INFLUENCE OF SHAPE FACTOR OF PRISMATIC ELEMENT ON THE ENERGY OF RAYLEIGH WAVES

A MAJOR PROJECT
SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE AWARD OF THE DEGREE
OF
MASTER OF TECHNOLOGY
IN
GEOTECHNICAL ENGINEERING

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JUNE, 2019

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CANDIDATE'S DECLARATION

I, ABHISHEK BANSAL, 2K17/GTE/01 of M.Tech (Geotechnical Engineering), hereby declare that project dissertation titled "Influence of Shape Factor of Prismatic Element on the Energy of Rayleigh Waves" which is submitted by me to the Department of Civil Engineering, Delhi Technological University, Delhi in fulfillment of the requirement for the award of the Master of Technology, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

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CERTIFICATE

I hereby certify that the Project Dissertation entitled "INFLUENCE OF SHAPE FACTOR OF PRISMATIC ELEMENT ON THE ENERGY OF RAYLEIGH WAVES" which is submitted by ABHISHEK BANSAL, 2K17/GTE/01 of M.Tech (Geotechnical Engineering) Delhi Technological University, Delhi in fulfillment of the requirement for the award of the degree of Master of Technology, is a record of the project work carried out by the him under my supervision. To the best of my knowledge this work has not been submitted in part or full for any Degree or Diploma to this University or elsewhere and plagiarism of 9% is found in this report.

Place: Delhi — Dr. ASHUTOSH TRIVEDI

Date: — **(SUPERVISOR)**

ACKNOWLEDGEMENT

I thank god almighty for bestowing his blessings and grace in completion of this thesis.

I express my sincere thanks to the University Management.

As I write this acknowledgement, I must clarify that this is not just a formal acknowledgement but also a sincere note of thanks and regard from my side. I feel a deep sense of gratitude and affection for those who were associated with the project and without whose co-operation and guidance this project could not have been conducted properly.

Words fail me to express my regards towards my project guide, Prof. A. Trivedi and Dean (IRD) Department of Civil Engineering, Delhi Technological University, Delhi for giving me an opportunity to work under his guidance, which really instilled in me the requisite confidence.

I also express my deep gratitude to Prof. Naresh Kumar, and Prof. A.K. Shrivastava, Department of Civil Engineering and all the Faculty member of Civil Engineering Delhi Technological University, Delhi for the pains taken by him to go through the manuscript and giving his useful suggestions.

Last but not the least, I would like to thank my family and friends who stimulated me to bring this work to a successful close.

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CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1
INTRODUCTION

Seismic wave are the form of energy waves which are generated by either due to the sudden breaking of rocks inside the earth's surface or due to some explosion or can be even caused by some vibrational disturbance caused inside or on the earth and can travel several kilometres within some second just like tsunami travels through the ocean, or the sound travels through the air. The types of seismic waves are shown in Fig. No. 1.1

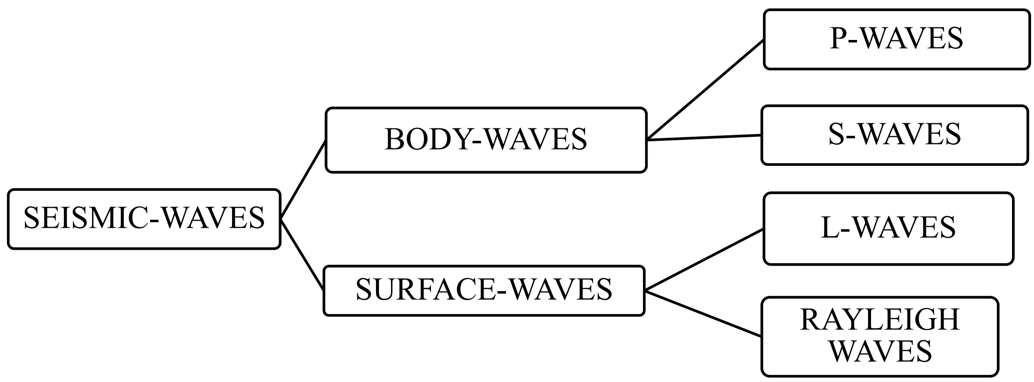


Figure 1.1 Types of Seismic Waves

Rayleigh wave carries about 66% of the total energy of seismic waves. Rayleigh wave is the most hazardous one as the vast majority of the destruction and shaking occurring from an earthquake is due to Rayleigh waves only, which is always greater than the other form of waves. Hence it is most important to understand the velocity of Rayleigh waves and the displacement produced by them. As energy is lost at every instant of time, therefore, an attenuation factor is to be considered which varies with many factors and one of them is found to shape factor.

CHAPTER 2
SHAPE FACTOR

Shape factor refers to a particular value which is affected by an object's shape and is independent of its dimensions and hence is also called as shape modifiers. It is a value by which physical properties (eg: the moment of inertia) of an object gets affected. In most of the design codes, the bearing capacity of two different loads like that for circular loads is assumed similar to square loads. In this way, the shape factors of the numerical axially symmetric (round) solution can be compared with the shape factors for square loads (B= L).

Various scientist like Terzaghi, Meyerhof, Hansen, Vesic, etc. have done various research related to shape factor. Terzaghi (1943) test are based on the results of the tests performed by Golder (1941), Vesic (1967) republished by Fang (1990), assumed failure surface was identical to Terzaghi's but at an angle which is inclined angle and results of all differs with one other all these have done their research considering soil to be elastic-plastic and some of them considered water table factor to calculate the various properties of soils. The work of above scientist cannot be used as they are limited to a particular case only or have not considered the effect of all the parameters. But Schmertmann et al. (1978) did all the work in the soil as elastic and his work is been used for an elastic type of soil since then.

However, the equations Schmertmann Et Al. (1978) used; are found to be more suitable results for shallow foundations. But among all of the above scientists, none have considered soil to purely elastic except Schmertmann et al. (1978). Hence using Schmertmann et al. (1978) shape factors values.

$$SF = f_1(\acute{m}, \acute{n}) + f_2(\mu, \acute{m}, \acute{n}) \tag{2.1}$$

$$SF = F_1 + F_2\left[\frac{(1-2\mu)}{(1-\mu)}\right] \tag{2.2}$$

$$f_1(\acute{m}, \acute{n}) = \frac{1}{\pi}(A_0 + A_1) \tag{2.3}$$

$$F_2 = \frac{\acute{n}}{\pi} tan^{-1} A_2 \tag{2.4}$$

$$A_0 = \acute{m}\, ln\left(\frac{\left(1+\sqrt{\acute{m}^2+1}\right)\sqrt{\acute{m}^2+\acute{n}^2}}{\acute{m}\left(1+\sqrt{\acute{m}^2+\acute{n}^2+1}\right)}\right) \tag{2.5}$$

$$A_1 = ln\left(\frac{\left(\acute{m}+\sqrt{\acute{m}^2+1}\right)\sqrt{1+\acute{n}^2}}{\acute{m}(1+\sqrt{\acute{m}^2+\acute{n}^2+1})}\right) \tag{2.6}$$

$$A_2 = \frac{\acute{m}}{\acute{n}(1+\sqrt{\acute{m}^2+\acute{n}^2+1})} \tag{2.7}$$

$$\acute{m} = \frac{L}{B} \tag{2.8}$$

$$\acute{n} = \frac{H}{B} \tag{2.9}$$

$$\mathrm{SF}=\frac{1}{\pi}\left(\frac{L}{B}\,ln\left(\frac{\left(1+\sqrt{\frac{L}{B}^2+1}\right)\sqrt{\frac{L}{B}^2+\frac{H}{B}^2}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^2+\frac{H}{B}^2+1}\right)}\right)+ln\left(\frac{\left(\frac{L}{B}+\sqrt{\frac{L}{B}^2+1}\right)\sqrt{1+\frac{H}{B}^2}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^2+\frac{H}{B}^2+1}\right)}\right)\right)+\frac{\frac{H}{B}}{\pi}tan^{-1}\left(\frac{\frac{L}{B}}{\frac{H}{B}\left(1+\sqrt{\frac{\acute{L}}{B}^2+\frac{H}{B}^2+1}\right)}\right) \tag{2.10}$$

For exact values of shape factor, see Appendix-II which provides value of shape factor for different values of $\acute{m}, \acute{n}, F_1, F_2$. One of the most important and commonly used values for shape factor for elastic medium is of Schmertmann as shown in Eqn. (2.10) which is a function of poisson’s ratio also and this equation or shape factor values are induced in the value of Rayleigh waves.

