

Field emission of electrons by taking into account the distribution of charge along the length of metallic carbon nanotubes (CNTs)

A PROJECT REPORT

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FOR THE AWARD OF DEGREE
OF

MASTER'S
IN
PHYSICS

Submitted by:
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2K19/MSCPHY/12

Under the supervision of

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We hereby certify that the work which is presented in the Major Project-II entitled **Field emission of electrons by taking into account the distribution of charge along the length of metallic carbon nanotubes (CNTs)** in fulfillment of the requirement for the award of the Degree of master's in physics by Rachana (2K19/MSCPHY/12) is submitted to the Department of Applied Physics, Delhi Technological University, Delhi is an authentic record of our own, carried out during a period from January 2021 to May 2021, under the supervision of Prof. Suresh Chand Sharma.

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I hereby certify that the M.Sc. project Dissertation titled **“Field emission of electrons by taking into account the distribution of charge along the length of metallic carbon nanotubes (CNTs)”** by Rachana (2K19/MSCPHY/12), Department of Applied Physics, Delhi Technological University, Delhi in fulfillment of the requirement for the award of the degree of masters in Physics, is a record of the project work carried out by the student under my supervision to the best of my knowledge this work has not been submitted in partial for any Degree or Diploma to this University or elsewhere.

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ABSTRACT

An mathematical expression has been derived and analysed for total potential energy V and field emission current density function ϕ of emitted electrons by taking into account the distribution of charge along the length of the carbon nanotubes (CNTs), matching to the representation for the tunnelling coefficient. This tunnelling coefficient is obtained from solving of the time independent Schrödinger's wave equation. Mathematical calculations for the potential energy V , tunnelling coefficient T and current density function ϕ have been worked out for a distinctive set of the CNTs factors. It is concluded that the potential energy in ergs declines with the radial distance r but increases with z -coordinate along the length of the CNT. Moreover, the current density function at the spherical CNT tip is substantially larger than along the length of cylindrical metallic CNT. In addition, the field emission current density function decreases with radius of CNTs in both the cases (i.e., cylindrical and spherical tip). Some of our theoretical results are in agreement with the current experimental observations.

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LIST OF SYMBOLS

S.No	Abbreviations	Full Forms
1	CNT	Carbon Nanotube
2	β	Field Enhancement Factor
3	r	Radius of CNT
4	d	Cathode –Anode Distance
5	V _a	Voltage at anode plate
6	$\Lambda(z)$	Charge density induced on surface metallic CNT
7	E_r	Radial Electric Field
8	E ₀	Electric field applied between cathode and anode plates
9	\mathcal{E}_ρ	Normalized radial energy
10	\mathcal{E}_r	Relative permittivity of the medium
11	ϕ	Potential due to the charge induced on CNT
12	V	Total potential energy
13	ψ	Eigen wave function
14	T	Tunneling Probabilty
15	ρ	Normalized radial distance

CHAPTER 1

1.1 INTRODUCTION

The phenomenon of the electric field emission from metallic particles and the field emission based carbon CNTs devices have been a great deal of interest for many years. Due to CNT's electronic structure, chemical inertness and high aspect ratio, they have extraordinary field emission properties.

Carbon nano-tubes have good emission stability, long emitter lifetime, and low work voltage. Hence due to the above mentioned properties, CNT emitters can handle potential applications in flat panel display, illumination and also handles bears potential applications in electron source for electron microscopy, other vacuum micro-electronic and nano-electronic devices. Choi *et al.* examined and reported the fabrication along with the performance of a high-brightness CNT field emission display. The method of electrons emission due to field from a carbon nanotube is understood because of external applied electric field experiencing an increase of electrons emission at the tip of the CNT, often denoted to be as the field enhancement factor β . For a solitary CNT, the value of the field enhancement factor is supposed to dependent on length z , radius r , and type of structure, i.e., single walled (SWNT), multi-walled (MWNT), open or closed cap.

Bonard et al. reported the field emission properties of single-wall carbon nanotube films, with emphasis on current-voltage (I-V) characteristics curve and current stability. Carbon nanotube films are magnificent field emitters, hence yield current densities more than the value 10 mA cm^2 with operating voltages which are much lesser than operating voltages for other film emitters. But indicate too a significant amount of declination in their performances with respect to time. In this case, divergence from Fowler-Nordheim behavior in the current-voltages (I-V) characteristics suggest that the emission behavior may be notably influenced by the electronic properties of the SWNT, and point towards non-metallic density of states at the tip of the carbon nano-tubes.

Nilsson et al. have examined the field emission properties (FE) of the carbon nanotube films (CNT) by a scanning anode FE apparatus which discloses that field emission properties is strongly influenced by the density and morphology of the CNT deposit. Large variation between the microscopic and macroscopic current and the field emission site densities are observed, and described in variation of field enhancement factor β 's terms. Films which have intermediate densities (of order 10^7 emitters/cm², according to electrostatic calculations) show the maximum emitted current densities.

Jo et al. investigate the field emission properties influenced by length and spacing of vertically aligned carbon nanotubes. In this situation, in general it is presented that, the macroscopic electric field E_{mac} , defined as electric field when emission current density touches the value upto 1 mA/cm², can be decreased by increasing the length z and the spacing of the CNTs. For the very large density CNT films, length is proportional to E_{mac} , with small extent whereas for the very small CNT films, the increase of spacing does not efficiently decrease E_{mac} .

Effect of radius to length ratio (aspect ratio) and cathode-anode distance on the field emission properties of a single tip based emitter has been reported by Smith et al. In this case, they have shown that the field enhancement factor β is only dependent on the emitter's height h and radius r by using computational simulation, when cathode to anode distance d is more than three times the height of the emitter measured from the CNT's tip.

Zhu et al. have observed that those field emitters which are made up of carbon nanotubes show good macroscopic emission properties which can operate at a very large current density, as large as 4A/cm². At small electric fields valued as small as 4-7 V/ μ m, they emit industrially useful current densities of 10mA/cm². Electron-field emission properties have been investigated by Zhong et al. analytically for carbon nanotubes (CNTs)

fabricated on metal tip. The threshold field is as low as $0.7 \text{ V}/\mu\text{m}$ with a vacuum gap of 0.7 mm and the current density reaches $10 \text{ mA}/\text{cm}^2$ at electronic field of $1.0 \text{ V}/\mu\text{m}$.

Ahmad and Tripathi have derived an expression for field enhancement factor β of CNTs under any positional scattering of CNTs, using a model having floating sphere is in between anode-cathode plates. Effect of anode-cathode distance d on the field enhancement factor β has been found out by this model. Mayer has suggested that some of the induced charges are distributed along the length of the CNT.

In this paper, we derive an expression for the field emission current density function by taking into account the distribution of charge along the length of the carbon nanotubes (CNTs), corresponding to the expression for the transmission coefficient obtained from the solution of the time independent Schrödinger wave equation. In Sec. II we develop a model for potential energy of emitted electrons using Gaussian surface in the form of cylinder of length l from the midpoint of CNT and radius a ($a \ll l$). In Sec. III we use the time independent Schrödinger wave equation and solved for tunneling probability of electrons using Wentzel-Kramers-Brillouin (WKB) approximation method. The field emission current density expression at 0K has been obtained in Sec. 2.6 using free electron model for a metallic case and ignoring the thermionically emitted electrons, i.e., at absolute temperature 0K . Numerical calculations for the potential energy, transmission coefficient and field emission current density function have been carried out for a typical set of the CNTs parameters. Results and discussions are given in chapter 3.

