Project Dissertation

"A Study on Estimating Accuracy of Value at Risk Models in Evaluating Risk in Equity Investments"

Submitted by:

Amit Kumar

2K14/MBA/06

Under the Guidance of

Prof. G.C. Maheshwari

DELHI SCHOOL OF MANAGEMENT

Delhi Technological University

Bawana Road, Delhi – 110042

Jan – May 2016

CERTIFICATE FROM INSTITUTE

This is to certify that the Project Dissertation titled **"A Study on Estimating Accuracy of Value at Risk Models in Evaluating Risk in Equity Investments"**, is a bonafide work carried out by **Mr. Amit Kumar** of MBA 2014-16 and submitted to Delhi School of Management, Delhi Technological University, Bawana Road, Delhi-42 in partial fulfillment of the requirements for the award of the Degree of Masters of Business Administration.

Signature of Guide   Signature of Head (DSM)

Seal of Head (DSM)

Place:

Date:

DECLARATION

I **Amit Kumar**, student of MBA 2014-16 of **Delhi School of Management, Delhi Technological University, Bawana Road, Delhi-42** declare that Project Dissertation Report on **“A Study on Estimating Accuracy of Value at Risk Models in Evaluating Risk in Equity Investments”** submitted in partial fulfillment of Degree of Masters of Business Administration is the original work conducted by me.

The information and data given in the report is authentic to the best of my knowledge.

This Report is not being submitted to any other University for award of any other Degree, Diploma and Fellowship.

AMIT KUMAR

Place:

Date:

ACKNOWLEDGEMENT

I would like to thank and express my sincere gratitude to **Prof. G.C. Maheshwari** for his valuable guidance and suggestions throughout the course of this project dissertation. I am thankful to him for providing me with useful references and information, which were of significant importance for the completion of this study. His constant encouragement, valuable insights and attention to detail made this exercise a great learning experience. I perceive this opportunity as a big milestone in my career development. I strive to use skills and knowledge gained here in the best possible way and will continue to work on further improvement, in order to attain my desired career objectives.

I am also thankful to **Prof. P.K. Suri, Head of Department** and all the faculty members of Delhi School of Management, DTU, Delhi for extending necessary support.

I extend my gratitude to **Delhi School of Management (Delhi Technological University)**, for giving me this opportunity.

Lastly, I thank Almighty, my family and friends for their constant encouragement.

Amit Kumar

2K14/MBA/06

EXECUTIVE SUMMARY

The dissertation aims to investigate the efficiency and accuracy of various methods available for computation of Value at Risk (VaR) for equity investments in India. The quoting of Value-at-Risk (VaR) has become a standard practice of measure of market risk adopted by banks, trading firms, mutual funds and others, including even the non-financial firms. But any risk measure is useful and reliable only insofar as it can be verified for its accuracy.

Value at Risk (VaR) is defined as the maximum potential change in value of a portfolio of financial instruments with the given probability over a certain time horizon. VaR measures can have various applications, such as in risk management, to evaluate the performance of risk takers and for regulatory requirements. Hence it is very important to develop methodologies that provide accurate estimates.

The main objective of this study is to evaluate the performance of the most popular VaR methodologies such as Historical Simulation, Exponentially Weighted Moving Average (EWMA) model and the Monte Carlo simulation using Geometric Brownian Motion (GBM) model, paying particular attention to their underlying assumptions and to their logical flaws. For this purpose, the historical data on 25 stocks listed on National Stock Exchange (NSE) and other popular Equity Indexes have been considered. The evaluated VaR value has been then back-tested with the actual returns and the hypothesis been tested using Chi-Square Test to evaluate the accuracy of VaR model. Also limitations of the VaR models have been examined.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER-1: INTRODUCTION

1.1 Introduction of the Project

1.1.1 Risk Management

"The essence of risk management lies in maximizing the areas where we have some control over the outcome, while minimizing the areas where we have absolutely no control over the outcome and linkage between effect and cause is hidden from us."[1]

Sampling is an important element of risk taking technique and the samples of the present and of the past are consistently being taken to make estimates about the future. We all have to make decisions based on the limited data and therefore the question arises, how accurate is the sample data we refer to?

Managing risk has always been at the center of every financial institution's activities as their ability to survive adverse economic cycles and phases of high volatility is highly correlated to both the quality of its risk selection and its capital endowment.

In 1952, both Markowitz and Roy independently published different types of VaR measures that attempted to develop a method of portfolio selection which incorporated covariance between risk factors based on optimizing rewards for a given level of risk. Both measures proved to be remarkably similar from a mathematically point of view; however Markowitz used a variance of simple return metric while Roy used a metric of shortfall risk.

Financial institutions are subject to many sources of risk. Risk can be broadly defined as the degree of uncertainty about future net returns. According to the fundamental sources, risk can be categorized broadly into following four types:

- **Credit Risk** which relates to the potential loss due to the inability of a counterpart to meet its obligations. It has three basic components: credit exposure, probability of default and loss in the event of default.
- **Operational Risk** takes into account the errors that can be made in instructing payments or settling transactions, and includes the risk of fraud and regulatory risks.

- **Liquidity Risk** is caused by an unexpected large and stressful negative cash flow over a short period. If a firm has highly illiquid assets and suddenly needs some liquidity, it may be compelled to sell some of its assets at a discount.
- **Market Risk** estimates the uncertainty of future earnings, due to the price change of marketable assets such as stocks, bonds, foreign exchange, futures & options etc.

1.1.2 Value at Risk (VaR) Measure

The most prominent of the risks discussed above in trading of stocks is the Market Risk, since it reflects the potential economic loss caused by the decrease in the market value of a portfolio. Value at Risk (VaR) has become the standard measure that financial analysts use to quantify this risk. It was introduced by JP Morgan in the 1980s, is a methodology that attempts to summarize the risk of an investment portfolio or even an entire institution. It is defined as the maximum potential loss in value of a portfolio of financial instruments with a given probability (level of significance) over a certain time horizon and for a given portfolio or position of instruments, under normal market conditions, attributable to changes in the market prices of financial instruments. In simpler words, it is the amount that indicates how much an investment can lose with a significance level of "α" over a given time horizon. Mathematically, it can be expressed as (Hull, John C.):

$$\mathbf{P\ [abs\ (loss) > VaR] < \alpha\ \%}$$

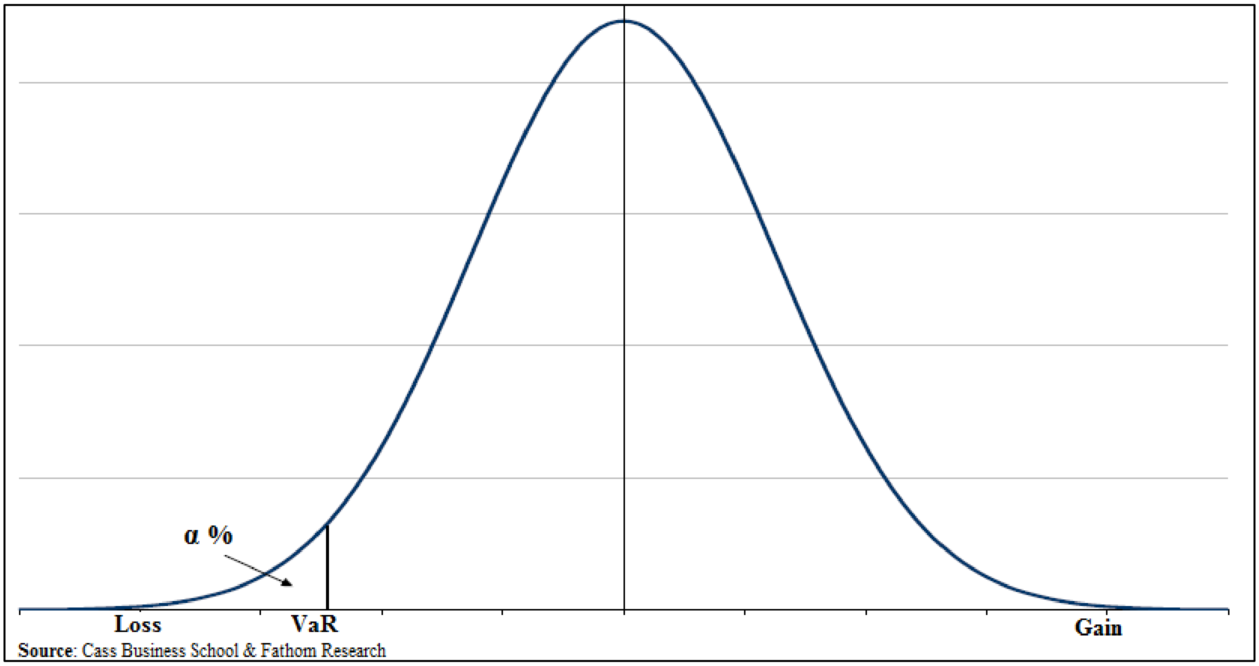

Fig. 1.1 - VaR at a particular significance level

For example, if the VaR on an asset is ₹1000 at a one-week horizon and 95% confidence interval, there is a only a 5% chance that the value of the asset will drop more than ₹1000 over any given week.

