EVENT-TRIGGERED CONTROL SYSTEM IN THE PRESENCE OF NON-LINEARITY AND OUTPUT QUANTIZATION

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CERTIFICATE

I, Tuhina Bhandari, Roll No. 2K19/C&I/18, a student of M.Tech. Control and Instrumentation (C&I), hereby declare that the dissertation/project titled **"EVENT-TRIGGERED CONTROL SYSTEM IN THE PRESENCE OF NON-LINEARITY AND OUTPUT QUANTIZATION"** under the supervision of Prof. Anup Kumar Mandpura of Electrical Engineering Department, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Technology and has not been submitted elsewhere for the award of any Degree.

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ABSTRACT

Event-triggered control is used for reduction in communication over the feedback link without compromising the stability of the system. It is a recent approach in control theory which aims at reduction of the communication load, by exchanging information only when a certain threshold event condition has been met, over the feedback link. This method of control achieves reduction of the communication overhead, by exchanging feedback information only when a certain threshold event condition has been met. The non-linearity due to integrator windup, causes performance degradation and can lead to an unstable system. Therefore, in this paper we study an event-triggered control system with continuous input along with saturation non-linearity and dead-zone non-linearity. The results with various input signals demonstrate that event triggered system achieves desired performance in the presence of non-linearities and output quantization. Stable operation at large values of quantization steps demonstrates that the desired system performance can also be achieved with fewer bits. The results are illustrated by simulations in MATLAB/Simulink.

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NOTATIONS

Throughout all the chapters in the thesis, a scalar is defined by $(x \in \mathcal{R})$, vector is defined by $(\boldsymbol{x} \in \mathcal{R}^n)$, a matrix is defined as $(A \in \mathcal{R}^{n\times n})$ and a signal at time $t \in \mathcal{R}$ by $\boldsymbol{x}(t)$, wherein the preliminary value of the signal at time $t = 0$ is given by x_0. Furthermore, identity matrix of order $n \times n$ is depicted as I_n.

Additionally, the $\boldsymbol{i}$-th element of vector $\boldsymbol{x}$ is depicted by $\boldsymbol{x}_{(\boldsymbol{i})}$, $\boldsymbol{A}_{(\boldsymbol{i})}$ denotes the general $\boldsymbol{i}$ th row of matrix $\boldsymbol{A}$, and transpose of the matrices and vectors is given by $()^{\boldsymbol{T}}$.
Absolute value of a scalar quantity is given by $|\boldsymbol{x}|$, the Euclidean vector norm is given by $||\boldsymbol{x}||$ and the norm of the matrix is given by $||\boldsymbol{A}||$.

Furthermore, any positive definite matrix $\boldsymbol{A}$ is defined as $\boldsymbol{A} > 0$ $(\boldsymbol{A} \geq 0)$ and a negative definite matrix $\boldsymbol{A}$ is defined as $\boldsymbol{A} < 0$ $(\boldsymbol{A} \leq 0)$. Additionally, the sum of the diagonal matrix i.e., trace of the matrix $\boldsymbol{A}$ is depicted by $trace(\boldsymbol{A})$.

The ellipsoids or the elliptical sets denoted by $\varepsilon(\boldsymbol{P}, \eta)$ which is defined by $x^T\boldsymbol{P}x \leq \eta^{-1}$ wherein the term $\boldsymbol{P} > \boldsymbol{0}$.

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

In recent times, almost all the control systems are coded digitally using a digital platform, termed a computer-based control system, wherein the controller used in a computer-based control system is software-based which is used to manipulate the plant present in the control system in a way its desired. Prior to the development of such a system, mostly analog controllers were used. Herein, the analog controllers continuously keep a check on the parameters of the plant while keeping constant communication within the closed-loop control system.

The main disadvantage of a closed-loop control system is that giving continuous feedback is integral in the way communication occurs between the processor and the sensors, and the computation and execution of the controller by the actuator happens at regular time intervals. Typically, continuous feedback is provided to the system disregarding the fact whether the changes in the plant output requires an adjustment in the parameters or not via the feedback. Furthermore, there may be continuous implementation of control action by the controller for maintenance of the performance and stability of the plant, even when the plant does not have any requirement to do so. For instance, in a zero-disturbance plant regulation system, a continuous feedback closed-loop system would continue updating the controller parameters even after the plant reaches steady state. This might become a critical problem in systems wherein limited energy is available to the processors and sensors in the form of batteries. Hence, an optimum usage of resources is of utmost importance.

Additionally, continuous feedback is quite problematic when used for remote locations i.e., when the processors and sensors are topographically distributed. In such a case, the exchange of information occurs with the help of wireless control system. Beneficially, using a wireless control system offers benefits such as reduced cost of wiring, reduction in power and/or energy requirements and being extremely reliable. However, wireless control system also offers disadvantages, for instance packet dropouts and transmission delays.

A basic approach to deal with the disadvantages mentioned regarding wireless control system and to prevent or minimize wastage of energy is to lower the amount of data transmitted continuously via feedback. Notably, this can be achieved when data is transmitted only when a certain event has occurred i.e., during event-triggering of a control system, which occurs in an aperiodic manner.

The elementary idea in event-triggered control system is that data is transferred to the controller only when it is required. Fundamentally it may be observed that an event-triggered control mechanism comprises the natural methodology for the control of particular systems. An example of a simple event-triggered control system may be that of a plant operated by the control action of on-off relay. In such a system, value of the control action is not altered unless the error surpasses a threshold value. Furthermore, event-triggered control system may also be used in systems where costly control action is used. An example for this may be, volumetric production control in a chemical processing plant. Herein, the rate of production cannot be changed often on a regular basis and should be prevented. Hence, an event-trigger may be applied in such a system, wherein the rate of production will not be changed unless the volume of production advances either towards the upper limit or towards the lower limit of a tank present in the chemical processing plant.

Mathematically, an exchange of information in an event-triggered control system does not take place until an event-triggering condition is reached. The event-triggering condition has varied forms and changes according to the nature of the control system. In an example, in case of a power grid, a fault occurring on the said power grid may be regarded as an event, which will eventually trigger a response in the control system. In general terms, in numerous systems, an event may occur just as a control error surpasses a certain event-triggering threshold. In this context, an event detector must be created so that an event may be interrupted and the information be released.

As stated previously, besides the efficient utilization of computation, communication and energy and/or power resources, event-triggered control system leads to less traffic in data transmission, compared to classical closed-loop control system, computer-based control system or wireless control system.

1.2 EVENT-TRIGGERED CONTROL SYSTEM

Control systems of modern control theory are usually realized digitally with the use of a computer in order to implement the controller. Typically, sample and hold devices provide the user interface with the plant which is analogous in nature. In the classical modern control theory approach, data is transmitted between the components of the system, such as for example sensor, plant and actuator, periodically, irrespective of the fact that whether or not the changes observed in the plant output requires a new controller output or not. However, the exchange or transfer of information occurring periodically offers unnecessary demands in communication that might become critical in various systems, such as networked or distributed systems, wherein optimum usage of the capacity of communication network is of utmost importance.

Notably, an alternative approach, called the *event-triggered control system* is offered in place of continuous time-driven closed-loop system. Herein, the controller will generate a control action only when changes are observed in the plant output, only when the observed change surpasses a pre-defined threshold value. Event-triggered control systems have been an active research area for the past decade. Herein, the fundamental feature of event-triggered based controllers is that they can provide the same functioning as a classical control theory approach with the additional benefit of reduction of transmission information between the plant of the control system and the controller. The significance of this additional benefit is given proof through various applications such as control systems operated by batteries and wireless transmission used between the controller and the plant. Hence, this kind of system often possess a limited amount of energy and memory.

In a situation where event-triggering is used, the control system determines when to update the output of the control, based on an event-triggering condition which occurs real time on the signal to be measured. This often leads to communication occurring in an aperiodic manner between the controller and the plant, and takes place on a purely requirement basis. Furthermore, the components of the control system do not communicate unless and until an event-triggering threshold condition is reached. In such a case, this event-triggering condition may be defined in various forms and changes according to the nature of the control system.

Generally, the event-triggering control mechanism is fabricated to update the actuators of the control system in case the measured error surpasses the pre-defined threshold value. Herein, the measured error is defined as the difference between the current value of the output and a recent value of the output. Furthermore, the threshold value may be a function of output of the system, or a constant, or a combination of the two. Henceforth, an event generator is required to provide the event-triggering condition, which should ideally be connected at the output of the plant.

There are two important facets of an event-triggering control system:

1) the design of the event-triggered control system should partially resemble the performance of a closed-loop control system, and

2) should make sure that while executing and triggering events successively, enough time separation must be present to avoid sampling in excess.

The second point is quite important to any control system using event-triggering mechanism. The main motivation to use event-triggering mechanism is to reduce the redundant communication happening between the controller and the plant. However, the event-triggering condition should be designed in such a way so as to prevent excessive triggering, as the time of execution of the control action is dependent on the occurrence of every new event. Therefore, the control system should be analyzed properly, and the event-triggering mechanism must be adjusted accordingly.

1.3 LITERATURE REVIEW

In a wired closed loop control system for stable operation, continuous feedback is provided. The studies conducted in [1] [2] demonstrated that the stable operation can be achieved through limited feedback. The studies in [3] [4] on event-triggered control system aim to reduce the communication overhead over the network required for feedback while maintaining a stable operation of the plant. Furthermore, by reducing the information exchange to the minimum communication which is necessary to ensure the required system performance, an overload on the communication network can be avoided. As compared to wired systems, wireless control systems provide advantages such as easy installation and maintenance and/or a high degree of flexibility. Furthermore, the shortcomings experienced in Networked Control Systems are removed in event-triggered control system and allows more efficient usage of bandwidth when there is limited bandwidth network.

The authors in [5] [6] have studied an event-triggered control system using a proportional controller. The assumption of a proportional controller enables the study to be focused on the stability and the communication properties of the event-triggered control system. Furthermore, an event-triggered control system is not always linear in nature. In this context, event-triggered control system with a Proportional-Integral controller (PI controller) model developed in [7] which achieves reduction of communication and computational effort with only a slight degradation in performance. Furthermore, the event-triggered control system with PI controller provides a framework for better analysis of stationary behavior of the event-triggered control system. Therefore, in this paper we have also considered a PI controller.

Additionally, the actuators used in the plant of the wired closed loop control system saturates because of limits, such as for example the maximum limit and the minimum limit which acts as a physical constraint. Although, the occurrence of saturation non-linearity in actuators is a common phenomenon noticeable in practical wired closed loop system, its effect on event-triggered control system was observed recently in linear systems [8]. Herein, upon simulation of saturation non-linearity in event-triggered control loop system, it was observed that the presence saturation non-linearity led to drastic results as compared to the ideal event -triggered control loop with only the PI controller added in the closed loop control system.

Furthermore, dead zone non-linearity is another such non-linearity found commonly in practical wired closed loop control system and has a certain influence on the performance of the wired closed loop control system. Additionally, compensation of dead zone non-linearity is still an outstanding issue [9]. However, none of the studies have considered saturation non-linearity along with dead-zone linearity along with their compensation in event-triggered control system.

Recently, event-triggered control system modelling was studied with quantization in [10]. The authors considered an event-triggered control, wherein the linear system is quantized. However, the authors do not take into account the non-linearity due to saturation in the actuator. In this paper, we consider a continuous-time event-triggered control system in presence of saturation non-linearity and output quantizer. The results demonstrate the desired performance can be achieved in the presence of actuator saturation and output quantization.

1.4 OBJECTIVES OF PRESENT WORK

The focus of the present work is

1) to develop an event-triggering condition to satisfactorily provide results of a control system,

2) to develop a simulation model using MATLAB-Simulink to observe the functioning and response of an event-triggered control system,

3) to subject the control system to non-linearities in order to simulate a real-time plant and controller

4) to observe the results obtained during simulation of the control system when a quantizer is connected at the output.

CHAPTER 2

CONTROLLER IN PRESENCE OF NON-LINEARITIES

2.1 INTRODUCTION

This chapter observes a continuous-time control system when subjected to non-linearities. Herein, the non-linearities introduced in the control system are saturation non-linearity and dead zone non-linearity.

2.2 SYSTEM MODEL

In this section, a suitable continuous-time control system is developed which will later on be subjected to non-linearities in the following sections. Herein, a general equation for continuous-time controller having feedback is given by

$$\dot{x}_p(t) = ax_p(t) + bu(t) + b_n n(t), \;\; x_p(0) = \; x_{p0} \tag{2.1a}$$

$$y(t) = \; x_p(t) \tag{2.1b}$$

2.3 SATURATION NON-LINEARITY

In this section, a suitable continuous-time control system representation is shown when subjected to saturation non-linearity due to the actuator present in the control system. Herein, the controller shown in the Figure 2.1 is part of an extended figure of the plant present in the control system. Therefore, general techniques for the analysis of a closed loop system are applied.

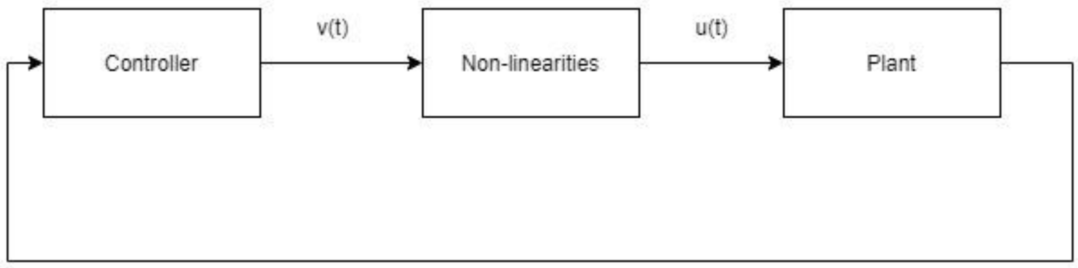

Figure 2.1 Continuous-time closed loop control system in presence of saturation non-linearity

Conventionally, saturation non-linearity present in the actuator is virtually present in all types of practical control systems. Furthermore, the saturation non-linearity acts like a constraint within which the control system works and yields maximum result. Herein, the saturation linearity may be a physical constraint or a safety constraint. For example, a valve is present in a control system. The output from this valve within the control system is operating between fully open and fully closed. In case, a controller present in the control system wants more or less flow of any fluid, more that permissible levels of the valve, saturation non-linearity occurs in the actuator. Hence, saturation non-linearity in the actuator leads to restrictions in achieving an optimum performance of the control system. Furthermore, saturation of actuators leads to stability issues as well within the feedback loop of the control system.

2.3.1 FEEDBACK CONTROLLER

The general formula for a feedback controller acting dynamically in a control system is given by

$$\begin{aligned} \dot{\boldsymbol{x}}_{\boldsymbol{c}}(t) &= \breve{\boldsymbol{A}}\,\boldsymbol{x}_{\boldsymbol{c}}(t) + \breve{\boldsymbol{B}}_{\boldsymbol{c}}\boldsymbol{y}(t) + \breve{\boldsymbol{B}}_{\boldsymbol{cr}}\boldsymbol{r}(t), \quad x_c(0) = 0 \\ \boldsymbol{v}(t) &= \breve{\boldsymbol{C}}\,\boldsymbol{x}_{\boldsymbol{c}}(t) + \breve{\boldsymbol{D}}\,\boldsymbol{y}(t) + \breve{\boldsymbol{D}}\,\boldsymbol{r}(t) \end{aligned} \tag{2.2}$$

wherein, the state of the controller $x_c \in \mathcal{R}$ and the reference signal $r \in \mathcal{R}$. $\breve{\boldsymbol{A}}$, $\breve{\boldsymbol{B}}$, $\breve{\boldsymbol{C}}$ and $\breve{\boldsymbol{D}}$ are real matrices having appropriate dimensions.

