# STUDY OF FCM WITH VARIOUS OPTIMIZATION ALGORITHMS FOR IMAGE SEGMENTATION

#### A DISSERTATION

SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF DEGREE OF

> MASTER OF TECHNOLOGY IN SOFTWARE ENGINEERING

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I, Dhairya Pankaj Savlekar, 2K20/SWE/08, student of Master of Technology (Software Engineering), hereby declare that the Major Project-II Dissertation titled "Study of FCM With Various Optimization Algorithms For Image Segmentation" which is submitted by me to the Department of Software Engineering, Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master Of Technology (Software Engineering) is original and not copied from any source without proper citation. This work has not been previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship, or other similar title or recognition.

i

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#### CERTIFICATE

I hereby certify that the Project Dissertation titled "Study of FCM With Various Optimization Algorithms For Image Segmentation" which is submitted by Dhairya Pankaj Savlekar, (2K20/SWE/08) to the Department of Software Engineering, Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Technology is a record of project work carried out by the student under my supervision. To the best of my knowledge, this work has not been submitted in part or full for any Degree or Diploma to this University or elsewhere.

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# **ACKNOWLEDGEMENT**

The successful completion of any task would be incomplete without accomplishing the people who made it all possible and whose constant guidance and encouragement secured us the success.

First of all, I would like to thank the Almighty, who has always guided me to follow the right path in life. My greatest thanks are to my parents who bestowed the ability and strength in me to complete this work.

My thanks are addressed to my mentor **Dr. Sonika Dahiya**, Department of Software Engineering who gave me this opportunity to work on a project under her supervision. It was her enigmatic supervision, unwavering support, and expert guidance which has allowed me to complete this work in due time. I humbly take this opportunity to express my deepest gratitude to her.

#### ABSTRACT

With computer-aided diagnostics, medical pictures are acquired utilizing electronic instruments such as CT scanners and MRI machines. Typically, computed tomography (CT)/magnetic resonance imaging (MRI) pictures obtained are restricted spatial resolution, low contrast, noise, and nonuniform variation because of environmental impacts. As a result, the peculiarities of the objects are fuzzy and twisted, and their meanings are not precise. Fuzzy sets and fuzzy logic are best suited for dealing with ambiguity and uncertainty. For segmentation, the fuzzy clustering approach has been widely employed and assorted photos during the previous decade. Fuzzy clustering is a significant challenge that is being actively researched in various real-world applications. One of the algorithms is the fuzzy c-means (FCM). since it is the most prevalent fuzzy clustering approach, clear and simple to implement. FCM on the other hand can stuck in local optima and is sensitive to initialization. This study compares seven fuzzy-clustered image segmentation methods utilized in CT scan and MRI brain image segments. The algorithms were examined using CT scan/MRI brain images in this investigation. The methods were statistically assessed in the research using two validity metrics, partition coefficient and partition entropy.

# **TABLE OF CONTENTS**

Content	Page No.
Candidate's Declaration	i
Certificate	ii
Acknowledgment	iii
Abstract	iv
Table of Contents	v-vi
List of Figures	vii
List of Tables	viii
List of Abbreviations	ix
CHAPTER-1: INTRODUCTION	1-3
1.1 Overview	1
1.1.1 Fuzzy Clustering	1
1.1.2 Optimization Algorithms	2
1.2 Motivation	2
1.3 Problem Statement	2
1.4 Organization of The Thesis	3
CHAPTER-2: LITERATURE REVIEW	4-18
2.1 Clustering	4
2.2 Fuzzy C-Means Algorithm	5
2.3 Literature Survey of Optimization Algorithms	6
2.3.1 Particle Swarm Optimization Algorithms	6
2.3.2 Cuckoo Search Optimization Algorithms	7
2.3.3 Genetic Algorithm Optimization Algorithms	9
2.3.4 Whale Optimization Algorithms	10
2.3.5 Grey Wolf Optimization Algorithms	12
2.3.6 Differential Evolution Optimization Algorithms	14
2.4 FCM With Optimization Algorithms	15
2.4.1 FCM-PSO Algorithm	15
2.4.2 FCM-CSO Algorithm	15
2.4.3 FCM-GA Algorithm	16

2.4.4 FCM-WOA Algorithm	16
2.4.5 FCM-GWO Algorithm	17
2.4.6 FCM-DE Algorithm	18
CHAPTER-3: RESULTS & ANALYSIS	19-25
3.1 Dataset Used	19
3.2 Hardware and Software Requirements	19
3.2.1 Hardware Requirements	19
3.2.2 Software Requirements	19
3.3 Quantitative Measures	20
3.4 Results and Analysis	20
3.4.1 Experimental Results	20
<b>CHAPTER-5: CONCLUSION &amp; FUTURE WORK</b>	25
CHAPTER-6: REFERENCES	26

# LIST OF FIGURES

S.NO	FIGURE NAME	PAGE NO.	
1.	Figure 4.1: MRI/CT scan brain images	21	
2.	Figure 4.2: Segmentation results for figure 4.1a	22	
3.	Figure 4.3: Segmentation results for figure 4.1b	23	
4.	Figure 4.4: Segmentation results for figure 4.1c	24	

# LIST OF TABLES

S.NO	TABLE NAME	PAGE NO.
1.	Table 1: Performance of algorithms based on $V_{PC}$ , validity index	25
2.	Table 2: Performance of algorithms based on $V_{PE}$ , validity index	25

# LIST OF ABBREVIATIONS

- FCM : Fuzzy C-Means
- PSO : Particle Swarm Optimizer
- CSO : Cuckoo Search Optimizer
- GA : Genetic Algorithm
- WOA : Whale Optimization Algorithm
- GWO : Grey Wolf Optimizer
- DE : Differential Evolution

# **CHAPTER-1 INTRODUCTION**

Clustering is the practice of arranging data items into distinct groups called clusters such that objects in one cluster are more similar to those in other clusters. Clustering methods are utilized in a plethora of areas, including as segmenting an image, crime location detection, tissue differentiation in medical images, software quality prediction [17]. Clustering methods are mainly categorized into following typese: hard and fuzzy [4].

