STRATEGIC PLANNING AND DECISION MAKING PROBLEMS IN THE BILEVEL PROGRAMMING FRAMEWORK

A thesis submitted to

DELHI TECHNOLOGICAL UNIVERSITY

in partial fulfilment of the requirements of the Award of Degree of

DOCTOR OF PHILOSOPHY

in

MATHEMATICS

by

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March 2021 **Enroll. No.: 2K14/PhD/AM/02**

DECLARATION

I declare that the research work reported in this thesis entitled **"Strategic Planning and Decision Making Problems in the Bilevel Programming Framework"** for the award of the degree of *Doctor of Philosophy in Mathematics* has been carried out by me under the supervision of *Prof. Anjana Gupta*, Department of Applied Mathematics, Delhi Technological University, Delhi, India.

The research work personified in this thesis, except where otherwise indicated, is my original research. This thesis has not been submitted by me earlier in part or full to any other University or Institute for the award of any degree or diploma. This thesis does not contain other person's data, graph or information unless specifically acknowledged.

Date: 04.03.2021 **(AKHILESH KUMAR)**

CERTIFICATE

This is to certify that the thesis entitled **"Strategic Planning and Decision Making Problems in the Bilevel Programming Framework"** submitted by **Mr. Akhilesh Kumar** in the Department of Applied Mathematics, Delhi Technological University, Delhi, India for the award of degree of Doctor of Philosophy in Mathematics, is a record of bonafide research work carried out by him under my supervision.

I have read this thesis and that, in my opinion, it is fully adequate in scope and quality as a thesis for the degree of Doctor of Philosophy.

To the best of my knowledge the work reported in this thesis is original and has not been submitted to any other Institution and University in any form for the award of any degree or diploma.

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ACKNOWLWDGEMENTS

In the accomplishment of this thesis there is contribution of many people in terms of their support, guidance, and assistance at multiple occasions and phases of my research work. I take this opportunity to acknowledge their countless contributions with gratitude.

I wish to express my sincere gratitude to my supervisor Prof. Anjana Gupta, Department of Applied Mathematics, Delhi Technological University (DTU), Delhi for her inspiring guidance and support during my research work and preparation of this thesis. The duration of my research work under her supervision is going to be a memorable phase of my life.

I earnestly thank Prof. S. Sivaprasad Kumar, Head of Department, Department of Applied Mathematics, DTU, for her all time support during the progress of the work.

I extend my sincere thanks to Prof. H. C. Taneja, Chairman of Departmental Research Committee, Department of Applied Mathematics, DTU for his everlasting support and guidance. I extend my gratitude to Prof. S. K. Garg, Department of Mechanical Engineering, DTU for his suggestions and administrative support during his tenure as Pro Vice Chancellor. The encouragement of all faculty members of the Department of Applied Mathematics and other Departments of DTU is acknowledged. I am thankful to all the members of DRC and SRC for their appreciation on my research work.

I am extremely grateful to Prof. Aparna Mehra, Department of Mathematics, Indian Institute of Technology, Delhi for her extended and valuable guidance, and encouragement for my Ph.D. work.

The suggestions of Prof. S. K. Neogy, Head, Indian Statistical Institute, Delhi Centre, Prof. C. K. Jaggi, Department of Operational Research, University of Delhi, and Prof. Uma Nangia, Head, Department of Electrical Engineering, DTU, as part of my Student Research Committee are sincerely thanked.

Staff members of the academic branch and administration of DTU gratefully acknowledged for providing the environment and facilities to carry out my research work. I express my thanks and best wishes to the office staff of the Department of Applied Mathematics for their all kinds of support.

A sincere gratitude is due to the administration of U.T. of Puducherry, especially the officials of Directorate of Higher and Technical Education, Puducherry for according permission to continue the Ph.D. programme as Part-Time along with the service.

I take this opportunity to thank the former and present Principal of Arignar Anna Government Arts & Science College (AAGASC), Karaikal, U.T. of Puducherry, for providing me a constructive environment to accomplish my Ph.D. work in coordination with my job responsibilities. I express my sincere thanks to Shri. V. Karuppaiya Pillai, Head, Department of Mathematics, AAGASC, for his extended support and encouragement to continue my Ph.D. work during the tenure of my service under his visionary leadership. I gratefully thank Dr. Manoj Pillai, Head, Department of Commerce, AAGASC, and Shri. Nimain Charan Swain, Asst. Professor, Department of Economics, AAGASC, for their valuable suggestions through extended discussions on industrial and business related terminologies and phenomena.

I owe a sincere thanks to experts from industry, Shri N. K. Panda and Shri Sudhir K. Varma, for their inputs from field to accomplish this interdisciplinary research work.

I wish to record my profound gratitude to my parents, relatives and friends for their all kinds of support and help for my academic achievements. Thank you all.

(AKHILESH KUMAR)

Dedicated to

my Parents

Shri. Omprakash Shastri and Smt. Promila Shastri

CONTENTS

Preface

The journey of this research work got initiated through identification of recent challenges in strategic planning faced by the decision-makers at managerial levels. Through interactions with practitioners of various sectors of the Industry, it started becoming clear that a peculiar situation is predominantly encountered by decision-makers. During a majority of situations of strategic planning, the selection of a move as a course of action by a decision-maker gets a reaction from one or more concerned parties, which in turn affects the objective of the decision-maker under consideration. The introspection of the literature in mathematical programming enabled us to realize that the mathematical modelling of such situations is possible through bilevel programming framework. The industrial interactions and literature review provided the motivation to address these challenges of strategic planning by modelling such decision-making issues as bilevel programming problems. For meticulously and precisely modelling the problems and to test the models with appropriate data, we narrowed down our study to the problems of *Railways* and *supply chain management*, due to approachability to the practitioners in these two sectors.

Subsequent to the task of modelling the addressed issues, another challenge which we faced during our research is the unavailability of solution algorithms to solve the problems modelled as variants of bilevel programming framework. Wherever any algorithms were available for solving such a problem, we discovered those as incapable of handling the problems of a practical scale of ours. This motivated us to work towards the development of solution algorithms for the variants of bilevel programming problems being dealt with, and thus achieve success in our main objective.

In our study, we have addressed both of these challenges collectively and contributed towards the development of decision support for some of the identified challenges of decision-makers which can be categorized within the scope of modelling through the bilevel programming. Further, we have supported our study by an implementation of developed algorithms on the relevant data obtained for appropriate cases from firms facing such problems. This has enabled us to contribute to the society through an optimal utilization of available opportunities.

The thesis entitled **"Strategic Planning and Decision Making Problems in the Bilevel Programming Framework"** comprises of five chapters followed by the bibliography and the list of publications.

The precursory **Chapter 1** manifests strategic planning and decision-making, and decision-support for the same. The concept of bilevel programming along with its variants is then introduced. A survey of literature on decision-making models using bilevel programming framework developed for assisting managerial decisions of firms from various sectors is presented thereafter. Noting some practical issues in the approaches followed in strategic planning, a scope of research for developing a decision-support is observed to fix the objective of thesis along with the plan of research work.

Preliminary concepts from different areas are used in our research work for development of solution algorithms. They need to be introduced with a bit detailed explanation before using them in the presentation of our work in subsequent chapters. All of such relevant concepts are presented in **Chapter 2** for providing the readers with a clear understanding of our interdisciplinary work. Additionally, an independent discussion on a special case of a variant of bilevel programming problem is explicated as a ground work for developing a GA-based solution methodology in a later chapter.

In **Chapter 3**, problem of railways is studied for decision-making on an operational issue of running special trains to tackle higher demand on specific routes during seasons of festivals and holidays. The study includes development of decision support for operational decisions on optimal utilization of rolling-stocks and determining optimal fare-price structure in a competitive environment coerced by other travelling service providers. The influence on the demand-shares by the competitors of railways is incorporated in decision making to utilize the rolling-stock accordingly. The problem is modelled as a mixed integer single-leader-multi-follower bilevel programming problem. A diversified-elitist genetic algorithm is introduced to solve the constructed model. The suggested methodology is illustrated by taking a test situation from Indian Railways. The work presented in this chapter has been published as a research paper entitled **"A Bilevel Programming Model for Operative Decisions on Special Trains: An Indian Railways Perspective"**, in *Journal of Rail Transport Planning & Management* (Elsevier), **8**, (2018), 184-206. doi: 10.1016/j.jrtpm.2018.03.001.

Chapter 4 develops a decision support for strategic pricing and aggregate production distribution planning for a small scale supplier intending to penetrate into a potential market engendered by a single buyer. A novel mixed integer single-leader-single-follower bilevel programming model is developed to formulate the problem in which the supplier is considered as a leader and the buyer as a follower. The proposed model subsumes the assessment of demand share against the price quotation, enabling the supplier to prepare an aggregate production distribution plan accordingly. An integer coded genetic algorithm is developed to solve the model and its implementation is exhibited through a test scenario. The work presented in this chapter is published as a research paper entitled **"A Bilevel Programming Model for a Cohesive Decision Making on Strategic Pricing and Production Distribution Planning for a Small Scale Supplier"**, in the journal *International Game Theory Review* (World Scientific Publishing Company), 22(2), 2020, doi: 10.1142/S0219198920400095.

Chapter 5 studies a strategic problem of price negotiation of the buyer with its multiple suppliers in an oligopolistic-monopsony market. The problem is studied to develop a decision support for identifying target prices for negotiation through which the common goal of all stakeholders *viz* maintaining a sustainable business environment can be achieved. For this purpose it is suggested for the buyer to identify the Nash-equilibrium prices of the suppliers' oligopolistic-competition as target prices, as adopting this strategy helps in avoiding adverse actions from either side. In order to develop a decision support for this strategic issue a mathematical model is formulated as a multi-leader-single-follower bilevel programming problem. A GA-based solution approach is proposed to solve such a bilevel programming problem. The proposed methodology is demonstrated by an implementation of a case of a manufacturing firm of the FMCG sector. The work presented in this chapter is communicated as a research paper entitled **"A Bilevel Game Model for Ascertaining Competitive Target Prices for a Buyer in Negotiation with Multiple Suppliers"** to the Journal *Omega* (Elsevier).

A summary followed by future scope of the research work is evinced to conclude the thesis. Finally, two independent results on convex optimization are presented in Appendix, which are referred in Chapter 3 for developing a methodology to solve a problem modelled there.

The thesis culminates in the bibliography and list of author's publication.

List of Abbreviations

APDP	Aggregate production-distribution planning
BCGA	Binary coded genetic algorithm
BLP	Bilevel programming
DC	Decision centre
DEGA	Diversified-Elitist-Genetic Algorithm
DL	Delivery location
FMCG	Fast-moving consumer goods
FOCs	First order conditions
GA	Genetic algorithm
INR	Indian Rupee
IR	Indian Railways
KKT	Karush-Kuhn-Tucker
LPP	Linear programming problem
MOBLP	Multi-objective BLP
MNL	Multinomial logit
PC	Production centre
RCGA	Real coded genetic algorithm

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Chapter 1

Introduction

Strategic planning and decision-making play a vital role in the progress of an organization and are main deciding factors of its future. Multiple issues in strategic planning and decision-making are faced by managers, in which a decision-maker requires including the reaction of other concerned parties to assess each possible course of action for choosing an optimal one. Bilevel programming (BLP) is an optimization framework which enables to mathematically model such decision-making situations. This introductory chapter is intended to give a brief understanding of structure of various types and variants of BLP and its applications into the strategic decision making. In this chapter, the foundations of BLP are presented followed by a description of its variants. The subsequent part of the chapter provides a review of literature on the design of models developed so far to handle managerial decision-making situations using the BLP framework. The chapter is concluded through a listing of objectives of this thesis and a brief plan of organizing our research work into this thesis.

The importance of strategic planning and decision-making can be understood from the quote of General Robert E. Wood "Business is like a war in one respect, if its grand strategy is correct, any number of tactical errors can be made and yet the enterprise proves successful". A company may overcome inefficient internal resource use if its basic strategy is brilliant, but it is not likely to overcome the wrong strategies even with excellent production and distribution performance [1]. In the current scenario of competitive business environment the importance of game-theoretic aspects of decision-support systems is highly realized due to the necessity of assessing the effects of others' action too. At the same time, situations are prevalently seen where there is a need of incorporating the reactions of other associated decision-makers while taking an action. Situations of decision-making on such strategic planning issues can be appropriately addressed by mathematically modelling them using the BLP framework. This chapter is aimed at providing an introduction to strategic planning and decision-making using the BLP framework.

1.1 Strategic planning and decision-making

Decision-making

Decision-making is an art which needs the skill of understanding the situation in terms of available alternatives and possible consequences in response to the choice of each available alternative. Decision-makers generally use their past experience and intuition to assess the available alternatives and cognitively choose one which they understand to be best.

In perplexed situations, where the future state of an organization depends on present decisions, the factual but precise information and scientific decision-support improve the decision-making ability of decision-makers. For such type of situations, data driven inputs help towards the reliability and validity of the information. Based on these data inputs, mathematical tools for decision-support suggest the best of available options to the decision-maker. Various optimization frameworks and their tools are capable of providing a decision-support to almost every data-driven decision-making situation. Just the need is to develop an appropriate optimization model which captures all the relevant aspects associated with the environment of the situation addressed.

Depending upon the purview, duration of implementation, and realization of its results the decision-making situations in any organization are broadly classified as strategic, operational, and tactical. Strategic decisions are prevalently for long-term durations and involve most crucial factors responsible for the growth and stability of an organization. Operational decisions often involve the planning of regular operations for mid-term durations ranging from a week to a month. Operational decisions are generally taken in pursuance of the strategic decisions. Whereas, tactical decisions, which are known as administrative decisions also, correspond to short-term duration of a day or two and generally cover the daily scheduling of tasks targeting efficient implementation of operational decisions. Among all the three categories discussed above, the strategic decisions are seen as most crucial, experienced to be complex, and require a systematic and rigorous planning on multiple aspects. Taking decision of strategic issues thus requires a proper decision-support. It is therefore appropriate here to know about the strategic planning and its role in management.

Strategic planning and decision-making

Every enterprise or organization has its pre-defined goals and mission to be achieved in a long-term. Each department of an enterprise plans a path of execution of operations for achieving these goals. But, it has been observed from the audits and introspective analysis of enterprises across the globe that there remains a lack of coordination among various departments. This results in some inefficient practices, some of them can be listed here (a) costly duplication of resources across departments, (b) interdepartmental conflicts, (c) difficulties in identifying profitable and unprofitable products. This defies the cohesiveness in planning of activities of various departments and therefore results in disinclination towards the vision and mission of the enterprise. In the current competitive business environment such inefficiencies give a significant setback to the growth of a company and eventually pose a threat to its survival.

Strategists and analysts have realized that the traditional setups of functional or divisional type of organizational structures are responsible for this lack of interdepartmental coordination. Thus, business firms are gradually moving from a disconnected divisional structure to the matrix structure of organization for promoting inter-divisional collaborations while recognizing vertical and formal structures. This improves lateral communication and cooperation between various departments of an organization. Consequently, the managers of various departments incline to cohesively address broader issues of the company, and thus plan their actions in an economically efficient manner. The planning and decision-making on crucial issues in such a coordinated organization setup improves the enterprise positioning and escalates their progress towards the goal of a company. The planning of long-term initiatives done in such a setup results as a strategic planning and provides a better framework for operational planning and action by various departments of the organization. Consequently, the coordinated decision-making for issues of strategic planning involves a large number of decision variables due to multiple aspects considered all together. This is experienced as one of challenging tasks to handle the resulting large-scale optimization problem.

Further, for building a competitive advantage over other competitor firms, the stakeholders of each enterprise attempt using innovative ideas to achieve company's goals through a coordinated and optimally efficient strategic plan. This innovation in strategic planning demands the development of new decision-support accordingly.

Some of important issues of business firms addressed under the strategic planning include minimizing the procurement costs and maximizing the resilience in inbound supply chain, strategic outsourcing through supplier selection and order allocation, an optimal utilization of available resources, competitive pricing of new products, and re-pricing of existing products. The decision-making on such strategic issues of planning requires a proper decision-support involving mathematical procedures of optimization framework. This is discussed under the next heading.

Decision-support for strategic planning

In recent competitive business environment, the planning of most of strategic issues converges to financial centric aim. Consequently, minimization of operational expenditures and maximization of sales driven through competitive pricing are considered to be of strategic importance. Decisions on such issues of strategic planning get a mathematical support from optimization framework.

Optimization problems are aimed at seeking those decisions which give the optimal value to the objective function of the decision-maker while restraining within limits of the constraints posed by external factors or agents. In some situations the constraints posed by one or more of these external agents are not static and come as a response depending on the choice of decision of the decision-maker. Such a situation makes the search for optimal decision convoluted for the decision-maker, due to the requirement of incorporating the reaction of external agent(s) corresponding to each possible decision. For the situation, when the reaction of these external agent(s) is an optimal solution of their respective optimization problem(s), such an optimization problem of a decision-maker is categorized as a BLP problem.

1.2 Bilevel programming (BLP)

This concept was originally proposed by H. Von Stackelberg [2] and thus a BLP problem is also termed as Stackelberg game. The framework of Stackelberg games is phenomenally different from the framework of Nash games. The problems discussed as Stackelberg games consider the situation when among two individuals or two groups of decision-makers, one takes the decision first and upon observing the same the other take decision. Whereas, in the Nash game framework all the decision-makers take their decisions simultaneously or subsequently without any knowledge of others decision. Henceforth, we fix to use the term BLP problem for a Stackelberg game. For theoretical developments on BLP framework one can refer to monographs [3,4].

BLP problems are mathematical programming problems involving sets of variables controlled by multiple decision-makers categorized at two levels. The hierarchy of levels involved in the structure of BLP is governed by the order of the decision-making. Decision-maker(s) who take the decision first are categorized as leader(s), whereas those who take a decision in response to the decision of these leader(s) are called followers. Leaders(s) are also called upper level decision-maker(s) and follower(s) as lower level decision-maker(s). The link between decision-maker(s) at two levels is established by the interdependence of their decisions on each other. It is assumed during the discussion of this framework that complete information is available, at least to the leader, about the constraints and objectives of both the leader and follower as well. This information makes it possible for the leader to assess the reaction of the follower for each of its action. One more assumption, which is considered except for a specific category of BLP, is that there is no cooperation among the leader and follower.

The mathematical formulation of BLP problem with most basic and general structure is presented below. Let the decision variables of the leader be $x_1, x_2, \dots, x_n$. For given values of $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$, the response of the follower is an optimal solution $y = (y_1, y_2 \dots, y_m) \in \mathbb{R}^m$ of his optimization problem.

i.e., for a given $x \in \mathbb{R}^n$, the response of the follower is a solution of the optimization problem

$$\max_{y} f_2(x, y)$$
$$\text{s.t. } g_2(x, y) \leq 0. \tag{1.2.1}$$

(Here, x is a parameter in the follower's optimization problem.)

The optimization problem of the leader is therefore expressed as given below while considering the reaction of the follower as a constraint to former's problem.

$$\boxed{\begin{array}{c} \max_{x \in \mathbb{R}^n} f_1(x, y) \\ \text{s.t. } g_1(x, y) \leq 0 \\ \text{where, } y \text{ is an optimal solution of follower's problem for a given } x \\ \max_{y \in \mathbb{R}^m} f_2(x, y) \\ \text{s.t. } g_2(x, y) \leq 0. \end{array}} \tag{1.2.2}$$

The optimization problem (1.2.2) expressed above is termed as a BLP problem. In the follower's problem (1.2.1) and the BLP problem (1.2.2) functions $f_1, f_2 \colon \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$, $g_1 \colon \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^r$, $g_2 \colon \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^s$. For a well-posed BLP problem, it is assumed that in (1.2.2), for any action of leader (x) the reaction of the follower (optimal solution y of (1.2.1) parameterized in x) is unique[1]. The sequential nature of decision-making depicted in BLP problem (1.2.2) infers that y can be considered as a function of x, as for any given values of vector x, the value of vector y can be obtained by solving the optimization problem (1.2.1) for y.

BLP problems can be classified into many categories depending upon the structure and properties of the constraints and the variables involved. The solution algorithms have been developed in the literature for some categories of these problems accordingly. A basic listing of categories is specifically detailed here for their clear identification and to

[1] The case when this assumption does not hold is reviewed in a later subsection as a separate category of BLP problem.

distinguish the problem of each category from the other. This categorization of BLP problems becomes a base for distinguishing solution approaches of one category from the other.

1.2.1 Classifications of BLP problems

The classifications are based on variable types, constraints, objectives and the understanding between the leader and the follower. Some basic classifications of problems categorized under the BLP framework are given below.

1. **Linear and non-linear BLP problems**

   This classification is done on the basis of presence of linearity in constraints and objectives included in the problem. A problem with general structure given by (1.2.2) is termed as linear BLP problem if all the constraints and objectives at both the levels are linear in the variables x and y. Thus a general linear BLP problem will have a structure given as following. For given $c_1, c_2 \in \mathcal{M}_{1\times n}(\mathbb{R})$, $d_1, d_2 \in \mathcal{M}_{1\times m}(\mathbb{R})$, $A_1 \in \mathcal{M}_{r\times n}(\mathbb{R})$, $B_1 \in \mathcal{M}_{r\times m}(\mathbb{R})$, $A_2 \in \mathcal{M}_{s\times n}(\mathbb{R})$, $B_2 \in \mathcal{M}_{s\times m}(\mathbb{R})$, and for $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$, $y = (y_1, y_2, \dots, y_m) \in \mathbb{R}^m$ the representation of a linear BLP problem is given as following.

$$\boxed{\begin{array}{c} \max\limits_{x\in\mathbb{R}^n} c_1 x + d_1 y \\ \text{s.t. } A_1 x + B_1 y \leq 0 \\ \text{where, } y \text{ is an optimal solution of follower's problem for a given } x \\ \max\limits_{y\in\mathbb{R}^m} c_2 x + d_2 y \\ \text{s.t. } A_2 x + B_2 y \leq 0. \end{array}} \qquad (1.2.3)$$

   (For any $p, q \in \mathbb{N}$, $\mathcal{M}_{p\times q}(\mathbb{R})$ denotes the set of all matrices over $\mathbb{R}$ of order $p \times q$.)

   In case, any of the functions $f_1(x,y)$, $f_2(x,y)$, $g_1(x,y)$, and $g_2(x,y)$ in (2) are non-linear in x or y, it is considered as non-linear BLP problem. One special type under this categorization is considered for the case when the objective function of the follower problem (1.2.1) is a convex function in y. In case of non-linear objective function of follower's problem the presence of convexity provides the uniqueness of optimal solution of follower's problem.

2. **Continuous and discrete BLP problems**

This classification is done on the basis of the type of variables involved. For the case when all the variables involved in a BLP problem are real and continuous, the problem is categorized as a continuous BLP problem. Confirming to class of discrete optimization problems, if any of the variables, at any level of decision-making, are restrained to belong to a subset of integers, the optimization problem is termed as a discrete BLP problem. One special type under the classification of discrete BLP problem is of binary BLP problem, in which each variable involves only two possibilities to attain a value (0 or 1). In case, both continuous and discrete variables are involved at any level of the hierarchy, the problem is termed as mixed-integer BLP problem.

3. **Single and multi-objective BLP problems**

This classification is done on the basis of number of objectives of each decision-maker. In the most fundamental form of a BLP problem it is assumed that each of the two decision-makers involved in a hierarchical decision-making has single objective only. Thus a BLP problem mathematically expressed in (1.2.2) is a single objective BLP problem. Whereas, for if any of the decision-makers has multiple objectives, the BLP problem is classified as a multi-objective BLP problem.

In many practical situations of hierarchical decision-making, leader and/or follower aim for multiple objectives. This situation is modelled as multi-objective BLP (MOBLP) problem. A general MOBLP problem with the leader aiming for p objectives and follower aiming for q objectives is expressed mathematically as following.

$$
\boxed{
\begin{array}{c}
\max\limits_{x \in \mathbb{R}^n} f_1(x,y) = \left(f_{11}(x,y), \dots, f_{1p}(x,y)\right) \\
\text{s.t. } g_1(x,y) \le 0 \\
\text{where, } y \text{ is an optimal solution of follower's problem for a given } x \\
\max_{y \in \mathbb{R}^m} f_2(x,y) = \left(f_{21}(x,y), \dots, f_{2q}(x,y)\right) \\
\text{s.t. } g_2(x,y) \le 0.
\end{array}
}
\tag{1.2.4}
$$

The case of $p = q = 1$ reduces the above problem to a single objective BLP problem (1.2.2). Thus for discussing a MOBLP problem it is assumed that $p > 1$ or $q > 1$.

1.2.2 BLP problems involving non-unique optimal response

This particular case is discussed when multiple optimal solutions are possible for the follower's optimization problem in response to some values of leader's variables. In this case, it becomes a challenge for the leader to subsume one among multiple optimal responses of the follower for computing a solution, as there is no surety that such particular solution will be realized as a response to an action of the leader. Further, if it is assumed that the follower also has complete knowledge of the leader's objective and constraints, this leads to formulation of optimistic and pessimistic BLP problems. This classification depends basically on the (business) relations between leader and follower.

If the leader can convince the follower for choosing that particular optimal solution of follower's parametric optimization problem which is favorable to the leader, in such a situation the BLP problem is termed as optimistic one. The practicality of this category of BLP problem lies in the fact that, principally, it is possible for the follower to choose such an optimal response as this does not amount to any compromise to his objective. If the long term goals of both the decision-makers are aligned and there is a harmony between them such case may be realized. Opposite to this is discussed the pessimistic BLP problem. Herein, it is assumed that the follower is completely aware of the leader's objective and selects that particular optimal response which is least favourable to the leader.

The literature in this context is a bit detailed and is out of the scope of this thesis as the problems discussed in our research work do not pertain to this category of BLP problems. Although interested users may refer to the monograph by Dempe [3] with the same nomenclature and monograph by Bard [5] which describes it as ill-posed BLP problem under the discussion of general BLP.

Remark 1.2.1: From this point onwards in this thesis, it is considered that the lower level parametric optimization problem has a unique solution for each given vector of values of leaders variables supplied as parameters there.

All the combinations of these types are also possible to model practical situations of optimization problems pertaining to BLP framework. To exemplify, continuous-linear BLP problem, continuous-non-linear BLP problem, discrete-linear BLP problem, discrete-non-linear BLP problem, multi-objective-discrete-linear BLP problem, etc. Through this classification, one can identify the traits of a BLP problem and this enables to understand the approaches of solution algorithms. In fact, these basic traits can be discussed even for the variants of the BLP discussed below.

1.2.3 Some other variants of BLP problems

In the most fundamental framework of BLP, the hierarchical structure of interactive decision-making among only two decision-makers is modelled, with one of them posed as leader and other as follower. But in some practical situations it is observed that multiple decision-makers participate at same level of the hierarchy by taking decisions simultaneously. In case of competitive and simultaneous decision-making of multiple decision-makers at same levels, the situation of a Nash game arises among them. The Nash game situation present in the hierarchical decision-making is discussed under the name multi-leader-follower BLP problem. Based on the situations of multiple decision-makers acting simultaneously as leaders or followers in a hierarchical decision-making situation, this type of BLP problems are classified into following three variants.

1. **Single-leader-multi-follower BLP problem**

   This type of game situation in a hierarchical decision-making arises when two or more followers react to the decisions made by a single leader. Due to the competition involved at the lower level of the hierarchy, the reaction from multiple followers corresponding to a decision of the leader is identified as a Nash-equilibrium point of the game[2] situation among the follower. The following Figure 1.1 depicts the structure of this variant of BLP problem.

[2] When two or more decision makers take decision competitively and simultaneously keeping into the consideration the strategies of other competitors, such a situation is specified as Nash game and the equilibrium is specified as Cournot equilibrium or Nash equilibrium. Whereas the competition with a hierarchical structure present in the bilevel programming due to asynchronous decision-making is termed as Stackelberg game. Thus, the solution of a bilevel programming problem is also termed as Stackelberg solution.

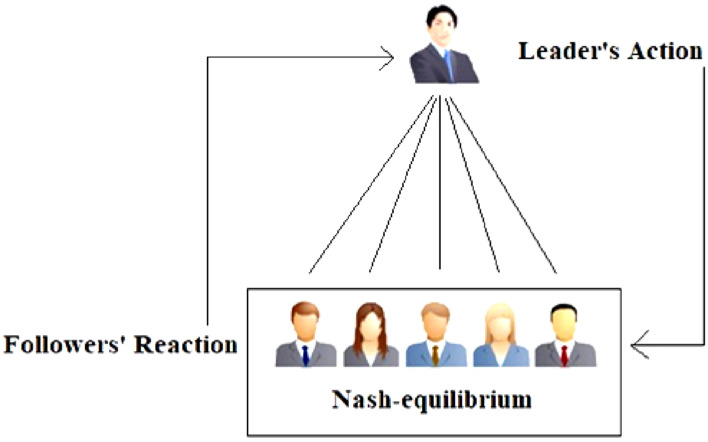

Figure 1.1: Multi-leader-single-follower BLP problem

The mathematical formulation of a problem pertaining to this variant of BLP is expressed as following. Let there be K followers which respond to the actions of a leader. If for any value of the vector of leader's decision variables $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$, followers $(k = 1, 2, \dots, K)$ react competitively to settle at a constrained Nash-equilibrium point comprising of the values of their decision variables $y_1 = (y_{11}, y_{12}, \dots, y_{1m_1})$, $y_2 = (y_{21}, y_{22}, \dots, y_{2m_2})$, …, $y_K = (y_{K1}, y_{K2}, \dots, y_{Km_K})$, and if $y = (y_1, y_2, \dots, y_K) = (y_{11}, y_{12}, \dots, y_{1m_1}, y_{21}, y_{22}, \dots, y_{2m_2}, \dots, y_{K1}, y_{K2}, \dots, y_{Km_K})$, then the single-leader-multi-follower BLP problem is expressed in general as following problem.

$$\max_{x \in \mathbb{R}^n} f_1(x, y)$$

$$\text{s.t. } g_1(x, y) \leq 0$$

where, for a given decision x of leader, the reaction y of followers is obtained a Nash-equilibrium of the game comprising of the optimization problem of each of the follower (parameterized in x) as given below. (1.2.5)

$$\max_{y_k \in \mathbb{R}^{m_k}} f_{2_k}(x, y)$$

$$\text{s.t. } g_{2_k}(x, y) \leq 0 \ (k = 1, 2, \dots, K).$$

2. **Multi-leader-single-follower BLP problem**

This type of game situation arises in a hierarchical decision-making when there are multiple leaders who choose their strategies simultaneously and competitively while incorporating the response of a single follower. In this game situation, the strategic action of each leader is influenced by the reaction of the follower. Due to the competition involved at the upper level of the hierarchy, leaders settle at a Nash-equilibrium point. The following Figure 1.2 depicts the structure of this variant of BLP problem.

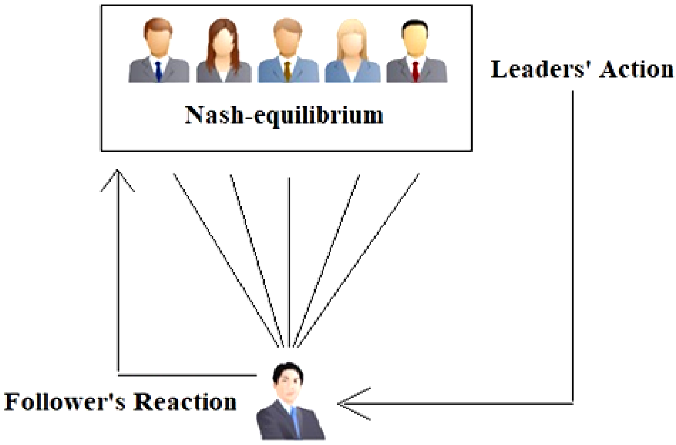

Figure 1.2: Multi-leader-single-follower BLP problem

The mathematical formulation of a problem pertaining to this variant of BLP is expressed as following. Let us consider the situation where there are L leaders. For the index l $(l = 1, 2, \dots, L)$ associated with each leader, if their decision variables are $x_1 = (x_{11}, x_{12}, \dots, x_{1n_1})$, $x_2 = (x_{21}, x_{22}, \dots, x_{2n_2})$, … , $x_L = (x_{L1}, x_{L2}, \dots, x_{Ln_L})$, respectively, with

$x = (x_1, x_2, \dots, x_L) = (x_{11}, x_{12}, \dots, x_{1n_1},\ x_{21}, x_{22}, \dots, x_{2n_2}, \dots, x_{L1}, x_{L2}, \dots, x_{Ln_L}) \in \mathbb{R}^{n_1+\cdots+n_L}$ as the vector of all the leader's decision variables, and if follower's response to any given x be in terms of the vector of variables given by $y = (y_1, y_2, \dots, y_m)$, then the multi-leader-single-follower BLP problem is expressed in general as a constrained Nash-equilibrium game with the problem of each leader as a player of the game as given below.

$$
\begin{aligned}
&\max_{x_l \in \mathbb{R}^{n_l}} f_{1l}(x_l, y) \\
&\text{s.t. } g_{1_l}(x_l, y) \leq 0 \\
&\text{where, } y \text{ is optimal response of the follower corresponding to given values of all leader variables', } x \\
&\max_{y \in \mathbb{R}^m} f_2\,(x, y) \\
&\text{s.t. } g_2(x, y) \leq 0.
\end{aligned}
\tag{1.2.6}
$$

In the multi-leader-single-follower BLP problem expressed above it is evinced that each leader needs to take the decision competitively while keeping the response of follower into consideration.

3. Multi-leader-multi-follower BLP problem

This type of game situation arises in a hierarchical decision-making when there are two or more competitive leaders who choose their strategies simultaneously and competitively, to which multiple followers react simultaneously and competitively. In this game situation, the strategic action of each leader is influenced by a collectively reaction of all the followers. Due to the competition involved at the upper level of the hierarchy, leaders settle at a Nash-equilibrium point while incorporating the response again as a Nash-equilibrium point of followers' competition. The following Figure 1.3 depicts the structure of this variant of BLP problem.

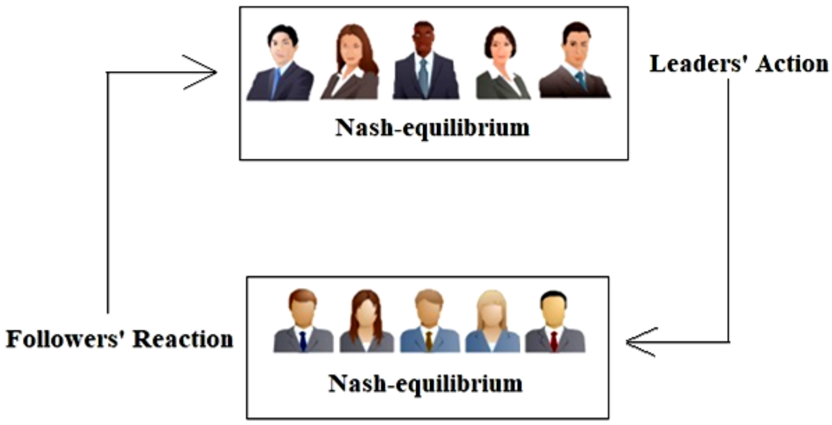

Figure 1.3: Multi-leader-multi-follower BLP problem

The mathematical formulation of a problem pertaining to this variant of BLP is explained below. Let us consider a situation where there are L leaders. For index $l\ (l = 1, 2, \dots, L)$ associated with leaders, if their decision variables are $x_1 = \left(x_{11}, x_{12}, \dots, x_{1n_1}\right)$, $x_2 = \left(x_{21}, x_{22}, \dots, x_{2n_2}\right)$, $\dots$, $x_L = \left(x_{L1}, x_{L2}, \dots, x_{Ln_L}\right)$, with $x = (x_1, x_2, \dots, x_L) = \left(x_{11}, x_{12}, \dots, x_{1n_1},\ x_{21}, x_{22}, \dots, x_{2n_2}, \dots, x_{L1}, x_{L2}, \dots, x_{Ln_L}\right) \in \mathbb{R}^{n_1+\cdots+n_L}$ as the vector of all leader's decision variables, and if each of the followers $(k = 1, 2, \dots, K)$ reacts competitively to settle at a constrained Nash-equilibrium point comprising of the values of their decision variables $y_1 = \left(y_{11}, y_{12}, \dots, y_{1m_1}\right)$, $y_2 = \left(y_{21}, y_{22}, \dots, y_{2m_2}\right)$, …, $y_K = \left(y_{K1}, y_{K2}, \dots, y_{Km_K}\right)$, with $y = (y_1, y_2, \dots, y_K) = \left(y_{11}, y_{12}, \dots, y_{1m_1},\ y_{21}, y_{22}, \dots, y_{2m_2}, \dots, y_{K1}, y_{K2}, \dots, y_{Km_K}\right)$, then the multi-leader-multi-follower BLP problem is expressed in general as a constrained Nash-equilibrium game with individual problem of each leader as a player of the game given by

$$\max_{x_l \in \mathbb{R}^{n_l}} f_{1l}(x_l, y)$$

$$\text{s.t. } g_{1_l}(x_l, y) \leq 0$$

where, for a given decision x of all leaders, the reactions of followers is obtained a Nash-equilibrium of the game comprising of the optimization problem of each of the follower (parameterized in x) as given below. (1.2.7)

$$\max_{y_k \in \mathbb{R}^{m_k}} f_{2_k}(x, y)$$

$$\text{s.t. } g_{2_k}(x, y) \leq 0\ (k = 1, 2, \dots, K).$$

Through this subsection it is explained that different variants of BLP problems can be expressed depending on multiple players competing in Nash game as leaders or followers. The BLP problem (1.2.2) discussed initially in previous subsection is thus distinguished from these variants by designating a specific term, single-leader-single-follower BLP problem. Each of the variants of BLP discussed in this subsection can be further classified into different types depending upon the type of variables and constraints involved, in a similar way as discussed for Single-leader-single-follower BLP problem in the previous subsection 1.2.1.

The categorization of BLP problems into the variants enables a learner to understand the structure of competition, whereas the classification of a BLP problem of each variant into different types enables to identify the appropriateness of the approaches for solution algorithms. In this chapter the nitty-gritty of the structure of various variants and types of BLP is presented, whereas a review of literature on methodologies to solve these problems is presented in the next chapter. With the acquaintance of structure of different types and variants of BLP, it is appropriate to review the literature addressing decision-making problems using the BLP framework.

1.3 An overview of real life applications of BLP

This section presents a brief review of the literature on applications of BLP for addressing real life decision-making problems. Application based works handling such modelling issues are found in three major areas *viz* government sector, engineering and technology, and business management.

Government Sector

The decision-support using BLP framework is developed for some typical decision-making issues of toll setting, transport network design, environmental economics, and defence strategies in government sector.

- **Toll Setting Problems:** The toll setting problems have been studied using BLP framework with an objective of maximizing revenue for the utilization of the available road network [6–13].
- **Transportation/Transit network design:** Transportation network management problems have been addressed in literature using BLP framework. Issues of transit network design including transit frequency optimization have been addressed in studies [14–17]. Transit network design problems in general are studied [18–26]. Management of traffic signalling is addressed in studies [27,28].
- **Environmental Economics:** Studies are available in literature which have addressed environmental policy making problems using BLP framework, to list some; optimal pollution control policies [29], water quality management [30], regulating norms for mining companies [31], agri-environmental policy framing [32].

- **Defence applications:** Studies on defence strategies like attacker-defender Stackelberg games have been studied (c.f. [33–37]). Research on developing missile defence and counter-attack systems using BLP framework include [38–42].

Engineering and Technological Design

The decision-support using BLP framework is developed for decision-making in following areas of engineering and technological designs.

- **Optimal Design:** Optimal design problems with issues like structural design [43–45] and shape optimization [46,47] have been studied in the field of materials management using BLP framework.
- **Optimal Control:** Studies on optimal control of robots have also used BLP framework for addressing various control issues of the field [48–51].
- **Chemical Industry:** BLP framework has been used to address problems of chemical industry like chemical reaction equilibrium analysis [52], design of steady-state chemical process [53], chemical process engineering [54], optimization of multi-component chemical systems [55].
- **Machine Learning:** Researchers have used BLP framework in the area of machine learning also for proper tuning of parameters to achieve computational accuracy and efficiency of evolutionary algorithms [56–59].

Business Planning and Management

The decision-support using BLP framework is developed for the decision-making in following areas of business planning and management.

- **Facility Location Problems:** Long-term planning problems of identifying facility locations [60] and locations of logistics distribution centres (DCs) [61] have been addressed using BLP framework.
- **Principal-agent Problems:** The principal-agent paradigm [62] has been addressed using BLP framework [63] and used to address problems of decision on executive compensation [64].
- **Electricity markets:** Pricing issues in electricity markets for handling competition have been addressed using BLP framework [65–67].

- **Supplier Chain Planning:** BLP framework has been used by researchers for addressing problems of supply chain network planning [68,69] as well as planning of supply chain operations [70,71]

1.4 Literature review of some planning problems relevant to our work

In this section we present in detail some mathematical concepts and results which are required as a fundamental knowledge to understand the work presented in subsequent chapters.

1.4.1 Price setting problems

Price setting is one of most crucial decisions of every business firm which sell their products or services. The decision on pricing is considered to very important as it is a major factor which influences the profit (or turnover) in a short term and governs the market-share as well as brand positioning in a long run. Especially in case of business-to-business dealings as prevalent in oligopolistic-monopsony market situations pricing decisions influence business-relations among the organizations which in turn, in a long-term, affect their performance also. This indicates a requirement of proper planning for making pricing decisions while focusing on the strategic vision and mission of the organization.

An optimal price setting problem is generally aimed to determine the prices of commodities/ services so as to maximize the total profit, when addressed from a purview of short-term planning. General pricing strategies (including high and low pricing strategies), adjustable pricing strategies (like market segmentation), differential pricing strategies, price skimming, competitive pricing strategies like penetration pricing and revenue management-based pricing have been reviewed in the literature by Dolgui and Proth [72].

Mathematical model of a pricing problem considering the linkage between prices and demand to influence the turnover was studied for the first time by Labbe et al. [73]. The model studies a toll setting problem in a BLP framework. As the soul of this modelling has remained an inspiration across our research work, it is appropriate to discuss this model here for a better understanding of our research work.

The price setting model formulated by Labbe et al. [73] considers the toll setting decision problem from the perspective of a taxation authority with an aim of maximizing the revenue. Activities considered available to users are categorized as taxed and untaxed. Vector of variables representing taxes, denoted by T, represents taxation authority's action, whereas vectors of variables x and y are used to denote the extent of use of taxed and untaxed activities as a response to minimize total cost. As the authority declares the toll rates first and users respond only upon knowing the same, therefore earlier is posed as leader and later as follower in the formulation of BLP problem. Thereby, this formulation enables the leader decide on the toll vector T by assessing the follower's response so as to maximize the revenue. Formulation of this problem modelling this action-and-reaction mechanism given by

$$\boxed{\begin{aligned} &\max_{T} \;\; Tx \\ &\text{where, } x \text{ is obtained by solving} \\ &\qquad \min_{x,y} \;\; (c_1 + T)x + c_2 y \\ &\qquad\quad \text{s.t. } A_1 x + A_2 y = b \\ &\qquad\qquad\qquad x, y \geq 0. \end{aligned}} \tag{1.4.1}$$

Here, c_1 and c_2 are considered as costs of taxed and untaxed activities incurred to user.

This generic modelling is useful to address pricing problems on purchase of commodities as well as services. Further studies on toll-setting problems which have based their studies on this modelling can be referred as [7,74–76].

Remark 1.4.1: (a) The approach used here for modelling the pricing problem is aimed at revenue maximization. This approach is appropriate for modelling the situation involving no significant operational costs for providing the products or services to the user. On the other hand, for situations involving larger costs the profit maximization approach is required to be followed for appropriately modelling of the pricing problem. Such a problem is prevalently faced during business-to-business price-negotiations, but has never been addressed for developing decision support.

(b) For example, it is inappropriate to apply this model directly to any situation of strategic pricing and planning of supply chain operations. For addressing this strategic pricing problem it requires a cohesive assessment of cost to fulfil the replicating demand. This requires integrating the operational planning problem along with the pricing problem in a BLP framework. A study addressing this issue is accomplished in our research work presented in Chapter 4.

The literature on operational planning in supply chain is reviewed below.

1.4.2 Operational planning problems in supply chain

Planning of various tasks to fulfil the demand of end-users over a short-term horizon is termed operational planning under the supply chain management [77]. In this context, aggregate production-distribution planning (APDP) problem is used to model the planning of these supply chain operations. An optimization problem modelled for collective planning of all the operations activities and arrangements namely,

- production in regular-time and over-time (in each plant),
- outsourcing,
- shipping volumes from the production facility or stack buffers to warehouses,
- shipping volumes from warehouses and stack buffers to end-users,
- inventory levels of finished products in warehouses to be maintained,

for each period with the aim of minimizing the total cost of all these operations is categorized as APDP problem [78]. The literature in the field of APDP problems is extensive and classified into seven categories [78]. Some of the significant contributions worth citing in APDP are [79–82]. Recent research developments in operational planning include [83,84].

Remark 1.4.2: As noted in previous subsection on pricing problems, a need is felt to address pricing issues in cohesion with operational planning problems. In our extensive review of literature we haven’t come across any noteworthy study addressing these strategic issues cohesively.

Another important area of decision-making for the management of supply chain focuses on the inbound supply. Among many important issues addressed in this context, those concerned about costs and performance are of supplier selection and order allocation of the demand for a pre-fixed planning horizon.

1.4.3 Supplier selection and order allocation

The supplier selection is a crucial strategic decision-making process for the procurement of the required products or the raw material. The primary literature on it begins with the work of Dickson [85]. The next accompanying vital decision is on the order allocation under multiple sourcing. Gaballa [86] and Jayaraman et al. [87] developed decision support for the order allocation problem to minimize the total procurement cost. The prevalent use of just-in-time approach for inventory procurement resulted in the recommendation of multiple factors for supplier selection [88–90]. Several researchers investigated the order allocation in concurrence with supplier selection [91–93]. The supplier selection and order allocation issues have been modelled as multi-objective decision-making problems also [94–96]. One can refer to the recent literature review by Aouadni et al. [97] on supplier selection and order allocation techniques.

Remark 1.4.3: The literature on the discussed strategic aspects of supply chain management is extensive, but has never been studied in connection with price-negotiations. This research gap is present despite that fact that price-negotiation is an important strategic issue which deeply influences the buyer-supplier business relationship. It further requires a decision-support mechanism to discern the demand-order allocations in connection with the price-quotations, enabling a decision-support for negotiations. A study addressing this issue is accomplished in our research work presented in Chapter 5.

1.4.4 Planning of railways operations

The planning and management of railway operations need a robust support system for decision making. The monograph by Caprara et al. [98] can be referred for detailed incite of planning issues of railway operations and available mathematical models to support the decision process. Two widely explored areas in the literature are train-scheduling and rolling-stock management. Some of recent research in the area of train scheduling includes [99,100]. The studies in [101–109] focus on the management of rolling-stock considering a fixed demand for each type of train. Another challenging area of interest for railways is to determine the fare-price structure for running the trains in a competitive state of oligopoly. Limited work is available in this context to cite (c.f. [110–112]). Li et al. [112] has developed a model in a BLP framework featuring fare-price Nash-equilibrium between the high-speed railways and civil aviation as leaders' problem and minimizing the total travelling-cost incurred by all passengers as a follower problem.

Apart from these studies there is another setup where the railway operators declare the running schedule, the number as well as classes of coaches in the train, and their fare-prices, much in advance of running the train. Models proposed in literature are not appropriate for addressing the fare-pricing strategies in such setups. Reason behind this is that the time window created due to prior declaration of fixed fare-prices provides ample opportunity to the competitors to react to the decision of railways by adjusting their fare-prices and hence affecting the demand-shares of railways. No research work is seen to address the strategic issue of fare-pricing decisions in this setup.