Furthermore, the equation 1.1 for controller and the augmented state vector is given by

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_p \\ \boldsymbol{x}_c \end{bmatrix} \tag{2.3}$$

Hence, the equation of the plant becomes

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\,\boldsymbol{x}(t) + \boldsymbol{B}\,\boldsymbol{u}(t) + \boldsymbol{B}\,\boldsymbol{n}(t) + \boldsymbol{B}\,\boldsymbol{r}(t), \quad x_c(0) = 0 \tag{2.4}$$

$$y(t) = \boldsymbol{C}\,x(t) \tag{2.5}$$

wherein,

$$\boldsymbol{A} = \begin{bmatrix} \breve{\boldsymbol{A}} & \boldsymbol{0} \\ \breve{\boldsymbol{B}}_{\boldsymbol{c}}\breve{\boldsymbol{C}} & \breve{\boldsymbol{A}}_{\boldsymbol{c}} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \breve{\boldsymbol{B}}_{c} \\ 0 \end{bmatrix}, \boldsymbol{B}_{\boldsymbol{n}} = \begin{bmatrix} \breve{\boldsymbol{B}}_{\mathrm{n}} \\ 0 \end{bmatrix}, \boldsymbol{B}_r = \begin{bmatrix} \boldsymbol{0} \\ \breve{\boldsymbol{B}}_{\boldsymbol{cr}} \end{bmatrix}, \quad \boldsymbol{C} = [\breve{\boldsymbol{C}} \quad \boldsymbol{0}] \tag{2.6}$$

Furthermore, by evaluating the equation (2.2) and the transition matrix given by the equation (2.3), the derived output for the controller is given by

$$\boldsymbol{v}(t) = \boldsymbol{K}x(t) + \boldsymbol{K}_r r(t) \tag{2.7}$$

wherein, $\boldsymbol{K}$ and $\boldsymbol{K}_r$ are given by the matrices

$$\boldsymbol{K} = [\breve{\boldsymbol{D}}_{\boldsymbol{c}}\breve{\boldsymbol{C}} \quad \breve{\boldsymbol{C}}_{\boldsymbol{c}}], \boldsymbol{K}_r = \breve{\boldsymbol{D}}_{\boldsymbol{cr}} \tag{2.8}$$

Herein, the variation is attainable for a plant which is linear in nature, where the plant is further controlled by a continuous-feedback controller which provides output dynamically. It further results in a general representation of a system where the parameters of the controller are encompassed in the description of the plant equation

(2.4). Thereafter, a control law which is proportional in nature results in the equation (2.7).

2.3.2 PROPORTIONAL-INTEGRAL CONTROLLER

Equating the matrices of the plant and controller to each other, and instead of using the equations 2.1 and 2.2, the following equation

$$\dot{\boldsymbol{x}}_c(t) = \boldsymbol{y}(t) - \boldsymbol{r}(t), \qquad \boldsymbol{x}_c(0) = 0$$

along with

$$\boldsymbol{v}(t) = \boldsymbol{K}_I \boldsymbol{x}_c + \boldsymbol{K}_P(\boldsymbol{y}(t) - \boldsymbol{r}(t))$$

is used in order to include PI controllers in the general equation for system representation, wherein $n_p = n_c = s$. Therefore, the matrix in equation 2.6 and the matrix in equation 2.8 is given by

$$\boldsymbol{A} = \begin{bmatrix} \breve{\boldsymbol{A}} & \boldsymbol{0} \\ \boldsymbol{C} & \boldsymbol{0} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \breve{\boldsymbol{B}} \\ 0 \end{bmatrix}, \boldsymbol{B_n} = \begin{bmatrix} \breve{\boldsymbol{B}}_{\mathbf{n}} \\ 0 \end{bmatrix}, \boldsymbol{B_n} = \begin{bmatrix} \boldsymbol{0} \\ \breve{\boldsymbol{I}}_{\boldsymbol{n_p}} \end{bmatrix}, \qquad (2.9)$$
$$\boldsymbol{C} = [\breve{\boldsymbol{C}} \quad \boldsymbol{0}], \boldsymbol{K} = [\boldsymbol{K}_P \boldsymbol{C} \quad \boldsymbol{K}_I], \boldsymbol{K_n} = -\boldsymbol{K}_P$$

2.3.3 GENERAL DEPICTION OF CONTINUOUS-TIME CLOSED LOOP CONTROL SYSTEM

The function for saturation non-linearity is given by the equation $\boldsymbol{u}(t) = \boldsymbol{sat}(\boldsymbol{v}(t))$, the following depiction of continuous-time closed loop control system is written, wherein the actuator comprises saturation non-linearity

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}\, sat(\boldsymbol{K}x(t) + \boldsymbol{K}_r\, r(t)) + \boldsymbol{B_n}\, n(t) + \boldsymbol{B_r}\, r(t),$$
$$\boldsymbol{x}(0) = \boldsymbol{x}_0, \qquad (2.10)$$
$$\boldsymbol{y}(t) = \boldsymbol{C}\, x(t)$$

Furthermore, this depiction is shown in Figure 2.2. Hereinafter, all system representations and considerations inside this chapter shall be based on the general system as depicted by equation 2.10, wherein suitable dimensions of $x \in \mathcal{R}$, $n = n_p + n_c$, $u \in \mathcal{R}$, $n \in \mathcal{R}$, $r \in \mathcal{R}$, $y \in \mathcal{R}$ and matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{B}_n, \boldsymbol{B}_r, \boldsymbol{K}, \boldsymbol{K}_r$ and $\boldsymbol{C}$ are considered.

2.4 DEAD ZONE NON-LINEARITY

As discussed in the previous sub-topic, the function for saturation non-linearity is defined the equation 2.10. In order to generalize the system further to include other non-linearities i.e., the dead zone non-linearity, the equation 2.10 is changed further using

$$\Phi\big(v(t)\big) = sat(v(t)) - v(t) \qquad (2.11)$$

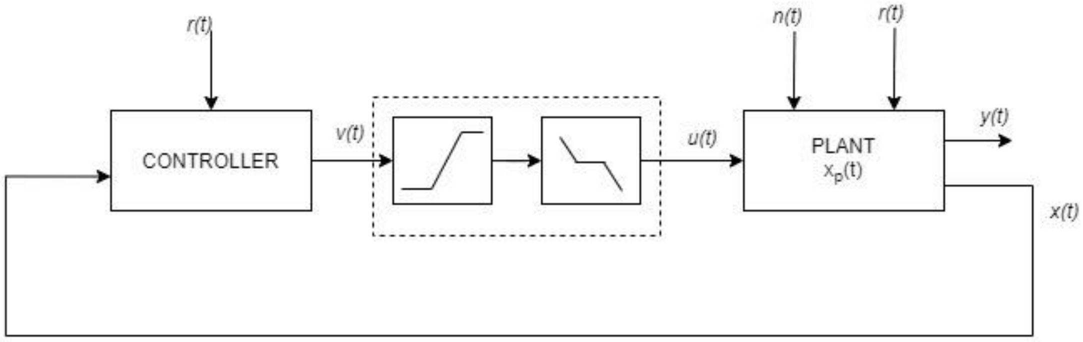

Figure 2.2 Representation of continuous time control system

with the term

$$\phi_{(i)}(v) = \phi\big(v_{(i)}\big) = \begin{cases} u_{0(i)} - v_{(i)} \; for \; v_{(i)} > u_{(i)} \\ 0 \; for \; -u_{min(i)} \leq v_{(i)} \leq u_{min(i)} \\ -u_{0(i)} - v_{(i)} \; for \; v_{(i)} < -u_{0(i)} \end{cases} \qquad (2.12)$$

wherein, $i \in \{1, \ldots, m\}$. Furthermore, the dead zone non-linearity $\phi\big(v_{(i)}\big)$ is shown in FIGURE 2.3,

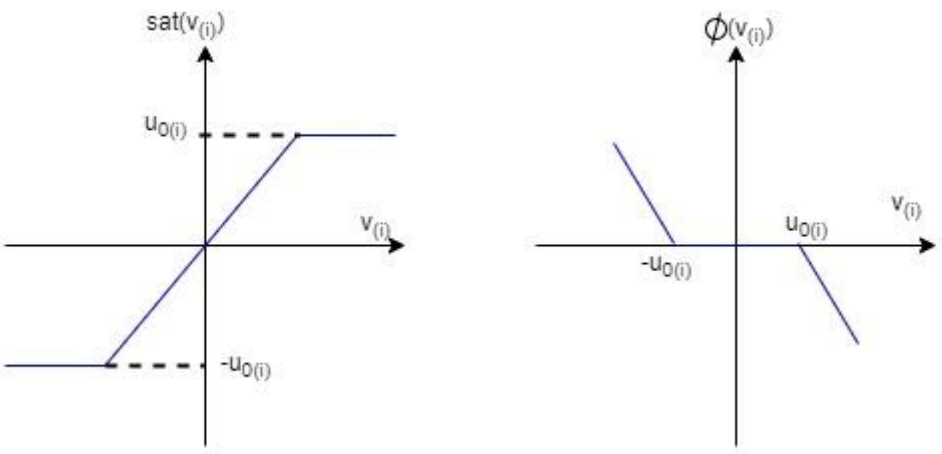

Figure 2.3 Graph of non-linearities

Thereafter, the dead zone non-linearity may be depicted using the general depicted equation 2.10 as shown

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \overline{\boldsymbol{A}}x(t) + \overline{\boldsymbol{B}}\boldsymbol{\phi}\big(\boldsymbol{K}x(t) + \boldsymbol{K_r}\boldsymbol{r}(t)\big) + \boldsymbol{B}_n n(t) \\ &+ (\boldsymbol{B}_r + \boldsymbol{B}\boldsymbol{K_r})\boldsymbol{r}(t), \\ x(0) &= x_0, \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t)\end{aligned} \tag{2.13}$$

wherein

$$\overline{\boldsymbol{A}} = \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K}, \overline{\boldsymbol{B}} = \boldsymbol{B}$$

The equation (2.13) may be used for the derivation of stability criterion for continuous time and can be extended to the derivation of stability criterion for event-triggered closed loop control system.

2.5 ANALYSIS FOR STABILITY

The equation 2.10 is further analyzed using asymptotic stability. Notably, $\boldsymbol{n}(t) = 0$ and $\boldsymbol{r}(t) = 0$ and equation from the state representation equation 2.10 becomes

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}sat\big(\boldsymbol{K}x(t)\big), \tag{2.14}$$

$$x(0) = x_0$$

or can equally be written as

$$\dot{\boldsymbol{x}}(t) = \overline{\boldsymbol{A}}x(t) + \overline{\boldsymbol{B}}\boldsymbol{\phi}\big(\boldsymbol{K}x(t)\big), \quad x(0) = x_0 \tag{2.15}$$

Furthermore, a quadratic Lyapunov function is used

$$V(x) \ = \ \boldsymbol{x^T P x}, \boldsymbol{P} = \boldsymbol{P^T} \tag{2.16}$$

in conjunction with the equations present as described above, a criterion for stability is obtained. Herein, a symmetric positive definite matrix is defined wherein $\boldsymbol{W} \in \mathcal{R}$, a positive definite diagonal matrix is defined wherein $\boldsymbol{S} \in \mathcal{R}$ and a matrix is defined wherein $\boldsymbol{Z} \in \mathcal{R}$. The symmetric positive definite matrix, the positive definite diagonal matrix and the matrix should satisfy the equation in the matrix given by

$$\begin{bmatrix} \boldsymbol{W}\overline{\boldsymbol{A}}^T + \overline{\boldsymbol{A}}\boldsymbol{W} & \overline{\boldsymbol{B}}\boldsymbol{S} - \boldsymbol{W}\boldsymbol{K}^T - \boldsymbol{Z}^T \\ \boldsymbol{S}\overline{\boldsymbol{B}}^T - \boldsymbol{K}\boldsymbol{W} - \boldsymbol{Z} & -2\boldsymbol{S} \end{bmatrix} \tag{2.17}$$

$$\begin{bmatrix} \boldsymbol{W} & \boldsymbol{Z}_{(i)}^T \\ \boldsymbol{Z}_{(i)} & \eta u_{0(i)^2} \end{bmatrix} \tag{2.18}$$

Hereinafter, if the matrices satisfy the equations 2.17 and 2.18, then an ellipsoid $\varepsilon(\boldsymbol{P}, \eta)$, with $\boldsymbol{P} = \boldsymbol{W}^{-1}$ and provides the asynptotic stability for the equations 2.14 and 2.15.

In order to prove equations 2.17 and 2.18, it is assumed that $\boldsymbol{v} = \boldsymbol{Kx}$ and $\boldsymbol{\omega} = \boldsymbol{Kx} + \boldsymbol{Gx}$ and is able to satisfy the first inequality

$$\phi(\boldsymbol{K}x)^T\boldsymbol{T}(\phi(\boldsymbol{K}x) + \boldsymbol{K}x + \boldsymbol{G}x) \leq 0 \tag{2.19}$$

Considering equation 2.16, an ellipsoid is the region which is to be calculated for stability. Furthermore, there is a second inequality which can be satisfied simultaneously while choosing an appropriate Lyapunov function present in the equation 2.16. If $\boldsymbol{W}$ is made equivalent to the inverse of $\boldsymbol{P}$ i.e., $\boldsymbol{W} = \boldsymbol{P}^{-1}$, and $\boldsymbol{Z}$ is made equivalent to the product of $\boldsymbol{G}$ and $\boldsymbol{W}$, then the first inequality given by the equation 2.19 can be rewritten in order to procure the equation 2.18.

Additionally, the derivative of Lyapunov function with respect to time needs to fulfil

$$\dot{V}(x) = \dot{x}^T P x + x^T P \dot{x} < 0$$

Moreover, by using equation 2.19, a term in ϕ which is quadratic in nature can be added according to the equation shown below

$$\dot{V}(x) \leq \dot{V}(x) - 2\phi^T \boldsymbol{T}(\phi + \boldsymbol{K}\boldsymbol{x} + \boldsymbol{G}\boldsymbol{x}) < \boldsymbol{0}$$

Herein, the equation is valid for all $\boldsymbol{x} \in \varepsilon(\boldsymbol{P}, \eta)$.

Furthermore, by using equation 2.15, the first equality can be rewritten as

$$\begin{bmatrix} x \\ \phi \end{bmatrix}^T \begin{bmatrix} \overline{\boldsymbol{A}}^T \boldsymbol{P} + \boldsymbol{P}\overline{\boldsymbol{A}} & \boldsymbol{P}\overline{\boldsymbol{B}} - \boldsymbol{K}^T \boldsymbol{T}^T - \boldsymbol{G}^T \boldsymbol{T}^T \\ \overline{\boldsymbol{B}}^T \boldsymbol{P} - \boldsymbol{T}\boldsymbol{K} - \boldsymbol{T}\boldsymbol{G} & -2\boldsymbol{T} \end{bmatrix} \begin{bmatrix} x \\ \phi \end{bmatrix} < 0$$

Additionally, non-linearities are present in the decision variables of the first inequality

$$\begin{bmatrix} x \\ \phi \end{bmatrix}^T = \begin{bmatrix} x\boldsymbol{P}\boldsymbol{P}^{-1} \\ \phi\boldsymbol{T}\boldsymbol{T}^{-1} \end{bmatrix}^T, \begin{bmatrix} x \\ \phi \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}^{-1}\boldsymbol{P}x \\ \boldsymbol{T}^{-1}\boldsymbol{T}\phi \end{bmatrix}$$

Also, assuming $\boldsymbol{W} = \boldsymbol{P}^{-1}$, $\boldsymbol{S} = \boldsymbol{T}^{-1}$ and $\boldsymbol{Z} = \boldsymbol{G}\boldsymbol{W}$, equation 2.17 is procured.

Overall, the asymptotic stability of the continuous-time closed loop wherein the actuator comprises saturation and dead zone non-linearities ensures that for any $\boldsymbol{x} \in \varepsilon(\boldsymbol{P}, \eta)$, since $\dot{V}(x) < 0$ is true for all values of $t \geq 0$.