### **1.1 OVERVIEW**

#### 1.1.1 Fuzzy Clustering

It is a type of grouping where datum factor existing house can be a period more noteworthy than one bunch essentially founded on specific participation values. In non-fuzzy grouping (additionally perceived as hard clustering), records are partitioned into superb bunches, the spot every data point can have a place with precisely one bunch. In fuzzy bunching, measurement variables can without a doubt have a place with numerous groups. Be that as it may, they don't imply probabilities, so the two qualities would rather not amount to 1. moreover saw as smooth bundling, is a sort of collection wherein every datum factor existing in the house can be a time of more conspicuous than one pack primarily established on unambiguous enlistment values. In non-cushy gatherings (also seen as hard bundling), records are divided into sublime gatherings, the spot each datum point can have a spot with exactly one gathering. In cushioned gathering, estimations factors can absolutely have a spot with various bundles. For instance, an apple can be red or green (hard gathering), however, an apple can additionally be pink AND green (cushioned grouping). Here, the apple can be red to a positive authentication as charmingly as green to some extent. Instead of the apple having a spot with green [green = 1] and now not ruby [red = 0], the apple can have a spot with green [green = 0.5] and red [red = 0.5]. These characteristics have normalized some place in the scope of nothing and 1; nevertheless, they don't indicate probabilities, so the two characteristics would prefer not to add up to 1.

Segmentation an image is the way of splitting a photograph in a range of classes based on unambiguous components such as variety, surface, pixel size, and so on. Clinical imaging, object localization, and content-based imaging are some of their applications. The use of solo computations is used to divide a picture other than any preceding skill of the result. Such computations examine themselves by sorting over designs in the provided data set. An image may be divided into multiple sections using similar computations (or bunches).

#### **1.1.2 Optimization Algorithms:**

Optimization of a problem refers to simply minimizing the objective cost or to maximizin the the performance of objective. An optimization procedure that is run by choosing different outcomes till an best is provided by algorithm.

#### **1.2 Motivation:**

Clustering is the method of arranging input items into different groups such that input data in one cluster are more comparable to those in other clusters. Image segmentation, criminal hot spot identification, tissue separation in medical imaging, software quality prediction, and more applications employ clustering algorithms [17]. In literature, most of the clustering methods are categorized into two types: Hard and soft. In first, every input totally or partially member of a group. In second, each data point gets a chance of being in a group. The most often used fuzzy clustering technique is Bezdek's fuzzy c-means (FCM) in 1974 [5] and now it is commonly used. This technique is successful, however of center points causes the iterative process to quickly slip in optimum outcome. There are various methods to improve the performance of FCM such as initialization of parameters, modified fcm, preprocessing input, etc. In this work, we have applied six optimization algorithms to the fuzzy c-means algorithm.

#### **1.3 Problem Statement**

With computer-aided diagnostics, medical pictures are acquired utilizing electronic instruments such as CT scanners and MRI machines. Due to environmental factors, acquired computed tomography (CT)/magnetic resonance imaging (MRI) pictures often exhibit poor fluctuation. As a result, object differences are muddled and twisted, and object meanings are not well defined. They are ideal for dealing with ambiguity and vagueness. Throughout the previous decade, the Clustering by fuzzy mechanism was commonly employed for image segmentation. The fuzzy c-means clustering technique is successful, however, the random selection of center points causes the iterative process to quickly fall into the local optimum

solution. We used optimization algorithms in such a way to overcome this issue of FCM. In this, a few optimization algorithms are used to improve the performance of FCM for image segmentation such as Particle Swarm Optimizer (PSO), Cuckoo Search Optimizer (CSO), Genetic Algorithm (GA), Whale Optimization Algorithm (WOA), Grey Wolf Optimization (GWO), and Differential Evolution (DE) for image segmentation.

#### 1.4 Organization of the thesis

Chapter 2 describes the literature review behind this study. Chapter 3 FCM with optimizers for image segmentation. Chapter 4 provides an explanation of the results and analysis that were obtained in the report. Chapter 5 contains an explanation of the conclusion and the future scope of this study.

# **CHAPTER-2 LITERATURE REVIEW**

# **2.1 CLUSTERING**

It is a procedure to categorize collection of input points such that input points in one group are more likely to input points in other groups.

Clustering generally classified into following categories:

- **Hard Clustering**: In hard clustering, each data point belongs to a cluster entirely or partly.
- **Soft Clustering**: Rather assigning each data point to a distinct cluster, soft clustering provides the probability of each data point to be in that particular group.

In a variety of applications, such as picture dividing, illegal activity detection, quality prediction in softwares, etc [17]. Image segmentation is a method of dividing a image multiple subgroups called as segments of image to decrease the complexity of image and ease further actions on image. In simple language, segmentation is the process of labeling pixels. All elements of picture that correspond to the same group are given the similar name. Consider the following example: For object detection, a picture must be provided as input. A segmentation approach may be used to send data to the detector rather than examining the complete picture. As a consequence, the detector will not analyze the complete picture, reducing the time required for inference. The technique of extracting ROIs from 3D image data, like MRI or CT scans, is known as medical image segmentation. The primary intention of dividing this data is to identify areas of interest that are needed for specialized studies, such as replicating physical characteristics or properly situating inside a patient. Image dividing in the field of medical is a time taking operation, to complete routine tasks. Most significant benefits of medical image segmentation is that we can get more accurate examination just the parts that are necessary. Certain surgeries, such as implant design, need the segmentation of certain structures, such as the hip or knee. Furthermore, segmentation enables the elimination of unwanted scan elements such as air, as well as the separation of various tissues such as bone and soft tissues.

