NON-STATIONARY SIGNAL ANALYSIS USING EIGEN VALUE DECOMPOSITION OF THE HANKEL MATRIX & WAVELET TRANSFORM

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CERTIFICATE

This is to certify that the dissertation title " Non-stationary Signal Analysis using Eigen value Decomposition of the Hankel Matrix & Wavelet Transform" submitted by **Charul Arora**, Roll. No. 2K12/SPD/25, in partial fulfilment for the award of degree of Master of Technology in Signal Processing & Digital Design at **Delhi Technological University, Delhi**.

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CANDIDATE'S DECLARATION

I hereby declare that the work, which is being presented in the dissertation, entitled " Non-stationary Signal Analysis using Eigen value Decomposition of the Hankel Matrix & Wavelet Transform", in partial fulfillment of the requirements for the award of the degree of Master of Technology in Signal Processing & Digital Design and submitted to the Department of Electronics and Communication Engineering of Delhi Technological University, Delhi.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

ABSTRACT

This work presents the time-frequency analysis of multi-component non-stationary signal. The method uses the iterative eigen value decomposition of Hankel matrix and analytic signal. The proposed work uses the wavelet transform to obtain the analytic signal. This analytic signal is used to find out the instantaneous parameters. The proposed method compares the instantaneous parameters obtained using Wavelet Transform (WT) method with the Hilbert Transform (HT) method.

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CHAPTER 1

INTRODUCTION

The illustration of non-stationary signals in each time and frequency domain is incredibly helpful for signal analysis in varied engineering applications like speech signal analysis and processing, medical specialty signals processing, telecommunication engineering, applied science & seismal signals process, The signal used in these applications are non-stationary signals i.e amplitude and frequency parameters vary with reference to time. Various time-frequency representations method are developed in literature for non-stationary signal analysis. Generally used time-frequency analysis techniques include spectrogram (squared magnitude of the short-time Fourier Transform (STFT)), scalogram (squared magnitude of the continual Wavelet Transform(CWT)), Wigner-Ville distribution (WVD), and Hilbert-Huang Transform (HHT) etc.

The basic technique for time-frequency analysis of nonstationary signals is spectrogram that comes from STFT. STFT assumes that the signal are stationary signal inside the window of study. The spectrum analysis supported Fourier transformation is applied on these short-time windowed signals. Also, in STFT, size of the analysis window is directly connected with the frequency resolution. The short analysis window provides poor resolution in frequency-domain whereas the employment of large size ofwindow offers higher resolution in frequency-domain.
The limitations of STFT technique are overcome by wavelet transform based technique for non-stationary signal analysis.

The wavelet basis functions are adaptative window functions and provide a new way of signal analysis in which signal analysis can be performed for a given point of time using many scales. This multi-resolution property of wavelet transform solves the problem of fixed resolution in time and frequency domain because of use of variable analysis window. The scalogram primarily based time-frequency method that is employed for analysis of nonstationary signals additionally needs choice of mother wavelet.

Another method used for time-frequency representation of signal is Wigner-Ville distribution (WVD). The WVD method offers excellent resolutions each in time-domain and frequency-domain. The WVD is appropriate for mono-component linear frequency modulated signals. But, for nonlinear frequency modulated signals and multi-component non-stationary signals, the WVD method generates cross-terms which may be thought-about as a significant disadvantage of WVD method. Numerous cross-terms reduction techniques such as, Fourier-Bessel series expansion, tunable-Q wavelet transform based method and kernel functions based methods are used.

Another time-frequency technique that has been used for non-stationary and non-linear signal analysis is Hilbert Transform (HT). The method is recursive in nature and doesn't need design of basis functions like in STFT and CWT strategies. In the Hilbert Transform (HT) technique, a multi-component non-stationary signal is decomposed using the empirical mode decomposition (EMD) technique.

The obtained components from EMD technique are called intrinsic mode functions(IMFs). The Hilbert Transform (HT) has been applied to those IMFs so as to compute their amplitude envelope and fast frequency functions .These amplitude envelope and fast frequency functions of IMFs are used for getting the Hilbert Transform (HT) based mostly time-frequency representation.

However, IMFs obtained from the EMD technique, suffer from mode commixture issue that results in improper time-frequency representation. To overcome the problems of EMD, the Hilbert Transform as an ensemble empirical mode decomposition (EEMD) has been proposed in for noisy signal analysis. The EEMD technique helps to cut back the mode commixture issue. The synchro squeezing transform based mostly the time-frequency analysis has been planned in.

This work presents the time-frequency analysis of multi-component non-stationary signal. The method uses the iterative eigen value decomposition of Hankel matrix and analytic signal. The proposed work uses the wavelet transform to obtain the analytic signal. This analytic signal is used to find out the instantaneous parameters. The proposed method compares the instantaneous parameters obtained using Wavelet Transform (WT) method with the Hilbert Transform (HT) method.

Time–frequency representation (TFR) is a view of a signal over both time and frequency. Time–frequency analysis means analysis into the time–frequency domain also called as "Time–Frequency Distribution (TFD)". TFRs are complex-valued fields over time & frequency, where the modulus of the field represents either amplitude or "energy density" i.e. the concentration of the root mean square over time and frequency, and the argument of the field represents the phase.

A signal, as a function of time, may be considered as a representation with perfect time resolution. In contrast, the magnitude of the Fourier transform (FT) of the signal may be considered as a representation with perfect spectral resolution but with no time information because the magnitude of the FT conveys frequency content but it fails to convey when, in time, different events occur in the signal.

TFRs provide a bridge between these two representations that they provide some temporal information and some spectral information simultaneously. Thus, TFRs are useful for the representation and analysis of signals containing multiple time-varying frequencies.

One form of TFR (or TFD) can be formulated by the multiplicative comparison of a signal with itself, expanded in different directions about each point in time. Such representations and formulations are known as quadratic or "bilinear" TFRs or TFDs (QTFRs or QTFDs) because the representation is quadratic in the signal (see Bilinear time–frequency distribution).

In engineering,the digital signal process techniques got to be fastidiously chosen consistent with the characteristics of the signals of interest. The frequency-based and time-frequency techniques are often mentioned in some literature (Cohen, 1995). The frequency based techniques (FBTs) are wide used for stationary signal analysis. For non-stationary signals, the time-frequency techniques (TFTs) in common use, like the short time Fourier transform (STFT), the wavelet transform (WT), ambiguity function (AF) and wigner-ville distribution (WVD),etc.,are sometimes performed for extracting transient features of the signals. These techniques use different algorithms to produce a time frequency illustration for a signal.
The STFT uses a customary Fourier transform over many forms of windows. The wavelet based techniques apply the mother wavelet with either discrete or continuous scales to a waveform to resolve the fixed time-frequency resolution problems inherent in STFT. In applications, the fast

version of wavelet transform, that's attributed to a pair of mirror filters with variable sampling rates,is sometimes used for reducing the amount of calculations to be done, thereby saving computer running time. AF and WVD are quadratic time-frequency representations, that use advanced techniques to combat these resolution difficulties. they need higher resolution than STFT however suffer from cross-term interference and turn out results with coarser graininess than wavelet techniques do. Of the wavelet-based techniques, the discrete wavelet transform (DWT), particularly its fast version, is sometimes used for encryption and decryption signals, whereas wavelet packet analysis (WPA)are self-made in the signal recognition and characteristic extraction. AF and WVD with excessive trans formation durations are clearly unacceptable within the development of time period observation systems.

In applications, the FBTs were usually utilized in noise and vibration engineering (Brigham, 1988). they supply the time-averaged energy info from a signal phase in frequency domain, however stay nothing in time domain. For non-stationary signals like vehicle noises, some implementation examples are the STFT (Hodges & Power, 1985), WVD, ironed pseudo-WVD (Baydar & Ball, 2001) and WT (Chen, 1998). especially, the WT as “mathematical microscope” in engineering permits the dynamical spectral composition of a non-stationary signal to be measured and given within the type of a time-frequency map and therefore, was advised as an efficient tool for nonstationary signal analysis.

Research Objective

1. Design and implementation of Non-Stationary signal analysis using Eigenvalue decomposition of the Hankel Matrix with Hilbert Transform and Wavelet Transform.
2. Comparative analysis of Hilbert Transform and Wavelet Transform Technique.

Thesis Organization

1. Chapter 1 includes the introduction of the Non-Stationary Signal Analysis & Time-frequency representation of signal with the research objectives of the thesis.

2. Chapter 2 is having the literature survey related to the Non-Stationary Signals, Time-frequency representation, Hilbert Transform & Wavelet Transform related methods.
3. Chapter 3 presents brief theory of Hilbert Transform.
4. Chapter 4 presents brief theory of Hankel Matrix.
5. Chapter 5 presents brief theory of Continues & Discrete Wavelet Transform.
6. Chapter 6 & 7 is related to the results and discussion, comparative analysis shown between the Hilbert and the wavelet transform technique.
7. Chapter 8 is the conclusion chapter where in the summary of the proposed work done and all its analysis is explained in a brief.

CHAPTER 2
Literature Review

Akbar M. Sayeed, Douglas *L.* Jones, et al (1994), [1], In this article, we determine the optimal kernel for mmse estimation of an arbitrary TFR (characterized by a kernel, &) of a realization of a process from the corresponding realization of a correlated process. The properties of the optimal kernel solution clearly demonstrate the limitations of the quasi-stationarity assumption.

Branka Jokanovi´c, Moeness Amin and Traian Dogaru, et al (2015), [2], Time-frequency (TF) representations are a powerful tool for analyzing Doppler and micro-Doppler signals. These signals are frequently encountered in various radar applications. Data interpolators play a unique role in TF signal representations under missing samples. When applied in the instantaneous autocorrelation domain over the time variable, the low-pass filter characteristic underlying linear interpolators lends itself to cross-terms reduction in the ambiguity domain. This is in contrast to interpolation performed over the lag variable or a direct interpolation of the raw data. We demonstrate the interpolator performance in both the time domain and the time-lag domain and compare it with sparse signal reconstruction, which exploits the local sparsity property assumed by most Doppler radar signals.

Akbar M. Sayeed, Douglas L. Jones, et al (1994), [3], Bilinear time-frequency representations (TFRs) and time-scale representations (TSRs) are potentially very useful for detecting a non-stationary signal in the presence of nonstationary noise or interference. As quadratic signal representations, they are promising for situations in which the optimal detector is a quadratic function of the observations. All existing time-frequency formulations of quadratic detection either implement classical optimal detectors equivalently in the time-frequency domain, without fully exploiting the structure of the TFR, or attempt to exploit the nonstationary structure of the signal in an ad hoc manner. We identify several important nonstationary quadratic detection scenarios which are “naturally” suited for TFR/TSR-based detectors; that is, in which TFR/TSR-based detectors are both optimal and exploit the many degrees of freedom available in the TFR/TSR. We also derive explicit expressions for the corresponding optimal TFR/TSR kernels. The proposed TFR/TSR detectors are directly applicable to many important radar/sonar detection problems.

MARIUSZ SULIMA, et al (2017) [4], This work presents a new DHT impulse response function based on the proposed nonlinear equation system obtained as a result of combining the DHT and IDHT equation systems. In the case of input time series with selected characteristics, the DHT results obtained using this impulse response function are characterized by a higher accuracy compared to the DHT results obtained based on the convolution using other known DHT impulse response functions. The results are also characterized by a higher accuracy than the DHT results obtained using the popular indirect DHT method based on discrete Fourier transform (DFT). Analysis of these example time series with selected characteristics was performed based on the signal to noise ratio.

Y Morales, et al (2017) [5], In the present investigation, a mathematical algorithm under MATLAB platform using Radial Hilbert Transform and Random Phase Moask for encrypting digital image is implemented. the algorithm is based on the use of the conventional Fourier transform and two random phase masks, which provide security and robustness to the system implemented. Random phase masks used throughout encryption and decryption are the keys to improve security and make the system immune to attacks by program generation phase masks.

Prabhjot kour, et al (2015) [6], The quality and the size of image data is constantly increasing. With the advancement in technology, many products in the market use images for control and display. Image compression is one of the primary image processing techniques that are embedded in all electronic products. Fast and optimally interactive post processing of these images is a major concern. E.g., reduce the redundancy of the image data in order to be able to store or transmit data in an efficient form is difficult task to be performed. This paper presents a frame work for an Image processing based Discrete Wavelet Transform System The approach helps the end user to generate images using DWT at a high level without any knowledge of the low-level design styles and architectures.

YANG Hang, et al (2016), [7],

A novel sensor deployment method utilize Discrete wavelet transform (DWT) is propose, and the DWT is used to calculate the subband energy entropy to ascertain the coverage cavities in Wireless sensor networks (WSNs). We address the problem of deploying a limit number of sensors to optimize the coverage ratio in 3D surface, while it is a complex surface in space and sensors can be deployed only onto it. Another novel aspect of this paper is that the method followed utilizes an Artificial bee colony algorithm with dynamic search strategy(ABC-DSS), which mimics the behavior of bees, and the new modified ABC-DSS algorithm matches the sensor deployment problems on 3D surface well. The extensive simulations illustrate that comparing with the deployment method based on Particle swarm optimization (PSO) and ABC, the ABC-DSS which utilizes the wavelet sub-band energy entropy is functional and efficient on 3D surface deployment problems.