CHAPTER 2

2.1 MODEL

Wang *et al.* and Ahmad *et al.* have developed a model for the calculation of the field enhancement factor for a single carbon nanotube (CNT). They took a CNT standing perpendicularly on cathode plate's plane, with height h and surmounted by a semi-spherical cap having radius ρ . The distance between cathode – anode plates is taken to be d . Since cathode plate is grounded, hence whole surface of metallic CNT become zero. In their model, they have assumed that almost all the charges exist at and near the top of the CNT.

In the present model, we have assumed that the induced charge is distributed along the length (i.e., along z -axis) of cylindrical metallic Carbon nanotube of length l from the midpoint of CNT of radius a placed between two plates of separation d (cf. figure 1). The voltage applied at the cathode plate is kept zero and the voltage applied at the anode plate is V_a . Let $\lambda(z)$ be the charge per unit length induced on the surface of the metallic CNT, and z -is the coordinate along the axis of the CNT measured from its midpoint.

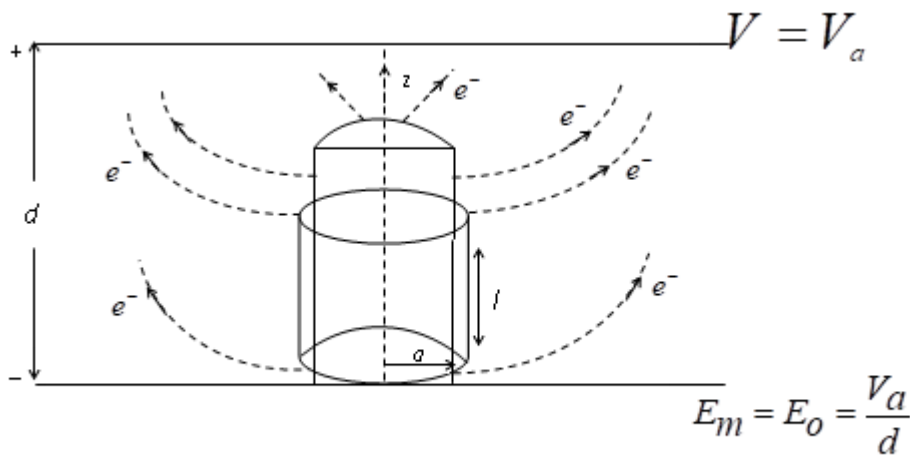


Figure .1

Consider a Gaussian surface in the form of cylinder of length l from the midpoint of the CNT and radius a . The radial electric field is given by

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{q_{encl}}{\epsilon_{r'}}$$

$$\oint |\mathbf{E}| 2\pi l dr = \frac{\lambda l}{\epsilon_{r'}}$$

$$(2\pi r) E_r(z) = \frac{\lambda(z)}{\epsilon_{r'}} \quad (1)$$

where $\epsilon_{r'}$ is the relative permittivity of the medium. Following Landau and Lifshitz [21] [cf. Page no.18], the charge per unit length induced on the surface of the cylindrical conducting CNT is given by

$$\lambda(z) = \frac{E_0 z}{\left[\log_e \left\{ 4 \frac{(l^2 - z^2)}{a^2} \right\} - 2 \right]} \quad (2)$$

where $E_0 (= V_a/d)$ is the applied electric field between the cathode and anode plates, z is the coordinate along the axis of carbon nanotube. In Eq. 1 $\lambda(z)$, (the charge per unit length induced on the surface of the conducting cylindrical CNT) is constant for a given value of z as we have considered values of z from $0.50 \mu\text{m}$ to $0.90 \mu\text{m}$.

2.2 Radial Electric field

Using the value of Eq. (2) into Eq. (1), we get the radial electric field as

$$E_r(z) = \frac{E_0 z}{2\pi \epsilon_{r'} r \left[\log_e \left\{ 4 \frac{(l^2 - z^2)}{a^2} \right\} - 2 \right]} \quad (3)$$

Potential at the surface of the conducting cylindrical CNT is zero because CNT considered to be perpendicularly standing on cathode plate which is grounded already, the cathode potential is kept at zero over the whole surface of the CNT. Hence CNT is considered the equipotential surface of CNTs, i.e., $\phi_{01}(r, z) = 0$. Consider a Gaussian surface in the form of a cylindrical conducting CNT of length l from the midpoint with cylinder of charge and having radius equal to

the distance of the point p at which the electric field is to be determined as shown in figure 2. Now the electric field from the surface of the CNT to a point p at distance r

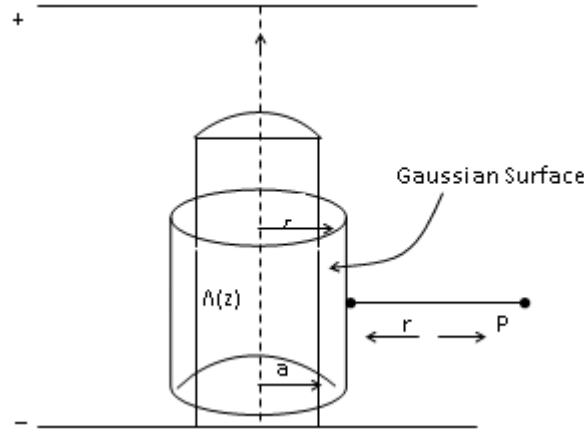


Figure 2

$$E(r, z) = \frac{\lambda(z)}{(2\pi r)\epsilon_r}$$

$$E_r(z) = \frac{E_0 z}{2\pi\epsilon_r r \left[\log_e \left\{ 4 \frac{(l^2 - z^2)}{a^2} \right\} - 2 \right]} \quad (4)$$

Now the potential outside the surface of conducting CNT can be written as

$$\phi_{02}(r, z) = - \int_a^r E(r, z) dr = \frac{E_0 z \log_e \left(\frac{a}{r} \right)}{2\pi\epsilon_r \left[\log_e \left\{ 4 \frac{(l^2 - z^2)}{a^2} \right\} - 2 \right]} \quad (5)$$

Total potential [i.e., potential on the surface of cylindrical metallic CNT + potential outside the surface of CNT at distance r] is given by

$$\phi_0(r, z) = \phi_{01}(r, z) + \phi_{02}(r, z) = \frac{E_0 z \log_e \left(\frac{a}{r} \right)}{2\pi\epsilon_r \left[\log_e \left\{ 4 \frac{(l^2 - z^2)}{a^2} \right\} - 2 \right]}$$

The value of potential at $r = a$ is zero, i.e., $\phi_0(a, z) = 0$, potential at the surface of CNT is zero.

2.3 TOTAL POTENTIAL ENERGY for a cylindrical metallic CNT can be written as

$$V(r, z) = e\phi_0(r, z) \quad (7)$$

$$V(r, z) = \frac{E_0 z}{2\pi\epsilon_r \left[\log_e \left\{ 4 \frac{(l^2 - z^2)}{a^2} \right\} - 2 \right]} \left[\log_e \left(\frac{a}{r} \right) \right] \quad \text{for } r > a \quad \dots (8)$$

Total potential energy at $r=a$, $V(a,z)=0$

2.4 SCHRÖDINGER'S WAVE EQUATION

The time independent Schrödinger's wave equation for an electron in the region $r > a$ may be written as

$$\nabla^2 \psi(r, \theta, z) + \frac{2m}{\hbar} \left[E - \frac{E_0 z}{2\pi\epsilon_r \left[\log_e \left\{ 4 \frac{(l^2 - z^2)}{a^2} \right\} - 2 \right]} \left[\log_e \left(\frac{a}{r} \right) \right] \right] \psi(r, \theta, z) = 0 \quad \dots(9)$$

where $\hbar \left(= \frac{h}{2\pi} \right)$ and h is the Planck's constant, m is the mass of electron and E is the total energy of the electron.

