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FRACTIONAL ORDER OSCILLATORS USING OTAs

A DISSERTATION

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Submitted by:

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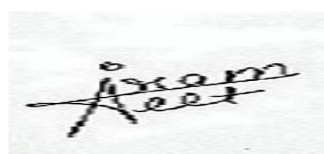

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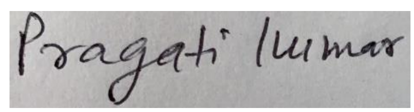

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ABSTRACT

Fractional order harmonic oscillators are sinusoidal oscillators in which, the reactive element, instead of being an integer order capacitor/inductor, is a fractional order reactive element, also referred as a constant phase element (CPE). The interest in these fractional order oscillators have stemmed from the fact that, unlike a conventional RC oscillator, the frequency and phase relationships in a fractional order oscillator are functions of the fractional order parameters α, β, γ… defining the CPE. Though the concept of a fractional order oscillator was introduced in the context of an FM demodulation system long back, it could not gather much attention till the early 2000s. The detailed theoretical framework for the general fractional order oscillators, however, was developed during the early years of the last decade, when wherein the design equations for CO and FO of fractional order oscillators were presented. Since then, many fractional order oscillator circuits employing different types of amplifiers and other active building blocks have been presented. Out of the fractional order oscillators reported, those oscillators which employ the OTA, are of special interest, as the control over the FO, CO and phase in these oscillator circuits can be implemented very easily by changing the bias currents of the OTAs. Very little work is available in open literature, wherein, fractional order oscillators have been realized with OTAs. In this work, we have presented OTA based fractional order oscillators, which employ two/three fractional order capacitors and five OTAs, providing two/three output voltages with electronically controllable phase difference between them. The dependence of FO, CO and phase difference between the output voltages on α, β and γ has been investigated in detail.

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LIST OF ABBREVIATIONS

OTA	Operational Transconductance Amplifier
CPE	Constant Phase Element
CPZ	Constant Phase Zone
FOC	Fractional Order Capacitor
CFE	Continued Fraction Expansion
CFOA	Current Feedback Operational Amplifier
OTRA	Operational Trans-resistance Amplifier
CDBA	Current Differencing Buffered Amplifier
CCII	Current Conveyor Second Generation
FC	Fractional Capacitor
FO	Frequency of oscillation
CO	Condition of oscillation
FOE	Fractional order Element
PSK	Phase shift Keying

CHAPTER 1

INTRODUCTION

1.1 Origin of Fractional Calculus

This dissertation deals with realization of fractional order oscillators using operational transconductance amplifiers (OTA).

Fractional order calculus and conventional (integer order) calculus are more than three centuries old mathematical tools. Fractional order calculus is, in a way, more comprehensive than integer order calculus to the extent that the latter is a special case of the former. The possibility of existence of non-integer derivative of a function was discussed between Leibniz and L'Hospital. Several other mathematicians, notable among them being, Euler, Laplace, Fourier, Lacroix, Abel, Riemann and Liouville, have contributed to the development of fractional order calculus. In 1819, the first paper was published on fractional derivative by mathematician Lacroix [1]. Beginning with $y = x^m$, where m is a positive integer, Lacroix found the n^{th} derivative of y as:

$$\frac{d^n y}{dx^n} = \frac{m!}{(m-n)!} x^{m-n} \quad m \geq n \qquad 1.1$$

and using Legendre's symbol Γ, for the generalized factorial, he expressed

$$\frac{d^n y}{dx^n} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)} x^{m-n} \quad m \geq n \qquad 1.2$$

1.2 Definition of Fractional Calculus

Fractional calculus does not imply fraction of any conventional differentiation, integration, or calculus of variations. It may be defined as the theory of integrations and derivatives of arbitrary order. The non-integer integration or derivative have been defined over the years. Several mathematicians have used their own notations and approaches. A popular version that has been most widely accepted in the world of fractional calculus is the Riemann-Liouville definition. The same result were arrived at, by Riemann-Liouville definition of a fractional derivative that were obtained by Lacroix. Some of the fractional order calculus definition are given as follow:

(i) Riemann-Liouville:

Integral:

$$J_C^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_c^{\tau} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau \qquad 1.3$$

Derivative:

$$D^{\alpha} f(t) = \frac{d^m}{dt^m}\left[\frac{1}{\Gamma(m-\alpha)} \int_0^{\tau} \frac{f(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau\right] \quad , m \in \mathbb{Z}^{+}, \; m-1 < \alpha \le m \qquad 1.4$$

(ii) Caputo

$$D^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^{\tau} \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau \qquad 1.5$$

Some other tools of interest for engineers are the classical transforms of Laplace and Fourier, that are valid and used in order to simplify operations like convolution and can be applied to solve fractional calculus differential equations.

1.2.1 Laplace Transform of Fractional Order Integral

The fractional integral of y(t) of order ν is defined as:

$$D^{-\nu} y(t) = \frac{1}{\Gamma(\mathrm{V})} \int_0^t (t-z)^{\nu-1} y(z) dz \quad , \nu > 0 \qquad 1.6$$

Using Laplace transform in above equation 1.6

$$\mathcal{L}\{D^{-\nu} y(t)\} = \frac{1}{\Gamma(V)} \mathcal{L}\{t^{\nu-1}\} \mathcal{L}\{y(t)\} = s^{-\nu} Y(s) \qquad v > 0 \qquad 1.7$$

Equation (1.7) is the Laplace transform of the fractional integral. As an example, we see for $\nu > 0$, $\mu > -1$

$$\mathcal{L}\{D^{-\mathcal{V}} t^{\mu}\} = \frac{\Gamma(\mu - 1)}{s^{\mu+\mathcal{V}+1}} \text{ and } \mathcal{L}\{D^{-\mathcal{V}} e^{at}\} = \frac{1}{s^{\mathcal{V}}(s-a)} \qquad 1.8$$

1.2.2 Laplace Transform of Fractional Order Derivative

The fractional derivative operator can be defined using the definition of the fractional integral. To this end, suppose that $\mathcal{V} = n - u$, where $0 < \mathcal{V} < 1$ and n is the smallest integer greater than u. Then, the fractional derivative of f(x) of order is u.

$$D^u f(x) = D^n [D^{-v} f(x)] \qquad 1.9$$

We recall that in the integer order operations, the Laplace transform of y(n) is given by

$$\mathcal{L}\{D^n y(n)\} = s^n Y - s^{n-1} y(0) - s^{n-2} y'(0) - s^{n-3} y''(0) \dots \dots . y^{n-1}(0)$$

$$= s^n Y(s) - \sum_{k=0}^{n-1} s^{n-k-1} y^k(0) \qquad 1.10$$

Using above equation, we can write, $D^v f(x) = D^n \left[D^{-(n-v)} f(x)\right]$ 1.11

Now, if we assume that the Laplace transform of y(t) exists, then by the use of (1.10) we have

$$\mathcal{L}\{D^v y(t)\} = \mathcal{L}\left\{D^n \left[D^{-(n-v)} y(t)\right]\right\}$$

$$= s^n \mathcal{L}\left[D^{-(n-v)}y(t)\right] - \sum_{k=0}^{n-1} s^{n-k-1} D^k \left[D^{-(n-v)}y(t)\right]_{t=0}$$

$$= s^n\left[s^{-(n-v)}Y(s)\right] - \sum_{k=0}^{n-1} s^{n-k-1} D^{k-n+v} y(0)$$

$$= s^v Y(s) - \textstyle\sum_{k=0}^{n-1} s^{n-k-1} D^{k-n+v} y(0) \qquad 1.12$$

1.3 Engineering Applications

The systems with higher-order dynamics and complex nonlinearities can be modeled easily by fractional order calculus using few coefficients [3-5], since it offers additional degree of freedom, in the choice of arbitrary order of the derivatives, to fit an specific behavior. Another important quality of the fractional order derivatives is that it depends not only on local conditions of the evaluated time, but also on all the history of the function.

Fractional order calculus has been employed to design analog signal processing and generation circuits during the last two decades. The following classes of analog circuits have been designed using (i) either the fractional order immittance element, popularly known as the constant phase element (CPE), or (ii) realizing a rational transfer function resulting from substitution of an approximation for the fractional order transfer function:

(i) Fractional order filters- The general methods for realization of first and second order filters in fractional order form were presented in [6-7]. Since then, fractional order filters using various active building block like the operational amplifier [8], current feedback operational amplifier [9], current conveyors [10], operational trans-resistance amplifier [11] and various other types of amplifiers have been presented in the open literature. Most of these filter circuits have been realized using one of the two approached mentioned above. the characterizing parameters of these filters have been shown to be controlled by appropriately choosing the order, ‘α’ of the fractional order transfer function.

(ii) Fractional order universal filters- A universal filter is a filter circuit in which all the generic filtering functions, namely, low pass, band pass, high pass, band reject and all pass, are available simultaneously. In open literature, various fractional order universal filters realized with different active building blocks viz. the classical operational amplifiers [12], operational transconductance amplifier [13], balanced output transconductance amplifier, adjustable current amplifier [14], voltage differencing transconductance amplifier [15] and the current differencing buffered amplifier [16], etc.

(iii) Fractional order Oscillator-Several realizations of fractional order oscillators have also appeared in open literature in recent past [17-22]. It has been shown in these works that it is possible to initiate and maintain sustained oscillations at a given frequency in an autonomous circuit with more than one fractional order capacitors. The frequency of oscillation and the condition of oscillation can be controlled over a much wider range because of the extra degree of freedom provided by the order α the fractional order elements.

(iv) Fractional order inverse filters- Inverse filters are frequency selective circuits whose frequency response is reciprocal of the frequency response of a conventional filter [23]. Several fractional order inverse filter circuits have been reported previously, realized with various active building blocks viz. operational amplifier [24], current feedback operational amplifier [25], operational trans-resistance amplifier [26], and second generation current conveyor [27] etc.

1.4 Objectives

On the basis of literature survey, it has been noted that compare to fractional order filters and inverse filters, relatively less work has been reported on the realization of fractional order oscillators. We have, thus framed the following objectives for this work:

i. Review of various methods of design of fractional order capacitors in general and implementation of fractional order capacitors for different values of the order of the capacitor using the Oustaloup, Levron, Mathieu and Nanot method.

ii. Design, simulation and experimental validation of an electronically tunable fractional order oscillator with two fractional order capacitors and OTAs.

iii. Design, simulation and experimental validation of an electronically tunable fractional order oscillator with three fractional order capacitors and OTAs.

1.5 Thesis Organization

Contents of the dissertation have been presented in five chapters as given below:

i. Chapter 1 provides us with the general introduction of fractional order systems and a brief review of some of the research works carried out in this area.

ii. Chapter 2 gives a detailed description about definitions and methods of designing fractional element used in engineering. Simulation results for the design of fractional order capacitor using the Oustaloup, Levron, Mathieu and Nanot method have also been included.

iii. In Chapter 3, a fractional order oscillator realized with two fractional order capacitors and three operational transconductance amplifiers has been presented. Simulation and experimental results have also been included.

iv. In Chapter 4, we have presented a new realization of an electronically tunable fractional order oscillator realized with three fractional order capacitors and five operational transconductance amplifiers.

Chapter 5 includes summary and scope for future work.

REFERENCES

[1] B. ed. Ross, 2006. Fractional calculus and its applications: proceedings of the international conference held at the University of New Haven, June 1974 (Vol. 457). Springer.

[2] C. Ma and Y. Hori, "Fractional order control and its application of PIαD controller for robust two inertia speed control," in Proceedings of the 4th International Power Electronics and Motion Control Conference (IPEMC '04), vol. 3, pp. 1477–1482, August 2004.

[3] J. De Esp´ındola, C. Bavastri, and E. De Oliveira Lopes, "Design of optimum systems of viscoelastic vibration absorbers for a given material based on the fractional calculus model," Journal of Vibration and Control, vol. 14, no. 9-10, pp. 1607–1630, 2008.

[4] F. B. M. Duarte and J. A. T. Machado, "Fractional dynamics in the describing function analysis of nonlinear friction," in Proceedings of the 2nd IFAC Workshop on Fractional Differentiation and Its Applications, vol. 2, July 2006.

[5] P. J. Torvik and R. L. Bagley, "On the appearance of the fractional derivative in the behaviour of real materials," Journal of Applied Mechanics, vol. 51, no. 2, pp. 294–298, 1984.

[6] A. G. Radwan, A. M. Soliman, and A. S. Elwakil. "First-order filters generalized to the fractional domain," Journal of Circuits, Systems, and Computers, 17, no. 1, pp. 55-66, 2008.

[7] A. G. Radwan, A. S. Elwakil, and A. M. Soliman. "On the generalization of second-order filters to the fractional-order domain." Journal of Circuits, Systems, and Computers, 18, no. 2, pp. 361-386, 2009.

[8] A. S. Ali, A. G. Radwan, and A. M. Soliman, "Fractional order Butterworth filter: active and passive realizations," IEEE journal on emerging and selected topics in circuits and systems, 3, no. 3, pp. 346-354, 2013.

[9] G. Tsirimokou, S. Koumousi, and C. Psychalinos, "Design of fractional-order filters using current feedback operational amplifiers," Journal of Engineering Science and Technology Review, 9, no. 4, pp. 71-81, 2016.

[10] F. Khateb, D. Kubánek, G. Tsirimokou, and C. Psychalinos, “Fractional-order filters based on low-voltage DDCCs,” Microelectronics Journal, 50, pp. 50-59, 2016.

[11] G.Kaur, A. Q. Ansari, and M. S. Hashmi, “Fractional order high pass filter based on operational transresistance amplifier with three fractional capacitors of different order,” Advances in Electrical and Electronic Engineering, 17, no. 2, pp. 155-166, 2019.

[12] M. C. Tripathy, K. Biswas, and S. Sen, “A design example of a fractional-order Kerwin–Huelsman–Newcomb biquad filter with two fractional capacitors of different order,” Circuits, Systems, and Signal Processing, 32, no. 4, pp. 1523-1536, 2013.

[13] G. Singh, Garima, and P. Kumar, “Fractional Order Capacitors Based Filters Using Three OTAs,” In 2020 6th International Conference on Control, Automation and Robotics (ICCAR), 2020, pp. 638-643.

[14] J. Dvorak, J. Jerabek, Z. Polesakova, D. Kubanek, and P. Blazek, “Multifunctional Electronically Reconfigurable and Tunable Fractional-Order Filter,” Elektronika ir Elektrotechnika, 25, no. 1, 2019, pp. 26-30.

[15] P. Rani, and R. Pandey, “Voltage differencing transconductance amplifier based fractional order multiple input single output universal filter,” Solid State Electronics Letters, 1, no. 2, pp. 110-118, 2019.

[16] A. Q. Ansari, G. Kaur, and M. S. Hashmi, “Current Differencing Buffered Amplifier (CDBA) based Current Mode Universal Fractional Order Filter,” Proc. Nat. Con. on Adv. in Microelectronics, Instrumentation and Communication (MICOM), BITS Pilani, India, 2015.

[17] A. G. Radwan, A. S. Elwakil, and A. M. Soliman, “Fractional-order sinusoidal oscillators: design procedure and practical examples,” IEEE Transactions on Circuits and Systems I, Regular Papers, 55, no. 7, pp. 2051-2063, 2008.

[18] W. Ahmad, R. El-Khazali, and A. S. Elwakil, “Fractional-order Wien-bridge oscillator,” Electronics Letters, 37, no. 18, pp. 1110-1112, 2001.

[19] A. G. Radwan, A. M. Soliman, and A. S. Elwakil, “Design equations for fractional-order sinusoidal oscillators: Four practical circuit examples,”

International Journal of Circuit Theory and Applications, 36, no. 4, pp. 473-492, 2008.

[20] V. Singh, and P. Kumar, “Fractional Order Oscillator Using OTAs,” In 2020 6th International Conference on Control, Automation and Robotics (ICCAR), 2020, pp. 27-32.

[21] L. A. Said, A. G. Radwan, A. H. Madian, and A. M. Soliman, “Fractional order oscillators based on operational transresistance amplifiers,” AEU-International Journal of Electronics and Communications, 69, no. 7, pp. 988-1003, 2015.

[22] A. Kartci, N. Herencsar, J. Koton, L. Brancik, K. Vrba, G. Tsirimokou, and C. Psychalinos, “Fractional-order oscillator design using unity-gain voltage buffers and OTAs,” In 2017 IEEE 60th International Midwest Symposium on Circuits and Systems (MWSCAS), 2017, pp. 555-558.

[23] S. S. Gupta, D. R. Bhaskar and R. Senani, “New analogue inverse filters realised with current feedback op-amps,” international Journal of electronics, 98, no. 8, pp.1103-1113, 2011.

[24] D. R. Bhaskar, Manoj Kumar, and Pragati Kumar, “Fractional order inverse filters using operational amplifier,” Analog Integrated Circuits and Signal Processing, 97, no. 1, pp. 149-158, 2018.

[25] E. M. Hamed, L. A. Said, A. H. Madian, and A. G. Radwan, “On the approximations of CFOA-based fractional-order inverse filters," Circuits, Systems, and Signal Processing, 39, no. 1, pp. 2-29, 2020.

[26] N. A. Khali, L. A. Said, A. G. Radwan, and A. M. Soliman, “Multifunction Fractional Inverse Filter Based on OTRA,” In 2019 Novel Intelligent and Leading Emerging Sciences Conference (NILES), vol. 1, 2019, pp. 162-165.

[27] N. A. Khalil, L. A. Said, A. G. Radwan, and A. M. Soliman, “Fractional Order Inverse Filters Based on CCII Family,” In 2019 Fourth International Conference on Advances in Computational Tools for Engineering Applications (ACTEA), 2019, pp. 1-4.

CHAPTER 2

METHODS OF REALIZING OF FRACTIONAL ORDER ELEMENT

2.1 Introduction

Fractional order elements (FOE) are the most important component of a fractional order system. These elements, however, are not yet available commercially, as standard components for different values of the fractional order parameters. As a result, these fractional order elements are realized using various approximation methods, which approximate the admittance of a fractional order element by a rational function in the complex variable ‘s’. The methods for realization of fractional order elements are categorized into: (a) single component-based realizations and (b) Multi-component -based realizations. Both of these methods have their advantages and limitations. In the present chapter, we present a brief review of these methods. We have also simulated fractional order capacitors for different values of the fractional order parameter using one of the widely used methods available in the literature.

2.2 Fractional Order Element

A fractional order immittance element follows fractional order differential equation which relates the voltage and current in it. The impedance of a fractional order element in the complex ‘s’ plane may be represented by any one of the following expressions [1]-[4]:

$$Z_F(s) = \frac{R}{(\tau s)^{\alpha}} \qquad 2.1$$

$$Z_F(s) \equiv \frac{1}{C_F(s)^{\alpha}} \qquad 2.2$$

$$Z_F(s) \equiv \frac{1}{Fs^{\alpha}} \qquad 2.3$$

Units of R and τ are Ω and second, α is a real number in equation (2.1). A fractional order capacitor, sometimes referred as fractional capacitor, C_F used in equation (2.2) has the unit of Farad/$s^{1-\alpha}$. The FOE is sometimes also referred as a fractor whose fractance 'F' expressed in equation (2.3) has the unit of ℧s^{α}. In all the above cases, the unit of the impedance of the FOE must be in Ohm.