# 2.2 FUZZY C-MEANS ALGORITHM:

It divides a set of input data points,  $o = \{o_1, o_2, ..., o_n\}$ , into c clusters having centers as  $z_1$  to  $z_c$ .

$$\mu_{ij} \in [0, 1] \tag{1}$$

 $\mu$  is membership matrix having dimensions (n, c), total number of data points represented by n and total number of clusters by c. Characteristics of  $\mu$  are as follows:

$$\sum_{j=1}^{c} \mu_{ij} = 1 \quad \forall i = [1, n]$$
(2)

$$0 < \sum_{i=1}^{n} \mu_{ij} < n \quad \forall j = [1, c]$$
(3)

The goal of the FCM algorithm is to minimize the Eq. (4):

$$J_m = \sum_{j=1}^{c} \sum_{i=1}^{n} \mu_{ij}^m d_{ij}$$
(4)

, where

$$d_{ij} = ||o_i - z_j|| \tag{5}$$

Scalar is m and designated as scaling factor and governs the softness of the generated clusters and  $d_{ij}$  is a euclidian distance.  $z_j$  is a center of cluster for the cluster  $j^{th}$  and we can get acc to eq. (6).

$$z_j = \frac{\sum_{i=1}^n \mu_{ij}^m o_i}{\sum_i^n \mu_{ij}^m} \tag{6}$$

The FCM algorithm is given below:

- 1. Select m (m > 1).
- 2. Initialize the membership values  $\mu_{ij}$ , i = 1, 2, ..., n; j = 1, 2, ..., c.
- 3. Obtain cluster centers acc to eq. (6).
- 4. Obtain  $d_{ij}$ .
- 5. According to Eq. (7) calculate membership function.

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{c} (\frac{d_{ij}}{d_{ik}})^{\frac{2}{m-1}}}$$
(7)

6. Go to step 2 if not converged.

\* Converges when eq. (4) can not be reduced more.

## 2.3 LITERATURE SURVEY OF OPTIMIZATION ALGORITHMS

#### 2.3.1 Particle Swarm Optimization Algorithm

It is a population-based probabilistic improvement strategy based on emphasess and a large number of birds. Kennedy and Eberhart introduced it in 1995.

The algorithmic stream begins with a population of particles whose placements address appropriate replies for the problem under consideration and whose speeds are inserted in the search space indiscriminately. Every molecule's health value is digested in every focus. Every molecule's speed is adjusted based on two factors: individual fittetst location and overall fittest location. The fittest private situation since underlying time step pbest is the best place the molecule has visited, and gbest is the best position the multitude has visited. Every molecule's speed and area are calculated as follows:

$$V(t+1) = w \cdot V(t) + c_1 r_1 (pbest(t) - X(t)) + c_2 r_2 (gbest(t) - X(t)); \quad k$$
  
= 1,2, ... P (8)

$$X(t+1) = X(t) + V(t+1)$$
(9)

here X is the position and V represents velocity of the candidate. w is inertia weight,  $c_1$  and  $c_2$  are constants, known as acceleration coefficients that control the impact of pbest and gbest

on the search, number of particles defined by P in the swarm,  $r_1$  and  $r_2$  are random values in the range zero and one.

The steps of PSO algorithm are given below:

- 1. Initialization
  - 1.1. Perform following steps for every particle in a swarm:
    - 1.1.1. Initialize  $X_i$  randomly
    - 1.1.2. Initialize  $V_i$  randomly
    - 1.1.3. Get fitness  $f(X_i)$
    - 1.1.4. Initiate  $pbest_i$  as  $X_i$
  - 1.2. Make a copy of  $X_i$  and assign it to *gbest*.
- 2. Repeat until convergence:
  - 2.1. For every particle:
    - 2.1.1. Update  $V_i^t$  and  $X_i^t$  according to Eq. (8) and (9)
    - 2.1.2. Get fitness  $f(X_i^t)$
    - 2.1.3.  $pbest_i \leftarrow X_i^t$  if  $f(pbest_i) < f(X_i^t)$
    - 2.1.4. gbest  $\leftarrow X_i^t$  if  $f(gbest) < f(X_i^t)$

#### 2.3.2 Cuckoo Search Optimization Algorithm

It is a nature-inspired algorithm, developed by Yang and Deb [13]. The behavior of brood parasitism of some cuckoo species is used by this algorithm. Levy flights are used to enhance the performance of this algorithm rather than using simple isotropic random walks [14].

Cuckoo birds use other host birds' nests for hatching purposes. Cuckoo birds are capable to select nests that contain eggs that are recently laid and to improve the probability of hatching their eggs can remove current eggs in the nest. Some host birds might respond to cuckoo parasitism by throwing out found other eggs or building a other nest at a different place. The CS algorithm was developed using the cuckoo breeding analogy. The evolution process of CSO is defined by the following mechanisms:

#### *Levy flight:*

Levy flight is used to generate a new candidate solution by adding constants in the current X with,

$$X_{new} = X + randn * c \tag{10}$$

Where c is a change of position and represented as,

$$c = 0.01 \cdot s \cdot (X - best) \tag{11}$$

Where s represents a step and is created using asymmetri Levy distribution as,

$$s = \frac{u}{|v|^{1/\beta}} \tag{12}$$

Where  $\beta = 1.5$  and  $u \sim N(0, \sigma_u^2)$ ,  $v \sim N(0, \sigma_v^2)$ 

$$\sigma_{u} = \left(\frac{\gamma(1+\beta) \cdot \sin(\frac{n\beta}{2})}{\gamma(\frac{1+\beta}{2}) \cdot \beta \cdot 2^{(\beta-1)/2}}\right)^{1/\beta}$$
$$\sigma_{v} = 1$$
$$u = randn * \sigma_{u}$$
$$v = randn$$

Replace some nests with obtained solution:

Under this, the host may throw the egg out of nest or can leave the nest (with a fraction  $p_a \epsilon$  [0, 1]), and build a completely new nest. Select randomly r  $\epsilon$  [0, 1], if r < p<sub>a</sub>, X is selected and modified using following equation,

$$X_{new} = X + rand \, \cdot (X_{d_1} - X_{d_2}) \tag{13}$$

where  $d_1$  and  $d_2$  are from 1 to N and random.