Let $2 \frac{m_e V_0 a^2}{\hbar^2} = \beta_1, \quad \dots\dots\dots(i)$

And $\frac{E_1 \rho}{V_0} = \mathcal{E}_\rho \quad \dots\dots\dots(ii)$

And $\frac{eE_0}{2\pi\epsilon_r V_0 \left[\log_e \left\{ 4 \frac{(l^2 - z^2)}{a^2} \right\} - 2 \right]} = \eta_1 \quad \dots\dots\dots(iii)$

So by inserting (i), (ii), (iii) in equation (9), we get,

$$\frac{d^2 W(\rho, z)}{d\rho^2} + \beta_1 \left[\mathcal{E}_\rho - \eta_1 z \left\{ \log_e \left(\frac{1}{\rho} \right) \right\} \right] W(\rho, z) = 0 \quad (10)$$

2.5 TUNNELING PROBABILITY OF ELECTRONS :-

Using the Wentzel- Kramers- Brillouin (WKB) approximation for evaluation of the tunneling probability of electrons with normalized radial energy \mathcal{E}_ρ is obtained as

$$T(\mathcal{E}_\rho) = \exp \left[-2 \int_{\rho_1}^{\rho_2} \beta_1 \left[\eta_1 z \left\{ \log_e \left(\frac{1}{\rho} \right) \right\} - \mathcal{E}_\rho \right]^{\frac{1}{2}} d\rho \right] \quad (11)$$

where $\rho_1 = r_1/a$ and $\rho_2 = r_2/a$. Equation (11) is valid when the radial energy \mathcal{E}_ρ is less than the potential barrier height $V(r, z)$, a range required for the electron field emission.

Solution of transmission equation

We know,

$$T(\mathcal{E}_\rho) = \exp \left[-2 \int_{\rho_1}^{\rho_2} \beta_1 \left[\eta_1 z \left\{ \log_e \left(\frac{1}{\rho} \right) \right\} - \mathcal{E}_\rho \right]^{\frac{1}{2}} d\rho \right]$$

In which ,

$$2 \frac{m_e V_0 a^2}{\hbar^2} = \beta_1,$$

And
$$\frac{E_1 \rho}{V_0} = \mathcal{E}_\rho ,$$

And
$$\frac{eE_0}{2\pi \mathcal{E}_r V_0 \left[\log_e \left\{ 4 \frac{(l^2 - z^2)}{a^2} \right\} - 2 \right]} = \eta_1$$

Now taking integration in transmission probability term as I and solving it further we get,

$$T(\mathcal{E}_\rho) = \exp[-2\beta_1 I]$$

$$I = \int_{\rho_1}^{\rho_2} 1. \left[\eta_1 z \left\{ \log_e \left(\frac{1}{\rho} \right) \right\} - \mathcal{E}_\rho \right]^{\frac{1}{2}} d\rho$$

using ILATE form of integration, we get,

$$I = \rho \left[\eta_1 z \left\{ \log_e \left(\frac{1}{\rho} \right) \right\} - \mathcal{E}_\rho \right]^{\frac{1}{2}} - \int \frac{-\eta_1 z \rho \cdot \rho \left(\frac{1}{\rho^2} \right)}{2 \left[\eta_1 z \left\{ \log_e \left(\frac{1}{\rho} \right) \right\} - \mathcal{E}_\rho \right]^{\frac{1}{2}}} d\rho$$

$$I = \rho \left[\eta_1 z \left\{ \log_e \left(\frac{1}{\rho} \right) \right\} - \varepsilon_\rho \right]^{\frac{1}{2}} + \int \frac{\eta_1 z}{2 \left[\eta_1 z \left\{ \log_e \left(\frac{1}{\rho} \right) \right\} - \varepsilon_\rho \right]^{\frac{1}{2}}} d\rho$$

Now by letting the 2nd integration as I2

I2=

$$I2 = \int \frac{\eta_1 z}{2 \left[\eta_1 z \left\{ \log_e \left(\frac{1}{\rho} \right) \right\} - \varepsilon_\rho \right]^{\frac{1}{2}}} d\rho$$

Let some other variable u as,

$$u = \left[\eta_1 z \left\{ \log_e \left(\frac{1}{\rho} \right) \right\} - \varepsilon_\rho \right]^{\frac{1}{2}}$$

So

$$u^2 = \left[\eta_1 z \left\{ \log_e \left(\frac{1}{\rho} \right) \right\} - \varepsilon_\rho \right]$$

$$\frac{1}{\eta_1 z} [u^2 + \varepsilon_\rho] = \log \left(\frac{1}{\rho} \right)$$

$$\rho = \exp \left[\frac{-1}{\eta_1 z} [u^2 + \varepsilon_\rho] \right]$$

Hence I2 become,

$$I2 = - \int \exp \left[\frac{-1}{\eta_1 z} [u^2 + \varepsilon_\rho] \right] du$$

$$I2 = - \int \exp \left[\frac{-1}{\eta_1 z} [u^2] \right] \exp \left[\frac{-1}{\eta_1} \varepsilon_\rho \right] du$$

$$I2 = - \exp \left[\frac{-1}{\eta_1 z} \varepsilon_\rho \right] \int \exp \left[\frac{-1}{\eta_1} [u^2] \right] du$$

Let another variable t as

$$t = \sqrt{\frac{u^2}{\eta_1 z}}$$

$$t^2 = \frac{u^2}{\eta_1 z}$$

$$t dt = \frac{u du}{n_1 z}$$

$$du = \sqrt{n_1 z} dt$$

So
$$I2 = -\exp\left[\frac{-1}{\eta_1 z} \mathcal{E}_\rho\right] \int \exp[-t^2] \sqrt{n_1 z} dt$$

Now since we know that value of below integration is

$$I3 = \int \exp[-t^2] dt = \sqrt{\pi} \frac{1}{2} \text{erf}(t)$$

So
$$I2 = -\exp\left[\frac{-1}{\eta_1 z} \mathcal{E}_\rho\right] \sqrt{n_1 z} \text{erf}\left[\frac{[\eta_1 z \{\log_e(\frac{1}{\rho})\} - \mathcal{E}_\rho]^{\frac{1}{2}}}{\sqrt{\eta_1 z}}\right]$$

Now put I and I2 in transmission equation

$$T(\mathcal{E}_\rho) = \exp\left[-2\beta_1 \left[\rho \left[\eta_1 z \left\{\log_e\left(\frac{1}{\rho}\right)\right\} - \mathcal{E}_\rho\right]^{\frac{1}{2}} - \exp\left[\frac{-1}{\eta_1 z} \mathcal{E}_\rho\right] \sqrt{n_1 z} \text{erf}\left[\frac{[\eta_1 z \{\log_e(\frac{1}{\rho})\} - \mathcal{E}_\rho]^{\frac{1}{2}}}{\sqrt{\eta_1 z}}\right]\right]\right]$$

Is our required equation

2.6 FIELD EMISSION CURRENT DENSITY AT A TEMPERATURE OF 0K

From the free electron model for a metal and neglecting the electrons with emitted due to temperature (i.e., taking into account at absolute zero temperatures), current density is expressed as

$$J(\mathcal{E}_\rho, \mathcal{E}_f, T(\mathcal{E}_\rho)) = \frac{4\pi m_e e}{h^3} V_0^2 \Phi((\mathcal{E}_\rho, \mathcal{E}_f, T(\mathcal{E}_\rho))),$$

Where,

$$\Phi(\mathcal{E}_f, \mathcal{E}_\rho, T(\mathcal{E}_\rho)) = \int_0^{\mathcal{E}_f} (\mathcal{E}_f - \mathcal{E}_\rho) T(\mathcal{E}_\rho) d\mathcal{E}_\rho$$

(12)

Here $\Phi(\mathcal{E}_f, \mathcal{E}_\rho, T(\mathcal{E}_\rho))$ is the field emission current density function.

