M. Tech. Structural Engineering

Atul Shakya

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DAMAGE DETECTION IN BUILDING USING MODE SHAPE SLOPE AND CURVATURE

A DISSERTATION

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FORTHE AWARD OF THE DEGREE OF

MASTER OF TECHNOLOGY

IN

STRUCTURAL ENGINEERING

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DECLARATION

I, Atul Shakya, Roll No. 2K20/STE/06 student of M.Tech (Structural engineering), hereby declare that the project dissertation titled "**Damage Detection in Building Using Mode Shape Slope and Curvature**" which is submitted by me to the Department of Civil, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Technology, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

Place: Delhi Atul Shakya

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CERTIFICATE

I hereby certify that the project dissertation titled "**Damage Detection in Building Using Mode Shape Slope and Curvature**" which is submitted by Atul Shakya, Roll No. 2K20/STE/06, Department of Civil, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Technology, is a record of the project work carried out by the students under my supervision. To the best of my knowledge this work has not been submitted in part or full for any Degree or Diploma to this University or elsewhere.

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Date: 30.05.2022 DR. SHILPA PAL

ABSTRACT

Structural Health Monitoring is a field focusing to enhance structural serviceability by periodic safety evaluation of the structural strength and stability. If the damages in the structure can be detected early, the structure can be saved from collapsing by using the proper retrofitting techniques. In this thesis, mode shape slope and curvature method has been used to detect damage and understand which mode's derivative are capable of detect damage in the building at different story. Three different height of building has been considered to understand the efficiency of the damage detection technique for the first five mode shape derivative of the building. First, damage is introduced in the building by reducing the stiffness of that story where damage is need to be detect by using the mode shape slope and curvature method. Modal analysis is done, to obtain the natural frequency and mode shape of the undamaged and damaged building. With the help of the central difference method, the derivatives of the mode shape have been calculated for the both buildings. Forward difference method is used at the fixed end of the building while backward difference method is used at the free end of the building. After obtaining the derivatives (i.e. 1^{st} derivative and 2^{nd} derivative), difference between undamaged and damaged mode shape slope/curvature is calculated and then normalized them to unity. Normalized mode shape slope and curvature is plotted to detect the damaged location in the building, A MATLAB programme has been written for the analysis of the undamaged and damaged building. It has been observed from the result show mode shape based methods are capable of detecting the damages. The difference between undamaged and damaged mode shape slope shows a higher value between damaged story and story before it. A sudden rise in function values such function known as Dirac-delta function in mathematical terms. The difference between

undamaged and damaged mode shape curvature changes it sign at the damaged location and reaches very large positive value at damaged location. Results shows mode shape curvature is capable of detect damage easily as compared to mode shape slop. It has been observed from the results that few higher mode of the building start contributing in damage detection as the height of the building increases. Low level damage can be also detected from the mode shape slope and curvature method.

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Signature

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ABBREVIATIONS

Abbreviation	Description
5D1	20% Damage Introduced at 1^{st} Story in 5 Story Building
5D3	20% Damage Introduced at 3^{rd} Story in 5 Story Building
5D4	20% Damage Introduced at 4^{th} Story in 5 Story Building
10D2	20% Damage Introduced at 2^{nd} Story in 10 Story Building
10D5	20% Damage Introduced at 5^{th} Story in 10 Story Building
10D9	20% Damage Introduced at 9^{th} Story in 10 Story Building
16D2	20% Damage Introduced at 2^{nd} Story in 16 Story Building
16D7	20% Damage Introduced at 7^{th} Story in 16 Story Building
16D13	20% Damage Introduced at 13^{th} Story in 16 Story Building

CHAPTER 1

INTRODUCTION

1.1 STRUCTURAL HEALTH MONITORING

Structural Health Monitoring is a field focusing to enhance structural serviceability by periodic safety evaluation of the structural strength and stability. If the damages in the structure can be detected early, the structure can be saved from collapsing by using the proper retrofitting techniques. Numerous techniques have been proposed in the last decades for structural damage detection. Detection of damage in the structure is the most important part of the structural health monitoring. Damage may be occurs in structure due to improper construction management, temperature variations, cracks due to cyclic loading etc.

1.2 DAMAGE DETECTION

Numerous techniques have been proposed in the last decades for structural damage detection. Detecting the damage in the structure will help in predicting the strength, durability, structure's life and most important the building's behaviour to any force acting on it. Damage in the structure changes its dynamic properties, the changes can be observed in the model parameters i.e., natural frequency, mode shape associates with the natural frequency [6]

1.2.1 Vibration-Based Methods

The damaged structure shows change in dynamic properties as compare to the undamaged structure. This change has been observed in the modal parameters like natural frequencies, mode shape of the structure. Any type of the change in the physical properties resulting from the damage can be helps in detect the damage of the structure with the help of modal parameter. Various techniques have been developed for the damage detection by using modal parameter. Some of vibration based damage detection techniques [2] are

1.2.1.1 Mode Shape Slope Method

The change between undamaged and damaged mode shape slopes shows a very large value at the location of damage for the first few lower modes also known as Dirac-delta function in mathematical terms as given in Eqn. 1.1 [9]

$$f'^{(i)}(x) = -\varphi \sum_{j=1}^{n} cos\left[(2j-1)\frac{\pi(x-x_p)}{2L}\right]$$

$$= \begin{cases} -\infty, & x = x_p \\ 0, & otherwise \end{cases} \qquad (1.1)$$

Where, $\varphi = constant\ value$, L= length of an element, x_p =distance of damaged location.

Therefore, the difference in mode shape slopes attains ∞ at the location of damage and at other location, this value is zero. One can also imply that $f'^{(i)}(x)$ is a Dirac delta function about $x=x_p$.

1.2.1.2 Mode shape curvature method

The change in mode shape curvatures changes its sign at the location of damage and attains very large positive and negative values on either side of the damage for first few lower modes. The difference in mode shape curvature between undamaged and damaged mode shape can be expressed as in Eqn. (1.2) [9]

$$f'^{(i)}(x) = -\gamma \sum_{j=1}^{n} (2j-1) sin\left[(2j-1)\frac{\pi(x-x_p)}{2L}\right]$$

$$= \begin{cases} \infty, & x = x_p^{+} \\ -\infty, & x = x_p^{+} \\ 0, & otherwise \end{cases} \qquad (1.2)$$

1.2.1.3 Based on Frequency Change

Since modal frequency is a global parameter, its value depends upon the sum total of properties at each point of the structure. On the other hand, damage is a local phenomenon, limited only to specific region of the structure. In case of local damage it is unable to detect the damage location.

1.3 OBJECTIVE OF THE STUDY

The objectives of the work are as follows.

1. To identify the efficiency of damage detection technique for different height of building using mode shape slope and curvature method.
2. To check the effectiveness of damage detection technique (i.e., modal slope and curvature) for building having damage at different height of building.
3. To understand the contribution of higher mode's derivative in damage detection for low, medium and high rise building using modal slope and curvature method.

1.4 ORGANIZATION OF THE THESIS

Chapter 1 deals with the introduction which involves the description of need of structural health monitoring, damage detection and vibration based method to detect damage.

Chapter 2 discusses some of the literature and their work on the damage detection techniques based on modal parameters.

Chapter 3 discusses about methodology and the validation of numerical model [7] with the help of MATLAB programme.

Chapter 4 discusses about the numerical models and methodology used to detect the damage in the building.

Chapter 5 deals with the result obtained after the analysis of the models and

Chapter 6 deals with the conclusions of the present study and provides suggestion for further work.

CHAPTER 2

LITERATURE REVIEW

2.1 GENERAL

This chapter gives an introduction on literature of damage detection techniques used to detect damage in structural element (i.e., simply supported, cantilever beam), bridges, building etc. Some of the literatures are discussed in the following sections.

2.2 LITERATURE ON DAMAGE DETECTION

Pandey et.al (1991) discussed about the damage detection in a cantilever and a simply supported beam using change in curvature mode shape. Analytical beam model of a cantilever and a simply supported beam is considered and change in curvature mode shape method is used to detect damage. The change in the curvature mode shape increase with increasing size of damage. Finite analysis was used to calculate the displacement of a cantilever and a simply supported beam. Curvature mode shapes were calculated from the displacement by using a central difference approximation and capable to detect damage in both type of beam.

Pandey et.al (1994) used the change in flexibility method to detect damage. To verified this method three type of beam were considered i.e., simply supported beam, free-free beam and cantilever beam. This method is give better results when the damage is located where the bending moment is maximum like in cantilever it easy to detect damage at the support end as compare to free end because bending moment is maximum at support in case of cantilever beam. In case of simply supported beam, damage can be easily detected at the mid span because bending moment is maximum at mid span. This method is verified by both analytical and experimental work.

Wahab and Roeck (1999) studied about the modal curvature method to detect damage in real damage scenario. For real damage scenario prestressed concrete bridge, bridge Z24 lies over the highway A1 (Bern/Zurich) is considered. To introduce damage in the bridge some concrete part was removed and replaced by still fill plates. Modal curvature method is able to detect damage in the structure.

Alvandi & Cremona (2006), discussed about the various type of method of detecting damage. Vibration-based damage detection techniques has been discussed i.e., mode shape curvature method, change in flexibility method, change in flexibility curvature method, change in frequency method. All the methods are capable of detect damage.

Dawari & Vesmawal (2013) discussed about the special type of damage which is generally occurs in the concrete due to poor consolidation i.e., “honeycomb damage”. Two types of methods has been used to detect damage i.e. modal curvature method and modal flexibility method. Eigen value analysis is performed out on the finite element method of reinforced concrete beam and eigen vectors extracted. The damage is introduced by reducing the stiffness in the beam. The finite element model is modeled on ansys software.

Roy and Chaudhuri (2013) discussed about the formulation of mathematical expression for cantilever beam to detect the damage by change in mode shape slope and curvature. Two numerical models has been consider and introduced damage at a story. Damage in numerical model is identified with the help of mode shape slope and curvature method.

Agarwal and Chaudhuri (2015) studied about the two damage detection technique i.e. mode shape slope and change in mode shape curvature to detect damage in truss bridge which is modeled on a SAP 2000 software. Damage has been introduced by reducing the area of cross-section by 40-50%.Modal analysis has been performed to calculated mode shape for undamaged and damaged structure both. Change in mode shape slope and curvature methods are capable to detect damage in the truss bridge.

Frans et.al (2017) compared the two methods of damage detection i.e., mode shape curvature method &damage locating vector methods (DLVM) for the damage detection of structures. Mode shape curvature method is used based on the change in second degree derivative of mode shape of damage and undamaged case whereas DLVM is based on the change in flexibility matrix of the damaged and undamaged case. The damage is easily detectable by above two methods. After checking three structure by the above two methods, it has been concluded by the author that the damage in shear building and beam type structures can be easily predicted by the mode shape curvature method. But for the plane truss system DLVM predicts the damaged members but the mode shapes curvature methods predicts the nodes of the truss not members.

Roy (2017) formulated the mathematical expressions of the derivatives of the mode shapes and used to detect damage in the structure. Numerical model of 16 story building has been considered and damage is introduced by reducing the stiffness of the story. Difference in the mode shape slope reaches maximum value at the damaged location (i.e. Dirac-delta function) and difference in mode shape curvature is discontinuous at the damage location.

2.3 GAP OF THE STUDY

Most of the researchers studied about the damage detection in the different types of structural elements mainly in beam by using various techniques. So, the gap of the study is to check the damage detection technique on different height of building having damage at different height of the building. Mode shape slope and curvature damage detection technique has been used to detect damage in building and considering first few modes of building to understand its various in ability to detect damage as the height of building changes.

CHAPTER 3

VALIDATION OF DAMAGE DETECTION METHODS

3.1 GENERAL

This chapter discusses about the methodology of damage detection and to check the damage detection technique that is used for validation. A validation work has been done considering a numerical problem from paper titled “Structural damage identification using mode shape slope and curvature” by Koushik Roy [9]. The remaining section of this chapter, discusses about the steps required to detect damage in building and the result comparison.

3.2 METHODOLOGY

Structure of 16 story building has been modeled and property of the building has been defined in the first step of the methodology. Damage is introduced in the building at the 7th story which should be identified by the help of the damage detection method. A free vibration analysis or modal analysis has been carried out to obtain the mode shape and natural frequency of the building. Forward, backward and central difference method has been used to calculate mode shape slope and curvature.

3.3 VALITATION OF DAMAGE DETECTION

3.3.1 Problem Statement

To demonstrate the effectiveness of modal slope and curvature for damage detection and location, a simulation study was carried out considering a numerical model of a 16-story shear building shown in Fig 3.1. A uniform mass and story stiffness distribution along the height of the structure is considered; i.e. k_1= k_2 =k_3 =k_{16}=250kN/m and m_1= m_2 =m_3 =m_{16}=200kg. Damage has introduced in the shear building by reducing the stiffness by 20% at 7th story. So, the damage has to be identified in the building at 7th story [7]. For this following steps are followed to solve this problem.

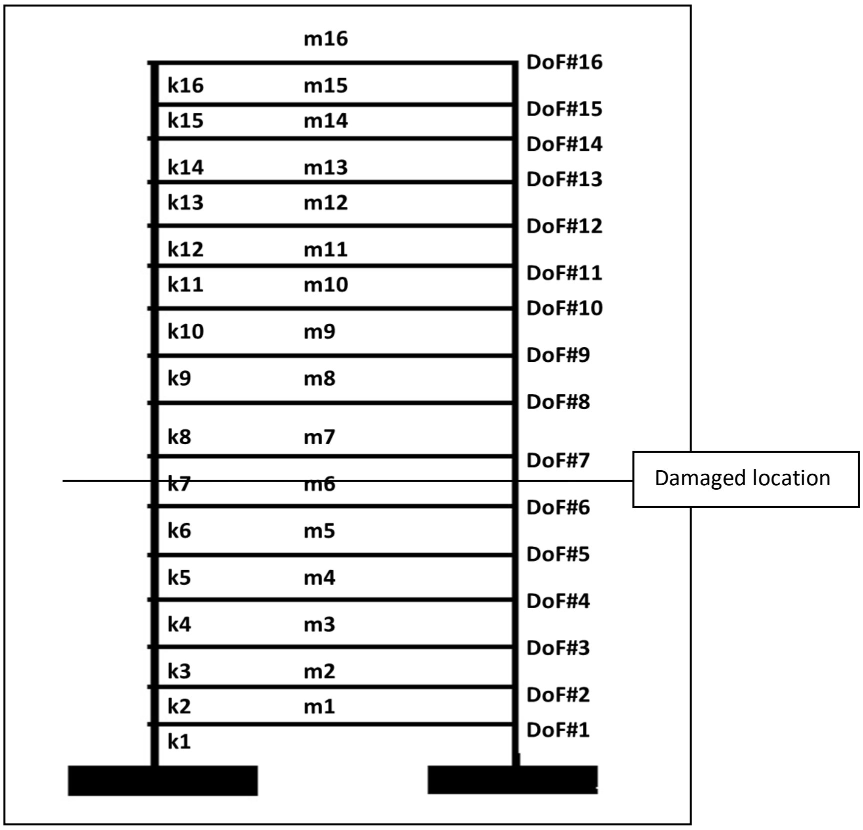


Fig 3.1 Building Model for Validation

3.3.2 Procedure and Analysis

Data required for the analysis of the 16 story building is given in the numerical problem. Steps followed to detect the damage in the building at the 7th story are as follows:

1. Create damage: Damage is introduced at 7th story by reducing the stiffness by 20%. So, the stiffness at 7th story becomes 200kN/m and on the other floor will be 250kN/m. Therefore there are two buildings one is undamaged building and another is damaged building.

2. Formation of mass and stiffness matrix: The mass matrix will be same for both undamaged and damaged building, 16*16 will be the size of the mass matrix for both building as shown in Table 3.1. Stiffness matrix for the undamaged and damaged building is shown in Table 3.2 and 3.3.

Table 3.2 Mass Matrix of 16 Story Building

200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	200	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	200	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	200	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	200	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	200	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	200	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	200	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	200	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	200	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	200	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	200	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	200	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	200	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	200

Table 3.3 Undamaged Stiffness Matrix of 16 Story Building

500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-250000	250000

Table 3.4 Damaged Stiffness Matrix of 16 Story Building

500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-250000	450000	-200000	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-200000	450000	-250000	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-250000	250000

3. Modal analysis: Perform a modal analysis of undamaged and damaged building using MATLAB programme and obtained mode shape as given in Table 3.4 and 3.5 respectively.

Table 3.5 Undamaged Mode Shape of 16 Story Building

No.	1st Mode	2nd Mode	3rd Mode
0	0	0	0
1	-0.00234	-0.00694	-0.01128
2	-0.00466	-0.01331	-0.02005
3	-0.00694	-0.01861	-0.02437
4	-0.00915	-0.02239	-0.02326
5	-0.01128	-0.02437	-0.01699
6	-0.01331	-0.02437	-0.00694
7	-0.01522	-0.02239	0.004659
8	-0.01699	-0.01861	0.015218
9	-0.01861	-0.01331	0.022394
10	-0.02005	-0.00694	0.02459
11	-0.02132	9.22E-17	0.02132
12	-0.02239	0.006936	0.01331
13	-0.02326	0.01331	0.00234
14	-0.02392	0.018605	-0.00915
15	-0.02437	0.022394	-0.01861
16	-0.02459	0.024368	-0.02392

Table 3.6 Damaged Mode Shape of 16 Story Building

No.	1st Mode	2nd Mode	3rd Mode
0	0	0	0
1	-0.00229	-0.007	-0.01106
2	-0.00456	-0.01344	-0.01974
3	-0.00679	-0.0188	-0.02416
4	-0.00896	-0.02263	-0.02336
5	-0.01105	-0.02463	-0.01753
6	-0.01304	-0.02465	-0.00791
7	-0.01538	-0.02217	0.006244
8	-0.01712	-0.0184	0.01622
9	-0.01871	-0.01315	0.022696
10	-0.02013	-0.00683	0.024275
11	-0.02137	4.66E-05	0.020617
12	-0.02242	0.006915	0.01251
13	-0.02328	0.013224	0.001704
14	-0.02392	0.018464	-0.00947
15	-0.02436	0.022212	-0.0186
16	-0.02458	0.024165	-0.02372

4. Normalized Mode shape: Normalized result of mode shapes are shown in Table 3.6 and 3.7 for undamaged and damaged building respectively.

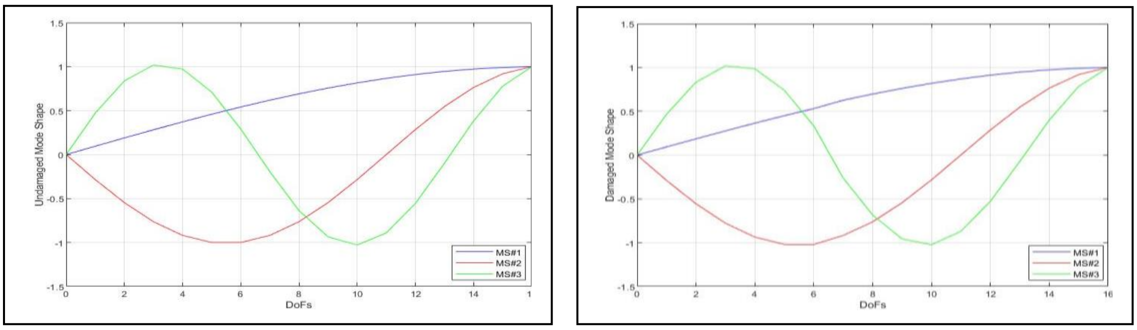
Table 3.7 Normalized Undamaged Mode Shape of 16 Story Building

No.	1st Mode	2nd Mode	3rd Mode
0	0	0	0
1	0.095164	-0.28463	0.471518
2	0.189466	-0.5462	0.838204
3	0.282052	-0.76352	1.018532
4	0.372084	-0.91899	0.972412
5	0.458746	-1	0.710095
6	0.541254	-1	0.289905
7	0.61886	-0.91899	-0.19474
8	0.690862	-0.76352	-0.63609
9	0.756607	-0.5462	-0.93602
10	0.8155	-0.28463	-1.02784
11	0.867007	3.78E-15	-0.89115
12	0.910663	0.28463	-0.55632
13	0.946072	0.5462	-0.09781
14	0.972914	0.763521	0.382443
15	0.990944	0.918986	0.777671
16	1	1	1

Table 3.8 Normalized Damage Mode Shape of 16 Story Building

No.	1st Mode	2nd Mode	3rd Mode
0	0	0	0
1	0.093174	-0.28987	0.466442
2	0.18552	-0.55631	0.832244
3	0.27622	-0.77779	1.018481
4	0.364468	-0.93641	0.98497
5	0.44948	-1.01935	0.738942
6	0.530502	-1.01991	0.33348
7	0.625893	-0.91758	-0.26329
8	0.69665	-0.76155	-0.68389
9	0.761222	-0.54397	-0.95694
10	0.819037	-0.28244	-1.02352
11	0.869581	0.001927	-0.86927
12	0.912405	0.286135	-0.52746
13	0.94713	0.547218	-0.07184
14	0.973447	0.764075	0.399272
15	0.991123	0.919181	0.78424
16	1	1	1

Plot the graph of normalized mode shape of undamaged and damaged building with the help of MATLAB programme are show in Fig 3.1 (a) and (b).



. (a) (b)

Fig 3.2 Normalized Mode Shape of (a) Undamaged Building (b) Damaged Building

5. Modal slope and curvature: Modal slope or mode shape slope is defined as the first derivative of the mode shape while second derivative of mode shape is known as modal curvature or mode shape curvature. To calculate the derivative of any function $m(x)$, the formulae given by (Grewal 2002) stated in Table 3.8 have been used.

