

# **METAMATERIAL BASED NANO ANTENNAS**

A PROJECT REPORT

SUBMITTED IN PARTIAL FULFILLMENT  
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DEGREE OF MASTER IN SCIENCE  
IN  
PHYSICS

Submitted by:  
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May, 2021

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### **Candidate's declaration**

I hereby certify that the work which is presented in the Major Project-II entitled “**Metamaterial Based Nano Antennas**” in fulfillment of the requirement for the award of the degree of Master in Science in Physics and submitted to the Department of Applied Physics, Delhi Technological University, Delhi is an authentic record of my own, carried out during a period from January to may 2020, under the supervision of Principle supervisor **DR. Yogita Kalra** & Care taking supervisor **Dr. kamal Kishor**.

The matter presented in this report /thesis has not been submitted by me for the award of any other degree of this or any other Institute/University. The work has been communicated in peer reviewed conference with following details:

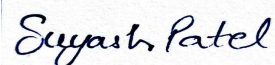
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## CERTIFICATE

This is to certified that the study report entitled “METAMATERIAL BASED NANO ANTENNAS” is a record of genuine study, conducted by Mr. Suyash Patel, as a part of the Dissertation-II for the forth semester of the batch 2019-21 of M.Sc. Physics Degree, offered by Delhi Technological University, under my guidance and supervision.

Place: Delhi

Date: 01/06/2021



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*Suyash Patel*

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## **ABSTRACT**

In this report we shall see that metamaterial which are extremely useful for subwavelength operations because of their material structural dependent properties, improves the directivity of radiation produced by an antenna. This report also highlights the resonant behavior of a DPS slab when it is matched with a DNG, ENG or MNG media. Because of their zero refraction property the whole structure of materials exhibits same phase oscillations of the molecules at all point, in the presence of electric field. Here the oscillations refers to the oscillations of polarized molecules of material along with direction of electric field. To explain these oscillation the Lorentz models and its special cases has been covered to some brief extent, which are the equivalent models to damped harmonic oscillators. These models leads the frequency of resonance and later we will see how the structure made by the matched two slabs could be considered as  $L - C$  equivalent circuits which is the most generalized analogy for the study of metamaterials.

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**List of abbreviation:**

<b>EM</b>	Electromagnetic
<b>DNG</b>	Double negative
<b>MNG</b>	Mu negative
<b>ENG</b>	Epsilon negative
<b>SNG</b>	Single negative
<b>DPS</b>	Double positive
<b>2TDLM</b>	Two time dependent Lorentz model

# Chapter 1

## **1.1 Introduction:**

The very 1st attempt for artificial materials was in nineteenth century when Jagdish Chuder Bose conducted the microwave experiment on artificial chiral elements at that time known as twisted structures[1]. The kock[3] in 1948 managed to construct lightweight microwave lenses using conducting bodies like sphere, strips, disks by arranging them periodically. In 1967 team of Veselago theoretically assumed a material having simultaneous negative permittivity and permeability[4]. The result of the investigation showed, for a monochromatic plane wave in such material the direction of poynting vector was antiparallele to the phase velocity, unlike to the conventional simple media. Since then the interest of researchers in metamaterials increased drastically.

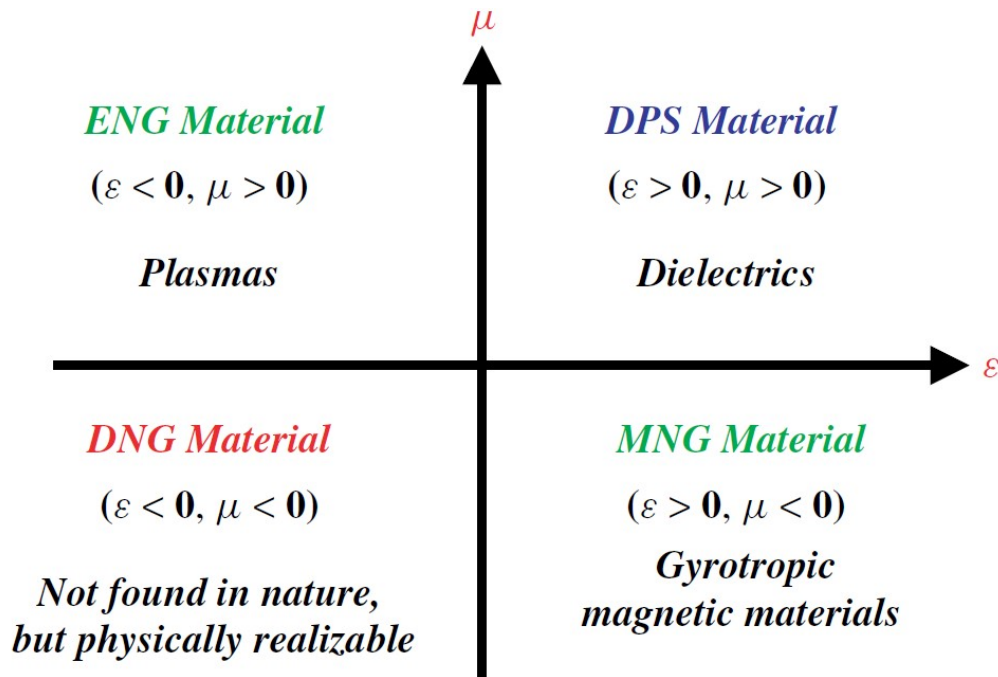
In a simple definition metamaterials are advanced type of materials which take their property from the chemistry of material as well as from the composition and arrangement of material. But the size, shape, composition of inclusion, density, arrangement and alignment of inclusion all play a vital role but it is to be mentioned that desired outcome is not guaranteed for all values of input parameters. The basic difference between metamaterials and photonic crystals is that metamaterials require sub-wavelength structures of inclusion while in photonic crystals periodicity of inclusion is same of the order of operating their wavelength. In recent developments new synthesis and fabrication techniques (molding sintering method, cutting molding method, cavity injection method, template self injection method and drilling method for microwave operation, Deep reactive ion etching and laser micromatching method, 3D direct writing method are some of the examples for terahertz operations) have overcome the problem of physical realization of metamaterials.

