M.TECH [CONTROL AND INSTRUMENTATION] [GOPAL SINGH] 2020

REALIZATION OF FRACTIONAL ORDER ANALOG UNIVERSAL FILTERs

A DISSERTATION

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CANDIDATE'S DECLARATION

I, Gopal Singh, Roll No. 2K18/C&I/08 student of M. Tech. (Control & Instrumentation), hereby declare that the Dissertation titled "**Realization of Fractional order Analog Universal Filters**" which is submitted by me to the Department of Electrical Engineering, Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Technology, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

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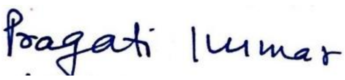
CERTIFICATE

I hereby certify that the Dissertation titled "**Realization of Fractional order Analog Universal Filters**" which is submitted by Mr. Gopal Singh, Roll No 2K18/C&I/08 Electrical Engineering Department, Delhi Technological University, Delhi in partial fulfillment of the requirement for the award of the degree of Master of Technology, is a record of the project work carried out by the students under my supervision. To the best of my knowledge this work has not been submitted in part or full for any Degree or Diploma to this University or elsewhere.



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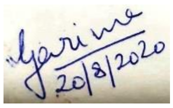

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ABSTRACT

Fractional order circuits incorporating fractional calculus concept have various applications in many fields namely, bio-medical engineering, control system, analog signal processing/generation, etc. In the present dissertation, along with a brief review of different methods of approximations used for the fractional order differentiator and integrator operator, fractional order analog universal filter circuits using operational transconductance amplifier and LT 1228 ICs have been presented. The workability of all the fractional order filter circuits along with fractional operator have been verified through PSPICE simulation and MATLAB simulation. Also, stability of all the designed fractional order filters have been discussed briefly.

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GOPAL SINGH

CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

ABB	Active Building Blocks
FoE	Fractional order Element
CFE	Continued Fraction Expansion
FoF	Fractional order Filter
VDTA	Voltage Differencing Transconductance Amplifier
OTA	Operational Transconductance Amplifier
MOTA	Multi Output Transconductance Amplifier
ACA	Adjustable Current Amplifier
UVC	Universal Voltage Conveyor
BOTA	Balanced Output Transconductance Amplifier
VM	Voltage Mode
FLPF	Fractional low-pass Filter
FHPF	Fractional high-pass Filter
FBPF	Fractional band-pass Filter
FBRF	Fractional band-reject Filter
FAPF	Fractional All-pass Filter
FoI	Fractional order Inductor
FoC	Fractional order Capacitor
SISO	Single Input Single Output
SIMO	Single Input Multi Ouput
MISO	Multi Input Single Ouput
CPE	Constant Phase Element
DVCCS	Differential Voltage Controlled Current Source
CFA	Current Feedback Amplifier

Chapter 1

INTRODUCTION

1.1 Introduction to fractional calculus

Fractional calculus is defined as a branch of mathematical analysis that deals with several different possibilities of real number power and complex number power and developing a calculus that generalizes the classical one. Likewise classical calculus, fractional calculus is also divided into two categories namely, fractional integrals and fractional derivatives which are the generalized version of classical integral and classical derivative.

Authors related to this topic usually cite this date as a birth date of fractional calculus as in a letter dated 16 September 1695, L'Hopital wrote to Leibniz asking for a notation he had used in his publications for the linear function $\frac{d^u y}{dx^n} = D^n y$ for the nth-derivative. L'Hopital questioned, what the result would be if n = ½, thereby Leibniz's responses: “An apparent paradox, from which one-day useful consequences will be drawn”. This marks the born of fractional calculus [1].

It is almost three centuries old as classical calculus and since its adds another dimension to understand or describes the nature in a better way, it finds numerous application in science and engineering community, for example, electric transmission lines, ultrasonic wave propagation in human cancellous bone, speech signals modeling, cardiac tissue electrode interface modeling, sound waves propagation in rigid porous materials, lateral and longitudinal control of autonomous vehicles, application in the theory of viscoelasticity, application in fluid mechanics, etc.

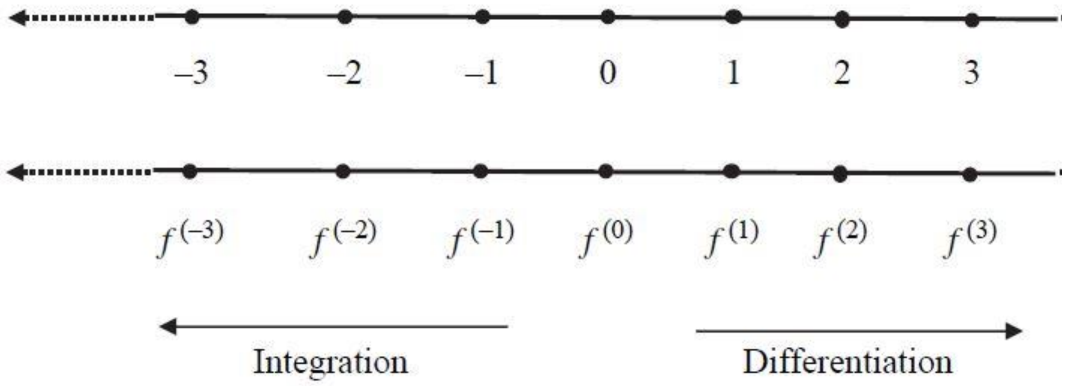

Figure 1.1 Interpolation of number line to fractional calculus

1.2 Literature Survey

Though fractional calculus is almost three centuries old [2], it has become more popular among researchers in the last few years. There is a vast scope for designing a fractional-order circuit and the researchers have mainly focused on designing fractional order element i.e. fractional-order inductor or fractional-order capacitor and fractional order filters using different active building blocks (ABB).

1.2.1 Fractional order Element (FoE)

In the open literature, various rational approximations are used for determining the fractional-order element. Some of these approximations are: Continued Fraction Expansion (CFE) [3], Carlson and Halijak approximation [4-5], Matsuda and Fujii approximation [6], Oustaloup, Levron, Mathieu, and Nanot , recursive approximation [7], Valsa and Vlach approximation [8], Charef, Sun, Tsao, and Onaral approximation [9], Modified Oustaloup [10] and El-Khazali reduced-order approximation [11]. It is found in the literature that fractional-order capacitor designed using CFE has a very small frequency range as compared to Valsa and Vlach approximation and Oustaloup, Levron, Mathieu, and Nanot recursive approximation approximation.

1.2.2 Fractional order Filters (FoF)

In the open literature, variety of fractional order analog filters are introduced using various active building blocks along with some passive components such as operational amplifier [12-13], current conveyors [14], current feedback amplifiers [15], voltage differencing transconductance amplifier (VDTA) [16], Operational Transconductance Amplifier (OTA) [17], Multi-Output Transconductance Amplifier (MOTA) with Adjustable Current Amplifier (ACA) [18], Balanced Output Transconductance Amplifier (BOTA) with ACA [19], Universal Voltage Conveyor (UVC) [20].

1.3 Thesis outline

1.3.1 Chapter 1

This chapter is further divided into three parts, where the first part briefly describes the history, evolution, and further improvements in fractional calculus along with its application in various fields. The second part mainly focuses on open literature surveys related to fractional-order elements, list of various approximation methods, their advantages and disadvantages and fractional order filters, different methods, and techniques for designing fractional-order filters using different active building blocks and trying to minimize the use of passive components. The third part briefly describes the organization of the thesis.

1.3.2 Chapter 2

This chapter is further divided into seven parts. The first and second part give us a brief introduction to the fractional operator and the list of definitions given by various researchers in this domain. In the third part, various methods for finding s^{α} approximation have been discussed in the brief and finally, one of the methods for finding s^{α} approximation viz. continued fraction expansion (CFE) has been discussed in detail along with its simulation results and comparison of two different methods that come under CFE approximation in the remaining parts.

1.3.3 Chapter 3

This chapter is further divided into 4 parts. The first part gives us an introduction to fractional order filter circuits using an operational transconductance amplifier (OTA) and many other active elements. The second part provides us a detailed description of various active and passive components used while designing the proposed universal Biquad voltage mode (VM) fractional order filter using OTAs. The third and fourth parts talk about the stability and simulation results of various filters namely fractional-order low pass filter (FLPF), fractional-order high pass filter

(FHPF), fractional-order bandpass filter (FBPF), fractional-order band-reject filter (FBRF) and fractional order all-pass filter (FAPF).

1.3.4 Chapter 4

This chapter is again divided into 4 parts. The first part gives us an introduction to filtering circuits using commercially available IC LT1228 and many other active elements. The second part provides us a detailed description of various active and passive components used while generalizing the modified biquad voltage mode (VM) fractional order filter using LT1228 IC. The third and fourth part talks about the stability and simulation results along with tunability of various filters namely, fractional-order low pass filter (FLPF), fractional-order high pass filter (FHPF), fractional-order bandpass (FBPF) filter, and fractional order band-reject filter (FBRF)

1.3.5 Chapter 5

This chapter summarizes the work presented in this thesis and also discusses the future work.

REFERENCES

[1] K. Oldham, and J. Spanier, "The fractional calculus theory and applications of differentiation and integration to arbitrary order," Elsevier, 1974.

[2] Y. Q. Chen, I. Petras, and D. Xue, "Fractional order control-a tutorial," In 2009 American control conference, 2009, pp. 1397-1411.

[3] S. C. D. Roy, "On the realization of a constant-argument immittance or fractional operator," IEEE Transactions on Circuit Theory, 14, no. 3, pp. 264-274, 1967.

[4] G. E. Carlson, and C. A. Halijak, "Simulation of the fractional derivative operator and the fractional integral operator," PhD diss., Kansas State University, 1960.

[5] G. Carlson, and C. Halijak, "Approximation of fractional capacitors (1/s)^(1/n) by a regular Newton process," IEEE Transactions on Circuit Theory, 11, no. 2, pp. 210-213, 1964.

[6] K. Matsuda, and H. Fujii, "H (infinity) optimized wave-absorbing control-Analytical and experimental results," Journal of Guidance, Control, and Dynamics, 16, no. 6, pp. 1146-1153, 1993.

[7] A. Oustaloup, F. Levron, B. Mathieu, and F. M. Nanot, "Frequency-band complex noninteger differentiator: characterization and synthesis," IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 47, no. 1, pp. 25-39, 2000.

[8] J. Valsa, and J. Vlach, "RC models of a constant phase element," International Journal of Circuit Theory and Applications, 41, no. 1, pp. 59-67, 2013.

[9] A. Charef, , H. H. Sun, Y. Y. Tsao, and B. Onaral, "Fractal system as represented by singularity function," IEEE Transactions on automatic Control, 37, no. 9, pp. 1465-1470, 1992.

[10] D. Xue, C. Zhao, and Y. Q. Chen, "A modified approximation method of fractional order system," In 2006 International Conference on Mechatronics and Automation, 2006, pp. 1043-1048.

[11] R. El-Khazali, "On the biquadratic approximation of fractional-order Laplacian operators," Analog Integrated Circuits and Signal Processing, 82, no. 3, pp. 503-517, 2015.

[12] A. S. Ali, A. G. Radwan, and A. M. Soliman, "Fractional order Butterworth filter: active and passive realizations," IEEE journal on emerging and selected topics in circuits and systems, 3, no. 3, pp. 346-354, 2013.

[13] B. Maundy, A. S. Elwakil, and T. J. Freeborn, "On the practical realization of higher-order filters with fractional stepping," Signal Processing, 91, no. 3, pp. 484-491, 2011.

[14] F. Khateb, D. Kubánek, G. Tsirimokou, and C. Psychalinos, "Fractional-order filters based on low-voltage DDCCs," Microelectronics Journal, 50, pp. 50-59, 2016.

[15] T. Suksang, V. Pirajnanchai, C. Suppitaksakul, and W. Loedhammacakra, "Design and improve the performance of OTA low pass filter with Fractional-order step," In 2012 IEEE International Conference on Electron Devices and Solid State Circuit (EDSSC), 2012, pp. 1-4.

[16] P. Rani, and R. Pandey, "Voltage differencing transconductance amplifier based fractional order multiple input single output universal filter," Solid State Electronics Letters, 1, no. 2, pp. 110-118, 2019.

[17] G. Tsirimokou, C. Psychalinos, and A. S. Elwakil, "Fractional-order electronically controlled generalized filters," International Journal of Circuit Theory and Applications, 45, no. 5, pp. 595-612, 2017.

[18] J. Dvorak, J. Jerabek, L. Langhammer, S. Kapoulea, and C. Psychalinos, "Design of Fully-Differential Frequency Filter with Fractional-Order Elements," In 2018 41st International Conference on Telecommunications and Signal Processing (TSP), 2018, pp. 1-7.

[19] J. Dvorak, J. Jerabek, Z. Polesakova, D. Kubanek, and P. Blazek, "Multifunctional Electronically Reconfigurable and Tunable Fractional-Order Filter," Elektronika ir Elektrotechnika, 25, no. 1, 2019, pp. 26-30.

[20] J. Koton, D. Kubanek, O. Sladok, K. Vrba, A. Shadrin, and P. Ushakov, "Fractional-order low-and high-pass filters using UVCs," Journal of Circuits, Systems and Computers, 26, no. 12, pp. 1750192, 2017.

Chapter 2

DESIGN AND SIMULATION OF FRACTIONAL ORDER INDUCTOR AND CAPACITOR

2.1 Introduction to Fractional Operator:

The term fractional operator was firstly coined by Riemann in 1838. For an integer order system, its Laplacian operator is represented by s^n, where n is an integer number while for a fractional-order system, its Laplacian operator is represented by s^α where $m<\alpha<m+1$; $m=0,\pm1,\pm2$. Although approximation of fractional order Laplacian operator can be done by various methods, it has a great significance in determining fractional-order Inductor and Capacitor. Accuracy of these approximations is measured by the flatness of phase response i.e. less ripple factor.

2.2 Definition of Fractional Calculus:

2.2.1 The Riemann-Liouville definition [1] for fractional-order integral can be written as:

$$aD_t^{-\alpha}f(t) = \frac{1}{\Gamma(\alpha)}\int_{\alpha}^{t}(t-\tau)^{\alpha-1}f(\tau)d\tau \qquad (2.1)$$

where $0<\alpha<1$, and a is the initial value. Putting a=0 in the above equation, the simplified integral becomes $D_t^{-\alpha}f(t)$, and for fractional-order derivative, it can be written as:

$$\frac{d^\alpha}{dt^\alpha}v(t) \triangleq D^\alpha v(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_{0}^{t}(t-\tau)^{-\alpha}v(\tau)d\tau \qquad (2.2)$$

2.2.2 The Grünwald-Letnikov definition [1] uses a single equation for fractional order differentiation and integral:

$$aD_t^{\alpha}f(t) = \lim_{h\to 0}\frac{1}{h^\alpha}\sum_{j=0}^{[(t-a)/h]}(-1)^j\binom{\alpha}{j}f(t-jh) \qquad (2.3)$$

where $\left(\frac{\alpha}{j}\right)$ are the binomial coefficients. Depending upon using the above equation as differentiation or integration, the values of α is decided whether non-integer positive or negative.

2.2.3 The Cauchy integral formula [1] for fractional-order is given as:

$$D^{\alpha}f(t) = \frac{\Gamma(\alpha+1)}{j2\pi}\oint \frac{f(\tau)}{(\tau-t)^{\alpha+1}}d\tau \tag{2.4}$$

where the closed path encircles the poles of the function $f(t)$.

2.2.4 The Caputo definition [1] for fractional-order differentiation is:

$$oD_t^{\alpha}f(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{f^{(m+1)}(\tau)}{(t-\tau)^{\alpha}}d\tau \tag{2.5}$$

where α=m+γ, m is an integer and 0<γ≤1. Also, its definition for fractional-order integral is:

$$oD_t^{-\gamma}f(t) = \frac{1}{\Gamma(\gamma)}\int_0^t \frac{f(\tau)}{(t-\tau)^{1-\gamma}}d\tau, \gamma > 0 \tag{2.6}$$

2.3 Various methods for finding the approximation to s^{α}:

2.3.1 **Continued Fraction Expansion (CFE)** [2]:

This fractional-order approximation is based on approximating a biquadratic second-order transfer function. This biquadratic form which is used to approximate sα yields 2nd order transfer function of equal orders, which can be further cascaded with other biquadratic forms to increase the operating ranges.

2.3.2 **Valsa and Vlach Approximation** [3]:

In this approximation, we simply derive the mathematical or network model called the constant phase element (CPE). An ideal constant phase element is defined as an element whose impedance or admittance phase response is constant over a range of frequency. Its impedance can be written as:

$$Z(s) = \Psi s^{\alpha} = \Psi(j\omega)^{\alpha} = \Psi\omega^{\alpha}\left(cos\left(\frac{\alpha\pi}{2}\right) + jsin\left(\frac{\alpha\pi}{2}\right)\right) \tag{2.7}$$

2.3.3 **Oustaloup, Levron, Mathieu, and Nanot approximation** [4]:

This method approximates fractional-order Laplacian operator s^{α} in the form:

$$s^{\alpha} = C \prod_{k=1}^{k=N} \frac{1 + s/\omega'_k}{1 + s/\omega_k} \tag{2.8}$$

where N is the order up to which RC network has to be formed, C is the gain adjustment parameter, ω'_k and ω_k are frequency terms related to minimum and maximum frequency [4]. One of the biggest constraint using this approximation is the effective range over which the phase response remains constant i.e $[10\omega_{min}, \omega_{max}/10]$, though its maximum and minimum operating frequency is $[\omega_{max}, \omega_{min}]$ respectively

2.3.4 **Matsuda and Fujii Approximation** [5]:

This method was originally referred to as the first form of Thiele's continued fraction (T-CF1). Since this method was firstly used by Matsuda in his work, thereafter it is known by his name. By approximating the original function into a set of equally logarithmic spaced frequencies $\boldsymbol{\omega^{\alpha}}$ can be written as:

$$\omega^{\alpha} = d_0 + \cfrac{\omega - \omega_0}{d_1 + \cfrac{\omega - \omega_1}{d_2 + \cfrac{\cdots}{\cdots + \cfrac{\omega - \omega_k}{d_{k+1} + \cdots}}}} \tag{2.9}$$

where $d_0, d_1, d_2 \dots \dots . d_{k+1}$are the notation whose values can be obtained from [6].