2.3 Frequency Response of the Fractional Order Element

The fractional order element in frequency domain can be written as shown in equation below:

$$Z_F = \frac{1}{F(j\omega)^{\alpha}} = \frac{1}{F\omega^{\alpha}} \angle -\frac{\alpha\pi}{2} \qquad 2.4$$

The phase of impedance is independent of frequency relation. The magnitude can be varied by fractional number α. the phase, however, will remain constant for any value of frequency. For this reason, the fractional order element is also known as constant phase element (CPE) as, unlike a conventional reactive element, the phase of the element does not change with frequency (like lossy reactive elements). An interesting analogy between a conventional, lossless, capacitor/inductor and a fractor may be drawn in terms of the phase angle; in case of lossless capacitor/inductor, the phase angle is always fixed (±90°), whereas in case of a fractor, the phase angle is given by ± α90°[5].

2.4 Pseudo-CPE and Constant Phase Zone

In ideal case [6], the fractional order element is having a constant phase for all frequency values, but practically it does not happen. The phase remains constant in a particular frequency range only, which is known as constant phase zone. The practically realized FOE is called as pseudo-CPE. There is some oscillation in that constant phase zone in phase of fractional order element that is called ripple. The amount of ripple in phase and the frequency range of the approximation are the design parameters for several approximation techniques used to realize the CPE.

2.5 Realization of Fractional Order Element

Several methods of realization of fractional order element have been proposed in literature [2]. They may be categorized in two groups:

i. Multi component realization
ii. Single component realization

2.5.1 Multi Component Realization

Phase shifting networks are used for realizing a fractional order element in constant phase zone. It is the best possible way of determining phase differencing function in [7]. It works in two sections, first one is to achieve constant phase with the help of phase shifter and second one is to define frequency range by using all pass sections. It requires large number of inductors and capacitors.

RC Ladder - based fractional order element: Several approximatins of the fractional order operator s^{α}, namely, Valsa Dobrak and Friedel Approximation [8], Oustaloup, Levron, Mathieu, and Nanot Recursive approximation [9], Continued Fraction Expansion (CFE) [10], El-Khazali reduced-order approximation [11], Carlson and Halijak approximation [12], Matsuda and Fujii approximation [13], Modified Oustaloup, Laveron, Mathieu and Nanot [14], Charef, Sun, Tsao, and Onaral approximation [15], and Squared Magnitude Function [16-17] have been used to realize an immittance function which resembles a Foster/Cauer like network. The element values of the network are determined, based on the frequency range and the amount of ripple specified. Higher the number of series/parallel branches, in RC network, the more accurate is the approximation.

The concept of the generalized impedance converter (GIC) has been used to propose realization of a fractional order inductor [18].

2.5.2 Single Component Realization

Several attempts are being made to realize the fractional order element as a single component device, available off-the shelf, for desired values of the parameter α so that this element could be used in real circuit applications. Physical CPE using electrochemical methods, fractal structure on silicon, lithographic processes have been reported in [4], [5], [20]-[25].

2.5.3 Oustaloup, Levron, Mathieu and Nanot Approximation Method

We have used, the approximation method proposed by, Oustaloup, Leveron, Mathieu and Nanot [9] to realize the fractional order capacitors used in the oscillator circuits proposed in this work. In the following, we explain this method with a design example.

The method approximates the operator s^{α} for non-integer values of the exponent 'α' by a rational function given below:

$$s^{\alpha} = c \prod_{k=1}^{k=N} \frac{1 + s/\omega'_k}{1 + s/\omega_k} \qquad 2.5$$

Where N, is the order of the approximation and 'c' is a constant, whose value depends upon the maximum frequency upto which the approximation is valid.

$$c = \left(\omega_{\max}\right)^{\alpha} \qquad 2.6$$

ω_{max} =maximum frequency range,

ω_{min} =minimum frequency range

$$\omega'_k = \omega_{\min}\left(\frac{\omega_{\max}}{\omega_{\min}}\right)^{(2k-1-\alpha)/(2N)} \qquad 2.7$$

$$\omega_k = \omega_{min}\left(\frac{\omega_{max}}{\omega_{min}}\right)^{(2k-1+\alpha)/(2N)} \quad 2.8$$

The fractional order operator s^{α}, thus is expressed by the following rational function, if the values of α, minimum and maximum frequency are specified.

$$s^{\alpha} = \frac{N(s)}{D(s)} \quad 2.9$$

The above rational function, when representing the immittance of an FOE, can be realized either by partial fraction expansion (Foster-I/II, forms) or continued fraction expansion (Cauer-I/II). An exemplary realization in the Foster form has been shown in fig 2.1 whose driving point impedance is given by:

$$Z(s) = \frac{1}{s^{\alpha}C_{\alpha}} = R_0 + \sum_{n=1}^{N}\frac{\frac{1}{C_n}}{s+\frac{1}{C_nR_n}} \quad 2.10$$

R_o = series resistor

R_n, C_n= resistors and capacitors in parallel network

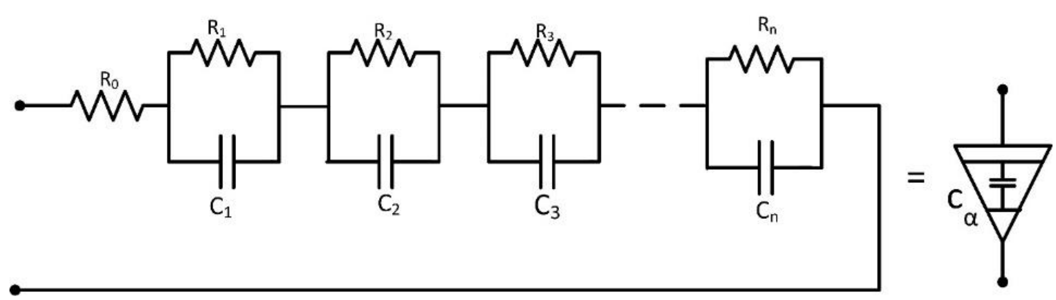


Figure 2.1 Fractional capacitor network

Design example: Let α=0.5 ω_{min}=10Hz, ω_{max}=1MHz, C_α=1uf, N=5

$$Z(s)=\frac{1}{C_\alpha s^\alpha}=\frac{1}{1\times10^{-6}\times s^\alpha}$$

Putting Laplacian operator s^α equation (2.5) in above equation

$$Z(s)=\frac{1}{1\times10^{-6}\times s^\alpha}=\frac{1}{10^{-6}c\prod_{k=1}^{k=N}\frac{1+s/\omega'_k}{1+s/\omega_k}}=\frac{1}{10^{-6}c}\prod_{k=1}^{k=N}\frac{1+s/\omega_k}{1+s/\omega'_k}$$

Using equation (2.6)-equation (2.8) , given , ω_{min}=10Hz, ω_{max}=1MHz, N=5, α=0.5,

we obtain c=1000, $\omega'_k=10\left(\frac{10^6}{10}\right)^{(2k-1-0.5)/(2N)}$ and $\omega_k=10\left(\frac{10^6}{10}\right)^{(2k-1+0.5)/(2N)}$

Thus, $\omega'_1=10\left(\frac{10^6}{10}\right)^{0.5/2}$ =17.78, $\omega'_2=10\left(\frac{10^6}{10}\right)^{(2.5)/(4)}$ =177.82, $\omega'_3=10\left(\frac{10^6}{10}\right)^{(4.5)/(6)}$

=1.7*10^3, $\omega'_4=10\left(\frac{10^6}{10}\right)^{(6.5)/(8)}$ =17.7*10^3, $\omega'_5=10\left(\frac{10^6}{10}\right)^{(8.5)/(10)}$ =177.7*10^3

$\omega_1=10\left(\frac{10^6}{10}\right)^{(1.5)/(2)}$ =56.23, $\omega_2=10\left(\frac{10^6}{10}\right)^{(3.5)/(4)}$ =562.3, $\omega_3=10\left(\frac{10^6}{10}\right)^{(5.5)/(6)}$ =56.23,

$\omega_4=10\left(\frac{10^6}{10}\right)^{(7.5)/(8)}$ =5.623*10^3, $\omega_5=10\left(\frac{10^6}{10}\right)^{(9.5)/(10)}$ =562.3*10^3

Then driving point impedance thus turns out to be:

Z(s) =

$$\frac{1000s^5+6.248\times10^8s^4+3.549\times10^{13}s^3+1.996\times10^{17}s^2+1.11\times10^{20}s+5.623\times10^{21}}{s^5+1.976\times10^5s^4+3.549\times10^9s^3+6.311\times10^{12}s^2+1.111\times10^{15}s+1.778\times10^{16}}$$

The values of Resistors and capacitors of the ladder has been found as:

R_o=1000Ω, R_1=1.6kΩ R_2=5.4kΩ, R_3=17.4kΩ, R_4=56.3kΩ,R_5=234.5kΩ

C_1=3.5nF, C_2=10nF, C_3=32nF, C_4=99.8nF, C_5=0.23uF

We have simulated the magnitude and phase response of the RC impedance network realized with the above values of components in PSPICE. The AC responses are shown below:

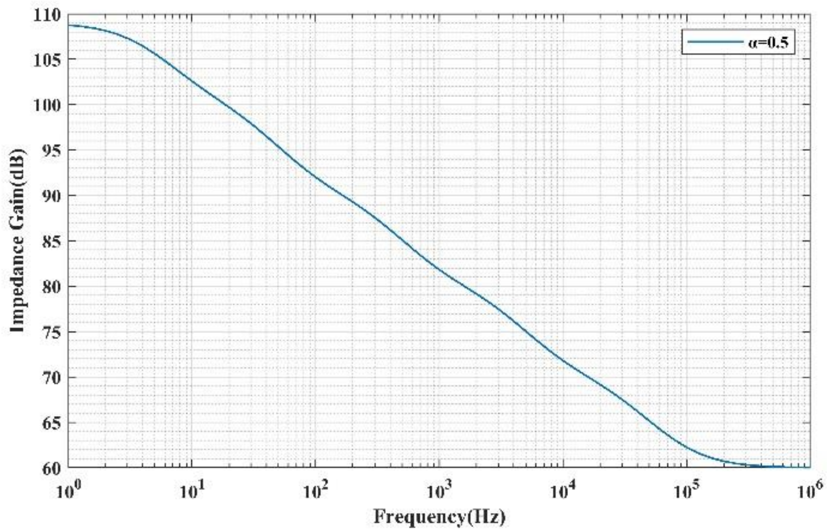


Figure 2.2 Impedance gain response

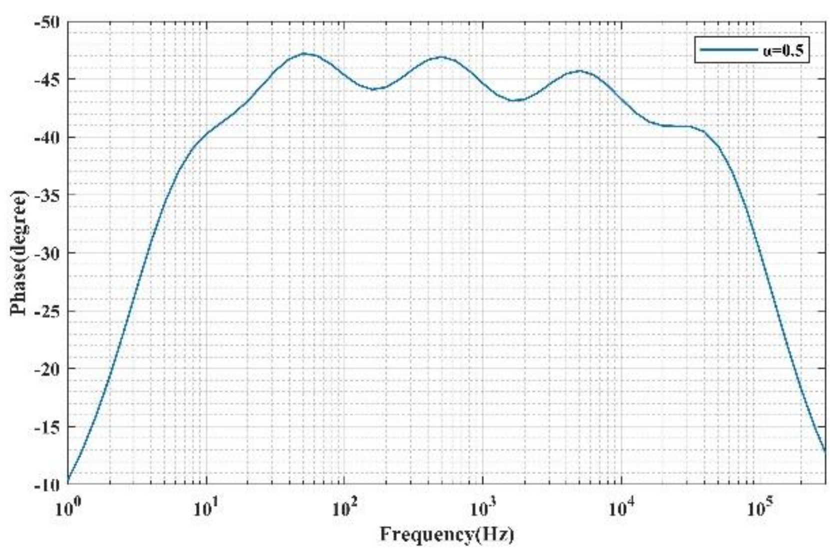


Figure 2.3 Phase response

From the phase response it is noted that there are ripples in the response around the desired phase value (45° as α =0.5) but we can reduce the ripple amplitude by increasing the value of N, the order of approximation. It was found that the decrease in the amplitude was not significant for N> 8. The complexity of ladder structure increase for increase in N. We have computed the values of all the passive components for ladder for which N =8 and values of α = 0.1-0.9, in steps of 0.1 and the desired capacitor value is 0.01uF.

Table 2.1 Value of resistors and capacitor of FC

$\boldsymbol{\alpha = 0.1}$ **N=8**		$\boldsymbol{\alpha = 0.2}$ **N=8**		$\boldsymbol{\alpha = 0.3}$ **N=8**	
$\boldsymbol{R_0(\Omega)}$ =16M		$R_0(\Omega)$=2.75M		$R_0(\Omega)$=0.45M	
R_n(MΩ)	C_n(pF)	R_n(MΩ)	C_n(pF)	R_n(MΩ)	Cn(pF)
4.16	0.013	1.49	0.042	0.40	0.17
5.36	0.10	2.47	0.25	0.843	0.84
6.76	0.835	3.93	1.612	1.69	4.20
8.51	6.63	6.23	10.171	3.37	21.05
10.72	52.69	9.87	64.16	6.74	105.5
13.50	418.47	15.66	404.64	13.46	528.35
17.04	3315.3	24.97	2538.0	27.13	2621.4
22.09	25577.0	42.33	14973.0	60.8	11685.0

Table 2.2 Values of resistors and capacitor of FC for alpha 0.4 to 0.6

$\boldsymbol{\alpha = 0.4}$ **N=8**		$\boldsymbol{\alpha = 0.5}$ **N=8**		$\boldsymbol{\alpha = 0.6}$ **N=8**	
$\boldsymbol{R_0(\Omega)}$ =76K		$\boldsymbol{R_0(\Omega)}$ =12K		$\boldsymbol{R_0(\Omega)}$ =2.09K	
R(MΩ)	C(pF)	R(MΩ)	C(pF)	R(MΩ)	C(pF)
0.093	0.84	0.020	4.42	0.0040	24.8
0.25	3.16	0.068	13.01	0.017	57
0.63	12.5	0.21	40.85	0.069	14.3
1.59	49.88	0.69	129.1	0.27	36.1
4.0	198.5	2.19	408.1	1.10	906.5

10.11	789.2	6.95	1288.8	4.42	2270.8
25.7	3094.1	22.47	3984	18.16	5531.3
77.8	10249.0	93.55	9571	108.1	9291.8

Table 2.3 Values of resistors and capacitor of FC for alpha 0.6 to 0.9

$\alpha = 0.7$ N=8		$\alpha = 0.8$ N=8		$\alpha = 0.9$ N=8	
$R_0(\Omega)$ =347.8		$R_0(\Omega)$ =57.76		$R_0(\Omega)$ =9.59	
R(MΩ)	C(nF)	R(MΩ)	C(nF)	R(MΩ)	C(nF)
0.000738	0.15	0.00011	1.06	0.000014	10.1
0.004	0.27	0.00081	1.54	0.00012	11.5
0.02	0.55	0.005	2.43	0.0009	14.4
0.10	1.1	0.03	3.85	0.007	18.2
0.512	2.19	0.207	6.10	0.06	22.8
2.57	4.37	1.31	9.62	0.4	28.6
13.47	8.36	8.75	14.42	4.2	33.5
121.8	9.25	134.77	9.38	142.2	9

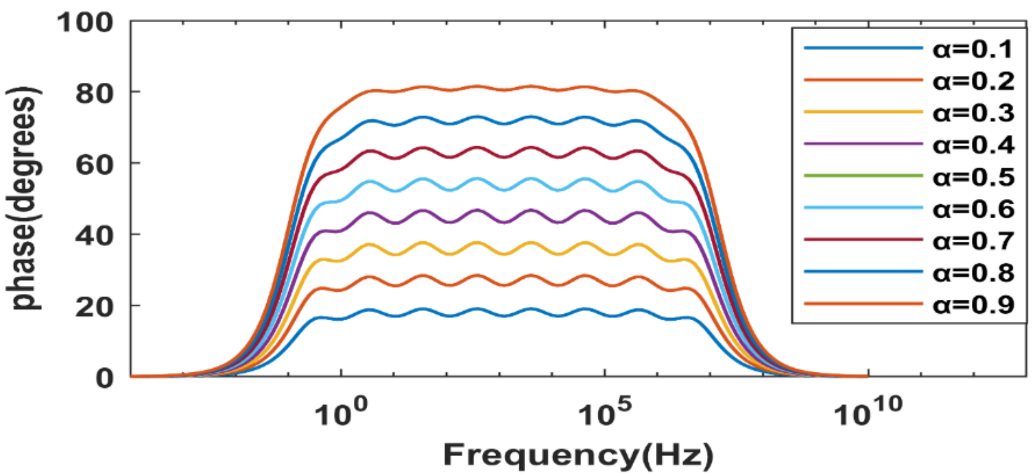


Figure 2.4 Phase response of FC for alpha 0.1 to 0.9

2.5.4 Conclusion

In the present chapter, a brief account of the fractional order element and various approaches for its realization has been presented. We have also reviewed the Oustaloup, Leveron, Mathieu and Nanot method which has been used to realize the fractional order capacitors used in the oscillator circuits proposed in this work. The values of passive components used in the realization of an 8^{th} order approximation of the FOC of 0.01uF, for values α = 0.1-0.9, in steps of 0.1.

REFERENCES

[1] K. Oldham and J. Spanier, “The fractional calculus theory and applications of differentiation and integration to arbitrary order”. Elsevier, 1974.

[2] K. S Cole and R. H. Cole, “Dispersion and absorption in dielectrics I. Alternating current characteristics,”. The Journal of chemical physics, 9(4), pp.341-351,1941.

[3] S. Westerlund and L. Ekstam, Capacitor theory. IEEE Transactions on Dielectrics and Electrical Insulation, 1(5), pp.826-839, 1994.

[4] A. Adhikary, M. Khanra, S. Sen and K. Biswas, May. “Realization of a carbon nanotube based electrochemical fractor,”. In 2015 IEEE International Symposium on Circuits and Systems (ISCAS) (pp. 2329-2332),2015.

[5] K. Biswas, S. Sen, and P. K. Dutta, “Realization of a constant phase element and its performance study in a differentiator circuit,”. IEEE Transactions on Circuits and Systems II: Express Briefs, 53(9), pp.802-806,2006.

[6] S. Saha, S. Das, R. Ghosh, B. Goswami, R. Balasubramanian, A. K Chandra, A. Gupta, “Design of a fractional order phase shaper for iso-damped control of a PHWR under step-back condition,”. IEEE Transactions on Nuclear Science.” pp:1602-12,2010.

[7] S. Darlington, “Realization of a constant phase difference”. The Bell System Technical Journal, 29(1), pp.94-104,1950.

[8] J. Valsa, P. Dvorak, and M. Friedl, “Network Model of the CPE," Radioengineering,” 20, no. 3, pp. 619–626, 2011.

[9] A. Oustaloup, F. Levron, B. Mathieu, and F. M. Nanot, “Frequency-band complex noninteger differentiator: characterization and synthesis,” IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 47, no. 1, pp. 25-39, 2000.

[10] S. C. D. Roy, “On the realization of a constant-argument immittance or fractional operator,” IEEE Transactions on Circuit Theory, 14, no. 3, pp. 264-274, 1967.

[11] R. El-Khazali, “On the biquadratic approximation of fractional-order Laplacian operators,” Analog Integrated Circuits and Signal Processing, 82, no. 3, pp. 503-517, 2015.

[12] G. Carlson, and C. Halijak, “Approximation of fractional capacitors (1/s)^(1/n) by a regular Newton process,” IEEE Transactions on Circuit Theory, 11, no. 2, pp. 210-213, 1964.

[13] K. Matsuda, and H. Fujii, “H (infinity) optimized wave-absorbing control-Analytical and experimental results,” Journal of Guidance, Control, and Dynamics, 16, no. 6, pp. 1146-1153, 1993.