The Steps of CSO algorithm are given below:

- 1. Initialize n size of host nest, probability  $P_{n}$ , and iteration limit  $M_{axt}$
- 2. For each nest Generate an initial population, evaluate fitness function for each nest
- 3. Generate a new solution  $X_i(t + 1)$  randomly
- 4. Evaluate fitness function for new position
- 5. Choose a nest among n solutions randomly
- 6. If selected nest has greater fitness value than preceding fitness then replace previous solution with newer solution

- 7. Abandom fraction  $P_a$  of worst nest
- 8. At new place build new nest
- 9. Keep the best solution
- 10. Rank the solution and find current best solution
- 11. Repeat steps 3 to 10 until  $M_{axt}$
- 12. Produce the best solution

#### 2.3.3 Genetic Algorithm Optimization Algorithm

The strategy of natural selection is employed in Genetic Algorithm (GA) that is metaheuristic and evolutionary algorithm. In evolutionary algorithms the strongest candidates become stronger while the weakest candidates are eliminated. The main idea behind this optimization algorithm is survival of the fittest ([15], [16]).

In this algorithm there are mainly three phases, selection, crossover, and mutation. Selection phase selects the fittest candidates among all the candidates. Two sets of candidates (parent<sub>1</sub> and parent<sub>2</sub>) are picked according to their fitness values. Candidate with large fitness value has more probability to be picked for a generation. In crossover compute offspring as,

$$x_1^{new} = 0.5[(1+\beta)x_1 + (1-\beta)x_2]$$
  

$$x_2^{new} = 0.5[(1-\beta)x_1 + (1+\beta)x_2]$$
(14)

where,

$$\beta = \begin{cases} (2u)^{\frac{1}{\eta_c + 1'}} & \text{if } u \le 0.5\\ (\frac{1}{2(1-u)})^{\frac{1}{\eta_c + 1'}} & \text{otherwise} \end{cases}$$
(15)

in which  $u \in [0, 1)$  and  $\eta_c$  is the distribution index.

For mutation add delta to each offspring based on probability,

$$\delta = \begin{cases} (2r)^{\frac{1}{\eta_m + 1}}, & \text{if } r < 0.5\\ 1 - (2(1 - r))^{\frac{1}{\eta_m + 1}}, & \text{if } r \ge 0.5 \end{cases}$$
(16)

The Steps of GA algorithm are given below:

- 1. Generate initial population of n genes
- 2. Select fittest genes for mating
- 3. Perform crossover
- 4. Perform mutation
- 5. Repeat steps 4 to 6 until convergence

#### 2.3.4 Whale Optimization Optimization Algorithm

One of the largest whales is the humpback whale. This algorithm is motivated by the hunting behavior of the humpback whale. The hunting strategy is divided into three phases:

#### Encircling Prey:

Prey can be sensed and surrounded by whales. Since area of ideal floor in the hunting room has been obscurely deduced, the WOA approach accepts that the continuous arrangement for the good solution is either desired victim or is close to it. After choosing best hunting specialist, the other research specialists will make an effort to coordinate their situation with the best hunting specialist. Accompanying conditions give meaning to this policy:

$$\vec{D} = \left| \vec{C} \cdot \vec{X_p}(t) - \vec{X}(t) \right| \tag{17}$$

$$\vec{X}(t+1) = \vec{X_p}(t) - \vec{A}.\vec{D}$$
(18)

current iteration is indicated by *t*, coefficient vectors are  $\vec{A}$  and  $\vec{C}$ ,  $\vec{X_p}$  is prey's position vector, and position vector of a whale is  $\vec{X}$ .  $\vec{A}$  and  $\vec{C}$  by following formula:

$$\vec{A} = 2\vec{a}.\vec{r_1} - \vec{a} \tag{19}$$

$$\vec{C} = 2.\,\vec{r_2} - \vec{a} \tag{20}$$

Various characteristics of  $\vec{a}$  are reduced from two to zero during consecutive generations.

#### *Bubble-net attacking method (exploitation phase)*

The exploitation phase is modeled using the position approach that is spirally updated. This method initially computes the distance between the whale at (X, Y) and the prey at  $(X^*, Y^*)$ . To simulate the helix-shaped movement of humpback whales, a spiral equation is established between the location of whale and prey, as seen below:

$$\vec{X}(t+1) = \vec{D'}e^{bt}\cos(2\pi t) + \vec{X^*}(t)$$
(21)

where distance of current whale is indicated by  $\vec{D'} = |\vec{X'}(t) - \vec{X}(t)|$  from victim.

#### Finding victim (exploration phase)

The Exploration phase depends on the  $\vec{A}$  vector's variation that can be used to find the victim. Whales find victim randomly based on their location. Therefore, we initialize  $\vec{A}$  by values that are greater than one or lesser than minus one to enforce whale to go away from a reference whale. In current situation search agent's position is modified as per randomly selected search agent rather than the best search agent. Below is the technique is defined mathematically:

$$\vec{D} = \left| \vec{C} \, \vec{X_{rand}} - \vec{X} \right| \tag{22}$$

$$\vec{X}(t+1) = \overrightarrow{X_{rand}} - \vec{A}.\vec{D}$$
<sup>(23)</sup>

 $\overrightarrow{X_{rand}}$  is a random vector.