VaR measures can have many applications, and is used both for risk management and for regulatory purposes. In particular, the Basel Committee on Banking Supervision (1996) at the Bank for International Settlements imposes to financial institutions such as banks and investment firms to meet capital requirements based on VaR estimates. Providing accurate estimates is of crucial importance. If the underlying risk is not properly estimated, this may lead to a sub-optimal capital allocation with consequences on the profitability or the financial stability of the institutions.

1.1.3 Characteristics of VaR Measure

The definition of VaR provides us with the following two important characteristics of this method of estimating market risk:

- VaR is a summary measure of risk that takes into account all possible sources of market risk in an integrated framework. This characteristic makes VaR particularly appealing and easy to communicate to senior management, non-financial executives, regulators and the public investors.
- VaR requires that it is possible to express future profits & losses on a portfolio in stochastic terms so that each future expected profits & losses (or intervals of profits & losses) can be associated with its probability to occur.

1.1.4 Parameters of VaR

The two most important components of value-at-risk models are:

- Length of time over which market risk is to be measured, i.e., **Time Horizon**. It is typically measured in days, weeks, months or year.
- **Confidence level (α)** at which market risk is measured.

The choice of these components by risk managers greatly affects the nature of the value-at-risk model. Bank regulators require banks to calculate VaR for market risk with a time horizon of 10 days and confidence level of 99%. The one day VaR calculated using various methods is converted to N-day VaR using the formula (Hull, John C.):

$$\textbf{N-day VaR} = \textbf{1-day VaR} * \sqrt{N}$$

1.1.5 Approaches to calculate VaR

There are three basic approaches that are used to compute Value at Risk, having numerous variations within each approach. The measure can be computed analytically by making assumptions about return distributions for market risks, and by using the variances in and covariances across these risks. It can also be estimated by running hypothetical portfolios through historical data or from Monte Carlo simulations.

Variance-Covariance Approach

Variance-Covariance method is also known as Linear VaR or Delta-Normal VaR. This approach is relatively simple and is widely used. This method includes parts of modern portfolio theory of Harry Markowitz, by taking account of correlation coefficients between assets. Historical data is used to calculate main parameters: mean, standard deviation, correlation. This method calculates VaR by assuming some theoretical distribution of asset returns. Usually normal distribution is used. This assumption allows volatility to be described in terms of standard deviations (SD). Another advantage of normal distribution is that it can be described by its first two moments: mean, and standard deviation. This distribution is symmetrical, so the skewness is zero and kurtosis is 3. If we want to find position of a random variable (X) in a normal distribution we use standard value of variable Z (Z-score). Every variable can be transformed to standard variable with formula:

$$Z = \frac{X - \mu}{\sigma}$$

where, μ is the mean and σ is the standard deviation (SD) of the normal distribution.

When measuring VaR only downward price changes are considered, or price changes that exceed some multiple of SD. For a given confidence level, the price varies between $X-\sigma*Z$ and $X+\sigma*Z$. The value of Z can be calculated in Microsoft Excel using formula NORM.INV (1-Confidence Level, Mean, SD). For a confidence interval of 99%, the corresponding Z-value is -2.33 and for a confidence level of 95% is -1.65.
VaR for an asset or portfolio can be calculated as VaR (α%) = Z * σ * P, where P is the position value in the asset or portfolio.

In practice portfolio VaR can be calculated using the following matrix formula (Jorion, Phillippe):

$$VaR = (V * C * V^T)^{1/2}$$

Where V is row vector of VaRs for each individual position in assets, C is the correlation matrix of assets; V^T is the transpose of matrix V.

Using the perfectly normal distribution for calculation of VaR also has a limitation. It can underestimate risk in tail of distribution at high levels of confidence. Returns don't always follow normal distribution, especially in shocks and crises, and variance and covariance can vary extremely over times.

Historical Simulation Approach

Historical Simulation is a popular way of estimating VaR. It involves using past data as a guide to what might happen in future. This method is computationally more intensive than the parametric-normal methods. It is a non-parametric method because it makes no assumptions about the statistical distribution of returns. This approach involves three step simulation technique: -

- Identify the market variables or risk factors affecting the portfolio. These will typically be exchange rates, equity prices, interest rates and so on.
- Data is then collected on the movement in these market variables and the changes are observed over the specified time horizon (let 500 days). Then apply each of these changes to portfolio (through model defining relationship between the market factor changes and single asset prices) to determine the series of daily changes in portfolio value that would have been realized had the current portfolio been held unchanged throughout those 500 trading days. The daily change in equity prices are calculated as the continuously compounded return as: -

$$\boldsymbol{u_i = ln(\frac{P_i}{P_{i-1}})}$$

  where, P_i is the closing price today and P_{i-1} is the closing price a day before.

- The last step is to sort the 500 changes in portfolio value in ascending order to arrive at an observed distribution of changes in portfolio value. Then the VaR number is read from this distribution. The VaR number will be equal to the percentile associated with the specified level of confidence. For a 95% level of confidence, the VaR number equals the 5th percentile of the distribution of changes in portfolio value. In our case this number will be 25th worst value for 95% level of confidence and 5th worst for 99% level of confidence.

Since, Historical Simulation approach assumes that the changes in historical prices are going to repeat themselves and values the recent and older data equally, it can cause bad estimates if there are recent trends, like higher volatility. For this simulation, it is important to have sufficient data, and that is the problem when dealing with new assets and risks. Typical trade-off for historical simulation is that we would like to have more data in order to observe the rare events, and on the other hand, we do not want to build our current risk estimates on very old market data.

The approach to historical simulation described above is simple one where all the volatilities (SD) over the specified time horizon are given equal weights. Another variation of historical simulation, where volatilities of the assets are estimated by assigning greater weights to the more recent observation than the previous one, can be used in estimation of a more accurate value of VaR. Two such models are Exponentially Weighted moving Average (EWMA) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH), where weights assigned to observations decrease exponentially as they become older. This study attempts to calculate VaR using HS-EWMA (Historical Simulation-EWMA) method and then estimate its accuracy against the simple historical simulation.

Exponentially Weighted Moving Average to estimate volatility is given by the formula (Hull, John C.):

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1\text{-}\lambda)\, u_{n-1}^2$$

where, σ_n is the volatility for the day 'n' which is calculated using the previous day's volatility σ_{n-1} and the decaying most recent return, i.e., u_{n-1}. Lambda (λ) is a constant between 0 and 1 known as decay constant. The weights for the previous value of returns u_i decline at the rate of λ as we move forward through the time. Square root of the variance σ_n^2 gives the standard deviation for the asset.

The EWMA approach has the attractive feature that relatively little data need to be stored. It is because at any given time the variance depends only on the current estimate of variance rate and the most recent observation on the market price. When a new observation of market price is obtained, a new daily percentage change is calculated and the above EWMA equation is updated with the new estimate of variance rate. The old value of the variance rate and the old value of the market price can then be discarded. The EWMA approach is designed to track changes in the volatility. The value of $\boldsymbol{\lambda}$ governs how responsive the estimate of daily volatility is to the most recent daily percentage change. A low value of $\boldsymbol{\lambda}$ leads to a great deal of weight being given to the $\boldsymbol{u_{n-1}^2}$ when $\boldsymbol{\sigma_n}$ is calculated. In this case, the estimates produced for the volatility on successive days are themselves highly volatile. A high value of $\boldsymbol{\lambda}$ (close to 1) produces estimates of the daily volatility that respond relatively slowly to new information provided by the daily percentage change.

