M.TECH [CONTROL AND INSTRUMENTATION] [JYOTI SRIVASTAVA] 2020

FRACTIONAL ORDER CAPACITOR BASED REALIZATION OF INVERSE FILTERS AND FILTERS

A DISSERTATION

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF

MASTER OF TECHNOLOGY
IN
CONTROL & INSTRUMENTATION

Submitted by:

[JYOTI SRIVASTAVA]

(Roll No. 2K18/C&I/10)

Under the supervision of

[PROF. PRAGATI KUMAR]
&
[MR. RAM BHAGAT]

DEPARTMENT OF ELECTRICAL ENGINEERING
DELHI TECHNOLOGICAL UNIVERSITY
(Formerly Delhi College of Engineering)
Bawana Road, Delhi-110042

AUGUST, 2020

DELHI TECHNOLOGICAL UNIVERSITY
(Formerly Delhi College of Engineering)
Bawana Road, Delhi-110042

CANDIDATE'S DECLARATION

I, Jyoti Srivastava, Roll No. 2K18/C&I/10 student of M.Tech. (Control & Instrumentation), hereby declare that the Dissertation titled "**Fractional order Capacitor Based Realization of Inverse Filters and Filters**" which is submitted by me to the Department of Electrical Engineering, Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Technology, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

jyoti srivastava

Place: Delhi **JYOTI SRIVASTAVA**

Date: 20-08-2020

DEPARTMENT OF ELECTRICAL ENGINEERING

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

Bawana Road, Delhi-110042

CERTIFICATE

I hereby certify that the Dissertation titled "**Fractional order Capacitor Based Realization of Inverse Filters and Filters**" which is submitted by Ms. Jyoti Srivastava, Roll No 2K18/C&I/10 Electrical Engineering Department, Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Technology, is a record of the project work carried out by the students under my supervision. To the best of my knowledge this work has not been submitted in part or full for any Degree or Diploma to this University or elsewhere.

Place: Delhi

Prof. PRAGATI KUMAR
SUPERVISOR

Mr. Ram Bhagat
SUPERVISOR

Date:20-08-2020

ABSTRACT

Fractional calculus, i.e. fractional order integration and differentiation, which is considered as the backbone of fractional order circuits, has numerous applications in science and engineering fields namely, bio-medical engineering, control system, analog signal processing/generation, fluid mechanics, etc. In this dissertation after briefly reviewing various methods employed in the realization of simulated fractional order capacitors novel circuits of operational transconductace amplifier-based inverse filters have been presented. As a second problem, the performance of an existing, VDTA-based biquad filter in fractional order domain has evaluated. The electronic tunability of the various parameters of the VDTA-based fractional order filters has also been examined. The workability of the inverse fractional order filter circuits along with fractional operator have been verified through Cadence virtuoso simulation tool using 0.18um CMOS technology parameter and generalized universal fractional order filter configuration using PSPICE and MATLAB simulation. Also, stability of the fractional order filter designed have been discussed briefly.

ACKNOWLEDGEMENT

First and foremost, I express my deep sense of gratitude to my supervisor, counselor and advisor **Prof. Pragati Kumar and Mr. Ram Bhagat**, Department of Electrical Engineering for their constant guidance, support, motivation and encouragement throughout the duration of this work. Their readiness for consultation at all times, educative comments, concern and assistance have been invaluable.

I want to thank **Dr. Uma Nangia,** Head of Electrical Department for building this platform and creating such an energetic environment for research work.

I want to thank **Mr. Ajishek Raj** (Research Scholar) for his help and support and all the staff of the Analog Signal Processing Lab, Electrical Engineering Department for their fullest cooperation.

I would like to thank my friends and all those who have directly or indirectly helped me in completion of the thesis well in time.

Finally, I wish to thank my parents for their moral support and confidence showed in me to pursue M. Tech at an advanced stage of my academic career.

JYOTI SRIVASTAVA

CONTENTS

CANDIDATE'S DECLARATION ii
CERTIFICATE iii
ABSTRACT iv
ACKNOWLEDGEMENT v
LIST OF FIGURES viii
LIST OF TABLES x
LIST OF ABBREVIATIONS xi
Chapter 1 INTRODUCTION 1
1.1 Overview 1
1.2 Literature review on Fractional order Systems 2
1.2.1 Fractional order immitance function or constant phase element 2
1.2.2 Fractional order Filters 2
1.2.3 Fractional order Inverse filter 3
1.2.4 Fractional order Universal filter 3
1.2.5 Fractional order Oscillator 3
1.3 Objective 4
1.4 Thesis Organization 4
Chapter 2 Fractional order immittance element 9
2.1 Introduction 9
2.2 Fractional Calculus 9
2.3 Definitions 10
2.4 Fractance Device 12
2.4.1 Methods of designing fractance element 13
2.5 Simulation Results 14
2.6 Application 19
2.7 Conclusion 20
Chapter 3 Fractional order Analog inverse filter using otas 23
3.1 Introduction 23
3.2 Proposed Fractional Order Voltage-Mode Inverse Filter Configurations 24
3.2.1 Fractional order inverse high pass filter: 26
3.2.2 Fractional order inverse band pass filter: 26
3.2.3 Fractional order inverse band elimination filter: 26
3.3 Simulation Results 27

3.4 Conclusion 31
Chapter 4 VDTA based fractional order universal filter 35
4.1 Introduction 35
4.2 Circuit Description 36
4.2.1 Designing Fractional-order Capacitor using Oustaloup, Laveron, Mathieu and Nanot Approximation 36
4.2.2 Steps for determining the fractional capacitor using Oustaloup, Laveron, Mathieu and Nanot Approximation 36
4.2.3 The Voltage Differencing Transconductance Amplifier 38
4.3 The Fractional order VDTA Filter 39
4.4 Stability Analysis 42
4.5 Simulation Results 44
4.5.1 Tunability with α 45
4.5.2 Tunability with I_{bias} 47
4.6 Conclusion 50
Chapter 5 Summary and future scope 53
5.1 Summary 53
5.2 Future Scope 54
PUBLICATION 55
APPENDICES 56
APPENDIX 1 56
APPENDIX 2 57

LIST OF FIGURES

Figure 2.1 Classification of fractional-order element [10] 12
Figure 2.2 Resulting scheme of Constant Phase Element 15
Figure 2.3 Phase response of fractional order element..19
Figure 2.4 Fractional order low pass filter..19
Figure 2.5 Fractional order low pass filter frequency response.................................20
Figure 3.1 Symbolic representation of OTA 25
Figure 3.2 Fractional order inverse high pass filter...25
Figure 3.3 Fractional order inverse band pass filter 25
Figure 3.4 Fractional inverse band elimination filter 26
Figure 3.5 CMOS implementation of OTA..28
Figure 3.6 Frequency responses of (a) fractional order inverse high pass filter, (b) fractional order inverse band pass filter and (c) fractional order inverse band elimination filter. 29
Figure 3.7 Tunability of gain for fractional order inverse high pass filter 29
Figure 3.8 Tunability of Q for fractional order inverse band pass filter.....................30
Figure 3.9 Tunability of Q for fractional order inverse band elimination filter 30
Figure 4.1 Foster I canonical RC structure...37
Figure 4.2 Phase response for different α 37
Figure 4.3 VDTA structure...38
Figure 4.4 VDTA using LM13700 IC 39
Figure 4.5 The modified fractional order current mode filter....................................39
Figure 4.6 W-plane (Region of stability)...42
Figure 4.7 Stability plot of FLPF C=0.382μF and α=0.9 43
Figure 4.8 Stability plot of FHPF C=0.382μF and α=0.9...43
Figure 4.9 Stability plot of FBPF C=0.382μF and α=0.9...43
Figure 4.10 Stability plot of FBSF C=0.382μF and α=0.9.......................................44
Figure 4.11 Stability plot of FAPF C=0.382μF and α=0.9.......................................44
Figure 4.12 Tunability of FLP with respect to α……………………………………45
Figure 4.13 Tunability of FHP with respect to α……………………………………45
Figure 4.14 Tunability of FBP with respect to α……………………………………46

Figure 4.15 Tunability of FBS with respect to α……………………………………46
Figure 4.16 Tunability of FAP with respect to α……………………………………46
Figure 4.17 Tunability of FLP with respect to I_{bias}…………………………………47
Figure 4.18 Tunability of FLP with respect to I_{bias}…………………………………47
Figure 4.19 Tunability of FLP with respect to I_{bias}…………………………………48
Figure 4.20 Tunability of FLP with respect to I_{bias}…………………………………48
Figure 4.21 Tunability of FLP with respect to I_{bias}…………………………………48

LIST OF TABLES

Table 2.1 Value of resistance and capacitance for α= 0.25 16
Table 2.2 Value of resistance and capacitance for α= 0.35 16
Table 2.3 Value of resistance and capacitance for α= 0.5 17
Table 2.4 Value of resistance and capacitance for α= 0.6 17
Table 2.5 Value of resistance and capacitance for α= 0.8 18
Table 2.6 Value of resistance and capacitance for α= 0.9 18
Table 2.7 Cut-off frequencies with respect to different values of alpha.....................20
Table 3.1 Cut off Frequency, Gain and Pole Quality Factor for α=1 27
Table 3.2 Cut-off Frequencies (in Hz) of Filters for Different α................................31
Table 4.1 Condition for Universal filter responses...40
Table 4.2 Filter parameters when α=1 42
Table 4.3 Different cases of stability, roots, ω_o and Q 44
Table 4.4 Cut-off frequencies (f_h) (in khz) of fractional filters for different α……...49
Table 4.5 Max/min frequencies (f_m) (in khz) of fractional filters for different α…...49
Table 4.6 Cut-off frequencies (f_h) (in khz) of fractional filters for different I_{bias}…...49
Table 4.7 Max/min frequencies (f_m) (in khz) of fractional filters for different I_{bias}....49
Table 4.8 Slope of attenuation in passband/stopband (dB/decade)…………………50

LIST OF ABBREVIATIONS

OTA	Operational Transconductance Amplifier
VDTA	Voltage Differencing Transconductance Amplifier
CMOS	Complementary Metal Oxide Semiconductor
CPE	Constant Phase Element
CFE	Continued Fraction Expansion
CFOA	Current Feedback Operational Amplifier
OTRA	Operational Trans-resistance Amplifier
CDBA	Current Differencing Buffered Amplifier
CCII	Current Conveyor Second Generation
BOTA	Balanced Output Transconductance Amplifier
ACA	Adjustable Current Amplifier
MISO	Multi Input Single Ouput
FDNR	Frequency Dependent Negative Resistor
FC	Fractional Capacitor
ADE	Analog Design Environment
VM	Voltage Mode
FLPF	Fractional low-pass Filter
FHPF	Fractional high-pass Filter
FBPF	Fractional band-pass Filter
FBRF	Fractional band-reject Filter
FAPF	Fractional All-pass Filter

Chapter 1

INTRODUCTION

1.1 Overview

The present dissertation deals with the realization of fractional order inverse filters and filters. In comparison to classical integer order models, fractional calculus gives us a better instrument for memory depiction and hereditary properties of systems and processes. The modelling of properties of real materials for mechanical, electrical and electro-mechanical uses the benefits of fractional derivative. The application of fractional element can vary from determination of frictionless curve shape to study of electrical transmission lines (developed by Heaviside in 1892) [1].

Fractional order calculus finds application in various fields of science and engineering viz. modelling of speech signals [2], modelling of cardiac tissue electrode interface [3], propagation of sound waves in rigid porous materials [4] control of autonomous vehicles (Lateral and longitudinal) [5] fluid Mechanics [6] edge detection [7], also.

Apart from the above application areas, fractional order immittance element, popularly known as a constant phase element (CPE), finds vast application in electrical and electronic engineering. During the last two decades, hundreds of research papers have appeared in open literature dealing with analog circuit implementation of fractional order circuit elements and signal processing circuits.

Analog signal processing circuits like filters, inverse filters, multipliers and oscillators using fractional order elements have gained more importance as using a fractional order circuit element instead of conventional reactive elements gives more flexibility in tuning of various parameters of these circuits. Using fractional order capacitor instead of integer order capacitors along with active building blocks gives us even more variations in terms of bandwidth, frequency, quality factors and gain of filters and inverse filters.

1.2 Literature review on Fractional order Systems

Over the last few years, research interest has increased in designing different fractional order circuits by using various approximation methods, although fractional calculus is three hundred years old [8]. In this thesis work, we have mainly focused on designing fractional operator (s^{α}) using different approximation methods, thereby using it as an application for designing different circuits namely fractional order inverse filter and fractional order universal filter. There is vast scope for designing a fractional order circuits and the researchers have mainly focused on fractional order element designing. In our thesis work we have mainly focused on designing part as well as finding its applications in filter circuits.

1.2.1 Fractional order immitance function or constant phase element (CPE)

The CPE is the basic unit for designing the fractional order analog circuits. There are various approximation techniques for fractional order operator that are available in open literature. Some of these are: Valsa and Vlach Approximation [9], Oustaloup, Levron, Mathieu, and Nanot Recursive approximation [10], Continued Fraction Expansion (CFE) [11], El-Khazali reduced-order approximation [12], Carlson and Halijak approximation [13], Matsuda and Fujii approximation [14], Modified Oustaloup, Laveron, Mathieu and Nanot [15], Charef, Sun, Tsao, and Onaral approximation [16], Squared Magnitude Function [17-18].

In most of these approximations, a Foster/Cauer like network is used to emulate the magnitude /constant phase behavior of the CPE in a specified frequency range. Higher the number of series/parallel branches, in RC network, the more accurate is the approximation. A physical CPE using electrochemical method has also been proposed in literature. [19]. The CPE is the basic unit for designing any fractional order analog circuits.

1.2.2 Fractional order Filters

Ever since the appearance of the general methods for realization of first and second order filters in fractional order form [20-21], fractional order filters using various active building block like the operational amplifier [22], current feedback operational amplifier [23], current conveyors [24], operational trans-resistance amplifier [25] and various other types of amplifiers have been presented in the open literature. In most of these filter circuits, the CPE used has been realized using the approximation methods suggested in [9-18]. It has been shown that the characterizing parameters of these filters can be controlled by appropriately choosing the order, 'α' the fractional order elements.

1.2.3 Fractional order Inverse filter

Inverse filters are a kind of frequency selective circuit in which the frequency response is reciprocal of the frequency response of the corresponding normal filter. Thus, an inverse low pass filter has its zeros lying in the LHS of the s-plane while its poles lie at infinity. Various fractional order inverse filter circuits have been reported in open literature using various active building blocks along with some passive components such as operational amplifier [26], current feedback operational amplifier (CFOA) [27], operational trans-resistance amplifier (OTRA) [28], current difference buffered amplifier (CDBA) [29], current conveyor second generation (CCII) [30], etc.

1.2.4 Fractional order Universal filter

Similarly, in open literature various fractional order universal filters have been reported using different active building blocks along with some passive components such as operational amplifier [31], Operational Transconductance Amplifier (OTA) [32], Balanced Output Transconductance Amplifier (BOTA) with ACA [33], voltage differencing transconductance amplifier (VDTA) [34], Current differencing Buffered Amplifier (CDBA) [35], etc.

1.2.5 Fractional order Oscillator

Several realizations of fractional order oscillators have also appeared in open literature in recent past [36-41]. It has been shown in these works that it is possible to initiate and maintain sustained oscillations at a given frequency in an autonomous circuit with more than one fractional order capacitors. The frequency of oscillation and the condition of oscillation can be controlled over a much wider range because of the extra degree of freedom provided by the order α the fractional order elements.

1.3 Objective

On the basis of literature survey and the outlined scope in the previous sub-section, the objectives are listed as below:

i. Design of fractional order capacitors using different methods of realization.
ii. Analysis and design of tunable fractional order analog inverse filter using OTAs.
iii. Generalization of a current mode multiple input single output (MISO) type biquad filter, employing one voltage differencing transconductance amplifier (VDTA), into fractional order domain examine the electronic tunability of its various parameters.

1.4 Thesis Organization

This thesis work consists of five chapters the objective of which is shown below:

i. Chapter 1 provides us with the general introduction of fractional order systems and a brief review of some of the research works carried out in this area.
ii. Chapter 2 gives a detailed description about definitions and methods of designing fractional element used in engineering. Simulation results for the design of fractional order capacitor using the Valsa and Vlach method have also been included in this chapter.
iii. In Chapter 3 fractional order inverse high pass, fractional order inverse low pass, fractional order inverse band pass, and fractional order inverse band reject filters are proposed using two operational transconductance amplifiers and fractional capacitors. Layout of all the circuits have also been included in this chapter.
iv. Chapter 4 is concerned with the electronic tunability of a current mode multiple input single output (MISO) type filter employing one voltage differencing transconductance amplifier (VDTA) and two fractional order capacitors.
v. Chapter 5 includes summary and scope for future work.

REFERENCES

[1] M. Dalir and M. Bashqur, "Applications of Fractional Calculus," Applied Mathematical Sciences, 4, no. 21, pp. 1021-1032, 2010.

[2] K. Assaleh and W. M. Ahmad, "Modeling of speech signals using fractional calculus," In 2007 9th International Symposium on Signal Processing and Its Applications, 2007, pp. 1-4.