2.3.5 **Carlson and Halijak Approximation** [7]:

Carlson and Halijak make use of well-known third-order Newton process for approximating $(\boldsymbol{1/s})^{\boldsymbol{1/n}}$ or $\boldsymbol{s^{1/n}}$. Considering transfer function T(s) of a system as:

$$T(s) = \frac{1 + MB}{1 + A^2 B} * A \tag{2.10}$$

For simulating $1/\sqrt{s}$, consider $M(s) = 1/s \;\; and \; B(s) = B_0/s$ and if feedback gain is taken very large, the transfer function becomes:

$$T(s)' = \lim_{B_0 \to \infty} T(s) = 1/As \tag{2.11}$$

Now to have a transfer function of $1/\sqrt{s}$, put $A = 1/\sqrt{s}$ in the above equation and by applying Taylor's expansion formula we can easily approximate it. Similarly, we can approximate $\sqrt{s}$ also by following the above steps.

2.4 Continued Fraction Expansion (CFE):

In our work FoI and FoC have been designed using Continued Fraction Expansion Approximation. In this approximation, two different methods were proposed:

2.4.1 Equal Ripple approximation method:

Firstly, this method was presented in [8] and latterly modified in [9] by adding a tuning parameter 'β' to minimize the error ripple around the normal operating region. In this method, s^{α} can be approximated for the realization of FoI as:

$$s^{\alpha} \approx \frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0} \equiv \frac{a_0 N(s)}{a_2 D(s)} \equiv H_d(s) \qquad (2.12)$$

where

$$a_0 = (\alpha^2 + 3\alpha + 2), a_2 = (\alpha^2 - 3\alpha + 2), a_1 = \{\beta(1 - \alpha^2) + 6\} \qquad (2.13)$$

are the real constants and for the realization of FoC, the above approximation can be written as $H_i(s) \equiv \frac{a_2 D(s)}{a_0 N(s)} = \frac{1}{H_d(s)}$. Since $0 < \alpha < 1$ as already defined, then $a_0 > a_2 > 0$ and for obtaining stable approximation $\beta > 6/(\alpha^2 - 1)$.

2.4.2 Exact phase approximation method:

In this method, the second-order approximation given by equation (2.12) is improved, thereby giving zero phase error of $\boldsymbol{H_d(s)}$. It is improved by changing the substitution of $\boldsymbol{a_0}, \boldsymbol{a_1}\ \boldsymbol{and}\ \boldsymbol{a_2}$ i.e.

$$a_1 = (a_2 - a_0)\left(\tan\frac{(2+\alpha)\pi}{4}\right), a_0 = \beta_1 + \beta_2\alpha^v + (\beta_1 + \beta_2)\alpha, \qquad (2.14)$$
$$a_0 = \beta_1 + \beta_2\alpha^v - (\beta_1 + \beta_2)\alpha$$

i.e. adding some more tuning parameters $\beta_1, \beta_2\ and\ v$ as compared to the previous method.

However, to increase the bandwidth of both the approximation methods namely equal ripple and exact phase response, cascading is the only technique we can apply i.e. cascading two or more transfer functions normalized at $\omega = 1rad/s$, $\omega = 100rad/s$, $\omega = 10000rad/s$ and so on i.e.:

$$H_{d2}(s) = H_{d1}(s)H_{d100}(s/100)$$
$$= \left(\frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0}\right)\left(\frac{a_0(s/100)^2 + a_1(s/100) + a_2}{a_2(s/100)^2 + a_1(s/100) + a_0}\right)$$
$$or\ H_{d3}(s) = \left(\frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0}\right)\left(\frac{a_0(s/100)^2 + a_1(s/100) + a_2}{a_2(s/100)^2 + a_1(s/100) + a_0}\right)\left(\frac{a_0(s/10000)^2 + a_1(s/10000) + a_2}{a_2(s/10000)^2 + a_1(s/10000) + a_0}\right) \quad (2.15)$$

2.5 Steps for the realization of FoI

(i) The current-voltage relationship for fractional-order inductor in Laplace domain can be represented as:

$$Z(s) = Ls^{\alpha} \approx L\frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0} \qquad \text{(from equation (2.12))}$$

(ii) The above equation can be re-written in terms of poles and zeros as:

$$L\frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0} = L\frac{a_0(s+z_1)(s+z_2)}{a_2(s+p_1)(s+p_2)} = L\frac{a_0 N(s)}{a_2 D(s)} \quad (2.16)$$

where $a_0, a_1\ and\ a_2$ have different values according to the methods chosen i.e. either exact phase method or equal ripple method. The substitution of these parameters which depends upon 'α' has been given in equation (2.14) for the exact phase method and equation (2.13) for equal ripple method.

(iii) By doing the partial fraction of equation (2.16), we get:

$$\frac{Z_L(s)}{s} = L\left(\frac{a_0}{a_2}\right)\frac{(s+z_1)(s+z_2)}{s(s+p_1)(s+p_2)} = L\left(\frac{a_0}{a_2}\right)\left\{\frac{k_0}{s} + \frac{k_1}{(s+p_1)} + \frac{k_2}{(s+p_2)}\right\} \quad (2.17)$$

where $k_o = \frac{z_1 z_2}{p_1 p_2},\ k_1 = \frac{(z_1-p_1)(z_2-p_1)}{-p_1(p_2-p_1)}\ and\ k_2 = \frac{(z_1-p_2)(z_2-p_2)}{-p_2(p_1-p_2)}$

(iv) Finally, $z_L(s)$ can be written in terms of the resistor (R) and the inductor (L) i.e.:

$$z_L(s) = L\left(\frac{a_0}{a_2}\right)\left\{k_0 + \frac{sk_1}{(s+p_1)} + \frac{sk_2}{(s+p_2)}\right\} \quad (2.18)$$

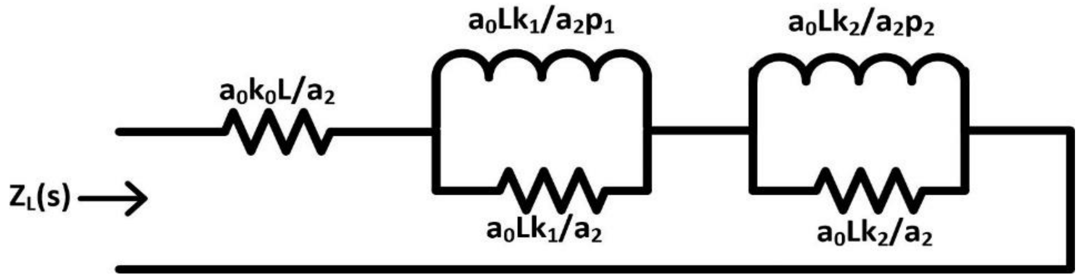

Figure 2.1 Single-stage realization of FoI

Considering β=3.8382 for equal ripple method and β_1= 0.5 and β_2= 1.5 for exact phase method, phase responses for single stage and double stage FoI are shown for different values of α.

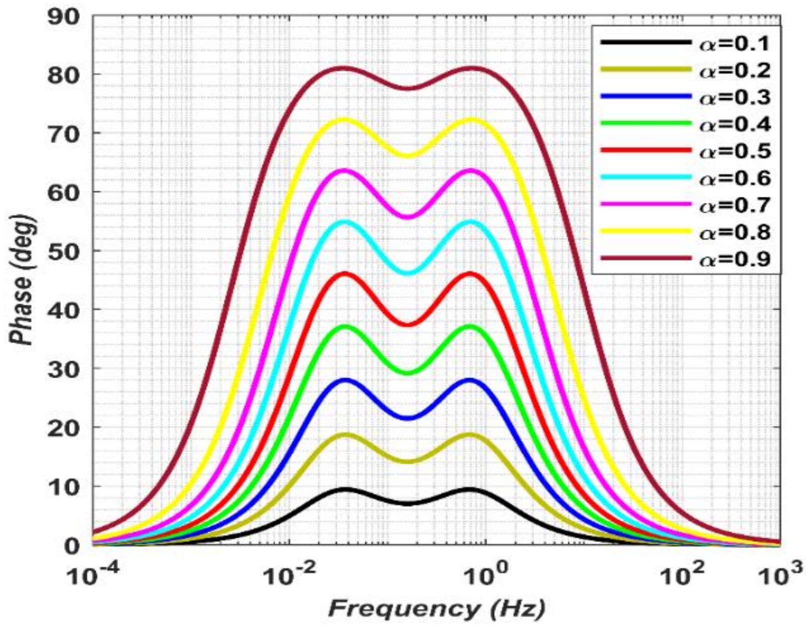

Figure 2.2 FoI first stage using equal ripple for different α

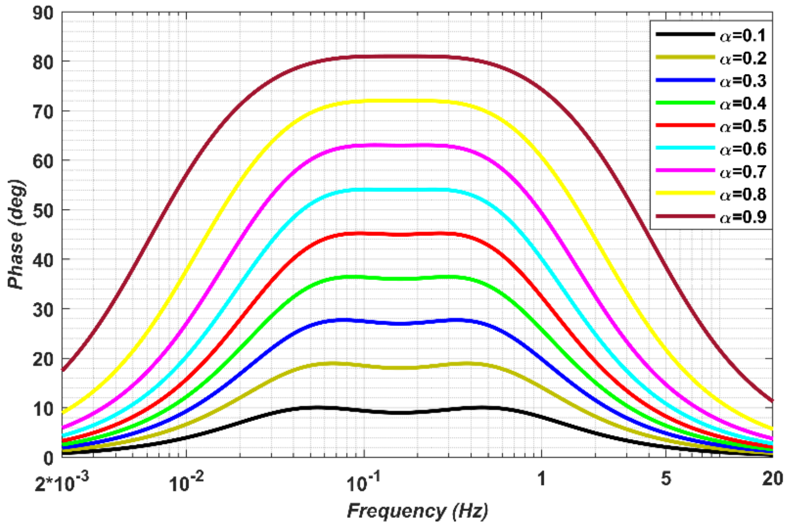

Figure 2.3 FoI first stage using the exact phase for different α

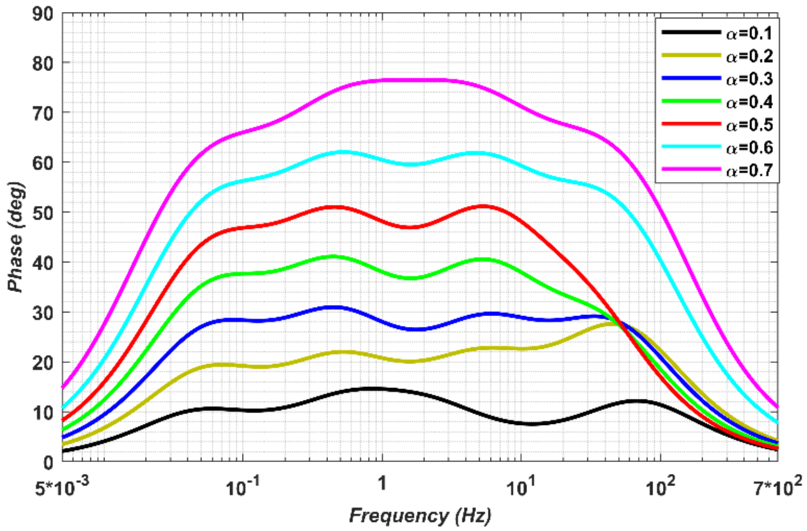

Figure 2.4 FoI second stage using the exact phase for different α

2.6 Steps for the realization of FoC

(i) The current-voltage relationship for the fractional-order capacitor in Laplace domain can be represented as:

$$Z_c(s) = \frac{1}{s^{\alpha}c} \approx \frac{a_2(s+p_1)(s+p_2)}{Ca_0(s+z_1)(s+z_2)} = \frac{a_2 D(s)}{Ca_0 N(s)} \tag{2.19}$$

(ii) By doing the partial fraction of equation (7), we get

$$Z_c(s) = \frac{a_2}{Ca_0}\left\{1 + \frac{\lambda_1}{(s+z_1)} + \frac{\lambda_2}{(s+z_2)}\right\} \tag{2.20}$$

where $\lambda_1 = \frac{(p_1-z_1)(p_2-z_1)}{(z_2-p_1)}$, $\lambda_2 = \frac{(p_1-z_2)(p_2-z_2)}{(z_1-p_2)}$ i.e. $Z_c(s)$ can be re-written in terms of the resistor (R) and capacitor (C).

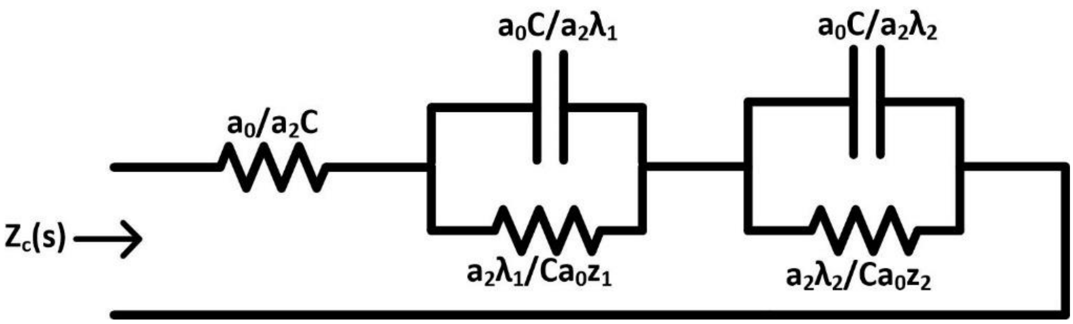

Figure 2.5 Single-stage realization of FoC

Considering β=3.8382 for equal ripple method and β_1= 0.5 and β_2= 1.5 for exact phase method, phase responses for single stage and double stage FoC are shown for different values of α.

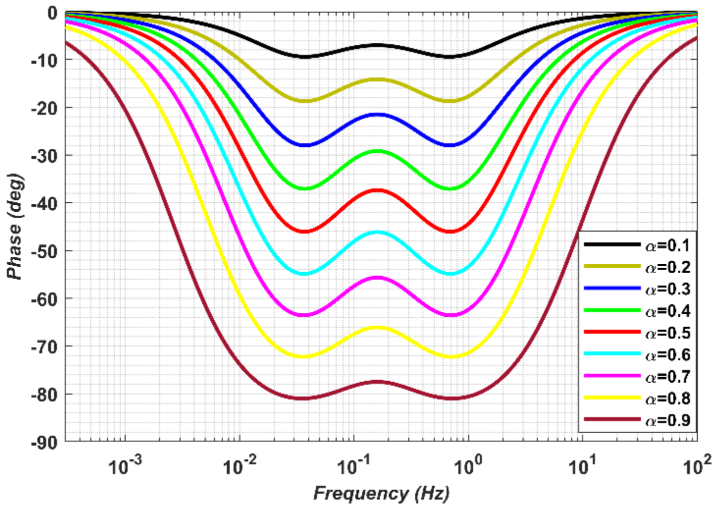

Figure 2.6 FoC first stage using equal ripple for different α

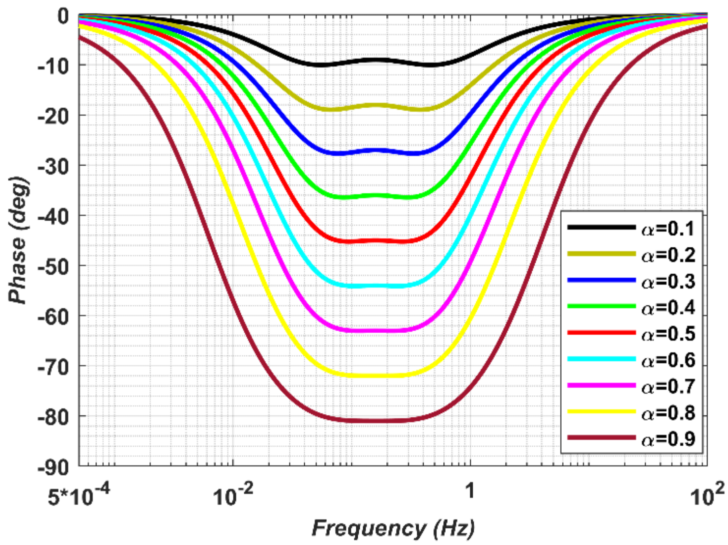

Figure 2.7 FoC first stage using the exact phase for different α

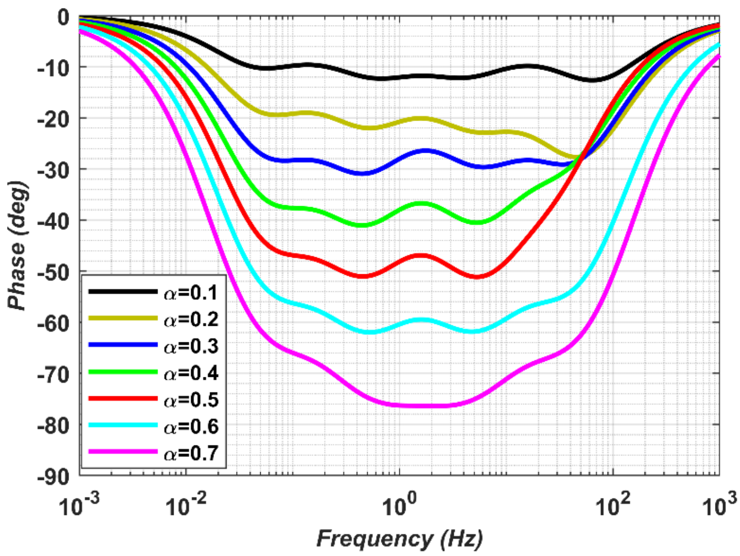

Figure 2.8 FoC second stage using the exact phase for different α

2.7 Comparison between the two methods namely, exact phase and equal ripple method:

On comparing the phase responses of FoI and FoC for two different methods for α=0.1, α=0.5 and α=0.9, we conclude that the ripples are less in exact phase as compared to equal ripple because of more tuning factors i.e. β_1 and β_2 in exact phase method.