So what happens with EM waves in such materials could be understood as, In a particulate composite media, EM-wave interacts with composites of host, hence electric and magnetic moment, that in reaction affects the properties of material at macroscopic level such as effective permittivity & permeability of bulk and As we know, the reaction of a system in

interacting with electric & magnetic field is dependent on the epsilon and mu properties of the material, which is shown below.-

$\epsilon > 0, \mu > 0$ (DPS)	Double positive media	Almost all the natural material(e.g. dielectrics) comes under this designation.
$\epsilon < 0, \mu > 0$ (ENG)	Epsilon negative media	In certain frequency regime plasmas behave to meet this characteristic.
$\epsilon > 0, \mu < 0$ (MNG)	Mu negative media	Some Gyrotropic materials exhibit this characteristic in certain frequency span.
$\epsilon < 0, \mu < 0$ (DNG)	Double negative media	Till now, no natural material has been found with this characteristic. These type of materials has only been realized with artificial constructs.

**Table 1.1: classification of materials**



**Figure 1: Metamaterial classification diagram.**

When we describe the material by frequency independent parameters, we don't consider the fact that in actual manner these properties are highly dependent on the operation of frequency. There has been many trials to describe the material with different models with frequency response. Since the magnetic field is small in comparison to the electric field by its wave impedance, so we generally think of describing such the model on the basis of electron motion with nucleus that creates a dipole, and in the changes that are produced by electric field. This phenomenon, leads to susceptibility model and permittivity. Also, media, for which the magnetic field is dominant, leads to permeability as a function of frequency with the help of these models. There have been many attempts in which the Lorentz model is very popular.

## 1.2 Lorentz model for permittivity and permeability:

In this model we describe the electron and nucleus motion in material as damped oscillator driven by electric field. This model correlates the domain response of an element to polarizing fields in media. The equation for damped oscillator could be written as-

$$\frac{d^2}{dt^2} P_i + \Gamma_L \frac{d}{dt} P_i + \omega_0^2 P_i = \epsilon_0 \chi_L E_i$$

From left 1st term accounts for accelerations of the changes, the second accounts damping mechanism with damping coefficient  $\Gamma_L$  in the system, and third is for restoring force. Where characteristic frequency  $f_o = \frac{\omega_o}{2\pi}$ . The driving term shows the coupling coefficient  $\chi_L$ . In the frequency domain, the response assuming the  $e^{(+j\omega t)}$  time dependence is given by expression-

$$P_i(\omega) = \frac{\chi_L}{-\omega^2 + j\Gamma_L\omega + \omega_o^2} \varepsilon_o E_i(\omega)$$

where small losses  $\frac{\Gamma_L}{\omega_o} \ll 1$ , the material exhibits resonating behavior at  $f_o$ . Where  $f_o$  is natural frequency. The relation between electric field and polarization in terms of electric susceptibility is given as-

$$\chi_{e,Lorentz}(\omega) = \frac{P_i(\omega)}{\varepsilon_o E_i(\omega)} = \frac{\chi_L}{-\omega^2 + j\Gamma_L\omega + \omega_o^2}$$

Then permittivity is-

$$\varepsilon_{Lorentz}(\omega) = \varepsilon_o [1 + \chi_{e,Lorentz}(\omega)]$$

The special scenarios for the Lorentz model are also obtained when one consider the acceleration term ignorable then it obtains Debye model-

$$\Gamma_d \frac{d}{dt} P_i + \omega_o^2 P_i = \varepsilon_o \chi_d E_i \quad \chi_{e,Debye}(\omega) = \frac{\chi_d}{j\Gamma_d\omega + \omega_o^2}$$

For restoring force  $\ll 1$ , one obtains the Drude model:

$$\frac{d^2}{dt^2} P_i + \Gamma_D \frac{d}{dt} P_i = \varepsilon_o \chi_D E_i \quad \chi_{e,Drude}(\omega) = \frac{\chi_D}{-\omega^2 + j\Gamma_D\omega}$$

Here the coupling coefficient in terms of plasma frequency is given as  $\chi_D = Wp^2$ .

If coupling coefficient is positive, then negative permittivity is given by Lorentz and Drude model only. Since the Lorentz model is resonant, susceptibility's real part and hence the permittivity become negative in a narrow frequency span, ahead of resonance frequency. In contrast to Lorentz model, the Drude model yields a negative real part of permeability over a

wide span i.e., for:  $w < \sqrt{(Wp^2 - \Gamma_D)}$

Similarly for magnetic response. Using-

$$E_i \rightarrow H_i$$

$$\frac{P_i}{\epsilon_0} \rightarrow M_i$$

It is the wide area of Metamaterials studies and complexity that requires the introduction for generalized models. For example, in the most general second-order theoretical models that has been introduced, The Two Time Derivative Lorentz Metamaterial (2TDLM) model is best suited.

$$\frac{d^2}{dt^2} P_i + \Gamma_L \frac{d}{dt} P_i + \omega_0^2 P_i = \epsilon_0 \chi_\alpha \omega_p^2 E_i + \epsilon_0 \chi_\beta \omega_p \frac{d}{dt} E_i + \epsilon_0 \chi_\gamma \frac{d^2}{dt^2} E_i$$

$$\chi_{e,2TDLM}(\omega) = \frac{\chi_\alpha \omega_p^2 + j \chi_\beta \omega_p \omega - \chi_\gamma \omega^2}{-\omega^2 + j \Gamma_L \omega + \omega_0^2}$$

