the total requirement $\sum b_j$ that is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \, .$$

This is known as balanced problem.

PROPOSED ALGORITHM

Step1: Calculate in 1st approximation the penalty by taking the difference between lowest and next lowest cost in each row or columns.

Mathematically
$|c_{12} - c_{11}|$ Considering c_{12} is smallest and c_{11} is next smallest cost in row and $|c_{21} - c_{11}|$ $_{21}$ is smallest and c_{11} is next smallest cost in column.

Step2: Taking largest difference in column and smallest difference in row (b_j)& (a_i) assuming a_i minimum cost in row and $b_j$maximum cost in column. Reduce the matrix (row or Column) with zero supply or demand.

Step3: Calculate in 2nd approximation the smallest cost in the transportation table $x_{ij} = \min(a_i b_j)$, assuming a_i minimum cost in row or b_j minimum cost in column. Reduce the matrix with zero supply or demand.

Step4: compute 1st& 2nd approximation alternative especially to get a desired minimum cost.

Step5: Minimizing total cost of transportation

$\boldsymbol{Z} = x_{11}c_{11} + x_{12}c_{12} \dots \dots \quad x_{mn}c_{mn}$ Where

$\sum_{j=1} x_{ij=a_i} \quad ,\sum_{i=1}^{m} x_{ij=b_j}$

NUMERICAL ILLUSTRATION

In this paper, consider four different-size cost minimizing transportation problems, selected form literature. We also use these examples to perform a comparative study of proposed algorithm with northwest corner and least cost methods. We solve example 2 step-by-step continuous.

Eaxmple-1 (273)

Destination Source	D1	D2	D3	D4	D5	Supply
S1	4	1	2	4	4	60
S2	2	3	2	2	2	35
S3	3	5	2	4	4	40
Demand	22	45	20	18	30	$\sum 135$

Example-2 (143)

Destination Source	D1	D2	D3	Supply
S1	6	8	4	14
S2	4	9	8	12
S3	1	2	6	5
Demand	6	10	15	$\sum 31$

Example-3 (415)

Destination Source	D1	D2	D3	D4	Supply
S1	7	5	9	11	30
S2	4	3	8	6	25
S3	3	8	10	5	20
S4	2	6	7	3	15

	Demand	30	30	20	10	$\sum 90$	
Example-4	Destination Source	D1	D2	D3	D4	Supply	2850
	S1	3	1	7	4	300	
	S2	2	6	5	9	400	
	S3	8	3	3	2	500	
	Demand	250	350	400	200	$\sum 1200$	
Example-5	Destination Source	D1	D2	D3		Supply	555
	S1	6	4	1		50	
	S2	3	8	7		40	
	S3	4	4	2		60	
	Demand	20	95	35		$\sum 150$	

Table 3: Examples

Example illustration

We present here the step-wise solution of one of these problems for better understanding of the reader. Considering this, step by step allocations in various cost cells are explained belowonly for Ex-2 from Table 3

Destination Source	D1	D2	D3	Supply
S1	6	8	4	14
S2	4	9	8	12
S3	1	2	6	5
Demand	6	10	15	$\sum 31$

Table 4: Solve step by step problem 2

STEP:1 Calculate the penalty by taking the difference between lowest and next lowest cost ineach row or columns Table 4.1.

Destination Source	D1	D2	D3	Supply	Row penalty
S1	6	8	4	14	2
S2	4	9	8	12	4
S3	1	5 2	6	0	1

Demand	6	5	15	∑31	
Column penalty	3	6	2		

Table 4.1

STEP2: Using Table 4.1, Taking smallest difference in row is min{2,4,1}=1 and largest difference in columns max{3,6,2}=6,x32=min{5,10}=5 then delete the S3 row because its zero supply. |D2-S3=|10-5|=5 is reaming demand

Destination Source	D1	D2	D3	Supply
S1	6	8	4	14
S2	4	9	8	12
Demand	6	5	15	∑26

Table 4.2

STEP3: Now we computing approximation the smallest cost in the transportation Table 4.2 is 4

Destination Source	D1	D2	D3	Supply
S1	6	8	14 4	0
S2	4	9	8	12
Demand	6	5	1	∑12

Table 4.2 is 4

X13= min{14, 15}=14 then delete the S1 row because its zero supply |D3-S1|=|15-14|=1 isreaming demand.

Destination Source	D1	D2	D3	Supply	Row penalt y
S2	4	9	1 8	12-1	4
Demand	6	5	0	∑11	
Colum penalty	2	4	7		

Table 4.3

STEP4: Similarly repeat the above steps, Using Table 4.3 taking the smallest and next smallest penalty in each row{4} and each column {2,4,7} now taking minimum penalty inrow is 4 and maximum penalty in column is 7 x23=min{12,1}=1 then delete D3 column.

Destination Source	D1	D2	Supply
S2	6 4	9	11-6=5
Demand	0	5	$\sum 5$

Table 4.4

Now we compute minimum cost in transportation Table 4.4 is 4, x21=min{6,11}=6 thendelete the D1colums because its zero demand |11-6|=5 is remand supply.

Destination Source	D2	Supply
S2	9 5	5-5=0
Demand	0	$\sum 0$

Table 4.5

In last cell same penalty row and column, x22= min{5,5}=5 then delete the whole matrixbecause its zero supply and demand.

STEP5: Using Table 4 Total allocates the point for obtained minimize cost

Destination Source	D1	D2	D3	Supply
S1	6	1 8	14 4	14
S2	6 4	5 9	1 8	12
S3	1	5 2	6	5
Demand	6	10	15	$\sum 31$

Table 4.6: Z= 4*14+4*6+9*5+8*1+2*5=143

RESULTS AND DISCUSSION

We test the performance of proposed method in comparison to NWCM and LCM by considering five different-size examples. Table-3 shows that the result obtained using proposed method in examples1, 2, 4 and 5 is same as the optimal result, while in example3 it is close to the optimal solution. It can be clearly seen that the proposed method gives more effective results in contrast to the existing methods – NWCM and LCM.

No: of example	Type ofproblem	Result ofNWCM	Result ofLCM	Result ofproposed Method	Optimal Solution
Example-1	3*5	363	278	273	273
Example-2	3*3	228	163	143	143
Example-3	4*4	540	435	415	410
Example-4	3*4	4400	2900	2850	2850
Example-5	3*3	730	555	555	555

Table 5

CONCLUSION

In this paper, we have developed an improved algorithm for obtaining the initial basicfeasible solution of transportation problems. The proposed algorithm is also tested for optimality. A comparison of proposed algorithm is made with Least Cost Method and North West Corner Method by considering 5 numerical examples. It is observed that the proposed algorithm yields more reliable results in contrast to the conventional methods.

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