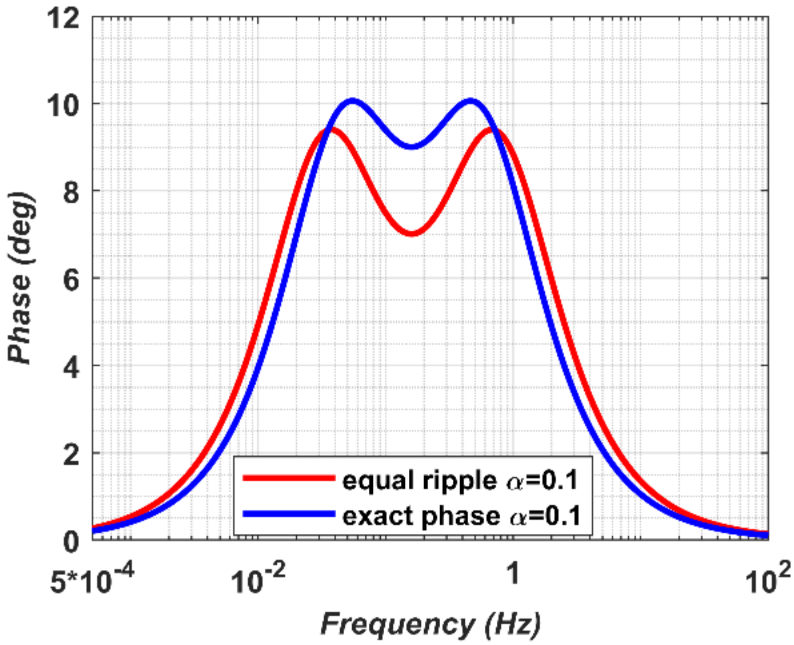

Figure 2.9 Comparing of FoI first stage for α=0.1

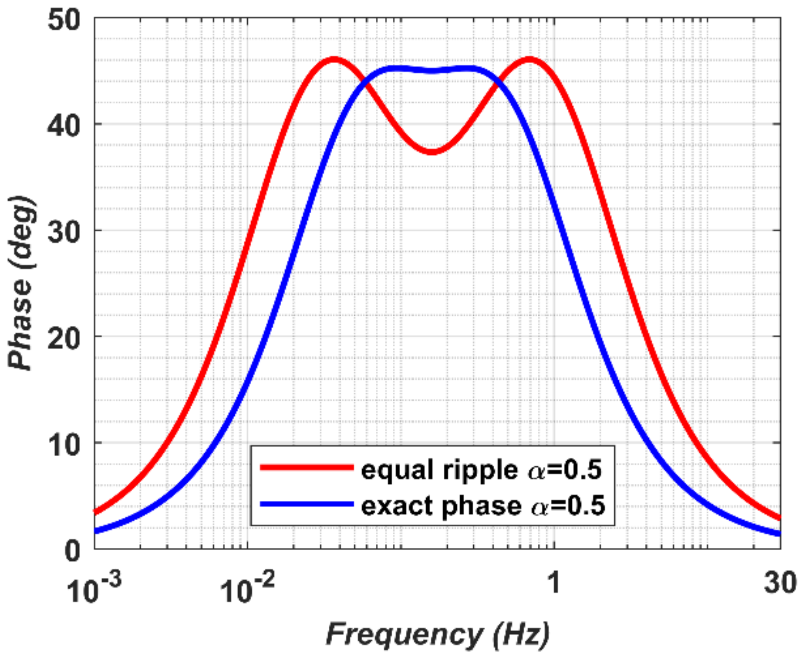

Figure 2.10 Comparing of FoI first stage for α=0.5

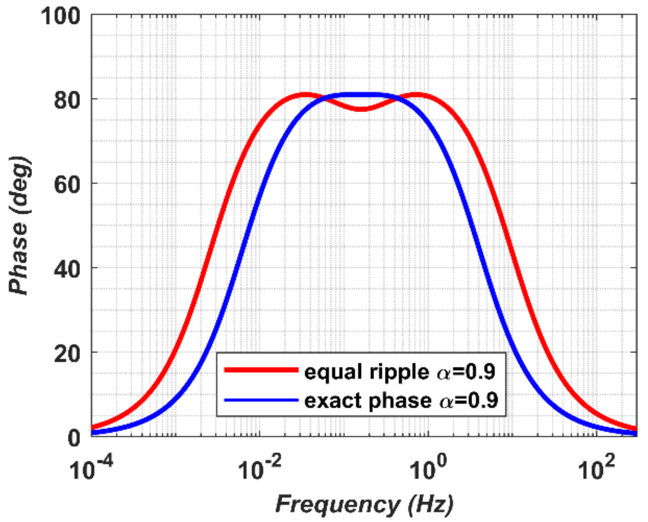

Figure 2.11 Comparing of FoI first stage for α=0.9

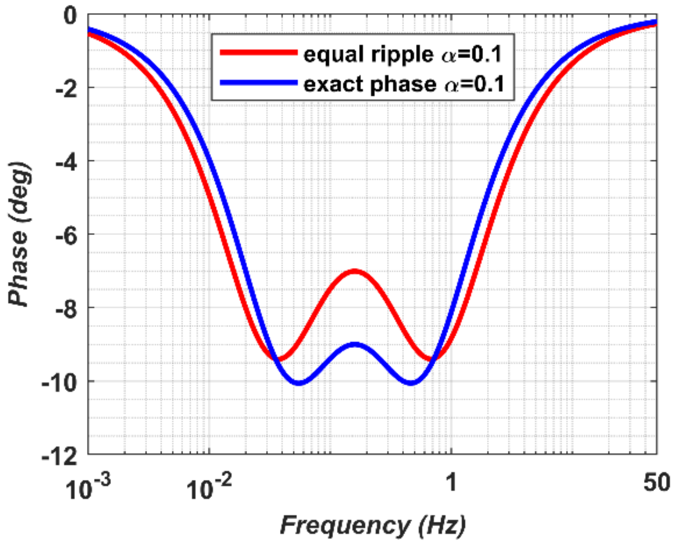

Figure 2.12 Comparing of FoC first stage for α=0.1

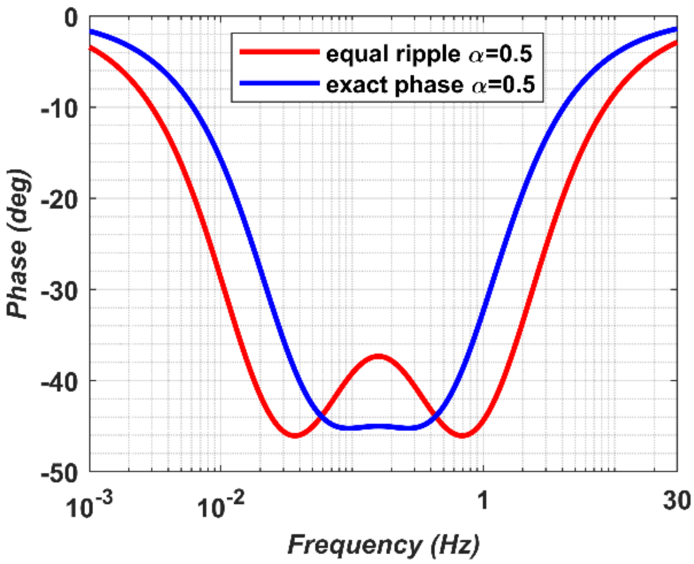

Figure 2.13 Comparing of FoC first stage for α=0.5

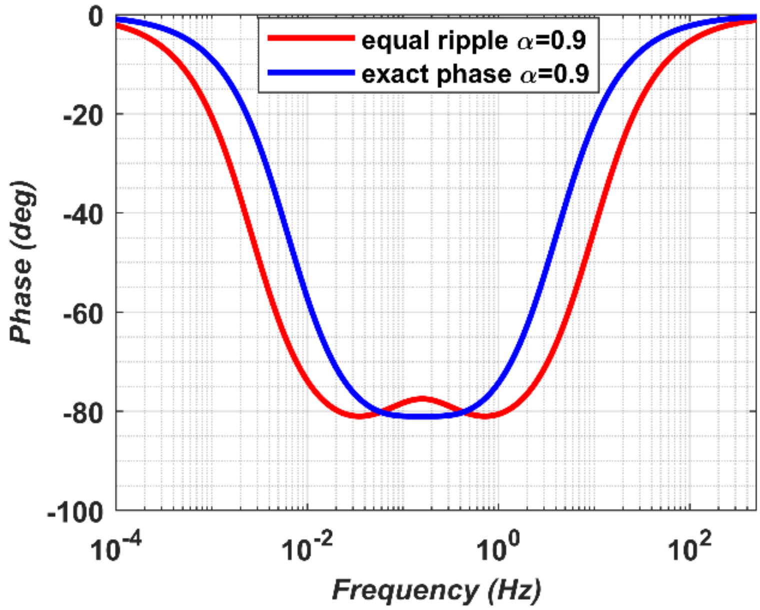

Figure 2.14 Comparing of FoC first stage for α=0.9

2.8 Conclusion

While realizing FoI or FoC using both the method, the major disadvantage or limitation, is their frequency range i.e. both methods obtains FoI and FoC at very less frequency which is quite not acceptable for practical purposes. Also, frequency response using an equal ripple method for the second stage realization of FoI or FoC is not possible and needs some more manipulation to obtain the correct graph.

REFERENCES

[1] C. R. C. Press, Solving applied mathematical problems with MATLAB, CRC Press, 2008.

[2] R. El-Khazali and Nabeel Tawalbeh, “Realization of fractional-order capacitors and inductors,” In 5th-IFAC Symposium on Fractional Diff. and its Applications, Nanjing, China, 2012.

[3] J. Valsa, and Jiri Vlach. "RC models of a constant phase element." International Journal of Circuit Theory and Applications, 41, no. 1 (2013): 59-67.

[4] A. Oustaloup, F. Levron, B. Mathieu, and F. M. Nanot, “Frequency-band complex noninteger differentiator: characterization and synthesis,” IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications 47, no. 1 (2000), 25-39.

[5] M. Khoichi, and F. Hironori, “H (infinity) Optimized wave absorbing control: Analytical and experimental result,” Journal of Guidance, Control, and Dynamics 16, no. 6 (1993): 1146-1153.

[6] O. Elwy, S. H. Rashad, L. A. Said, and A. G. Radwan, “Comparison between three approximation methods on oscillator circuits,” Microelectronics Journal 81 (2018): 162-178.

[7] G. Carlson, and C. Halijak, “Approximation of fractional capacitors (1/s)^(1/n) by a regular Newton process,” IEEE Transactions on Circuit Theory 11, no. 2 (1964): 210-213.

[8] B. T. Krishna, and K. V. V. S. Reddy, “Active and passive realization of fractance device of order ½,” Active and passive electronic components 2008 (2008).

[9] T. J. Freeborn, B. Maundy, and A. Elwakil, “Second-order approximation of the fractional Laplacian operator for equal-ripple response,” In 2010 53rd IEEE International Midwest Symposium on Circuits and Systems, pp. 1173-1176, IEEE, 2010.

Chapter 3

FRACTIONAL ORDER CAPACITOR BASED FILTER USING THREE OTAs[1]

3.1 Introduction

Ever since the generalization of the classical first-order and second-order filters came into fractional-order domain [1]-[2], a multitude of fractional order filters with different properties have appeared in the open literature. In a fractional order filter, the designers have more precise control over the pass-band/ stop-band characteristics because of the additional degree of freedom provided by the fractional-order parameter ‘α’. These filters have been realized using different types of active elements like operational amplifiers [3]-[4], current conveyors [5], current feedback amplifiers [6], operational transconductance amplifiers [7]-[19] and variety of other active elements. As the fractional-order capacitors of arbitrary values and order are not available commercially, in most of the works presented on fractional-order filters, the fractional-order capacitor(s) have been simulated by semi-infinite RC networks in which the values of the RC components have been computed using some approximations [20]-[21]. Of the various active elements used for the realization of fractional order filters, operational trans-conductance amplifiers (OTAs) are more versatile as the gain of this amplifier can be controlled externally by controlling its bias current. This feature is very useful in tuning the various parameters of the realized filters. A detailed review of the various fractional-order filters realized with OTAs [7]-[19] has revealed the following:

[1] The content and results of the following paper has been reported in this chapter: **G. Singh**, Garima, and P. Kumar, “Fractional Order Capacitors Based Filters Using Three OTAs,” In 2020 6th International Conference on Control, Automation and Robotics (ICCAR), 2020, pp. 638-643. https://doi.org/10.1109/ICCAR49639.2020.9108100 **Indexing:** SCOPUS and EI Compendex

(i) Single-input-single-output (SISO) type of fractional order filters utilizing six to eleven number of OTAs has been presented in [7]-[13].

(ii) Single-input-multiple-output (SIMO) type of fractional order filters have been realized in [14]-[15].

(iii) Multi-input-single-output (MISO) type of fractional order filter has been realized in [14]. This structure utilizes 5 OTAs and two fractional-order capacitors while realizing different types of filters.

(iv) OTAs along with some other additional active elements [16]-[19] have been used to realize fractional-order filters with different properties.

From the above discussion, it emerges that very little work has been done on the realization of multi-input-single-output type of fractional order filters using OTAs. These types of filter structures are very useful as different filter outputs may be obtained without changing the nature of the elements in different branches. Therefore, the main aim of this chapter is to present a new multi-input-single-output type fractional order filter utilizing only three single-output OTAs and two fractional-order capacitors.

3.2 Circuit Description

3.2.1 Designing Fractional-order Capacitor using Valsa and Vlach Method [21]

This method introduces the concept of constant phase element (CPE) which is defined as an element whose impedance or admittance phase response is constant over a range of frequency. Its impedance can be written as:

$$Z(s) = \Psi s^{\alpha} = \Psi (j\omega)^{\alpha} = \Psi \omega^{\alpha}\left(cos\left(\frac{\alpha\pi}{2}\right) + jsin\left(\frac{\alpha\pi}{2}\right)\right) \tag{3.1}$$

This method presents a fractance capacitor CPE model that can be constructed using resistors and capacitors and its phase varies from -90 to 0 degree.

3.2.2 Steps for determining the fractance capacitor CPE model:

1) Starting with given values of α (between 0 to 1), $\Delta\varphi$ (ripple factor in degrees), ω_{min} and ω_{max}.

2) Calculate the values of R_1 or C_1 by using the equation:

$$R_1 C_1 = \frac{1}{\omega_{min}} \tag{3.2}$$

3) Find the product of the constant term ‘ab’ using:

$$ab = \frac{0.24}{1 + \Delta\varphi} \tag{3.3}$$

4) Values of parameters a and b individually can be found using:

$$\emptyset_{av} = 90\alpha = 90\frac{\log a}{\log ab} \tag{3.4}$$

5) Determine the necessary number of sections (m) using:

$$m = 1 - \frac{\log\left(\frac{\omega_{max}}{\omega_{min}}\right)}{\log(ab)} \tag{3.5}$$

6) Finally, the branch elements can be calculated using:

$$\begin{aligned} R_k &= R_1 a^{k-1} \; where \; k = \pm 1, \pm 2, \dots m \\ C_k &= C_1 b^{k-1} \; where \; k = \pm 1, \pm 2, \dots m \end{aligned} \tag{3.6}$$

7) Correction elements (R_P and C_P) can be calculated as:

$$\begin{aligned} R_p &= R_1 \frac{1-a}{a} \\ C_p &= C_1 \frac{b^m}{1-b} \end{aligned} \tag{3.7}$$

8) Average frequency (ω_{av}) can be obtained using:

$$\omega_{av} = \frac{1}{R_1 C_1 (ab)^{k-1}} \sqrt{a} \; where \; k = int(m/2) \tag{3.8}$$

9) Input admittance of the required structure as shown in fig. 1 can be obtained using:

$$Y(j\omega_{av}) = \frac{1}{R_p} + j\omega_{av} C_p + \sum_{k-1}^{m} \frac{j\omega_{av} C_k}{1 + j\omega_{av} R_k C_k} \tag{3.9}$$

10) Finally, the slope of modulus D can be calculated as:

$$D = Z_{av} \omega_{av}^{-\alpha} \; where \; Z_{av} = \frac{1}{|Y(j\omega_{av})|} \tag{3.10}$$

Finally, the resulting Foster-II canonical RC structure as proposed in [21] is shown in fig 3.1.

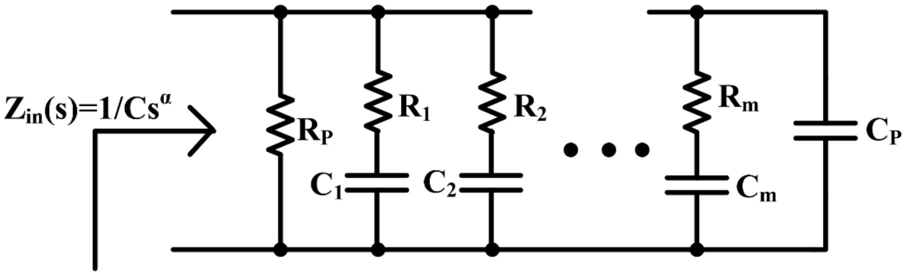

Figure 3.1 Foster II canonical RC structure

By considering f_{min}=1mHz, f_{max}= 1MHz and m=5, the phase response of the resultant structure shown above is plotted in fig 3.2 for different values of α ranging between 0 to 1.

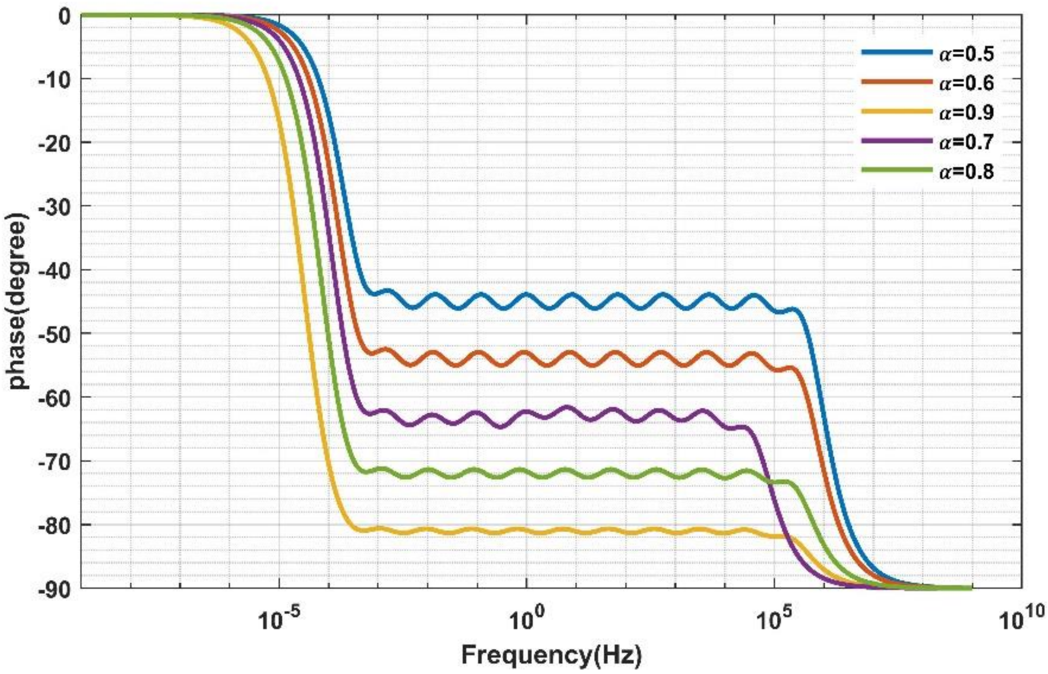

Figure 3.2 Phase response for different α

3.2.3 Operational Transconductance Amplifier (OTA)

The operational transconductance amplifier is a differential voltage-controlled current source (DVCCS). Ideally in OTAs, the output current is a function of differential input voltage and is expressed as:

$$I_0 = g_m(V_2 - V_1) \tag{3.11}$$

where I_0 is the output current, g_m is the transconductance gain, V_1 and V_2 are inverting and non-inverting terminal voltages.