Remark 1.4.4: Research gaps noted in this section indicate need of establishing an inter-linkage between strategic planning and operational decision-making issues. Also, these decision-making situations are appropriate to be handled using BLP framework. This motivates to develop decision-support by mathematically modelling these strategic issues. A study addressing this issue is accomplished in our research work and is presented in Chapter 3.

1.5 Objectives of research work and organization of the thesis

Adopting an integrated approach for addressing strategic planning issues facilitates an efficient utilization of internal capacities and better handling of competition, and thereby is seen as aligned towards the long term goals of the organization, as described in section 1.1. From the review of literature on decision-making models using BLP framework presented in section 1.3, it is observed that an appreciable amount of research work has addressed decision-making issues at higher-level management in context of Stackelberg-type-competition. But, a few among those consolidate the decision-making on operational aspects of different departments within the organization simultaneously while addressing competition in the market. In this context, some of decision-making issues faced particularly in the supply chain management and railways planning are noted in section 1.4 which can be addressed in cohesion due to their inter-linkage and inter-dependence. A lack of modelling these decision-making aspects in conjunction indicates a serious research gap in the literature on mathematical modelling approaches for addressing the strategic planning and decision-making problems. On this account, a prime objective of this thesis is to develop a decision-support for some critical issues of strategic planning and decision-making in a competitive business environment using the BLP framework.

Furthermore, while pursuing the research work towards our prime objective, we observed that the mathematical models developed using BLP framework for addressing such practical problems of strategic planning, involve 1000s of variables and 100s of constraints. And, the solution algorithms[3] available in the literature for solving BLP problems and their variants, resulted as incapable of handling problems of the scale mentioned here. Even for some variants of BLP problems no algorithmic developments could be found. This may be a major reason behind the limited use of the BLP framework for addressing those strategic planning and decision-making problems which require incorporating the response of other decision-makers. For accomplishing our first objective, it is therefore compelling to develop solution algorithms capable of handling the problems formulated in our research work using BLP framework. Accordingly, the algorithmic development for such problems is identified as the second objective of our thesis.

Comprehensively, a broad objective of this thesis is to develop a BLP based decision-support for some of the strategic planning issues which require incorporating a response assessment mechanism into decision-making. Here, it is aimed to develop such a decision-support which is capable of handling large-scale instances of real situations from industry. This broad objective can be clearly detailed as following constituent sub-objectives of the study.

[3] For maintaining the flow of the thesis, the review of solution algorithms available in literature for different variants and types of BLP problems is separately presented in the next chapter, along with some other fundamentals of metaheuristic algorithms.

1. To identify and model some strategic issues of decision-making which require response assessment mechanism into decision-making, and appropriately design mathematical model using the BLP framework.
2. To develop such methodologies for solving the problems taken up for study which are capable of handling practical instances of large scale.
3. To implement both of the above developments on real cases from relevant industry for demonstrating the capability of proposed solution methodologies.
4. To verify the success of developed decision-support through comparison of obtained results with appropriate situations.

In this thesis, some pivotal issues of strategic planning have been addressed which are faced by business firms of some important sectors of the economy of any nation. In each part of our study, we have primarily focused on a crucial aspect of decision making under the strategic planning *viz* strategic pricing in conjunction with cost budgeting and planning of operational arrangements.

For developing the decision support in line with the objectives our research, in each part of our study, a schematic research design is adopted as outlined below.

1. An appropriate strategic planning and decision-making situation considered for the study is addressed in detail by modelling through the BLP framework.
2. A solution methodology is developed using metaheuristic approach for handling large scale instances of addressed problem. For this purpose, particularly the genetic algorithm (GA) based approach is adopted.
3. The developed GA-based solution method is coded into a computer program using the MATLAB software.
4. Relevant data is obtained from appropriate organizations and used as inputs to implement the developed decision-support system.
5. Comparison analysis of obtained results is performed to verify the predominance of the decision-support system.

Organization of Thesis

The forthcoming chapter presents solution algorithms existing in literature for various types and variants of BLP, fundamentals of GA for solving optimization problems and specifically the GA-based approaches for solving BLP problems, and finally some other concepts to understand the work discussed in subsequent chapters.

We start from the interest of public sector, among which the railways is an important service provider for commuting of general public of countries with connected landmasses. In Chapter 3, the strategic issue of decision on running special trains is addressed for the seasons of heavy demand overflowing the capacities of regularly running trains. The strategic planning includes decisions on optimal utilization of operational capacities of rolling-stock, and revenue management through competitive pricing with service to be provided to the general public as far as possible. A diversified-elitist GA is proposed for solving a mixed-integer single-leader-multi-follower BLP problem thus formulated. A case of Indian Railways is studied to capably solve the problem addressed in the research work.

Small and medium scale enterprises play a crucial role in a developing economy. Such an enterprise initially approaches a single buyer to sell its products, and experience a challenge to penetrate into this potential market due to some suppliers already selling those products to the buyer. With a competitive quality of its product a small scale supplier needs to quote smart prices of its products to the buyer so as to gain a profitable share of demand-order. This decision-making on pricing, at the same time, needs to be assessing on the capacity of the enterprise to fulfil the demand-orders in response to the prices to be quoted. The decision-support for this strategic pricing is developed in the Chapter 4 through a mixed-integer BLP problem with two objectives at lower level. The bi-objective programming problem at lower level is handles using the weighted-sum method. A GA-based approach is developed to solve the problem which internally handles bi-objective programming problem at lower level using the weighted-sum method. The success of the developed decision-support mechanism is illustrated through a data-set of a scenario of a small-scale enterprise from the manufacturer sector.

Procurement of raw material or outsourced spares parts is another very important financial activity for any enterprise. For the case of oligopolistic-monopsony market, negotiations precede the actual procurement deal and managers meet for negotiation with a target price in mind. In this market situation it is crucial to identify the scope price negotiation upto which mutual financial interests are respected for the buyer as well as suppliers of the products. The Chapter 5 explores the target prices for such a price negotiation with focus on a long term goal of maintaining sustained business environment. For addressing this strategic issue, a multi-leader-single-follower BLP problem is formulated to mathematically model the problem of identification of target prices negotiation of the buyer with the suppliers. A GA-based solution methodology combining a theoretical approach for computing stationary points is presented. The efficacy of the proposed concept and the solution algorithm is demonstrated through a data-set obtained from a manufacturing firm of fast-moving consumer goods (FMCG) sector. Further a demonstration indicates the capability of the proposed tool to aware the buyer about possible cartels or market sweeping pricing strategies of suppliers during the actual practice of negotiation.

The thesis is finally summarized with findings and conclusions of the research work, followed by a discussion on the scope of further research in future.

Chapter 2

Preliminary Concepts

In this chapter, we review the solution algorithms available in the literature for solving different types and variants of BLP problems. Also, we discuss herein some preliminary facts which are necessary for understanding the algorithms proposed in our work to solve problems formulated as BLP models. First, the solution algorithms available in the literature for solving the variants of BLP problems are listed. Thereafter, a brief introduction of GAs is presented along with a detailed description of genetic operators used in our research work. A brief of GA-based methodologies available in literature for solving BLP problems is also included to summarize the same with a classification of such types. Finally, Nash-games and their role in BLP problems involving multiple leaders and/or multiple followers are presented with discussion on theoretical developments. Also, a theoretical development is discussed which is used in developing a GA-based solution methodology.

2.1. A survey of literature on methodologies for solving BLP problems

BLP problems are non-convex in nature due to the hierarchical structure. Even the simpler types are also experienced as difficult to handle mathematically. BLP problems have been proved to be NP-hard[4] [113,114]. The complexity issues indicating the nonexistence of a polynomial time algorithm even for linear BLP problem have been discussed by Deng [115]. Due to the convoluted structure of BLP there is no well-established direct solution procedure and therefore a BLP problem is usually modified into a single level optimization problem, which is solved to obtain a solution [116].

A review of the literature on solution methodologies is presented for each variant of BLP problems, first for basic one involving single leader and single follower and then for the other variants involving multiple leaders or followers. Solution methodologies have been developed by researchers majorly for single-leader-follower BLP problems and a little attention has been paid towards BLP problems with multiple leaders and/ or multiple followers. Similar to any optimization problem, the solution methodologies can be classified as classical and metaheuristic, so we present this literature review accordingly.

2.1.1 Methods for solving single-leader-follower BLP problems

Surprisingly, there is no such approach available in literature which directly paves a theoretical procedure for solving a general single-leader-follower BLP problem. Some classical methods based on deterministic approaches have theoretically been developed in the literature and researchers have demonstrated these methods by solving problems involving a few variables. These approaches majorly suggest obtaining a system of conditions equivalent to the lower level problem to annex these as constraints to the upper level problem, and then solving thus obtained single level optimization problem through well known classical optimization techniques [116].

Whereas, the randomized search based metaheuristic methods have been developed to handle practical BLP problems involving a comparatively higher number of variables. The literature in this context is observed to tackle solving BLP problems with three types of approaches. First one employs random exploration at both levels. Herein, the lower

[4] NP-Hardness is a term frequently used to indicate the Non-deterministic Polynomial time hardness in computational complexity of a problem [246].

level optimization problem is solved corresponding to each candidate value of upper level variables and which in turn is explored for the optimal solution of the overall BLP problem through exploration for optimal value of leader's objective. The second one is a nested approach which suggests exploring for upper level variables through an evolutionary algorithm while evaluating the response by directly solving the lower level optimization problem. The third approach suggests rewriting the BLP problem as a single level optimization problem, and then solving the resulting problem using classical or evolutionary approaches. With this general introduction on solution approaches we get into the details presented subsequently.

The presentation of the literature review on solution methods is organized with further classifications into its types as discussed in section 1.2. For each type of single-leader-follower BLP problems, first the classical approaches are reviewed followed by evolutionary methods.

The sequential nature of decision-making and the structure of the BLP (as depicted in (1.2.2)) indicates, that the follower's variables y can be viewed as a function of x. *i.e.*, $y = y(x)$. In this line of thought, some definitions are used in the literature [4,117] to discuss about the structure and solution of BLP problem (1.2.2). These can be considered as an initial step towards the discussion of solution algorithms and are presented below.

(a) The constrained region of the BLP problem:

$$S = \{(x, y): x \in \mathbb{R}^n, y \in \mathbb{R}^m, g_1(x, y) \leq 0, g_2(x, y) \leq 0\}$$

(b) Feasible set for the follower for each fixed $x \in \mathbb{R}^n$:

$$S(x) = \{y \in \mathbb{R}^m: g_2(x, y) \leq 0\}$$

(c) Projection of S onto the leader's decision space:

$$S(X) = \{x \in \mathbb{R}^n: \exists y \in \mathbb{R}^m, g_1(x, y) \leq 0, g_2(x, y) \leq 0\}$$

(d) Follower's rational reaction set for $x \in S(X)$:

$$P(x) = \{y \in \mathbb{R}^m: y \in \arg\min\{f_2(x, w): w \in S(x)\}\}$$

(e) Inducible region:

$$\mathcal{IR} = \{(x, y): x \in S(X), y \in P(x)\}$$

Definition 2.1.1: $(x^*, y^*) \in \mathbb{R}^{n+m}$ is said to be an optimal solution of the BLP problem (1.2.2), for if it gives the maximum value to the leader's objective function over the inducible region ($\mathcal{IR}$).

In terms of the definitions stated above, the BLP problem (1.2.2) can be re-expressed as the following.

$$\max_{(x,y)} f_1(x,y)$$

$$\text{s.t. } (x,y) \in \mathcal{IR} \tag{2.1.1}$$

When we express the BLP problem (1.2.2) as an optimization problem (2.1.1), we must note that the leader has no direct control over the follower's variables y, but can indirectly control the follower by choosing such an x which maximizes his objective function $f_1(x,y)$ with y as the follower's rational response (*i.e.*, $y \in P(x)$).

To ensure that the BLP problem (2) is well-posed, following assumptions are needed to be considered.

1. S is a non-empty compact set.
2. For any decision on x taken by the leader, the follower should have some room to respond. Mathematically, it is expressible as $P(x) \neq \emptyset$.
3. In fact, for the leader to assess the response $y \in \mathbb{R}^m$ on behalf of the follower for each possible action ($x \in \mathbb{R}^n$ such that $P(x) \neq \emptyset$), the uniqueness of response is ensured through the assumption that $P(x)$ is a singleton set in $\mathbb{R}^m$.

Remark 2.1.1: The third assumption listed above indicates that the function P is a point-to-point map. For the situation where this assumption is not fulfilled the concept of cooperative and non-cooperative BLP is discussed accordingly depending on the behavior of the follower

With this fundamental discussion on what a solution of a single-leader-follower BLP problem means, the development of solution algorithms further depends upon the particular types of BLP problem. Henceforth, we get into the solution approaches for each type of BLP problems distinguished from others as per the classifications made in subsection 1.2.1.

Linear BLP problems with continuous variables

A problem with general structure given by (2) is termed as linear BLP problem if all the constraints and objectives at both the levels are linear in the variables x and y. Thus a general linear BLP problem will have a structure given as following. For given $c_1, c_2 \in \mathcal{M}_{1\times n}(\mathbb{R})$[5], $d_1, d_2 \in \mathcal{M}_{1\times m}(\mathbb{R})$, $A_1 \in \mathcal{M}_{r\times n}(\mathbb{R})$, $B_1 \in \mathcal{M}_{r\times m}(\mathbb{R})$, $A_2 \in \mathcal{M}_{s\times n}(\mathbb{R})$, $B_2 \in \mathcal{M}_{s\times m}(\mathbb{R})$, and for $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$, $y = (y_1, y_2, \ldots, y_m) \in \mathbb{R}^m$ the representation a linear BLP problem is given as following.

$$\max_{x\in\mathbb{R}^n} c_1 x + d_1 y$$

$$\text{s.t. } A_1 x + B_1 y \leq 0$$

where, y is an optimal solution of follower's problem for a given x

$$\max_{y\in\mathbb{R}^m} c_2 x + d_2 y$$

$$\text{s.t. } A_2 x + B_2 y \leq 0. \tag{2.1.2}$$

In terms of the terminologies discussed above and the optimization problem (2.1.1) explained as an equivalent of BLP problem (1.2.2), an equivalent of linear BLP problem (2.1.2) can be expressed as following.

$$\max_{(x,y)} c_1 x + d_1 y$$

$$\text{s.t. } (x, y) \in \mathcal{IR} \tag{2.1.3}$$

(Here, $\mathcal{IR}$ represents the inducible region corresponding to the BLP problem (2.2.2).)

With this prelude of mathematical structure of linear BLP problem, we present the review of literature on solution methods to solve linear BLP problems.

Classical methods available in the literature for solving a linear BLP problem can broadly be classified into three categories depending upon their approaches to handle the follower's problem. First category is of vertex enumeration approach which is based on extreme point ranking method for solving linear programming problems [118]. The K-th best algorithm is suggested by Bialas & Karwan [119] following this approach. Authors base this algorithm on (a) the result that the solution of a linear BLP problem occurs at a vertex of S (in $\mathbb{R}^{n+m}$) (proved explicitly later by Bard [120] also), and (b) the fact that inducible region is a subset of the constrained region ($IR \subseteq S$). Algorithm suggests to sequentially explore vertices of constrained region S for optimality of the leader's

[5] For any $p, q \in \mathbb{Z}^+$, $\mathcal{M}_{p\times q}(\mathbb{R})$ denotes the set of all matrices over $\mathbb{R}$ of order $p \times q$.

objective function (*i.e.*, solve the following linear programming problem (2.1.4)) and then test whether that particular point belongs to the inducible region ($\mathcal{IR}$) or not.

$$\max_{(x,y)} c_1x + d_1y$$
$$\text{s.t. } (x,y) \in S \tag{2.1.4}$$

where, $S = \{(x,y): x \in \mathbb{R}^n, y \in \mathbb{R}^m, A_1x + B_1y \leq 0,\ A_2x + B_2y \leq 0\}$.

In case, of an affirmative result, the optimal solution of BLP problem (2.1.2) is achieved. If such an optimal solution does not belong to $\mathcal{IR}$, then the algorithm suggests exploring for next best solution of (2.1.4) and then test again that one for being element of $\mathcal{IR}$.

The second approach is based on exploiting the Karush-Kuhn-Tucker (KKT) conditions for the lower-level problem. Methods following this approach [121–124] suggest obtaining the constraint qualifications based on KKT conditions corresponding to the lower-level optimization problems, annex them in the upper-level problem, and then handle the complimentarity problem through different ways.

The third approach suggested in works of Anandalingam and White [125,126] is based on penalty function defined by appending the duality gap of the follower's problem into the leader's objective function. The theoretical support behind this approach lies in the fact that the duality gap for the follower's problem becomes zero for if $(x,y) \in \mathcal{IR}$. Some other theoretical approaches for solving BLP problems by expressing them as single-level optimization problems are [127,128].

The only evolutionary approach available in the literature for solving a linear BLP problem is developed by Hejazi *et al*. [129]. Authors suggest first converting the linear BLP problem to a single level optimization problem by annexing the KKT conditions equivalent to the lower level linear programming problem (LPP), and then solve the resulting problem using a binary coded GA.

Nonlinear BLP problems with continuous variables

The problem with general structure (1.2.2) is specified as non-linear BLP problem for if at least one of the functions $f_1(x,y)$, $f_2(x,y)$, $g_1(x,y)$, and $g_2(x,y)$ are non-linear in (x,y).

Classical methods available in the literature for solving a nonlinear BLP problem can broadly be classified into different categories depending upon their approaches to handle the follower's problem. Starting from the most fundamental one, first category is based on exploiting the KKT conditions for the lower-level problem. Method following this approach [130] suggest obtaining the constraint qualifications based on KKT conditions corresponding to the lower-level optimization problems, annex them in the upper-level problem, and then handle the complimentarity problem through different ways. The next category of methods is based on the descent direction approach which extracts derivative information for deciding on the direction of exploration [114,131,132]. Another category of methods based on penalty function approach are suggested in [133–135]. Methods following the trust region approach for solving nonlinear BLP problems with continuous variables are [136–138]. One more method based on branch-and-bound approach is available in literature for solving BLP problems specifically with leader's objective function linear in (x, y), follower's objective function quadratic in (x, y) and leader's constraints independent of y (c.f., [139]).

There are only two evolutionary approaches available in the literature for solving a nonlinear BLP problem with continuous variables. Both are Nested approaches which suggest applying GA at upper level and directly solving the parameterized lower level problem. Mathieu *et al*. [140] suggest the approach for the case in which lower level parametric optimization problem is a linear programming problem in follower's variables. Whereas, Yin [141] suggest applying Frank-Wolfe algorithm for solving non-linear programming problem at lower level.

Discrete BLP problems

The problem with general structure (1.2.2) is specified as discrete BLP problem for if variables x and y are discrete. These problems are also called integer or binary BLP problems, depending on the specific types of variables involved. Specific scenarios of discrete BLP problems are discussed theoretically as

- Discrete-Discrete BLP problem: For if $x \in \mathbb{Z}^n$ and $y \in \mathbb{Z}^m$.
- Discrete-Continuous BLP problem: For if $x \in \mathbb{Z}^n$ and $y \in \mathbb{R}^m$.
- Continuous-Discrete BLP problem: For if $x \in \mathbb{R}^n$ and $y \in \mathbb{Z}^m$.

In case, some of variables among (x, y) are continuous and other discrete, then such a hierarchical problem is also termed as mixed-integer BLP problem.

The review of literature is presented here together for both linear and nonlinear discrete BLP problems, as the available solution approaches focus mainly on the discreteness instead of linearity or nonlinearity. Fundamental work by Vicente *et al*. [142] demonstrates that the inducible region for each scenario can be different for same problem. Authors demonstrate further that a discrete variable at any level of the problem can lead to a disconnected inducible region.

Most commonly used branch-and-bound technique and branch-and-cut to handle discreteness in optimization problems remain the base for work on classical methods for solving discrete BLP problems. Methods available in the literature mostly use the nested approaches or KKT-based single level reduction approach (for the case when lower level problem involves all continuous variables) to handle the hierarchical structure of discrete BLP problems [116].

Vicente *et al*. [142] address discrete linear BLP problems to analyze the properties and existence of the optimal solution for different scenarios listed above. Moore and Bard [143] propose a branch-and-bound approach with a nested structure for solving mixed-integer linear BLP problem. But the approach is non-scalable beyond a few integer variables. Same authors handle binary BLP problem in [144]. Bialas and Karwan [121] use branch-and-cut technique seeking incremental improvements in the upper level objective function to solve the continuous-discrete linear BLP problem. Some authors use Bender-decomposition-based techniques along with KKT-based reduction techniques to solve mixed-integer BLP problems in studies [145–147].

Sinha *et al*. [116] in their review on methodologies to solve BLP problems indicate a scope of algorithmic development by pointing out the shortcoming of existing methodologies to solve discrete BLP problems involving a large number of variables. Due to this drawback of existing methodologies the evolutionary methods have been attempted to solve discrete BLP problems in [148–154]. By far, no evolutionary algorithm is developed for efficiently solving mixed-integer BLP problems by utilizing the properties of discreteness and the problem structure.

Multi-objective BLP problems

Conforming to terminologies discussed for defining a solution of a BLP problem (presented in the beginning of section 2.1.1), the concept of "weak efficiency set of solutions to the lower level problem" is defined for MOBLP problems in place of "follower's reaction set".

Weak efficiency set of solutions to the lower level problem for $x \in S(X)$:

$$P(x) = \{y \in \mathbb{R}^m : y \in \text{argmin}\{f_2(x, w) : w \in S(x)\}\}$$

A solution of a multi-objective BLP problem is discussed through following definitions [155].

Definition 2.1.2: For a fixed $x \in S(X)$, if y is a Pareto-optimal[6] solution to the lower level problem, then (x, y) is a feasible solution to the problem (1.2.4).

Definition 2.1.3: If (x^*, y^*) is a feasible solution to the problem (1.2.4) and there are no $(x, y) \in IR$, such that $f_1(x, y) \succ f_1(x^*, y^*)$, then (x^*, y^*) is a Pareto-optimal[7] solution to the problem (1.2.4).

Remark 2.1.2:

(1) For $p > 1$, the solution set to the overall problem (1.2.4) is thus referred as the leader's Pareto-optimal frontier.

(2) For the follower's problem to be multi-objective (*i.e.*, $q > 1$), corresponding to any given value of leader's variables x, there is a set of Pareto-optimal solutions of the follower's multiobjective optimization problem parameterized in x. Thus in this case the lower level optimization problem as a part of BLP problem (1.2.4) represents a set-valued map defined as $\Psi: S(X) \rightarrow \wp(\mathbb{R}^m)$ such that $\Psi(x) = \text{argmax}\ \{f_2(x, y): g_2(x, y) \leq 0\}$, which is the Pareto-optimal set of the follower's optimization problem parameterized in x.

[6] For multi-objective lower level problem (*i.e.*, $q > 1$), a response of the follower is considered in terms of Pareto-optimal solution. Whereas, for if lower level problem involves single objective (*i.e.*, $q = 1$), a response of the follower is considered in terms of an optimal solution.

[7] For multi-objective upper level problem (*i.e.*, $p > 1$), a solution of the leader's problem is considered in terms of Pareto-optimal solution. Whereas, for if upper level problem involves single objective (*i.e.*, $p = 1$), a solution of the leader's problem is considered in terms of an optimal solution.

A more practical approach to tackle the MOBLP problems is possible if the leader is able to identify the choice function or pattern of the follower regarding the trade-off between multiple objectives of the later. Such patterns can be learnt through the identification of weights of objective functions of the lower level problem as decided by the follower.

Eichfelder [156] proposes a theoretical approach to solve MOBLP problems by blending a numerical optimization technique to solve the lower level optimization problem with an adaptive exhaustive search method to handle the upper level optimization problem. Shi and Xia [157] use ϵ-constraint method at both levels of MOBLP problem to convert the problem into an ϵ-constraint BLP problem. Authors further replace this ϵ-constrained BLP problem with a single level optimization problem by replacing the lower level problem with equivalent KKT conditions. Both the solution methodologies for MOBLP problems have been developed by considering multiple assumptions and demonstrated only on minuscule illustrations. Computational expensiveness and non-scalability to large-scale problems is noted for these methodologies by Sinha *et al*. [116].

For some theoretical results on optimality conditions for MOBLP problems research articles [158–160] can be referred. A more recent development by Lafhim [161] on necessary conditions for local weak efficient solution of MOBLP problem indicates the scope of emergence of research in the context of solution methodologies for MOBLP problems.

Evolutionary approaches like particle swarm optimization (PSO), hybridized evolutionary algorithms, GA have been used in literature to solve MOBLP problems with different variations through linearity [55], discreteness of variables [162]. The concerns of computational expense on solution methodologies for MOBLP problem which are noted above are addressed by Sinha [163]. The author proposes a progressively interactive evolutionary algorithm by considering interaction of lower level decision-maker with upper level decision-maker for obtaining the most preferred response at upper level. The algorithm is thus suggested for a particular case of MOBLP problem similar to the optimistic BLP problem.

2.1.2 Methods for solving multi-leader-follower BLP problems

BLP problems which involve multiple leaders or multiple followers have been addressed to a very limited extent for solution algorithms. A little research in this area is therefore listed in a single section only, but under separate headings of single-leader-multi-follower, multi-leader-single-follower, and multi-leader-multi-follower BLP problems. For a BLP problem, where there are multiple leaders or followers which compete with each other for optimizing their individual objectives while incorporating the reaction or action from the other level, the complexity of the decision-making is observed to be very high. This situation is attributively termed as a Stackelberg-Nash game [164]. For understanding theoretically such variants of BLP problems, it is essential to understand the fact that at whichever level there are multiple decision-makers, the competition among them is appropriately modelled as a Nash game.

Single-leader-multi-follower BLP problems

In case of single-leader-multi-follower BLP problem (1.2.4), for any given value of leader's variables x, the reaction of followers ($y = (y_1, y_2, \dots y_K)$) needs to be computed by solving a constrained Nash-equilibrium game problem of K players. The players in this situation are refer to the followers of BLP problem (1.2.4) and their optimization problems constituting the Nash-game precisely comprise the lower level problem of (1.2.4) parameterized in the values of the leader's variables x. It should be noted that in this framework, corresponding any given value of leader's variables x, the reaction y refers to a Nash-equilibrium point of the corresponding constrained Nash-equilibrium problem of the followers. For any given value of leader's variables x, once this Nash-equilibrium point is computed, then only the leader's objective function value can be calculated. As solving a constrained Nash-equilibrium problem is itself a taxing task, the complexity involved in solving a single-leader-multi-follower BLP problem can be easily realized.

Anandalingam [165] suggests a penalty function based approach for solving the linear case of single-leader-multi-follower BLP problem. Lu [166] suggests a method based on KKT approach for solving again the linear case of this variant of BLP problem. Calvete and Galé [167] proves that single-leader-multi-follower linear BLP problem with independent followers can be converted into a single-leader-follower linear BLP problem.

A GA-based approach for solving single-leader-multi-follower BLP problem is developed by Liu [164]. Author demonstrates the methodology by solving a few problems involving a maximum of eight variables, but the computational time reported by the author is surprisingly high even for such miniscule problems.

Multi-leader-single-follower BLP problems

The multi-leader-single-follower BLP problem addresses the situation of multiple leaders competing with each other in which the reaction of a follower in response to their actions affects their decisions-making. This competition among them amounts to a constrained Nash-game problem in which the follower's optimization problem is included as a part of constraints of each leader's optimization problem. This indicates that multi-leader-single-follower BLP problems also involve a complex mathematical structure, making it difficult to develop some solution methodology. This variant of BLP problems is not seen to be properly addressed in literature for solution algorithms.

Zhang et al. [168] present nine cases of multi-leader-single-follower BLP problem, and corresponding decision models depending upon various relationships between multiple leaders. All these models consider the follower's problem as a linear programming problem. Authors further propose use nested approach by applying particle swarm optimization algorithm at both levels for solving the problem.

No more work is available in literature specifically addressing this variant of BLP problems. Some researchers have addressed this problem as a particular case of multi-leader-multi-follower BLP problems with response of follower considered as optimal solution of single follower's optimization problem solved for any given vector of values for leaders' variables [169,170].

Multi-leader-multi-follower BLP problems

Even more perplexed situation is of a BLP problem where there are multiple leaders competing among themselves as well as multiple followers competing among themselves. The situation involves Nash-games at both levels of action and reaction, wherein to the environment created due to a Nash-equilibrium point of competition among leaders, the followers react with another Nash-equilibrium point settled for the competition among them. General BLP problem involving multiple leaders and/or multiple followers involving continuously differentiable functions is addressed by Leyffer and Munson

[169], Hori and Fukushima [170] for developing solution algorithms through iterative approaches.

Sinha *et al*. [171] suggest an evolutionary algorithm for solving the linear multi-leader-multi-follower BLP problem in which GA is used for exploration of variables at both levels. As this approach suggests using evolutionary exploration at both levels for approximating a solution of the problem, therefore these approaches are computationally too expensive to be applied to the problems involving large number of variables. Nie [172] handles a special type of multi-leader-follower BLP problem in which some among a group of players become leaders and rest as followers, and this selection of leaders keep on changing dynamically.

To summarize, there is a limited research on algorithms for solving BLP problems involving multiple leaders and/ or multiple followers. Over that, algorithms existing in the literature in this purview have strong limitations and are incapable of handling large-scale problems [173].

2.2. Genetic algorithms for solving optimization problems

For handling NP-hard or nondeterministic polynomial time hard optimization problems or large-scale practical optimization problems the classical methods fail to find even a local optimal solution or a near-optimal solution [174]. For such problems it is aimed to look for a method capable of finding a good feasible solution in reasonable amount of time. Metaheuristic methods are observed to work successfully for producing a good-quality solution within a practical time frame most of the time. The mind map shown in Figure 2.1 lists various metaheuristic algorithms developed so far for solving various optimization problems.

All these metaheuristic algorithms are basically inspired from the nature and use a blend of global exploration and local exploitation of the search space. There are no agreed guidelines available in literature for choosing an algorithm for solving large-scale nonlinear optimization problems. Especially for those problems which are nondeterministic polynomial time hard or NP-hard, any particular algorithm cannot be identified as all time efficient one [175]. Thus, it is difficult to justify the choice of one particular artificially intelligent metaheuristic algorithm over another for solving such problems. Among all these metaheuristic algorithms, genetic algorithms (GAs) are the most incipient one and have been widely used for solving various optimization problems, especially the complex and NP-hard ones.

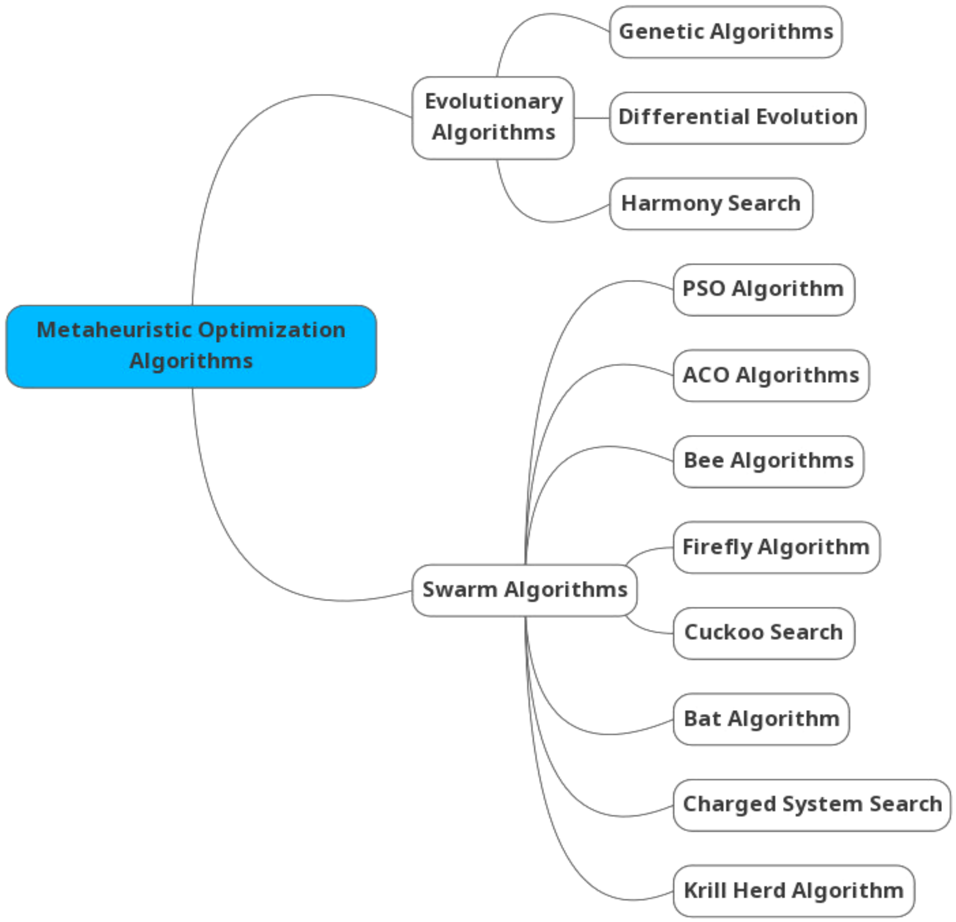

Figure 2.1: Various metaheuristic algorithms for solving optimization problems

GA approaches are practically relevant till date, as are being successfully used by researchers for solving practical problems in emerging areas of optimization. The area of BLP is also indifferent in this context. As the research work presented in subsequent chapters of this thesis include design of GA-based methodologies for solving variants of BLP problems, therefore it is impelling to present here fundamentals of GAs.

2.2.1 Fundamentals of genetic algorithms

The concept of GA as originally conceived by John Holland, Professor at University of Michigan, Ann Arbor, and developed with his students, codes the chromosomes as binary numbers. For fundamental details one can refer to well documented texts [176,177]. Since then the GAs with its variants have broadly evolved as a class of methodologies, which the researchers have used innovatively for solving various types of optimization problems formulated for almost every area of application. GA involves some basic terminologies which are explained below.

- **Chromosome:** Chromosome is a set of numbers ordered generally in a form of string or sequence, and representing a candidate solution to the problem under consideration. There are multiple chromosomes involved in a practical execution of GA and all are of a fixed length.
- **Gene:** A gene is an element position of a chromosome.
- **Allele:** Allele is a value which a gene takes in a chromosome.
- **Population:** A set of certain number of chromosomes taken together during any iterative stage of the GA.
- **Generation:** Each iterative stage of GA is termed as a generation.
- **Population size:** The number of chromosomes in any population of GA is termed as population size. It is decided prior to the execution of GA and kept as fixed across generations. This depends on the nature and the scale of the problem. There has not been any particular mechanism for deciding the population size, and there remained a practice to fine-tuned it depending upon the size of the problem being solved and complexity involved in computations (for e.g., evaluation of fitness function). But very recent research has come up to support a decision on this [178].
- **Parent and offspring chromosomes:** Chromosomes of any particular generation on which genetic operators[8] are applied are termed as parent chromosomes whereas those obtained as outcome of these operators are termed as offspring.

The execution of GA involves an iterative computational procedure performed in a sequential manner which is described through a flow chart depicted in Figure 2.2. Steps mentioned in the flow chart are further explained hereafter.

Initialization

Initial population of chromosomes is generated randomly with an attempt to cover a diverse range of candidate solutions. This involves an encoding mechanism to represent

[8] Genetic operators are explained subsequently with detailed explanation on GA.

candidate solutions as chromosomes. There are multiple ways available in literature for genetic representation of candidate solutions as chromosomes. Apart from the originally developed binary coding [176], some other representation used to encode chromosomes include real encoding, integer encoding, encoding as matrices, encoding as tree. A GA encoding chromosomes using binary numbers is termed as binary-coded genetic algorithm (BCGA), whereas the one using real numbers as real-coded genetic algorithm (RCGA), and one the using integers only as integer-coded GA. With no superiority of a particular way of encoding over the other [179], users choose one depending on its appropriateness to the problem being solved.

Fitness evaluation

The quality of candidate solutions represented as chromosomes is assessed by its fitness function value. For this sake, a fitness function is ingeniously defined so that with the improvement in value computed for chromosomes across generations, the corresponding candidate solutions move towards becoming a feasible solution with better value of the objective function. The fitness function is generally seen to be user defined as specific to the problem. Also, some constraint handling techniques for GA suggest using penalty functions against infeasibility and associate them with fitness function [180,181].

Selection

From the population of each particular generation some chromosomes are selected to breed a new generation. The selection process is so designed to prefer the fitter chromosomes (*i.e.*, those with better fitness function value). There are multiple procedures suggested in literature for random selection of chromosomes. Most basic one is roulette-wheel selection [176] which considers the probability of selecting a chromosome proportional to its fitness[9]. Another such procedure is of tournament selection, in which fittest chromosome is selected from a randomly selected subset of population and this process is repeated till the pool of chromosomes required for breeding is complete [182]. The procedure of selecting from top a proportion of chromosomes ranked per their fitness to generate the offspring population is termed as truncation selection [183].

[9] Fitness is a relative term which indicates the qualitative status of a chromosome compared with others in terms of the fitness function value.

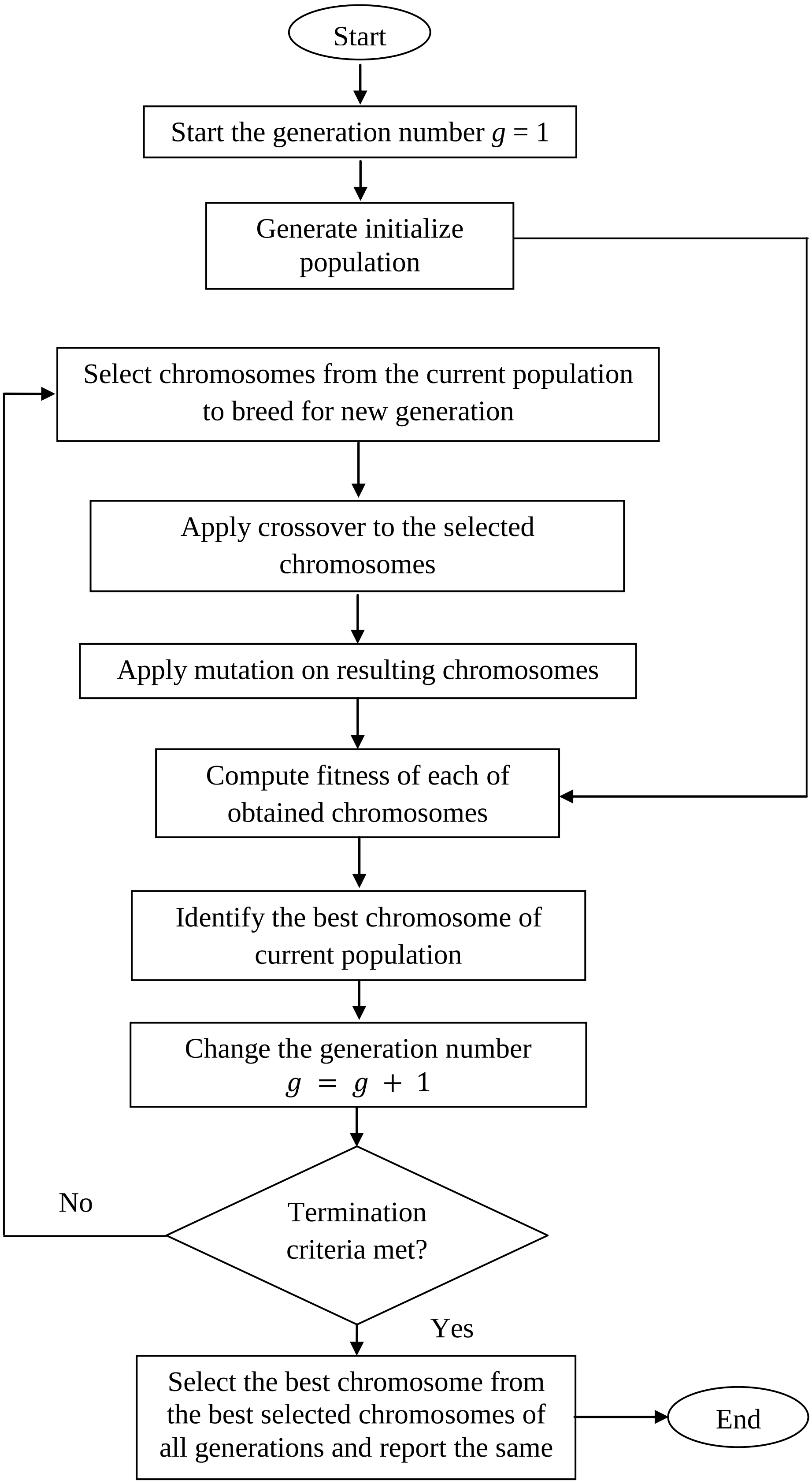

Figure 2.2: Flowchart of GA for solving optimization problems

Elitism is a strategy which is used by annexing it to various selection techniques with an intension of improving convergence speed of the GA. The strategy is about carrying over a small proportion of best chromosomes of the previous generation to the next one [184]. Experimental studies (e.g., [185,186]) have examined the efficacy of use of elitism into GAs.

Genetic operators

Chromosomes selected using any one of procedures mentioned above are operated using crossover operator followed by mutation operator. This process generates new chromosomes from those which are selected using any of above discussed procedures. As the application of these operators generates a new population from that of previous generation, the process is usually termed as breeding or mating. Crossover operator is designed to generate a new pair of chromosomes from any pair of selected chromosomes. There are application-dependent as well as application-independent crossovers suggested in literature, some review papers worth referring in this context are [187–189]. Some application specific crossover operators have recently been developed [190]. There are many crossover operators available in literature which are application independent in general but can be used appropriately depending upon the situation, to name some one-point crossover and two-point crossover [191], multi-section crossover [192], Laplace crossover [193].

On the other hand, the mutation operator is designed to alter alleles of one or more randomly selected genes thereby generating a new chromosome. There are many mutation operators available in literature, to name some Gaussian mutation operator [194], Cauchy mutation operator [195], mean mutation operator [196], power mutation [197]. Review articles on mutation operators suggested in literature can be referred in this context [198,199]. Some application specific mutation operators can be referred [200–202].

The literature on the genetic operators is enormous and it is practically impossible to present here an exhaustively review of it. We explain in subsequent sections only those genetic operators which we have specifically used in the research work explained, *viz*, Laplace crossover and power mutation.

Termination

The sequential computational process continues from one generation to the next until a termination condition is met. Conditions generally used as termination criteria are:

- a solution is found that satisfies minimum criteria,
- pre-fixed number of generations is reached,
- pre-fixed computation time spent,
- best fitness value across generations after reaching a plateau does not improve any more for many successive iterations,
- a combinations of above criteria.

Designing of GA for solving any particular optimization problem requires the user to decide on (a) type of encoding of chromosomes (*i.e.*, genetic representation), (b) choice of genetic operators, and (c) defining a problem specific fitness function. In case of practical problems involving a large number of variables and large real number values associated with those variables, the use of BCGA is faced with some issues like hamming cliff, uneven variations due to changes in alleles of genes representing significant and insignificant bits. Such challenges are not faced with RCGA. The chromosome encoding is also simple and straightforward for RCGA as compared with BCGA. In case of mixed integer programming problems (MIPP) the encoding of chromosomes can be done with the genes representing real variables to have a real allele and those representing integer variables to have integer allele. Such algorithms are termed in literature as RCGAs for MIPPs.

2.2.2 Metaheuristic-based approaches for solving BLP problems

Majorly two metaheuristic-based approaches are available in the literature for solving a BLP problem. First one applies metaheuristic algorithm at both levels of BLP problem, one for exploration of leader's variables and the other for approximating the corresponding reaction of follower. The second approach suggests applying a metaheuristic algorithm for exploration of leader's variables only, while directly solving the follower's parameterized optimization problem for obtaining the reaction corresponding to any action of leader encoded for exploration through the metaheuristic algorithm. This approach is commonly termed as Nested approach for solving BLP problems.

The fitness function in these approaches is mostly seen to be considered as leader’s objective function or a composite of the same with some real function. As a BLP problem is posed to assess best action of the leader while incorporating the follower’s reaction mechanism, therefore defining the fitness function for corresponding to the leader’s objective function justifies the comparison of chromosomes of any population and appropriately reflects the progress of the algorithmic computations over the generations.

For the situations in which solving the follower’s problem for any given values of leader’s variables is a challenging task, the first approach is observed to be appropriate to apply. On the other hand, if computing the response of follower(s) by directly or indirectly solving the follower’s optimization problem is viable, the nested approach gives an advantage over the first approach. The advantage of nested approach in this situation is that herein each chromosome representing vector of values of leader’s variables along with corresponding reaction of follower(s) corresponds to a point in inducible region of the BLP problem. Thereby, the exploration by such an algorithm remains confined to feasible solution of the BLP problem. This provides superiority to the nested approach over the other metaheuristic approach in terms of computational efficiency and closeness to the realization of the implementation of the solution due to exactness of the computed response.

In our research work we have adopted GA-based nested approach for solving the BLP problems formulated to address some strategic decision-making issues. The execution of this approach is described through the flow chart depicted in Figure 2.3.

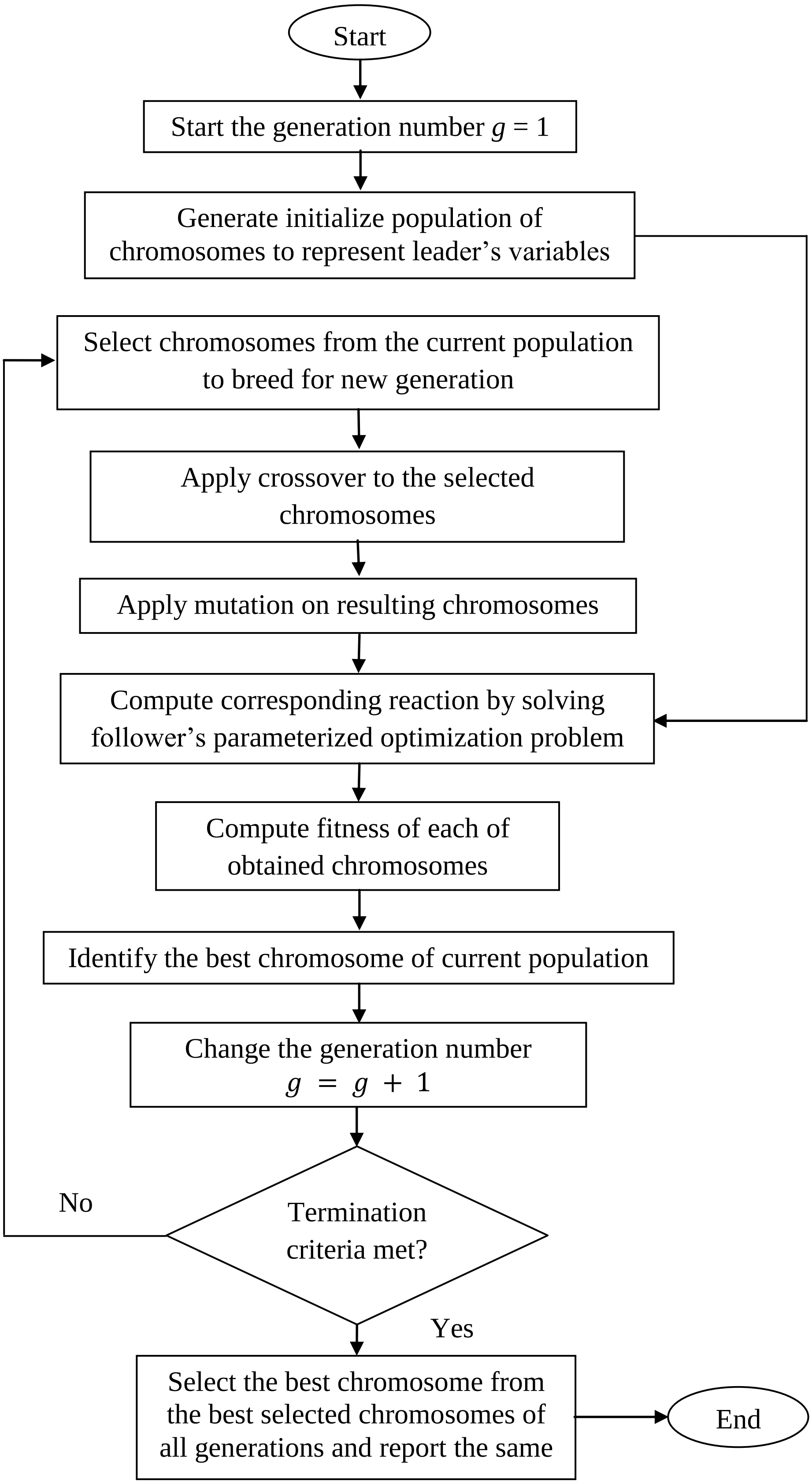

Figure 2.3: Flow Chart of GA-based nested approach for solving BLP problems

2.2.3 Detailed explanation of genetic operators used in our work

In our research work included in subsequent chapters, the modelled optimization problems involve a mix of integer and real variables. Also, these problems are of a practically large scale, requiring the use of metaheuristic algorithms like GAs for solving them. Therefore, we have designed problem specific RCGAs with chromosome-encoding through real numbers or integers. As a second important step towards designing such a GA, we have used Laplace crossover [193] and power mutation [197] as genetic operators. The reason behind using these operators is that they have been appropriately developed to use in RCGAs for MIPPs and preserve the feasibility of the problem further the offspring chromosomes generated as a result. The success of these operators is demonstrated in a later development [203]. Popular use in further research developments of GA indicates their acceptance to use appropriately [204–207]. These genetic operators are accordingly explained below.

