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5th SEMESTER | **B. Tech.**
SUPPLEMENTARY EXAMINATION | **(FEBRUARY-2019)**

MC 315: Modern Algebra

Time: 3:00 Hours | **Max. Marks: 50**

Note: All questions are compulsory. Attempt **any two parts** from each question. Assume suitable missing data, if any.

Q1.a) Define Quaternion group and show that it is a non-abelian group of order 8.

b) Prove that the set of all n n^{th} roots of unity forms a cyclic group w.r.t. multiplication.

c) Find all the left cosets of <H,+> in <G,+>, where G = **Z** and $H = \{5x : x \in Z\}$.

Q2.a) If G is a group and H is a subgroup of index 2 in G, prove that H is normal subgroup of G.

b) If G is a cyclic group and N is a normal subgroup of G, then show that G/N is cyclic. Also show by an example that the converse need not be true.

c) State and prove Fundamental Theorem of group homomorphism.

Q3.a) Show that the set of Gaussian integers $Z[i]$ is a commutative ring with unity.

b) Show by an example that union of two subrings of ring may not be a subring.

c) If F is a field of characteristic p (p is a prime), then show that $(a+b)^p = a^p + b^p \ \forall a, b \in F$.

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Q4.a) Let $f: R \rightarrow R'$ be a homomorphism and A be an ideal of R. Show that $f(A)$ is an ideal of $f(R)$.

b) Find all maximal ideals of Z_{12}, the ring of integers modulo 12.

c) Show that every field F is a Euclidean domain.

Q5.a) Prove that a finite integral domain is a field.

b) In a PID, prove that any two greatest common divisors of *a* and *b* are associates.

c) Prove that $Z\left[\sqrt{-3}\right]$ is not a U.F.D.

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