The steps of WOA are given below:

- 1. Input data, max iterations, and population size P.
- Create a population with P whales, each whale can be represented as P(X) where X is n x c matrics.
- 3. Initialize a, A, C, t, b, p.
- 4. Get the best search agent.
- 5. Perform steps 8 and 9 for every whale.
- 6. If probability is less than half then if absolute value of A is less than one then modify the position of current whale acc. to equation (9) else absolute value of A is greater or equal

to one then select a random whale and modify the position of current whale acc. to equation (14).

- 7. Else if probability is greater than or equal half then modif the position of the current whale acc. to equation (12).
- 8. Update the best search agent if any better solution available.
- 9. Update a, A, C, t, b, p.
- 10. Go to step 7 if the terminating condition is not exhausted.

### 2.3.5 Grey Wolf Optimization

It is based on community and contextual program that imitates natural leadership structure and it is simililar to how grey wolves hunt prey. Grey wolves are crown hunters, it defines that in the food chain they are on top. Grey wolves wonder in groups, with every pack including an average of 5 - 12 individuals.

Leading wolf is the alpha wolf, and other members must obey the leading wolf's instructions. Betas are auxiliary wolves that assist the leading wolves in taking decisions and are termed as the best prospect for the position of alpha. Delta wolves must be subject to alpha and beta wolves. Omega wolves are the pack's scapegoat, are the least important members, and are only permitted to eat at the end.

### Social hierarchy:

Alpha wolf ( $\alpha$ ) is the best wolf, beta is a second good, and delta is a third good, Others are omega.

Circling the victim:

$$\vec{D} = \left| \vec{C} \cdot \vec{X_p}(t) - \vec{X}(t) \right| \tag{24}$$

$$\vec{X}(t+1) = \vec{X_p}(t) - \vec{A}.\vec{D}$$
<sup>(25)</sup>

*t* defines current loop,  $\vec{A}$  and  $\vec{C}$  coefficient vectors, position vector of the victim is defined as  $\vec{X_p}$ , and position vector of a grey wolf indicated by  $\vec{X}$ . Calculation of  $\vec{A}$  and  $\vec{C}$  are given below:

$$\vec{A} = 2\vec{a}.\vec{r_1} - \vec{a}$$

$$\vec{C} = 2.\vec{r_2}$$
(26)

Where peripherals of  $\vec{a}$  are linearly decreased from two to zero over the course of iterations and  $r_1$ ,  $r_2$  are random vectors within zero and one.

#### Hunting:

In every iteration, as per the location of top three fit wolf, omegas update their positions because Alpha, beta, and delta have a better understanding of the optimum position of the victim.

$$\overrightarrow{D_{\alpha}} = |\overrightarrow{C_{1}}.\overrightarrow{X_{\alpha}} - \overrightarrow{X}|$$

$$\overrightarrow{X_{1}} = \overrightarrow{X_{\alpha}} - \overrightarrow{A_{1}}.(\overrightarrow{D_{\alpha}})$$
(27)

$$\overrightarrow{D_{\beta}} = |\overrightarrow{C_2}, \overrightarrow{X_{\beta}} - \overrightarrow{X}|$$

$$\overrightarrow{X_2} = \overrightarrow{X_{\beta}} - \overrightarrow{A_{2}}, (\overrightarrow{D_{\beta}})$$
(28)

$$\overrightarrow{D_{\delta}} = |\overrightarrow{C_{3}}.\overrightarrow{X_{\delta}} - \overrightarrow{X}| 
\overrightarrow{X_{3}} = \overrightarrow{X_{\delta}} - \overrightarrow{A_{3}}.(\overrightarrow{D_{\delta}})$$
(29)

#### *Prey attack (exploitation):*

Grey wolf completes the hunt by attacking the prey once it is ideal and models that we reduce  $\vec{a}$  value.  $\vec{A}$  is a random value withing -2a to 2a, where a is reduced to zero from two during period of iterations. Absolute value of A less than one make wolves to hunt the victim.

#### Victim finding (exploration):

Absolute value of A greater than one makes the grey wolves to bifurcate from the victim to find a better victim. Athor parameter of GWO that favors exploration is  $\vec{C}$ . It has values within zero and two. C greater than 1 emphasizes the attack and C less than one does not promote the attack.

The steps of GWO are given below:

- 1. Create the initial inputs of n candidate wolves
- 2. Set a=2, A, and C acc to the equation (17)
- 3. Update the position of the omega wolf using equations (18), (19), (20).
- 4. Update a, A, C.
- 5. Update  $X_{\alpha}$ ,  $X_{\beta}$ , and  $X_{\delta}$ .
- 6. Go to step 3 if the terminating condition is not exhausted.
- 7. Return  $X_{\alpha}$ .

#### 2.3.6 Differential Evolution Optimization Algorithm

It is a global optimization method of continuity and consistency discrete domain functions. The starting vector input is selected at random and span the whole space of parameter. This algorithm creates new vectors of parameters by summing the weighted differences of two input vectors. Let us name this procedure a mutation. The parameters of altered vector added in the parameters of other preset vector, the aiming vector, to produce the trial vector. This method is known as selection.

For every target vector a mutant vector is generated according to following,

$$V_{i,G+1} = x_{r_1,G} + F \cdot (x_{r_2,G} - x_{r_3,G})$$
(30)

with indexes that are random integer  $r_1, r_2, r_3 \in \{1, 2, ..., NP\}$ , different from each other, and F greater than zero. r1, r2, and r3 other than the current value that are random. F is a real and constant factor  $\epsilon$  [0, 2] that controls the differential variation's amplification ( $x_{r_2,G} - x_{r_3,G}$ ).