2.6 EXEMPLARY MODEL

An exemplary model is used to explain the methods discussed in this chapter. Herein, the equations 2.1(a) and 2.1(b) are transformed into equation 2.4 and equation 2.7. An augmented state vector $x = [x_p \quad x_c]^T$ is used to modify the equations, thereby giving

$$\dot{x}(t) = \begin{bmatrix} a & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} b \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} b_n \\ 0 \end{bmatrix} r(t) + \begin{bmatrix} 0 \\ -1 \end{bmatrix} r(t), \quad x(0) = x_0,$$

$$y(t) = [1 \quad 0]\, x(t),$$

$$v(t) = [k_P \quad k_I]\, x(t) - k_P r(t)$$

Herein, the numerical values used are $a = 0.1$, $b = 1$, $bn = 0.1$, $k_P = -1.6$ and $k_I = -1$.

Furthermore, a function denoting saturation non-linearity is introduced $u(t) = sat(v(t))$, thereby modifying the above equations

$$\dot{x}(t) = \begin{bmatrix} a & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} b \\ 0 \end{bmatrix} sat([k_P \quad k_I]x(t) - k_P r(t) + \begin{bmatrix} b_n \\ 0 \end{bmatrix} r(t) + \begin{bmatrix} 0 \\ -1 \end{bmatrix} r(t),$$

$$x(0) = x_0,$$

And the following matrices are procured using equation 2.10

$$\boldsymbol{A} = \begin{bmatrix} a & 0 \\ 1 & 0 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} b \\ 0 \end{bmatrix}, \boldsymbol{K} = [k_P \quad k_I], \boldsymbol{K}_r = -k_P, \boldsymbol{B}_n = \begin{bmatrix} b_n \\ 0 \end{bmatrix}, \boldsymbol{B}_r = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

Herein, the signals for noise and reference were not included i.e., $n(t) = r(t) = 0$. Thereafter, the above equations are written as

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} a & 0 \\ 1 & 0 \end{bmatrix} x(t)\, sat\,([k_P\, k_I]x(t)) \\ x(0) &= x_0 \end{aligned} \qquad (2.20)$$

CHAPTER 3

EVENT-TRIGGERED CONTROL IN PRESENCE OF NON-LINEARITIES

The derivations from Chapter 2 are used to develop a criterion for stability control systems which are event-triggered, wherein the actuator comprises saturation non-linearity in this chapter.

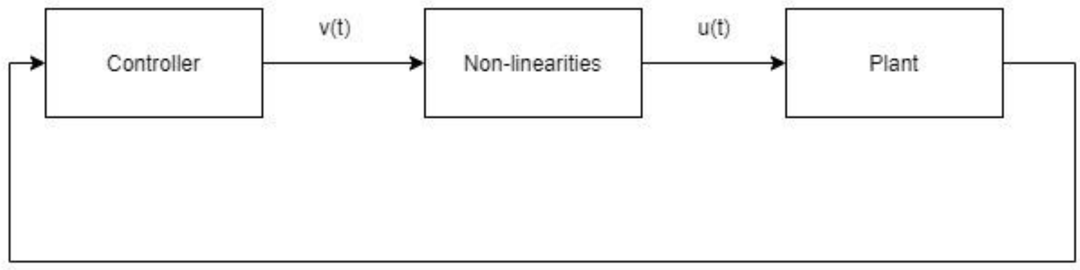

Figure 3.1 Representation of control system with non-linearities

Figure 3.1 shows the system which is to be considered. Similar to the previous chapter, firstly a suitable depiction of the control system is shown.

3.1 SYSTEM MODEL

For a control system which is event-triggered, a suitable depiction of the control system needs to be derived. Again, the dynamics of the controller will be added by usage of an extended description of the plant so that the methods derived for analysis of stability in Chapter 2 is applicable.

3.1.1 FEEDBACK CONTROLLER

The equation of the plant in consideration is given by,

$$\dot{\boldsymbol{x}}_p(t) = \breve{\boldsymbol{A}}\boldsymbol{x}_P(t) + \breve{\boldsymbol{B}}u(t) + \breve{\boldsymbol{B}}_n n(t), x_p(0) = x_{p0} \tag{3.1}$$

$$y(t) = \breve{\boldsymbol{C}}x_p(t) \tag{3.2}$$

Herein, the plant is controlled by a generalized feedback controller which is dynamic and linear in nature. However, the feedback controller keeps receiving information about the output of the plant $y(t)$ only at times when an event occurs t_k, wherein $k \in \{0,1,2,...\}$. Therefore, the description of controller during the time interval $[t_k, t_{k+1})$ is given by

$$\dot{\boldsymbol{x}}(t) = \breve{\boldsymbol{A}}_c\boldsymbol{x}_c(t) + \breve{\boldsymbol{B}}_c\boldsymbol{y}(t_k) + \breve{\boldsymbol{B}}_{cr}\boldsymbol{r}(t), \boldsymbol{x}_c(t_k) = \boldsymbol{x}_{ck} \tag{3.3}$$

$$v(t) \ = \breve{\boldsymbol{C}}_c\boldsymbol{x}_c(t) + \breve{\boldsymbol{D}}_c\boldsymbol{y}(t_k) + \breve{\boldsymbol{D}}_{cr}\boldsymbol{r}(t) \tag{3.4}$$

wherein, the state of the integrator used in the controller is $x_c \in \mathcal{R}$ is and the reference signal $r \in \mathcal{R}$. Furthermore, $\breve{\boldsymbol{A}}_c$, $\breve{\boldsymbol{B}}_c$, $\breve{\boldsymbol{B}}_{cr}, \breve{\boldsymbol{C}}_c, \breve{\boldsymbol{D}}_c$ and $\breve{\boldsymbol{D}}_{cr}$ are real matrices having appropriate dimensions.

However, this depiction of the controller is valid only during the time interval $[t_k, t_{k+1})$. Therefore, it is essential to create a model which is valid for all time $t \geq 0$ so that the methods presented herein may be utilized for analyzing the stability of the control system. Hence, an error is introduced at the output given by

$$\boldsymbol{e}(t) = \boldsymbol{y}(t_k) - \boldsymbol{y}(t) \tag{3.5}$$

with the usage of this error, the controller may be modelled with the purpose of holding for all times $t \geq 0$ as shown in the following equations

$$\dot{\boldsymbol{x}}(t) = \breve{\boldsymbol{A}}_c\boldsymbol{x}_c(t) + \breve{\boldsymbol{B}}_c\boldsymbol{y}(t) + \breve{\boldsymbol{B}}_c\boldsymbol{e}(t) + \breve{\boldsymbol{B}}_{cr}\boldsymbol{r}(t), x_c(0) = x_{c0} \tag{3.6}$$

$$v(t) \ = \breve{\boldsymbol{C}}_c\boldsymbol{x}_c(t) + \breve{\boldsymbol{D}}_c\boldsymbol{y}(t) + \breve{\boldsymbol{D}}_c\boldsymbol{e}(t) + \breve{\boldsymbol{D}}_{cr}\boldsymbol{r}(t) \tag{3.7}$$

Furthermore, an augmented vectorized state matrix

$$x = \begin{bmatrix} \boldsymbol{x}_p \\ \boldsymbol{x}_C \end{bmatrix} \tag{3.8}$$

along with equation 3.6 can be used to rewrite the equations 3.1 and 3.2 of the plant as,

$$\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t) + \boldsymbol{B}_n n(t) + \boldsymbol{B}_r r(t) + \boldsymbol{B}_e e(t), x(0) = x_0 \quad (3.9)$$

$$y(t) = \boldsymbol{C}x(t) \quad (3.10)$$

along with

$$\boldsymbol{A} = \begin{bmatrix} \breve{\boldsymbol{A}} & 0 \\ \breve{\boldsymbol{B}}_c\breve{\boldsymbol{C}} & \breve{\boldsymbol{A}} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \breve{\boldsymbol{B}} \\ 0 \end{bmatrix}, \boldsymbol{B}_n = \begin{bmatrix} \breve{\boldsymbol{B}}_n \\ 0 \end{bmatrix}, \boldsymbol{B}_r = \begin{bmatrix} 0 \\ \breve{\boldsymbol{B}}_{cr} \end{bmatrix}, \boldsymbol{B}_e = \begin{bmatrix} 0 \\ \boldsymbol{B}_c \end{bmatrix} \quad (3.11)$$

Additionally, the output of the controller given by equation 3.7 may now be written as

$$v(t) = \boldsymbol{K}\boldsymbol{x}(t) + \boldsymbol{K}_r\boldsymbol{r}(t) + \boldsymbol{K}_e\boldsymbol{e}(t) \quad (3.12)$$

wherein

$$K = [\breve{\boldsymbol{D}}_c\breve{\boldsymbol{C}} \quad \breve{\boldsymbol{C}}_c], K_r = \breve{\boldsymbol{D}}_{cr}, K_e = \breve{\boldsymbol{D}}_C \quad (3.13)$$

Therefore, a generalized depiction of the plant system comprising the dynamics of the controller is procured. Herein, the control law of equation 3.12 is proportional. Notably, for every plant which is linear in nature, the transformation achieved above is possible, wherein the controller is dynamic and triggered by an event.

3.1.2 PROPORTIONAL-INTEGRAL CONTROLLER

Equating the matrices of the plant and controller to each other, and instead of using the equations 3.1 and 3.2, the following equation

$$\dot{\boldsymbol{x}}_c(t) = \boldsymbol{y}(t) - \boldsymbol{r}(t), \qquad \boldsymbol{x}_c(t_k) = \boldsymbol{x}_{ck}$$

along with

$$\boldsymbol{v}(t) = \boldsymbol{K}_I\boldsymbol{x}_c + \boldsymbol{K}_P(\boldsymbol{y}(t_k) - \boldsymbol{r}(t))$$

is used in order to include PI controllers in the general equation for system representation, wherein $n_p = n_c = s$. Additionally, with the introduction of error in the equation 3.5 and with the usage of augmented vectorized state matrix as shown in the equation 3.8, the generalized depiction of system denoted by the equations (3.9, 3.12) are procured once more. Hence, the respective matrix of equations 3.11 and 3.13 become

$$\boldsymbol{A} = \begin{bmatrix} \breve{\boldsymbol{A}} & 0 \\ \breve{\boldsymbol{C}} & 0 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \breve{\boldsymbol{B}} \\ 0 \end{bmatrix}, \boldsymbol{B}_n = \begin{bmatrix} \breve{\boldsymbol{B}}_n \\ 0 \end{bmatrix}, \boldsymbol{B}_r = \begin{bmatrix} 0 \\ -\boldsymbol{I}_{n_p} \end{bmatrix}, \boldsymbol{B}_e = \begin{bmatrix} 0 \\ \boldsymbol{I}_{n_p} \end{bmatrix}, \quad (3.14)$$
$$\boldsymbol{C} = [\breve{\boldsymbol{C}} \quad 0], \boldsymbol{K}_r = [\boldsymbol{K}_P\boldsymbol{C} \quad \boldsymbol{K}_I], K_r = -K_P, K_e = K_P$$

3.1.3 GENERAL DEPICTION OF EVENT-TRIGGERED CONTROL SYSTEM

With the introduction of the saturation non-linearity using the function $u(t) = sat(v(t))$, a generalized version of the constant feedback control system representation for plants is developed which is controlled by a controller which is triggered by events and is subjected to non-linearities, which is shown by the following equation

$$\begin{aligned} \dot{x}(t) = Ax(t) + Bsat\big(Kx(t) + K_r r(t) + K_e e(t)\big) + B_n n(t) \\ + B_r r(t) + B_e e(t), \\ x(0) = x_0, \\ y(t) = Cx(t) \end{aligned} \quad (3.15)$$

Subsequently, Figure 3.2 is a block diagram representing the equation 3.15. Hereinafter, all system representations and considerations inside this chapter shall be based on the general system as depicted by equation 3.15, wherein suitable dimensions of $x \in \mathcal{R}$, $n = n_p + n_c$, $u \in \mathcal{R}$, $n \in \mathcal{R}$, $r \in \mathcal{R}$, $y \in \mathcal{R}$ and matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{B}_n, \boldsymbol{B}_r, \boldsymbol{B}_e$, $\boldsymbol{K}, \boldsymbol{K}_r, \boldsymbol{K}_e$ and $\boldsymbol{C}$ are considered. In spite of the supplementary terms of error $e(t)$, the general system of the control system is same as the depiction of continuous-time control systems subjected to saturation non-linearity as given by equation 2.10.

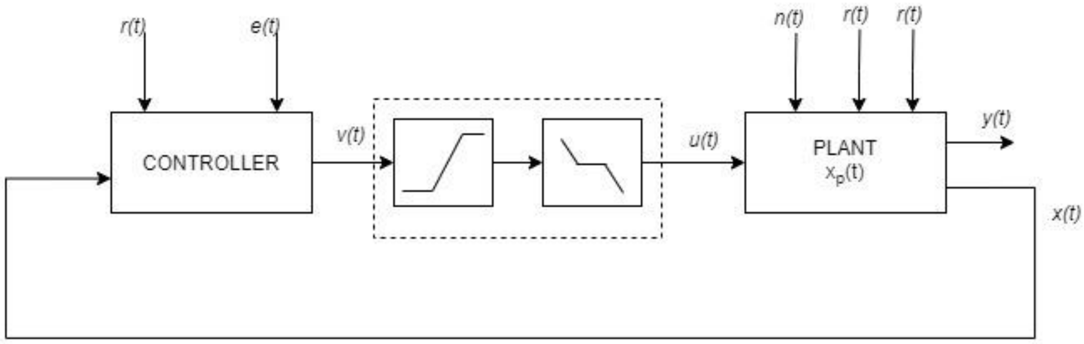

Figure 3.2 Representation of event-triggered control loop

Additionally, the dead-zone non-linearity is added as shown by the equation 2.11. With the addition of the dead-zone non-linearity, the previous depiction is transformed into a new equation given by,

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \overline{\boldsymbol{A}}x(t) + \overline{\boldsymbol{B}}\boldsymbol{\phi}\big(\boldsymbol{K}x(t) + \boldsymbol{K_r r}(t) + K_e e(t)\big) + \boldsymbol{B}_n n(t) \\ &\quad + (\boldsymbol{B}_r + \boldsymbol{BK_r})\boldsymbol{r}(t) + (B_e + Bk_e)e(t), \\ x(0) &= x_0, \\ \boldsymbol{y}(t) &= \boldsymbol{Cx}(t) \end{aligned} \tag{3.16}$$

along with

$$\overline{\boldsymbol{A}} = \boldsymbol{A} + \boldsymbol{BK}, \overline{\boldsymbol{B}} = \boldsymbol{B}.$$

3.2 ANALYSIS FOR STABILITY

A criterion for stability is derived for a control system which is event-triggered in the presence of non-linearities. In order to develop such a criterion for stability, the quadratic Lyapunov function, represented by equation 2.16 is used. In this section, among all of the exogenous signals, only the error $e(t)$ will be considered. Hence $\boldsymbol{d}(t) = \boldsymbol{w}(t) = 0$. Therefore, the state equations from the equations 3.15 and 3.16 is transformed into

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}sat\big(\boldsymbol{K}x(t) + \boldsymbol{K}_e e(t)\big) + \boldsymbol{B}_e e(t), x(0) = x_0 \tag{3.17}$$

$$\dot{\boldsymbol{x}}(t) = \overline{\boldsymbol{A}}x(t) + \overline{\boldsymbol{B}}\phi\big(Kx(t) + \boldsymbol{K}_e e(t)\big) + (\boldsymbol{B}_e + \boldsymbol{B}\boldsymbol{K}_e)e(t), \quad x(0) = x_0 \tag{3.18}$$

The equations derived in this section may be used to cover additional noise signals or reference signals. This will be further explained in subsequent sections.

In equation 3.17, only those errors are considered which appear either only outside or only inside the non-linearity term (either K_e or B_e terms are zero). For a control system which is event-triggered in presence of non-linearities, both K_e and B_e terms are to considered. Hence, the criterion of stability has to be modified to accommodate more terms.