CHAPTER 3
NUMERICAL SOLUTION OF ENERGY OF RAYLEIGH WAVES

Rayleigh waves exist close to the limit of an elastic-half space and have both longitudinal and transverse movements which reduce exponentially as the distance from surface increases. A plane wave passing from an elastic prismatic member is considered. In this z is positive towards downward direction as zero is assumed on the surface of the earth and the boundary of elastic-half space is assumed to be x-y plane. Here, w and u are the displacements due to Rayleigh waves in the direction z and x, respectively. Both the displacements are independent of y. Therefore,

$$\begin{cases} u = \frac{\partial \emptyset}{\partial x} + \frac{\partial \varphi}{\partial z} \\ w = \frac{\partial \emptyset}{\partial z} - \frac{\partial \varphi}{\partial x} \end{cases} \tag{3.1}$$

where ψ and ϕ are two potential functions. The dilation $\bar{\varepsilon}$ can be defined as

$$\bar{\epsilon} = \epsilon_x + \epsilon_y + \epsilon_z = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \tag{3.2}$$

As we know,

$$\frac{\partial v}{\partial y} = 0 \tag{3.3}$$

Therefore,

$$\bar{\epsilon} = \left(\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \varphi}{\partial x \partial z}\right) + (0) + \left(\frac{\partial^2 \emptyset}{\partial z^2} - \frac{\partial^2 \varphi}{\partial x \partial z}\right) \tag{3.4}$$

$$\bar{\epsilon} = \left(\frac{\partial^2 \emptyset}{\partial x^2}\right) + \left(\frac{\partial^2 \emptyset}{\partial z^2}\right) = \nabla^2 \emptyset \tag{3.5}$$

Rotating the x-z plane, Eqn. can be written as,

$$2\bar{\omega}_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = \left(\frac{\partial^2 \varphi}{\partial x^2}\right) + \left(\frac{\partial^2 \varphi}{\partial z^2}\right) = \nabla^2 \varphi \tag{3.6}$$

From the eqn. of motion for compression wave,

$$\begin{cases} \rho\left(\frac{\partial^2 u}{\partial t^2}\right) = (\lambda + G)\frac{\partial \bar{\varepsilon}}{\partial x} + G(\nabla^2 u) \\ \rho\left(\frac{\partial^2 w}{\partial t^2}\right) = (\lambda + G)\frac{\partial \bar{\varepsilon}}{\partial z} + G(\nabla^2 w) \end{cases} \tag{3.7}$$

Substituting Eqns. (3.1), (3.5) in (3.7) yields

$$\rho\frac{\partial}{\partial x}\left(\frac{\partial^2\emptyset}{\partial t^2}\right)+\rho\frac{\partial}{\partial z}\left(\frac{\partial^2\varphi}{\partial t^2}\right)=(\lambda+2G)\frac{\partial}{\partial x}(\nabla^2\emptyset)+G\frac{\partial}{\partial z}(\nabla^2\varphi) \tag{3.8}$$

In a similar manner, substituting (3.2), (3.7) in (3.9)

$$\rho\frac{\partial}{\partial z}\left(\frac{\partial^2\emptyset}{\partial t^2}\right)-\rho\frac{\partial}{\partial x}\left(\frac{\partial^2\varphi}{\partial t^2}\right)=(\lambda+2G)\frac{\partial}{\partial z}(\nabla^2\emptyset)-G\frac{\partial}{\partial x}(\nabla^2\varphi) \tag{3.9}$$

Equations (3.10) and (3.11) will be satisfied if

$$(1)\ \left(\frac{\partial^2\emptyset}{\partial t^2}\right)=(\lambda+2G)\frac{\partial}{\partial x}(\nabla^2\emptyset)=\nu^2{}_p\nabla^2\emptyset \tag{3.10}$$

$$(2)\ \left(\frac{\partial^2\varphi}{\partial t^2}\right)=\left(\frac{G}{\rho}\right)(\nabla^2\varphi)=\nu^2{}_s\nabla^2\varphi \tag{3.11}$$

Considering sinusoidal wave which is moving in positive x-direction.

Assuming the solution be expressed of $\emptyset\ and\ \varphi$ as

$$\begin{cases}\emptyset=F(z)\exp[i(\omega t-fx)]\\ \varphi=G(z)\exp[i(\omega t-fx)]\end{cases} \tag{3.12}$$

where G(z) and F(z) are considered to be functions of depth

$$f=\frac{2\pi}{wavelength} \tag{3.13}$$

$$i=\sqrt{-1} \tag{3.14}$$

Substituting Eq. (3.14) in (3.12) and solving, we get

$$-\omega^2F(\mathrm{z})=\nu^2{}_p[F''(\mathrm{z})-f^2F(z)] \tag{3.15}$$

Similarly,

$$-\omega^2G(\mathrm{z})=\nu_s{}^2[G''(\mathrm{z})-f^2G(z)] \tag{3.16}$$

Solving further gives,

$$\begin{cases}F(z)=A_1e^{-qz}+A_2e^{qz}\\ G(z)=B_1e^{-sz}+B_2e^{sz}\end{cases} \tag{3.17}$$

where A_1, A_2, B_1, B_2 are constants and

$$\frac{q^2}{f^2}=1-\frac{\omega^2}{\nu_p{}^2} \tag{3.18}$$

$$\frac{s^2}{f^2}=1-\frac{\omega^2}{\nu_s{}^2} \tag{3.19}$$

as F(z) and G(z) cannot approach infinity therefore $A_2, B_2=0$ therefore,

$$\begin{cases}F(z)=A_1e^{-qz}\\ G(z)=B_1e^{-sz}\end{cases} \tag{3.20}$$

therefore,

$$\begin{cases}\emptyset=A_1e^{-qz}\exp[i(\omega t-fx)]\\ \varphi=B_1e^{-sz}\exp[i(\omega t-fx)]\end{cases} \tag{3.21}$$

Applying boundary conditions for retaining wall at z=0, $\sigma_z = 0, \tau_{zx} = 0, \tau_{zy} = 0$

We know that,

$$\sigma_z = \lambda\bar{\varepsilon} + 2G\varepsilon_z = \lambda\bar{\varepsilon} + 2G\left(\frac{\partial w}{\partial z}\right) = 0 \tag{3.23}$$

$$\tau_{zx(z=0)} = G\gamma_{zx} = G\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) = 0 \tag{3.24}$$

From (3.1)- (3.2), (3.11)- (3.29) put values of all and solving, we get

$$16\left(1 - \frac{\omega^2}{\nu_p{}^2 f^2}\right)\left(1 - \frac{\omega^2}{\nu_s{}^2 f^2}\right) = \left[2 - \left(\frac{\lambda+2G}{G}\right)\frac{\omega^2}{\nu_p{}^2 f^2}\right]^2 \left[2 - \frac{\omega^2}{\nu_s{}^2 f^2}\right] \tag{3.25}$$

$$\text{Wavelength} = \frac{velocity\ of\ wave}{\omega/_{2\pi}} = \frac{\nu_r}{\omega/_{2\pi}} \tag{3.26}$$

Therefore,

$$\begin{cases} \frac{\nu_r{}^2}{\nu_s{}^2} = V^2 \\ \frac{\nu_r{}^2}{\nu_p{}^2} = \alpha^2 V^2 \\ \nu_p{}^2 = \frac{\lambda+2G}{\rho} \\ \nu_s{}^2 = \frac{G}{\rho} \\ \lambda = \frac{(2\mu G)}{(1-2\mu)} \end{cases} \tag{3.27}$$

$$\alpha^2 = \frac{\nu_s{}^2}{\nu_p{}^2} = \frac{G}{\lambda+2G} = \frac{(1-2\mu)}{(2-2\mu)} \tag{3.28}$$

Substituting above values in Eqn. (3.30)

$$V^6 - 8V^4 - (16\alpha^2 - 24)V^2 - 16(1-\alpha^2) = 0 \tag{3.29}$$

$$V^6 - 8V^4 - \left(16\left[\frac{(1-2\mu)}{(2-2\mu)}\right] - 24\right)V^2 - 16\left(1 - \left[\frac{(1-2\mu)}{(2-2\mu)}\right]\right) = 0 \tag{3.30}$$

Equation 3.29 is a cubic equation in V^2. For a given value of Poisson's ratio, the value of V^2 can be found and hence ν_r can be determined in terms of velocity of p-waves (ν_p)or velocity of s-waves(ν_s).