This model not only includes all standard behaviors of Lorentz model, like resonance for natural frequency but also permits the additional driving mechanism. Which is important in the calculation of time dependent phenomenas. This model satisfies a generalized Kramer's Krönig relation that is for casual  $\chi_\gamma > -1$  and for limiting behavior

$$\lim_{\omega \rightarrow 0} \chi_{e,2TDLM}(\omega) \rightarrow \chi_\alpha \text{ and } \lim_{\omega \rightarrow \infty} \chi_{e,2TDLM}(\omega) \rightarrow \chi_\gamma .$$

# Chapter 2

## 2.1 Wave parameters in DNG media:

Ziolkowski and Heyman studied this mathematical concept and shown that DNG media are possible. Where  $\varepsilon < 0, \mu < 0$  are the properties of a DNG media. for small losses-

$$\sqrt{\varepsilon} = \sqrt{\varepsilon_r \varepsilon_0 - j\varepsilon''} \approx -j \left( |\varepsilon_r \varepsilon_0|^{1/2} + j \frac{\varepsilon''}{2|\varepsilon_r \varepsilon_0|^{1/2}} \right)$$

$$\sqrt{\mu} = \sqrt{\mu_r \mu_0 - j\mu''} \approx -j \left( |\mu_r \mu_0|^{1/2} + j \frac{\mu''}{2|\mu_r \mu_0|^{1/2}} \right)$$

And hence the wave number and impedance can be expressed as-

$$k = \omega \sqrt{\varepsilon} \sqrt{\mu} \approx -\frac{\omega}{c} |\varepsilon_r|^{1/2} |\mu_r|^{1/2} \left[ 1 + j \frac{1}{2} \left( \frac{\varepsilon''}{|\varepsilon_r| \varepsilon_0} + \frac{\mu''}{|\mu_r| \mu_0} \right) \right]$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\varepsilon}} \approx \eta_0 \frac{|\mu_r|^{1/2}}{|\varepsilon_r|^{1/2}} \left[ 1 + j \frac{1}{2} \left( \frac{\mu''}{|\mu_r| \mu_0} - \frac{\varepsilon''}{|\varepsilon_r| \varepsilon_0} \right) \right]$$

Where  $c = 1/\sqrt{\varepsilon_0 \mu_0}$  (speed of light) and free space wave impedance  $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ .

Also refractive index-

$$n = \frac{kc}{\omega} = \sqrt{\frac{\varepsilon}{\varepsilon_0}} \sqrt{\frac{\mu}{\mu_0}} = - \left[ \left( |\varepsilon_r| |\mu_r| - \frac{\varepsilon'' \mu''}{\varepsilon_0 \mu_0} \right) + j \left( \frac{\varepsilon'' |\mu_r|}{\varepsilon_0} + \frac{\mu'' |\varepsilon_r|}{\mu_0} \right) \right]^{1/2}$$

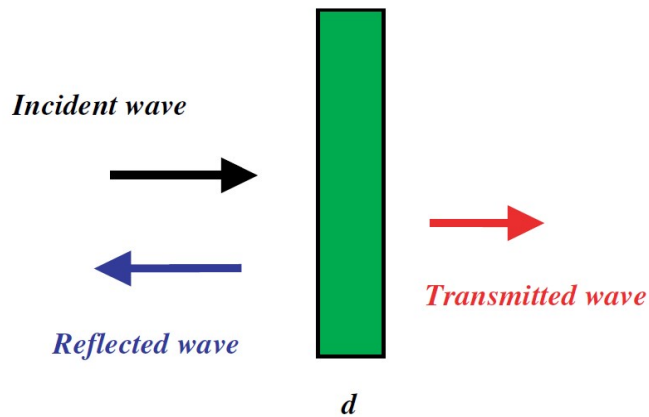
$$\approx -|\varepsilon_r|^{1/2} |\mu_r|^{1/2} \left[ 1 + j \frac{1}{2} \left( \frac{\varepsilon''}{|\varepsilon_r| \varepsilon_0} + \frac{\mu''}{|\mu_r| \mu_0} \right) \right]$$

possesses a negative real part.

The refractive index with negative value in a DNG Metamaterial has also been successfully explained theoretically and achieved experimentally by several studies such as compensating the phase and in small resonators, negative angle refraction, sub-wavelength waveguide.

## 2.2 Scattering from a DNG slab:

For a normally incident plane wave the reflectivity and transmittivity can be calculated. For



the slab displayed in figure-

For the slab having an infinite length in transverse direction and width  $d$  in the direction of propagating wave the media is characterised by  $\epsilon_1, \mu_1$  and slab be characterised by  $\epsilon_2, \mu_2$ .

For normal incidence-

$$R = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \frac{1 - e^{-j2k_2 d}}{1 - [(\eta_2 - \eta_1)/(\eta_2 + \eta_1)]^2 e^{-j2k_2 d}}$$

$$T = \frac{4\eta_2 \eta_1}{(\eta_2 + \eta_1)^2} \frac{e^{-jk_2 d}}{1 - [(\eta_2 - \eta_1)/(\eta_2 + \eta_1)]^2 e^{-j2k_2 d}}$$

Eq...1.12

For a matched DNG medium,  $\eta_2 = \eta_1$ , so that  $R = 0$  and  $T = e^{-jk_2 d} = e^{+j\|k_2\|d}$ .

Thus the medium adds a positive phase to the wave transmitting inside the slab. Unlike the DPS media, phase variation becomes negative.

Hence a matched DNG slab is mostly placed to counter the changes caused by propagation of planewave through DPS media, then  $k_{DPS}d_{DPS} + k_{DNG}d_{DNG} = 0$ .



