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Ist Semester END SEMESTER EXAMINATION B.Tech.

PAPER CODE - MA 101 Supplementary-~~Odd Nov./Dec. 2018~~ Feb. 2019

TITLE OF PAPER - Mathematics-1

TIME: 03 HRS MAX. MARKS: 50

Note: Attempt any FIVE questions. Each question carry equal marks.

Assume suitable missing data, if any.

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1. (a) Discuss the convergence and divergence of the following (5)

$$\text{(i)} \sum_{n=1}^{\infty} \frac{5^n}{4^n+3} \qquad \text{(ii)} \sum_{n=1}^{\infty} n e^{-n^2}.$$

(b) Find the radius and interval of convergence for $\sum_{n=0}^{\infty} \frac{(x-\sqrt{2})^{2n+1}}{2^n}$. (5)

For what values of x does the series converges (a) absolutely, (b) conditionally?

2. (a) Find the radius of curvature at the point $(3a/2, 3a/2)$ of the Folium $x^3+y^3=3axy$. (5)

(b) Find the length of the curve $y=(x/2)^{2/3}$ from $x=0$ to $x=2$. (5)

3. (a) If $w=f(x,y)$ where $x=r\cos\theta$ and $y=r\sin\theta$ show that (5)

$$\left(\frac{\partial w}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2=\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2.$$

(b) Find the point $P(x,y,z)$ closest to the origin on the plane (5)
$2x+y-z-5=0$.

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4. (a) Evaluate $\int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2} dy\, dx$. (5)

(b) Find the average value of $F(x, y, z) = xyz$ over the cube bounded by the coordinate planes and the planes $x = 2$, $y = 2$, and $z = 2$ in the first octant. (5)

5. (a) Suppose that the height of a hill above sea level is given by $z = 1000 - 0.01x^2 - 0.02y^2$. If you are at the point $(60, 100)$ in what direction is the elevation changing fastest? What is the maximum rate of change of the elevation at this point? (5)

(b) Define gradient, divergence and curl. Hence, discuss their physical significance. (5)

6. (a) Evaluate the line integral $\int_c (x-y)\, dx + (x+y)\, dy$ counterclockwise around the triangle with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$. (5)

(b) Verify Green's theorem for $\int_c (xy + y^2)\, dx + x^2\, dy$ where C is the region bounded by $y = x$ and $y = x^2$. (5)

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