Chapter 3

3.1 RESULTS AND DISCUSSIONS

Here, we have used typical carbon nanotube (CNT) parameters, for evaluating potential energy $V(r,z)$, tunneling probability T and the field emission current density function ϕ , in our calculation. Fig. 3 show the deviation in potential energy $V(r, z)$ (ergs). Working with Eq. (8), with radial distance r (in nanometers) for different values of $z=0.90\mu\text{m}$, $0.80\mu\text{m}$, $0.70\mu\text{m}$ and $0.50\mu\text{m}$ for a fixed value of radius of cylindrical metallic CNT, i.e., $a=0.4\text{ nm}$. From figure 3 we can say that the potential energy of an electron decreases with r . It also shows the increase in potential energy with z (where z is the coordinate along the axis of cylindrical CNT) i.e., the electron has greater potential energy at the centre of cylindrical metallic CNT. The potential energy for spherical CNT tip is given by Ze^2/r , where Z is the charged state and e is the electronic charge. In our case $Z=1$. The plot for potential energy (for spherical tip) is well known and is given by Sodha *et al.*

Using Eq.11, we have plotted in figure 4, the variation of transmission probability of an electron with the normalized radial energy ϵ_p for different values of z , e.g., $z = 0.90\mu\text{m}$, $0.80\mu\text{m}$, $0.70\mu\text{m}$ and $0.50\mu\text{m}$ and for the following typical parameters: radius of CNT $a=0.4\text{nm}$, length of CNT $l=1\mu\text{m}$, applied electric field between the plates $E_0=3.9\text{V}/\mu\text{m}$, $\rho_1=0.5(=r_1/a)$, $\rho_2=0.95(=r_2/a)$ as limits of integration. It can be seen from figure 4 that for a particular value of the normalized radial energy, transmission probability decreases with z and the variation of transmission probability is validated by the variation of potential energy with z because as $V(r)$ increases with z , transmission probability must decrease with z .

In figure 5, we have plotted the variation of Transmission probability of electrons with the normalized radial energy for different CNT radius e.g., $a=0.4\text{nm}$, 0.6nm and 0.8nm , for a fixed value of $z=0.90\mu\text{m}$. From figure 5 we can

say that with the increase in value of CNT radius a , the Transmission probability decreases. Similar variation (cf. figure 9) has been observed for spherical CNT tip. The dependence of the current density function Φ on the normalized Fermi energy ε_f corresponding to the dependence of $T(\varepsilon_\rho)$ on ε_ρ for the same parameters as fig. 4 and 5 is shown in fig. 6 and 7. The trend of the dependence of current density function Φ on ε_f follows the trend of dependence of $T(\varepsilon_\rho)$ on ε_ρ as displayed by figures 4 and 5. Figure 6 shows the increase in current density function Φ with decrease of z . figure 7 shows that the current density function Φ decreases with the CNT radius for a fixed value of z . A Similar type of variation is seen for spherical CNT tip (cf. figure 11).

Zhou *et al.* have concluded that by increasing the carbon nanotube' (CNTs) radius at low emission currents, all field emission properties deteriorate. Moreover, Xu *et al.* in their experimental observations reveal that the field enhancement factor decreases with tube radius. Hence our theoretical results (cf. figures 7&11) are in accordance with the experimental observations of Zhou *et al.* and Xu *et al.*

We have also compared the current density function of spherical CNT tip with the side walls of CNT (i.e., along the length of cylindrical CNT) [cf. figures 6 &10]. It can be seen that the current density function at the tip of CNT is substantially larger than along the length of the CNT. Shang *et al.* concluded the emission dominance due to CNT tip. These theoretical calculations suggest that the enhanced field emission from the tip of the CNT could be attributed to the lowered potential barrier at the tip of nanotube compared with the side walls of carbon nanotube. Another possible reason for the enhanced field emission from the tip of the CNT is due to accumulation of charges at the CNT tip.

Recently, Sharma and Tewari have examined the influence of plasma parameters on growth and field emission properties of spherical carbon nanotube tip and cylindrical surfaces and found out that due to increase in the number of CNT's per unit volume and plasma parameters, the radius of the

spherical CNT tip and cylindrical surfaces decreases, and consequently the field emission factor for both increases. It is also reported that the field emission from spherical tip is larger than from cylindrical CNT surfaces.

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CHAPTER 4

4.1 Variation of potential energy $V(r, z)$ of an electron in ergs with radial distance r for $a=0.4\text{nm}$ and for different values of z i.e., $0.50\mu\text{m}$, $0.70\mu\text{m}$, $0.80\mu\text{m}$ and $0.90\mu\text{m}$, respectively.