The hybrid is applied to the paints when collecting the assigned border vectors. So the initial vector is:

$$u_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, \dots, u_{Di,G+1})$$
(31)

Is formed where,

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & if \ (randb(j) \le CR) \ or \ j = rnbr(i) \\ x_{ji}, G & if \ (randb(j) > CR) \ and \ j \ne rnbr(i) \\ j = 1, 2, \dots D \end{cases}$$
(32)

- 1. Create the initial input of n candidates
- 2. Calculate mutant vectors for each candidate
- 3. Calculate trial vectors for each candidate
- 4. Perform selection greedily
- 5. Repeat steps 2 to 4 until convergences

### 2.4 FCM WITH OPTIMIZATION ALGORITHMS

### 2.4.1 FCM-PSO

FCM-PSO algorithm processes N candidate solutions to obtain the best solution. Each candidate solution is termed as particle and it is represented as P(X, pbest, V).

where *X* is  $n \times c$  matrix and can be described as,

$$X = \begin{bmatrix} \mu_{11} & \cdots & \mu_{1c} \\ \vdots & \ddots & \vdots \\ \mu_{n1} & \cdots & \mu_{nc} \end{bmatrix}$$
(33)

*pbest* is a local best position, V is a velocity matrix, and both are also  $n \times c$  matrices.

- 1. Initialize parameters *P*, *c*<sub>1</sub>, *c*<sub>2</sub>, *w*, maximum iterations.
- 2. Create a swarm with P particles, each particle is defined as: P(X, pbest, gbest, V).
- 3. For every particle create initial X, V, pbest and for the swarm gbest.
- 4. According to Eq. (6) calculate cluster centers for each particle.
- 5. According to Eq. (4) calculate fitness values for each particle.
- 6. For every particle calculate pbest.
- 7. For swarm get gbest.
- 8. According to Eq. (8) for every particle update the velocity matrix.
- 9. According to Eq. (9) for every particle update the position matrix.
- 10. Go to step 4 if ending condition is not exhausted.
- 11. Perform iterations of FCM on the best particle until termination condition\*.

<sup>\*</sup> Terminates when number of iterations exhaust.

# 2.4.2 FCM-CSO

FCM-CSO algorithm processes N candidate solutions to obtain the best solution. Each candidate solution is termed as cuckoo and it is represented as N(X).

where X is same as eq (33).

- 1. Create initial inputs of n nests, nest is represented as N(X), where X is  $n \times c$  matrix.
- 2. By using levy flight randomly select cuckoo.
- 3. Get fitness value of selected cuckoo acc to eq (4).
- 4. Randomly select a nest.
- 5. Get its fitness value  $(F_n)$  acc to eq (4).
- 6. If randomly selected cuckoo's fitness is less than randomly selected nest's then replace with cuckoo.
- 7. Nests of the fraction  $P_a$  are replaced with new nests.
- 8. Get fitness and keep nests with high fitness.
- 9. Give numbering to solutions to get the current best.
- 10. Save the nest that is most promising and call it as best fitness.
- 11. If the terminating condition is not met go to step 2.
- 12. Perform iterations of FCM on the best nest until termination condition\*.

\* Terminates when number of iterations exhaust.

## 2.4.3 FCM-GA

FCM-GA algorithm processes N candidate solutions to obtain the best solution. Each candidate solution is termed as gene and it is represented as G(X).

where X is same as eq (33).

- 1. Create initial input of n genes, each gene is represented as G(X), where X is  $n \times c$  matrix.
- 2. According to Eq. (6) calculate cluster centers for each gene.
- 3. According to Eq. (4) calculate fitness value for each gene.
- 4. Select genes for mating.
- 5. According to Eq. (14) perform crossover.
- 6. According to Eq. (16) perform mutation.
- 7. Repeat steps 4 to 6 until convergence\*.
- 8. Perform iterations of FCM on the best gene until termination condition\*.

\* Convergence condition is a maximum number of iterations.

# 2.4.4 FCM-WOA

FCM-WOA algorithm processes N candidate solutions to obtain the best solution. Each candidate solution is termed as whale and it is represented as P(X).

where X is same as eq (33).

- 1. Input data, max iterations, and population size *P*.
- 2. Create a population with *P* whales, each whale is representeas P(X) where *X* is *n* x *c* matrics.
- 3. Initialize a, A, C, t, b, p.
- 4. Get cluster centers for each whale acc. to equation (6).
- 5. Get fitness values for each whale acc. to equation (4).
- 6. Get the best search agent.
- 7. Perform steps 8 and 9 for every whale.
- 8. If probability is less than half then if absolute value of A is less than one then modify the position of current whale acc. to equation (18) else absolute value of A is greater or equal to one then select a random whale and modify the position of current whale acc. to equation (4).
- 9. Else if probability is greater than or equal half then modify the position of the current whale acc. to equation (21).
- 10. Calculate fitness values for every whale acc. to equation (4).
- 11. Update the best search agent if better solution available.
- 12. Update a, A, C, t, b, p.
- 13. If the terminating condition is not met, go to step 7.
- 14. Perform iterations of FCM on the best candidate until termination condition\*.

## 2.4.5 FCM-GWO

<sup>\*</sup> Terminates when number of iterations exhaust.

FCM-GWO algorithm processes N candidate solutions to obtain the best solution. Each candidate solution is termed as wolf and it is represented as W(X).

where X is same as eq (33).

- Create the initial input of n wolves, each wolf is represented as W(X), where X is the n × c matrix.
- 2. Set value, a=2, A, and C using the equation (24).
- 3. Get fitness of each member of the input and initialize  $X_{\alpha}$ ,  $X_{\beta}$ , and  $X_{\delta}$ .
- 4. Update the position of the omega wolf using equations (25), (26), (27).
- 5. Update a, A, C.
- 6. Get the fitness for every wolves acc to eq (4).
- 7. Update  $X_{\alpha}$ ,  $X_{\beta}$ , and  $X_{\delta}$ .
- 8. If the terminating condition\* is not met go to step 3.
- 9. Return  $X_{\alpha}$ .
- 10. Perform iterations of FCM on the best candidate until termination condition\*.