Monte Carlo Simulation Approach

A Monte Carlo simulation is an attempt to predict the future many times over. It is based on the generation, or simulation, of large number of scenarios to represent possible future price changes that could affect the value of the portfolio. In each trial or scenario, an independent random number for each of the market factors is selected from a particular distribution. At the end of the simulation, thousands or millions of "random trials" produce a distribution of outcomes that can be analyzed. These random moves are constrained to reflect historical market volatility and correlation. The portfolio is then valued under each of these scenarios. The resulting changes in portfolio value are then analysed to arrive at a single VaR number.

This method requires the following steps to calculate VaR number: -

- **Specify the Model for Simulation:** This study uses the Geometric Brownian motion (GBM), which is technically a Markov process. This means that the stock price follows a random walk and is consistent with (at the very least) the weak form of the efficient market hypothesis (EMH), i.e., past price information is already incorporated and the next price movement is "conditionally independent" of past price movements. The formula for GBM is (Ross, Sheldon M.):

$$\frac{\Delta S}{S} = \mu . \Delta t + \sigma . \varepsilon . \sqrt{\Delta t}$$

  where, "S" is the stock price, " μ " is the expected return, " σ " (Greek sigma) is the standard deviation of returns, "t" is time, and " ε " is the random variable.

If we rearrange the formula to solve just for the change in stock price, we see that GBM says the change in stock price is the stock price "S" multiplied by the two terms:

$$\Delta S = S(\mu.\Delta t + \sigma.\varepsilon.\sqrt{\Delta t})$$

The first term is a "drift" and the second term is a "shock". For each time period, our model assumes the price will "drift" up by the expected return. But the drift will be shocked (added or subtracted) by a random shock. The random shock will be the standard deviation "σ" multiplied by a random number "ε". This is simply a way of scaling the standard deviation.

- **Generate Random Trials:** A large number of random draws are simulated from the estimated statistical distribution of the market factor returns. This study uses 500 numbers of random draws.
- **Process the Output:** The simulation produces a distribution of hypothetical future outcomes. If we want to estimate VaR with 95% confidence, then we only need to locate the twenty-fifth worst outcome.

The Monte-Carlo process permits analysis of the impact of events that were not infact observed over the historical period but that are just as likely to occur as events that were observed.

1.1.6 Comparison of VaR Approaches

Each approach/methodology of VaR calculation has its own strengths and weaknesses and is best adapted to a different environment.

- The Variance-Covariance approach has its principal virtue speed in computation as one needs to estimate only 2 parameters: average / expected return and standard deviation of returns. However, the quality of VaR estimates degrades with the portfolios of non-linear instruments. Departures from normality in the portfolio return distribution also represent a problem for this approach.
- Historical simulation is free from distributional assumptions, it is easy to implement and conceptually simple, which makes it easier to explain to senior management. But it is computationally intensive as it requires the portfolio be revalued once for every day in

the historical sample period. Another drawback is that only one sample path is used, which may not adequately represent future distributions.

- Monte Carlo VaR is not limited by price changes observed in the sample period, because revaluations are based on sampling from an estimated distribution of price changes. Monte Carlo simulation usually involves many more re-pricings of the portfolio and allows its users to tailor ideas about future patterns that depart from historical patterns. But this method requires computer time and a good understanding of the stochastic process used is therefore, most expensive and time consuming approach.

Thus, every method can be best chosen as per the type of portfolio and demands of the situation in terms of speed, simplicity, ease of implementation and reliability of results.

1.2 Objectives of the Study

Primary Objective:

The main objective of the study is to assess the accuracy of various Value at Risk models, i.e., Historical Simulation, EWMA model and Monte-Carlo Simulation, in calculating the market risk for equity investments, through statistical tests. The VaR values have been calculated using these models and then back-tested against the next series of historical data to ascertain its accuracy.

Secondary Objectives:

1. To know whether the asset returns and market returns follow normal distribution.
2. To study the limitations of each of the tested VaR models.

CHAPTER-2: LITERATURE REVIEW

Value at Risk was initially developed to measure financial risk exposure and communicate this data in a simplistic form to the stakeholders and was later upgraded to provide a common benchmark to control and compare total risk across risk taking units. Moving forward form its initial purpose, it has been used to calculate the optimal level of capital to be held in reserve by a financial institution.

Thompson & McCarthy [2008] say that while VaR is being based on firm scientific foundations, it is very easily understood and has become widely used by corporate treasurers and fund managers as well as by financial institutions however there is no general consensus on how to actually calculate it.

One of the early models employed in capturing volatility is the equally weighted moving average model. This framework assumes that the N-period historic estimate of variance is based on an equally weighted moving average of the N-past one-period squared returns. However, under this formulation all past squared returns that enter the moving average are equally weighted and this may lead to unrealistic estimates of volatility. In this respect the exponentially weighted moving average (EWMA) framework proposed by J.P Morgan's "RiskMetrics" assigns geometrically declining weights on past observations with the highest weight been attributed to the latest (i.e. more resent) observation. By assigning the highest weight to the latest observations and the least to the oldest the model is able to capture the dynamic features of volatility.

Varma, J.R. [2000] in a SEBI committee report titled 'Value at Risk Models in the Indian Stock Market' suggested National Stock Exchange (NSE) and Bombay Stock Exchange (BSE) to use EWMA method to report VaR for their market indices. Maximum likelihood estimation yielded an estimate for λ as 0.923 for the Nifty and 0.929 for the Sensex, which was not statistically significantly different from the value of 0.94 for λ used in J. P. Morgan's RiskMetrics system for daily horizons. EWMA model was tested using historical data from July 1, 1990 to June 30, 1998, on the Indian stock market indices - the NSE-50 Index (Nifty) and the BSE-30 Index (Sensex), using a λ of 0.94 to permit easier comparability and facilitate further extensions to the model. The number of VaR violations was found to be well within the allowable limits of sampling error, suggesting EWMA as an accurate model for Indian markets.

According to Perignon and Smith [2006] survey, 73% of banks among 60 US, Canadian and large international banks over 1996-2005 have reported that their VaR methodology used was historical simulation. The Monte Carlo (MC) simulation was the second most popular method.

The strength of the Monte Carlo simulation approach is the flexibility it offers users to make different distributional assumptions and deal with various types of risk, but it can be painfully slow to run. Glasserman, Heidelberger and Shahabuddin [1997] used approximations from the variance-covariance approach to guide the sampling process in Monte Carlo simulations and report a substantial savings in time and resources, without any appreciable loss of precision.

Engle, R.F. & Manganelli, Simone [2011] in their working paper titled 'Value at Risk Models in Finance' concluded that Monte Carlo simulations outperformed every other VaR approach at common confidence levels of 95% and 99%.

The Basel Accord [1996] Amendment describes the form of back-tests that must be undertaken by firms wishing to use a VaR model for calculation of market risk capital rule (MRR). Regulators recommend using the last 250 days of profit & loss (P&L) data to back test the 1% 1-day VaR that is predicted by and internal VaR model. The model should be back-tested against both the theoretical and actual P&L. Whether or not actual P&L gives rise to more exceptions during back-tests than theoretical P&L will depend on the nature of trading. If the main activity is hedging one should expect fewer exceptions, but if traders are undertaking more speculative trades that increase the P&L volatility, then the opposite will be observed. The Basel committee have chosen a confidence level of 99% and a 10 day time horizon to determine the minimum capital level for commercial banks and the resulting VaR must be multiplied by a factor of at least 3 to account for non-normality or model errors.

Beder [1995] applies eight common VaR methodologies to three hypothetical portfolios. The results show the differences among these methods can be very large, with VaR estimates varying by more than 14 times for the same portfolio. Clearly, there is a need for a statistical approach to estimation and model selection.