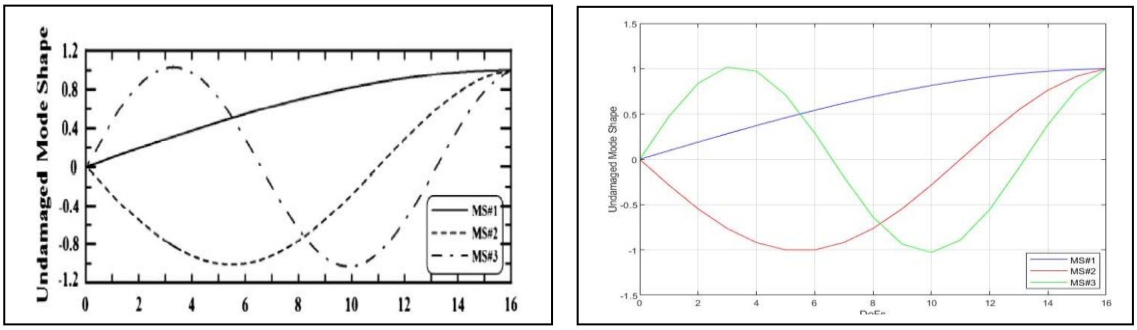
Table 3.9 Expression of Forward, Central and Backward Difference Method

Derivative	Modal Slope	Modal Curvature
Forward Difference Method	$m^{'}(x) = \frac{m(x+h) - m(x)}{h}$	$m^{''}(x) = \frac{m(x+2h) - 2m(x+h) + m(x)}{h * h}$
Central Difference Method	$m^{'}(x) = \frac{m(x+h) - m(x-h)}{2h}$	$m^{''}(x) = \frac{m(x+2h) - 2m(x) + m(x-h)}{h * h}$
Backward Difference Method	$m^{'}(x) = \frac{m(x) - m(x-h)}{h}$	$m^{''}(x) = \frac{m(x) - 2m(x-h) + m(x-2h)}{h * h}$

Central difference method needs one preceding and one succeeding value of the function i.e. $m(x-h)$ and $m(x+h)$, to find the derivative of any function. However, at boundaries (at the fixed and free ends) function does not have any of these values. Therefore, to calculate the modal slope and curvature for undamaged and damaged cases, the forward method is used at the fixed end and backward method is used at the free end.

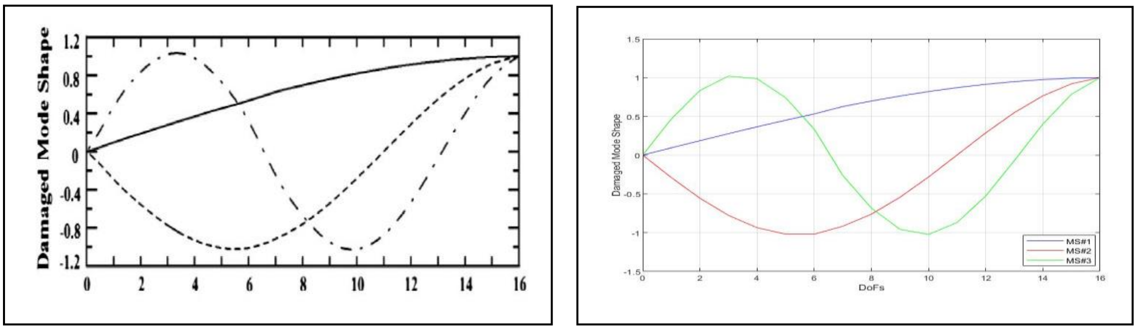
3.3.3 Validation of Results

The results obtained after the analysis of the 16 story building are almost similar to result of Roy [7].



(a) (b)

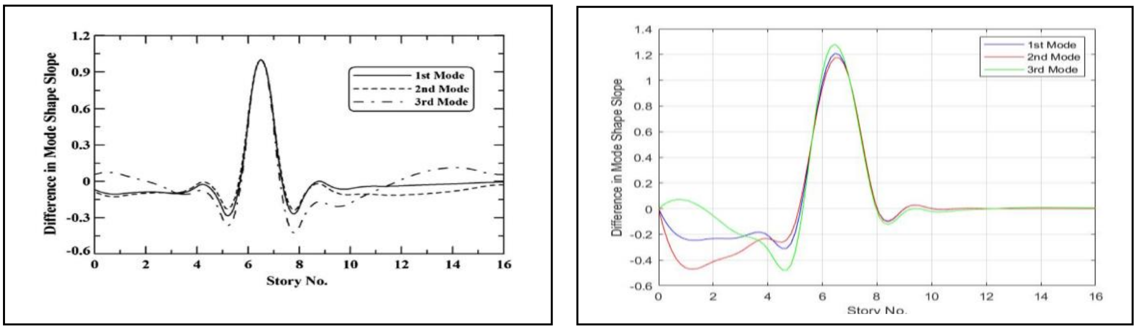
Fig 3.3 Undamaged Mode Shape (Normalized), results (a) Research Paper (b) Simulation in Matlab



(a) (b)

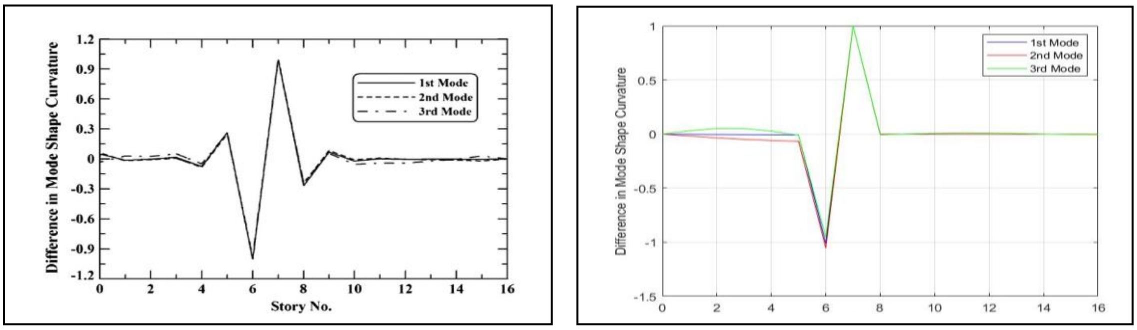
Fig 3.4 Damaged Mode Shape (Normalized), results (a) Research Paper (b) Simulation in Matlab

Normalized undamaged and damage mode shape obtained after the simulation in MATLAB is almost similar to the results of research paper [9] considered for the validation as shown in Fig 3.3 and 3.4 respectively. Its shows that the technique is working properly for the normalized undamaged and damaged mode shape.



(a) (b)

Fig 3.5 Change in modal slope, results (a) Research Paper (b) Simulation in Matlab



(a) (b)

Fig 3.6 Change in modal curvature, results (a) Research Paper (b) Simulation in Matlab

Change in modal slope are shown in Fig 3.5, the result obtained after the simulation in MATLAB are similar to the results of research paper [9] and similarly change in modal curvature also shows almost similar result to the research paper [9] as shown in Fig 3.6.So, from the comparison of results, it is observed that the results are almost similar and the technique is working properly. This technique will be used for some other problems and expansion of the technique will be implemented to check the efficiency of the method for different height of building in further chapters.

CHAPTER 4

METHODOLOGY

4.1 GENERAL

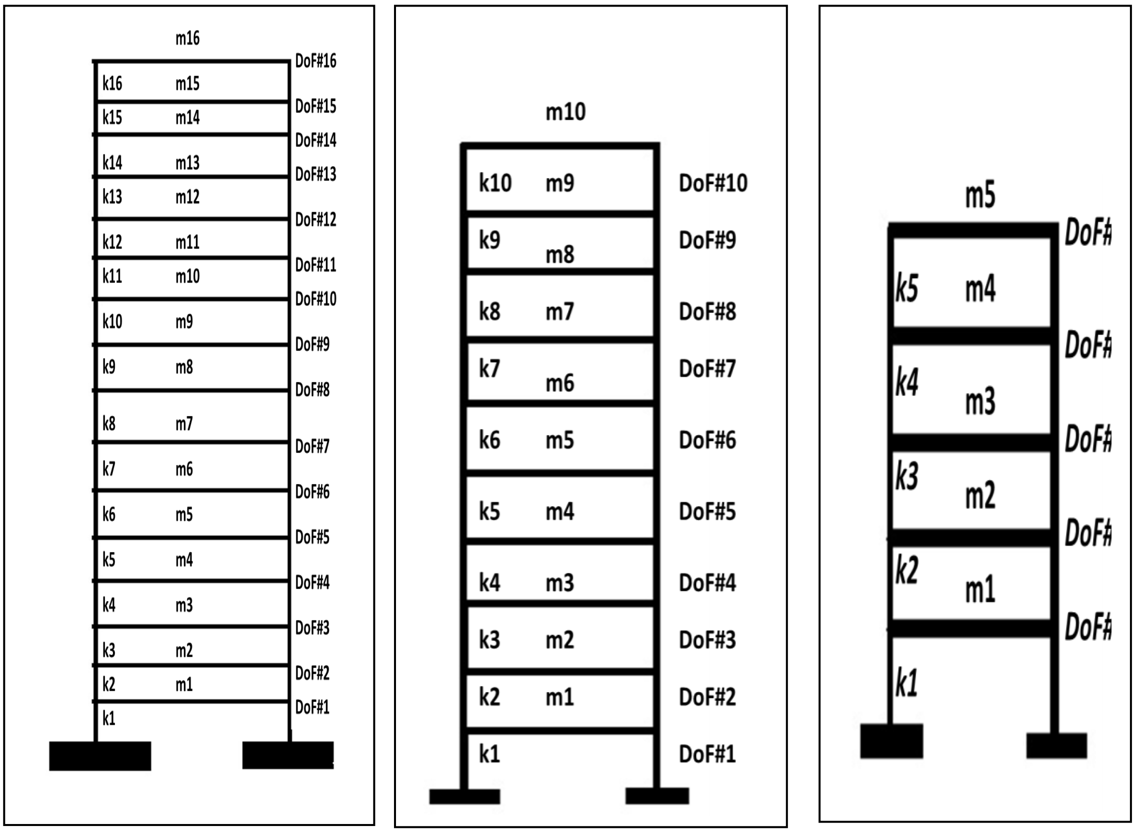
In this chapter discussed about the methodology used to detect damage in the building. Different height of Building models have been considered to understand the effectiveness of the damage detection technique i.e., mode shape slope and curvature, capable to detect damage in various building. Damage has been introduced in different level of story and its procedure has been discussed. Three different height of buildings are considered and damaged has been introduced at different level and damage detection method is used to detect damage.

4.2 BUILDING MODELS

To understand the difference in effectiveness of the damage detection in low, medium and high rise building by using modal slope and curvature, a study was carried out considering three different numerical models i.e. 16-story, 10-story and 5-story having uniform mass and stiffness of story which is distributed uniformly along the height of building as shown in Fig 4.1. Detail of the damage introduced in building and respective numerical model name is shown in Table 1. Damage has been introduced in building by reducing the stiffness of story by 20%.A modal analysis has been carried out for each numerical model to obtain natural frequency and mode shape. Obtained mode shapes are normalized to unity for both undamaged and damaged building. The forward, backward and central difference method has been used to obtained the mode shape slope and curvature method. Difference between undamaged and damaged mode shape slope and curvature is calculated and plotted with respect to number of story. To understand the efficiency of the damage detection method, a low level damage, i.e., reducing stiffness by 5%, is introduced in a building and compare the results with the 20% damaged story.

Table 4.1 Location of 20% Damage Introduced in the Buildings.

No of story in building (H)	Detail	Damage introduced at Story		
		Below 25% of H	At 50% of H	Above 75% of H
5 Story	Damaged Story	1	3	4
	Numerical Model	5D1	5D3	5D4
10 Story	Damaged Story	2	5	9
	Numerical Model	10D2	10D5	10D9
16 Story	Damaged Story	2	7	13
	Numerical Model	16D2	16D7	16D13



(a) (b) (c)

Fig 4.1 Diagram of Numerical model of (a) 16 Story Building (b) 10 Story building (c) 5 Story building

4.3. METHODOLOGY

4.3.1 Stiffness Matrix and Mass Matrix

First, obtained the mass matrix and a stiffness matrix for both undamaged and damaged building. The mass matrix will be same for the both undamaged and damaged building.

4.3.2. Modal Analysis

Modal analysis was carried for undamaged and damaged models to obtain the modal parameter of the structure i.e. mode shape, natural frequencies with the help of MATLAB.

4.3.3 Normalization of Mode Shape

After obtaining the mode shape for undamaged and damaged structure, normalized them to unity and plotted in graph.

4.3.2. Mode Shape slope and Curvature

Modal slope or mode shape slope is defined as the first derivative of the mode shape while second derivative of mode shape is known as modal curvature or mode shape curvature. To calculate the derivative of any function $m(x)$, the formulae given by (Grewal 2002) stated in Table 1 have been used.

Table 4.2 Expression of Forward, Central and Backward Difference Method

Derivative	Mode Shape Slope	Mode Shape Curvature
Forward Difference Method	$m'(x) = \frac{m(x+h) - m(x)}{h}$	$m''(x) = \frac{m(x+2h) - 2m(x+h) + m(x)}{h * h}$
Central Difference Method	$m'(x) = \frac{m(x+h) - m(x-h)}{2h}$	$m''(x) = \frac{m(x+2h) - 2m(x) + m(x-h)}{h * h}$
Backward Difference Method	$m'(x) = \frac{m(x) - m(x-h)}{h}$	$m''(x) = \frac{m(x) - 2m(x-h) + m(x-2h)}{h * h}$

Central difference method needs one preceding and one succeeding value of the function i.e. $m(x-h)$ and $m(x+h)$, to find the derivative of any function. However, at boundaries (at the fixed and free ends) function does not have any of these values. Therefore, to calculate the modal slope and curvature for undamaged and damaged cases, the forward method is used at the fixed end and backward method is used at the free end.

4.3.3 Differences of Mode Shape Slope and Curvature

To detect the damage and its location in the damage structure, the difference between undamaged and damaged mode shape slopes/curvature are determined and are normalized to unity. Graphs are plotted between "differences in mode shape slope/curvature" vs. "Story No." for each mode.

CHAPTER 5

RESULTS AND DISCUSSION

5.1 GENERAL

In this chapter, the results obtained from the analysis of the undamaged and damaged building models are summarized in the tables and for better understanding modal slope and curvature have been plotted on the graph with respect to number of story (or DoFs). Analysis of the each model (i.e., undamaged and damaged building) has been done with the help of MATLAB programme.

5.2 ANALYSIS OF 5 STORY BUILDING

To understanding the effectiveness of the mode shape slope and curvature as damage detection techniques for low rise building, a 5 story building model has been considered. Damage has been introduced by reducing the stiffness of story by 20% at three different height of the building. First damage is introduced at story below 25% of height of the building, second damage at 50% of height of the building and third damage at story above 75% of height of the building as shown in Table 4.1.

Table 5.1 Location of Damage Introduced in 5 Story Building

Detail H (height of building)	Damage introduced at Story		
	Below 25% of H	At 50% of H	Above 75% of H
Damaged Story	1	3	4
Numerical Model	5D1	5D3	5D4
Damage introduced (in %)	20%	20%	20%

5.2.1 Undamaged Building of 5 Story

Mass matrix and a stiffness matrix obtained for undamaged 16 story building is shown in Table 4.2 (a) and 4.2 (b). The mass matrix will be same for the undamaged and damaged building but stiffness matrix will be different. First five natural frequencies obtained after the modal analysis of undamaged building are 1.6016, 4.6751, 7.3698, 9.4674 and 10.7981 Hz.

Table 5.2 (a) Mass Matrix of 5 Story Building

200	0	0	0	0
0	200	0	0	0
0	0	200	0	0
0	0	0	200	0
0	0	0	0	200

Table 5.2(b) Stiffness Matrix of Undamaged 5 Story Building

500000	-250000	0	0	0
-250000	500000	-250000	0	0
0	-250000	500000	-250000	0
0	0	-250000	500000	-250000
0	0	0	-250000	250000

First five mode shapes obtained after the modal analysis of the building and then normalized them to unity shown in Table 4.3 and these mode shapes are plotted in graph shown in Fig 4.1

Table 5.3 Normalized Undamaged Mode Shape of 5 Story Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	0.28463	-0.83083	1.309721	-1.68251	1.918986
2	0.5462	-1.08816	0.372786	1.397877	-3.22871
3	0.763521	-0.59435	-1.20362	0.521109	3.513337
4	0.918986	0.309721	-0.71537	-1.83083	-2.68251
5	1	1	1	1	1

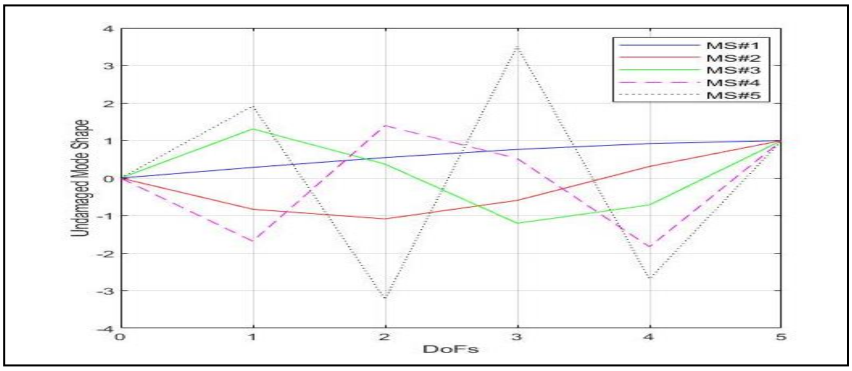


Fig 5.1 Normalized Undamaged Mode Shape of 5 Story Building

5.2.2 Damaged Building Model 5D1

Damage has been introduced at 1st story by reducing the stiffness by 20%. A stiffness matrix obtained for undamaged 5 story building is shown in Table 4.4. First five natural frequencies obtained after the modal analysis of 5D1 damaged building model are 1.5339, 4.5177, 7.2129, 9.3726 and 10.7703 Hz. Mode shape corresponding to these five natural frequencies have been calculated and then normalized them to unity with the help of MATLAB programme. Normalized mode shapes are summarized in Table 5.4 and graphical representation of these mode shapes is shown in Fig 5.5.

Table 5.4 Stiffness Matrix of 5D1 Damaged Building

450000	-250000	0	0	0
-250000	500000	-250000	0	0
0	-250000	500000	-250000	0
0	0	-250000	500000	-250000
0	0	0	-250000	250000

Table 5.5 Normalized Damaged Mode Shape of 5D1 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	0.336866	-0.91559	1.302438	-1.52378	1.633665
2	0.581325	-1.05788	0.204328	1.484732	-3.0445
3	0.782584	-0.51827	-1.22952	0.374035	3.431172
4	0.925687	0.35541	-0.64312	-1.77438	-2.6636
5	1	1	1	1	1

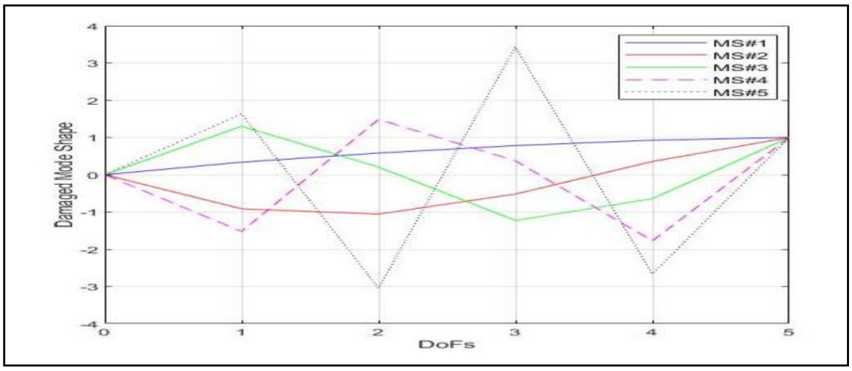


Fig 5.2 Normalized Damaged Mode Shape of 5D1

By the help of central difference method, forward difference method and backward difference method, obtain the modal slope and curvature of the damage (5D1) and undamaged building by the help of their mode shape. Difference of the undamaged and damaged (5D1) building modal slope (and curvature) is calculated, and then normalized them to unity are summarised in Table 5.6 (a) and 5.6 (b) respectively.

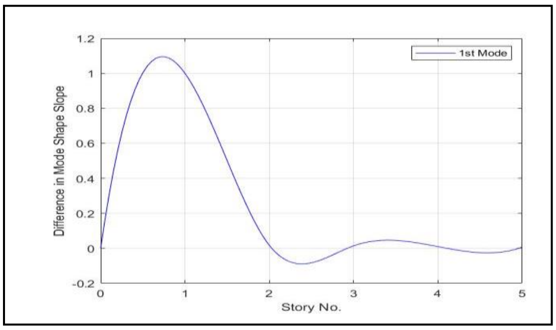
Table 5.6(a) Change in Modal Slope of Undamaged and 5D1 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	1	1	1	1	1
2	0.016587	-0.08042	0.009309	0.152902	-0.10158
3	0.014212	-0.00771	-0.12035	0.015201	0.08265
4	0.009531	0.038039	-0.01295	-0.07354	-0.04108
5	0.006701	0.045688	0.072251	0.056454	0.018905

Table 5.6(b) Change in Modal Curvature of Undamaged and 5D1 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	1	1	1	1	1
2	-0.00105	0.069229	-0.30373	0.162053	0.03057
3	-0.0037	0.076193	0.044404	-0.43746	-0.25491
4	-0.00566	0.0153	0.170404	0.259981	0.226668
5	-0.00566	0.0153	0.170404	0.259981	0.119976

Change in mode shape slope is plotted in Fig 5.3 for first five modes. Fig 5.3 (a) to (e) shows plot having maximum positive value between fixed story and story#1 indicating the damage location at the third story. Such type of pattern known as Dirac delta function [9]. Modal slope differences of all five modes have been plotted in Fig 5.3 (f). It has been observed from the results that damage can be detect with the help of mode shape slope method in low rise building having damage at lower story.



