

MC 404: Matrix Computation

Time: 1:30 Hours

Max. Marks: 25

Note: All questions are compulsory.

- Consider the following floating point number system.
Base $\beta = 2$, length of mantissa $m = 3$ and range of exponent $\{-1,0,1,2\}$. Find all the decimal numbers which can be represented by this system. [5]
- Explain ill conditioning of a matrix. Let $A = \begin{bmatrix} 0.1a & 0.1a \\ 1 & 1.5 \end{bmatrix}$. Determine a such that $cond(A)$ is minimized. Use the maximum norm. [5]
- Consider the following system of equations.
$$0.0030x_1 + 59.14x_2 = 59.17$$
$$5.291x_1 - 6.130x_2 = 46.78$$

Solve the system using Gaussian elimination and use four digit arithmetic with rounding. Is it a good approximation to the exact solution? If no, then why it so and demonstrate how can you get a good approximation? [5]
- Find the Cholesky decomposition of the matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Hence, find A^{-1} . Write two limitations of Cholesky decomposition. [5]
- Find the QR factorization of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and use this factorization to solve the least squares problem $Ax = [1 \ 2 \ 3 \ 4]^T$. [5]

~All the Best~