CHAPTER 3
HILBERT TRANSFORM

Hilbert Transform (HT) is widely used in signal and network theory and it has very practical applications in numerous fields. Such as communication systems, radio detection and ranging systems and medical imaging. Hilbert transform provides a ±90° phase shift to the input signal, therefore if we choose the input signal to be a sine function then calculative its Hilbert Transform can offer cosine function. The Hilbert Transform is simply like any of the special phase adjusted filters that are attainable. In Fourier transform we alter the time domain signal to the frequency domain signals, however in Hilbert Transform domain of operation remains identical. Hilbert Transform are often used to design digital filters which may be infinite impulse response (IIR) and Finite impulse response (FIR) filters. In FIR filters there's no feedback whereas in IIR filters the feedback is present. Hilbert Transform acts as a causative sequence and relates the important part of Fourier transform to the imaginary part of Fourier transform. The Fourier Transforms need complete information of each Real and imagined parts of the magnitude and phase for all frequencies within the range $-\pi < \omega < \pi$. Hilbert Transforms applied to causative signals takes advantage of the actual fact that Real sequences have parallel Fourier transforms.

Hilbert Transform (HT) is an analytical tool that's helpful for the illustration of varieties of signals like band pass signals. This transform is additionally used for various varieties of modulation schemes as series side band AM modulation. Hilbert Transform is totally different from the other transform that's employed in signal process because in this no domain change is needed. If we are taking the sign to be within the time domain then by using the Hilbert Transform we get the signaling in time domain solely. This special property of Hilbert Transform is true for frequency domain signal additionally, that additional helps within the complexness reduction. currently we take the generalized form of the Hilbert Transform in equation (2.1).

$$X^{\wedge}\ (t) = 1/\pi \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau}$$

Where x^(t) is the Hilbert Transform of x(t). The Hilbert Transform involves the convolution of the input signal and the mpulse response. Most importantly we take the transfer function of the Linear Filter as it satisfies superposition principle and it can be only represented in transfer function form. This linear filter will work to phase shift all frequency components by –π/2 radians. The magnitude characteristics of the filter are 1 for all frequencies whereas the real signals have positive as well as negative frequencies. As HILBERT TRANSFORM introduces a 90° phase shift twice causes a 180° phase shift, which can cause a phase reversal of the original signal. The considered amplitudes of all frequency components in the signal, however are unaffected by transmission through the device. Such an ideal device is referred as Hilbert transformers. And for the inverse Hilbert Transform (IHT) as shown in equation.

$$X(t) = -1/\pi \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau$$

The input and the output x(t) is termed as Hilbert Transform pairs.

3.1 APPLICATIONS OF HILBERT TRANSFORM

Hilbert Transform possesses a large vary of applications within the analysis of system design. This Transform is so helpful for various functions like latency analysis in neuro-physiological signals, style of freaky stimuli for psychoacoustic experiments, speech data compression for communication, regularization of convergence issues in multi-channel acoustic echo cancellation, and signal process for exteroception prostheses. Here we are finding out very well regarding the 3 vital areas during which mathematician Transform will be used and enforced to induce the required results.

3.1.1 HILBERT TRANSFORM in Image Processing

The various television images that are of continuous value are transmitted using modulation techniques for video and audio signals. HILBERT TRANSFORM acts as a promising algorithm also for the earth images. HILBERT TRANSFORM is considered to be useful in manipulating images since the transmission bandwidth is efficiently reduced. And the HILBERT

TRANSFORM is also proposed as one of the many coding techniques that can be used in practical fields for the imaging materials.

For the image processing we consider the analytical function as the sum of the real and imaginary parts in terms of x and y nominations of the images as they are considered for the 2D domain. For example, we take the Fourier transform algorithm which is often applied to the same number sequences in the time and frequency domains. Stand still image signals are non-fluctuations materials and information. So, the digitized and stored continuous images for evaluations and verifications can be derived from continuous images at one instant of time. HILBERT TRANSFORM is used for the continuous signals for the images that adopt such algorithm in modulation systems particularly.

3.1.2 HILBERT TRANSFORM in Edge Detection.

Edges represent the discontinuities in the intensity in an image. Edges created by occlusions, shadows, roofs, textures, etc. may have the coherent local intensity. Edge detection is a process that measures, detects, and localizes the changes in intensity. Edge detection is an important step in the process of segmentation also.

In this a new method for edge detection using one dimensional processing is used which is the Gaussian function. The image is smoothed using 1 D Gaussian along the horizontal (or vertical) scan lines to reduce noise. Detection is then used in the orthogonal direction i.e., along vertical (or horizontal) scan lines to detect the edges.

This method is based on the 2 D operators in the sense that smoothing is done along one direction in the sense that smoothing is done along one direction and the detection is applied along the orthogonal direction. But it also results in some loss of edge information.

3.2.3 HILBERT TRANSFORM in Signal Analysis:

One of the important application of HT is creating a Analytic Signal. For signal s(t), given its Hilbert Transform s^(t) it is defined as a composition:

sA(t)=s(t)+js^(t)sA(t)=s(t)+js^(t)

The Analytic Signal that we have a tendency to get is complicated, thus we are able to categorical it in exponential notation:

sA(t)=A(t)ejψ(t)sA(t)=A(t)ejψ(t)

where:

A(t)A(t) is the instantaneous amplitude

ψ(t)ψ(t) is the instantaneous phase.

So however these area unit helpful?

The instant amplitude are often helpful in several cases(it is wide used for locating the envelopeof easy harmonic signals). Here is an example for impulse response)

:

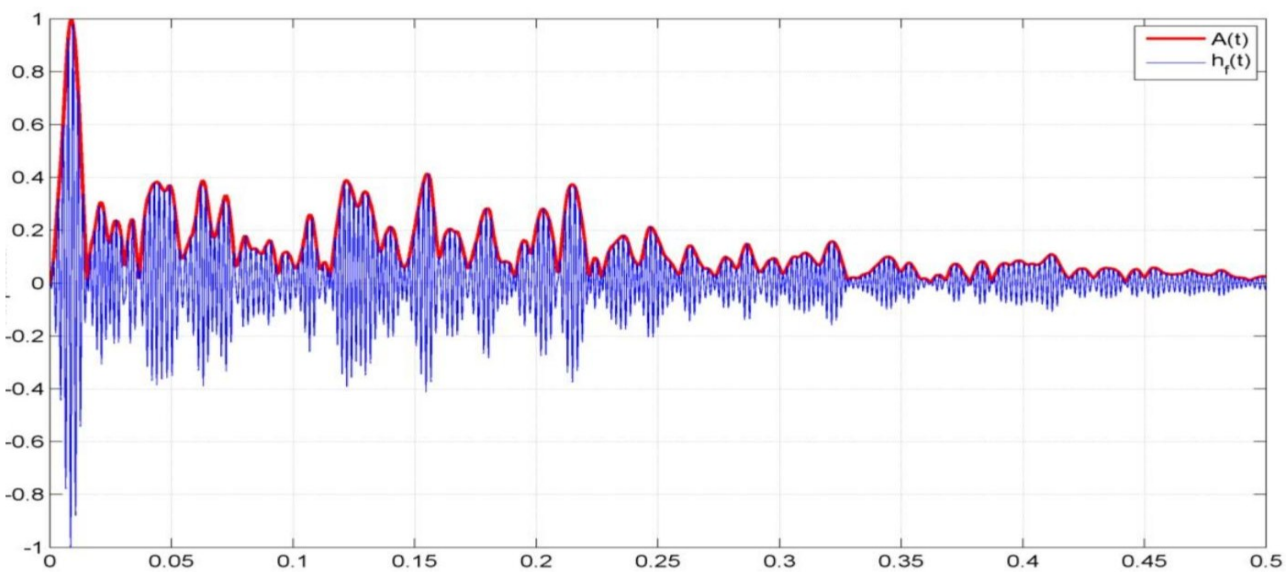


Figure 1

Now, based on the phase(ψ), we can evaluate the instantaneous frequency(IF):

f(t)=12πdψdt(t)f(t)=12πdψdt(t)

Which is again useful in several applications, like frequency detection of a sweeping tone, rotating engine, etc.

Other samples of usage include:

Sampling of narrowband signals in telecommunications (mostly exploitation Hilbert filters).

Medical imaging.

Array process for Direction of Arrival.

System response analysis.

The analytic signal made by the Hilbert transform is beneficial in several signal analysis applications. If you bandpass filter the signal 1st, the analytic signal illustration provide you info regarding the native structure of the signal:

phase indicates the native symmetry at the purpose, wherever zero is positive bilaterally symmetric (peak), ππ is negative bilaterally symmetric (trough), and ±π/2±π/2 is anti-symmetric (rising / falling edge).

amplitude indicates the strength of the structure at the purpose, freelance of the symmetry (phase).

This illustration has been used for

feature detection via native energy (amplitude)

feature classification exploitation section

feature detection via section congruency

It has additionally been extending to higher dimensions exploitation the Riesz remodel, as an example the heritable signal.

Implementing a Hilbert Transform allows us to make Associate in Nursing analytic signal supported some original real-valued signal. And within the comms world we are able to use the analytic signal to simply and accurately reason the fast magnitude of the initial real-valued signal. That method is employed in AM reception. additionally from the analytic signal we are able to simply and accurately reason the fast section of the initial real-valued signal. That method is employed in each section and FM reception. Your prof is correct in covering the Hilbert Transform as a result of it is so damn helpful in comms systems.

The Hilbert Transform once used on real knowledge, provides "a true (instantaneous) amplitude" (and some more) for stationary phenomena, by turning them into "specific" advanced knowledge. for example, a trigonometric function cos(t)cos(t) is inherently of amplitude one, that you are doing not see directly, since it visually wiggles between −1−1 and eleven, and sporadically vanishes.

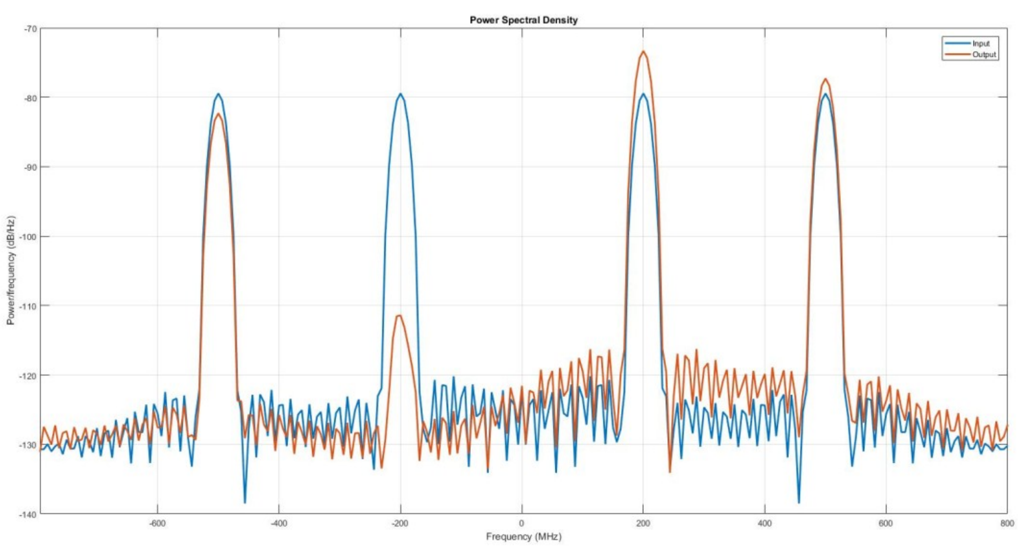
Another way of trying Hilbert Transform is in frequency domain. As real signal have identical positive and negative frequency parts, thus in analysis this info is redundant.

Hilbert Transform is employed to eliminate the negative frequency half and double the magnitude of positive frequency half (to keep power same).

Here, the designed Hilbert Transform filter is band pass in nature that passes frequencies from 50 MHz to 450 megacycle per second. The input is add of 2 curving signals having frequencies capable 200MHz and 500MHz.

From the PSD plot, we are able to see the negative frequency element of 200MHz signal gets attenuated whereas 500MHz signal passes in and of itself.

Figure 2:



CHAPTER 4
HANKEL MATRIX

A squared Hankel matrix, which is denoted by HxK of size K×K, can be prepared using 2K−1 samples of signal x[k], can be represented as follows [10]:

$$H_K^x = \begin{vmatrix} x[0] & x[1] & . & . & x[K-1] \\ x[1] & x[2] & . & . & x[K] \\ . & . & . & . & . \\ . & . & . & . & . \\ x[K-1] & x[K] & . & . & x[2K-2] \end{vmatrix} \quad (1)$$

The parameter K can be computed for the following two cases as follows:

Case 1: When, a multi-component non-stationary signal contains amplitude modulated (AM) components, and fi denotes the frequency of ith mono-component, Fs denotes sampling rate and normalized frequency,

$f_l = \frac{1}{N_l} = \frac{F_l}{F_s}$,

, in this case, the size of Hankel matrix can be given as follows:

K ≥ MLCM, (2)

where MLCM is the least common multiple (LCM) of Mi, i = 1, 2, ...,N.