For oblique incident equation (1.12) is clearly modified. By putting longitudinal wavenumber elements & transverse impedance. If one consider this incident wave as evanescent, i.e. transverse element of wave vector in incidence is larger comparing to wave number in medium ( $k_t^2 > w^2\mu_1\varepsilon_1$  and  $k_t^2 > w^2\mu_2\varepsilon_2$ ). then the transverse wave impedance in both media become purely imaginary. Here we are assuming the no loss condition, that is,  $\eta_{1,transverse} = jX_{1,transverse}$  and  $\eta_{2,transverse} = jX_{2,transverse}$ , and longitudinal element of wave in both media also turns to be imaginary, i.e.,  $k_{1,longitudinal} = j\alpha_1$  and  $k_{2,longitudinal} = j\alpha_2$  [6,7].

(The choice of the signs for  $\alpha_1, \alpha_2$  remains to be discussed.). The transverse impedance of the wave in DPS- DNG medium possess unlike signs. Meaning that if one works as a capacitor, the other works as inductor. Hence  $sgn[X_{1,transverse}] = -sgn[X_{2,transverse}]$ , where  $sgn(x) = +1(-1)$  for  $for x > 0(x < 0)$ [6,7]. Choosing  $\mu_2 = -\mu_1$  and  $\varepsilon_2 = -\varepsilon_1$ , we can show that  $X_{1,transverse} = -X_{2,transverse}$ . Inserting these values into eqn 1.12. one sees  $R = 0$  but now  $T = e^{-jk_{2,longitudinal}} = e^{j\alpha_2 d}$ . Now What should be the sign for  $\alpha_2$ ? As we know that at the boundary of a DPS and DNG media, the tangential elements of the EM fields needs to be continuous. but, the epsilon and mu of both media are with unlike signs, the normal spatial derivative of these tangential elements are discontinuous at this boundary [8]. Meaning that, if the tangential element of EM field decreases with observation point getting close to boundary from the double positive side, the same element must increase as we recedes from the boundary in the double negative region. It is to be remind that, according to Eq. (1.12) the total reflection for “incident” evanescent wave for this scenario is  $R = 0$ . That’s why, when the evanescent wave reaches the DNG part of 1<sup>st</sup> boundary in matched slab from DPS side, it decays, i.e.  $\alpha_1 < 0$ , and no “reflected” evanescent wave will be there. But, as we cross the 1<sup>st</sup> boundary of slab into DNG part, the tangential element of field around the domain of the boundary in the DNG region should “grow” for satisfying the discontinuity conditions. (Remark, if the evanescent wave decays in double negative slab, the tangential elements at DPS–DNG boundary will show same slopes, without being consistent boundary conditions discussed before.) That’s why, in the transmission coefficient value  $T = e^{\alpha_2 d}$  we should take  $\alpha_2 > 0$ . that’s how, a matched DNG slab counter and compensate the decay of tangential waves in DPS region by growing the evanescent waves in the DNG slab. This was firstly revealed by Pendry [7] and is the basic concept for the applications of subwavelength focusing as well as perfect lenses [7].

A question may arise here: If we get the growth of evanescent waves inside the slab, then what about a semi-infinite matched Double negative media?? What about “incident” evanescent waves approaching this boundary? This would be a completely different problem. In that scenario, we deal with one boundary, and the reflection and transmission coefficients would be-

$$R_{DPS-DNG} = \frac{\eta_{2,transverse} - \eta_{1,transverse}}{\eta_{2,transverse} + \eta_{1,transverse}}$$

$$T_{DPS-DNG} = \frac{2\eta_{2,transverse}}{\eta_{2,transverse} + \eta_{1,transverse}}$$

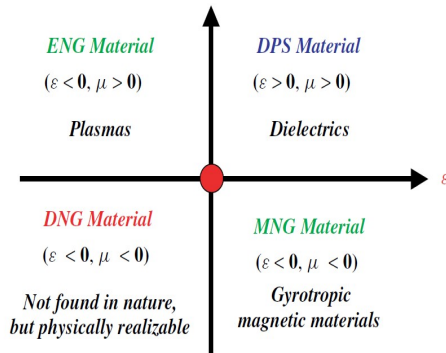
For the matched condition, as described before, we have  $\eta_{1,transverse} = jX_{1,transverse}$  and have  $\eta_{1,transverse} = jX_{2,transverse}$  with  $X_{1,transverse} = X_{2,transverse}$ . That's why,  $R_{DPS-DNG} = \infty$  and  $T_{DPS-DNG} = \infty$ , which shows the boundary resonance the boundary. This is ofcourse one other proof for this boundary nourishing a surface plasmons wave, that is a relevant factor for understanding the interactions of this boundary [6,7]. This is similar to excite a resonance circuit at its resonance frequency, which also lead towards infinitely extended fields in the composed material when no losses are there. Hence we say, whenever there is a source facing the boundary of two semi-infinite joint DNG and DPS media, a resonating wave at the surfaces excites parallel to the boundary, giving a very large value of fields. But, the fields at both sides near this boundary, decay exponentially with moving away from boundary; hence, field distribution shows the generation of surface waves along the boundary.

Hence the presence of  $R_{DPS-DNG} = \infty$  and  $T_{DPS-DNG} = \infty$  is justified.

# Chapter 3

## 3.1 Zero refractive index:

Metamaterial possessing permeability and permittivity near zero and the index of refraction, much smaller than one, offers very important application. We can locate their position in  $\epsilon - \mu$  diagram with a red dot. Metamaterials with negative and positive values of negative refractive index has been realized both theoretically and experimentally, by researchers. Since, Propagation constant continuously passes through zero (zero index) with slope  $\neq 0$  ( $v_g \neq 0$ ) in its span from a DNG to DPS region. By considering a source in a ZIM, electromagnetic band gap with a excitation frequency which lies in span of the EBG's pass band researchers got extremely narrow antenna patterns.