[14] D. Xue, C. Zhao, and Y. Q. Chen, “A modified approximation method of fractional order system,” In 2006 International Conference on Mechatronics and Automation, 2006, pp. 1043-1048.

[15] A. Charef, , H. H. Sun, Y. Y. Tsao, and B. Onaral, “Fractal system as represented by singularity function,” IEEE Transactions on automatic Control, 37, no. 9, pp. 1465-1470, 1992.

[16] M. Khanra, J. Pal, and K. Biswas, “Rational approximation of fractional order SISO System using squared-magnitude function,” In Mathematical Methods in Engineering International Symposium, pp. 21-24. 2010.

[17] C. P. Chen, and Y. T. Tsay, “A squared magnitude continued fraction expansion for stable reduced models,” International Journal of Systems Science, 7, no. 6, pp. 625-634, 1976.

[18] A. Adhikary, S. Choudhary and S. Sen, "Optimal Design for Realizing a Grounded Fractional Order Inductor Using GIC," in *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 65, no. 8, pp. 2411-2421, Aug. 2018, doi: 10.1109/TCSI.2017.2787464.

[19] B. Sapoval,R. Gutfraind, P. Meakin, M. Keddam and H. Takenouti, “Equivalent-circuit, scaling, random-walk simulation, and an experimental study of self-similar fractal electrodes and interfaces”. Physical Review E, 48(5), p.3333,1993.

[20] T. C. Haba, G. Ablart, T. Camps, F. Olivie,“Influence of the electrical parameters on the input impedance of a fractal structure realised on silicon”. Chaos, Solitons, Fractals 24(2):479–490, 2005.

[21] G. W. Bohannan, “Application of fractional calculus to polarization dynamics in solid dielectric materials” (Doctoral dissertation, Montana State University-Bozeman, College of Letters & Science),2000.

[22] D. Mondal, ‘‘Fabrication and performance studies of PMMA -coated fractional order elements’’. Master’s thesis, Indian Inst. Of Technol. Kharagpur, India, Dept. Electrical Eng,2012.

[23] I. S. Jesus nd J. T. Machado, “Development of fractional order capacitors based on electrolyte processes”. Nonlinear Dynamics, 56(1-2), pp.45-55,2009.

[24] A. M. Elshurafa, M. N. Almadhoun, K.N. Salama and H. N. Alshareef, “Microscale electrostatic fractional capacitors using reduced graphene oxide percolated polymer composites”. Applied Physics Letters, 102(23), p.232901,2013.

[25] R. Caponetto, S. Graziani, F. L. Pappalardo and F. Sapuppo, “Experimental characterization of ionic polymer metal composite as a novel fractional order element”. Advances in Mathematical Physics, 2013.

CHAPTER 3

SECOND ORDER OSCILLATOR USING OTAs[1]

3.1 Introduction

In the previous chapter we have presented various approached of realization of a fractional order capacitor. In the present chapter we have proposed a fractional order oscillator using two fractional order capacitors

Analog circuit realization of fractional order signal processing and signal generation circuits have started receiving renewed interest during the last decade [1]-[38]. Several fractional order filters and fractional order oscillators have been introduced in the open-literature wherein different types of amplifiers and fractional order capacitors have been used to realize these circuits. As fractional order capacitors are yet not available as standard circuit elements, these fractional order capacitors, have been realized using various approximation techniques which give a rational function approximation of the operator $s^{\alpha}(0<\alpha\leq 1)$ resulting in a Foster-like network emulating the behavior of a fractional order capacitor. Compared to fractional order filters, relatively less research work has been carried out on the realization of fractional order oscillators [3]-[4], [6], [13], [16-18], [21]-[33], [36]-[38]. In a fractional order oscillator, unlike a conventional oscillator, the FO, CO and the phase relationship between different voltages can be controlled with the help of the fractional order parameter ‘α’. A control over the phase relationship is a very useful feature in applications like PSK modulation-demodulation, music synthesizers etc. Fractional order oscillators have been realized with various active building blocks like operational amplifiers [3]-[4], [6], [13], [16], [24]-[25], [32], [38], operational

[1] The contents of this chapter presented in the 6th International conference on Control, Automation and Robotics (ICCAR), Singapore, 2020. **V. Singh, and P. Kumar, “Fractional Order Oscillators Using OTAs,” 2020, ,27-32].**

transresistance amplifiers [17], [32] current conveyors and their different variants [18], [21-23], [33], [29] current feedback amplifiers [28], voltage differencing inverting buffered amplifier [36], operational transconductance amplifiers [24] and current mirrors [30]. In most of these realizations the conventional capacitor(s) in an integer order realization of an existing oscillator has (ve) been replaced by fractional order capacitor(s). From the study of various fractional order oscillator circuits, it has been observed that very little work has been reported on the realization of fractional order oscillator circuits using operational transconductance amplifiers (OTAs). In such oscillator using control over the CO and FO of the fractional order oscillator can very easily be implemented.

3.2 Proposed Fractional Order Oscillator Circuit

The proposed circuit uses five OTAs and two fractional order capacitors. An OTA is a differential voltage controlled current source whose circuit symbol is shown below in Fig.3.1 while its terminal relationships are given in equation (3.1).

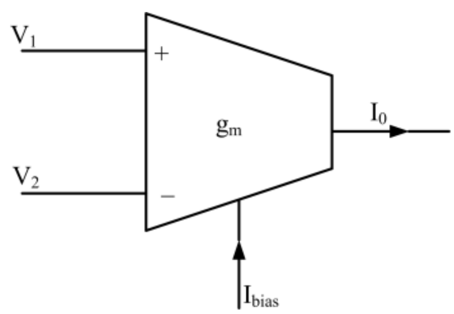


Figure 3.1 OTA symbol

$$I_0 = g_m (V_1 - V_2),\ g_m = \frac{I_{Bias}}{2V_T} \qquad 3.1$$

A fractional order capacitor, on the other hand, whose circuit symbol is shown below in Fig. 3.2 is characterized by the driving point impedance

$$Z(s) = \frac{1}{s^{\alpha} C_{\alpha}} \qquad 3.2$$

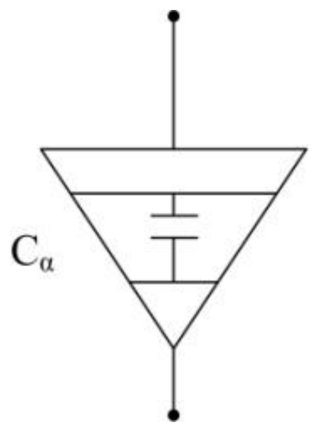


Figure 3.2 Fractional order capacitor symbol

The proposed fractional order oscillator is shown below in Fig. 3.3. The state equations of the oscillator circuit can be described by equation (3.3):

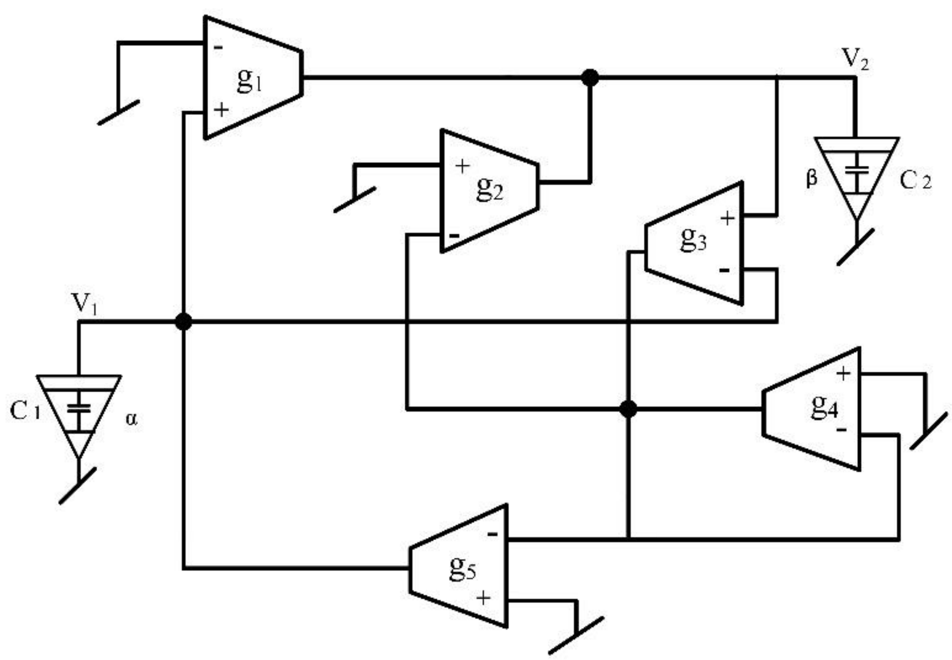


Figure 3.3 Fractional order oscillator

$$\begin{bmatrix} D^{\beta}V_2 \\ D^{\alpha}V_1 \end{bmatrix} = \begin{bmatrix} \dfrac{-g_2g_3}{g_4c_2} & \dfrac{g_1g_4+g_2g_3}{g_4c_2} \\ \dfrac{-g_5g_3}{g_4c_1} & \dfrac{g_5g_3}{g_4c_1} \end{bmatrix} \begin{bmatrix} V_2 \\ V_1 \end{bmatrix} \qquad 3.3$$

where D^{α} and D^{β} represent the fractional order derivative operator. By using the basic definition of the Laplace transform applicable to the fractional order derivative operator [4] the characteristic equation (CE) of the autonomous circuit represented by the fractional order state equations given above is as follows:

$$s^{\alpha+\beta} + s^{\alpha}\frac{g_2g_3}{g_4c_2} - s^{\beta}\frac{g_3g_5}{g_4c_1} + \frac{g_1g_5g_3}{g_4c_1c_2} = 0 \qquad 3.4$$

Putting   in equation (3.4) and separating real and imaginary part with help of Euler's relation we get the following two relationships:

$$\omega^{\alpha+\beta}\cos\left(\frac{(\alpha+\beta)\pi}{2}\right) + \omega^{\alpha}\frac{g_2g_3}{g_4c_2}\cos\left(\frac{\alpha\pi}{2}\right) - \omega^{\beta}\frac{g_3g_5}{g_4c_1}\cos\left(\frac{\beta\pi}{2}\right) + \frac{g_1g_5g_3}{g_4c_1c_2} = 0 \qquad 3.5$$

$$\omega^{\alpha+\beta}\sin\left(\frac{(\alpha+\beta)\pi}{2}\right) + \omega^{\alpha}\frac{g_2g_3}{g_4c_2}\sin\left(\frac{\alpha\pi}{2}\right) - \omega^{\beta}\frac{g_3g_5}{g_4c_1}\sin\left(\frac{\beta\pi}{2}\right) = 0 \qquad 3.6$$

The CO and FO of the fractional order oscillator must satisfy both (3.5) and (3.6) and are expressed as follows:

$$g_2 = \frac{-\omega^{\alpha+\beta}\sin\left(\frac{(\alpha+\beta)\pi}{2}\right) + \omega^{\beta}\frac{g_5g_3}{g_4c_1}\sin\left(\frac{\beta\pi}{2}\right)}{\omega^{\alpha}\frac{g_3}{g_4c_2}\sin\left(\frac{\alpha\pi}{2}\right)} \qquad 3.7$$

$$\omega^{\alpha+\beta}\sin\left(\frac{\beta\pi}{2}\right)+\omega^{\beta}\frac{g_3g_5}{g_4c_1}\sin\left(\frac{(\alpha-\beta)\pi}{2}\right)-\frac{g_1g_5g_3}{g_4c_1c_2}\sin\left(\frac{\alpha\pi}{2}\right)=0 \quad 3.8$$

From the above two equations (3.7) and (3.8) it may be observed that the transconductance g_2 may be selected in such a way that the circuit oscillates for a specified value of ω rad/sec for predefined values of α and β. We have shown below in Table 3.1 the CO, FO and the phase relationships between the two capacitor voltages for different values of α and β.

Table 3.1 Different cases of generalized relation of CO, FO and phase

Oscillator parameter			
Cases	***CO(g_2)***	***FO(ω)***	***Phase (ϕ)***
1. α=β=1	$g_2=g_5$	$\omega=\sqrt{\frac{g_1g_3g_5}{g_4c_1c_2}}$	$\frac{\pi}{2}$
2. $\alpha=\beta\neq 1$	$g_2=\frac{-\omega^{2\alpha}\sin(\alpha\pi)+\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_5g_3\omega^{\alpha}}{g_4c_1}}{\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_3\omega^{\alpha}}{g_4c_2}}$	$\omega=\left(\frac{g_1g_5g_3}{g_4c_1c_2}\right)^{\frac{1}{2\alpha}}$	$\frac{\alpha\pi}{2}$
3. $\alpha=1$ $0<\beta<1$	$g_2=\frac{-\omega^{1+\beta}\cos\left(\frac{\beta\pi}{2}\right)+\sin\left(\frac{\beta\pi}{2}\right)\frac{g_5g_3\omega^{\beta}}{g_4c_1}}{\frac{g_3\omega}{g_4c_2}}$	$\omega^{1+\beta}\sin\left(\frac{\beta\pi}{2}\right)+\omega^{\beta}\frac{g_3g_5}{g_4c_1}\cos\left(\frac{\beta\pi}{2}\right)-\frac{g_1g_5g_3}{g_4c_1c_2}=0$	$\frac{\pi}{2}$
4. $\beta=1$ $0<\alpha<1$	$g_2=\frac{-\omega^{\alpha+1}\cos\left(\frac{\alpha\pi}{2}\right)+\frac{g_5g_3\omega}{g_4c_1}}{\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_3\omega^{\alpha}}{g_4c_2}}$	$\omega^{\alpha+1}+\omega\frac{g_3g_5}{g_4c_1}\cos\left(\frac{\alpha\pi}{2}\right)-\frac{g_1g_5g_3}{g_4c_1c_2}\sin\left(\frac{\alpha\pi}{2}\right)=0$	$\frac{\alpha\pi}{2}$

The electronic tunability of CO and FO for different values of 'α'by changing 'g_3' have been illustrated in Fig3.4 –Fig3.6

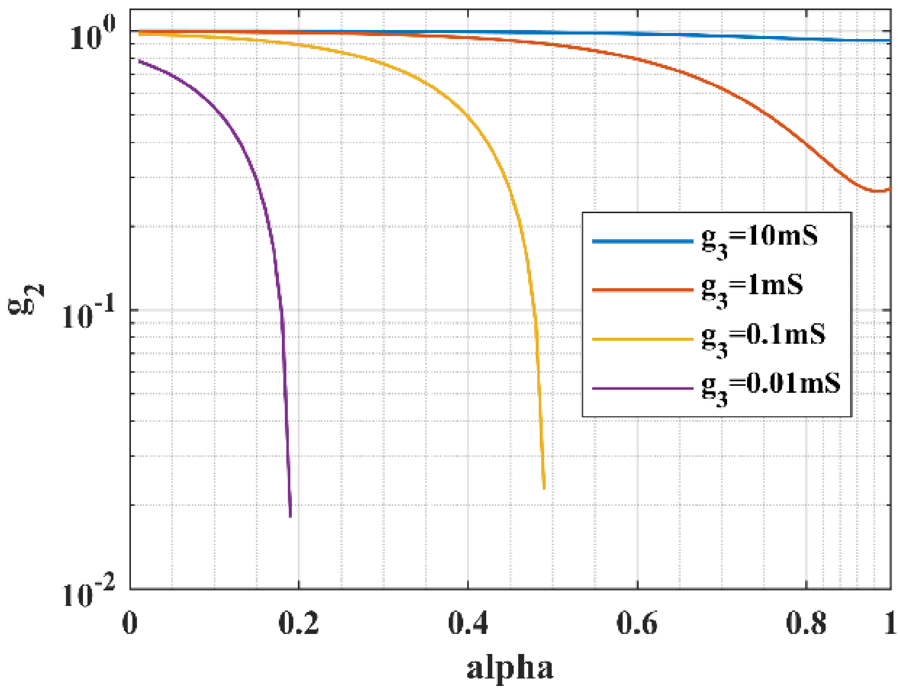


Figure 3.4 Tunability of CO with g_3 and α for case 2

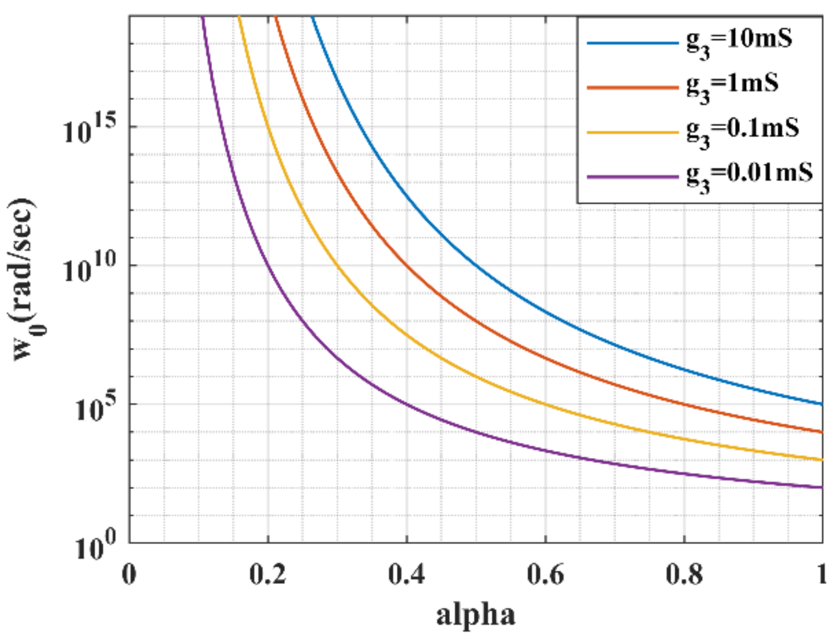


Figure 3.5 Tunability of FO with g_3 and α for case 2

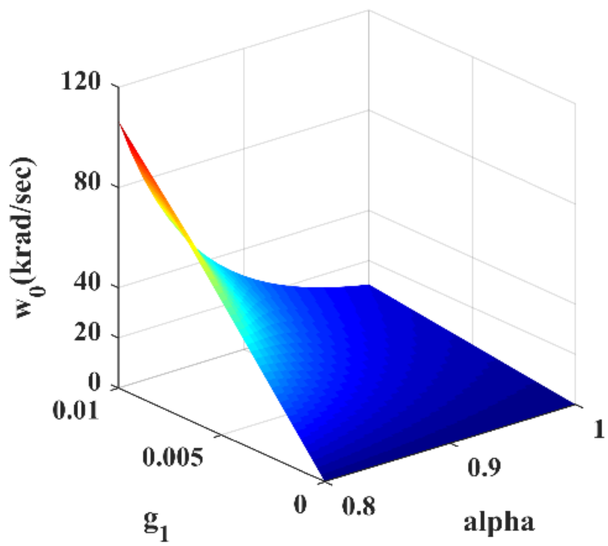


Figure 3.6 Tunability of FO with g_1 and α for case 2

3.3 Stability Analysis

Stability of fractional order systems has been studied in detail [7], [20]. As the stability of any linear dynamic system depends on the location of the roots of the CE in the s-plane. It is necessary to know about the location of the poles in the s-plane. For analysis of the stability of fractional order systems the CE of the system is transformed into W-plane using the transformation $W = s^{\frac{1}{k}}$ where k is a positive integer related to the non-integer order 'α' by the relation α = n/k (n being another positive integer). The stable and unstable regions for system in W-PLANE are defined as $\pi / 2k < |\phi_W| < \pi / k$ and $|\varphi_w| < \pi / 2k$ respectively where φ_w represents the angle of the roots in the W-plane measured from the +ve real axis of the W-plane. Stability of fractional order oscillators have been studied in the W-plane for different values of 'α' by finding the location of the roots of the CE in W-plane. At least one pole must lie on the line $|\phi_W| = \pi / 2k$ for system to be oscillatory and remaining poles should lie in the stable region as shown in Fig.3.7. If even one pole lies in the region $|\phi_W| < \pi / 2k$ the system become unstable.