Case 2: When,a multi-component non-stationary signal contains frequency modulated (FM) components, in which Δfmin represents the minimum frequency separation between components, and Fs denotes sampling rate used for discretizing the signal, then for this case, the size of Hankel matrix can be specified in the range K ≫Fs/Δfmin [18].

The eigenvalue matrix λx having eigenvalues in its diagonal and real eigenvector matrix, Vx for the matrix HxK, are related as follows:

$H_N^x = V_x \Lambda_x V_x^T$

The dominant eigenvalue pairs are selected based on the significant threshold point (STP) criteria mentioned in [18], which are on the basis of the 10% of the maximum eigenvalue. Each dominant eigenvalue pair has been selected to prepare a new eigenvalue matrix. This new eigenvalue matrix corresponding to ith eigenvalue pair can be represented as follows:

$$\bar{\lambda}_x^i = \begin{vmatrix} 0 & 0 & 0 & . & 0 \\ . & . & . & . & . \\ 0 & -\lambda_i & . & . & 0 \\ . & . & . & . & . \\ 0 & 0 & . & +\lambda_i & 0 \\ . & . & . & . & . \\ 0 & 0 & . & . & 0 \end{vmatrix} \qquad (4)$$

A reconstruction matrix for ith component is formed using (4) and the matrix Vx as follows:

$H_N^x = V_x \Lambda_x V_x^T$

The average of skew diagonal elements of reconstruction matrix HxiN is considered to determine the ith decomposed component. The obtained components need to satisfy the mono-component signal criteria (MSC) mentioned in.

The MSC is based on the following two rules:

Rule 1: The absolute value of the difference between the number of zero-crossings and the number of local extrema of the obtained component is either zero or one.

Rule 2: The number of significant eigenvalue pairs which are obtained by performing eigen value decomposition (EVD) on the Hankel matrix constructed using the samples of the obtained component is 1. The components, which do not satisfy the MSC such components are subjected to next iteration. This process will be repeated till all the obtained components follow the MSC.

In the end, the components, which fulfill the MSC are considered as mono-component signals and applied to the next merging step. After last iteration, the components which perform overlapping of 1 dB bandwidth, are merged together. After this stage, the obtained components are considered as a set of decomposed components using IEVD-HM method [10]. These obtained final components are also considered as a set of mono-component non-stationary signals.

CHAPTER 5

WAVELET TRANSFORM

In recent years, the wavelet transforms emerged within the field of image/signal process as an alternate to the well-known Fourier transform (FT) and its connected transforms, namely, the discrete cos transform (DCT) and also the discrete sin transform (DST). within the Fourier theory, a signal (an image is taken into account as a finite 2-D signal) is expressed as a add, in theory infinite, of sines and cosines, creating the ft appropriate for infinite and periodic signal analysis. For many years, the ft dominated the sphere of signal process, however, if it succeeded well in providing the frequency info contained within the analyses signal; it did not provide any info regarding the prevalence time. This disadvantage, however not the sole one, actuated the scientists to scrutinize the transform horizon for a “messiah” transform. the primary step in this long analysis journey was to cut the signal of interest in many components then to analyses every part severally. The concept at a primary ook looked as if it would be very promising since it allowed the extraction of your time info and the localization of various frequency elements. This approach is thought because the Short-Time Fourier transform (STFT). the basic question, that arises here, is the way to cut the signal? The simplest resolution to the present perplexity was after all to find a completely scalable modulated window within which no signal cutting is required any longer. This goal was achieved with success by the utilization of the wavelet transform.

Formally, the wavelet transform is outlined by several authors as a mathematical technique within which a specific signal is analyses (or synthesized) within the time domain by using totally different versions of a expanded (or contracted) and translated (or shifted) basis perform known as the wavelet paradigm or the mother wavelet. However, in reality the wavelet transforms found its essence and emerged from totally different discipline &wasn't, as declared by Mallat, all new to mathematicians operating in Fourier analysis, orto pc vision researchers finding out multiscale image process. At the start of the twentieth century, Haar, a German man of science introduced the primary wavelet transform named once him (almost a century once the introduction of the linear unit, by the French J. Fourier). The Haar wavelet basis function has compact support and integer coefficients. Later, the Haar basis was used in physics to review pedesis. Since then, totally different works are meted out either within the development of the speculation associated with the Wavelet, or towards its application in several fields. within the field of signal process, the good achievements reached in several studies by Mallat, Meyer and Daubechies have allowed the emergence of a good vary of wavelet based applications. In fact, impressed by the work developed by Mallat on the relationships between the construction Mirror Filter (QMF), pyramid algorithms and orthonormal Wavelet bases (, Meyer made the primary non-trivial wavelets. However, the foremost necessary work was meted out by In grid Daubechies. Supported Mallat's work, Daubechies succeeded to construct a group of Wavelet orthonormal basis functions, that became the cornerstone of the many application Few years later, an equivalent author, together with others, given a group of Wavelet biorthogonal basis perform, that later found their use in several applications, particularly in image committal to writing. Recently, JPEG2000, a biorthogonal wavelet-based compression has been adopted because the new compression customary.

The Wavelet Transform is analogous to the Fourier Transform (or rather more to the windowed Fourier transform) with a totally totally different advantage perform. the most distinction is this: Fourier Transform decomposes the signal into sines and cosines, i.e.the functions localized in Fourier house; in contrary the Wavelet Transform uses functions that area unit localized in each the $64000 and Fourier space. Generally, the Wavelet Transform may be expressed by the subsequent equation:

$$F(a,b) = \int_{-\infty}^{\infty} f(x)\, \psi^{*}_{(a,b)}(x)\, \mathrm{d}x$$

Where the * is the complex conjugate symbol and function ψ is some function. This function may be chosen arbitrarily providing it obeys bound rules.

As it is seen, the wavelet transform is indeed an infinite set of assorted transforms, betting on the advantage function used for its computation. this is often the main reason, why we are able to hear the term "wavelet transform" in very different situations and applications. There also are some ways a way to sort the types of the wavelet transforms. Here we show solely the division based on the wavelet orthogonality. we are able to use orthogonal wavelets for discrete wavelet transform development and non-orthogonal wavelets for continuous wavelet transform development. These 2 transforms have the subsequent properties:

1. The discrete wavelet transform returns a data vector of a similar length because the Input is. Usually, even in this vector several information are nearly zero. This corresponds to the actual fact that it decomposes into a group of wavelets (functions) that are orthogonal to its translations and scaling. so we decompose such a signal to a same or lower range of the wavelet coefficient spectrum as is that the number of signal information points. Such a wavelet spectrum is incredibly sensible for signal process and compression, for instance, as we get no redundant info here.

2. The continual wavelet transform in contrary returns an array one dimension larger than the input file. For a 1D information we get an image of the time-frequency plane. We are able to simply see the signal frequencies evolution throughout the period of the signal and compare the spectrum with different signals spectra. As here is employed the non-orthogonal set of wavelets, information are extremely related, therefore huge redundancy is seen here. This helps to examine the leads to a a lot of humane type.

For a lot of details on wavelet transform see any of the thousands of wavelet resources on the net, or for instance.

Within Gwyddion processing library, each these transforms are enforced and therefore themodules exploitation wavelet transforms may be accessed among information method → Integral Transforms menu.

The idea of wavelet transforms is that the transform shall allow only change in time extension, but not the shape. It is affected by choosing the suitable basis function that allow for this. Changes in time extension are expected to conform the corresponding analysis frequency of the basis function. Based on uncertainty the principle of signal processing,

$$\Delta t \Delta \omega \geqq \frac{1}{2}$$

where t represents time and ω angular frequency (ω = 2πf, where f is temporal frequency).

The higher is the required resolution in time, the lower is the resolution in frequency to be. The larger is the extension of the analysis windows is chosen, the larger is the value of Δt

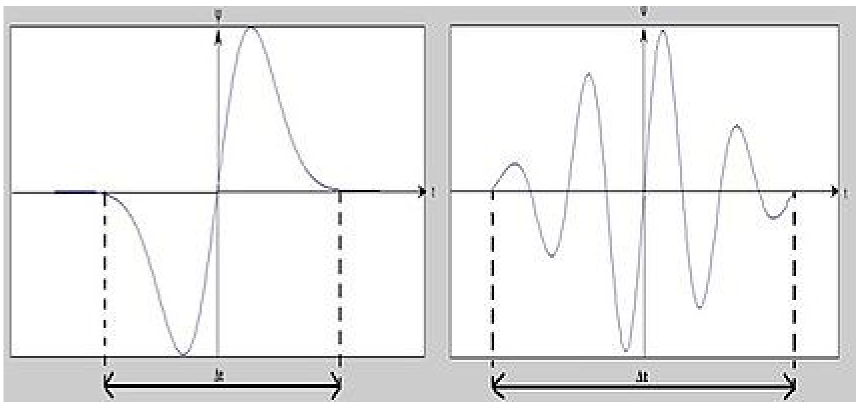


Figure 3

When Δt is large,

1. Bad time resolution
2. Good frequency resolution
3. Low frequency, large scaling factor

When Δt is small

1. Good time resolution
2. Bad frequency resolution
3. High frequency, small scaling factor

In other words, the basis function Ψ can be regarded as an impulse response of a system with which the function x(t) has been filtered. The transformed signal provides information about the time and the frequency. Therefore, wavelet-transformation contains information similar to the short-time-Fourier-transformation, but with additional special properties of the wavelets, which show up at the resolution in time at higher analysis frequencies of the basis function. The difference in time resolution at ascending frequencies for the Fourier transform and the wavelet transform is shown below.

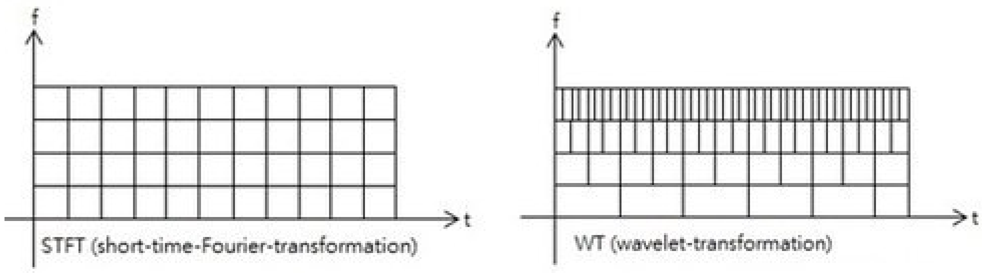


Figure 4

This shows that wavelet transformation is good in time resolution of high frequencies, while for slowly varying functions, the frequency resolution is remarkable.

Another example: The analysis of three superposed sinusoidal signals with STFT and wavelet-transformation.

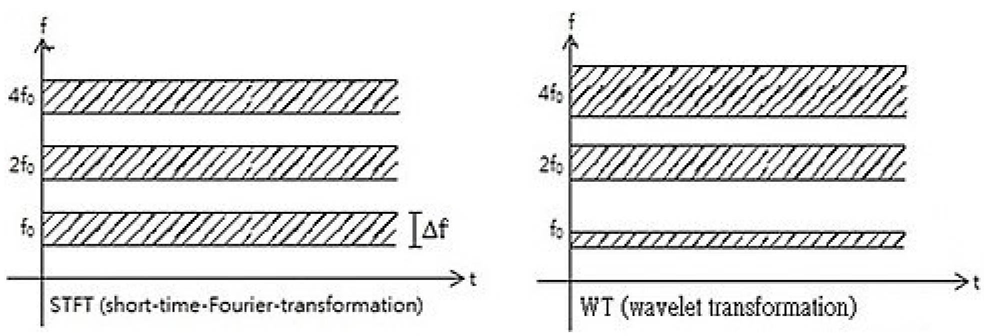


Figure 5

Wavelet compression

Wavelet compression is a form of data compression well suited for image compression (sometimes also video compression and audio compression). Notable implementations are JPEG

2 , DjVu and ECW for still images, Cine Form, and the BBC's Dirac. The goal is to store image data in as little space as possible in a file. Wavelet compression can be either lossless or lossy.[1]

Using a wavelet transform, the wavelet compression methods are adequate for representing transients, such as percussion sounds in audio, or high-frequency components in two-dimensional images, for example an image of stars on a Hilbert Transform sky. This means that the transient elements of a data signal can be represented by a smaller amount of information than would be the case if some other transform, such as the more widespread discrete cosine transform, had been used.

Discrete wavelet transform has been successfully applied for the compression of electro cardio graph (ECG) signals In this work, the high correlation between the corresponding wavelet coefficients of signals of successive cardiac cycles is utilized employing linear prediction.