Laplace crossover [193]

It generates two offspring chromosomes $y^1 = (y_1^1, y_2^1, \dots, y_s^1)$ and $y^2 = (y_1^2, y_2^2, \dots, y_s^2)$ from the parent chromosomes $x^1 = (x_1^1, x_2^1, \dots, x_s^1)$ and $x^2 = (x_1^2, x_2^2, \dots, x_s^2)$. A random number is uniformly generated in the interval $(0, 1)$ and if it is greater than the probability of crossover (*pc*), then $y^1 = x^1$ and $y^2 = x^2$, else otherwise Laplace crossover operates on pair x^1 and x^2 to generate y^1 and y^2 as follows.

Step 1. Generate uniformly distributed random numbers $u_i, r_i \in [0, 1]$

Step 2. Generate a random number ρ_i satisfying the Laplace distribution as:

$$\rho_i = \begin{cases} a - b\log(u_i), r_i \le 1/2; \\ a + b\log(u_i), r_i \ge 1/2. \end{cases}$$

where a is location parameter and $b > 0$ is a scaling parameter which is taken as an integer in present study.

Step 3. Obtain offspring y^1 and y^2 with

$$y_i^1 = x_i^1 + \rho_i|x_i^1 - x_i^2|$$
$$y_i^2 = x_i^2 + \rho_i|x_i^1 - x_i^2|.$$

Step 4. If any of the y_i^1 and/ or y_i^2 is not an integers, then it is truncated to an integer by using ceiling or floor functions with probability 0.5.

Power mutation [197]

This operator generates the chromosome $y = (y_1, y_2, \dots, y_s)$ from the parent chromosome $x = (x_1, x_2, \dots, x_s)$. For each $i = 1, \dots, s$, a random number is uniformly generated in the interval $(0, 1)$ and if it is found to be greater than the probability of mutation (pm), then $y_i = x_i$, else otherwise mutation is carried out in the i^{th} gene as follows.

Step 1. Generate a random number $\upsilon_i \in [0, 1]$ following the uniform distribution.

Step 2. Obtain $\eta_i = (\upsilon_i)^p$, where p is considered as an integer in case of integer restriction on the i^{th} component of the chromosome.

Step 3. Generate a random number $r \in [0, 1]$ uniformly distributed.

Step 4. Take $\tau = \frac{(x_i - x_i^l)}{(x_i^u - x_i)}$, x_i^l and x_i^u are lower and upper bounds respectively on the i^{th} component of the decision variable, and obtain y_i from x_i as

$$y_i = \begin{cases} x_i - \eta_i(x_i - x_i^l), \tau < r; \\ x_i + \eta_i(x_i^u - x_i), \tau \geq r. \end{cases}$$

Step 5. If y_i is not an integer, then it is truncated to an integer by using ceiling or floor functions with probability 0.5.

2.3. Nash games and their role in multi-leader-follower BLP problems

2.3.1 Nash games

A competitive situation is termed as a Nash game when two or more decision-makers choose their strategies simultaneously or consequently but without any knowledge on opponents' actual decision. Although herein, it is assumed that each decision-maker has complete information about all possible strategies available to other opponents. Another assumption considered in this context is that all of them behave non-cooperatively. In the context of this Nash game the decision-makers are customarily called as players and sets of values of decision variables under their control are termed as strategies.

Unconstrained Nash Game

Definition 2.3.1: The solution of a Nash-game, termed as a **Nash-equilibrium point**, is identified as the vector of values corresponding to decision variables of each player such that no player can improve the individual payoff by altering its strategy with other players adhering to their strategies specified by this point.

Herein, it is assumes that:

- The payoff of each player is influenced by the choice of that particular player's strategy and that of the other players too.
- All the players aim to maximize their own payoffs.
- All the players can control on their individual strategy.

Mathematically, a Nash-equilibrium point can be expressed as following.

Definition 2.3.2: For a Nash game among K players, let $f_k(x_1, \dots, x_K)$ denote the payoff of player k $(k = 1, 2, \dots, K)$, with all possible values of x_k as the strategies available to player k. A point $x^* = (x_1^*, \dots, x_K^*)$ is a Nash-equilibrium point of this game if

$$f_k(x_1^*, \dots, x_k^*, \dots, x_K^*) \geq f_k(x_1^*, \dots, x_k, \dots, x_K^*), \quad \forall x_k, \forall k. \tag{2.3.1}$$

Remark 2.3.1: (Necessary condition for a Nash-equilibrium point)

For an unconstrained Nash-game with strategy vector $x = (x_1, \dots, x_K)$ as continuous and payoff functions f_k continuously differential *w.r.t.* x_k, above condition (2.3.1) implies that the Nash-equilibrium point x^* is a stationary point of each player's payoff function f_k, *i.e.*, x^* satisfies

$$\frac{\partial f_k}{\partial x_k} = 0, \qquad \forall\, k. \tag{2.3.2}$$

These conditions (2.3.2) are famously termed as "first order conditions (FOCs)".

Further, for developing a method to solve an unconstrained Nash-equilibrium problem, sufficient conditions should be established to ascertain a Nash-equilibrium point. Following result discuss about when these conditions become sufficient to conclude the existence of Nash-equilibrium point.

Result 2.3.2: (Sufficiency of FOCs in case of concavity of payoff functions [208])
If the payoff f_k of each player k $(k = 1, 2, \dots, K)$ is concave functions *w.r.t.* the strategy x_k, then the FOCs (2.3.2) become sufficient for concluding a point $x^* = (x_1^*, \dots, x_K^*)$ is a Nash-equilibrium point.

Result 2.3.3: (Sufficiency of FOCs in case of quasi-concavity of payoff functions [209])
FOCs (2.3.2) are sufficient for existence of Nash-equilibrium even for the case when payoffs f_k quasiconcave.

Remark 2.3.1: If the profit function is concave with respect to price, FOCs (2.3.2) become sufficient for Nash-equilibrium point of a Nash game among players competing to maximize individual profit. However, in the case of non-concavity, and in the situation when no sufficient condition is satisfied, then the solutions obtained by solving the conditions (2.3.2) must be verified using the Nash equilibrium definition post-hoc.

Further, a specific situation where the uniqueness of Nash equilibrium can be assured is discussed in following result by Anderson et al. [209].

Result 2.3.4: For strictly quasi-concave payoff function defined to model the profit using the logit demand results in a unique Nash price equilibrium.

A basic method for solving a Nash game stems from the above discussed results which is known as FOC method [208].

Some other methods discussed in literature for solving unconstrained Nash games are: relaxation method [210], projection method [211], nonlinear complementarity problem approaches [212], fixed point iteration method [213].

Constrained Nash Game

Mathematically, a constrained Nash game can be expressed as following.

Definition 2.3.3: For a Nash game among K players, with $f_k(x_1, \dots, x_K)$ denoting the payoff to player k $(k = 1, 2, \dots, K)$, and values of x_k as the strategies available to player k. If the strategies available to each player k is restricted by a set of equality constraints represented by $h_k(x_1, \dots, x_K) = 0$ and a set of inequality constraints represented by $g_k(x_1, \dots, x_K) \leq 0$, then the constrained Nash game can be mathematically expressed as a set of following K constrained optimization problems considered together.

$$\left.\begin{array}{l} \max\limits_{x_k} f_k(x_1, \dots, x_K) \\ \text{subject to} \\ \qquad h_k(x_1, \dots, x_K) = 0 \\ \qquad g_k(x_1, \dots, x_K) \leq 0 \end{array}\right\} \quad \forall\, k = 1, 2, \dots, K \qquad (2.3.3)$$

Definition 2.3.4: Nash-equilibrium point of such a game refers to a point $(x_1^*, \dots, x_K^*)$ satisfies (2.3.1) and all the constraints (of each player) given in (2.3.3).

In the case of a constrained Nash-game, the FOCs obtained for the Lagrangian function together with additional inequality constraints represent the KKT-necessary conditions [214] of Nash equilibrium for regular points [215]. Whereas, Friedman [208] suggested sufficient conditions in this case are as following.

Result 2.3.5: For a non-cooperative game with complete information, Nash equilibrium exists if,

(1) the strategy set is nonempty, compact, and convex for each player;
(2) the payoff function is defined, continuous, and bounded; and
(3) each individual payoff function f_k is concave with respect to individual strategy x_k.

Result 2.3.2: Anderson et al. [209] proves the existence of unique (price) equilibrium point of a constrained Nash game with strictly quasi-concave payoff functions representing the profit functions modelled under the logit demand. Thus, for such a case, solution of KKT-necessary conditions would uniquely determine the Nash equilibrium prices.

Next two subsections demonstrate theoretical development of conditions for handling Nash game at lower and upper levels of BLP problem, in case of multiple players (decision-makers in context of BLP problem) at respective levels.

2.3.2 Nash game situation in single-leader-multi-follower BLP problems

The literature in this context is minuscule and is listed in section 2.1.2. Majority of methods suggest ascertaining Nash equilibrium point based on approaches inspired from the above theoretical discussion. The same approach is used by Leyffer and Munson [169] in a development towards solution methodology for solving multi-leader-follower BLP problem. Authors suggest to, first obtain FOCs for followers' problems as a system of equations, which equivalently represents optimal response of followers competing in a Nash game among each other. These FOCs are then suggested to be annexed to the leader's constraints for obtaining single level optimization problem. Thus, using this procedure, a single level optimization problem is obtained as an equivalent of the single-leader-multi-follower BLP problem.

2.3.3 Nash game situation in multi-leader-single-follower BLP problems

Theoretical developments for solution methodologies of multi-leader-single-follower BLP problems can only be taken as a special case from the literature in the context of multi-leader-multi-follower BLP problems. Among limited developments, a major break-through has come through the work of Leyffer and Munson [169], Hori and Fukushima [170].

In our research work presented in Chapter 5, we have developed a methodology for solving a large scale multi-leader-single-follower BLP problem with bilinear objective functions. This methodology is proposed on the basis of a theoretical development appropriately derived from the work of Leyffer and Munson [169] for the case discussed. First we present here the part of work of Leyffer and Munson [169] used for our theoretical development and the same is followed by our theoretical development derived for using the same in solution methodology.

Approach of Leyffer and Munson [169]

A multi-leader-single-follower BLP problem is considered as expressed below for k leaders (indexed $i = 1,2,\dots,k$) competing with each other.

(MLSF-BLP)

$$
\begin{array}{ll}
(\mathrm{LDMP}-i) & \min\limits_{x_i \geq 0} f_i(x_i, y) \\
& \text{subject to} \\
& \quad g_i(x_i, y) \geq 0, \\
& \quad G_i(x_i, y) = 0, \\
& \text{where, } y \text{ is optimal response of the follower corresponding} \\
& \text{to the leaders' variables, } x = \{x_i : i = 1,2,\dots,k\}, \\
(\mathrm{FDMP}) & \min\limits_{w} b(x, w) \\
& \text{subject to} \\
& \quad c(x, w) \geq 0, \\
& \quad w > 0.
\end{array}
\tag{2.3.4}
$$

Here, $(\mathrm{LDMP} - i)$ is optimization problem of leader i, $(i = 1, 2, \dots, k)$ in which the follower's optimization problem (FDMP) is incorporated as a constraint.

Definition 2.3.1: If, for any given vector $x = \{x_i : i = 1, 2, \dots, k\}$, y is an optimal solution of (FDMP) in (MLSF-BLP), such that the vector (x, y) satisfy constraints $g_i(x_i, y) \geq 0$ and $G_i(x_i, y) = 0$, $(i = 1, 2, \dots, k)$, then (x, y) is a *feasible solution* of (MLSF-BLP).

For the (MLSF-BLP) problem described above the penalty approach is described in following the steps.

Step 1: The KKT equivalent of (FDMP) (in variables w), is expressed as following with multipliers denoted by z

$$0 \leq w \ \perp \ \nabla_w b(x, w) - \nabla_w c(x, w) z \geq 0 \qquad (2.3.5)$$

$$0 \leq z \ \perp \ c(x, w) \geq 0. \qquad (2.3.6)$$

Step 2: **(a)** Redefining y as $y = (w, z)$, and defining

$$h(x, y) = \begin{bmatrix} \nabla_w b(x, w) - \nabla_w c(x, w) z \\ c(x, w) \end{bmatrix} \qquad (2.3.7)$$

(b) Introducing slack variables s, the conditions become

$$h(x, y) - s = 0 \qquad (2.3.8)$$

$$0 \leq y \ \perp s \geq 0 \qquad (2.3.9)$$

Step 3: Incorporating these conditions reduces (MLSF-BLP) to the following equilibrium problem with equilibrium constraints (EPEC).

For each leader $i = 1, 2, \dots, k$

$$\min_{x_i \geq 0} f_i(x_i, y)$$

subject to

$$g_i(x_i, y) \geq 0,$$

$$G_i(x_i, y) = 0,$$

$$h(x, y) - s = 0,$$

$$0 \leq y \ \perp s \geq 0.$$

The above equilibrium problem with equilibrium constraints is rewritten as following single level constrained Nash game problem.

$$
\text{(SLNG)} \qquad \left.\begin{aligned}
&\min f_i(x_i, y) \\
&\text{subject to} \\
&\quad -g_i(x_i, y) \leq 0, \\
&\quad G_i(x_i, y) = 0, \\
&\quad s - h(x, y) = 0, \\
&\quad -x_i \leq 0, \\
&\quad -y \leq 0, \\
&\quad -s \leq 0, \\
&\quad Ys \leq 0.
\end{aligned}\right\} \qquad (2.3.10)
$$

(Here, $Y = diag(y_1, y_2, \dots, y_r)$.)

Step 4: A desired solution to (SLNG) is a solution of the following strong-stationarity conditions

$$\nabla_{x_i} f_i(x_i, y) - \lambda_i' \nabla_{x_i} g_i(x_i, y) - \nu_i' \nabla_{x_i} G_i(x_i, y) - \mu_i' \nabla_{x_i} h(x_i, y) - \chi_i = 0 \qquad \dots (2.3.11)$$

$$\nabla_y f_i(x_i, y) - \lambda_i' \nabla_y g_i(x_i, y) - \nu_i' \nabla_y G_i(x_i, y) - \mu_i' \nabla_y h(x_i, y) - \psi_i + S\xi_i = 0 \qquad \dots (2.3.12)$$

$$\mu_i' \nabla_s \big(s - h(x, y)\big) - \sigma_i' \nabla_s(s) - \xi_i' \nabla_s(Ys) = 0 \qquad (2.3.13)$$

$$0 \leq g_i(x_i, y) \perp \lambda_i \geq 0 \qquad (2.3.14)$$

$$G_i(x_i, y) = 0 \qquad (2.3.15)$$

$$h(x, y) - s = 0 \qquad (2.3.16)$$

$$0 \leq x_i \perp \chi_i \geq 0 \qquad (2.3.17)$$

$$0 \leq y \perp \psi_i \geq 0 \qquad (2.3.18)$$

$$0 \leq s \perp \sigma_i \geq 0 \qquad (2.3.19)$$

$$0 \leq -Ys \perp \xi_i \geq 0 \qquad (2.3.20)$$

(Here, $S = diag(s_1, s_2, \ldots, s_r)$.)

Definition 2.3.2: A feasible solution (x, y) of (MLSF-BLP) is called a *strong-stationarity point* if there exist multipliers λ_i, χ_i, ψ_i, σ_i, ξ_i, ν_i, μ_i which satisfy strong-stationarity conditions (2.3.11) – (2.3.20).

Step 5: Among the strong-stationarity conditions written above, as (2.3.14), (2.3.17)-(2.3.20) are complementarity conditions, therefore the process of solving the system of conditions (2.3.11) – (2.3.20) can be eased out by solving instead the following nonlinear programming problem. If an optimal solution of (2.3.21) – (2.3.29) gives the objective function value zero *i.e.*, $C_{penalty} = 0$, then that optimal solution satisfies the strong-stationarity conditions (2.3.11) – (2.3.20).

(Pen – NLP)

$$\min C_{penalty} = \sum_{i=1}^{k} (x_i'\chi_i + t_i'\lambda_i + y'\psi_i + s'\sigma_i) + y's \tag{2.3.21}$$

subject to

$$\nabla_{x_i} f_i(x_i, y) - \lambda_i'\nabla_{x_i} g_i(x_i, y) - \nu_i'\nabla_{x_i} G_i(x_i, y) - \mu_i'\nabla_{x_i} h(x_i, y) - \chi_i = 0, \quad \forall\, i = 1, 2, \ldots, k, \tag{2.3.22}$$

$$\nabla_y f_i(x_i, y) - \lambda_i'\nabla_y g_i(x_i, y) - \nu_i'\nabla_y G_i(x_i, y) - \mu_i'\nabla_y h(x_i, y) - \psi_i + S\xi_i = 0, \quad \forall\, i = 1, 2, \ldots, k, \tag{2.3.23}$$

$$\mu_i - \sigma_i + \mathrm{Y}\xi_i = 0, \quad \forall\, i = 1, 2, \ldots, k, \tag{2.3.24}$$

$$-g_i(x_i, y) + t_i = 0, \quad \forall\, i = 1, 2, \ldots, k, \tag{2.3.25}$$

$$G_i(x_i, y) = 0, \quad \forall\, i = 1, 2, \ldots, k, \tag{2.3.26}$$

$$h(x, y) - s = 0, \tag{2.3.27}$$

$$x_i \geq 0, y \geq 0, s \geq 0, t_i \geq 0, \lambda_i \geq 0, \chi_i \geq 0, \psi_i \geq 0, \sigma_i \geq 0, \xi_i \geq 0, \tag{2.3.28}$$

ν_i, μ_i unrestricted in sign. (2.3.29)

(Here, $Y = \mathrm{diag}(y_1, y_2, \ldots, y_r)$.)

Result 2.3.6: If $(x^*, y^*, s^*, t_i^*, \lambda_i^*, \chi_i^*, \psi_i^*, \sigma_i^*, \xi_i^*, \nu_i^*, \mu_i^*)$ is a local optimal solution of the non-linear programming problem (2.3.21) – (2.3.29) with $C_{penalty} = 0$, then (x^*, y^*) is a strong-stationary point of (MLSF-BLP).

From all this theoretical development suggested by Leyffer and Munson we extract the following conclusion.

For testing a feasible solution (x^*, y^*) of (MLSF-BLP) to be a strong-stationary point, it is sufficient to solve the (Pen-NLP) parameterized in $(x, y) = (x^*, y^*)$ for optimal values of s, t_i, λ_i, χ_i, ψ_i, σ_i, ξ_i, ν_i, μ_i and check whether the objective function value $C_{penalty} = 0$.

Remark 2.3.2: Nash games situation in multi-leader-multi-follower BLP problems is addressed also addressed by Leyffer and Munson in the same study [169]. For addressing the situation of Nash game at lower level of such a variant of BLP problem authors suggest using FOCs as equivalent of Nash-equilibrium problem of followers and adopting them in place of conditions (2.3.8) and (2.3.9). The same procedure as presented above is suggested by authors to be followed subsequently. The convexity is assumed here for individual optimization problems of followers with leaders' variables considered as parameters therein.

2.4. A special case of multi-leader-single-follower BLP problem

In this section, a special case of (MLSF-BLP) is discussed and a practical method to test the strong-stationarity of a feasible solution is observed for this case. We consider the case of a multi-leader-single-follower BLP problem formulated as (MLSF-BLP) with the upper level constraint functions g_i, G_i are linear in (x, y), lower level constraints are linear in (x, y); and upper level objective functions f_i are bilinear in (x_i, y), lower level objective is bilinear in (x, w). Henceforth, we term this special case of BLP problem as "bilinear multi-leader-single-follower BLP problem".

Result 2.4.1: For a feasible solution (x^*, y^*) of a bilinear multi-leader-single-follower BLP problem expressed as (MLSF-BLP) (numbered as (2.3.4)) with functions involved in objectives and constraints considered as mentioned above, the optimization problem (2.3.21) – (2.3.29) becomes the linear programming problem $(\mathrm{Para-LP})$ (numbered as

(2.4.1)) in variables $\lambda_i, \chi_i, \psi_i, \sigma_i, \xi_i, \nu_i, \mu_i$, and parameterized in corresponding values $x^*, y^*, s^*, t_i^*, S^*, Y^*$, where,

(1) s^* and t_i^* are values of s and t_i, respectively obtained by using conditions (2.3.27) and (2.3.25), respectively, and

(2) $S^* = diag(s_1^*, s_2^*, \dots, s_r^*)$ and $Y^* = \mathrm{diag}(y_1^*, y_2^*, \dots, y_r^*)$.

$(\text{Para} - \text{ LP})$ … (2.4.1)

$$\min_{\lambda_i,\chi_i,\psi_i,\sigma_i,\xi_i,\nu_i,\mu_i} C_{pp} = \sum_{i=1}^{k} (x_i^*{}'\chi_i + t_i^*{}'\lambda_i + y^*{}'\psi_i + s^*{}'\sigma_i)$$

subject to

$$\nabla_{x_i} f_i(x_i^*, y^*) - \lambda_i^{'} \nabla_{x_i} g_i(x_i^*, y^*) - \nu_i^{'} \nabla_{x_i} G_i(x_i^*, y^*) - \mu_i^{'} \nabla_{x_i} h(x_i^*, y^*) - \chi_i = 0,$$

$$\forall\, i = 1, 2, \dots, k,$$

$$\nabla_y f_i(x_i^*, y^*) - \lambda_i^{'} \nabla_y g_i(x_i^*, y^*) - \nu_i^{'} \nabla_y G_i(x_i^*, y^*) - \mu_i^{'} \nabla_y h(x_i^*, y^*) - \psi_i + S^* \xi_i = 0,$$

$$\forall\, i = 1, 2, \dots, k,$$

$$\mu_i - \sigma_i + Y^* \xi_i = 0, \quad \forall\, i = 1, 2, \dots, k,$$

$$\lambda_i \geq 0, \chi_i \geq 0, \psi_i \geq 0, \sigma_i \geq 0, \xi_i \geq 0,$$

ν_i, μ_i unrestricted in sign.

Therefore, by using the conclusion of result 2.3.6 with 2.4.1, the following deduction can be made.

Deduction 2.4.1: For testing a feasible solution (x^*, y^*) of bilinear multi-leader-single-follower BLP problem considered as a special case of (MLSF-BLP) to be a strong-stationary point, it reduces to:

(1) obtain values of t_i^* and s^* using (2.3.25) and (2.3.27), respectively, such that $s^* \perp y^*$, with a check for non-negativity conditions, then

(2) solve the linear programming problem (Para-LP) given in (2.4.1) parameterized in (x^*, y^*, s^*, t_i^*), for $\lambda_i, \chi_i, \psi_i, \sigma_i, \xi_i, \nu_i, \mu_i$, and then

(3) check if the objective function value $C_{pp} = 0$ for an optimal solution.

Theoretical development for solving bilinear multi-leader-single-follower BLP problem

For a bilinear multi-leader-single-follower BLP problem discussed above, if a vector of values for the variable x is chosen randomly, then the lower level problem (FDMP) in (2.3.4) becomes a linear programming problem in variables w, parametrized in the values of x. This LPP can be easily solved for obtaining an optimal solution y. And therefore, the corresponding vector of values for (x, y) would satisfy the KKT conditions (2.3.5) and (2.3.6). Accordingly, the vector of values for the variable s can be obtained using (2.3.8) (same as (2.3.27)). Further, the values of each variable t_i can be obtained using (2.3.25) and tested for non-negativity. If, through all these computations a vector of values for (x, y) is obtained which satisfies the testing criteria just discussed, then by Definition 2.3.1, it is a feasible solution of (MLSF-BLP). In such a case, for the obtained values of (x, y, s, t_i), the (Para-LP) can be solved to obtain an optimal solution. If the objective function value $C_{pp} = 0$, then we get (x, y) as a strong stationary point of (MLSP-BLP).

Through this discussion it is learnt that if we randomly generate a vector of values for leaders' variables x, we can obtain follower's reaction y and test for strong-stationary point of this special case of (MLSP-BLP). Thus, if we use a real-coded GA by coding the chromosomes as vectors of values for x, obtaining corresponding values for y, and taking the fitness value of each chromosome as objective function value C_{pp} to be computed by the above procedure, then we can achieve a stationary-point in some generation obtained through reproduction operators of GA.

Stemmed from the theoretical developments presented above, a GA-based approach is proposed in Chapter 5 for obtaining strong-stationarity points of the bilinear multi-leader-single-follower BLP problem, which is a special case of (MLSF-BLP) involving linear constraints and bilinear objective functions.

Remark 2.4.1: A strong-stationarity point (x^*, y^*) for $x^* = \{x_i^*: i = 1, 2, \ldots, k\}$, thus obtained can further be tested for a strong-stationary Nash-equilibrium point of the (MLSF-BLP) by repeatedly solving the single-leader-single-follower BLP problem (2.4.2) as given below for each leader i $(i = 1, 2, \ldots, k)$.

(LDMP $- i^{-1}$) $\min_{x_i \geq 0} f_i(x_i, y)$

subject to

$g_i(x_i, y) \geq 0,$

$G_i(x_i, y) = 0,$

where, y is optimal response of the follower corresponding to above values of x_i and keeping values of other leaders' variables fixed as $x_{\bar{\imath}} = x_{\bar{\imath}}^*: \bar{\imath} = 1,2, \dots, k, \bar{\imath} \neq i,$ (2.4.2)

(FDMP) $\min_{w} b(x, w)$

subject to

$c(x, w) \geq 0$

$w \geq 0.$

Remark 2.4.2: Solving such a single-leader-single-follower bilevel programming problem having large number of variables further requires a heuristic algorithm. One such algorithm is proposed in Chapter 3.

With this necessary backdrop of concepts introduced by now our research work is presented in subsequent chapters.

Chapter 3

Decision Support to Railways on Running Special Trains

In this chapter[10], we develop a decision support for railways on operational decisions of running special trains to tackle higher demand on specific routes during seasons of festivals and holidays. These operational decisions comprise of utilizing rolling-stocks and determining optimal fare-price structure in a competitive environment coerced by other travelling service providers. The influence on the demand-shares by the competitors of railways is incorporated in decision making on utilizing the rolling-stock accordingly. A novel mixed-integer single-leader-multi-follower bilevel programming model is proposed in which the railways is considered a leader and a group of all competitors to railways is a follower. The model is designed with the objective of railways as the leader to maximize the expected revenue by deciding on routes, rolling-stock assembly planning and fare-pricing for special trains subject to constraints on resources and the anticipated demand arising out of Nash-equilibrium fares of the follower. A diversified-elitist genetic algorithm is introduced to solve the proposed model. The proposed methodology is illustrated by taking a test situation from Indian Railways (IR). The empirical analysis demonstrates the success of the proposed model in strategically addressing the fare-price competition and preparing the operational plan for running the special trains.

[10] The contents of this chapter are based on research paper: "A bilevel programming model for operative decisions on special trains: An Indian Railways perspective", Journal of Rail Transport Planning and Management (Elsevier). 8(3) 2018, 184–206. doi:10.1016/j.jrtpm.2018.03.001.

3.1 Introduction

Worldwide, apart from the regular demand of travelling services across the year, a significantly higher demand is observed on specific routes during seasons of holidays and festivals. In a majority of countries, the railways being the largest state-owned enterprise for mass public transport, take initiatives to fulfil this demand. This happens so due to one more fact that, in comparison to other modes of public-transport, railways have a scope of enhancing the capacity of its already running fleet by attaching a few more train units (commonly known as passenger coaches).

However, when a relatively high volume of passengers is expected to be received, the railway operators plan to run some special trains to clear the rush which may also result in generating additional revenue for the railways. It is frequently the case that the rolling-stock which is retired into the yards due to high maintenance costs is temporarily brought in back for use in providing service under the exercise of running special-trains. In such a seasonal situation, the efficient usage of existing rolling-stock is considered as a viable option instead of blocking a considerable investment in procuring a new one which may later remain unused for long. Thus, in addition to running regular trains, railway operators can strategically plan to run special trains on specific potential routes where a higher mass of passengers is expected during specific seasons. The decisions on fare-pricing and rolling-stock assembly are the other two significant components embedded in this decision problem.

During such seasons of higher demands, the competitors to railways perceive this situation as an opportunity of earning more revenue, and therefore adjust their fare-prices competitively while considering the pre-fixed fare-prices of railways[11] into their pricing mechanism. Such price adjustment by these public transport service providers influences the demand-shares of railways, and further perplexes the decision-making problem of later on an efficient utilization of available resources.

A railway operator is thus posed with a challenge to ingeniously decide on the choice of routes for running special trains, associated fare-pricing, and an efficient assembly of rolling-stock for its special trains according to its demand-share resulting out

[11] It is a conventional practice by state-owned railway operators to announce the full fare-price structure in advance for its mass-transit trains. The reason behind this are considered as socio-economic status of the general public and some practical issues in accessing the real-time fare-prices are travelers.

of the fare-price competition of other transport service providers. As all these decision-making aspects are needed to be addressed cohesively, so this posses the overall problem before the railway authorities as an issue of strategic planning.

Although the literature is available on some associated aspects decision-making problems of railways, but no such work is found to address this challenging and perplexed problem of railways which requires assessing the competitive response of other travelling service providers. This has motivated us to develop a decision-support for railways for the issue of strategically planning discussed above.

The facts noted below verify the practicality of the described strategic planning problem and encourage for the development of a decision-support for this problem.

- In China, during the National Day week October 1-7, Labour Day week May 1-7, and Chinese New Year period in January/ February, and during the fall season in Japan, a very high footfall of travellers is observed. India, the culturally and seasonally diverse country, witnesses such overshooting demand on different routes at different times throughout the year. Although no information is available on special trains for China Railway Corporation, Japan Railways Group [216] and Indian Railways [217] run special trains to clear the rush.
- The Government of India has recently approved the formation of Rail Development Authority to thrive for decision support on some strategic issues including pricing of services, efficient allocation of resources, and promoting competition as well as protection of customer interests [218]. It had been realized to design a realistic program of fare revision to eliminate the losses on passenger services and to stop the subsidy for non-suburb trains [219].
- There does not seem to exist any noteworthy study explicating the mathematical rigor of the decision-making problem on running special trains.
- Moreover, in our literature research regarding the decision support of railway operations, narrated in section 1.4.4, we could not find any research on 'the price dependent demand' together with 'demand-dependent allocation of rolling-stock' for railways.

These research gaps have motivated us to develop a decision-support in this context, the relevance of which is indicated by the initiative of the Government of India cited above for mentoring the IR on its operations. To address this issue of decision-support, we contemplate a model which enables the railways for demand-dependent assembly planning while dictating the fare-pricing rather than being at a receiving end of its competitors'. This model formulates the above-discussed decision-making situation as single-leader-multi-follower BLP problem involving mixed integer variables and bilinear objective functions considering railways as the leader and their competitors as followers. Herein, the leader has to take the necessary decisions on running special trains to maximize its total expected revenue in anticipation of the reaction of the follower described through Nash-equilibrium. Further, due to lack of theoretical developments for methods to solve a single-leader-multi-follower BLP problem involving mixed integer variable problems, we design a GA based methodology for a better search of a solution of the proposed bilevel programming model. Thereby, two of research contributions are presented in this chapter. A case is further studied using the designed model and proposed solution methodology on an instance taken from IR to demonstrate a pattern of outcomes for decision-support.

This chapter is arranged into six sections. The subsequent section 3.2 demonstrates the design of model for the addressed decision-making issue of railways. Section 3.3 presents a GA based methodology developed for solving the modelled problem. A case study is presented in subsequent section 3.4 to demonstrate the implementation of devised solution methodology for the modelled problem. Following this, a comparison analysis is presented in section 3.5 to demonstrate the success of proposed model against the one in which the competition from other transport service providers on the concerned routes is ignored into the modelling. Also, the efficiency of developed solution methodology is also tested in comparison to the simple GA. The chapter is conclusively summarized in section 3.6.

3.2 Formulation of mathematical model

3.2.1 Assumptions and notations

To build the mathematical model, a finite planning horizon is partitioned into multiple time periods. Also, on each route, we assume the following to hold.

- The travelling-mode-choice behaviour of passengers is not influenced by departure and arrival times of transport services. During seasons of festivals and holidays, when only a limited number of seats remain available against a high demand, passengers are more concerned about the availability of seat rather than the departure and arrival times. Also, through this assumption, we intend to communicate that the proposed study includes only the decision making on operational planning and pricing for special trains, and it does not include scheduling and time-tabling or runtime planning of special trains.
- The competitors of railways remain same in each period on the route. The planning horizon for running special trains is typically not so long for observing a significant change in the number of service providers in the competition.
- The price sensitivity of passengers on each route is identical for all service providers on the route. The price sensitivity is a behavioural aspect of passengers and not affected by service providers.
- The demand arising on intermediate halts in a route is not explicitly considered but integrated into the expected demand on that route; the fare-prices of the special trains are observed to remain same on its entire route irrespective of boarding and de-boarding points of passengers.
- The demand shares received by the service providers are probabilistic; the total profit to be earned through operational planning is an expected profit only. To this effect, we apply the multinomial Logit model to determine the demand shares of the railways and its competitors.

The indices, parameters, and variables used to describe the mathematical formulation of our model are listed below.

Indices

I	potential route to run train(s); $i = 1, 2, \ldots, I$
R	type of train; $r = 1, 2, \ldots, R$
J	class/ type of coach; $j = 1, 2, \ldots, J$
T	number of time periods[12]; $t = 1, 2, \ldots, T$
K_i	the number of competitor transporters[13] on route i; $(k_i = 1, 2, \ldots, K_i)$

[12] The total planning horizon is discretized into equal subintervals enumerated by $t = 1, 2, \ldots, T$.

Leader's parameters and variables

Parameters

a_j	number of seats in each coach of class j
f_{ir}	fixed cost of running a train of type r on route i
g_{irj}	cost of adding one coach of class j to train type r on route i
B	total budget to run special trains on various proposed routes
D_{it}	total demand in terms of number of passengers to travel on route i in period t
c_{irj}	cost incurred per passenger travelling in class j of train type r on route i (INR/passenger)
pl_{irj}	the minimum reservation price of coach of class j in train type r on route i (INR/ passenger)
pu_{irj}	the maximum reservation price of coach of class j in train type r on route i (INR/ passenger)
α_{irj}	the coefficient of non-monetary factors in services of a coach of class j in train type r on route i
M_{rj}	maximum number of coaches of class j that can be assembled to a train type r
M_{ir}	maximum number of coaches that can be attached in a train type r on route i
M_j	number of coaches of class j that are available for use in running special trains on various routes

Variables

z_L	the expected gross profit of leader
X_{ir}	binary variable taking value 1 if route i is initiated with a train type r and else zero
x_{irj}	number of coaches of class j to be assembled in train type r on route i
Y_{irj}	binary variable taking value 1 if at least one coach of class j is assembled to train type r initiated on route i and else zero
p_{irjt}	fare of a seat in class j of train type r on route i in period t (INR/seat)
$P_{irjt}\,(p_{it})$	the probability of choosing to travel in class j of train type r on route i in period t when the vector of fares is $p_{it} = \left\{\{p_{irjt} : r, j\}, \{p_{k_i t} : k_i\}\right\}$
dr_{irjt}	number of passengers to be offered seats in class j of train type r on route i in period t

[13] The assumption that the number of competitors on the route remain same during the planning horizon is reflected in the model here when this parameter is taken independent of t.

U_{irjt} utility of travelling in class j of train type r on route i in period t

Follower's parameters and variables

Parameters

c_{k_i} cost incurred to competitor transporter k_i on route i (INR/passenger)

α_{k_i} coefficient for non-monetary factors in services of competitor k_i on route i

ξ_{k_i} the minimum proportion of cost per passenger to be recovered by k_i

Av_{k_i} availability of seats with k_i

Variables

$z_{F_{k_i}}$ the expected gross profit of all competitors on route i

p_{k_it} fare to travel on route i through competitor transporter k_i on route i in period t (INR/person)

$P_{k_it}(p_{it})$ the probability of choosing to travel with k_i on route i in period t when the vector of fares is $p_{it} = \left\{\{p_{irjt} : r, j\}, \{p_{k_it} : k_i\}\right\}$

dr_{k_it} number of passengers to be offered a seat by the competitor k_i on route i in period t

U_{k_it} utility of travelling through k_i on route i in period t

Global parameter for both Leader and Follower

β_i Price sensitivity factor for all participating decision makers[14] on route i

3.2.2 Estimating demand shares through multinomial logit model

The multinomial logit (MNL) model is typically used in literature to learn the discrete choice behavior of travellers [220–228]. In our proposed model, once we have an information on the fare-prices of competitors in response to the fare-prices of railways, the MNL model determines the probabilistic demand share of railways which in turn facilitates in assembly planning of rolling stocks for running special trains.

It is important to realize that the fare-price competition is a crucial factor affecting the demand-shares and revenues of all the transport service providers. For any given price structure of railways declared in advance, the fare-price competition among the

[14] The assumption that the price sensitivity of passengers on each route is identical is reflected in this parameter which is taken to be independent of all service providers including the railways and its competitors K_i.

competitors of railways settles at a Nash-equilibrium point. In this competition, each contender aims to maximize the individual revenue[15] with a binding limit on its seating capacities. The problem fits well in a BLP framework with the railways considered as the leader decision maker, and each of its competitors on each route considered as a follower decision maker. On each route i and time period t, let the vector of fares of railways and its competitors be

$$p_{it} = \left\{ \{ p_{irjt} : r = 1, \dots, R, j = 1, \dots J \}, \{ p_{k_i t} : k_i = 1, \dots K_i \} \right\}. \quad (3.2.1)$$

Then, as per the MNL model, the utility of travelling in the class j of train type r on route i in time t is given by

$$U_{irjt} = \alpha_{irj} - \beta_i \, p_{irjt} + \epsilon_{irjt} \;, \quad \forall \, i, \forall \, r, \forall \, j, \quad (3.2.2)$$

while the utility of travelling through an alternative k_i at the same time t, is given by

$$U_{k_i t} = \alpha_{k_i} - \beta_i \, p_{k_i t} + \epsilon_{k_i t} \;, \quad \forall \, k_i \;, \quad (3.2.3)$$

and the utility of not to travel at all on the route i is given by

$$U_t^0 = \epsilon_t^0, \quad \forall \, i, \forall \, r, \forall \, j. \quad (3.2.4)$$

The parameters α_{irj} , α_{k_i} amount for non-monetary factors like comfort, travelling time, the number of intermediate halts, personal preference, ambiance, and branding, and the variables ϵ_{irj} , ϵ_{k_i} and ϵ_t^0 are assumed to be independently and identically distributed random variables following the Gumbel distribution.

Given a price vector in (3.2.1) and the values of variables Y_{irj} , indicating the classes of coaches and types of trains available on route i, the probability that a passenger choosing the class j of train type r to travel on route i at time t, is given by

[15] The transport service providers identified as competitor to railways are assumed to have fixed capacities to accommodate passengers unlike a flexible capacity of railways. Thereby it is considered that these competitors bear no significant operational cost per passenger. And that, only a fixed cost is incurred for operating their service, which is irrespective of the number of passengers travelling with them. Therefore, for modelling their fare-pricing mechanism it is appropriate to consider their objective as maximization of total revenue instead of total profit.

$$P_{irjt}\,(p_{it}) = \frac{Y_{irj}\ e^{\alpha_{irj} - \beta_i\, p_{irjt}}}{1\ +\ \sum_{r=1}^{R}\sum_{j=1}^{J} Y_{irj}\, e^{\alpha_{irj} - \beta_i\, p_{irjt}}\ +\ \sum_{k_i=1}^{K_i} e^{\alpha_{k_t} - \beta_i\, p_{k_i t}}}\ . \tag{3.2.5}$$

The probability that a passenger chooses the competitor k_i as an alternative, is given by

$$P_{k_i}(p_{it}) = \frac{e^{\alpha_{k_i} - \beta_i\, p_{k_i t}}}{1\ +\ \sum_{r=1}^{R}\sum_{j=1}^{J} Y_{irj}\ e^{\alpha_{irj} - \beta_i\, p_{irjt}}\ +\ \sum_{k_i=1}^{K_i} e^{\alpha_{k_t} - \beta_i\, p_{k_i t}}}\ , \tag{3.2.6}$$

and the probability that a passenger drops the idea of travelling is given by

$$P_t^0(p_{it}) = \frac{1}{1\ +\ \sum_{r=1}^{R}\sum_{j=1}^{J} Y_{irj}\ e^{\alpha_{irj} - \beta_i\, p_{irjt}}\ +\ \sum_{k_i=1}^{K_i} e^{\alpha_{k_t} - \beta_i\, p_{k_i t}}}. \tag{3.2.7}$$

Corresponding to the total demand on route i at time t, the demand-shares of railways (for class j and train type r) and competitor transporters (k_i) are given by $P_{irjt}\,(p_{it})\ D_{it}$ and $P_{k_i}(p_{it})\ D_{it}$, respectively.[16]

3.2.3 Components of the problem related to railways

Objective function of expected profit

The leader (railways) has to identify the routes to run trains, decide on the type of trains to be run on these routes, the number of coaches of each relevant class to be assembled in these trains, and the fare-prices for each class of coach so as to maximize the total profit.

Profit is obtained by subtracting the total fixed cost of running trains of various types on various routes and the fixed cost of assembling coaches of various classes of trains being decided to run from the net profit earned from passengers during the entire planning horizon. For this, the railways decides on binary variables X_{ir}, integer variables x_{irj}, and prices p_{irjt}.

[16] For estimating the expected number of passengers to choose each of these transport service providers, expressions of their demand-shares can be rounded to the nearest integer.

$$\max z_L = \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{r=1}^{R}\sum_{j=1}^{J}\left(p_{irjt} - c_{irj}\right)\min\left\{P_{irjt}\left(p_{it}\right) D_{it},\ a_j\, x_{irj}\right\} - \sum_{i=1}^{I}\sum_{r=1}^{R} f_{ir}\, X_{ir} - \sum_{i=1}^{I}\sum_{r=1}^{R}\sum_{j=1}^{J} g_{irj}\, x_{irj} \qquad (3.2.8)$$

Price bounds

The fare-prices are surmised to be bounded; lower bounds are the minimum acceptable fares to railways, and the oligopolistic market enforces upper bounds. These bounds on fare-prices restrain the railways from moving out of affordable limits of fares in a fray of competition and profitability.

$$pl_{irj} \le p_{irjt} \le pu_{irj}, \quad \forall i, \quad \forall r, \quad \forall j, \quad \forall t. \qquad (3.2.9)$$

Budget constraint

The railways in capacity has to perform all operations of running various special trains and assembling various coaches to these trains within the available budget.

$$\sum_{i=1}^{I}\sum_{r=1}^{R} f_{ir}\, X_{ir} + \sum_{i=1}^{I}\sum_{r=1}^{R}\sum_{j=1}^{J} g_{irj}\, x_{irj} \le B. \qquad (3.2.10)$$

Constraints on recovery of operational cost

Each train, if decided to run, should generate a turnover which recovers the investment on running it.

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\left(p_{irjt} - c_{irj}\right)\min\left\{P_{irjt}\left(p_{it}\right) D_{it},\ a_j\, x_{irj}\right\} \ge f_{ir}\, X_{ir} + \sum_{j=1}^{J} g_{irj}\, x_{irj}, \quad \forall i, \quad \forall r \qquad \dots (3.2.11)$$

Constraints on assembly of coaches

Coaches are assembled in types of trains (decided to run on the route) for which they are appropriate, that is, if $X_{ir} = 1$ and $Y_{irj} = 1$, then $0 \le x_{irj} \le M_{rj}$. Equivalently,

$$0 \le x_{irj} \le M_{rj}\, X_{ir}\, Y_{irj}, \quad \forall i, \quad \forall r, \quad \forall j. \qquad (3.2.12)$$

Upper bound on the length of trains posses a constraint on assembly of coaches as following.

$$\sum_{j=1}^{J} x_{irj} \leq M_{ir}\ , \quad \forall i, \quad \forall r. \tag{3.2.13}$$

Maximum availability of coaches of each type also governs the assembly.

$$\sum_{i=1}^{I}\sum_{r=1}^{R} x_{irj} \leq M_j\ , \quad \forall j. \tag{3.2.14}$$

3.2.4 Components of problem related to competitors of railways

Objective function of expected profit

In each period $t = 1, \dots, T$, and each route $i = 1, \dots, I$, the competitors $k_i = 1, \dots, K_i$, compete to settle at Nash-equilibrium price vector $\{p_{k_i t}: k_i = 1, \dots, K_i\}$, with the individual objective of maximizing their revenue in this price-competition.

$$\max\ z_{F_{k_i}} = p_{k_i t}\ \min\{P_{k_i t}(p_{it}) D_{it},\ Av_{k_i}\}. \tag{3.2.15}$$

Reservation price constraint

Each competitor of railways decides its price while following the “cost plus” policy, that is, fare-price in any duration should be at least some percentage more than the cost incurred for providing services to the passengers.

$$p_{k_i t} \geq (1 + \xi_{k_i}) c_{k_i}\ , \quad \forall k_i, \quad \forall t, \tag{3.2.16}$$

where $\xi_{k_i} > 0$ are predetermined constants.

Remark 3.2.1: Since the capacity of each competitor of railways to accommodate passengers is fixed and the same is included in the formulation of the objective function (3.2.15), the there are no more aspects other than the reservation price constraint to be appropriately included for modelling their fare-price decision mechanism.

3.2.5 Decision-making problem of railways on special trains

The bilinear bilevel mixed integer programming problem (RBPP) for railways is described as follows.

To determine $\left\{\{X_{ir}\},\{x_{irj}\},\{Y_{irj}\},\{p_{irjt}\}: i,r,j,t\right\}$ which solve (LDMP).