The trans-conductance of this source can be controlled by an external bias current (I_{bias}) as given by the equation below:

$$g_m = \frac{I_{bias}}{2V_T} \tag{3.12}$$

where V_T is the thermal equivalent voltage.

The symbolic representation of OTA along with its ideal small-signal model is shown in fig. 3.3 and fig. 3.4.

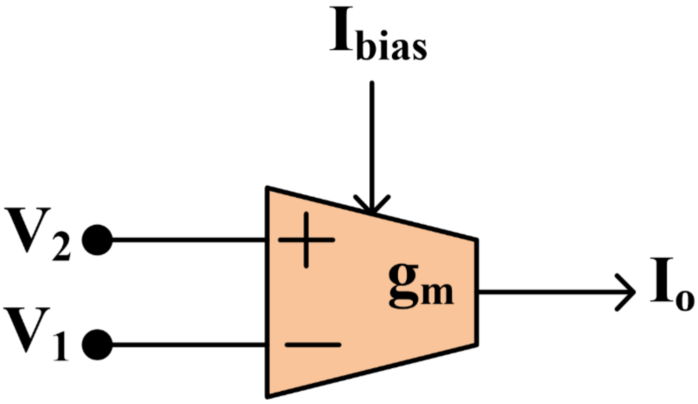

Figure 3.3 OTA representation

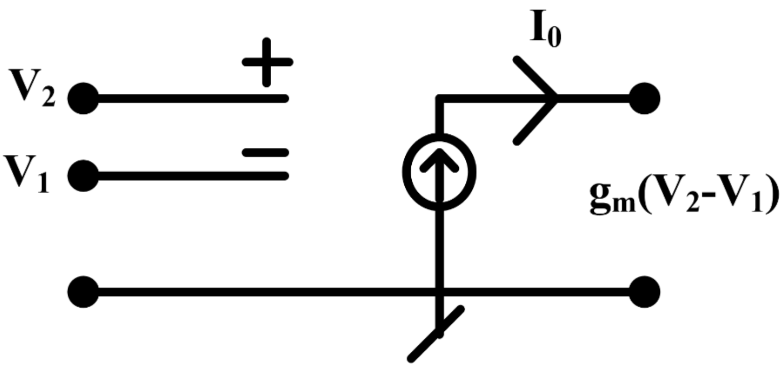

Figure 3.4 Small signal equivalent model of ideal OTA

3.3 Proposed Universal Biquad Fractional order Filter Configuration

The proposed universal biquad filter configuration is shown in fig. 3.5. It belongs to the class of multi-input single-output (MISO) type of filter structures where, by appropriately choosing different inputs, various types of filter responses can be obtained. We have used two identical fractional-order capacitors (C_α) whose driving point impedance is given by $1/Cs^\alpha$.

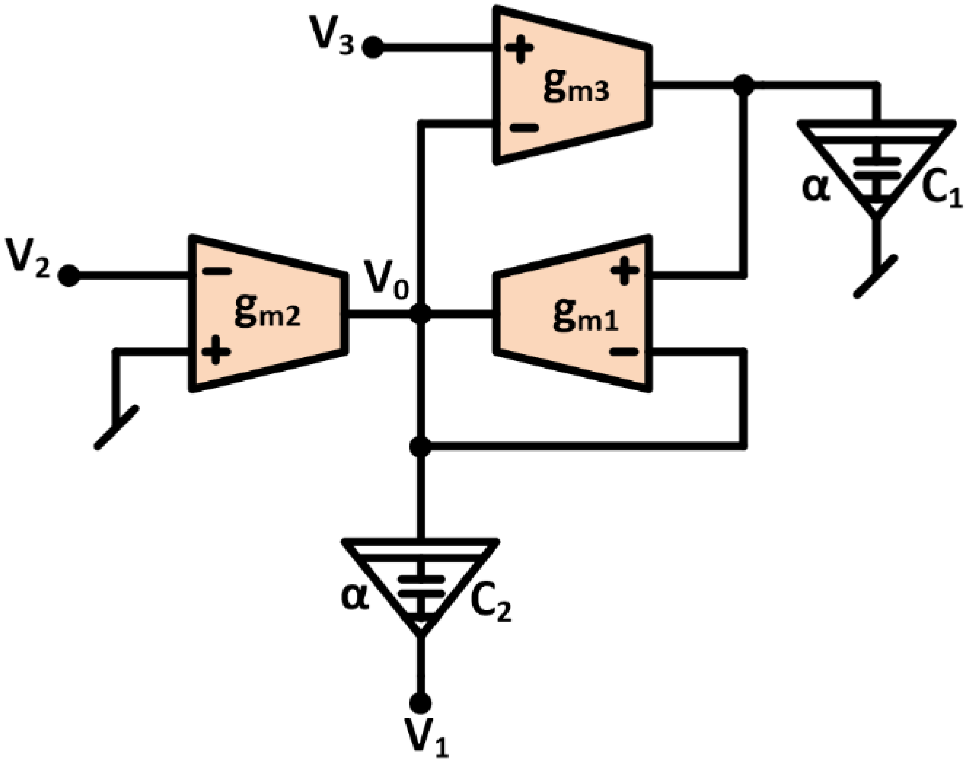

Figure 3.5 Proposed filter configuration

A routine analysis of the circuit given above results in the following output function:

$$V_0(s) = \frac{s^{2\alpha}(C_1C_2)V_1(s) - s^{\alpha}(C_2 g_{m2})V_2(s) + (g_{m1}g_{m3})V_3(s)}{s^{2\alpha}C_1C_2 + s^{\alpha}C_2 g_{m1} + g_{m1}g_{m3}} \tag{3.13}$$

where g_{m1}, g_{m2} and g_{m3} are the trans-conductance's of the different OTAs. For any fractional-order filters with a transfer function T(s), the following important parameters should be determined [2]:

a) ω_m : defined as the frequency at which the magnitude response either has maxima or minima and can be evaluated by solving the equation i.e., $\left(d|T(j\omega)|/d\omega\right)_{\omega=\omega_m} = 0$.

b) ω_h : defined as the frequency at which the power drops to half the pass-band power, known as half-power frequency (also known as cutoff frequency) and can be evaluated by solving the equation i.e., $|T(j\omega_h)| = \left(1/\sqrt{2}\right)|T(j\omega_{passband})|$. The bandwidth of any filter can also be calculated using this half-power frequency.

3.3.1 Fractional Order Low-Pass Filter (FLPF)

If we select $V_{in} = V_3$ and $V_1 = V_2 = 0$, the general transfer function given in equation (3.13) represents a fractional-order low-pass filter (FLPF) transfer function described as:

$$\frac{V_0(s)}{V_{in}(s)} = \frac{g_{m1}g_{m3}}{s^{2\alpha}C_1C_2 + s^{\alpha}C_2g_{m1} + g_{m1}g_{m3}} \tag{3.14}$$

Using the definition of cutoff frequency given above, the value of cutoff frequency for FLPF for any order ‘α’ can be obtained from equation (3.15):

$$Y^4 + 2\cos\left(\frac{\alpha\pi}{2}\right)Y^3 + (1 + 2k\cos(\alpha\pi))Y^2 + 2k\cos\left(\frac{\alpha\pi}{2}\right)Y - k^2 = 0 \tag{3.15}$$

where $Y = \frac{\omega_h^{\alpha}}{a}$, $k = \frac{b}{a}$, $a = \frac{g_{m1}}{c_1}$ and $b = \frac{g_{m3}}{c_2}$

3.3.2 Fractional order High-Pass Filter (FHPF)

By selecting $V_{in} = V_1$ and $V_2 = V_3 = 0$, the general transfer function represented by equation (3.13) can be converted into a fractional-order high-pass filter (FHPF) transfer function described as:

$$\frac{V_0(s)}{V_{in}(s)} = \frac{s^{2\alpha}C_1C_2}{s^{2\alpha}C_1C_2 + s^{\alpha}C_2g_{m1} + g_{m1}g_{m3}} \tag{3.16}$$

The value of cutoff frequency for FHPF for any order ‘α’ can be obtained from equation (3.17):

$$\omega_{mFLPF} \cdot \omega_{mFHPF} = \omega_{hFLPF} \cdot \omega_{hFHPF} = (ab)^{\frac{1}{\alpha}} \tag{3.17}$$

3.3.3 Fractional order Band-Pass Filter (FBPF)

If we select $V_{in} = V_2$ and $V_1 = V_3 = 0$ then the general transfer function given in equation (3.13) gives the fractional-order band-pass filter (FBPF) transfer function described as:

$$\frac{V_0(s)}{V_{in}(s)} = \frac{-s^{\alpha}C_2g_{m2}}{s^{2\alpha}C_1C_2 + s^{\alpha}C_2g_{m1} + g_{m1}g_{m3}} \tag{3.18}$$

The value of maximum frequency for FBPF with order ‘α’ can be obtained from equation (3.19):

$$(X^2 - k)\left(X^2 + \cos\left(\frac{\alpha\pi}{2}\right)X + k\right) \tag{3.19}$$

where $X = \frac{\omega_m^{\alpha}}{a}$

The value of half-power frequency for FBPF for any order α can be obtained from equation (3.20):

$$Y^4 + 2\cos\left(\frac{\alpha\pi}{2}\right)Y^3 + \left(-1 + 2k\cos(\alpha\pi) - 8\cos^2\left(\frac{\alpha\pi}{2}\right) - 8\cos\left(\frac{\alpha\pi}{2}\right)k^{\frac{1}{2}}\right)Y^2 + 2k\cos\left(\frac{\alpha\pi}{2}\right)Y + k^2 = 0 \tag{3.20}$$

3.3.4 Fractional order Band-Reject Filter (FBRF)

When $V_{in} = V_1 = V_3$ and $V_2 = 0$, the general transfer function given in equation (3.13) represents a fractional-order band-reject filter (FBRF) transfer function described as:

$$\frac{V_0(s)}{V_{in}(s)} = \frac{s^{2\alpha}C_1C_2 + g_{m1}g_{m3}}{s^{2\alpha}C_1C_2 + s^{\alpha}C_2g_{m1} + g_{m1}g_{m3}} \tag{3.21}$$

The value of minimum frequency for FBRF for any order α can be obtained from equation (3.22):

$$\left(X^2 - k\right)\left(\begin{array}{l} X^4\cos\left(\frac{\alpha\pi}{2}\right) + X^3 + \left[4k\cos\left(\frac{\alpha\pi}{2}\right) - 2k\cos(\alpha\pi)\cdot\cos\left(\frac{\alpha\pi}{2}\right)\right]X^2 + kX \\ + k^2\cos\left(\frac{\alpha\pi}{2}\right) \end{array}\right) \tag{3.22}$$

The value of half-power frequency for FBRF for any order α can be obtained from equation (3.23):

$$Y^4 - 2\cos\left(\frac{\alpha\pi}{2}\right)Y^3 - (1 - 2k\cos(\alpha\pi))Y^2 - 2k\cos\left(\frac{\alpha\pi}{2}\right)Y + k^2 \tag{3.23}$$

3.3.5 Fractional order All-Pass Filter (FAPF)

If we select $V_{in} = V_1 = V_2 = V_3$, the general transfer function given in equation (3.13) represents a fractional-order all-pass filter (FAPF) transfer function described as:

$$\frac{V_0(s)}{V_{in}(s)} = \frac{s^{2\alpha}C_1C_2 - s^{\alpha}C_2g_{m2} + g_{m1}g_{m3}}{s^{2\alpha}C_1C_2 + s^{\alpha}C_2g_{m1} + g_{m1}g_{m3}} \tag{3.24}$$

The important frequency parameters of different fractional-order filters, namely, maximum or minimum frequency (ω_m) and half-power frequency (ω_h) for the special case when α = 1 have been shown in Table 3.1. For non-integer values of α, their values may be computed by solving the system of non-linear equations given in equation (3.15)-(3.23) for different filters by approximately mapping the values of g_{m1}, g_{m2} and g_{m3} into 'a', 'b' and k as defined therein.

Table 3.1 Parameters when α =1

Types of filters	ω_m	ω_h
FLPF	--	$\left(a\sqrt{\left(k \pm \frac{\sqrt{8k^2-4k+1}}{2} - \frac{1}{2}\right)}\right)$
FHPF	--	$\frac{ab}{\left(a\sqrt{\left(k \pm \frac{\sqrt{8k^2-4k+1}}{2} - \frac{1}{2}\right)}\right)}$
FBPF	$a\sqrt{k}$	$a\left(\frac{\sqrt{4k+1}}{2} \pm \frac{1}{2}\right)$
FBRF	$a\sqrt{k}$	$a\left(\frac{\sqrt{4k+1}}{2} \pm \frac{1}{2}\right)$

3.4 Stability Analysis

A detailed analysis of stability for the fractional-order system has been presented in [22]. For a fractional-order system, the stability graph is plotted in W-plane as shown in Fig. 3. In our work, the stability depends on the coefficients of $s^{i\alpha}$ ($0 < \alpha < 1$) as given in equation (3.25). By considering the coefficients of equation (3.25) positive, different cases for stability are shown in Table 3.2 and have been adopted from [2]. It may be also noted from Table 3.2, that fractional-order filters are stable if $\delta > \left|\frac{\alpha\pi}{2}\right|$ where $0 < \alpha < 1$. The stability plot of FLPF, FHPF, FBPF and FBRF is shown in Fig. 3.7 – 3.10 for α = 0.7 respectively, using MATLAB command forlocus.

$$D(s) = s^{2\alpha}C_1C_2 + s^{\alpha}C_2 g_{m1} + g_{m1}g_{m3} \qquad (3.25)$$

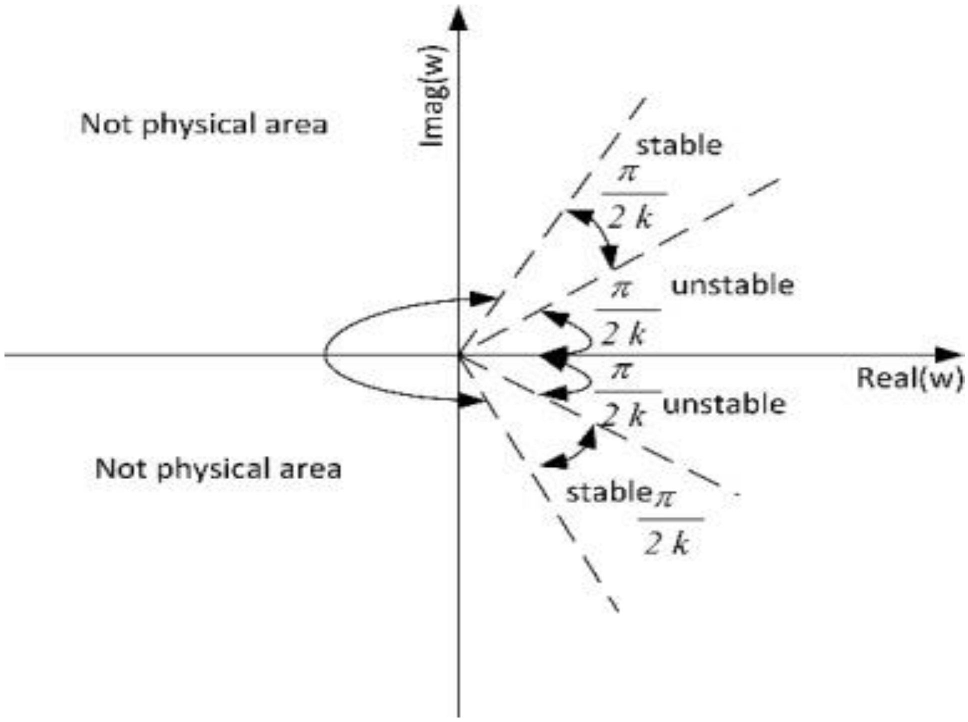

Figure 3.6 W-plane

Table 3.2 Relations, stability conditions, roots, pole frequency ω_0 and pole quality factor Q for different cases

Cases	Relations	Condition for stability and roots	ω_0, Q
1	$\frac{g_{m1}}{C_1} \geq 4\frac{g_{m3}}{C_2}$	$\alpha < 2$, $r_{1,2} = \frac{-\frac{g_{m1}}{C_1} \pm \sqrt{\left(\frac{g_{m1}}{C_1}\right)^2 - 4\left(\frac{g_{m1}g_{m3}}{C_1C_2}\right)}}{2}$ $= g_{1,2}e^{j\pi}$	$\omega_{01,2} = g_{1,2}^{1/\alpha}$, $Q = \frac{-1}{2\cos(\pi/\alpha)}$
2	$\frac{g_{m1}}{C_1} < 4\frac{g_{m3}}{C_2}$	$\alpha < \frac{2\delta}{\pi}$, $\delta = \cos^{-1}\left(\frac{-\frac{g_{m1}}{C_1}}{\sqrt{\frac{g_{m1}g_{m3}}{C_1C_2}}}\right) > \frac{\pi}{2}$, $r_{1,2} = \sqrt{\frac{g_{m1}g_{m3}}{C_1C_2}}e^{\pm j\delta}$	$\omega_0 = \left(\sqrt{\frac{g_{m1}g_{m3}}{C_1C_2}}\right)^{1/\alpha}$ $Q = \frac{-1}{2\cos(\delta/\alpha)}$