Wavelet compression is not good for all kinds of data: transient signal characteristics mean good wavelet compression, while smooth, periodic signals are better compressed by other methods, particularly traditional harmonic compression (frequency domain, as by Fourier transforms and related).

See Diary Of An x264 Developer: The problems with wavelets (2010) for discussion of practical issues of current methods using wavelets for video compression.

Method

First a wavelet transform is applied. This produces as many coefficients as there are pixels in the image (i.e., there is no compression yet since it is only a transform). These coefficients can then be compressed more easily because the information is statistically concentrated in just a few coefficients. This principle is called transform coding. After that, the coefficients are quantized and the quantized values are entropy encoded and/or run length encoded. A few 1D and 2D applications of wavelet compression use a technique called "wavelet footprints".

Comparison with Fourier transform and time-frequency analysis

Transform	Representation	Input
Fourier transform	$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi}\, dx$	ξ, frequency
Time-frequency analysis	$X(t, f)$	t, time; f, frequency
Wavelet transform	$X(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \overline{\Psi\left(\frac{t-b}{a}\right)} x(t)\, dt$	a, scaling; b, time

Wavelets have some slig Hilbert Transform benefits over Fourier transforms in reducing computations when examining specific frequencies. However, they are rarely more sensitive, and indeed, the common Morlet wavelet is mathematically identical to a short-time Fourier transform using a Gaussian window function. The exception is when searching for signals of a known, non-sinusoidal shape (e.g., heartbeats); in that case, using matched wavelets can out perform standard STFT/Morlet analyses.

Discrete Wavelet Transform

The discrete wavelet transform (DWT) is an implementation of the wavelet transform using a discrete set of the wavelet scales and translations obeying some defined rules. In other words, this transform decomposes the signal into mutually orthogonal set of wavelets, which is the main difference from the continuous wavelet transform (CWT), orits implementation for the discrete time series sometimes called discrete-time continuous wavelet transform (DT-CWT).

The wavelet can be constructed from a scaling function which describes its scaling properties. The restriction that the scaling functions must be orthogonal to its discrete translations implies some mathematical conditions on them which are mentioned everywhere, e.g. the dilation equation

$$\phi(x) = \sum_{k=-\infty}^{\infty} a_k \phi(Sx - k)$$

where S is a scaling factor (usually chosen as 2). Moreover, the area between the function must be normalized and scaling function must be orthogonal to its integer translations, i.e.

$$\int_{-\infty}^{\infty} \phi(x)\,\phi(x+l)\,\mathrm{d}x = \delta_{0,l}$$

After introducing some more conditions (as the restrictions above does not produce a unique solution) we can obtain results of all these equations, i.e. the finite set of coefficients a_k that define the scaling function and also the wavelet. The wavelet is obtained from the scaling function as N where N is an even integer. The set of wavelets then forms an orthonormal basis which we use to decompose the signal. Note that usually only few of the coefficients a_k are nonzero, which simplifies the calculations.

In the following figure, some wavelet scaling functions and wavelets are plotted. The most known family of orthonormal wavelets is the family of Daubechies. Her wavelets are usually denominated by the number of nonzero coefficients a_k, so we usually talk about Daubechies 4, Daubechies 6, etc. wavelets. Roughly said, with the increasing number of wavelet coefficients the functions become smoother. See the comparison of wavelets Daubechies 4 and 2 below. Another mentioned wavelet is the simplest one, the Haar wavelet, which uses a box function as the scaling function.

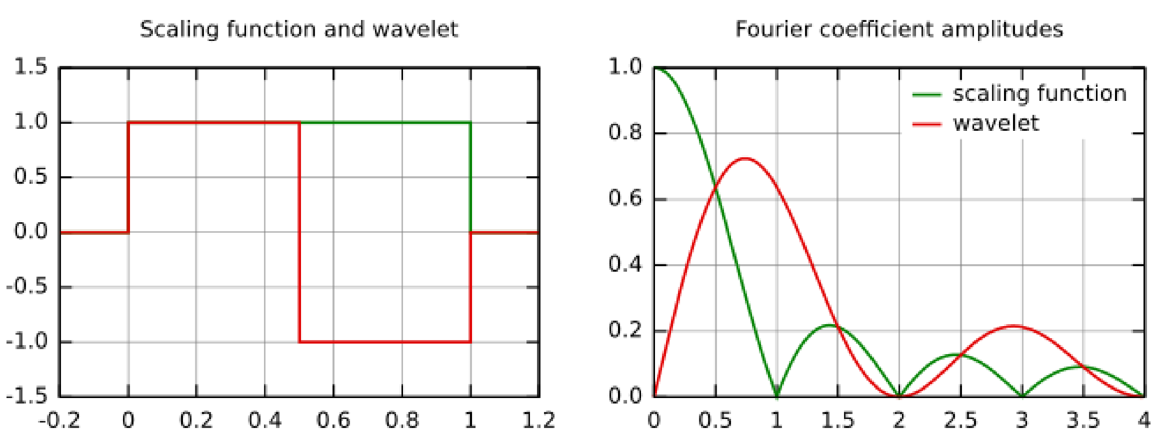


Figure 6. Haar scaling function and wavelet (left) and their frequency content (right).

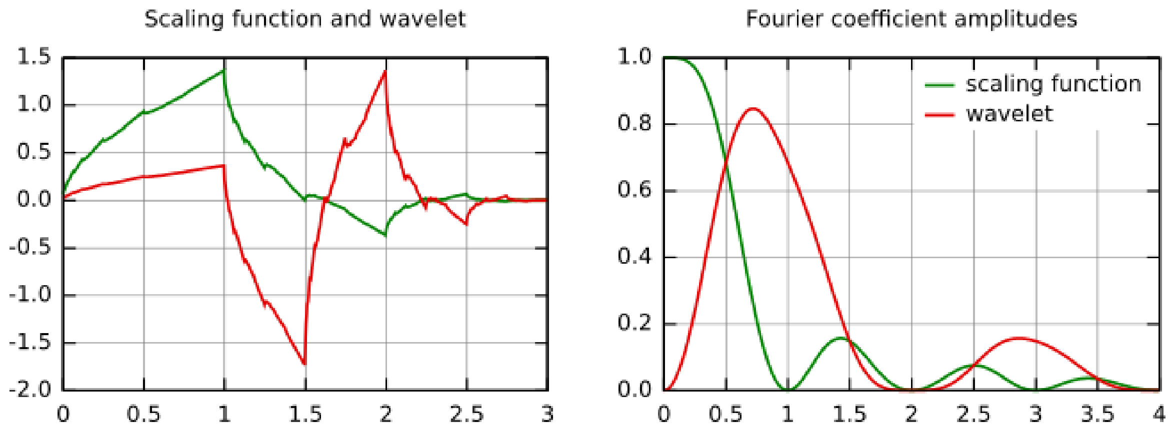


Figure 7. Daubechies 4   caling function and wavelet (left) and their frequency content (right).

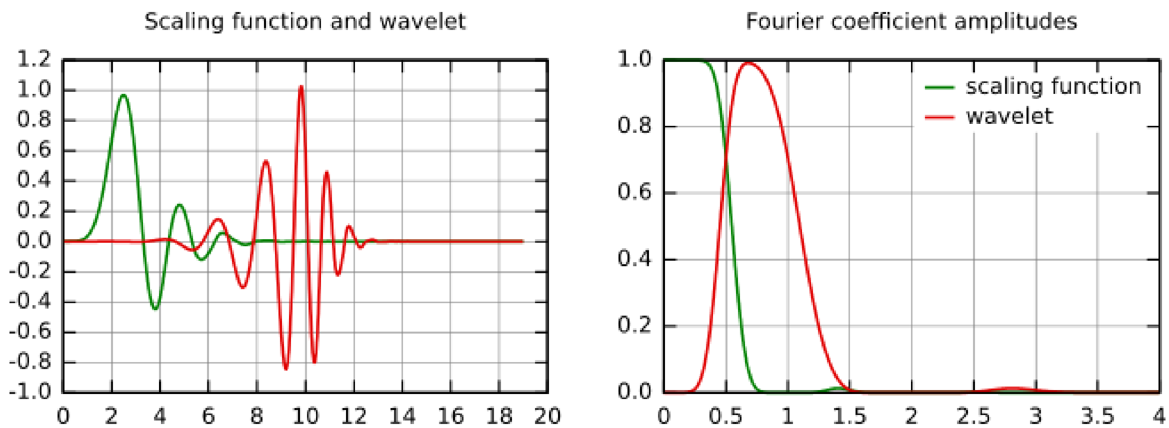


Figure 8. Daubechies 20 scaling function and wavelet (left) and their frequency content (right).

There are several types of implementation of the DWT algorithm. The oldest and most known one is the Mallat (pyramidal) algorithm. In this algorithm two filters – smoothing and non-smoothing one are constructed from the wavelet coefficients and those filters are recurrently used to obtain data for all the scales. If the total number of data $D = 2^N$ is used and the signal length is L, first D/2 data at scale $L/2^{N-1}$ are computed, then (D/2)/2 data at scale $L/2^{N-2}$, … up to finally obtaining 2 data at scale L/2. The result of this algorithm is an array of the same length as the input one, where the data are usually sorted from the largest scales to the smallest ones.

Within Gwyddion the pyramidal algorithm is used for computing the discrete wavelet transform. Discrete wavelet transform in 2D can be accessed using DWT module.

Discrete wavelet transform can be used for easy and fast denoising of a noisy signal. If we take only a limited number of highest coefficients of the discrete wavelet transform spectrum, and we perform an inverse transform (with the same wavelet basis) we can obtain more or less denoised signal. There are several ways how to choose the coefficients that will be kept. Within Gwyddion, the universal thresholding, scale adaptive thresholding [2] and scale and space adaptive thresholding [3] is implemented. For threshold determination within these methods we first determine the noise variance guess given by

$$\hat{\sigma} = \frac{\text{Median}\,|Y_{ij}|}{0.6745}$$

where Y_{ij} corresponds to all the coefficients of the highest scale sub band of the decomposition (where most of the noise is assumed to be present). Alternatively, the noise variance can be obtained in an independent way, for example from the AFM signal variance while not scanning. For the highest frequency subband (universal thresholding) or for each subband (for scale adaptive thresholding) or for each pixel neighborhood within subband (for scale and space adaptive thresholding) the variance is computed as

$$\hat{\sigma}_Y^2 = \frac{1}{n^2} \sum_{i,j=1}^{n} Y_{ij}^2$$

Threshold value is finally computed as

$$T(\hat{\sigma}_X) = \hat{\sigma}^2 / \hat{\sigma}_X$$

where

$$\hat{\sigma}_X = \sqrt{\max(\hat{\sigma}_Y^2 - \hat{\sigma}^2, 0)}$$

When threshold for given scale is known, we can remove all the coefficients smaller than threshold value (hard thresholding) or we can lower the absolute value of these co-efficient by threshold value (soft thresholding).

DWT denoising can be accessed with Data Process → Integral Transforms → DWT Denoise.

Continuous Wavelet Transform

Continuous wavelet transform (CWT) is an implementation of the wavelet transform using arbitrary scales and almost arbitrary wavelets. The wavelets used are not orthogonal and the data obtained by this transform are highly correlated. For the discrete time series we can use this transform as well, with the limitation that the smallest wavelet translations must be equal to the data sampling. This is sometimes called Discrete Time Continuous Wavelet Transform (DT-CWT) and it is the most used way of computing CWT in real applications.

In principle the continuous wavelet transform works by using directly the definition of the wavelet transform, i.e. we are computing a convolution of the signal with the scaled wavelet. For each scale we obtain by this way an array of the same length N as the signal has. By using M arbitrarily chosen scales we obtain a field N×M that represents the time-frequency plane directly.

The algorithm used for this computation can be based on a direct convolution or on a convolution by means of multiplication in Fourier space (this is sometimes called Fast Wavelet Transform). The choice of the wavelet that is used for time-frequency decomposition is the most important thing. By this choice we can influence the time and frequency resolution of the result. We cannot change the main features of WT by this way(low frequencies have good frequency and bad time resolution; high frequencies have good time and bad frequency resolution), but we can some how increase the total frequency of total time resolution. This is directly proportional to the width of the used wavelet in real and Fourier space. If we use the Morlet wavelet for example (real part –damped cosine function) we can expect high frequency resolution as such a wavelet is very well localized in frequencies. In contrary, using Derivative of Gaussian(DOG) wavelet will result in good time localization, but poor one in frequencies.CWT is implemented in the CWT module that can be accessed with Data Process → Integral Transforms → CWT.

CHAPTER -6

PROPOSED APPROACH

HILBERT & WAVELET TRANSFORM BASED SPECTRUM REPRESENTATION

Steps of implementation

1). Firstly, consider a multicomponent non-stationary signal y(t) with constant amplitude/ frequency parameters.

2). Extract the components from a multi-component signal into mono-components signal.