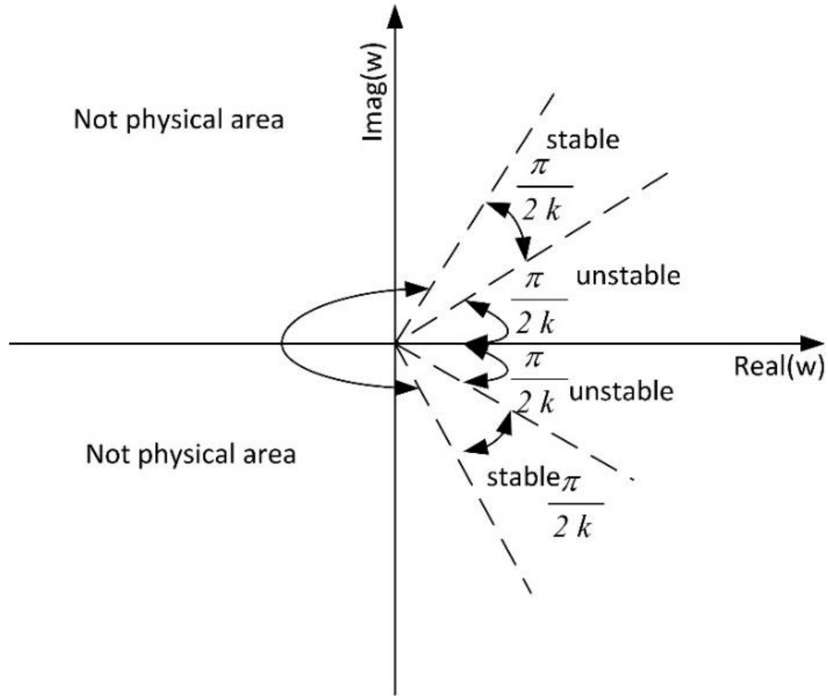


Figure 3.7 W-plane

We have selected the value of k = 10 and plotted the location of the roots of the CE of the fractional order oscillator in the W-plane for different values of 'α'. And shown them in Fig.3 8-3.10. It is noted from Fig. 3.8 that the fractional order oscillators presented in this paper are indeed stable!

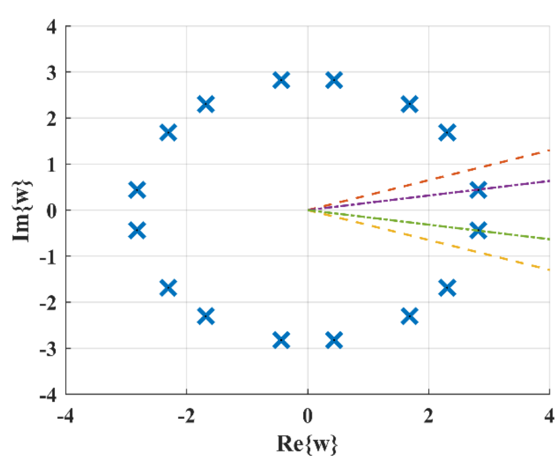


Figure 3.8 Poles in W-plane of oscillator for α=β=0.8

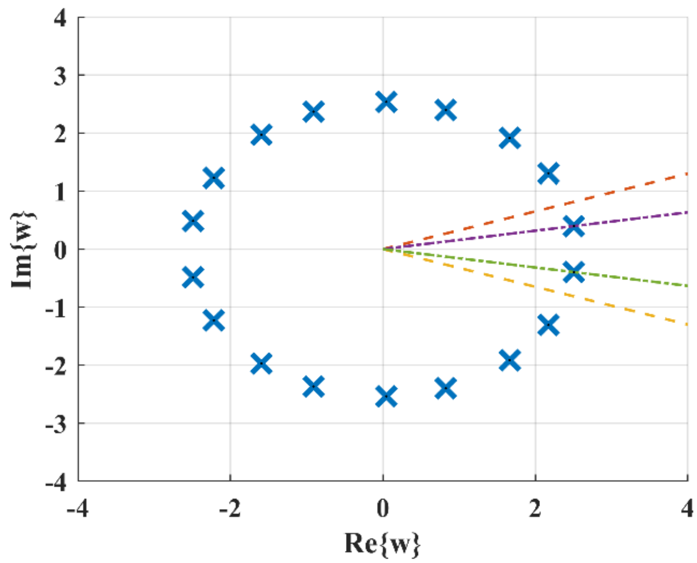


Figure 3.9 Poles in W-plane of oscillator for α=β=0.

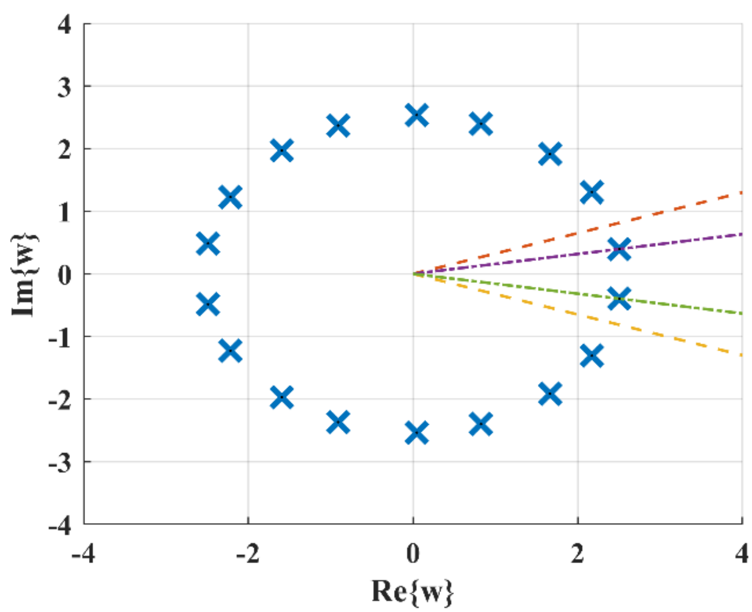


Figure 3.10 Poles in W-plane of oscillator for α=β=0.7

3.4 Simulation and Experiment Results

Workability of the fractional order oscillator presented in this paper has been verified by PSPICE simulations and experimental results. We have used CMOS implementation of an OTA [10] to test the design of the fractional order oscillator. Two identical valued fractional order capacitors of value 0.95μF(rad/sec)$^{(1-\alpha)}$ designed using the Oustaloup approximation method were used in simulation studies ($\alpha = \beta$= 0.9, 0.8 0.7). We have simulated the fractional order oscillator represented by case 2 from Table 3.1 (equal fractional-order capacitors) with (i) $\alpha = \beta$ =0.7 (ii) $\alpha = \beta$ =0.8 (iii) $\alpha = \beta$ =0.9 .The designed FO and the corresponding values of transconductances meeting the CO and FO are given below in Table 3.2.

Table 3.2 Values of transconductance at different order

C=0.95uF α=β	Values of transconductance (mS)					Theoretical value of FO(ω_0)
	g_1	g_2	g_3	g_4	g_5	
0.7	3.98	0.26	3.98	3.98	3.98	23.7KHz
0.8	4.13	1.33	4.13	4.13	4.13	5.61KHz
0.9	4.13	2.80	4.13	4.13	4.13	1.75KHz

Fig 3.11-Fig 3.13 show the output voltage waveforms for different values of the fractional order parameters.

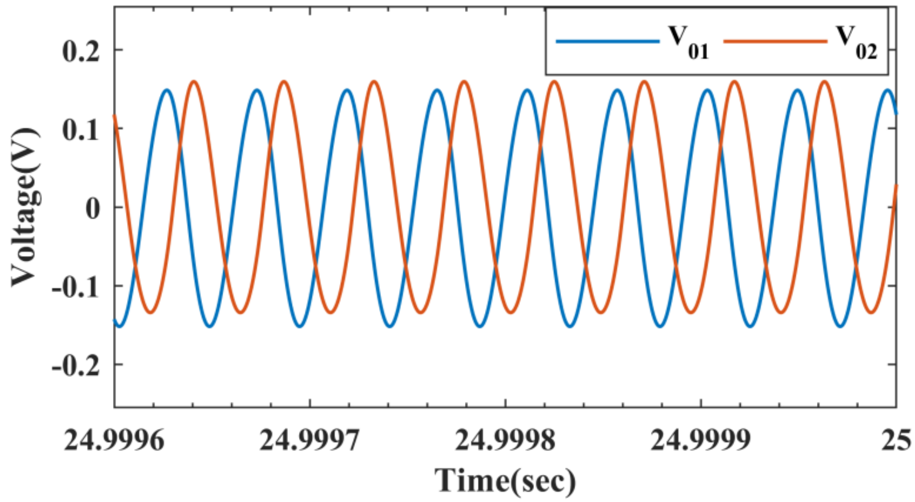


Figure 3.11 α=β=0.7 PSPICE Simulation of oscillator

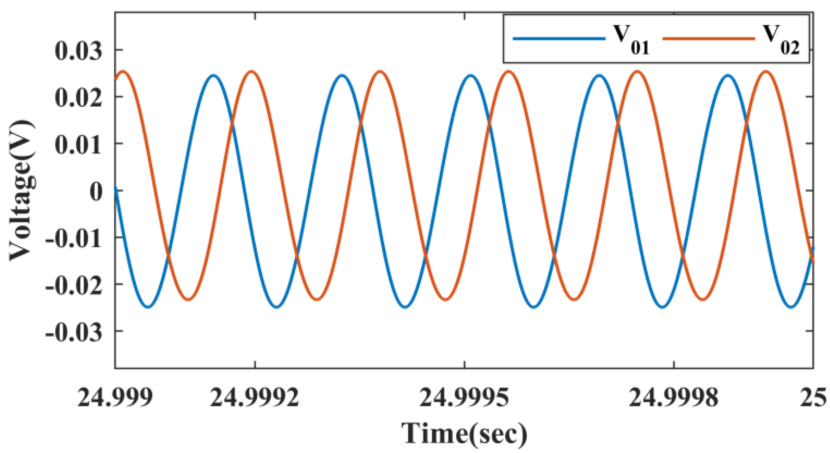


Figure 3.12 α=β =0.8 PSPICE Simulation of oscillator

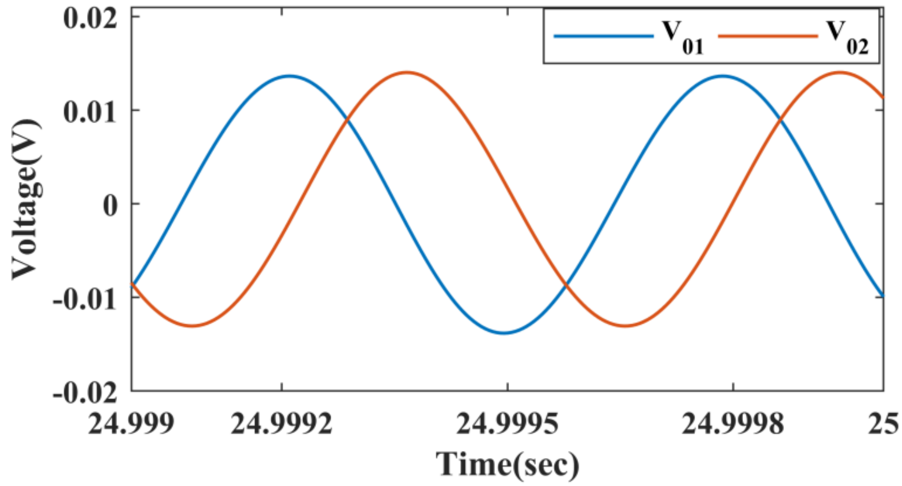


Figure 3.13 α=β =0.9 PSPICE Simulation of oscillator

In Table 3.3 we have shown the values of FO measured in simulation along with the values of theoretical FO for different values of the fractional order parameters. From this table it is observed that the values obtained from simulation are very close to their theoretical counterparts.

Table 3.3 PSPICE and Theoretical values of FO at different α=β

C=0.95uFr(ad/sec) α=β	PSPICE value of FO(ω_0)	Theoretical value of FO(ω_0)
0.7	22KHz	23.7KHz
0.8	5.42KHz	5.61KHz
0.9	1.74KHz	1.75KHz

We have also realized a fractional order oscillator using LM13700 IC OTA biased with ± 15V. A fractional order oscillator with a nominal frequency of oscillation of 1.4MHz was designed by selecting two identical valued fractional order capacitors for α=β=0.8, with C_1=C_2=10nF(rad/sec)$^{(0.2)}$. An eighth order approximation of the fractional order capacitor of value 10nF (rad/sec)$^{0.2}$ corresponding to ‘α’ = 0.8 using

Oustaloup, Levron, Mathieu and Nanot method [2] resulting into the circuit given below in Fig.3.14 was used to realize these capacitors. Values of different capacitors and resistors used in Fig 3.14 are given in Table 3.4.

Table 3.4 RC ladder values using Oustaloup approximation

R(Ω) R_0=7Ω	10	56	330	1.7K	8.2K	39K	220K	3.3M
C(nF)	0.6	1	1.5	2	3	4	6.5	3.8

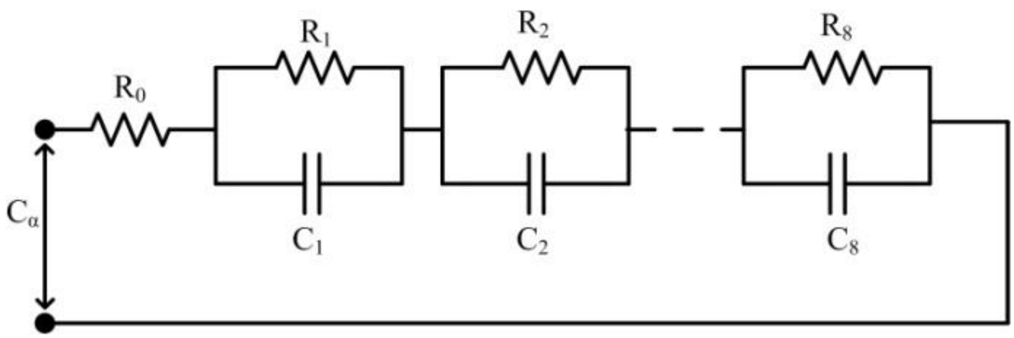


Figure 3.14 RC ladder approximating the fractional order capacitor

The required values of different transconductances g_1=5.5mS, g_3=g_4=g_5=1.2mS, g_2=0.8mS were set by controlling the bias current of all the OTAs. The experimental set up of the oscillator circuit is shown below in Fig.3.15. The output waveform of the oscillator is shown in Fig. 3.16. Fig. 3.17 shows the Lissajous pattern showing the phase relationship between the two output voltages while Fig.3.18 shows the frequency spectrum of the voltage V_1. It may be observed from the experimental results indicated on the output waveform (obtained on Keysight DSOX3034T) that the values of experimentally obtained FO is equal to 1.456 MHz and the phase difference between the two output voltages is equal to 71.728°) that these results are in very close agreement with the corresponding designed value (1.4MHz and 72°).

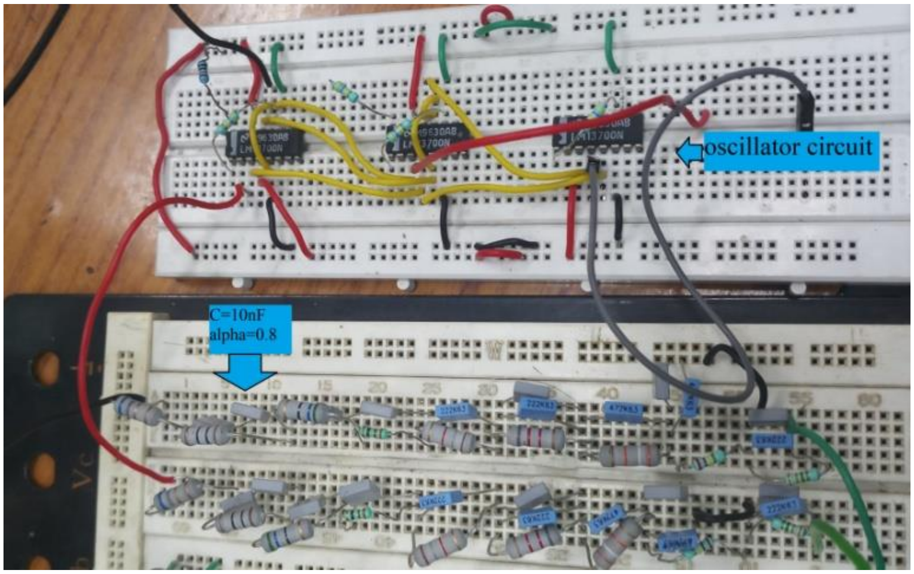


Figure 3.15 Circuit implementation

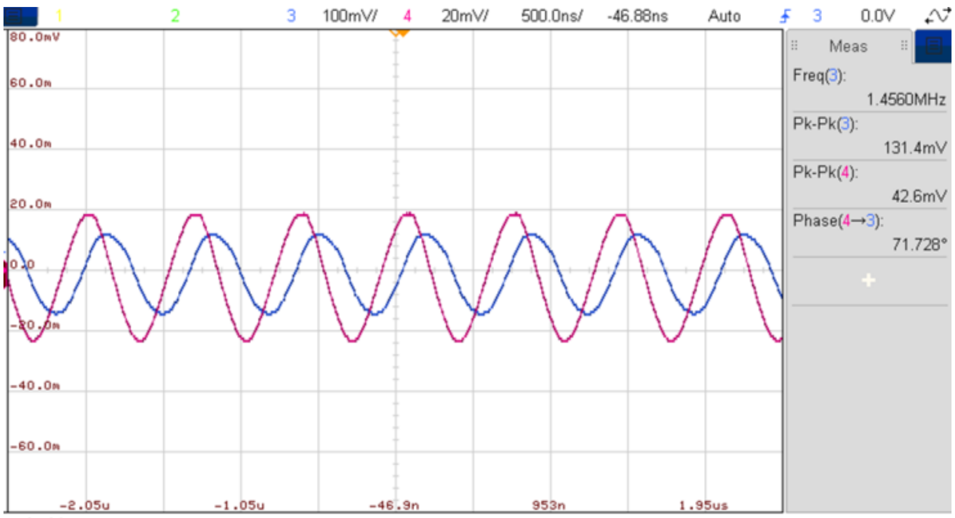


Figure 3.16 Experiment result for α=β=0.8 of fractional order oscillator

Figure 3.17 Lissajous pattern for α=β= 0.8 of fractional order oscillator

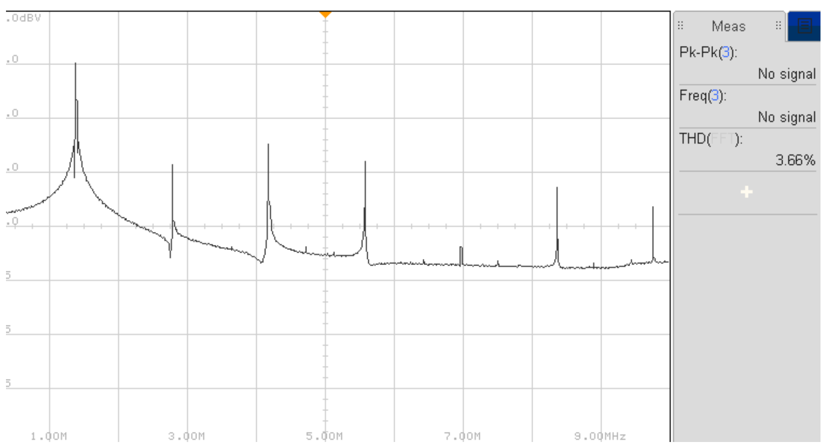


Figure 3.18 Frequency spectrum for α=β=0.8 of fractional order oscillator

3.5 Conclusion

In this chapter, an OTA based fractional order oscillator with electronic tunability of CO and FO has been presented. PSPICE simulation results and experimental results validating the operation of the presented circuit have also been include.