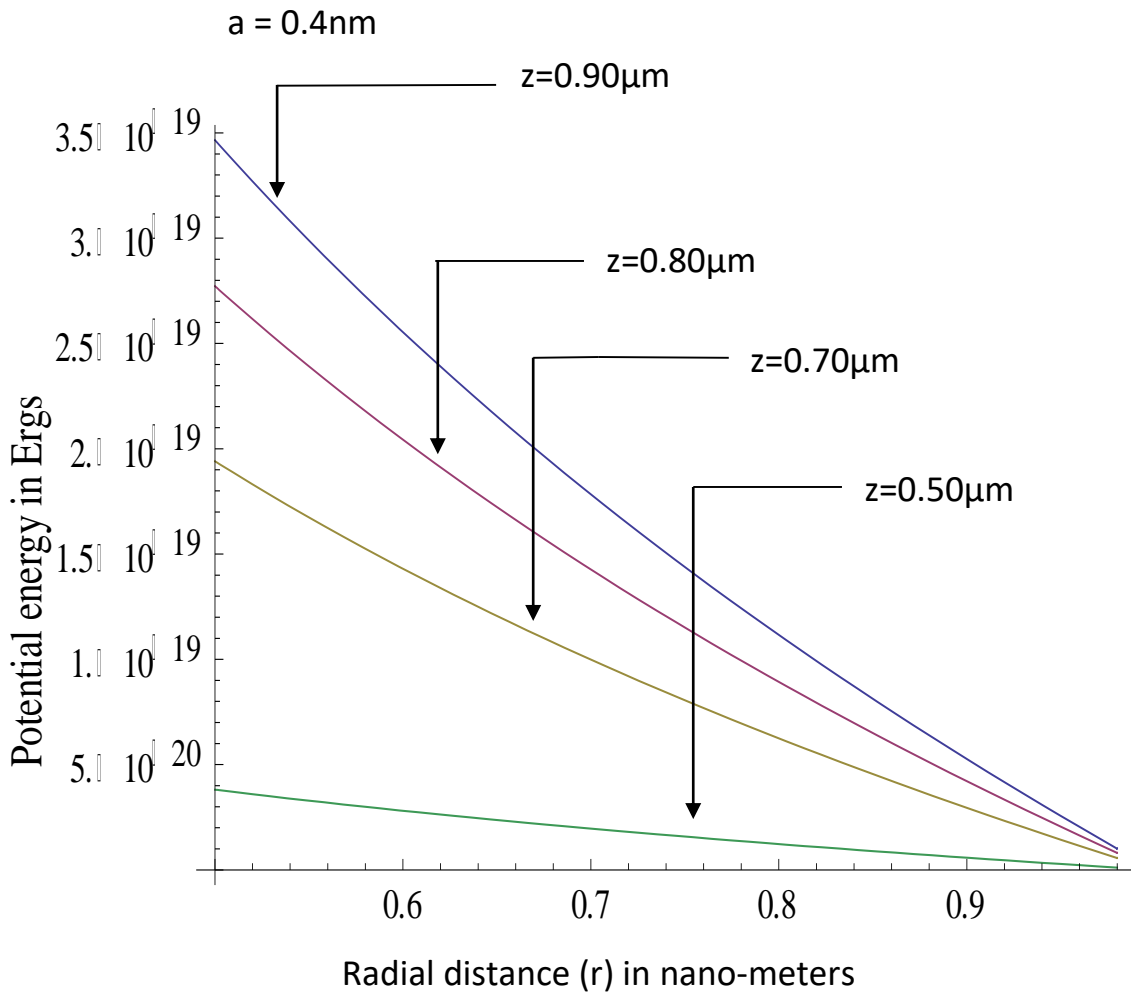


Figure 3.

4.2 Transmission probability $T(\mathcal{E}_\rho)$ as a function of the normalized radial energy \mathcal{E}_ρ along the length of CNT for different values of z i.e., $z= 0.50\mu\text{m}$, $0.70\mu\text{m}$, $0.80\mu\text{m}$ and for a fixed tube radius $a=0.4\text{nm}$.

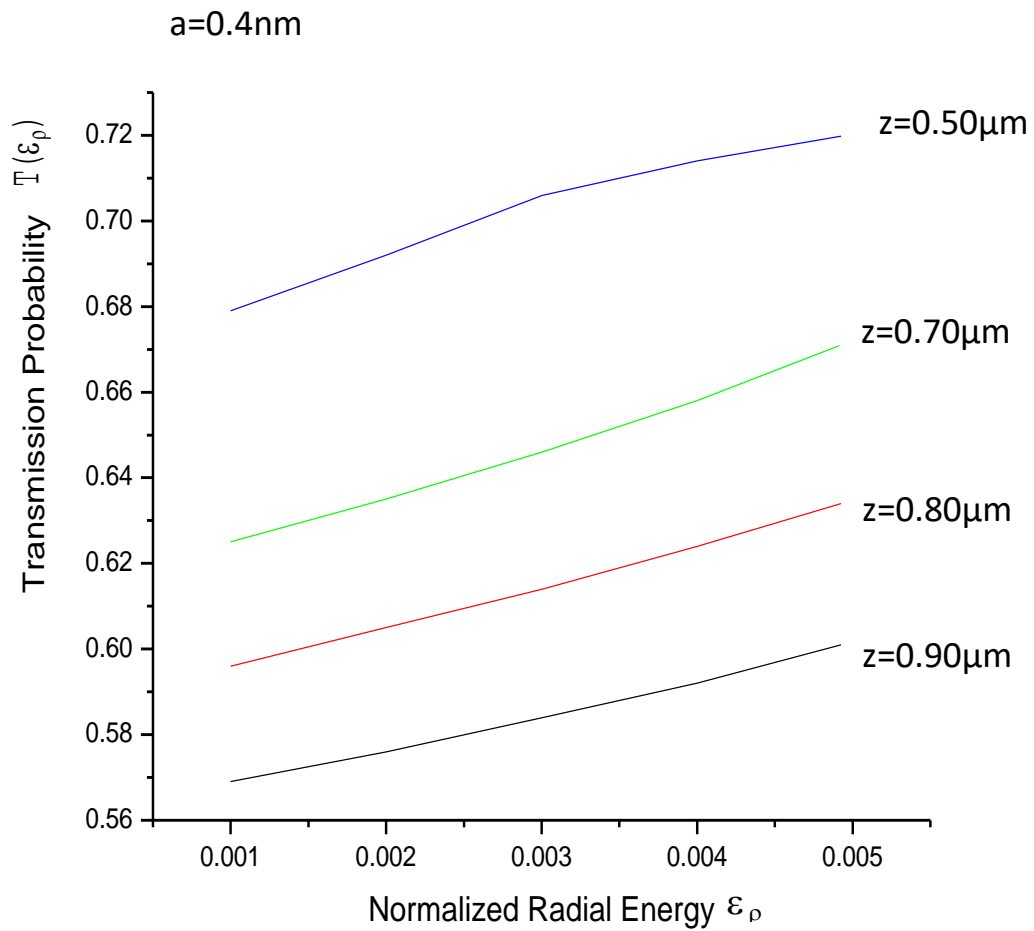


Figure. 4

4.3 Transmission probability $T(\mathcal{E}_\rho)$ as a function of the normalized radial energy \mathcal{E}_ρ along the length of the CNT for different values of a (radius of CNT), i.e., $a=0.4\text{nm}$, 0.6nm and 0.8nm and for a fixed value of $z = 0.90\mu\text{m}$.