Bao, Lee & Saltoglu [2006] evaluated the predictive power of VaR models in emerging markets. Through their research, they applied traditional VaR models, conditional autoregressive VaR models and also applied their models to extreme value theory. Their results showed that their benchmark, the RiskMetrics model developed by J.P. Morgan, produced good results in tranquil periods, whereas in crisis periods VaR approaches based on extreme value theory produced better results. The authors also discovered that while filtering can improve the predictive results using Extreme Value Theory, it can make the other models less useful.

Kuesters, Mittnik & Paolella [2006] applied both a conditional and an unconditional VaR model to NASDAQ-composite data and concluded that most of the models were unable to produce accurate results due to a tendency to underestimate market risk. However, they did find that although the conditional VaR models do produce an increased level of volatility in their estimates, if heteroskedasticity is factored into the calculation, then the model will provide a satisfactory output. The author's final conclusion was that mixed normal GARCH, extreme value theory and filtered historical stimulation models usually provide the most accurate forecasts.

VaR has tail risk when it fails to summarise the relative choice between portfolios as a result of its underestimation of the risk of portfolios with fat-tailed properties and a high potential for large losses. The tail risk of VaR emerges since it measures only a single quartile of the profit/loss distributions and disregards any loss beyond the VaR level. In the world of portfolio management, the existence of fat tails can, in part, be linked to behavioural finance due to excessive optimism or pessimism from the investor, causing large market movements and ultimately leads to additional risk exposure.

Mandelbrot (1963) and Fama (1965) found some empirical facts of financial markets as mentioned below:

- Financial return distributions are leptokurtic, i.e., they have heavier tails and a higher peak than a normal distribution.
- Equity returns are typically negatively skewed.
- Squared returns have significant autocorrelation, i.e. volatilities of market factors tend to cluster. This is a very important characteristic of financial returns, since it allows the researcher to consider market volatilities as quasi-stable, changing in the long run, but stable in the short period. Most of the VaR models make use of this quasi-stability to evaluate market risk.

Tail risk is technically defined as a higher-than-expected risk of an investment moving more than three standard deviations (3σ) away from the portfolio's mean distribution. VaR models are usually based on normal asset returns and do not work under extreme price fluctuations. This point is emphasised through the financial market crisis of 2008. Concerning this crisis a large amount of occurrences was found to be in the tails of the distributions and as a result VaR models were useless for measuring and monitoring market risk.

Yasuhiro Yamai and Toshinao Yoshiba [2002] in their study titled ‘Comparative analyses of Expected Shortfall and Value-at-Risk under Market Stress’ concluded that VaR and expected shortfall may underestimate the risk of securities with fat-tailed properties and a high potential for large losses.

CHAPTER-3: RESEARCH METHODOLOGY

3.1 Data Collection Sources

The study is done with special reference to the Indian equities market and it is secondary in nature. For calculating VaR of individual assets, daily closing prices (adjusted) of 25 securities trading on NSE Nifty-50 have been collected and converted into daily returns for a period of 500 trading days spanning from 2nd January 2012 to 15th January, 2014. Also, daily adjusted closing prices of 4 major stock indices, i.e. BSE SENSEX, S&P CNX NIFTY, BSE 500 and NSE 500 has been taken to evaluate VaR on well diversified portfolios.

For evaluating the accuracy of VaR forecasts, returns on these assets have been observed (back-tested) for next 500 days spanning from 16th January, 2014 to 29th January, 2016.

The daily adjusted closing prices of the individual stocks and the market indices have been taken from the NSE website (www.nseindia.com), BSE website (www.bseindia.com) and Moneycontrol website (www. moneycontrol.com).

3.2 Tools & Techniques Used

Microsoft Excel 2010 and IBM SPSS (Statistical Package for Social Sciences) version 20 were used primarily as the tools for analysis of data collected for this study. The Data Analysis Toolpak Add-in in Excel 2010 for performing various statistical tests was also used.

Log Returns for daily change in market prices have been calculated in Excel by using the following formula (Hull, John C.):

$$u_i = ln(\frac{P_i}{P_{i-1}})$$

where, P_i represents the today's closing price and P_{i-1} represents the previous day closing price. Then the PERCENTILE () function was used to arrive at VaR number as percentage.

The Historical Simulation was performed in Excel while SPSS was used for the purpose of calculating volatility using Exponentially Weighted Moving Average (EWMA) technique.

Monte Carlo simulation has been performed using the Geometric Brownian Motion (GBM) method using the formula (Ross, Sheldon M.):

$$\Delta S = S\,(\mu.\Delta t + \sigma.\varepsilon.\ \sqrt{\Delta t})$$

where, the first term represents $\boldsymbol{\mu\Delta t}$ represents drift and the second term $\boldsymbol{\sigma\varepsilon\sqrt{\Delta t}}$ represents the shock. It assumes that for each period the price will drift up by expected return but the drift is shocked by the random shock.

Random trials were generated through Excel using the function:
= P*(1+ (NORM.INV (RAND (), Mean, STDEV)))
Where

- P - Initial price / previous day price,
- Mean - Expected return of the stock,
- STDEV – Standard Deviation of the return series,
- RAND() – This function returns an evenly distributed random real number greater than or equal to 0 and less than 1, which is essentially the required probability term, and
- NORM.INV() – This function returns the inverse of the normal cumulative distribution for the specified mean and standard deviation.

For the purpose of this study, 100 random trials with daily steps for 500 days have been generated. The final prices arrived at are converted to daily returns. Then these returns have been sorted in ascending order to calculate the VaR number.

VaR has been calculated with following parameters:

- one day time horizon, and
- 95% and 99% confidence level.

After VaR is calculated, back-testing has been performed to measure its accuracy by counting the number of losses exceeding VaR number. Back-testing systematically checks whether the frequency of losses exceeding VaR is in line with ‘p=1-c’, or alternatively, it checks whether the frequency of losses below VaR is in line with ‘c’, i.e. the confidence level of the VaR measure. In its simplest form, the back-testing procedure consists of just calculating the percentage of times that the observed returns fall below the negative VaR estimate, and comparing that number to the confidence level used.

3.3 Formulation of Hypothesis

Accuracy of the calculated VaR estimates has been assessed through Hypothesis Testing, that our computed VaR estimates truly forecasts the returns observed on all the 25 NSE NIFTY securities as well as on 4 stock indices – S&P CNX Nifty, BSE Sensex, NSE 500 and BSE 500.

Accuracy of VaR estimates will be established if the frequency of losses exceeding computed VaR for each of the 25 securities and for 4 indices is equal to 5% of the total number of days observed (500 days) for 95% VaR and 1% of the total number of days for 99% VaR. The Hypothesis has been formulated as:

Null Hypothesis: The proportion of the losses as determined by the VaR method is not significant with the expected number of losses, i.e.

H_0 : $P_1 = P_2 = P_3 = ... = P_k$

where,

- k is the number of securities or stock indices for which VaR is calculated and evaluated for accuracy, and
- P_k is the proportion of losses exceeding VaR estimates for k^{th} stock.

Alternate Hypothesis: Consequently the alternate hypothesis is that the proportion of the losses as determined by the VaR method is significant with the expected number of losses and the VaR model does not adequately estimates the actual value at risk.

H_1: $P_1 \neq P_2 \neq P3 \neq ... \neq P_k$.

This hypothesis has been tested by using Chi-square tests used for testing the equality of several population proportions. Chi-square statistic (χ^2) has been computed by using the following equation (Keller, Gerald):

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_{oi} - f_{ei})^2}{f_{oi}}$$

where,

f_{oi} = observed frequency of losses beyond computed VaR for every i^{th} stock.
f_{ei} = expected frequency of losses beyond computed VaR for every i^{th} stock.

For 95% VaR, f_{ei} is 5% of the observed number of days, i.e., 5% of 500 = 25, and for 99% VaR, f_{ei} is 1% of the observed number of days, i.e., 1% of 500 = 5.