For displacement of Rayleigh Waves, put (3.21) in (3.1) and put B_1in terms of A_1 we get,

$$u = ifA_1(-e^{-qz} + \frac{2qs}{s^2+f^2}e^{-sz})\exp\left[i(\omega t - fx)\right] \tag{3.31}$$

$$w = A_1 q(-e^{-qz} + \frac{2f^2}{s^2+f^2}e^{-sz})\exp\left[i(\omega t - fx)\right] \tag{3.32}$$

Rate of attenuation of displacement along x-direction with depth z will depend on the factor U where,

$$U = (-e^{-qz}) + \left(\frac{2qs}{s^2+f^2}e^{-sz}\right) = (-e^{-\left(\frac{q}{f}\right)(fz)}) + \left[\frac{2\left(\frac{q}{f}\right)\left(\frac{s}{f}\right)}{\frac{s^2}{f^2}+1}\right](e^{-\left(\frac{s}{f}\right)(fz)}) \quad (3.33)$$

Rate of attenuation of displacement along z-direction with depth z will depend on the factor W where,

$$W = (-e^{-qz}) + \left(\frac{2f^2}{s^2+f^2}e^{-sz}\right) = (-e^{-\left(\frac{q}{f}\right)(fz)}) + \left[\frac{2}{\frac{s^2}{f^2}+1}\right](e^{-\left(\frac{s}{f}\right)(fz)}) \quad (3.34)$$

where,

$$\frac{q^2}{f^2} = 1 - \frac{\omega^2}{{v_p}^2} = 1 - \frac{{v_r}^2}{{v_p}^2} = 1 - \alpha^2 V^2 \quad (3.35)$$

$$\frac{s^2}{f^2} = 1 - \frac{\omega^2}{{v_s}^2} = 1 - \frac{{v_r}^2}{{v_s}^2} = 1 - V^2 \quad (3.36)$$

Put Eqn. (3.13) in Eqn. (3.33) and (3.34), we get

$$U = (-e^{-\left(\frac{q}{f}\right)\left(\frac{2\pi Z}{wavelength}\right)}) + \left[\frac{2\left(\frac{q}{f}\right)\left(\frac{s}{f}\right)}{\frac{s^2}{f^2}+1}\right](e^{-\left(\frac{s}{f}\right)\left(\frac{2\pi Z}{wavelength}\right)}) \quad (3.37)$$

$$W = (-e^{-\left(\frac{q}{f}\right)\left(\frac{2\pi z}{wavelength}\right)}) + \left[\frac{2}{\frac{s^2}{f^2}+1}\right](e^{-\left(\frac{s}{f}\right)\left(\frac{2\pi z}{wavelength}\right)}) \quad (3.38)$$

we know that,

Intensity of load transmitted to the subgrade can be given by $Q = \frac{W}{A}$ (3.39)

The coefficient of subgrade reaction k_s can be given by $k = \frac{Q}{z_s}$ (3.40)

Therefore from Eqn. (3.48) and (3.49) we get,

$$wavelength = \sqrt{\frac{k}{m}} = \sqrt{\frac{Q}{(z_s)m}} = \sqrt{\frac{W}{A(z_s)m}} = \sqrt{\frac{mg}{A(z_s)m}} = \sqrt{\frac{g}{A(z_s)}} \quad (3.41)$$

Here,

g= acceleration due to gravity= 9.81m/s^2

A= area of the shape under consideration = Shape factor * unit area = SF

Therefore Eqn. (3.46) and (3.47) becomes

$$U = (-e^{-\left(\frac{q}{f}\right)\left(\frac{2\pi z\sqrt{(SF)(z_S)}}{\sqrt{g}}\right)}) + \left[\frac{2\left(\frac{q}{f}\right)\left(\frac{s}{f}\right)}{\frac{s^2}{f^2}+1}\right](e^{-\left(\frac{s}{f}\right)\left(\frac{2\pi z\sqrt{(SF)(z_S)}}{\sqrt{g}}\right)}) \quad (3.42)$$

$$W = (-e^{-\left(\frac{q}{f}\right)\left(\frac{2\pi z\sqrt{(SF)(z_S)}}{\sqrt{g}}\right)}) + \left[\frac{2}{\frac{s^2}{f^2}+1}\right](e^{-\left(\frac{s}{f}\right)\left(\frac{2\pi z\sqrt{(SF)(z_S)}}{\sqrt{g}}\right)}) \quad (3.43)$$

Energy= Potential Energy + Kinetic Energy (3.44)

Energy= Work done for displacement+ Kinetic Energy (3.45)

Energy= Force * displacement + Kinetic Energy (3.46)

Energy= Mass * Acceleration*Displacement + Kinetic Energy (3.47)

$$\text{Energy= Mass} * \frac{\text{Velocity}}{\text{Time}} * \text{Displacement} + \frac{1}{2} * \text{Mass} * (\text{velocity})^2 \quad (3.48)$$

$$\frac{\text{Energy}}{\text{Mass}} = \frac{\text{Velocity}}{\text{Time}} * \text{Displacement} + \frac{1}{2} * (\text{velocity})^2 \quad (3.49)$$

Rate of attenuation of energy per unit mass is given by $\ddot{E}$

$\ddot{E} = V * (\mathrm{U}) + \frac{1}{2} * (V)^2$ and $\ddot{E} = V * (\mathrm{W}) + \frac{1}{2} * (V)^2$ along x-direction and z-direction respectively. So, the final equations with effect of shape factor on energy with the help of Eqn. (4.50) and (4.51) will become:

$$\ddot{\mathrm{E}}_x = \mathrm{V} * ((-\mathrm{e}^{-\left(\frac{\mathrm{q}}{\mathrm{f}}\right)\left(\frac{2\pi z\sqrt{(\mathrm{SF})(z_S)}}{\sqrt{g}}\right)}) + \left[\frac{2\left(\frac{\mathrm{q}}{\mathrm{f}}\right)\left(\frac{\mathrm{s}}{\mathrm{f}}\right)}{\frac{\mathrm{s}^2}{\mathrm{f}^2}+1}\right](\mathrm{e}^{-\left(\frac{\mathrm{s}}{\mathrm{f}}\right)\left(\frac{2\pi z\sqrt{(\mathrm{SF})(z_S)}}{\sqrt{g}}\right)})) + \frac{1}{2} * (\mathrm{V})^2 \quad (3.50)$$

$$\ddot{\mathrm{E}}_z = \mathrm{V} * ((-\mathrm{e}^{-\left(\frac{\mathrm{q}}{\mathrm{f}}\right)\left(\frac{2\pi z\sqrt{(\mathrm{SF})(z_S)}}{\sqrt{g}}\right)}) + \left[\frac{2}{\frac{\mathrm{s}^2}{\mathrm{f}^2}+1}\right](\mathrm{e}^{-\left(\frac{\mathrm{s}}{\mathrm{f}}\right)\left(\frac{2\pi z\sqrt{(\mathrm{SF})(z_S)}}{\sqrt{g}}\right)})) + \frac{1}{2} * (\mathrm{V})^2 \quad (3.51)$$

Here, Eqn. (3.50) is Rate of attenuation of energy per unit mass with depth and shape factor variation in horizontal-direction and Eqn. (3.51) is for rate of attenuation of energy per unit mass with depth and shape factor variation in vertical-direction.

Using the value of shape factor of Schmertmann et al. (1978) from Eqn. (2.10) in Eqn. (3.50) and (3.51).

Energy Ratio $\ddot{\mathrm{E}}_x = \frac{\ddot{\mathrm{E}}_x\, at\ z}{\ddot{\mathrm{E}}_x\, at\ z=0}$ and $\ddot{\mathrm{E}}_z = \frac{\ddot{\mathrm{E}}_z\, at\ z}{\ddot{\mathrm{E}}_z\, at\ z=0}$ (3.54)

$$\ddot{\mathrm{E}}_x = V * \left\{ \left\{ -e^{-\left(\frac{q}{f}\right)\left[\frac{2\pi z}{\sqrt{g}}\sqrt{\left[\frac{1}{\pi}\left(\frac{L}{B}\ln\left(\frac{\left(1+\sqrt{\frac{L}{B}^{2}+1}\right)\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}+1}\right)}\right)+\ln\left(\frac{\left(\frac{L}{B}+\sqrt{\frac{L}{B}^{2}+1}\right)\sqrt{1+\frac{H}{B}^{2}}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}+1}\right)}\right)\right)+\left[\frac{(1-2\mu)}{(1-\mu)}\right]\left(\frac{H}{\pi B}\left(\tan^{-1}\left(\ln\left(\frac{\left(\frac{L}{B}+\sqrt{\frac{L}{B}^{2}+1}\right)\sqrt{1+\frac{H}{B}^{2}}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}+1}\right)}\right)\right)\right)\right)\right](z_S)}\right]} \right\} +$$

$$\left(\left[\frac{2\left(\frac{q}{f}\right)\left(\frac{s}{f}\right)}{\frac{s^2}{f^2}+1}\right]\left\{ e^{-\left(\frac{s}{f}\right)\left[\frac{2\pi z}{\sqrt{g}}\sqrt{\left[\frac{1}{\pi}\left(\frac{L}{B}\ln\left(\frac{\left(1+\sqrt{\frac{L}{B}^{2}+1}\right)\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}+1}\right)}\right)+\ln\left(\frac{\left(\frac{L}{B}+\sqrt{\frac{L}{B}^{2}+1}\right)\sqrt{1+\frac{H}{B}^{2}}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}+1}\right)}\right)\right)+\left[\frac{(1-2\mu)}{(1-\mu)}\right]\left(\frac{H}{\pi B}\left(\tan^{-1}\left(\ln\left(\frac{\left(\frac{L}{B}+\sqrt{\frac{L}{B}^{2}+1}\right)\sqrt{1+\frac{H}{B}^{2}}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}+1}\right)}\right)\right)\right)\right)\right](z_S)}\right]} \right\}\right) \right\} + \frac{1}{2} * (V)^2 \tag{3.52}$$