It is important that the error signal $e(t)$ be a bounded signal. Herein, it is realized that the error signal $e(t)$ is bounded via amplitude by using a quadratic norm. hence, the error signal $e(t)$ is a part of the following set

$$\mathcal{W}(\mathcal{R}, \delta) = \{e(t) \in \mathcal{R}^P e^T \text{Re} \leq \delta^{-1}\}, R = R^T > 0, \delta > 0 \tag{3.19}$$

By properly defining the event generator, the limitations of bounds of the error signal may be procured. Notably, an event generator makes sure that the error signal $e(t)$ is a part of the set defined by the equation 3.19 for all the times $t \geq 0$. Furthermore, the event generator is defined by the following condition of event triggering

$$e^T \text{Re} = \delta^{-1} \tag{3.20}$$

Furthermore, a symmetric positive definite matric is defined wherein $\boldsymbol{W} \in \mathcal{R}$, a positive definite diagonal matrix is defined wherein $\boldsymbol{S} \in \mathcal{R}$ and a matrix is defined wherein $\boldsymbol{Z} \in \mathcal{R}$. The symmetric positive definite matrix, the positive definite diagonal matrix and the matrix should satisfy the equation in the matrix given by

$$\begin{bmatrix} W\bar{A}^T + \bar{A}W + \tau_1 W & \bar{B}S - WK^T - Z^T & B_e + BK_e \\ S\bar{B} - KW - Z & -2S & -K_e \\ (B_e + BK_e)^T & -K_e^T & -\tau_2 R \end{bmatrix} < 0 \tag{3.21}$$

$$\begin{bmatrix} W & Z_{(i)}^T \\ Z_{(i)} & \eta u_{0(i)}^2 \end{bmatrix} \geq 0, i \in \{1, \ldots., m\} \tag{3.22}$$

$$-\tau_1 \delta + \tau_2 \eta < 0 \tag{3.23}$$

Herein, when the error signal $e(t) = 0$, then the ellipsoid $\varepsilon(\boldsymbol{P}, \eta)$, with $\boldsymbol{P} = \boldsymbol{W}^{-1}$ and provides the asynptotic stability for the equations 3.17 and 3.18. Furthermore, for any value of the error signal $e(t) \in \mathcal{W}(\boldsymbol{R}, \delta)$ and $x_0 \epsilon\ \varepsilon(\boldsymbol{P}, \eta)$, the trajectories of the non-linear system of equation 3.17 and equation 3.18 act as bounded signals and are bound to the ellipsoid $\varepsilon(\boldsymbol{P}, \eta)$.

In order to prove that the ellipsoid $\varepsilon(\boldsymbol{P}, \eta)$ is a positive invariant set with respect to the error signal $e(t)$, the condition $\dot{V}(x) < 0$ may be extended as per the following equation

$$\dot{V}(x) + \tau_1(x^T P x - \eta^{-1}) + \tau_2(\delta^{-1} - e^T \mathrm{Re}) < 0 \tag{3.24}$$

wherein $\tau_1, \tau_2 > 0$. The relation shown in equation 3.24 makes sure that $\dot{V}(x) < 0$ is true for all values of $\boldsymbol{x}$ which satisfies the condition of $\boldsymbol{x}^T \boldsymbol{P} \boldsymbol{x} \geq \eta^{-1}$ for any value of the error signal $e(t) \in \mathcal{W}(\boldsymbol{R}, \delta)$.

The relation in the equation 3.24 is to be verified at the boundary of the ellipsoid of $\varepsilon(\boldsymbol{P}, \eta)$ i.e., at $x(t_1) \in \partial\varepsilon(\boldsymbol{P}, \eta)$. In case the error signal $e(t) \in \mathcal{W}(\boldsymbol{R}, \delta)$ is true, it will follow $\dot{V}(x(t_1)) < 0$.

In order to analyze the equations further, the inequality present in the equation 3.24 is divided into two inequality equations

$$-\tau_1 \delta + \tau_2 \eta < 0 \tag{3.25}$$

$$\dot{V}(x) + \tau_1(x^T P x) - \tau_2(e^T Re) < 0 \tag{3.26}$$

Furthermore, the state equation in equation 4.18 is used and assuming $\boldsymbol{W} = \boldsymbol{P}^{-1}$, $\boldsymbol{S} = \boldsymbol{T}^{-1}$ and $\boldsymbol{Z} = \boldsymbol{GW}$, equation 3.21 is procured.

3.3 MINIMUM TIME BETWEEN EVENTS

In a control system which is event-triggered, it is vital to exclude Zeno behaviour. Hence it is essential to calculate a lower limitation of the bounded signal for the time elapsed between two consecutive events. Herein, the term Zeno behaviour refers to the time elapsed between multiple events which may be triggered in a certain timeframe, wherein the timeframe is infinitesimal in quantity.

Furthermore, the lower limitations of the bounded signal for the time elapsed between two events occurring consecutively is termed as minimum inter-event time and is denoted by

$$T_{min} = min\{t_{k+1} - t_k\}$$

Herein, the plant given by the equations (2.1, 2.2). Furthermore, the noise signal of the plant is not considered. Hence, for $n(t) = 0$, the trajectory of the plant output will be given by

$$y(t) = \breve{\boldsymbol{C}} e^{\breve{\boldsymbol{A}}t} \boldsymbol{x}_0 + \breve{\boldsymbol{C}} \int_0^t e^{\breve{\boldsymbol{A}}(t-\tau)} \breve{\boldsymbol{B}} u(\tau) d\tau$$

Additionally, the approximation equation is used

$$e^T Re \leq \|e\|^2 \|R\|$$

along with the norm of the error signal $e(t)$

$$\|e(t)\| = \|y(t_k) - y(t)\|$$

Hence, the error signal $e(t)$ has the upper limits

$$\|e(t)\| = \|\breve{\boldsymbol{C}}\left(e^{\breve{\boldsymbol{A}}t} - \boldsymbol{I}_n\right) x_0 + \breve{\boldsymbol{C}} \int_0^t e^{\breve{\boldsymbol{A}}(t-\tau)} \breve{\boldsymbol{B}} \boldsymbol{u}(\tau) \| \leq \breve{e}(t)$$

whenever $t_k = 0$ and $\breve{e}(t)$ is given by the equation

$$\breve{e}(t) = \max_t \|\breve{\boldsymbol{C}}(e^{\breve{\boldsymbol{A}}t} - \boldsymbol{I}_n)\| x_{\max} + \int_0^t \|\breve{\boldsymbol{C}} e^{\breve{\boldsymbol{A}}(t-\tau)} \breve{\boldsymbol{B}}\| \, d\tau u_{0max}$$

wherein

$$x_{\max} = \max_t \|x_p\|, u_{0max} = \max_{i \in \{1,2,\ldots,m\}} u_{0(i)}$$

Therefore, it is observed that $\|u\| \leq u_{0max}$ and is not dependent on the output of the controller $v(t)$ because of the limitations of the actuator. The events in the control system occur when the event condition $e^T Re = \delta^{-1}$ is reached and satisfied. Furthermore, the equation

$$T_{\min} \geq \bar{T} = \arg \min_t \left\{ e(t) = \sqrt{\frac{1}{\delta \|\boldsymbol{R}\|}} \right\}$$

defines the lower limits of the minimum time taken between the events for occurrence. Therefore, this shows that the exchange of information using the feedback loop in the control system depends on the event generator, which is defined by the terms $\boldsymbol{R}$ and δ. However, these terms may affect the stability of the control loop as well.

3.4 EXTENSION OF STABILITY ANALYSIS

In this section, the extension of stability criteria obtained in Chapter 2 is extended. Herein, the extended version of stability criterion in the present chapter includes a different version of the event generator in presence of the noise signal and reference signals i.e., $n(t) \neq 0$ and $r(t) \neq 0$.

3.4.1 EVENT GENERATOR ALTERNATIVE

It is proposed in Section 3.2m, that the error signal $e(t)$ is bounded by upper and lower limits to create a criterion for stability. Furthermore, the event generator is triggered whenever the condition for occurrence of an event is met according to equation 3.20.

Hence, an alternative approach to define an event generator must be obtained. Herein, the individual output from the plant of the control system $y_{(i)} (i \in \{1, \ldots, p\})$ should be bounded by the amplitude, which is useful for practical applications. thereby, definitions such as

$$e_{(1)}^2 \leq \delta_1^{-1}, e_{(2)}^2 \leq \delta_2^{-1}, \ldots, e_{(p)}^2 \leq \delta_p^{-1}$$

defines a cuboid which is p-dimensional and is represented by the figure 3.3,

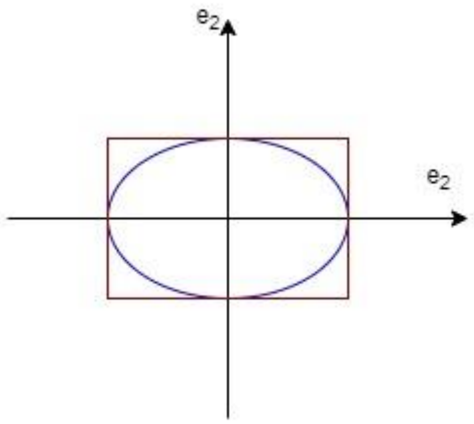

Figure 3.3 Event conditions based on ellipse and cuboid

Herein, in the given figure, the elliptical area is introduced by the event condition given by the equation (3.20). Furthermore, the event generator is triggered whenever the error signal $e(t)$ reaches the boundary of the cuboid.

Additionally, Zeno behaviour may be excluded when event conditions are reached using the cuboid. In order to calculate the minimum inter-event time, the equations in Section 3.3 are used along with the revised terms

$$R = \begin{bmatrix} \delta_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & \delta_p \end{bmatrix}, \delta = 1.$$

3.4.2 NON-ZERO NOISE SIGNALS

In real time control systems, noises and disturbances affect the plant of the control system and cannot be neglected. Henceforth, $n(t) \neq 0$ is considered to find the new criterion of stability. Herein, the state equations are given by

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}sat\big(\boldsymbol{K}x(t) + \boldsymbol{K}_e e(t)\big) + \boldsymbol{B}_n n(t) + \boldsymbol{B}_e e(t) \quad (3.27)$$

Or

$$\begin{aligned} \dot{\boldsymbol{x}}(t) = \overline{\boldsymbol{A}}x(t) + \overline{\boldsymbol{B}}\phi\big(Kx(t) + \boldsymbol{K}_e e(t)\big) + \boldsymbol{B}_n n(t) \\ + (\boldsymbol{B}_e + \boldsymbol{B}\boldsymbol{K}_e)e(t), \end{aligned} \quad (3.28)$$

wherein, $x(0) = x_0$ for both the equations. Additionally, analogous to the error signal $e(t)$, the noise signal $n(t)$ is bounded by upper and lower limits to derive criterion for stability. Furthermore, it is assumed that the noise signal $n(t)$ is bounded by a quadratic norm. Hence, the noise signal $n(t)$ is part of the set

$$\mathrm{V}_n(Q_n, \epsilon_n) = \{n \in R^q, n^T Q_n n < \epsilon_n^{-1}\}, Q_n = Q_n^T > 0, \epsilon_n > 0 \quad (3.29)$$

which leads to the inequality

$$\dot{V}(x) + \tau_1(x^T P x - \eta^{-1}) + \tau_2(\delta^{-1} - e^T \mathrm{Re}) + \tau_3(\epsilon_n^{-1} - n^T Q_n n) < 0$$

wherein $\tau_1, \tau_2, \tau_3 > 0$.

Additionally, Zeno behaviour is excluded in case the noise signals $n(t) \neq 0$ exist. However, the minimum time between occurrence of the events needs to be revised, since the output of the plant $y(t)$ and is affected by the noise signals $n(t)$.

Herein, the plant given by the equations (2.1, 2.2), giving the trajectory of the plant output will be given by

$$y(t) = \breve{\boldsymbol{C}}e^{\breve{A}t}\boldsymbol{x}_0 + \breve{\boldsymbol{C}}\int_0^t e^{\breve{A}(t-\tau)}(\breve{\boldsymbol{B}}u(\tau) + \breve{\boldsymbol{B}}_{\boldsymbol{n}}\boldsymbol{d}(\tau))d\tau.$$

Furthermore, the norm of the error signal $e(t)$ has the upper limits

$$\|e(t)\| = \|\breve{\boldsymbol{C}}\left(e^{\breve{A}t} - \boldsymbol{I}_n\right)x_0 + \breve{\boldsymbol{C}}\int_0^t e^{\breve{\boldsymbol{A}}(t-\tau)}(\breve{\boldsymbol{B}}\boldsymbol{u}(\tau) + \breve{\boldsymbol{B}}_{\boldsymbol{n}}\boldsymbol{d}(\tau))d\tau\,\| \;\leq\; \breve{e}(t)$$

For every value of time $t_k = 0$, with $\breve{e}_n(t)$ is given by

$$\breve{e}(t) \;=\; \max_t \|\breve{\boldsymbol{C}}\left(e^{\breve{A}t} - \boldsymbol{I}_n\right)\|x_{\max} \;+ \int_0^t \|\breve{\boldsymbol{C}}e^{\breve{A}(t-\tau)}\|d\tau(\|\breve{\boldsymbol{B}}\,\|u_{0max} \;+ \|\breve{\boldsymbol{B}}_{\boldsymbol{n}}\|d_{\max}\; d\tau$$

wherein,

$$d_{\max} \;=\; \max_{\mathrm{n}\in\mathrm{V}_n(Q_n,\epsilon_n)} \|n\|$$

Finally, the equation

$$T_{\min} \geq \bar{T}_n = \arg\;\min_t\left\{e_n(t) = \sqrt{\frac{1}{\delta\|\boldsymbol{R}\|}}\right\}$$

Hence, the error signals and/or the noise signals which are limited within the upper and lower limits may be considered in a straightforward manner as discussed in Section 3.4.1. Herein, the criterion for stability and the minimum time taken between the occurrence of events is changed according to that.

3.4.3 NON-ZERO REFERENCE SIGNALS

In real time control systems, possible reference signals are considered, wherein $r(t) \neq 0$. In this section, it is assumed that the noise signals are negligible i.e., $n(t) \;=\; 0$. The equation is given by

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}sat\left(\boldsymbol{K}x(t) + \boldsymbol{K}_{\boldsymbol{r}}r(t) + \boldsymbol{K}_e e(t)\right) + \boldsymbol{B}_r r(t) + \boldsymbol{B}_e e(t) \qquad (3.30)$$

$$\dot{\boldsymbol{x}}(t) = \overline{\boldsymbol{A}}x(t) + \overline{\boldsymbol{B}}\phi\big(Kx(t) + \boldsymbol{K_r}r(t) + \boldsymbol{K}_e e(t)\big) + (\boldsymbol{B}_r + \boldsymbol{B}\boldsymbol{K}_r)r(t) + (\boldsymbol{B}_e + \boldsymbol{B}\boldsymbol{K}_e)e(t) \tag{3.31}$$

wherein, $x(0) = x_0$ in both the equations. The equations for the event generator remain the same as is defined in the Section 3.2.

For derivation of criterion of stability in the presence of reference signal $r(t)$, the reference signal $r(t)$ needs to be bounded. Hence, is it assumed that the reference signal $r(t)$ is limited with the help of a quadratic norm, wherein $r(t)$ belongs to a set

$$\mathrm{V}_r(Q_r, \epsilon_r) = \{r \in R^{n_c}, r^T Q_r r < \epsilon_r^{-1}\}, Q_r = Q_r^T > 0, \epsilon_r > 0 \tag{3.32}$$

Therefore, the inequality given by the equation

$$\dot{V}(x) + \tau_1(x^T Px - \eta^{-1}) + \tau_2(\delta^{-1} - e^T \mathrm{Re}) + \tau_3(\epsilon_r^{-1} - r^T Q_r r) < 0$$

wherein $\tau_1, \tau_2, \tau_3 > 0$ is derived.