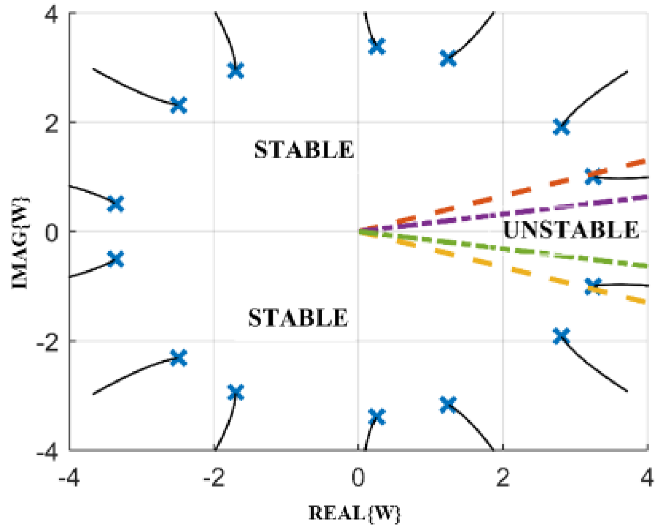

Figure 3.7 Stability plot of FLPF C=0.382µF and α=0.7

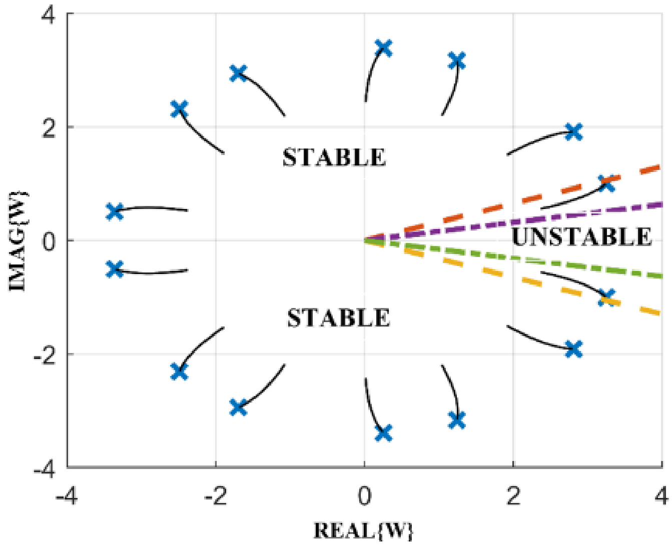

Figure 3.8 Stability plot of FHPF C=0.382µF and α=0.7

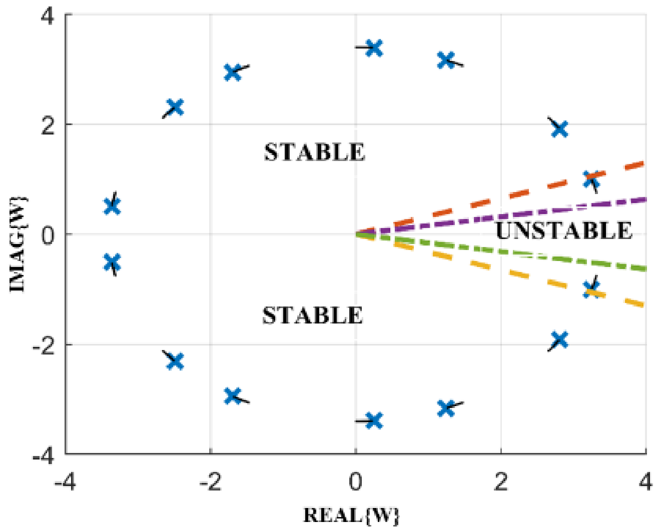

Figure 3.9 Stability plot of FBPF C=0.382µF and α=0.7

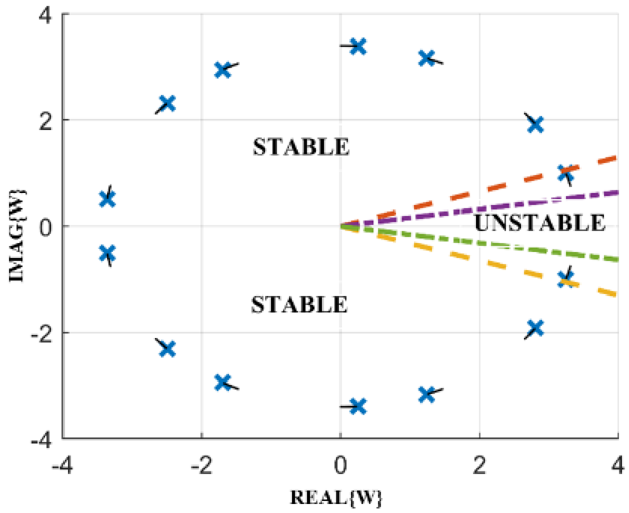

Figure 3.10 Stability plot of FBRF C=0.382µF and α=0.7

3.5 Simulation Results

We have verified the workability of the proposed fractional-order filters through PSPICE simulation using the macro model of OTA IC LM 13700. Equal valued fractional-order capacitors ($C_1 = C_2$ =0.382uF/(rad/sec)$^{(1-\alpha)}$) are used in PSPICE simulations. These fractional-order capacitors for different values of α (0.7, 0.8. 0.9) were designed using Valsa and Vlach approximation method [21] of order 6, resulting in Foster type network shown in Fig. 3.11.

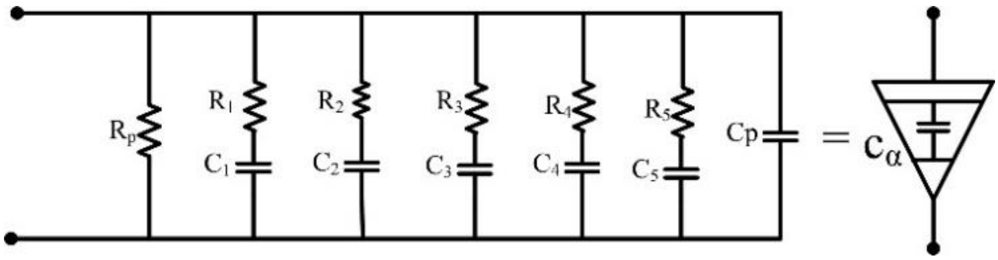

Figure 3.11 Fractional-order Capacitor

All the filters presented in this chapter were designed with values of $g_{m1} = g_{m2} = g_{m3}$ =5.48mA/V and R_{bias} =100kΩ.

The different frequency parameters for all the filters for different values of α = 0.7 to α = 1 have been listed in Table no. 3.3 and 3.4. From these tables, it may be noted that the error between the theoretical values and the values obtained from PSPICE simulations is very small in case of all the filters (3.9%).

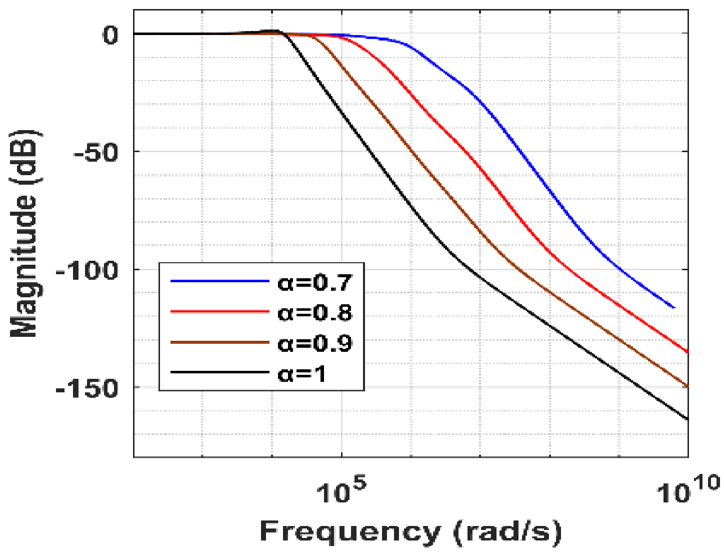

Figure 3.12 Magnitude response of FLPF for different α

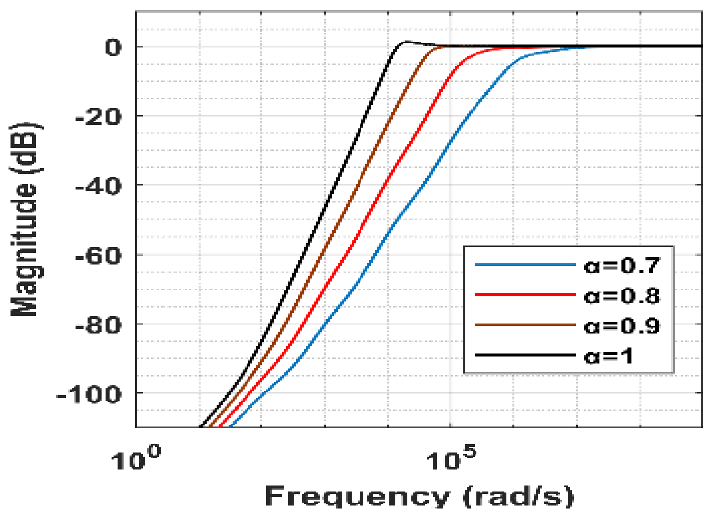

Figure 3.13 Magnitude response of FHPF for different α

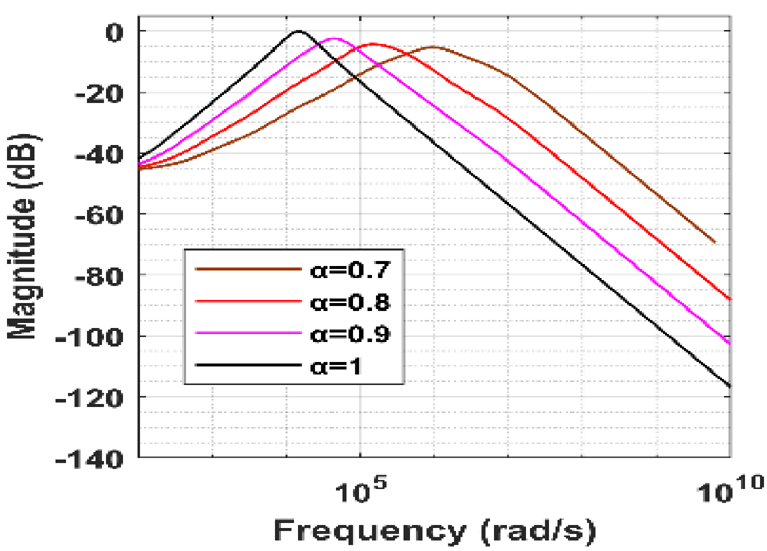

Figure 3.14 Magnitude response of FBPF for different α

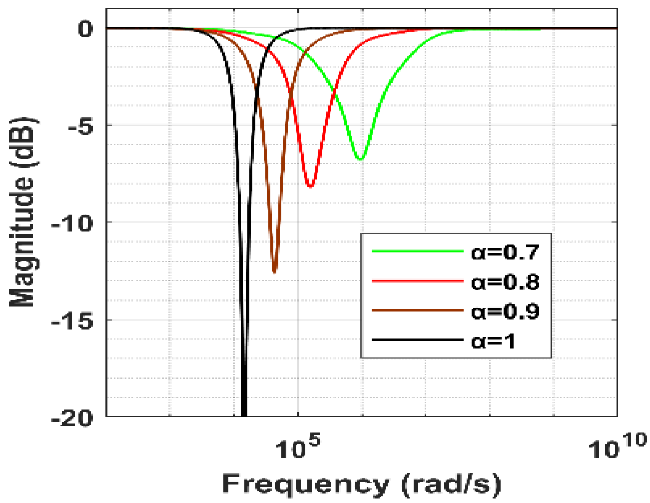

Figure 3.15 Magnitude response of FBRF for different α

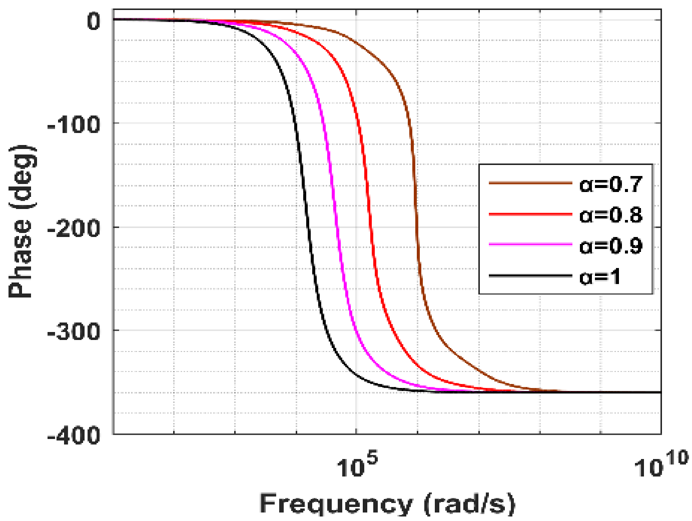

Figure 3.16 Phase response of FAPF for different α

Table 3.3 Cutoff Frequency (ω_h) (in krad/s)

α	FLPF		FHPF	
	PSPICE (ω_h)	Theoretical (ω_h)	PSPICE (ω_h)	Theoretical (ω_h)
0.7	486.63	487.81	1480.14	1500.46
0.8	129.67	130.13	202.35	202.87
0.9	46.12	46.87	38.63	38.33
1.0	14.9	14.34	14.1	14.34

Table 3.4 Maximum/Minimum Frequency (ω_m) (in krad/s)

α	FBPF		FBRF	
	PSPICE (ω_m)	Theoretical (ω_m)	PSPICE (ω_m)	Theoretical (ω_m)
0.7	864.71	866.81	864.56	866.81
0.8	154.87	156.12	156.37	156.12
0.9	41.43	42.87	42.43	42.87
1.0	14.07	14.34	14.17	14.34

3.6 Conclusion

In this chapter, two equal valued fractional-order capacitors based filter using OTAs with electronic tunability of important frequency parameters have been presented and validated using PSPICE simulation results. When fractional order filter circuits gain momentum, more broad and difficult situations can be taken up by choosing different capacitors of different orders. Besides that, the advantages are:

1) Considering the available filter design, they impose the constraint on filter responses which can be easily removed in fractional order filters.
2) Stability range and other important designing parameters are the functions of α, which provides an extra degree of freedom to us.

REFERENCES

[1] A.G. Radwan, A. M. Soliman, and A. S. Elwakil, “First-order filters generalized to the fractional domain,” Journal of Circuits, Systems, and Computers, 17, no. 01, pp. 55-66, 2008.

[2] A. G. Radwan, A. S. Elwakil, and A. M. Soliman, “On the generalization of second-order filters to the fractional-order domain,” Journal of Circuits, Systems, and Computers, 18, no. 02, pp. 361-386, 2009.

[3] A. S. Ali, A. G. Radwan, and A. M. Soliman, “Fractional-order Butterworth filter: active and passive realizations,” IEEE journal on emerging and selected topics in circuits and systems, 3, no. 3, pp. 346-354, 2013.

[4] B. Maundy, A. S. Elwakil, and T. J. Freeborn, “On the practical realization of higher-order filters with fractional stepping,” Signal Processing, 91, no. 3, pp. 484-491, 2011.

[5] F. Khateb, D. Kubánek, G. Tsirimokou, and C. Psychalinos, “Fractional-order filters based on low-voltage DDCCs,” Microelectronics Journal, 50, pp. 50-59, 2016.

[6] T. Suksang, V. Pirajnanchai, C. Suppitaksakul, and W. Loedhammacakra, “Design and improve the performance of OTA low pass filter with Fractional-order step,” In 2012 IEEE International Conference on Electron Devices and Solid State Circuit (EDSSC), 2012, pp. 1-4.

[7] G. Tsirimokou, C. Psychalinos, and A. S. Elwakil, “Fractional-order electronically controlled generalized filters,” International Journal of Circuit Theory and Applications, 45, no. 5, pp. 595-612, 2017.

[8] M. R. Dar, N. A. Kant, F. A. Khanday, and C. Psychalinos, “Fractional-order filter design for ultra-low frequency applications,” In 2016 IEEE International Conference on Recent Trends in Electronics, Information & Communication Technology (RTEICT), 2016, pp. 1727-1730.

[9] R. Verma, N. Pandey, and R. Pandey, “Electronically tunable fractional order all pass filter,” In IOP Conference Series: Materials Science and Engineering, vol. 225, no. 1, p. 012229, IOP Publishing, 2017.

[10] D. V. Kamath, S. Navya, and N. Soubhagyaseetha, “Fractional order OTA-C current-mode all-pass filter,” In 2018 Second International Conference on

Inventive Communication and Computational Technologies (ICICCT), 2018, pp. 383-387.

[11] M. V. Bhat, S. B. SS, and D. V. Kamath, "G m-C Current Mode Fractional All Pass Filter of order α (0< α< 1)," In 2019 3rd International conference on Electronics, Communication and Aerospace Technology (ICECA), 2019, pp. 240-245.

[12] P. Bertsias, C. Psychalinos, A. S. Elwakil, and B. J. Maundy, "Simple Multi-Function Fractional-Order Filter Designs," In 2019 8th International Conference on Modern Circuits and Systems Technologies (MOCAST), 2019, pp. 1-4.

[13] L. E. Sacu, and M. Alci, "Low-power OTA-C Based Tuneable Fractional Order Filters," Electronic Components and Materials, 48, no. 3, pp. 135-144, 2018.

[14] R. Verma, N. Pandey, and R. Pandey, "Electronically tunable fractional order filter," Arabian Journal for Science and Engineering, 42, no. 8, pp. 3409-3422, 2017.

[15] E. Kaskouta, T. Kamilaris, R. Sotner, J. Jerabek, and C. Psychalinos, "Single-input multiple-output and multiple-input single-output fractional-order filter designs," In 2018 41st International Conference on Telecommunications and Signal Processing (TSP), 2018, pp. 1-4.

[16] R. Arya, G. Souliotis, S. Vlassis, and C. Psychalinos, "A 0.5 V tunable complex filter for Bluetooth and Zigbee using OTAs," Analog Integrated Circuits and Signal Processing, 79, no. 1, pp. 73-81, 2014.