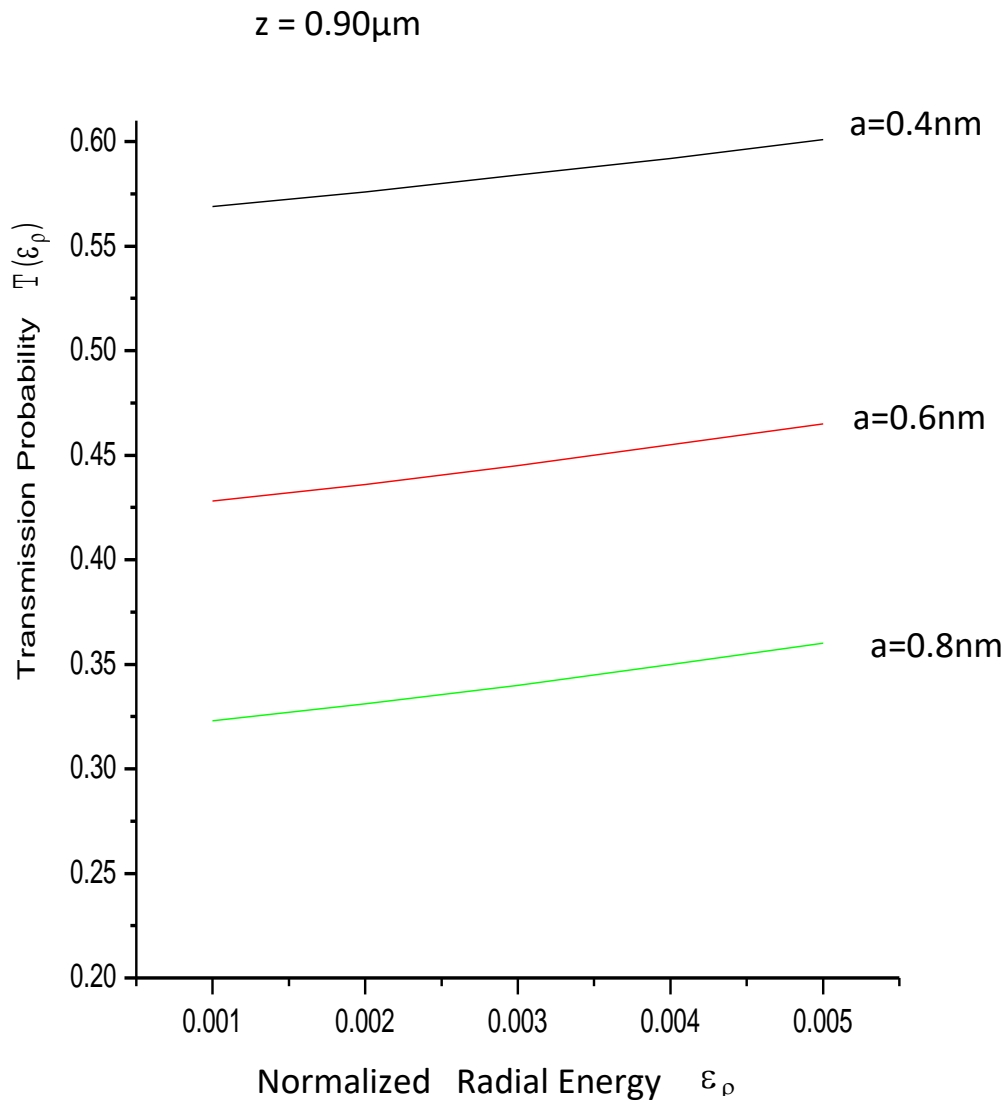


Figure 5.

4.4 Electric field emission current density function Φ as a function of the normalized Fermi energy \mathcal{E}_f along the length of the CNT for different values of z , i.e., $0.50\mu\text{m}$, $0.70\mu\text{m}$, $0.80\mu\text{m}$, $0.90\mu\text{m}$, and for a fixed tube radius $a=0.4\text{nm}$.

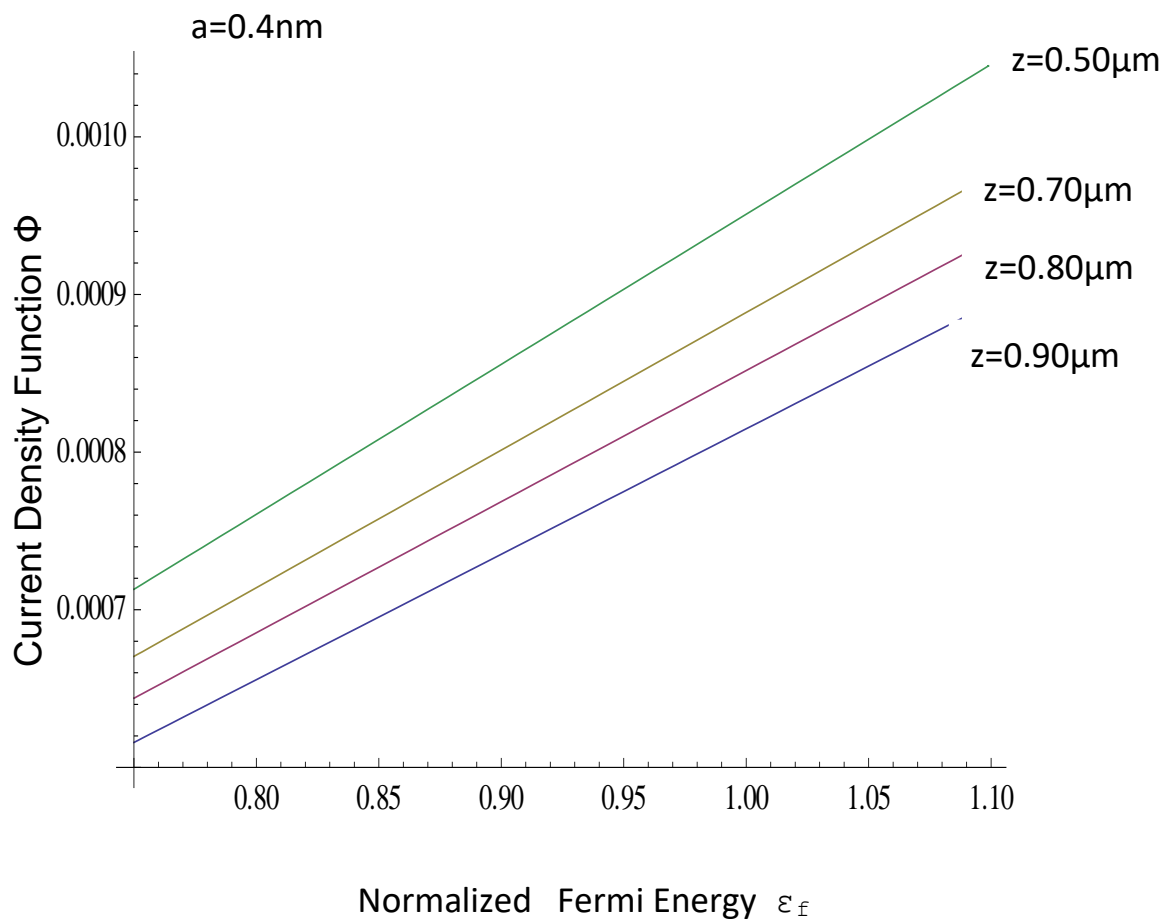


Figure 6.

4.5 Electric field emission current density function Φ as a function of the normalized Fermi energy \mathcal{E}_f along the length of the CNT for different values of tube radius a i.e., 0.4nm, 0.6nm and 0.8 nm, and for a fixed value of $z = 0.90\mu\text{m}$.