Alù et al. has described that by enclosing a subwavelength very small aperture in any plane screen which is a perfect conductor with slab of  $\mu < \mu_0$ , the power transmission from this hole, due to coupling of wave of incidence with the leaky wave, power increases significantly. By covering both side of hole the power can not only be increased but also it can be converged and directed as a sharp beam in a any required direction.

The properties the propagating & scattering, of dispersive material which is comparable to free space, with zero refractive index, was studied theoretically by Ziolkowski using FDTD simulations. It was described that the EM field in matched ZIM ((e.g.  $\epsilon_{real}(w_0) \cong$

$0, \mu_{real}(w_o) \cong 0$ ), hence  $(w_o) = Z_o$  and  $\eta_{real}(w_o) \approx 0$ ) takes static feature in space, even when remaining dynamic in time, such that their underlying physics remains undisturbed .

To illustrate this nature, consider Maxwell's equation-

$$\nabla \times E_w = -j\omega\mu H_w$$

$$\nabla \cdot \varepsilon E_w = \rho_w$$

$$\nabla \times H_w = J_w$$

$$\nabla \cdot \mu H_w = 0$$

Where  $\varepsilon_{real}(w_o) \cong 0, \mu_{real}(w_o) \cong 0$  above equations reduce to-

$$\nabla \times E_w = 0$$

$$\nabla \cdot \varepsilon E_w = 0$$

$$\nabla \times H_w = J_w$$

$$\nabla \cdot \mu H_w = 0$$

If the fields are finite right side of equation automatically remains consistent[1].

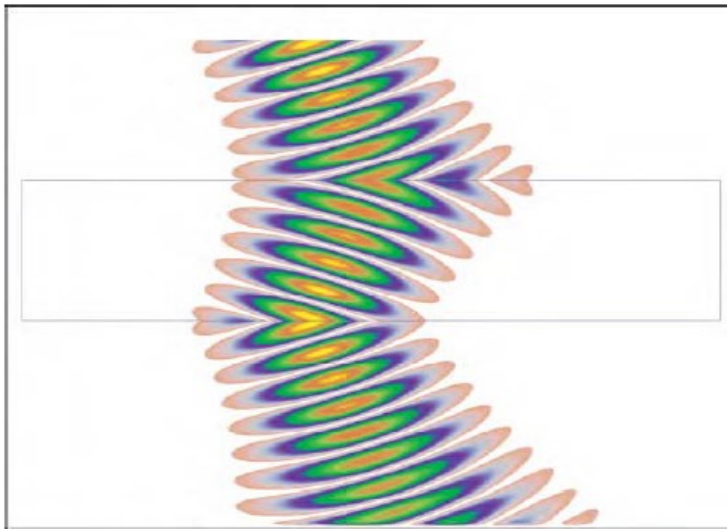
### 3.2 Basics of waveguide and antenna applications:

If one pairs a DNG substance with a DPS substance or ENG, MNG layer, it may experience that poynting vector becomes antiparallel with phase velocity, it is also to be mentioned that even after the interference between two mediaum with atleast one of them with opposite sign parameter, plays a major role for anomalous behavior due to combined structure. Continuity tangential equation at the boundary in between two medium is given as -

$$\frac{1}{-j\omega\mu_1} \frac{\partial E_{1,tan}}{\partial n} \Big|_{\text{Interface}} = \frac{1}{-j\omega\mu_2} \frac{\partial E_{2,tan}}{\partial n} \Big|_{\text{Interface}}$$

$$\frac{1}{j\omega\varepsilon_1} \frac{\partial H_{1,tan}}{\partial n} \Big|_{\text{Interface}} = \frac{1}{j\omega\varepsilon_2} \frac{\partial H_{2,tan}}{\partial n} \Big|_{\text{Interface}}$$

Here  $\frac{\partial}{\partial n}$  is for normal derivative and  $\epsilon_i, \mu_i, i = 1, 2$  are the permittivity and permeability for both medium. Here we can see that normally derived tangential elements need not be continuous and further more if  $\mu_1$  and  $\mu_2$  or  $\epsilon_1$  and  $\epsilon_2$  have different signs then the derived tangential elements or both side in interference will have opposite signs. Therefore we see a V shaped discontinuity.



**Figure 3: The NIR behavior has been confirmed with FDTD simulations, from[22] copyright @2003 The optical society of America.**

The tangential element of the fields at the interference of two media[1] that implies a intense resonant phenomena at boundary is quite alike to current and voltage distribution at boundary of  $L - C$  circuit. Also it is to be noted that these “boundary resonances” is actually independent of the width of paired components. because it is arising parallel to discontinuity of these two materials[2]. The phenomenon is described in many methods, one of them is equivalent circuit consideration[9].

# Chapter 4

## 4.1 Subwavelength cavities and waveguides:

As we know the boundary resonance near the junction of two conjugative material with one of them having oppositely signed characteristic parameter provides concentrated resonant phenomenon. For the purpose of design of thin, cavity resonators & parallel plate waveguides, pairing of a thin layers of BNG media with layers of DPS media or a ENG's and MNG's together or with DPS material provides antiparallel behavior of phase velocity and poynting vectors in DNG slab.

The figure 2.1 shows the structure in this scenario-

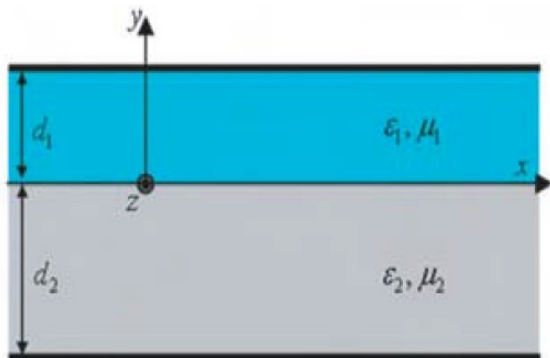


Figure for parallel plate waveguides in 1D composed of any pair of two different materials having alike epsilon and mu values and signs. @ IEEE[5].