The null hypothesis is tested by comparing the observed and expected frequencies of losses exceeding VaR forecasts to see if two are in agreement. It is obvious that observed frequencies cannot be exactly the same as the corresponding expected frequencies and discrepancies are bound to exist to some extend due to unexpected variations. These discrepancies are measured by computing the χ^2 value, which gives the magnitude of the overall relative discrepancy. In testing the null hypothesis, H_0 is expected to be true if the discrepancies between f_{oi} and f_{ei} as measured by computed χ^2 value can be explained as only being variation due to sampling error. At a particular significance level and the degrees of freedom the computed χ^2 value is compared with the tabulated χ^2 value known as critical value (χ_c^2). The decision rule is that we reject the Null Hypothesis, H_0 at a level of significance when $\chi^2 > \chi_c^2$, otherwise H_0 is accepted.

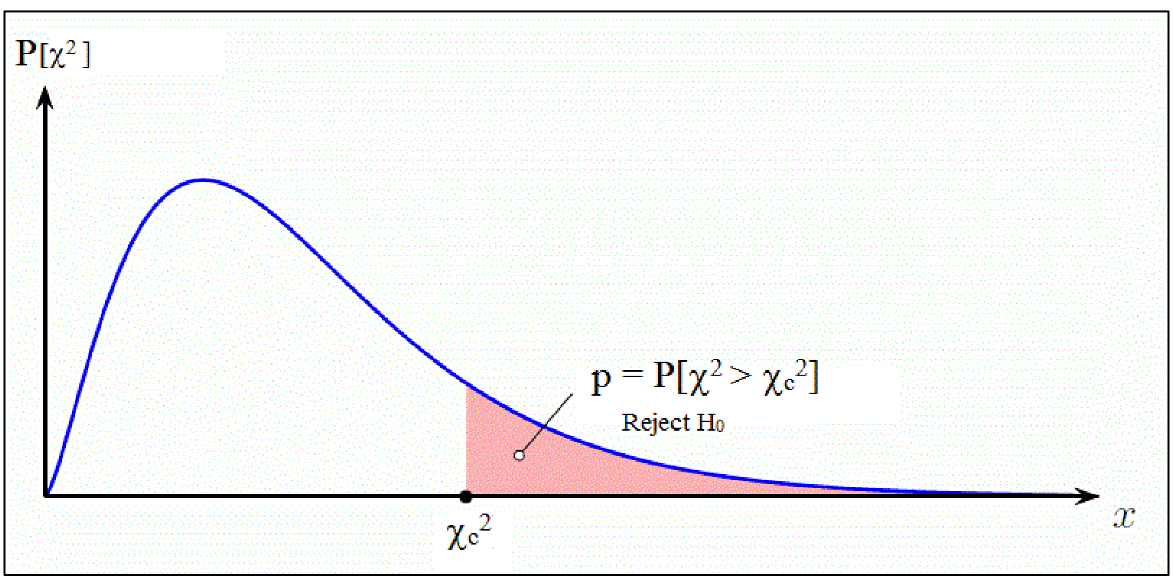

Fig. 2.1 - Chi-Square Distribution

<u>CHAPTER-4</u>: <u>DATA ANALYSIS</u>

4.1 Value at Risk – Historical Simulation

<u>Individual Stocks</u>

Stocks	Average Return	Standard Deviation	1-day 95% Absolute VaR	95% VaR Violations	1-day 99% Absolute VaR	99% VaR Violations
ACC Ltd.	0.05%	1.61%	2.37%	24	3.30%	5
Asian Paints	0.11%	1.79%	2.77%	19	4.30%	1
Axis Bank	0.14%	2.02%	3.13%	47	4.92%	5
BHEL	0.03%	2.53%	3.71%	47	6.23%	10
Bharti Airtel	0.01%	1.79%	2.73%	19	4.12%	4
Bosch Ltd.	0.16%	1.92%	2.87%	33	4.12%	2
Cipla	0.12%	1.75%	2.71%	80	4.03%	7
Coal India	0.02%	2.02%	3.13%	33	4.85%	7
DRREDDY	0.12%	1.60%	2.66%	52	4.19%	11
GAIL	-0.02%	1.97%	2.92%	24	4.64%	5
HCL Tech	0.07%	1.99%	3.00%	47	4.47%	2
HDFC Bank	0.11%	1.23%	1.67%	33	2.56%	7
HINDALCO	-0.08%	2.61%	4.22%	38	6.18%	8
HUL	0.07%	1.51%	2.40%	56	3.64%	6
ITC Ltd.	0.01%	1.53%	2.36%	33	5.11%	7
ICICI Bank	0.05%	1.89%	2.73%	61	4.60%	13
IDEA Cellular	-0.04%	2.27%	4.03%	19	5.07%	4
Infosys Ltd.	0.06%	1.69%	2.35%	33	5.06%	7
L&T	0.06%	1.78%	2.66%	42	4.67%	5
Lupin	0.17%	1.67%	2.59%	52	4.39%	11
Maruti	0.21%	1.60%	1.94%	71	3.92%	15
ONGC	-0.03%	2.12%	3.28%	56	4.57%	8
Reliance Ind.	0.01%	1.62%	2.54%	47	3.75%	7
SBI	0.07%	1.98%	2.86%	61	4.79%	13
Tata Steel	-0.12%	2.36%	3.82%	28	5.85%	6

Table 4.1 - VaR values for 25 NIFTY stocks

Stock Indices

Indices	Average Return	Standard Deviation	1-day 95% Absolute VaR	95% VaR Violations	1-day 99% Absolute VaR	99% VaR Violations
S&P CNX NIFTY	0.05%	0.93%	1.46%	26	2.12%	7
SENSEX	0.05%	0.93%	1.44%	32	2.12%	9
NIFTY 500	0.07%	0.95%	1.57%	23	2.15%	8
BSE 500	0.07%	0.94%	1.55%	36	2.13%	7

Table 4.2 - VaR values for Indices

Interpreting the VaR numbers obtained in the Tables 4.1 and 4.2, it can be said that for any of these securities or stock indices, losses are expected to exceed the corresponding VaR number in only 25 times out of 500 times for 95% confidence level and 5 times out of 500 times for 99% confidence level. The number of VaR violations can be clearly seen exceeding the expected number of losses in most of the cases. It can be seen that the Historical Simulation approach overestimates the loss in case of 21 securities and 3 of the 4 stock indices for both the levels of significance. Only in case of 3 securities, i.e., Asian Paints, Bharti Airtel and Idea Cellular Ltd., that the numbers of losses observed are within the expected number of VaR violations. While in other cases losses are found to be equal to the expected number of losses.

Hypothesis Testing – Chi-Square Statistic

	Individual Stocks		**Indices**	
	95% VaR	**99% VaR**	**95% VaR**	**99% VaR**
Computed Chi-Square ($\chi 2$)	180.64	81.82	7.37	6.67
Degrees of freedom (df)	24	24	3	3
Significance Level (α)	.05	.01	.05	.01
Critical Value (χ_c^2)	36.42	43	7.82	11.3
p-Value	5.52E-26	3.11E-08	0.06	0.08
Null Hypothesis	Rejected	Rejected	Accepted	Accepted

Table 4.3 - Chi-Square Test Results for Historical Simulation

Chi-square test have been used for testing the hypothesis that VaR number, as calculated in tables 4.1 and 4.2 above, accurately forecasts the worst likely daily losses on 25 NSE securities with 95% and 99% confidence levels. Table 4.3 shows the calculated values of Chi-squares for both confidence levels and the corresponding degrees of freedom, critical value, p-value and whether the null hypothesis is accepted or not.

It can be seen from this table that in case of individual securities, the p-values calculated for both the confidence levels are much less than their respective significance levels of 0.05 and 0.01. This is because of the fact that the computed chi-square values for individual securities are greater than respective critical values. Hence the null hypothesis that the proportion of the losses as determined by the Historical Simulation model is not significant with the expected number of losses is rejected.

But in case of stock indices, the p-values calculated are greater than their respective significance levels and therefore the null hypothesis is accepted. Hence the proportion of losses found through Historical Simulation model in case of indices are not significant and the variation can be just due to sampling error.