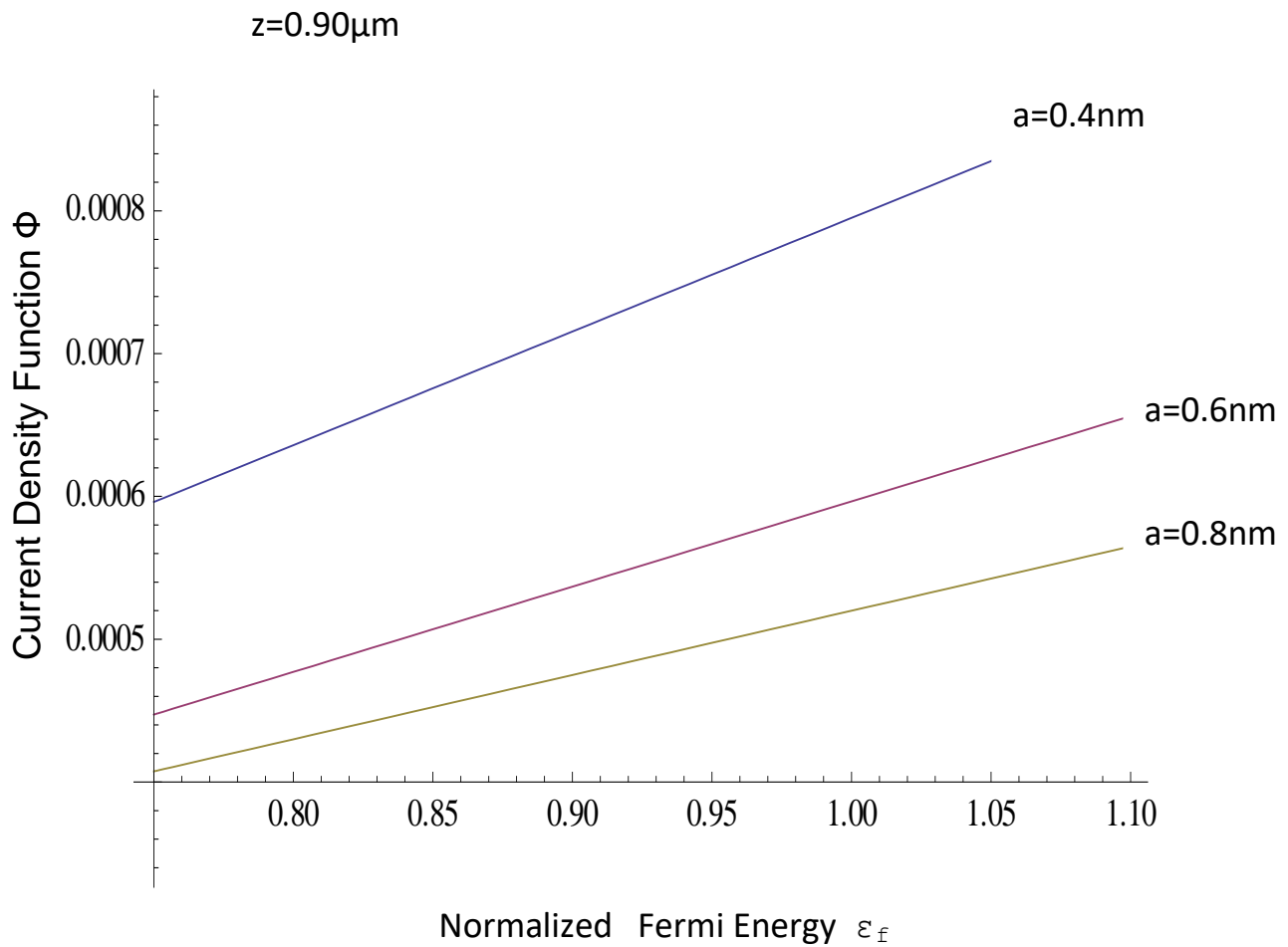


Figure 7.

4.6 Transmission probability $T(\mathcal{E}_\rho)$ as a function of the normalized radial energy ε_ρ for the spherical tip of radius $a = 0.4\text{nm}$

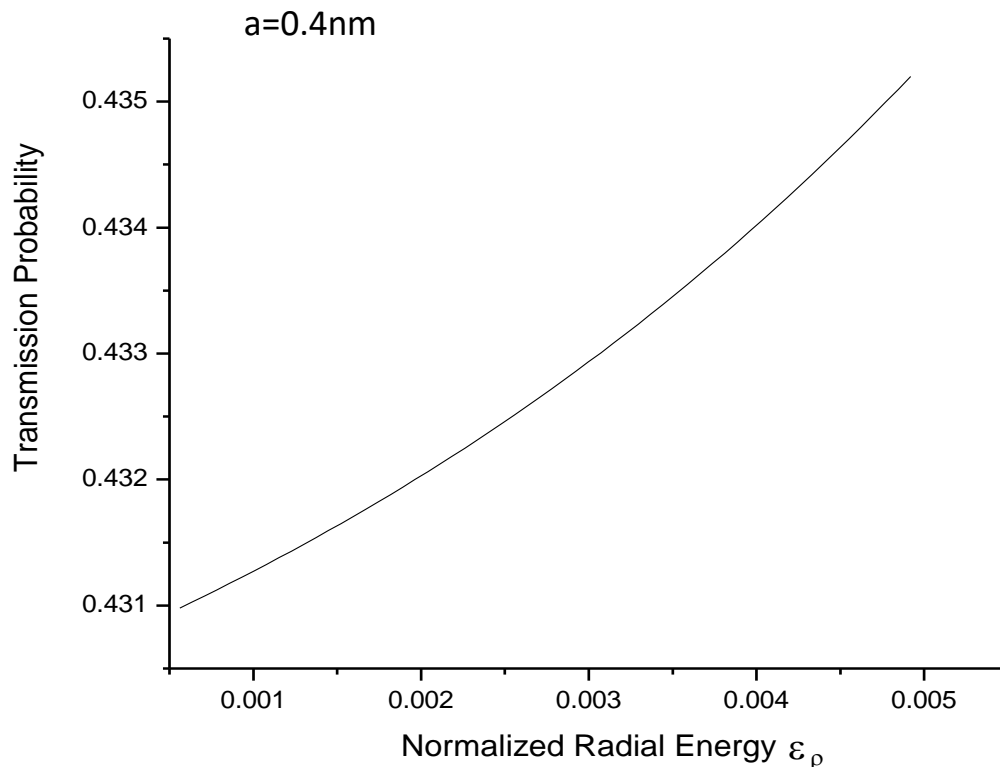


Figure 8.

4.7 Transmission probability $T(\epsilon_\rho)$, function of the normalized radial energy ϵ_ρ for the distinct values of spherical tip radius $a = 0.4\text{nm}$, 0.6nm and 0.8 nm .

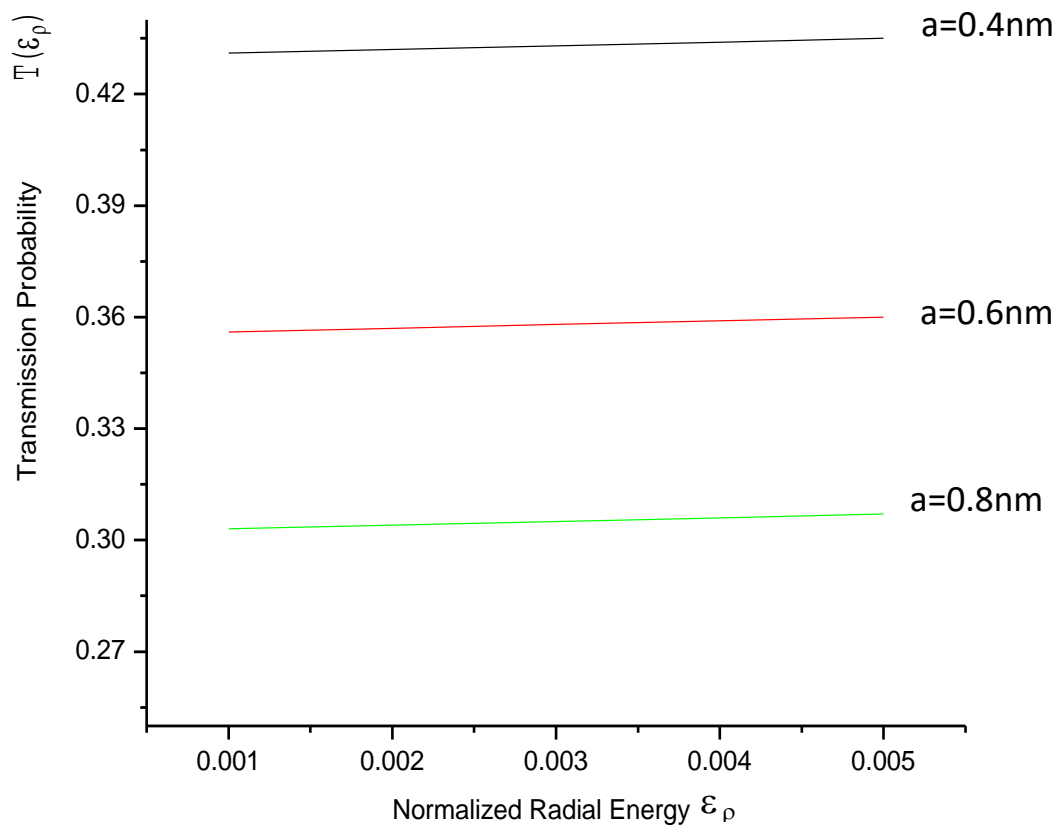


Figure 9.