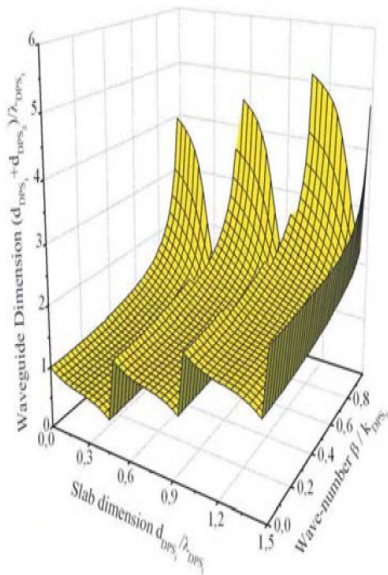
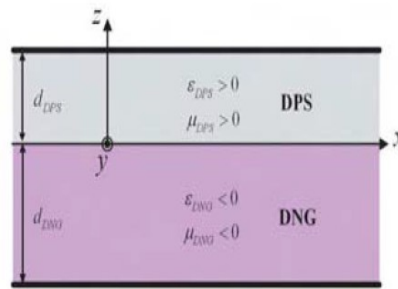
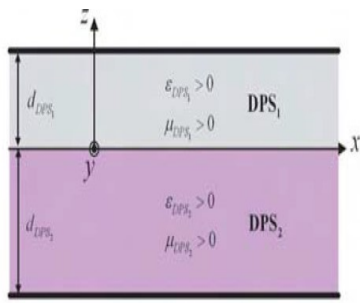
It is composed of two parallel layers, perfectly conductors (PECs) composed two rectangular slabs of homogeneous & isotropic media with  $\epsilon_1, \mu_1$  and  $\epsilon_2, \mu_2$  and thickness  $d_1, d_2$ . In the thin waveguide limit, like the conventional DPS-DPS scenario, dispersion for supporting modes is not related to totals width of waveguide  $d = d_1 + d_2$  but related to the fraction  $\frac{d_1}{d_2}$ , which leads to theoretical possibilities of constructing waveguide with supportive resonant mode. Even if the total thickness approaches very small values comparing to electric field

this mechanism persists [10]. This is shown in figure 2.2. where both the scenarios DPS-DNS and DPS-DNG waveguides has been compared. The dispersion curves for transverse electric ( $TE_x$ ) polarization sketches exhibiting minimum thickness for waveguide,  $d_1 + d_2$ , is necessary to get a supportive mode propagation with factor  $e^{-j\beta x}$ . for any combination  $(d, \beta)$  dispersion relation  $TE_x$  could be expressed as-

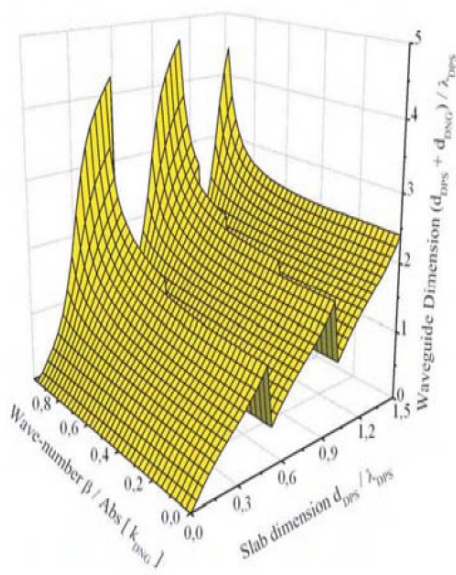
$$\frac{\mu_1}{k_{t1}^{TE}} \tan(k_{t1}^{TE} d_1) = -\frac{\mu_2}{k_{t2}^{TE}} \tan(k_{t2}^{TE} d_2)$$

With-

$$k_{t1} = \sqrt{\omega^2 \varepsilon_1 \mu_1 - \beta^2}$$



(a)



(b)

**Figure 4: dispersion plots (a) in case of DPS-DPS matching and (b) for DPS-DNG matching**  
**From[7] @ IEEE.**

and also  $w$  is a monochromatic radial frequency of excitation. Figure 2.2 clearly distinguishes between these two scenarios. The minimum required thickness for a conventional DPS-DPS waveguide is a finite value (which is the cut off thickness), on the other hand in case of DPS-DNG waveguide, thickness is independent of  $\beta$ , a mode is supportive if the ratio  $\frac{d_1}{d_2}$  is sufficiently close to the value of  $-\frac{\mu_2}{\mu_1}$ . The interesting fact is we can describe this phenomena is present at the boundary in form of compact resonance. The resonance that occurs “spatially” in DPS-DPS waveguide, might be replaced with DPS-DNG waveguide. This permits, a significant decrease in the waveguide lateral dimension. Reduction in lateral dimensions has been shown experimentally (see, e.g., [11]). As expected in [1], the possibilities of resonating cavities having subwavelength dimension loaded with conjugate materials, opens up many values and applications. Similarly, 2D, 3D cavities (subwavelength) or Fabry–Perot elements can be improvised. These cavities allow the realistic possibilities of applications beyond the diffraction limits [12,13].

#### 4.2 EFFICIENT, ELECTRICALLY SMALL DIPOLE ANTENNAS:

Now the question is : Can we use a DNG (or SNG) layer for modifying the input impedance of antennas or make it equal to the free space impedance. In order to improve the overall antenna performance, Ziolkowski and Kipple has studied this problem [14]. According to their study if one surrounds an small electric dipole with DNG shell then the system produces larger radiated power compared to the currently working antennas [14].