[3] R. L. Magin and M. Ovadia, "Modeling the Cardiac Tissue Electrode Interface Using Fractional Calculus," Journal of Vibration and Control, 14, no. 9-10, pp. 1431-1442, 2008.

[4] Z. E. A. Fellah, C. Depollier, and M. Fellah, "Application of fractional calculus to the sound waves propagation in rigid porous materials: Validation via ultrasonic measurement," Acta Acustica united with Acustica, 88, no. 1, 2002, pp. 34-39.

[5] J. I. Su´arez , B. M. Vinagre , A. J. Calder´on , C. A. Monje and Y. Q. Chen, "Using Fractional Calculus for Lateral and Longitudinal Control of Autonomous Vehicles," In International Conference on Computer Aided Systems Theory, Springer, Berlin, Heidelberg, 2003, pp. 337-348.

[6] V. V. Kulish and J. L. Lage, "Application of Fractional Calculus to Fluid Mechanics," J. Fluids Eng, 124, no. 3, pp. 803-806, 2002.

[7] B. Mathieu, P. Melchior, A. Oustaloup and Ch. Ceyral, "Fractional differentiation for edge detection," Signal Processing, 83, no. 11, pp. 2421-2432, 2003.

[8] Y. Q. Chen, I. Petras, and D. Xue, "Fractional order control-a tutorial," In 2009 American control conference, 2009, pp. 1397-1411.

[9] J. Valsa, and J. Vlach, "RC models of a constant phase element," International Journal of Circuit Theory and Applications, 41, no. 1, pp. 59-67, 2013.

[10] A. Oustaloup, F. Levron, B. Mathieu, and F. M. Nanot, "Frequency-band complex noninteger differentiator: characterization and synthesis," IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 47, no. 1, pp. 25-39, 2000.

[11] S. C. D. Roy, "On the realization of a constant-argument immittance or fractional operator," IEEE Transactions on Circuit Theory, 14, no. 3, pp. 264-274, 1967.

[12] R. El-Khazali, “On the biquadratic approximation of fractional-order Laplacian operators,” Analog Integrated Circuits and Signal Processing, 82, no. 3, pp. 503-517, 2015.

[13] G. Carlson, and C. Halijak, “Approximation of fractional capacitors (1/s)^(1/n) by a regular Newton process,” IEEE Transactions on Circuit Theory, 11, no. 2, pp. 210-213, 1964.

[14] K. Matsuda, and H. Fujii, “H (infinity) optimized wave-absorbing control-Analytical and experimental results,” Journal of Guidance, Control, and Dynamics, 16, no. 6, pp. 1146-1153, 1993.

[15] D. Xue, C. Zhao, and Y. Q. Chen, “A modified approximation method of fractional order system,” In 2006 International Conference on Mechatronics and Automation, 2006, pp. 1043-1048.

[16] A. Charef, , H. H. Sun, Y. Y. Tsao, and B. Onaral, “Fractal system as represented by singularity function,” IEEE Transactions on automatic Control, 37, no. 9, pp. 1465-1470, 1992.

[17] M. Khanra, J. Pal, and K. Biswas, “Rational approximation of fractional order SISO System using squared-magnitude function,” In Mathematical Methods in Engineering International Symposium, pp. 21-24. 2010.

[18] C. P. Chen, and Y. T. Tsay, “A squared magnitude continued fraction expansion for stable reduced models,” International Journal of Systems Science, 7, no. 6, pp. 625-634, 1976.

[19] M. S. Krishna, S. Das, K. Biswas, and B. Goswami, “Fabrication of a fractional order capacitor with desired specifications: a study on process identification and characterization,” IEEE Transactions on Electron Devices, 58, no. 11, pp. 4067-4073, 2011.

[20] A. G. Radwan, A. M. Soliman, and A. S. Elwakil. “First-order filters generalized to the fractional domain,” Journal of Circuits, Systems, and Computers, 17, no. 1, pp. 55-66, 2008.

[21] A. G. Radwan, A. S. Elwakil, and A. M. Soliman. “On the generalization of second-order filters to the fractional-order domain.” Journal of Circuits, Systems, and Computers, 18, no. 2, pp. 361-386, 2009.

[22] A. S. Ali, A. G. Radwan, and A. M. Soliman, “Fractional order Butterworth filter: active and passive realizations,” IEEE journal on emerging and selected topics in circuits and systems, 3, no. 3, pp. 346-354, 2013.

[23] G. Tsirimokou, S. Koumousi, and C. Psychalinos, “Design of fractional-order filters using current feedback operational amplifiers,” Journal of Engineering Science and Technology Review, 9, no. 4, pp. 71-81, 2016.

[24] F. Khateb, D. Kubánek, G. Tsirimokou, and C. Psychalinos, “Fractional-order filters based on low-voltage DDCCs,” Microelectronics Journal, 50, pp. 50-59, 2016.

[25] G. Kaur, A. Q. Ansari, and M. S. Hashmi, “Fractional order high pass filter based on operational transresistance amplifier with three fractional capacitors of different order,” Advances in Electrical and Electronic Engineering, 17, no. 2, pp. 155-166, 2019.

[26] D. R. Bhaskar, Manoj Kumar, and Pragati Kumar, “Fractional order inverse filters using operational amplifier,” Analog Integrated Circuits and Signal Processing, 97, no. 1, pp. 149-158, 2018.

[27] E. M. Hamed, L. A. Said, A. H. Madian, and A. G. Radwan, “On the approximations of CFOA-based fractional-order inverse filters," Circuits, Systems, and Signal Processing, 39, no. 1, pp. 2-29, 2020.

[28] N. A. Khali, L. A. Said, A. G. Radwan, and A. M. Soliman, “Multifunction Fractional Inverse Filter Based on OTRA,” In 2019 Novel Intelligent and Leading Emerging Sciences Conference (NILES), vol. 1, 2019, pp. 162-165.

[29] R. Bhagat, D. R. Bhaskar, and P. Kumar, “Multifunction Filter/Inverse Filter Configuration Employing CMOS CDBAs,” International Journal of Recent Technology and Engineering, 8, no. 04, pp. 8844-8853, 2019.

[30] N. A. Khalil, L. A. Said, A. G. Radwan, and A. M. Soliman, “Fractional Order Inverse Filters Based on CCII Family,” In 2019 Fourth International Conference on Advances in Computational Tools for Engineering Applications (ACTEA), 2019, pp. 1-4.

[31] M. C. Tripathy, K. Biswas, and S. Sen, “A design example of a fractional-order Kerwin–Huelsman–Newcomb biquad filter with two fractional capacitors of different order,” Circuits, Systems, and Signal Processing, 32, no. 4, pp. 1523-1536, 2013.

[32] G. Singh, Garima, and P. Kumar, "Fractional Order Capacitors Based Filters Using Three OTAs," In 2020 6th International Conference on Control, Automation and Robotics (ICCAR), 2020, pp. 638-643.

[33] J. Dvorak, J. Jerabek, Z. Polesakova, D. Kubanek, and P. Blazek, "Multifunctional Electronically Reconfigurable and Tunable Fractional-Order Filter," Elektronika ir Elektrotechnika, 25, no. 1, 2019, pp. 26-30.

[34] P. Rani, and R. Pandey, "Voltage differencing transconductance amplifier based fractional order multiple input single output universal filter," Solid State Electronics Letters, 1, no. 2, pp. 110-118, 2019.

[35] A. Q. Ansari, G. Kaur, and M. S. Hashmi, "Current Differencing Buffered Amplifier (CDBA) based Current Mode Universal Fractional Order Filter," Proc. Nat. Con. on Adv. in Microelectronics, Instrumentation and Communication (MICOM), BITS Pilani, India, 2015.

[36] A. G. Radwan, A. S. Elwakil, and A. M. Soliman, "Fractional-order sinusoidal oscillators: design procedure and practical examples," IEEE Transactions on Circuits and Systems I, Regular Papers, 55, no. 7, pp. 2051-2063, 2008.

[37] W. Ahmad, R. El-Khazali, and A. S. Elwakil, "Fractional-order Wien-bridge oscillator," Electronics Letters, 37, no. 18, pp. 1110-1112, 2001.

[38] A. G. Radwan, A.. M. Soliman, and A. S. Elwakil, "Design equations for fractional-order sinusoidal oscillators: Four practical circuit examples," International Journal of Circuit Theory and Applications, 36, no. 4, pp. 473-492, 2008.

[39] V. Singh, and P. Kumar, "Fractional Order Oscillator Using OTAs," In 2020 6th International Conference on Control, Automation and Robotics (ICCAR), 2020, pp. 27-32.

[40] L. A. Said, A. G. Radwan, A. H. Madian, and A. M. Soliman, "Fractional order oscillators based on operational transresistance amplifiers," AEU-International Journal of Electronics and Communications, 69, no. 7, pp. 988-1003, 2015.

[41] A. Kartci, N. Herencsar, J. Koton, L. Brancik, K. Vrba, G. Tsirimokou, and C. Psychalinos, "Fractional-order oscillator design using unity-gain voltage buffers and OTAs," In 2017 IEEE 60th International Midwest Symposium on Circuits and Systems (MWSCAS), 2017, pp. 555-558.

Chapter 2

FRACTIONAL ORDER IMMITTANCE ELEMENT

2.1 Introduction

Fractional order immittance function, also popularly known as the constant phase element (CPE) is the most important constitutive element of any fractional order analog processing circuit. The driving point impedance of a CPE is expressed as Z(s) = $Ks^{\pm\alpha}$. The exponent ‘α’ may have a non-integer value. The introduction of the non-integer power of the complex frequency variables brings the picture of concept of non-integer differentiation and integration. In the following, we briefly present the fundamental definitions and concepts of fractional order calculus.

2.2 Fractional Calculus [1]

The concept of fractional calculus was first brought into light by Leibniz and L'Hospital in 1695. However, Riemann first coined the term fractional operator in 1838. Having characteristics to fulfill the drawbacks of integer order, this system brings more possibilities into research area. Referring to Wallis's infinite product for $\pi/2$ Leibnitz used the term $d^{\frac{1}{2}}$ quoting that it might be possible that similar results could be used for it. In 1819, the French mathematician named S.F. Lacroix, mentioned the derivative of arbitrary order for the very first time. Less than two pages were devoted on this topic in his 700 page long text on differential and integral calculus. Beginning with $y = x^p$, where p is a positive integer, he found the qth derivative to be:

$$\frac{d^q y}{dx^q} = \frac{p!}{(p-q)!} x^{p-q} \tag{2.1}$$

By putting q equal to ½ and p by any positive real integer m, and using Legendre's symbol Γ (generalized factorial symbol) Lacroix obtained the following formula:

$$\frac{d^{1/2}y}{dx^{1/2}} = \frac{\Gamma(m+1\,)}{\Gamma(m+1/2)}x^{m-1/2} \tag{2.2}$$

Equation (2.2) gives the derivative of an arbitrary order ½ of the expression x^m, where the gamma function Γ can be defined by equation (2.3) as:

$$\Gamma(\mathrm{z}) = \int_0^{\infty} e^{-u}u^{z-1}du, \qquad for\ all\ z\ \in \mathbb{R} \tag{2.3}$$

He also introduces the example for y=x and gave different examples for more understanding of readers. Different arbitrary order derivations were made by Fourier and Euler, but no examples or applications were given by them.

Fractional calculation does not imply the calculus of fractions, nor the fraction of any calculus of variations. It is the theory of integrations and derivatives of any random order, which bring together and sum up documentation of integer order differentiation and integration.

The beauty of this subject is that nature's reality is better depicted by fractional derivatives and integrals. So the credit belongs to Niels Henrik Abel [2] for providing its application in Tautochrone problem, which attracted many researchers in this field among whom Liouville made the first big attempt in defining fractional derivative which is listed in section 2.3.

In spite of the fact that, as far as 300 years old, this field was on the mathematicians intrigue, just the most recent couple of years this is showed up in a few applied fields of science, for example, control theory, dispersion theory, electromagnetic, biomedicine, and signal and processing of images [3-8].

2.3 Definitions

The fractional derivatives can be defined by using two main approaches as described below:

(i) Grunwald-Letnikov definitions: In the first approach limits of finite differences are considered as differentiation and integration. As defined in [9] this approach is followed by Grunwald-Letnikov definition depicted in equation (2.4).

$$aD_t^{\alpha} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\alpha}{j} f(t - jh) \tag{2.4}$$

where $w_j^{\alpha} = (-1)^j \binom{\alpha}{j}$ are the binomial coefficients of $(1 - z)^{\alpha}$. The values of α can be non-integer positive or negative depending upon the differentiation or integration. The subscripts to the left and right side of D are the lower and upper limits of the integral.

(ii) Riemann-Liouville: In the second approach, a convolution type representation of repeated integration is generalized. This approach is employed by Riemann-Liouville and Caputo definitions [9]. These are fundamentally related to fractional integration operator, making it more popular than other approaches. This approach is significant in handling with real-world problem, since it allows traditional initial and boundary conditions for the formulation of the problem. Resulting the following expression as given in equation (2.5)

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(\mathrm{n} - \alpha)} \int_{\alpha}^{t} \frac{f^{(n)}(\tau)}{(t - \tau)^{\alpha+1-n}} d\tau \tag{2.5}$$

where $\alpha \in (n - 1, n)$, and $\Gamma(\cdot)$ is defined as the gamma function.

Laplace transform is a significant tool to design and study the electronic circuits, it transforms circuit system from the time domain into the frequency domain. The analysis of circuits can be done algebraically with this transformation, minimizing the complexity of solving lengthy differential equations. Hence applying Laplace transform to (2.5) gives us equation (2.6)

$$L\{D_t^{\alpha} f(t)\} = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f(0)^{(k)} \tag{2.6}$$

where $f(0)$ is the initial condition.

s^{α} is known as the fractional Laplacian operator the most important mathematical tool is being applied to various applications in fractional domain.

(iii) Caputo's definition of Fractional Order differentiation: This comes under other approaches apart from first two as mentioned

$$oD_t^{\alpha} f(t) = \frac{1}{\Gamma(1 - \gamma)} \int_0^t \frac{f^{(m+1)}(\tau)}{(t - \tau)^{\gamma}} d\tau \tag{2.7}$$

where $\alpha = m + \gamma$, m is an integer and $0 < \gamma \leq 1$.Also, its definition for fractional-order integral is defined as [9]:

$$ {}_{0}D_t^{\gamma} f(t) = \frac{1}{\Gamma(-\gamma)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1+\gamma}} d\tau, \gamma > 0 \quad (2.8)$$

2.4 Fractance Device

A fractional order impedance function may be characterized in the frequency domain using the above mentioned definitions.

$$Z(s) = ks^{\alpha} = (k\omega)^{\alpha} e^{j\frac{\alpha\pi}{2}} \quad (2.9)$$

where k is constant and α is a fractional order. The analog behavior of the element can be clearly explained from equation (2.9).

$$\alpha = \begin{cases} -1 < \alpha < 0\,, element\ behaves\ as\ fractional\ order\ capacitor \\ 0 < \alpha < 1, element\ behaves\ as\ fractional\ order\ inductor \end{cases} \quad (2.10)$$

For special cases of α:

$$\alpha = \begin{cases} -1, it\ represents\ capacitor \\ -2\,, it\ represents\ the\ FDNR \\ 1, it\ represents\ inductor \end{cases} \quad (2.11)$$

where FDNR stands for Frequency Dependent Negative Resistor. Figure 1.1 classifies these elements briefly.

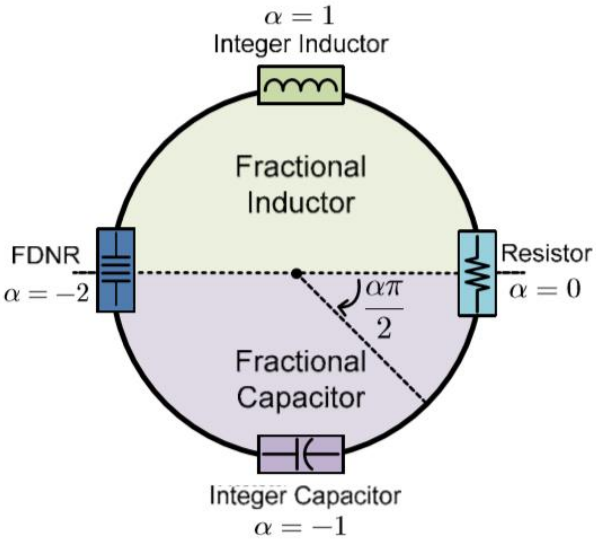


Figure 2.1 Classification of fractional-order element [10]

However, for physical realization of fractional order systems the fractional order capacitors and inductors are not yet available commercially. Hence, until the device is available for use there are numerous techniques to simulate the behavior of these fractional order impedances. In the following we describe some of these techniques which have been employed by different researchers in realization of the CPE used in various analog signal processing circuits.