$$\ddot{E}_z = V * \left\{ \left\{ -e^{-\left(\frac{q}{f}\right)\frac{2\pi z}{\sqrt{g}}\left[\sqrt{\left[\frac{1}{\pi}\left(\frac{L}{B}\ln\left(\frac{\left(1+\sqrt{\frac{L}{B}^{2}+1}\right)\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}+1}\right)}\right)+\ln\left(\frac{\left(\frac{L}{B}+\sqrt{\frac{L}{B}^{2}+1}\right)\sqrt{1+\frac{H}{B}^{2}}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}+1}\right)}\right)\right)+\left[\frac{(1-2\mu)}{(1-\mu)}\right]\left(\frac{H}{\pi B}\left(\tan^{-1}\left(\ln\left(\frac{\left(\frac{L}{B}+\sqrt{\frac{L}{B}^{2}+1}\right)\sqrt{1+\frac{H}{B}^{2}}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}+1}\right)}\right)\right)\right)\right)\right]}(z_S)\right]} \right\} +$$

$$\left(\left[\frac{2}{\frac{s^2}{f^2}+1}\right]\left\{ e^{-\left(\frac{s}{f}\right)\frac{2\pi z}{\sqrt{g}}\left[\sqrt{\left[\frac{1}{\pi}\left(\frac{L}{B}\ln\left(\frac{\left(1+\sqrt{\frac{L}{B}^{2}+1}\right)\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}+1}\right)}\right)+\ln\left(\frac{\left(\frac{L}{B}+\sqrt{\frac{L}{B}^{2}+1}\right)\sqrt{1+\frac{H}{B}^{2}}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}+1}\right)}\right)\right)+\left[\frac{(1-2\mu)}{(1-\mu)}\right]\left(\frac{H}{\pi B}\left(\tan^{-1}\left(\ln\left(\frac{\left(\frac{L}{B}+\sqrt{\frac{L}{B}^{2}+1}\right)\sqrt{1+\frac{H}{B}^{2}}}{\frac{L}{B}\left(1+\sqrt{\frac{L}{B}^{2}+\frac{H}{B}^{2}+1}\right)}\right)\right)\right)\right)\right]}(z_S)\right]} \right\}\right)\right\} + \frac{1}{2} * (V)^2 \qquad (3.53)$$

Eqn. (3.52) and (3.53) represents the rate of attenuation of energy per unit mass with depth and shape factor variation in horizontal-direction and in vertical-direction with the value of shape factor of Schmertmann et al. (1978) respectively.

CHAPTER 4
RESULTS & DISCUSSION

A Cohesionless elastic soil is being considered whose Poisson's Ratio is varied or changed as per Table 4.1 and foundation window or prismatic element is considered for different L/B and H/B ratios i.e. 1, 5, 10, 100 and for different poisson's ratio also i.e. 0.25, 0.29, 0.33, 0.40, 0.50.
Displacement in x and z direction is considered and an elastic half-space and include both longitudinal and transverse motions.
Neglecting complex values of V, different values of different properties obtained are listed in tables below.

Table 4.1 Lame's constant for different Poisson's ratio

Poisson's Ratio	Lame's Constant/ Shear Modulus
0.25	1
0.29	1.38095238
0.33	1.94117647
0.4	4
0.5	INFINITY

Table 4.2 Velocity Ratio for different Poisson's Ratio

Poisson's Ratio	0.25	0.29	0.33	0.4	0.5
V_1	-0.919401687	-0.925843	-0.932023	-0.942195	-0.955313
V_2	0.919401687	0.925843	0.932023	0.942195	0.955313
V_3	-1.776147669	-1.89697 - 0.1645921i	-1.90698 - 0.266301i	1.92765 - 0.399607i	-1.96606 - 0.567196i
V_4	1.776147669	1.89697 + 0.1645921i	1.90698 + 0.266301i	-1.92765 + 0.399607i	1.96606 + 0.567196i
V_5	-1.999999999	1.89697 - 0.1645921i	1.90698 - 0.266301i	-1.92765 - 0.399607i	1.96606 - 0.567196i
V_6	1.999999999	-1.89697 + 0.1645921i	-1.90698 + 0.266301i	1.92765 + 0.399607i	-1.96606 + 0.567196i

Table 4.3 Shape Factor for different L/B & H/B ratio for Poisson's Ratio = 0.25

Poisson's Ratio= 0.25	n=H/B=1	n=H/B=5	n=H/B=10	n=H/B=100
m=L/B = 1	0.15583333	0.13333333	0.13166667	0.13083333
m=L/B = 5	0.44216667	0.5705	0.55733333	0.54516667
m=L/B = 10	0.50066667	0.76983333	0.78666667	0.76133333
m=L/B = 100	0.55533333	1.02133333	1.21166667	1.51783333

Table 4.4 Shape Factor for different L/B & H/B ratio for Poisson's Ratio = 0.29

Poisson's Ratio= 0.29	n=H/B=1	n=H/B=5	n=H/B=10	n=H/B=100
m=L/B = 1	0.15427465	0.13104225	0.12933803	0.12848592
m=L/B = 5	0.44158451	0.56841549	0.55470423	0.54221831
m=L/B = 10	0.5003662	0.7685	0.78456338	0.7583662
m=L/B = 100	0.55529577	1.0211831	1.2113662	1.51571127

Table 4.5 Shape Factor for different L/B & H/B ratio for Poisson's Ratio = 0.33

Poisson's Ratio= 0.33	n=H/B=1	n=H/B=5	n=H/B=10	n=H/B=100
m=L/B = 1	0.15252985	0.12847761	0.12673134	0.12585821
m=L/B = 5	0.44093284	0.56608209	0.55176119	0.53891791
m=L/B = 10	0.50002985	0.76700746	0.78220896	0.75504478
m=L/B = 100	0.55525373	1.02101493	1.21102985	1.51333582

Table 4.6 Shape Factor for different L/B & H/B ratio for Poisson's Ratio = 0.4

Poisson's Ratio= 0.4	n=H/B=1	n=H/B=5	n=H/B=10	n=H/B=100
m=L/B = 1	0.14891667	0.12316667	0.12133333	0.12041667
m=L/B = 5	0.43958333	0.56125	0.54566667	0.53208333
m=L/B = 10	0.49933333	0.76391667	0.77733333	0.74816667
m=L/B = 100	0.55516667	1.02066667	1.21033333	1.50841667

Table 4.7 Shape Factor for different L/B & H/B ratio for Poisson's Ratio = 0.5

Poisson's Ratio= 0.5	n=H/B=1	n=H/B=5	n=H/B=10	n=H/B=100
m=L/B = 1	0.142	0.113	0.111	0.11
m=L/B = 5	0.437	0.552	0.534	0.519
m=L/B = 10	0.498	0.758	0.768	0.735
m=L/B = 100	0.555	1.02	1.209	1.499

Table 4.8 Alpha Value for different Poisson's Ratio

Poisson’s Ratio	Square of Alpha	Alpha
0.25	0.333333333	0.577350269
0.29	0.295774648	0.543851678
0.33	0.253731343	0.503717523
0.4	0.166666667	0.40824829
0.5	0	0

Table 4.9 s/f and q/f for different Poisson's Ratio

Poisson's Ratio	(s^2/f^2)=(1-V^2)	q^2/f^2=(1-(ALPHA*V)^2)
0.25	0.154700538	0.718233513
0.29	0.142814739	0.746466331
0.33	0.131333127	0.779591988
0.4	0.112268582	0.852044764
0.5	0.087377072	1

All the graphs show the variation of energy with depth for five different Poisson's ratios (0.50, 0.40. 0.33, 0.29. 0.25) and with different shape factors i.e. for different L/B and H/b ratios. Here we will get four different components of the energy of Rayleigh waves among which two represents horizontal displacement and the other two represents vertical displacements. Further, among these two displacement components, they are divided on the basis of velocity components which can either be negative or positive.