REFERENCES

[1] A. Oustaloup, “Fractional order sinusoidal oscillators: optimization and their use in highly linear FM modulation,” IEEE Transactions on Circuits and Systems 28.10, pp. 1007-1009,1981.

[2] A. Oustaloup, F. Levron, B. Mathieu and F. M. Nanot, “Frequency-band complex noninteger differentiator: characterization and synthesis,” IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 47(1), pp.25-39,2000.

[3] W. Ahmad, R. El-Khazali, and A. S. Elwakil. “Fractional-order Wien-bridge oscillator,” Electronics Letters 37.18, pp. 1110-1112,2001.

[4] A. G. Radwan, A. M. Soliman, and A. S. Elwakil, “Design equations for fractional-order sinusoidal oscillators: four practical circuits examples,” Int. J .Circ. Theory Appl., vol. 36, pp. 473-492, 2007.

[5] A. G. Radwan, A. M. Soliman, and A. S. Elwakil, “First-order filters generalized to the fractional domain,” Journal of Circuits, Systems, and Computers 17, no. 01, pp.55-66, 2008.

[6] A. G. Radwan, A. S. Elwakil, & A. M. Soliman, “Fractional-order sinusoidal oscillators: design procedure and practical examples,” IEEE Transactions on Circuits and Systems I: Regular Papers, 55(7), pp. 2051-2063, 2008.

[7] A. G Radwan, et al, “On the stability of linear systems with fractional-order elements,” Chaos, Solitons & Fractals 40.5, pp. 2317-2328,2009.

[8] A. G. Radwan, A. S. Elwakil, and A. M. Soliman, “On the generalization of second-order filters to the fractional-order domain,” Journal of Circuits, Systems, and Computers 18.02, pp. 361-386, 2009.

[9] T. J. Freeborn, B. Maundy, and A. S. Elwakil, “Field programmable analogue array implementation of fractional step filters,” IET circuits, devices & systems 4.6, pp. 514-524, 2010.

[10] E. W. Greeneich, “Analog integrated circuits,” Springer Science & Business Media, 2012.

[11] A. Soltan, A. G. Radwan, and A. M. Soliman, “Fractional order filter with two fractional elements of dependant orders,” Microelectronics Journal 43.11, pp. 818-827, 2012.

[12] T. J. Freeborn, B. Maundy and A. Elwakil, “Fractional-step Tow-Thomas biquad filters,” Nonlinear Theory and Its Applications, IEICE 3.3, pp.357-374, 2012.

[13] M. Brent, A. Elwakil, and S. Gift, “On the realization of multiphase oscillators using fractional-order all pass filters,” Circuits, Systems, and Signal Processing 31.1, pp. 3-17,2012.

[14] M. C. Tripathy, K. Biswas, and S. Sen, “A design example of a fractional-order Kerwin–Huelsman–Newcomb biquad filter with two fractional capacitors of different order,” Circuits, Systems, and Signal Processing 32.4, pp.1523-1536, 2013.

[15] A. Soltan, A. G. Radwan, and A. M. Soliman, “CCII based fractional filters of different orders,” Journal of advanced research 5.2, pp.157-164, 2014.

[16] A. L Said, A. G. Radwan, A.H. Madian, and A. M. Soliman, “Fractional order two port network oscillator with equal order,” In 2014 26th International Conference on Microelectronics (ICM), pp. 156-159, 2014.

[17] L. A. Said, A. G. Radwan A. H. Madian, and A. M. Soliman, “Fractional order Oscillators Based on Operational Transresistance Amplifiers,” Int.J. Electron. Comm. (AEÜ), vol.69, pp. 988-1003, 2015.

[18] M. E. Fouda, A. Soltan, A. G. Radwan, A. M. Soliman, “Fractional-order multi-phase oscillators design and analysis suitable for higher-order PSK applications,” Analog Integrated Circuits and Signal Processing, pp. 301-12, 2016.

[19] D. Kubanek, J. Koton, J. Jerabek, P. Ushakov and A. Shadrin, “Design and properties of fractional-order multifunction filter with DVCCs,” 2016 39th International Conference on Telecommunications and Signal Processing (TSP), pp.620-624, 2016.

[20] M. S Semary, A. G. Radwan, and H. N. Hassan, “Fundamentals of fractional-order LTI circuits and systems: number of poles, stability, time and frequency responses,” International Journal of Circuit Theory and Applications 44.12, pp. 2114-2133,2016.

[21] D. Kubánek, F. Khateb, G. Tsirimokou, and C. Psychalinos, “Practical design and evaluation of fractional-order oscillator using differential voltage current conveyors,” Circuits, Systems, and Signal Processing 35, no. 6, pp. 2003-2016, 2016.

[22] L. A. Said, A. G. Radwan, A. H. Madian and A. M. Soliman, “Fractional order oscillator design based on two-port network,” Circuits Syst. Signal Process. 35 pp. 3086–3112,2016.

[23] L. A. Said, A. G. Radwan, A. H. Madian, A. M. Soliman, “Fractional-order oscillator based on single CCII,” in IEEE Int. Conf. Telecommun. Signal Process, pp. 603-606, 2016.

[24] M. R Dar, N. AliKant, F. A. Khanday, G. Tsirimokou, and C. Psychalinos,“Design of fractional-order multiphase sinusoidal oscillators,” In 3rd International Conference on Electrical, Electronics, Engineering Trends, Communication, Optimization and Sciences (EEECOS 2016), pp. 1-5, 2016.

[25] L. A. Said, A. G. Radwan, A. H. Madian, and A. M. Soliman, “Three fractional-order-capacitors-based oscillators with controllable phase and frequency,” Journal of Circuits, Systems and Computers 26, no. 10 (2017): 1750160, 2017.

[26] A. M. EL-Naggar, L. A. Said, A. G. Radwan, A. H. Madian, A. M. Soliman, “Fractional order four-phase oscillator based on double integrator topology,” in IEEE Int. Conf. Modern Circ. and Syst. Technologies, pp. 1-4, 2017.

[27] A. Kartci, N. Herencsar, J. Koton & C. Psychalinos, “Compact MOSRC Voltage-Mode Fractional-Order Oscillator Design,” in IEEE European Conf. Circ. Theor. Design, pp. 1-4, 2017.

[28] A. L. Said, A. G. Radwan, A. H. Madian, and A. M. Soliman, “Generalized family of fractional-order oscillators based on single CFOA and RC network,” In 2017 6th International Conference on Modern Circuits and Systems Technologies (MOCAST), pp. 1-4, 2017.

[29] A. Pradhan, K. S. Subhadhra, N. Atique, R. K. Sharma, & S. S. Gupta, “MMCC-based current-mode fractional-order voltage-controlled oscillators,” In 2018 2nd International Conference on Inventive Systems and Control (ICISC), pp. 763-768,2018.

[30] G. Tsirimokou, C. Psychalinos, A. Elwakil , & B. Maundy, “Fractional-Order Multiphase Sinusoidal Oscillator Design Using Current-Mirrors,” In 2018 41st International Conference on Telecommunications and Signal Processing (TSP) ,pp. 1-4, 2018.

[31] A. Pradhan, and R. K. Sharma, “Generalised Fractional-Order Oscillators using OTA,” In 2018 5th International Conference on Signal Processing and Integrated Networks (SPIN), pp. 402-406, 2018.

[32] O. Elwy, L. A. Said, A. H. Madian & A. G. Radwan, “Fractional-Order Relaxation Oscillators Based on Op-Amp and OTRA,” In 2018 30th International Conference on Microelectronics (ICM), pp. 212-215 ,2018.

[33] S. K. Mishra, M. Gupta, & D. K. Upadhyay, “Design and implementation of DDCC-based fractional-order oscillator,” International Journal of Electronics 106.4, pp.581-598, 2019.

[34] S. Mahata, R. Kar, & D. Mandal, "Optimal fractional-order highpass butterworth magnitude characteristics realization using current-mode filter," AEU-International Journal of Electronics and Communications, 102, pp.78-89, 2019.

[35] R. Verma, N. Pandey, & R. Pandey, " CFOA based low pass and high pass fractional step filter realizations," AEU-International Journal of Electronics and Communications, 99, pp. 161-176, 2019.

[36] A. Kartci, N. Herencsar, J. Dvorak, and K. Vrba, "VDIBA-Based Fractional-Order Oscillator Design," In 2019 42nd International Conference on Telecommunications and Signal Processing (TSP), pp. 744-747, 2019.

[37] K. Bingi, R. Ibrahim, M. N. Karsiti, S. M. Hassan, I. Elamvazuthi, & A.M. Devan, "Design and Analysis of Fractional-order Oscillators Using Scilab," In 2019 IEEE Student Conference on Research and Development (SCOReD) ,pp. 311-316, 2019.

[38] G. M. Ahmed, L. A. Said, A. H. Madian, & A. G. Radwan, "Fractional-Order Oscillators Based on Double Op-Amp," In 2019 Fourth International Conference on Advances in Computational Tools for Engineering Applications (ACTEA), pp. 1-4, 2019.

CHAPTER 4

FRACTIONAL ORDER OSCILLATOR USING THREE FRACTIONAL ORDER CAPACITORS[2]

4.1 Introduction

In the previous chapter we had presented an introductory fractional order oscillator employing five OTAs and two fractional order capacitors. In this chapter we have extended the concept to realize fractional order oscillators using three fractional order capacitors in which the phase relationship between different output voltages can be controlled electronically by changing the bias currents of different OTAs as per specified tuning relationships.

Fractional order harmonic oscillators are sinusoidal oscillators, in which, the reactive element, instead of being an integer order capacitor/inductor, is a fractional order reactive element, also referred as a constant phase element (CPE). The interest in these fractional order oscillators have stemmed from the fact that, unlike a conventional RC oscillator, the frequency and phase relationships in a fractional order oscillator are functions of the fractional order parameters α, β, γ… defining the CPE. Though the concept of a fractional order oscillator was introduced in the context of an FM demodulation system long back [1], it could not gather much attention till the early 2000s. A fractional order Wein-Bridge oscillator was proposed in [2], wherein, the conventional capacitors in the classical Wein-Bridge oscillator circuit were replaced by fractional order capacitors. It was shown that sustained oscillations were possible in this circuit and the FO and CO were dependent, not only on the values of the resistors and capacitors, but also, on the values of α, β, the parameters, defining the fractional order capacitors. Numerical simulations were carried out for the solution of the fractional order differential equations to verify the theoretical propositions. This work was followed by yet another work [3] in which the necessary conditions for latch up in sinusoidal oscillators were presented. The detailed theoretical framework for the general fractional order oscillators, however, was developed in [4], [5] wherein the design equations for CO and FO of fractional order oscillators were presented. Since then, many fractional order oscillator circuits employing different types of amplifiers and other active building blocks have been presented [6-20]. It has also been noted from the literature survey that, most of the fractional order oscillator circuits presented above, are generalizations of an existing conventional oscillator circuits.

Out of the fractional order oscillators reported above, those oscillators which employ the OTA, are of special interest, as the control over the FO, CO and the phase in these oscillator circuits can be implemented very easily by changing the bias currents of the OTAs. Very little work is available in open literature, wherein, fractional order oscillators have been realized with OTAs [19], [21].

Recently, an OTA based realization of a fractional order oscillator utilizing two fractional order capacitors was reported [21]. The circuit, unlike the previously reported fractional order oscillator circuits [6]-[20] was not derived from an existing operational transconductance amplifier based oscillator circuit. It had employed 3 OTAs and two fractional order capacitors. It was shown that for a given value of FO, α and β, suitable values of the transconductances could be set by varying the bias currents of the OTAs, resulting in sustained oscillations in the circuit. In this paper, we present yet another OTA based fractional order oscillator, which employs three fractional order capacitors and five OTAs, providing three output voltages with electronically controllable phase difference between them. The dependence of FO, CO and phase difference between the output voltages on α, β and γ has been investigated in detail.

4.2 Fractional Order Oscillator using three fractional order capacitor

4.2.1 Fractional order oscillator theory

The general theory and design equations for fractional order oscillators with 2/3 fractional order capacitors have been presented in [4-5]. The dynamics of a linear autonomous system with three fractional order capacitors may be expressed by the following fractional order state equations:

$$\begin{bmatrix} D^{\alpha}V_1 \\ D^{\beta}V_2 \\ D^{\gamma}V_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = [A] \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \qquad 4.1$$

The CO and FO can be obtained by simultaneously solving the following set of equations [16]:

$$\omega^{\alpha+\beta+\gamma}\cos\left(\frac{(\alpha+\beta+\gamma)\pi}{2}\right) - a_{33}\omega^{\alpha+\beta}\cos\left(\frac{(\alpha+\beta)\pi}{2}\right) - a_{11}\omega^{\beta+\gamma}\cos\left(\frac{(\beta+\gamma)\pi}{2}\right)$$
$$-a_{22}\omega^{\alpha+\gamma}\cos\left(\frac{(\alpha+\beta)\pi}{2}\right) + \omega^{\alpha}|A_{\alpha}|\cos\left(\frac{\alpha\pi}{2}\right) + \omega^{\beta}|A_{\beta}|\cos\left(\frac{\beta\pi}{2}\right) + \omega^{\gamma}|A_{\gamma}|\cos\left(\frac{\gamma\pi}{2}\right) - |A| = 0$$

4.2

$$\omega^{\alpha+\beta+\gamma}\sin\left(\frac{(\alpha+\beta+\gamma)\pi}{2}\right) - a_{33}\omega^{\alpha+\beta}\sin\left(\frac{(\alpha+\beta)\pi}{2}\right) - a_{11}\omega^{\beta+\gamma}\sin\left(\frac{(\beta+\gamma)\pi}{2}\right)$$
$$-a_{22}\omega^{\alpha+\gamma}\sin\left(\frac{(\alpha+\gamma)\pi}{2}\right) + \omega^{\alpha}|A_{\alpha}|\sin\left(\frac{\alpha\pi}{2}\right) + \omega^{\beta}|A_{\beta}|\sin\left(\frac{\beta\pi}{2}\right) + \omega^{\gamma}|A_{\gamma}|\sin\left(\frac{\gamma\pi}{2}\right) = 0$$

4.3

Where $|A_{\alpha}| = a_{22}a_{33} - a_{23}a_{32}$, $|A_{\beta}| = a_{11}a_{33} - a_{13}a_{31}$ and $|A_{\gamma}| = a_{11}a_{22} - a_{12}a_{21}$

4.2.2 Proposed Fractional Order Oscillator

The proposed fractional order oscillator with three fractional order capacitors is shown in Fig.4 1. A routine analysis of the circuit using the classical relationship between the voltage and current for the fractional order capacitor and the OTA [21], we obtain the following set of fractional order state equations:

$$\begin{bmatrix} D^{\alpha}V_1 \\ D^{\beta}V_2 \\ D^{\gamma}V_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{g_1}{C_2} & -\frac{g_2}{C_2} \\ 0 & 0 & -\frac{g_5}{C_1} \\ \frac{g_3}{C_3} & -\frac{g_3}{C_3} & -\frac{g_4}{C_3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \qquad 4.4$$

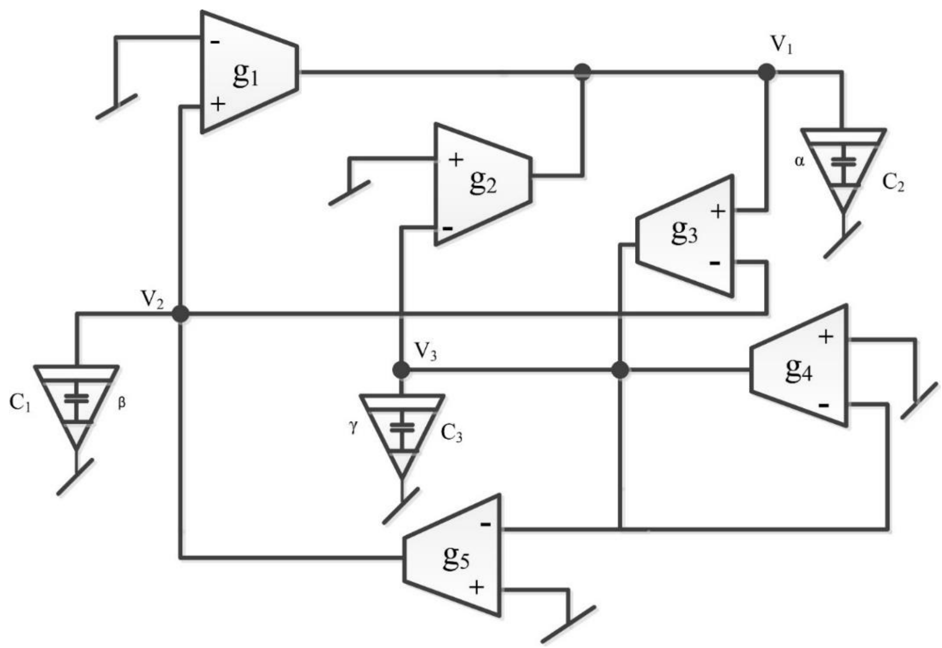


Figure 4.1 Proposed fractional order oscillator

The characteristic equation derived from the above equations is expressed as follows:

$$s^{\alpha+\beta+\gamma} + s^{\alpha+\beta}\frac{g_4}{C_3} - s^{\alpha}\frac{g_5 g_3}{C_1 C_3} + s^{\beta}\frac{g_2 g_3}{C_2 C_3} + \frac{g_1 g_5 g_3}{C_1 C_2 C_3} = 0 \qquad 4.5$$

After putting s = jω, and separating the real part and imaginary parts, we obtain two design equations given below in Eq. 4.6 and 4.7:

$$g_2 = \frac{\omega^{\alpha+\beta+\gamma}\left(-\sin\left(\frac{(\alpha+\beta+\gamma)\pi}{2}\right)\right) + \omega^{\alpha}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_5 g_3}{C_3 C_1} - \omega^{\alpha+\beta}\sin\left(\frac{(\alpha+\beta)\pi}{2}\right)\frac{g_4}{C_3}}{\omega^{\beta}\sin\left(\frac{\beta\pi}{2}\right)\frac{g_3}{C_2 C_3}} \tag{4.6}$$

$$\omega^{\alpha+\beta+\gamma}\sin\left(\frac{(\alpha+\gamma)\pi}{2}\right) + \omega^{\alpha}\sin\left(\frac{(\beta-\alpha)\pi}{2}\right)\frac{g_5 g_3}{C_1 C_3} + \omega^{\alpha+\beta}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_4}{C_3} - \sin\left(\frac{\beta\pi}{2}\right)\frac{g_1 g_5 g_3}{C_1 C_2 C_3} = 0 \tag{4.7}$$

From the above equations, it may be noted that it is possible to design the oscillator for a given value of α, β, γ and ω, by choosing suitable values for various transconductances (g_1-g_5). We have used different combinations of these three fractional order capacitor parameters to realize eight cases which are summarized in Table 4.1 including the most general case ($\alpha \neq \beta \neq \gamma \neq 1$) described in Eq. 4.6 and 4.7.