If we take an small electric dipole antenna of length  $\ell$  (ideal case) with driving current  $I_0$ , frequency  $f_0$ , wavelength  $\frac{1}{f_0\sqrt{\epsilon\mu}}$  and it is embedded in double positive media such that wavenumber  $k = w\sqrt{\epsilon\mu} > 0$  with wave impedance  $\eta = \sqrt{\mu}/\sqrt{\epsilon}$ . Then it produces the complexity in power for radius  $r$  given by [15]

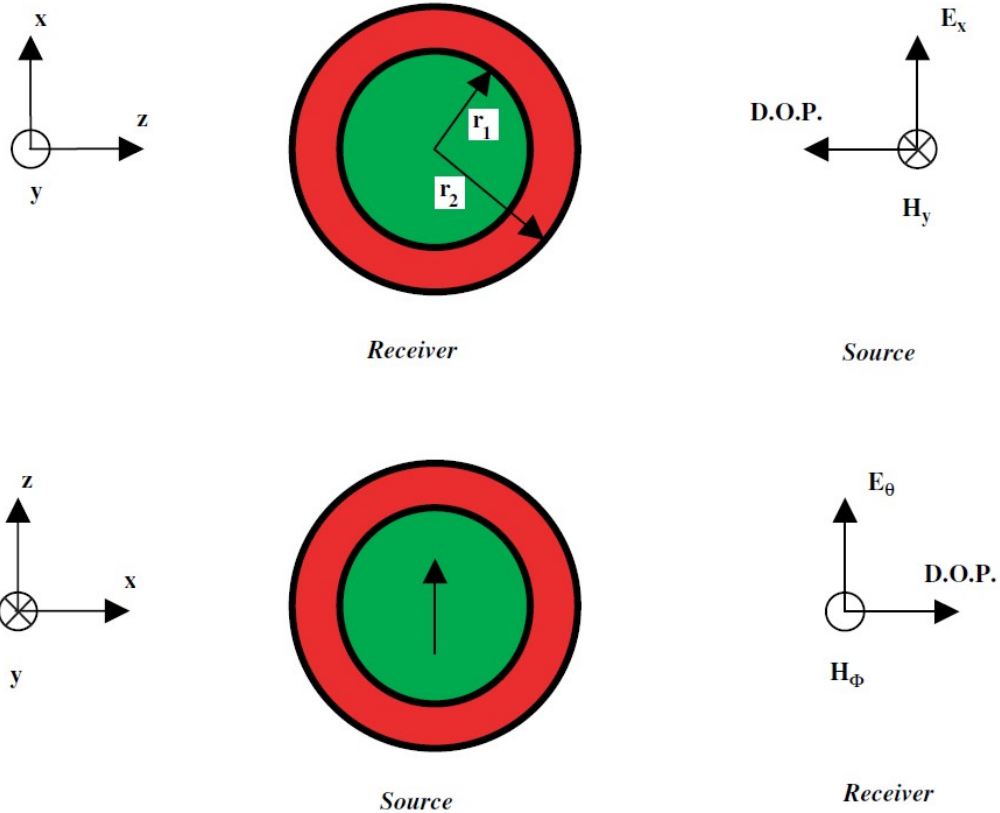
$$P = \iint_S \left( \frac{1}{2} \mathbf{E}_\omega \times \mathbf{H}_\omega^* \right) \cdot \hat{r} \, dS = \eta \left( \frac{\pi}{3} \right) \left| \frac{I_0 \ell}{\lambda} \right|^2 \left[ 1 - j \frac{1}{(kr)^3} \right] = P_{\text{rad}} + jP_{\text{reac}}$$



[using harmonic signals of time:  $\exp(+j\omega t)$ ]. The power of capacitive reactive element in equation above is very high near the electrically small antennas (e.g., for  $r = a$ ). Here  $a$  is radius for smallest sphere which surrounds the dipole antenna and highly restrains the efficiency as a radiation producer. Reactance ratio  $\frac{P_{react}}{P_{rad}} \gg 1$  for  $ka \ll 1$  indicates that antennas radiated power is very small in comparison to reactive power of electrically small radiators. Since the reactive power of a dipole dominates according to the energy of the electric field, that is,  $P_{reacc} = \omega(W_m - W_e) \approx -\omega W_e$ , where  $W_e$  &  $W_m$  are respective time-averaged EM-field energies. Later studies found that this capacitive reactance turns into an inductive reactance with DNG media applications. Where the wavenumber becomes  $k = \omega\sqrt{\epsilon}\sqrt{\mu} = -\omega\sqrt{\|\epsilon\|}\sqrt{\|\mu\|} < 0$  & wave Impedance  $\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}} = \frac{\sqrt{\|\mu\|}}{\sqrt{\|\epsilon\|}} > 0$  [14].

But in actual, this negative permittivity loading of the capacitor helps it to act like an inductor. That's how, the path for this antenna applications with hollow DNG shell was came into picture.

Most of the time, the 3-region (two-nested-sphere) geometry is used for most of the study that is displayed in Figure below. The antenna or in particular dipole antenna is placed along z-axis at centre in Nested sphere. The unknowns in above relation can be found by forcing the boundary conditions. With  $E_\theta, E_\phi$  continuous at boundary of the shell, the resulting equation, present in [14], were easy to obtain which can be solved numerically. They are also suited for including passive losses in parameters of the media; that is, parameters are already set to  $\epsilon = \epsilon_r\epsilon_o - j\epsilon''$  and  $\mu = \mu_r\mu_o - j\mu''$ , where  $\epsilon_r, \mu_r < 0$  and  $\epsilon'', \mu'' > 0$ . The electrically tiny dipole Antenna  $\ell = 100\mu m = \lambda_o/300$  driven at  $f_o = 10GHz$