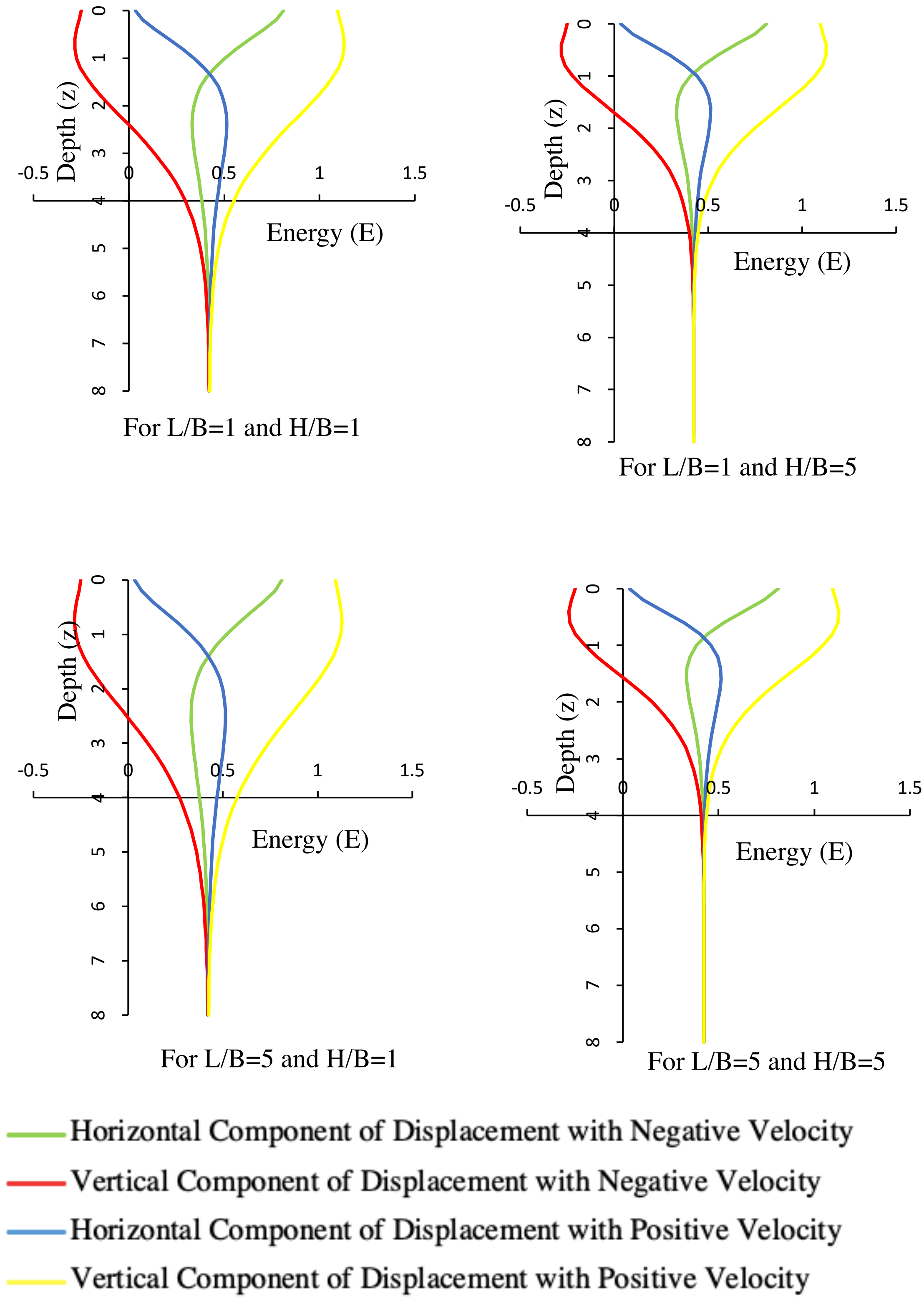


Figure No 4.1 Energy of Rayleigh Waves Vs Depth for $\mu = 0.25$

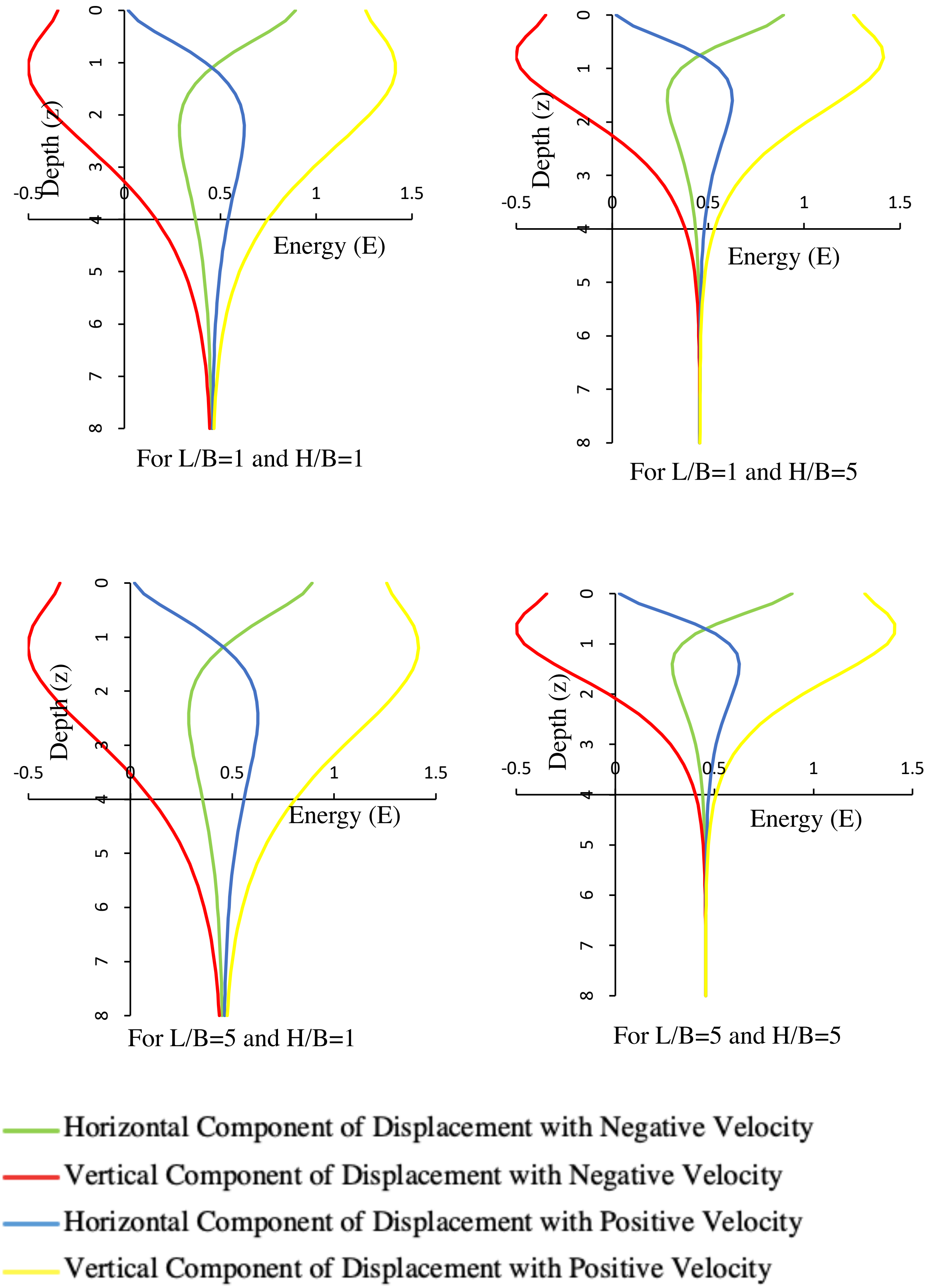
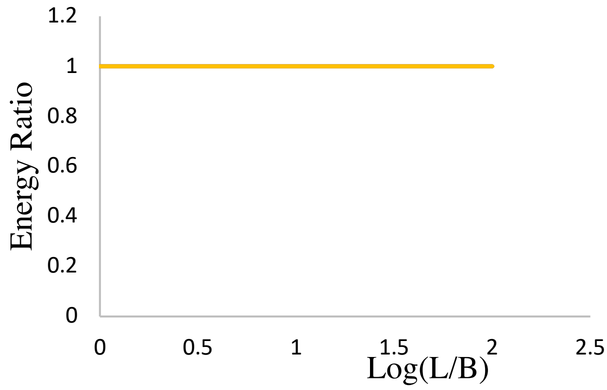
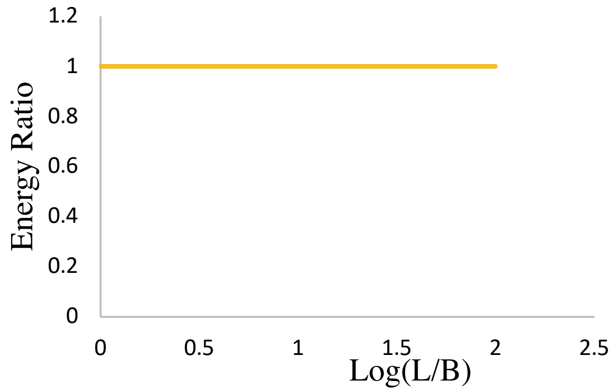