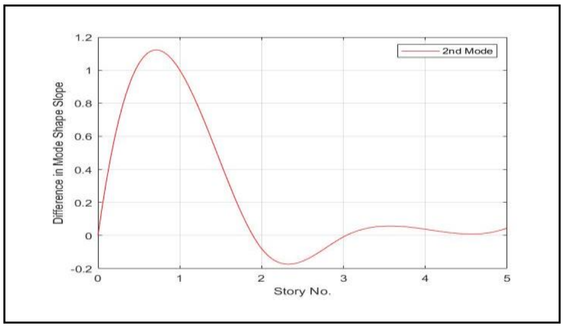
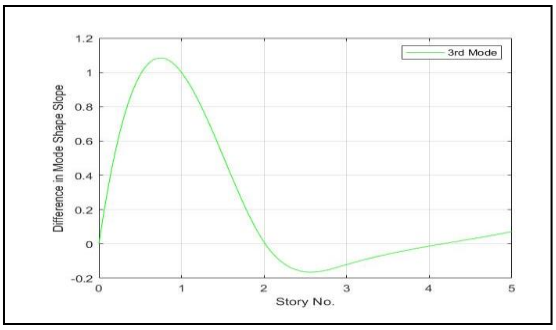


Fig. 5.3 (a) First Modal Slope of 5D1

Fig. 5.3 (b) Second Modal Slope of 5D1



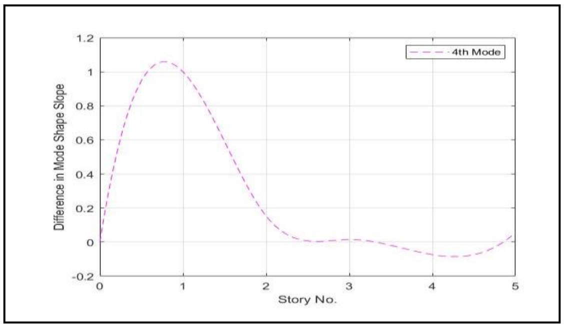
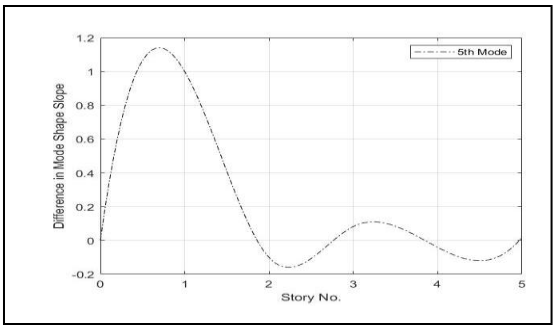


Fig. 5.3 (c) Third Modal Slope of 5D1

Fig. 5.3 (d) Fourth Modal Slope of 5D1



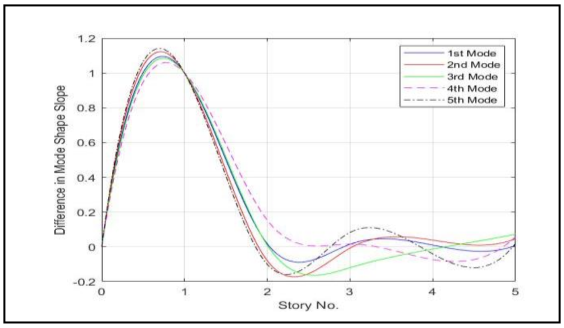
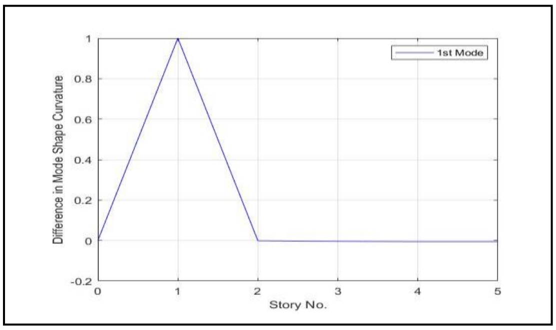


Fig. 5.3 (e) Fifth Modal Slope of 5D1

Fig. 5.3 (f) Combined Modal slope of 5D1

Change in mode shape curvature is plotted in Fig 5.4 for first five modes. Fig 5.4 (a) to (e) shows plot having maximum positive at the damaged location in all five. Modal curvature differences of all five modes have been plotted in Fig 5.4 (f). It has been observed from the results that damage can be detect with the help of mode shape curvature method in low rise building having damage at lower story.



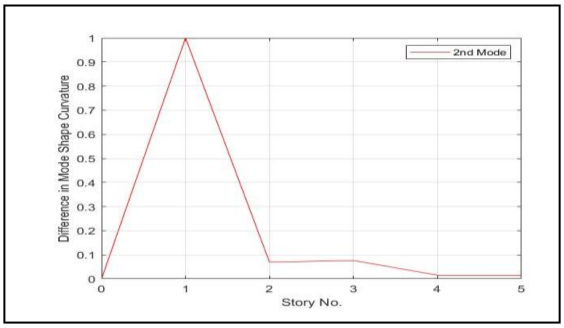
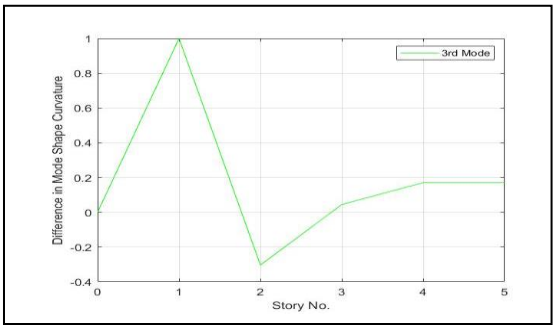


Fig. 5.4 (a) First Mode Curvature of 5D1

Fig. 5.4 (b) Second Mode Curvature of 5D1



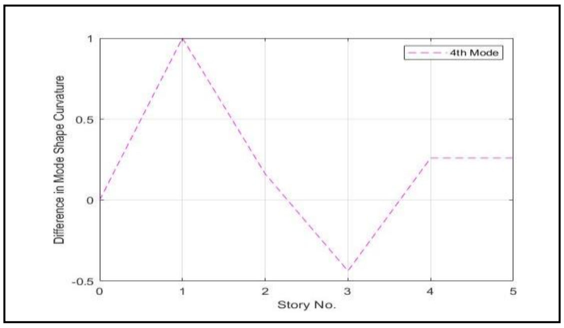
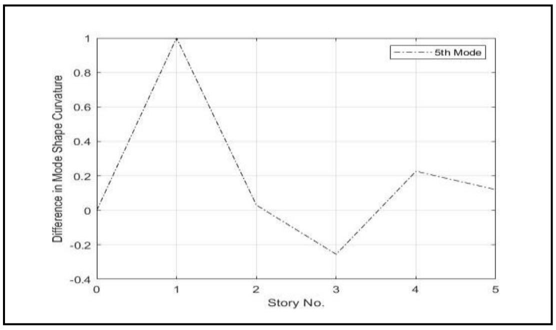


Fig. 5.4 (a) Third Mode Curvature of 5D1

Fig. 5.4 (b) Fourth Mode Curvature of 5D1



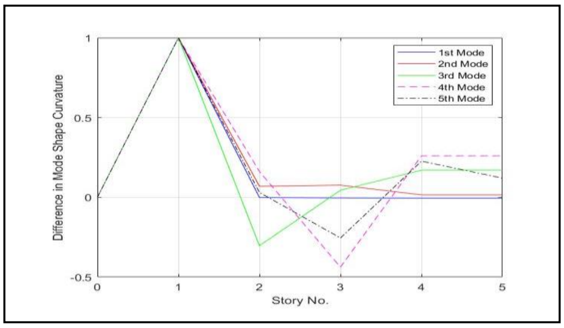


Fig. 5.4 (a) Fifth Mode Curvature of 5D1

Fig. 5.4 (b) Combined Mode Curvature of 5D1

5.2.3 Damaged Building Model 5D3

Damage has been introduced at 3st story by reducing the stiffness by 20%. A stiffness matrix obtained for undamaged 5 story building is shown in Table 5.7. First five natural frequencies obtained after the modal analysis of 5D1 damaged building model are 1.5612, 4.6135, 7.1131, 9.4333 and 10.4401 Hz. Mode shape corresponding to these five natural frequencies have been calculated and then normalized them to unity with the help of MATLAB programme. Normalized mode shapes are summarized in Table 5.8 and graphical representation of these mode shapes is shown in Fig 5.5.

Table 5.7 Stiffness Matrix of 5D3 Damaged Building

500000	-250000	0	0	0
-250000	450000	-200000	0	0
0	-200000	450000	-250000	0
0	0	-250000	500000	-250000
0	0	0	-250000	250000

Table 5.8 Normalized Damaged Mode Shape of 5D3 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	0.267998	-0.9082	1.080046	-2.06416	1.474315
2	0.515366	-1.20591	0.434212	1.67288	-2.12655
3	0.774984	-0.56476	-1.2404	0.467259	2.522919
4	0.923019	0.327792	-0.59797	-1.81044	-2.4424
5	1	1	1	1	1

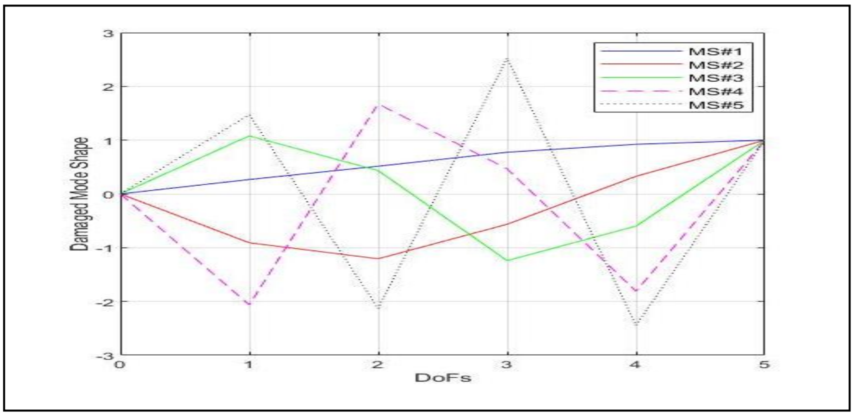


Fig 5.5 Normalized Damaged Mode Shape of 5D3

By the help of central difference method, forward difference method and backward difference method, obtain the modal slope and curvature of the damage (5D3) and undamaged building by the help of their mode shape. Difference of the undamaged and damaged (5D3) building modal slope (and curvature), normalized them to unity are summarised in Table 5.9 (a) and 5.9 (b) respectively.

Table 5.9(a) Change in Modal Slope of Undamaged and 5D3 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.88433	-0.86695	1.097387	-1.08007	-1.27853
2	0.80573	0.787545	3.445987	-1.28743	0.633082
3	1	1	1	1	1
4	0.005731	0.014796	-0.01839	-0.02692	-0.49521
5	0.004033	0.018071	0.117401	0.020388	0.240107

Table 5.9(b) Change in Modal Curvature of Undamaged and 5D3 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.04883	-0.23289	2.063299	2.575863	1.102887
2	-1.13622	-1.18164	-1.54245	-2.44488	-1.10037
3	1	1	1	1	1
4	-0.0034	0.00655	0.271589	0.094624	1.225221
5	-0.0034	0.00655	0.271589	0.094624	1.470632

The difference between undamaged mode shape slope and 5D3 damaged mode shape slope are shown in Fig. 5.6 for first five modes. It is observed from the results that Fig 5.6 (a) and (b) shows, plot having maximum positive value between story#2 and story#3 indicating the damage location at the third story.

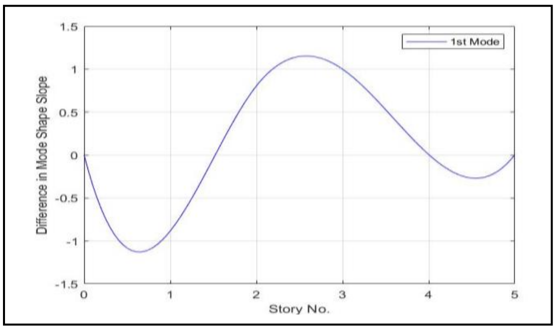


Fig. 5.6 (a) First Mode Slope of 5D3

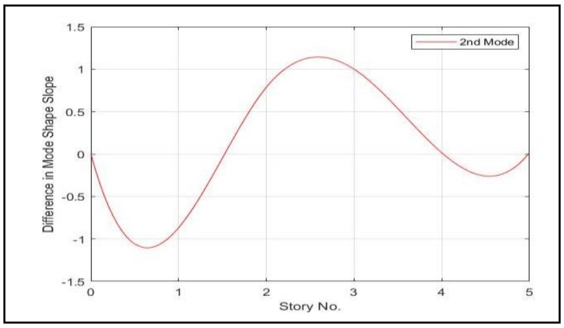


Fig. 5.6 (b) Second Mode Slope of 5D3

Fig 5.6 (c) and (d) shows plot having maximum positive value at story#2 and after story#3 respectively. Fig 5.6 (e) shows plot having higher absolute negative value as compare to maximum positive value. It observed from the result that changes in modal slope of mode 1 and mode 2 are capable to detect damage while others are not.

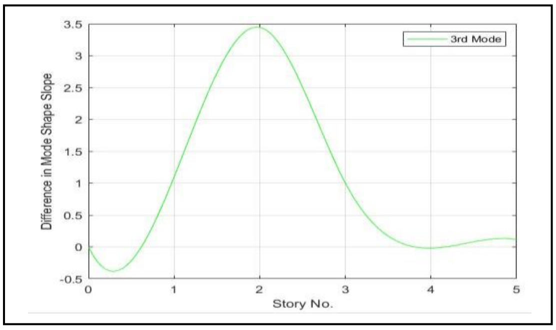


Fig. 5.6 (c) Third Mode Slope of 5D3

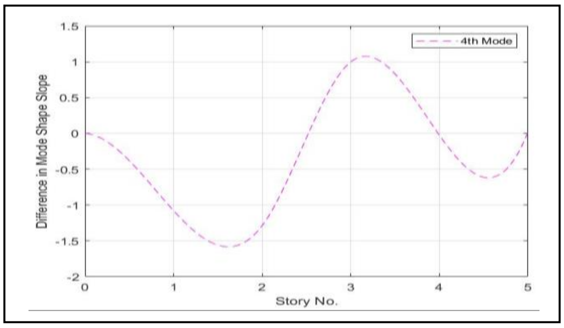
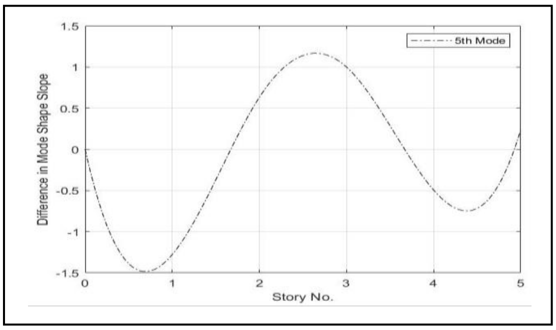


Fig. 5.6 (d) Fourth Mode Slope of 5D3



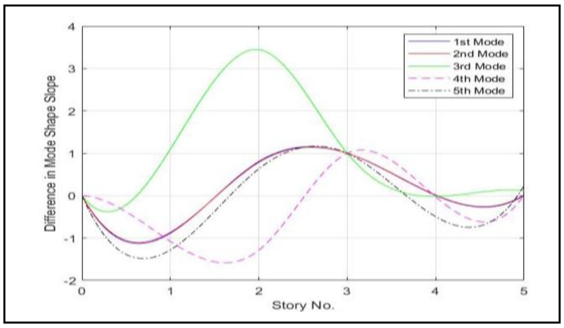
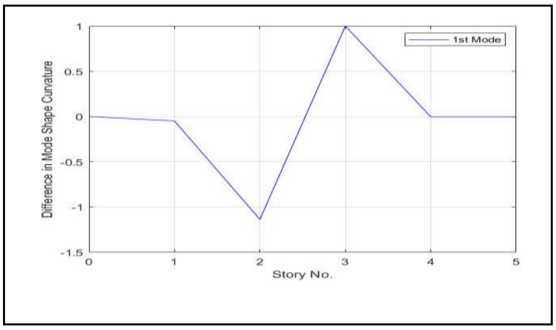


Fig. 5.6 (e) Fifth Mode Slope of 5D3

Fig. 5.6 (f) Combined Mode Slope of 5D3

The difference between undamaged mode shape curvature and 5D3 damaged mode shape curvature are shown in Fig. 5.7**.** It has been observed from the Fig 5.7(a) and (b) that plot changed sign between story#2 and story#3 with the maximum value at story#3 while higher modes shows inappropriate pattern shown in Fig 5.7 (c), (d) and (e). Derivatives of lower modes (i.e., mode 1 and mode 2) are capable to detect the damage location while higher modes are not. The change in mode shape curvature of undamaged and 5D3 damaged building of all five mode shape is plotted, shown in Fig 5.7 (f).



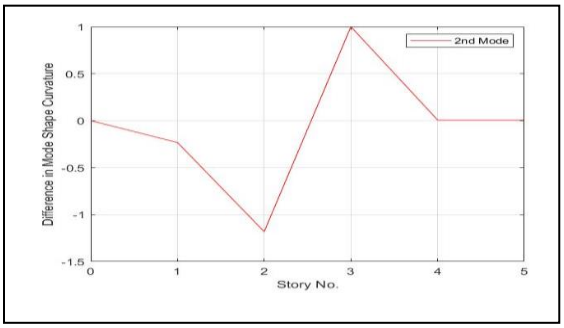
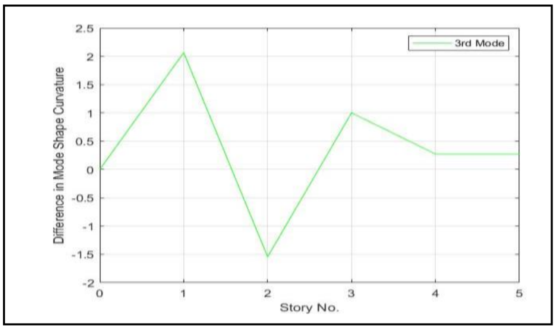


Fig. 5.7 (a) First Modal Curvature of 5D3

Fig. 5.7 (b) Second Modal Curvature of 5D3



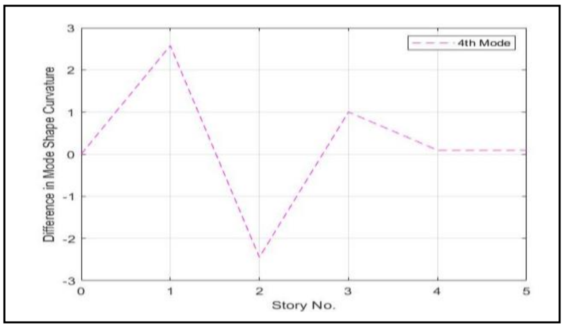
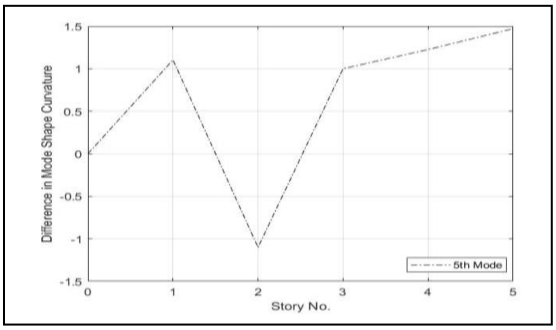


Fig. 5.7 (c) Third Modal Curvature of 5D3

Fig. 5.7 (d) Fourth Modal Curvature of 5D3



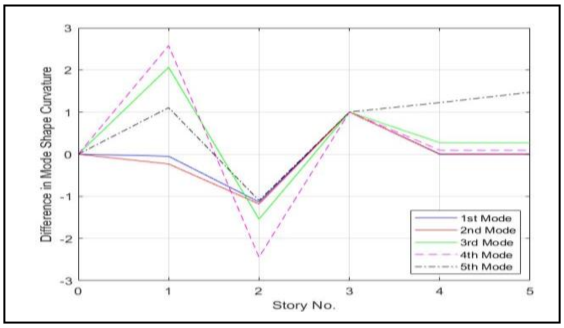


Fig. 5.7 (e) Fifth Mode Curvature of 5D3

Fig. 5.7 (f) Combined Modal Curvature of 5D3

5.2.4 Damaged Building Model 5D4

Damage has been introduced at 4th story by reducing the stiffness by 20%. A stiffness matrix obtained for undamaged 5 story building is shown in Table 5.10. First five natural frequencies obtained after the modal analysis of 5D4 damaged building model are 1.5804, 4.4790, 7.3442, 9.2224 and 10.5238 Hz. Mode shape corresponding to these five natural frequencies have been calculated and then normalized them to unity with the help of MATLAB programme. Modal slope and curvature corresponding to these mode shapes can be calculated with the help of forward, backward and central difference method. The difference between undamaged and damaged mode shape slope is calculated and normalized them to unit shown in Table 5.11(a) and mode shape curvature in Table 5.11(b).

Table 5.10 Stiffness Matrix of 5D4 Damaged Building

500000	-250000	0	0	0
-250000	500000	-250000	0	0
0	-250000	450000	-200000	0
0	0	-200000	450000	-250000
0	0	0	-250000	250000

Table 5.11(a) Change in Modal Slope of Undamaged and 5D4 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.74712	-0.32823	0.466068	6.161959	3.070364
2	-0.6012	-1.04359	-2.17194	4.316737	1.489923
3	0.813947	0.794994	-0.37554	-7.6559	-3.41382
4	1	1	1	1	1
5	0.002128	0.056681	0.011883	0.144622	0.184687

Table 5.11(b) Change in Modal Curvature of Undamaged and 5D4 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.04453	0.214851	1.589938	8.448882	-3.81091
2	-0.08419	0.155126	0.64365	-9.37711	3.41325
3	-1.16412	-1.10603	-2.16466	3.354321	-2.34224
4	1	1	1	1	1
5	0.0361	0.234795	0.155019	0.192438	-0.16835

The difference between undamaged mode shape slope and 5D4 damaged mode shape slope are shown in Fig. 5.8 for first five modes. It is observed from the results that Fig 5.8 (a) and (b) shows, plot having maximum positive value between story#2 and story#3 indicating the damage location at the third story. Modal slope of mode 1 and mode 2 capable of detect damage in 5 story building having damage at 4th story. Change in modal slope of all modes is plotted and shown in Fig 5.8(f).