Moreover, the minimum time taken between the occurrence of events $T_{\min}$ is unaffected when the reference signal $r(t) \neq 0$ as only the output of the controller gets affected by the reference signal $r(t)$, wherein the output of the controller is bounded by upper and lower limits of the non-linearities present in the control system.

3.5 EXEMPLARY MODEL

The exemplary model discussed in Chapter 2 is used to observe the results presented here. Therein, a threshold for event is introduced, from where information is passed on only from the event generator to the controller to the control system, only when the event condition is met $|e(t)| = |y(t_k) - y(t)| = \bar{e}$. Furthermore, the event generator is synthesized based on $R = 1$ and $\delta = \bar{e}^2$

Hence, the equation 2.8 and equation 2.9 of the controller are given by

$$\dot{x}(t) = y(t_k) - r(t), x_C(k) = x_{\boldsymbol{ck}}$$

$$v(t) = k_I x_c(t) + k_P\big(y(t_k) - r(t)\big)$$

wherein the time is given by $t \in [t_k, t_{k+1})$. Additionally, with the introduction of the error signal at the output $e(t) = y(t_k) - y(t) = x_{\boldsymbol{p}}(t_k) - x_{\boldsymbol{p}}(t)$, the augmented state vector $x = [x_P \; x_c]^T$ as well as the function for saturation non-linearity $u(t) = sat(v(t))$, the state equation presented in equation 3.15 reads

$$x(t) = \begin{bmatrix} a & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} b \\ 0 \end{bmatrix} \text{sat}([k_P k_I]) x\,(t) - \; k_P r(t) + k_P e(t) + \begin{bmatrix} b_n \\ 0 \end{bmatrix} n(t) \\ + \begin{bmatrix} 0 \\ -1 \end{bmatrix} r(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e(t), \qquad x(0) = x_0$$

which is true for all the times $t \geq 0$. Herein, $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{K}, \boldsymbol{K}_r, \boldsymbol{B}_n$ and $\boldsymbol{B}_r$ are the same matrix as mentioned in Section 2.6. The error signal $e(t)$ comprises matrices given by

$$k_e = k_P, B_e = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The numerical values of the system parameters are same as mentioned in Section 2.6.

CHAPTER 4

NON-LINEARITY COMPENSATION

As shown in exemplary models in the previous chapters, it has been observed that practical control systems comprise integrator windup and the effect due to dead zone non-linearity cannot be ignored while using a control system which is event-triggered. Hence, this chapter deals with methods to overcome the windup and dead zone present in the integrator of the controller.

4.1 INTRODUCTION

Integrator windup is a detrimental effect which occurs whenever an integrator is present in the actuators and either the plant or controller of the control systems. Furthermore, the non-linearities present in the system creates a difference between the output of the actuator $u(t)$ and the output of the controller $v(t)$. Herein, the various non-linearities impeded the response of the feedback in the control system and causes the state of the integrator x_c to windup. Notably, the presence of saturation non-linearity in the state of the controller is a major reason for the occurrence of severe windup, leading to volatile behaviour of the system. Additionally, windup of integrator in the control system may have detrimental impact when event triggering is used in the control system as only a limited amount of information is exchanged between the controller and plant of the control system. Hence, past information saved in the controller is used by the controller in the absence of new information.

A solution to deal with such problems is to add an anti-windup compensation block to the loop of the controller, which helps to minimize or possibly prevent windup of the integrator present in the controller. Typically, the anti-windup compensation is mainly constructed when difference between the output of the actuator $u(t)$ and the output of the controller $v(t)$ is fed back. Herein, in case saturation of the actuator does not occur, then the between the output of the actuator $u(t)$ and the output of the controller $v(t)$ is zero. Otherwise, between the output of the actuator $u(t)$ and the output of the controller $v(t)$ has some arbitrary value and remedial steps are taken to minimize windup in the integrator portion of the controller.

Hereinafter, in the subsequent sections, an anti-windup structure which is static in nature is constructed which is used to augment the performance of the control system.

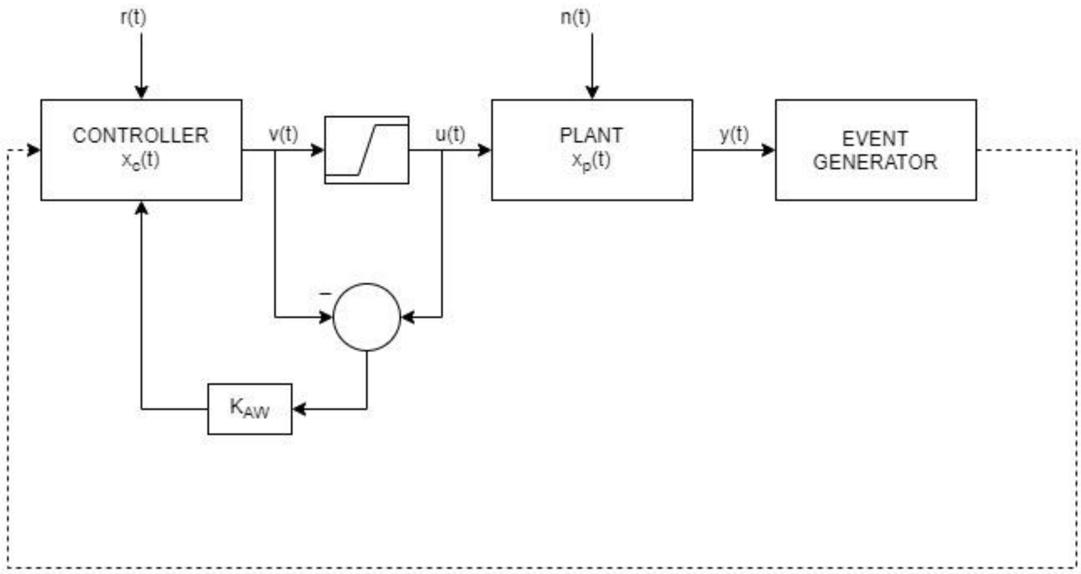

Figure 4.1 Representation of control system comprising an event generator with a static anti-windup block

Dead zone non-linearity is memory-less and static which outlines the insensitive nature of components when the signal given as input is a small signal. Furthermore, it might be observed as a static relation between u(t) and v(t) which has no value or a zero value for a specific input range. Thereafter, after that particular range of input values is crossed, u(t) and v(t) showcases a linear relationship.

Furthermore, the dead zone non-linearity limits the performance of a CLCS, both static and dynamic, and leads to the controller not being highly precise. Henceforth, in order to compensate the dead zone non-linearity, an inverse dead zone module (IDM) of the said non-linearity is implemented, as shown in

Figure 4.2 shows that an IDM of the non-linearity is connected before the non-linearity in order to compensate its effect. Additionally, the compensation completely cancels the effects observed by the non-linearity. Therefore, dead-zone non-linearity in the CLCS is fully compensated, and the input signal u(t) is found equivalent to e(k), wherein u(t) is the signal going into the IDM and e(k) is the signal coming out of the dead zone block after getting compensated. Herein, it is considered that both saturation non-linearity and dead zone non-linearity comprises a singular non-linearity. Hence, the output from the dead zone non-linearity after compensation shall remain $u(t)$.

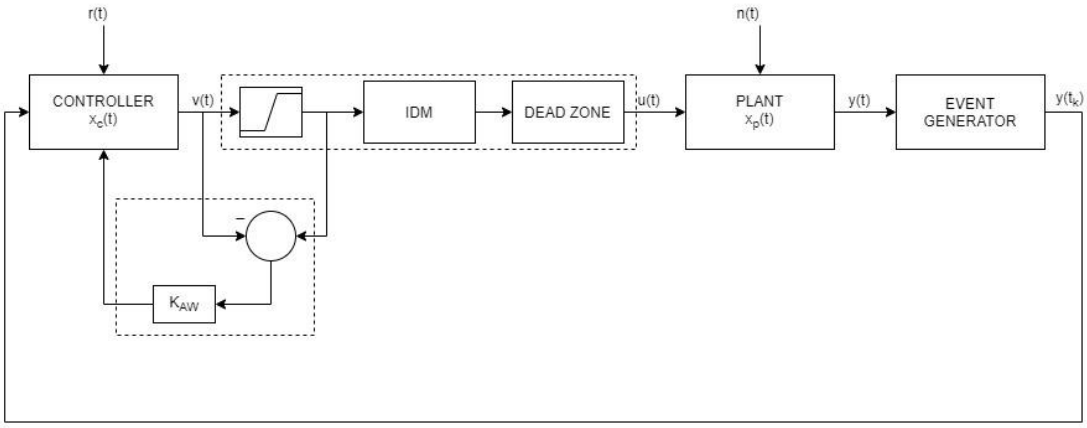

Figure 4.2 Representation of control system comprising an event generator with Inverse Dead zone module (IDM)

4.2 COMPENSATION MODEL

The system considered in this section is the same as the one considered in Section 3.1 of the previous chapter. The difference between the output of the actuator and the controller corresponds to the dead zone non-linearity which is given by

$$u(t) - v(t) = \sin(v(t) - v(t)) = \phi\big(v(t)\big).$$

Furthermore, a static gain $\boldsymbol{K}_{AW}$ is used, and the difference is fed back as input to the controller. Therefore, the general equation written for the controller in equation (3.3) is written further as

$$\dot{\boldsymbol{x}}_c(t) = \breve{\boldsymbol{A}}_c \boldsymbol{x}_c(t) + \breve{\boldsymbol{B}}_c \boldsymbol{y}(t) + \breve{\boldsymbol{B}}_c \boldsymbol{e}(t) + \breve{\boldsymbol{B}}_{cr} r(t) + \boldsymbol{K}_{AW}\, \phi\big(v(t)\big),\ x_c(0) = x_{c0}$$

Additionally, an adapted state equation is used to depict the controller, and the following general depiction is presented

$$\dot{\boldsymbol{x}}(t) = \overline{\boldsymbol{A}}\boldsymbol{x}(t) + \overline{\boldsymbol{B}}\phi\big(\boldsymbol{K}x(t) + \boldsymbol{K}_r\boldsymbol{r}(t) + \boldsymbol{K}_e\boldsymbol{e}(t)\big) + \boldsymbol{B}_n\boldsymbol{n}(t) + (\boldsymbol{B}_r + \boldsymbol{B}\boldsymbol{K}_r)\boldsymbol{r}(t) + (\boldsymbol{B}_e + \boldsymbol{B}\boldsymbol{K}_e)\boldsymbol{e}(t),$$
$$x(0) = x_0, \tag{4.1}$$
$$y(t) = C\,x(t)$$

wherein

$$\overline{\boldsymbol{A}} = \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K}, \overline{\boldsymbol{B}} = \begin{bmatrix} \breve{\boldsymbol{B}} \\ \boldsymbol{K}_{AW} \end{bmatrix}.$$

The main difference between the depiction of the control system when it is triggered by an event in the presence of anti-windup compensation and IDM as shown in equation (4.1) and the depiction of the control system without any non-linearity compensation as shown in equation 3.16 is the different description of the matrix term $\overline{\boldsymbol{B}}$ which comprises supplementary degrees of freedom for creation of the closed-loop control system through the feedback gain term $\boldsymbol{K}_{AW}$. These supplementary degrees of freedom may be used to enhance the performance of the controller. For instance, the region of stability may be increased by selecting a suitable value of $\boldsymbol{K}_{AW}$ as shown in the subsequent sections.

After the adaptation of matrix term $\overline{\boldsymbol{B}}$, all the methodologies mentioned in Chapter 3 can be used as it is in order to calculate the stability regions for loops of the controller that are an extension of the anti-windup structure which is static in nature.

Herein, a symmetric positive definite matrix is defined wherein $\boldsymbol{W} \in \mathcal{R}$, a positive definite diagonal matrix is defined wherein $\boldsymbol{S} \in \mathcal{R}$ and a matrix is defined wherein $\boldsymbol{Z} \in \mathcal{R}$. The symmetric positive definite matrix, the positive definite diagonal matrix, the matrix and positive scalar terms τ_1, τ_2, η should satisfy the equation in the matrix given by

$$\begin{bmatrix} \boldsymbol{W}\overline{\boldsymbol{A}}^T + \overline{\boldsymbol{A}}\boldsymbol{W} + \tau_1\boldsymbol{W} & \overline{\boldsymbol{B}}\boldsymbol{S} + \boldsymbol{W}\boldsymbol{K}^T - \boldsymbol{Z}^T & \boldsymbol{B_e} + \boldsymbol{B}\boldsymbol{K_e} \\ \boldsymbol{S}\overline{\boldsymbol{B}}^T - \boldsymbol{K}\boldsymbol{W} - \boldsymbol{Z} & -2\boldsymbol{S} & -\boldsymbol{K_e} \\ (\boldsymbol{B_e} + \boldsymbol{B}\boldsymbol{K_e})^{\boldsymbol{T}} & -\boldsymbol{K_e}^{\boldsymbol{T}} & -\tau_2\boldsymbol{R} \end{bmatrix} < 0 \tag{4.2}$$

$$\begin{bmatrix} \boldsymbol{W} & \boldsymbol{Z}^T{}_{(i)} \\ \boldsymbol{Z}_{(i)} & \eta u_{0(i)}^2 \end{bmatrix} \geq 0\,, i \in \{1, \ldots., m\} \tag{4.3}$$

$$\tau_1\delta + \tau_2\eta < 0 \tag{4.4}$$

Then, when $e = 0$ an ellipsoid $\varepsilon(\boldsymbol{P}, \eta)$, with $\boldsymbol{P} = \boldsymbol{W}^{-1}$ and provides the asynptotic stability for the system given by the equation (4.1). Furthermore, for any value of the error signal $e(t) \in \mathcal{W}(\boldsymbol{R}, \delta)$ and $x_0 \epsilon\ \varepsilon(\boldsymbol{P}, \eta)$, the trajectories of the non-linear system of equation (4.1) and equation (4.18) act as bounded signals and are bound to the ellipsoid $\varepsilon(\boldsymbol{P}, \eta)$.

Beneficially, the anti-windup compensation presented in this chapter does not modify the minimum time between the occurrence of events, given by T_{min}. Herein, only the controller gets affected by the presence of the non-linearities, wherein the state $\boldsymbol{x}_c(t)$ is not used for the generation of any event and the output of the controller $v(t)$ is bounded within the upper and lower limits of the saturation non-linearity.

4.3 EXEMPLARY MODEL

The exemplary model from chapter 3 is modified by the presence anti-windup structure in which is static in nature. The difference $k_{AW}\big(u(t) - v(t)\big)$ is communicated as feedback to the controller. Furthermore, with the introduction of dead zone $\phi\big(v(t)\big) = sat\big(v(t)\big) - v(t)$, the state equation is rewritten as,

$$\dot{x}(t) = \begin{bmatrix} a + bk_P & bk_I \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} b \\ k_{AW} \end{bmatrix} \Phi([k_P k_I])x(t) - k_p r(t) + \begin{bmatrix} b_n \\ 0 \end{bmatrix} n(t)$$
$$+ \begin{bmatrix} -bk_P \\ -1 \end{bmatrix} r(t) + \begin{bmatrix} bk_P \\ 1 \end{bmatrix} e(t), x(0) = x_0$$

wherein the same matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{K}, \boldsymbol{K}_r, \boldsymbol{B}_n, \boldsymbol{B}_r, \boldsymbol{K}_e$ and $\boldsymbol{B}_e$ as mentioned in Section 4.5 and

$$\bar{\boldsymbol{A}} = \begin{bmatrix} a + bk_P & bk_I \\ 1 & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} b \\ k_{AW} \end{bmatrix}, \boldsymbol{B}_r + \boldsymbol{B}\boldsymbol{K}_r = \begin{bmatrix} -bk_P \\ -1 \end{bmatrix}, \boldsymbol{B}_e + \boldsymbol{B}\boldsymbol{K}_e = \begin{bmatrix} bk_P \\ 1 \end{bmatrix}.$$

In order to calculate the estimated for the regions of stability, the optimization problem is given by

$$\min\{-\mathrm{trace}(\mathbf{W})\}$$

which is used again and solved with the help of YALMIP toolbox for $\tau_1 = \tau_2 = 0.1$.