4.2 Value at Risk - EWMA Model

Individual Stocks

Stocks	Average Return	EWMA Standard Deviation	1-day 95 % Absolute VaR	95% VaR Violations	1-day 99% Absolute VaR	99% VaR Violations
ACC Ltd.	0.05%	1.08%	1.72%	29	2.46%	9
Asian Paints	0.11%	1.79%	2.84%	18	4.06%	1
Axis Bank	0.14%	2.60%	4.14%	21	5.91%	3
BHEL	0.03%	2.34%	3.82%	49	5.41%	10
Bharti Airtel	0.01%	1.89%	3.09%	17	4.38%	3
Bosch Ltd.	0.16%	1.74%	2.71%	19	3.89%	3
Cipla	0.12%	1.41%	2.20%	49	3.16%	12
Coal India	0.02%	1.87%	3.06%	34	4.33%	7
DRREDDY	0.12%	1.69%	2.67%	45	3.82%	10
GAIL	-0.02%	2.08%	3.44%	26	4.86%	5
HCL Tech	0.07%	2.39%	3.86%	11	5.49%	1
HDFC Bank	0.11%	0.98%	1.50%	43	2.17%	9
HINDALCO	-0.08%	2.72%	4.55%	29	6.40%	6
HUL	0.07%	1.19%	1.89%	24	2.70%	12
ITC Ltd.	0.01%	1.72%	2.82%	21	3.99%	3
ICICI Bank	0.05%	1.97%	3.19%	30	4.53%	7
IDEA Cellular	-0.04%	2.34%	3.89%	22	5.48%	4
Infosys Ltd.	0.06%	1.53%	2.46%	31	3.51%	6
L&T	0.06%	1.90%	3.06%	18	4.35%	3
Lupin	0.17%	1.96%	3.06%	29	4.40%	6
Maruti	0.21%	1.56%	2.35%	43	3.41%	10
ONGC	-0.03%	2.10%	3.49%	31	4.92%	7
Reliance Ind.	0.01%	1.70%	2.78%	28	3.93%	6
SBI	0.07%	1.70%	2.74%	57	3.90%	14
Tata Steel	-0.12%	2.57%	4.35%	23	6.11%	4

Table 4.4 - VaR Calculation for Stocks using EWMA

Stock Indices

Indices	**Average Return**	**EWMA Standard Deviation**	**1-day 95 % Absolute VaR**	**95% VaR Violations**	**1-day 99% Absolute VaR**	**99% VaR Violations**
S&P CNX NIFTY	-0.04%	0.88%	1.48%	28	2.08%	7
SENSEX	-0.05%	0.91%	1.54%	23	2.15%	6
NIFTY 500	-0.05%	0.85%	1.44%	31	2.01%	9
BSE 500	-0.05%	0.84%	1.44%	36	2.01%	10

Table 4.5 – VaR Calculation for Indices using EWMA

Interpreting the VaR numbers obtained in the Tables 4.4 and 4.5, it can be said that for any of these securities or stock indices, losses are expected to exceed the corresponding VaR number in only 25 times out of 500 times for 95% confidence level and 5 times out of 500 times for 99% confidence level. The number of VaR violations can be clearly seen exceeding the expected number of losses in most of the cases. It can be observed that the EWMA model approach overestimates the loss in case of 16 securities and all of the 4 stock indices for both the levels of significance. While in case of 9 securities, the number of losses observed is within the expected number of VaR violations. Hence it can be interpreted that the EWMA model is efficiently predicting the VaR value than the Historical Simulation approach up to a certain extent.

Hypothesis testing - Chi-Square Statistic

	Individual Stocks		**Indices**	
	95% VaR	**99% VaR**	**95% VaR**	**99% VaR**
Computed Chi-Square(χ2)	164.21	71.92	7.16	9.29
Degrees of freedom (df)	24	24	3	3
Significance Level (α)	.05	.01	.05	.01
Critical Value (χ_c^2)	36.42	43	7.82	11.3
p-Value	5.53E-26	1.12E-06	0.07	0.08
Null Hypothesis	Rejected	Rejected	Accepted	Accepted

Table 4.6 - Chi-Square Test Results for EWMA model

Chi-square test have been used for testing the hypothesis that VaR number, as calculated in tables 4.4 and 4.5 above, accurately forecasts the worst likely daily losses on 25 NSE securities with 95% and 99% confidence levels. Table 4.6 shows the calculated values of Chi-squares for both the confidence levels and the corresponding degrees of freedom, critical value, p-value and whether the null hypothesis is accepted or not.

It can be seen from this table that in case of individual securities, the p-values calculated for both the confidence levels are much less than their respective significance levels of 0.05 and 0.01. Although the EWMA model was determining VaR estimate better than the Historical Simulation approach but it is still not performing good. Hence the null hypothesis that the proportion of the losses as determined by the EWMA model is not significant with the expected number of losses, is rejected.

But in case of stock indices, the p-values calculated are greater than their respective significance levels and therefore the null hypothesis is accepted. Hence the proportion of losses found through EWMA model in case of indices are not significant and the variation can be just due to sampling error.

4.3 Monte Carlo Simulation Approach

The Monte Carlo simulation was done on the stock indices using the Geometric Brownian Motion model. It is a non-parametric test that is designed to examine whether the price changes are normally distributed. The mean and standard deviation of historical prices are taken to predict the future prices. The final prices arrived at are converted to daily returns. Then these returns have been sorted in ascending order to calculate the VaR number.

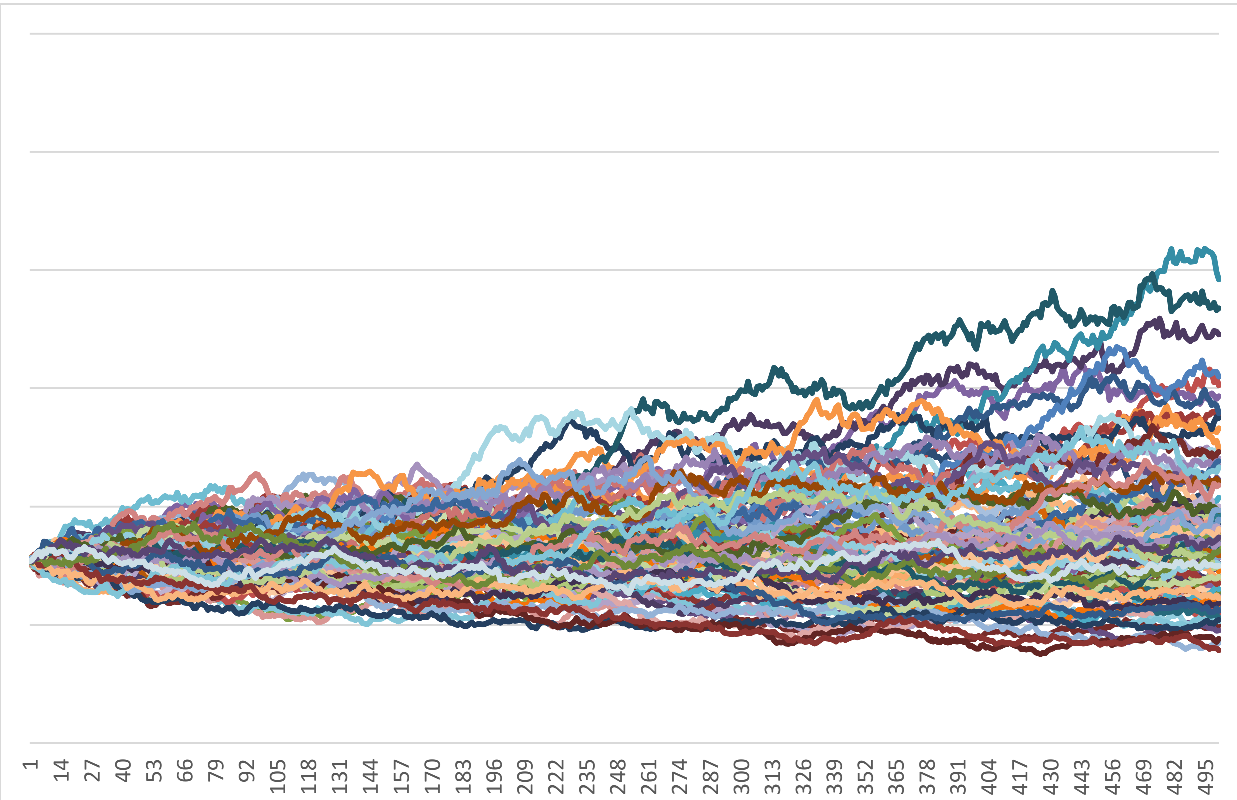

Fig. 4.1 – Brownian Motion for S&P CNX NIFTY

The VaR estimates calculated using Monte Carlo simulation is shown below:

Indices	Average Return	Standard Deviation	1-day 95% Absolute VaR	95% VaR Violations	1-day 99% Absolute VaR	99% VaR Violations
S&P CNX NIFTY	0.05%	0.98%	2.09%	27	3.02%	6
SENSEX	0.08%	1.39%	1.64%	31	2.42%	8
NIFTY 500	0.06%	1.09%	1.97%	24	2.96%	9
BSE 500	0.08%	1.11%	1.95%	30	2.57%	7

Table 4.7 - Monte Carlo VaR Estimates

We can see from the above table that the observed number of VaR violations is very close to the expected number of violations, i.e., 25 for 95% confidence level and 5 for 99% confidence levels.

