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Supplementary Examination
III Semester, February 2019
Discrete Mathematics
(MC-201) (NEW SCHEME)

Max Marks: 50 Time: 3.00 Hours

Note:

- **Attempt all Questions and do any two parts out of the three in each Question.**
- **All Questions carry equal marks.**
- **Assume suitable missing data if any.**

Q 1. (a) Let $A_1, A_2, \cdots, A_n$ be any n sets. Show by mathematical induction that

$$\overline{\left(\bigcup_{i=1}^{n} A_i\right)} = \bigcap_{i=1}^{n} \overline{A_i}$$

(b) Write short note on

i. Bipartite Graph

ii. Euler Graph

(c) i. Give a direct proof of "If 'm' and 'n' are odd integers, then mn is an odd integer".

ii. Prove or disprove that the union of two subgroups of a group G is a subgroup if and only if one is contained in the other.

Q 2. (a) Rewrite the following argument using quantifiers. Prove the validity also:

i. If a number is odd then its square is odd. K is a particular number that is odd. Therefore K^2 is odd.

ii. All healthy people eat an apple a day. You do not eat apple a day. You are not a healthy person.

(b) Show that (by using rule of Inference) the hypothesis "If you send me e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program then I will wake up feeling refreshed".

(c) State and prove the Lagrange's theorem. Also discuss the theorem with a suitable example.

Q 3. (a) For any Lattice L, prove the following:

$$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$

P.T.O.

(b) If in a group G, $x^5 = e$, $xyx^{-1} = y^2$ for $x, y \in G$ then show that $O(y) = 31$.

(c) If R is an equivalence relation on a set X and $|X| = |R|$. What must the relation look like? Explain.

Q 4. (a) Let n be a positive integer, and $p^2/n; p$ is a prime number, then D_n where $D_n = \{x : x|n \ \forall \ x \in N\}$ will not be a boolean algebra.

(b) Let

$$p(x, y, z) = (x \wedge y) \vee (x \vee (x \vee (y' \wedge z)))$$

be a Boolean polynomial. Obtain the truth table for the Boolean function $f : B_3 \to B$ as determined by this Boolean polynomial.

(c) i. Prove that intersection of two sublattices is a sublattice. What is about union of two sublattices?

ii. Let $X = \{a, b, c\}$. Define $f : X \to X$ such that $f = \{(a, b), (b, a), (c, c)\}$. Determine (i) f^{-1} (ii) f^2 (iii) f^3 (iv) f^4

Q 5. (a) $D_n = \{x : x|n \ \forall \ x \in N\}$
Consider $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$.

i. Find all the lower bounds of 10 and 15.
ii. Determine GLB of 10 and 15.
iii. Find all the upper bounds of 10 and 15.
iv. Determine LUB of 10 and 15.
v. Find greatest element of D_{30}.
vi. Find least element of D_{30}.

(b) Prove that "An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices".

(c) Solve the recurrence relation

$$a_n = 5a_{n-1} - 6a_{n-2} + 7^n$$