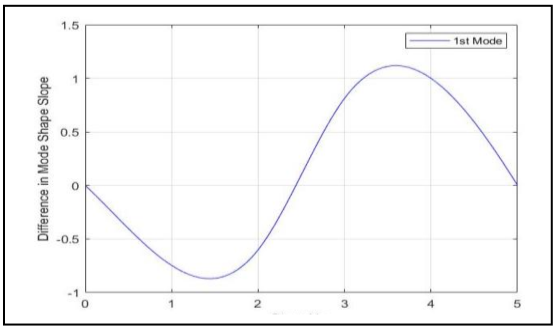


Fig. 5.8 (a) First Modal Slope of 5D4

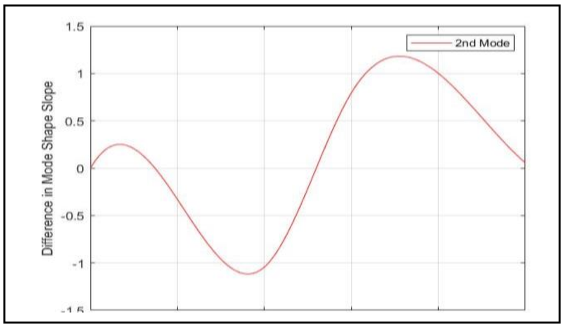


Fig. 5.8 (b) Second Modal Slope of 5D4

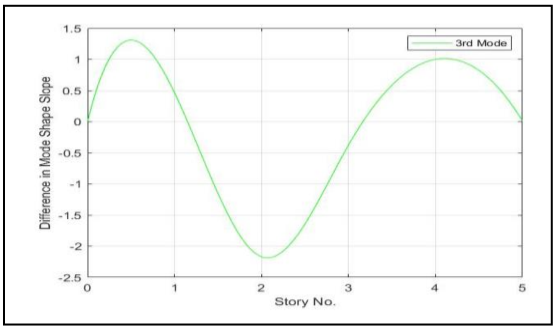


Fig. 5.8 (c) Third Modal Slope of 5D4

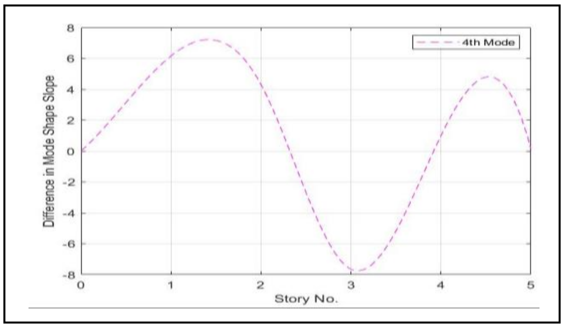


Fig. 5.8 (d) Fourth Modal Slope of 5D4

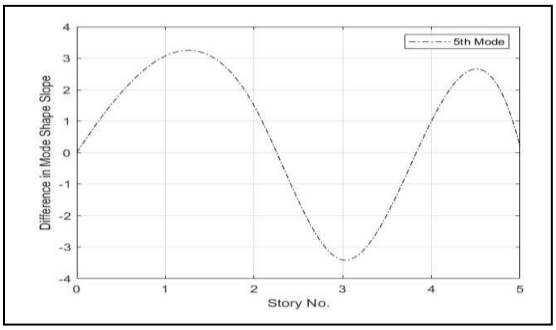


Fig. 5.8 (e) Fifth Modal Slope of 5D4

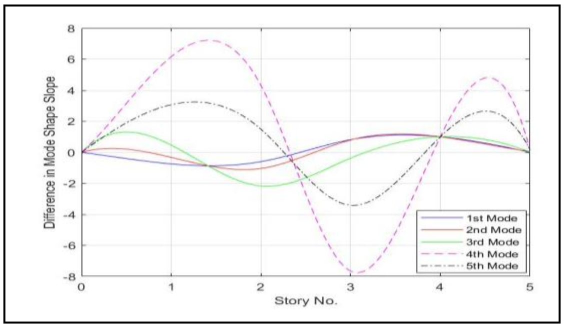


Fig. 5.8 (f) Combined Modal Slope of 5D4

The difference between undamaged mode shape curvature and 5D4 damaged mode shape curvature are shown in Fig. 5.9**.** It has been observed from the result that Fig 5.9 (a) and (b) shows, plot changed sign between story#2 and story#3 with the maximum value at story#3 while higher modes shows inappropriate pattern shown in Fig 5.9 (c), (d) and (e). Derivatives of lower modes are capable to detect the damage location while higher modes are not. The change in mode shape curvature of undamaged and 5D3 damaged building of all five mode shape is plotted, shown in Fig 5.9 (f).

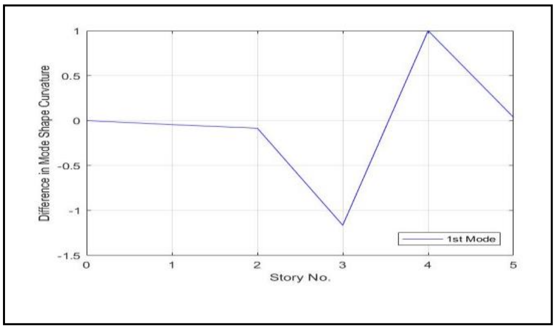


Fig. 5.9 (a) First Modal Curvature of 5D4

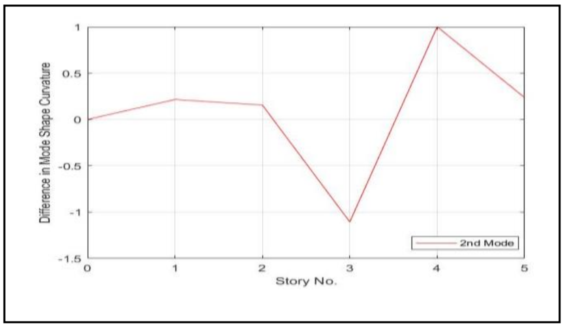


Fig. 5.9 (b) Second Modal Curvature of 5D4

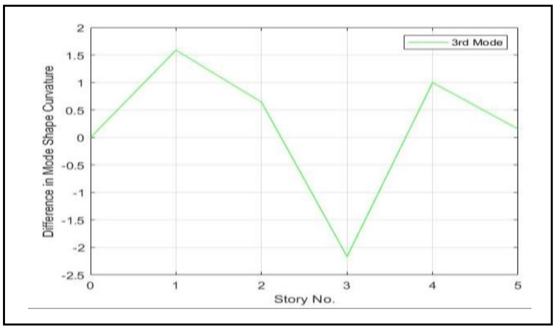


Fig. 5.9 (c) Third Modal Curvature of 5D4

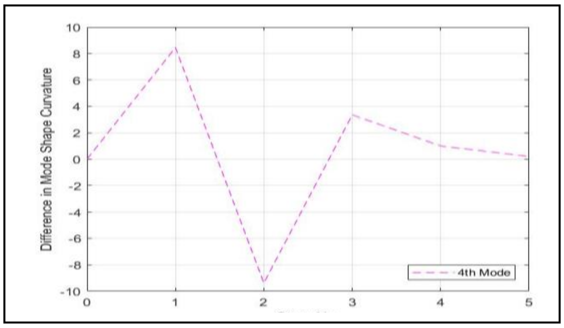


Fig. 5.9 (d) Fourth Modal Curvature of 5D4

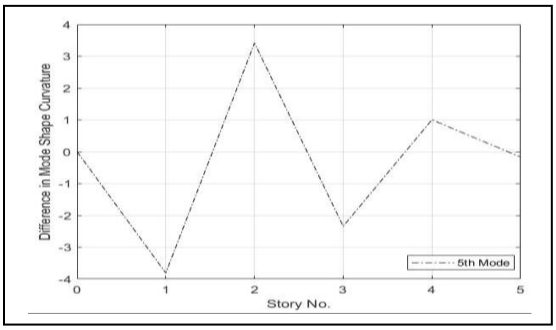


Fig. 5.9 (e) Fifth Modal Curvature of 5D4

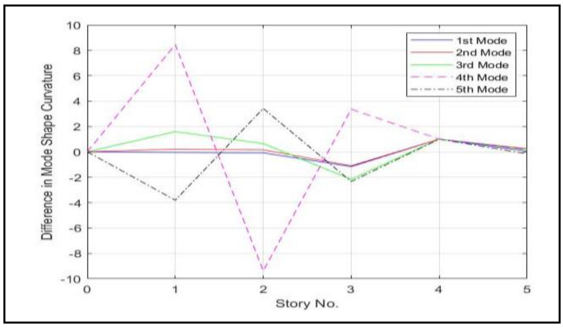


Fig. 5.9 (f) Combined Modal Curvature of 5D4

Result obtained from the analysis of three models of 5 story building are summarized in Table 12 , that shows which modal slope and curvature are capable to detect damage in buildings.

Table 5.12 Derivative of Different Mode of Numerical Models of 5 Story Building

Numerical model	Derivatives	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
5D1	1st derivative	✓	✓	✓	✓	✓
	2nd derivative	✓	✓	✓	✓	✓
5D3	1st derivative	✓	✓	×	×	×
	2nd derivative	✓	✓	×	×	×
5D4	1st derivative	✓	✓	×	×	×
	2nd derivative	✓	✓	×	×	×

5.3 ANALYSIS OF 10 STORY BUILDING

To understanding the effectiveness of the mode shape slope and curvature as damage detection techniques for medium rise building, a 10 story building model has been considered. Damage has been introduced by reducing the stiffness of story by 20% at three different height of the building. First damage is introduced at story below 25% of height of the building, second damage at 50% of height of the building and third damage at story above 75% of height of the building as shown in Table 5.13.

Table 5.13 Location of Damage Introduced in10 Story Building

Detail H (height of building)	Damage introduced at Story		
	Below 25% of H	At 50% of H	Above 75% of H
Damaged Story	1	3	4
Numerical Model	10D2	10D5	10D9
Damage introduced (in %)	20%	20%	20%

5.3.1 Undamaged Building of 10 Story

Mass matrix and a stiffness matrix obtained for undamaged 10 story building is shown in Table 5.14 (a) and 5.14 (b). The mass matrix will be same for the undamaged and damaged building but stiffness matrix will be different. First five natural frequencies obtained after the modal analysis of undamaged building are 0.8410, 2.5042, 4.1115, 5.6270 and 7.0167 Hz. Similar procedure will be follow for the analysis 10 story building as done for 5 story building. MATLAB programme has been used for analysis.

Table 5.14 (a) Mass Matrix of 10 Story Building

200	0	0	0	0	0	0	0	0	0
0	200	0	0	0	0	0	0	0	0
0	0	200	0	0	0	0	0	0	0
0	0	0	200	0	0	0	0	0	0
0	0	0	0	200	0	0	0	0	0
0	0	0	0	0	200	0	0	0	0
0	0	0	0	0	0	200	0	0	0
0	0	0	0	0	0	0	200	0	0
0	0	0	0	0	0	0	0	200	0
0	0	0	0	0	0	0	0	0	200

Table 5.14(b) Stiffness Matrix of Undamaged 10 Story Building

500000	-250000	0	0	0	0	0	0	0	0
-250000	500000	-250000	0	0	0	0	0	0	0
0	-250000	500000	-250000	0	0	0	0	0	0
0	0	-250000	500000	-250000	0	0	0	0	0
0	0	0	-250000	500000	-250000	0	0	0	0
0	0	0	0	-250000	500000	-250000	0	0	0
0	0	0	0	0	-250000	500000	-250000	0	0
0	0	0	0	0	0	-250000	500000	-250000	0
0	0	0	0	0	0	0	-250000	500000	-250000
0	0	0	0	0	0	0	0	-250000	250000

First five mode shapes obtained after the modal analysis of the building and then normalized them to unity shown in Table 5.15 and these mode shapes are plotted in graph shown in Fig 5.10.

Table 5.15 Normalized Undamaged Mode Shape of 10 Story Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	0.14946	-0.44504	0.730682	-1	1.24698
2	0.295582	-0.80194	1.071256	-1	0.554958
3	0.4351	-1	0.83989	1.12E-15	-1
4	0.5649	-1	0.16011	1	-1
5	0.68208	-0.80194	-0.60515	1	0.554958
6	0.784024	-0.44504	-1.04733	-7.1E-16	1.24698
7	0.868454	-1.4E-15	-0.93034	-1	-1.3E-15
8	0.933484	0.445042	-0.31664	-1	-1.24698
9	0.977662	0.801938	0.466104	6.06E-16	-0.55496
10	1	1	1	1	1

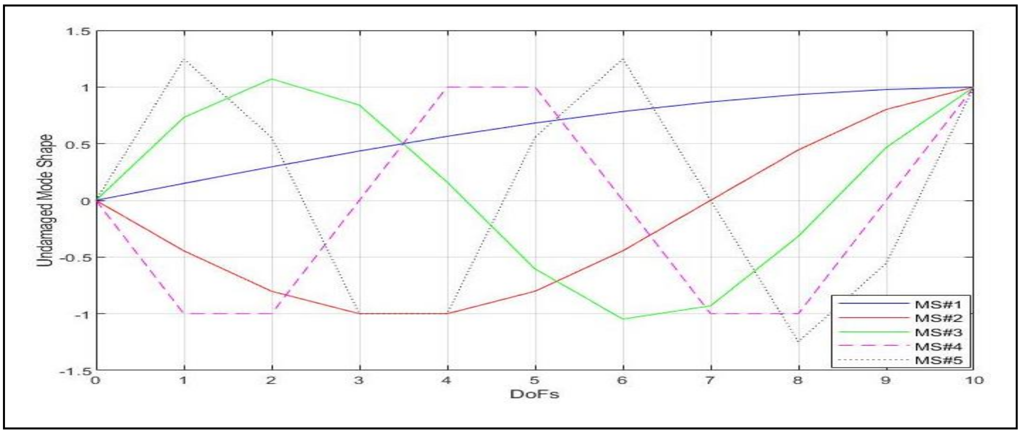


Fig 5.10 Normalized Undamaged Mode Shape of 10 Story Building

5.3.2 Damaged Building Model 10D2

Damage has been introduced at 2nd story by reducing the stiffness by 20%. A stiffness matrix obtained for undamaged 16 story building is shown in Table 5.16. First five natural frequencies obtained after the modal analysis of 10D2 damaged building models are 0.8225, 2.4697, 4.0942, 5.6270 and 6.9818 Hz. Similarly, mode shape corresponding to these five natural frequencies has been calculated and then normalized them to unity with the help of MATLAB programme. Change in mode shape slope of building has been calculated and shown in Table 5.17(a) and mode shape curvature in

Table 5.17(b). Graphical representation of normalized result of change in modal slope and modal curvature is shown in Fig 5.11 and 5.12 repectively.

Table 5.16 Stiffness Matrix of 10D2 Damaged Building

450000	-200000	0	0	0	0	0	0	0	0
-200000	450000	-250000	0	0	0	0	0	0	0
0	-250000	500000	-250000	0	0	0	0	0	0
0	0	-250000	500000	-250000	0	0	0	0	0
0	0	0	-250000	500000	-250000	0	0	0	0
0	0	0	0	-250000	500000	-250000	0	0	0
0	0	0	0	0	-250000	500000	-250000	0	0
0	0	0	0	0	0	-250000	500000	-250000	0
0	0	0	0	0	0	0	-250000	500000	-250000
0	0	0	0	0	0	0	0	-250000	250000

Table 5.17(a) Change in Modal Slope of Undamaged and 10D2 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	1.001559	0.838563	0.030003	-1.5	0.31793
2	1	1	1	1	1
3	0.004571	-0.02166	0.012085	-2.7E-15	-0.06291
4	0.004509	-0.01605	-0.00088	-1.6E-15	-0.04509
5	0.004231	-0.00788	-0.01013	7.77E-16	0.030134
6	0.003756	0.000296	-0.01129	2E-15	0.041605
7	0.00311	0.006243	-0.00575	2.22E-16	-0.00669
8	0.002325	0.008574	0.001197	-8.9E-16	-0.02542
9	0.001437	0.007077	0.004362	0	-0.00073
10	0.000972	0.005425	0.004504	1.11E-16	0.01545

Table 5.17(b) Change in Modal Curvature of Undamaged and 10D2 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.99884	-1.07287	-1.49036	1.121189	0.184348
2	1	1	1	1	1
3	5.03E-05	0.003704	-0.01302	-5.1E-16	4.09E-05
4	-0.00017	0.00752	-0.01291	2.53E-15	-0.05153
5	-0.00038	0.008813	-0.00558	2.21E-15	-0.06197
6	-0.00057	0.007546	0.003251	1.4E-16	-0.01797
7	-0.00072	0.004348	0.007843	-3.6E-15	0.030093
8	-0.00085	0.000313	0.006045	1.36E-15	0.037386
9	-0.00093	-0.00331	0.000285	2.7E-16	0.010468

10	-0.00093	-0.00331	0.000285	2.7E-16	0.03236

The difference between undamaged mode shape slope and 10D2 damaged mode shape slope are shown in Fig. 5.11 for first five modes. It is observed from the results that mode 1 and 3 showing the Dirac delta function [7] and capable to detect damage while other modes are not showing positive results to detect damage. Change in modal slope of all modes is plotted which are shown in Fig 5.11(f).

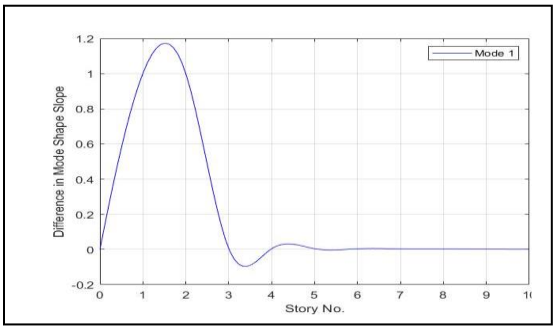


Fig. 5.11 (a) First Modal Slope of 10D2

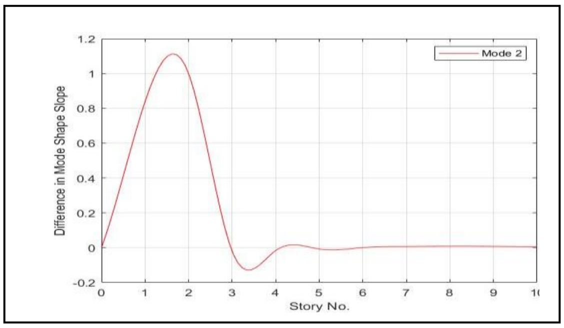


Fig. 5.11 (b) Second Modal Slope of 10D2

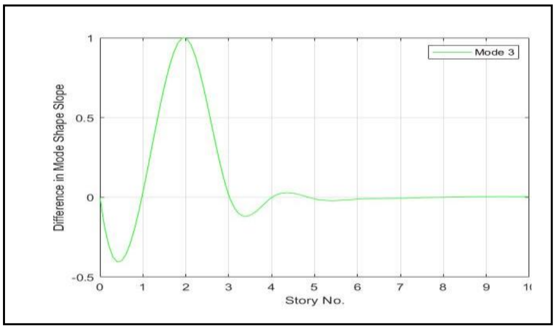


Fig. 5.11 (c) Third Modal Slope of 10D2

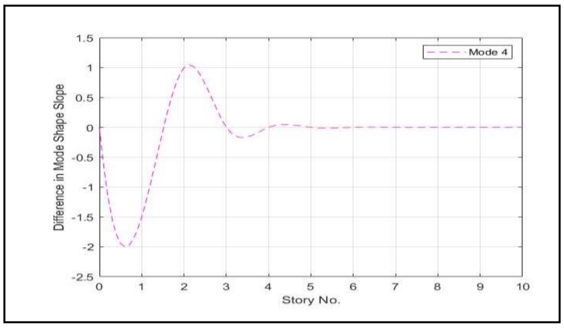


Fig. 5.11 (d) Fourth Modal Slope of 10D2

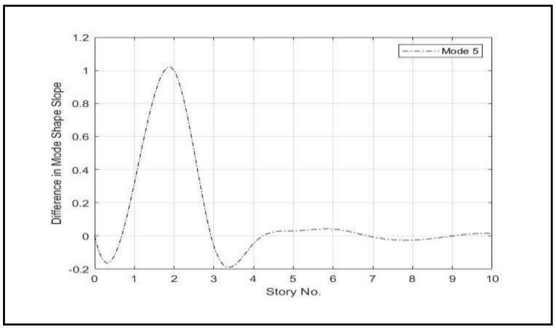


Fig. 5.11 (e) Fifth Modal Slope of 10D2

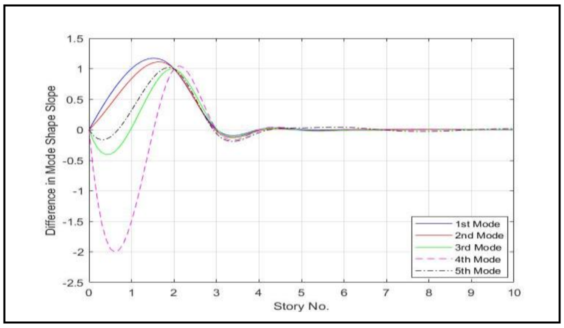


Fig. 5.11 (f) Modal Slope of 10D2

The difference between undamaged mode shape slopes and 10D2 damaged mode shape slopes are shown in Fig. 5.12**.** It has been observed from the result that Fig 5.12 (a), (b) and (c) shows, plot changed sign between story#1 and story#2 with the maximum value at story#2 while higher modes shows inappropriate pattern shown in Fig 5.12 (d) and (e). The change in mode shape curvature of undamaged and 10D2 damaged building of all five mode shape is plotted, shown in Fig 5.12 (f).