CHAPTER 5

QUANTIZATION

This chapter emphasizes on quantization of the signal from the plant of the control system and using the quantized signal as feedback into the controller. Herein, the information given by $y(t_k)$ experiences a certain delay in procuring information causing the controller of the control system to depend on prior information for a certain amount of time

5.1 NEED FOR QUANTIZATION

Typically, a signal such as for example, $x(t)$ are usually continuous-time signals. The value of time t consists of real values over some definite interval such that the value of the signal $x(t)$ is allowed to have real values according to the time t. Usually, systems comprising such signals, other wise known as analog signals are used for feasibility, but there are specific situations where digital signals may be used. Herein, the digital signals comprise discrete signals which has a finite domain and a range. Furthermore, in digital signals, digitization of domain of the discrete signals is performed, which is termed as *sampling* and processing of the digitized range is termed as *quantization*.

Most of the devices make use of both analog and digital signals. Moreover, digital signals provide robustness to disturbances, high efficiency and are very versatile in nature. However, at times the analog signals are suitable and might be even necessary. Herein, it is preferred that actuators or sensors in a given control system is interfaced with analog signals for convenience of communication. Hence, deriving digital signals from analog signals are important for processing of information in many control systems.

Let intervals between samples of an arbitrary signal be given by T, wherein the samples are equally spaced and are picked off from the analog signal at suitable times. Therefore, the sampled or the discrete-time signal $x[n]$ has the following relation with the continuous time signal

$$x[n] = x(nT)$$

However, a practical control system comprises two shortcomings. Firstly, the sensor which obtains the samples is not able to collect a value at a single time. In this case, some integration occurs over a specific time interval, which means that the sampling actually comprises an average value of the analog signal in a definite interval. Secondly, a practical control system comprises noise. Hence, even with the integration of all the values, the exact value of the analog signal will not be obtained at a time.

The consequence of quantization are extremely important to select the thresholds of event triggering properly. It is possible to design error signals based on the variables of quantization which provide with asymptotic stability as compared to non-quantized event-triggered control system. Herein, in a control system which is triggered by the occurrence of events, communication between the controller and the plant of the control system may comprise time delays and loss of information. Furthermore, quantization is required for all measurements requiring a sensor and control communication that are sent over a wireless digital network.

The measured state variables are quantized to represent them by a finite number of bits in order to use it for operations by the controller and to communicate them over wireless digital network. Additionally, there are some important inferences of using quantization in control system triggered by events. Firstly, it is typical to set the plant error signal as zero whenever an occurrence of event is updated. However, with the use of quantization at the output end of the control system, this step no longer holds true as the quantized value of the state of the plant is used for updating the state of the controller and the value is generally not the same as the original plant state. Secondly, events are typically triggered when compared to the norm of the state of the plant to the norm of the error signal. However, the control system stability is dependent on the computations of the measurements which are non-quantized in nature and are supposed to be identified with certainty.

Herein, it is aimed to find conditions for event-triggering based on quantization variables that are available and also ensure asymptotic stability in existence of quantization errors. Notably, a static quantizer is used commonly as it is easier to implement in any control system. Hence, a logarithmic quantizer is used as described in the following section. Furthermore, Figure 5.1 shows the control system with a quantizer at the output of the event-triggered control system

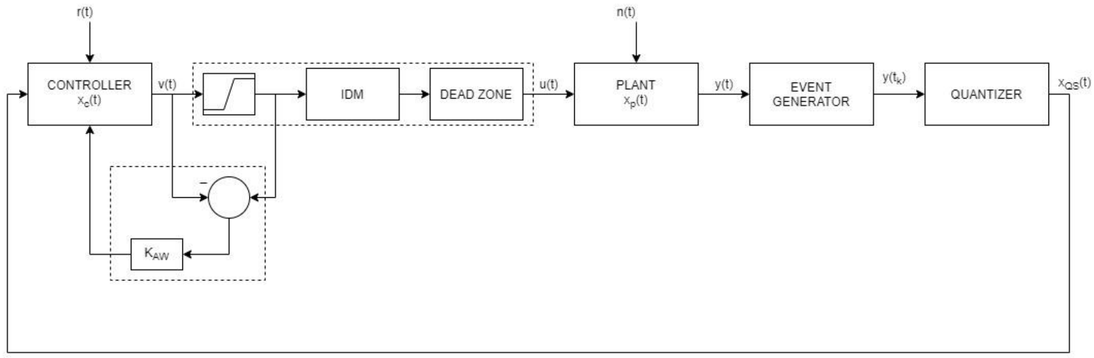

Figure 5.1 Representation of event-triggered control system with quantizer at the output

5.2 QUANTIZATION MODEL

As discussed, a logarithmic quantizer is used to procure quantized variables of the control system after triggering of an event in the control system, wherein the logarithmic quantizer is defined as a function $q: \mathcal{R}^n \rightarrow \mathcal{R}^n$ comprising the property

$$|x - q(x)| \leq \delta|x|, x \in R, \delta > 0 \tag{5.1}$$

At instants when the event is triggered and gets updated at instants of time given by t_i, wherein $i \in Z^+$, the state of the controller is updated using the measured quantized value given by

$$q\big(x(ti)\big) \rightarrow \hat{x}(t_i) \tag{5.2}$$

Furthermore, the quantization error is defined as

$$e_q(t) = \hat{x}(t) - q\big(x(t)\big) \tag{5.3}$$

wherein, $q\big(x(t)\big)$ denotes the quantized value of $x(t)$ for any value of time, specifically $t \geq 0$ using the quantizer as given by equation 5.1. Notably, the variables $q\big(x(t)\big)$ and $e_q(t)$ are used for computation of condition for triggering of an event in the control system. Additionally, $e_q(t_i) = 0$ i.e., the quantization error is set to zero at the instance of updating the triggering of an event according to equation 5.2.

CHAPTER 6

PRACTICAL IMPLEMENTATION

In this chapter, a set of initial conditions $\varepsilon(\boldsymbol{P},\ \eta)$ are used to test whether they provide a result showcasing the stable behaviour of the control system. In order to carry this out, $\boldsymbol{P}\ and\ \ \eta$ are given significant values for the verification of the conditions of stability. Herein, two optimization problems are used in order to avoid using random values for the initial conditions and preventing inefficiency

- Maximizing the size of the set of initial conditions $\varepsilon(\boldsymbol{P},\ \eta)$ which provides with a control system comprising a stable behaviour for a given event generator in addition with exogenous signal; and
- Maximizing the size of the error signal $\mathcal{W}(\boldsymbol{R},\delta)$ along with (or not) exogenous signals $\mathrm{V}_n(Q_n,\epsilon_n)/\mathrm{V}_r(Q_r,\epsilon_r)$ providing with a stable behaviour of the control system.

Section 6.1 discusses algorithms related to both the optimization problems described above.

6.1 ALGORITHMS

The decision variables denoted by $\boldsymbol{W},\boldsymbol{S},\boldsymbol{Z}$ and η are included in the conditions for determining stability when there is a fixed time constant τ_i. Additionally, parameters describing the error signal $(\boldsymbol{R},\delta)$, the noise signals (Q_n,ϵ_n) and the reference signals (Q_r,ϵ_r) are considered to be supplementary variables.

Herein, exogenous signals $\mathrm{V}_n(Q_n,\epsilon_n),\mathrm{V}_r(Q_r,\epsilon_r)$ are fixed in both of the algorithms as this thesis focuses more on the event-triggering aspect of the control system.

As discussed above, the first algorithm may be used to derive the maximum stability region $\varepsilon(\boldsymbol{P},\ \eta)$ for a given event generator wherein $(\boldsymbol{R},\delta)$ are fixed.

Algorithm 1: Steps to maximize of initial conditions $\varepsilon(\boldsymbol{P},\ \eta)$

1. $\boldsymbol{R}\ and\ \delta$ are already specified, a suitable objective function is chosen $f(\varepsilon(\boldsymbol{P},\ \eta))$.
2. τ_1, τ_2, τ_3 and τ_4 is fixed.
3. The optimization problem is solved for $\boldsymbol{W}, \boldsymbol{S}, \boldsymbol{Z}$ and η as shown

$$min\{f(\varepsilon(\boldsymbol{P}, \eta))\}$$

Thereafter, Algorithm 2 may be used to derive the maximum set of error signal $\mathcal{W}(\boldsymbol{R}, \delta)$. Furthermore, the values of $\boldsymbol{R}\ and\ \delta$ may be used to find conditions of an event which provide with a stable behaviour of the control system

Algorithm 2: Steps to maximize error signal $\mathcal{W}(\boldsymbol{R}, \delta)$

1. An objective function is chosen $f(\mathcal{W}(\boldsymbol{R}, \delta))$.
2. τ_1, τ_2, τ_3 and τ_4 is fixed.
3. The optimization problem is solved for $\boldsymbol{W}, \boldsymbol{S}, \boldsymbol{Z}$ and η as shown

$$min\{f(\varepsilon(\boldsymbol{P}, \eta))\}$$

6.2 FUNCTIONS FOR OPTIMIZATION

A suitable criterion for deciding the size has to be found for maximization of the respective set with the use of algorithms are described previously. There are many possibilities which renders into various objective functions $f(\varepsilon(\boldsymbol{P},\ \eta))$ out of which two examples are:

- $f(\varepsilon(\boldsymbol{P},\ \eta))\ =\ n\log(\eta) + \log(\det(\boldsymbol{P}))$ which refers to the maximization of volume.
- $f(\varepsilon(\boldsymbol{P},\ \eta))\ =\ \beta_0\eta + \beta_1 trace(\boldsymbol{P})$ comprising the weight parameters β_0 and β_1 which constructs homogenous ellipsoids which are homogenous in all directions.

Still, the transformation $\boldsymbol{P} = \boldsymbol{W}^{-1}$ leads to the non-linearity of the objective functions. Consequently, the objective functions are transformed:

- $f(\varepsilon(\boldsymbol{W^{-1}},\ \eta))\ =\ n\,log(\eta) + log(det(\boldsymbol{W}))$
- $f(\varepsilon(\boldsymbol{W^{-1}},\ \eta))\ = \beta_0\eta + \beta_1 trace(\boldsymbol{W})$

6.3 IMPLEMENTATION

The implementation of the algorithms is performed in MATLAB/Simulink software, wherein the software comprises tools which conveniently help to simulate the behaviour of the loop in the control system. However, the tools offered by the software are based on LMI conditions are inconvenient for solving of optimization problems. Herein, a toolbox like YALMIP or CVX may be used to get rid of the issues and allow an implementation which is simple in nature and executes both the algorithms as described previously in this chapter.

Herein, the YALMIP toolbox is used for the derivation of stability regions of the control system.

6.4 SIMULATION

In this section, the results obtained for various objective functions are discussed for different time constants τ_i of the control system

ALGORITHM 1: The results for the first algorithm is shown in Figure 6.1 for various objective functions. Furthermore, the error signal belongs to the set $e(t) \in \mathcal{W}(\mathcal{R}, \delta)$, the noise signals and the reference signal is not considered, $n(t) = r(t) = 0$ and the values of the time constants are given by $\tau_1 = \tau_2 = 0.1$. Additionally, it is observed that the functions including the following terms $-trace(\boldsymbol{W})$ or $-log(det(\boldsymbol{W}))$ provide almost synonymous results. Moreover, the objective function comprising η provides conventional result. It is also observed that $\beta_0\eta + \beta_1 trace(\boldsymbol{W})$ provides with the same result as the objective function $-trace(\boldsymbol{W})$ when the weight parameters β_0 and β_1 are made equivalent to 1 i.e., $\beta_0 = \beta_1 = 1$. Though, the result becomes conventional with the increment of β_0. However, those objective functions comprising the term $n \log(\eta)$ are not solvable with the use of YALMIP toolbox in MATLAB/Simulink.

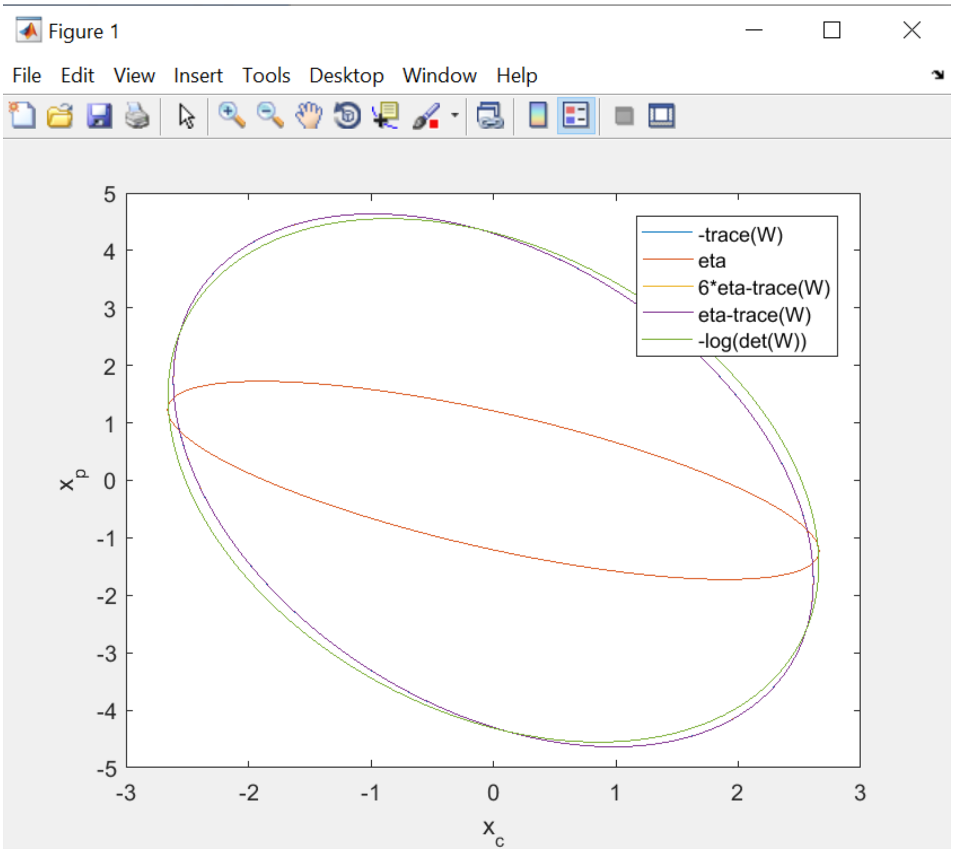

Figure 6.1 Results of Algorithm 1 using various objective functions

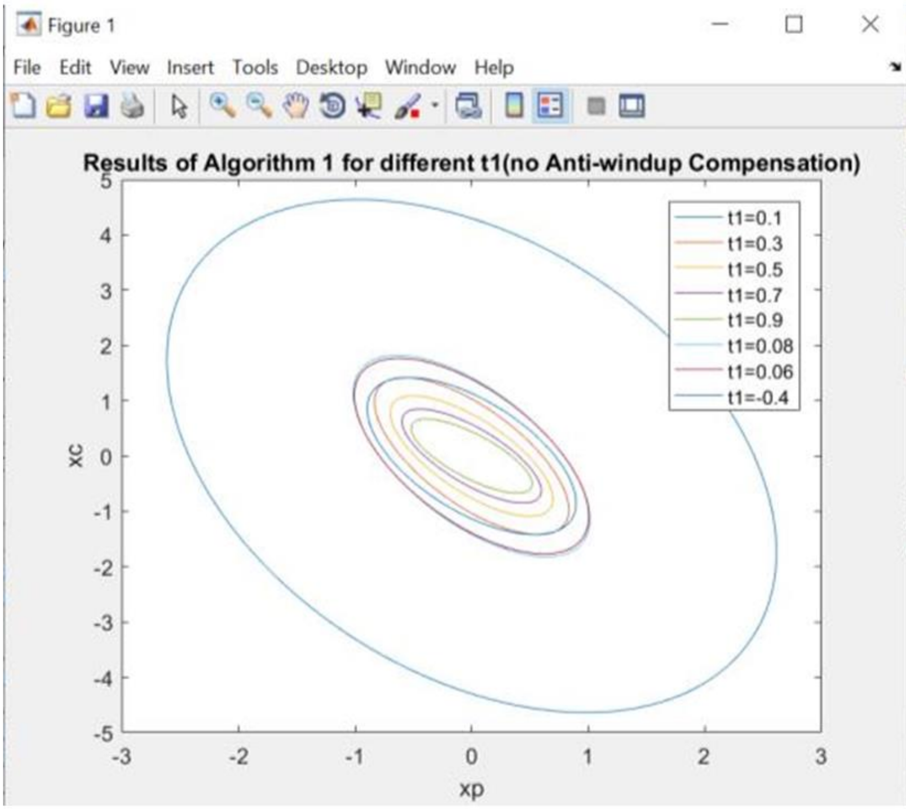

Figure 6.2 Results of Algorithm 1 for different τ_1 (with no anti-windup compensation)