\* Terminates when number of iterations exhaust.

## 2.4.6 FCM-DE

FCM-DE algorithm processes N candidate solutions to obtain the best solution. Each candidate solution is represented as C(X, M, T).

where X is same as eq (33).

- 1. Create the initial input of n candidates, each candidate is represented as C(X, M, T), where M is a mutant vector, T is a trial vector and X, M, and T are  $n \times c$  matrix.
- 2. Calculate mutant vectors for each candidate using equation (30).
- 3. Calculate the trial vector for each candidate using equation (32).
- 4. Perform selection greedily.
- 5. Repeat steps 2 to 4 until convergence.
- 6. Perform iterations of FCM on the best candidate until termination condition\*.

# **CHAPTER-3**

<sup>\*</sup> Convergence condition maximum number of iterations exhaust.

# **RESULTS AND ANALYSIS**

# **3.1 DATASET USED**

Brainweb (http://www.bic.mni.mcgill.ca/brainweb) is a simulated brain database of realistic MRI brain images created using an MRI simulator.

# **3.2 HARDWARE AND SOFTWARE REQUIREMENTS**

**3.2.1 Hardware requirements:** All the algorithms are implemented using python 3.8 with 64-bit ubuntu operating system.

### 3.2.2 Software requirements:

**Python:** Python is the main deciphered general purpose programming language. The justification for his plan strains the intelligibility of the code in the redundant space. Python uses a choice of garbage and dynamic creation. It is compatible with a various standards, including organized, object-related, and hands-on programming.

**Jupyter Notebook:** Jupyter Notebook is the leading web-based computer archive creation and sharing program. It offers a simple and fluid record-based user interface.

### Some Standard Libraries That Are Used:

**Numpy:** NumPy is a Python bundle that serves as the logon logic. The Python library contains versatile cluster objects, specific objects (covered exposures, grids, etc.) and fast clusters such as numerical computation, sensitivity, shape control, positioning, selection, I / O, discrete Fourier transform, and basics. It includes various schedules for performing tasks such as polynomial math, basic factual activities, and even irregular reproductions.

**Pandas:** It is a fast, versatile, and easy-to-use open source information discovery and control tool based on the python.

**Matplotlib:** Matplotlib is a python library that offers static, animated, and intuitive detection. Matplotlib is easy to think of hard things and based on simple things. Create a package suitable for distribution. Create smart figures that can be zoomed in, moved and updated. Change format and visual plan. Products for various document designs. You must use JupyterLab and a graphical user interface. Use variable decisions from external devices based on Matplotlib.

**OpenCV:** OpenCV is a large library for various computations related to images and videos, AI, and modifying images. OpenCV works with Python, C++ and Java. You can recognize objects, people, and surprisingly human handwriting with photos and recordings. Combined with additional libraries like Numpy, a very efficient library for mathematical tasks, OpenCV can also do all the activities that Numpy should be able to do, thus increasing the amount of weapons in your inventory.

#### **3.3 QUANTITATIVE MEASURES:**

For quantitative comparison, cluster validity functions used to assess the performance of various clustering techniques. For fuzzy partition partition coefficient  $V_{PC}$  [18] and partition entropy  $V_{PE}$  [19] are used to assess the performance. They are as follows:

$$V_{pc}(U) = \frac{1}{n} \left( \sum_{k=1}^{n} \sum_{i=1}^{c} u_{ik}^2 \right)$$
(33)

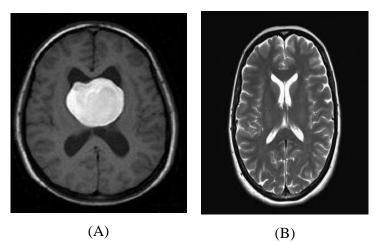
$$V_{pe}(U) = -\frac{1}{n} \left( \sum_{k=1}^{n} \sum_{i=1}^{c} u_{ik} log u_{ik} \right)$$
(34)

We can get better performance when value of V<sub>PC</sub> is maximum or V<sub>PE</sub> is minimum.

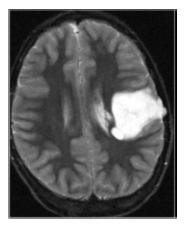
### **3.4 EXPERIMENTAL ANALYSIS**

The outcomes of all optimizers are shown and assessed using appropriate metrics. The experiment is carried out using common pictures from the BrainWeb collection. To demonstrate the efficiency of the optimizers, they are applied to multiple brain MRI pictures from the BrainWeb dataset.

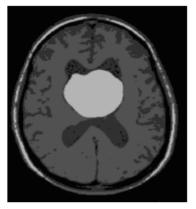
**3.4.1 Experimental Results:** 7 algorithms are examined in this section with help of real brain CT scan images, Figure 4.1 given below.