(RBPP)

(LDMP) $\quad \max\ z_L$

$$= \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{r=1}^{R}\sum_{j=1}^{J}(p_{irjt} - c_{irj})\ \min\{P_{irjt}(p_{it})\,D_{it},\ a_j\, x_{irj}\} - \sum_{i=1}^{I}\sum_{r=1}^{R} f_{ir}\, X_{ir} - \sum_{i=1}^{I}\sum_{r=1}^{R}\sum_{j=1}^{J} g_{irj}\, x_{irj}$$

s.t. $\quad pl_{irj} \leq p_{irjt} \leq pu_{irj} \qquad \forall i,\ \forall r,\ \forall j,\ \forall t$

$$\sum_{i=1}^{I}\sum_{r=1}^{R} f_{ir}\, X_{ir} + \sum_{i=1}^{I}\sum_{r=1}^{R}\sum_{j=1}^{J} g_{irj}\, x_{irj} \leq B$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}(p_{irjt} - c_{irj})\ \min\{P_{irjt}(p_{it})\,D_{it},\ a_j\, x_{irj}\} \geq f_{ir}\, X_{ir} + \sum_{j=1}^{J} g_{irj}\, x_{irj} \qquad \forall i, \forall r$$

$$x_{irj} \leq M_{rj}\, X_{ir}\, Y_{irj} \qquad \forall i,\ \forall r,\ \forall j$$

$$\sum_{j=1}^{J} x_{irj} \leq M_{ir} \qquad \forall i,\ \forall r$$

$$\sum_{i=1}^{I}\sum_{r=1}^{R} x_{irj} \leq M_j \qquad \forall j$$

$p_{irjt} \geq 0$, $X_{ir}, Y_{irj} \in \{0,1\}$, x_{irj} are non-negative integers

where, in each period t and each route i, competitors k_i aim to compute the Nash-equilibrium price vector $\{p_{k_i t}: k_i = 1, \dots, K_i\}$,

$$(\text{FDMP} - k_i) \qquad \max\ z_{F_{k_i}} = p_{k_i t} \min\{P_{k_i t}(p_{it}) D_{it},\ Av_{k_i}\}$$

$$\text{s.t. } p_{k_i t} \geq (1 + \xi_{k_i}) c_{k_i} \qquad \forall k_i,\ \forall t$$

Remark 3.2.2: In the BLP problems formulated above for decision-making of railways, the binary variables Y_{irj} are implicitly involved in the objective functions at both levels through the factor $P_{irjt}\,(p_{it})$.

The BLP problem (RBPP) formulated above to model the decision-making situation of Railways involves peculiar expressions in the objective functions at both levels which are complicated to handle while solving the problem. Therefore, it is suggested to perform the following simplifications in the original form of the problem.

Let dr_{irjt} denote the number of passengers to be served by the railways in coach class j of train type r on route i in period t, and $dr_{k_i t}$ the number of passengers to be served by the competitor k_i in time period t. Then, $dr_{irjt} = \min\{P_{irjt}\,(p_{it})\, D_{it}, a_j\, x_{irj}\}$ and $dr_{k_i t} = \min\{P_{k_i t}(p_{it}) D_{it}, Av_{k_i}\}$.

Thus, the problem (RBPP) is equivalently expressed below for determining $\{\{X_{ir}\},\{x_{irj}\},\{Y_{irj}\},\{p_{irjt}\},\{dr_{irjt}\}: i,r,j,t\}$; where X_{ir} and Y_{irj} are binary variables, x_{irj} and dr_{irjt} are non-negative integer variables, and p_{irjt} are price variables.

(LDMP)

$$\max\ z_1 = \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{r=1}^{R}\sum_{j=1}^{J}(p_{irjt} - c_{irj})\, dr_{irjt} - \sum_{i=1}^{I}\sum_{r=1}^{R} f_{ir}\, X_{ir} - \sum_{i=1}^{I}\sum_{r=1}^{R}\sum_{j=1}^{J} g_{irj}\, x_{irj}$$

$$\text{s.t.} \qquad pl_{irj} \leq p_{irjt} \leq pu_{irj}, \quad \forall i,\ \forall r,\ \forall j,\ \forall t$$

$$\sum_{i=1}^{I}\sum_{r=1}^{R} f_{ir}\, X_{ir} + \sum_{i=1}^{I}\sum_{r=1}^{R}\sum_{j=1}^{J} g_{irj}\, x_{irj} \leq B$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}(p_{irjt} - c_{irj})\, dr_{irjt} \geq f_{ir}\, X_{ir} + \sum_{j=1}^{J} g_{irj}\, x_{irj}, \quad \forall i, \quad \forall r,$$

$$dr_{irjt} - P_{irjt}(p_{it})\, D_{it} \le 0, \quad \forall i,\ \forall r,\ \forall j,\ \forall t,$$

$$dr_{irjt} - a_j\, x_{irj} \le 0, \quad \forall i,\ \forall r,\ \forall j,\ \forall t,$$

$$x_{irj} - M_{rj}\, X_{ir}\, Y_{irj} \le 0, \quad \forall i,\ \forall r,\ \forall j,$$

$$\sum_{j=1}^{J} x_{irj} \le M_{ir}, \quad \forall i,\ \forall r,$$

$$\sum_{i=1}^{I}\sum_{r=1}^{R} x_{irj} \le M_j, \quad \forall j,$$

$$dr_{irjt} \ge 0, \quad \forall i,\ \forall r,\ \forall j,\ \forall t,$$

$p_{irjt} \ge 0$, X_{ir}, $Y_{irj} \in \{0, 1\}$, x_{irj}, and dr_{irjt} are non-negative integers,

where, in each period t and for each route i, competitors k_i aim to compute the Nash equilibrium price vector $\{p_{k_i t}: k_i = 1, \dots, K_i\}$,

$$(\mathrm{FDMP} - k_i) \qquad \max\ z_{2k_i} = p_{k_i t}\, dr_{k_i t}$$

$$\text{s.t.} \quad p_{k_i t} \ge \left(1 + \xi_{k_i}\right) c_{k_i},$$

$$dr_{k_i t} \le P_{k_i t}(p_{it})\, D_{it}\ ,$$

$$dr_{k_i t} \le Av_{k_i}\ ,$$

$dr_{k_i t} \ge 0$, $dr_{k_i t}$ are non-negative integers.

3.3 Solution methodology

Though some alterations and relaxations can be worked out in (RBPP) model, yet this in no way will reduce the computational challenges associated with the (RBPP) model. Some of such concerns are addressed in the following remark.

Remark 3.3.1: Following the approach of [146,229] in applying Bender's decomposition, and setting $Z_{irj} = X_{ir} Y_{irj}$, we can resolve the nonlinear implicative constraints (12) into the linear constraints by introducing additional binary variables. However, this transformation will add to the computational difficulty in solving (LDMP) problem due to increase in the number of binary variables. Moreover, it will neither simplify the bilinear structure of the objective function.

3.3.1 A theoretical development for computing Nash-equilibrium fare-prices of competitors

The lower level problem amounts to solve a constrained Nash-equilibrium problem of determining equilibrium prices $\{p_{k_i t}: k_i\}$, for each route i and in each period t. For each competitor k_i, in each period t, the problem $(\mathrm{FDMP} - k_i)$ is a constrained bilinear optimization problem of the form (A.1) in Appendix. Therefore, using part (a) of Result A.1 of Appendix, the optimization problem of each follower $(\mathrm{FDMP} - k_i)$ can be rewritten as below.

$$\max\ \pi_{k_i t}\left(p_{k_i t}: k_i = 1, \dots, K_i\right) = D_{it}\ \frac{p_{k_i t}\ e^{\alpha_{k_i} - \beta\ p_{k_i t}}}{\left(1 + K + \sum_{k_i=1}^{K_i} e^{\alpha_{k_t} - \beta\ p_{k_i t}}\right)} \tag{3.3.1}$$

$$\text{s.t.}\ \ D_{it}\ \frac{e^{\alpha_{k_i} - \beta\ p_{k_i t}}}{\left(1 + K + \sum_{k_i=1}^{K_i} e^{\alpha_{k_t} - \beta\ p_{k_i t}}\right)} \le Av_{k_i}\,, \tag{3.3.2}$$

$$p_{k_i t} \ge \left(1 + \xi_{k_i}\right) c_{k_i}\,, \tag{3.3.3}$$

where $K = \sum_{r=1}^{R} \sum_{j=1}^{J} Y_{irj}\ e^{\alpha_{irj} - \beta\ p_{irjt}}$ is a constant for values of variables p_{irjt} and Y_{irj} passed by (LDMP).

The Lagrangian function corresponding to this optimization problem is

$$L_{k_i}\left(p_{k_i t}, s_{k_i}, s'_{k_i}, \lambda_{k_i}, \mu_{k_i}\right)$$

$$= D_{it}\ \frac{p_{k_i t}\ e^{\alpha_{k_i} - \beta\ p_{k_i t}}}{(1+F)} - \lambda_{k_i}\left(D_{it}\ \frac{e^{\alpha_{k_i} - \beta\ p_{k_i t}}}{(1+F)} - Av_{k_i} + s_{k_i}^2\right)$$
$$- \mu_{k_i}\left(\left(1 + \xi_{k_i}\right) c_{k_i} - p_{k_i t} + {s'}_{k_i}^{2}\right)$$

$$= D_{it}\ \frac{\left(p_{k_i t} - \lambda_{k_i}\right) e^{\alpha_{k_i} - \beta\ p_{k_i t}}}{(1+F)} - \lambda_{k_i}\left(-Av_{k_i} + s_{k_i}^2\right)$$
$$- \mu_{k_i}\left(\left(1 + \xi_{k_i}\right) c_{k_i} - p_{k_i t} + {s'}_{k_i}^{2}\right) \tag{3.3.4}$$

Here, $F \equiv F\left(p_{1t}, p_{2t}, \dots, p_{K_i t}\right) = K + \sum_{k_i=1}^{K_i} e^{\alpha_{k_t} - \beta\, p_{k_i t}}$ and s_{k_i} and s'_{k_i} are slack variables for constraints (3.3.2) and (3.3.3), respectively.

The profit function (3.3.1) is strictly quasiconcave with respect to individual price $p_{k_i t}$ (by referring to part (b) of Result A.1 in Appendix) over a convex feasible set determined by (3.3.2) – (3.3.3). Therefore, it suffices to solve the KKT conditions (with complementary slackness conditions) for obtaining Nash-equilibrium prices [209,230].

The KKT conditions for optimal fare-prices p_{k_i} of the competitor k_i are

$$\frac{\partial L_{k_i}}{\partial p_{k_i}} = 0, \frac{\partial L_{k_i}}{\partial \lambda_{k_i}} = 0, \frac{\partial L_{k_i}}{\partial \mu_{k_i}} = 0, \frac{\partial L_{k_i}}{\partial s_{k_i}} = 0, \frac{\partial L_{k_i}}{\partial s'_{k_i}} = 0 \quad (k_i = 1, \dots, K_i), \tag{3.3.4}$$

yielding the following system of equations, for each $k_i = 1, \dots, K_i$.

$$\boxed{\begin{aligned}
& D_{it}\, \lambda_{k_i}\, e^{\alpha_{k_i} - \beta\, p_{k_i t}} (1 + F) \\
& \qquad\qquad + \beta D_{it} \left(\lambda_{k_i} - p_{k_i t}\right) e^{\alpha_{k_i} - \beta\, p_{k_i t}} \left(1 + F - e^{\alpha_{k_i} - \beta\, p_{k_i t}}\right) \\
& \qquad\qquad + \mu_{k_i} (1 + F) = 0 \\
& D_{it}\, e^{\alpha_{k_i} - \beta\, p_{k_i t}} + \left(s_{k_i}^2 - A v_{k_i}\right)(1 + F) = 0 \\
& \left(1 + \xi_{k_i}\right) c_{k_i} - p_{k_i t} + s'^2_{k_i} = 0 \\
& s_{k_i} \lambda_{k_i} = 0 \\
& s'_{k_i} \mu_{k_i} = 0
\end{aligned}} \tag{3.3.5}$$

For each route i and in each period t, the lower level (Nash equilibrium) problem of (RBPP) reduces to solving the system of equations (3.3.5).

Remark 3.3.2: With this development in hand, we can express the BLP problem (RBPP) into a single level optimization problem by inserting the system of nonlinear equations (3.3.5) as constraints into the upper level (LDMP) problem. However, the resulting optimization problem still includes a bilinear objective function and a mix of nonlinear and linear constraints along with continuous and discrete variables. As there does not seem to exist a computationally tractable algorithm for solving such a class of optimization problems, therefore the approach of converting the BLP problem into a single level optimization problem and solving the same is not viable.

We utilize this mechanism of solving equations (3.3.5) for obtaining followers' response rather for developing a GA-based nested approach for solving the modeled problem (RBPP). Such a solution methodology is suggested below under the name Diversified-Elitist-genetic algorithm (DEGA).

3.3.2 Diversified-Elitist-genetic algorithm for solving problem of railways

The conventional solution methodologies for solving bilevel programming problems often postulate smoothness, linearity or convexity set-up in the models. However, many real-life situations of decision-making do not meet these aspects, and same is the case for the situation addressed in this study. Also, the proposed model is an instance of the class of bilevel programming problems having a single leader and multiple followers, and the problem at the follower level demands computing constrained Nash-equilibrium solution. This class of bilevel programming problems seems to have been overlooked and not investigated with rigor in the literature. As a result, there exists no algorithm to solve such problems. Acknowledging these challenges associated with the solution procedures, we propose DEGA to solve (RBPP) model. The DEGA embeds in itself, addressing the system of equations in (3.3.5) for computation of Nash-equilibrium fare-prices of the followers. An elite-preserving diversification operator endeavors to maintain diversity in the population by avoiding accumulation of chromosomes near the fittest chromosome, which in turn prevent a premature convergence of the algorithm to a locally optimal solution.

Chromosome encoding

The chromosomes of the population are encoded as three-part array. The first part comprises of an array of length $I \times R$, of binary numbers indicating the values of variables $\{X_{ir} : i = 1, \dots, I,\ r = 1, \dots, R\}$. The second part corresponds to a two-dimensional array of order $(I \times R,\ J)$ of binary numbers indicating the values of variables $\{Y_{irj} : i = 1, \dots, I, ;\ r = 1, \dots, R;\ j = 1, \dots, J\}$. And the third part corresponds to a two-dimensional array of order $(I \times R,\ J \times T)$ representing fare-prices of various coaches of various trains on various potential routes $\{p_{irjt} : i = 1, \dots, I, ;\ r = 1, \dots, R;\ j = 1, \dots, J; t = 1, \dots, T\}$ in the range of reservation prices $[pl_{irj}, pu_{irj}]$.

The search space for fare-prices could be enormous for applying the heuristic elitist diversified GA algorithm. To enhance the search efficiency, and thereby to reduce the search time, we have discretized the search space to consider only the integer grid points. This step is also practically viable since the fare prices in the coaches' class typically vary by some multiple of a constant factor of currency.

A general chromosome structure used in the implementation of the algorithm is shown in Figure 3.1. The GA parameters used in the proposed algorithm are population size (*popsize*); number of generations (G); current generation ($g,\ \ g = 1, 2, \dots, G$); crossover rate (*pcX* for $\{X_{ir}\}$ and *pc* for $\{p_{irjt}\}$); mutation rate (*pmX* for $\{X_{ir}\}$ and *pm* for $\{p_{irjt}\}$).

Initialization

The initial population is randomly generated as follows. The first part of a chromosome is randomly generated as a one-dimensional array of length $I \times R$ with binary entries. The second part of this chromosome is a two-dimensional array of order $(I \times R,\ J)$ with binary entries in each row depending on the corresponding value in the first part of the chromosome. If the entry in the first part is 0, then the corresponding row in the second part of a chromosome is taken all zeros to indicate that assembly of coaches is superfluous in a not running train; else the corresponding row of binary numbers is generated randomly with at least one entry as 1. The third part of a chromosome is randomly generated as a two-dimensional array of order $(I \times R,\ J \times T)$ with integer values ranging in the interval $[pl_{irj}, pu_{irj}]$.

Genetic operators

Crossover operator: We use the conventional single point crossover operator in the first part of the chromosomes with the probability *pcX*. If it happens, then the crossover in the second part of the chromosome is carried out at the same position. The crossover in the third part of the chromosome is performed with the probability *pc* using the Laplace crossover operator [193], detailed in Chapter 2.

$i=1$				$i=2$				…		$i=I$			
$j=1$	$j=2$	…	$j=J$	$j=1$	$j=2$	…	$j=J$	…	…	$j=1$	$j=2$	…	$j=J$
X_{11}	X_{12}	…	X_{1R}	X_{21}	X_{22}	…	X_{2R}	…	…	X_{I1}	X_{I2}	…	X_{IR}

		$j=1$	$j=2$	…	$j=J$
$i=1$	$r=1$	Y_{111}	Y_{112}	…	Y_{11J}
	$r=2$	Y_{121}	Y_{122}	…	Y_{12J}
	…	…	…	…	…
	$r=R$	Y_{1R1}	Y_{1R2}	…	Y_{1RJ}
$i=2$	$r=1$	Y_{211}	Y_{212}	…	Y_{21J}
	$r=2$	Y_{221}	Y_{222}	…	Y_{22J}
	…	…	…	…	…
	$r=R$	Y_{2R1}	Y_{2R2}	…	Y_{2RJ}
$i=I$	$r=1$	Y_{I11}	Y_{I12}	…	Y_{I1J}
	$r=2$	Y_{I21}	Y_{I22}	…	Y_{I2J}
	…	…	…	…	…
	$r=R$	Y_{IR1}	Y_{IR2}	…	Y_{IRJ}

		$t=1$				…			$t=T$			
		$j=1$	$j=2$	…	$j=J$	…	…	…	$j=1$	$j=2$	…	$j=J$
$i=1$	$r=1$	p_{1111}	p_{1121}	…	p_{11J1}	…	…	…	p_{111T}	p_{112T}	…	p_{11JT}
	$r=2$	p_{1211}	p_{1221}	…	p_{12J1}	…	…	…	p_{121T}	p_{122T}	…	p_{12JT}
	…	…	…	…	…	…	…	…	…	…	…	…
	$r=R$	p_{1R11}	p_{1R21}	…	p_{1RJ1}	…	…	…	p_{1R1T}	p_{1R2T}	…	p_{1RJT}
$i=2$	$r=1$	p_{2111}	p_{2121}	…	p_{21J1}	…	…	…	p_{211T}	p_{212T}	…	p_{21JT}
	$r=2$	p_{2211}	p_{2221}	…	p_{22J1}	…	…	…	p_{221T}	p_{222T}	…	p_{22JT}
	…	…	…	…	…	…	…	…	…	…	…	…
	$r=R$	p_{2R11}	p_{2R21}	…	p_{2RJ1}	…	…	…	p_{2R1T}	p_{2R2T}	…	p_{2RJT}
$i=I$	$r=1$	p_{I111}	p_{I121}	…	p_{I1J1}	…	…	…	p_{I11T}	p_{I12T}	…	p_{I1JT}
	$r=2$	p_{I211}	p_{I221}	…	p_{I2J1}	…	…	…	p_{I21T}	p_{I22T}	…	p_{I2JT}
	…	…	…	…	…	…	…	…	…	…	…	…
	$r=R$	p_{IR11}	p_{IR21}	…	p_{IRJ1}	…	…	…	p_{IR1T}	p_{IR2T}	…	p_{IRJT}

Figure 3.1: Chromosome structure

Mutation operator: A bit-flip mutation is performed on the first part of the chromosome with the probability *pmX* for each gene. If a gene in first part of the chromosome does not go through the mutation, then the corresponding row in the second part of the chromosome remains unchanged. In case a gene in the first part of chromosome undergoes mutation to become 1, then the corresponding row in the second part of the chromosome is randomly generated with at least one entry as 1, otherwise it is taken to be all zeros. The mutation in the third part of the chromosome is performed with the probability *pm* using the power mutation operator [197], detailed in Chapter 2.

The use of Laplace crossover and power mutation operators on the third part of chromosomes (which represents prices of railways) provides an advantage that the feasibility of chromosomes is not disturbed in terms of reservation price bounds in problem (LDMP).

Incorporating followers reaction and fitness evaluation

Nash-equilibrium: For each chromosome in population representing leader's variables, the followers' response in terms of the competitors' prices is obtained by solving the system of equations (3.3.5). Nash-equilibrium prices thus obtained for competitors on each route and each period are supplied to (LDMP). This provides complete information about the vector of fares $p_{it} = \left\{\{p_{irjt} : r, j\}, \{p_{k_i t} : k_i\}\right\}$.

Fitness evaluation: For each chromosome, with its values $\{X_{ir}\}, \{Y_{irj}\}, \{p_{irjt}\}$ and obtained values $P_{irjt}\,(p_{it})$, problem (LDMP) is solved to obtain the optimal number of coaches of each class in each train and the number of seats in these coaches. The fitness value of each chromosome is taken as the optimal value of z_1. The chromosome with the highest fitness value is taken to be the elitist of that population.

Updating the new population

The new population obtained from the parent population P^g is adopted to be a population of the next generation P^{g+1} only if it's maximum fitness value, in comparison to the maximum fitness value of the previous generation, does not decrease. Otherwise, the elitist chromosome of population P^g is added to the new population by removing one randomly selected chromosome from it. The population, thus obtained is then adopted to form population for the next generation P^{g+1}.

Elite-preserving diversification operator

For any generation g, before sending the population P^{g+1} for further reproduction to P^{g+2}, a check of diversity is suggested to be applied on the population P^{g+1} as detailed below. Firstly, distance of each chromosome from the elitist one are measured using the computation rule (3.3.6), and then the variance of these distances is obtained to determine the density of the population. The distance between two chromosomes $\{X_{ir}^1, Y_{irj}^1, p_{irjt}^1\}$ and $\{X_{ir}^2, Y_{irj}^2, p_{irjt}^2\}$ of the population is defined as

$$\frac{\sum_{i=1}^{I}\sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{t=1}^{T}\left(Y_{irj}^1\, p_{irjt}^1 - Y_{irj}^2\, p_{irjt}^2\right)^2}{pu_{irj} - pl_{irj}} + \frac{\sum_{i=1}^{I}\sum_{r=1}^{R}\left|X_{ir}^1 - X_{ir}^2\right|}{I \times R} \qquad (3.3.6)$$

In the above expression, first term is the rationalized Euclidean distance between fare-prices and second term is the rationalized Hamming distance between binary vectors.

If variance is found to be less than a pre-specified threshold, then the elitist chromosome of the population is retained, and the diversification operator is repeatedly applied on other chromosomes until the population density overshoots the threshold or the upper limit on the number of attempts $\overline{D}$ of diversification is reached. The diversification operator explained below is applied to each chromosome other than the elitist one as follows.

(i) A chromosome is randomly generated afresh (as in the initialization step) with a probability 0.3.

(ii) Otherwise,

- a bit-flip mutation operator is applied on the first part of the chromosome with probability 0.5. If the gene in the first part of the chromosome undergoes mutation to become 1, then the corresponding row in the second part of the chromosome is randomly generated with at least one entry 1; else it is taken as row of zeros.
- the first and second parts of the chromosome are retained and power mutation is applied to the third part with probability 0.5.

This procedure preserves the elitist chromosome in the population P^{g+1}. This operator is developed to avoid a premature convergence of the algorithm to a local solution of the problem. Similar strategies were adopted in [231–233].

Termination criterion

Maximum number of generations is prefixed and the execution of algorithm is terminated upon reaching this point.

The algorithm detailed above is summarized through a flowchart given as following Figure 3.2.

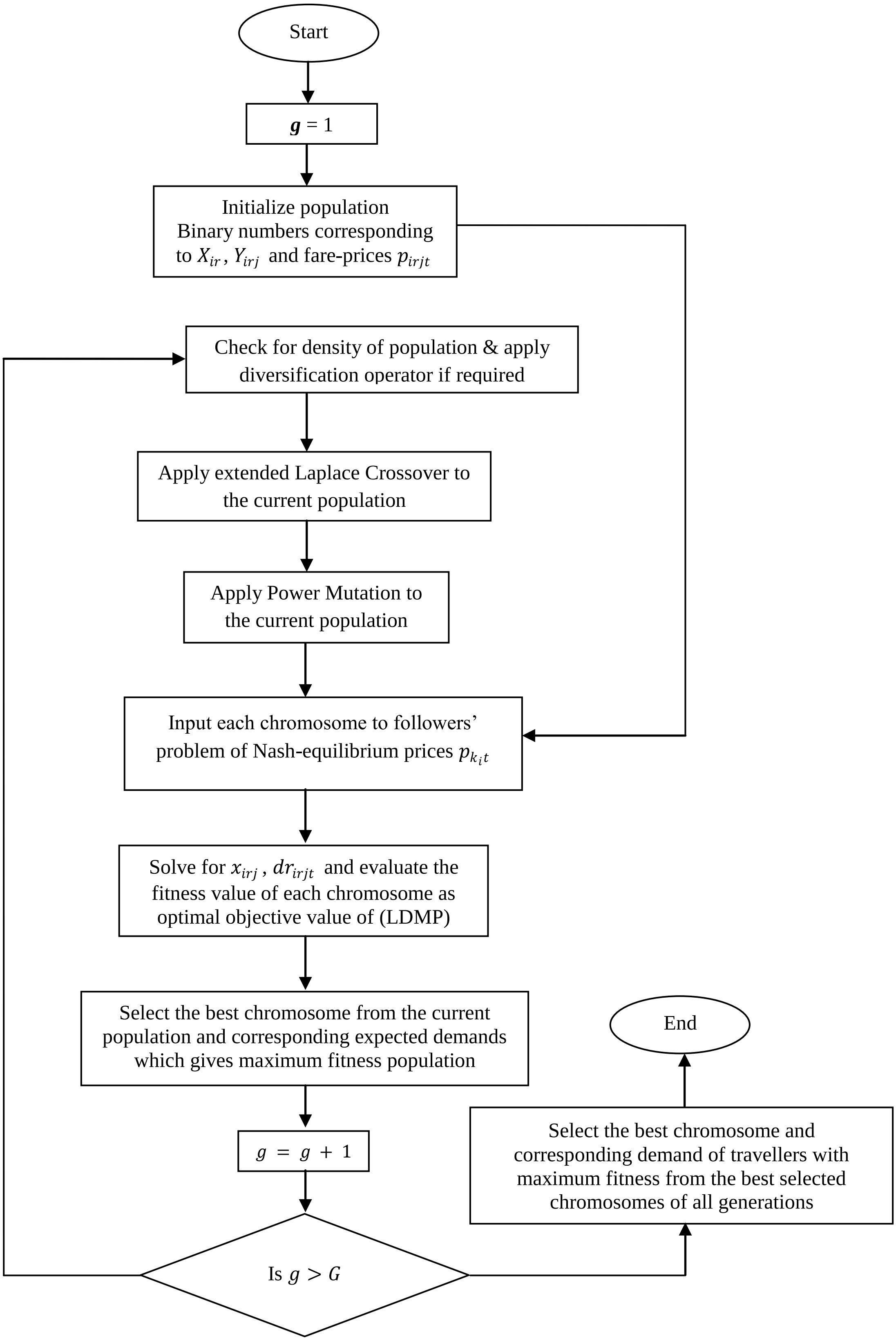

Figure 3.2: Flow Chart of DEGA for solving (RBPP) problem

3.4 An experimental study from Indian Railways

3.4.1 Relevant information and terminology of Indian Railways

Indian Railways (IR) is fourth most sizable voluminous railway network in the world comprising of 119,630 kilometres of total track and 92,081 kilometres of running track covering routes of 66,687 kilometres with 7,216 stations as on March 31, 2016 [234]. In 2015-16, IR carried 8.107 billion passengers and transported 1.101 billion tons of freight [235]. A total of 13,313 passenger trains ran daily during 2016 – 17. IR is the single largest provider of transportation services and is rightly called the lifeline of India. The widespread infrastructure of IR is geographically divided into 17 zones, each responsible for maintaining and managing its own rolling-stock. The decision on running special trains through major junctions is considered independently by each zone. The trains run by IR are categorized into 26 types. This classification is based on the average speed, number of stoppages (halts), distance covered, and some special features. Out of these, three types of trains namely "Superfast Express/ Mail", "Express/ Mail", and "Passenger Trains" are majorly run to serve the general public. These three types of trains are assembled with five classes of passenger coaches, namely AC-I, AC-II, AC-III, sleeper class (SL), second class (II-class). These coaches differ in the type of accommodation provided to the passengers and have seating/ accommodation capacity (per coach) as 28, 48, 64, 72, and 90, respectively. The special trains are also considered among these three categories because of much availability of the rolling-stock in these segments.

The special trains face competition from private airlines and luxury buses; nowadays some private airlines in India offer fares as low as comparable not only with AC-I class but also with AC-II class, and infrequently with AC-III (on long routes). Also, studies have revealed that road transportation may be detrimental to the railways industry.

3.4.2 Data inputs

We consider the test case of the Northern Railway Zone with 4 potential routes, namely Delhi – Mumbai, Delhi – Chandigarh, Delhi – Ranchi, and Jaipur – Lucknow (depicted by $i = 1, 2, \dots, 4$, respectively) for making a decision on running special trains with the planning horizon comprising of 4 weeks ($t = 1, 2, \dots, 4$). On these routes, 3 types of trains can run by appropriately assembling 5 classes of coaches. For coaches of 5 types, the indices used are $j = 1$: AC-I, $j = 2$: AC-II, $j = 3$: AC-III, $j = 4$: sleeper class (SL), $j = 5$: second class (II-class). The availability of numbers of coaches in respective classes is taken as 10, 10, 30, 50, and 50. The price sensitivity coefficients (β_i) of the four routes are 0.0009, 0.001, 0.0045, and 0.004, respectively. The profit margin coefficients $\left(\xi_{k_i}\right)$ for each alternative transport service provider on the four routes are 0.8, 0.6, 0.7, and 0.5, respectively. The sanctioned budget for running special trains on potential routes is assumed to be INR 20,00,000. The forecasted demand and other input parameters are listed in Table 3.1 to Table 3.6.

	time (t) in week			
routes (i)	1	2	3	4
1	2000	3000	4000	3000
2	1000	2500	2000	1000
3	3000	3200	3300	1500
4	3000	3500	4000	3500

Table 3.1: Demand on various routes (D_{it})

	routes (i)			
train type (r)	1	2	3	4
1	(25, 20)	(25, 20)	(25, 20)	(25, 20)
2	(25, 15)	(25, 15)	(25, 15)	(25, 15)
3	(20, 10)	(20, 10)	(20, 10)	(20, 10)

Table 3.2: Length of train, fixed cost (in INR with multiple 10^4) (ml_{ir}, f_{ir})

train type (r)	coach class (j)				
	1	2	3	4	5
1	(1, 85)	(3, 55)	(25, 45)	(25, 30)	(25, 15)
2	(1, 80)	(3, 50)	(25, 40)	(25, 25)	(25, 10)
3	(0, 80)	(0, 50)	(0, 40)	(0, 25)	(25, 9)

Table 3.3: Maximum number possible to assemble, cost of adding a coach (INR with multiple of 10^2) $\left(M_{rj}, g_{irj}\right)$

route (i)	train type (r)	coach type (j) 1	2	3	4	5
1	1	(0.4, 250)	(0.3, 150)	(0.1, 100)	(0.05, 50)	(0.02, 30)
	2	(0.3, 300)	(0.2, 140)	(0.09, 90)	(0.04, 40)	(0.01, 30)
	3	(0.001, 210)	(0.001, 130)	(0.001, 80)	(0.001, 30)	(0.001, 30)
2	1	(0.85, 150)	(0.8, 100)	(0.2, 60)	(0.05, 40)	(0.02, 10)
	2	(0.81, 130)	(0.7, 80)	(0.1, 50)	(0.04, 30)	(0.02, 10)
	3	(0.001, 120)	(0.001, 70)	(0.001, 40)	(0.001, 20)	(0.001, 10)
3	1	(0.4, 250)	(0.3, 150)	(0.1, 100)	(0.05, 60)	(0.01, 35)
	2	(0.3, 300)	(0.2, 140)	(0.09, 90)	(0.04, 50)	(0.01, 35)
	3	(0.001, 210)	(0.001, 130)	(0.001, 80)	(0.001, 40)	(0.001, 35)
4	1	(0.3, 180)	(0.4, 120)	(0.2, 80)	(0.03, 40)	(0.02, 10)
	2	(0.4, 200)	(0.3, 100)	(0.095, 70)	(0.02, 30)	(0.02, 10)
	3	(0.001, 140)	(0.001, 90)	(0.001, 50)	(0.001, 20)	(0.001, 10)

Table 3.4: Coefficient of non-monetary factors, Cost incurred to railways per passenger-(INR/passenger) (α_{irj}, c_{irj})

		$\boldsymbol{pl_{irj}}$					$\boldsymbol{pu_{irj}}$				
routes	train type	coach type (j)					coach type (j)				
(i)	(r)	1	2	3	4	5	1	2	3	4	5
1	1	3100	2000	1000	410	150	6000	3000	1950	900	400
	2	3000	1950	950	380	120	5900	2950	1900	870	370
	3	300	300	300	300	90	300	300	300	300	340
2	1	900	610	410	170	100	1500	800	600	350	150
	2	800	560	360	140	70	1400	780	550	320	120
	3	300	300	300	300	50	300	300	300	300	100
3	1	3100	2000	1000	410	150	6000	3000	1950	900	400
	2	3000	1950	950	380	120	5900	2950	1900	850	370
	3	300	300	300	300	90	300	300	300	300	340
4	1	3100	2000	1000	410	150	6000	3000	1950	900	380
	2	3000	1950	950	380	120	5900	2950	1900	850	350
	3	300	300	300	300	100	300	300	300	300	330

Table 3.5: Reservation prices of railways (INR/ passenger) for each category of coach in each type of train on each route, pl_{irj} and pu_{irj}

route (i)	parameters	competitors ($\boldsymbol{k_i}$)							
		1	2	3	4	5	6	7	8
1	c_{k_i}	700	540	470	680	530	440	630	400
	Av_{k_i}	150	350	350	150	350	350	150	550
	α_{k_i}	1	1.1	1.2	0.9	0.95	1.3	0.8	1
2	c_{k_i}	100	250	100	180	250	500	450	550
	Av_{k_i}	150	80	200	100	900	150	150	150
	α_{k_i}	0.02	0.1	0.015	0.06	0.4	1	0.8	1.1
3	c_{k_i}	380	280	380	600	450	750	-	-
	Av_{k_i}	150	350	150	550	150	150	-	-
	α_{k_i}	0.02	0.1	0.015	0.06	0.4	1	-	-
4	c_{k_i}	310	510	420	810	320	610	660	620
	Av_{k_i}	150	120	200	100	500	150	150	150
	α_{k_i}	0.01	0.06	0.015	0.05	0.2	1	0.9	1.1

Table 3.6: Parameters pertaining to competitor transporters

Remark 3.4.1: For considering the befitting instance of our problem and its scale (regarding the number of routes and planning horizon), we referred to the webpage of special trains on the website of IR ("National Train Enquiry System - Special Trains, Indian Railways," 2017). The webpage lists all special trains scheduled to run in the following two months. On May 03, 2017, a total of 412 trains were listed to run, out of which 290 trains were scheduled in May 2017. Due to different train numbers assigned to the same train while up and down journey, the actual number of trains running was 145. About 15 trains were scheduled only for a single run on specific but different dates, so we do not account for them. The remaining 130 trains in 26 zones result on an average five trains per zone. A further re-look indicate that Northern Railway Zone in IR had run only four special trains in May 2017.

3.4.3 Implementation of Diversified-Elitist-genetic algorithm

We solved the proposed bilevel programming problem (RBPP) applying the above input data. We used MATLAB 2015b to code the program. The parameters involved in Laplace crossover and power mutation along with the population size ($popsize$) are tuned for various combinations of probabilities of crossover, mutation and tournament selection. The initial experiments of DEGA with different sets of parameters enabled us to tune the maximum number of generations to $G = 150$ (*i.e.*, each run of DEGA is stopped with maximum 150 iterations). The best result obtained with a specific set of GA parameters shows no significant improvement in fitness value in the last 100 iterations (depicted later in Figure 3.3), establishing stability of DEGA. The best one found are $popsize = 50$, $a = 0$, $b = 1$, and $p = 1$ or 0.5. We set the maximum number of generations to 150 after tuning the parameters up to 500 iterations. The threshold for population density is tuned to 0.8 and $\overline{D}$ is set at 50. For each combination of the parameters (Table 3.7), we performed a set of 10 experiments of DEGA. Table 3.7 tabulates the relative error of the best solution obtained for each combination of parameters in comparison to the overall best solution. Figure 3.3, shows variation in the maximum fitness attained in various generations of a DEGA run. Among the 320 solutions generated (32 combinations with ten runs each) for (RBPP), we finally pick the one yielding the best fitness value in the results and analysis.

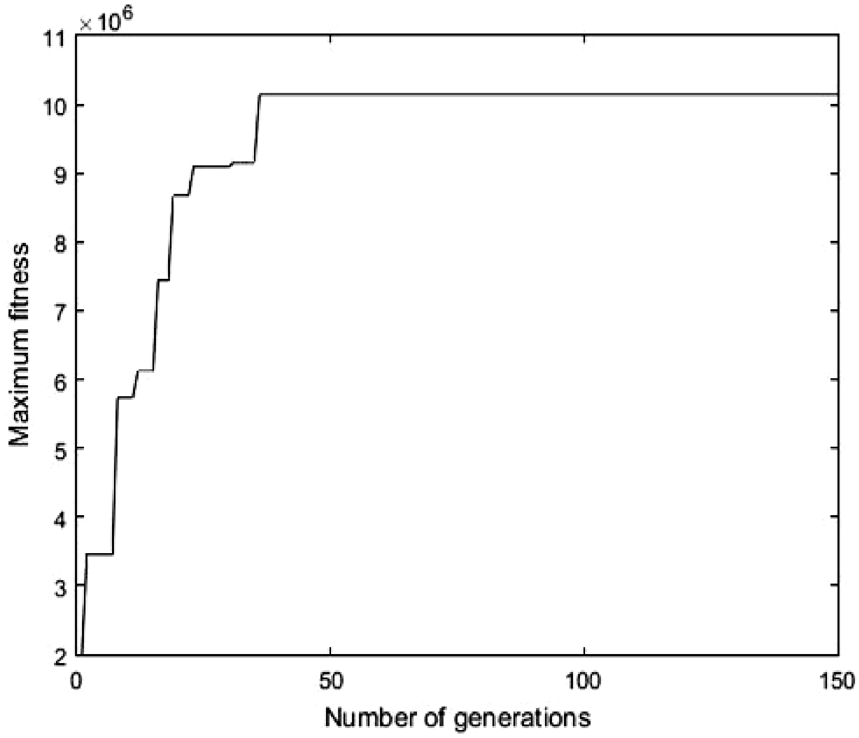

Figure 3.3: Maximum fitness vs. generations of best solution by DEGA

pcX	*pc*	*pmX*	*pm*	*p*	relative error	mean error
0.5	0.9	0.2	0.4	1	0.173	0.027
0.5	0.9	0.2	0.6	1	0.177	0.022
0.5	0.9	0.4	0.4	1	0.227	0.041
0.5	0.9	0.4	0.6	1	0.099	0.119
0.5	0.8	0.2	0.4	1	0.172	0.028
0.5	0.8	0.2	0.6	1	0.161	0.041
0.5	**0.8**	**0.4**	**0.4**	**1**	**0.000**	**0.242**
0.5	0.8	0.4	0.6	1	0.210	0.019
0.2	0.9	0.2	0.4	1	0.099	0.119
0.2	0.9	0.2	0.6	1	0.105	0.111
0.2	0.9	0.4	0.4	1	0.247	0.065
0.2	0.9	0.4	0.6	1	0.147	0.059
0.2	0.8	0.2	0.4	1	0.225	0.037
0.2	0.8	0.2	0.6	1	0.211	0.020
0.2	0.8	0.4	0.4	1	0.105	0.111
0.2	0.8	0.4	0.6	1	0.173	0.027
0.5	0.9	0.2	0.4	0.5	0.211	0.020
0.5	0.9	0.2	0.6	0.5	0.267	0.090
0.5	0.9	0.4	0.4	0.5	0.173	0.027
0.5	0.9	0.4	0.6	0.5	0.243	0.060
0.5	0.8	0.2	0.4	0.5	0.186	0.011
0.5	0.8	0.2	0.6	0.5	0.210	0.019
0.5	0.8	0.4	0.4	0.5	0.218	0.029
0.5	0.8	0.4	0.6	0.5	0.240	0.056
0.2	0.9	0.2	0.4	0.5	0.275	0.099
0.2	0.9	0.2	0.6	0.5	0.273	0.097
0.2	0.9	0.4	0.4	0.5	0.211	0.020
0.2	0.9	0.4	0.6	0.5	0.230	0.045
0.2	0.8	0.2	0.4	0.5	0.246	0.063
0.2	0.8	0.2	0.6	0.5	0.268	0.091
0.2	0.8	0.4	0.4	0.5	0.259	0.080
0.2	0.8	0.4	0.6	0.5	0.186	0.010

Table 3.7: Error analysis for different combination of parameters in DEGA; the best one is highlighted in bold

3.4.4 Results and analysis

The output decisions are as follows. Table 8 shows the decision of IR on types of trains. Table 3.9 tabulated the optimal number of coaches in the running trains. Table 3.10 provides the best fare-prices for different category of coaches in various types of trains. Table 3.11 lists the expected number of passengers in response to the arrangement for IR. Table 3.12 to Table 3.15 present the fare-price reaction of competitors on various routes in different periods, and also their expected passenger turn-ups. The projected total profit to IR is INR 1,01,38,195. Here, we observe that the fares listed in Table are compatible with the fare-chart of IR for the regularly running trains [236].

train type (r)	routes (i)			
	1	2	3	4
1	1	1	0	1
2	1	0	1	0
3	0	0	0	0

Table 3.8: Decision on running trains suggested for IR (X_{ir})

route (i)	train type (r)	coach type (j)				
		1	2	3	4	5
1	1	1	0	12	12	0
	2	1	3	12	9	0
	3	0	0	0	0	0
2	1	0	3	5	0	0
	2	0	0	0	0	0
	3	0	0	0	0	0
3	1	0	0	0	0	0
	2	0	0	1	0	8
	3	0	0	0	0	0
4	1	0	0	0	8	17
	2	0	0	0	0	0
	3	0	0	0	0	0

Table 3.9: Decision on number of coaches suggested for IR (x_{irj})

route	train type	time (t)									
		1					2				
		coach type (j)					coach type (j)				
(i)	(r)	1	2	3	4	5	1	2	3	4	5
1	1	5946	-	1082	621	-	4515	-	1076	893	-
	2	3400	2477	1026	462	0	4174	2083	1550	417	0
	3	-	-	-	-	-	-	-	-	-	-
2	1	-	720	584	-	-	-	697	593	-	-
	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
3	1	-	-	-	-	-	-	-	-	-	-
	2	-	-	1022	-	354	-	-	1823	-	122
	3	-	-	-	-	-	-	-	-	-	-
4	1	-	-	-	410	150	-	-	-	430	150
	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-

		time (t)									
		3					4				
1	1	5465	-	1847	707	-	3820	-	1932	412	-
	2	3458	2399	1024	645	-	5826	2874	1534	385	-
	3	-	-	-	-	-	-	-	-	-	-
2	1	-	676	482	-	-	-	790	508	-	-
	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
3	1	-	-	-	-	-	-	-	-	-	-
	2	-	-	996	-	158	-	-	1752	-	245
	3	-	-	-	-	-	-	-	-	-	-
4	1	-	-	-	749	151	-	-	-	835	161
	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-

Table 3.10: Fare-prices suggested for IR (INR/ Passenger) (p_{irjt})

		time (t)									
		1					2				
route	train type	coach type (j)					coach type (j)				
(i)	(r)	1	2	3	4	5	1	2	3	4	5
	1	1	-	71	44	-	8	-	479	661	-
1	2	10	22	74	0	-	28	144	499	0	-
	3	-	-	-	-	-	-	-	-	-	-
	1	0	102	64	0	0	0	144	273	0	0
2	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
	1	-	-	-	-	-	-	-	-	-	-
3	2	0	0	29	0	554	0	0	35	0	657
	3	-	-	-	-	-	-	-	-	-	-
	1	0	0	0	434	677	0	0	0	506	1419
4	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
		time (t)									
		3					4				
	1	11	-	760	863	-	8	-	487	701	-
1	2	27	144	768	647	-	27	144	506	148	-
	3	-	-	-	-	-	-	-	-	-	-
	1	0	144	294	0	0	0	117	73	0	0
2	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
	1	-	-	-	-	-	-	-	-	-	-
3	2	0	0	34	0	646	0	0	14	0	241
	3	-	-	-	-	-	-	-	-	-	-
	1	0	0	0	576	1530	0	0	0	505	705
4	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-

Table 3.11: Number of passengers expected to travel through IR (dr_{irjt})

	time (t)	competitors ($\boldsymbol{k_i}$)							
		1	2	3	4	5	6	7	8
$\boldsymbol{p_{k_i t}}$	1	1265	973	854	1225	957	800	1137	720
	2	26930	25861	2703	27554	25580	2814	27177	25717
	3	26346	24796	24945	22873	24751	3411	26645	24321
	4	3416	26581	26755	26996	2419	26914	3194	26007
$\boldsymbol{d_{k_i t}}$	1	150	215	265	140	188	307	137	245
	2	0	0	349	0	0	349	0	0
	3	0	0	0	0	0	349	0	0
	4	150	0	0	0	350	0	150	0

Table 3.12: Nash-equilibrium fare-prices of competitors in INR ($p_{k_i t}$) and expected passenger demand ($d_{k_i t}$) on route $i = 1$

	time (t)	competitors ($\boldsymbol{k_i}$)							
		1	2	3	4	5	6	7	8
$\boldsymbol{p_{k_i t}}$	1	160	400	160	290	400	800	720	880
	2	1005	1714	712	1451	400	1985	1785	2085
	3	1080	19509	788	1526	17502	19170	1860	2160
	4	201	433	160	311	400	800	720	21893
$\boldsymbol{d_{k_i t}}$	1	82	69	81	75	94	115	102	117
	2	150	79	200	99	401	150	150	150
	3	150	0	199	99	0	0	150	150
	4	90	77	93	84	108	132	117	0

Table 3.13: Nash-equilibrium fare-prices of competitors in INR ($p_{k_i t}$) and expected passenger demand ($d_{k_i t}$) on route $i = 2$

	time (t)	competitors ($\boldsymbol{k_i}$)					
		1	2	3	4	5	6
$\boldsymbol{p_{k_it}}$	1	647	5940	646	3097	3228	9500
	2	4756	4862	4810	4788	4708	4948
	3	5434	4714	5369	5315	765	5377
	4	6526	480	5264	5694	7368	5372
$\boldsymbol{d_{k_it}}$	1	149	0	149	0	0	0
	2	0	0	0	0	0	0
	3	0	0	0	0	0	0
	4	0	169	0	0	0	0

Table 3.14: Nash-equilibrium fare-prices of competitors in INR (p_{k_it}) and expected passenger demand (d_{k_it}) on route $i = 3$

	time (t)	competitors ($\boldsymbol{k_i}$)							
		1	2	3	4	5	6	7	8
$\boldsymbol{p_{k_it}}$	1	671	3759	9855	4838	488	919	5039	944
	2	5830	5825	5832	6071	5908	6018	1002	6044
	3	804	6042	6091	5119	5433	6210	1026	5942
	4	4684	770	631	4575	488	949	7402	974
$\boldsymbol{d_{k_it}}$	1	149	0	0	0	376	149	0	149
	2	0	0	0	0	0	0	149	0
	3	149	0	0	0	0	0	150	0
	4	0	119	199	0	425	149	0	149

Table 3.15: Nash-equilibrium fare-prices of competitors in INR (p_{k_it}) and expected passenger demand (d_{k_it}) on route $i = 4$

routes (i)	time (t)			
	1	2	3	4
1	1869	2517	3569	2671
2	901	1796	1186	891
3	881	692	830	424
4	1934	2074	2405	2251

Table 3.16: Total demand expected to be satisfied by all travelling service providers including IR

3.5 Comparison analysis

Among three comparisons presented below, the first one is intended to verify the performance of the developed solution methodology, whereas other two comparisons justify the model formulation and results of computation.