[17] Jerabek, R. Sotner, D. Kubánek, J. Dvorak, L. Langhammer, N. Herencsar, and K. Vrba, "Fractional-order low-pass filter with electronically adjustable parameters," In 2016 39th International Conference on Telecommunications and Signal Processing (TSP), 2016, pp. 569-574.

[18] J. Dvorak, J. Jerabek, Z. Polesakova, D. Kubanek, and P. Blazek, "Multifunctional Electronically Reconfigurable and Tunable Fractional-Order Filter," Elektronika ir Elektrotechnika, 25, no. 1, 2019, pp. 26-30.

[19] J. Dvorak, Z. Polesakova, J. Jerabek, L. Langhammer, A. Kartci, and J. Koton, "Non-Integer-Order Low-Pass Filter with Electronically Controllable Parameters," In 2018 IEEE International Symposium on Circuits and Systems (ISCAS), 2018, pp. 1-5.

[20] A. Oustaloup, F. Levron, B. Mathieu, and F. M. Nanot, “Frequency-band complex non-integer differentiator: characterization and synthesis,” IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 47, no. 1, pp. 25-39, 2000.

[21] J. Valsa, and J. Vlach, “RC models of a constant phase element,” International Journal of Circuit Theory and Applications, 41, no. 1, pp. 59-67, 2013.

[22] A. G. Radwan, A. M. Soliman, A. S. Elwakil, and A. Sedeek, “On the stability of linear systems with fractional-order elements,” Chaos, Solitons & Fractals, 40, no. 5, pp. 2317-2328, 2009.

Chapter 4

UNIVERSAL BIQUAD FRACTIONAL ORDER FILTER USING SINGLE LT1228 IC[2]

4.1 Introduction

In integer order analog filters available in open literature [1-3], the transition from pass band to stop band depends on integer multiples of a fixed slope (±20n decibels/decade). This usually results in the order of a filter being more than the minimum required to realize a given attenuation characteristics. In fractional order filters on the other hand, an attenuation of 20(n+α) where α varies from 0 to 1, provides more precise control over transition from pass band to stop band because of additional degree of freedom 'α' provided by the fractional order 'α' in the filter transfer function. With the availability of some fractional order immittances (both fractional order capacitors and fractional order inductors) as experimental prototypes, the possibility of these fractional order immittances becoming available as standard passive elements is ever increasing. As a result, research work on fractional order analog circuits and other related topics [4-17] has accelerated during the last decades.

Fractional order filters have been realized using different active elements like operational amplifier [4-9], current conveyors [10], current feedback followers [11], operational transconductance amplifiers [12] and various other active elements. Since the fractional order capacitor of any non-integer order is not available commercially [13-14], most of the paper presented on fractional order circuits use fractional order capacitor, designed using some integer order approximation of the term s^{α} resulting in a Foster like RC network approximating the fractional order capacitor.

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[2] The content and results of the following paper has been reported in this chapter: **G. Singh** and Garima, "Universal Biquad Fractional order Filter Using Single LT1228 IC," In 2020 International Conference for Innovation in Technology (INOCON), 2020. (Accepted) **Indexing:** SCOPUS and EI Compendex

Out of the various filters realized with different active elements, filters realized with commercially available IC LT1228 are more versatile as they can be easily implemented using current feedback amplifier (CFA) and operational transconductance amplifier (OTA). A detailed review of filters realized using LT1228 IC [15-17] reveals the following:

(i) Single input multiple output (SIMO) type second order filter utilizing two LT1228 IC and two OTAs have been presented [15].

(ii) Single input single output (SISO) type first order all pass filter utilizing single LT1228 IC structure [16].

(iii) Multi input single output (MISO) type universal second order filter using single LT1228 IC structure [17]

From the above discussion, it thus emerges that, very little work has been done on realization of voltage mode (VM) multi input single output (MISO) fractional order filter using single LT1228 IC. These MISO type of structures, are very useful to get different responses without disturbing the structure. In this chapter, thus, we have, generalized the design of an existing MISO type VM biquad filter [1] realized with LT1228 IC by replacing the integer order capacitors with fractional order capacitors and examined the tunability of the different parameters of the realized filters with the fractional order parameter 'α' and the bias current I_{bias}. Also, the stability of the realized filters has been studied in detail.

4.2 Circuit Description

4.2.1 Designing Fractional-order Capacitor using Oustaloup, Levron, Mathieu, and Nanot Approximation [13]

This approximation method mainly focuses on characteristics and synthesis of frequency band complex non-integer differentiator. Using this method, the fractional operator s^α can be synthesis in the frequency band of interest [ω_{min}, ω_{max}]. The approximated fractional operator s^α can be written as:

$$s^{\alpha} = C\prod_{k=1}^{k=N}\frac{1+ s/\omega_k'}{1+ s/\omega_k} \tag{4.1}$$

4.2.2 Steps for determining the fractional capacitor using Oustaloup, Levron, Mathieu, and Nanot Approximation

1) Starting with given values of α (between 0 to 1), ω_{min} and ω_{max} (desired minimum and maximum frequency) and N (no. of order).
2) Unity gain frequency (ω_u) can be obtained as:

$$\omega_u = \sqrt{\omega_{\min} \times \omega_{\max}} \tag{4.2}$$

3) Gain adjustment parameter 'C' can be calculated as:

$$C = \left(\frac{\omega_u}{\omega_{\min}} \right)^{\alpha} \tag{4.3}$$

4) By calculating all the parameters listed above, s^{α} can be approximated as:

$$s^{\alpha} = C \prod_{k=1}^{k=N} \frac{1 + s/\omega_k'}{1 + s/\omega_k} \tag{4.4}$$

where $\omega_k' = \omega_{\min} \left(\frac{\omega_{\max}}{\omega_{\min}} \right)^{(2k-1-\alpha)/(2N)}$ and $\omega_k = \omega_{\min} \left(\frac{\omega_{\max}}{\omega_{\min}} \right)^{(2k-1+\alpha)/(2N)}$

5) By taking the partial fraction of the equation (4.4) it can be generalized as:

$$z(s) = \frac{1}{s^{\alpha} C} = R_0 + \sum_{n=1}^{N} \frac{\frac{1}{C_n}}{s + \frac{1}{R_n C_n}} \tag{4.5}$$

and thereby applying network synthesis to convert it into foster I circuit as shown in fig 1.

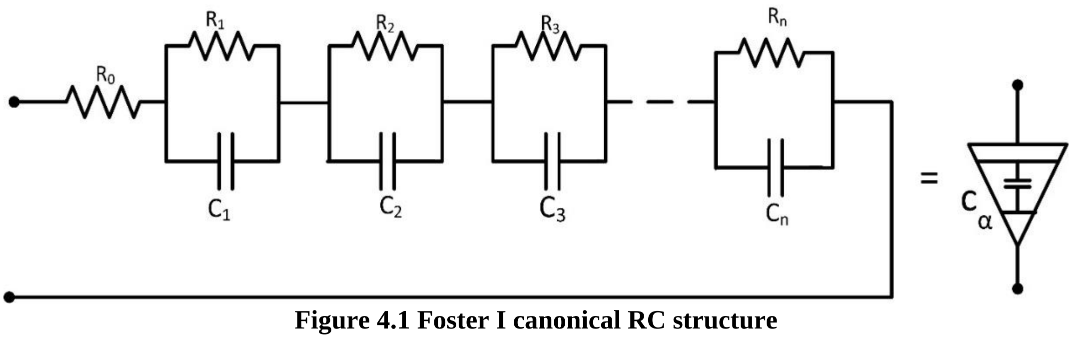


Figure 4.1 Foster I canonical RC structure

By considering f_{min}=0.1Hz, f_{max}= 100 MHz and N=8, the phase response of the resultant structure shown above is plotted in fig 4.2 for different values of α ranging between 0 to 1.

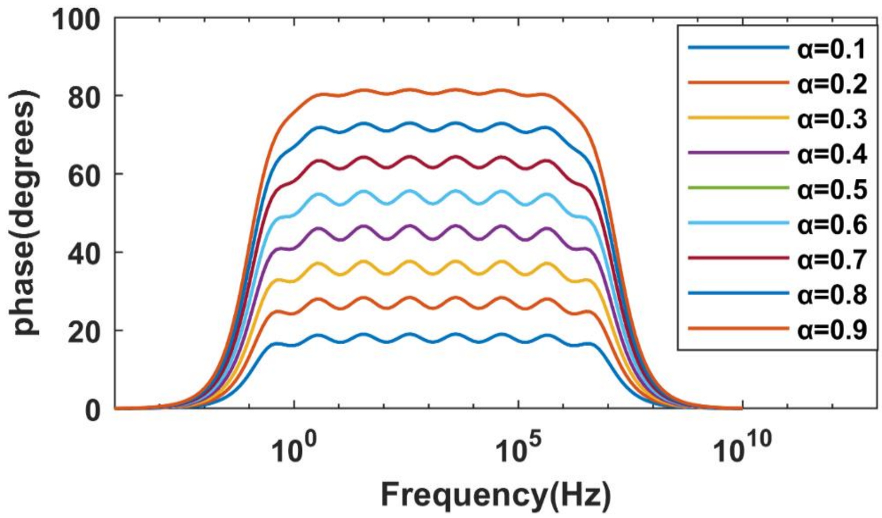
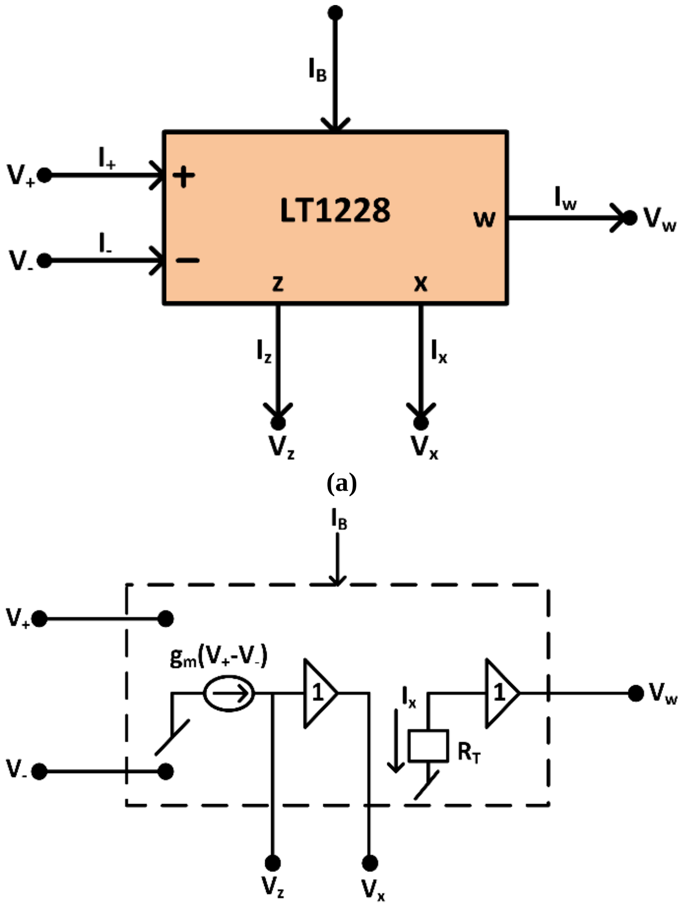


Figure 4.2 Phase response for different α

4.2.3 LT1228 IC

The LT1228 IC schematic symbol along with its equivalent circuit is shown in fig. 4.3.



(a)

(b)

Figure 4.3 a) Schematic symbol b) Equivalent circuit of LT1228 IC

Ideally, its characteristics equation can be described as (1):

$$\begin{bmatrix} I_+ \\ I_- \\ I_z \\ V_x \\ V_w \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ g_m & -g_m & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & R_T & 0 \end{bmatrix} \begin{bmatrix} V_+ \\ V_- \\ V_z \\ I_x \\ I_w \end{bmatrix} \tag{4.6}$$

where V_x, V_w, V_+, V_- and V_z are the output and input port voltages, I_{V+}, I_{V-}, I_Z, I_x and I_w are the output and input port currents, g_m is the transconductance gain whose relation with I_{bias} is given in equation (4.7) and R_T is the transresistance gain which is ideally very high. Also, impedances at terminals V_+, V_- and Z are high while impedances at terminals X and W are low.

LT1228 is a commercially available IC manufactured by Linear Technology Inc. [18]. It is a combination of CFA and OTA which has an external bias current to control its gain externally (as given in equation (4.7)) which is further used for tuning the filter responses.

$$g_m = 10 I_{bias} \tag{4.7}$$

where I_{bias} is the external bias current.

4.3 Modified Universal Biquad Fractional order Filter Configuration

The modified filter circuit, obtained by replacing the integer order capacitors of the filter circuit presented in [1] with fractional order capacitor (C_α) whose driving point impedance is given by $1/Cs^\alpha$, is shown in fig. 4.4. Like the integer order filter circuit presented in [1], which belongs to the class of multi input single output (MISO) type of configuration, the proposed circuit also belongs to the MISO class, wherein by selecting appropriate input signal, various filter responses along with electronic tunability can be obtained.

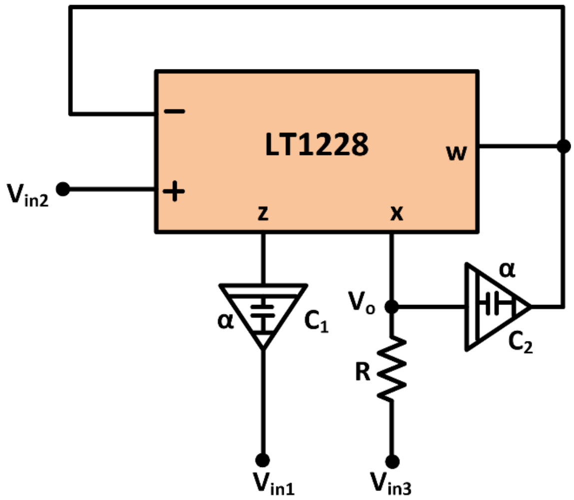


Figure 4.4 Modified filter circuit

Analyzing the fractional filter circuit as shown above the following output function is obtained:

$$V_0(s) = \frac{s^{2\alpha}V_{in1}(s) + s^{\alpha}(g_m / C_1)V_{in2}(s) + (g_m / C_1C_2R)V_{in3}(s)}{D(s)} \quad (4.8)$$

where $D(s) = s^{2\alpha} + (g_m / C_1) + (g_m / C_1C_2R)$ (4.9)

For any fractional order filter, with transfer function T(s), the various important parameters need to be determined [4].

4.3.1 Fractional Low Pass Filter

By selecting $V_{in} = V_{in3}$ and $V_{in1} = V_{in2} = 0$ we get the required FLPF transfer function:

$$\frac{V_0(s)}{V_{in}(s)} = \frac{(g_m / C_1C_2R)}{D(s)} \quad (4.10)$$

The DC gain of FLPF is 1 while the high frequency gain is 0. Its cutoff frequency, maximum frequency and right phase frequency for any order alpha can be obtained by equations (4.11), (4.12) and (4.13).

$$2X^3 + 3\cos\left(\frac{\alpha\pi}{2}\right)X^2 + 2[1 + k\cos(\alpha\pi)]X + k\cos\left(\frac{\alpha\pi}{2}\right) = 0 \quad (4.11)$$

$$Y^4 + 2\cos\left(\frac{\alpha\pi}{2}\right)Y^3 + (1 + 2k\cos(\alpha\pi)Y^2 + 2k\cos\left(\frac{\alpha\pi}{2}\right)Y - k^2 = 0 \quad (4.12)$$

$$\omega_{rp} = \left(\frac{-a\cos\left(\frac{\alpha\pi}{2}\right) - \sqrt{a^2\cos^2\left(\frac{\alpha\pi}{2}\right) - 4ab\cos(\alpha\pi)}}{2\cos(\alpha\pi)}\right)^{\frac{1}{\alpha}} \quad (4.13)$$

where $X=\frac{\omega_m^\alpha}{a}, Y=\frac{\omega_h^\alpha}{a}, k=\frac{b}{a}, a=\frac{g_m}{C_1}$ and $b=\frac{1}{C_2 R}$

4.3.2 Fractional High Pass Filter

If we select $V_{in}=V_{in1}$ and $V_{in2}=V_{in3}=0$ we get the required FHPF transfer function:

$$\frac{V_0(s)}{V_{in}(s)}=\frac{s^{2\alpha}}{D(s)} \tag{4.14}$$

Similarly, the DC gain of FHPF is 0 while the high frequency gain is 1. The important frequency parameters for any order alpha can be calculated using relation (4.15):

$$\begin{aligned}&\omega_{mFLPF}\cdot\omega_{mFHPF}=\omega_{hFLPF}\cdot\omega_{hFHPF}\\&=\omega_{rpFLPF}.\omega_{rpFHPF}=(ab)^{\frac{1}{\alpha}}\end{aligned} \tag{4.15}$$

4.3.3 Fractional Band Pass filter

By selecting $V_{in}=V_{in2}$ and $V_{in1}=V_{in3}=0$ we get the required FBPF transfer function:

$$\frac{V_0(s)}{V_{in}(s)}=\frac{s^{\alpha}(g_m/C_1)}{D(s)} \tag{4.16}$$

Both high frequency gain and DC gain is 0. For any order alpha, its 3dB frequency, maximum frequency and right phase frequency parameters can be calculated using equation (4.17), (4.18) and (4.19).

$$\left(X^2-k\right)\left(X^2+\cos\left(\frac{\alpha\pi}{2}\right)X+k\right) \tag{4.17}$$

$$\begin{aligned}&Y^4+2\cos\left(\frac{\alpha\pi}{2}\right)Y^3+\left(-1+2k\cos(\alpha\pi)-8\cos^2\left(\frac{\alpha\pi}{2}\right)-8\cos\left(\frac{\alpha\pi}{2}\right)k^{\frac{1}{2}}\right)Y^2\\&+2k\cos\left(\frac{\alpha\pi}{2}\right)Y+k^2=0\end{aligned} \tag{4.18}$$

$$\omega_{rp}=\left(\frac{-a\pm\sqrt{a^2-4ab\cos^2\left(\frac{\alpha\pi}{2}\right)}}{2\cos\left(\frac{\alpha\pi}{2}\right)}\right)^{\frac{1}{\alpha}} \tag{4.19}$$

4.3.4 Fractional Band Reject Filter

If we select $V_{in}=V_{in1}=V_{in3}$ and $V_{in2}=0$ we get the required FBRF transfer function:

$$\frac{V_0(s)}{V_{in}(s)} = \frac{s^{2\alpha} + (g_m / C_1C_2R)}{D(s)} \tag{4.20}$$

Both DC gain and high frequency gain is 1. For any order alpha, its minimum frequency and 3dB frequency parameters can be determined using equation (4.21) and (4.22).

$$\left(X^2 - k\right)\left(X^4 \cos\left(\frac{\alpha\pi}{2}\right) + X^3 + \begin{bmatrix} 4k\cos\left(\frac{\alpha\pi}{2}\right) \\ -2k\cos(\alpha\pi)\cdot\cos\left(\frac{\alpha\pi}{2}\right) \end{bmatrix} X^2 + kX + k^2\cos\left(\frac{\alpha\pi}{2}\right) \right) \tag{4.21}$$

$$Y^4 - 2\cos\left(\frac{\alpha\pi}{2}\right)Y^3 - \left(1 - 2k\cos(\alpha\pi)\right)Y^2 - 2k\cos\left(\frac{\alpha\pi}{2}\right)Y + k^2 = 0 \tag{4.22}$$

By putting alpha=1, all the important frequency parameter for different filters namely FLPF, FHPF, FBPF and FBRF have been given in table 1. For values of alpha other than 1, these important parameters of different filter can be obtained using equation (4.10)-(4.22).

