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SEVENTH SEMESTER | B.TECH (MC)

SUPPLEMENTARY EXAMINATION | FEBRUARY 2019

MC 405 GRAPH THEORY

Time: 3 Hours | Max.Marks: 40

Note: Answer **ALL** by selecting any two parts from each. All questions carry equal marks.

Q1(a) Let $G = (p, q)$ graph having p vertices and q edges all of whose vertices have degree k or $k + 1$. If G has $p_k > 0$ vertices of degree k and p_{k+1} vertices of degree $k + 1$ then show that

$$p_k = (k + 1)p - 2q.$$

(b) Define the Ring sum of two graphs and complement of a graph. Show that a graph is self complementary if it has 4n or 4n+1 vertices.

(c) Prove that isomorphism of simple graphs is an equivalence relation.

Q2(a) Prove that a simple graph with n vertices and k components can have at most (n-k) (n-k+1)/2 edges.

(b) Prove that in a complete graph with n vertices, there are (n-1)/2 edge disjoint Hamiltonion circuits, if n is an odd number greater than or equal to 3.

(c) Prove that a connected graph G is an Euler graph iff it can be decomposed into circuits.

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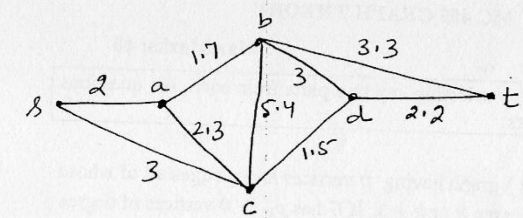
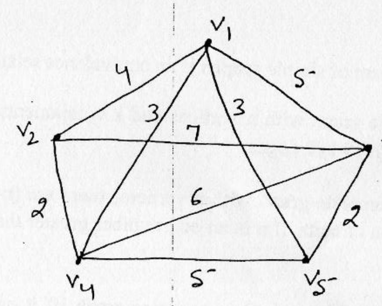
Q3(a) Define binary tree. Prove that the maximum number of vertices in a binary tree of height h is $(2^h - 1)$, $h \geq 0$.

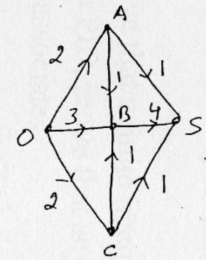
(b) Apply Dijkstra algorithm to find shortest path from s to t in the graph given below:



(c) Find the minimal spanning tree for the weighted graph given below:



Q4(a). Find a maximal flow in the network shown below:



(b) Prove that a vertex v of a connected graph G is a cut vertex iff there exist two vertices x and y in G such that every path between them passes through v.

(c) Show that every cycle in a graph has an even number of edges in common with any cut-set.

Q5 (a) Define a k-chromatic graph. Prove that every tree with two or more vertices is 2- chromatic. Find an example of a 2- chromatic graph which is not a tree.

(b) Define edge connectivity of a graph. Show that the edge connectivity of a graph G cannot exceed the minimum degree of a vertex in G.

(c) Define Perfect Matching in a graph. Find the number of perfect matching in the complete bipartite graph $K_{n,n}$.

END