2.4.1 Methods of designing fractance element (s^{α})[1]

I. Valsa and Vlach Approximation [11]:

Valsa and Vlach have described the analysis of the fractal systems for creating the analog model for the Constant Phase element (CPE). Their paper provides the methodology to simulate the CPE behavior. The effect of component tolerances for resultant responses is also described well in the paper. The impedance of CPE is defined as:

$$Z(s) = D(j\omega)^{\alpha} = Dj^{\alpha}\omega^{\alpha} = D\omega^{\alpha}e^{j\alpha} = D\omega^{\alpha}(\cos\varphi + \sin\varphi) \qquad (2.12)$$

where $\varphi = \alpha\frac{\pi}{2}$ for φ in radians or $\varphi = 90\alpha$ for φ in degrees. The character of impedance $Z(s)$ is decided by exponent α, as explained in previous section. This approximation is derived using basic network model of RC branches. The simulation results for this method will be shown in the later section.

II. Continued Fraction Expansion (CFE) [12]:

This approximation is based on the biquadratic second order transfer function of equal orders obtained by approximating s^{α}, which can be further used to increase the operating ranges by cascading with other biquadratic forms.

III. Matsuda and Fujii Approximation [13]:

This method was originally referred to as first form of Thiele's continued fraction (T-CF1). However, Matsuda and Fujii used this method for the very first time and henceforth this method is now referred by their name. By approximating the original function into set of equally logarithmic spaced frequencies, ω^{α} can be written as:

$$\omega^{\alpha} = d_0 + \cfrac{\omega - \omega_0}{d_1 + \cfrac{\omega - \omega_1}{d_2 + \cfrac{\cdots}{\cdots + \cfrac{\omega - \omega_k}{d_{k+1} + \cdots}}}} \qquad (2.13)$$

IV. Oustaloup, Levron, Mathieu, and Nanot approximation [14]:

[1] In contrast to the prevalent practice of naming a method by the name of the first authors of the paper in which the method was introduced, we have consciously put the name of all the authors of the paper in the name of the method.

This approximation is obtained via fractional operator s^{α} as shown in the following equation

$$s^{\alpha} = C\prod \frac{1 + s/\omega'_k}{1 + s/\omega_k} \quad (2.14)$$

where C is parameter for adjusting.

V. Carlson and Halijak Approximation [15]:

This approximation deals with the well-known third order newton process for approximating $(1/s)^{1/n}$ or $s^{1/n}$. Considering a system whose transfer function T(s) is given by:

$$T(s) = \frac{1 + MB}{1 + A^2B} * A \quad (2.15)$$

For simulating $1/\sqrt{s}$, consider $M(s) = 1/s \; and \; B(s) = B_0/s$ and by considering large feedback gain, the new transfer function can be re-written as :

$$T(s)' = \lim_{B_0 \to \infty} T(s) = 1/As \quad (2.16)$$

Now to obtain transfer function $1/\sqrt{s}$, simply substitute $A = 1/\sqrt{s}$ in the above equation and apply Taylor's expansion formula to approximate it. Similarly, we can approximate $\sqrt{s}$ also.

2.5 Simulation Results

In this dissertation, we have performed the approximation of fractional capacitors for different value of α and different frequency ranges using mainly following two methods

i. Valsa and Vlach method

ii. Oustaloup, Laveron, Mathieu and Nanot method

In the following we present the details of the first method with the help of an example and PSPICE simulations. For simulating the characteristics of ideal CPE, the resulting electrical scheme model is shown in Fig 2.2.

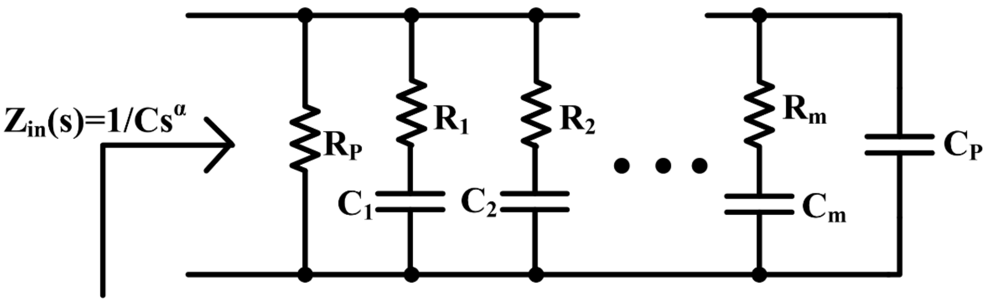


Figure 2.2 Resulting scheme of Constant Phase Element

For designing a constant phase element (CPE) in the frequency range and ripple factor as given below:

f_{min} (minimum frequency) = 1mHz, f_{max} (maximum frequency) = 1MHz and $\Delta\boldsymbol{\varphi}$ (ripple factor in degree) =±1°, following steps need to follow:

Step 1: We calculate the value of

$$ab = \frac{0.24}{1 + \Delta\varphi} \tag{2.17}$$

and from the product of ab we can find out individual values of a and b using formula $ab = 10^{\alpha \log(ab)}$. For different value of α, different value of a and b are obtained. From the f_{min} and f_{max} we calculate ω_{min} and ω_{max} using the formulae:

$$\omega_{min} = 2 * \pi * f_{min} \; and \; \omega_{max} = 2 * \pi * f_{max} \tag{2.18}$$

Step 2: We can find the value of time constant:

$$\tau = 1/\omega_{min} \tag{2.19}$$

and by assuming the value of first capacitor C_1 to determine the value of R_1 from the relation:

$$R_1 = \tau/C_1 \tag{2.20}$$

Step 3: We find the number of branches (m) from equation:

$$\omega_{max} = \frac{\omega_{min}}{ab^m} \tag{2.21}$$

$where\ m = \left(log10\,\frac{\omega_{max}}{\omega_{min}}\right)/(log10\ (ab))$, giving m as number of branches which can be further used for finding out the values of capacitors C_p, C_k and resistors R_p, R_k of the branches:

$$R_k = R_1 a^{k-1}, k = 1,2, \dots m \; and \; R_p = R_1 \frac{1-a}{a} \tag{2.22}$$

$$C_k = C_1 b^{k-1}, k = 1,2, \dots m \; and \; C_p = C_1 \frac{b^m}{1-b} \tag{2.23}$$

Step 4: The input impedance for the selected value of R_1 and C_1 can be calculated from the equation:

$$Y(j\omega_{av}) = \frac{1}{R_p} + j\omega_{av}C_p + \sum_{k-1}^{m} \frac{j\omega_{av}C_k}{1 + j\omega_{av}R_kC_k} \tag{2.24}$$

For finding the average frequency we use formulae:

$$\omega_{av} = z_k\sqrt{a} = \frac{1}{R_1C_1(ab)^{k-1}}\sqrt{a} \tag{2.25}$$

$where\ k = int(m/2).$

Step 5: The slope of modulus (D) is proportional to α, and can be calculated as:

$$D = Z_{av}\omega_{av}{}^{-\alpha} \tag{2.26}$$

$where\ Z_{av} = \frac{1}{|Y(j\omega_{av})|}.$

We have computed the values of the resistances and capacitances for different values α using MATLAB. These values are tabulated below in Table-2.1-Table 2.6.

Table 2.1 Value of resistance and capacitance for α= 0.25

$R_1 = 4740.2\ K\Omega$	$C_1 = 0.03359\ \mu F$
$R_2 = 2789.9\ K\Omega$	$C_2 = 6.849\ \mu F$
$R_3 = 1642.1\ K\Omega$	$C_3 = 1.396\ \mu F$
$R_4 = 966.46\ K\Omega$	$C_4 = 0.2847\ \mu F$
$R_5 = 568.83\ K\Omega$	$C_5 = 0.0581\ \mu F$
$R_6 = 334.79\ K\Omega$	$C_6 = 0.0118\ \mu F$
$R_7 = 197.05\ K\Omega$	$C_7 = 2.4130\ nF$
$R_8 = 115.98\ K\Omega$	$C_8 = 0.49198\ nF$
$R_9 = 68.259\ K\Omega$	$C_9 = 100.31\ pF$
$R_{10} = 40.175\ K\Omega$	$C_{10} = 20.451\ pF$
$R_P = 3.316\ M\Omega$	$C_P = 5.2375\ pF$

Table 2.2 Value of resistance and capacitance for α= 0.35

$R_1 = 12.138\ M\Omega$	$C_1 = 13.118\ \mu F$
$R_2 = 5.7793\ M\Omega$	$C_2 = 3.3063\ \mu F$
$R_3 = 2.7516\ M\Omega$	$C_3 = 0.833\ \mu F$

$R_4 = 1.3101\ M\Omega$	$C_4 = 0.210\mu F$
$R_5 = 623.76\ K\Omega$	$C_5 = 0.0529\mu F$
$R_6 = 296.98\ K\Omega$	$C_6 = 13.342\ \mu F$
$R_7 = 141.40\ K\Omega$	$C_7 = 3.363\ nF$
$R_8 = 67.322\ K\Omega$	$C_8 = 847.52\ pF$
$R_9 = 32.053\ K\Omega$	$C_9 = 213.61\ pF$
$R_{10} = 15.261\ K\Omega$	$C_{10} = 53.838\ pF$
$R_P = 13.356\ M\Omega$	$C_P = 18.142\ pF$

Table 2.3 Value of resistance and capacitance for α= 0.5

$R_1 = 57.71\ M\Omega$	$C_1 = 2.759\ \mu F$
$R_2 = 19.99\ M\Omega$	$C_2 = 0.956\ \mu F$
$R_3 = 6.93\ M\Omega$	$C_3 = 0.331\ \mu F$
$R_4 = 2.39\ M\Omega$	$C_4 = 0.115\ \mu F$
$R_5 = 831.08\ K\Omega$	$C_5 = 39.73\ nF$
$R_6 = 287.89\ K\Omega$	$C_6 = 13.763\ nF$
$R_7 = 99.73\ K\Omega$	$C_7 = 4.768\ nF$
$R_8 = 34.55\ K\Omega$	$C_8 = 1.652\ nF$
$R_9 = 11.968\ K\Omega$	$C_9 = 0.572\ nF$
$R_{10} = 4.1457\ K\Omega$	$C_{10} = 0.198\ nF$
$R_P = 108.89\ M\Omega$	$C_P = 0.105\ nF$

Table 2.4 Value of resistance and capacitance for α= 0.6

$R_1 = 175.34\ M\Omega$	$C_1 = 0.908\ \mu F$
$R_2 = 49.134\ M\Omega$	$C_2 = 0.389\ \mu F$
$R_3 = 13.769\ M\Omega$	$C_3 = 0.167\ \mu F$
$R_4 = 3.858\ M\Omega$	$C_4 = 71.316\ nF$
$R_5 = 1.081\ M\Omega$	$C_5 = 30.539\ nF$
$R_6 = 302.98\ K\Omega$	$C_6 = 13.078\ nF$
$R_7 = 84.903\ K\Omega$	$C_7 = 5.600\ nF$
$R_8 = 23.792\ K\Omega$	$C_8 = 2.398\ nF$

$R_9 = 6.667\ K\Omega$	$C_9 = 1.027\ nF$
$R_{10} = 1.868\ K\Omega$	$C_{10} = 0.4398\ nF$
$R_P = 450\ M\Omega$	$C_P = 0.329\ \mu F$

Table 2.5 Value of resistance and capacitance for α= 0.8

$R_1 = 2.075\ G\Omega$	$C_1 = 76.74\ nF$
$R_2 = 380.50\ M\Omega$	$C_2 = 50.21\ nF$
$R_3 = 69.775\ M\Omega$	$C_3 = 32.86\ nF$
$R_4 = 12.795\ M\Omega$	$C_4 = 21.51\ nF$
$R_5 = 2.346\ M\Omega$	$C_5 = 15.073\ nF$
$R_6 = 430.26\ K\Omega$	$C_6 = 9.209\ nF$
$R_7 = 78.90\ K\Omega$	$C_7 = 6.026\ nF$
$R_8 = 14.47\ K\Omega$	$C_8 = 3.943\ nF$
$R_9 = 2.65\ K\Omega$	$C_9 = 2.580\ nF$
$R_{10} = 486.54\ \Omega$	$C_{10} = 1.688\ nF$
$R_P = 9.2403\ G\Omega$	$C_P = 3.198\ nF$

Table 2.6 Value of resistance and capacitance for α= 0.9

$R_1 = 9.997\ G\Omega$	$C_1 = 15.92\ nF$
$R_2 = 1.483\ G\Omega$	$C_2 = 12.884\ nF$
$R_3 = 220.0\ M\Omega$	$C_3 = 10.423\ nF$
$R_4 = 32.635\ M\Omega$	$C_4 = 8.431\ nF$
$R_5 = 4.841\ M\Omega$	$C_5 = 6.820\ nF$
$R_6 = 718.14\ K\Omega$	$C_6 = 5.517\ nF$
$R_7 = 106.53\ K\Omega$	$C_7 = 4.463\ nF$
$R_8 = 15.803\ K\Omega$	$C_8 = 3.610\ nF$
$R_9 = 2.344\ K\Omega$	$C_9 = 2.920\ nF$
$R_{10} = 347.75\ \Omega$	$C_{10} = 2.363\ nF$
$R_P = 57.398\ G\Omega$	$C_P = 10\ nF$

The phase response of the fractional order capacitor for different values of α have been obtained in PSPICE and are given below in Fig. 2.3.

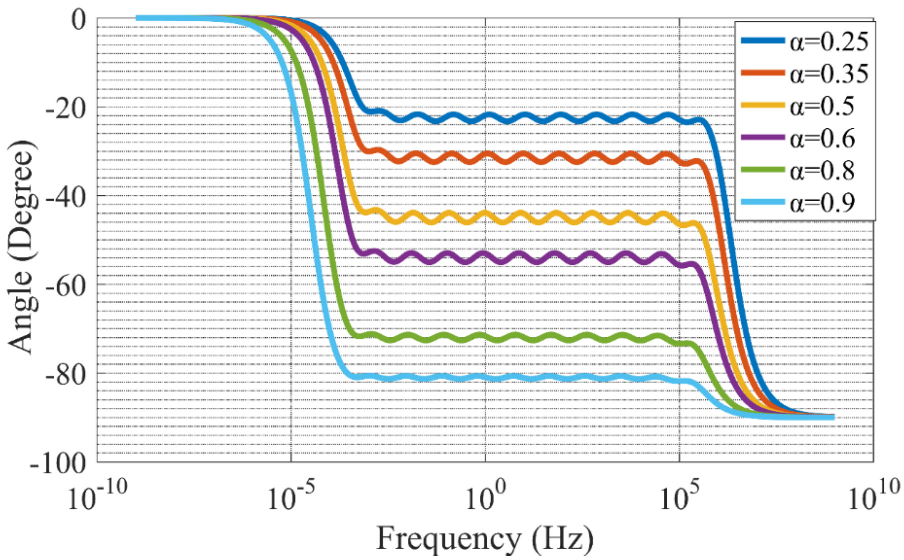


Figure 2.3 Phase response of fractional order element

2.6 Application

The workability of the fractional order capacitor realized using the Valsa and Vlach method have been verified by realizing a low pass filters using fractional order capacitors realized by the RC network of order 10. The values of RC components used for approximating fractional capacitor have been given in the previous section. The lowpass filter of order α with R=1kΩ and C=0.382uf/(rad/sec)$^{(1-\alpha)}$ is shown below in Fig. 2.4. Likewise for fractional order, we can say we are designing α order circuit as shown below.

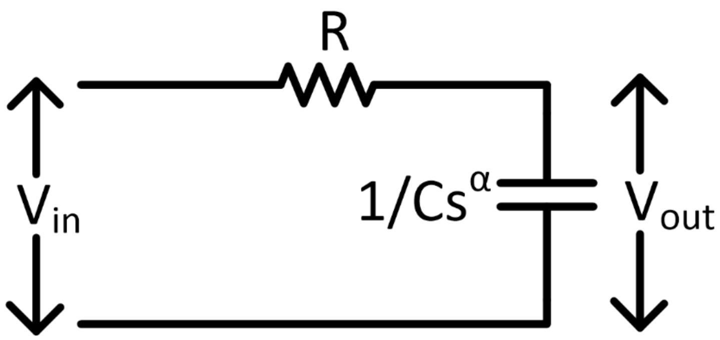


Figure 2.4 Fractional order low pass filter

Equation 2.27 describes the characteristics of above system:

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + s^{\alpha}CR} \tag{2.27}$$

We have implemented the above α order circuit for different values of α in our PSpice for which the frequency responses are obtained as shown below:

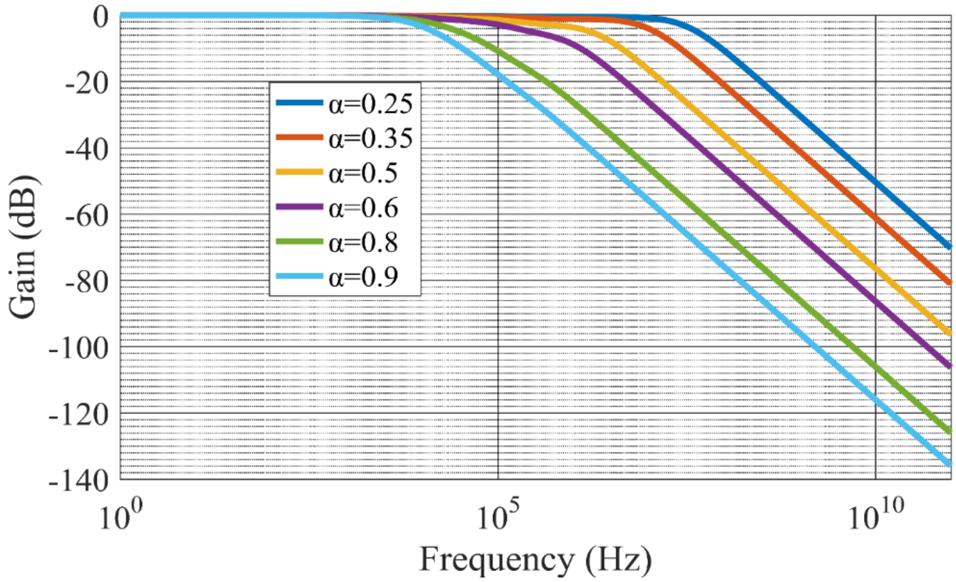


Figure 2.5 Fractional order low pass filter frequency response

Cut-off frequencies for various values of α is given in Table 2.1 by determining their frequencies at -3dB gain

Table 2.7 Cut-off frequencies with respect to different values of α

α	Cut-off frequency (Hz)
0.25	28.480 M
0.35	7.4989 M
0.5	547.818 K
0.6	108.166 K
0.8	16.876 K
0.9	9.0063 K

2.7 Conclusion

This chapter illustrates a relatively simple and acceptably faithful network model of the CPE, built from passive resistors and capacitors. In comparison to other presented models in research domain, its design does not need complicated optimization steps. Different values of gain can be obtained within specific frequency range.