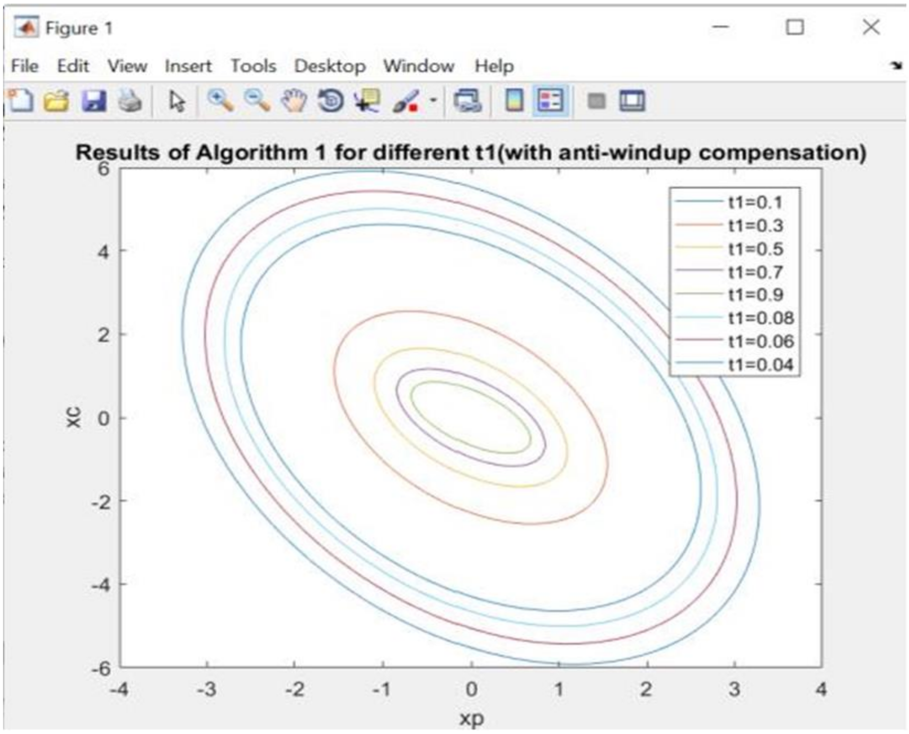

Figure 6.3 Results of Algorithm 1 for different τ_1 (with anti-windup compensation)

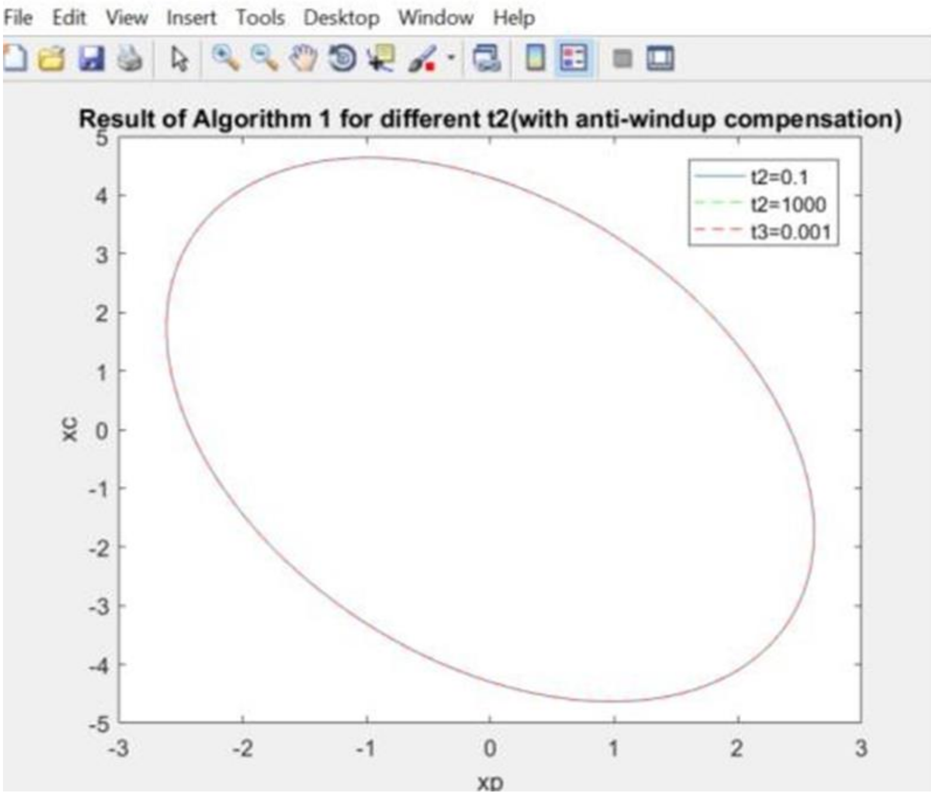

Figure 6.4 Result of Algorithm for different τ_2

All of the ellipsoids as seen in Figures 6.2, 6.3 and 6.4 were derived by considering $-trace(\boldsymbol{W})$ as the objective function. Herein, Figure 6.2 and 6.3 portrays the significance of τ_1 in both with and/or without anti-windup compensation. As observed, the first algorithm is feasible for $\tau_1 \in [0.04, 1.4]$. Furthermore, as seen, the size of the stability region keeps varying according to the different values of τ_1. For instance, in figure 6.3, the maximum is observed to be around $\tau_1 \approx 0.04$.

However, in Figure 6.4 it is observed that the stability region is the same for a varied range of τ_2, wherein the values of τ_1 vary in the range $\tau_1 \in [0.04, 1.4]$,. Hence, it is feasible for the first algorithm to be unaffected. This happens because, a visible change in τ_2 steers towards a different scaling between $\boldsymbol{P}\ and\ \eta$. Herein, both the terms are related to each other based on the inequality, $x^T \boldsymbol{P} x \leq \eta^{-1}$, thereby successfully describing the ellipsoid $\varepsilon(\boldsymbol{P},\ \eta)$.

ALGORITHM 1: The second algorithm is used for the maximization of $\mathcal{W}(\mathcal{R}, \delta)$ so as to recover $\mathcal{R}\ and\ \delta$ which are used for the definition of event generator. As discussed in Section 3.5, an event generator which is scalar in nature is used. Herein, $\boldsymbol{R} = 1$ and δ is denoted as the target function which is to be used. Furthermore, the relation $|e(t)| \leq \bar{e}$ is held with respect to $\bar{e} = \sqrt{\delta^{-1}}$.

Figures 6.5 and 6.6 shows the significance of τ_1 on the size of the stability region for both with anti-windup compensation and without anti-windup compensation. Herein, it is observed that the second algorithm is feasible when the value of τ_1 lies in the range, $\tau_1 \in [10^{-5}, 1.4]$. Table 6.1 shows the values for resulting $\bar{e}$. As shown in the table, it is observed that there is a maximum for $\bar{e}$ at around $\tau_1 \approx 0.5$.

τ_1	0.005	0.01	0.05	0.1	0.5	0.9	1.4
$\bar{e}$	0.038	0.053	0.112	0.149	0.215	0.201	0.120

TABLE 6.1

Additionally, in Figure 6.7, it is observed that the results are unaffected for a varied range of τ_2 when $\tau_1 = 0.1$. Furthermore, the threshold of the events i.e., $\bar{e}$ is unaffected with the change in τ_2 as observed in Table 6.2

τ_1	0.005	0.005	0.005	0.1	0.1	0.1	0.9	0.9	0.9
τ_2	0.001	0.1	1000	0.001	0.1	1000	0.001	0.1	1000
$\bar{e}$	0.038	0.038	0.038	0.149	0.149	0.149	0.112	0.112	0.112

TABLE 6.2

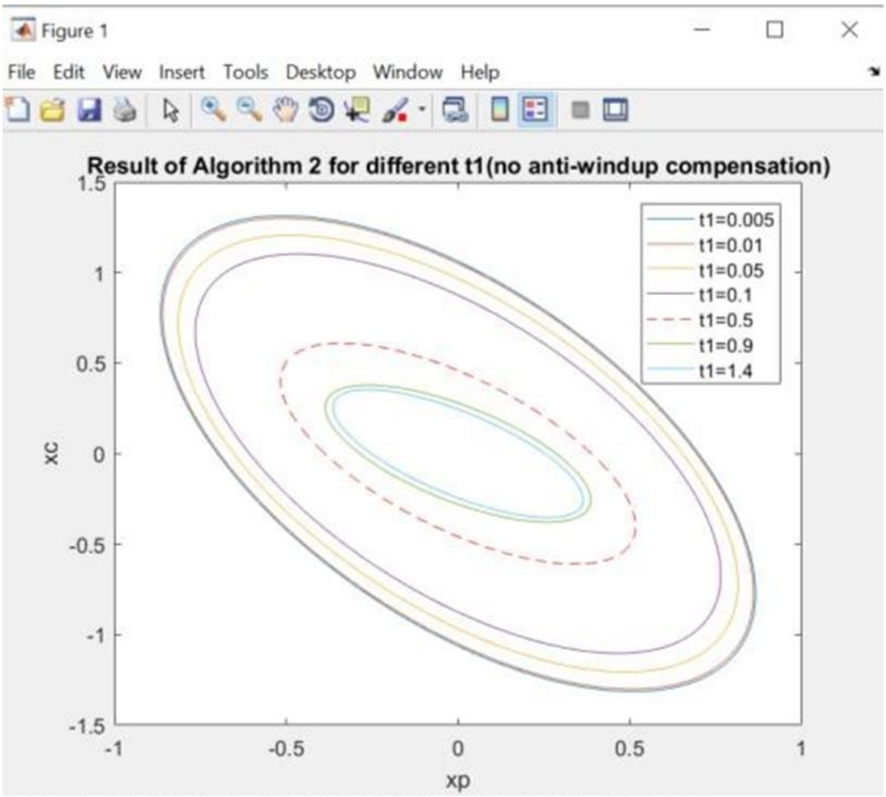

Figure 6.5 Result of Algorithm 2 for different τ_1 (no anti-windup compensation)

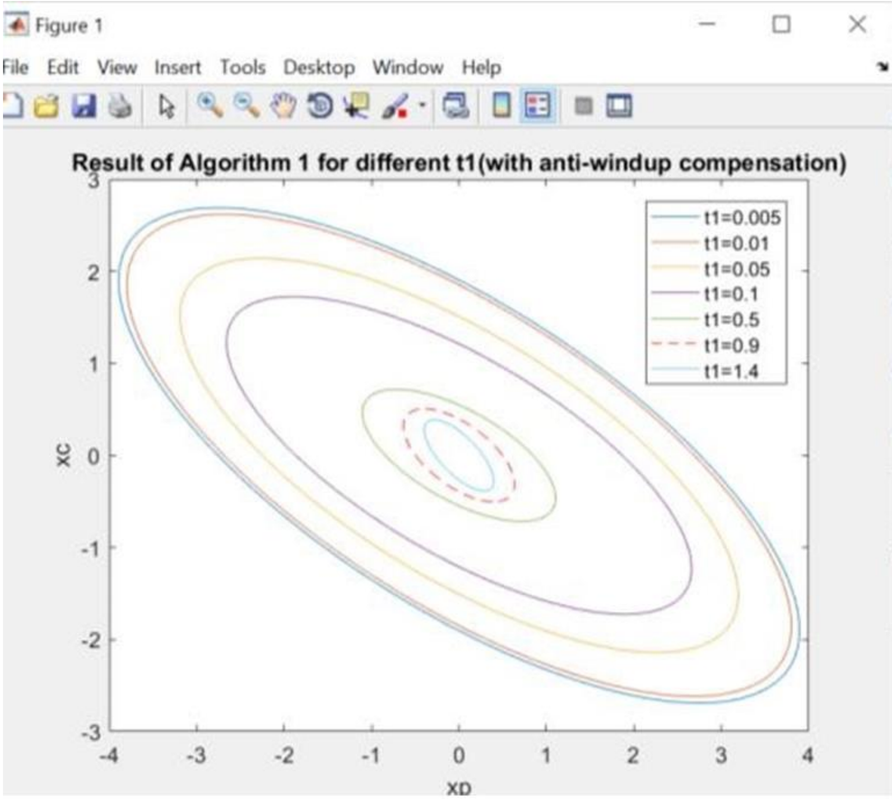

Figure 6.6 Result of Algorithm 2 for different τ_1 (with anti-windup compensation)

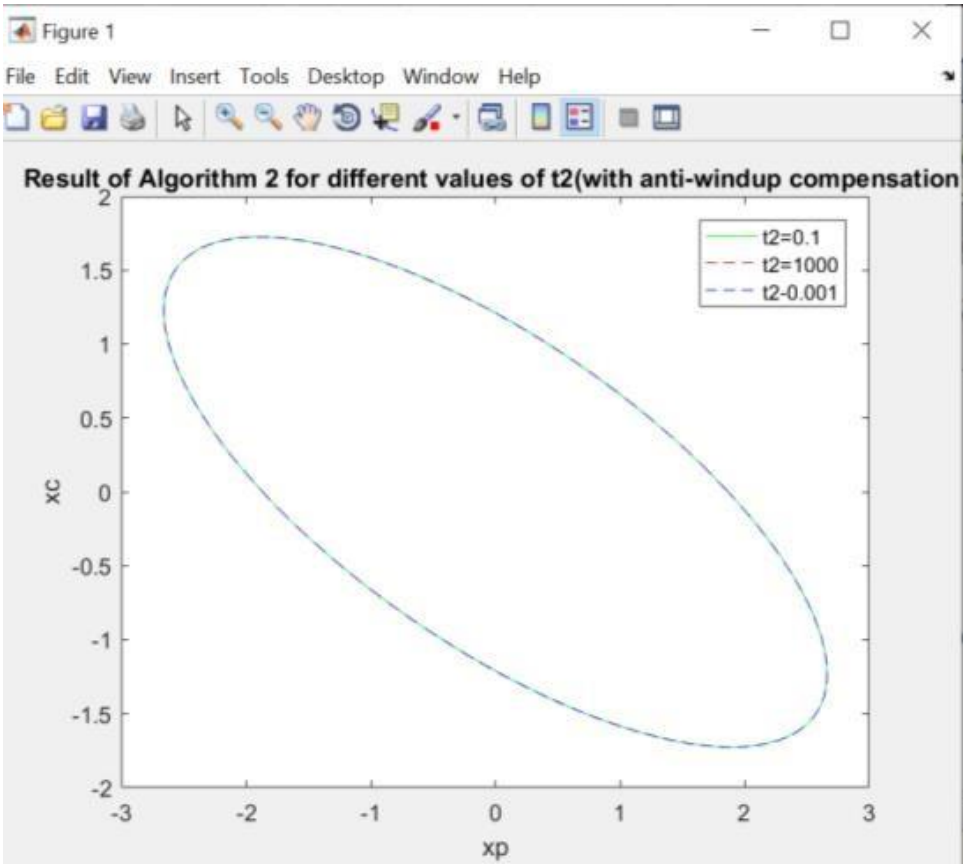

Figure 6.7 Result of Algorithm 2 for different values of τ_2 (with anti-windup compensation)

DESIGN OF THE CONTROLLER: The stability region of the ellipsoids is dependent on the design of the controller as well as shown in Figure 6.8, wherein the values of k_P and k_I are varied. As observed from the figure, the size of the stability region increases as the ratio of $\frac{k_P}{k_I}$ increases. This is because the influence of k_I is lower because of the potential windup of the integrator. Hence, suitable designs of the controller may also be used to make the stability region size bigger.