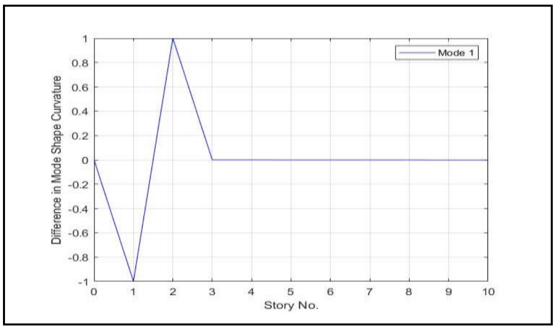


Fig. 5.12 (a) First Modal Curvature of 10D2

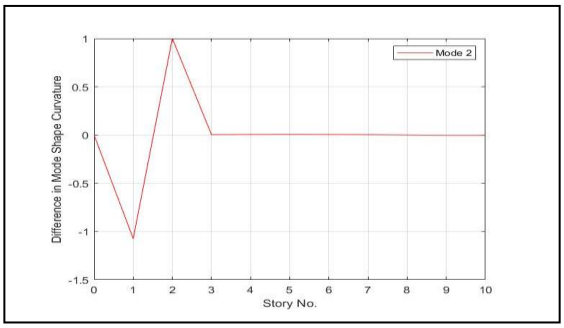


Fig. 5.12 (b) Second Modal Curvature of 10D2

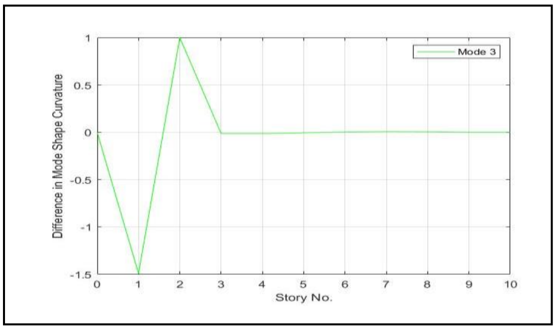


Fig5.12 (c) Third Modal Curvature of 10D2

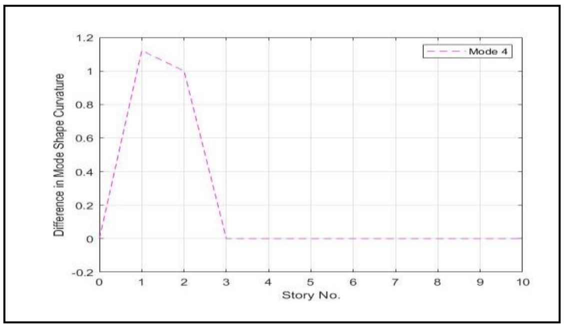


Fig. 5.12 (d) Fourth Modal Curvature of 10D2

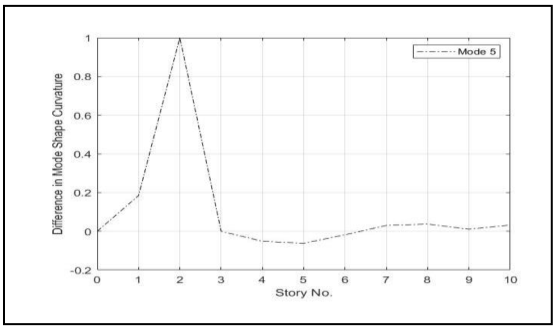


Fig.5.12 (e) Fifth Modal Curvature of 10D2

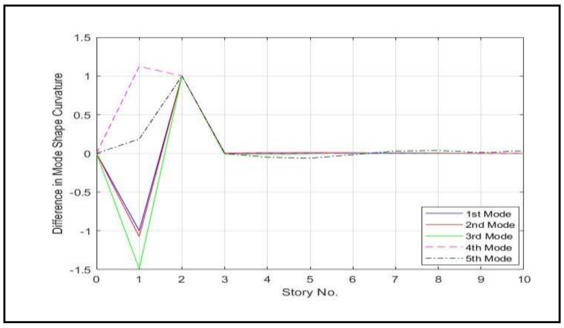


Fig. 5.12 (f) Combined Modal Curvature of 10D2

5.3.3 Damaged Building Model 10D5

Damage has been introduced at 5th story by reducing the stiffness by 20%. A stiffness matrix obtained for 10D5 building is shown in Table 5.18. First five natural frequencies obtained after the modal analysis of 10D5 damaged building models are 10.8290, 2.4390, 4.0215, 5.6270 and 6.8227 Hz. Mode shape corresponding to these five natural frequencies have been calculated and then normalized them to unity with the help of MATLAB programme. In similar way, normalized change in modal slope of undamaged and 10D5 damaged building is calculated shown in Table 5.20(a) and plotted graph is shown in Fig. 5.13, and normalized modal curvature difference shown in Table 5.20(b) and plotted graph in Fig 14.

Table 5.18 Stiffness Matrix of 10D5 Damaged Building

500000	-250000	0	0	0	0	0	0	0	0
-250000	500000	-250000	0	0	0	0	0	0	0
0	-250000	500000	-250000	0	0	0	0	0	0
0	0	-250000	450000	-200000	0	0	0	0	0
0	0	0	-200000	450000	-250000	0	0	0	0
0	0	0	0	-250000	500000	-250000	0	0	0
0	0	0	0	0	-250000	500000	-250000	0	0
0	0	0	0	0	0	-250000	500000	-250000	0
0	0	0	0	0	0	0	-250000	500000	-250000
0	0	0	0	0	0	0	0	-250000	250000

Table 5.19 Normalized Damaged Mode Shape of 10D5 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	0.144692	-0.4632	0.68413	-1	1.313593
2	0.286243	-0.83548	1.018826	-1	0.673306
3	0.421582	-1.04377	0.833133	-5.2E-16	-0.96848
4	0.547771	-1.04718	0.221899	1	-1.16972
5	0.690646	-0.7945	-0.68382	1	0.753577
6	0.789957	-0.4364	-1.05912	1.16E-15	1.171318
7	0.872123	0.007352	-0.89345	-1	-0.1532
8	0.93536	0.449664	-0.27143	-1	-1.24984
9	0.978296	0.803711	0.489227	-5.6E-16	-0.48743
10	1	1	1	1	1

Table 5.20(a) Change in Modal Slope of Undamaged and 10D5 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.40493	-0.60104	0.712562	-0.42857	1.258277
2	-0.37941	-0.45882	-0.54084	-0.92857	-0.3731
3	-0.33779	-0.24418	-1.55231	0.571429	-3.06271
4	0.957629	0.917524	0.977303	3	1.776584
5	1	1	1	1	1
6	0.002449	4.45E-05	-0.05778	2E-15	0.175908
7	0.002029	0.002009	-0.0285	1.05E-15	-0.0364
8	0.001517	0.002789	0.006883	2.78E-16	-0.11036
9	0.000938	0.002311	0.022607	-1.1E-16	-0.00143
10	0.000635	0.001774	0.023124	-1.2E-15	0.067526

Table 5.20(b) Change in Modal Curvature of Undamaged and 10D5 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.007	-0.05196	0.196176	0.055362	-2.40163
2	-0.01378	-0.09664	0.248642	0.131432	-2.30559
3	-0.0201	-0.12763	0.11032	-0.73344	0.429949
4	-1.03449	-1.08618	-1.00805	-0.20074	-1.21034
5	1	1	1	1	1
6	-0.00037	0.002485	0.018195	1.19E-15	-0.069
7	-0.00047	0.001443	0.040355	-2.7E-15	0.152302
8	-0.00055	0.000118	0.030416	1.05E-15	0.1761
9	-0.00061	-0.00107	0.001033	-2E-15	0.04911
10	-0.00061	-0.00107	0.001033	-2E-15	0.137915

The difference between undamaged mode shape slope and 10D2 damaged mode shape slope are shown in Fig. 5.13 for first five modes. It is observed from the results that mode 1 and 2 showing the Dirac delta function [7] and capable to detect damage while other modes are not showing positive results to detect damage. Change in modal slope of all modes is plotted which are shown in Fig 5.13(f).

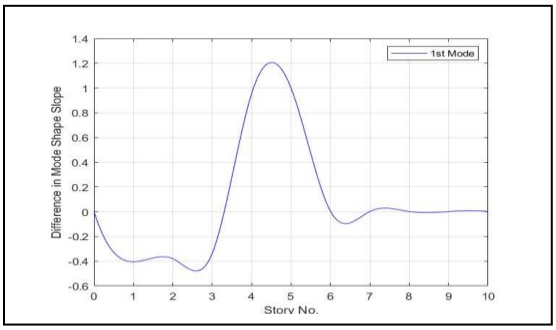


Fig. 5.13 (a) First Modal Slope of 10D5

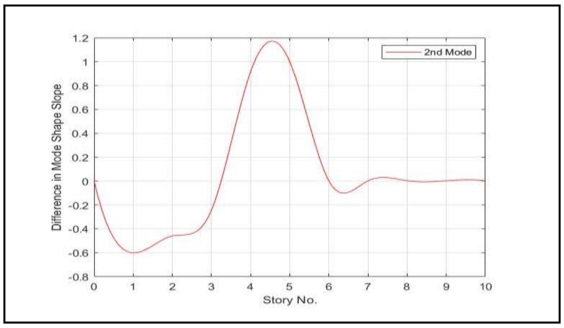


Fig. 5.13 (b) Second Modal Slope of 10D5

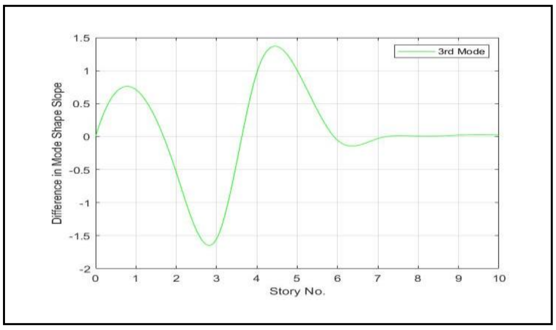


Fig. 5.13 (c) Third Modal Slope of 10D5

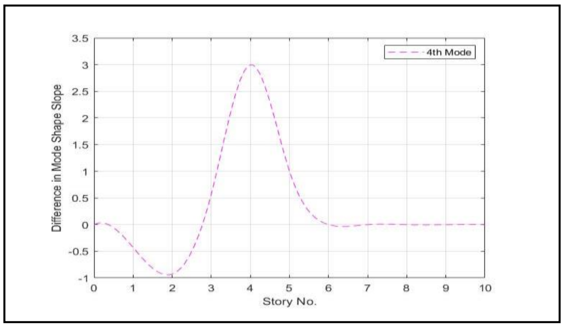


Fig. 5.13 (d) Fourth Modal Slope of 10D5

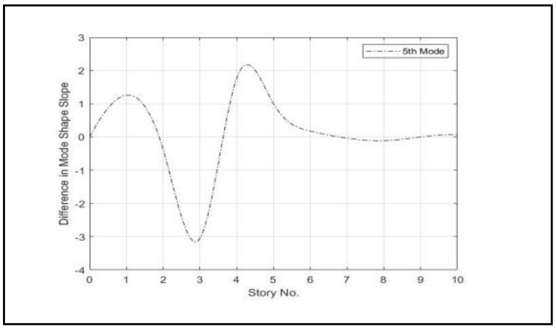


Fig. 5.13 (e) Fifth Modal Slope of 10D5

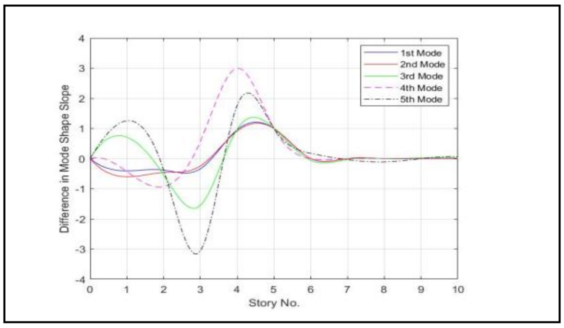


Fig. 5.13 (f) Combined Modal Slope of 10D5

The difference between undamaged mode shape slopes and 10D2 damaged mode shape slopes are shown in Fig. 5.14**.** It has been observed from the result that Fig 5.14 (a), (b) and (c) shows, plot changed sign between story#4 and story#5 with the maximum value at story#5 while higher modes shows inappropriate pattern shown in Fig 5.14 (d) and (e). The change in mode shape curvature of undamaged and 10D2 damaged building of all five mode shape is plotted, shown in Fig 5.14 (f).

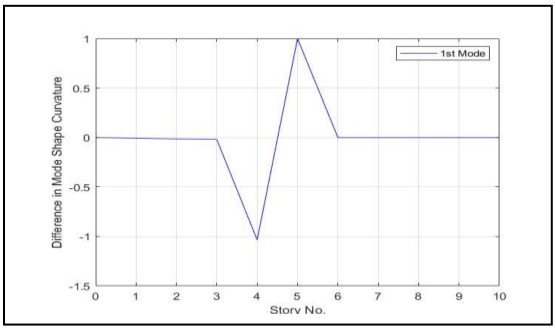


Fig. 5.14 (a) First Modal Curvature of 10D5

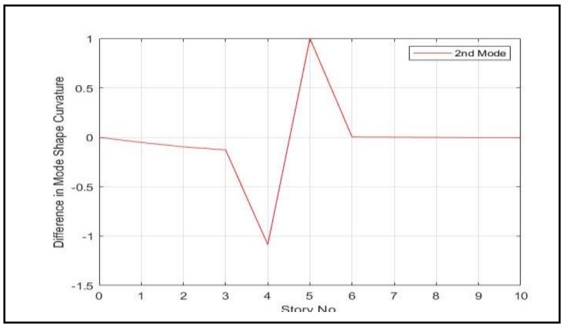


Fig. 5.14 (b) Second Modal Curvature of 10D5

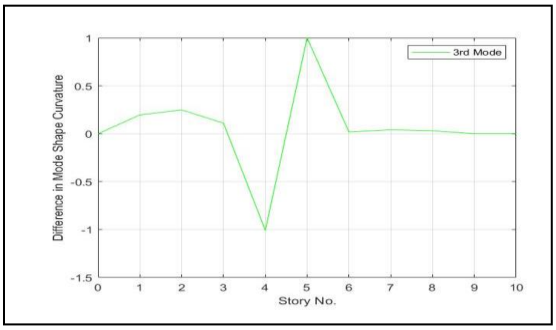


Fig. 5.14 (c) Third Modal Curvature of 10D5

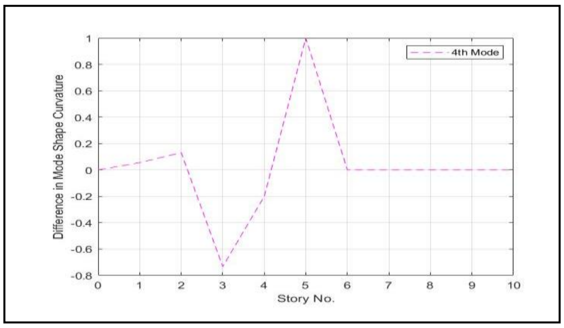


Fig. 5.14 (d) Fourth Modal Curvature of 10D5

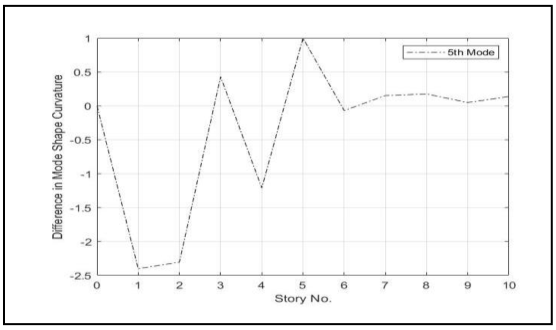


Fig. 5.14 (e) Fifth Modal Curvature of 10D5

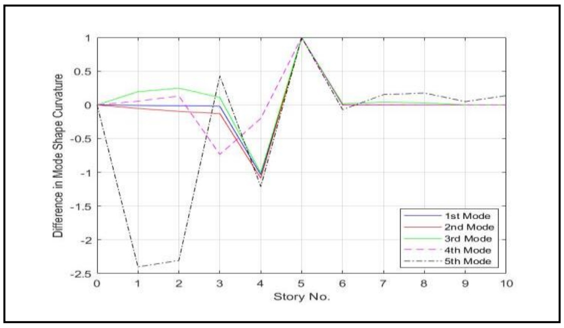


Fig. 5.14 (f) Combined Modal Curvature of 10D5

5.3.4 Damaged Building Model 10D9

Damage has been introduced at 9^{th} story by reducing the stiffness by 20%. A stiffness matrix obtained for 10D9 building is shown in Table 5.21. First five natural frequencies obtained after the modal analysis of 10D9 damaged building models are 0.8393, 2.4671, 4.0164, 5.5362 and 6.9883 Hz. Mode shape corresponding to these five natural frequencies have been calculated and then normalized them to unity with the help of MATLAB programme. In similar way, normalized change in modal slope of undamaged and 10D5 damaged building is calculated is shown in Table 4.23 (a) and normalized modal curvature in Table 5.23 (b).

Table 5.21 Stiffness Matrix of 10D9 Damaged Building

500000	-250000	0	0	0	0	0	0	0	0
-250000	500000	-250000	0	0	0	0	0	0	0
0	-250000	500000	-250000	0	0	0	0	0	0
0	0	-250000	500000	-250000	0	0	0	0	0
0	0	0	-250000	500000	-250000	0	0	0	0
0	0	0	0	-250000	500000	-250000	0	0	0
0	0	0	0	0	-250000	500000	-250000	0	0
0	0	0	0	0	0	-250000	450000	-200000	0
0	0	0	0	0	0	0	-200000	450000	-250000
0	0	0	0	0	0	0	0	-250000	250000

Table 5.22 Normalized Damaged Mode Shape of 10D9 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	0.147583	-0.41534	0.711613	-1.11999	1.441435
2	0.291882	-0.75084	1.060679	-1.15585	0.659628
3	0.429688	-0.942	0.86936	-0.07288	-1.13958
4	0.557936	-0.95207	0.235126	1.080641	-1.18112
5	0.673772	-0.77912	-0.5189	1.18812	0.599073
6	0.77462	-0.45639	-1.00856	0.145525	1.455268
7	0.858235	-0.04593	-0.98439	-1.03794	0.066885
8	0.922759	0.373364	-0.4587	-1.2167	-1.42466
9	0.977754	0.807763	0.490529	0.032022	-0.54238
10	1	1	1	1	1

Table 5.23(a) Change in Modal Slope of Undamaged and 10D9 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.34496	0.712838	-0.07445	-0.71921	0.589093
2	-0.32955	0.394814	0.341692	0.217403	-1.87996
3	-0.30433	-0.04421	0.602532	1.091344	-1.60845
4	-0.27001	-0.49083	0.399737	1.204431	1.033833
5	-0.22756	-0.827	-0.25517	0.299426	2.191626
6	-0.17814	-0.95909	-0.98768	-1.04319	0.128152
7	-0.12315	-0.84163	-1.27291	-1.67156	-2.17226
8	0.961422	0.722045	0.552428	0.322833	-0.30565
9	1	1	1	1	1

10	9.27E-05	0.005825	0.024425	0.032022	0.012577

Table 5.23(b) Change in Modal Curvature of Undamaged and 10D9 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.0051	0.099625	-0.14438	-0.29965	-0.63042
2	-0.01006	0.173936	-0.16528	-0.4233	-0.33564
3	-0.01473	0.203706	-0.02881	-0.25127	0.59767
4	-0.019	0.180471	0.179713	0.163984	1.083717
5	-0.02274	0.108697	0.307614	0.534569	0.641601
6	-0.02584	0.004923	0.237458	0.501764	-0.14772
7	-0.02822	-0.10596	-0.02522	-0.01674	-0.61232
8	-1.03792	-1.23909	-1.33304	-1.52269	-1.60497
9	1	1	1	1	1
10	0.01091	0.083328	0.190905	0.28074	0.202835

The difference between undamaged mode shape slope and 10D2 damaged mode shape slope are shown in Fig. 5.15 for first five modes. It is observed from the results that mode 1 and 2 showing the Dirac delta function [7] and capable to detect damage while other modes are not showing positive results to detect damage. Change in modal slope of all modes is plotted which are shown in Fig 5.15(f).

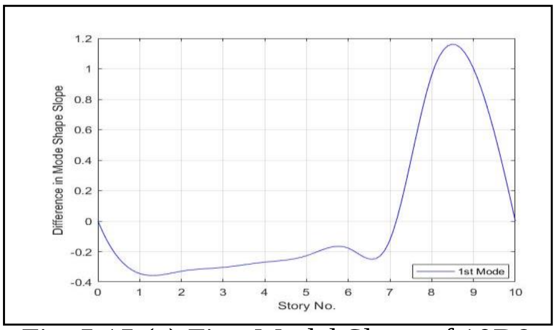


Fig. 5.15 (a) First Modal Slope of 10D9

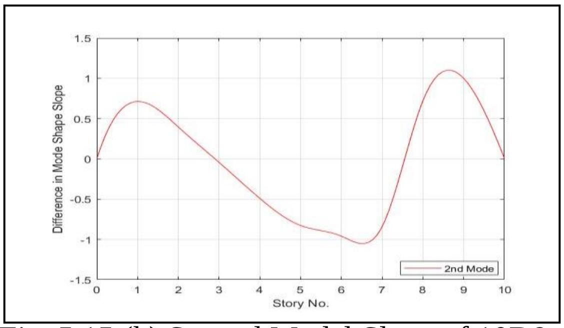


Fig. 5.15 (b) Second Modal Slope of 10D9

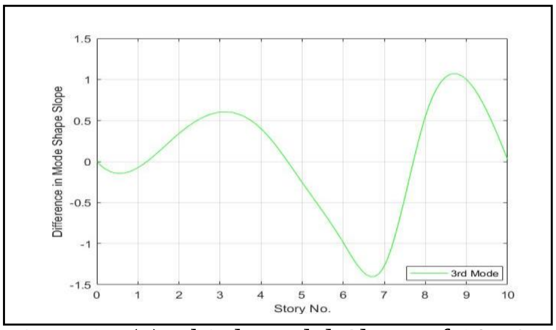


Fig. 5.15 (c) Third Modal Slope of 10D9

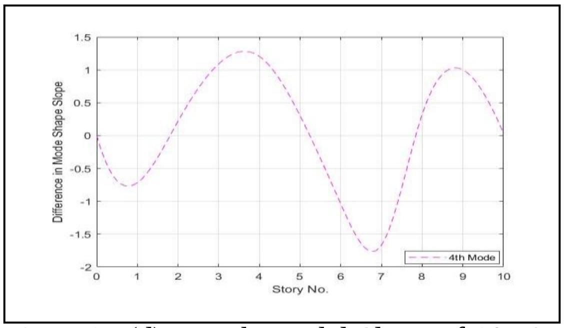


Fig. 5.15 (d) Fourth Modal Slope of 10D9

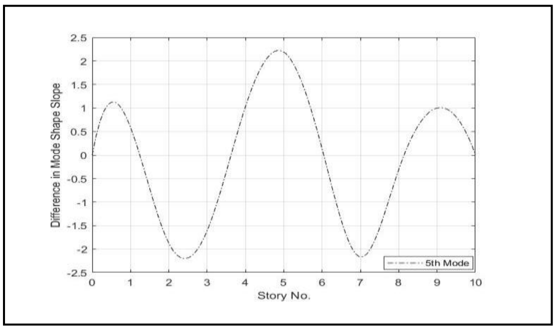


Fig. 5.15 (e) Fifth Modal Slope of 10D9

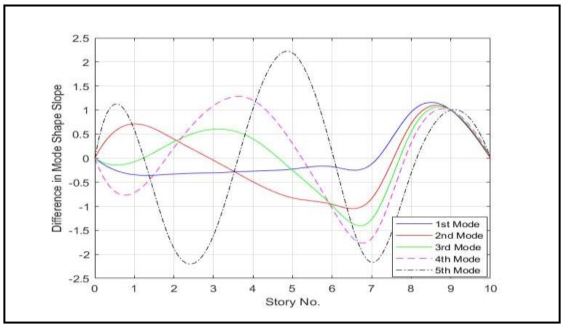


Fig. 5.15 (f) Combined Modal Slope of 10D9

The difference between undamaged mode shape slopes and 10D2 damaged mode shape slopes are shown in Fig. 5.16**.** It has been observed from the result that Fig 5.16 (a), (b) and (c) shows, plot changed sign between story#8 and story#9 with the maximum value at story#9 while higher modes shows inappropriate pattern to detect damage shown in Fig 5.16 (d) and (e). The change in mode shape curvature of undamaged and 10D9 damaged building of all five mode shape is plotted, shown in Fig 5.16 (f).