3). All the decomposed components are mono-component non-stationary signals.

4). These components have well defined and meaningful instantaneous frequencies

in time-frequency plane.

5). We compute the HT for all the decomposed components to determine their instantaneous amplitude and frequency functions. [10]

$$\mathcal{H}[\mathrm{y}(t)] = \frac{1}{\pi}\ \mathrm{p.v.} \int_{-\infty}^{-\infty} \frac{\mathrm{y}(u)}{(t-u)} du$$

p.v. indicates the integral corresponding to its Cauchy principle value.

6). The HT of a real signal can be used to determine its complex form which is also known as analytic signal and such signal can be represented as follows:

$$\mathrm{z}(t) = \mathrm{y}(t) + j\mathcal{H}[\mathrm{y}(t)]$$

7). The instantaneous amplitude, phase, and frequency of obtained mono component non-stationary signals can be obtained as follows:

$$A(t) = \sqrt{y^2(t) + (\mathcal{H}[y(t)])^2}$$
$$\theta(t) = \arctan(\mathcal{H}[y(t)]/y(t))$$
$$\omega(t) = \frac{d\theta(t)}{dt}$$

8). Compute the instantaneous parameters using continues wavelet transform by using:

$$W_x(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} s(t)\psi^*(\frac{t-b}{a})$$

9). The arrangement of computed instantaneous frequency and squared instantaneous amplitude of all mono-component non-stationary signals, provides the time-frequency representation of the multicomponent non-stationary signal in the proposed method.

The HILBERT TRANSFORM (HT) is applied to extract the instantaneous amplitude and instantaneous frequency functions of the decomposed components obtained from IEVD-HM method. The HT H[y(t)] of a real valued decomposed mono-component. [10]

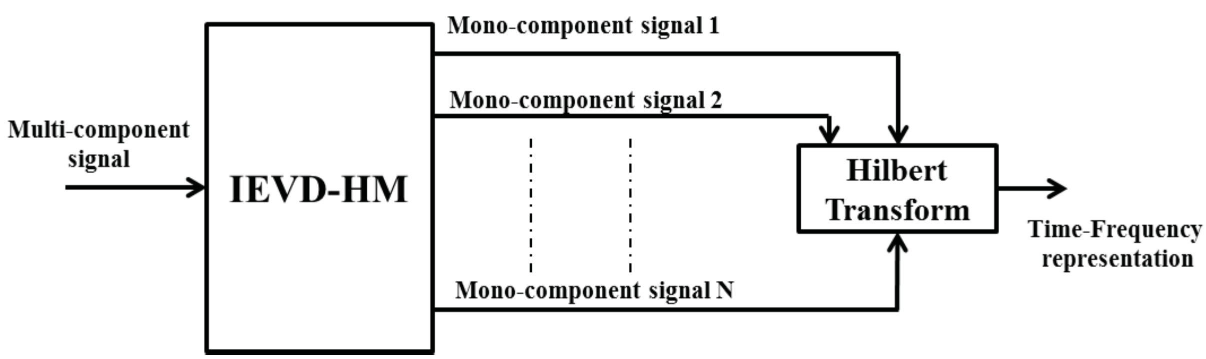


Figure 9

non-stationary signal y(t) is computed as follows:

$$\mathcal{H}[y(t)] = \frac{1}{\pi} \text{ p.v.} \int_{-\infty}^{-\infty} \frac{y(u)}{(t-u)} du \qquad (6)$$

In the above expression, the p.v. indicates the integral corresponding to its Cauchy principle value. The Hilbert Transform of a real signal can be used to determine its complex form which is also known as analytic signal and such signal can be represented as follows:

z(t) = y(t) + jH[y(t)] (7)

The instantaneous amplitude, phase, and frequency of obtained mono-component non-stationary signals can be obtained as follows:

$$A(t) = \sqrt{y^2(t) + (\mathcal{H}[y(t)])^2}$$

$$\theta(t) = \arctan(\mathcal{H}[y(t)]/y(t))$$

$$\omega(t) = \frac{d\theta(t)}{dt}$$

These instantaneous amplitude and frequency parameters of the obtained components are arranged in order to design a framework of proposed method of time-frequency representation.

Extraction of components from multi-component signal with constant amplitude & frequency parameters:

Let H_N^x be the sq. Hankel matrix of size N*N consisting of 2N-1 elements formed from a real signal x[n] having Q no. of samples expressed as:

$$H_N^x = \begin{bmatrix} x[0] & x[1] & \cdot & \cdot & \cdot & x[N-1] \\ x[1] & x[2] & \cdot & \cdot & \cdot & x[N] \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x[N-1] & x[N] & \cdot & \cdot & \cdot & x[2N-2] \end{bmatrix}$$

where n = 0,1,2,…;Q-1, Q>=2N-1

Now, the Eigen Value Decomposition (EVD) of the square matrix H_N^x is:

$$H_N^x = V_x \Lambda_x V_x^T$$

Λ_x is a diagonal matrix with N real eigen values

V_x is an orthogonal matrix having real eigen vectors as its columns & each column consisting of N elements.

Let $x[n]$ be a multi-component signal consisting of L constant amplitude-frequency mono-components signals as

$$x[n] = \sum_{l=1}^{L} X_l\,[n] = \sum_{l=1}^{L} A_l \text{ COS } (2\pi f_l n\ + \theta_l)$$

$n = 0 , 1 , .., Q - 1$

$f_l = \frac{1}{N_l} = \frac{F_l}{F_s}$, where F_s & F_l are the sampling frequency and frequency of $x_l[n]$ respectively.

A_l, θ_l, & f_l represent the amplitude, phase and normalized frequency of $x_l[n]$ respectively.

N_l represents the period of $x_l[n]$ in samples.

Using above equations, H_N^x can be represented as follows:

$H_N^x = \sum_{l=1}^{L} H_N^{x_l}$ where $H_N^{x_l} = (H_N^{x_l})^T$

The characteristic equation of H_N^x is given by

$$\det(H_N^x - \lambda I) = \lambda^N - Tr(H_N^x)\lambda^{N-1} + + \det(H_N^x) = 0$$

$Tr(\cdot)$ and $\det(\cdot)$ are the trace and the determinant of a matrix respectively.

We now derive the conditions of the Hankel matrix of size N to enable separation of monocomponent signals of $x[n]$ using EVD of H_N^x as below.

If $N = \sigma N_{LCM}$, *the* $2L$ *eigenvalues and corresponding eigenvectors of* $H_{\sigma N_{LCM}}^x$ *are equal to the set consisting of non-zero eigenvalues and eigenvectors of* $H_{\sigma N_{LCM}}^{x_l}$.

$$Tr(H_{\sigma N_{LCM}}^{x/}) = A_l \sum_{n=}^{\sigma N_{LCM}-1} \cos\ (2\pi f_l 2n + \theta_l)$$

Compute $H_N^{x_k}$ as follows:

$$\widetilde{H} = V_x \widetilde{\Lambda_{xk}} V_x^T$$

The average of elements of $2N - 1$ skew diagonals of $\widetilde{H}$ provide values of the kth component of *x[n]*, denoted by $\tilde{x}[n]$; where $n = 0, 1, ..., 2N - 2$.

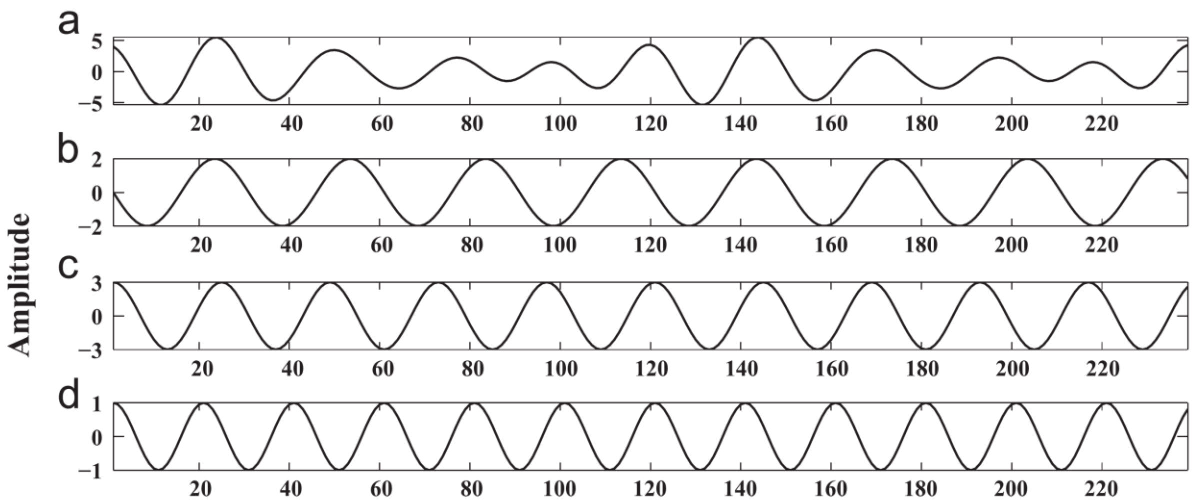


Figure 9. (a) Multi-component signal $x[n]$, (b) mono-component signals $x_1[n]$ and $X_1[n]$, (c) mono- component signal $x_2[n]$ & $X_2[n]$, (d) mono-component signal $x3$ $[n]$ and X3 $[n]$.

xn, denoted $X_k[n]_;$ $k = 1; 2; ..., L$, are the same as the original mono-component signals of xn, denoted $x_k[n]$, $8k$; i.e, for $N = \sigma N_{LCM}$, $\widetilde{H} = H_N^{x_k}$ and $\tilde{x}[n] = x_k[n]$,

The frequency resolution that can be achieved by performing EVD of H_N^x increases non-monotonically with N.

In this case, the modified eigenvalue diagonal matrix preserving the kth non-zero eigenvalue pair ofΛ_x is given by

Λ_{kx} =diag $(0, \dots, 0, \lambda_{x,k}, 0, \dots, 0, \lambda_{x,N-k+1}, 0, \dots, 0)$

The Hankel matrix formed by preserving the kth eigenvalue pair $of H_N^x$, denoted $\widetilde{H}$ is computed using $\widetilde{\Lambda_x}$ as follows:

$$H_N^{x_k} = V_x \Lambda_x V_x^T = \lambda vv_{x,k} + \lambda vv_{x,N-k+1}$$

The kth mono-component signal of $x[n]$ is extracted by taking the mean of elements of skew diagonals of $\widetilde{H}$. Let the kth original and extracted component of $x[n]$ be denoted by $x_k[n]$ and

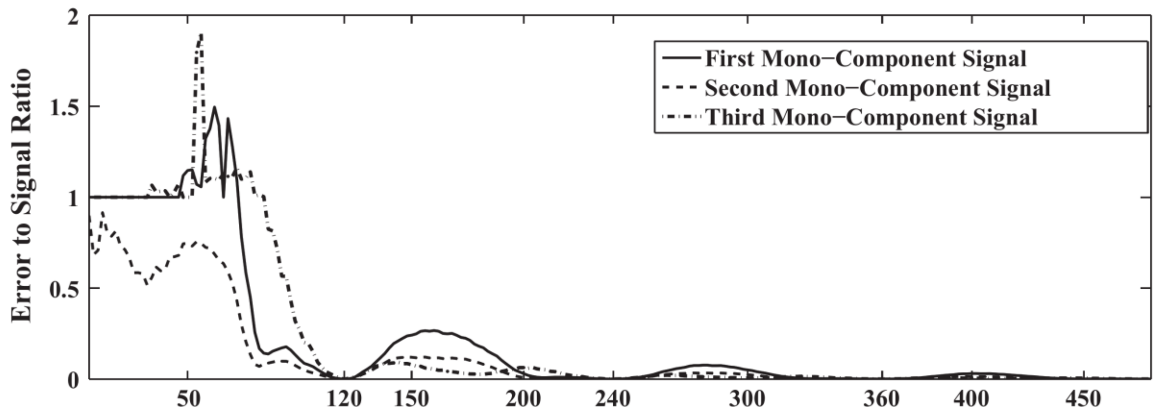


Figure 10. Error to signal ratio (N/S) for the three mono-component signals of the multi-component signal $x[n]$ with respect to the Hankel matrix size (N) , computed after the first *Iteration*.

Analytical Signal Extraction via Wavelet Transform [15]:

Considering an analytic wavelet function denoted by $g(t)$ and its Fourier transform by $\hat{g}(t)$ satisfying:

for a given signal $s(t) \in L^2(R, dt)$ the WT of $s(t)$ wrt wavelet $g(t)$ is defined as

$$S(b,a) \; = \frac{1}{a} J^{\infty} - \infty s(t) \overline{g}(\frac{t-b}{a}) \, dt$$

where $t, b \in R,\; R$ is the real number set and $a > \; 0$, $g(t)$ is complex conjugate of $g(t)$.