REFERENCES

[1] G. A. Anastassiou, “Fractional differentiation inequalities,” vol. 68, New York: Springer, 2009.

[2] C. Houzel, “The Work of Niels Henrik Abel,” The Legacy of Niels Henrik Abel-The Abel Bicentennial, Oslo 2002, Springer-Verlag, 2004.

[3] M. D. Ortigueira, “An introduction to the fractional continuous-time linear systems: the 21st century systems”, IEEE Circuits and Systems Magazine , 8, no. 3, pp. 19–26, 2008.

[4] I. Podlubny, I. Petras, B. M. Vinagre, P. O’ Leary, and L. Dorcak, “Analogue realizations of fractional-order controllers”, Nonlinear Dynamics, 29, no. 1-4, pp. 281–296, 2002.

[5] T. Haba, G. Loum, J. Zoueu, and G. Ablart, “Use of a component with fractional impedance in the realization of an analogical regulator of order ½”, Journal of Applied Sciences , 8, no. 1, pp. 59–67, 2008.

[6] J. A. T. Machado, I. S. Jesus, A. Galhano, and J. B. Cunha, “Fractional order electromagnetics”, Signal Processing , 86, no. 10, pp. 2637–2644, 2006.

[7] M. F. M. Lima, J. A. T. Machado, and M. Crisostomo, “Experimental signal analysis of robot impacts in a fractional calculus perspective”, Journal of Advanced Computational Intelligence and Intelligent Informatics, 11, pp. 1079-1085, 2007.

[8] N. Ferreira, F. Duarte, M. Lima, M. Marcos, and J. Machado, “Application of fractional calculus in the dynamical analysis and control of mechanical manipulators”, Fractional Calculus and Applied Analysis , 11, no. 1, pp. 91–113, 2008.

[9] C. R. C. Press, Solving applied mathematical problems with MATLAB, CRC Press, 2008.

[10] G. Tsirimokou, “Design of fractional-order analog integrated circuits,” PhD dissertation, 2017.

[11] J. Valsa, P. Dvorak, and M. Friedl, “Network Model of the CPE," Radioengineering, 20, no. 3, pp. 619–626, 2011.

[12] Y. Q. Chen, B. M. Vinagre, and I. Podlubny, “Continued fraction expansion approaches to discretizing fractional order derivatives - an expository review”, Nonlinear Dynamics, 38, no. 1, pp. 155–170, 2004.

[13] K. Matsuda, and H. Fujii, "H (infinity)–optimized wave-absorbing control: analytical and experimental results", Journal of Guidance, Control, and Dynamics, 16, no. 6, pp.1146–1153, 1993.

[14] A. Oustaloup, F. Levron, B. Mathiew, F. M. Nanot, "Frequency band complex noninteger differentiator: Characterization and synthesis", IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 47, no. 1, pp. 25–39, 2000.

[15] G. Carlson, and C. Halijak, "Approximation of fractional capacitors (1/s)^1/n by a regular Newton process", IEEE Transactions on Circuit Theory, 11, no. 2, pp. 210-213, 1964.

Chapter 3

FRACTIONAL ORDER ANALOG INVERSE FILTER USING OTAs[2]

3.1 Introduction

In the previous chapter, we have very briefly summarized the fundamental definitions of fractional order derivatives and integrals. Various approximation methods used for simulating the behaviour of a constant phase element were also presented. A network model of the CPE using the Valsa and Vlach method was also developed. In the present chapter we present fractional order inverse filter circuits realizing fractional order, inverse high pass, inverse bandpass and inverse band reject responses.

An analog inverse filter provides frequency selective characteristics which are inverse of the conventional analog filter. Inverse filters remove the consequences of distortion in the signal while processing it through system. Analog inverse filters have potential usages in communication systems, control systems, instrumentation and measurement systems. Over the years, many inverse filter circuits realized with different active elements have been proposed in open literature [1]-[20]. Among these reported inverse filter configurations, only few utilise FCs [1], [7], [8]. In [1], two fractional order inverse filter circuits employing operational amplifier were proposed, in which one of the circuits used one operational amplifier and five admittances whereas the other circuit employed one operational amplifier and seven admittances to realize different fractional order inverse filter responses, namely, fractional order inverse high pass response, fractional order inverse band pass response, and fractional order inverse

[2] The content and results of the following paper has been reported in this chapter: **J. Srivastava**, R. Bhagat, and P. Kumar, "Analog Inverse Filters using OTAs," In 2020 6th International Conference on Control, Automation and Robotics (ICCAR), 2020, pp. 638-643. https://doi.org/10.1109/ICCAR49639.2020.9108048 **Indexing:** SCOPUS and EI Compendex.

low pass response. In [7], three fractional order inverse filters, namely, fractional order inverse low pass filter, fractional order inverse high pass filter, and fractional order inverse band pass filter employing two current feedback operational amplifiers (CFOAs) were presented. The fractional order inverse low pass filter circuit used four resistors and two FCs, fractional order inverse high pass filter circuit used two resistors, four FCs and fractional order inverse band pass filter circuit utilized three resistors and three FCs. Two operational trans resistance amplifiers (OTRAs) and eight admittances based fractional order inverse filter was reported in [8]. The circuits presented therein can provide fractional order inverse low pass filter response, fractional order inverse high pass filter response, fractional order inverse band pass filter response and fractional order inverse notch filter response. From the above literature survey, it may be noted that there are very few inverse filter configurations reported in the open literature which use FCs. Also, the reported circuits generally use a very large number of passive elements and do not have any provision for control of different parameters of the realized circuits.

In this chapter three new tunable voltage-mode analog inverse filter circuits using operational transconductance amplifiers (OTAs) and fractional capacitors (FCs) are presented. The proposed structures realize fractional order inverse high pass filter response, fractional order inverse band pass filter response and fractional order inverse band elimination filter response employing three single output OTAs and two FCs only. The maximum/minimum frequency (ω_m) and bandwidth of the presented fractional order inverse band pass filter and fractional order inverse band elimination filter can be tuned orthogonally by changing the transconductance of different OTAs. The high frequency gain of fractional order inverse high pass filter can be changed independently while its cut-off frequency (ω_h) and pole quality factor (Q) can be changed orthogonally. The performance of all the presented circuits has been evaluated using Cadence Virtuoso simulation tools with 0.18µm technology parameters.

3.2 Proposed Fractional Order Voltage-Mode Inverse Filter Configurations

All the proposed fractional order inverse filter circuits employ three OTAs and two FCs. An OTA represented symbolically in Fig. 3.1, and equation (3.1) characterizes the terminal relationship between its input differential voltage and output current:

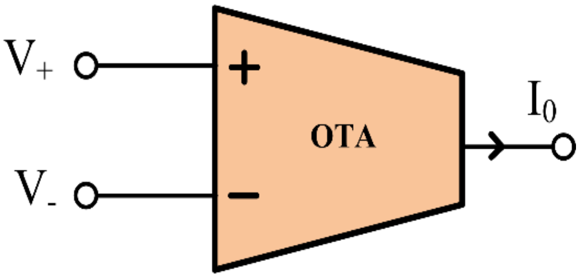


Figure 3.1 Symbolic representation of OTA

$$I_{out} = g_m(V_+ - V_-) \tag{3.1}$$

The fractional order capacitor on the other hand is characterized by the following driving point impedance :

$$Z(s) = \frac{1}{Cs^{\alpha}}\Omega \tag{3.2}$$

The Fig. 3.2-3.4 depict the different fractional order inverse filter circuits proposed in this chapter.

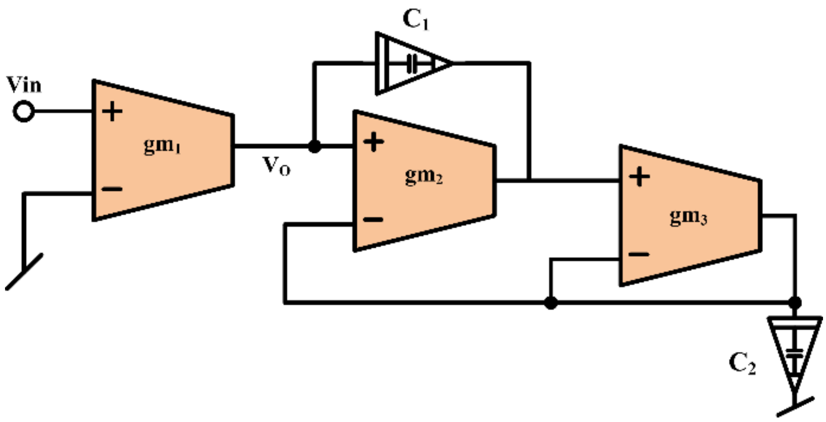


Figure 3.2 Fractional order inverse high pass filter

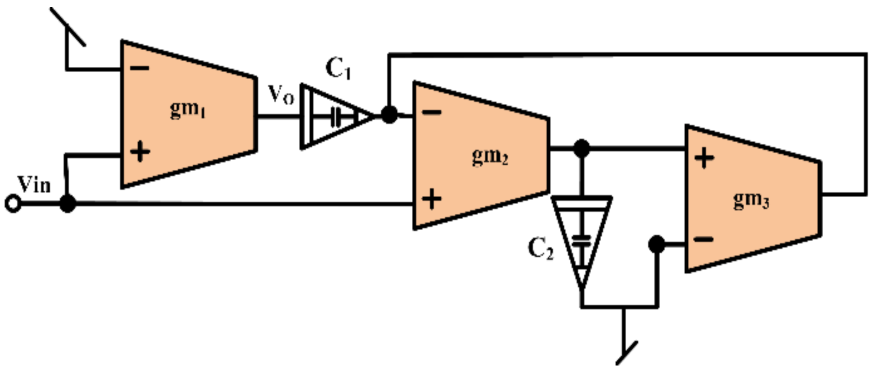


Figure 3.3 Fractional order inverse band pass filter

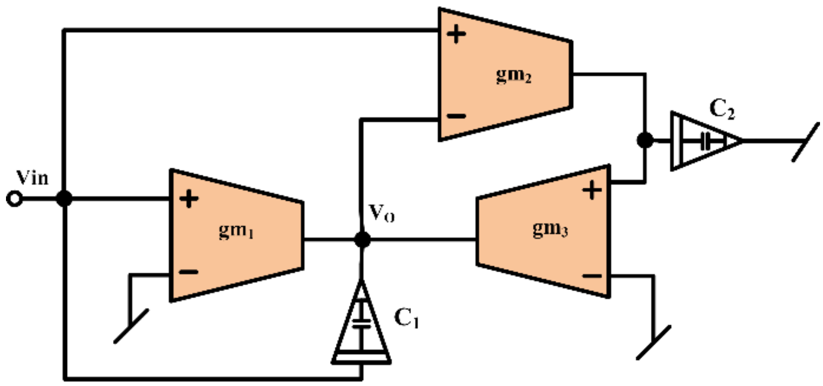


Figure 3.4 Fractional inverse band elimination filter

Analysing the circuits shown above using terminal characteristics of the OTAs and the fractional order capacitors given in equations (3.1)-(3.2) the transfer functions (3.3)-(3.8) for the various fractional order inverse filter circuits are obtained as:

3.2.1 Fractional order inverse high pass filter:

$$\frac{V_0(s)}{V_{IN}(s)} = \frac{s^{2\alpha} + a_0 s^{\alpha} + b_0}{s^{2\alpha} c_0} \qquad (3.3)$$

where $a_0 = g_3 / C_2$, $b_0 = g_2 g_3 / C_1 C_2$ and $c_0 = g_2 / g_1$.

3.2.2 Fractional order inverse band pass filter:

$$\frac{V_0(s)}{V_{IN}(s)} = \frac{s^{2\alpha} + a_1 s^{\alpha} + b_1}{s^{\alpha} c_1} \qquad (3.4)$$

where $a_1 = g_2 g_3 / g_1 C_2$, $b_1 = g_2 g_3 / C_1 C_2$ and $c_1 = g_2 g_3 / g_1 C_2$.

3.2.3 Fractional order inverse band elimination filter:

$$\frac{V_0(s)}{V_{IN}(s)} = \frac{s^{2\alpha} + a_2 s^{\alpha} + b_2}{s^{2\alpha} + c_2} \qquad (3.5)$$

where $a_2 = g_1 / C_1$, $b_2 = g_2 g_3 / C_1 C_2$ and $c_2 = g_2 g_3 / C_1 C_2$.

The cut off frequency (ω_h) and maximum/minimum frequency (ω_m) of fractional order inverse high pass filter, fractional order inverse band pass filter and fractional order inverse band elimination filter, correspond to these frequencies for their normal filter counterparts and may be found by solving the following non-linear equations [24]:

$$-\omega_h^{4\alpha} + 2a_0 \cos\frac{\pi\alpha}{2}\omega_h^{3\alpha} + a_0^{\,2}(1 + 2\cos\pi\alpha)\omega_h^{2\alpha} + 2a_0^{\,3}\cos\frac{\pi\alpha}{2}\omega_h^{\alpha} + a_0^{\,4} = 0 \qquad (3.6)$$

$$\left(\omega_m^{2\alpha} - b_1\right)\left(\omega_m^{2\alpha} + b_1 + a_1 \cos\frac{\pi\alpha}{2}\right) = 0 \tag{3.7}$$

$$\left(\omega_m^{2\alpha} - b_2\right)\left(\begin{array}{l}\cos\frac{\pi\alpha}{2}\omega_m^{4\alpha} + a_2\omega_m^{3\alpha} + \\ \left\{4b_2\cos\frac{\pi\alpha}{2} - 2b_2\cos\pi\alpha\cos\frac{\pi\alpha}{2}\right\}\omega_m^{2\alpha} + a_2 b_2 \omega_m^{\alpha} + b_2^2\cos\frac{\pi\alpha}{2}\end{array}\right) = 0 \tag{3.8}$$

The theoretical value of maximum/minimum frequency of band pass and band elimination filter can be calculated from $\omega_m = (b_1)^{1/\alpha}$ and $\omega_m = (b_2)^{1/\alpha}$ respectively.

Table 3.1 shows the values of the $\omega_{m/h}$, H and Q for $\alpha = 1$. From Table 3.1, it may be observed that H for fractional order inverse high pass filter can be varied by changing the value of g_1 while its ω_h and Q can be tuned orthogonally. On the other hand ω_m and BW of fractional order inverse band pass filter and fractional order inverse band elimination filter can be tuned orthogonally.

Table 3.1 Cut off Frequency, Gain and Pole Quality Factor for α=1

	H	**Frequency**	**BW**	**Q**
Inverse high pass filter	$\frac{g_2}{g_1}$	$\omega_h = \sqrt{\frac{g_3 g_2}{C_2 C_1}}$	-	$\sqrt{\frac{g_2 C_2}{C_1 g_3}}$
Inverse band pass filter	1	$\omega_m = \sqrt{\frac{g_3 g_2}{C_2 C_1}}$	$\frac{g_3 g_2}{g_1 C_2}$	$g_1\sqrt{\frac{C_2}{g_2 g_3 C_1}}$
Inverse band elimination filter	1	$\omega_m = \sqrt{\frac{g_3 g_2}{C_2 C_1}}$	$\frac{g_1}{C_1}$	$\frac{1}{g_1}\sqrt{\frac{C_1 g_2 g_3}{C_2}}$

3.3 Simulation Results

The performance of the proposed circuits has been validated through Analog Design Environment (ADE) tool of Cadence Virtuoso with 0.18µm CMOS technology parameters. The CMOS OTA [23] used in this chapter is shown in Fig. 3.5 and aspect ratio used are taken from [25]. Four types of aspect ratios are used in this combination. The DC power supply of ±0.9V were used in the simulations.