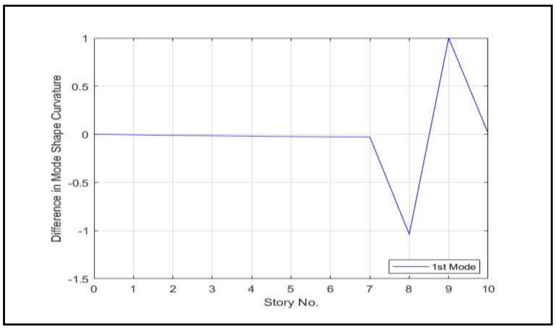


Fig. 5.16 (a) First Modal Curvature of 10D9

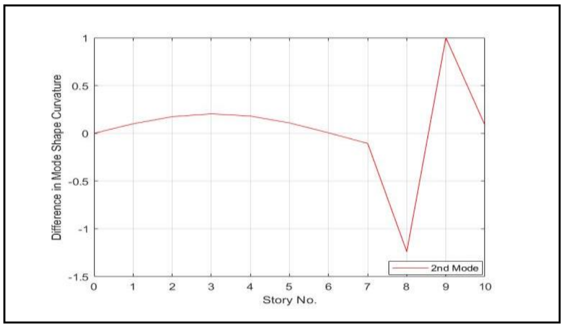


Fig. 5.16 (b) Second Modal Curvature of 10D9

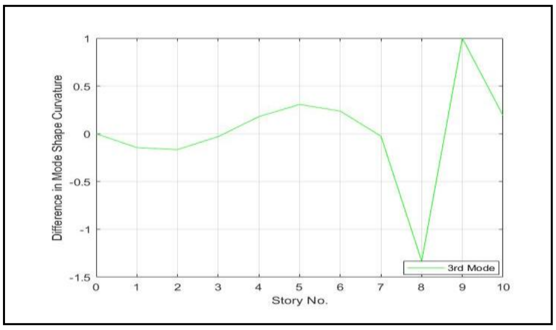


Fig. 5.16 (a) Third Modal Curvature of 10D9

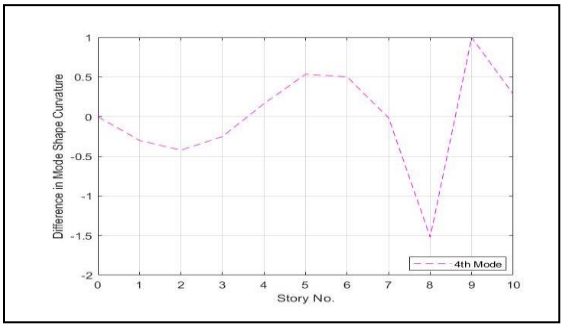


Fig. 5.16 (b) Fourth Modal Curvature of 10D9

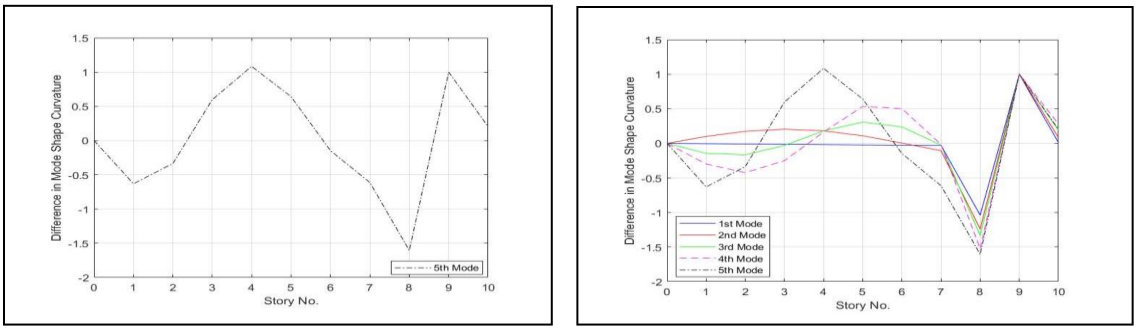


Fig. 4.15 (a) Fifth Modal Curvature of 10D9

Fig. 4.15 (b) Combined Modal Curvature of 10D9

Result obtained from the analysis of three models of 5 story building are summarized in Table 5.24 , that shows which modal slope and curvature are capable to detect damage in buildings.

Table 5.24 Derivatives of Different Mode of Numerical Models of 10 Story Building

Numerical model	Derivatives	1st mode	2nd mode	3rd mode	4th mode	5th mode
10D2	1st derivative	✓	✓	✗	✗	✗
	2nd derivative	✓	✓	✓	✗	✗
10D5	1st derivative	✓	✓	✗	✗	✗
	2nd derivative	✓	✓	✓	✗	✗
10D9	1st derivative	✓	✓	✗	✗	✗
	2nd derivative	✓	✓	✓	✗	✗

5.4 ANALYSIS OF 16 STORY BUILDING

To understanding the effectiveness of the mode shape slope and curvature as damage detection techniques for high rise building, a 16 story building model has been considered. Damage has been introduced by reducing the stiffness of story by 20% at three different height of the building. First damage is introduced at story below 25% of height of the building, second damage at 50% of height of the building and third damage at story above 75% of height of the building as shown in Table 5.25.

Table 5.25 Location of Damage Introduced in16 Story Building

Detail H (height of building)	Damage introduced at Story		
	Below 25% of H	At 50% of H	Above 75% of H
Damaged Story	2	7	13
Numerical Model	16D2	16D7	16D13
Damage introduced (in %)	20%	20%	20%

5.4.1 Undamaged Building of 16 Story

Mass matrix and a stiffness matrix obtained for undamaged 16 story building is shown in Table 5.26 (a) and 5.26 (b). The mass matrix will be same for the undamaged and damaged building but stiffness matrix will be different. First five natural frequencies obtained after the modal analysis of undamaged building are 0.5355, 1.6016, 2.6532, 3.6808 and 4.6751 Hz.

Table 5.26 (a) Mass Matrix of 16 Story Building

200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	200	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	200	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	200	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	200	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	200	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	200	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	200	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	200	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	200	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	200	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	200	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	200	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	200	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	200	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	200

Table 5.26(b) Stiffness Matrix of Undamaged 16 Story Building

500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-250000	250000

First five mode shapes obtained after the modal analysis of the undamaged 16 story building and then normalized them to unity shown in Table 4.27 and these mode shapes are plotted in graph shown in Fig 4.16

Table 5.27 Normalized Undamaged Mode Shape of 16 Story Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	0.095164	-0.28463	0.471518	-0.65414	0.83083
2	0.189466	-0.5462	0.838204	-1.02837	1.088156
3	0.282052	-0.76352	1.018532	-0.96257	0.594351
4	0.372084	-0.91899	0.972412	-0.4849	-0.30972
5	0.458746	-1	0.710095	0.200266	-1
6	0.541254	-1	0.289905	0.799734	-1
7	0.61886	-0.91899	-0.19474	1.057002	-0.30972
8	0.690862	-0.76352	-0.63609	0.861984	0.594351
9	0.756607	-0.5462	-0.93602	0.298129	1.088156
10	0.8155	-0.28463	-1.02784	-0.39329	0.83083
11	0.867007	3.78E-15	-0.89115	-0.91643	-1.5E-16
12	0.910663	0.28463	-0.55632	-1.04743	-0.83083
13	0.946072	0.5462	-0.09781	-0.73024	-1.08816
14	0.972914	0.763521	0.382443	-0.10059	-0.59435
15	0.990944	0.918986	0.777671	0.572106	0.309721
16	1	1	1	1	1

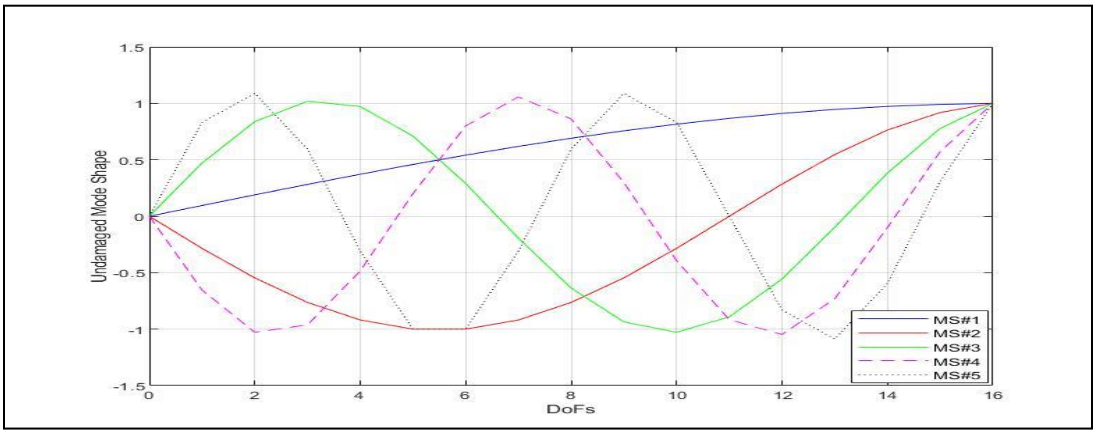


Fig 5.17 Normalized undamaged mode shape of 16 story building

5.4.2 Damaged Building Model 16D2

Damage has been introduced at 2st story by reducing the stiffness by 20%. A stiffness matrix obtained for 16D2 damaged building is shown in Table 5.28. First five natural frequencies obtained after the modal analysis of 16D2 damaged building models are 0.5277, 1.5822, 2.6315, 3.6656 and 4.6697 Hz. Mode shape corresponding to these five natural frequencies have been calculated and then normalized them to unity with the help of MATLAB programme. Normalized mode shapes are summarized in Table 5.29

Table 5.28 Stiffness Matrix of 16D2 Damaged Building

450000	-200000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-200000	450000	-250000	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-250000	250000

Table 5.29 Normalized Damaged Mode Shape of 16D2 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	0.093447	-0.27345	0.440496	-0.60326	0.78136
2	0.209227	-0.58824	0.87069	-1.03733	1.085405
3	0.300012	-0.79356	1.024417	-0.94439	0.581121
4	0.388159	-0.93614	0.954092	-0.45069	-0.32338
5	0.472892	-1.0047	0.675098	0.234266	-1.00517
6	0.553467	-0.99383	0.248453	0.819808	-0.9947
7	0.629174	-0.90438	-0.23253	1.057457	-0.29917
8	0.699349	-0.74343	-0.66266	0.846366	0.60239
9	0.763373	-0.5237	-0.94786	0.276111	1.089088
10	0.820685	-0.26256	-1.02575	-0.41131	0.825729
11	0.870779	0.019336	-0.8793	-0.92419	-0.00631
12	0.913216	0.299704	-0.54054	-1.04488	-0.834
13	0.947623	0.556376	-0.08355	-0.72217	-1.08732
14	0.973696	0.769058	0.391703	-0.093	-0.59179
15	0.991206	0.920936	0.78129	0.575641	0.311299
16	1	1	1	1	1

In similar way, normalized change in modal slope of undamaged and 10D5 damaged building is calculated is shown in Table 5.30 (a) and normalized modal curvature in Table 5.30 (b).

Table 5.30(a) Change in Modal Slope of Undamaged and 16D2 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	1.00427	1.019891	0.88026	0.274022	-0.07592
2	1	1	1	1	1
3	0.001843	-0.01244	0.025403	-0.02158	0.005454
4	0.001907	-0.01267	0.020441	-0.00791	-0.00403
5	0.001931	-0.01166	0.011566	0.007067	-0.00948
6	0.001916	-0.00965	0.001397	0.016772	-0.00786
7	0.001863	-0.00696	-0.00744	0.017846	-0.00137
8	0.001774	-0.00395	-0.01298	0.011237	0.004807
9	0.001651	-0.00099	-0.01433	0.0012	0.00657
10	0.001497	0.001583	-0.01184	-0.00713	0.003621
11	0.001316	0.003497	-0.00685	-0.01028	-0.00096
12	0.001111	0.00458	-0.00121	-0.00792	-0.00357
13	0.000885	0.004769	0.003264	-0.00252	-0.00287
14	0.000644	0.004113	0.005321	0.002269	-0.00037
15	0.000391	0.002769	0.00463	0.003796	0.001278
16	0.000262	0.00195	0.003619	0.003535	0.001577

Table 5.30(b) Change in Modal Curvature of Undamaged and 16D2 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.99639	-0.98743	-1.04904	-1.2729	-0.4644
2	1	1	1	1	1
3	8.4E-05	-0.00089	-0.0024	0.011119	0.02818
4	4.38E-05	0.000436	-0.00753	0.01623	-0.01824
5	4.08E-06	0.001578	-0.01022	0.01372	-0.06164
6	-3.5E-05	0.002438	-0.01011	0.005691	-0.08828
7	-7.2E-05	0.002946	-0.00756	-0.00354	-0.08892
8	-0.00011	0.003077	-0.00351	-0.00968	-0.06462
9	-0.00014	0.002844	0.000798	-0.0104	-0.02662
10	-0.00017	0.0023	0.004177	-0.00626	0.009566
11	-0.00019	0.001529	0.005816	-5.5E-05	0.031972
12	-0.00022	0.000637	0.005466	0.004783	0.037048
13	-0.00023	-0.00026	0.003473	0.006007	0.029138
14	-0.00025	-0.00105	0.000641	0.003578	0.016105
15	-0.00026	-0.00164	-0.00202	-0.00052	0.004681
16	-0.00026	-0.00164	-0.00202	-0.00052	0.000598

The difference between undamaged mode shape slope and 16D2 damaged mode shape slope are shown in Fig. 5.18 for first five modes. It is observed from the results that mode 1, 2 and 3 showing the Dirac delta function [7] and capable to detect damage while other modes are not showing positive results to detect damage. Change in modal slope of all modes is plotted which are shown in Fig 5.18(f).

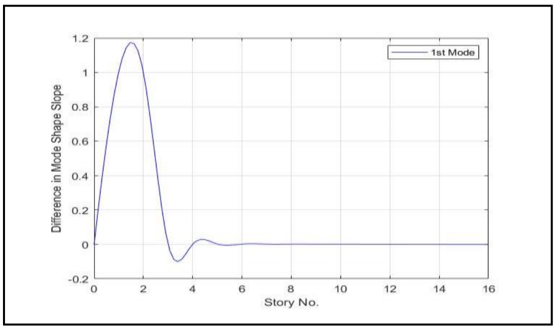


Fig. 5.18 (a) First Modal Slope of 16D2

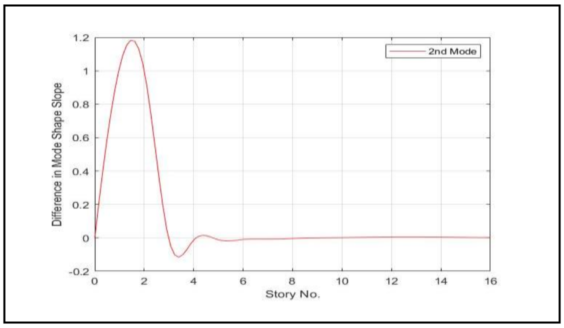


Fig. 5.18 (b) Second Modal Slope of 16D2

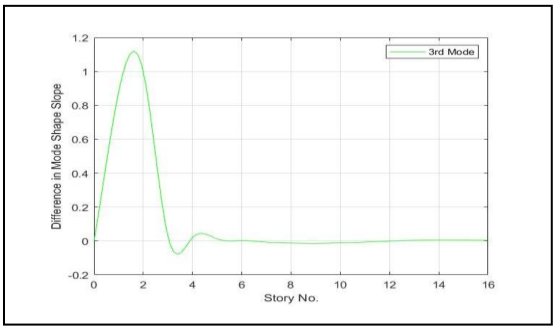


Fig. 5.18 (c) Third Modal Slope of 16D2

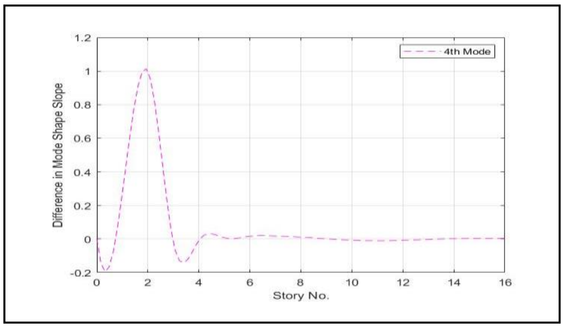


Fig. 5.18 (d) Fourth Modal Slope of 16D2

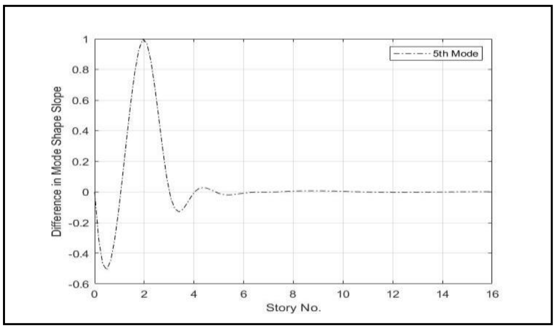


Fig. 5.18 (e) Fifth Modal Slope of 16D2

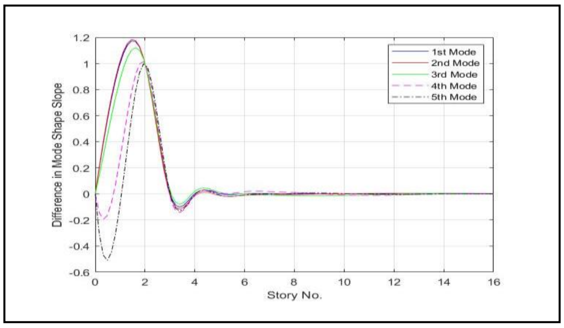


Fig. 5.18 (f) Combined Modal Slope of 16D2

The difference between undamaged mode shape slopes and 16D2 damaged mode shape slopes are shown in Fig. 5.19**.** It has been observed from the result that Fig 5.19 (a), (b) and (c) shows, plot changed sign between story#1 and story#2 with the maximum value at story#2 while higher modes (i.e., mode 3 and 4) shows inappropriate pattern to detect damage shown in Fig 5.19 (d) and (e). The change in mode shape curvature of undamaged and 10D9 damaged building of all five mode shape is plotted, shown in Fig 5.19 (f).

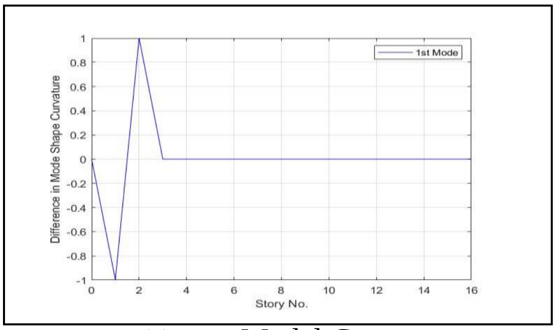


Fig. 5.19 (a) First Modal Curvature of 16D2

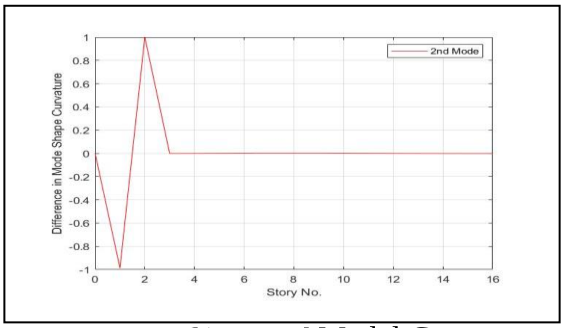


Fig. 5.19 (b) Second Modal Curvature of 16D2

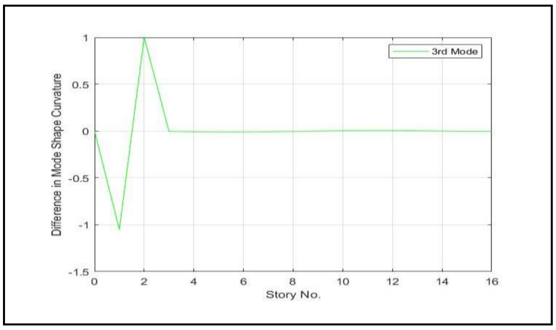


Fig. 5.19 (c) Third Modal Curvature of 16D2

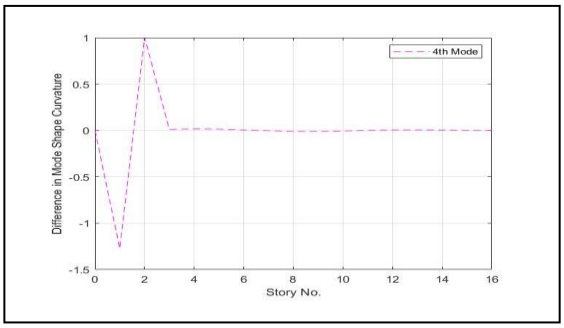


Fig. 5.19 (d) Fourth Modal Curvature of 16D2

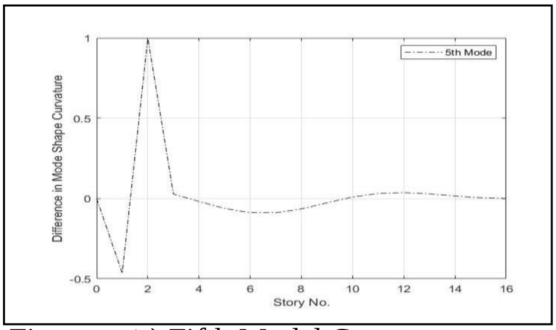


Fig. 5.19 (e) Fifth Modal Curvature of 16D2

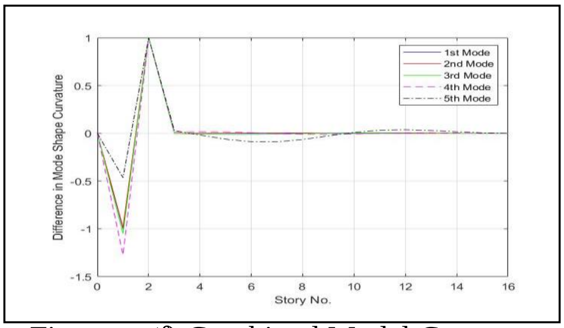


Fig. 5.19 (f) Combined Modal Curvature of 16D2

5.4.3 Damaged Building Model 16D7

Damage has been introduced at 7th story by reducing the stiffness by 20%. A stiffness matrix obtained for 16D7 damaged building is shown in Table 5.31. First five natural frequencies obtained after the modal analysis of 16D7 damaged building models are

0.5302, 1.5997, 2.6137, 3.6732 and 4.6342 Hz. Mode shape corresponding to these five natural frequencies have been calculated and then normalized them to unity with the help of MATLAB programme. Normalized mode shapes are summarized in Table 5.32

Table 5.31 Stiffness Matrix of 16D7 Damaged Building

500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-250000	450000	-200000	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-200000	450000	-250000	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-250000	250000

Table 5.32 Normalized Damaged Mode Shape of 16D7 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	0.093174	-0.28987	0.466442	-0.61525	0.920344
2	0.18552	-0.55631	0.832244	-0.96832	1.216447
3	0.27622	-0.77779	1.018481	-0.90876	0.687473
4	0.364468	-0.93641	0.98497	-0.46195	-0.30779
5	0.44948	-1.01935	0.738942	0.181717	-1.09429
6	0.530502	-1.01991	0.33348	0.747945	-1.13857
7	0.625893	-0.91758	-0.26329	1.057326	-0.22859
8	0.69665	-0.76155	-0.68389	0.85427	0.654432
9	0.761222	-0.54397	-0.95694	0.287182	1.093579
10	0.819037	-0.28244	-1.02352	-0.40228	0.790986
11	0.869581	0.001927	-0.86927	-0.92032	-0.04811
12	0.912405	0.286135	-0.52746	-1.04618	-0.85457
13	0.94713	0.547218	-0.07184	-0.72623	-1.08141
14	0.973447	0.764075	0.399272	-0.09681	-0.57476
15	0.991123	0.919181	0.78424	0.573868	0.321732
16	1	1	1	1	1

In similar way, normalized change in modal slope of undamaged and 16D7 damaged building is calculated is shown in Table 5.33 (a) and normalized modal curvature in Table 5.33 (b).