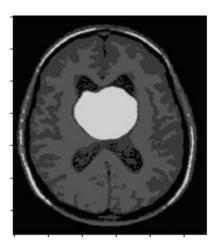
(B)



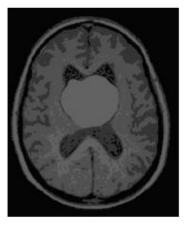
(C) FIGURE 4.1 MRI/CT scan brain images



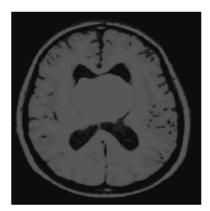
FCM



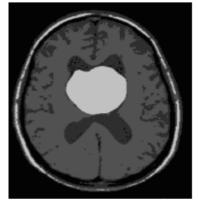
FCM-PSO



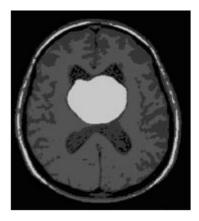
FCM-CSO



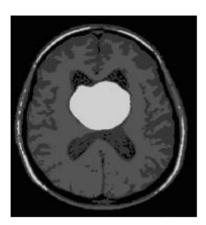
FCM-GA



FCM-WOA

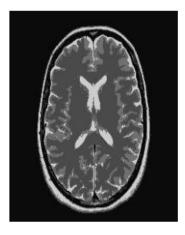


FCM-GWO

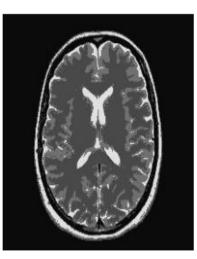


FCM-DE

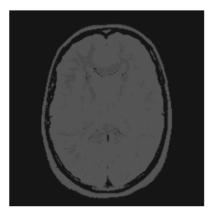
FIGURE 4.2 Segmentation results for figure 4.1a



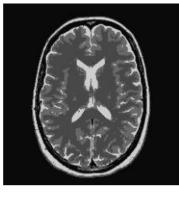
FCM



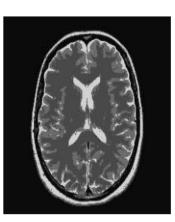
FCM-PSO



FCM-GA

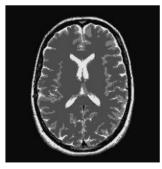


FCM-WOA



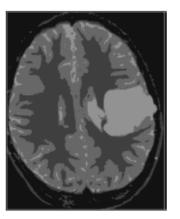
FCM-CSO

FCM-GWO

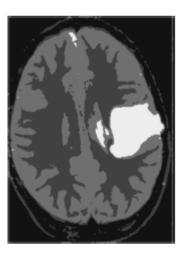


FCM-DE

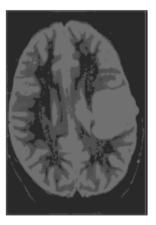
FIGURE 4.3 Segmentation results for figure 4.1b



FCM



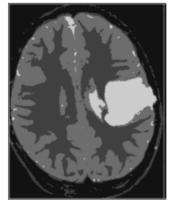
FCM-PSO



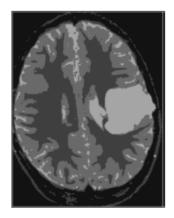
FCM-CSO



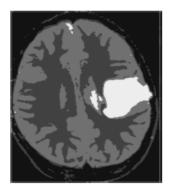
FCM-GA



FCM-WOA



FCM-GWO



FCM-DE

FIGURE 4.4 Segmentation results for figure 4.1c

Images	FCM	PSO	CSO	GA	WOA	GWO	DE
4.1a	0.838	0.842	0.756	0.481	0.832	0.793	0.840
4.1b	0.895	0.891	0.849	0.412	0.888	0.896	0.895
4.1c	0.782	0.836	0.614	0.447	0.815	0.780	0.835

TABLE 1 Segmentation quality based on VPC

TABLE 2 Segmentation quality based on  $V_{PE}$ 

Images	FCM	PSO	CSO	GA	WOA	GWO	DE
4.1a	0.307	0.292	0.459	0.919	0.323	0.312	0.293
4.1b	0.202	0.199	0.284	1.078	0.214	0.198	0.192
4.1c	0.421	0.317	0.732	1.025	0.362	0.424	0.319

Figure 4.2, 4.3, and 4.4 show the segmentation result of figures 4.1(a), 4.1(b), and 4.1(c). Table 1 and Table 2 show the values of the  $V_{PC}$  and  $V_{PE}$  validity index respectively. It is noted from figure 4.2 that FCM-PSO and FCM-DE have given the best results, with 0.842 and 0.840  $V_{PC}$  respectively. In the case of figure 4.1b, FCM-GWO outperformed all the algorithms as shown in figure 4.3 with a validity index  $V_{PC}$  of 0.896 and  $V_{PE}$  of 0.198, FCM and FCM-DE are the second-best algorithms with a validity index  $V_{PC}$  of 0.895. For figure 4.1c FCM-PSO and FCM-DE outperformed all other algorithms with a validity index  $V_{PC}$  of 0.835 respectively.

# CHAPTER-4 CONCLUSION AND FUTURE WORK

The technique of extracting regions of interest (ROIs) from images, such as MRI or CT scans, is known as medical image segmentation. The primary aim of dividing this data is to know about areas of interest that are needed for specialized studies, such as replicating physical characteristics or properly situating designed implants inside a patient. Dividing an image in the field of medical may be time taking operation, but recent improvements in Artificial Intelligence (AI) techniques have made it simple to complete daily tasks. Significant advantage of segmentating an image in medical field is that we can get more detailed and precise outcomes based on the input. Certain surgeries, such as implant design, need the segmentation of certain structures, such as the hip or knee. Furthermore, segmentation enables the elimination of unwanted scan elements such as air, as well as the separation of various tissues such as bone and soft tissues. By combining several options for processings, people in this field may build a multiple segmented masks.

It is very benificial to enhance of segmentation algorithms in the medical field. The proposed optimizers tries to optimize the FCM algorithm by minimizing the cost function so that it does not fall into local minima. FCM-DE and FCM-PSO has achieved the best performance with the average value of validity index  $V_{PC}$  as 0.856. Proposed optimizers are compared with the FCM algorithm and DE and PSO outperformed the FCM algorithm. As a part of future work can be done to further enhance performance of the optimizers by finding method to preprocess the image so optimizers can give more accurate results.

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