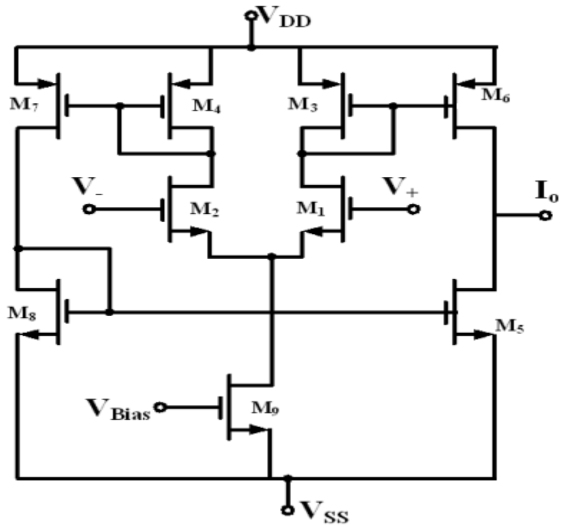
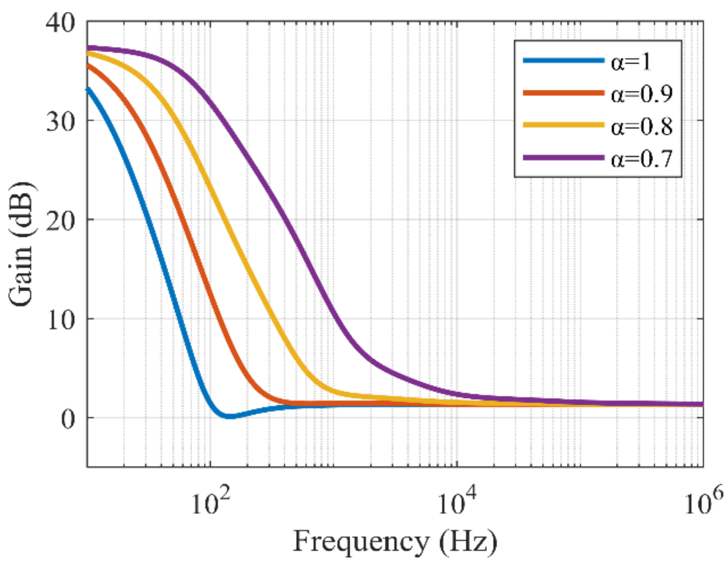
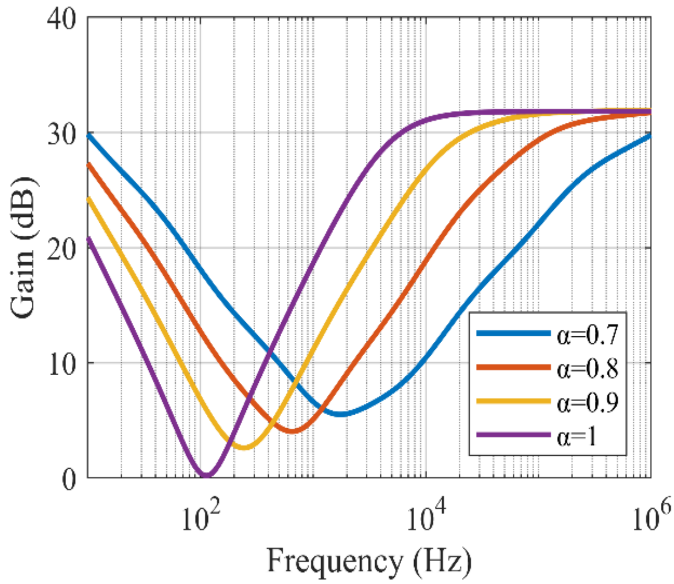


Figure 3.5 CMOS implementation of OTA

We have used two identical valued fractional order capacitors of 0.382 μF (rad/sec)$^{(1-\alpha)}$ designed using Valsa and Vlach method [24]. Different values of α (0.7, 0.8, 0.9 and 1) have been used in simulations. The fractional order inverse filters were designed for a ω_h/ω_m of 113 Hz ($\alpha = 1$). The value of α was changed from 0.7 to 1.0 in step of 0.1. The frequency response of fractional order inverse high pass filter, fractional order inverse band pass filter and fractional order inverse band elimination filter have been displayed in Fig. 3.6(a)-(c) respectively.



(a)



(b)

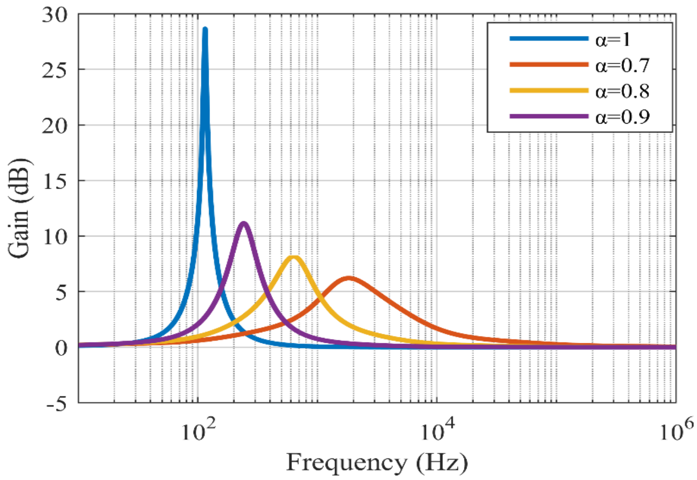


Figure 3.6 Frequency responses of (a) fractional order inverse high pass filter, (b) fractional order inverse band pass filter and (c) fractional order inverse band elimination filter.

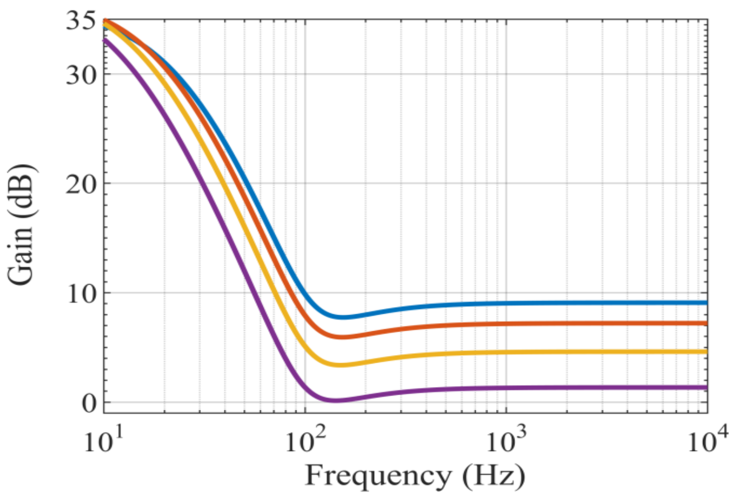


Figure 3.7 Tunability of gain for fractional order inverse high pass filter

In Fig. 3.7, the electronic tunability of gain for fractional order inverse high pass filter has been shown by varying transconductance g_1 for 715.549µS, 516.164µS, 275.038μS and 152.162μS. The Q of the fractional order inverse band pass filter and inverse band elimination filter have been varied for constant frequency 130 Hz by varying transconductance g_1 (715.549μS, 516.164μS, 275.038μS and 152.162μS) and their respective results have been illustrated in Fig. 3.8 and Fig. 3.9.

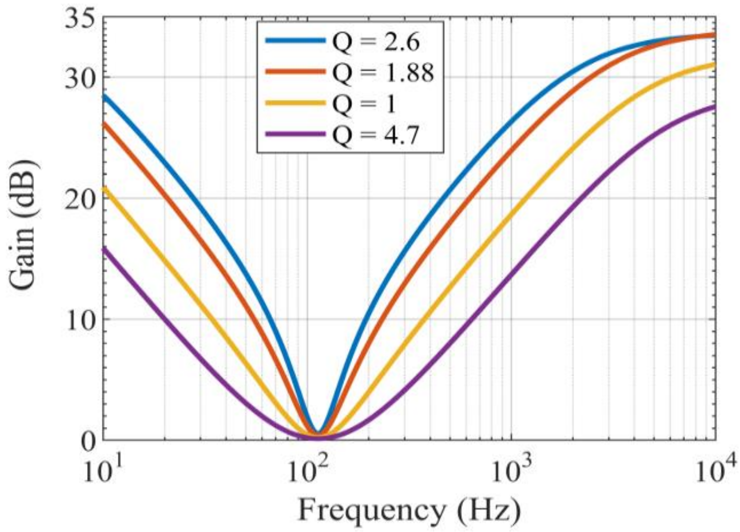


Figure 3.8 Tunability of Q for fractional order inverse band pass filter

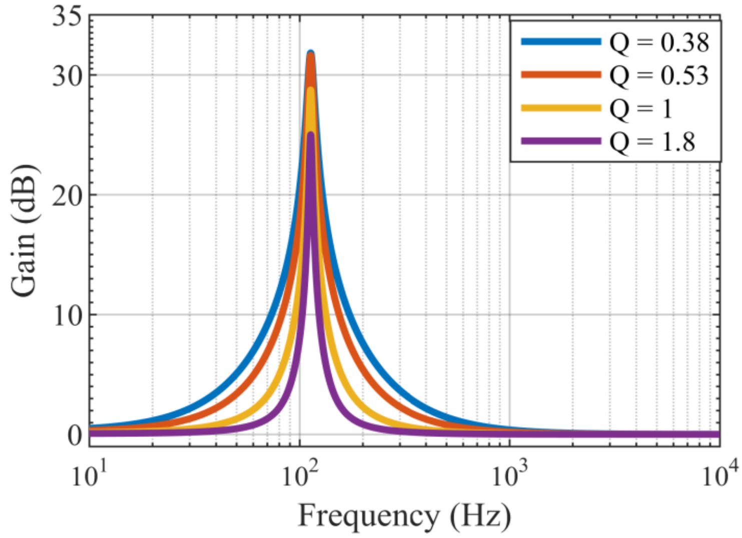


Figure 3.9 Tunability of Q for fractional order inverse band elimination filter

Table 3.2 gives the values of the ω_h, ω_m of the presented fractional order inverse filters measured through simulations and calculated theoretically for different values of α.

Table 3.2 Cut-off Frequencies (in Hz) of Filters for Different α

α	**1**	**0.9**	**0.8**	**0.7**
Inverse high pass (Simulated)	131.8	204.2	676	3715
Inverse high pass (Theoretical)	113.05	217.293	709.056	3349.88
Inverse band pass (Simulated)	112.2	234.4	660.7	1738
Inverse band pass (Theoretical)	113.05	234.479	583.63	1885.08
Inverse band elimination (Simulated)	114	239.9	631	1820
Inverse band elimination (Theoretical)	113.05	234.479	583.63	1885.08

3.4 Conclusion

Three new VM analog inverse filter configurations employing three OTAs and two fractional capacitors have been presented. The proposed circuits are capable of providing fractional order inverse high pass response, fractional order inverse band pass response and fractional order inverse band elimination response. The presented fractional order inverse band pass filter and fractional order inverse band elimination filter circuits have orthogonal tunability of ω_m and bandwidth. The gain of the fractional order inverse high pass filter can be tuned independently while its ω_h and Q can be changed orthogonally. Simulation results have confirmed the working of these fractional order inverse filter circuits.

REFERENCES

[1] D. R. Bhaskar, M. Kumar and P. Kumar, "Fractional order inverse filters using operational amplifier," Analog Integrated Circuits and Signal Processing, 97, no. 1, pp. 149-158, 2018.

[2] S. S. Gupta, D. R. Bhaskar, and R. Senani, and A.K. Singh, "Inverse active filters employing CFOA," Electrical Engineering, 91, no. 1, pp 23-26, 2009.

[3] S. S. Gupta, D. R. Bhaskar, and R. Senani, "New analogue inverse filters realized with current feedback op-amp," International Journal of Electronic, 98, no. 8, pp. 1103–1113, 2011.

[4] H. Y. Wang, S. H. Chang, T. Y. Yang, and P. Y. Tsai, "A novel multifunction CFOA based inverse filter," Circuits and Systems, 2, pp. 14–17, 2011.

[5] V. Patil, and R. K. Sharma, "Novel inverse active filters employing CFOA", International Journal for Scientific Research & Development, 3, no. 7, pp. 359–360, 2015.

[6] T. Tsukutani, Y. Kunugasa, and N. Yabuki, "CCII-Based Inverse Active Filters with Grounded Passive Components," Journal of Electrical Engineering, 6, no. 4, pp. 212-215, 2018.

[7] E. M. Hamed, L. A. Said, A. H. Madian, and A. G. Radwan. "On the Approximations of CFOA-Based Fractional-Order Inverse Filters," Circuits, Systems, and Signal Processing, 39, no. 1, pp. 2-29, 2020.

[8] N. A. Khali, L. A. Said, A. G. Radwan, and A. M. Soliman. "Multifunction Fractional Inverse Filter Based on OTRA," In 2019 Novel Intelligent and Leading Emerging Sciences Conference (NILES), vol. 1, 2019, pp. 162-165.

[9] R. Bhagat, D. R. Bhaskar, and P. Kumar, "Inverse Band Reject and All Pass Filter Structure Employing CMOS CDBAs," International Journal of Engineering Research & Technology (IJERT), pp. 39-44, 2019.

[10] K. Garg, R. Bhagat, and B. Jaint, "A novel multifunction modified CFOA based inverse filter," In 2012 IEEE 5th India International Conference on power electronics (IICPE), 2012, pp. 1-5.

[11] N. A. Shah, and M. F. Rather, "Realization of voltage-mode CCII based all pass filter and its inversion version," Indian Journal of Pure & Applied Physics, 44, pp. 269–271, 2006.

[12] B. Chipipop, and W. Surakampontorn, “Realization of current-mode FTFN-based inverse filter,” Electronic Letters, 35, no. 9, pp. 690–692, 1999.

[13] H. Y. Wang, and C.T. Lee, “Using nullors for realisation of current mode FTFN based inverse filters,” Electronic Letters, 35, no. 22, pp. 1889-1890, 1999.

[14] A. K. Singh, A. Gupta, and R. Senani, “OTRA-based multi-function inverse filter configuration,” Advances in Electrical and Electronic. Engineering, 15, no. 5, pp. 846-856, 2018.

[15] A. Pradhan, and R. K. Sharma, “Generation of OTRA-Based Inverse All Pass and Inverse Band Reject Filters,” Proceedings of the National Academy of Sciences, India Section A: Physical Sciences, pp. 1-11, 2019.

[16] A. R. Nasir, and S. N. Ahmad, “A new current mode multifunction inverse filter using CDBA,” International Journal of Computer Science and Information Security, 11, no. 12, pp. 50-52, 2013.

[17] R. Pandey, N. Pandey, T. Negi, and V. Garg, “CDBA based universal inverse filter,” ISRN Electronics, pp. 1-6, 2013.

[18] A. Sharma, A. Kumar, and P. Whig, “On performance of CDTA based novel analog inverse low pass filter using 0.35 μm CMOS parameter,” International Journal of Science, Technology & Management, 4, no. 1, pp. 594–601, 2015.

[19] N. A. Shah, M. Quadri, and S. Z. Iqbal, “High output impedance current-mode all pass inverse filter using CDTA,” Indian Journal of pure and applied physics, 46, pp. 893–896, 2008.

[20] T. Tsukutani, Y. Sumi, and N. Yabuki, “Electronically tunable inverse active filters employing OTAs and grounded capacitors,” International Journal of Electronics Letters, 4, no. 2, pp. 166–176, 2016.

[21] R. Verma, N. Pandey, and R. Pandey. “Electronically tunable fractional order filter,” Arabian Journal for Science and Engineering, 42, no. 8, pp. 3409-3422, 2017.

[22] A. G. Radwan, A. S. Elwakil, and A. M Soliman, “On the generalization of second-order filters to the fractional-order domain,” Journal of Circuits, Systems, and Computers, 18, no. 2, pp. 361-386, 2009.

[23] E.W. Greeneich, Analog Integrated Circuits, Chapman & Hall, 1941, pp. 192.

[24] J. Valsa, P. Dvorak, and M. Friedl, "Network Model of the CPE," Radioengineering, 20, no. 3, pp. 619–626, 2011.

[25] D. R. Bhaskar, A. Raj and P. Kumar, "Mixed Mode Universal Biquad Filter Using OTAs," Journal of Circuits, Systems and Computers, 2019.

Chapter 4

VDTA BASED FRACTIONAL ORDER UNIVERSAL FILTER[3]

4.1 Introduction

In the previous chapter we presented circuit configurations realizing inverse high pass filter, inverse bandpass filter and inverse band reject filter in fractional order domain. In the present chapter, we have generalized a VDTA based integer order filter in the non-integer (fractional order) domain and examined the electronic tunability of various parameters of the filter.

Analog signal processing applications frequently employ active filters as important basic building blocks. Ever since the introduction of the VDTA [1], it has been extensively utilized for realization of filter circuits and sinusoidal oscillators [2–5]. In like manner, use of fractional order circuits are gaining more attention, in research areas of biomedical, control systems, and instrumentation, as it explains dynamics of natural system more closely. Analog filter, designed using fractional order elements, gives a better control over attenuation gradient in the pass/stop band. Several fractional order filter circuits utilizing different active building blocks (ABBs) [4-15] have been presented recently. Many of these circuits have generalized an existing integer order filter circuit into fractional order domain. A careful perusal of the existing literature on fractional order filters reveals that very few circuits of fractional order filters utilizing VDTAs have been presented [4-5].

