

Note: Attempt any five questions and all questions carry equal marks.

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- (1) (a) Find the inverse Laplace transforms of

$$\frac{s-1}{s^2-6s+25}$$

- (b) Find the Laplace transforms of

$$\frac{1}{t}(\cos at - \cos bt)$$

- (2) (a) Find the  $A^{-1}$  by Gauss-Jordan method

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

- (b) Test for consistency the following system and find solution, if it exists:

$$\begin{aligned} 4x - 2y + 6z &= 8 \\ x + y - 3z &= -1 \\ 15x - 3y + 9z &= 21. \end{aligned}$$

- (3) (a) Apply convolution theorem to evaluate

$$L^{-1} \left\{ \frac{s}{(s^2 + a^2)^2} \right\}$$

- (b) Using Laplace transforms, find the solution of the initial value problems

$$\frac{d^2y}{dx^2} + y = 3 \cos 2x, \text{ at } y = \frac{dy}{dx} = 0, \text{ where } x = 0.$$

- (4) (a) Evaluate by change the order of integration in the integral

$$\int_x^{\sqrt{2-x^2}} \frac{xy dx}{\sqrt{x^2+y^2}}$$

- (b) Evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} dz dy dx.$$

- (5) (a) Show that

$$\sin^{-1} x = x + \frac{1^2}{3!}x^3 + \frac{1^2 3^3}{5!}x^5 + \frac{1^2 3^2 5^2}{7!}x^7 + \dots$$

Hence find the value of  $\pi$ .

- (b) Find the length of curve  $y^3 = x^3$  from origin to the point (1, 1).

- (6) (a) Find a series of cosines of multiple of  $x$  which will represent  $x \sin x$  in the interval  $(0, \pi)$ .

- (b) Find the Fourier series for the function  $f(x)$  in the interval  $(-\pi, \pi)$ , where

$$f(x) = \begin{cases} \pi + x, & -\pi < x < 0, \\ \pi - x, & 0 < x < \pi. \end{cases}$$

- (7) (a) Evaluate the integral

$$\int_0^1 \frac{x^2}{1+x^3} dx$$

by using Simpson's 1/3rd rule.

- (b) Find by Newton-Raphson iterative method, the real root of the equation  $3x = \cos x + 1$ .

- (8) (a) Use Runge-Kutta method of fourth order to solve the differential equation

$$\frac{dy}{dx} = x - y^2,$$

with  $y(0) = 1$  to find  $x = 0.2$  and  $h = 0.1$ .

- (b) Solve the equations:

$$\begin{aligned} 20x + y - 2z &= 17; \\ 3x + 20y - z &= -18; \\ 2x - 3y + 20z &= 25. \end{aligned}$$

by Gauss-Seidal iteration method.