Comparison 1: Comparison of the performance of DEGA against simple GA

Keeping all other parameters and input data same, when simple GA is run on the test case, it provides a solution which yields a profit of INR 74,52,482 to IR. As this profit is much lower than the one obtained from DEGA, it indicates the significance of the Diversification operator used in our solution methodology.

The proposed (RBPP) model incorporates influences of competitors on demand-share through the fare-price reaction. To test the acclaimed prominence of our model we attempt to compare the results of the previous section with two cases when either the reaction of competitors is completely ignored in the model or the railways assumes a monopoly in the market by setting the escalated fare-prices in special trains.

Comparison 2: Comparison with a case in which competition is ignored

For this case, we consider a single level optimization model as (LDMP) (without considering competitors' reaction on prices thorough $(\mathrm{FDMP}-k_i)$) for determining variables $\{dr_{irjt}\}$, $\{x_{irj}\}$ by fixing values of variables $\{X_{ir}\}$ and $\{p_{irjt}\}$ given in Table 3.8 and Table 3.10, respectively, as parameters. Further, to omit the influence of competitors we use the following expression for estimating the expected number of passengers to prefer the class j $(j=1,2,\dots,J)$ of train type r $(r=1,2,\dots,R)$ to travel on route i $(i=1,2,\dots,I)$ at time t $(t=1,2,\dots,T)$.

$$P_{irjt}\,(p_{it}) = \frac{Y_{irj}\ e^{\alpha_{irj}-\beta_i\,p_{irjt}}}{1\ +\ \sum_{r=1}^{R}\sum_{j=1}^{J} Y_{irj}\,e^{\alpha_{irj}-\beta_i\,p_{irjt}}}. \tag{3.5.1}$$

Solving this optimization problem, the optimal values obtained for variables $\{dr_{irjt}\}$, and $\{x_{irj}\}$ are listed in Table 3.17 and Table 3.18, respectively. These tables depict the number of passengers expected to travel with IR on considered routes and the train-assembly arrangements required accordingly, for the discussed case.

		time (t)									
		1					2				
route	train type	coach type (j)					coach type (j)				
(i)	(r)	1	2	3	4	5	1	2	3	4	5
1	1	4	0	249	359	0	6	0	400	577	0
	2	28	78	260	411	0	28	126	417	660	0
	3	0	0	0	0	0	0	0	0	0	0
2	1	0	144	246	0	0	0	144	611	0	0
	2	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	27	0	506	0	0	22	0	413
	3	0	0	0	0	0	0	0	0	0	0
4	1	0	0	0	340	954	0	0	0	401	1123
	2	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0
		time (t)									
		3					4				
1	1	9	0	557	802	0	6	0	405	585	0
	2	27	144	576	864	0	28	128	423	669	0
	3	0	0	0	0	0	0	0	0	0	0
2	1	0	144	471	0	0	0	144	248	0	0
	2	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	23	0	449	0	0	11	0	230
	3	0	0	0	0	0	0	0	0	0	0
4	1	0	0	0	496	1391	0	0	0	443	1246
	2	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0

Table 3.17: Number of passengers expected to travel with IR in new arrangement (dr_{irjt})

route	train type	coach type (j)				
(i)	(r)	1	2	3	4	5
	1	1	0	9	12	0
1	2	1	2	8	12	0
	3	0	0	0	0	0
	1	0	3	10	0	0
2	2	0	0	0	0	0
	3	0	0	0	0	0
	1	0	0	0	0	0
3	2	0	0	1	0	6
	3	0	0	0	0	0
	1	0	0	0	7	16
4	2	0	0	0	0	0
	3	0	0	0	0	0

Table 3.18: Decision of IR on number of coaches for the new arrangement (x_{irj})

With this arrangement, the total profit of IR is deceptively expected to be INR 1,06,16,014. At first look, this profit appears lucrative than the one obtained from (RBPP), but it is elusive as explained below through a comparison with the solution of our model.

Overlooking the competitors in terms of their fare-price response (and thereby overestimating the preference of passengers), the IR obtains inappropriate estimates the expected demand and hence quixotically plan for the number of coaches of various types to be assembled in its trains. For example, Table 3.18 suggests for 9 coaches of type $j = 3$ in train type $r = 1$ on route $i = 1$, which is 3 lesser than the same directed in Table 3.9. The 9 coaches of this type have a total capacity for 576 passengers only, and for the situation to be actually realized due to the influence of competitors, this is insufficient to accommodate the expected passenger turnout in period $t = 3$ (*i.e.*, 760, see Table 3.11), resulting in an opportunity loss. On the other hand, Table 3.18 suggests for 10 coaches of type $j = 3$ in train type $r = 1$ on route $i = 2$, which is 5 more than the same suggested in

Table 3.9. Therefore, with this arrangement of trains, the capacity of additional 5 coaches is expected to remain unused during actual runs of these special trains.

Comparison 3: Comparison with an optimistic approach

We compare the results of our model with one more similar decision-making situation, when the IR decides for highest possible fare-prices to encash on the opportunity of rising demand and overlooks its competitors. The solutions of the single level optimization problem with fare-prices fixed as upper reservation prices[17], *i.e.*, $p_{irjt} = pu_{irj}$, $\forall\, t$, are reported in Table 3.19 to Table 3.21. The total profit to this arrangement is expected as INR 71,29,662, indicating a significant decrease in profit due to lower passenger ridership. It betokens that if the IR ignores its competitors and monopolizes the market by declaring its highest fare-prices, then the public reacts with a small passenger turnout and thereby the objective of earning a maximum profit gets deified. The above comparisons indicate the superiority of our proposed bilevel model which incorporates competition while deciding on the optimal fare-prices and assembly planning of special trains.

Train decision (X_{ir})	routes (i)			
train type (r)	1	2	3	4
1	1	1	0	1
2	1	0	1	0
3	0	0	0	0

Table 3.19: Decision on running trains of IR

[17] Instead of upper reservation prices, following the approach of [247], we may consider demand share based discrete set of fare points and solve the resultant single level operational planning problem.

		time (t)									
		1					2				
route	train type	coach type (j)					coach type (j)				
(i)	(r)	1	2	3	4	5	1	2	3	4	5
	1	1	-	71	44	-	8	-	479	661	-
1	2	10	22	74	0	-	28	144	499	0	-
	3	-	-	-	-	-	-	-	-	-	-
	1	0	102	64	0	0	0	144	273	0	0
2	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
	1	-	-	-	-	-	-	-	-	-	-
3	2	0	0	29	0	554	0	0	35	0	657
	3	-	-	-	-	-	-	-	-	-	-
	1	0	0	0	434	677	0	0	0	506	1419
4	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
		time (t)									
		3					4				
	1	11	-	760	863	-	8	-	487	701	-
1	2	27	144	768	647	-	27	144	506	148	-
	3	-	-	-	-	-	-	-	-	-	-
	1	0	144	294	0	0	0	117	73	0	0
2	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
	1	-	-	-	-	-	-	-	-	-	-
3	2	0	0	34	0	646	0	0	14	0	241
	3	-	-	-	-	-	-	-	-	-	-
	1	0	0	0	576	1530	0	0	0	505	705
4	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-

Table 3.20: Number of passengers expected to travel in response to prices $p_{irjt} = pu_{irj}$ of IR (dr_{irjt})

route	train type	coach type (j)				
(i)	(r)	1	2	3	4	5
1	1	1	0	5	11	0
	2	1	3	5	10	0
	3	0	0	0	0	0
2	1	0	2	10	0	0
	2	0	0	0	0	0
	3	0	0	0	0	0
3	1	0	0	0	0	0
	2	0	0	0	0	5
	3	0	0	0	0	0
4	1	0	0	0	2	8
	2	0	0	0	0	0
	3	0	0	0	0	0

Table 3.21: Decision of IR on number of coaches x_{irj} for passengers' turn-up corresponding to prices $p_{irjt} = pu_{irj}$

3.6 Conclusions

In this chapter, we addressed a strategic planning problem of railways for decisions on running special-trains, including the decision on optimal fare-pricing and assembly planning, in a competitive environment. On that front, we proposed a bilinear bilevel mixed integer programming model featuring a constrained Nash-equilibrium problem at the follower's level to capture the fare-price response of the competitors of railways. A diversified elitist genetic algorithm is suggested to solve the proposed problem. The model as well as the solution methodology, is demonstrated with an experimental study of scenario extracted from the IR. The empirical results are further compared with two situations for verifying the efficacy of the suggested model. It is demonstrated though comparison analysis that using the proposed approach for decision on arranging trains the IR can generate an additional revenue to a maximum possible extent by running special trains in a financially viable way. The developed decision support system is useful for every such railway authority for which it is obligatory to declare the fixed fare-prices in advance at the time of announcing the schedule of special trains. The proposed conceptions of the model and its solution procedure are incipient and novel.

Here we have taken an opportunity to address the issue of paucity of solution methods to efficiently handle the single-leader-multi-follower BLP problems of large scale. To address this issue, we have suggested a diversified-elitist GA based solution methodology. This attempt is the first initiative to design a solution technique for handling large scale single-leader-multi-follower BLP problems.

This chapter can be summarized by noting two of our contributions. The first is to address a strategic planning problem of railways by designing a mathematical model for the same, and the second is to suggest a GA based solution methodology for solving a case of single-leader-multi-follower BLP problem.

Chapter 4

Decision Support to Small Scale Supplier for Strategically Negotiating Prices with a Buyer

In this chapter[18], we develop a decision support for a small scale supplier to identify target prices for negotiation with potential buyer in order to clinch a deal with maximum profit. In this study, the supplier is known to adopt multi-sourcing strategy for the required products. Whereas, negotiations are considered in the situation where the prices with other existing suppliers of the buyer are already fixed and the small scale supplier considered is preparing to enter into business with this buyer. For strategic pricing decisions of the small scale supplier in such a situation, it is imperative to identify the demand order allocation mechanism of the buyer while cohesively estimating the cost of the production-and-logistic operations to fulfil the replicating demand. In this context a novel mixed integer bilevel programming model is proposed to formulate the problem in which the supplier is considered as a leader and the buyer is a follower. The proposed model subsumes the assessment of demand share against the price quotation, enabling the supplier to prepare an aggregate production distribution plan accordingly. An integer coded genetic algorithm is used to solve the model and its implementation is exhibited through a test scenario. Through the analysis of results it is demonstrated that the prices suggested by the decision-support developed enable the small scale supplier to successfully penetrate into the considered potential market. The comparison analysis demonstrates that targeting the suggested prices during negotiation is better than adopting penetration pricing strategy.

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[18] The contents of this chapter are based on research paper: "A Bilevel Programming Model for a Cohesive Decision Making on Strategic Pricing and Production Distribution Planning for a Small Scale Supplier", International Game Theory Review (World Scientific Publishing Co.). 22(2) (2020) 204009-1–34. doi: 10.1142/S0219198920400095.

4.1 Introduction

Small scale suppliers, due to limited resources, customarily start their business with a single buyer. It is intended by an ingenious small scale supplier to negotiate prices of its products so as to penetrate into the market while receiving a profitable business from the potential buyer. Once the prices get settled as a result of negotiation, the buyer allocates demand orders to all the suppliers including the new small scale supplier in order to receive a supply of required products at it delivery location(s) for a span of certain planning horizon, with a minimum total procurement cost. The supplier then discerns the production, stocking and distribution plan, considering its limited resources for fulfilling the demand received from the buyer in response to the prices settled at. By adhering to this sequential decision making, the supplier may receive a demand share beyond its capacity to fulfil in a pursuit of acquiring maximum demand share through penetration pricing[19]. If such a situation is encountered, it would consequent not only in an opportunity loss, but also adversely affect future business prospects of the supplier due to the loss of credibility. Therefore, for any supplier with limited operational capacities, it is suggested to adopt a profit maximization strategy for pricing while contemplating the capacities to fulfil the replicating demand. For this purpose it is imperative for such a supplier to concurrently discern the total cost of production-and-logistics and the operational capacities for fulfilling the demand to be replicated.

For such a significant problem, there does not seem to exist any noteworthy study explicating strategic pricing by estimating the replicating demand and cohesively assessing the operational plans in accordance. In this regard, a novel mixed integer bilinear BLP model is presented to address the strategic pricing problem from the perspective of a small-scale supplier approaching a potential buyer. The proposed model poses the supplier as a leader and the potential buyer as a follower. The problem before the leader is of price-setting for its various products and coherent assessment of production-and-logistics arrangements to fulfil the replicating demand-orders from various delivery locations of the buyer as a follower. The proposed model in the BLP framework involves bilinear objective function for leader and bi-objective integer programming problem for formulating the follower's reaction and further involves binary and integer variables.

[19] Penetration pricing consists of setting competitive prices to obtain a larger market share [72].

Further, no algorithm is available in the literature for solving a class of bilinear BLP problems with multiple objectives at follower's level and involving binary and integer variables at both the levels. To cover this research gap, at first hand, a variant of GA is proposed to solve such a class of problems. Thus this chapter can be comprehended to include dual contribution in research, first to develop a decision-support through modelling the problem being addressed, and the second to develop a GA-based solution methodology for an unattended classed of BLP problems.

This chapter is structured into six sections. Section 4.2 introduces the bilevel optimization model for a cohesive decision-making for the small scale supplier. Section 4.3 explains the devised solution methodology based on GA. Section 4.4 presents an experimental evaluation of the proposed approach through an implementation of proposed solution methodology on a test scenario. Section 4.5 presents comparative analysis to demonstrate efficiency of the proposed approach for decision-support on price negotiation. Section 4.6 presents managerial implications of the accomplished research work and Section 4.7 gives a summary of the work presented in this chapter.

4.2 Formulation of mathematical model

4.2.1 Problem description

To design a model for the strategic pricing problem of a small scale supplier, it is considered that the supplier intends to quote prices of N products manufactured at its production centre (PC) so as to receive a profitable demand-order for initiating a business with a specific buyer. We consider the situation where the suppliers have to deliver these products at multiple delivery locations (DLs) of the buyer firms situated at J different geographical sites. This setup is considered for a pre-defined planning horizon discretized into a total of T periods. This dictates that, the prices of a product will remain same in the planning horizon, but can vary from one DL to the other.

On the other hand, the buyer firm is already in business with K other suppliers for purchasing similar products. The price-negotiation of the buyer with these K suppliers is concluded. Upon an analysis of outcomes of negotiation with existing suppliers the buyer is keen to induce a new supplier for improving the performance of inbound supply chain by intensifying the competition among the suppliers in a long term in a context of prices and quality of the products. In this situation, we consider the case when a small-scale supplier, upon an assessment of quality performance records, is called by this buyer for a negotiation on prices.

Observing this available opportunity, a perspicacious supplier tries to target such prices for negotiation which enable penetrating into the prospective market with most profitable demand share. For ascertaining such target prices the supplier is assumed to have an estimate of the prices negotiated with the existing suppliers. Acquiring such information is quite possible through market intelligence. The supplier under consideration, further needs to simulate the buyer's decision behaviour in terms of demand-order allocations in a response to the prices negotiated with this supplier and those already fixed by other K suppliers. In this context, it is considered that the buyer allocates these demand-orders to each of these suppliers aiming to minimize total purchase cost and maximize the economic qualitative score of procurement over the entire planning horizon. The knowledge of this business environment enables the small-scale supplier to discern the production-and-logistics arrangements and associated costs of fulfilling the replicating demand-orders, so as to assess the resulting profit. This mechanism is depicted in Figure 4.1.

A cost optimal arrangement as referred above for fulfilling the demand-orders includes decisions on regular and overtime production volumes, transportation volumes from PC to its warehouses/ distribution centres (DCs), transportation volumes from DCs to buyer's DLs and inventories in each of its warehouse for each period of the planning horizon. The situation is described in Figure 4.2.

In nutshell, the demand and hence the production and distribution decisions depend on the prices offered to the supplier. Therefore, the decision on prices of the products is primary, and this mechanism involves the pricing decision to be taken first, followed by the buyer's decision on demand-order allocation as a response to it. Observing this chronology of decision-making, the bilevel programming framework seems to be appropriate for modelling the pricing problem of the supplier considering the small scale supplier as leader and the buyer as follower. As the prices already settled with existing suppliers are known and fixed, therefore there is no role of them as decision-makers in this problem, except that their fixed prices are taken as constant coefficients while modelling the response of the buyer.

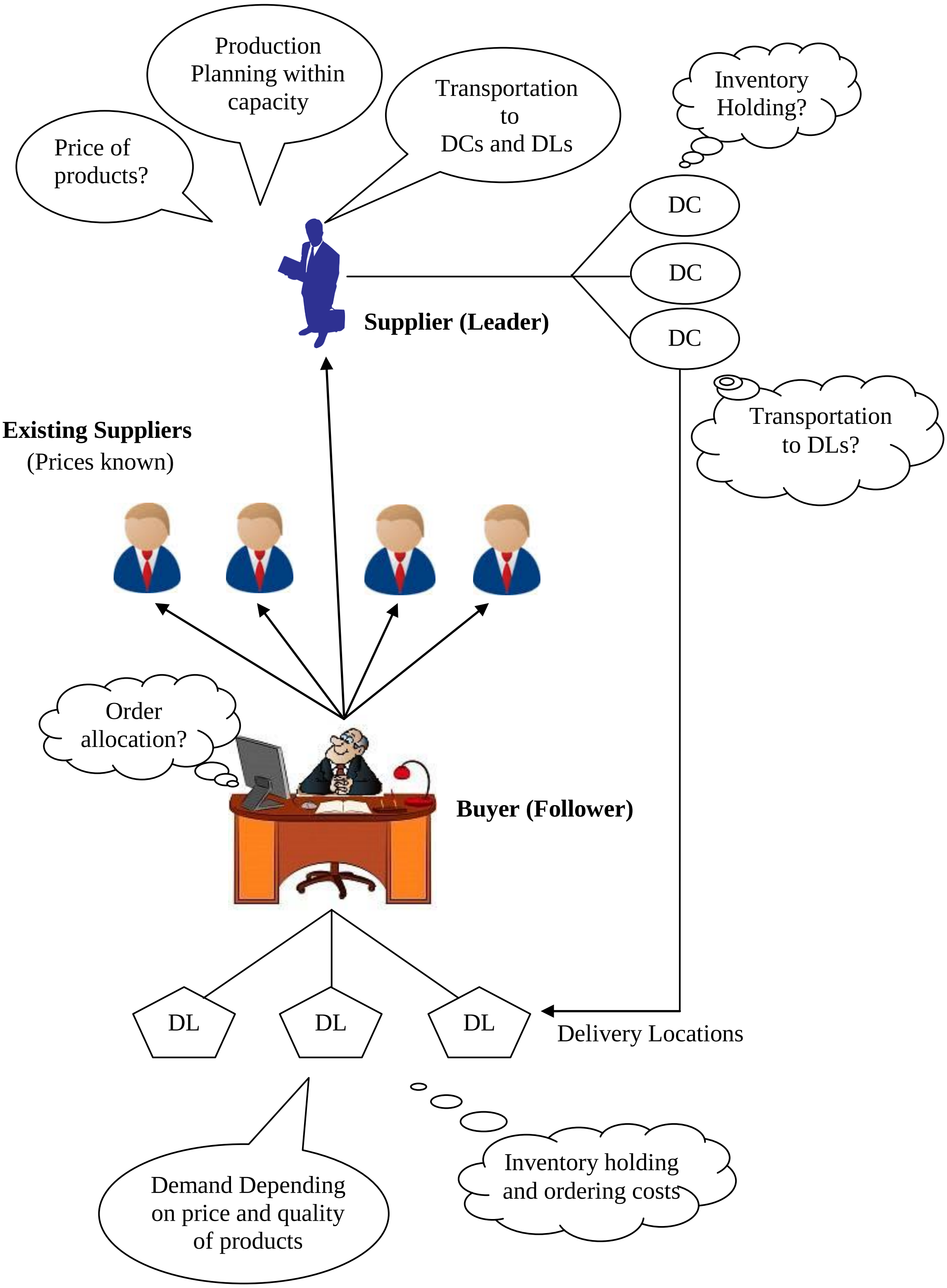

Figure 4.1: Depiction of problem structure

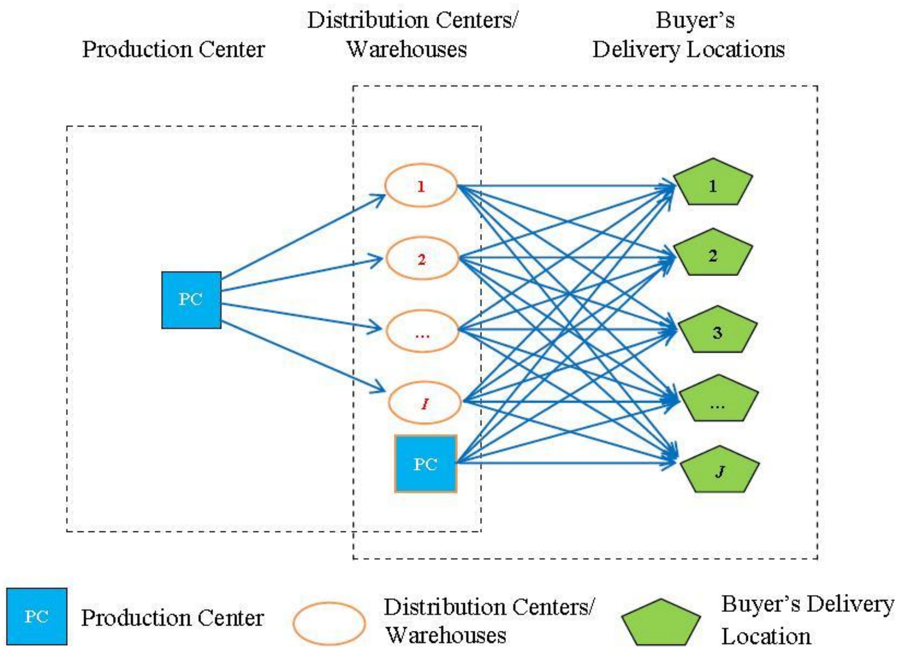

Figure 4.2: Production and distribution structure

4.2.2 Assumptions and notations

- To build the mathematical model, a finite planning horizon is partitioned into T multiple periods $(t = 1, 2, \ldots, T)$.
- The small-scale supplier has limited production capacity, in sense of labour/ machine capacity in planning horizon T.
- The small-scale supplier can produce more number of units than those demanded in a period and keep them as inventory in I warehouses/ DCs for the next period. The DCs have limited storage capacities.
- Products can be supplied either directly from the PC of the small-scale supplier or through DCs to buyer's DLs. But, at the end of each period of the planning horizon, it is ensured that nothing is left as stock at PC due to no storage capacity available there.
- The cost of transportation to deliver the product at DLs is to be borne by the suppliers themselves and not by the buyer. So, the prices of products depend on DLs also.
- The small scale supplier or any other supplier is not allowed to backorder any quantity in any period.

- A fix ordering cost is incurred to the buyer every time when he purchases any product from a supplier for a DL.
- The buyer has a storage capacity at its delivery locations for keeping quantities more than the demand in any period, so that the ordering cost for next period can sometimes be compensated by the inventory cost.
- Buyer keeps a bound on maximum number of defective units in the purchase while deciding of purchase volumes from various suppliers.
- For a given pricing vector of the products by the small scale supplier, the buyer's problem, on conversion to a scalar problem by weighted sum approach, has a unique optimal order allocation vector.

The small scale supplier henceforth is called as "the supplier" and differentiated from other suppliers with the name "existing suppliers" throughout this discussion. Indices, parameters, and variables used to describe the mathematical formulation of our model are listed below.

Indices and sets

N	number of different type of products; $n = 1, 2, \dots, N$
I	number of DCs of the supplier; $i = 1, 2, \dots, I$
	$i = 0$ stands for the PC as a source to transport products to directly to DLs.
K	number of existing suppliers; $k = 1, 2, \dots, K$
J	number of buyer's DLs; $j = 1, 2, \dots, J$
T	planning horizon (number of periods); $t = 1, 2, \dots, T$

Leaders parameters and variables

Parameters

lp_{nj}	minimum reservation price of product n for its demand at buyer's DL j (INR/ unit) [20]
Lp_{nj}	maximum reservation price of product n for its demand at buyer's DL j (INR/ unit)
a_{nt}	regular time production cost of product n in period t (INR/unit)
b_{kn_kt}	overtime production cost of product n in period t (INR/unit)
r_{nt}	machine-hours required for production of per unit of product n in period t

[20] This is practically interpreted as break-even price of the supplier for a particular product.

tcp_{int} cost of transportation of product n from PC to DC i of the supplier in period t (INR/ unit)

tc_{ijnt} cost of transportation of product n from DC i of the supplier to buyer's DL j in period t (INR/ unit)

d_{int} inventory carrying cost of product n at DC i of the supplier in period t (INR/unit)

v_n space occupied by per unit of product n (cu-ft/unit)

MR_t maximum regular machine-hours (man-hours) available with the supplier in period t

M_t maximum total machine-hours (man-hours) available with the supplier in period t

V_{it} maximum space available in DC i of the supplier in period t (cu-ft)

Variables

z_L Gross profit of the supplier

mp_{nj} per unit price of product n from the supplier for its demand at DL j (INR/unit)

Q_{nt} regular time production volume of product n of the supplier in period t (units)

O_{nt} overtime production volume of product n of supplier in period t (units)

SS_{int} inventory level (safety stock) of product n at DC i of supplier in period t (units)

I_{int} consignment volume of product n sent from PC to DC i of the supplier in period t (units)

x_{ijnt} consignment volume of product n from DC i of the supplier to buyer's DL j in period t (units)

Follower's parameters and variables

Parameters

z_{F_1} total cost of procurement and holding products at various DLs (INR/unit)

z_{F_2} total economic qualitative score of procurement

p_{njk} price of product n fixed by supplier k at DL j (INR/unit)

O_j fixed cost of ordering a delivery at DL j from the supplier (INR/unit)

O_{jk} fixed cost of ordering a delivery at DL j from existing supplier k (INR/unit)

H_{nj} Inventory carrying cost per unit of product n incurred at delivery at DL j (INR/unit)

D_{jnt} total forecasted demand of product n at buyer's DL j in period t (units)

Myo_{jnk} maximum purchase volume of product n from existing supplier k for DL j in any period

Mxo_n maximum purchase volume of product n from the supplier for various DLs in any period

VF_j maximum inventory carrying space at DL j of the buyer (cu-ft)

mq_n Economic qualitative score of the supplier for product n

q_{nk} Economic qualitative score of existing supplier k for product n

d_n average number of defects expected in product n to be purchased from the supplier (in %)

d_{nk} average number of defects expected in product n to be purchased from existing supplier k (in %)

df_n maximum acceptable average number defects in product n (in %)

Variables

y_{jnt} number of units of product n to be purchased from the supplier for DL j in period t

yo_{jnkt} number of units of product n to be purchased from existing supplier k for DL j in period t

Y_{jt} binary variable: takes value 1 if at least one unit of any of N products is purchased by the buyer from the supplier for delivery at DL j in period t; and 0 otherwise

YO_{jkt} binary variable: takes value 1 if at least one unit of any of N products is purchased by the buyer from existing supplier k for delivery at DL j in period t; and 0 otherwise

With all the details on the problem, assumptions and notations given by now, the mathematical model in BLP framework is presented below from the supplier's perspective for strategically ascertaining prices to negotiate with the buyer. Herein, the supplier is posed as leader and the buyer as follower.

4.2.3 Components of the problem related to the supplier

Objective function of expected profit

The objective of the supplier is to maximize the total profit through the decision on values of price variables (mp_{nj}) for each product n and each DL $j = 1, 2, \dots, J$, and there upon values of variables of production, inventory, and transportation ($Q_{kn_kj}, O_{kn_kj}, SS_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt}$) based on the demand shares received as a response to the prices quoted by all the suppliers. The objective function is given as following.

$$\max z_L = \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{j=1}^{J} mp_{nj}\, y_{jnt} - \sum_{t=1}^{T}\sum_{n=1}^{N}(a_{nt}\,Q_{nt} + b_{nt}\,O_{nt})$$
$$- \left(\sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=1}^{I} d_{int}\,SS_{int} + \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=1}^{I} tcp_{int}\,I_{int} + \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=0}^{I}\sum_{j=1}^{J} tc_{ijnt}\,x_{ijnt}\right).$$

… (4.2.1)

Price bounds

The prices are speculated within bounds; lower bounds are the minimum acceptable prices to the supplier. Upper bounds are enforced due to the competition imposed by other suppliers, in a sense that for prices more than these prices would not replicate any demand to the supplier.

$$lp_{nj} \le mp_{nj} \le Lp_{nj}, \quad \forall n_k, \forall j. \tag{4.2.2}$$

Regular time production hours

The production volumes for various products are restricted by the regular time production hours.

$$\sum_{n=1}^{N} r_{nt}\,Q_{nt} \le MR_t, \quad \forall t. \tag{4.2.3}$$

Total production hours

The total production volumes obtainable through the provisions of overtime engagement of labour/ machines along with the regular time production are also restricted by the total available production hours.

$$\sum_{n=1}^{N} r_{nt}(Q_{nt} + O_{nt}) \leq M_t , \quad \forall t. \tag{4.2.4}$$

Inventory balancing constraints

The demand orders received for each product are fulfilled through the total production volumes together with the available inventory volumes maintained in the previous period while maintaining required inventory volumes for the current period.

$$Q_{nt} + O_{nt} + \sum_{i=1}^{I} SS_{in(t-1)} - \sum_{i=1}^{I} SS_{int} = \sum_{j=1}^{J} y_{jnt} , \quad \forall n, \forall t. \tag{4.2.5}$$

Space constraints at DC

At any period, the consignment volumes of various products to be received at each DC (I_{int}) along with the products already available there as inventory maintained during the previous period ($SS_{in(t-1)}$) should be capacitated in the available space at the DC particular.

$$\sum_{n=1}^{N} v_n\left(I_{int} + SS_{in(t-1)}\right) \leq V_{it} , \quad \forall i, \forall t. \tag{4.2.6}$$

Transport plan for delivery at each DL

For each period, the consignment volumes of various products from PC to DC(s), from DC(s) to DLs, and directly from PC to DLs are to be planned to fulfil the demand of each DL for each product. Following three constraints govern this requirement. First two of the following constraints describe the transportation plan from DC(s) to DLs, whereas the third one describes transportation plan directly from PC to DLs.

$$\sum_{i=0}^{I} x_{ijnt} \geq y_{jnt} , \quad \forall j, \forall n, \forall t, \tag{4.2.7}$$

$$\sum_{j=1}^{J} x_{ijnt} \leq I_{int} + SS_{in(t-1)} - SS_{int}\,, \quad \forall i \neq 0, \forall n, \forall t, \tag{4.2.8}$$

$$\sum_{j=1}^{J} x_{0jnt} = Q_{nt} + O_{nt} - \sum_{i=1}^{I} I_{int}\,, \quad i = 0, \forall n, \forall t. \tag{4.2.9}$$

4.2.4 Components of the problem related to the buyer

Based on the price quotes received from all the suppliers, the buyer solves the cost-optimal demand order allocation problem.

First objective function

The buyer decides on allocating demand shares (y_{jnt} , yo_{jnkt}) for various products in response to the received price quotes (mp_{nj}) from the supplier while considering the prices already settled with existing suppliers (p_{njk}) for minimum total procurement cost.

$$\begin{aligned}\min z_{F_1} = & \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{j=1}^{J}\left\{mp_{nj}\, y_{jnt} + \sum_{k=1}^{K}\{p_{njk}\, yo_{jnkt}\}\right\} + \sum_{t=1}^{T}\sum_{j=1}^{J} O_j Y_{jt} \\ & + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K} O_{jk}\, YO_{jkt} \\ & + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N} H_{nj}\left\{\sum_{t=1}^{t'}\left(y_{jnt} + \sum_{k=1}^{K} yo_{jnkt}\right) - \sum_{t=1}^{t'} D_{jnt}\right\}\end{aligned}$$

… (4.2.10)

Second objective function

In each period $t = 1, \ldots, T$, and each route $i = 1, \ldots, I$, the competitors $k_i = 1, \ldots, K_i$, aim to compute Nash-equilibrium price vector $\{p_{k_i t}: k_i = 1, \ldots, K_i\}$, so as to maximize their profit through a competition.

$$\max z_{F_1} = \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{j=1}^{J}\left\{mq_n\, y_{jnt} + \sum_{k=1}^{K} q_{nk}\, yo_{jnkt}\right\} \tag{4.2.11}$$

Upper bound on demand-order to existing suppliers

This constraint of FDMP gives a prefixed upper bound on the demand orders of each product to be assigned to the existing suppliers (depending upon the production capacities, transportation carrier capacities and profitability of existing suppliers)

$$yo_{jnkt} \leq Myo_{jnk}, \quad \forall j, \forall n, \forall k, \forall t, \tag{4.2.12}$$

Upper bound on demand-order to the suppliers

This constraint bounds an upper limit on the total demand-order of each product to the supplier, according to prior information by the later. At this initial stage of business, the supplier can give only one upper bound on the total order of each product as the price is yet to decided and negotiable.

$$\sum_{j=1}^{J} y_{jnt} \leq Mxo_n, \quad \forall n, \forall t, \tag{4.2.13}$$

Order-allocation against demand fulfilment

The following two constraints ensure that in any period, the buyer purchases products for each DL not less than what is required while keeping a room for purchasing additional quantities to reduce the total ordering cost. Whereas, in the final period there should not be any inventory remaining at any DL.

$$\sum_{t=1}^{t'} \left(y_{jnt} + \sum_{k=1}^{K} yo_{jnkt} \right) \geq \sum_{t=1}^{t'} D_{jnt}, \quad \forall j, \forall n, t' = 1, \dots, T-1, \tag{4.2.14}$$

$$\sum_{t=1}^{T} \left(y_{jnt} + \sum_{k=1}^{K} yo_{jnkt} \right) = \sum_{t=1}^{T} D_{jnt}, \quad \forall j, \forall n, \tag{4.2.15}$$

Inventory capacity constraints

At each DL, total inventory volumes of various products to be stocked for the next period after their consumption as per the demand in any period should not exceed the storage capacity available there.

$$\sum_{n=1}^{N} v_n \left\{ \sum_{t=1}^{t'} \left(y_{jnt} + \sum_{k=1}^{K} yo_{jnkt} \right) - \sum_{t=1}^{t'} D_{jnt} \right\} \le VF_j \,, \quad \forall j, \forall n, \forall t', \qquad (4.2.16)$$

Constraint to limit average number of defects

This constraint is employed to limit the average number of defects in various products up to a certain tolerance limit.

$$\sum_{t=1}^{T} \sum_{n=1}^{N} \sum_{j=1}^{J} \left\{ d_n y_{jnt} + \sum_{k=1}^{K} \{ d_{nk} \, yo_{jnkt} \} \right\} \le \sum_{n=1}^{N} df_n \sum_{t=1}^{T} \sum_{j=1}^{J} D_{jnt} \,, \qquad (4.2.17)$$

Inventory capacity constraints

The following sets of constraints ensure that the ordering cost is accounted only on the purchase of the product. First set of constraints

$$\left(\sum_{t=1}^{T} D_{jnt} \right) Y_{jt} - y_{jnt} \ge 0, \quad \forall j, \forall n, \forall t, \qquad (4.2.18)$$

$$\left(\sum_{t=1}^{T} D_{jnt} \right) YO_{jkt} - yo_{jnkt} \ge 0, \quad \forall j, \forall n, \forall k, \forall t. \qquad (4.2.19)$$

4.2.5 Decision-making problem of the supplier on pricing

The strategic problem of the supplier to identify prices for negotiation is summarized as following. Herein, the part of optimization problem directly controlled by supplier (leader) is identified as (LDMP) and the part of optimization problem considered on behalf of buyer (follower) is identified as (FDMP). The overall bilevel price setting problem (BPSP) is modelled as following.

To determine $\left\{ \{mp_{nj}\}, \{Q_{nt}\}, \{O_{nt}\}, \{SS_{int}\}, \{I_{int}\}, \{x_{ijnt}\} : i, j, n, t \right\}$ by solving (LDMP) with reaction of the buyer in terms of optimal solution of his decision-making problem, formulated as (FDMP), incorporated into the problem.

(BPSP)

(LDMP) $\max z_L$

$$= \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{j=1}^{J} mp_{nj}\, y_{jnt} - \sum_{t=1}^{T}\sum_{n=1}^{N}(a_{nt} Q_{nt} + b_{nt} O_{nt})$$

$$- \left(\sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=1}^{I} d_{int}\, SS_{int} + \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=1}^{I} tcp_{int}\, I_{int}\right.$$

$$\left. + \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=0}^{I}\sum_{j=1}^{J} tc_{ijnt}\, x_{ijnt}\right)$$

s.t. $\quad lp_{nj} \le mp_{nj} \le Lp_{nj}\ ,\ \forall n_k, \forall j,$

$$\sum_{n=1}^{N} r_{nt} Q_{nt} \le MR_t\ , \qquad \forall t,$$

$$\sum_{n=1}^{N} r_{nt}(Q_{nt} + O_{nt}) \le M_t\ , \qquad \forall t,$$

$$Q_{nt} + O_{nt} + \sum_{i=1}^{I} SS_{in(t-1)} - \sum_{i=1}^{I} SS_{int} = \sum_{j=1}^{J} y_{jnt}\ , \qquad \forall n, \forall t,$$

$$\sum_{n=1}^{N} v_n\big(I_{int} + SS_{in(t-1)}\big) \le V_{it}\ , \qquad \forall i, \forall t,$$

$$\sum_{i=0}^{I} x_{ijnt} \ge y_{jnt}\ , \qquad \forall j, \forall n, \forall t,$$

$$\sum_{j=1}^{J} x_{ijnt} \le I_{int} + SS_{in(t-1)} - SS_{int}\ , \qquad \forall i \ne 0, \forall n, \forall t,$$

$$\sum_{j=1}^{J} x_{0jnt} = Q_{nt} + O_{nt} - \sum_{i=1}^{I} I_{int}\ , \qquad i = 0, \forall n, \forall t,$$

$mp_{nj} \ge 0$, $Q_{nt}, O_{nt}, SS_{int}, I_{int}, x_{ijnt} \ge 0$ are integers, $\forall i, \forall j, \forall n, \forall t,$

where, $\{y_{jnt} : j, n, t\}$ are obtained by solving the following bi-objective programming problem in variables y_{jnt} , yo_{jnkt} , Y_{jt} , YO_{jkt} $\forall j, n, k, t$.

(FDMP) $\min z_{F_1}$

$$= \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{j=1}^{J}\left\{mp_{nj}\, y_{jnt} + \sum_{k=1}^{K}\{p_{njk}\, yo_{jnkt}\}\right\} + \sum_{t=1}^{T}\sum_{j=1}^{J} O_j\, Y_{jt}$$

$$+ \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K} O_{jk}\, YO_{jkt}$$

$$+ \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N} H_{nj}\left\{\sum_{t=1}^{t'}\left(y_{jnt} + \sum_{k=1}^{K} yo_{jnkt}\right) - \sum_{t=1}^{t'} D_{jnt}\right\}$$

$$\max z_{F_1} = \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{j=1}^{J}\left\{mq_n\, y_{jnt} + \sum_{k=1}^{K} q_{nk}\, yo_{jnkt}\right\}$$

s.t. $yo_{jnkt} \le Myo_{jnk}\,,\ \forall j, \forall n, \forall k, \forall t,$

$$\sum_{j=1}^{J} y_{jnt} \le Mxo_n, \qquad \forall n, \forall t,$$

$$\sum_{t=1}^{t'}\left(y_{jnt} + \sum_{k=1}^{K} yo_{jnkt}\right) \ge \sum_{t=1}^{t'} D_{jnt} \quad \forall j, \forall n, t' = 1, \dots, T-1,$$

$$\sum_{t=1}^{T}\left(y_{jnt} + \sum_{k=1}^{K} yo_{jnkt}\right) = \sum_{t=1}^{T} D_{jnt} \quad \forall j, \forall n,$$

$$\sum_{n=1}^{N} v_n\left\{\sum_{t=1}^{t'}\left(y_{jnt} + \sum_{k=1}^{K} yo_{jnkt}\right) - \sum_{t=1}^{t'} D_{jnt}\right\} \le VF_j\,, \forall j, \forall n, \forall t'$$

$$\sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{j=1}^{J}\left\{d_n y_{jnt} + \sum_{k=1}^{K}\{d_{nk}\, yo_{jnkt}\}\right\} \le \sum_{n=1}^{N} df_n \sum_{t=1}^{T}\sum_{j=1}^{J} D_{jnt}\ ,$$

$$\left(\sum_{t=1}^{T} D_{jnt}\right) Y_{jt} - y_{jnt} \ge 0, \qquad \forall j, \forall n, \forall t,$$

$$\left(\sum_{t=1}^{T} D_{jnt}\right) YO_{jkt} - yo_{jnkt} \geq 0, \qquad \forall j, \forall n, \forall k, \forall t,$$

$$y_{jnt}\,, yo_{jnkt} \geq 0, Y_{jt}\,, YO_{jkt} \in \{0, 1\},$$

$$y_{jnt}\,, yo_{jnkt} \text{ are integers.}$$

This mathematical model is formulated using BLP framework to address the issue of decision on identifying prices for the supplier to target during negotiation with the buyer. The formulated model involves integer variables and a bilinear objective functions at both levels with the follower's reaction problem as a bi-objective programming problem.

4.3 Solution methodology

As noted already, no algorithm is available in the literature to solve this class of BLP problem. Taking this opportunity, a GA based solution methodology for solving such a BLP problem is developed in our research work as presented below. As the discussed BLP problem involves a bi-objective programming problem at follower's level, herein we adopt the weighted sum approach after normalizing two objectives different scales.

4.3.1 Handling the follower's bi-objective problem using weighted sum approach

For any given prices of the supplier corresponding to variables $\{mp_{nj}\}$, the bi-objective follower's problem is solved using weighted sum approach. As in our problem, first objective of the follower's problem involves total cost and the second objective involves economic qualitative score, so before converting the problem into single objective using weighted sum method, it is requires to normalize these objectives first using the technique of Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) proposed by Lai and Hwang [237].

4.3.2 GA-based approach for solving problem of the supplier

For developing a methodology to solve the BLP problem (BPSP), mimicking the price-negotiation mechanism between the supplier and the buyer helps understanding the chronology of an interlinked decision-making at both the levels. A price offer of the supplier is responded by the buyer in terms of demand-order allocations as a solution to (FDMP). Consequently the supplier discerns the aggregate production-distribution plan to estimate the total cost of fulfilling these demand-orders. This indicates that variables representing prices of the supplier $\left(mp_{nj}\right)$ are the most basic and independent variables and values of rest of the variables depend on these only. Therefore, it is imperative to encode the price variables of the supplier as chromosomes in the GA.

Further, it is noteworthy that due to the currency systems followed across the globe, the prices of the products can be quoted up to two to three decimal places. Even, as all variables except mp_{nj} are integers, hence, without loss of generality, we propose to use an integer coded GA to solve (BPSP), with price variables encoded as integers. Also it further reduces the search space for variables mp_{nj} through an adoption of integer-coded GA. Therefore, this scheme is observed to be appropriateness due to noted practicality and computational efficiency.

Based on these practical implications and theoretical developments discussed by far, we propose a GA-based solution methodology which is summarized through pseudocode presented in Algorithm 4.1 and Algorithm 4.2. Other details of the GA are presented subsequently.

Chromosome encoding

As the very purpose of the problem is to obtain the prices which can give maximum profit to leader, the chromosomes of population are taken as an array of prices of various products for various delivery locations $\{mp_{nj} : n = 1, 2, \dots, N, j = 1, 2, \dots, J\}$. A general chromosome structure is shown in Figure 4.3. The number of genes in a chromosome is *NJ*. First *J* genes represent prices of product 1 for supplying at *J* locations, so on, the last *J* genes represent price of product *N* for supply at *J* locations. Each allele is an integer value representing respective prices.

Data: Input data and GA parameters

$g \leftarrow 0$;

Initialize population (The supplier's prices $\{mp_{nj} : n, j\}$);

Evaluate fitness of population members (using Algorithm 4.2);

while $g < G$ **do**

> tournament selection (retaining best-fit chromosome);
>
> generate new individuals through extended Laplace crossover and power mutation;
>
> evaluation fitness of population members (using Algorithm 4.2);
>
> update new population for next generation.
>
> $g \leftarrow g + 1$;
>
> select best-fit chromosome of the new generation (along with corresponding response y);

end

Return best-fit chromosome (along with corresponding response y) over all generations

Algorithm 4.1: Genetic Algorithm for solving (BPSP)

Input: GA population of chromosomes

for $i \leftarrow 1$ **to** $popsize$ **do**

> substitute the supplier's prices mp_{nj} (represented by chromosome) in (FDMP) to solve the follower's bi-objective mixed-integer linear programming problem using weighted-sum approach (detailed in Section 4.3.1) to obtain corresponding demand order vector y_{jnt} ;
>
> evaluate the fitness value as objective function value of (LDMP);

end

Output: Fitness values of all chromosomes of the population

Algorithm 4.2: Fitness evaluation of population members

mp_{11}	mp_{12}	...	mp_{1J}	mp_{21}	mp_{22}	...	mp_{2J}	...	...	mp_{N1}	mp_{N2}	...	mp_{NJ}
$j = 1$	$j = 2$	...	$j = J$	$j = 1$	$j = 2$	...	$j = J$	...	...	$j = 1$	$j = 2$	...	$j = J$
prices of product1 ($i = 1$) at various J locations				prices of product1 ($i = 2$) at various J locations				...		prices of product1 ($i = k$) at various J locations			

Figure 4.3: Chromosome structure

Initialization

The following GA parameters are used in the proposed algorithm: population size *popsize*; number of generations G; current generation g, $(g = 1, 2, \dots, G)$; crossover rate *pc*; mutation rate *pm*; location parameter a and scaling parameter $b > 0$ for Laplace crossover; index of power mutation p. The initial population size *popsize* is randomly generated for prices $\{mp_{nj} : n, j\}$ with integer values in the range of reservation prices $[lp_{nj}, Lp_{nj}]$.

Genetic operators

Crossover operator: We use a single point crossover namely the Laplace crossover [203] on each chromosome with the probability *pc*. The same is explained in Section 2.2.2.

Mutation operator: The mutation is performed on a chromosome with the probability *pm* using the power mutation operator [203]. The same is explained in Section 2.2.2.

We note that Laplace crossover and the power mutation operators do not disturb feasibility of chromosomes in terms of reservation price bounds in problem (LDMP).

Selection

A tournament selection mechanism with tournament size 2, is adopted for selecting better fitness chromosomes for reproduction phase. The best-fit (elitist) chromosome is, although, preserved.

Incorporating follower's reaction and fitness evaluation

The fitness of each chromosome is measured as the corresponding objective value z_L of (LDMP). For this, a chromosome is supplied to (FDMP) which is solved by the above-mentioned procedure using the weighted sum approach. The optimal demand (*i.e.*, optimal solution of (FDMP)) received from it is supplied to (LDMP). Then (LDMP) is solved to generate the optimal aggregate-production-distribution plan. The objective function value z_L of (LDMP) is thereby computed and is taken as the fitness value of the chromosome considered.

Updating the new population

The new population obtained from the parent population P^g is adopted to be a population of the next generation P^{g+1} only if it's maximum fitness value, in comparison to the maximum fitness value of the previous generation, does not decrease. Otherwise, the population P^g is preserved as population P^{g+1} for regenerating the next generation.

Termination criterion

The execution of the algorithm is terminated after the completion of pre-defined maximum number of generations G. The value of G may be tuned by observing stability in the fitness value through various combinations of GA parameters.

4.4 An experimental study on a relevant case of a small-scale supplier

In this section, the effectiveness of the proposed model and its solution algorithm are demonstrated through a test instance generated in consultation with some industry experts. Also a comparison analysis between the proposed approach and the low price strategy is shown to exhibit the superiority of the former. The formulation of our model is inspired by a scenario of a small-scale supplier.