Table 4.1 Parameters when α = 1

Types of filters	ω_m	ω_h	ω_{rp}
FLPF	$a\sqrt{(k-1)}$	$\left(a\sqrt{\left(k \pm \frac{\sqrt{8k^2 - 4k + 1}}{2} - \frac{1}{2}\right)}\right)$	$\sqrt{ab}$
FHPF	$\frac{b}{\sqrt{(k-1)}}$	$\frac{ab}{\left(a\sqrt{\left(k \pm \frac{\sqrt{8k^2 - 4k + 1}}{2} - \frac{1}{2}\right)}\right)}$	$\sqrt{ab}$
FBPF	$a\sqrt{k}$	$a\left(\frac{\sqrt{4k+1}}{2} \pm \frac{1}{2}\right)$	0,∞
FBRF	$a\sqrt{k}$	$a\left(\frac{\sqrt{4k+1}}{2} \pm \frac{1}{2}\right)$	$\sqrt{ab}$

4.4 Stability Analysis

Fractional order system stability has been studied in detail [19] and its stability graph is plotted in W-plane as shown in fig. 4.5. For any fractional order system, stability depends on the coeffficients of equation (4.9).

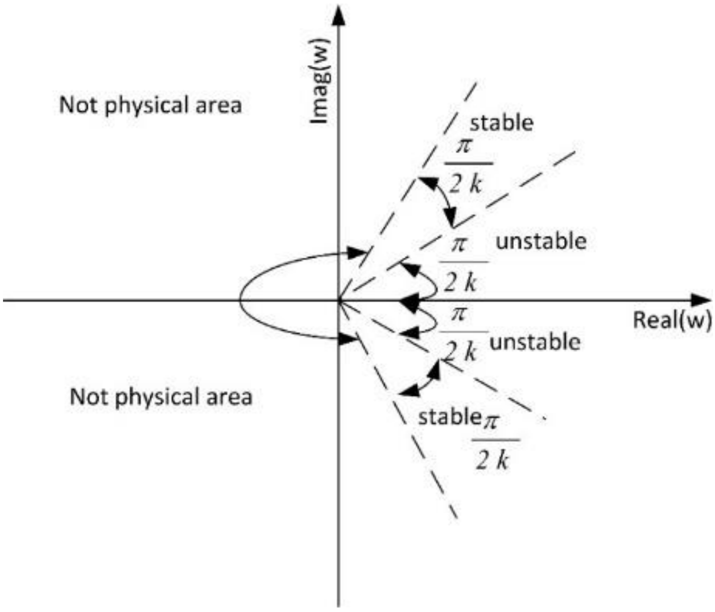


Figure 4.5 W-plane

In this chapter, considering all the coefficients of equation positive, two different cases arises that are given in table II and has been adopted from [4]. Stability graph of different filters namely FLPF, FHPF, FBPF, and FBRF are shown in fig 4-7 respectively.

Table 4.2 Pole frequency (ω_0) and pole quality factor (Q) for different cases

Cases	Relations	Condition for stability and roots	ω_0, Q
1	$a \geq 4b$	$\alpha < 2$, $r_{1,2} = \frac{-a \pm \sqrt{(a)^2 - 4(ab)}}{2} = g_{1,2} e^{j\pi}$	$\omega_{01,2} = g_{1,2}^{1/\alpha}$, $Q = \frac{-1}{2\cos(\pi/\alpha)}$
2	$a < 4b$	$\alpha < \frac{2\delta}{\pi}$, $\delta = \cos^{-1}\left(\frac{-a}{2\sqrt{ab}}\right) > \frac{\pi}{2}$, $r_{1,2} = \sqrt{ab} e^{\pm j\delta}$	$\omega_0 = \left(\sqrt{ab}\right)^{1/\alpha}$, $Q = \frac{-1}{2\cos(\delta/\alpha)}$

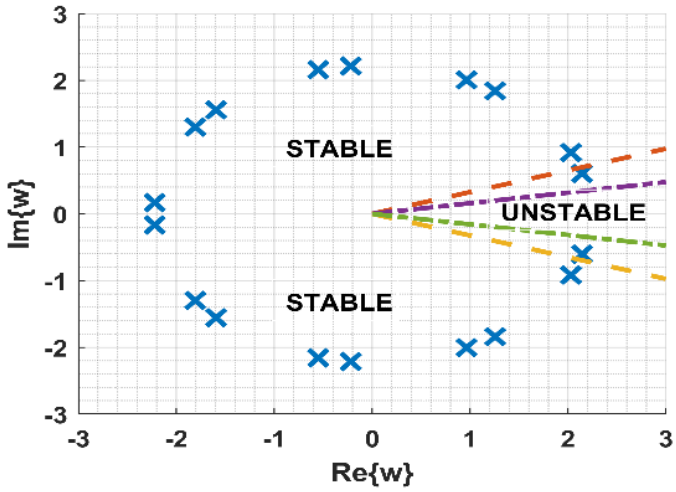


Figure 4.6 Stability plot of FLPF C=0.382µF and α=0.9

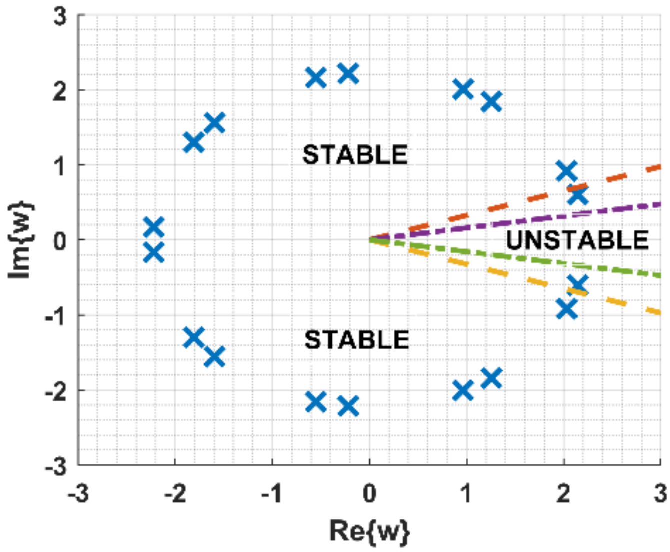


Figure 4.7 Stability plot of FHPF C=0.382µF and α=0.9

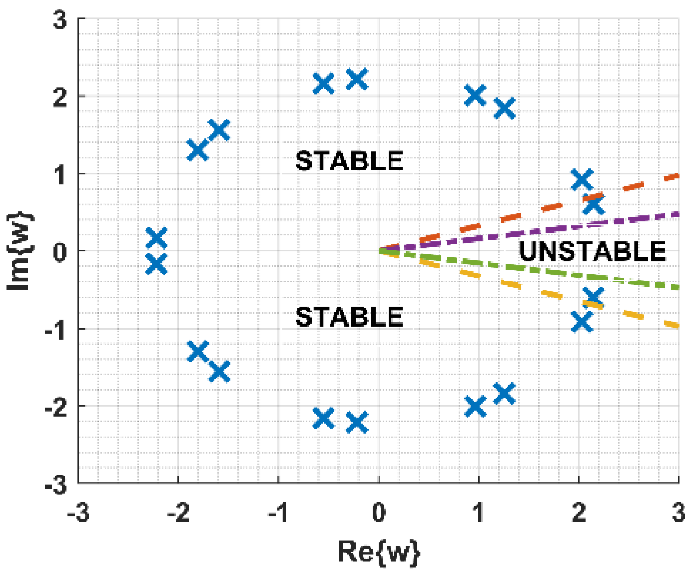


Figure 4.8 Stability plot of FBPF C=0.382µF and α=0.9

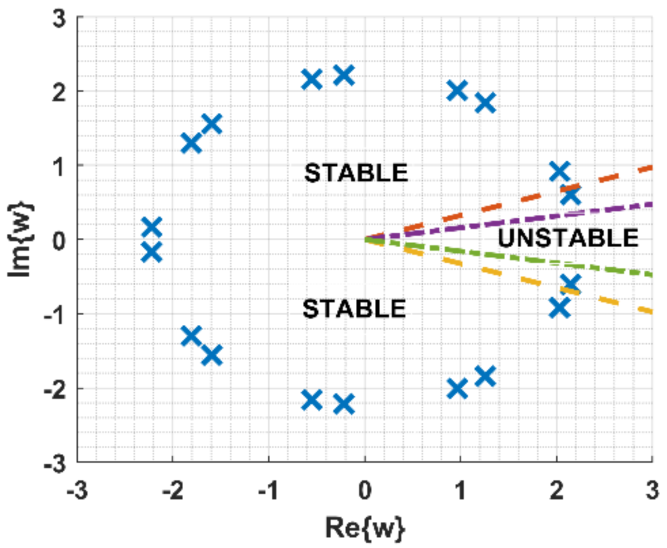


Figure 4.9 Stability plot of FBRF C=0.382µF and α=0.9

4.5 Simulation Results

The workability of the modified circuit have been validated through PSPICE simulation using the macro model of IC LT1228. The DC power supply voltage is taken as ± 12v and value of passive component R_1=1kΩ. The simulation results obtained matches with theoretical analysis and it is further divided into two categories:

4.5.1 Tunability with α:

It can be achieved by replacing each ordinary capacitors with fractional order capacitors ($C_1=C_2=0.382uF/(rad/sec)^{(1-\alpha)}$) designed using Oustaloup, Levron, Mathieu, and Nanot method for α=0.6, α=0.7, α=0.8, α=0.9 of order 8 resulting in foster type network as shown in fig 4.1.

All the filters designed in this section have values I_{bias}=0.226mA (R_{bias}=100kΩ) as shown in figure 4.10-4.13. The different frequency parameters for filters namely FLPF, FHPF, FBPF and FBRF for α=0.6 to α=1.0 have been listed in Table III. The difference between simulation values and theoretical values is very small and the error is as minimum as 4.4%.

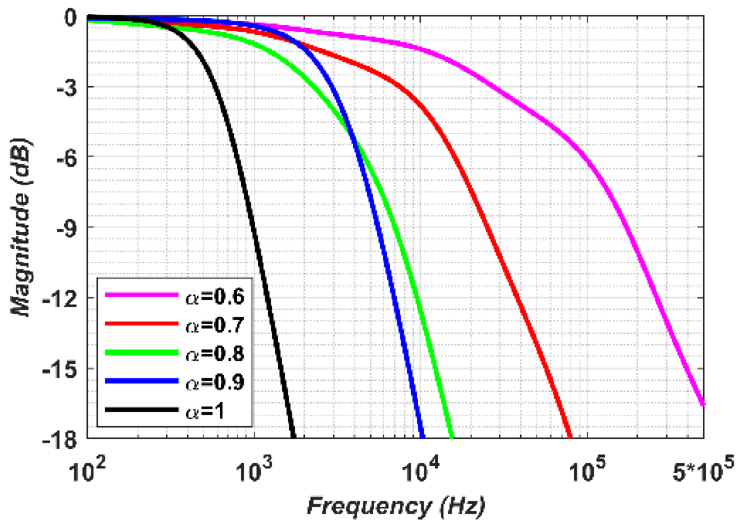


Figure 4.10 Magnitude response of FLPF for different α

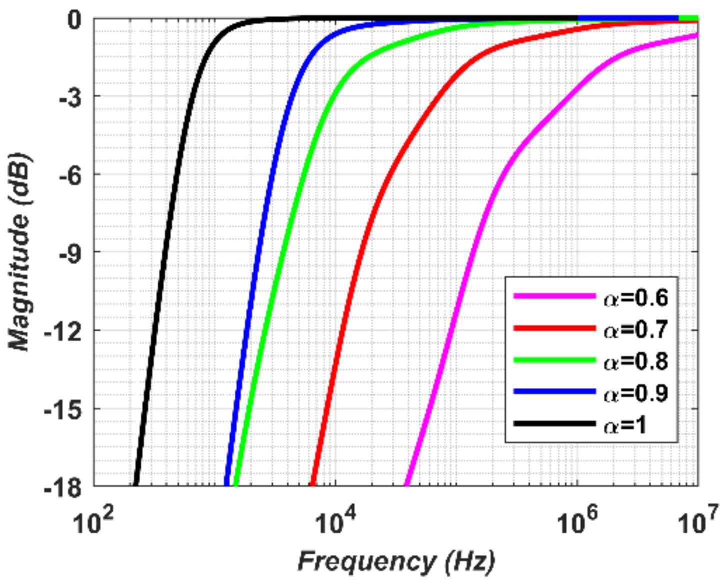


Figure 4.11 Magnitude response of FHPF for different α

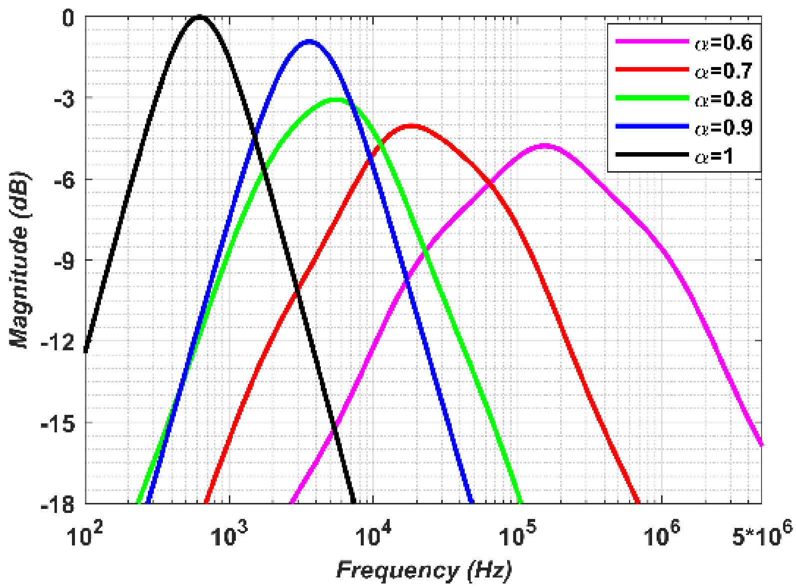


Figure 4.12 Magnitude response of FBPF for different α

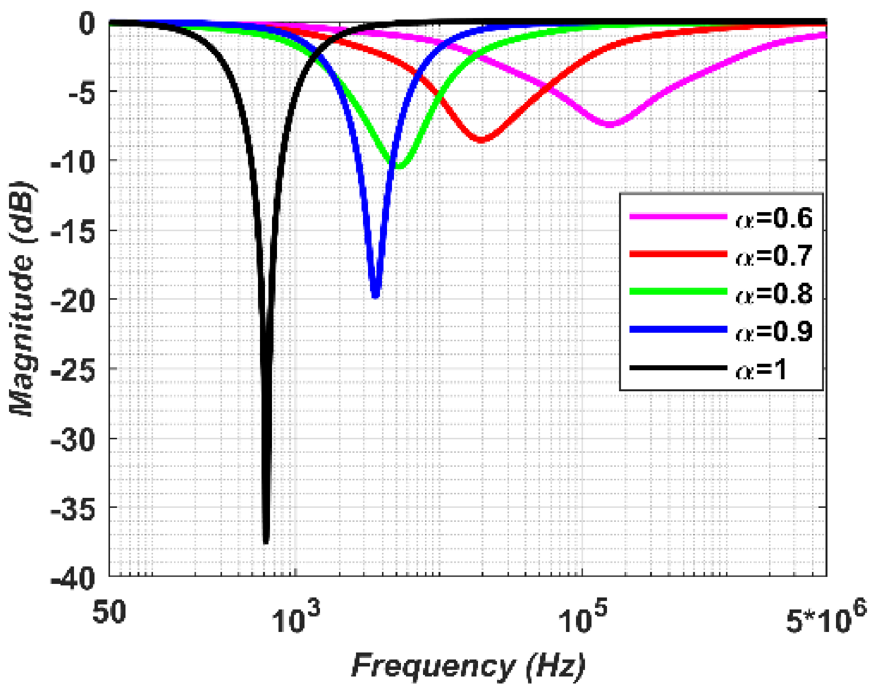


Figure 4.3 Magnitude response of FBRF for different α

Table 4.3 Cutoff Frequency (f_h) of FLPF and FHPF for different α

α	Cutoff Frequency (f_h) (in kHz)			
	FLPF		FHPF	
	PSPICE	Theoretical	PSPICE	Theoretical
0.6	28.857	30.244	841.954	807
0.7	7.285	7.01	70.58	68.030
0.8	2.380	2.45	9.635	10.124
0.9	1.15	1.108	2.13	2.23
1.0	0.585	0.587	0.659	0.668

Table 4.4 Maximum/Minimum frequency (f_h) of FBPF and FBRF for different α

α	Maximum/Minimum Frequency (f_m) (in kHz)			
	FBPF		FBRF	
	PSPICE	Theoretical	PSPICE	Theoretical
0.6	154.882	156.3	154.882	156.3
0.7	20.73	21.77	20.82	21.77
0.8	5.219	5.01	5.248	5.01
0.9	1.54	1.572	1.55	1.572
1.0	616.6	0.626	0.616	0.626