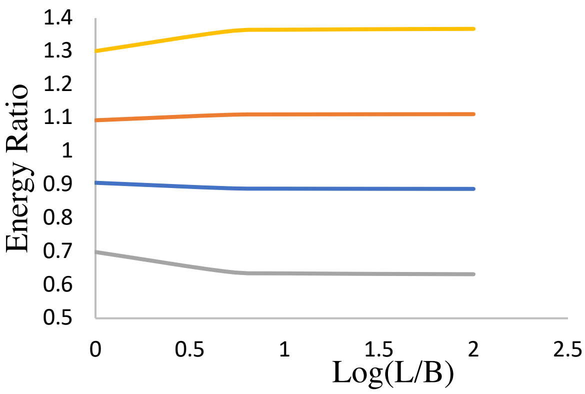
Figure No 4.2 Energy of Rayleigh Waves Vs Depth for $\mu = 0.5$



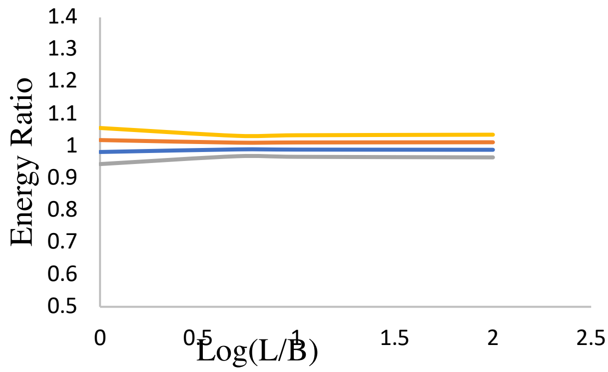
For H/B=1 at depth = 0 m



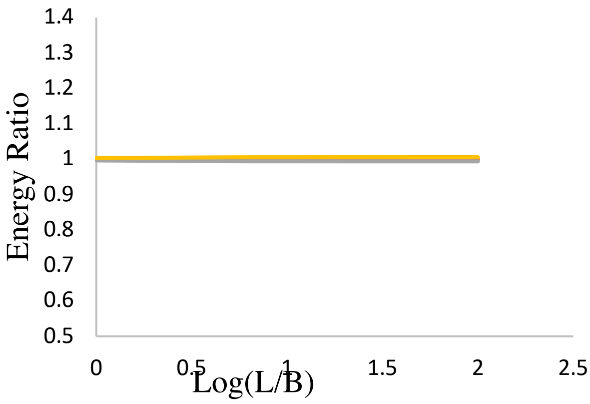
For H/B=5 at depth = 0 m



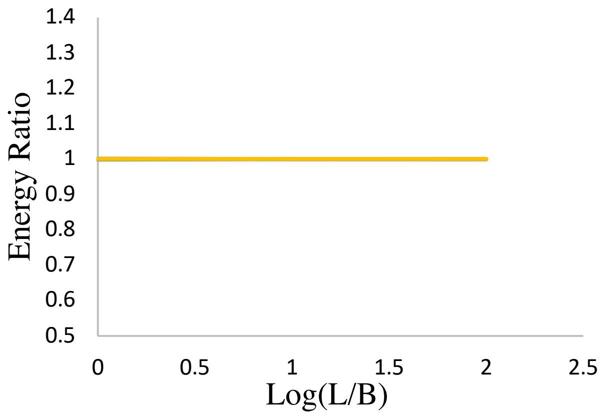
For H/B=1 at depth = 4 m



For H/B=1 at depth = 4 m



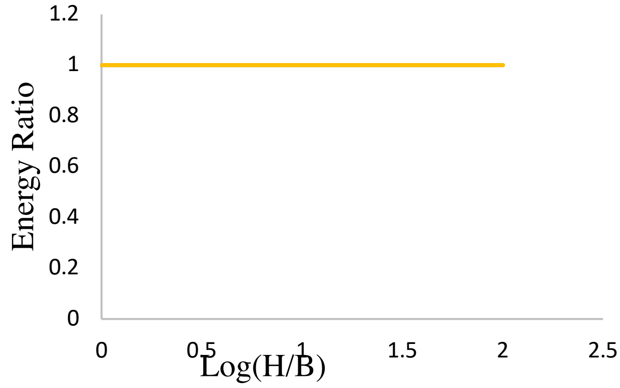
For H/B=1 at depth = 8 m



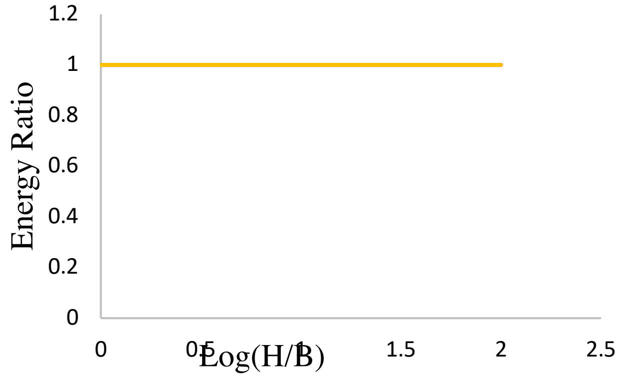
For H/B=1 at depth = 6 m

Horizontal Component of Displacement with Negative Velocity
Vertical Component of Displacement with Negative Velocity
Horizontal Component of Displacement with Positive Velocity
Vertical Component of Displacement with Positive Velocity

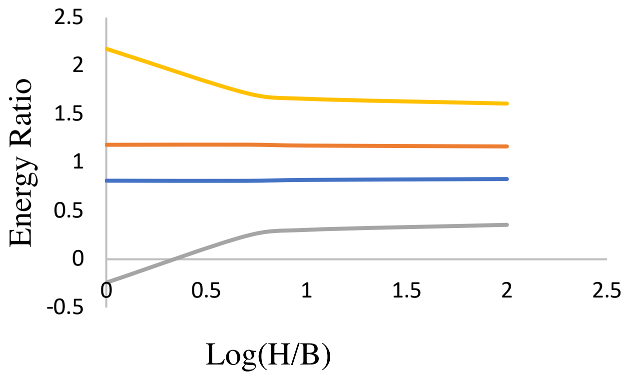
Figure No 4.3 Energy Ratio VS Log(L/B) for poisson's ratio = 0.25 at different depths.



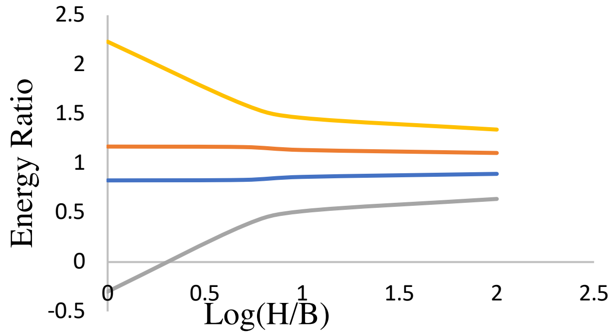
For L/B=1 at depth =0



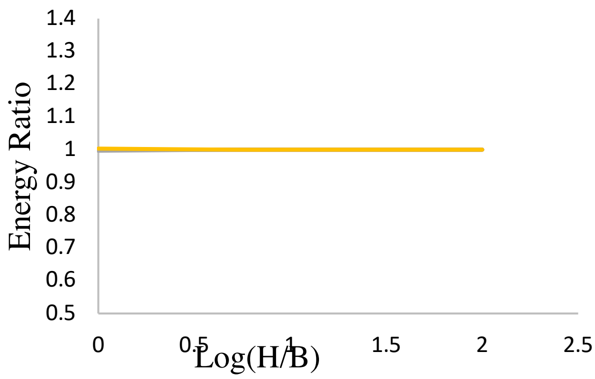
For L/B=5 at depth =0



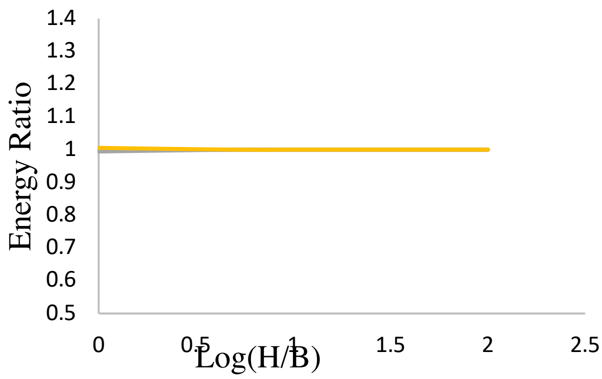
For L/B=1 at depth =2



For L/B=1 at depth =2



For L/B=1 at depth =8



For L/B=1 at depth =8

Horizontal Component of Displacement with Negative Velocity
Vertical Component of Displacement with Negative Velocity
Horizontal Component of Displacement with Positive Velocity
Vertical Component of Displacement with Positive Velocity

Figure No 4.4 Energy Ratio VS Log(H/B) for poisson's ratio = 0.25 at different depths.