If $s(t)$ is a signal with finite energy and $g(t)$ is an analytic wavelet function, then $S(b,a)$, the WT of $s(t)$, wrt $g(t)$, is a complex function with respect to real-valued variable b and the scale factor $a(a > \; 0\,)$. For a fixed value of a, the imaginary part of this complex function is the HT of the real part; i.e., $S(b,a)$ is an analytic function wrt b.

If $g(t)$ is an analytic wavelet function with its real part $g_R(t)$ being even and $C_g = j_0^1 \; (\hat{g}t) \; dt$, with $0 < C_g < 1$. Then for an arbitrary real $s(t) \in L^2(R, dt)$, we have

$$\frac{1}{C_g} J_0^{\infty} S(t,a) \frac{da}{a} = s(t) + jH[s(t)]$$

where $S(t,a)$ is defined above, $H[s(t)]$ is the HT of $s(t)$.

Definition of $IP's$:

Define the IP's of a real-valued signal $s(t)$ as follows:

$$e(t) = \sqrt{s^2(t) + H^2[s(t)]}$$

$$\theta(t) = \arctan\left(\frac{H[s(t)]}{s(t)}\right)$$

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}[a_\iota rcta_\iota n(\frac{H[s(t)]}{s(t)})]$$

where $e(t)$, $\theta(t)$ and $f(t)$ is the instantaneous amplitude (IA , in- stantaneous phase (IPh , and instantaneous frequency (IF) of $s(t)$, respectively.

formula for IF estimation is

$$f(t) = \frac{1}{2\pi}\frac{s(t)\frac{dH[s(t)]}{dt} - H[s(t)]\frac{ds(t)}{dt}}{e^2(t) + \in^2 e_{\max}^2}$$

where

$$e_{\max}^2 = \max\left(e^2(t)\right)$$

and $< \varepsilon < 1$.

Now, comparisons of IF estimation are presented as

$$s(t) = e^{-(1/2)(c_1 t)^2}\cos(m_1 t) + e^{-(1/2)(c_2 t)^2}\cos(m_2 t)$$

where $c_1 = \sqrt{2}\sigma m_1/(2\pi\tau_1)$, $c_2 = \sqrt{2}\sigma m_2/(2\pi\tau_2)$, $\sigma = 5$, $\tau_1 = \tau_2 = 4$, $m_1 =$ 28:28, $m_2 =$ 42:43, we may have

$$H[s(t)] = e^{-(1/2)(c_1 t)^2}\sin(m_1 t) + e^{-(1/2)(c_2 t)^2}\sin(m_2 t) .$$

IV. TECHNIQUE BASED ON IEVD-HM AND HILBERT TRANSFORM [10]

In order to get the time-frequency illustration of a multi-component non-stationary signal, the projected IEVDHM mathematician rework technique has been shown in Fig. Firstly, a multicomponent non-stationary signal is rotten exploitation IEVDHM technique, that provides the elements on the idea of eigenvalue pairs of Hankel matrix as explained in section II. Every eigenvalue try represents the strength of corresponding element. All the rotten elements are thought-about to be mono-component non-stationary signals. These elements have well outlined and significant instant frequencies in time-frequency plane. We have a tendency to cypher the mathematician rework for all the rotten elements to see their instant amplitude and frequency functions. The arrangement of computed instant frequency and square instant amplitude of all mono-component non-stationary signals, provides the time frequency illustration of the multicomponent non-stationary signal with in the projected technique.

Non-stationary Signal Analysis Using Wavelet Transform

Wavelet analysis is an approach which decomposes a time-domain signal into components in different time windows and different frequency bands and presents the resulting information in the form of a surface in the time-frequency plane, sometimes referred to as a scalogram. The scalogram is similar in concept to the spectrogram but differs from in that the frequency resolution of the scalogram is logarithmic rather than linear, as is the case for the spectrogram. Because of the nature of the frequency resolution, the wavelet approach is more effective in analyzing both the long-time, low-frequency and the short-time, high-frequency content of a time signal. This characteristic is very useful for analyzing pulse-like and non-stationary signals. The continuous wavelet transform of a square-integrable, continuous time signal s(t) is the inner product between and the analyzing wavelet Ãa;b(t), which gives the wavelet coefficients

$$W_x(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} s(t)\psi^*(\frac{t-b}{a})$$

CHAPTER -7

SIMULATION AND RESULTS

Example 1.

The proposed method, IEVD-HM-HILBERT TRANSFORM [10]and WAVELET TRANSFORM for non-stationary signal analysis has been studied for three representative multi-component non-stationary signals.

The brief description of these synthetic signals denoted by $s_1[n]$, $s_2[n]$, and $s_3[n]$ is given below.

A. Signal 1:

The signal $s_1[n]$ is the summation of three sinusoidal signals with constant amplitude and frequency parameters

[redacted]

Where, [redacted]

f_2 = 850 Hz, f_3 = 1190 Hz, and sampling frequency (F_s) is 2800 Hz. The signal length considered is 799 samples. The plot of signal $s_1[n]$ is shown in Fig. 11(a). The Fig. 11 shows its three mono-component signals (Fig. 11(b)-(d)). The time-frequency representation of this signal $s_1[n]$ using IEVD-HM-HILBERT TRANSFORM method [10] is shown in Fig. 12.

We have also shown the spectrogram of signal $s_1[n]$ in Fig. 12. It is clear that the proposed method provides the similar number of frequency components (three) like spectrogram but with good resolution.

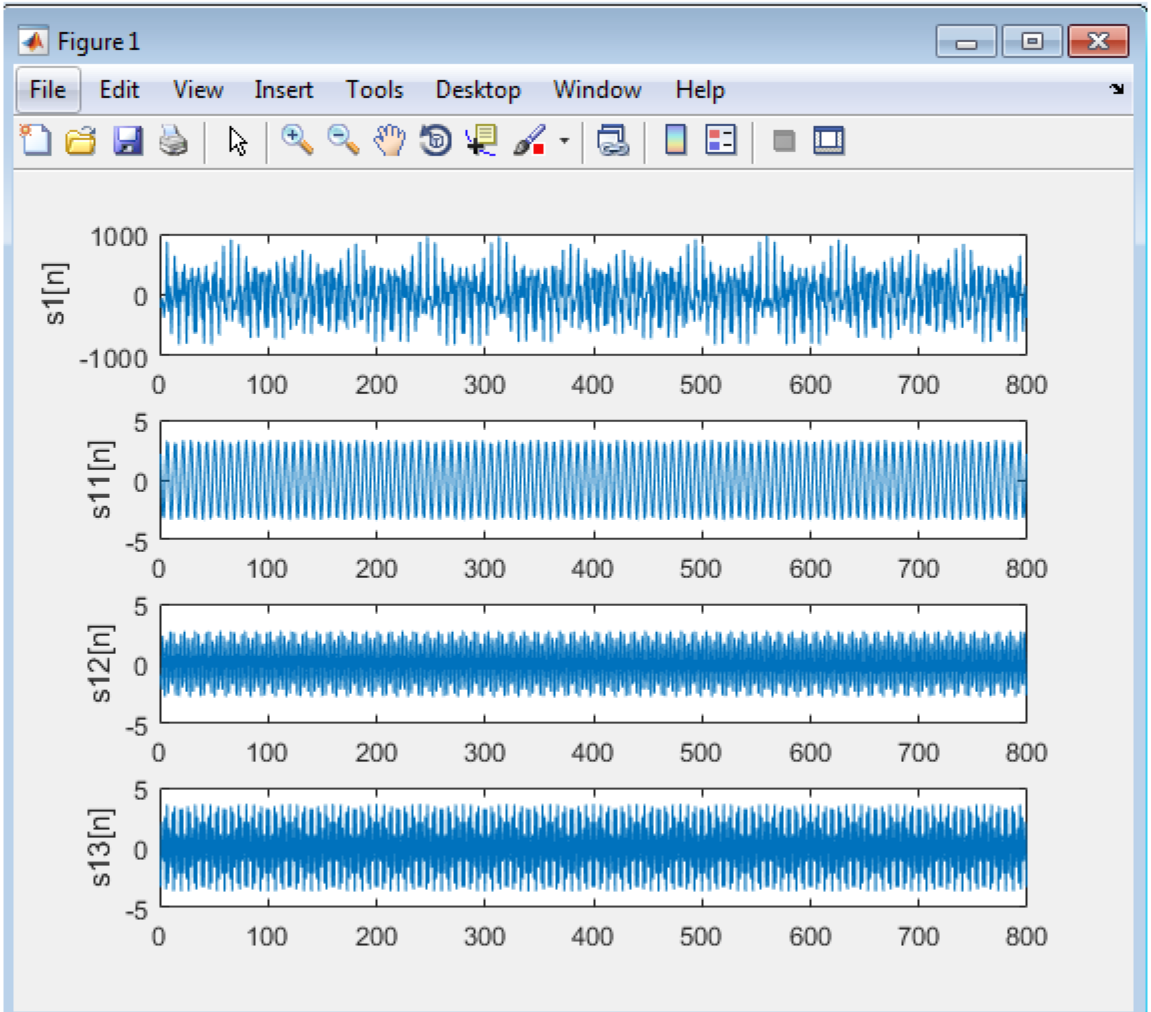


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Figure 11. Signal s1[n]; components of signal s1[n]

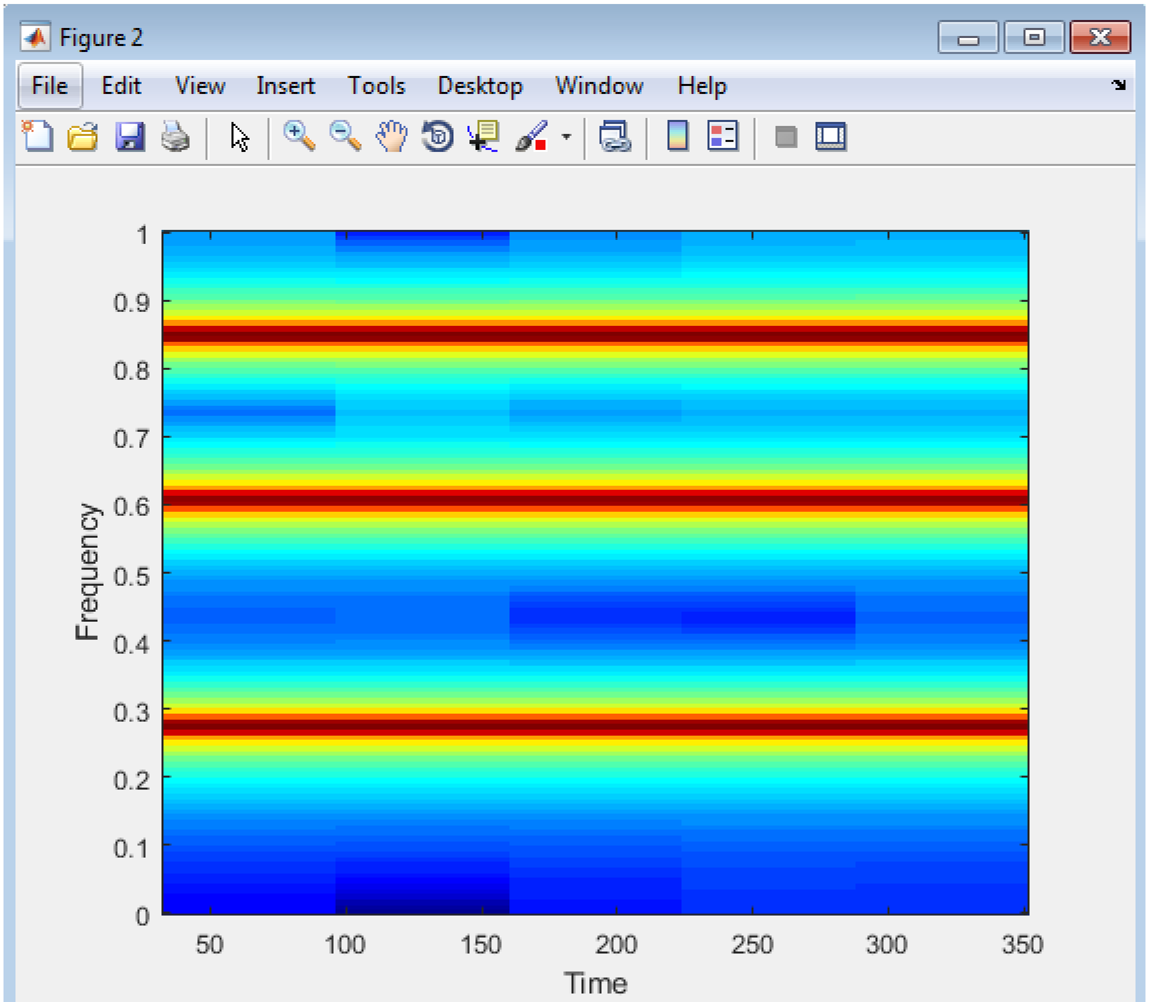

Figure 12. Time-frequency representation of signal s1[n] using IEVD-HM-HILBERT TRANSFORM method

B. Signal 2:

C. The signal $s_2[n]$ [5] has been obtained by concatenation of two constant frequency components, given as: $x_1[n] = 2.1\cos(2\pi f_1 n/F_s)$ and $x_2[n] = (7/3)\cos(2\pi f_2 n/F_s)$ where, f_1 =415 Hz, f_2 =541 Hz and F_s = 2100 Hz. The considered signal length for this signal is 599 samples. The plot of signal $s_2[n]$ has been presented in Fig. 5(a). The mono-component signals associated with this multicomponent non-stationary signal are shown in Fig. 5(b) and (c), respectively. The time-frequency representation of signal $s_2[n]$ using IEVD-HM-HILBERT TRANSFORM method has been shown in the Fig. 6. The Fig. 7 shows the spectrogram of this signal $s_2[n]$. For this signal also, our proposed method provides better time-frequency representation than spectrogram based method.

