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III^{rd} SEMESTER **B. Tech. (MC)**

SUPPLEMENTARY EXAMINATION February 2019

$MC-207$ ***Differential Equations and Applications***

Time: 3 Hours Max. Marks: 40

*Note: Attempt **all** the questions by selecting **any two parts** from each question.*

(1) (a) Find the general solution of the homogeneous linear system

$$\mathbf{x}' = \begin{pmatrix} 7 & 4 & 4 \\ -6 & -4 & -7 \\ -2 & -1 & 2 \end{pmatrix}\mathbf{x}.$$

(b) Solve the initial-value problem

$$\mathbf{x}' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}\mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

(c) Find all solutions the equation

$$\mathbf{x}' = \begin{pmatrix} 7 & -1 & 6 \\ -10 & 4 & -12 \\ -2 & 1 & -1 \end{pmatrix}\mathbf{x}.$$

(08)

(2) (a) Use separation of variables and find product solution of partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial u}{\partial y}.$$

(b) Define the *Regular Sturm-Liouville problem*, and show that the following Bounded Value Problems(BVPs) is a Sturm-Liouville problem

(i) $x^2y'' + xy' + \lambda y = 0;\ \ y'(1) = 0, y'\left(e^{2\pi}\right) = 0, \lambda > 0$

(ii) $y'' + \lambda y = 0;\ \ y(0) = 0, y(\pi) = 0, \lambda > 0$

(c) show that the set of eigenvalues corresponding to the set of eigenvalues is orthogonal with respect to the weight function $p(x)$ on the interval $[a, b]$. (10)

(3) (a) Eliminate the arbitrary function from $z = f\left(x^2 + y^2\right)$ to obtain a first order partial differential equation.

(b) Find the general solution of the partial differential equation

$$(y + z)\,p + (z + x)\,q = x + y.$$

(c) Find the general solution of the partial differential equation

$$2xz - px^2 - 2qxy + pq = 0,$$

by using *Charpit's equation.* (08)

(4) (a) Solve $\left(D^3 - 4D^2D' + 4DD'^2\right)z = 4\sin(2x + y).$

(b) Solve

$$\left(D^2D' - 2DD'^2 + D'^3\right)z = \frac{1}{x^2}.$$

(c) Find the general solution of equation

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2} = \left(x^2 + y^2\right)^{n/2}$$

(08)

(5) (a) Solve the *Neumann problem*

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

for a rectangular plate subject to the boundary conditions:

$$u_x(0, y) = 0 = u_x(a, y),\ u(x, 0) = x,\ \textit{and}\ u(x, b) = 0.$$

(b) A bar AB of length 10 cm has its ends A and B kept at 30^0 and 100^0 temperatures respectively, until steady-state condition is reached. Then the temperature at A is lowered to 20^0 and that at B to 40^0 and these temperatures are maintained. Find the subsequent temperature distribution in the bar.

P.T.O