4.4.1 Relevant information about the market ecosystem

A scenario of a supplier firm is considered which produce 5 types of products at their PC and is enter into a negotiation on prices with a potential buyer requiring the delivery of these products at 10 DLs. There are 3 other existing suppliers who also supply these 5 products to the buyer. The planning horizon of 4 months is considered for month-wise planning.

4.4.2 Data inputs

The data input including various costs, prices, demand and other parameters is given in Tables 4.1 to 4.8. The initial period available inventory and the pre-decided final period inventory levels are not listed explicitly but are included in Table 4.13. Let the regular time machine hours and a total machine hours including overtime available for production be 700 hours and 1050 hours, respectively. Also, the available storage space at DC 1 and DC 2 are 15000 cu-ft and 7500 cu-ft, respectively. The maximum acceptable percentage of defects by the buyer for the 5 products be 4, 2.5, 6, 2 and 2.7 respectively. Fixed cost of ordering products for each location from the supplier is Rs. 2700 and from each of existing suppliers (indexed by $k = 1, 2, 3$) is Rs. 2000, 3000 and 4000, respectively.

period (t)	product (n)	delivery location (j)									
		1	2	3	4	5	6	7	8	9	10
1	1	8000	9000	5000	5000	3000	8000	8500	4500	5000	2500
	2	5000	1000	2000	7000	1000	4500	1500	2500	6500	1500
	3	6000	6800	3800	3800	2300	6000	6400	3400	3800	1900
	4	800	1000	400	400	200	900	1000	500	600	1600
	5	4500	900	1800	6300	900	4100	1400	2300	5900	1400
2	1	4000	5000	2000	2000	1000	4500	5000	2500	3000	8000
	2	1500	1500	1000	3000	800	2000	2000	1500	3500	1200
	3	3600	720	1440	5040	720	3280	1120	1840	4720	1120
	4	1000	1100	600	600	400	1000	1100	600	600	1300
	5	5200	5900	3300	3300	2000	5200	5600	3000	3300	1700
3	1	5000	5500	2500	3000	1600	5500	6000	2500	3500	2400
	2	7000	2500	1000	6000	2500	6500	2000	1500	6500	2000
	3	3200	600	1300	4500	600	3000	1000	1700	4200	1000
	4	1100	1000	700	500	400	900	1200	500	700	1200
	5	5700	5300	3600	3000	2200	4700	6200	2700	3600	1500
4	1	9000	10000	4000	4000	2500	8500	9000	3500	3000	3500
	2	5000	1700	3000	8000	3000	5500	1500	2500	7500	5000
	3	5400	6100	3400	3400	2100	5400	5800	3100	3400	1700
	4	700	900	400	400	200	800	900	500	500	1400
	5	4100	800	1600	5700	800	3700	1300	2100	5300	1300

Table 4.1: Total demand of the buyer at various DLs

production costs	regular time (a_{nt}) product (n)					overtime (b_{nt}) product (n)				
Period (t)	1	2	3	4	5	1	2	3	4	5
1	10	20	15	25	18	15	25	20	30	24
2	11	21	13	26	19	13	26	18	31	25
3	10.5	21	12.5	26	18	12.5	26	17	31	24
4	10	20.5	17	27	17	17	26.5	22	33	23

Table 4.2: Production costs (INR/unit) at PC

tcp_{int}	DC 1 product (n)					DC 2 product (n)				
period (t)	1	2	3	4	5	1	2	3	4	5
1	0.2	0.4	0.3	0.5	0.35	0.42	0.84	0.63	1.05	0.74
2	0.2	0.4	0.3	0.5	0.35	0.42	0.84	0.63	1.05	0.74
3	0.2	0.4	0.3	0.5	0.35	0.42	0.84	0.63	1.05	0.74
4	0.2	0.4	0.3	0.5	0.35	0.42	0.84	0.63	1.05	0.74
d_{int} ($\forall\ t = 1, 2, 3, 4$)	0.1	0.19	0.15	0.2	0.18	0.2	0.38	0.3	0.4	0.36

Table 4.3: Transportation costs (INR/unit) from PC to DCs and inventory costs (INR/unit) at DCs

	product (n)	deliver location (j)									
		1	2	3	4	5	6	7	8	9	10
PC (i = 0)	1	2	2.5	3	3.3	1	2	2.5	3	3.3	1
	2	3	3	3	3.3	1	3	3	3	3.3	1
	3	2.5	3	3.1	3.1	1	2.5	3	3.1	3.1	1
	4	4	5	5	5.2	2.5	4	5	5	5.2	2.5
	5	2.8	2.9	3	3	2	2.8	2.9	3	3	2
DC 1 (i = 1)	1	1	2	5	3	3	1	2	5	3	3
	2	2	4	8	6	5	2	4	8	6	5
	3	1.5	2.5	5.5	3.5	3.5	1.5	2.5	5.5	3.5	3.5
	4	2.5	5	9	7	6	2.5	5	9	7	6
	5	1.8	3	6	4	4	1.8	3	6	4	4
DC 2 (i = 2)	1	3	1	1	4	5	3	1	1	4	5
	2	3.5	1.5	1.5	4.5	4.5	3.5	1.5	1.5	4.5	4.5
	3	4.5	2.5	2.5	5	5	4.5	2.5	2.5	5	5
	4	6	3	3.3	7	7	6	3	3.3	7	7
	5	4	2	2.2	4.8	4.8	4	2	2.2	4.8	4.8

Table 4.4: Transportation costs (INR/unit) to DLs (tc_{ijnt}) (same for each period t)

reservation prices	product (n)	delivery location (j)									
		1	2	3	4	5	6	7	8	9	10
lp_{nj}	1	231	220	230	240	245	230	235	238	240	245
	2	382	360	380	400	410	380	390	396	400	410
	3	347	348	363	378	386	363	371	375	378	386
	4	428	400	425	450	463	425	438	445	450	463
	5	316	296	314	332	341	314	323	329	332	341
Lp_{nj}	1	531	520	530	540	545	530	535	538	540	545
	2	682	660	680	700	710	680	690	696	700	710
	3	647	648	663	678	686	663	671	675	678	686
	4	728	700	725	750	763	725	738	745	750	763
	5	616	596	614	632	641	614	623	629	632	641

Table 4.5: Reservation prices (INR/unit) of products at various DLs for the supplier

	Product (n)				
	1	2	3	4	5
v_n	0.16	0.32	0.2	0.35	0.3
Mxo_n	10,000	5000	6600	4000	5500
r_{nt} ($\forall\, t = 1, 2, \ldots, 4$)	0.02	0.04	0.03	0.05	0.035
H_{nj} ($\forall\, j = 1, 2, \ldots, 10$)	40	80	60	90	70

Table 4.6: Volume/unit of product, maximum purchase volumes from the supplier, machine hours required per unit of product, and holding costs/unit incurred to the buyer

Product (n)	(d_n, mq_n)	(d_{nk}, q_{nk})		
		$k = 1$	$k = 2$	$k = 3$
1	(2.57, 0.25)	(3.47, 0.28)	(3.15, 0.26)	(4.25, 0.21)
2	(1.75, 0.3)	(6.74, 0.21)	(6.11, 0.27)	(2, 0.23)
3	(2.1, 0.28)	(5.205, 0.23)	(4.725, 0.25)	(6.375, 0.21)
4	(1.5, 0.32)	(1.735, 0.3)	(1.575, 0.28)	(2.125, 0.26)
5	(1.95, 0.29)	(5.25, 0.25)	(5.01, 0.26)	(2.1, 0.24)

Table 4.7: Average number of defects (in %), economic qualitative scores of suppliers

competitor	product	Delivery location (j)									
(k)	(n)	1	2	3	4	5	6	7	8	9	10
	1	(338, 3000)	(357, 3000)	(330, 2500)	(344, 2000)	(357, 1000)	(338, 3000)	(357, 3000)	(330, 2500)	(344, 2000)	(357, 1000)
	2	(455, 3000)	(462, 1000)	(447, 1500)	(462, 2000)	(473, 1000)	(455, 3000)	(462, 1000)	(447, 1500)	(462, 2000)	(473, 1000)
1	3	(482, 4000)	(495, 4500)	(475, 2000)	(492, 2500)	(490, 1000)	(482, 4000)	(495, 4500)	(475, 2000)	(492, 2500)	(490, 1000)
	4	(510, 1000)	(523, 1200)	(502, 1100)	(515, 1000)	(520, 900)	(510, 1000)	(523, 1200)	(502, 1100)	(515, 1000)	(520, 900)
	5	(440, 5000)	(455, 5200)	(435, 4800)	(444, 4700)	(450, 4800)	(440, 5000)	(455, 5200)	(435, 4800)	(444, 4700)	(450, 4800)
	1	(335, 4000)	(331, 4500)	(357, 2000)	(344, 2500)	(357, 1000)	(335, 4000)	(331, 4500)	(357, 2000)	(344, 2500)	(357, 1000)
	2	(452, 2500)	(451, 1000)	(473, 1500)	(462, 3000)	(462, 1500)	(452, 2500)	(451, 1000)	(473, 1500)	(462, 3000)	(462, 1500)
2	3	(478, 3000)	(480, 3500)	(468, 1500)	(488, 2000)	(480, 1100)	(478, 3000)	(480, 3500)	(468, 1500)	(488, 2000)	(480, 1100)
	4	(505, 1100)	(518, 1200)	(498, 1300)	(510, 1000)	(510, 800)	(505, 1100)	(518, 1200)	(498, 1300)	(510, 1000)	(510, 800)
	5	(435, 4500)	(448, 5000)	(428, 5000)	(438, 5000)	(445, 4800)	(435, 4500)	(448, 5000)	(428, 5000)	(438, 5000)	(445, 4800)
	1	(330, 4000)	(331, 4500)	(357, 3000)	(344, 3000)	(357, 2000)	(330, 4000)	(331, 4500)	(357, 3000)	(344, 3000)	(357, 2000)
	2	(447, 1800)	(448, 1000)	(462, 1000)	(462, 3500)	(473, 2000)	(447, 1800)	(448, 1000)	(462, 1000)	(462, 3500)	(473, 2000)
3	3	(470, 2500)	(475, 3200)	(462, 1500)	(480, 1800)	(470, 1000)	(470, 2500)	(475, 3200)	(462, 1500)	(480, 1800)	(470, 1000)
	4	(490, 1000)	(515, 1300)	(495, 800)	(505, 900)	(510, 800)	(490, 1000)	(515, 1300)	(495, 800)	(505, 900)	(510, 800)
	5	(435, 4800)	(448, 4800)	(425, 4500)	(438, 4900)	(440, 5000)	(435, 4800)	(448, 4800)	(425, 4500)	(438, 4900)	(440, 5000)

Table 4.8: Competitor suppliers' prices (INR/unit), maximum supply quantities (p_{njk} , Myo_{njk})

4.4.3 Implementation of proposed GA-based approach

We solved the proposed bilevel programming problem (BPSP) applying the above input data. The program was coded in MATLAB 2014a. The (FDMP) problem is solved by attaching weights 0.5 and 0.5 with two objectives. We have also presented the results for an ordered weights combinations (0.6, 0.4), (0.4, 0.6), (0.7, 0.3), (0.3, 0.7), in the Appendix at the end of the paper. The population size *popsize* is tuned for various combinations of probabilities of crossover, mutation and tournament selection, parameters of Laplace crossover a, b and that of power mutation p. It is found to be best at 10. The maximum number of generations is set to 6000 and the parameter α is tuned to 0. For each combination of the parameters (Table 9), a set of 10 experiments of GA are performed. The relative error of the best solution obtained from each combination of parameters in comparison to the overall best solution obtained is tabulated in Table 4.9.

Figure 4.4, shows the variation of the maximum fitness attained at various generations for a GA run that yields the best fitness value of those reported in Table 9. The best result obtained with a specific set of GA parameters shows no improvement in its fitness value in the last 500 iterations establishing its stability.

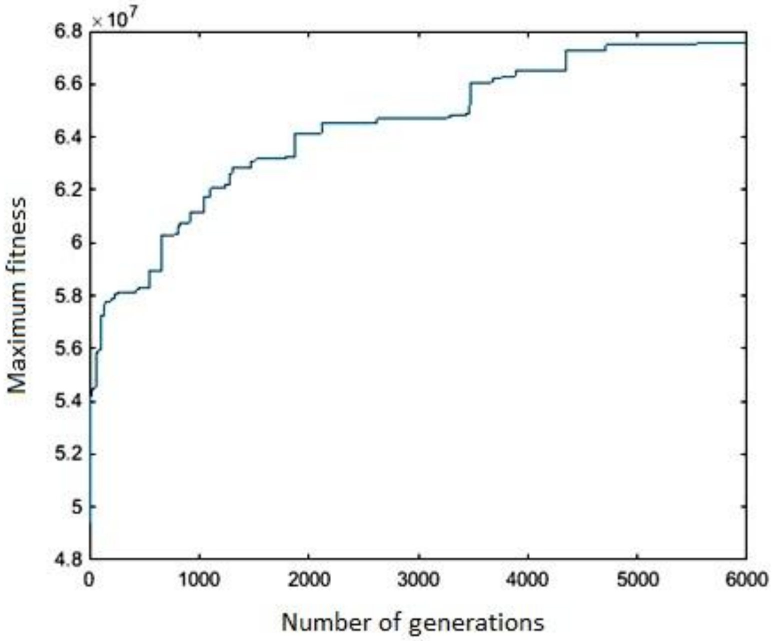

Figure 4.4: Maximum fitness vs. generations of best solution

pt	*pc*	*a*	*b*	*pm*	*p*	relative error (%)
0.95	0.9	0	50	0.05	10	0.071
0.95	0.8	0	50	0.05	10	0.095
0.95	0.6	0	50	0.05	10	0.05
0.95	0.9	0	50	0.01	10	0.067
0.95	0.8	0	50	0.01	10	0.083
0.95	0.6	0	50	0.01	10	0.109
0.95	0.9	0	50	0.01	0.1	0.06
0.95	0.9	0	50	0.01	1	0.089
0.95	0.9	0	50	0.05	1	0.047
0.8	0.9	0	50	0.05	10	0.061
0.8	0.8	0	50	0.05	10	0.047
0.8	0.6	0	50	0.05	10	0.069
0.8	0.9	0	50	0.01	10	0.047
0.8	0.8	0	50	0.01	10	0.086
0.8	0.6	0	50	0.01	10	0.063
0.8	**0.9**	**0**	**50**	**0.01**	**0.1**	**0**
0.8	0.9	0	50	0.01	100	0.133
0.8	0.9	0	500	0.01	0.1	0.091
0.8	0.9	0	5	0.01	100	0.109
0.8	0.9	0	500	0.01	100	0.126
0.8	0.9	0	500	0.01	10	0.081
0.8	0.9	0	5	0.01	10	0.091
0.8	0.9	0	5	0.01	0.1	0.015
0.8	0.9	0	1	0.01	0.1	0.029
0.8	0.9	0	5	0.01	0.01	0.088
0.8	0.9	0	1	0.01	0.01	0.054
0.8	0.9	0	50	0.01	0.01	0.03
0.8	0.9	0	50	0.01	1	0.024
0.8	0.9	0	50	0.05	1	0.063

Table 4.9: Results corresponding to different settings of the GA parameters

4.4.4 Results and analysis

Among the 290 optimal solutions generated (29 combinations with 10 run each) for problem (BPSP), the one yielding the best fitness value is finally picked and shown below in the results and analysis.

Using the developed model and designed solution methodology discussed by now, the best prices suggested for the supplier to target during negotiation are shown in Table 4.10

product	delivery location (j)									
(n)	1	2	3	4	5	6	7	8	9	10
1	385	356	403	540	293	526	535	538	461	545
2	600	360	380	557	710	599	690	696	616	561
3	647	437	662	678	686	663	579	675	678	686
4	728	700	604	575	763	425	738	745	643	762
5	616	596	415	632	341	610	623	329	629	641

Table 4.10: Prices suggested for the supplier (INR/unit)

Table 4.11 shows the demand orders assessed to be obtained by the supplier for different products in each period as a response to the price offer depicted in Table 4.10. Table 4.12 shows production volumes of products in various periods required to fulfil the replicated demand orders. Table 4.13 lists the transportation volumes from PC to the two DCs and the inventory volumes suggested to be maintained in the two DCs in periods $t = 1, 2, 3, 4$; row for period $t = 0$ shows the inventory volumes available at the beginning of the period $t = 1$ and the row for $t = 4$ shows the pre-decided inventory volumes to be maintained at final period. These two rows are the inputs to the problem. Table 4.14 shows the transport arrangements for the five products to be shipped from the DCs or PC to DLs in each period as a part of cost optimal aggregate production distribution plan. For example, in period $t = 1$, DL $j = 5$ is to be supplied with 1000 units of product 1 and 200 units of product 3, both from PC. Similarly, in period $t = 2$, 5700 units of product 1, 281 units of product 2, 720 units of product 3 and 3000 units of product 4 are planned to be supplied to DL $j = 10$, directly from PC ($i = 0$), and 4300 units of product 1 and 1000 units of product 4 are planned to be supplied to DL $j = 7$, from DC 2 ($i = 2$), and so on.

period (t)	Product (n)	delivery location (j)									
		1	2	3	4	5	6	7	8	9	10
1	1	1000	1500	500	500	1000	1000	1000	2500	500	500
	2	0	0	1000	2000	0	0	0	1500	500	0
	3	0	0	1900	0	200	0	0	1900	1800	800
	4	0	0	0	0	0	0	0	0	0	700
	5	0	0	0	4500	0	0	0	0	1000	0
2	1	0	0	0	0	0	0	4300	0	0	5700
	2	0	0	0	0	1800	0	0	1500	1419	281
	3	0	0	1140	2780	320	0	0	1640	0	720
	4	0	0	0	0	0	0	1000	0	0	3000
	5	0	0	783	4717	0	0	0	0	0	0
3	1	0	0	0	2500	0	0	7500	0	0	0
	2	0	0	0	1900	0	3100	0	0	0	0
	3	0	0	0	3360	0	620	0	0	2620	0
	4	1800	1900	0	0	0	0	300	0	0	0
	5	0	0	0	0	0	2583	2917	0	0	0
4	1	4000	0	0	0	0	5500	0	0	0	0
	2	3200	0	0	0	0	0	500	0	1300	0
	3	2700	0	0	500	0	0	0	0	3400	0
	4	0	0	0	0	0	0	0	0	0	0
	5	3517	800	0	0	0	0	1183	0	0	0

Table 4.11: Demand obtained by the supplier (y_{jnt})

period (t)	Q_{nt}					O_{nt}				
	product (n)					product (n)				
	1	2	3	4	5	1	2	3	4	5
1	12,800	2462	5100	0	5500	0	1538	0	5700	0
2	1175	7588	6600	0	5000	4525	0	0	3000	0
3	0	2537	14,700	0	4500	10,000	0	0	0	0
4	11,000	5875	0	0	7000	0	0	0	0	0

Table 4.12: Production volumes for the supplier

	period (t)	DC 1					DC 2				
		product (n)					product (n)				
		1	2	3	4	5	1	2	3	4	5
I_{int}	1	2000	0	0	1800	0	9300	1500	0	3200	0
	2	0	2588	0	0	0	0	1500	0	0	283
	3	2500	637	7720	0	1583	7500	0	1000	0	2917
	4	10,000	3075	0	0	4017	1000	1500	0	0	2983
SS_{int}	0	**1000**	**500**	**1000**	**500**	**1000**	**500**	**1000**	**500**	**1000**	**500**
	1	0	500	0	2300	1000	4300	0	0	4200	500
	2	0	3088	0	2300	1000	0	0	0	3200	0
	3	0	625	7100	500	0	0	0	1000	1000	0
	4	**500**	**500**	**500**	**500**	**500**	**1000**	**1000**	**1000**	**1000**	**1000**

Table 4.13: Transportation volumes from PC to DCs in each period and inventory volumes to be maintained at DCs in each period

source (i)	period (t)							
	1		2		3		4	
	(n, j)	x_{ijnt}	(n, j)	x_{ijnt}	(n, j)	x_{ijnt}	(n, j)	x_{ijnt}
PC (i = 0)	(1, 5)	1000	(1, 10)	5700	(2, 4)	1900	(2, 9)	1300
	(1, 10)	500	(2, 5)	1800	(3, 4)	3360	-	-
	(2, 4)	2000	(2, 9)	1419	(3, 9)	2620	-	-
	(2, 9)	500	(2, 10)	281	-	-	-	-
	(3, 3)	1400	(3, 3)	1140	-	-	-	-
	(3, 5)	200	(3, 4)	2780	-	-	-	-
	(3, 8)	1900	(3, 5)	320	-	-	-	-
	(3, 9)	800	(3, 8)	1640	-	-	-	-
	(3, 10)	800	(3, 10)	720	-	-	-	-
	(4, 10)	700	(4, 10)	3000	-	-	-	-
	(5, 4)	4500	(5, 4)	4717	-	-	-	-
	(3, 9)	1000	-	-	-	-	-	-
DC 1 (i = 1)	(1, 1)	1000	-	-	(1, 4)	2500	(1, 1)	4000
	(1, 4)	500	-	-	(2, 6)	3100	(1, 6)	5500
	(1, 6)	1000	-	-	(3, 6)	620	(2, 1)	3200
	(1, 9)	500	-	-	(4, 1)	1800	(3, 1)	2700
	(3, 9)	1000	-	-	(5, 6)	2583	(3, 4)	500
	-	-	-	-	-	-	(3, 9)	3400
	-	-	-	-	-	-	(5, 1)	3517
DC 2 (i = 2)	(1, 2)	1500	(1, 7)	4300	(1, 7)	7500	(2, 7)	500
	(1, 3)	500	(2, 8)	1500	(4, 2)	1900	(5, 2)	800
	(1, 7)	1000	(4, 7)	1000	(4, 7)	300	(5, 7)	1183
	(1, 8)	2500	(5, 3)	783	(5, 7)	2917	-	-
	(2, 3)	1000	-	-	-	-	-	-
	(2, 8)	1500	-	-	-	-	-	-
	(3, 3)	500	-	-	-	-	-	-

Table 4.14: Transportation volumes (x_{ijnt}) for DLs, (n, j) = (product, DL), transportation volumes not mentioned are 0

The demand orders obtained by other existing suppliers (for comparison) are shown in Tables 4.15 to Table 4.17.

period	Product	delivery location (j)									
(t)	(n)	1	2	3	4	5	6	7	8	9	10
1	1	3000	3000	2500	2000	1000	3000	3000	0	2000	1000
	2	3000	0	0	2000	0	3000	500	0	2000	0
	3	3000	3300	400	1800	1000	3000	2900	0	0	0
	4	800	1000	400	400	200	900	1000	500	600	900
	5	0	0	0	0	0	0	0	0	0	0
2	1	3000	3000	2500	2000	1000	3000	3000	2500	2000	1000
	2	3000	1000	1500	2000	1000	3000	1000	1500	2000	1000
	3	4000	4220	2000	2500	1000	4000	4500	2000	2500	1000
	4	1000	1100	1100	1000	900	1000	1200	1100	1000	900
	5	5000	1400	0	0	0	5000	0	0	0	0
3	1	3000	0	1000	2000	1000	3000	0	1000	0	0
	2	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0
4	1	0	3000	0	0	0	3000	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0

Table 4.15: Demand-orders of existing supplier $k = 1$

period	Product	delivery location (j)									
(t)	(n)	1	2	3	4	5	6	7	8	9	10
1	1	4000	4500	2000	2500	1000	4000	4500	2000	2500	1000
	2	2000	1000	0	3000	0	1500	1000	0	3000	0
	3	3000	3500	1500	2000	1100	3000	3500	1500	2000	1100
	4	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0
2	1	4000	4500	2000	2500	1000	0	4500	2000	2000	1000
	2	2500	1000	1500	3000	1500	2500	1000	1500	3000	1500
	3	3000	0	1500	2000	1100	3000	220	1500	2000	1100
	4	0	0	600	0	100	1100	700	500	800	0
	5	4500	5000	0	5000	0	4500	5000	0	5000	0
3	1	0	4500	0	0	100	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0
4	1	0	1000	0	0	0	0	0	0	2500	0
	2	0	700	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	1560	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0

Table 4.16: Demand-orders of existing supplier $k = 2$

period	product	delivery location (j)									
(t)	(n)	1	2	3	4	5	6	7	8	9	10
	1	0	0	0	0	0	0	0	0	0	0
	2	0	0	1000	0	1000	0	0	1000	1000	1500
1	3	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0
	5	4500	900	1800	1800	900	4100	1400	2300	4900	1400
	1	4000	4500	3000	0	2000	4000	700	3000	3000	2000
	2	1800	1000	1000	3500	2000	1800	1000	1000	3500	2000
2	3	2500	3200	1500	1800	1000	2500	3200	1500	1800	1000
	4	0	0	0	500	0	600	0	0	0	0
	5	780	4800	4500	373	3433	1517	4000	4500	4900	4000
	1	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0
	2	1200	1000	0	3500	0	1800	1000	0	2781	819
	3	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0
	5	1203	0	1617	0	767	0	0	1200	0	500
4	1	0	0	0	0	0	0	0	0	0	0
	2	1800	1000	1000	3100	0	1800	1000	0	3500	2000
	3	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0

Table 4.17: Demand-orders of existing supplier $k = 3$

Figure 4.5 shows the demand shares of each product received by the supplier against those to be received by other existing suppliers in the planning horizon.

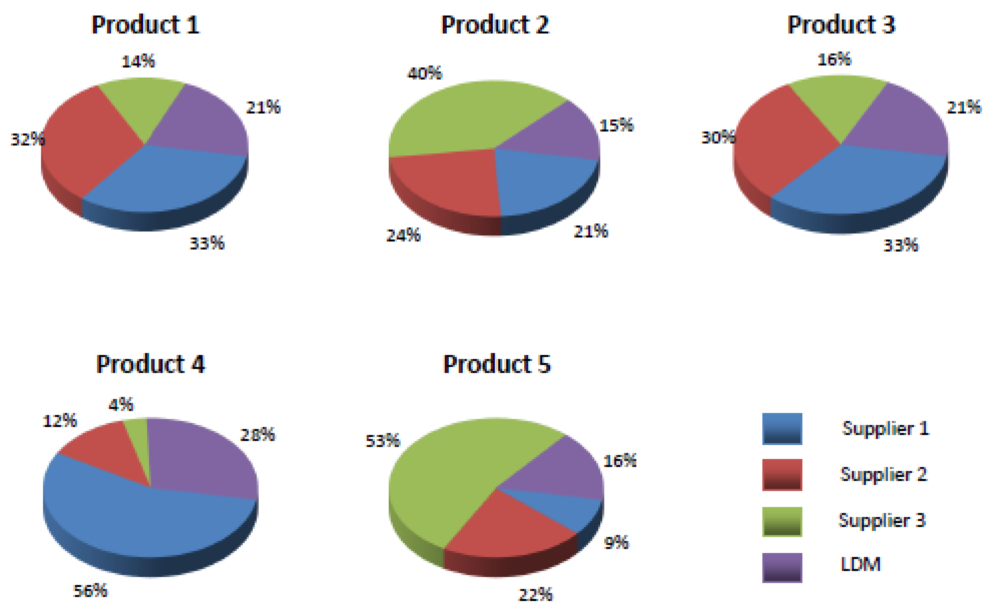

Figure 4.5: Comparison of demand shares of the supplier and existing suppliers

4.5 Comparison analysis

The efficacy of the proposed approach for taking a strategic decision on target prices for negotiation is highlighted in this section through a comparison of the results obtained in the previous section with two more experiments.

First, we attempt to anticipate the demand orders from the buyer in response to the lowest prices (minimum reservation prices lp_{nj} of the supplier). The demand shares are obtained by solving (FDMP) with the supplier's prices set at $mp_{nj} = lp_{nj}$. When we endeavour to solve the subsequent APDP problem, it is realized that the demand orders are out of the capacity of the supplier to fulfil. This confirms that adopting the strategy of targeting for lowest possible prices in pursuit of penetration into the market is not always effective and may even result in an opportunity loss by falling into an incapacitated situation to fulfil the replicating demand-orders allocations.

Second, we perform the experimental analysis as demonstrated in the previous section for each case of variation in weights of two objectives in (FDMP) as (0.4, 0.6), (0.6, 0.4), (0.3, 0.7) and (0.7, 0.3). The optimal prices thus opined by using the developed decision support for the supplier and corresponding demand-order allocations to be replicated by the buyer are shown in Tables 4.18 to Table 4.25. Also, demand shares corresponding to these weights are depicted in Figure 4.6 and Figure 4.7, exhibiting a significant market share to be received for each case of weights of two objectives of the buyer.

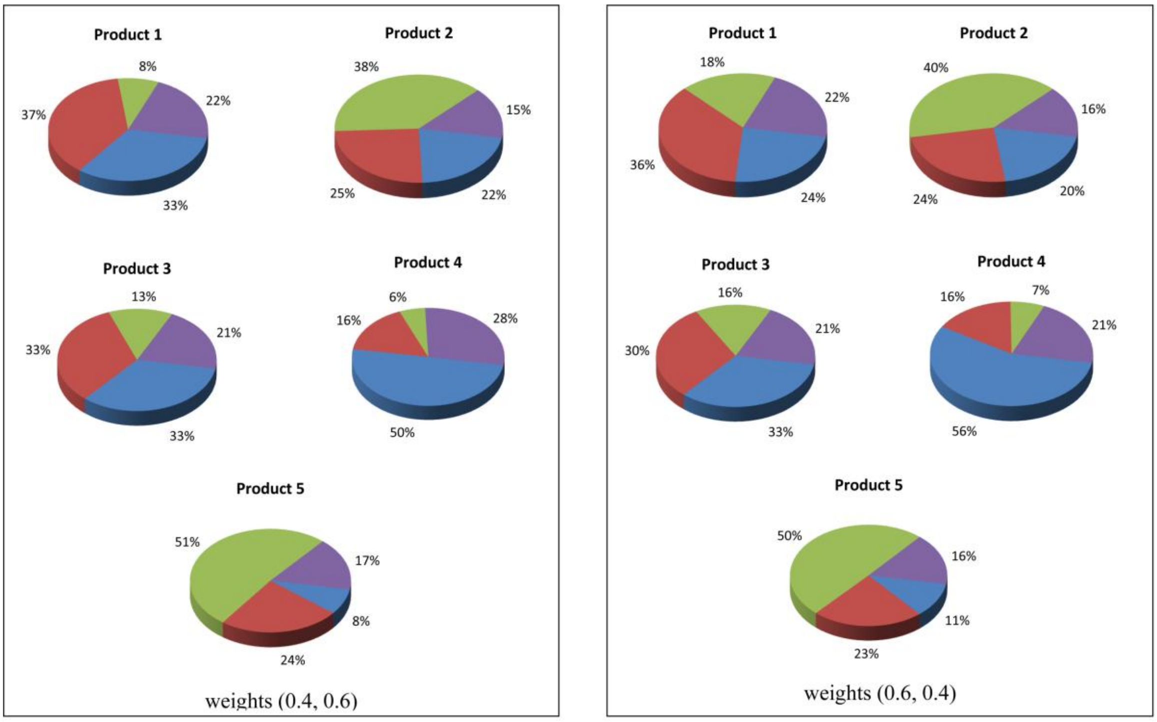

Figure 4.6: Comparison of demand-shares obtained by the supplier and existing suppliers for weights (0.4, 0.6) and (0.6, 0.4)

product	delivery location (j)									
(n)	1	2	3	4	5	6	7	8	9	10
1	501	491	408	429	500	518	252	536	493	398
2	541	467	635	557	611	513	466	522	686	621
3	643	466	661	541	476	588	670	490	678	681
4	606	616	714	621	717	665	733	700	710	705
5	444	563	553	622	516	407	407	446	468	626

Table 4.18: Prices suggested for the supplier (INR/unit) for weights (0.4, 0.6)

product	delivery location (j)									
(n)	1	2	3	4	5	6	7	8	9	10
1	372	501	470	394	545	377	476	461	364	545
2	641	392	680	700	710	664	465	696	498	710
3	647	648	663	678	423	663	371	663	678	686
4	669	700	654	750	463	668	738	670	750	763
5	616	296	314	632	341	470	399	629	579	641

Table 4.19: Prices suggested for the supplier (INR/unit) for weights (0.6, 0.4)

product	delivery location (j)									
(n)	1	2	3	4	5	6	7	8	9	10
1	262	459	286	490	534	523	480	362	521	545
2	679	658	672	691	641	658	466	668	444	546
3	446	374	367	569	681	599	390	675	588	672
4	605	487	673	588	463	682	438	743	624	726
5	536	305	533	625	351	441	623	489	625	475

Table 4.20: Prices suggested for the supplier (INR/unit) for weights (0.3, 0.7)

product	delivery location (j)									
(n)	1	2	3	4	5	6	7	8	9	10
1	337	520	430	540	323	497	451	440	357	543
2	622	660	385	400	463	534	624	601	657	570
3	645	531	663	650	597	555	387	628	652	446
4	496	700	496	459	562	725	728	445	529	739
5	600	298	532	471	617	556	345	556	559	620

Table 4.21: Prices suggested for the supplier (INR/unit) for weights (0.7, 0.3)

	product (n)	delivery location (j)									
		1	2	3	4	5	6	7	8	9	10
	1	(1000, 7000)	(1500, 7500)	(500, 4500)	(2500, 2500)	(1000, 2000)	(1000, 7000)	(0, 8500)	(1500, 3000)	(500, 4500)	(500, 2000)
	2	(0, 5000)	(0, 1000)	(1000, 1000)	(0, 7000)	(0, 1000)	(1000, 3500)	(0, 1500)	(1500, 1000)	(0, 6500)	(1500, 0)
$t = 1$	3	(3000, 3000)	(0, 6800)	(300, 3500)	(300, 3500)	(1200, 1100)	(0, 6000)	(0, 6400)	(0, 3400)	(1800, 2000)	(0, 1900)
	4	(0, 800)	(0, 1000)	(0, 400)	(0, 400)	(0, 200)	(0, 900)	(0, 1000)	(0, 500)	(0, 600)	(700, 900)
	5	(0, 4500)	(0, 900)	(0, 1800)	(4500, 1800)	(0, 900)	(0, 4100)	(0, 1400)	(0, 2300)	(1000, 4900)	(0, 1400)
	1	(0, 7000)	(0, 12,000)	(0, 7500)	(3831, 4500)	(0, 2000)	(0, 7000)	(0, 11,000)	(0, 4500)	(0, 4500)	(6169, 2000)
	2	(0, 7300)	(0, 3000)	(0, 4000)	(0, 8500)	(1641, 4500)	(0, 7300)	(259, 3000)	(0, 4000)	(0, 8500)	(3100, 4500)
$t = 2$	3	(0, 9500)	(0, 7420)	(1140, 5000)	(0, 6300)	(0, 3100)	(0, 9500)	(0, 7920)	(1490, 5000)	(3250, 6300)	(720, 3100)
	4	(0, 2297)	(0, 2823)	(0, 717)	(0, 1500)	(0, 900)	(0, 1000)	(0, 2663)	(500, 1100)	(800, 1000)	(2700, 1200)
	5	(0, 10,900)	(0, 9190)	(5500, 930)	(0, 9900)	(0, 5000)	(0, 13,000)	(0, 9800)	(0, 7800)	(0, 10,580)	(0, 1810)
	1	(0, 3000)	(2900, 1100)	(0, 0)	(0, 0)	(2100, 1000)	(500, 3000)	(0, 4500)	(4000, 0)	(500, 2000)	(0, 0)
	2	(2600, 1800)	(0, 2000)	(0, 0)	(2400, 2600)	(0, 0)	(0, 4300)	(0, 2000)	(0, 500)	(0, 1500)	(0, 0)
$t = 3$	3	(0, 0)	(0, 0)	(0, 0)	(3830, 1090)	(0, 320)	(0, 0)	(0, 0)	(0, 150)	(2770, 0)	(0, 0)
	4	(503, 0)	(177, 0)	(983, 0)	(0, 0)	(100, 0)	(1700, 0)	(537, 0)	(0, 0)	(0, 0)	(0, 0)
	5	(0, 0)	(2810, 0)	(0, 470)	(0, 0)	(0, 0)	(0, 0)	(0, 2000)	(0, 0)	(0, 0)	(2690, 0)
	1	(5000, 3000)	(0, 4500)	(0, 1000)	(0, 669)	(0, 0)	(5000, 3000)	(0, 4500)	(0, 0)	(0, 2500)	(0, 1531)
	2	(0, 1800)	(0, 700)	(0, 1000)	(0, 3500)	(0, 159)	(1000, 1400)	(0, 241)	(0, 1000)	(4000, 3500)	(0, 0)
$t = 4$	3	(2700, 1800)	(0, 700)	(0, 1000)	(1720, 3500)	(0, 159)	(2180, 1400)	(0, 241)	(0, 1000)	(0, 3500)	(0, 0)
	4	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
	5	(4100, 0)	(0, 0)	(1400, 200)	(0, 2100)	(0, 0)	(0, 600)	(0, 1300)	(0, 0)	(0, 1620)	(0, 0)

Table 4.22: Comparison of demand orders to be obtained by the supplier against the total to be obtained by existing suppliers for weights (0.4, 0.6) $\left(y_{jnt}, \sum_{k=1}^{K} yo_{jnkt}\right)$

	product (n)	delivery location (j)									
		1	2	3	4	5	6	7	8	9	10
	1	(4000,4000)	(0,9000)	(500,4500)	(500,4500)	(2000, 1000)	(1000,7000)	(1000,7500)	(0,4500)	(500,4500)	(500,2000)
	2	(3200,1800)	(0,1000)	(1000,1000)	(0,7000)	(0,1000)	(0,4500)	(0,1500)	(800,1700)	(0,6500)	(0,1500)
t = 1	3	(3000,3000)	(300,6500)	(300,3500)	(0,3800)	(1200, 1100)	(0,6000)	(0,6400)	(0,3400)	(1800, 2000)	(0,1900)
	4	(0,800)	(0,1000)	(0,400)	(0,400)	(0,200)	(0,900)	(0,1000)	(0,500)	(0,600)	(700,900)
	5	(0,4500)	(0,900)	(0,1800)	(4500,1800)	(0,900)	(0,4100)	(0,1400)	(0,2300)	(1000, 4900)	(0,1400)
	1	(0,11,000)	(0,10,500)	(0,6300)	(0,7500)	(3100, 2000)	(0,10,500)	(1200,12,000)	(0,7500)	(0,7500)	(5700,4000)
	2	(0,7300)	(0,3000)	(0,4000)	(5000,8500)	(0,4300)	(0,7300)	(0,3000)	(0,4000)	(0,8500)	(0,4500)
t = 2	3	(0,9500)	(0,7420)	(1140,5000)	(0,6300)	(320,3100)	(620,9500)	(0,7920)	(1640, 5000)	(2160, 6300)	(720,3100)
	4	(0,1669)	(320,2680)	(0,1700)	(0,1500)	(0,1000)	(0,2046)	(680,2500)	(0,1600)	(800,1000)	(2200,1700)
	5	(0,9500)	(0,9783)	(0,5923)	(0,5010)	(0,4980)	(0,9500)	(0,8300)	(1600, 4500)	(0,9707)	(3900,400)
	1	(0,3000)	(0,0)	(2200,0)	(0,0)	(0,0)	(0,4000)	(6800,0)	(1000,0)	(0,2000)	(0,0)
	2	(0,1800)	(0,2000)	(0,0)	(0,3500)	(0,0)	(1400,1800)	(500,1000)	(0,500)	(0,3500)	(3100,0)
t = 3	3	(0,0)	(0,0)	(0,0)	(6140,0)	(0,0)	(0,1560)	(0,0)	(0,0)	(460,0)	(0,0)
	4	(1131,0)	(0,0)	(0,0)	(0,0)	(0,0)	(654,0)	(0, 20)	(0,0)	(0,0)	(0,0)
	5	(5500,0)	(0,1417)	(0,977)	(0,1290)	(0,0)	(0,4100)	(0,4800)	(0,0)	(0,0)	(0,0)
	1	(0,4000)	(10,000,0)	(0,0)	(0,1500)	(0,0)	(0,4000)	(0,0)	(0,0)	(0,0)	(0,0)
	2	(3000,1400)	(0,700)	(0,1000)	(0,0)	(0,2000)	(0,3500)	(0,1000)	(0,1000)	(2000, 3500)	(0,0)
t = 4	3	(2700,0)	(0,0)	(0,0)	(500,0)	(0,0)	(0,0)	(0,0)	(0,0)	(3400,0)	(0,0)
	4	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
	5	(0,0)	(0,800)	(0,1600)	(5500,200)	(0, 20)	(0,0)	(0,0)	(0,1700)	(0,2493)	(0,200)

Table 4.23: Comparison of demand orders to be obtained by the supplier against the total to be obtained by existing suppliers for weights (0.6, 0.4) $\left(y_{jnt}, \sum_{k=1}^{K} yo_{jnkt}\right)$

	product (n)	delivery location (j) 1	2	3	4	5	6	7	8	9	10
t = 1	1	(1000, 7000)	(1500, 7500)	(500, 4500)	(500, 4500)	(1000, 2000)	(2500, 5500)	(1000, 7500)	(0, 4500)	(500, 4500)	(1500, 1000)
	2	(800, 4200)	(0, 1000)	(1000, 1000)	(500, 6500)	(1000, 0)	(200, 4300)	(0, 1500)	(1500, 1000)	(0, 6500)	(0, 1500)
	3	(0, 6000)	(0, 6800)	(300, 3500)	(400, 3400)	(200, 2100)	(3000, 3000)	(0, 6400)	(1900, 1500)	(0, 3800)	(800, 1100)
	4	(0, 800)	(0, 1000)	(0, 400)	(0, 400)	(0, 200)	(0, 900)	(0, 1000)	(0, 500)	(0, 600)	(700, 900)
	5	(0, 4500)	(0, 900)	(0, 1800)	(1400, 4900)	(0, 900)	(0, 4100)	(0, 1400)	(0, 2300)	(4100, 1800)	(0, 1400)
t = 2	1	(0, 7000)	(0, 7500)	(0, 4500)	(0, 4500)	(0, 1600)	(0, 7000)	(1300, 7500)	(0, 2500)	(0, 4500)	(6169, 2000)
	2	(0, 7300)	(0, 3000)	(0, 4000)	(312, 8500)	(1731, 4500)	(291, 7300)	(0, 3000)	(1500, 4000)	(0, 8500)	(3100, 4500)
	3	(1620, 9500)	(0, 7420)	(0, 5000)	(0, 6300)	(0, 3100)	(0, 9500)	(0, 7920)	(1640, 5000)	(3250, 6300)	(720, 3100)
	4	(471, 2100)	(229, 2400)	(0, 1300)	(500, 1000)	(0, 1000)	(600, 2100)	(0, 3200)	(0, 1600)	(0, 1800)	(2200, 1700)
	5	(0, 9500)	(0, 11,817)	(0, 8110)	(2010, 9600)	(0, 5000)	(0, 10,707)	(823, 9800)	(2667, 4500)	(0, 9400)	(0, 4123)
t = 3	1	(0, 7000)	(6500, 0)	(0, 2000)	(0, 2500)	(3500, 0)	(0, 4000)	(0, 6700)	(0, 4000)	(0, 2000)	(0, 1000)
	2	(4088, 0)	(0, 1000)	(0, 0)	(0, 3000)	(0, 0)	(0, 3909)	(0, 1500)	(0, 0)	(0, 6000)	(912, 1022)
	3	(660, 0)	(0, 0)	(0, 1140)	(5620, 0)	(320, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
	4	(0, 229)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
	5	(5110, 0)	(0, 183)	(0, 0)	(390, 0)	(0, 0)	(0, 2893)	(0, 1177)	(0, 0)	(0, 0)	(0, 0)
t = 4	1	(0, 4000)	(6500, 0)	(0, 2000)	(0, 2000)	(0, 0)	(3500, 4000)	(0, 4500)	(0, 2000)	(0, 0)	(0, 1000)
	2	(2112, 0)	(700, 1000)	(0, 1000)	(2188, 3000)	(0, 69)	(0, 2500)	(0, 1000)	(0, 0)	(0, 3000)	(0, 0)
	3	(0, 420)	(0, 0)	(0, 0)	(1020, 0)	(0, 0)	(2180, 0)	(0, 0)	(0, 0)	(3400, 0)	(0, 0)
	4	(0, 0)	(0, 371)	(0, 400)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
	5	(390, 0)	(0, 0)	(0, 390)	(0, 0)	(0, 0)	(0, 0)	(1300, 0)	(633, 0)	(2800, 0)	(377, 0)

Table 4.24: Comparison of demand orders to be obtained by the supplier against the total to be obtained by existing suppliers for weights (0.3, 0.7) $\left(y_{jnt}, \sum_{k=1}^{K} yo_{jnkt}\right)$

	product (n)	delivery location (j)									
		1	2	3	4	5	6	7	8	9	10
t = 1	1	(1000, 7000)	(1500, 7500)	(500, 4500)	(500, 4500)	(1000, 2000)	(1000, 7000)	(3500, 5000)	(0, 4500)	(500, 4500)	(500, 2000)
	2	(500, 4500)	(0, 1000)	(0, 2000)	(0, 7000)	(0, 1000)	(0, 4500)	(0, 1500)	(1000, 1500)	(3500, 3000)	(0, 1500)
	3	(0, 6000)	(3300, 3500)	(300, 3500)	(1800, 2000)	(200, 2100)	(0, 6000)	(1000, 5400)	(0, 3400)	(0, 3800)	(0, 1900)
	4	(0, 800)	(1000, 0)	(0, 400)	(0, 400)	(0, 200)	(900, 0)	(500, 500)	(0, 500)	(0, 600)	(1600, 0)
	5	(0, 4500)	(0, 900)	(0, 1800)	(1400, 4900)	(0, 900)	(3100, 1000)	(0, 1400)	(0, 2300)	(1000, 4900)	(0, 1400)
t = 2	1	(0, 8000)	(3000, 7500)	(0, 5500)	(800, 7500)	(0, 2000)	(500, 11000)	(0, 12000)	(0, 5500)	(0, 7500)	(5700, 4000)
	2	(1400, 5897)	(700, 3000)	(0, 1984)	(0, 8500)	(1800, 4500)	(0, 6000)	(0, 3000)	(0, 3406)	(0, 8500)	(1100, 4500)
	3	(0, 9500)	(0, 7420)	(1140, 5000)	(325, 6300)	(0, 3100)	(0, 9500)	(0, 7920)	(1640, 5000)	(2775, 6300)	(720, 3100)
	4	(0, 2800)	(0, 2003)	(0, 1700)	(0, 1500)	(0, 1000)	(0, 2700)	(900, 1200)	(0, 1600)	(0, 1454)	(3100, 800)
	5	(283, 9500)	(0, 10200)	(0, 8500)	(0, 9700)	(4713, 0)	(503, 9500)	(0, 10,200)	(0, 7800)	(0, 9700)	(0, 3127)
t = 3	1	(0, 7000)	(10,000, 0)	(0, 0)	(0, 0)	(0, 3000)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
	2	(1203, 2500)	(0, 1000)	(0, 3000)	(0, 5500)	(0, 0)	(0, 2500)	(500, 1000)	(0, 2094)	(3297, 703)	(0, 1000)
	3	(0, 0)	(0, 0)	(0, 0)	(2915, 0)	(320, 0)	(2180, 0)	(0, 0)	(0, 0)	(1185, 0)	(0, 0)
	4	(0, 0)	(997, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(1100, 0)	(0, 0)	(0, 0)	(0, 0)
	5	(1617, 0)	(0, 1800)	(0, 0)	(0, 2300)	(287, 0)	(3597, 0)	(0, 2900)	(0, 0)	(0, 600)	(0, 1373)
t = 4	1	(0, 3000)	(0, 0)	(0, 3000)	(0, 700)	(0, 100)	(0, 7000)	(8000, 0)	(0, 3000)	(2000, 0)	(0, 0)
	2	(0, 2500)	(0, 1000)	(0, 16)	(0, 3000)	(0, 0)	(0, 5500)	(0, 1000)	(0, 0)	(5000, 0)	(0, 1000)
	3	(2700, 0)	(0, 0)	(0, 0)	(1840, 1560)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(2060, 0)	(0, 0)
	4	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(346, 0)	(0, 0)
	5	(3600, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(1900, 0)	(0, 0)

Table 4.25: Comparison of demand orders to be obtained by the supplier against the total to be obtained by existing suppliers for weights (0.7, 0.3) $\left(y_{jnt}, \sum_{k=1}^{K} yo_{jnkt}\right)$

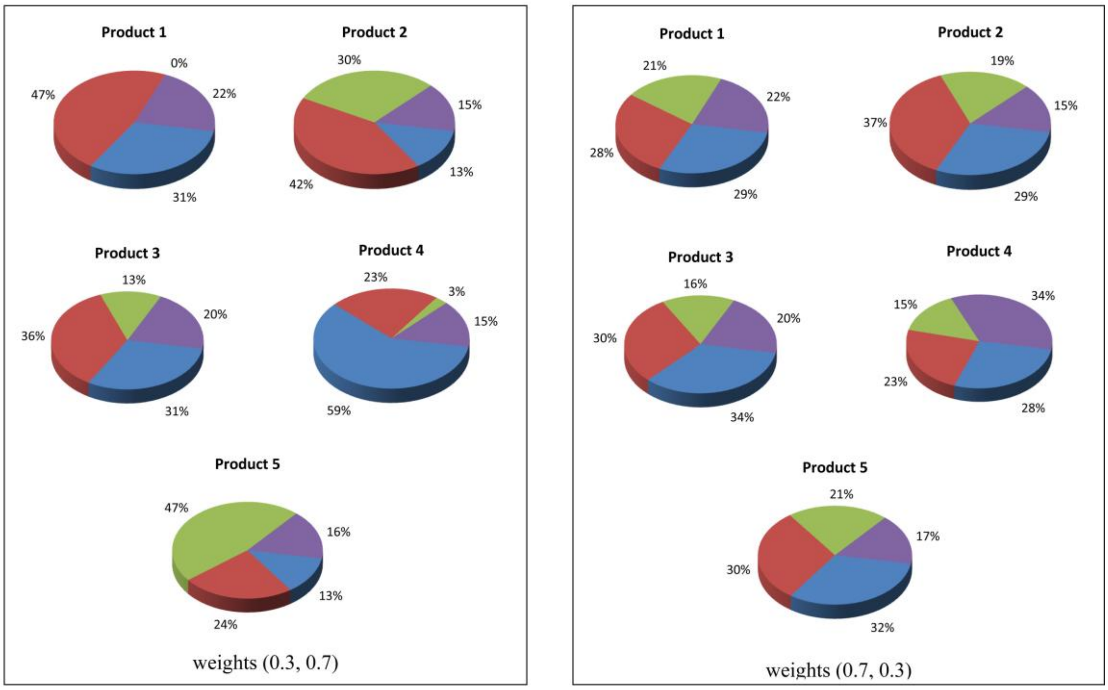

Figure 4.7: Comparison of total demand obtained by the supplier and existing suppliers weights (0.3, 0.7) and (0.7, 0.3)

The above analysis exhibits that the proposed approach yields a successful pricing strategy for penetration as well as profit based revenue management strategy for a small scale supplier intending to enter into a potential market engendered by a single buyer (Figures 4.5, 4.6 and 4.7).