This chapter is concerned with the electronic tunability of a current mode multiple input single output (MISO) type biquad filter employing one voltage

---

[3] The content and results of the following paper has been reported in this chapter: **J. Srivastava**, "VDTA based fractional order universal filter," In 2020 International Conference for Innovation in Technology (INOCON), 2020. (Accepted) **Indexing:** SCOPUS and EI Compendex.

differencing transconductance amplifier (VDTA) and two fractional order capacitors. The tunability of all the filters, namely, fractional high pass filter (FHPF), fractional low pass filter (FLPF), fractional band pass filter (FBPF), fractional band reject filter (FBRF), and fractional all pass filter (FAPF) with respect to α and I_{bias} is shown. The workability of the modified filter is verified using PSPICE simulations.

Here, we have generalized the modified single VDTA based three input and a single output current-mode electronically tuneable universal biquadratic filter [2] in fractional order domain and examined the electronic tunability of the various parameters of the filter. The pole frequency (ω_0) of the modified circuit can be electronically tuned by using two parameters α and I_{bias} independently. Also, the stability of the presented circuit has been analysed.

4.2 Circuit Description

4.2.1 Designing Fractional-order Capacitor using Oustaloup, Laveron, Mathieu and Nanot Approximation [16]

This approximation method mainly focuses on characteristics and synthesis of frequency band complex non-integer differentiator. Using this method, the fractional operator s^α can be synthesis in the frequency band of interest [ω_{min}, ω_{max}] The approximated fractional operator s^α can be written as:

$$s^{\alpha} = C\prod_{k=1}^{k=N}\frac{1+ s/\omega_k^{'}}{1+ s/\omega_k} \tag{4.1}$$

4.2.2 Steps for determining the fractional capacitor using Oustaloup, Laveron, Mathieu and Nanot Approximation

1) Starting with given values of α (between 0 to 1), ω_{min} and ω_{max} (desired minimum and maximum frequency) and N (no. of order).
2) Unity gain frequency (ω_u) can be obtained as:

$$\omega_u = \sqrt{\omega_{\min} \times \omega_{\max}} \tag{4.2}$$

3) Gain adjustment parameter 'C' can be calculated as:

$$C = \left(\frac{\omega_u}{\omega_{\min}}\right)^{\alpha} \tag{4.3}$$

4) By calculating all the parameters listed above, s^α can be approximated as:

$$s^{\alpha} = C\prod_{k=1}^{k=N}\frac{1+ s/\omega_k^{'}}{1+ s/\omega_k} \tag{4.4}$$

where $\omega_k^{'} = \omega_{\min}\left(\frac{\omega_{\max}}{\omega_{\min}}\right)^{(2k-1-\alpha)/(2N)}$ and $\omega_k = \omega_{\min}\left(\frac{\omega_{\max}}{\omega_{\min}}\right)^{(2k-1+\alpha)/(2N)}$

5) By taking the partial fraction of the equation (4.4) it can be generalized as:

$$z(s) = \frac{1}{s^{\alpha}c} = R_0 + \sum_{n=1}^{N}\frac{\frac{1}{c_n}}{s+\frac{1}{R_n c_n}} \tag{4.5}$$

and thereby applying network synthesis to convert it into foster I circuit as shown in fig 1.

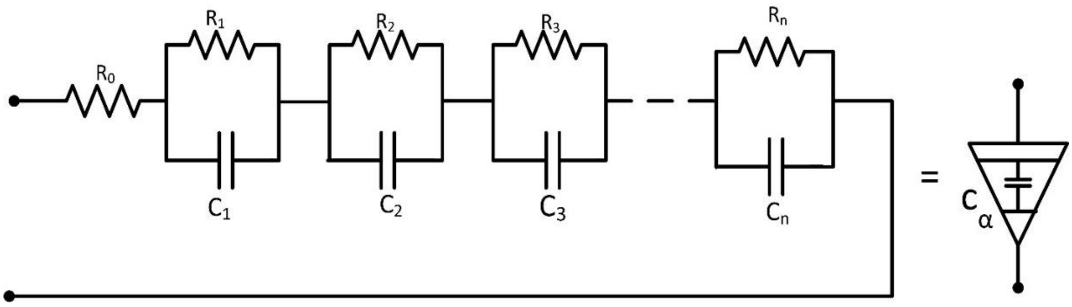


Figure 4.1 Foster I canonical RC structure

By considering f_{min}=0.1Hz, f_{max}= 100 MHz and N=8, the phase response of the resultant structure shown above is plotted in fig 4.2 for different values of α ranging between 0 to 1.

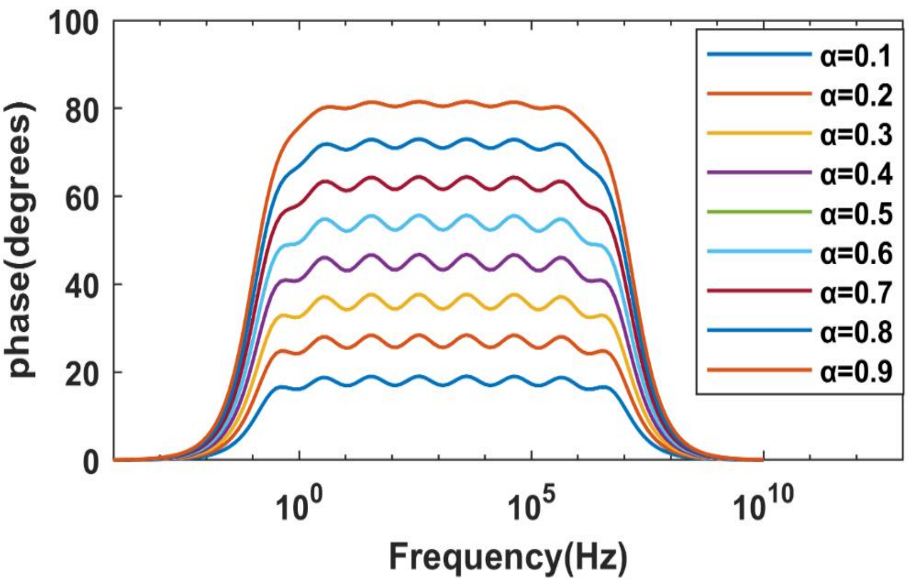


Figure 4.2 Phase response for different α

4.2.3 The Voltage Differencing Transconductance Amplifier (VDTA)

The VDTA is an adaptable ABB as shown in Fig. 4.1.

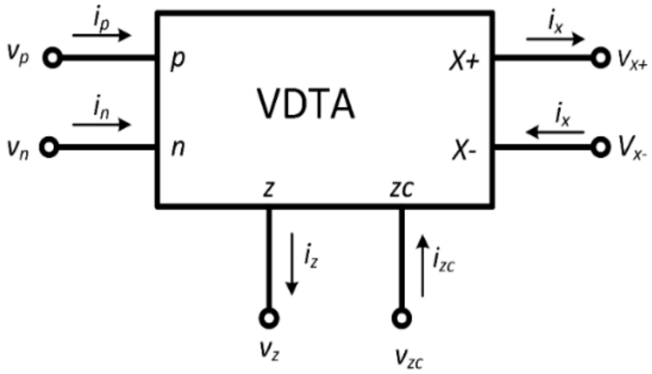


Figure 4.3 VDTA structure

Essentially, the VDTA device comprises of an input voltage subtractor where input voltage difference ($v_p - v_n$) is transferred through the z-terminal in form of current (i_z) by the first transconductance gain (g_{m1}), and a dual output transconductance amplifier that are responsible for converting the voltage at z-terminal to the currents at x-terminals by transconductance gain (g_{m2}). Also, zc terminal, gives the copy of current i_z as i_{zc}. Evaluating the fundamental operation of an ideal VDTA, the following matrix depicts its terminal relationship:

$$\begin{bmatrix} i_p \\ i_n \\ i_z \\ i_x \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ g_{m1} & -g_{m1} & 0 & 0 \\ 0 & 0 & 0 & g_{m2} \end{bmatrix} \begin{bmatrix} v_p \\ v_n \\ v_{zc} \\ v_z \end{bmatrix} \tag{4.6}$$

where g_{m1} and g_{m2} are the electronically controllable transconductances gain of the VDTA using OTAs. Its relation with external bias current (I_{bias}) is given in equation (4.2).

$$g_m = \frac{I_{bias}}{2V_T} \tag{4.7}$$

Since the VDTA is yet not available, there are several possibilities of its implementation, implementing it through ICs is commercially more feasible. In this chapter VDTA is implemented using three OTA ICs LM13700 at two transconductance gain stages and six input output terminals as shown in Fig 4.2.

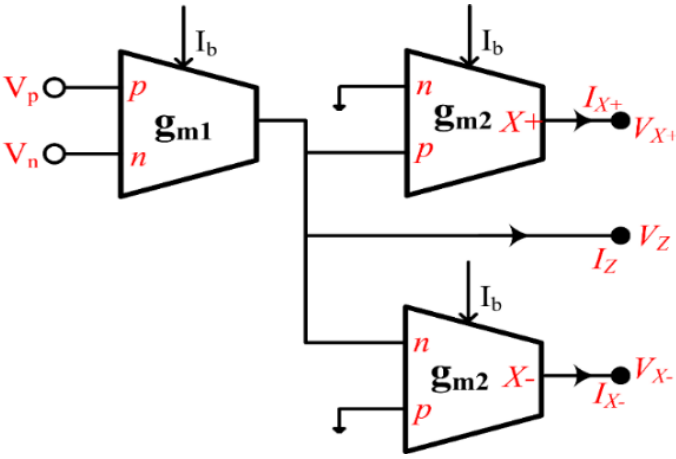


Figure 4.4 VDTA using LM13700 IC

4.3 The Fractional order VDTA Filter

The generalized fractional order universal filter circuit with multiple input, single output, comprising of single VDTA and two FCs is shown in Fig 4.3. The use of FCs [16-17] provides the flexibility of tuning through two parameters namely, transconductance gain (g_m) and α making it easier to be tuned and controlled.

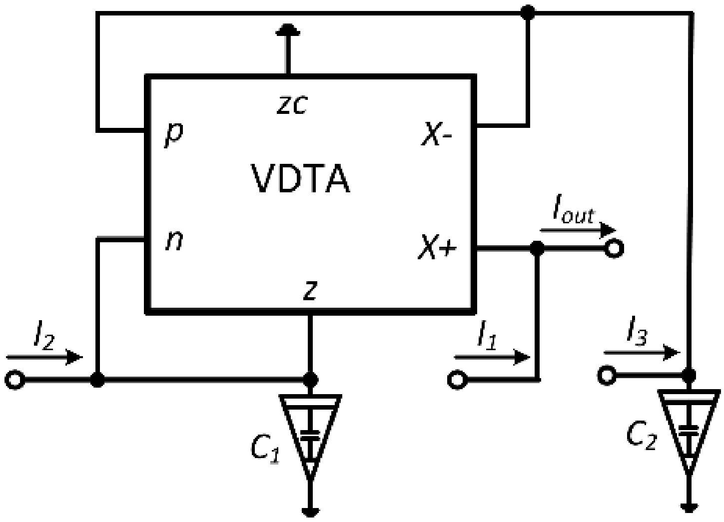


Figure 4.5 The modified fractional order current mode filter

Equations (4.3) and (4.4) give the output function of the above circuit.

$$I_{out} = \frac{D(s)I_1 + (g_{m2}/C_1)s^{\alpha}I_2 + (g_{m1}g_{m2}/C_1C_2)I_3}{D(s)} \quad (4.8)$$

$$D(s) = s^{2\alpha} + (g_{m1}/C_1)s^{\alpha} + (g_{m1}g_{m2}/C_1C_2) \quad (4.9)$$

Conditions for realization of FLPF, FHPF, FBPF, FBSF, and FAPF are shown in Table 4.1.

Table 4.1 Condition for Universal filter responses

Filters	Cases
FLP	$I_3 = I_{in}, I_1 = I_2 = 0$
FHP	$I_1 = -I_2 = -I_3 = I_{in}, g_{m1} = g_{m2}$
FBP	$I_2 = I_{in}, I_1 = I_3 = 0$
FBR	$I_1 = -I_2 = I_{in}, I_3 = 0, g_{m1} = g_{m2}$
FAP	$I_1 = -I_2 / 2 = I_{in}, I_3 = 0, g_{m1} = g_{m2}$

For any fractional order filter with transfer function T(s), the following important parameters should be determined [11]: ω_m (maxima or minima frequency), ω_h (pole cutoff frequency/half power frequency), and ω_{rp} (right phase frequency). The bandwidth of any filter can also be calculated using ω_h.

A. FLPF

$$\frac{I_{out}}{I_{in}} = \frac{\left(g_{m1}g_{m2} / C_1C_2\right)}{D(s)} I_{in} \tag{4.10}$$

For ω_m

$$2X^3 + 3\cos\left(\frac{\alpha\pi}{2}\right)X^2 + 2\left[1 + k\cos(\alpha\pi)\right]X + k\cos\left(\frac{\alpha\pi}{2}\right) = 0 \tag{4.11}$$

For ω_h

$$Y^4 + 2\cos\left(\frac{\alpha\pi}{2}\right)Y^3 + (1 + 2k\cos(\alpha\pi)Y^2 + 2k\cos\left(\frac{\alpha\pi}{2}\right)Y - k^2 = 0 \tag{4.12}$$

For ω_{rp}

$$\omega_{rp} = \left(\frac{-a\cos\left(\frac{\alpha\pi}{2}\right) - \sqrt{a^2\cos^2\left(\frac{\alpha\pi}{2}\right) - 4ab\cos(\alpha\pi)}}{2\cos(\alpha\pi)}\right)^{\frac{1}{\alpha}} \tag{4.13}$$

B. FHPF

$$\frac{I_{out}}{I_{in}} = \frac{s^{2\alpha}}{D(s)} I_{in} \tag{4.14}$$

For ω_m, ω_h and ω_{rp}

$$\omega_{mFLPF} \cdot \omega_{mFHPF} = \omega_{hFLPF} \cdot \omega_{hFHPF} = \omega_{rpFLPF} . \omega_{rpFHPF} = (ab)^{\frac{1}{\alpha}} \tag{4.15}$$

C. FBPF

$$\frac{I_{out}}{I_{in}} = \frac{(g_{m2}/C_1)s^{\alpha}}{D(s)} I_{in} \tag{4.16}$$

For ω_m

$$\left(X^2 - k\right)\left(X^2 + \cos\left(\frac{\alpha\pi}{2}\right)X + k\right) \tag{4.17}$$

For ω_h

$$Y^4 + 2\cos\left(\frac{\alpha\pi}{2}\right)Y^3 - Y^2 + 2k\cos(\alpha\pi)Y^2 - \left(8k\cos^2\left(\frac{\alpha\pi}{2}\right) + 8\sqrt{k}\cos\left(\frac{\alpha\pi}{2}\right)\right)Y^2 + 2k\cos(\alpha\pi)Y + k^2 = 0 \tag{4.18}$$

For ω_{rp}

$$\omega_{rp} = \left(\frac{-a \pm \sqrt{a^2 - 4ab\cos^2\left(\frac{\alpha\pi}{2}\right)}}{2\cos\left(\frac{\alpha\pi}{2}\right)}\right)^{\frac{1}{\alpha}} \tag{4.19}$$

D. FBSF

$$\frac{I_{out}}{I_{in}} = \frac{s^{2\alpha} + (g_{m1}g_{m2}/C_1C_2)}{D(s)} \tag{4.20}$$

For ω_m

$$\left(X^2 - k\right)\left(\begin{array}{l} X^4\cos\left(\frac{\alpha\pi}{2}\right) + X^3 + 4k\cos\left(\frac{\alpha\pi}{2}\right)X^2 \\ -2k\cos(\alpha\pi)\cdot\cos\left(\frac{\alpha\pi}{2}\right)X^2 + kX + k^2\cos\left(\frac{\alpha\pi}{2}\right) \end{array}\right) \tag{4.21}$$

For ω_h

$$Y^4 - 2\cos\left(\frac{\alpha\pi}{2}\right)Y^3 - (1 - 2k\cos(\alpha\pi))Y^2 - 2k\cos\left(\frac{\alpha\pi}{2}\right)Y + k^2 = 0 \tag{4.22}$$

where $X = \frac{\omega_m^{\alpha}}{a}, Y = \frac{\omega_h^{\alpha}}{a}, k = \frac{b}{a}, a = \frac{g_{m1}}{C_1}$ and $b = \frac{g_{m2}}{C_2}$

E. FAPF

$$\frac{I_{out}}{I_{in}} = \frac{s^{2\alpha} - (g_{m1}/C_1)s^{\alpha} + (g_{m1}g_{m2}/C_1C_2)}{D(s)} \tag{4.23}$$

By putting the value of $\alpha = 1$, all the important frequency parameters for different filters namely, FLPF, FHPF, FBPF, and FBRF are shown in Table II. For the values of α other than 1, these important parameters of different filter can be obtained using equation (4.5)-(4.17).