4.5.2 Tunability with I_{bias}

It can be achieved by changing the values of g_m and R by the same factor so that the ratio (g_m / R) remains constant which implies its cutoff frequency (ω_0) will remain same but quality factor (Q) changes while using standard capacitor as given in equation (4.23)-(4.24). All the filters designed in this section have values I_{bias1}=0.02825mA, I_{bias2}=0.0565mA, I_{bias3}=0.113mA, I_{bias4}=0.226mA and I_{bias5}=0.452mA respectively as shown in figure 13-16.

$$\omega_0 = \sqrt{\frac{g_m}{C_1 C_2 R}} \tag{4.23}$$

$$Q = \sqrt{\frac{C_1}{C_2 g_m R}} \tag{4.24}$$

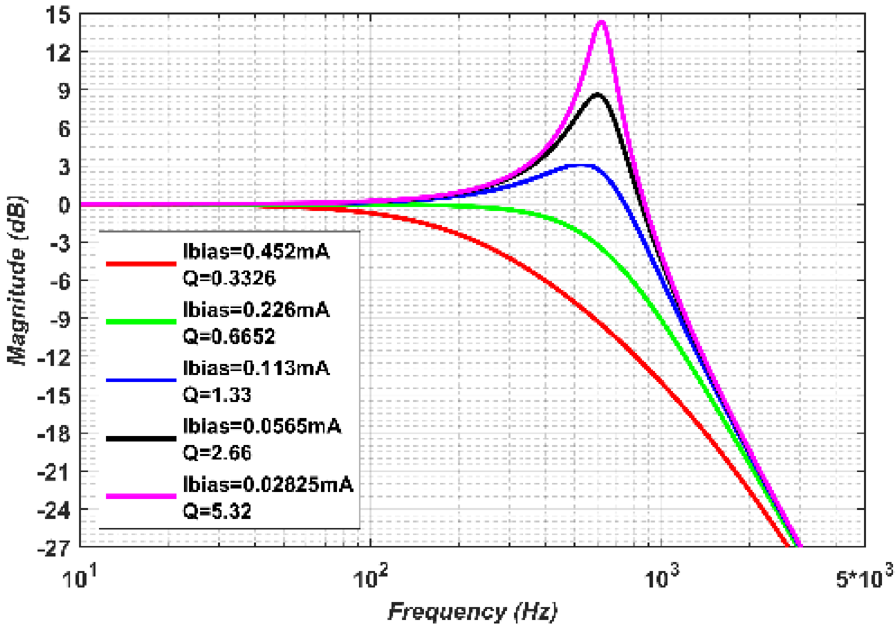


Figure 4.14 Magnitude response of FLPF for different I_{bias}

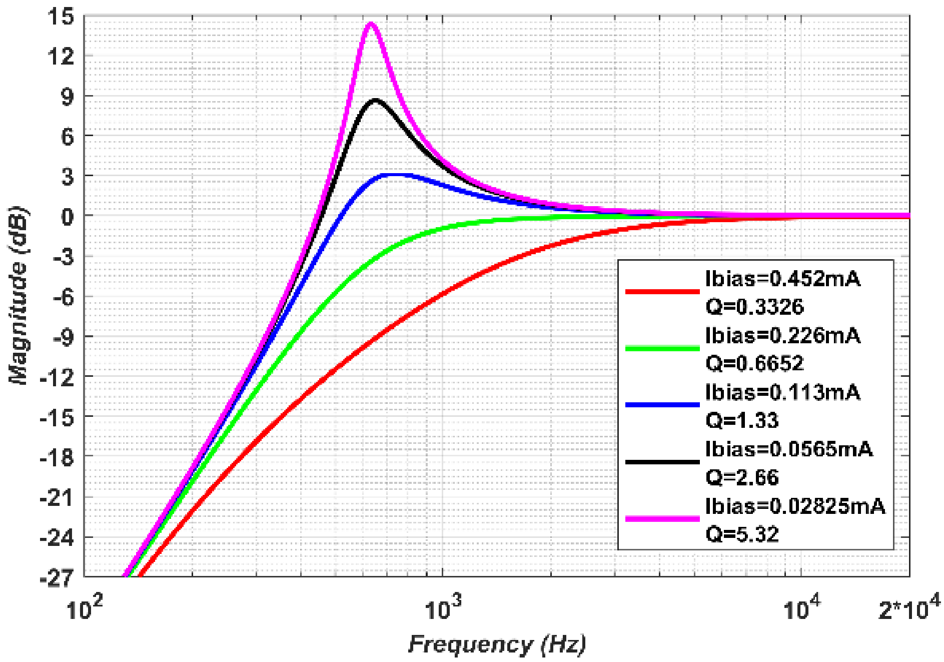


Figure 4.15 Magnitude response of FHPF for different I_{bias}

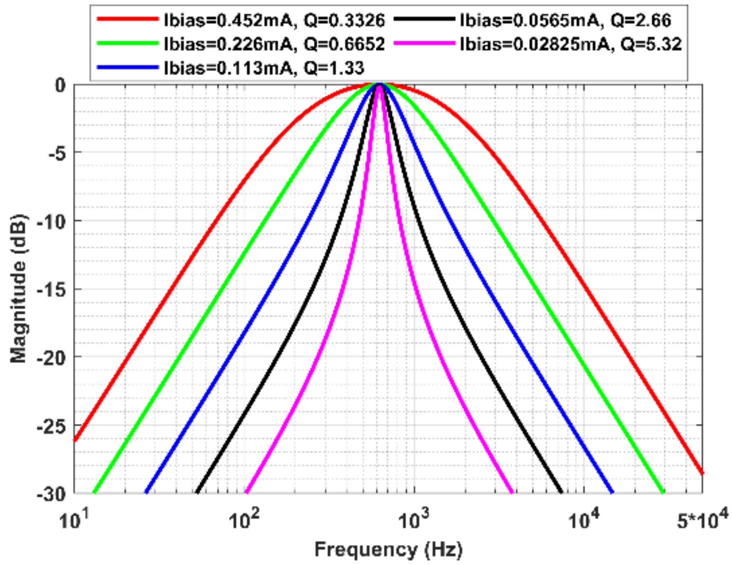


Figure 4.16 Magnitude response of FBPF for different I_{bias}

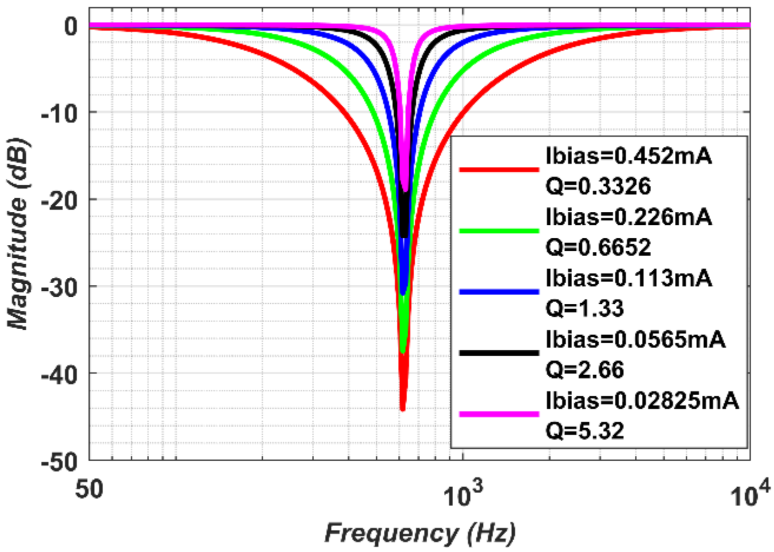


Figure 4.17 Magnitude response of FBRF for different I_{bias}

4.6 Conclusion

In our chapter, the main focus is to develop different techniques for electronic tuning of different types of filter. The first method suggests us how to tune with different values of alpha and second method suggests us how to tune with different values of Ibias. Some of the constraints or motivation for future work are:

1) When fractional order system gains momentum, two different valued fractional order capacitors can be used in order to obtain more precisely tuning of parameters.

2) While tuning with different values of I_{bias}, the quality factor is inversely proportional to product of g_m and R and cutoff frequency is directly proportional to (g_m / R) ratio for standard capacitor.

REFERENCES

[1] S. Klungtong, D. Thanapatay, and W. Jaikla, "Three-input single-output voltage-mode multifunction filter with electronic controllability based on single commercially available IC," Active and Passive Electronic Components, 2017.

[2] C. M. Chang, B. M. Al-Hashimi, and J. N. Ross, "Unified active filter biquad structures," IEE Proceedings Circuits, Devices and Systems, 151, no. 4, 2004, pp. 273-277.

[3] S. A. Mahmoud, M. A. Hashiesh, and A. M. Soliman, "Low-voltage digitally controlled fully differential current conveyor," IEEE Transactions on Circuits and Systems I, Regular Papers, 52, no. 10, pp. 2055-2064, 2005.

[4] A. G. Radwan, A. S. Elwakil, and A. M. Soliman. "On the generalization of second-order filters to the fractional-order domain." Journal of Circuits, Systems, and Computers, 18, no. 02, pp. 361-386, 2009.

[5] A. S. Ali, A. G. Radwan, and A. M. Soliman, "Fractional order Butterworth filter: active and passive realizations," IEEE journal on emerging and selected topics in circuits and systems, 3, no. 3, pp. 346-354, 2013.

[6] A. G. Radwan, A. S. Elwakil, and A. M. Soliman, "Fractional-order sinusoidal oscillators: design procedure and practical examples," IEEE Transactions on Circuits and Systems I: Regular Papers, 55, no. 7, pp. 2051-2063, 2008.

[7] L. A. Said, A. G. Radwan, A. H. Madian, and A. M. Soliman, "Fractional order oscillator design based on two-port network," Circuits, Systems, and Signal Processing, 35, no. 9, pp. 3086-3112, 2016.

[8] A. Soltan, A. G. Radwan, and A. M. Soliman, "Fractional order Sallen–Key and KHN filters: stability and poles allocation," Circuits, Systems, and Signal Processing, 34, no. 5, pp. 1461-1480, 2015.

[9] T. J. Freeborn, B. Maundy, and A. Elwakil. "Fractional-step Tow-Thomas biquad filters," Nonlinear Theory and Its Applications, IEICE, 3, no. 3, pp. 357-374, 2012.

[10] J. Koton, J. Jerabek, N. Herencsar, and D. Kubanek, "Current conveyors in current-mode circuits approximating fractional-order low-pass filter," In 2017 European Conference on Circuit Theory and Design (ECCTD), 2017, pp. 1-4.

[11] L. Langhammer, R. Sotner, J. Dvorak, J. Jerabek, and J. Polak. "Fully-differential tunable fractional-order filter with current followers and current amplifiers," In

2017 27th International Conference Radioelektronika (RADIOELEKTRONIKA), 2017, pp. 1-6.

[12] J. Srivastava, R. Bhagat, and P. Kumar. “Analog Inverse Filters Using OTAs,” In 2020 6th International Conference on Control, Automation and Robotics (ICCAR), 2020, pp. 627-631.

[13] A. Oustaloup, F. Levron, B. Mathieu, and F. M. Nanot, “Frequency-band complex noninteger differentiator: characterization and synthesis,” IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 47, no. 1, pp. 25-39, 2000.

[14] J. Valsa, and J. Vlach “RC models of a constant phase element,” International Journal of Circuit Theory and Applications, 41, no. 1, pp. 59-67, 2013.

[15] M. Olšák, and D. Biolek, “Controlled filters with LT1228 OTA amplifiers,” In Proceedings of 23rd the International Conference Telecommunications and Signal Processing (TSP), 2000, pp. 191-194.

[16] A. Chaichana, S. Siripongdee, and W. Jaikla, “Electronically Adjustable Voltage-mode First-order Allpass Filter Using Single Commercially Available IC,” In IOP Conference Series: Materials Science and Engineering, 559, no. 1, 2019.

[17] S. Siripongdee, and W. Jaikla, “Universal filter using single commercially available IC: LT1228,” In MATEC Web of Conferences, 95, 2017.

[18] http://www.linear.com/product/LT1228.

[19] A. G. Radwan, A. M. Soliman, A. S. Elwakil, and A. Sedeek, “On the stability of linear systems with fractional-order elements,” Chaos, Solitons & Fractals, 40, no. 5, pp. 2317-2328, 2009.

Chapter 5

SUMMARY AND FUTURE SCOPE

5.1 Summary

In chapter-1, there is a brief discussion on fractional calculus regarding its origin, history and application in various engineering and medical fields.

In chapter-2, introduction to fractional operator followed by various fractional calculus definition along with list of various approximation methods. have been discussed in brief. Finally, a detailed explanation with its simulation results of continued fraction expansion (CFE) method is carried out along with its advantages and disadvantages also.

In chapter-3, a novel universal biquad fractional order filter circuit using three OTAs and two grounded fractional order capacitors along with its stability and simulation results have been discussed. Here, the fractional order capacitor is designed using Valsa and Vlach approximation method.

In chapter-4, a generalized universal biquad fractional order filter circuit using single LT1228 IC and two fractional order capacitors along with its stability and simulation results have been discussed. Here, the fractional order capacitor is designed using Oustaloup, Levron, Mathieu, and Nanot approximation method.

5.2 Future Scope

There are several possibilities to extent the work presented in this dissertation in different directions. Some of these are:

I. The techniques that are used in this dissertation namely Valsa and Vlach, and Oustaloup, Levron, Mathieu, and Nanot for approximation of FoC. The work presented in this dissertation may be extended to carry out a relative comparison of performance of the circuits in which the FoCs used are realized with other methods suggested in the literature, to determine the best approximation for a particular filter.

II. The filters presented in the circuits may also be realized using a rational approximation of the operator s^{α} with integer order approximations suggested in literature and realizing the approximated rational transfer function of the filters using any active device.

PUBLICATIONS

PAPERS IN INTERNATIONAL CONFERENCES

[1] **G. Singh**, Garima, and P. Kumar, "Fractional Order Capacitors Based Filters Using Three OTAs," In 2020 6th International Conference on Control, Automation and Robotics (ICCAR), 2020, pp. 638-643. (SCOPUS) https://doi.org/10.1109/ICCAR49639.2020.9108100

[2] **G. Singh**, and Garima, "Universal Biquad Fractional order Filter Using Single LT1228 IC," In 2020 International Conference for Innovation in Technology (INOCON), 2020. (Accepted) (SCOPUS)

APPENDICES

APPENDIX 1

PSPICE model files used for LM13700 (OTA)

```
C1  6  4  4.8P
C2  3  6  4.8P
C3  5  6  6.26P
D1  2  4  DX
D2  2  3  DX
D3  11 21 DX
D4  21 22 DX
D5  1  26 DX
D6  26 27 DX
D7  5  29 DX
D8  28 5  DX
D10 31 25 DX
D11 28 25 DX
F1  4  3  POLY(1)   V6 1E-10 5.129E-2 -1.189E4 1.123E9
F2  11 5  V2        1.022
F3  25 6  V3        1.0
F4  5  6  V1        1.022
F5  5  0  POLY(2)   V3 V7 0 0 0 0 1
G1  0  33 5         0 .55E-3
I1  11 6  300U
Q1  24 32 31        QX1
Q2  23 3  31        QX2
Q3  11 7  30        QZ
Q4  11 30 8         QY
V1  22 24 0V
V2  22 23 0V
V3  27 6  0V
V4  11 29 1.4
V5  28 6  1.2
V6  4  32 0V
V7  33 0  0V
.MODEL QX1 NPN (IS=5E-16     BF=200 NE=1.15 ISE=.63E-16 IKF=1E-2)
.MODEL QX2 NPN (IS=5.125E-16 BF=200 NE=1.15 ISE=.63E-16 IKF=1E-2)
.MODEL QY  NPN (IS=6E-15     BF=50)
.MODEL QZ  NPN (IS=5E-16     BF=266)
.MODEL DX  D   (IS=5E-16)
.ENDS
```

PSPICE model files used for LT1228 using OTA and CFA

```
* THE OTA
Q11 5 5 21 QN 10
Q12 21 21 22 QN 10
VC 22 4 DC 0
F1 26 4 VC 0.375
F2 27 4 VC 0.25
F3 28 4 VC 0.375
F4 7 23 VC 1.6
F5 7 24 VC 1.6
VB 7 25 DC 1.4
CE1 23 7 11PF
CE2 24 7 11PF
RE13 23 32 120
RE14 24 33 120
Q13 29 25 32 QPI
Q14 1 25 33 QPI
Q15 23 3 28 QNI 9
Q16 23 3 27 QNI
Q17 23 3 26 QNI
Q18 24 2 26 QNI 9
Q19 24 2 27 QNI
Q20 24 2 28 QNI
VM 29 4 DC 1.4
FM 1 4 VM 1
DM 29 1 DC
C1 1 7 5PF
*
* THE CFA
Q2A 4 1 10 QP 0.5
Q3A 11 10 200 QN
Q4A 11 11 7 QP
Q5A 9 11 7 QP
Q6A 12 11 7 QP
Q7A 4 9 12 QP
Q8A 7 12 13 QN 10
RSCA 13 6 10
IBA 7 10 DC 300U
*
Q2B 7 1 110 QN 0.5
Q3B 111 110 200 QP
Q4B 111 111 4 QN
*
Q5B 9 111 4 QN
Q6B 112 111 4 QN
Q7B 7 9 112 QN
Q8B 4 112 113 QP 10
RSCB 6 113  10
```

```
IBB 110 4 DC 300U
*
RC 8 200 20
R9 9 0 201600
D1 9 6 DC
D2 6 9 DC
*
.MODEL DC D
.MODEL QNI NPN
.MODEL QN NPN (IS=168E-18 BF=150 ISC=40E-18 NC=1 RB=250 RE=8
RC=100
+CJE=0.37P VJE=0.65 MJE=0.33 FC=0.7 CJC=0.8P VJC=0.62 MJC=0.44
+TF=300P
.MODEL QPI PNP
.MODEL QP PNP (IS=230E-18 BF=150 ISC=113E-18 NC=1 RB=250 RE=8
RC=100
+CJE=0.34P VJE=0.75 MJE=0.40 FC=0.7 CJC=0.8P VJC=0.5 MJC=0.36
+TF=300P
*
.ENDS LT1228
```