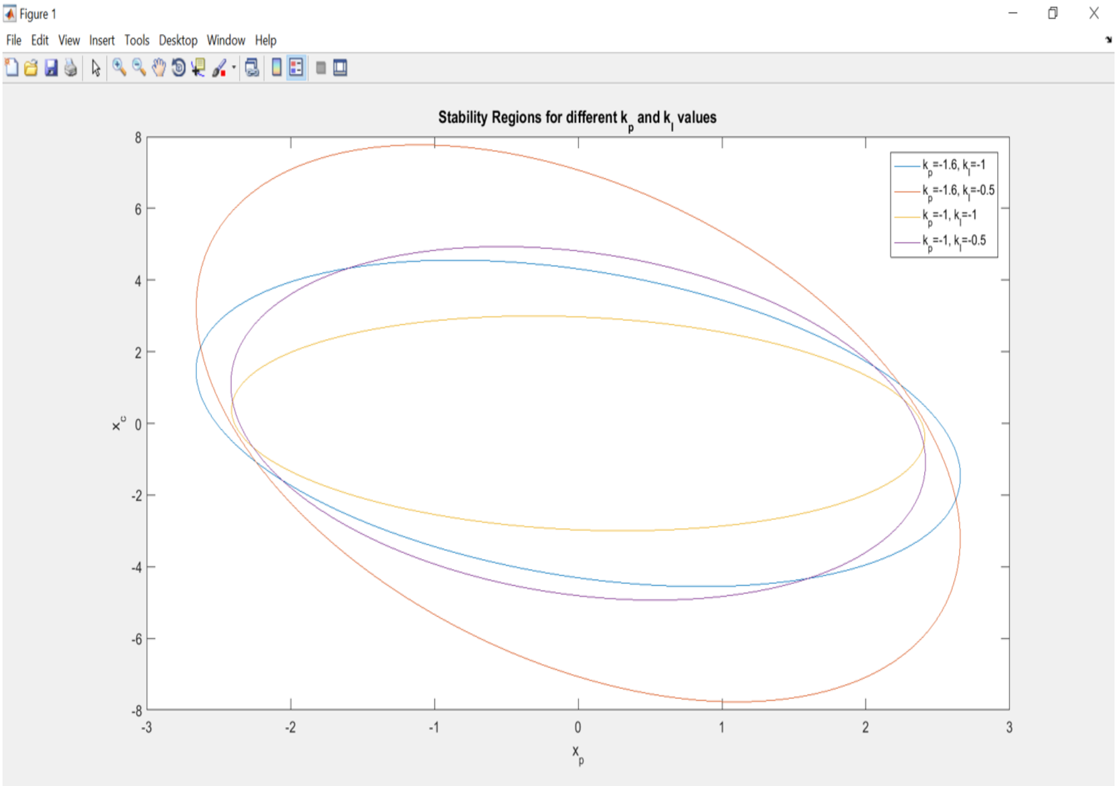

Figure 6.8 Regions of stability for different controller parameters

Furthermore, the following block diagram is constructed in MATLAB/Simulink which impersonates a real-time Tank reservoir system which is triggered by events. Herein, the controller, the actuator and the sensor are assumed to be connected wirelessly. The system further comprises an event generator, from where new information is carried forward to the controller only when event condition is met, wherein the event condition is given by

$$|\partial x_P(t_k) - \partial x_P(t)| = \bar{e}$$

Additionally, the controller sends the output $v(t)$ to the actuator after meeting the appropriate event conditions.

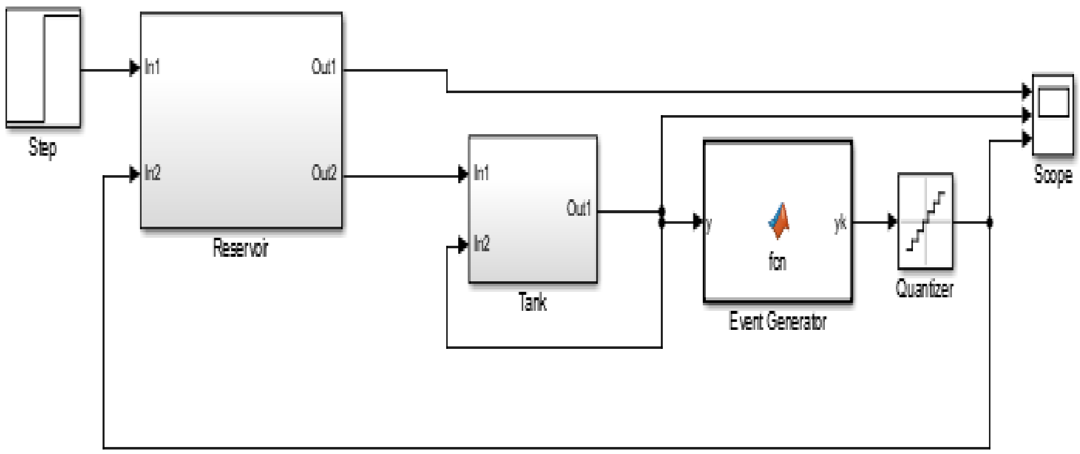

Figure 6.9 Simulation of Tank reservoir system

Herein, the reservoir fills the tank whenever the event generator reaches the event triggering condition ē. Herein, the system becomes a closed loop CLCS and starts providing continuous feedback to the reservoir to resume filling up of the tank.

The reservoir is given a step signal of 50 units and the tank is given a step output of 10 units. Furthermore, a saturation block is connected to denote the saturation non-linearity in the actuator, wherein the maximum and minimum saturation values are ±5 units. Subsequently, an Inverse Dead zone block is connected prior to the dead zone non-linearity block in order to compensate the non-linearity in the plant. Additionally, the dead zone non-linearity block has the units ±0.5 units. The simulation is performed with a quantizer connected at the output of the event-generator, wherein the quantizer has definite steps of quantization, for instance 0.005. Additionally, the threshold for event triggering is chosen to be 10 units. When the difference of controller input and input to the event generator is 10, then the event is triggered. **Error! Reference source not found.**.9 shows the simulation model for the event-triggered control system in MATLAB-Simulink.

6.5 RESULTS

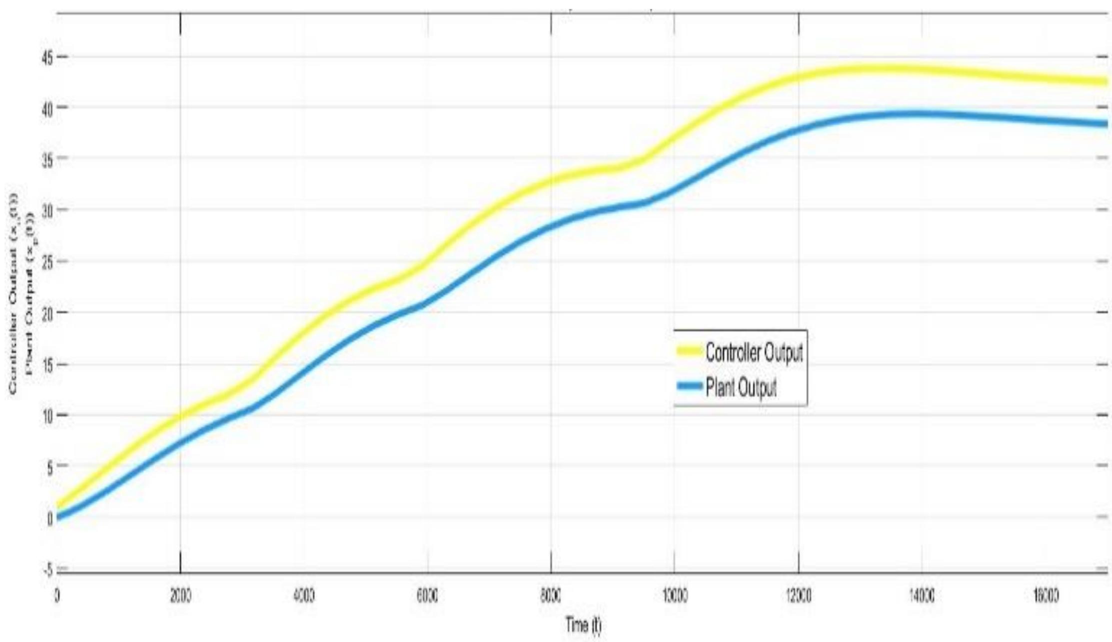

Figure 6.10 Event-triggered Controller Output vs Plant Output with saturation non-linearity and compensation

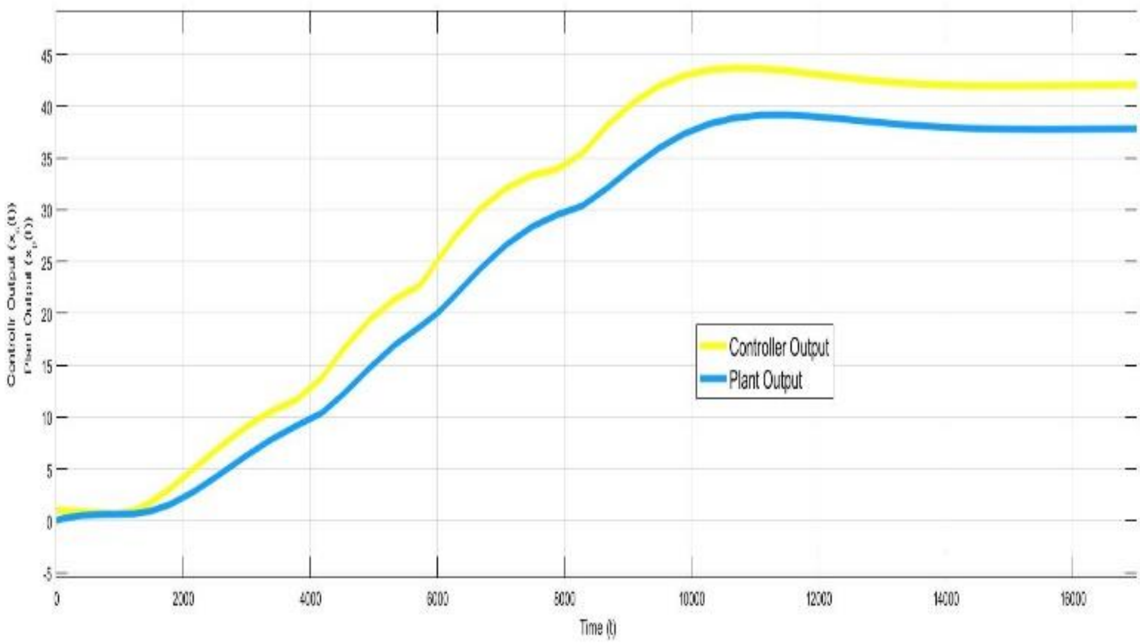

Figure 6.11 Event-Triggered CLCS with non-linearities (saturation and dead zone non-linearity) and their compensation

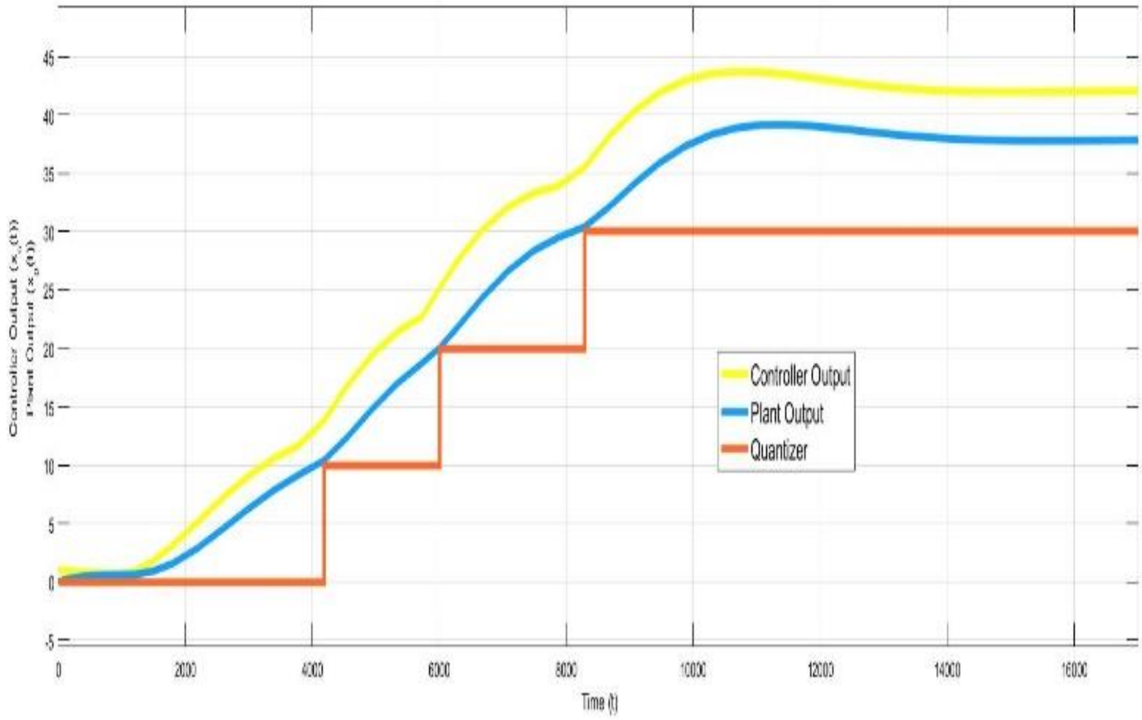

Figure 6.12 Event-triggered CLCS with quantizer output

In Figure 6.10 and Figure 6.11 the simulated results are observed for saturation and dead zone non-linearities along with its compensation, when the CLCS is subjected to event triggering. Herein, the reservoir produces the output of the controller and the tank produces output from the plant. From Figure 6.10, Figure 6.11 and Figure 6.12, it may be seen that the event is triggered whenever the event threshold reaches a difference of 10 units between the controller input and the output of the event generator. Meanwhile quantizing the output value obtained from the event generator helps in further reduction of the communication overload. Subsequently, event triggering at only those moments where the threshold is met helps in overcoming the feedback losses faced in a wired control system comprising a feedback loop.

CONCLUSION

This thesis was based on the influence of non-linearities of a practical system on stability of a closed loop system which is triggered by events. It comprises calculation of stability regions for the control loop which is based on triggering of events. Furthermore, in order to overcome the possibility of degradation of performance of the control system due the windup of the integrator, the conditions of stability are extended to add a structure for anti-windup structure and inverse dead zone non-linearity. Subsequently, improvement in the control behavior of the control system is observed and the size of the stability region may be respectively increased.

Furthermore, it is observed that exchange of information over feedback comprises upper limit and lower limit for all methodologies considered with the derivation of lower limits on the minimum time taken between two simultaneous events.

The size of the determined stability regions depends on the conditions of the event as well as the parameters of the controller, exogenous signals, quantization and optimization functions. The results are illustrated with the help of simulations.

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