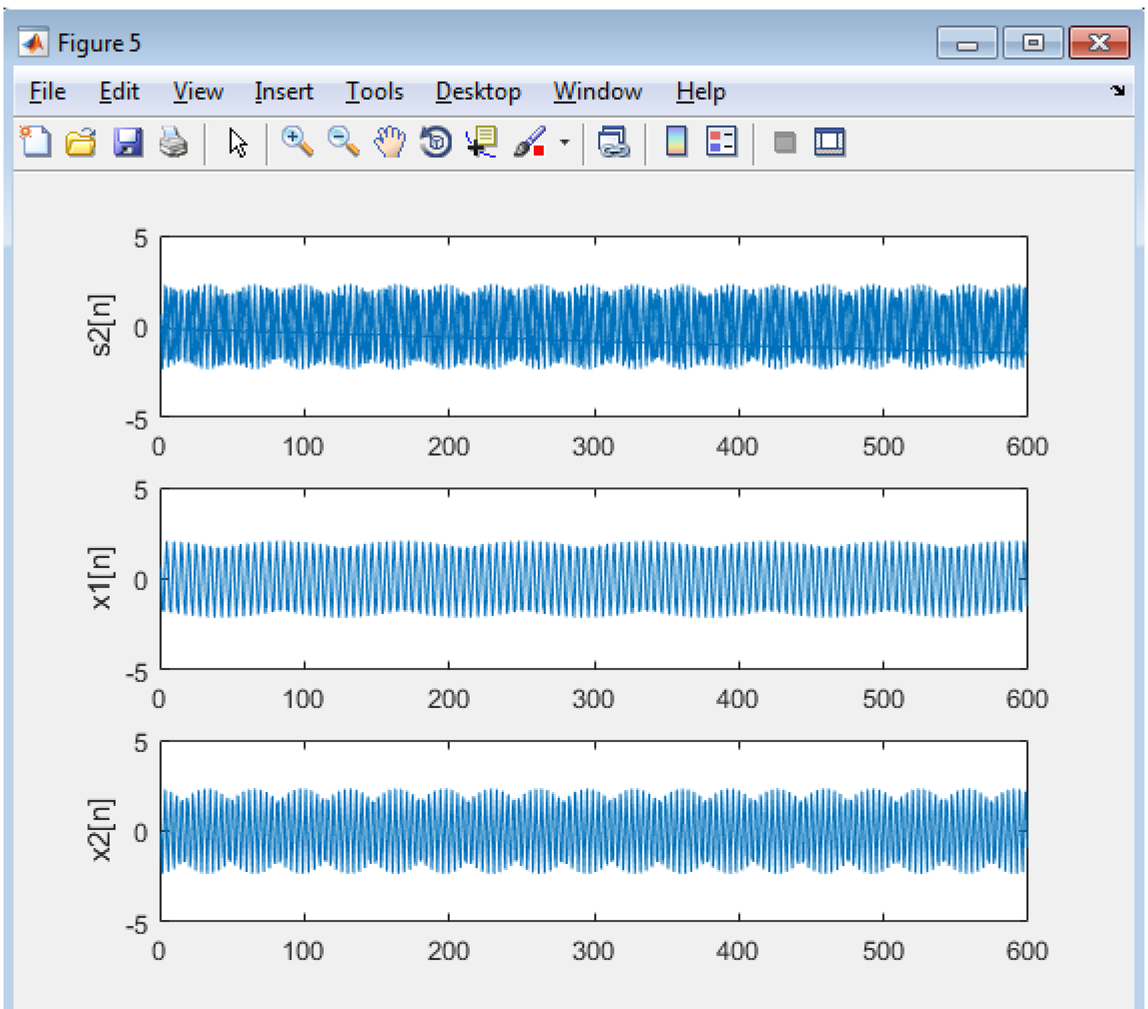

Figure 13. Signal $s2[n]$; (b)-(c) concatenating components of signal $s2[n]$

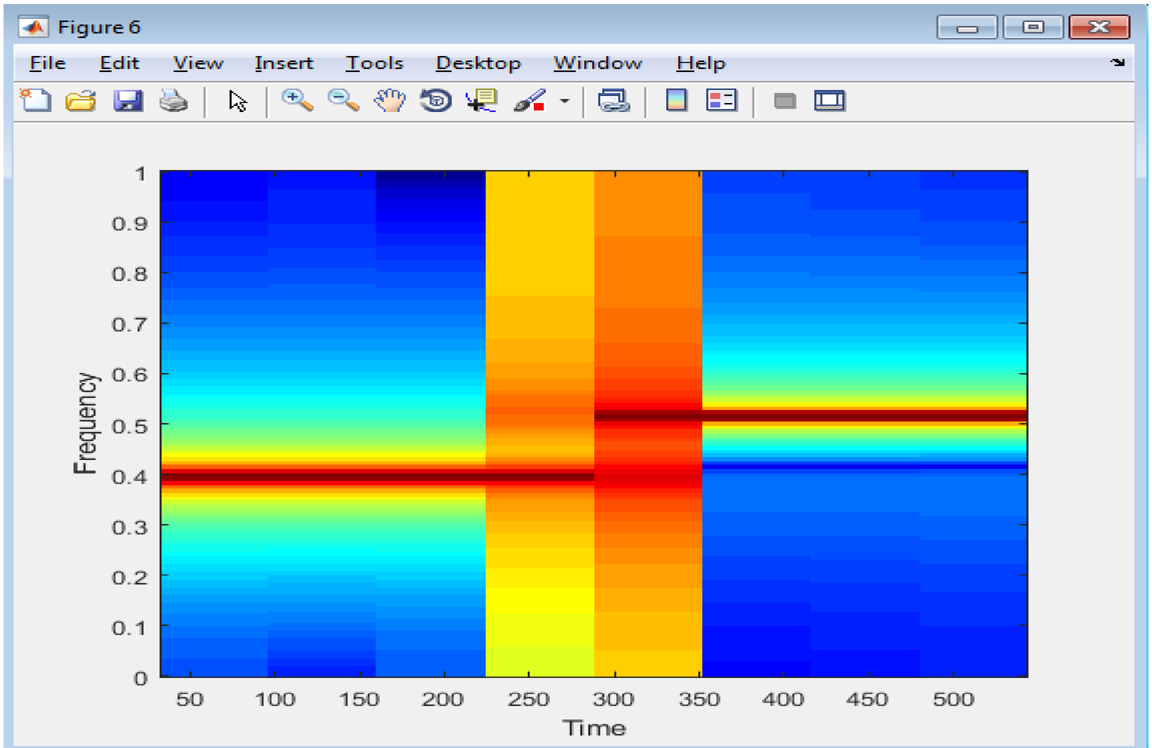

Figure 15. Time-frequency representation of signal $s2[n]$ using IEVD-HM-HILBERT TRANSFORM method

The multi-component non-stationary signal $s_3[n]$ [9], [21] has been obtained with the summation of one linearly increasing frequency chirp signal and one linearly decreasing frequency chirp signal. The mathematical expression for the signal $s_3[n]$ is given as follows:

$$s_3[n] = \cos\left(\frac{5.5\pi n^2}{40000} + 0.41n\right) + \cos\left(0.0004443\pi n^2 - \frac{7.5}{3}n\right)$$