Table 5.33(a) Change in Modal Slope of Undamaged and 16D7 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.23854	-0.46194	0.065221	1.362451	0.645819
2	-0.23228	-0.41268	-0.05499	0.338654	0.018165
3	-0.22193	-0.33433	-0.20265	-0.84178	-0.63611
4	-0.20763	-0.23229	-0.31624	-1.64179	-0.94344
5	-0.18958	-0.11359	-0.33942	-1.6957	-0.70726
6	0.985452	0.948799	1.065802	0.428212	0.883056
7	1	1	1	1	1
8	0.001209	-0.00041	-0.02381	0.005636	0.037852
9	0.001126	-0.00011	-0.02606	0.000638	0.049963
10	0.001021	0.00015	-0.0214	-0.00353	0.026766
11	0.000898	0.000344	-0.01227	-0.00512	-0.00805
12	0.000758	0.000455	-0.00205	-0.00395	-0.02743
13	0.000604	0.000476	0.006018	-0.00127	-0.02167
14	0.000439	0.000411	0.009701	0.001125	-0.00263
15	0.000267	0.000277	0.008414	0.00189	0.009797
16	0.000179	0.000195	0.006569	0.001762	0.01201

Table 5.33(b) Change in Modal Curvature of Undamaged and 16D7 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.00182	-0.01772	0.031556	0.294635	2.36224
2	-0.00362	-0.03422	0.05112	0.45552	3.319661
3	-0.00537	-0.04837	0.050441	0.409407	2.22369
4	-0.00705	-0.05919	0.027683	0.176771	-0.44853
5	-0.00864	-0.06593	-0.01174	-0.13727	-3.31584
6	-1.01264	-1.05397	-0.95474	-1.41896	-1.71167
7	1	1	1	1	1
8	-7.2E-05	0.000309	-0.00614	-0.00481	-0.18216
9	-9.4E-05	0.000287	0.001633	-0.00519	-0.06209
10	-0.00011	0.000233	0.007683	-0.00314	0.051319
11	-0.00013	0.000156	0.010574	-4.5E-05	0.107343
12	-0.00015	6.61E-05	0.009882	0.002376	0.09909
13	-0.00016	-2.4E-05	0.006246	0.002995	0.056088
14	-0.00017	-0.0001	0.001119	0.001788	0.014898
15	-0.00018	-0.00016	-0.00369	-0.00026	-0.00646
16	-0.00018	-0.00016	-0.00369	-0.00026	0.004426

The difference between undamaged mode shape slope and 16D7 damaged mode shape slope are shown in Fig. 5.20 for first five modes. It is observed from the results that first three mode are capable to detect damage while other modes are not showing positive results to detect damage. Change in modal slope of all modes is plotted, are shown in Fig 5.20(f).

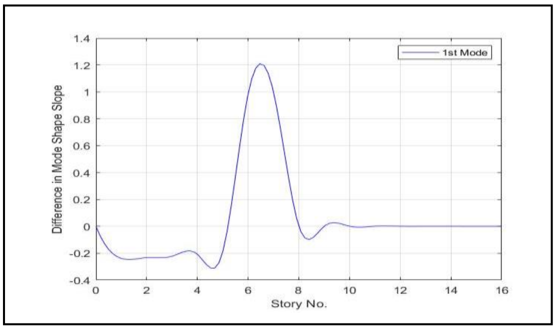


Fig. 5.20 (a) First Modal Slope of 16D7

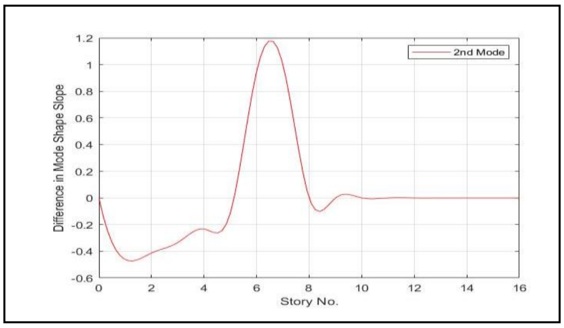


Fig. 5.20 (b) Second Modal Slope of 16D7

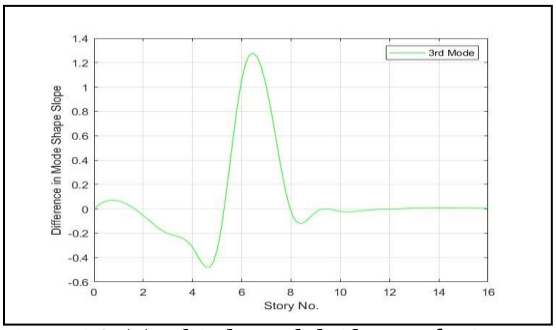


Fig. 5.20 (c) Third Modal Slope of 16D7

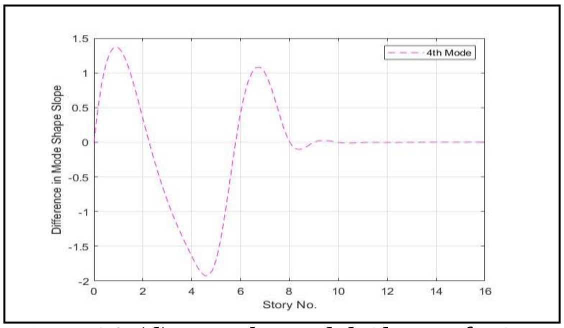


Fig. 5.20 (d) Fourth Modal Slope of 16D7

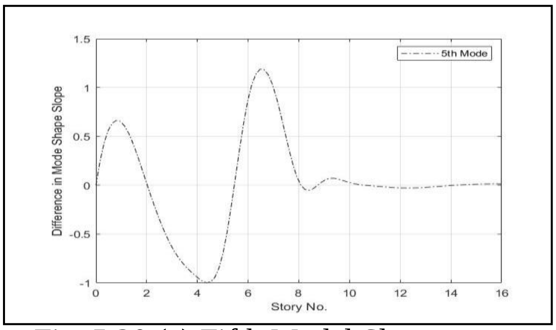


Fig. 5.20 (e) Fifth Modal Slope of 16D7

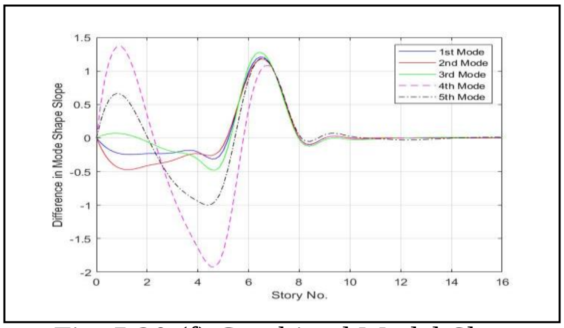


Fig. 5.20 (f) Combined Modal Slope of 16D7

The difference between undamaged mode shape slopes and 10D2 damaged mode shape slopes are shown in Fig. 5.21**.** It has been observed from the result that first four mode are capable to detect damage in 10D2 damaged building. It has been observed from the result that Fig 5.21 (a), (b), (c) and (d) shows, plot changed sign between story#6 and story#7 with the maximum value at story#7. The change in mode shape curvature of undamaged and 10D9 damaged building of all five mode shape is plotted, shown in Fig 5.21 (f).

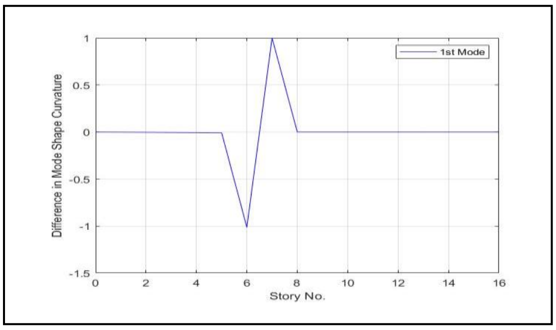


Fig. 5.21 (a) First Modal Curvature of 16D7

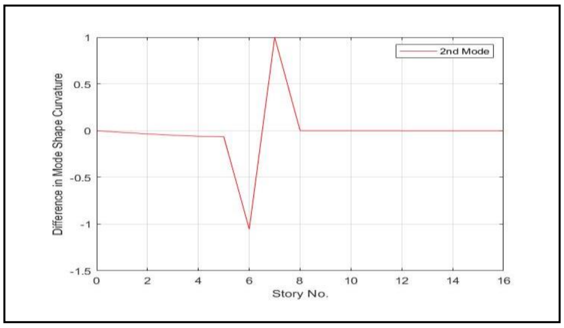


Fig. 5.21 (b) Second Modal Curvature of 16D7

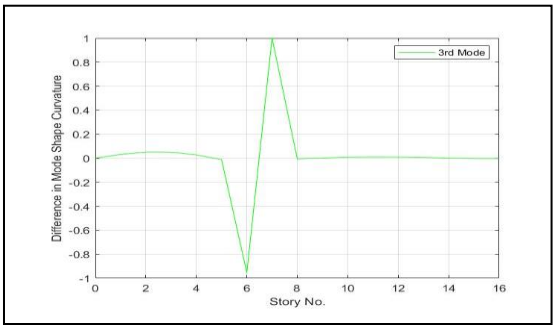


Fig. 5.21 (c) Third Modal Curvature of 16D7

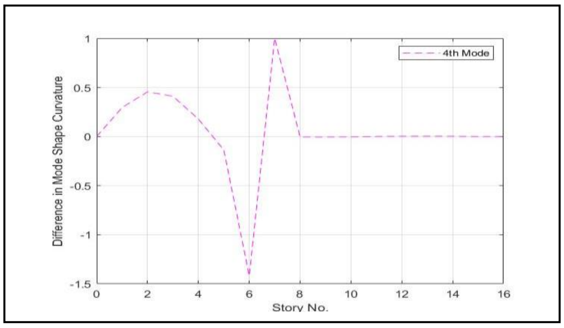


Fig. 5.21 (d) Fourth Modal Curvature of 16D7

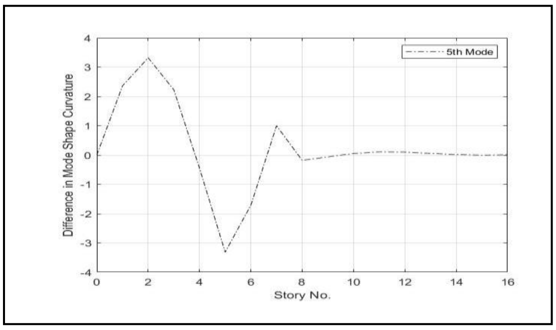


Fig. 5.21 (e) Fifth Modal Curvature of 16D7

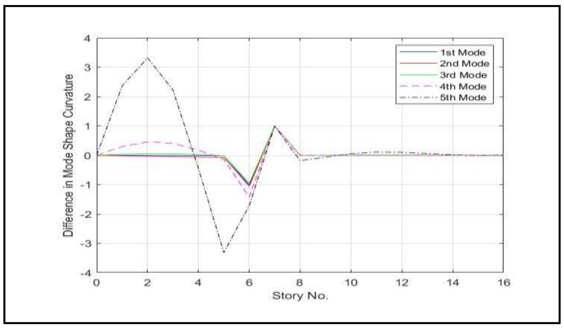


Fig. 5.21 (f) Combined Modal Curvature of 16D7

5.4.4 Damaged Building Model 16D13

Damage has been introduced at 13th story by reducing the stiffness by 20%. A stiffness matrix obtained for 16D13 damaged building is shown in Table 5.34. First five natural frequencies obtained after the modal analysis of 16D13 damaged building models are 0.5344, 1.5815, 2.6185, 3.6696 and 4.6693 Hz. Mode shape corresponding to these five natural frequencies have been calculated and then normalized them to unity with the help of MATLAB programme. Normalized mode shapes are summarized in Table 5.35

Table 5.34 Stiffness Matrix of 16D13 Damaged Building

500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-250000	450000	-200000	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-200000	450000	-250000	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-250000	500000	-250000
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-250000	250000

Table 5.35 Normalized Damaged Mode Shape of 16D13 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	0.094187	-0.27382	0.484074	-0.69891	0.779516
2	0.187525	-0.52601	0.86332	-1.10059	1.022281
3	0.279172	-0.73665	1.055611	-1.03421	0.561136
4	0.368301	-0.8891	1.019306	-0.528	-0.28639
5	0.454109	-0.97131	0.762266	0.202764	-0.93672
6	0.535821	-0.9768	0.340155	0.847293	-0.94205
7	0.612701	-0.90512	-0.15562	1.131483	-0.29872
8	0.684056	-0.76195	-0.61769	0.934475	0.550303
9	0.749242	-0.55858	-0.946	0.340052	1.020402
10	0.807671	-0.31109	-1.06945	-0.39899	0.787884
11	0.858816	-0.03903	-0.96131	-0.96835	0.012854
12	0.902217	0.236122	-0.64499	-1.12589	-0.77103
13	0.946297	0.556741	-0.075	-0.72429	-1.08725
14	0.973027	0.769257	0.397235	-0.09498	-0.59158
15	0.990982	0.921006	0.783446	0.574718	0.311431
16	1	1	1	1	1

In similar way, normalized change in modal slope of undamaged and 16D7 damaged building is calculated is shown in Table 5.36 (a) and normalized modal curvature in Table 5.36 (b).

Table 5.36(a) Change in Modal Slope of Undamaged and 16D13 Damaged Building

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.22672	0.372165	0.242767	-0.85912	1.155057
2	-0.22239	0.296093	0.237028	-0.31951	-0.31736
3	-0.21522	0.178801	0.210496	0.346406	-1.56416
4	-0.2053	0.033505	0.145872	0.881885	-1.69202
5	-0.19274	-0.12324	0.032442	1.07845	-0.607
6	-0.1777	-0.27327	-0.12612	0.85629	0.916674
7	-0.16034	-0.39869	-0.30787	0.296589	1.788461
8	-0.14088	-0.48382	-0.47465	-0.38731	1.380966
9	-0.11953	-0.51685	-0.58003	-0.93008	-0.01933
10	-0.09656	-0.49122	-0.58166	-1.11631	-1.41341
11	-0.07222	-0.40644	-0.45483	-0.86553	-1.80163
12	0.983106	0.913775	0.898677	0.688464	0.209449
13	1	1	1	1	1
14	9.32E-05	0.00426	0.008519	0.001672	-0.0004
15	5.66E-05	0.002868	0.007396	0.002804	0.001386
16	3.79E-05	0.00202	0.005776	0.002612	0.00171

Table5.36(b) Change in Modal Curvature of Undamaged and 16D13 Damaged Building.

Story	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
1	-0.00141	0.022373	-3.5E-05	-0.20451	-5.36861
2	-0.00281	0.04225	0.005004	-0.33067	-7.95155
3	-0.00418	0.05739	0.017966	-0.32977	-6.88584
4	-0.00549	0.066039	0.037982	-0.20131	-3.75978
5	-0.00674	0.067117	0.060221	0.006362	-1.36064
6	-0.00792	0.060331	0.077049	0.213973	-1.54675
7	-0.009	0.046217	0.080307	0.341132	-3.79661
8	-0.00997	0.026097	0.064082	0.337153	-5.51966
9	-0.01083	0.001959	0.027145	0.201156	-3.9548
10	-0.01156	-0.02373	-0.02573	-0.01646	1.671114
11	-0.01216	-0.04828	-0.08407	-0.23226	9.274484
12	-1.01647	-1.07325	-1.08772	-1.30898	11.27565
13	1	1	1	1	1
14	-3.6E-05	-0.00109	0.000994	0.002648	0.026965
15	-3.7E-05	-0.0017	-0.00324	-0.00038	0.008779
16	-3.7E-05	-0.0017	-0.00324	-0.00038	0.000648

The difference between undamaged mode shape slope and 16D13 damaged mode shape slope are shown in Fig. 5.22 for first five modes. It is observed from the results that first three mode are capable to detect damage while other modes are not showing positive results to detect damage. Change in modal slope of all modes is plotted which are shown in Fig 5.22(f).

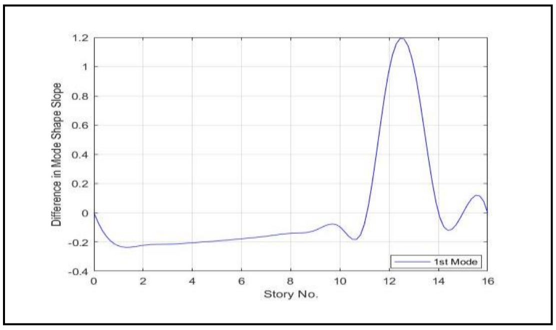


Fig. 5.22 (a) First Modal Slope of 16D13

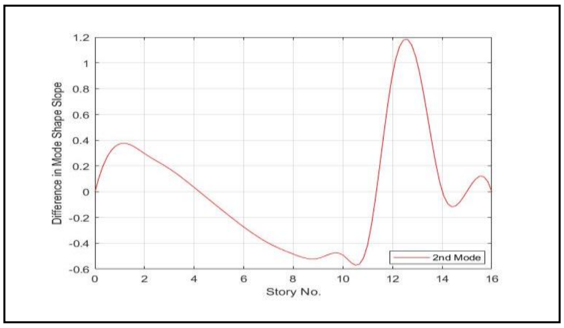


Fig. 5.22 (b) Second Modal Slope of 16D13

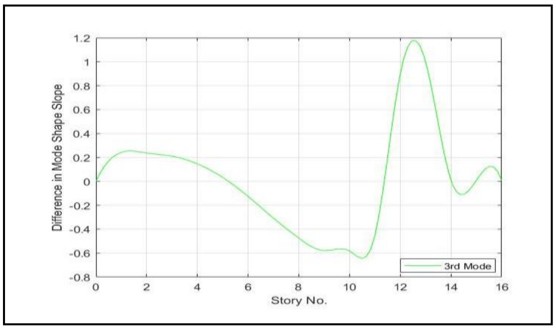


Fig. 5.22 (c) Third Modal Slope of 16D13

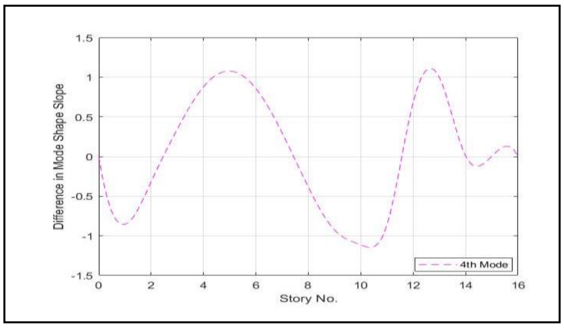


Fig. 5.22 (d) Fourth Modal Slope of 16D13

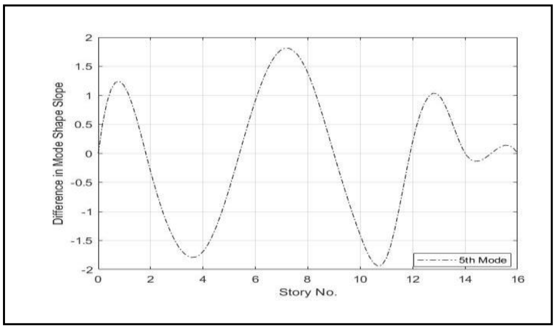


Fig. 5.22 (e) Fifth Modal Slope of 16D13

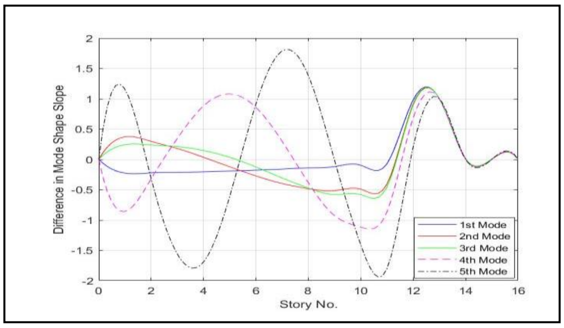


Fig. 5.22 (f) Combined Modal Slope of 16D13

The difference between undamaged mode shape slopes and 10D13 damaged mode shape slopes are shown in Fig. 5.23**.** It has been observed from the result that first four mode are capable to detect damage in 10D13 damaged building. It has been observed from the result that Fig 5.23 (a), (b), (c) and (d) shows, plot changed sign between story#12 and story#13 with the maximum value at story#13. The change in mode shape curvature of undamaged and 10D9 damaged building of all five mode shape is plotted, shown in Fig 5.23 (f).