4.6 Managerial implications

In this chapter, a decision support is developed to handle pricing decisions strategically maximizing profit while focusing to penetrate into a market engendered by single buyer and some existing suppliers. Through this study it is demonstrated that in case of business-to-business dealings the pricing strategy should be wisely used by adopting a proper decision-support rather than simply following penetration pricing in pursuit of acquiring greater market shares. It is further demonstrated that for negotiating with a buyer to get a profitable share of demand-orders a small scale supplier needs to assess the demand order allocation mechanism of the buyer and price structure of other suppliers already in business with the buyer.

The business situation described above is predominantly observed in sectors of small scale manufacturing. One example is of the non-branded apparel manufacturing sector, where small scale suppliers approach a potential buyer for seeking business through price negotiation for a range of apparels. Another example is of FMCG sector, where the small/ medium scale manufacturers target the marketing based companies (for their brand values) to get associated as third-party-manufacturer and supply their products. Furthermore, even the medium and large scale suppliers of any manufacturing industry also customarily identify and dedicate their specific production plant and/ or division and/ or DC to a particular buyer. This production plant and/ or DC forms a business division of the principal supplier. This business division acts as a small supplier in its own capacity for the decision making with the buyer. Therefore, a business division has to face the aforementioned planning problems independently.

4.7 Conclusions

In this chapter, a decision support is developed for the cohesive decision making on pricing strategy and operational planning of production and logistics. The problem is addressed from the perspective of a small scale supplier who is approaching a potential buyer to solicit a business through compelling prices in comparison to those from existing suppliers of the buyer. The proposed model accomplishes to devise a revenue management based penetration pricing strategy utilizing the capacity assessment through assessment of demand allocation from the buyer. The model is formulated in a bilevel programming framework, wherein the supplier is considered as leader while the buyer is considered as the follower. The problem enables the assessment of demand allocation from the buyer is considered as modelled as a bi-objective integer programming problem so as to minimize the total purchasing cost with maximum economic qualitative score.

A GA based approach is suggested to solve the proposed problem. The model as well as the solution methodology is demonstrated with an experimental study through a test scenario designed in consultation with some industry experts. The experimental results are further compared with two situations to test the efficacy of the suggested model. Results of test case along with the comparative analysis demonstrate the strength of the proposed model to successfully devise penetration strategy through revenue management.

Chapter 5

Decision Support to Buyer for Strategically Ascertaining Target Prices to Negotiate with Multiple Suppliers

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In this chapter[21], a decision-support is developed for a strategic problem of identifying target prices for the single buyer to negotiate with multiple suppliers in order to achieve common goal of maintaining sustained business environment. For this purpose, oligopolistic-competitive equilibrium prices of suppliers are suggested to be considered as target prices. The problem of identifying these prices is modeled as a multi-leader-single-follower BLP problem. Herein, suppliers are considered as leaders competing in a Nash game to maximize individual profit and the buyer as follower responding with demand-order allocations to minimize the total procurement-cost. In order to formulate individual profits of suppliers, assessment of respective operational cost to fulfil replicating demand-orders is achieved by integrating aggregate-production-distribution-planning mechanism into the problem. Additionally, a genetic-algorithm-based technique is designed to solve large-scale instances of the modelled problem. The proposed methodology is tested on the data of a leading FMCG manufacturing firm, which manufactures goods through multiple sourcing. Empirical analysis shows that target prices obtained through the proposed model outmatch negotiated rates in terms of company's procurement cost. Besides, our strategy inherits capability of identifying market-sweeping activities and possibility of supplier-cartel, which are potential threats to the business prospects of the buyer.

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[21] The contents of this chapter are based on research paper: "A bilevel game model for ascertaining competitive target prices for a buyer in negotiation with multiple suppliers", communicated to the Journal Omega (Elsevier).

5.1 Introduction

In an oligopolistic-monopsony market, suppliers compete on prices to attract a competitive demand share from their buyer, for maximizing their profits. The buyer, on the other side, intends to minimize overall procurement cost for receiving a regular supply of required products. For pursuing this objective, the buyer seeks to exploit the competition between suppliers and therefore negotiates with them to further lower down the prices. The status of being a sole buyer of an oligopolistic-monopsony market provides a bargaining power to entice suppliers with a larger share of demand-order in exchange for lowering the prices. In our problem, the overall demand of buyer is considered to be less than the combined output capacities of all the suppliers. In this situation, bargaining power of the buyer is further increased, and therefore suppliers are forced to renegotiate prices for obtaining demand-orders.

In such a scenario, the buyer sometimes adopts an opportunistic discriminatory approach for negotiation with suppliers to reduce the procurement cost to a minimum extent possible. For some suppliers prices negotiated in this manner may be well below a point where all the suppliers would have agreed upon due to complete awareness of competition in the discussed business environment. This develops a dissonance among such suppliers and a tendency of non-cooperation starts developing [238]. This antipathy adversely affects the buyer-supplier relationship [239–242], which, in turn impacts performance of their supply chain detrimentally [243]. On the other hand, if price-negotiations of the buyer with suppliers settle at a point which is well above an equilibrium point of suppliers' oligopolistic competition, financial interests of the buyer are adversely affected in this case in terms of a potentially higher procurement cost incurred. In such a situation, the buyer is compelled to look for alternative arrangements of sourcing as a long term initiative for intensifying the competition among suppliers. This, in turn, reduces the bargaining power of the suppliers and their demand-shares also in the future, resulting in reduced profit in long run.

This indicates that aggrieved stakeholders on either side of the considered market situation have an adverse action space that can thwart from maintaining a sustained and continuous business environment. Accordingly, the outcome of price-negotiations should ensure long term gains through sustained and continuous replenishment of products for the buyer vis-à-vis short term gains through opportunistic discrimination in price negotiation. Similarly, each supplier should prefer getting sustained and continuous replenishment orders from the buyer vis-à-vis creating an adverse action space which may affect their business prospects. Therefore, it is imperative to identify those prices for the business deal at which this common goal is achieved while pursuing the individual objectives. Consequently, it is appropriate for the stakeholders at both sides to settle at equilibrium prices of suppliers' oligopolistic-competition.

Identifying oligopolistic-competitive equilibrium prices of suppliers as target prices for negotiation imposes the competition among suppliers up to their individual production-distribution capabilities, and cost-efficiencies. As a result, the prices thus negotiated leave no space for any opportunistic discrimination on the part of buyer. Further, as these target prices for negotiation are based only on competitive capabilities of suppliers, therefore we appropriately term these as *competitive target prices*.

In an oligopolistic-monopsony market ecosystem, the sole buyer being a pivotal player is in a position to step forward for taking an initiative for maintaining a sustained business environment. Also, having complete information on costs and capacities of suppliers, the buyer is in a position to assess the oligopolistic-competitive equilibrium prices of suppliers and identify those as target prices for negotiation. Due to these reasons it is appropriate to address this problem from the buyer's perspective in this market situation. The situation of oligopolistic-monopsony market is common in many sectors, but the problem of identifying target prices for negotiations has never been studied to the best of our knowledge.

In this study, a decision support system developed for identifying competitive target prices for negotiations of the buyer with multiple suppliers in the oligopolistic-monopsony market situation is presented. We model the problem of identifying the competitive target prices as a multi-leader-single-follower BLP problem with linear constraints and bilinear objective functions. As suppliers make the first move by offering the prices, therefore they are considered as leaders; whereas the buyer responding in terms of the demand-order allocation is the follower, in this model. Price competition among the group of suppliers for receiving maximum profitable demand shares from the buyer leads to a game situation among these suppliers. In the considered environment of oligopolistic competition among suppliers, the assessment of prices requires integrated planning as total cost of fulfilling the demand-orders is related with different operational costs pertaining to production and distribution. In order to formulate their total operational costs as a part of respective profit functions and for keeping the production-and-logistics capacity constraints at suppliers' end into consideration, it is pressing to embed the APDP problem in the price-setting problem for replicating their decision behaviour in the model. The follower's reaction is embedded in the bilevel structure through a solution of the optimal demand-order allocation problem. As this model is designed to consider all the essential factors which are discussed above relating to the buyer and suppliers, the solution of this model would render competitive target prices for negotiation.

Furthermore, we experienced that the methodologies available in the literature for solving multi-leader-single-follower BLP problems are not adequate to handle large scale instances. We thus propose a GA-based procedure for solving general multi-leader-single-follower BLP problems specifically involving linear constraints and bilinear objective functions. The proposed algorithm is suitably modified to address the bilevel game problems with a specific structure as exhibited by our model. Thus, the contribution of this paper is two-fold, from the modelling as well as methodology development perspectives. The proposed model and solution algorithm are finally illustrated on a data set of an FMCG manufacturing firm concerned about the procurement cost of the ingredients of its products.

We demonstrate an implied advantage of using our model that upon identifying competitive target prices prior to the real negotiation the buyer is enabled to identify the possibility of cartels or price sweeping strategies. Such practices in oligopolistic-monopsony markets adversely affect the buyer's business immediately or in the long run. Suppliers collusively making a cartel[22] for price rigging turn out to be a threat to the buyer's business environment. Further, during the price negotiation, large-scale suppliers often offer extraordinarily low prices, sometimes even lower than the break-even-point of their business, to pose cut-throat competition among other suppliers. Such a strategy played by a supplier may, prima facie, indicate a reduction in the total procurement cost of the buyer. But, if the buyer is carried away by these offers, this may result in success of such a market sweeping strategy of a supplier, and a loss of business and business relations with other suppliers, thereby taking away the buyer's potential to negotiate in

[22] The practice of suppliers colluding as a cartel is unethical and illegal [248].

the long run. Such a strategy of a supplier can result in a more significant threat to the business environment of the buyer. The implicit advantage of our model to identify the possibility of these two threats as demonstrated during the case study presented in the later section.

This chapter is arranged into seven sections. The upcoming section 5.2 presents the problem description, some basic assumptions, and formulation of the problem of ascertaining competitive target prices as a multi-leader-single-follower BLP problem. Section 5.3 presents a GA-based solution methodology proposed for solving the discussed problem. Section 5.4 demonstrates an experimental study of a manufacturing firm. Comparative analysis of obtained results is presented in Section 5.5 to demonstrate the success of the decision-support. Managerial implications of adopting the suggested approach and utilizing the proposed decision-support are listed in Section 5.6. The chapter is concluded in section 5.7 with a summary of the work presented herein.

5.2 Formulation of mathematical model

5.2.1 Problem description

The negotiation process among buyer and suppliers is considered in the following setup.

- A buyer has already identified a set of suppliers based on their production and logistic infrastructural capacities and the quality standards of both products and services.
- Each of these suppliers can produce some or all of the required products and deliver them at various locations of the buyer.
- Considering the supply capacities of all the suppliers the total requirements of the buyer for various products can be fulfilled in each period.
- This setup is considered to be fixed for a pre-defined multi-period planning horizon.

The buyer wishes to negotiate on prices with these suppliers before entering into a new agreement. At this stage, suppliers have opportunity to review and agree on the prices of the goods considering production-and-logistics costs over the planning period and competition from the other suppliers. We have considered the business setup where the suppliers bear the cost of on-time delivery at various DLs of the buyer. It compels the suppliers to assess their costs of transportation, inventory, and production while negotiating on prices with the buyer.

The price-negotiation process of the buyer with each supplier begins with the supplier's offer on the prices of the products. The buyer tries to negotiate with each supplier to lower the prices of the products by offering a greater proportion of the demand-order. The supplier rethinks on the profit while discerning the production-distribution costs, and reviews the competition from those suppliers who deal with the buyer in these products particularly. The supplier then agrees for these prices or tries to negotiate further with the buyer. The process of price negotiation gets over once the supplier and the buyer arrive at the final prices. Figure 5.1 provides a schema for the discussed negotiation process. We develop a model for the buyer to identify competitive target prices for negotiations with a relatively competitive group of suppliers.

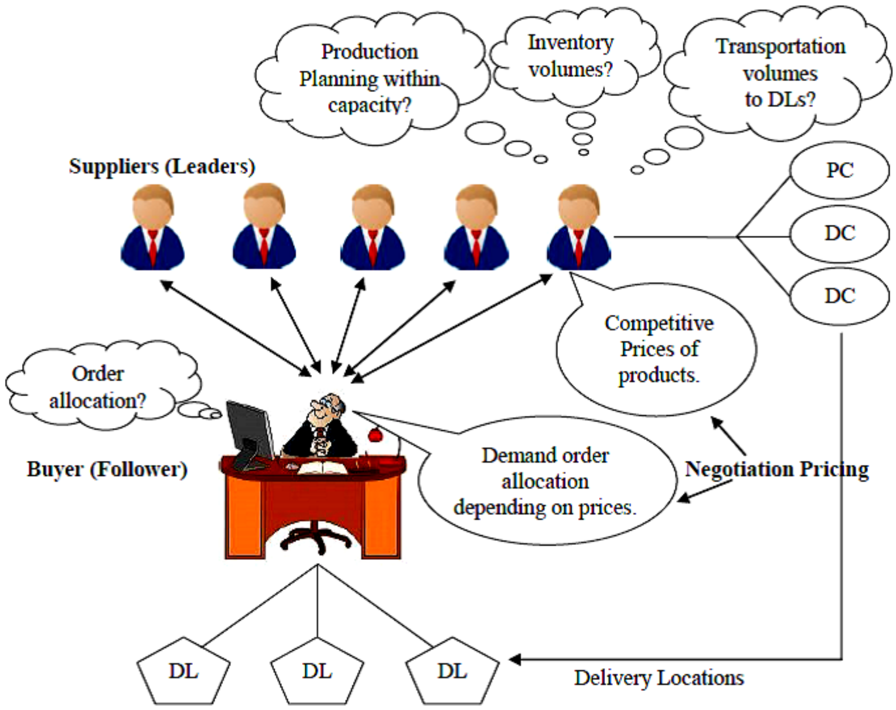

Figure 5.1: Depiction of structure

5.2.2 Assumptions and notations

For modelling the negotiation process described above some basic assumptions are considered about the products under consideration, suppliers and their supply arrangements, planning horizon, and some demand related technicalities of the business environment surrounding both the buyer and suppliers. These assumptions are listed below.

- The buyer has a knowledge of production and logistic resources and infrastructure of each supplier, to assess an aggregate-production-distribution plan on behalf of the later for any given demand order allocation.[23]
- Each supplier may deal in some or all products among those required by the buyer. A general situation is considered in which different suppliers can deal in subsets of the set of all products required by the buyer.
- Each product is homogenous in form, quality, quantity, size, across various suppliers dealing in that product.
- The prices of products are to be negotiated for a fixed time horizon which is discretized into equal subintervals. These subintervals are termed as periods. The demand orders are to be allocated to various suppliers for delivery of products at each delivery location, during every period of planning horizon.
- Each supplier has a single PC with no capacity to store any inventory over a period.
- All or some part of products manufactured in any period can be transported directly to various DLs of the buyer. In general, the products are transported first from the PC to DC for inventory and cross-docking, and from there transported to the different DLs. This transportation arrangement of each supplier is depicted in Figure 5.2.
- The DC can store each product that the supplier is dealing in. Likewise, all the DLs can accommodate all types of products.
- Suppliers are aware of the production-distribution capacities of their competitors.
- No significant changes in technology and business environment are expected during the planning horizon to impact the costs drastically.
- The demand fluctuations for the required goods shall be negligible during the planning horizon.[24]

[23] Such an assumption is practical in view of the assessment by the buyer during the supplier selection process.

[24] Since the planning horizon is short-term, it is reasonable to assume that the buyer can forecast demand encompassing fluctuations in it.

- The buyer knows the price for each supplier below which the supplier would no longer be able to bargain. These prices are identified through break-even prices of suppliers.
- Discounts or differential pricing are not provisioned in our model.

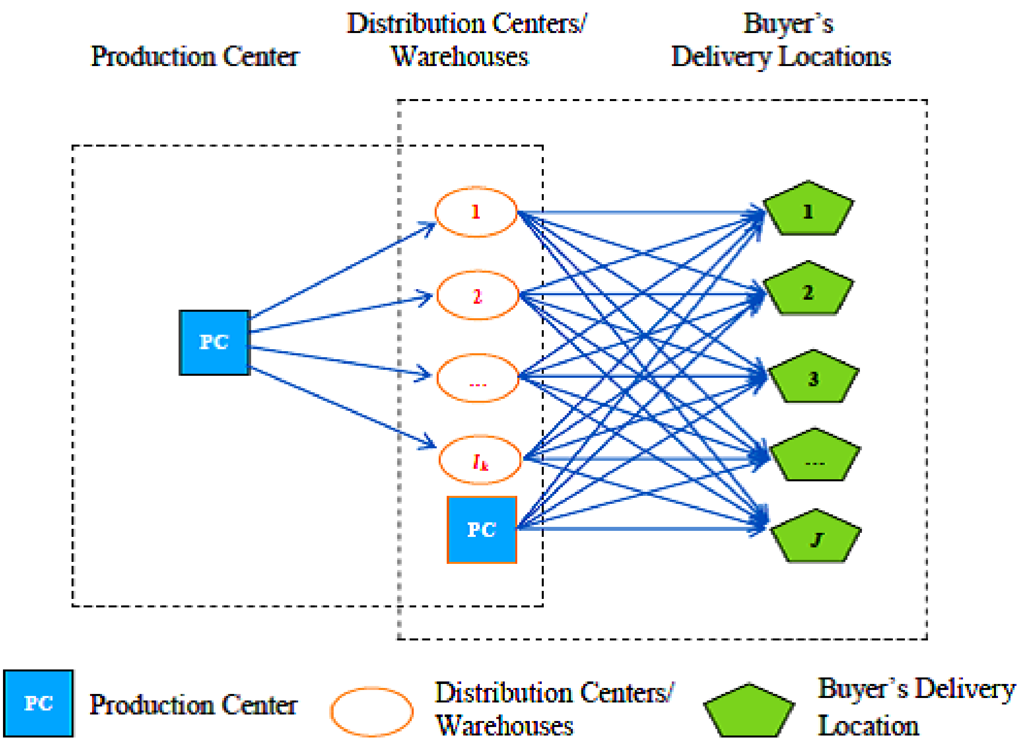

Figure 5.2: Production and distribution structure

The indices, parameters, and variables used to describe the mathematical formulation of our model are listed below.

Indices and sets

K	number of suppliers; $k = 1, 2, \dots, K$
N	number of different type of products; $n = 1, 2, \dots, N$
N_k	set of indices of the products that the supplier k deals in ($N_k \subseteq \{1, 2, \dots, N\}$); $n_k \in N_k$
J	number of buyer's DLs; $j = 1, 2, \dots, J$
T	planning horizon (number of periods); $t = 1, 2, \dots, T$

I_k	number of DCs of the supplier k; $i_k = 1, 2, \ldots, I_k$
	$i_k = 0$ stands for the PC of the supplier k, to act as a source to transport products to directly to DLs.

Leaders parameters and variables

Parameters

lp_{kn_kj}	minimum reservation price of product n_k from supplier k for its demand at buyer's DL j (INR/ unit) [25]
Lp_{kn_kj}	maximum reservation price of product n_k from supplier k for its demand at buyer's DL j (INR/ unit)
a_{kn_kt}	regular time production cost of product n_k for supplier k in period t (INR/unit)
b_{kn_kt}	overtime production cost of product n_k for supplier k in period t (INR/unit)
r_{kn_kt}	machine-hours required by supplier k for production of per unit of product n in period t
$tcp_{kn_ki_kt}$	cost of transportation of product n_k from PC to DC i_k of supplier k in period t (INR/ unit)
$tc_{kn_ki_kjt}$	cost of transportation of product n_k from DC $i_k$of supplier k to buyer's DL j in period t (INR/ unit)
$d_{kn_ki_kt}$	inventory carrying cost of product n_k at DC i_k of supplier k in period t (INR/unit)
v_n	space occupied by per unit of product n_k (cu-ft/unit)
MR_{kt}	maximum regular machine-hours (man-hours) available with supplier k in period t
M_{kt}	maximum total machine-hours (man-hours) available with supplier k in period t
V_{i_kt}	maximum space available in DC i_k of supplier k in period t (cu-ft)

Variables

z_{L_k}	Gross profit of supplier k
p_{kn_kj}	per unit price of product n_k from supplier k for its demand at DL j (INR/unit)
Q_{kn_kt}	regular time production volume of product n_k of supplier k in period t (units)
O_{kn_kt}	overtime production volume of product n_k of supplier k in period t (units)

[25] This is practically interpreted as break-even price of the supplier for a particular product.

$SS_{kn_ki_kt}$ inventory level (safety stock) of product n_k at DC i_k of supplier *k* in period *t* (units)

$I_{kn_ki_kt}$ consignment volume of product n_k to be sent from PC to DC i_k of the supplier *k* in period *t* (units)

$x_{kn_ki_kjt}$ consignment volume of product n_k from DC i_k of supplier *k* to buyer's DL *j* in period *t*

Follower's parameters and variables

Parameters

z_F total cost of procurement and holding products at various DLs (INR/unit)

D_{jnt} total forecasted demand of product *n* at buyer's DL *j* in period *t* (units)

My_{kn_k} maximum purchase volume of product n_k from supplier *k* in any period

VF_j maximum inventory carrying space at DL *j* of the buyer (cu-ft)

Variables

y_{kn_kjt} number of units of product n_k to be purchased from supplier *k* for DL *j* in period *t*

With all these details, the formulation of a mathematical model for ascertaining competitive target prices as a bilinear multi-leader-single-follower BLP problem is explained below.

5.2.3 Components of problem related to suppliers

For each supplier ($k = 1, 2, \dots, K$), the objective function and constraints which directly control the decision-behaviour are mathematically modelled below.

Objective function

The objective is to maximize the total profit through the decision on values of price variables (p_{kn_kj}) for each product $n_k \in N_k$ and each delivery location $j = 1, 2, \dots, J$, and there upon values of variables of production, inventory, and transportation ($Q_{kn_kj}, O_{kn_kj}, SS_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt}$) based on the demand shares received as a response to the prices quoted by all the suppliers. The objective function is given as following.

$$\max z_{L_k} = \sum_{t=1}^{T} \sum_{n_k \in N_k} \sum_{j=1}^{J} p_{kn_kj}\, y_{kn_kjt}$$
$$- \left\{ \sum_{t=1}^{T} \sum_{n_k \in N_k} \left(a_{kn_kt} Q_{kn_kt} + b_{kn_kt} O_{kn_kt} \right) \right.$$
$$+ \left(\sum_{t=1}^{T} \sum_{n_k \in N_k} \sum_{i_k=1}^{I_k} d_{kn_ki_kt} SS_{kn_ki_kt} + \sum_{t=1}^{T} \sum_{n_k \in N_k} \sum_{i=1}^{I} tcp_{kn_ki_kt} I_{kn_ki_kt} \right.$$
$$\left. \left. + \sum_{t=1}^{T} \sum_{n_k \in N_k} \sum_{i=0}^{I} \sum_{j=1}^{J} tc_{kn_ki_kjt}\, x_{kn_ki_kjt} \right) \right\}. \qquad (5.2.1)$$

Price bounds

The prices are speculated within bounds; lower bounds are the minimum acceptable prices to the supplier, whereas the upper bounds are enforced through the competition imposed by the other suppliers.

$$lp_{kn_kj} \;\le\; p_{kn_kj} \le Lp_{kn_kj}\;, \quad \forall n_k, \forall j. \qquad (5.2.2)$$

Regular time production hours

The production volumes for various products are restricted by the regular time production hours.

$$\sum_{n_k \in N_k} r_{kn_kt} Q_{kn_kt} \le MR_{kt}\;, \quad \forall t. \qquad (5.2.3)$$

Total production hours

The total production volumes obtainable through the provisions of overtime engagement of labour/ machines along with the regular time production are also restricted by the total available production hours.

$$\sum_{n_k \in N_k} r_{kn_kt} \left(Q_{kn_kt} + \; O_{kn_kt} \right) \le M_{kt}\;, \quad \forall t. \qquad (5.2.4)$$

Inventory balancing constraints

The demand orders received for each product are fulfilled through the total production volumes together with the available inventory volumes maintained in the previous period while maintaining required inventory volumes for the current period.

$$Q_{kn_kt} + O_{kn_kt} + \sum_{i_k=1}^{I_k} SS_{kn_ki_k(t-1)} - \sum_{i_k=1}^{I_k} SS_{kn_ki_kt} = \sum_{j=1}^{J} y_{kn_kjt}\ , \quad \forall n_k, \forall t. \qquad (5.2.5)$$

Space constraints at DC

At any period, the consignment volumes of various products to be received at each DC ($I_{kn_ki_kt}$) along with the products already available there as inventory maintained during the previous period ($SS_{kn_ki_k(t-1)}$) should be capacitated in the available space at the DC particular.

$$\sum_{n_k \in N_k} v_n\left(I_{kn_ki_kt} + SS_{kn_ki_k(t-1)}\right) \leq V_{i_kt}\ , \quad \forall t, \forall i_k. \qquad (5.2.6)$$

Transport plan for delivery at each DL

For each period, the consignment volumes of various products from PC to DC(s), from DC(s) to DLs, and directly from PC to DLs are to be planned to fulfil the demand of each DL for each product. Following three constraints govern this requirement. First two of the following constraints describe the transportation plan from DC(s) to DLs, whereas the third one describes transportation plan directly from PC to DLs.

$$\sum_{i=0}^{I} x_{kn_ki_kjt} \geq y_{kn_kjt}\ , \quad \forall j, \forall n_k, \forall t, \qquad (5.2.7)$$

$$\sum_{j=1}^{J} x_{kn_ki_kjt} \leq I_{kn_ki_kt} + SS_{kn_ki_k(t-1)} - SS_{kn_ki_kt}\ , \quad \forall i_k \neq 0, \forall n_k, \forall t, \qquad (5.2.8)$$

$$\sum_{j=1}^{J} x_{kn_k0jt} = Q_{kn_kt} + O_{kn_kt} - \sum_{i_k=1}^{I_k} I_{kn_ki_kt}\ , \quad i_k = 0, \forall n_k, \forall t. \qquad (5.2.9)$$

5.2.4 Components of the problem related to the buyer

Based on the price quotations received from various suppliers, the buyer solves the cost optimal demand allocation problem.

Objective function

The buyer decides on allocating demand shares (y_{kn_kjt}) corresponding to the received price quotes (p_{kn_kj}) from each supplier for various products for minimum total procurement cost.

$$\min z_F = \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n_k\in N_k}\sum_{t=1}^{T} p_{kn_kj}\, y_{kn_kjt} \qquad (5.2.10)$$

Constraint on maximum purchase volumes

The total of purchase volumes of each product over all DLs is restricted by a maximum value in any period by each supplier. Such restrictions are pre-decided by the suppliers due to the potential of their production line or sometimes the supplier managing business with multiple buyers.

$$\sum_{j=1}^{J} y_{kn_kjt} \le My_{kn_k}\,, \quad \forall k, \forall n_k, \forall t. \qquad (5.2.11)$$

Constraint on maximum purchase volumes

The total of purchase volumes of each product for each period from various suppliers must meet the demand of each DL.

$$\sum_{k=1}^{K} y_{kn_kjt} = D_{jnt}\,, \quad \forall j, \forall n_k\,, \forall t. \qquad (5.2.12)$$

Space constraint

The total of purchase volumes of all the products from various suppliers at each DL should be up to maximum inventory carrying capacity of the DL, for each period.

$$\sum_{k=1}^{K}\sum_{n_k\in N_k} v_n y_{kn_kjt} \le VF_j\,, \quad \forall j, \forall t. \qquad (5.2.13)$$

5.2.5 Decision-making problem of the buyer on finding target prices

The multi-leader-single-follower BLP problem (Price-BLP) for obtaining competitive target prices is summarized as follows. To determine values of the variables $\left\{\{p_{kn_kj}\},\{Q_{kn_kt}\},\ \{O_{kn_kt}\},\{SS_{kn_ki_kt}\},\{I_{kn_ki_kt}\},\{x_{kn_ki_kjt}\}: n_k, i_k, j, t\right\}$, by solving a constrained Nash-game of K suppliers depicted by optimization problems $(\mathrm{LDMP}-k)$ incorporating therein the response of buyer as an optimal solution of the problem (FDMP).

(Price-BLP)

$(\mathrm{LDMP}-k)$ $\max z_{L_k}$

$$
\begin{aligned}
&= \sum_{t=1}^{T}\sum_{n_k\in N_k}\sum_{j=1}^{J} p_{kn_kj}\, y_{kn_kjt} \\
&- \left\{\sum_{t=1}^{T}\sum_{n_k\in N_k}\left(a_{kn_kt}Q_{kn_kt} + b_{kn_kt}O_{kn_kt}\right)\right. \\
&+ \left(\sum_{t=1}^{T}\sum_{n_k\in N_k}\sum_{i_k=1}^{I_k} d_{kn_ki_kt}SS_{kn_ki_kt} + \sum_{t=1}^{T}\sum_{n_k\in N_k}\sum_{i=1}^{I} tcp_{kn_ki_kt}I_{kn_ki_kt}\right. \\
&+ \left.\left.\sum_{t=1}^{T}\sum_{n_k\in N_k}\sum_{i=0}^{I}\sum_{j=1}^{J} tc_{kn_ki_kjt}\, x_{kn_ki_kjt}\right)\right\}
\end{aligned}
$$

subject to

$$lp_{kn_kj} \ \le\ p_{kn_kj} \le Lp_{kn_kj}\ , \qquad \forall n_k, \forall j,$$

$$\sum_{n_k\in N_k} r_{kn_kt}Q_{kn_kt} \le MR_{kt}\ , \qquad \forall t,$$

$$\sum_{n_k\in N_k} r_{kn_kt}\left(Q_{kn_kt} +\ O_{kn_kt}\right) \le M_{kt}\ , \qquad \forall t,$$

$$Q_{kn_kt} + O_{kn_kt} + \sum_{i_k=1}^{I_k} SS_{kn_ki_k(t-1)} - \sum_{i_k=1}^{I_k} SS_{kn_ki_kt} = \sum_{j=1}^{J} y_{kn_kjt}\ , \qquad \forall n_k, \forall t,$$

$$\sum_{n_k \in N_k} v_n\left(I_{kn_k i_k t} + SS_{kn_k i_k (t-1)}\right) \le V_{i_k t}\,, \qquad \forall t, \forall i_k,$$

$$\sum_{i=0}^{I} x_{kn_k i_k jt} \ge y_{kn_k jt}\,, \qquad \forall j, \forall n_k, \forall t,$$

$$\sum_{j=1}^{J} x_{kn_k i_k jt} \le I_{kn_k i_k t} + SS_{kn_k i_k (t-1)} - SS_{kn_k i_k t}\,, \qquad \forall i_k \ne 0, \forall n_k, \forall t,$$

$$\sum_{j=1}^{J} x_{kn_k \mathbf{0} jt} = Q_{kn_k t} + O_{kn_k t} - \sum_{i_k=1}^{I_k} I_{kn_k i_k t}\,, \qquad i_k = 0, \forall n_k, \forall t,$$

$$p_{kn_k j}, Q_{kn_k t}, O_{kn_k t}, I_{kn_k i_k t}, SS_{kn_k i_k t}, x_{kn_k i_k jt} \ge 0\,, \qquad \forall n_k, \forall i_k, \forall j, \forall k, \forall t;$$

where, $\{y_{kn_k jt} : k, n_k, j, t\}$ is rational response obtained from following problem

(FDMP) $$Min\, z_F = \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{n_k \in N_k} \sum_{t=1}^{T} p_{kn_k j}\, y_{kn_k jt}$$

subject to

$$\sum_{j=1}^{J} y_{kn_k jt} \le M y_{kn_k}\,, \qquad \forall k, \forall n_k, \forall t$$

$$\sum_{k=1}^{K} y_{kn_k jt} = D_{jnt}\,, \qquad \forall j, \forall n_k\,, \forall t$$

$$\sum_{k=1}^{K} \sum_{n_k \in N_k} v_n y_{kn_k jt} \le VF_j\,, \qquad \forall j, \forall t$$

$$y_{kn_k jt} \ge 0\,, \qquad \forall j, \forall n_k, \forall k, \forall t\,.$$

Here, it is to be observed that all the constraints of the multi-leader-single-follower BLP problem modelled above are linear and objective functions of leaders and follower are bilinear. Therefore, this model is formulated specifically as a bilinear multi-leader-single-follower BLP problem, as discussed in section 2.4.

5.3 Solution methodology

The solution methodologies available in the literature suggest solving the multi-leader-common-follower BLP problems in terms of strong-stationary points [169] and strong-stationary equilibrium points [170]. Leyffer and Munson [169] suggest a direct method to obtain strong-stationary points by solving a nonlinear programming problem derived from strong-stationarity conditions by posing complementarity conditions as the objective function to be minimized to zero. The authors demonstrated the proposed approach through numerical experiments on randomly generated small-scale and medium-scale electricity market problems involving a maximum of 150 constraints and 160 variables. Hori and Fukushima [170] proposed a Gauss-Seidel method for numerical convergence to a strong-stationary equilibrium point, which involves solving a penalty-based quadratic optimization problem in each of the iterations. Authors demonstrate the proposed approach by solving five examples, involving a maximum of 12 variables and less than 10 constraints, are presented to demonstrate the process. Altogether, no instance of such a BLP problem involving more than 150 variables is seen to be solved in the literature.

From the literature review of methodologies to solve multi-leader-single-follower BLP problems, it is found that, till date, no such problem of a size more than 150 variables has been solved to obtain strong-stationary points. When we applied the approaches suggested by Hori and Fukushima [170] and Leyffer and Munson [169] for solving an instance of our practical problem with data obtained from a manufacturing firm which involve a total of 1000 variables (760 of leaders' and 240 of the follower), the nonlinear optimization problems formulated accordingly did not converge to any solution. This impels us to consider developing an algorithm capable of handling large scale multi-leader-single-follower BLP problems for obtaining strong-stationary points. As our model for identifying competitive target prices is formulated as a multi-leader-single-follower BLP problem with bilinear objective functions and linear constraints at both levels, we concentrate on developing an algorithm for solving this case of BLP problems. This case is termed as bilinear multi-leader-single-follower BLP problem (please see Section 2.4).

A GA-based approach is proposed here for solving bilinear multi-leader-single-follower BLP problems. Computationally, linear programming problems are experienced to be easier to solve in comparison to nonlinear programming problems of comparable scale. Considering this computational experience and the fact given below, a GA-based solution approach is developed which is explained subsequently.

For this purpose, the optimization problem derived as equivalent of the strong-stationarity conditions using penalty based approach of Leyffer and Munson [169], is considered as explained in Chapter 2 (c.f.: optimization problem (2.3.1) – (2.3.29)). Further, for a bilinear multi-leader-single-follower BLP problem, this optimization problem when parameterized in the leaders' and followers' variables becomes a linear programming problem in multipliers (c.f. (Para – LP) enumerated as (2.4.1)). This enables testing a feasible solution (x^*, y^*) of the bilinear multi-leader-single-follower BLP problem for a strong-stationarity point by simply solving a linear programming problem, as explained theoretically in Section 2.4. Whereas, a feasible solution of such a multi-leader-single-follower BLP problem can be obtained for any given values of leaders' variables x^*, by deterministically solving the follower's optimization problem (as the same is a linear programming problem parameterized in x^*). Therefore, a GA-based methodology is designed for (a) random exploration of values of leaders' variables, (b) obtain correspondingly a feasible solution, and (c) then test the same for strong stationarity by following the procedure given in deduction 2.4.1. The solution methodology developed on these lines is explained below.

5.3.1 GA-based approach for solving a general bilinear multi-leader-single-follower BLP problem

Based on the theoretical developments discussed in Section 2.4, we propose a GA-based solution methodology for obtaining strong-stationary points for a general bilinear multi-leader-single-follower BLP problem. The steps involved in the methodology are summarized through pseudocode presented in Algorithm 5.1 and Algorithm 5.2. Other details of the GA are presented subsequently.

Chromosome encoding

For leaders' variables $x = \{x_i : i = 1, 2, \dots, k\}$, where each $x_i = \{x_{i1}, x_{i2}, \dots, x_{ir_i}\}$, the chromosomes of the population are encoded as an array of length equal to the number of all the leaders' variables indicating the values $\{x_i : i = 1, 2, \ldots, k\}$. A general chromosome structure used for the proposed algorithm is shown in Figure 5.3.

Data: Input data and GA parameters
$g \leftarrow 0$;
Initialize population (values of leaders' variables $x = x_i : i = 1, 2, \ldots, k$);
Evaluate fitness of population members (using Algorithm 5.2);
while $g < G$ **do**
    tournament selection (retaining best-fit chromosome);
    generate new individuals through extended Laplace crossover
    and power mutation.
    evaluation fitness of population members (using Algorithm 5.2);
    update new population for next generation.
    $g \leftarrow g + 1$;
    select best-fit chromosome of the new generation (along with corresponding response y);
end
Return best-fit chromosome (along with corresponding response y) over all generations

Algorithm 5.1: Genetic Algorithm

Input: GA population of chromosomes
for $i \leftarrow 1$ **to** $popsize$ **do**
    substitute leaders' variables x (represented by chromosome) in (FDMP) to solve the follower's LPP (parameterized in x) for obtaining y;
    obtain $s = h(x, y)$ and $t_i = g_i(x_i, y)$ (following explanation of Section 2.4);
    solve the LPP (Pen – LP) (numbered 2.4.1) by supplying values of x, y, s, t_i.
    evaluate the fitness value as objective function value of (Pen – LP);
end
Output: Fitness values of all chromosomes of the population

Algorithm 5.2: Fitness evaluation of population members

$i = 1$				$i = 2$				...		$i = k$			
$j = 1$	$j = 2$	...	$j = r_1$	$j = 1$	$j = 2$	...	$j = r_2$	...	...	$j = 1$	$j = 2$	...	$j = r_k$
x_{11}	x_{12}	...	x_{1R}	x_{21}	x_{22}	...	x_{2r_2}	...	...	x_{k1}	x_{k2}	...	x_{kr_k}

Figure 5.3: Chromosome structure

Initialization

The following GA parameters are used in the proposed algorithm: population size *popsize*; number of generations G; current generation g, $(g = 1, 2, \dots, G)$; crossover rate *pc*; mutation rate *pm*; location parameter a and scaling parameter $b > 0$ for Laplace crossover; index of power mutation p.

Genetic operators

Crossover operator: We use a single point crossover for a chromosome, Laplace crossover operator with the probability *pc* [203]. The same is explained in Section 2.2.2.

Mutation operator: The mutation is performed on a chromosome with the probability *pm* using the power mutation operator [203]. The same is explained in Section 2.2.2.

We note that Laplace crossover and the power mutation operators do not disturb feasibility of chromosomes in terms of reservation price bounds in problem (LDMP – k).

Incorporating follower's reaction and fitness evaluation

Follower's reaction and inputs for (Pen – LP): For each chromosome in population (which corresponds to leaders' variables x), problem (FDMP) is solved. The optimal response y thus obtained along with value of x is supplied to tested for feasibility of each (LDMP – k) of (MLSF-BLP). Through this complete information about (x, y), the values of variables s and t_i are calculated using (2.3.25) and (2.3.27).

Fitness evaluation: For each chromosome, with its value $x = \{x_i : i = 1, 2, \dots, k\}$ and obtained values y, s, $t_i : i = 1, 2, \dots, k$, the problem (Pen – LP) numbered as (2.4.1) is solved. Corresponding value of the objective function $C_{penalty}$ is considered as the fitness value of that chromosome.

Updating the new population

The new population obtained from the parent population P^g is adopted to be a population of the next generation P^{g+1} only if it's maximum fitness value, in comparison to the maximum fitness value of the previous generation, does not decrease. Otherwise, the population P^g is preserved as population P^{g+1} for regenerating the next generation.

Termination criterion

The execution of the algorithm is terminated after the completion of pre-defined maximum number of generations G. The value of G may be tuned by observing stability in the fitness value through various combinations of GA parameters.

5.3.2 Some adaptations in the proposed algorithm for solving the problem of the buyer

As in the problem (Price-BLP), the values of Leaders' price variables $(\{p_{kn_kj}: k, n_k, j\})$ decide the demand allocations $\{y_{kn_kjt}: k, n_k, j, t\}$ by the follower, and thereafter the values of production-and-distribution planning variables $(Q_{kn_kj}, O_{kn_kj}, SS_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt}: k, n_k, i_k, j, t)$ can be obtained, depending on these demand allocations. This gives a specific structure to the bilevel programming problem (Price-BLP). A problem specific slight modification is thus presented for the proposed GA for solving the problem (Price-BLP). We encode the chromosomes as a row vector of values corresponding to Leaders' price variables (p_{kn_kj}) only, with each value ranging in the interval $[lp_{kn_kj}, Lp_{kn_kj}]$ for $k = 1, 2, \ldots, K$, $n_k = 1, 2, \ldots, N_k$, $j = 1, 2, \ldots, J$. Values of the rest of leaders' variables are obtained during the fitness evaluation. For a set of values of leaders' prices $\{p_{kn_kj}: k, n_k, j\}$ generated as a chromosome, we first evaluate the optimal response of the follower $\{y_{kn_kjt}: k, n_k, j, t\}$. Thereby, $(\text{LDMP} - k)$ is solved to obtain $Q_{kn_kt}, O_{kn_kt}, SS_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt}\ : k, n_k, i_k, j, t$ for each leader k.

The concatenated vector of values for $\{p_{kn\_kj}, Q_{kn_kj}, O_{kn_kj}, SS_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt}, y_{kn_kjt}: k, n_k, i_k, j, t\}$ thus becomes a feasible solution of the (Price-BLP), in concurrence with (x, y) as a feasible solution of the general bilinear multi-leader-single-follower BLP problem.

The algorithm modified for the problem (Price-BLP) is summarized through the pseudocode presented below.

Data: Input data and GA parameters
$g \leftarrow 0$;
Initialize population (values of suppliers' variables $p_{kn_kj} : k, n_k, j$);
Evaluate fitness of population members, and corresponding x and y (using Algorithm 5.4);
while $g < G$ **do**
    tournament selection (retaining best-fit chromosome);
    generate new individuals through extended Laplace crossover
    and power mutation;
    Evaluate fitness of population members, and corresponding x and y (using Algorithm 5.4);
    update new population for next generation;
    $g \leftarrow g + 1$;
    Select best-fit chromosome of the new generation (along with complete x and y, as obtained in Algorithm 5.4);
end

Algorithm 5.3: Genetic Algorithm for (Price-BLP)

Input: GA population of chromosomes
for $i \leftarrow 1$ **to** $popsize$ **do**
    substitute suppliers' prices p_{kn_kj} (represented by chromosome) in (FDMP) to solve the follower's LPP for obtaining y_{kn_kjt};
    for each leader k, solve $(\text{LDMP} - k)$ to obtain Q_{kn_kt}, O_{kn_kt}, $SS_{kn_ki_kt}$, $I_{kn_ki_kt}$, $x_{kn_ki_kjt}$
    obtain s and t_i following the rule $s = h(x, y)$ and $t_i = g_i(x_i, y)$ with $x = \{p_{kn\_kj}, Q_{kn_kj}, O_{kn_kj}, SS_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt}, y_{kn_kjt}\}$ and $y = \{y_{kn_kjt}\}$;
    solve (Pen – LP) with values of x, y, s, t_i supplied as parameters.
    evaluate the fitness value as objective function value of (Pen – LP);
end
Output: Fitness values of all chromosomes of the population

Algorithm 5.4: Fitness evaluation of population members - for (Price-BLP)

For further testing the strong stationarity points thus obtained for strong stationarity equilibrium points, one can refer to the description given in Remark 2.4.1.

5.4 An experimental study on a manufacturing firm

5.4.1 Relevant information about the firm

The formulation of our model is inspired by a scenario of a leading manufacturing firm of the FMCG sector. The firm has 5 production plants in the southern region of the Republic of India. Each of its production plants manufactures certain finished products, which require 5 ingredients of varied quantities over each week, depending upon the production plan. A production plan is prepared for a block of 4 weeks. The requirement arises for procuring different amounts of ingredients for each of the manufacturing plants by suppliers over each week. These ingredients are procured from a total of 8 suppliers who deal with some or all these ingredients. The manufacturing firm is therefore considered as buyer in the context of our study. The buyer has already identified these suppliers through various quality and potential parameters. As most of the required components involve frequently fluctuating production costs to vary over each month, the price-quotes must be invited from the suppliers. Based on their price-quotes, the demand allocation is done to these suppliers to supply the ingredients over the planning horizon of 4 weeks.