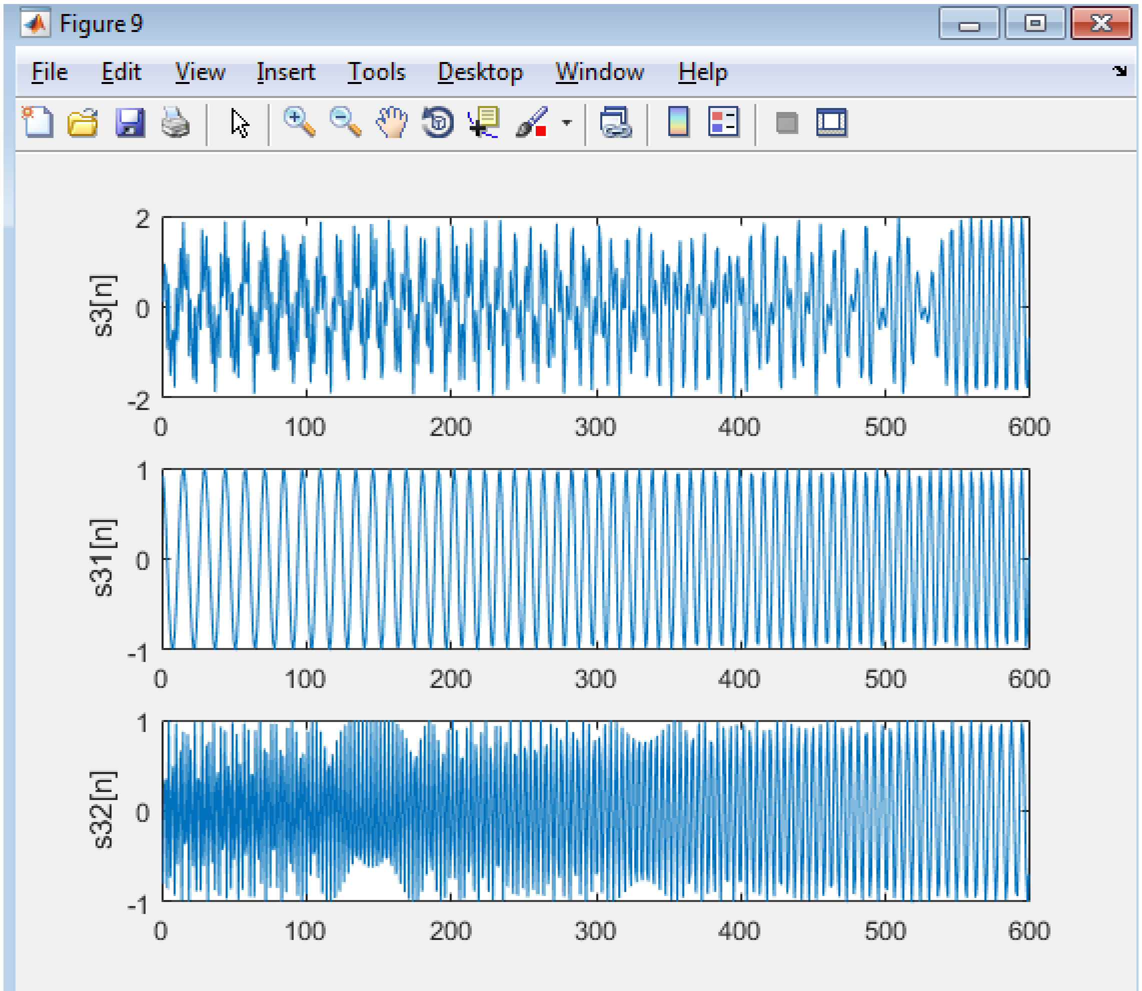

Figure 16. Signal $s3[n]$; (b)-(c) components of signal $s3[n]$

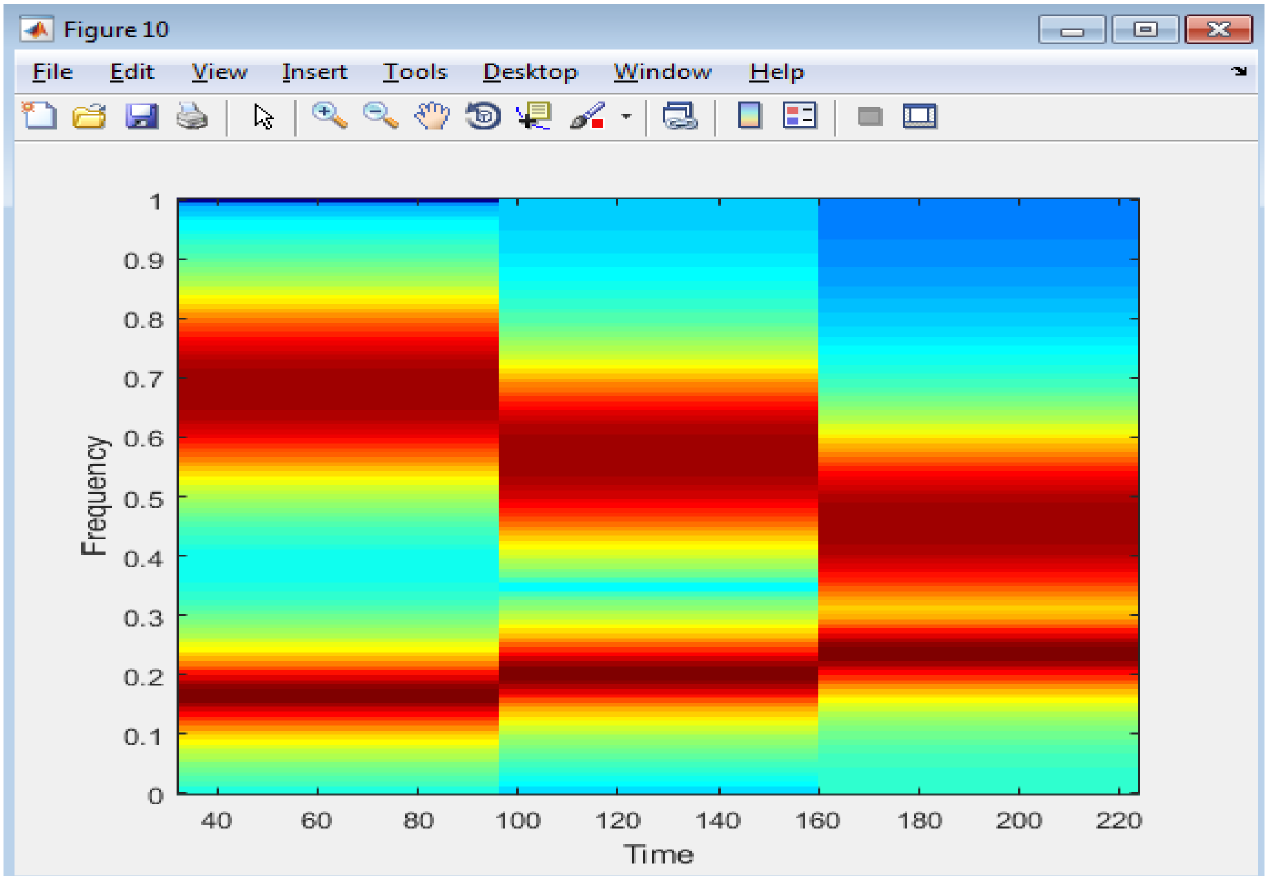

Figure 17. Time-frequency representation of signal $s3[n]$ using IEVD-HM-HILBERT TRANSFORM method

The signal $s_3[n]$ has been shown in Fig. 8(a) and its mono component signals are shown in Fig. 8(b) and 8(c). The time frequency representation of signal $s_3[n]$ using IEVD-HM-HILBERT TRANSFORM method is shown in Fig. 9 and spectrogram of signal $s_3[n]$ is shown in Fig . It clear from the Fig. 9 and 10 that the proposed method gives better resolution in time-frequency plane as compared to spectrogram method.

Example 2.

Let $x[n]$ be a multi-component signal shown in figure 1.

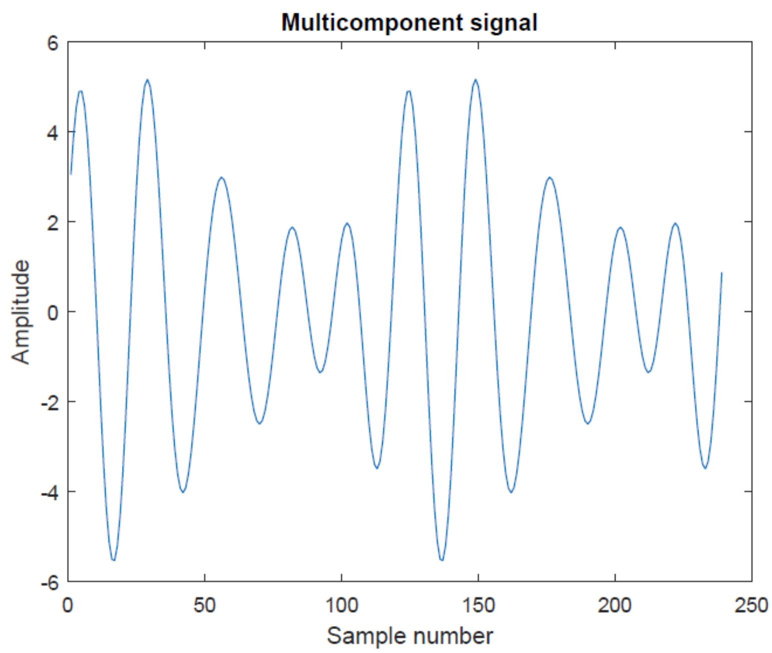

Figure 18. Shows the multicomponent signal.

$x[n] = \sum_{i=1}^{3} X_i\ [n] = \sum_{i=1}^{3} A_i \cos\ (2\pi f_i t\ + \theta_i)$

A1 = 2

A2 = 3

A3 = 1

f1 = 640/3

f2 = 800/3

f3 = 320

fs = 6400

Q1 = $\pi/2$

Q2 = 0

Q3 = 0

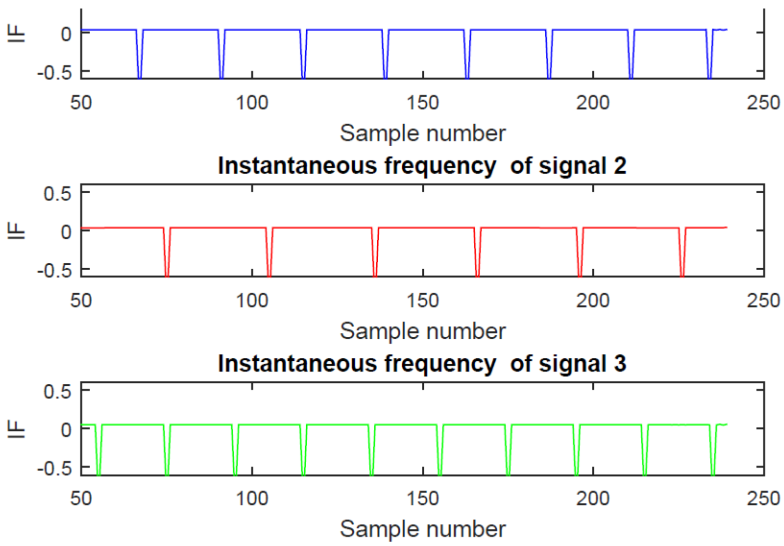

Figure 19. Shows the theoretical instantaneous frequency (IF) of mono component signals.

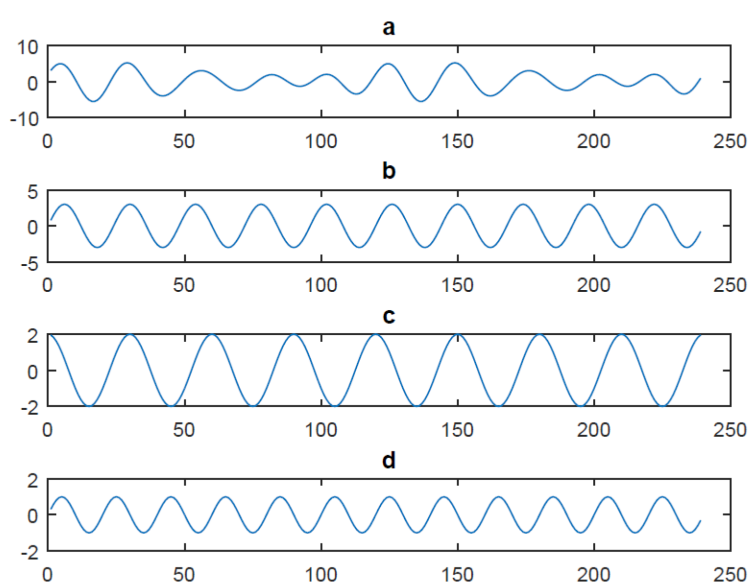

Figure 20. 'a' Shows the multicomponent and 'b' to 'd' show the mono-components obtained by EVD of Hankel Matrix.

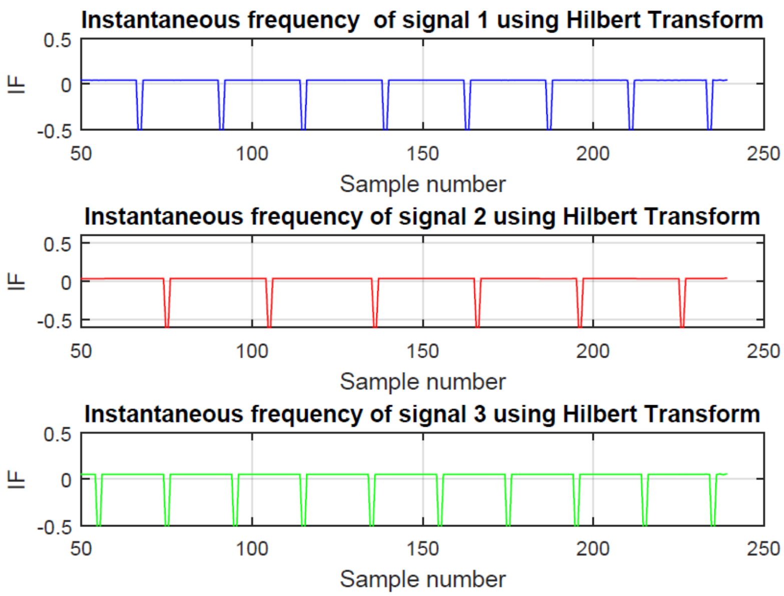

Figure 21. Shows the IF of mono-components obtained by HT method.

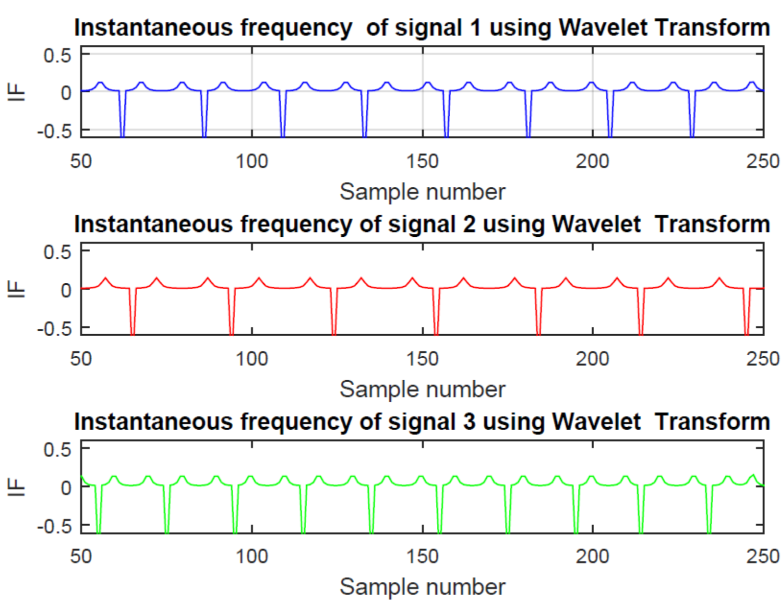

Figure 22. Shows the IF's of mono-components obtained by WT method.

Performance Evaluation

In this work, the performance of analyzed signal is computed in terms of Mean square error (MSE) & Peak signal to noise ratio (PSNR). The following measures of performance are used for quantitative estimation of the performance and analysis of the proposed technique.

Table 7.1 shows the comparative analysis of the proposed work using two evaluating parameters PSNR and MSE.

Table 7.1 Comparison of Performance on Signal x[n]

Algorithm	MSE	PSNR
Hilbert Transform	0.58	10.7618
Wavelet Transform	0.47	12.0161

It shows the PSNR of the wavelet is around 10% higher as compared to Hilbert algorithm due to time-frequency localization of wavelet transform. The MSE of both the algorithm also displaying the wavelet profound performance.

CHAPTER -8

CONCLUSION & FUTURE SCOPE

We have proposed a method for analysis of nonstationary signals based on IEVD-HMHT and Wavelet Transform. The simulation results presented in this paper show the potential of the proposed method for non-stationary signal analysis. This type of feature makes the proposed method suitable for real-time implementation for non-stationary signal analysis. The proposed time-frequency domain in this paper can be studied for the analysis of various non-stationary signals like as, speech signals, biomedical signals, mechanical signals, seismic signals, etc. The proposed method can be compared with other existing methods for non-stationary signal analysis in time-frequency domain. In this thesis work, Hilbert Transform and wavelet based transform techniques are used to extract different features from vibration data. Wavelet and multiresolution analysis techniques of signal transformation are already in practice in other signal processing fields like acoustics, digital image processing, etc. As morlet wavelet has been used as mother wavelet in this thesis, similar kind of work can be done using other wavelets to compare the efficiency in terms of quality of the features to be extracted. Work can also be done in expanding the available wavelets options to maximize its applications.

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