

**Note :** Answer any FIVE questions.  
Assume suitable missing data, if any.

1. [a] State and prove Stoke's theorem in vector analysis (7)  
 [b] Find the work done by the force  $\vec{f} = (2y + 3)\hat{i} + xz\hat{j} + (yz - x)\hat{k}$ , when it moves a particle from the point (0,0,0) to the point (2,1,1) along the curve  $x = 2t^2$ ,  $y = t$  and  $z = t^3$ . (3)
2. Define piezoelectric effect and converse piezoelectric effect. Discuss the Application of tensor analysis to the piezoelectricity and converse piezoelectricity. (10)
3. [a] State and prove the Cauchy-Riemann equations for a function of a complex variable to be analytic. (6)  
 [b] Find the residues of  $f(z) = \frac{ze^z}{(z-a)^3}$  at  $z=a$ . (4)
4. A thin rectangular plate whose surface is impervious to heat flow has arbitrary distribution of temperature  $f(x,y)$  at  $t=0$ . Its four edges  $x=0$ ,  $x=a$ ,  $y=0$  and  $y=b$  are kept at zero temperature. Determine the subsequent temperature of the plate after time 't'. (10)
5. [a] Apply Runge-Kutta method to the equation  $y' = x + y$ ,  $y(0) = 1$  to determine  $x=0.1$  and  $0.2$  correct to four decimal places. (6)  
 [b] Calculate the approximate value of  $\sin x$  for  $x=0.54$  using the following table:

x	0.5	0.7	0.9	1.1	1.3	1.5
$\sin x$	0.47943	0.64422	0.78333	0.89121	0.96356	0.99749

6. Answer any *four* of the following: (4×2.5 = 10)
- [a] Define Kronecker delta and prove that (a)  $\delta_k^j a^j = a^k$ .  
 [b] Find  $\text{div} \vec{f}$ , where  $\vec{f} = \text{grad} (x^3 + y^3 + z^3 - 3xyz)$ .  
 [c] Define Pole and residue of pole  
 [d] Prove that (i)  $E^{-1} = 1 - \nabla$  (ii)  $(1 - \Delta)(1 - \nabla) = 1$   
 [e] Separate  $\text{Log} (1+i)$  in to real and imaginary parts.