Table 4.2 Filter parameters when α=1

Types of filters	ω_m	ω_h	ω_{rp}
FLPF	$a\sqrt{(k-1)}$	$\left(a\sqrt{\left(k \pm \frac{\sqrt{8k^2-4k+1}}{2} - \frac{1}{2}\right)}\right)$	$\sqrt{ab}$
FHPF	$\frac{b}{\sqrt{(k-1)}}$	$\frac{ab}{\left(a\sqrt{\left(k \pm \frac{\sqrt{8k^2-4k+1}}{2} - \frac{1}{2}\right)}\right)}$	$\sqrt{ab}$
FBPF	$a\sqrt{k}$	$a\left(\frac{\sqrt{4k+1}}{2} \pm \frac{1}{2}\right)$	$0,\infty$
FBRF	$a\sqrt{k}$	$a\left(\frac{\sqrt{4k+1}}{2} \pm \frac{1}{2}\right)$	$\sqrt{ab}$

4.4 Stability Analysis

For any fractional order system, the stability can be obtained by mapping s-plane into W-planes [18]. The stability graph shown in fig 4.5-4.9 is plotted using matlab command forlocus by keeping the coefficients of characteristics equation (4.4) positive where value of α lies between 0 and 1. Table 4.3 shows the different cases of stability for fractional order and accordingly its pole quality factor Q and pole frequency ω_0. Also, while observing the graph carefully we can conclude that the number of poles in the stability plot will be equal to highest power of s multiple by 10.

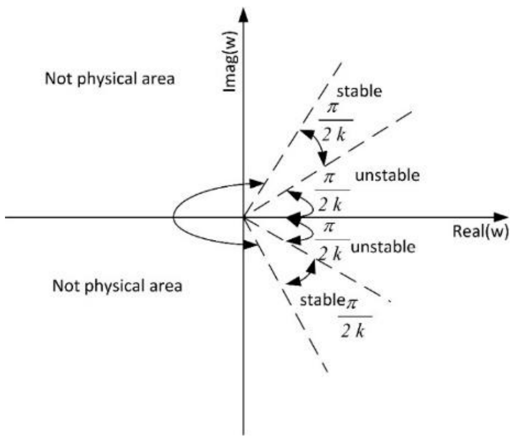


Figure 4.6 W-plane (Region of stability)

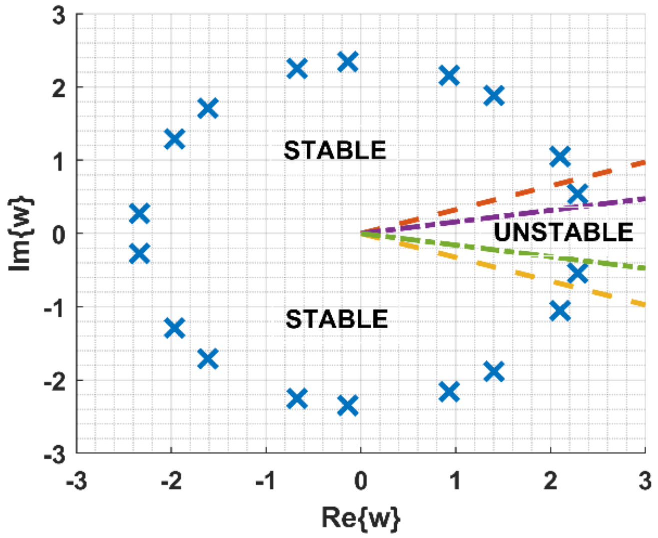


Figure 4.7 Stability plot of FLPF C=0.382μF and α=0.9

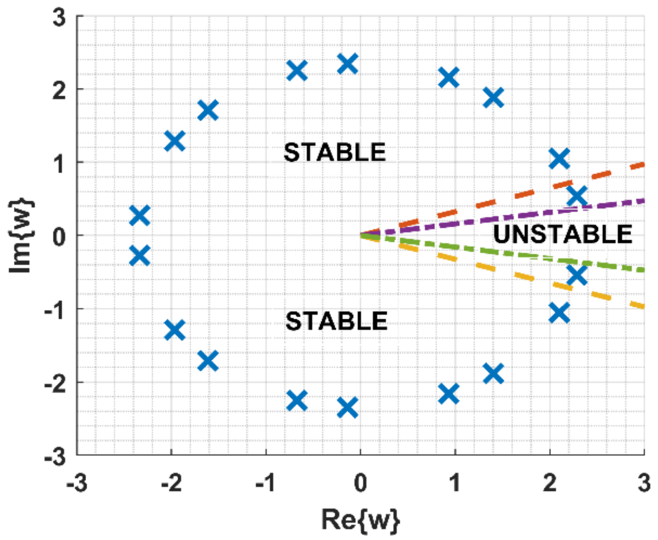


Figure 4.8 Stability plot of FHPF C=0.382μF and α=0.9

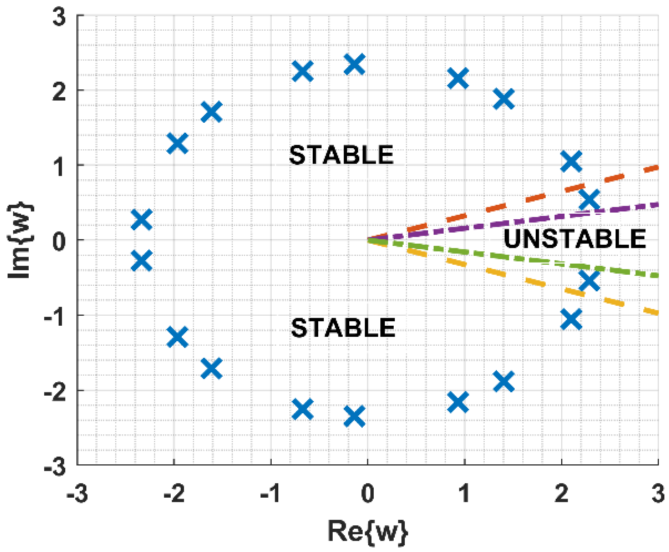


Figure 4.9 Stability plot of FBPF C=0.382μF and α=0.9

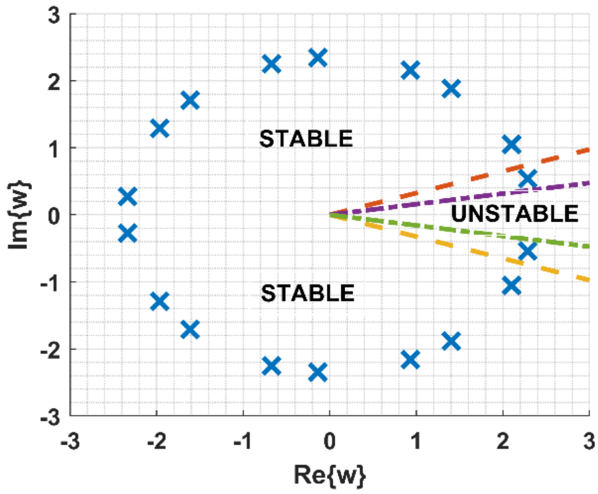


Figure 4.10 Stability plot of FBSF C=0.382μF and α=0.9

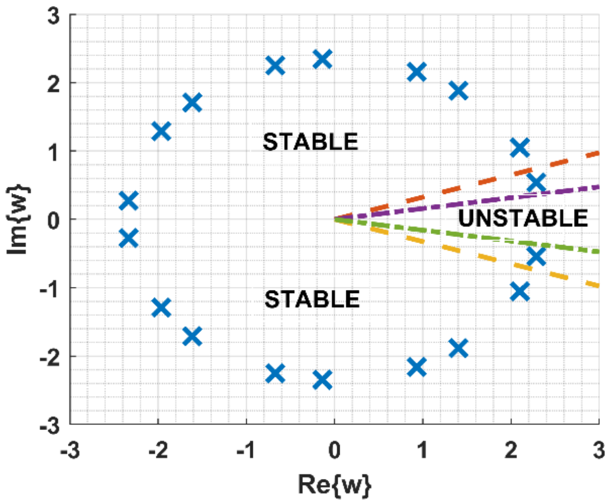


Figure 4.11 Stability plot of FAPF C=0.382μF and α=0.9

Table 4.3 Different cases of stability, roots, ω_o and Q

Cases	Relations	Condition for stability and roots	ω_0, Q
1	$a \geq 4b$	$\alpha < 2$, $r_{1,2} = \dfrac{-a \pm \sqrt{(a)^2 - 4(ab)}}{2} = g_{1,2}e^{j\pi}$	$\omega_{01,2} = g_{1,2}^{1/\alpha}$, $Q = \dfrac{-1}{2\cos(\pi/\alpha)}$
2	$a < 4b$	$\alpha < \dfrac{2\delta}{\pi}$, $\delta = \cos^{-1}\left(\dfrac{-a}{2\sqrt{ab}}\right) > \dfrac{\pi}{2}$, $r_{1,2} = \sqrt{ab}e^{\pm j\delta}$	$\omega_0 = \left(\sqrt{ab}\right)^{1/\alpha}$, $Q = \dfrac{-1}{2\cos(\delta/\alpha)}$

4.5 Simulation Results

The workability of the FO filters has been verified through PSPICE simulation using the macro model of OTA IC LM 13700.

4.5.1 Tunability with α:

It can be achieved by replacing each ordinary capacitors with equal valued FCs $(C_1 = C_2 = 0.382\ \mu F/(rad/sec)^{(1-\alpha)})$ designed using Oustaloup, Laveron, Mathieu and Nanot method [16] for α=0.6, α=0.7, α=0.8, α=0.9 in the frequency range of 0.1 Hz to 100 MHz. All the filters designed in this section have values $I_{bias1} = I_{bias2} = 0.1079mA$ (R_{bias}=265 KΩ).

The different frequency parameters for filters for α=0.6 to α=1.0 have been listed in Table 4.4-4.5 and their simulation results are shown in Fig (4.10) - (4.14). The simulation values and theoretical values differ in a very small range of error (less than 5%).

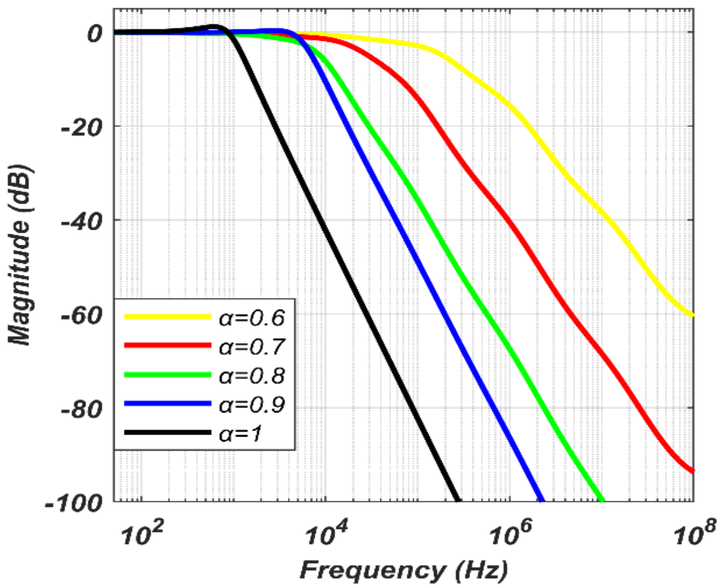


Figure 4.12 Tunability of FLP with respect to α

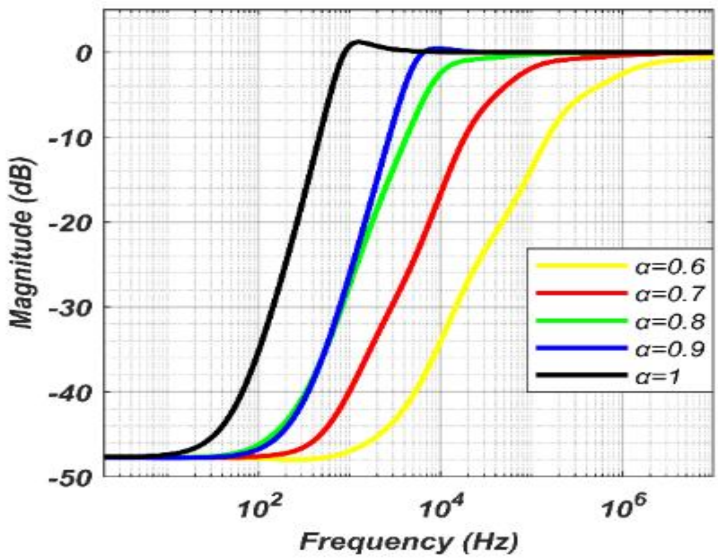


Figure 4.13 Tunability of FHP with respect to α

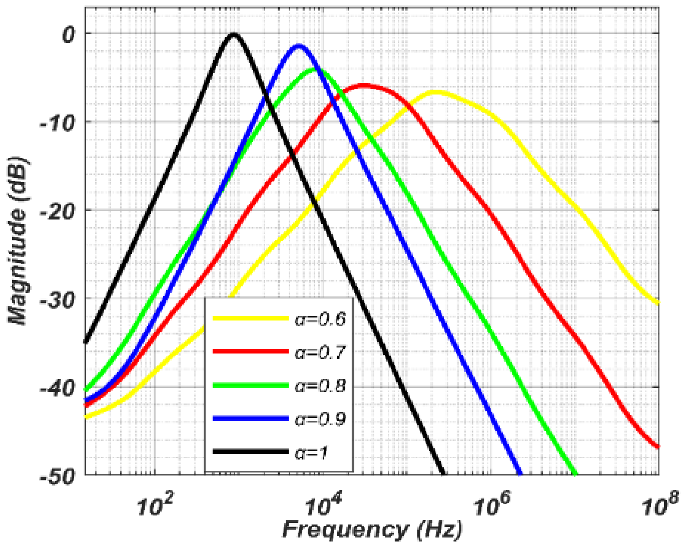


Figure 4.14 Tunability of FBP with respect to α

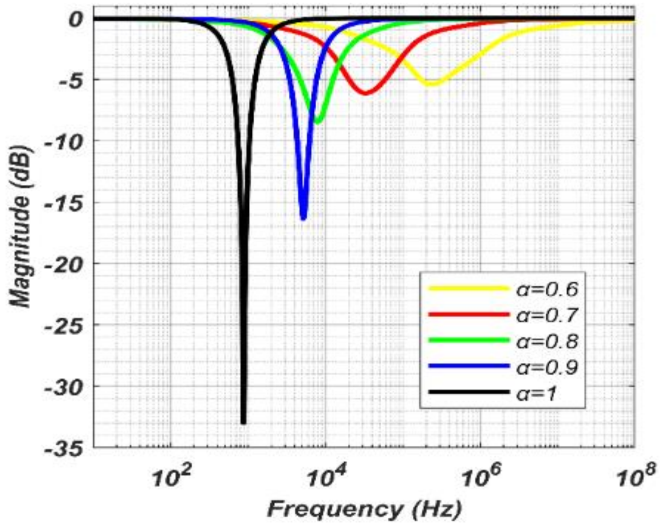


Figure 4.15 Tunability of FBS with respect to α

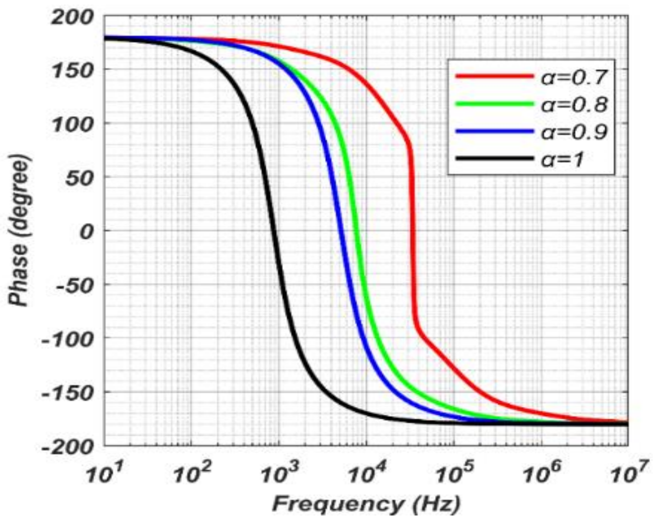


Figure 4.16 Tunability of FAP with respect to α

4.5.2 Tunability with I_{bias}

Tunability with I_{bias} is obtained by varying the external bias current of OTAs, here by keeping the value of I_{bias1} and I_{bias2} same for all the conditions $I_{bias1} = I_{bias2} = 0.1079\ mA, 0.40857\ mA, 1.1\ mA, and 1.9\ mA$ with the values of the bias resistors R_{bias} as 265 KΩ, 70 KΩ, 25 KΩ and 15 KΩ respectively. All the important frequency parameters for filters have been listed in Table 4.6-4.7 and their simulation results is shown in Fig 4.15-4.19. Again, the error between theoretical and simulated values are very less and acceptable (less than 5%).