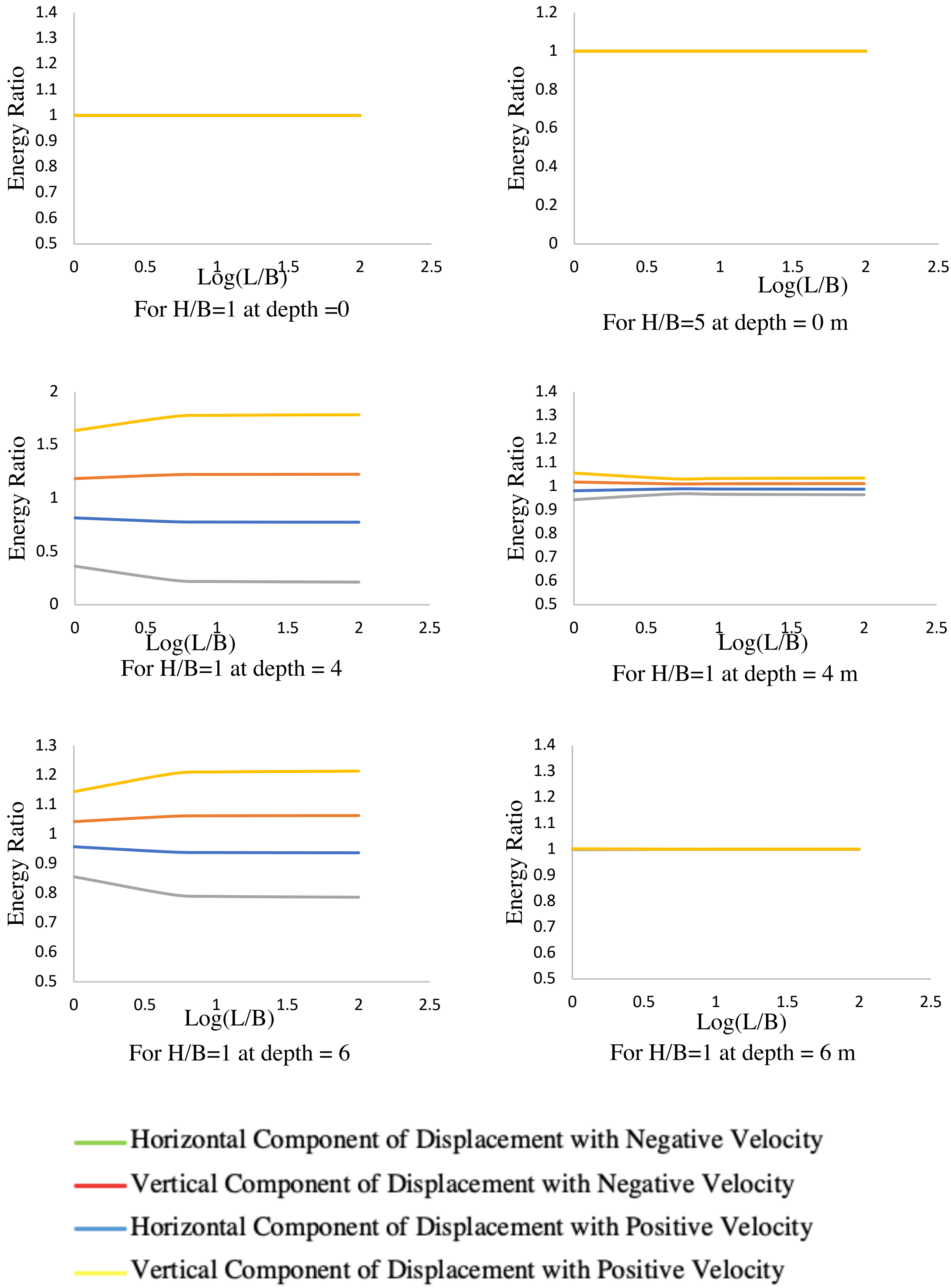


Figure No 4.5 Energy Ratio VS Log(L/B) for poisson's ratio = 0.5 at different depths.

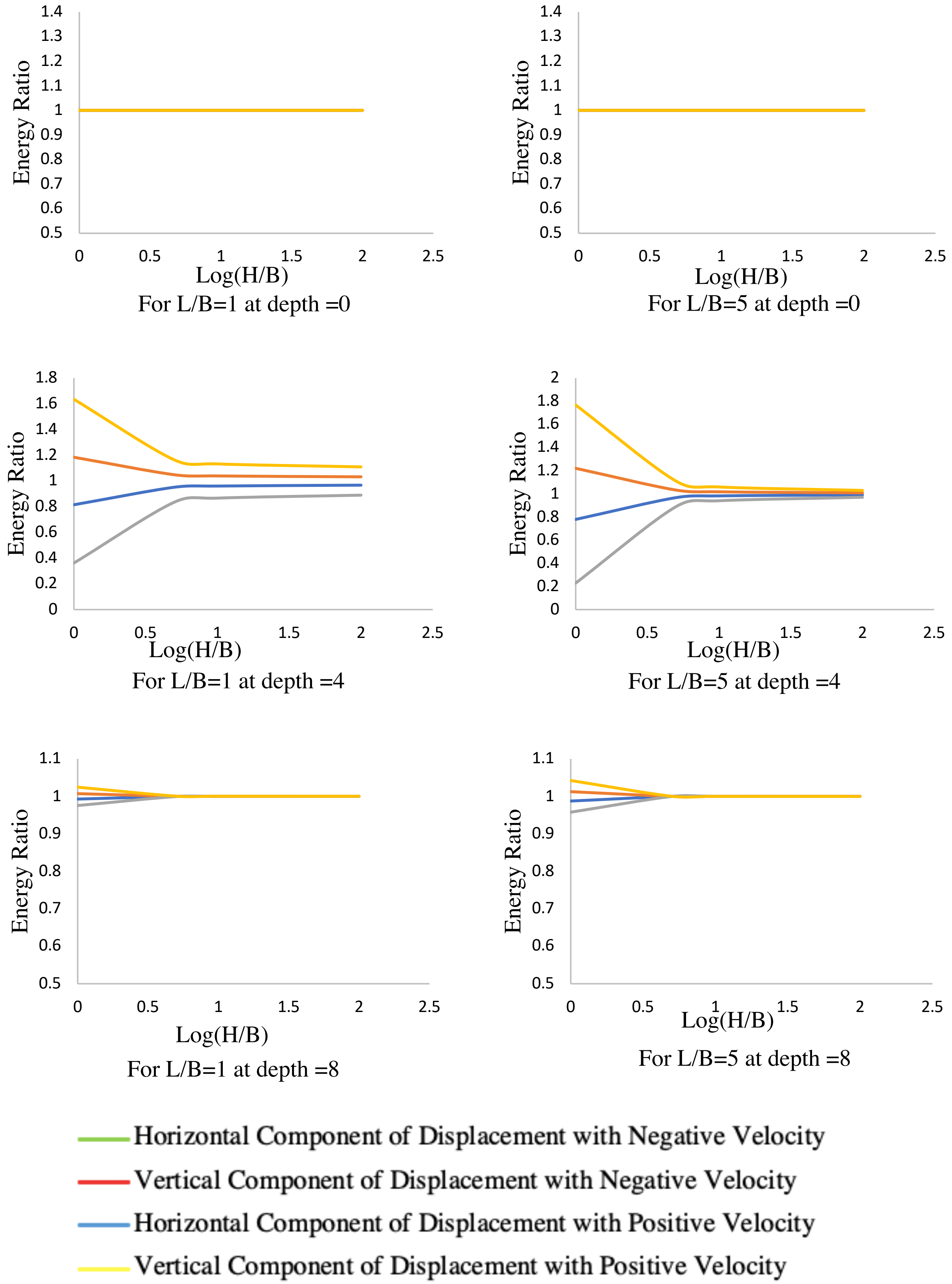
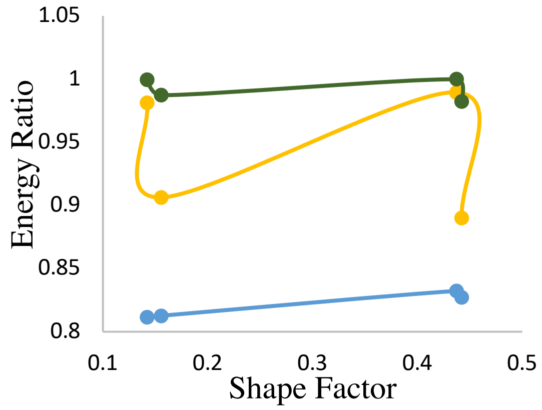
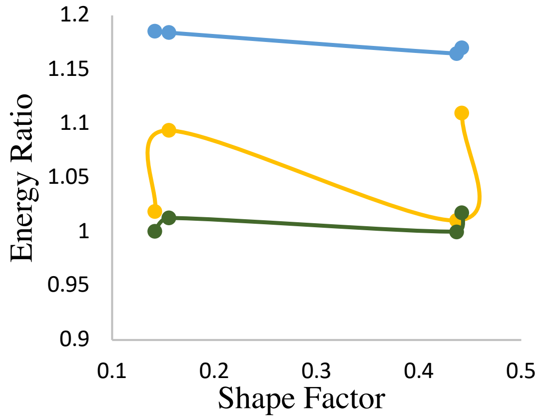