**Figure the propagating wave direction at source and the receiver is shown @ America physical society.**

was embedded in small sphere composed of DPS material (free space) having a radius  $r_1 = 100\mu m$ , which is also surrounded by a DNG shell with radius  $r_2$  with permittivity and permeability values  $(\epsilon_{2r}, \mu_{2r}) = (-3, -3)$  and equal electric and magnetic loss tangents  $LT = \epsilon''/|\epsilon_o|\epsilon_r$  set to the fixed values  $LT = 0, 0.00001, 0.0001, 0.001$ . The outer region  $r > r_2$  is considered as a free space. Here both the components of metamaterial antenna and the free-space reference dipole antenna were ideal. The driving current here is 1.0 A. The radiated power gain plots w.r.t. the outer radius of DNG shells are drawn in Figure 2.14. Here maximum for lossless scenario is at  $r_{2,max} = 185.8\mu m$ . This arrangement is a natural mode of the systems and produce an enhanced resonance, even after being enough tiny than a free space wavelength ( $r_{2,max} = \lambda_o/161$ ). This mode was earlier confirmed as natural mode..

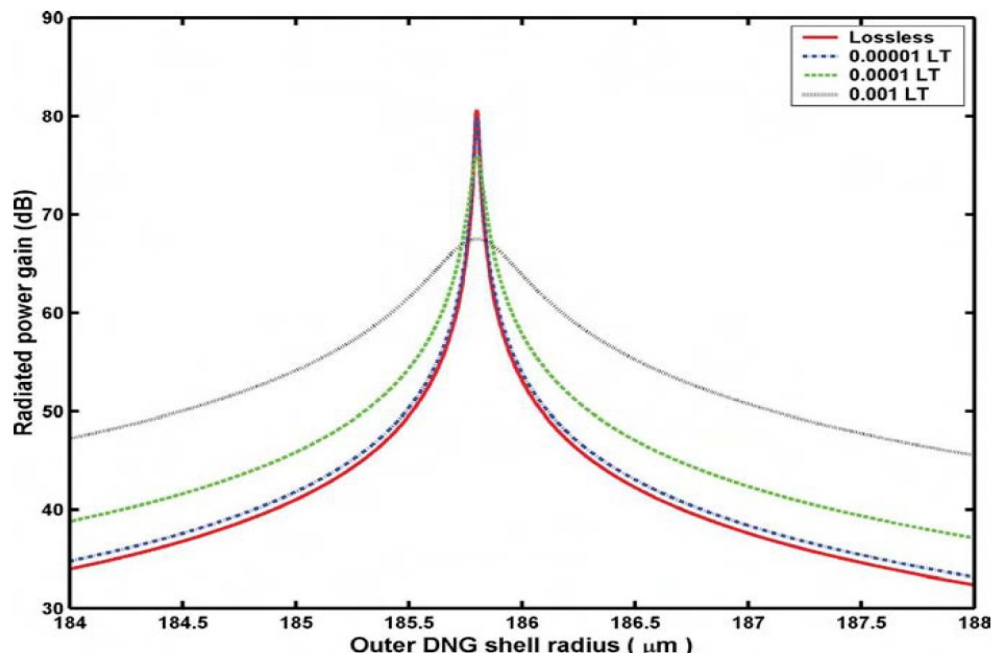


Figure 5: gain of radiated power for  $r_1 = 100\mu m$ .

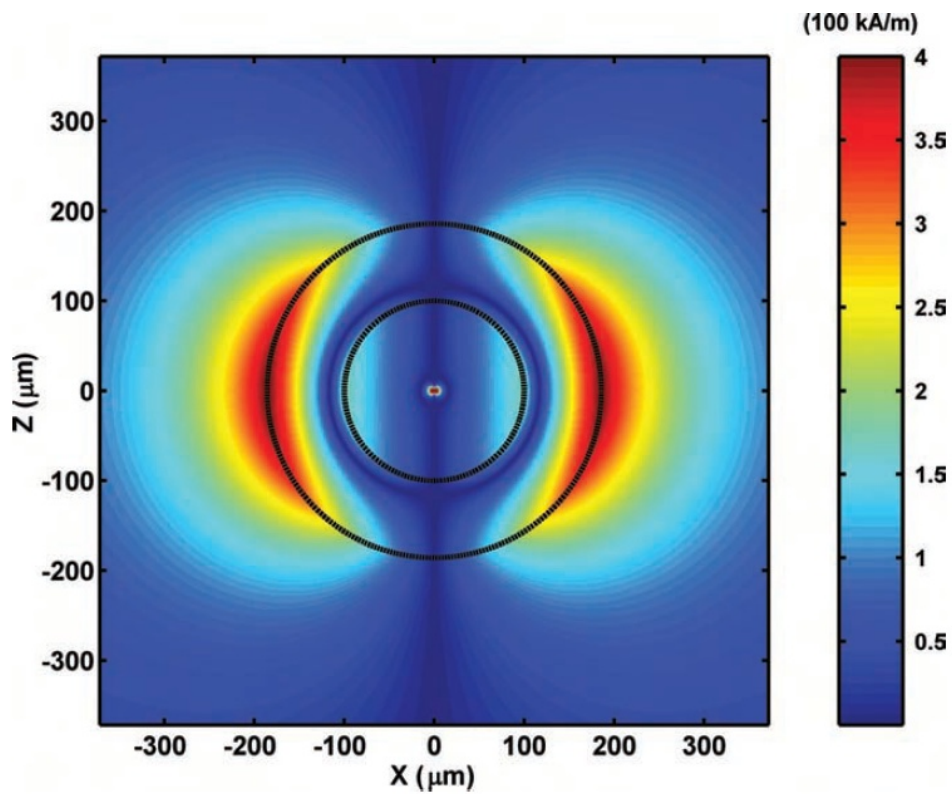


Figure 6 contour plots with  $r_1 = 100\mu m$  and  $r_2 = 185.8\mu m$ . @IEEE

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