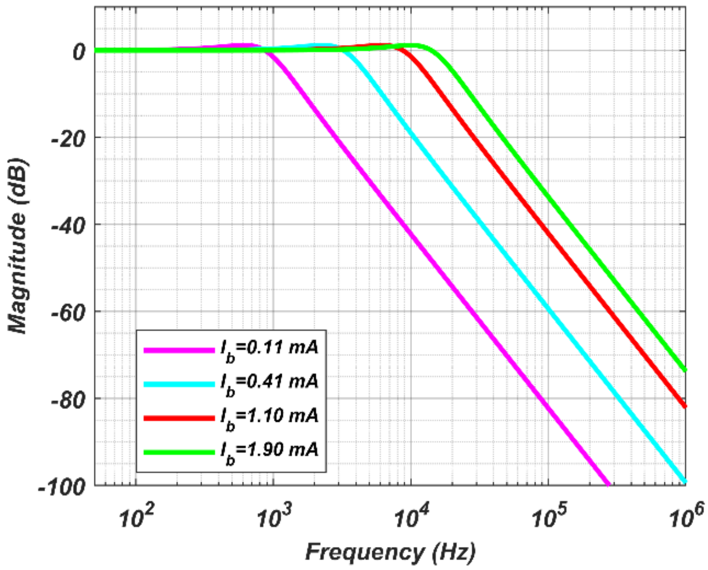


Figure 4.17 Tunability of FLP with respect to I_{bias}

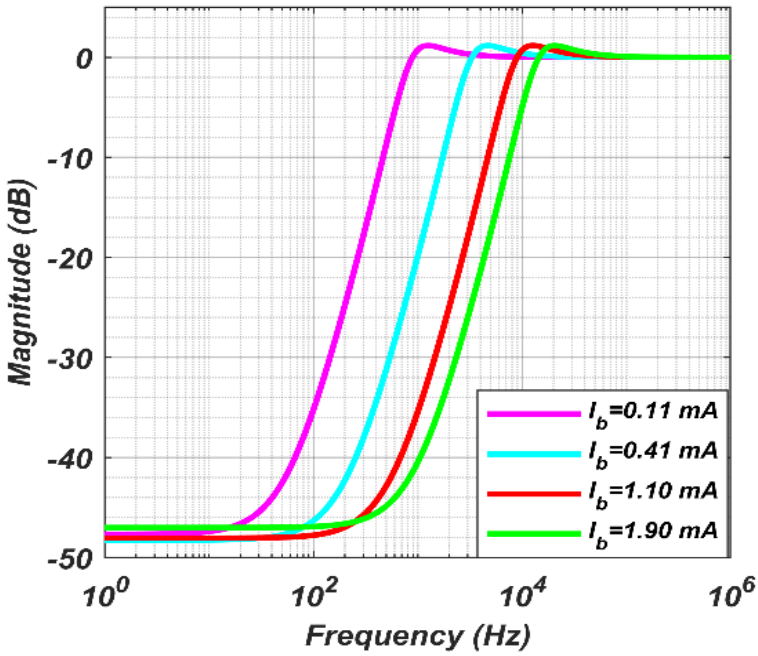


Figure 4.18 Tunability of FHP with respect to I_{bias}

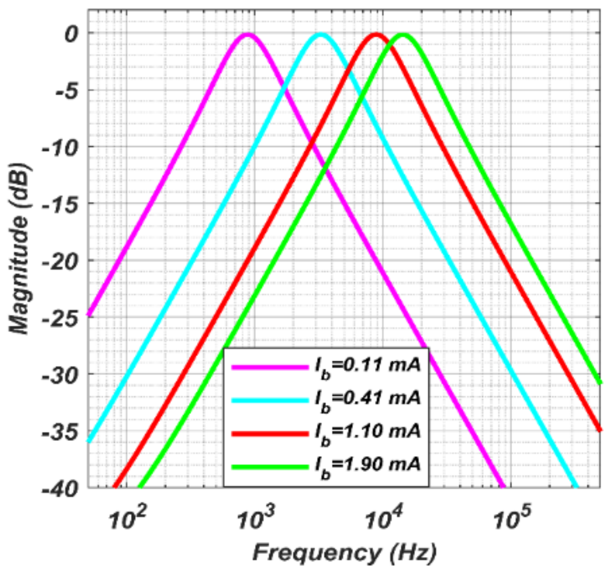


Figure 4.19 Tunability of FBP with respect to I_{bias}

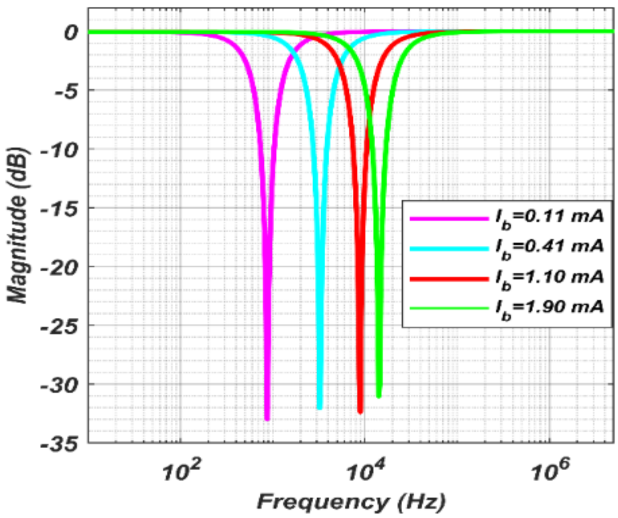


Figure 4.20 Tunability of FBS with respect to I_{bias}

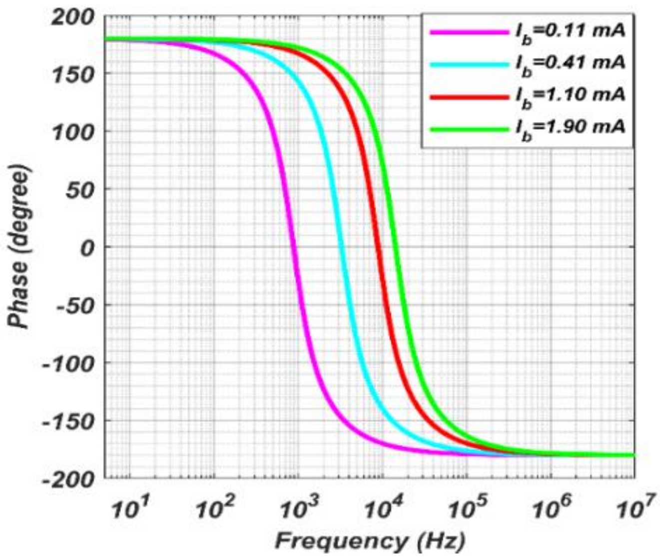


Figure 4.21 Tunability of FAP with respect to I_{bias}

Table 4.4 Cut-off frequencies (f_h) (in khz) of fractional filters for different α

α	1	0.9	0.8	0.7	0.6
FLPF (Simulated)	1.11	6.1054	6.3855	19.020	104.557
FLPF (Theoretical)	1.0999	2.426	6.110	19.405	94.190
FHPF (Simulated)	0.89	4.4173	9.0124	67.9611	824.383
FHPF (Theoretical)	0.88	2.0835	9.018	61.271	758.168

Table 4.5 Max/min frequencies (f_m) (in khz) of fractional filters for different α

α	1	0.9	0.8	0.7	0.6
FBPF (Simulated)	0.871	5.2481	8.1283	30.903	223.872
FBPF (Theoretical)	0.865	2.426	7.423	34.482	267.230
FBRF (Simulated)	0.871	5.1286	7.9433	33.113	234.423
FBRF (Theoretical)	0.865	2.426	7.423	34.482	267.230

Table 4.6 Cut-off frequencies (f_h) (in khz) of fractional filters for different I_{bias}

	265 k	70 k	25 k	15 k
FLPF (Simulated)	1.1103	4.1389	11.103	17.826
FLPF (Theoretical)	1.0996	4.1639	11.659	19.431
FHPF (Simulated)	0.6898	2.58767	7.001	11.262
FHPF (Theoretical)	0.6796	2.57315	7.2059	12.0009

Table 4.7 Max/min frequencies (f_m) (in khz) of fractional filters for different I_{bias}

	265 k	70 k	25 k	15 k
FBPF (Simulated)	0.871	3.2359	8.9125	14.453
FBPF (Theoretical)	0.864	3.2735	9.1659	15.276
FBRF (Simulated)	0.871	3.2359	8.9125	14.453
FBRF (Theoretical)	0.864	3.2735	9.1659	15.276

Table 4.8 Slope of attenuation in passband/stopband (dB/decade)

	1	**0.9**	**0.8**	**0.7**	**0.6**
FLPF	-40.01	-37.624	-31.47	-27.187	-23.778
FHPF	40.436	36.833	30.20	26.156	23.267

4.6 Conclusion

This chapter has presented a tunaeable configuration of VDTA based fractional order universal filter. Tuneability of the filter parameters with respect to value of α and g_m has been exhibited. Without changing the circuit configuration, using only one VDTA and two fractional capacitors, the circuit presented in this chapter can realize all the five standard biquadratic fractional filter functions. The condition for the stability of all the modified filters have also been presented.which gives us the absolute idea of the location of poles.

REFERENCES

[1] D. Biolek, R. Senani, V. Biolkova, and Z. Kolka, "Active elements for analog signal processing: classification, review, and new proposals," Radioengineering, 17, no. 4, pp. 15–32, 2008.

[2] J. Satansup, and W. Tangsrirat, "Compact VDTA-based current-mode electronically tunable universal filters using grounded capacitors," Microelectronics Journal, 45, no. 6, pp. 613-618, 2014.

[3] J. Satansup, and W. Tangsrirat, "Single VDTA-based current-mode electronically tunable multifunction filter," In 4th International Science, Social Science, Engineering and Energy Conference, 2012, pp. 1-8.

[4] P. Rani, and R. Pandey, "Voltage differencing transconductance amplifier based fractional order multiple input single output universal filter," Solid State Electronics Letters 1, no. 2, pp. 110-118, 2019.

[5] D. Prasad, M. Kumar, and M. W. Akram, "Current mode fractional order filters using VDTAs with Grounded capacitors," International Journal of Electronics and Telecommunications, 65, no. 1, pp. 11-17, 2019.

[6] A. S. Ali, A. G. Radwan, and A. M. Soliman, "Fractional order Butterworth filter: active and passive realizations," IEEE journal on emerging and selected topics in circuits and systems, 3, no. 3, pp. 346-354, 2013

[7] T. J. Freeborn, B. Maundy, and A. Elwakil, "Fractional-step Tow-Thomas biquad filters," Nonlinear Theory and Its Applications, IEICE, 3, no. 3, pp. 357-374, 2012.

[8] J. Koton, J. Jerabek, N. Herencsar, and D. Kubanek, "Current conveyors in current-mode circuits approximating fractional-order low-pass filter," In 2017 European Conference on Circuit Theory and Design (ECCTD), 2017, pp. 1-4.

[9] A. Soltan, A. G. Radwan, and A. M. Soliman, "CCII based fractional filters of different orders," Journal of advanced research, 5, no. 2, pp. 157-164, 2014.

[10] L. Langhammer, R. Sotner, J. Dvorak, J. Jerabek, and J. Polak, "Fully-differential tunable fractional-order filter with current followers and current amplifiers," In 2017 27th International Conference Radioelektronika (RADIOELEKTRONIKA), 2017, pp. 1-6.

[11] A. G. Radwan, A. S. Elwakil, and A. M. Soliman, "On the generalization of second-order filters to the fractional-order domain." Journal of Circuits, Systems, and Computers, 18, no. 2, pp. 361-386, 2009.

[12] R. Verma, N. Pandey, and R. Pandey, "Electronically tunable fractional order filter," Arabian Journal for Science and Engineering, 42, no. 8, pp. 3409-3422, 2017.

[13] E. Kaskouta, T. Kamilaris, R. Sotner, J. Jerabek, and C. Psychalinos, "Single-input multiple-output and multiple-input single-output fractional-order filter designs," In 2018 41st International Conference on Telecommunications and Signal Processing (TSP), 2018, pp. 1-4.

[14] J. Jerabek, R. Sotner, D. Kubánek, J. Dvorak, L. Langhammer, N. Herencsar, and K. Vrba, "Fractional-order low-pass filter with electronically adjustable parameters," In 2016 39th International Conference on Telecommunications and Signal Processing (TSP), 2016, pp. 569-574.

[15] J. Dvorak, Z. Polesakova, J. Jerabek, L. Langhammer, A. Kartci, and J. Koton. "Non-Integer-Order Low-Pass Filter with Electronically Controllable Parameters," In 2018 IEEE International Symposium on Circuits and Systems (ISCAS), 2018, pp. 1-5.

[16] A. Oustaloup, F. Levron, B. Mathieu, and F. M. Nanot, "Frequency-band complex noninteger differentiator: characterization and synthesis," IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications 47, no. 1, pp. 25-39, 2000.

[17] J. Valsa, P. Dvorak, and M. Friedl, "Network Model of the CPE," Radioengineering, 20, no. 3, pp. 619–626, 2011.

[18] A. G. Radwan, A. M. Soliman, A. S. Elwakil, and A. Sedeek, "On the stability of linear systems with fractional-order elements," Chaos, Solitons & Fractals, 40, no. 5, pp. 2317-2328, 2009.

Chapter 5

SUMMARY AND FUTURE SCOPE

To have a better understanding of natural phenomena happening around, study of fractional calculus plays an important role. As it adds an extra degree of freedom in terms of 'α', where m<α<m+; m=±1, ±2,..so on, it gives us more variations as compared to integer ones. In this work, step by step procedures are discussed about the methods to approximate of fractional operator s^{α}, followed by a fractional order inverse filter and universal fractional order filter using different ABB.

5.1 Summary

In chapter-1, we have discussed about the fractional calculus, its origin, history and most importantly its need to tackle the practical application problems. Along with that, how is it useful in present scenario and its application in electrical and electronics engineering have also been discussed briefly. A little discussion on comparison between fractional and integer one is also mentioned.

In chapter-2, different definitions related to fractional calculus that are available in open literature is discussed along with fractance device. A list of approximation methods to approximate fractance element is also mentioned and finally detailed discussion along with simulation results on Valsa and Vlach approximation is also carried out.

In chapter-3, a novel analog inverse fractional order filter structure using operational transconductance amplifier and fractional order capacitor is discussed in detail. Here, the fractional order capacitor is designed using Valsa and Vlach approximation.

In chapter-4, a generalized current mode universal biquad fractional order filter using VDTA is discussed in detail along with its stability. Here the fractional order capacitor is designed using Oustaloup, Laveron, Mathieu and Nanot approximation.

5.2 Future Scope

There is a vast scope of expansion in terms of fractional calculus and works that have carried out using fractional element. Some of these are:

I. In our work, we only talked about fractional order capacitor and its various approximation methods but there is a scope for designing and simulating fractional order inductor.

II. The approximation methods used in our work can be improved further with simple steps and more appropriate operating frequency.

III. Fabrication of fractional order capacitor or fractional order inductor is also an area where researchers need to work.

PUBLICATION

PAPERS IN INTERNATIONAL CONFERENCES

[1] **J. Srivastava**, R. Bhagat, and P. Kumar, "Analog Inverse Filters Using OTAs," In 2020 6th International Conference on Control, Automation and Robotics (ICCAR), 2020, pp. 627-631. (SCOPUS) https://doi.org/10.1109/ICCAR49639.2020.9108100

[2] **J. Srivastava**, "VDTA based fractional order universal filter," In 2020 International Conference for Innovation in Technology (INOCON), 2020. (Accepted) (SCOPUS)

APPENDICES

APPENDIX 1

PSPICE model files used for LM13700 (OTA)

```
C1  6  4  4.8P
C2  3  6  4.8P
C3  5  6  6.26P
D1  2  4  DX
D2  2  3  DX
D3  11 21 DX
D4  21 22 DX
D5  1  26 DX
D6  26 27 DX
D7  5  29 DX
D8  28 5  DX
D10 31 25 DX
D11 28 25 DX
F1  4  3  POLY(1)   V6 1E-10 5.129E-2 -1.189E4 1.123E9
F2  11 5  V2        1.022
F3  25 6  V3        1.0
F4  5  6  V1        1.022
F5  5  0  POLY(2)   V3 V7 0 0 0 0 1
G1  0  33 5         0 .55E-3
I1  11 6  300U
Q1  24 32 31        QX1
Q2  23 3  31        QX2
Q3  11 7  30        QZ
Q4  11 30 8         QY
V1  22 24 0V
V2  22 23 0V
V3  27 6  0V
V4  11 29 1.4
V5  28 6  1.2
V6  4  32 0V
V7  33 0  0V
.MODEL QX1 NPN (IS=5E-16     BF=200 NE=1.15 ISE=.63E-16 IKF=1E-2)
.MODEL QX2 NPN (IS=5.125E-16 BF=200 NE=1.15 ISE=.63E-16 IKF=1E-2)
.MODEL QY  NPN (IS=6E-15     BF=50)
.MODEL QZ  NPN (IS=5E-16     BF=266)
.MODEL DX  D   (IS=5E-16)
.ENDS
```

APPENDIX 2

Dimensions of CMOS Transistors used in Fig 3.5:

Table 1 Aspect ratio for OTA

MOS Transistors	Aspect ratio W/L
M1-M2	3.6/.36
M3-M4	1.44/.36
M5	1.44/.36
M6-M7	2.88/.36
M8	1.44/.36
M9	2.88/.36