Table 4.1 Cases of relation between CO and FO at different α, β and γ

CASES	Condition of oscillation	Frequency(ω)
1. $\alpha = \beta = \gamma \neq 1$	$g_2 = \frac{-\omega^{3\alpha}\sin\left(\frac{3\alpha\pi}{2}\right) - \omega^{2\alpha}\sin(\alpha\pi)\frac{g_4}{C_3} + \omega^{\alpha}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_5 g_3}{C_1 C_3}}{\omega^{\alpha}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_3}{C_2 C_3}}$	$\omega^{3\alpha}\sin(\alpha\pi) + \omega^{2\alpha}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_4}{C_3} - \sin\left(\frac{\alpha\pi}{2}\right)\frac{g_1 g_5 g_3}{C_1 C_2 C_3} = 0$
2. $\gamma = 1$ $\alpha = \beta$	$g_2 = \frac{-\omega^{2\alpha+1}\cos\alpha\pi - \omega^{2\alpha}\sin(\alpha\pi)\frac{g_4}{C_3} + \omega^{\alpha}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_5 g_3}{C_1 C_3}}{\omega^{\alpha}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_3}{C_2 C_3}}$	$\omega^{2\alpha+1}\cos\left(\frac{\alpha\pi}{2}\right) + \omega^{2\alpha}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_4}{C_3} - \sin\left(\frac{\alpha\pi}{2}\right)\frac{g_1 g_5 g_3}{C_1 C_2 C_3} = 0$
3. $\gamma = 1$ $\alpha \neq \beta$	$g_2 = \frac{-\omega^{\alpha+\beta+1}\cos\left(\frac{(\alpha+\beta)\pi}{2}\right) - \omega^{2\alpha}\sin\left(\frac{(\alpha+\beta)\pi}{2}\right)\frac{g_4}{C_3} + \omega^{\alpha}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_5 g_3}{C_1 C_3}}{\omega^{\beta}\sin\left(\frac{\beta\pi}{2}\right)\frac{g_3}{C_2 C_3}}$	$\omega^{\alpha+\beta+1}\cos\left(\frac{\alpha\pi}{2}\right) + \omega^{\alpha}\sin\left(\frac{(\beta-\alpha)\pi}{2}\right)\frac{g_5 g_3}{C_1 C_3} + \omega^{\alpha+\beta}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_4}{C_3} - \sin\left(\frac{\beta\pi}{2}\right)\frac{g_1 g_5 g_3}{C_1 C_2 C_3} = 0$
4. $\beta = 1$ $\alpha = \gamma$	$g_2 = \frac{-\omega^{2\alpha+1}\cos(\alpha\pi) - \omega^{\alpha+1}\cos\left(\frac{\alpha\pi}{2}\right)\frac{g_4}{C_3} + \omega^{\alpha}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_5 g_3}{C_1 C_3}}{\omega\frac{g_3}{C_3 C_3}}$	$\omega^{2\alpha+1}\sin(\alpha\pi) + \omega^{\alpha+1}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_4}{C_3} - \omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)\frac{g_5 g_3}{C_1 C_3} - \frac{g_1 g_5 g_3}{C_1 C_2 C_3} = 0$
5. $\beta = 1$ $\alpha \neq \gamma$	$g_2 = \frac{-\omega^{\alpha+\gamma+1}\cos(\alpha\pi) - \omega^{\alpha+1}\cos\left(\frac{\alpha\pi}{2}\right)\frac{g_4}{C_3} + \omega^{\alpha}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_5 g_3}{C_1 C_3}}{\omega\frac{g_3}{C_3 C_2}}$	$\omega^{\alpha+\gamma+1}\sin\left(\frac{(\alpha+\gamma)\pi}{2}\right) + \omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)\frac{g_5 g_3}{C_1 C_3} + \omega^{\alpha+1}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g_4}{C_3} - \frac{g_1 g_5 g_3}{C_1 C_2 C_3} = 0$

6. $\alpha = 1$ $\beta = \gamma$	$g_2 = \dfrac{-\omega^{2\gamma+1}\cos(\beta\pi) - \omega^{\beta+1}\cos\left(\frac{\beta\pi}{2}\right)\frac{g_4}{C_3} + \omega\frac{g_5 g_3}{C_1 C_3}}{\omega^{\beta}\sin\left(\frac{\beta\pi}{2}\right)\frac{g_3}{C_2 C_3}}$	$\omega^{2\beta+1}\cos\left(\frac{\gamma\pi}{2}\right) - \omega\cos\left(\frac{\beta\pi}{2}\right)\frac{g_5 g_3}{C_1 C_3} + \omega^{\beta+1}\frac{g_4}{C_3} - \sin\left(\frac{\beta\pi}{2}\right)\frac{g_1 g_5 g_3}{C_1 C_2 C_3} = 0$
7. $\alpha = 1$ $\beta \neq \gamma$	$g_2 = \dfrac{-\omega^{\beta+\gamma+1}\cos\left(\frac{(\beta+\gamma)\pi}{2}\right) - \omega^{\beta+1}\cos\left(\frac{\beta\pi}{2}\right)\frac{g_4}{C_3} + \omega\frac{g_5 g_3}{C_1 C_3}}{\omega^{\beta}\sin\left(\frac{\beta\pi}{2}\right)\frac{g_3}{C_2 C_3}}$	$\omega^{\beta+\gamma+1}\cos\left(\frac{\gamma\pi}{2}\right) - \omega\cos\left(\frac{\beta\pi}{2}\right)\frac{g_5 g_3}{C_1 C_3} + \omega^{\beta+1}\frac{g_4}{C_3} - \sin\left(\frac{\beta\pi}{2}\right)\frac{g_1 g_5 g_3}{C_1 C_2 C_3} = 0$

4.3 Phase Difference Between output Voltages

To obtain the phase difference between the three output voltages, first we have to determine the transfer functions between the three output voltages, which are taken across the three fractional order capacitors, and therefrom, determine the phase difference between these voltages [16]. We have presented the transfer functions and the corresponding phase difference in Table 4.2. It may be noted that the sum of the phase difference between the three voltages is 2π. The value of the phase angles, can, thus be changed by changing the values of α, β, γ.

Table 4.2 Transfer function and phase relation of voltages

Transfer Function	PHASE
$\dfrac{V_1}{V_2} = \dfrac{s^{\beta} g_2 C_1 + g_1 g_5}{s^{\alpha} C_2 g_5}$	$\phi_{12} = -\dfrac{\alpha\pi}{2} + tan^{-1}\left(\dfrac{\omega^{\beta}\, sin\left(\frac{\beta\pi}{2}\right) g_2 C_1}{\omega^{\beta}\, cos\left(\frac{\beta\pi}{2}\right) g_2 C_1 + g_1 g_5}\right)$
$\dfrac{V_3}{V_2} = \dfrac{-s^{\beta} C_1}{g_5}$	$\phi_{32} = -\dfrac{\beta\pi}{2}$
$\dfrac{V_3}{V_1} = \dfrac{s^{\beta} g_3 C_1}{C_3 C_1 s^{\gamma+\beta} + s^{\beta} g_4 C_1 - g_5 g_3}$	$\varphi_{31} = \dfrac{\beta\pi}{2} - \tan^{-1}\left(\dfrac{C_3 C_1 \omega^{\gamma+\beta}\sin\left(\frac{(\gamma+\beta)\pi}{2}\right) + \omega^{\beta}\sin\left(\frac{\beta\pi}{2}\right) g_4 C_1}{C_3 C_1 \omega^{\gamma+\beta}\cos\left(\frac{(\gamma+\beta)\pi}{2}\right) + \omega^{\beta} g_4 C_1 \cos\left(\frac{\beta\pi}{2}\right) - g_5 g_3}\right)$

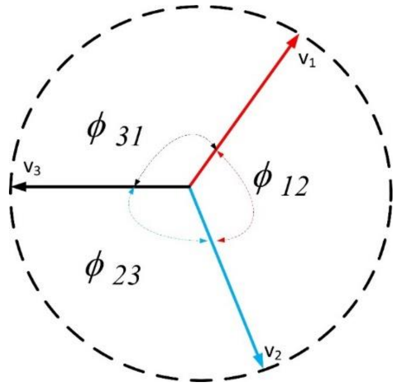


Figure 4.2 Phase relation of voltages

4.3.1 Design example

The procedure to determine the values of transconductances required for sustained oscillations in the proposed circuit for the given values of the FO, α, β, γ and C_1, C_2, C_3 is illustrated with an example. Let the values of α =0.9, β =0.8, γ=0.7, $C_1 = C_2 = C_3$= 0.95uF and FO =1.1 KHz, be specified. The values of different transconductances are to be determined so that sustained oscillations are produced in the circuit. Let us assume $g_1 = g_3 = g_4 = g_5 = g$, and $C_1 = C_2 = C_3 = C$. The CO and FO equations as expressed in Eq. 4.8 - 4.9 reduce to :

$$g_2 = \frac{\omega^{\alpha+\beta+\gamma}\left(-\cos\left(\frac{(\alpha+\beta+\gamma)\pi}{2}\right)\right)+\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)\frac{g^2}{C^2}-\omega^{\alpha+\beta}\cos\left(\frac{(\alpha+\beta)\pi}{2}\right)\frac{g}{C}-\frac{g^3}{C^3}}{\omega^{\beta}\cos\left(\frac{\beta\pi}{2}\right)\frac{g}{C^2}}$$

$$\omega^{\alpha+\beta+\gamma}\sin\left(\frac{(\alpha+\gamma)\pi}{2}\right)+\omega^{\alpha+\beta}\sin\left(\frac{\alpha\pi}{2}\right)\frac{g}{C}+\omega^{\alpha}\sin\left(\frac{(\beta-\alpha)\pi}{2}\right)\frac{g^2}{C^2}-\sin\left(\frac{\beta\pi}{2}\right)\frac{g^3}{C^3}=0$$

Solving these two equations, we may obtain the values of $g = g_1 = g_3 = g_4 = g_5$ and g_2 as1.7mS and 3.4mS respectively.

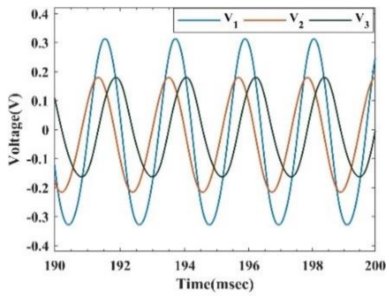
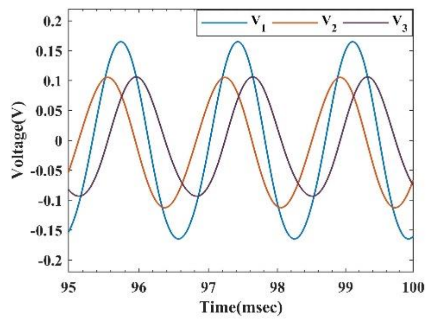
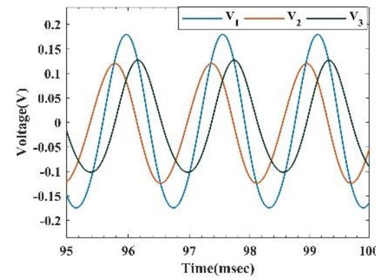
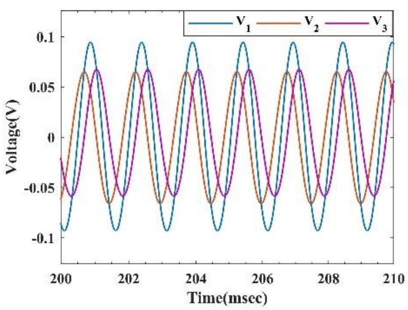
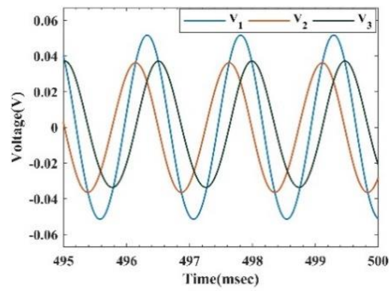
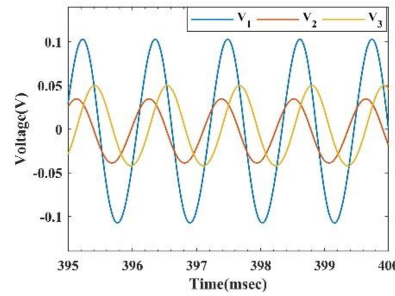
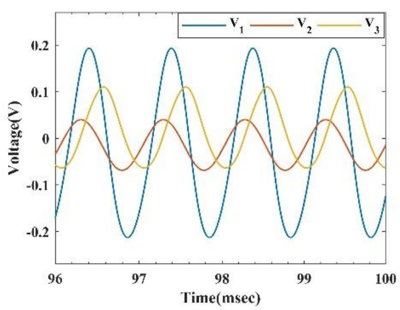
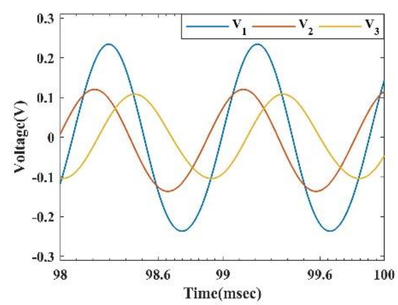
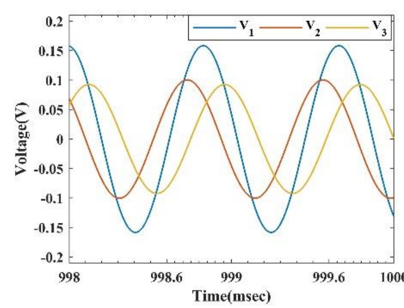
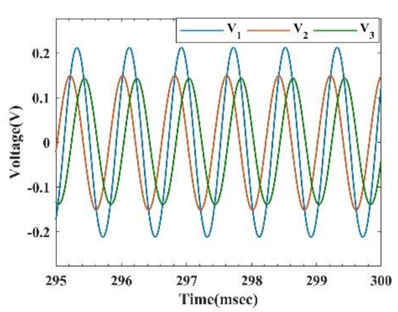
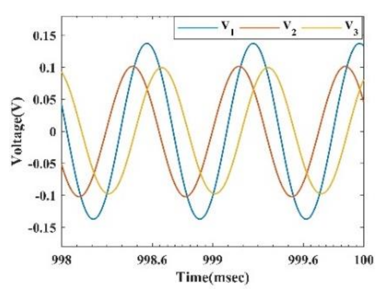
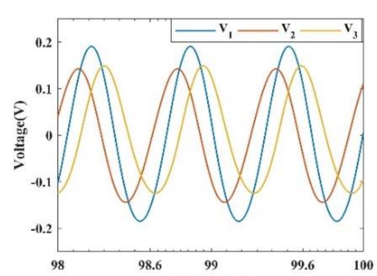
4.4 Simulation results

We now present PSPICE simulation results to establish the workability of the proposed fractional order oscillator circuit. We have used the CMOS implementation of the OTA [21]. The three fractional order capacitors were realized using the approximation method suggested by Oustaloup, Leveron, Mathieu and Lanot [22] and values were taken as 0.95μF(rad/sec) $^{(1-\alpha)}$. Table 4.3 shows the values of different transconductances used for various Combinations of α, β and γ. The time responses of the fractional order oscillators have been depicted in Table 4.4 given below.

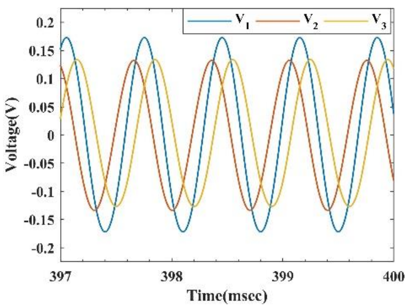
Table 4.3 Transconductance values of five OTA at different values for α, β and γ

α	β	γ	g_1(mS)	g_2(mS)	g_3(mS)	g_4(mS)	g_5(mS)
1	1	0.9	3.23	4.22	3.23	3.23	3.23
1	1	0.8	3.93	4.22	3.93	4.22	3.93
1	1	0.7	4.13	4.02	4.13	4.13	4.13
1	1	0.6	4.12	4.22	4.12	4.12	4.12
1	1	0.5	3.98	4.02	3.98	3.98	3.98
0.9	1	1	2.49	2.80	2.49	2.49	2.49
0.8	1	1	4.34	1.09	4.34	4.34	4.34
0.9	0.9	0.9	3.23	4.22	3.23	3.23	3.23
0.9	0.9	0.8	3.23	3.15	3.23	3.23	3.23
0.9	0.9	0.7	3.23	2.66	3.23	3.23	3.23
0.9	0.9	0.6	3.23	2.62	3.23	3.23	3.23
0.9	0.9	0.5	3.9	2.46	3.9	3.9	3.9
0.9	0.9	0.4	2.97	2.08	2.97	2.97	2.97
0.9	0.9	0.3	2.97	2.07	2.97	2.97	2.97
0.9	0.9	0.2	3.23	2.45	3.23	3.23	3.23
0.9	0.9	0.1	3.50	2.39	3.50	3.50	3.50
0.8	0.8	0.9	4.22	4.13	4.22	4.22	4.22
0.8	0.8	0.8	3.23	3.06	3.23	3.23	3.23
0.8	0.8	0.7	3.50	1.92	3.50	3.50	3.50
0.8	0.8	0.6	4.02	1.5	4.02	1.5	4.02
0.8	0.8	0.5	4.02	1.37	4.02	4.02	4.02
0.8	0.9	0.8	4.13	1.33	4.13	4.13	4.13
0.7	0.8	0.8	4.13	0.56	4.13	4.13	4.13
0.6	0.8	0.8	3.94	0.17	3.94	3.94	3.94
0.7	0.7	0.9	4.31	4.13	4.31	4.31	4.31
0.7	0.7	0.8	4.13	2.97	4.13	4.13	4.13
0.7	0.7	0.7	4.13	1.75	4.13	4.13	4.13
0.7	0.7	0.6	4.13	0.78	4.13	4.22	4.13
0.7	0.7	0.5	4.13	0.49	4.13	4.13	4.13
0.7	0.9	0.7	2.13	0.48	2.13	2.13	2.13
0.7	0.8	0.7	2.63	0.29	2.63	2.63	2.63
0.7	0.7	0.9	3.5	4.13	4.31	4.31	4.31

Table 4.4 Time response of the fractional order oscillators

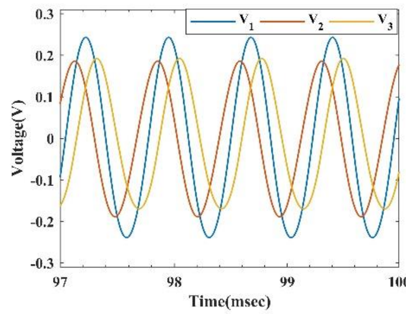
α=1,β=1,γ=0.9  ϕ_{12}=-35.903,ϕ_{31}=141.25,ϕ_{32}=-87.19	α=1,β=1,γ=0.8  ϕ_{12}=-42.41,ϕ_{31}=137.01ϕ_{32}=-89.43	α=1,β=1,γ=0.7  ϕ_{12}=-45.88,ϕ_{31}=134.17,ϕ_{32}=-90.04
α=1,β=1,γ=0.6  ϕ_{12}=-46.71,ϕ_{31}=134.49,ϕ_{32}=-91.12	α=1,β=1,γ=0.5  ϕ_{12}=-46.65,ϕ_{31}=134.15,ϕ_{32}=-90.80	α=0.9,β=1,γ=1  ϕ_{12}=-63.11,ϕ_{31}=118.14,ϕ_{32}=-91.02
α=0.8,β=1,γ=1  ϕ_{12}=-64.35,ϕ_{31}=136.11,ϕ_{32}=-69.54	α=0.9,β=0.9,γ=0.9  ϕ_{12}=-33.84,ϕ_{31}=119.65,ϕ_{32}=-85.50	α=0.9,β=0.9,γ=0.8  ϕ_{12}=-43.04,ϕ_{31}=124.9,ϕ_{32}=-81.22
α=0.9, β=0.9, γ=0.7  ϕ_{12}=-47.20, ϕ_{31}=128.1, ϕ_{32}=-81.38	α=0.9, β=0.9, γ=0.6  ϕ_{12}=-48.56, ϕ_{31}=136.98, ϕ_{32}=-82.54	α=0.9, β=0.9, γ=0.5  ϕ_{12}=-47.66, ϕ_{31}=136.24, ϕ_{32}=-89.90