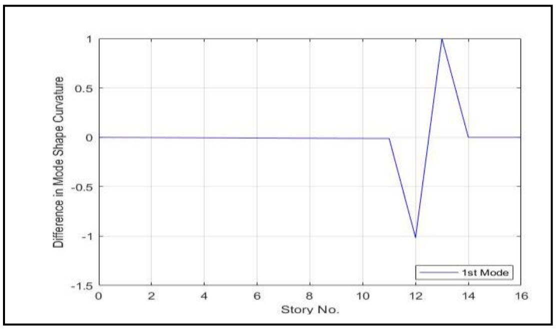


Fig. 5.23 (a) First Modal Curvature of 16D13

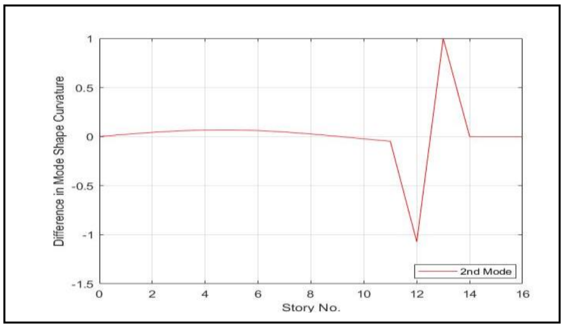


Fig. 5.23 (b) Second Modal Curvature of 16D13

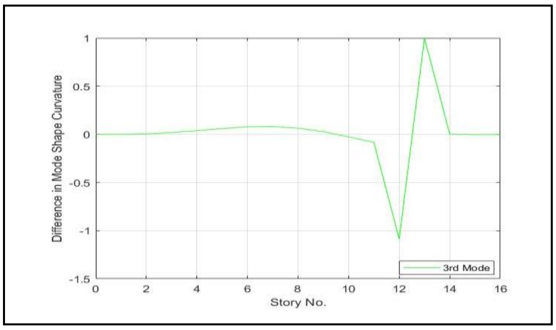


Fig. 5.23 (a) Third Modal Curvature of 16D13

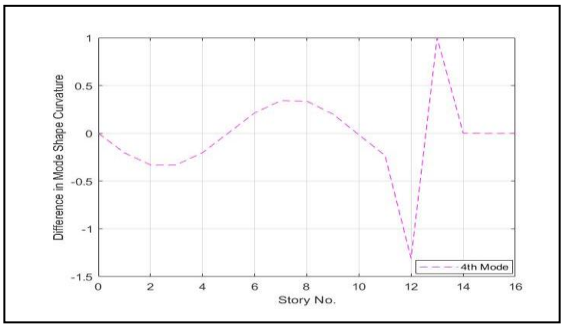


Fig. 5.23 (b) Fourth Modal Curvature of 16D13

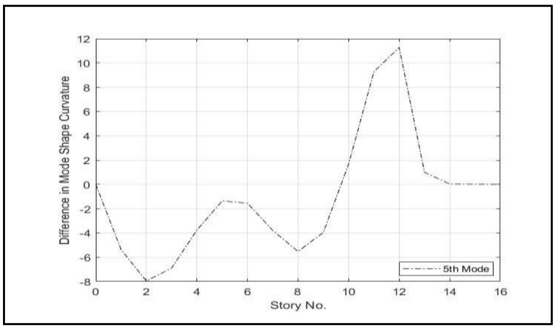


Fig. 5.23 (a) Fifth Modal Curvature of 16D13

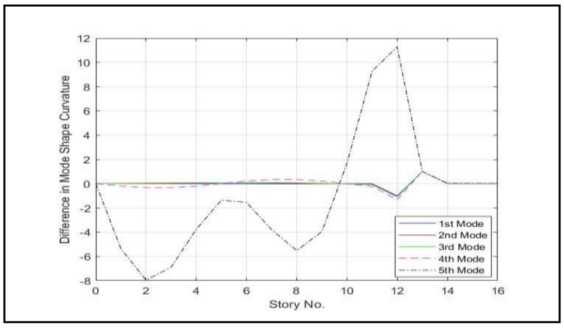


Fig. 5.23 (b) Combined Modal Curvature of 16D13

Result obtained from the analysis of three models of 5 story building are summarized in Table 5.37 , that shows which modal slope and curvature are capable to detect damage in buildings.

Table 5.37 Derivatives of Different Mode of Numerical Models of 16 Story Building

Numerical model	Derivatives	1st mode	2nd mode	3rd mode	4th mode	5th mode
16D2	Modal Slope	✓	✓	✓	✗	✗
	Modal Curvature	✓	✓	✓	✓	✓
16D7	Modal Slope	✓	✓	✓	✗	✗
	Modal Curvature	✓	✓	✓	✓	✗
16D13	Modal Slope	✓	✓	✓	✗	✗

CHAPTER 6

CONCLUSIONS

6.1 INTRODUCTION

In this chapter the conclusion drawn from the analysis of three different height building to detect damage at different height of building using mode shape slope and curvature method is reported.

6.2 CONCLUSIONS

1. The change between undamaged and damaged modal slope (i.e. 1^{st} derivative) shows a large value at the damage location for the first few modes.

2. The change between undamaged and damaged modal curvature (i.e., 2^{nd} derivative) changes its sign at the damage location from negative to positive and reaches very large positive at the damaged location.

3. The damage detection by modal curvature method is easier than that of modal slope method.

4. In low rise building (i.e., 5^{th} story building), the damage at lower story (i.e. story 1) can be detected with modal slope and curvature (i.e., 1^{st} derivative and 2^{nd} derivative) of all first five modes of the building.

5 Damage at mid and higher story in the low rise building can be detected with modal slope and curvature of mode 1 and mode 2 while higher mode's (i.e. mode 3, 4 and 5) derivatives are not capable to detect damage as it does not changes sign and not shows higher value at damage location for modal slope and curvature.

6. In medium rise building (i.e., 10 story building), the change in modal slope (i.e., first derivative) of lower modes (i.e., mode 1 and 2) are capable to detect damage while change in modal curvature of mode 1, 2 and 3 are capable to detect damage at any height of building (except top story).

7. In high rise building (i.e., 16 story building), the first three mode can be detect damage at any height of building (i.e., damage at lower, mid and higher story) using mode slope method (except top story).

8. In high rise building (i.e., 16 story building), the lower story damage (i.e. 2^{nd} story) can be detected by modal curvature of all five modes.

9. Damage at the mid and higher story (i.e. 7^{th} and 13^{th} story) in 16 story building is detected by first three mode in case of mode shape slope, shows larger value at damage location, and first four modes in case of mode shape curvature, shows the change in sign and higher value at damage location.

10. The height of building increases higher modes (i.e. mode 3 and 4) start contributing to detect damage in the building using mode shape slope and curvature method, as it shows larger value at damage location for modal slope and changes sign at damage location for modal curvature.

6.3 FUTURE SCOPE OF THE WORK

Study on the damage detection in higher building, vertical irregular building is needed to check the effectiveness of these techniques used in this thesis. Also study is to be extended for detecting multiple damages in the structure.

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APPENDIX

MATLAB Programme for change between undamaged and damaged mode shape slope and curvature.

Following program calculates the natural frequencies, stiffness matrix, normalized mode shape, modal slope (i.e., 1st derivative), modal curvature (i.e., 2nd derivative) for undamaged and damaged building model.

```
clc
clear all
%----INITIAL INPUT DATA---- %

N=input('No. of Story = ');
k(1:N)=input('Uniform story stiffness = ');
m(1:N)=input('Uniform mass distribution = ');
M=diag(m);
K=diag(0,N-1);
ds=input('damaged storey = ');
stiffness_reduced_perc=input('percentage of stiffness reduction = ');
k_ds=((100-stiffness_reduced_perc)/100)*k(ds);

                      %---UNDAMAGED BUILDING---%

%------- 1. STIFFNESS MATRIX  %

% Diagonal and upper triangular matrix element
for i=1:N-1
    K(i,i)=k(i)+k(i+1);
    K(i,i+1)=-k(i+1);
end
%lower Triangulat Matrix element
for i=2:N
    K(i,i-1)=-k(i);
end
%Last diagonal element
for i=N
    K(i,i)=k(i);
end
K;
xlswrite('Undamaged_Stiffness_Matrix.xlsx',K);


%------ 2. MODAL FREQUENCY AND MODE SHAPE NORMALIZE TO UNITY %
```

```
%Eigen vale and vector calculation
[v,d]=eig(K,M);
w=sqrt(d);
%Natural Frequency and Time period
%------The natural frequency of this structure are as follows
W(1:N)=0;
for i=1:N
W(i)=w(i,i)/(2*pi);
end
xlswrite('Undamaged_Natural_Frequency.xlsx',[W(:)]);

%------The time periods of this structure are as follows (sec)
T(1:N)=0;
for i=1:N
T(i)=(2*3.14)/w(i,i);
end
xlswrite('Undamaged_Time_Period.xlsx',[T(:)]);

%------UNDAMAGED MODE SHAPES--------%
d_m1=[0;v(:,1)];
d_m2=[0;v(:,2)];
d_m3=[0;v(:,3)];
d_m4=[0;v(:,4)];
m5=[0;v(:,5)];
xlswrite('UNDAMAGED general
Mode_Shape.xlsx',[d_m1(:),d_m2(:),d_m3(:),d_m4(:),m5(:)])


%NORMALIZATION OF MODE SHAPE VECTORS
for i=1:N
    v(:,i)=v(:,i)/v(N,i);
end

%-------3.UNDAMAGED MODE SHAPES, NORMALIZED TO UNITY

mode1=[0;v(:,1)];
mode2=[0;v(:,2)];
mode3=[0;v(:,3)];
mode4=[0;v(:,4)];
mode5=[0;v(:,5)];
xlswrite('UNDAMAGED
Mode_Shape.xlsx',[mode1(:),mode2(:),mode3(:),mode4(:),mode5(:)])



%----DAMAGED BUILDING----%
d_k=k;
d_k(ds)=k_ds;
d_K=diag(0,N-1);
```

```
%------- 1. STIFFNESS MATRIX  %

% Diagonal and upper triangular matrix element
for i=1:N-1
    d_K(i,i)=d_k(i)+d_k(i+1);
    d_K(i,i+1)=-d_k(i+1);
end

%lower Triangulat Matrix element
for i=2:N
    d_K(i,i-1)=-d_k(i);
end

%Last diagonal element
for i=N
    d_K(i,i)=d_k(i);
end
d_K;
xlswrite('Damaged_Stiffness_Matrix.xlsx',d_K);

%------ 2. MODAL FREQUENCY AND MODE SHAPE NORMALIZE TO UNITY %

%Eigen vale and vector calculation
[d_v,d_d]=eig(d_K,M);
d_w=sqrt(d_d);

%Natural Frequency and Time period
%------The natural frequency of this structure are as follows
d_W(1:0)=0;
for i=1:N
d_W(i)=d_w(i,i)/(2*pi);
end
xlswrite('Damaged_Natural_Frequency.xlsx',[d_W(:)]);

%------The time periods of this structure are as follows (sec)
d_T(1:N)=0;
for i=1:N
d_T(i)=(2*pi)/d_w(i,i);
end
xlswrite('Damaged_Time_Period.xlsx',[d_T(:)]);

%------DAMAGED MODE SHAPES--------%
d_m1=[0;d_v(:,1)];
d_m2=[0;d_v(:,2)];
d_m3=[0;d_v(:,3)];
d_m4=[0;d_v(:,4)];
d_m5=[0;d_v(:,5)];
```

```
xlswrite('DAMAGED general Mode_Shape.xlsx',[d_m1(:),d_m2(:),d_m3(:),d_m4(:),d_m5(:)])


%NORMALIZATION OF MODE SHAPE VECTORS
for i=1:N
    d_v(:,i)=d_v(:,i)/d_v(N,i);
end

% MODE SHAPES

d_mode1=[0;d_v(:,1)]
d_mode2=[0;d_v(:,2)]
d_mode3=[0;d_v(:,3)]
d_mode4=[0;d_v(:,4)]
d_mode5=[0;d_v(:,5)]

xlswrite('DAMAGED Mode_Shape.xlsx',[d_mode1(:),d_mode2(:),d_mode3(:),d_mode4(:),d_mode5(:)])


%------ DERIVATIVE OF UNDAMAGED MODE SHAPE--------%

% Function of undamaged mode shape
M1_fu=mode1;
M2_fu=mode2;
M3_fu=mode3;
M4_fu=mode4;
M5_fu=mode5;


% Degree of freedom and interval %
x=[0:1:N ];
h=1;

%---DIFINING ARRAY FOR FIRST DERIVATIVE OF UNDAMAGED MODE SHAPE---%
M1_fud=[0:1:N];
M2_fud=[0:1:N];
M3_fud=[0:1:N];
M4_fud=[0:1:N];
M4_fud=[0:1:N];
M5_fud=[0:1:N];

%----CALCULATING FIRST DERIVATIVE OF UNDAMAGED MODE SHAPE---%

%forward Difference%
for i=1
        M1_fud(i)=(M1_fu(i+h)-M1_fu(i))/h;
        M2_fud(i)=(M2_fu(i+h)-M2_fu(i))/h;
```

```
        M3_fud(i)=(M3_fu(i+h)-M3_fu(i))/h;
        M4_fud(i)=(M4_fu(i+h)-M4_fu(i))/h;
        M5_fud(i)=(M5_fu(i+h)-M5_fu(i))/h;
end

%Central Difference
for i=2:N
       M1_fud(i)=(M1_fu(i+h)-M1_fu(i-h))/(2*h);
       M2_fud(i)=(M2_fu(i+h)-M2_fu(i-h))/(2*h);
       M3_fud(i)=(M3_fu(i+h)-M3_fu(i-h))/(2*h);
       M4_fud(i)=(M4_fu(i+h)-M4_fu(i-h))/(2*h);
       M5_fud(i)=(M5_fu(i+h)-M5_fu(i-h))/(2*h);
end

%Backward difference
for i=N+1
        M1_fud(i)=(M1_fu(i)-M1_fu(i-h))/h;
        M2_fud(i)=(M2_fu(i)-M2_fu(i-h))/h;
        M3_fud(i)=(M3_fu(i)-M3_fu(i-h))/h;
        M4_fud(i)=(M4_fu(i)-M4_fu(i-h))/h;
        M5_fud(i)=(M5_fu(i)-M5_fu(i-h))/h;
end

xlswrite('UNDAMAGED
Mode_Shape_Slope.xlsx',[M1_fud(:),M2_fud(:),M3_fud(:),M4_fud(:),M5_fud(:)])

%------ DERIVATIVE OF DAMAGED MODE SHAPE--------%

M1_fd=d_mode1;
M2_fd=d_mode2;
M3_fd=d_mode3;
M4_fd=d_mode4;
M5_fd=d_mode5;


% Degree of freedom and interval %
x=[0:1:N ];
h=1;

%---DIFINING ARRAY---%
M1_fdd=[0:1:N];
M2_fdd=[0:1:N];
M3_fdd=[0:1:N];
M4_fdd=[0:1:N];
M4_fdd=[0:1:N];
M5_fdd=[0:1:N];

%----CALCULATING FIRST DERIVATIVE OF UNDAMAGED MODE SHAPE---%
```

```
%forward Difference%
for i=1
        M1_fdd(i)=(M1_fd(i+h)-M1_fd(i))/h;
        M2_fdd(i)=(M2_fd(i+h)-M2_fd(i))/h;
        M3_fdd(i)=(M3_fd(i+h)-M3_fd(i))/h;
        M4_fdd(i)=(M4_fd(i+h)-M4_fd(i))/h;
        M5_fdd(i)=(M5_fd(i+h)-M5_fd(i))/h;
end

%Central Difference
for i=2:N
       M1_fdd(i)=(M1_fd(i+h)-M1_fd(i-h))/(2*h);
       M2_fdd(i)=(M2_fd(i+h)-M2_fd(i-h))/(2*h);
       M3_fdd(i)=(M3_fd(i+h)-M3_fd(i-h))/(2*h);
       M4_fdd(i)=(M4_fd(i+h)-M4_fd(i-h))/(2*h);
       M5_fdd(i)=(M5_fd(i+h)-M5_fd(i-h))/(2*h);
end

%Backward difference
for i=N+1
        M1_fdd(i)=(M1_fd(i)-M1_fd(i-h))/h;
        M2_fdd(i)=(M2_fd(i)-M2_fd(i-h))/h;
        M3_fdd(i)=(M3_fd(i)-M3_fd(i-h))/h;
        M4_fdd(i)=(M4_fd(i)-M4_fd(i-h))/h;
        M5_fdd(i)=(M5_fd(i)-M5_fd(i-h))/h;
end

xlswrite('DAMAGED
Mode_Shape_Slope.xlsx',[M1_fdd(:),M2_fdd(:),M3_fdd(:),M4_fdd(:),M5_fdd(:)])


%-------DIFFERENCE BETWEEN UNDAMAGED AND DAMAGED MODE SHAPE SLOPE



    M1_FD= M1_fud- M1_fdd;
    M2_FD= M2_fud- M2_fdd;
    M3_FD= M3_fud- M3_fdd;
    M4_FD= M4_fud- M4_fdd;
    M5_FD= M5_fud- M5_fdd;


xlswrite('DIFFERENCE_undamaged_damaged_ModeShapeSlope.xlsx',[M1_FD(:),M2_FD(
:),M3_FD(:),M4_FD(:),M5_FD(:)])

% First_detivative of mode shape 1 , Normalized to unity
ds;
for i=1:N+1
    M1_FD(i)=(M1_FD(i))/(M1_FD(ds+1));
end
```

```
% First_detivative of Mode Shape 2 , Normalized to unity
for i=1:N+1
    M2_FD(i)=(M2_FD(i))/(M2_FD(ds+1));
end

% First_detivative of Mode Shape 3 , Normalized to unity
for i=1:N+1
    M3_FD(i)=(M3_FD(i))/(M3_FD(ds+1));
end

% First_detivative of Mode Shape 4 , Normalized to unity
for i=1:N+1
    M4_FD(i)=(M4_FD(i))/(M4_FD(ds+1));
end
% First_detivative of Mode Shape 5 , Normalized to unity
for i=1:N+1
    M5_FD(i)=(M5_FD(i))/(M5_FD(ds+1));
end

M1_FD(1)=0;
M2_FD(1)=0;
M3_FD(1)=0;
M4_FD(1)=0;
M5_FD(1)=0;


xlswrite('NORMALIZED_DIFFERENCE_undamaged_damaged_ModeShapeSlope.xlsx',[M1_F
D(:),M2_FD(:),M3_FD(:),M4_FD(:),M5_FD(:)])

%------Smoothing Spline-------%

  xx=linspace(0,N);
  pp1=spline(x,M1_FD);
  pp2=spline(x,M2_FD);
  pp3=spline(x,M3_FD);
  pp4=spline(x,M4_FD);
  pp5=spline(x,M5_FD);

  yy1=ppval(pp1,xx);
  yy2=ppval(pp2,xx);
  yy3=ppval(pp3,xx);
  yy4=ppval(pp4,xx);
  yy5=ppval(pp5,xx);

%PLOTTING OF DIFFERENCE IN MODE SHAPES SLOPE i.e, first derivative%

x;
plot(xx,yy1,'b-')
```

```
hold on

plot(xx,yy2,'r-')
hold on

plot(xx,yy3,'g-')
hold on

plot(xx,yy4,'m--')
hold on

plot(xx,yy5,'k-.')

grid on


legend({'1st Mode','2nd Mode','3rd Mode','4th Mode','5th Mode'})
xlabel('Story No.')
ylabel('Difference in Mode Shape Slope')

           %----SECOND DERIVATIVE---%

%forward Difference%
for i=1
        M1_sd(i)=(M1_f(i+2*h)-2*M1_f(i+h)+M1_f(i))/(h*h);
        M2_sd(i)=(M2_f(i+2*h)-2*M2_f(i+h)+M2_f(i))/(h*h);
        M3_sd(i)=(M3_f(i+2*h)-2*M3_f(i+h)+M3_f(i))/(h*h);
        M4_sd(i)=(M4_f(i+2*h)-2*M4_f(i+h)+M4_f(i))/(h*h);
        M5_sd(i)=(M5_f(i+2*h)-2*M5_f(i+h)+M5_f(i))/(h*h);
end

%Central Difference
for i=2:N
       M1_sd(i)=(M1_f(i+h)-2*M1_f(i)+M1_f(i-h))/(h*h);
       M2_sd(i)=(M2_f(i+h)-2*M2_f(i)+M2_f(i-h))/(h*h);
       M3_sd(i)=(M3_f(i+h)-2*M3_f(i)+M3_f(i-h))/(h*h);
       M4_sd(i)=(M4_f(i+h)-2*M4_f(i)+M4_f(i-h))/(h*h);
       M5_sd(i)=(M5_f(i+h)-2*M3_f(i)+M5_f(i-h))/(h*h);
end

%Backward difference
for i=N+1
        M1_sd(i)=(M1_f(i)-2*M1_f(i-h)+M1_f(i-2*h))/(h*h);
        M2_sd(i)=(M2_f(i)-2*M2_f(i-h)+M2_f(i-2*h))/(h*h);
        M3_sd(i)=(M3_f(i)-2*M3_f(i-h)+M3_f(i-2*h))/(h*h);
        M4_sd(i)=(M4_f(i)-2*M4_f(i-h)+M4_f(i-2*h))/(h*h);
        M5_sd(i)=(M5_f(i)-2*M5_f(i-h)+M5_f(i-2*h))/(h*h);
end
```

```
% Second_detivative of mode shape 1 , Normalized to unity
ds;
for i=1:N+1
    M1_sd(i)=(M1_sd(i))/(M1_sd(ds+1));
end

% Second_detivative of Mode Shape 2 , Normalized to unity
for i=1:N+1
    M2_sd(i)=(M2_sd(i))/(M2_sd(ds+1));
end

% Second_detivative of Mode Shape 3 , Normalized to unity
for i=1:N+1
    M3_sd(i)=(M3_sd(i))/(M3_sd(ds+1));
end

% Second_detivative of Mode Shape 4 , Normalized to unity
for i=1:N+1
    M4_sd(i)=(M4_sd(i))/(M4_sd(ds+1));
end

% Second_detivative of Mode Shape 5 , Normalized to unity
for i=1:N+1
    M5_sd(i)=(M5_sd(i))/(M5_sd(ds+1));
end

M1_sd(1)=0;
M2_sd(1)=0;
M3_sd(1)=0;
M4_sd(1)=0;
M5_sd(1)=0;

xlswrite('Mode_Shape_Cuvature.xlsx',[M1_sd(:),M2_sd(:),M3_sd(:),M4_sd(:)
,M5_sd(:)])

%PLOTTING OF DIFFERENCE IN MODE SHAPES CURVATURE i.e, second derivative
x;
plot(x,M1_sd,'b-')
hold on

plot(x,M2_sd,'r-')
hold on

plot(x,M3_sd,'g-')
hold on

plot(x,M4_sd,'m--')
hold on
```

```
plot(x,M5_sd,'k-.')

grid on

legend({'1st Mode','2nd Mode','3rd Mode','4th Mode','5th Mode'})
xlabel('Story No.')
ylabel('Difference in Mode Shape Curvature')
```