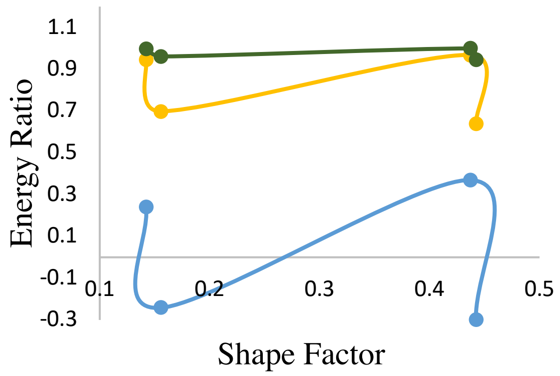
Figure No 4.6 Energy Ratio VS Log(H/B) for poisson's ratio = 0.5 at different depths.



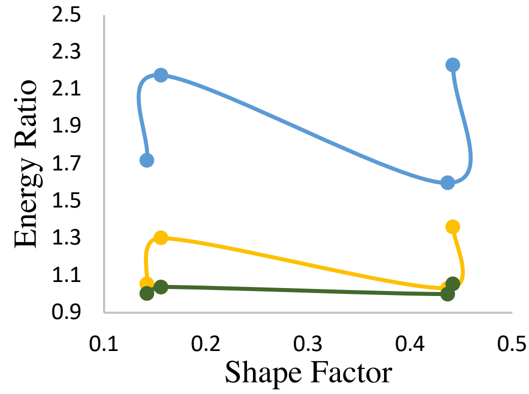
(a) Horizontal Component of Displacement with Negative Velocity



(b) Horizontal Component of Displacement with Positive Velocity



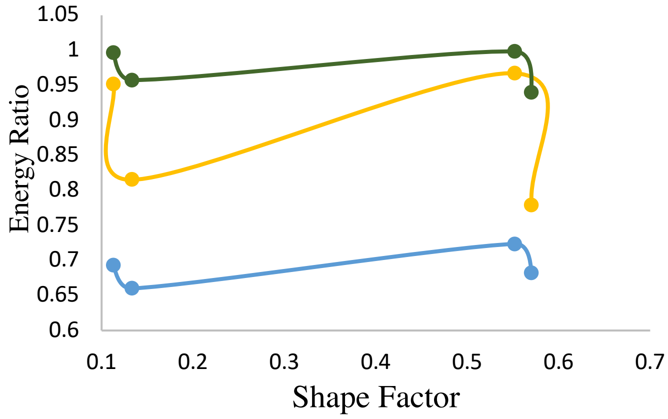
(c) Vertical Component of Displacement with Negative Velocity



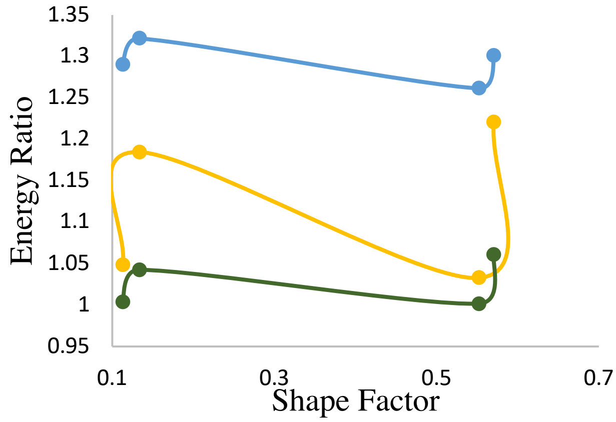
(d) Vertical Component of Displacement with Positive Velocity

At depth = 2 At depth = 4 At depth = 6

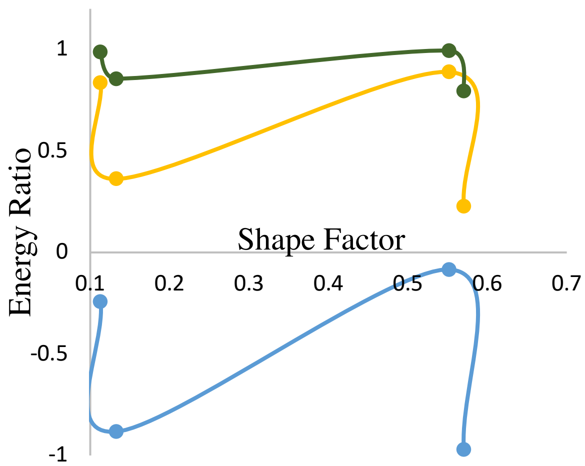
Figure No 4.7 Energy Ratio vs Shape Factor for passion's ratio of 0.25



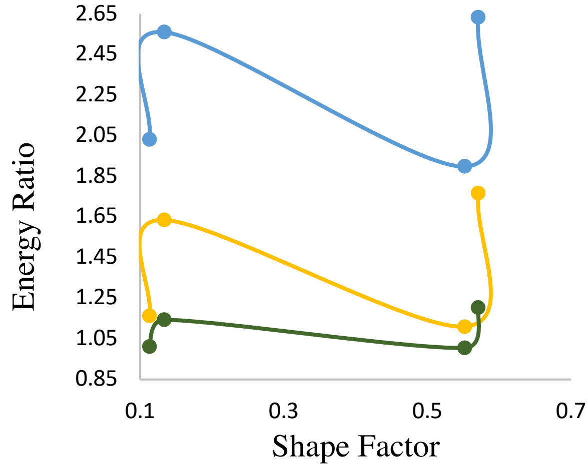
(a) Horizontal Component of Displacement with Negative Velocity



(b) Horizontal Component of Displacement with Positive Velocity



(c) Vertical Component of Displacement with Negative Velocity



(d) Vertical Component of Displacement with Positive Velocity

At depth = 2 At depth = 4 At depth = 6

Figure No 4.8 Energy Ratio vs Shape Factor for passion's ratio of 0.50

CHAPTER 5

CONCLUSION

Since P waves travel faster than all other waves, they arrive first, then S wave and then the Rayleigh waves arrive but the ground displacement produced by them is of much higher than any other waves. The disturbance amplitude of Rayleigh waves decreases gradually with distance. Actual Rayleigh wave velocity is calculated as a function of either P wave or S wave.

The point where the waves strike the prismatic member or element the energy will vary with depth as shown in the graphs. After a number of plots and comparison of each result various conclusions were drawn:

- Energy components either horizontal or vertical are a replica or mirror image of themselves and all of the four components converge about a depth of 4 meters i.e. with increases in-depth the effect of the energy of Rayleigh waves will be more. Hence for a particular Length to Width ratio, we need to have proper depth so that the effect of Rayleigh waves can be avoided.
- The Rayleigh waves moves radially outward inform of a cylindrical wave front. Equation 3.46, 3.47 shows that the path of the particle in motion is same to an elliptical motion with major axis normal to the surface.

With proper analysis for a high risked earthquake zone, with knowledge of soil properties, we can get the desired shape factor value and hence the dimension of the foundation or retaining wall can be chosen properly to ensure maximum safety.

APPENDIX-I

LIST OF SYMBOLS

Symbol	Abbreviations
z	Positive Downward
u	Displacements in The Directions X
w	Displacements in The Directions Z
$\emptyset$	Potential Function
φ	Potential Function
λ	Lame's Constant
k	Spring's Constant for Elastic Support
z_s	Static Deflection
W	Load
A	Area of The Foundation
G	Shear Modulus
ν_s	Shear Wave Velocity
ν_p	Compression Wave Velocity
ν_r	Rayleigh Wave Velocity
f	$\frac{2\pi}{wavelength}$
V	$\frac{\nu_r}{\nu_S}$
$F(z)$	Functions of Depth
$G(z)$	Functions of Depth
$\overline{\in}$	Dilation
U	Rate of attenuation of displacement in x-direction with depth
W	Rate of attenuation of displacement in z-direction with depth

$\acute{m}$	Length to Width ratio
$\acute{n}$	Height to Width ratio
L	Length of the prismatic element
B	Width of the prismatic element
H	Height of prismatic element
k_s	Coefficient of Subgrade Reaction

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