For its financial planning, the firm is concerned about assessing the total budget allocated for the procurement of ingredients before the invitation of price quotes from suppliers. The firm is also keen to observe whether its suppliers are competing for price quotes or colluding to settle at some higher prices. The latter situation may result in a dominance of its suppliers over the buyer. In such a case, the buyer firm would need to consider more suppliers for selection and demand order allocation to induce more competition to the existing suppliers. A manufacturer is vigilant for performing such analysis in pursuit of minimizing procurement cost, which comes as a significant component of production cost. At the same time, the firm is a production giant with a high brand value in the market of its finished products. Another priority concern of such a firm at this point of time is supplier integration to strengthen its inbound supply chain. In this line of thought, the firm is open to suggesting that any of its suppliers adjust the price quotes to avoid their opportunity losses for the same demand order allocation in case of raised prices. Further, through a prior assessment of prices, the firm would be better prepared to manage its inbound supply chain to be snag-free by allocating demand orders to its suppliers. As the concerns of this manufacturing firm match the theme of our framework therefore a case data from this firm is taken-up our study.

5.4.2 Data inputs

Henceforth, the manufacturing firm is the buyer, whereas the ingredients to be procured from its suppliers are products. Out of the five, one product is patent with a specific supplier, and the supplier can supply this product only, so there is no competition. We consider the remaining 4 products to be provided by 7 suppliers. Each of the seven suppliers has a single PC and a different number of DCs/ warehouses for inventory storage as well as for cross-docking of shipments of products for transportation from PC to DLs. For reference, the products, suppliers, and their DCs, and DLs numbered with corresponding indices.

The 4th product is a volatile product in a liquid state and is transported only through special containers, so neither its cross-docking nor maintaining inventory is considered practically feasible. This specification of the structural setup can easily be incorporated into the modelling by not defining any variable against the product for inventory and transportation to DCs for all suppliers dealing in this product. The details of the suppliers dealing in products and number of DCs are tabulated in Table 5.1. Reservation prices of all the suppliers are listed in Table 5.2 and other supply chain related data relevant to our study is tabulated in Table 5.3 to Table 5.9.

	supplier (k)						
	1	2	3	4	5	6	7
Product sets ($\boldsymbol{N_k}$)	$\{1\}$	$\{2,3\}$	$\{1\}$	$\{4\}$	$\{2,3\}$	$\{1,4\}$	$\{2,3,4\}$
Number of DC(s) ($\boldsymbol{I_k}$)	2	1	1	0	1	2	2

Table 5.1: Supplier's profile (products and DCs)

		supplier (k)											
		1	2		3	4	5		6		7		
	DL	n_1	n_2		n_3	n_4	n_5		n_6		n_7		
	(j)	1	2	3	1	4	2	3	1	4	2	3	4
$\boldsymbol{lp_{kn_kj}}$	1	25700	5560	5550	31000	89600	4750	4695	24250	90050	5790	6010	92700
	2	25800	5660	5700	31100	89300	4850	4795	24100	89800	6040	6260	92800
	3	25950	5810	5450	31175	89700	5000	4945	24200	89350	6365	6585	92875
	4	25950	5560	5050	30300	91000	5000	4945	24350	89600	6340	6560	92000
	5	25600	5160	4850	29800	90600	4650	4595	24250	89850	5640	5860	91500
$\boldsymbol{Lp_{kn_kj}}$	1	28700	5700	5700	32900	94500	5100	4900	27700	92500	6250	6350	97400
	2	28800	5800	5850	33000	94200	5200	5000	27550	92250	6500	6600	97500
	3	28950	5950	5600	33075	94600	5350	5150	27650	91800	6825	6925	97575
	4	28950	5700	5200	32200	95900	5350	5150	27800	92050	6800	6900	96700
	5	28600	5300	5000	31700	95500	5000	4800	27700	92300	6100	6200	96200
$\boldsymbol{My_{kn_k}}$		175	350	430	225	175	275	350	250	100	300	300	100

Table 5.2: Reservation prices and maximum purchase volumes

D_{jnt}	DL (j)																			
	1				2				3				4				5			
Product (n)	time period (t)				time period (t)				time period (t)				time period (t)				time period (t)			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	150	75	100	75	125	100	75	50	75	75	50	50	75	50	75	50	200	100	120	80
2	75	75	75	75	75	75	75	50	75	50	75	25	75	75	50	25	100	100	80	50
3	125	125	100	50	100	75	75	75	100	75	50	50	80	80	100	15	130	130	90	90
4	50	75	75	50	50	75	75	25	75	75	50	0	60	60	80	0	80	80	80	60

Table 5.3: Forecasted demands of products at each DL for each time period

	product (n)	supplier (k) 1	2	3	4	5	6	7
$\boldsymbol{a_{kn_kt}}$	1	20000	-	21000	-	-	20500	-
	2	-	3600	-	-	3500	-	3400
	3	-	3500	-	-	3450	-	3600
	4	-	-	-	81000	-	80500	83000
$\boldsymbol{b_{kn_kt}}$	1	20200	3650	21200	-	-	20700	-
	2	-	3550	-	-	3550	-	3450
	3	-	-	-	-	3500	-	3650
	4	-	-	-	82000	-	81500	84000
$\boldsymbol{r_{kn_kt}}$	1	0.75	0.5	0.75	-	-	0.75	-
	2	-	0.5	-	-	0.5	-	0.33
	3	-	-	-	-	0.5	-	0.33
	4	-	-	-	1.5	-	1.5	1.25

Table 5.4: Production costs - regular-time and overtime (INR per unit), and machine-hours required for production (hours per unit)

	supplier (k) 1	2	3	4	5	6	7
$\boldsymbol{MR_{kt}}$	168	144	168	168	288	336	456
$\boldsymbol{M_{kt}}$	168	168	168	168	336	336	504

Table 5.5: Maximum machine-hours available regular-time and total (including overtime)

$\boldsymbol{k}$	$\boldsymbol{n_k}$	$\boldsymbol{i_k}$	$\boldsymbol{tcp_{kn_ki_kt}}$	$\boldsymbol{d_{kn_ki_kt}}$	$\boldsymbol{V_{i_kt}}$
1	1	1	1200	10	4000
		2	900	10	6000
2	2	1	1000	10	4000
	3	1	1000	10	
3	1	1	6500	10	6000
5	2	1	400	9	4000
	3	1	400	9	
6	1	1	900	10	5000
		2	600	10	5000
7	2	1	850	10	4000
		2	950	10	4000
	3	1	850	10	
		2	950	10	

Table 5.6: Transportation costs from PC to DC(s) and costs of inventory at DC(s) (INR per unit), maximum space available at DC(s)

		$i_k = 0$					$i_k = 1$					$i_k = 2$				
supplier	product	DL (j)					DL (j)					DL (j)				
(k)	(n_k)	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	1	1000	1100	1250	1000	850	500	600	750	750	400	750	800	900	700	400
2	2	500	600	750	750	400	600	700	850	600	200	-	-	-	-	-
	3	500	600	750	750	400	600	700	850	600	200	-	-	-	-	-
3	1	5500	5000	4800	5500	6500	1400	1500	1575	700	200	-	-	-	-	-
4	4	500	200	600	1900	1500	-	-	-	-	-	-	-	-	-	-
5	2	900	1000	1000	1000	400	500	600	750	750	400	-	-	-	-	-
	3	900	1000	1000	1000	400	500	600	750	750	400	-	-	-	-	-
6	1	1500	1250	800	1050	1300	500	600	750	750	400	1100	950	200	1200	1100
	5	1500	1250	800	1050	1300	-	-	-	-	-	-	-	-	-	-
7	2	1400	1500	1575	700	200	600	800	1150	1550	850	1100	1350	1675	1650	950
	3	1400	1500	1575	700	200	600	800	1150	1550	850	1100	1350	1675	1650	950
	5	1400	1500	1575	700	200	-	-	-	-	-	-	-	-	-	-

Table 5.7: Costs of transportation from PC or DC(s) to DLs[26] ($tc_{kn_k i_k jt}$)

[26] As all the suppliers in the problem have DCs not more than 2, due to this fact the costs of transportation have been listed in a single table only. But this doesn't mean that all the suppliers have same DCs. It is simply an enumeration of DCs. They own or hire for their individual warehouses which are located even at different places.

product (n)			
1	2	3	4
8	8	8	12.5

Table 5.8: Space occupied per tonne (in sq. ft) (v_n)

DL (j)				
1	2	3	4	5
8000	8000	4000	5500	6000

Table 5.9: Maximum inventory carrying space at DLs of the buyer[27] (sq. ft) (VF_j)

5.4.3 Implementation of proposed GA-based approach

The (Price-BLP) problem modelled as a multi-leader-single-follower BLP problem, discussed in context of manufacturing firm with the input data tabulated above is solved using the modified algorithm proposed in the Section 5.3.2. The program is coded in MATLAB 2019a. The parameters of Laplace crossover and power mutation along with population size ($popsize$) are tuned for various combinations of probabilities of crossover, mutation, and tournament selection. The best found are $popsize = 20$, $a = 0$, $b = 0.15$, $p = 1$ or 10.We set the maximum number of generations to 1000 after tuning the parameters for attainment of the best fitness value equal to zeros for strong stationary point. For each combination of parameters (Table 5.10), we performed a set of 10 experiments of GA. Table 5.10 tabulates the relative error of the best solutions obtained for each combination of parameters against the ideal fitness value zero in comparison with the arithmetic mean of the best fitness values of various combinations. Figure 5.4 shows variation in the best fitness attained in various generations of a GA run. Among 240 solutions generated (24 combinations with 10 runs each) for (Price-BLP), 22 strong-stationary points (fitness value = 0) were obtained. The arithmetic mean of total

[27] This accounts for the capacity to accommodate all the products except product 4, which needs to be stored in a separately installed container (at each DL) having storage capacity which is more sufficient to accommodate the demand for each period.

procurement cost of the buyer corresponding to these 22 instances of prices (which correspond to strong-stationarity points) is INR 166,844,775, with a standard deviation of 266396.83, giving coefficient of variation 0.16 %.This indicates parity among all the 22 strong-stationarity points from buyer's perspective, in terms of the total procurement cost. The desired fitness value zero (for strong-stationarity points) is attained for the runs of 18 out of 24 combinations of GA parameters, whereas for rest of the combinations there no improvement in fitness value for more than last 300 generations.

pt	*pc*	*pm*	*P*	fitness value	relative error
0.8	0.7	0.001	1	**0**	0
0.8	0.7	0.001	10	6263	0.920
0.8	0.7	0.005	1	**0**	0
0.8	0.7	0.005	10	**0**	0
0.8	0.8	0.001	1	**0**	0
0.8	0.8	0.001	10	12780	1.877
0.8	0.8	0.005	1	**0**	0
0.8	0.8	0.005	10	**0**	0
0.8	0.9	0.001	1	**0**	0
0.8	0.9	0.001	10	1145	0.168
0.8	0.9	0.005	1	3500	0.514
0.8	0.9	0.005	10	**0**	0
0.9	0.7	0.001	1	**0**	0
0.9	0.7	0.001	10	3500	0. 514
0.9	0.7	0.005	1	**0**	0
0.9	0.7	0.005	10	**0**	0
0.9	0.8	0.001	1	**0**	0
0.9	0.8	0.001	10	**0**	0
0.9	0.8	0.005	1	**0**	0
0.9	0.8	0.005	10	**0**	0
0.9	0.9	0.001	1	197	0.029
0.9	0.9	0.001	10	**0**	0
0.9	0.9	0.005	1	**0**	0
0.9	0.9	0.005	10	**0**	0

Table 5.10: Error analysis for different combinations of parameters in GA; those giving strong-stationary points highlighted in bold

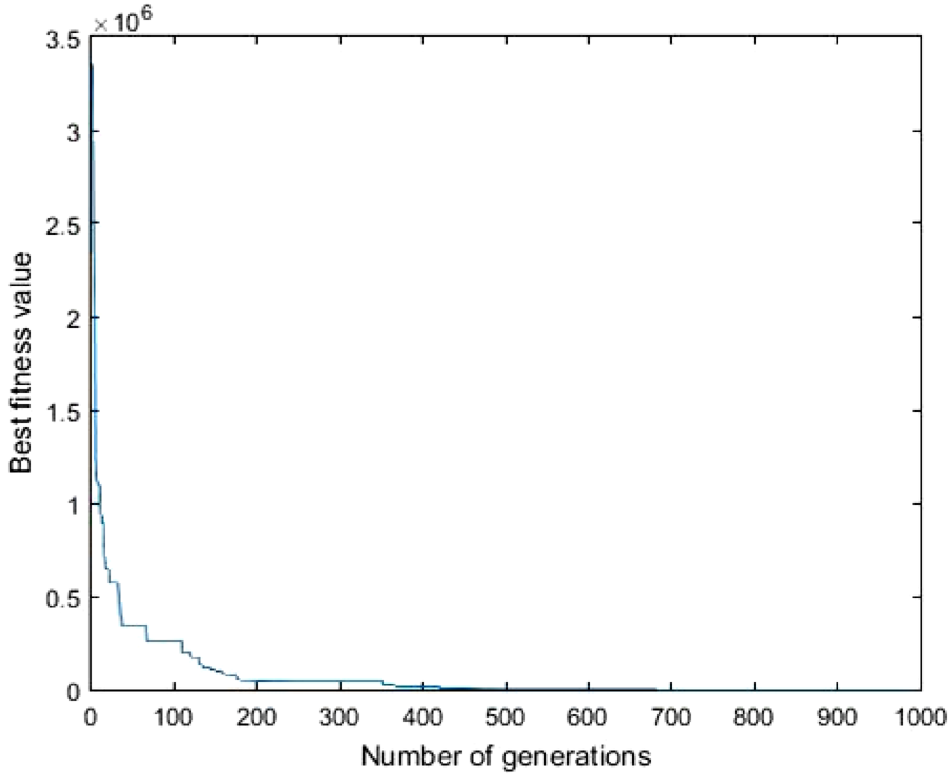

Figure 5.4: Best fitness value vs. Generation of a solution.

5.4.4 Results and analysis

The strong-stationarity point with the total procurement cost closest to the arithmetic mean of all the 22 ones obtained from our computation is detailed as following. Table 5.11 shows prices for the suppliers for various products they deal in to be delivered at each of 5 DLs. As a response to these prices, the consequent demand allocations from the buyer are tabulated in Table 5.12. Subsequently, the aggregate-production-distribution plans of each of 7 suppliers are presented in Table 5.13 to Table 5.32. The total procurement cost of the buyer in this instance is assessed as INR 166,871,150. It is noted that the inventory volumes of all the suppliers at each of their warehouses are obtained as zeros for all products during each period.

Remark 5.4.1: For the discussed scenario of the firm, a further testing of each of these 22 strong-stationarity points for strong-stationarity Nash-equilibrium point is not being taken-up for the following reasons.

1. There is parity among all the obtained strong-stationarity points in terms of total procurement cost of the buyer (the problem being studied from buyer's perspective only).
2. The decision-makers of the discussed firm (buyer here) confirmed the efficacy of the results obtained so far in the context of total procurement cost assessed corresponding to competitive prices obtained corresponding to all the 22 strong-stationarity points when compared with the actual costs incurred. From the perspective of business management, this indicates that for the buyer a scope of further negation on prices was there without any compromise on the cooperative relation with suppliers.
3. Thus, even if the buyer would have considered the target prices as any one of those obtained corresponding to these strong-stationarity points, and negotiated up to the same with suppliers, then the objective of strategic pricing as, discussed in section 1, would be satisfactorily achieved.
4. Over that, theoretically, there is no surety of obtaining a strong-stationarity Nash-equilibrium point, and the question of its existence remaining unconfirmed.

		DL (*j*)				
supplier (*k*)	product (n_k)	1	2	3	4	5
1	1	28700	25800	25950	28950	28599.92
2	2	5560	5560	5950	5560	5160
	3	5689.914	5850	5450	5050	4850
3	1	32899.92	33000	31175	30300	29800
4	4	89600	94200	93504.25	91000	90600
5	2	4750	4850	5000	5000	4650
	3	4695	4795	4945	5150	4595
6	1	24250	27550	24200	24350	24250
	4	92499.98	89800	89350	92050	93292.56
7	2	6250	6499.984	6824.942	6800	6100
	3	6350	6599.426	6925	6882.551	6200
	4	92707.78	92800	92875	96700	91500

Table 5.11: Target prices with suppliers (p_{kn_kj}) (INR per tonne)

		DL (j)																			
		1				2				3				4				5			
supplier (k)	product (n_k)	time period (t)				time period (t)				time period (t)				time period (t)				time period (t)			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	1	0	0	0	0	125	100	75	50	50	0	25	25	0	0	0	0	0	0	0	0
2	2	0	0	0	0	0	0	0	0	0	0	0	0	25	0	0	0	100	100	80	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	80	80	100	15	105	55	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	200	100	120	30
4	4	50	75	75	50	0	0	0	0	0	0	0	0	60	60	80	0	65	40	20	60
5	2	75	75	75	75	75	75	75	50	75	50	75	25	50	75	50	25	0	0	0	50
	3	125	125	100	50	100	75	75	75	100	75	50	50	0	0	0	0	25	75	90	90
6	1	150	75	100	75	0	0	0	0	25	75	25	25	75	50	75	50	0	0	0	50
	4	0	0	0	0	25	25	50	25	75	75	50	0	0	0	0	0	0	0	0	0
7	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	25	50	25	0	0	0	0	0	0	0	0	0	15	40	60	0

Table 5.12: Buyer's Demand allocation (y_{kn_kjt}) - (number of tonnes to be purchased)

		time period (t)			
		1	2	3	4
$\boldsymbol{Q_{kn_kt}}$	$n_k = 1$	175	100	100	75
$\boldsymbol{O_{kn_kt}}$	$n_k = 1$	0	0	0	0

Table 5.13: Production volumes (regular-time and over-time): Supplier 1

$\boldsymbol{I_{kn_ki_kt}}$	time period (t)			
DC ($\boldsymbol{i_k}$)	1	2	3	4
1	0	0	0	0
2	0	0	0	0

Table 5.14: Consignment volumes from PC to DC(s): Supplier 1

$x_{kn_ki_kjt}$	DC (i_k)											
	0				1				2			
	time period (t)				time period (t)				time period (t)			
J	1	2	3	4	1	2	3	4	1	2	3	4
1	0	0	0	0	0	0	0	0	0	0	0	0
2	125	100	75	50	0	0	0	0	0	0	0	0
3	50	0	25	25	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0

Table 5.15: Transportation volumes from PC to DC(s): Supplier 1 ($n_k = 1$)

		time period (t)			
	product (n_k)	1	2	3	4
$\boldsymbol{Q_{kn_kt}}$	1	103	100	80	0
	2	185	135	100	15
$\boldsymbol{O_{kn_kt}}$	1	22	0	0	0
	2	0	0	0	0

Table 5.16: Production volumes (regular-time and over-time): Supplier 2

$I_{kn_ki_kt}$	time period (t)			
product (i_k)	1	2	3	4
2	0	0	0	0
3	0	0	0	0

Table 5.17: Consignment volumes from PC to DC(s): Supplier 2 (DC: $i_k = 1$)

$x_{kn_ki_kjt}$		$i_k = 0$				$i_k = 1$			
Product	DL	time period (t)				time period (t)			
(n_k)	(j)	1	2	3	4	1	2	3	4
2	1	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
	4	25	0	0	0	0	0	0	0
	5	100	100	80	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
	4	80	80	100	15	0	0	0	0
	5	105	55	0	0	0	0	0	0

Table 5.18: Transportation volumes from PC to DC(s): Supplier 2

		time period (t)			
	product (n_k)	1	2	3	4
Q_{kn_kt}	1	200	100	120	30
O_{kn_kt}	1	0	0	0	0

Table 5.19: Production volumes (regular-time and over-time): Supplier 3

time period (t)			
1	2	3	4
0	0	0	0

Table 5.20: Consignment volumes from PC to DC(s) ($I_{kn_ki_kt}$;$n_3 = 1, i_3 = 1$): Supplier 3

$x_{kn_ki_kjt}$	$i_k = 0$				$i_k = 1$			
	time period (t)				time period (t)			
DL (j)	1	2	3	4	1	2	3	4
1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0
5	200	100	120	30	0	0	0	0

Table 5.21: Transportation volumes from PC to DC(s) ($n_3 = 1$): Supplier 3

	time period (t)			
	1	2	3	4
$\boldsymbol{Q_{kn_kt}}$	175	175	175	110
$\boldsymbol{O_{kn_kt}}$	0	0	0	0

Table 5.22: Production volumes (regular-time and over-time) ($n_k = 5$): Supplier 4

	time period (t)			
DL (j)	1	2	3	4
1	50	75	75	50
2	0	0	0	0
3	0	0	0	0
4	60	60	80	0
5	65	40	20	60

Table 5.23: Transportation volumes from PC to DC(s) ($i_k = 0, n_k = 5$): Supplier 4

		time period (t)			
	t	1	2	3	4
$\boldsymbol{Q_{kn_kt}}$	$n_k = 2$	226	226	261	225
	$n_k = 3$	350	350	315	265
$\boldsymbol{O_{kn_kt}}$	$n_k = 2$	49	49	14	0
	$n_k = 3$	0	0	0	0

Table 5.24: Production volumes (regular-time and over-time): Supplier 5

	time period (t)			
	1	2	3	4
$\boldsymbol{n_k = 2, i_k = 1}$	0	0	0	0
$\boldsymbol{n_k = 3, i_k = 1}$	0	0	0	0

Table 5.25: Consignment volumes from PC to DC(s): Supplier 5

$x_{kn_ki_kjt}$		$i_k = 0$				$i_k = 1$			
		time period (t)				time period (t)			
	DL(j)	1	2	3	4	1	2	3	4
$n_k = 2$	1	75	75	75	75	0	0	0	0
	2	75	75	75	50	0	0	0	0
	3	75	50	75	25	0	0	0	0
	4	50	75	50	25	0	0	0	0
	5	0	0	0	50	0	0	0	0
$n_k = 3$	1	125	125	100	50	0	0	0	0
	2	100	75	75	75	0	0	0	0
	3	100	75	50	50	0	0	0	0
	4	0	0	0	0	0	0	0	0
	5	25	75	90	90	0	0	0	0

Table 5.26: Transportation volumes from PC to DC(s): Supplier 5

		time period (t)			
	product (n_k)	1	2	3	4
$\boldsymbol{Q_{kn_kt}}$	1	250	200	200	200
	5	100	100	100	25
$\boldsymbol{O_{kn_kt}}$	1	0	0	0	0
	5	0	0	0	0

Table 5.27: Production volumes (regular-time and over-time): Supplier 6

$\boldsymbol{I_{kn_ki_kt}}$	time period (t)			
	1	2	3	4
$\boldsymbol{n_k = 2, i_k = 1}$	150	75	100	75
$\boldsymbol{n_k = 3, i_k = 1}$	0	0	0	0

Table 5.28: Consignment volumes from PC to DC(s): Supplier 6

$x_{kn_ki_kjt}$		DC (i_k)											
		0				1				2			
Product	DL	time period (t)				time period (t)				time period (t)			
(n_k)	(j)	1	2	3	4	1	2	3	4	1	2	3	4
1	1	0	0	0	0	150	75	100	75	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0	0	0
	3	25	75	25	25	0	0	0	0	0	0	0	0
	4	75	50	75	50	0	0	0	0	0	0	0	0
	5	0	0	0	50	0	0	0	0	0	0	0	0
5	1	0	0	0	0	-	-	-	-	-	-	-	-
	2	25	25	50	25	-	-	-	-	-	-	-	-
	3	75	75	50	0	-	-	-	-	-	-	-	-
	4	0	0	0	0	-	-	-	-	-	-	-	-
	5	0	0	0	0	-	-	-	-	-	-	-	-

Table 5.29: Transportation volumes from PC to DC(s): Supplier 6

		time period (t)			
	product (n_k)	1	2	3	4
$\boldsymbol{Q_{kn_kt}}$	1	0	0	0	0
	3	0	0	0	0
	5	40	90	85	0
$\boldsymbol{O_{kn_kt}}$	1	0	0	0	0
	3	0	0	0	0
	5	0	0	0	0

Table 5.30: Production volumes (regular-time and over-time): Supplier 7

		time period (t)			
product ($\boldsymbol{n_k}$)	DC (i_k)	1	2	3	4
2	1	0	0	0	0
	2	0	0	0	0
3	1	0	0	0	0
	2	0	0	0	0

Table 5.31: Consignment volumes from PC to DC(s) ($I_{kn_ki_kt}$): Supplier 7

$x_{kn_ki_kjt}$		DC (i_k)											
		0				1				2			
		time period (t)				time period (t)				time period (t)			
product (n_k)	DL (j)	1	2	3	4	1	2	3	4	1	2	3	4
1	1	0	0	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0	0	0
5	1	0	0	0	0	-	-	-	-	-	-	-	-
	2	25	50	25	0	-	-	-	-	-	-	-	-
	3	0	0	0	0	-	-	-	-	-	-	-	-
	4	0	0	0	0	-	-	-	-	-	-	-	-
	5	15	40	60	0	-	-	-	-	-	-	-	-

Table 5.32: Transportation volumes from PC to DC(s): Supplier 7

The results of our model have been shared with the concerned managers of the manufacturing firm. Upon the comparison of our results with the firm's actual procurement expenses and arrangements, it is shared by the decision-makers of the firm that the projected total procurement costs assessed through the instances of our results are significantly lower than their actual procurement costs. Hereupon the scope of improvement in the negotiation process is realized by the decision-makers of procurement department. It has come out from the discussion with decision-makers of the buyer firm that computing the competitive target prices through our decision-support would remarkably help their team participating in the negotiation process.

5.5 Comparison analysis

The proposed (Price-BLP) model identifies the prices to which the buyer should target for negotiation to create a constructive competition among the suppliers. The comparison of the results at one side provides the buyer firm with benchmark prices for negotiation. On the other side, due to the proposed model so designed, these prices respect the supplier's reservation prices thereby safeguarding financial interests of suppliers. To further demonstrate the inherited benefits of our model we compare the results of the previous subsection with two cases: when either there is a price sweeping strategy played by a supplier, and when some suppliers attempt to make a cartel for price rigging.

First, we consider a situation wherein the price-negotiation of the buyer with various suppliers results in final prices given in Table 5.33. This instance indicates that, while all the suppliers settle for the prices same as their competitive target prices (given in Table 5.11), the supplier 7 attempts to quote prices for products 2 and 3 much lower than what are corresponding estimates of competitive target prices. This abnormal quote of extraordinarily low prices in an oligopolistic-monopsony market is generally suspected as a plot of market-sweeping strategy. Prima facie such price offers are lucrative but eventually can result in a loss of business-relation with other suppliers thereby the buyer losing a potential to negotiate in future. We attempt to verify for this instance that whether the demand-order allocations in response to such an ambitious quote of prices by the supplier 7 can result in a market-sweep.

		DL (j)				
supplier (k)	product (n_k)	1	2	3	4	5
1	1	28700	25800	25950	28950	28599.92
2	2	5560	5560	5950	5560	5160
	3	5689.914	5850	5450	5050	4850
3	1	32899.92	33000	31175	30300	29800
4	4	89600	94200	93504.25	91000	90600
5	2	4750	4850	5000	5000	4650
	3	4695	4795	4945	5150	4595
6	1	24250	27550	24200	24350	24250
	4	92499.98	89800	89350	92050	93292.56
7	2	**4500**	**4500**	**4800**	**4800**	**4400**
	3	**4000**	**4000**	**4200**	**4200**	**4000**
	4	92707.78	92800	92875	96700	91500

Table 5.33: Suppliers' prices p_{kn_kj} (INR per tonne) - Market sweeping strategy by supplier 7 for products 2 and 3

For this verification we solve the lower level optimization problem (FDMP) of (Price-BLP) while considering the prices as parameters with values depicted in Table 5.33. The demand-orders so obtained are listed in Table 5.34, which confirm the surmised affect to a large extent as in this case the supplier 2 loses his business with the buyer in full and supplier 3 in part. In this situation, on the bases of prior knowledge of competitive target prices, a buyer with a long-term strategic vision would be able to restrain from getting carried away with such artificially lower price quotes for ephemeral benefits.

Further, we compare the results of our model with another illustrative scenario wherein suppliers 1, 3, and 6 adhere to same higher price for product 1 during the negotiation with the buyer, whereas other suppliers settle at competitive prices only. This instance is demonstrated through the prices given in Table 5.35. Prior knowledge of competitive target prices using the suggested model clearly indicates the possibility of a cartel, which may further be confirmed using appropriate statistical techniques (e.g., Padhi and Mohapatra [244]). The data analysis presented hereinafter in this context estimates the potential opportunity-loss to the buyer due to this cartel, in the situation of unavailability of a decision-support. This analysis counts further one more accomplishment of the suggested model.

		DL($\boldsymbol{j}$)																			
		1				2				3				4				5			
supplier	product	time period (t)				time period (t)				time period (t)				time period (t)				time period (t)			
($\boldsymbol{k}$)	(n_k)	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	1	0	0	0	0	125	100	75	50	50	0	25	25	0	0	0	0	0	0	0	0
2	2	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	3	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	200	100	120	30
4	4	50	75	75	50	0	0	0	0	0	0	0	0	60	60	80	0	65	40	20	60
5	2	0	0	0	0	0	0	0	0	50	29	33	0	50	46	22	0	0	0	0	0
	3	105	55	25	0	0	0	0	0	0	0	0	0	0	0	0	0	130	130	90	0
6	1	150	75	100	75	0	0	0	0	25	75	25	25	75	50	75	50	0	0	0	50
	4	0	0	0	0	25	25	50	25	75	75	50	0	0	0	0	0	0	0	0	0
7	2	**75**	**75**	**75**	**75**	**75**	**75**	**75**	**50**	**25**	**21**	**42**	**25**	**25**	**29**	**28**	**25**	**100**	**100**	**80**	**50**
	3	**20**	**70**	**75**	**50**	**100**	**75**	**75**	**75**	**100**	**75**	**50**	**50**	**80**	**80**	**100**	**15**	**0**	**0**	**0**	**90**
	4	0	0	0	0	25	50	25	0	0	0	0	0	0	0	0	0	15	40	60	0

Table 5.34: Buyer's demand order allocation y_{kn_kjt} (number of tons to be purchased) - Market sweeping strategy of supplier 7

If the final prices, given in Table 5.35, become an outcome of price negotiation with an uninformed buyer, it would incur a total procurement cost of INR 176,184,900 (through demand allocation given in Table 5.36), against INR 166,871,150 corresponding to the oligopolistic-competitive prices listed in Table 5.11 (as obtained using the proposed model). This would result in an additional cost of INR 9,313,750, which is 5.58% higher than the procurement cost against the competitive prices. Whereas the total profits of each supplier in both the situations discussed above are listed in the Table 5.37 to demonstrate an escalation in individual profit of suppliers 1, 3, and 6 with rest of suppliers remaining unaffected. This profit making price adherence by these three suppliers therefore indicates a possibility of a cartel and not just coincidence.

		DL ($\boldsymbol{j}$)				
supplier ($\boldsymbol{k}$)	product (n_k)	1	2	3	4	5
1	1	**32899.92**	**33000**	**31175**	**30300**	**29800**
2	2	5560	5560	5950	5560	5160
	3	5689.914	5850	5450	5050	4850
3	1	**32899.92**	**33000**	**31175**	**30300**	**29800**
4	4	89600	94200	93504.25	91000	90600
5	2	4750	4850	5000	5000	4650
	3	4695	4795	4945	5150	4595
6	1	**32899.92**	**33000**	**31175**	**30300**	**29800**
	4	92499.98	89800	89350	92050	93292.56
7	2	6250	6499.984	6824.942	6800	6100
	3	6350	6599.426	6925	6882.551	6200
	4	92707.78	92800	92875	96700	91500

Table 5.35: Suppliers' prices p_{kn_kj} (INR per tonne) - Possible cartel by suppliers 1, 3, and 6 for product 1

		DL (j)																			
		1				2				3				4				5			
supplier	product	time period (t)				time period (t)				time period (t)				time period (t)				time period (t)			
(***k***)	(n_k)	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	1	40	16	21	14	34	20	17	11	22	17	13	11	22	12	17	11	50	20	24	15
2	2	0	0	0	0	0	0	0	0	0	0	0	0	25	0	0	0	100	100	80	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	80	80	100	15	105	55	0	0
3	1	52	28	33	24	43	37	25	17	26	28	17	17	26	18	25	17	70	37	39	26
4	4	50	75	75	50	0	0	0	0	0	0	0	0	60	60	80	0	65	40	20	60
5	2	75	75	75	75	75	75	75	50	75	50	75	25	50	75	50	25	0	0	0	50
	3	125	125	100	50	100	75	75	75	100	75	50	50	0	0	0	0	25	75	90	90
6	1	58	31	46	36	48	43	33	22	28	31	20	22	28	20	33	22	80	43	57	39
	4	0	0	0	0	25	25	50	25	75	75	50	0	0	0	0	0	0	0	0	0
7	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	25	50	25	0	0	0	0	0	0	0	0	0	15	40	60	0

Table 5.36: Buyer's demand allocation y_{kn_kjt} (number of tonnes to be purchased) – possible cartel

	suppliers (k)						
profit against	1	2	3	4	5	6	7
cartel prices in Table 5.35	**42,36,125**	7,25,950	**29,26,200**	51,43,500	10,40,500	**97,90,375**	17,84,500
competitive prices in Table 5.11	21,15,000	7,25,950	10,35,000	51,43,500	10,40,500	48,13,750	17,84,500

Table 5.37: Comparison of profits of suppliers resulting from competitive prices and from cartel prices

The above comparison indicates the worthiness of our proposed model for a buyer to assess competitive target prices prior to a price negotiation with suppliers.

5.6 Managerial implications

Negotiations handled transparently using this decision support system will portray the buyer's intention of creating constructive competition and non-indulgence into opportunistic discrimination with the motive of minimizing procurement cost. Similarly it will ensure that the suppliers arrive at a settlement price for supply which have adequate margin of safety with regard to their profitability. Thus the use of developed decision support will ensure that there is no opportunistic discrimination from the buyer using his bargaining power as a single buyer and similarly the suppliers too cannot reap enormous surplus profits defying the competition. Hence, this model helps to arrive at that satisficing point of demand and supply which will be logically beneficial for both buyers and sellers. Further, the knowledge of competitive target prices enables the buyer to rule out any undue influence from suppliers' side.

In all, the decision support system developed for our study will help in creating healthy entrepreneurial platform in which the business interests of the buyer and sellers are protected. Eventually, in such a business relationship with the suppliers inculcates a support behaviour in them which proves to be helpful in combating sudden unforeseen contingency situations in the future. The use of this decision support system can help even in reducing the lengthy negotiation and communication time span as the target prices ascertained by the model can be used as a consensus base prices for fixing the deal.

Utilizing the proposed framework for ascertaining competitive target prices prior to negotiation further enables the buyer to

- surmise and accordingly intervene in case if any supplier attempts to adopt any market-sweeping techniques through abnormal low prices during the negotiation,
- identify the possibility of any cartel formation by the suppliers and take safeguarding measures through alternative channels of sourcing.

In the event of any market sweeping technique or cartel formation, the buyer with prior knowledge of competitive target prices will come to know about this at its inception and counter strategies can be formulated at the right time without causing any damage to the operational efficiencies of the buyer's organization.

The proposed model can be appropriately used by any buyer firm which is concerned for minimizing the total procurement costs by creating a healthy competition among its suppliers through a non-cooperative game for their price quotes. The scope of application of our model is listed below.

1. The model can be used by government authorities for assessment of competitive prices for the tenders invited for the purchase of required material. An indication can be obtained about the existing suppliers for any possible cartels, as these practices are prevalently found among those supplying products to government organizations.
2. In case of the setup of strategic business units by the suppliers dealing in multiple products, our model is capable of catering such a scenario due to consideration of transportation cost as product dependent.
3. This model can be used by non-trading and non-manufacturing organizations like Universities, hospitals which procure a sizable proportion of purchases locally.
4. The situation of suppliers having multiple PCs can also be catered through simple extension of our model by merely introducing additional index for the PCs.

5.7 Conclusions

This work addresses the problem of ascertaining competitive target prices for the buyer of an oligopolistic-monopsony market to negotiate with suppliers in line with a balanced approach of safeguarding the financial interests of each supply-chain partner. The problem is mathematically formulated as a multi-leader-single-follower bilinear BLP problem, featuring suppliers as leaders and the buyer as a follower, as it fits naturally to the problem. Annexing suppliers' operational planning with the bilevel game problem of price-setting enables the buyer to assess the capacities of suppliers for fulfilling the demand-orders. Motivated by lack of solution methodologies to handle large-scale instances, a GA-based methodology is proposed to solve a general multi-leader-single-follower BLP problem with bilinear objectives and linear constraints. Further, a modification to the proposed algorithm is suggested to solve the game problem having a specific bilevel structure, as present in our proposed model. The model is illustrated by a study on an FMCG manufacturing firm's procurement setup having similar concerns. Comparison of computational results with the firm's current configuration and test situations demonstrate the efficacy and prominence of the suggested model.

An interesting future research direction is to solve the discussed problem with a condition on the quantities of products as integers only. Another challenging potential problem can be addressed for the case of differential pricing. Similar problems may be studied for other market setups, for example, where multiple buyers also compete to fulfil their demands-orders.

Summary and Scope of Work in Future

Summary

In this thesis, we have studied strategic planning and decision-making problems in the purview of bilevel programming framework. Our main focus in this research work has been on cohesively considering pricing and operational planning issues for suggesting a better approach of handling strategic issues. Some of complex strategic issues of planning and decision-making of Railways and business organization managing supply chain are addressed by modelling the action and reaction setup.

The development of solution methodology for solving each of the addressed problems modelled in bilevel programming framework is another field of contribution of our research work. This part of our research contribution opens a door for developing metaheuristic-based algorithms for solving those variants of bilevel programming problems for which either a methodology is not available in literature or theoretically suggested direct methods are incapable of handling large scale instances of such problem.

With necessary groundwork presented in first two chapters, in Chapter 3 we have analyzed a decision-making problem of Railways to cohesively plan for running special trains on some routes along with a pricing strategy. The formulation of the model is so designed to handle the competitive response of other transport service providers plying on such routes. Thereby the model is formulated as a mixed integer single-leader-multi-follower bilevel programming problem. A GA-based solution methodology is proposed for solving the addressed problem. A detailed computational study of Indian Railways is presented in this context with comparison analysis demonstrating the consequences of ignoring the competition into modelling.

In Chapter 4, a strategic planning problem of a small-scale supplier is addressed. In the study presented in this chapter, a decision-support is developed to identify target prices for negotiation of such a supplier with a quality seeking buyer. The problem is addressed for maximizing the profit of the supplier through a successful penetration into the market engendered by the buyer and some existing suppliers with their known fixed prices for same products. The model is formulated in a bilevel programming framework involving integer and binary variables at both levels and bi-objective programming problem at follower's level. Due to absence of a solution methodology for solving such a bilevel programming problem, we have proposed a GA-based approach. An illustrative case study of an appropriate supplier firm is presented to demonstrate an implementation of the proposed algorithm for computations of the relevant problem fitting the modelled framework.

Further, Chapter 5 deals with the game theoretic mechanism of a variant of bilevel programming framework which involves multiple leaders and a single follower. A strategic problem of identifying target prices for negotiation of a buyer with multiple suppliers is addressed in the presented study. For this purpose, a mathematical model is formulated as a multi-leader-single-follower BLP problem. Our second major contribution presented in this chapter is to develop a GA-based solution methodology for handling large-scale instances of multi-leader-single-follower bilevel programming problem with bilinear objective functions at both levels. An appropriate case study of an Indian firm from FMCG sector is presented with computational analysis performed through an implementation of the proposed algorithm.

In a nut shell, the research work compiled in this thesis includes contributions in both the areas of optimization. The first one being application of BLP into managerial decision-making through modelling. Whereas, the second being algorithmic development for solving optimization problem, particularly in BLP framework.

Scope of work in future

The research work complied in this thesis is merely an initiation of the possible research work in this area. There are enormous opportunities for research waiting in terms addressing strategic planning problems which require incorporating reactions of other decision-makers while taking decisions on important issues. Developing solution methodologies capable of handling large-scale instances of variants of bilevel programming problems is identified as another direction of future research.

Although the results of computational experiments of case studies support the success and creditability of solution methodologies proposed in this work, still there is a room for increased success in terms of computational efficiency. One such possibility lies in designing a parallel genetic algorithm for the considered class of bilevel programming problems. Future work may also include developing other artificially intelligent techniques. The robustness issue of designed models, especially discovering how the small changes in the input parameters can affect the overall solution found as a result of optimization, can be studied in future.

In case of planning problem discussed for Railways, the uncertainty in the total demand, availability of rolling-stock and costs can be modeled as stochastic or fuzzy variables to design a more robust operational planning for the railways. Pricing strategies to be handled in a setup of differential pricing are another set of challenging problems, which can be addressed in future using the bilevel programming framework. One of our research work on studying the decision behaviour of sole supplier receiving contingent demand from multiple buyers [245] is noteworthy for incrementing further to develop a negotiation mechanism.

Appendix

Some results on convexity

Result A.1: Let $f(x)$ is a convex decreasing function in x defined as

$$f(x) = \delta \frac{e^{(a-bx)}}{(1 + K + e^{(a-bx)})} \qquad \forall\, x \in \mathbb{R}^+$$

where, δ, a, b, K are positive constants. Let x_0 and y_0 be fixed positive constants. Then,

(a) the constrained bilinear optimization problem (A.1)

$$\boxed{\begin{aligned} &\max_{x,y}\ xy \\ &\text{s.t.}\ \ x \geq x_0, \\ &\qquad y \leq f(x), \\ &\qquad 0 \leq y \leq y_0 \end{aligned}} \tag{A.1}$$

is equivalent to the optimization problem (A.2)

$$\boxed{\begin{aligned} &\max_{x,y}\ xy \\ &\text{s.t.}\ \ x \geq x_0, \\ &\qquad y \leq f(x), \\ &\qquad 0 \leq y \leq y_0; \end{aligned}} \tag{A.2}$$

(b) the objective function $xf(x)$ of optimization problem (A.2) is a strictly quasi-concave function.

Proof: (a) As the given function $f(x)$ is non-negative decreasing function with $\lim_{x\to\infty} f(x) = 0$, therefore for the case of non-redundancy of constraints of optimization problem (A.1) bounding the variable y from above, the curve $y = f(x)$ intersects the line $y = y_0$ at some point (say, A) in the first quadrant of x-y plane. Let $\tilde{x}$ be the ordinate of the point of intersection A of the line $y = y_0$ and the curve $y = f(x)$. In any of the two cases, when $0 < x_0 < \tilde{x}$ or $x_0 \geq \tilde{x}$, the optimal solution of problem (A.1) is attained at some point $B\big(x, f(x)\big)$ on the curve $y = f(x)$ with $f(x) \leq y_0$ and $x \geq x_0$. The two situations are depicted in Figure A.1. Therefore, the optimization problem (A.1) is equivalent to solving the optimization problem (A.2).

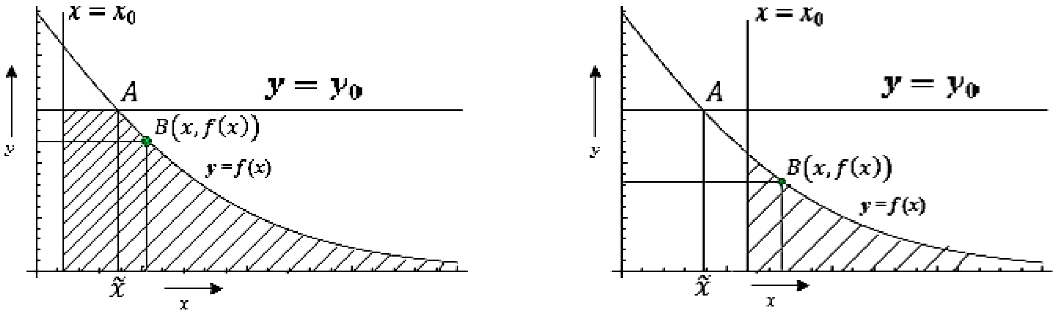

Figure A.1: Cases when problem (A.1) attains its optimal solution

(b) As $f(x)$ is convex non-negative decreasing function on $\mathbb{R}^+$, therefore $xf(x)$ is a strictly quasi-concave function by Result A.2 (proved below).

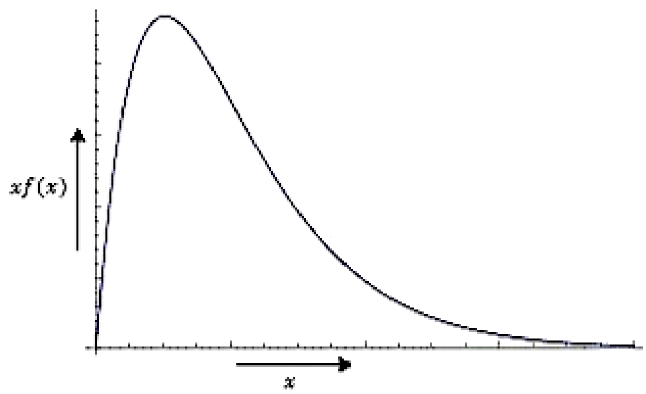

Figure A.2: Objective function of optimization problem (A.2)

Result A.2: For a non-negative decreasing convex function defined on $\mathbb{R}^+$, the function $xf(x)$ is strictly quasi-concave.

Proof: For a given non-negative decreasing convex function $f(x)$ defined on $\mathbb{R}^+$, let us define $g(x) = xf(x)\ \forall\ x \in \mathbb{R}^+$. Let $a, b \in \mathbb{R}^+$, $a \neq b$ and $\lambda \in (0, 1)$. Without loss of generality, let us consider $a < b$. Consider,

$$\begin{aligned}
g\big((1-\lambda)a + \lambda b\big) &= \big((1-\lambda)a + \lambda b\big)\, f\big((1-\lambda)a + \lambda b\big) \\
&\geq \big((1-\lambda)a + \lambda b\big)\, f(b) \qquad \text{(as } f(x) \text{ is a decreasing function)} \\
&= (1-\lambda)a\, f(b) + \lambda b\, f(b) \\
&= (1-\lambda)a\, f(b) + \lambda\, g(b) \\
&= \lambda\, g(b) + (1-\lambda)\big(a\, f(b) - b\, f(b) + b\, f(b)\big) \\
&= g(b) + (1-\lambda)(a-b)f(b) \\
&> g(b) \qquad \text{(as } a < b \text{ and } f(b) \geq 0\text{)} \\
&> \min\{g(a), g(b)\}.
\end{aligned}$$

This proves that,

$$g\big((1-\lambda)a + \lambda b\big) > \min\{g(a), g(b)\}\ \forall a, b \in \mathbb{R}^+, a \neq b, \lambda \in (0, 1).$$

Therefore, $g(x) = x\, f(x)$ is a strictly quasi-concave function.

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List of Publications

1. **Akhilesh Kumar**, Anjana Gupta, Aparna Mehra; *A bilevel programming model for a cohesive decision-making on strategic pricing and production distribution planning for a small-scale supplier*, International Game Theory Review, World Scientific Publishing Co., 22 (2) 204009-1–34 (2020). doi:10.1142/S0219198920400095.

2. **Akhilesh Kumar**, Anjana Gupta, Aparna Mehra; *A bilevel programming model for operative decisions on special trains: An Indian Railways perspective*, Journal of Rail Transport Planning and Management, Elsevier, 8, 184–206 (2018). doi:10.1016/j.jrtpm.2018.03.001.

3. **Akhilesh Kumar**, Anjana Gupta, Aparna Mehra; *Contingent demand fulfilment decision problem from supplier's perspective*, IEEE X-plore, 2271–2277 (2016). doi:10.1109/ICEEOT.2016.7755098.

4. **Akhilesh Kumar**, Anjana Gupta, Aparna Mehra; A bilevel game model for ascertaining target prices for a buyer in negotiation with multiple suppliers, communicated to Omega, Elsevier.