The Chi-square statistic is as shown below:

	95% VaR	99% VaR
Computed Chi-Square($\chi 2$)	2.78	6.06
Degrees of freedom (df)	3	3
Significance Level (α)	.05	.01
Critical Value (χ_c^2)	7.82	11.3
p-Value	0.43	0.11
Null Hypothesis	Accepted	Accepted

Table 4.8 – Chi-Square Test Results for Monte Carlo Simulation

The Chi-Square test shows a high p-value for both confidence levels. It is 0.43 for 95% VaR estimate which very high than the significance value 0.05 and hence the null hypothesis is accepted. Also p-value for 99% VaR is 0.11 which is greater than 0.01 significance level. Hence null hypothesis is accepted that there is no significant difference between the observed and expected level of losses determined by Monte Carlo simulation method.

4.4 Findings & Recommendations

The study investigates the accuracy of VaR Models used in equity markets in India. A sample of 25 stocks trading on NSE and 4 other major indices were taken for this purpose. The findings and recommendations of the study are as follows:

1. On the basis of results of our hypothesis testing, we reject the hypothesis that the proportion of losses exceeding VaR number for individual securities is equal to expected frequency, in case of Historical Simulation Model and EWMA model. In turn it implies that, our VaR estimate does not accurately measure the risk in equity investment in India. In this sense, the VaR forecasts appear to be quite conservative. But this does not mean that VaR as a risk measurement tool is not an accurate measure of risk. This may be because the sample size of our number of stocks is small.

2. In case of VaR for 4 indices (representing the portfolio of securities), we have not found sufficient evidence to reject our hypothesis at 5% and 1% level of significance and thus, our hypothesis that our VaR estimate for stock indices accurately forecast the portfolio risk of equity investment in India is accepted. This is despite of the fact that the return on the stock indices have been observed over the same period as for 25 NSE securities and VaR has been calculated with same assumption of normally distributed returns. But still, our VaR estimates for stock indices is better assessing the portfolio risk of equity investment as compared to that of individual securities.

3. EWMA Model and Monte Carlo Simulation provide far better estimates of VaR than the Historical Simulation Model. But still Historical Simulation is widely used owing to its simplicity.

4. This study is also highlighting an important finding relating to a shortcoming of VaR as a risk measurement tool. One of the shortcomings of VaR is that it conveys nothing about the size of violations when they do occur, i.e. VaR figure provides no indication of the magnitude of losses that may result if prices move by an amount which is more adverse than the amount dictated by the chosen confidence level. Thus there is a need for some

complementary technique that can complement VaR in telling us the worst expected loss with some predefined probability as well as the magnitude of losses which are expected to be worse than VaR.

5. An underlying assumption of VaR models is that returns are normally distributed. However, this assumption of normally distributed returns is not validated by the behaviour of returns on 25 NSE securities and 4 stock indices observed as shown by their Kurtosis and skewness values which are the measure of tailedness and asymmetry in distribution of returns. All the securities are showing excess kurtosis estimate indicating that returns are not normally distributed; rather returns are leptokurtic exhibiting higher peaks. Similarly, skewness estimates are showing negatively skewed returns for most of the securities, indicating that tail of the return distribution is longer to the left of the mean return. (The Kurtosis and Skewness values are given in the Annexure-I).

6. In can be concluded that no form of risk measurement (including VAR) is a substitute for good management. Risk management as a process encompasses much more than just risk measurement and it is recommended that VaR models need to be supplemented with stringent back testing procedures in order to maintain a realistic level of confidence in the model.

4.4 Limitations and Scope for Further Research

1. The study is limited to only three models for estimation of Value at Risk, i.e., Historical Simulation, EWMA model and Monte Carlo simulation, while there are numerous variations of these methods like GARCH and Maximum Likelihood techniques that attempt to evaluate better estimates for volatility.

2. The only takes into account the securities and indices in Indian Market. Further research may be done with data from other world markets to better assess the accuracy of VaR models.

3. This study does not takes into account Stress testing which involves examining how the portfolio would have performed under a scenario of more extreme market conditions, both positive and negative. For further research stress testing of the portfolios may be undertaken which account for extreme events that do not occur from time to time but are virtually impossible according to the probability distributions assumed for market variables. A five standard deviation daily move may be one such extreme event. Also Principal Component Analysis may be undertaken to define a set of components or factors that explain the historical movements in prices.

4. For further research it is recommended that the sample size can be increased to calculate VaR. The sample size is limited to 500 days for this study.

CHAPTER-5: BIBLIOGRAPHY & REFERENCES

1. Bernstein, Peter L., '*Against the Gods: The remarkable Story of Risk*', September 1998, Chapter 12 pp. 197.

2. Jorion Phillippe, *Value at Risk: The new benchmarking for managing financial risk*, 3rd edition, McGraw-Hill Trade, 2007.

3. Jorion, Philippe. *'Risk Management Lessons from Long Term Capital Management'*, Working Paper (June 1999).

4. Hull, John C., *'Options, Futures and Other Derivatives'* (Seventh Edition) - 2009.

5. Duffie, Darrell and J. Pan, "An overview of value at risk", *Journal of Derivatives*, 4, 7-49 (Spring 1997).

6. Hendricks, D. "*Evaluation off Value-at-Risk Models using Historical Data*", Economic Policy review, Federal Reserve Bank of Philadelphia, July/August 1996 : 19-29.

7. Dhesi, G., Shakeel, M. B., & Xiao, L., *Modified Brownian motion Approach to Modelling Returns Distribution*, 2016.

8. Linsmeier,Thomas and Neil Pearson, *"Risk Measurement: An Introduction to Value at Risk"*, Working Paper of the University of Illinois, 1996.

9. Manganelli, S. & Engle, R. F., "*Value at risk models in Finance*", European Central Bank Working Paper No.75, 2011.

10. Keller, Gerald, "*Statistics for Management & Economics*" (Tenth Edition) – 2014.

CHAPTER-6: ANNEXURES

Annexure-I

Stocks	Skewness	Kurtosis
ACC Ltd.	0.4038	0.4234
Asian Paints	0.0621	0.9534
Axis Bank	-0.0787	1.6190
BHEL	0.2589	2.6175
Bharti Airtel	0.2577	1.2337
Bosch Ltd.	0.3006	1.6779
Cipla	-0.3424	3.1196
Coal India	0.2383	4.3074
DRREDDY	-0.2651	1.4000
GAIL	-0.5068	4.8209
HCL Tech	-0.8372	6.0035
HDFC Bank	0.4327	1.3397
HINDALCO	0.1040	1.0066
HUL	0.3668	2.7338
ITC Ltd.	-0.8012	3.8471
ICICI Bank	0.0686	2.1559
IDEA Cellular	-0.1813	0.8894
Infosys Ltd.	-0.1891	6.5231
L&T	-0.1703	1.5445
Lupin	-0.2226	1.3265
Maruti	0.0238	4.3191
ONGC	0.1307	3.5383
Reliance Ind.	-0.1707	2.3945
SBI	0.2085	2.0670
Tata Steel	-0.4592	2.5304
Indices		
S&P CNX NIFTY	-0.8152	4.2161
SENSEX	-0.7980	4.3050
NIFTY 500	-1.1764	6.3557
BSE 500	-1.1998	6.5153