4.8 Electric Field emission current density function Φ as a variation of the normalized Fermi energy ε_f for the spherical tip of radius $a=0.4\text{nm}$.

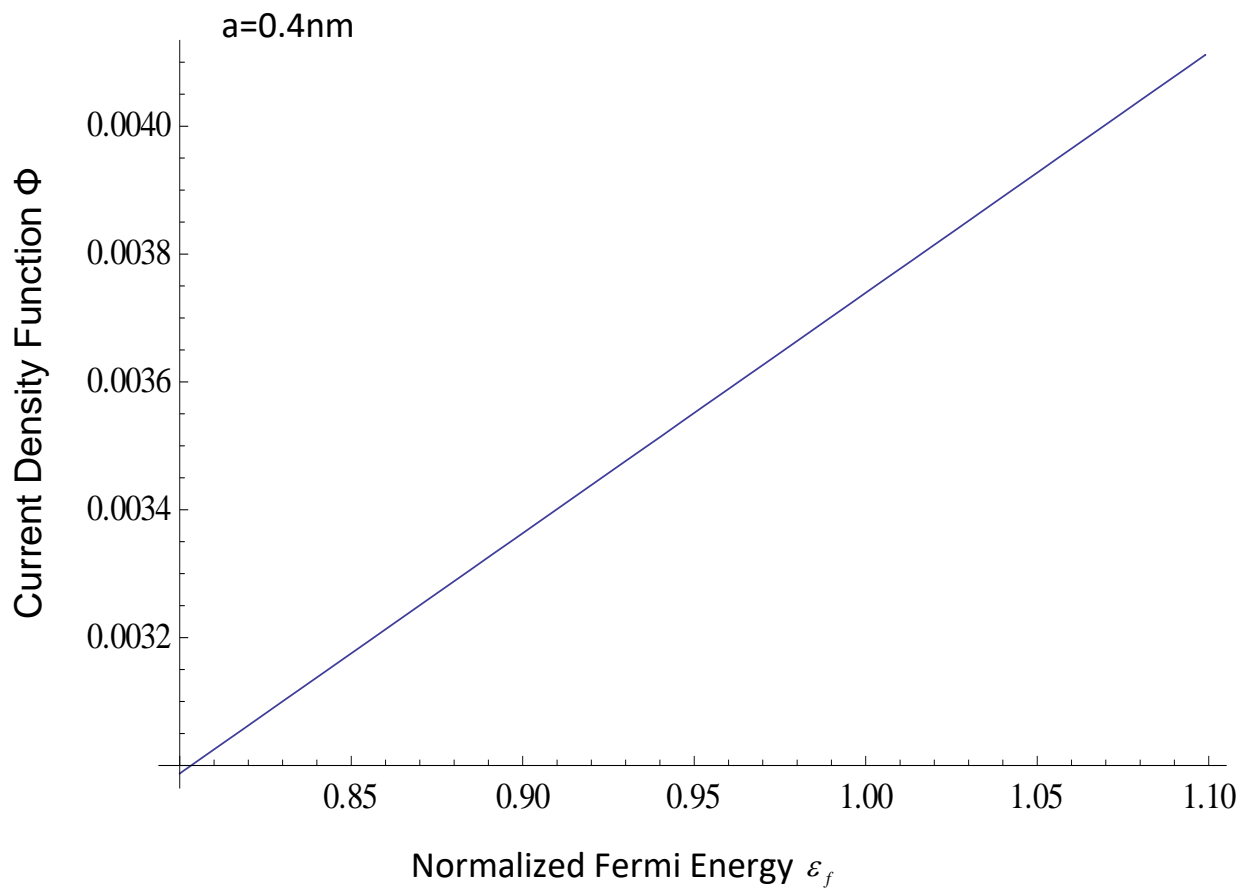


Figure 10.

4.9 Electric Field emission current density function Φ as a variation of the normalized Fermi energy ε_f for the different values of spherical tip radius $a=0.4\text{nm}$, 0.6nm and 0.8 nm .

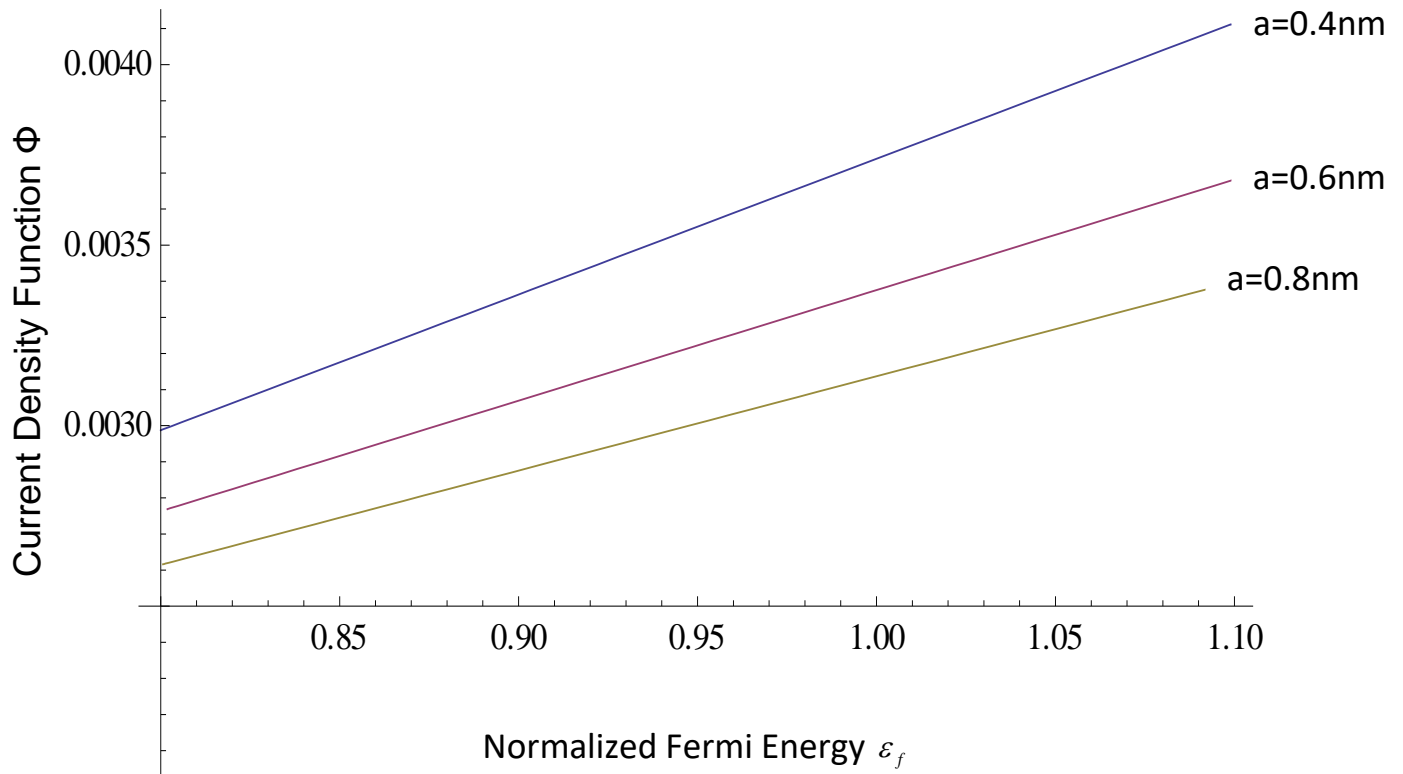


Figure 11.

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