$\alpha=0.9, \beta=0.9, \gamma=0.2$



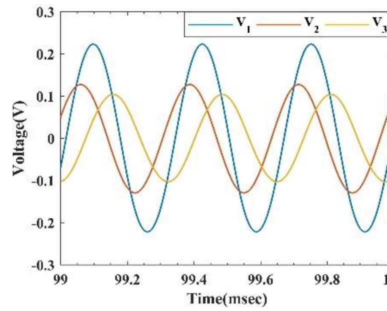
$\phi_{12}=-50.19, \phi_{31}=132.71, \phi_{32}=-89.91$

$\alpha=0.9, \beta=0.9, \gamma=0.1$



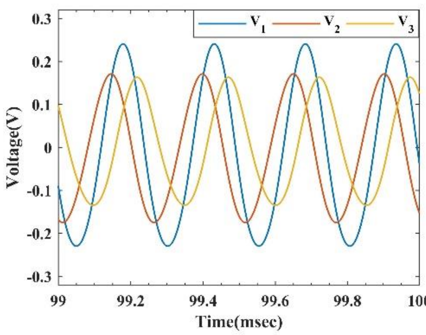
$\phi_{12}=-51.62, \phi_{31}=132.59, \phi_{32}=-80.21$

$\alpha=0.8, \beta=0.8, \gamma=0.8$



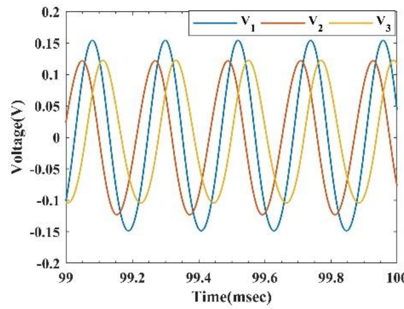
$\phi_{12}=-41.4, \phi_{31}=116.60, \phi_{32}=-74.04$

$\alpha=0.8, \beta=0.8, \gamma=0.7$



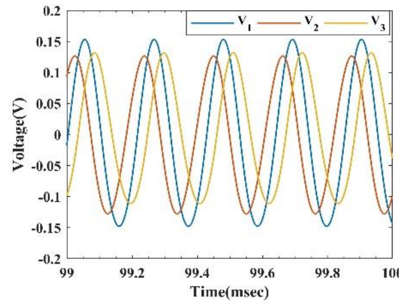
$\phi_{12}=-44.6, \phi_{31}=132.6, \phi_{32}=-88.03$

$\alpha=0.8, \beta=0.8, \gamma=0.6$



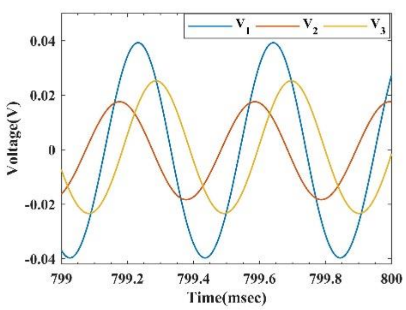
$\phi_{12}=-55.5, \phi_{31}=127.49, \phi_{32}=-71.02$

$\alpha=0.8, \beta=0.8, \gamma=0.5$



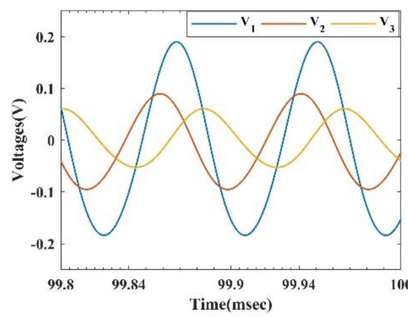
$\phi_{12}=-56.97, \phi_{31}=128.39, \phi_{32}=-71.36$

$\alpha=0.8, \beta=0.9, \gamma=0.8$



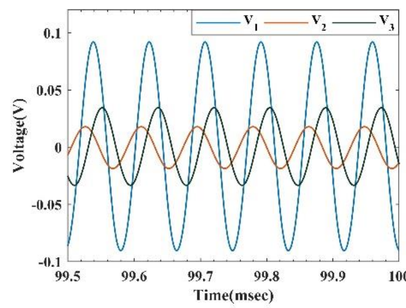
$\phi_{12}=-50.34, \phi_{31}=133.21, \phi_{32}=-82.02$

$\alpha=0.7, \beta=0.8, \gamma=0.8$



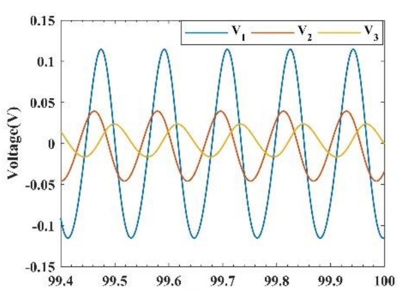
$\phi_{12}=-52.22, \phi_{31}=125.97, \phi_{32}=-72.19$

$\alpha=0.6, \beta=0.8, \gamma=0.8$



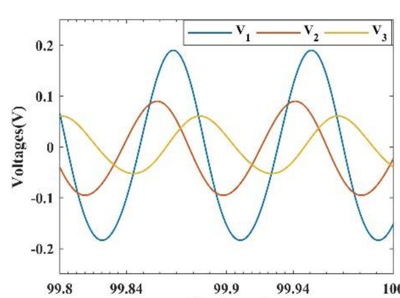
$\phi_{12}=-52.08, \phi_{31}=125.90, \phi_{32}=-71.9$

$\alpha=0.7, \beta=0.7, \gamma=0.9$



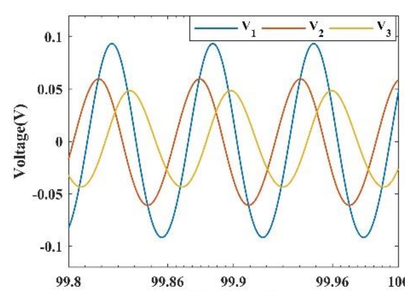
$\phi_{12}=-39.23, \phi_{31}=105.25, \phi_{32}=-68.49$

$\alpha=0.7, \beta=0.7, \gamma=0.8$



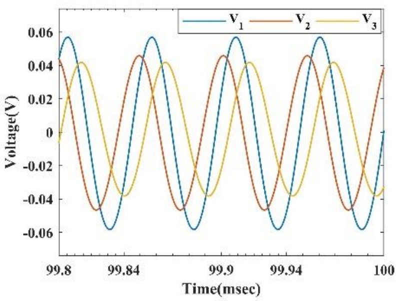
$\phi_{12}=-43.54, \phi_{31}=111.16, \phi_{32}=-68.70$

$\alpha=0.7, \beta=0.7, \gamma=0.7$



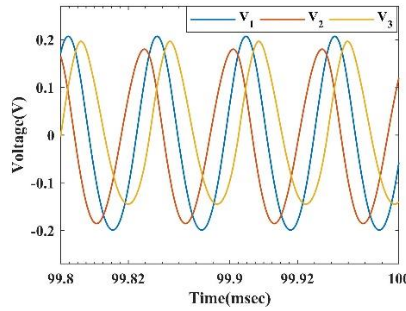
$\phi_{12}=-57.35, \phi_{31}=123.28, \phi_{32}=-66.6$

α=0.7,β=0.7,γ=0.6



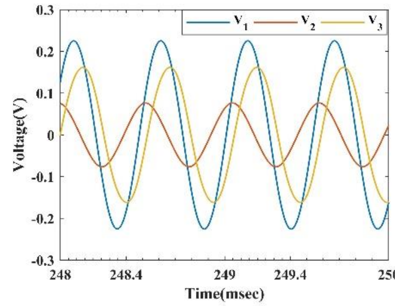
ϕ_{12}=-51.78,ϕ_{31}=118.34,ϕ_{32}=-78.75

α=0.7,β=0.7,γ=0.5



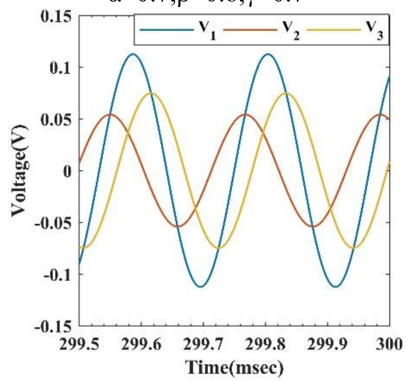
ϕ_{12}=-61.27,ϕ_{31}=126.47,ϕ_{32}=-66.75

α=0.7,β=0.9,γ=0.7



ϕ_{12}=-64.10,ϕ_{31}=146.22,ϕ_{32}=-81.32

α=0.7,β=0.8,γ=0.7



ϕ_{12}=-60.94,ϕ_{31}=133.42,ϕ_{32}=-72.36

The details of the variation of the FO and the phase difference between the three output voltages with the fraqctional order parameters α ,β and γ   are given below in Table. 4.5 and Table. 4.6 respectively

Table 4.5 Simulated and Theoretical values of FO for different values of α ,β and γ

α	β	γ	f_0 (Hz) (PSPICE)	f_0 (Hz) (Theoretical)
1	1	0.9	480	448.3
1	1	0.8	600	611.4
1	1	0.7	625	637.5
1	1	0.6	663	667.1
1	1	0.5	675	679.3
0.9	1	1	538	555
0.8	1	1	1.08k	1.29k
0.9	0.9	0.9	1.10k	0.7k
0.9	0.9	0.8	1.18k	1.11k
0.9	0.9	0.7	1.26k	1.32k
0.9	0.9	0.6	1.42k	1.81k
0.9	0.9	0.5	1.43k	1.35k

0.9	0.9	0.2	1.43k	1.73k
0.9	0.9	0.1	1.39k	1.44k
0.8	0.8	0.8	3.05k	3.51k
0.8	0.8	0.7	3.39k	3.72k
0.8	0.8	0.6	4.55k	4.45k
0.8	0.8	0.5	4.71k	5.30k
0.8	0.9	0.8	2.44k	2.50k
0.7	0.8	0.8	6.76k	5.64k
0.6	0.8	0.8	11k	9.28k
0.7	0.7	0.9	7.45k	3.38k
0.7	0.7	0.8	12.06k	10.34k
0.7	0.7	0.7	19.336k	17.69k
0.7	0.7	0.6	17.336k	20.9k
0.7	0.7	0.5	19.04K	18.9k
0.7	0.9	0.7	1.94k	1.61k
0.7	0.8	0.7	4.59k	4.17k

Table.4.6 The different order of α, β, γ given phase (ϕ) in simulation and MATLAB

α	β	γ	ϕ_{12}	ϕ_{32}	ϕ_{31}	ϕ_{12}	ϕ_{32}	ϕ_{31}
Different combinations			PSPICE			Theoretical		
1	1	0.9	-35.93	-87.19	141.25	-40.9	-90	124.9
1	1	0.8	-42.41	-89.43	137.01	-45.71	-90	129.77
1	1	0.7	-45.88	-90.04	134.17	-48.69	-90	131.34
1	1	0.6	-46.71	-91.12	134.49	-48.69	-90	133.56
1	1	0.5	-46.65	-90.80	134.15	-46.5	-90	134.26
0.9	1	1	-63.11	-91.02	118.14	-24.84	-90	115.84
0.8	1	1	-64.35	-69.54	136.11	-51.53	-90	114.87
0.9	0.9	0.9	-33.84	-85.50	119.65	-38.01	-81	115.20
0.9	0.9	0.8	-43.04	-81.22	124.9	-43.95	-81	122.79
0.9	0.9	0.7	-47.20	-81.383	128.1	-46.6	-81	127.43
0.9	0.9	0.6	-48.56	82.544	136.98	-44.33	-81	133.93
0.9	0.9	0.5	-47.66	-89.90	136.24	-54.93	-81	126.77
0.9	0.9	0.2	-50.19	-82.913	132.71	-41.68	-81	138.17
0.9	0.9	0.1	-51.62	-80.21	132.59	-49.86	-81	130.07
0.8	0.8	0.8	-41.4	-74.04	116.60	-39.91	-72	115.55
0.8	0.8	0.7	-44.6	-88.03	132.6	-50.80	-72	119.05
0.8	0.8	0.6	-55.5	-71.02	127.49	-56.59	-72	120.82
0.8	0.8	0.5	-56.97	-71.36	128.39	-57.44	-72	121.87
0.8	0.9	0.8	-50.34	-82.02	133.21	-50.19	-81	130.44
0.7	0.8	0.8	-52.22	-72.19	125.97	-54.9	-72	119.57
0.6	0.8	0.8	-52.08	-71.9	125.90	-49.78	-72	121.71
0.7	0.7	0.9	-39.23	-68.49	105.25	-46.46	-63	94.46
0.7	0.7	0.8	-43.54	-68.70	111.16	-45.20	-63	104.03
0.7	0.7	0.7	-57.35	-66.6	123.28	-49.26	-63	111.20
0.7	0.7	0.6	-51.78	-78.75	118.34	-56.00	-63	112.27
0.7	0.7	0.5	-61.27	-66.74	126.47	-58.33	-63	112.26
0.7	0.9	0.7	-64.10	-81.32	146.22	-62.98	-81	144.87
0.7	0.8	0.7	-60.94	-72.36	133.42	-62.9	-72	130.93

Fig. 4.3 shows the variation, of the FO with γ when $\alpha=\beta=1$, while Fig. 4.4 displays the variation of g_2 with γ when $\alpha=\beta=1$. Fig. 4.5 depicts the variation of g_2 and g_1 with γ when $\alpha=\beta=1$. Fig. 4.6 shows the variation in g_2 with α and β for $\gamma=1$. Fig. 4.7 show the variation in ϕ_{12} with α and β whereas variation in ϕ_{31} with β and γ has been depicted in Fig. 4.8.