Table 6.1 - Skewness & Kurtosis for Securities and Indices

Annexure-II

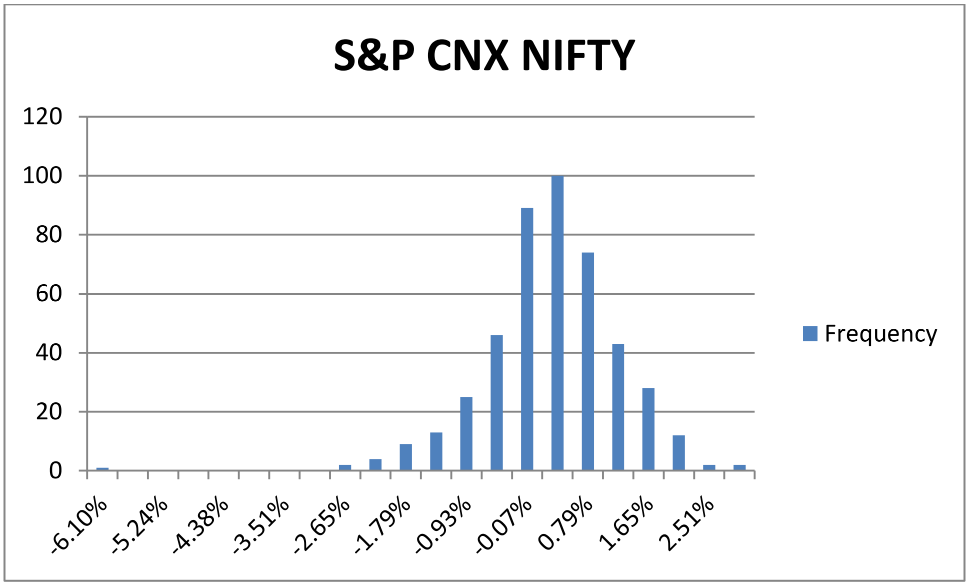

Fig. 6.1 – Histogram of Returns for S&P CNX NIFTY

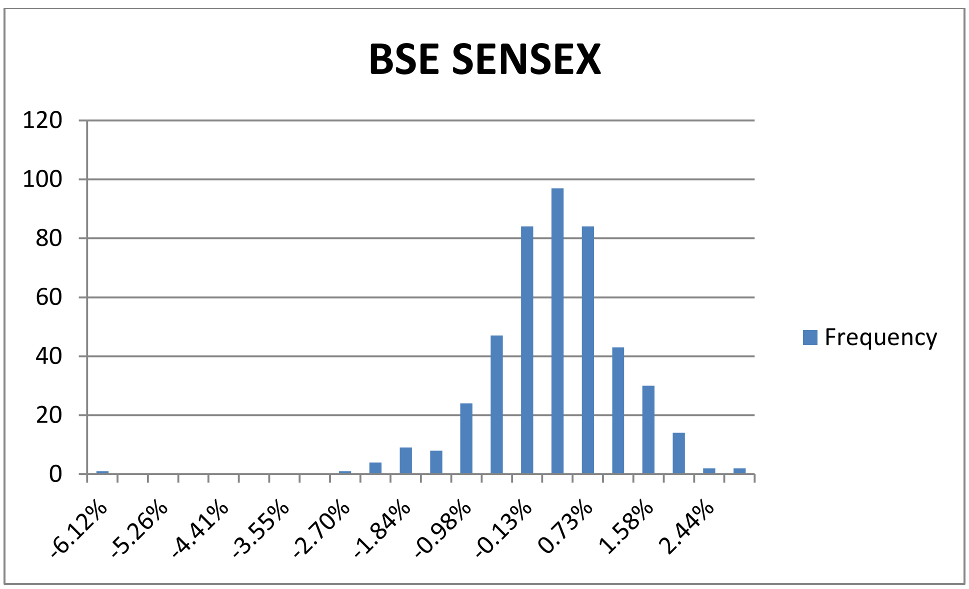

Fig. 6.2 – Histogram of Returns for BSE SENSEX

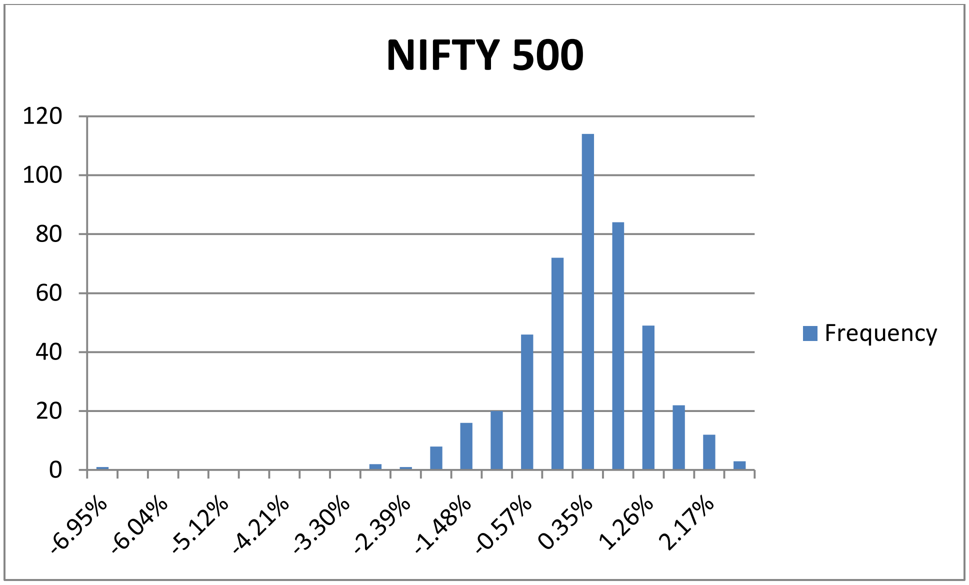

Fig. 6.3 – Histogram of returns for NIFTY 500

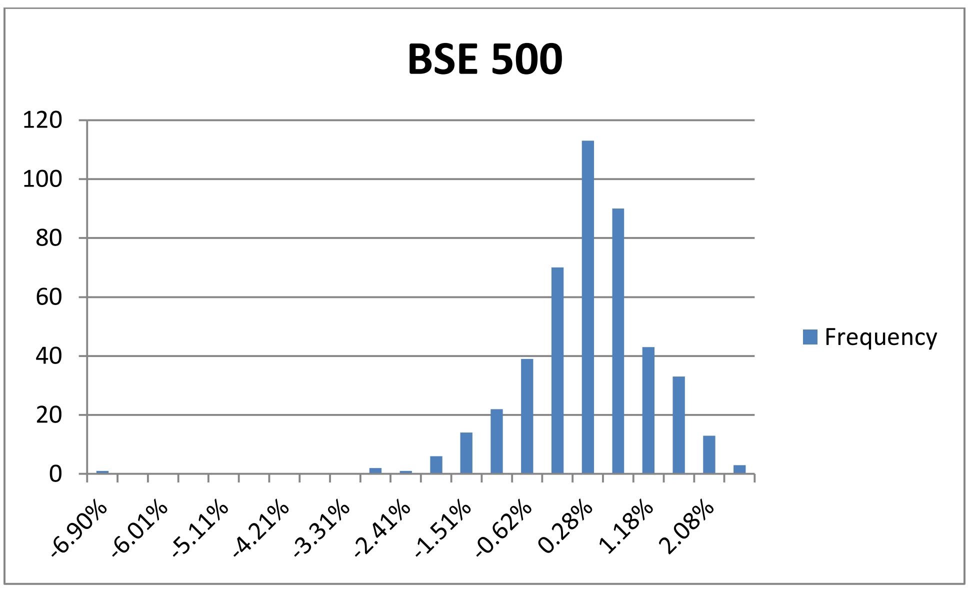

Fig. 6.4 – Histogram of Return for BSE 500