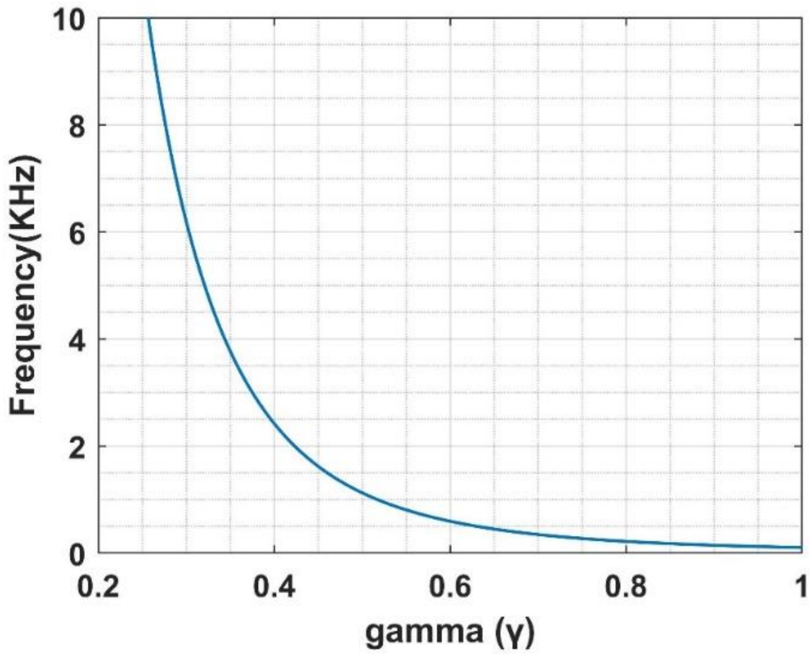


Figure 4.3 Variation of FO with α=β=1, γ

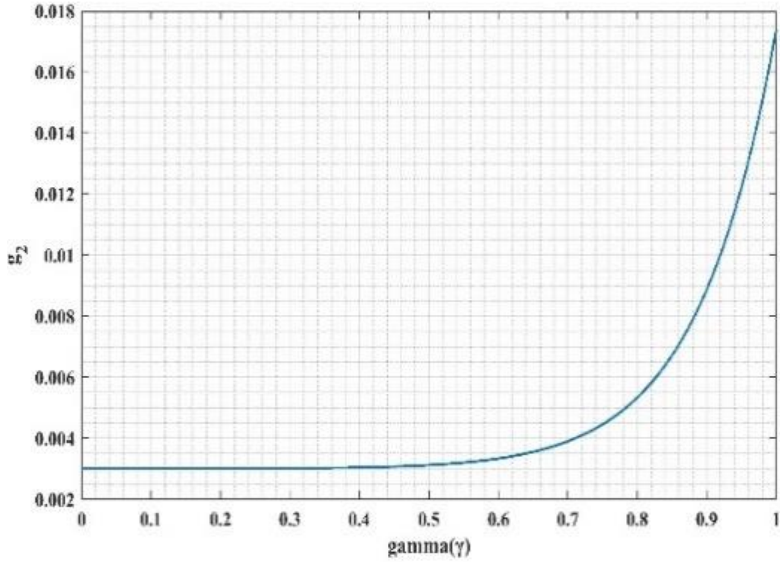


Figure 4.4 Variation of CO (g_2) with α=β=1, γ

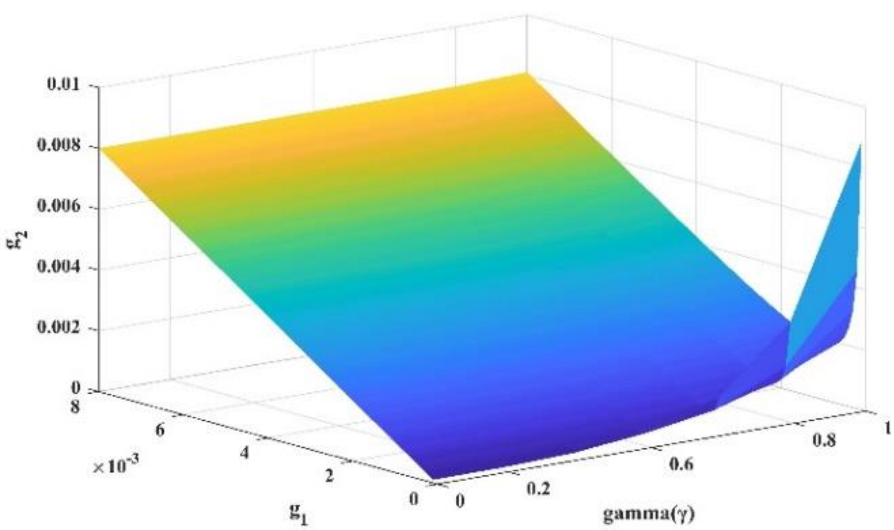


Figure 4.5 Variation of g_1, g_2 with γ

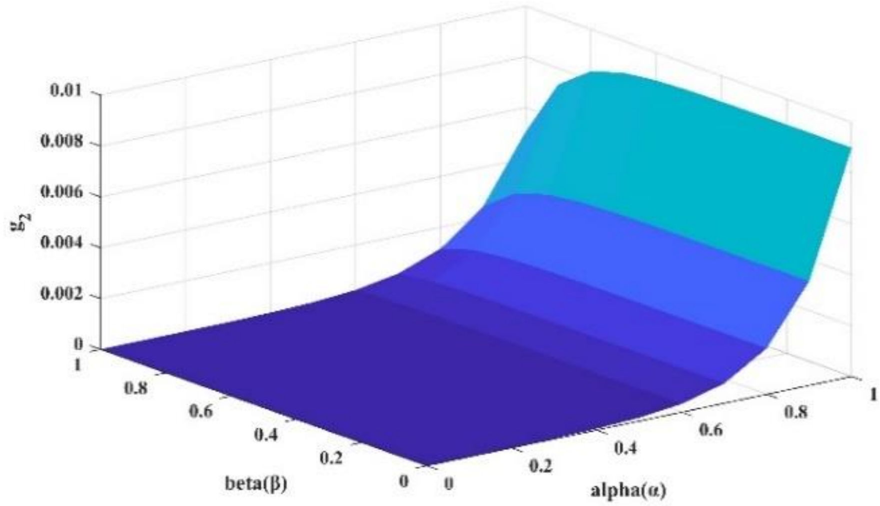


Figure 4.6 Variation in CO (g_2) , α and β

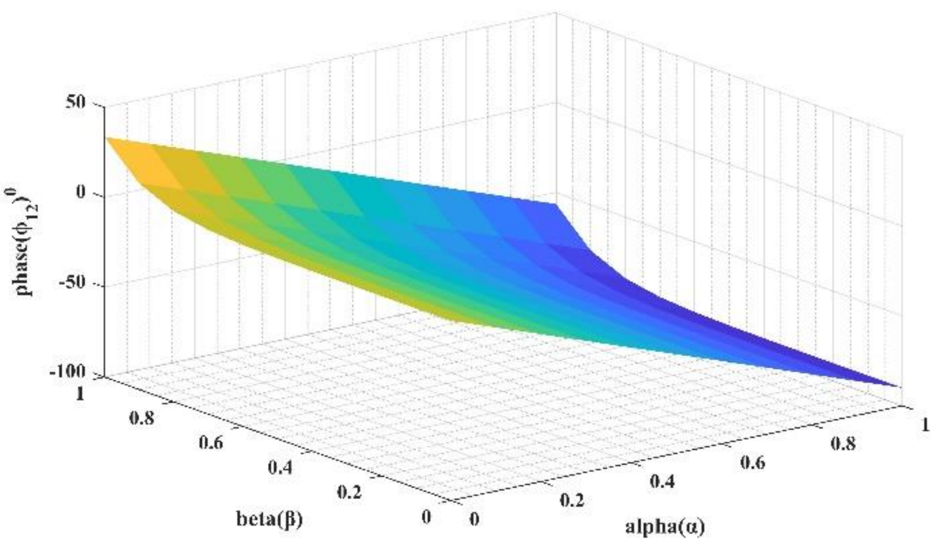


Figure 4.7 Variation of ϕ_{12} with α and β

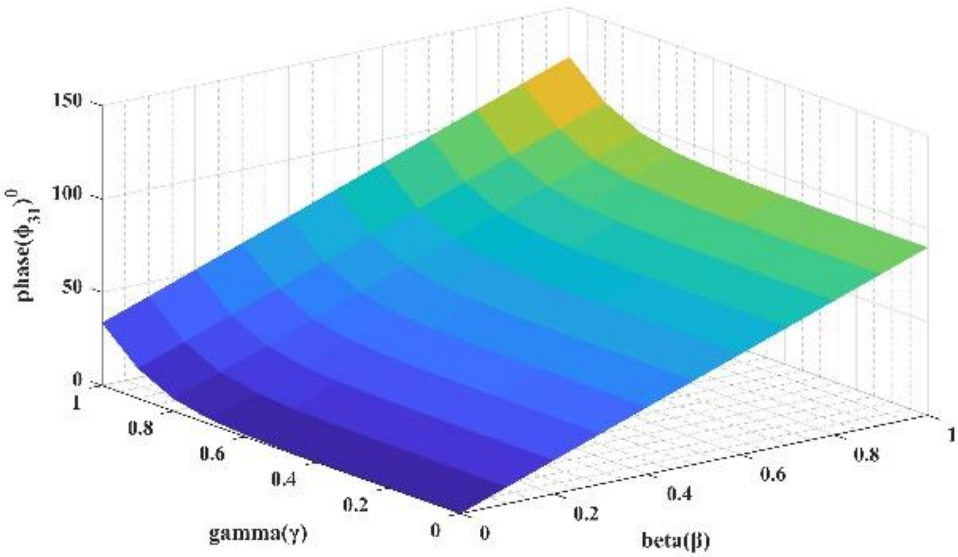


Figure 4.8 Variation of ϕ_{31}, with β and γ

4.5 Experiment Work

We have also tested the proposed fractional order oscillator experimentally, using LM13700 OTA IC biased with ± 15V and fractional order capacitors realized using the approximation method proposed by Oustaloup, Levron, Mathieu and Nanot [22]. A fractional order oscillator with a nominal frequency of oscillation of 1.2MHz was designed by selecting three identical valued fractional order capacitors for $\alpha = \beta = \gamma = 0.8$, with $C_1 = C_2 = C_3 = 10nF(rad/sec)^{(0.2)}$ g_1=3.5mS, g_2=3.9mS, g_3=3.5mS, g_4=3.5mS, g_5=3.5mS. The experiment and theoretical results of phase difference in voltages of oscillator is mentioned in Table 4.7.

Table 4.7 Experiment and Theoretical values in phase difference of voltages

Frequecy1.2MHz			Frequency=1.1MHz		
Experimental			Theoretical		
ϕ_{12}	ϕ_{32}	ϕ_{31}	ϕ_{12}	ϕ_{32}	ϕ_{31}
-9.906	-71.36	81.23	-10	-72	82

A snapshot of the experimental set up is shown in Fig. 4.9, while the output put waveforms obtained are shown in Fig. 4.10 respectively. The difference in the experimental values and the theoretical values may be attributed to the non-exact values of the realized fractional order capacitors.

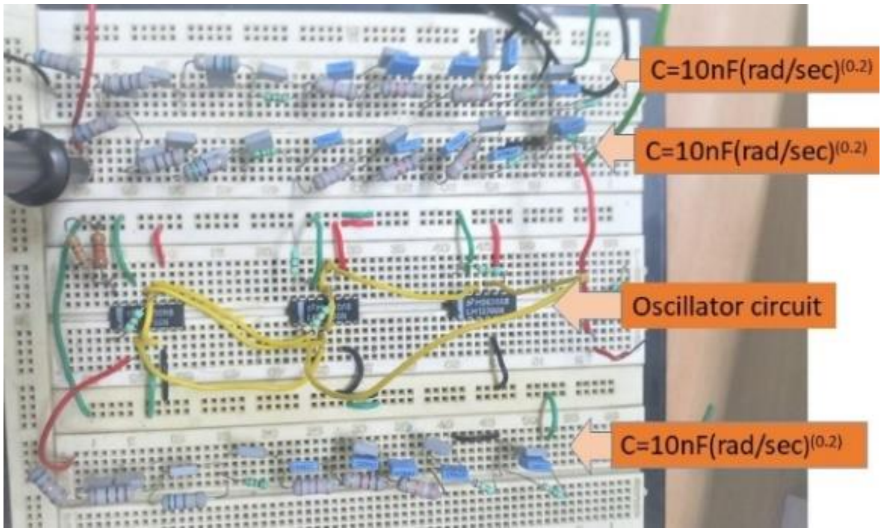


Figure 4.9 Circuit implementation of fractional order oscillator

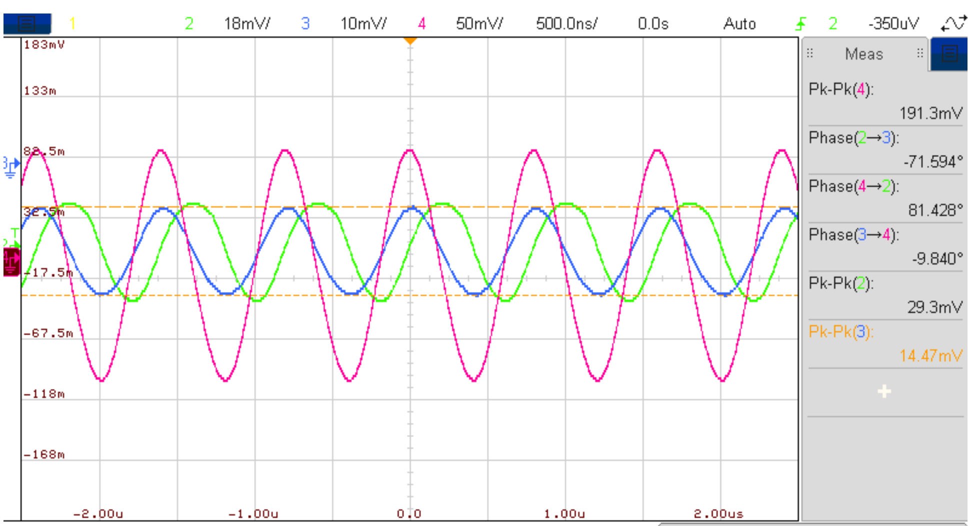


Figure 4.10 Experiment result at α=β=γ=0.8 of fractional order oscillator

4.6 Conclusion

In this chapter, a fractional order oscillator realized with five OTAs and three fractional order capacitors has been presented. General design equations representing eight different cases for α, β and γ have been derived. The CO, FO and the phase difference between the different output voltages can be tuned electronically by changing the values of transconductaces of different OTAs for specified values of α, β and γ. PSPICE simulations using CMOS OTAs [21] and fractional order capacitors designed using the approximation method proposed by Oustaloup, Levron, Mathieu and Nanot [22] and experimental results obtained using IC OTA LM13700 have also presented to establish the workability of the presented circuit.

REFERENCES

[1] A. Oustaloup, "Fractional order sinusoidal oscillators: optimization and their use in highly linear FM modulation," IEEE Transactions on Circuits and Systems 28 (10), 1007-1009,1981.

[2] W. Ahmad, R. El-Khazali, and A.S. Elwakil, "Fractional-order Wien-bridge oscillator. Electronics Letters,"37(18), 1110-1112, 2001.

[3] A. S. Elwakil, and W. M. Ahmad, "On the necessary and sufficient conditions for latch-up in sinusoidal oscillators," International Journal of Electronics, 89(3), 197-206, 2002.

[4] A.G. Radwan, A. M. Soliman, and A. S. Elwakil, "Design equations for fractional-order sinusoidal oscillators: four practical circuits examples," Int. J .Circ. Theory. Appl., vol. 36, 473-492, 2007.

[5] A. G. Radwan, A.S. Elwakil and A.M. Soliman, "Fractional-order sinusoidal oscillators: Design procedure and practical examples," IEEE Trans. Circuits Syst. 55 ,2051–2063,2008.

[6] M. Brent, A. Elwakil, and S. Gift, "On the realization of multiphase oscillators using fractional-order all pass filters," Circuits, Systems, and Signal Processing 31.1, 3-17,2012.

[7] L. A. Said, A.G. Radwan, A. H. Madian, and A. M. Soliman, "Fractional order Oscillators Based on Operational Transresistance Amplifiers," Int.J. Electron. Comm. (AEÜ), vol.69, 988-1003, 2015.

[8] M. E. Fouda, A. Soltan, A. G. Radwan and A. M. Soliman, "Fractional-order multi-phase oscillators design and analysis suitable for higher-order PSK applications," Analog Integrated Circuits and Signal Processing, 87(2), 301-312, 2016.

[9] D. Kubánek, F. Khateb, G. Tsirimokou, and C. Psychalinos, "Practical design and evaluation of fractional-order oscillator using differential voltage current conveyors," Circuits, Systems, and Signal Processing, 35(6), 2003-2016, 2016.

[10] L. A. Said, A. G. Radwan, A. M. Madian, and A. M. Soliman, "Fractional order oscillator design based on two-port network," Circuits, Systems, and Signal Processing, 35(9), 3086-3112, 2016

[11] L. A. Said, A. G. Radwan, A. H. Madian, A. M. Soliman, "Fractional-order oscillator based on single CCII," in IEEE Int. Conf. Telecommun. Signal Process, 603-606, 2016.

[12] M. R. Dar, N. AliKant, F. A. Khanday, G.Tsirimokou, and C. Psychalinos, "Design of fractional-order multiphase sinusoidal oscillators," In 3rd International Conference on Electrical, Electronics, Engineering Trends, Communication, Optimization and Sciences (EEECOS 2016),1-5, 2016.

[13] A. M. EL-Naggar., L. A. Said, A. G. Radwan, A.H. Madian, A.M. Soliman, "Fractional order four-phase oscillator based on double integrator topology," in IEEE Int. Conf. Modern Circ. and Syst. Technologies, 1-4, 2017.

[14] A. Kartci, N. Herencsar, J. Koton & C. Psychalinos, "Compact MOS-RC Voltage-Mode Fractional-Order Oscillator Design," in IEEE European Conf. Circ. Theor. Design, 1-4, 2017.

[15] A. L. Said, A. G. Radwan,A. M. Madian and A. M. Soliman, "Generalized family of fractional-order oscillators based on single CFOA and RC network," In 2017 6th International Conference on Modern Circuits and Systems Technologies (MOCAST),

[16] L. A. Said, A. G. Radwan, A. H. Madian and A. M. Soliman, "Three fractional-order-capacitors-based oscillators with controllable phase and frequency," Journal of Circuits, Systems and Computers, 26(10), 1750160, 2017.

[17] A. Pradhan, K. S. Subhadhra, N. Atique, R. K. Sharma and S. S. Gupta, "MMCC-based current-mode fractional-order voltage-controlled oscillators," In 2018 2nd International Conference on Inventive Systems and Control (ICISC),763-768, 2018.

[18] G. Tsirimokou, C. Psychalinos, A. Elwakil and B. Maundy, "Fractional-Order Multiphase Sinusoidal Oscillator Design Using Current-Mirrors," In 2018 41st International Conference on Telecommunications and Signal Processing (TSP),1-4, 2018.

[19] A. Pradhan, and R. K. Sharma, “Generalised Fractional-Order Oscillators using OTA,”In 2018 5th International Conference on Signal Processing and Integrated Networks (SPIN), 402-406, 2018.

[20] O. Elwy, L. A. Said, A. H. Madian, and A. G. Radwan, “Fractional-Order Relaxation Oscillators Based on Op-Amp and OTRA,” In 2018 30th International Conference on Microelectronics(ICM), 212-215, 2018.

[21] E. W. Greeneich, “Analog integrated circuits,” Springer Science & Business Media, 2012.

[22] A. Oustaloup, F. Levron, B. Mathieu and F. M. Nanot, “Frequency-band complex noninteger differentiator: characterization and synthesis,” IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 47(1) 25-39, 2000.

CHAPTER 5

SUMMARY AND FUTURE SCOPE

5.1 Summary

Chapter 1 provides us with the general introduction of fractional order systems and a brief review of some of the research works carried out in this area

In **Chapter 2**, a brief account of the fractional order element and various approaches for its realization has been presented. We have also reviewed the Oustaloup, Leveron, Mathieu and Nanot method which has been used to realize the fractional order capacitors used in the oscillator circuits proposed in this work. The values of passive components used in the realization of an 8^{th} order approximation of the FoCof 0.01uF, for values α = 0.1-0.9, in steps of 0.1.

In **Chapter 3**, after a very brief introduction of some of the important works on realization of fractional order analog signal processing circuits using different active building blocks, an OTA based fractional order oscillator with electronic tunability of CO and FO has been presented. Detailed expressions for CO, FO and stability of the oscillator have been discussed. PSPICE simulation results and experimental results validating the operation of the presented circuit have also been included in the chapter.

In **Chapter 4** a fractional order oscillator realized with five OTAs and three fractional order capacitors has been presented. General design equations representing eight different cases for α, β and γ have been derived. The CO, FO and the phase difference between the different output voltages can be tuned electronically by changing the values of transconductaces of different OTAs for specified values of α, β and γ. PSPICE simulations using CMOS OTAs and fractional order capacitors designed using the approximation method proposed by Oustaloup, Levron, Mathieu and Nanot

[22] and experimental results obtained using IC OTA LM13700 have also presented to establish the workability of the presented circuit.

5.2 Future scope

The work carried out in this dissertation may be extended in several directions. Some of these are listed below:

I. Different methods of realization of fractional order capacitor may be used in the realization of the same oscillator structure and the relative performance of fractional order oscillators thus realized may be compared.

II. In case of multi-phase fractional order oscillators, condition of stability may be studied in detail.

III. Methods for electronic tuning of the fractional order capacitor parameters (α, β and γ) by changing some voltage/current may be investigated.

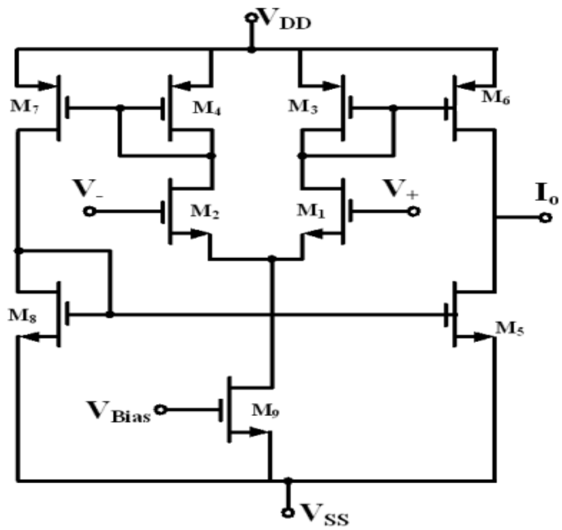
PUBLICATION

PAPER IN INTERNATIONAL CONFERENCE

1. **V. Singh,** and P. Kumar, (2020) "Fractional Order Oscillators Using OTAs,", In 2020 6th International conference on Control, Automation and Robotics (ICCAR),Singapore,27-32 . https://doi.org/10.1109/ICCAR49639.2020.9108095

APPENDIX

CMOS OTA PSPICE library file



CMOS OTA [1]

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+U0      = 280.5758609  UA      = -1.208176E-9 UB      = 2.159494E-18
+UC      = 5.340577E-11 VSAT    = 9.601364E4   A0      = 1.7852987
+AGS     = 0.4008594    B0      = -3.73715E-9  B1      = -1E-7
+KETA    = -1.136459E-3 A1      = 2.580625E-4  A2      = 0.9802522
+RDSW    = 105.472458   PRWG    = 0.5          PRWB    = -0.2
+WR      = 1            WINT    = 0            LINT    = 1.571909E-8
+XL      = 0            XW      = -1E-8        DWG     = -7.918114E-9
+DWB     = -3.223301E-9 VOFF    = -0.0956759   NFACTOR = 2.4447616
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+CDSCB   = 0            ETA0    = 2.489084E-3  ETAB    = -2.143433E-5
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+PRT     = 0            UTE     = -1.5         KT1     = -0.11
+KT1L    = 0            KT2     = 0.022        UA1     = 4.31E-9
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```

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m9 6 11 4 4 nmos w= 5.4u l=.36u
vdd 2 0 dc 0.9
vss 0 4 dc 0.9
.ends ota
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Reference

[1] E. W. Greeneich, "Analog integrated circuits," Springer Science & Business Media, 2012.